20 Thank You $\boxed{14}$

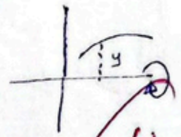
The University of Jordan, Mathematics Department

Calculus II, Second Exam, 12-5-2024, I

Student's Name: _ . Student's Number: _

Instructor's Name: Banan MA'AEAH Lecture Time: _

1) (4 points) Find the area of the surface obtained by rotating the curve $y = \sqrt{7-x}$, $0 \le x \le 3$ about the x-axis.



4

$$A = \int_0^3 2\pi r \left(\sqrt{1+\left(\frac{dy}{dx}\right)^2}\right) dx$$

$$y = \sqrt{7-x}$$

$$\left(\frac{dy}{dx}\right)^2 = \left(\frac{-1}{2\sqrt{7-x}}\right)^2$$

$$A = \int_0^3 2\pi (y) \left(\sqrt{\frac{1}{4(7-x)} + 1}\right) dx$$

$$\left(\frac{dy}{dx}\right)^2 + 1 = \frac{1}{4(7-x)} + 1$$

$$A = \int_0^3 2\pi \left(\cancel{\sqrt{7-x}}\right)\left(\frac{\sqrt{1+4(7-x)}}{2\cancel{\sqrt{7-x}}}\right) dx$$

$$= -\frac{\pi}{6}\left(\sqrt{(17)^3} - \sqrt{(29)^3}\right)$$

$$A = \frac{2\pi}{2}\int_0^3 \sqrt{1+28-4x}$$

$$A = \pi \int_0^3 (29-4x)^{1/2}$$

$$= \pi \left(29-4x\right)^{3/2} \cdot \frac{2}{3 \cdot -4}\Big]_0^3 = \left(\frac{-2\pi}{12}\sqrt{(29-4x)^3}\right)\Big]_0^3$$

2) (2 points) Test the sequence $\{ 2 + (0.53)^n \}_{n=1}^{\infty}$ for convergence.

$$\lim_{n\to\infty} \left(2 + (0.53)^n\right) = \lim_{n\to\infty} \left(2 + 0.53^{\infty}\right) = \boxed{2}$$

2

It is Convergance.

oints) Use geometric series to express the number $0.\overline{35} = 0.353535\cdots$
atio of integers

$$\frac{35}{100} + \frac{35}{10000} + \cdots = \frac{35}{(100)^1} + \frac{35}{(100)^2} + \cdots + \frac{35}{(100)^n}$$

$$= \sum_{n=1}^{\infty} \frac{35}{(100)^n} = \sum_{n=1}^{\infty} 35 \left(\frac{1}{100}\right)^n$$

Since $0.01 < 1$ the Series is Convergent.

$$\text{Sum} = \frac{a}{1-r} = \frac{35/100}{1 - \frac{1}{100}}$$

$$\text{Sum} = \frac{35}{\cancel{100}} \cdot \frac{\cancel{100}}{99} = \boxed{\frac{35}{99} = 0.\overline{35}}$$

3

4) (3 points) Determine whether the series is absolutely convergent, conditionally convergent, or divergent $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\,(\ln(n))^6}$ $= \sum_{n=2}^{\infty} \left| \frac{(-1)^n}{n(\ln(n))^6} \right| = \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^6}$

Divergance test

$$\lim_{n\to\infty} \frac{1}{\infty} = 0$$ Test fails by divergance test.

2

Integral test * impeper integral *

$$\lim_{n\to\infty} \int_2^{\infty} \frac{1}{n(\ln n)^6}\,dn = \lim_{t\to\infty} \int_2^{t} \frac{1}{n(\ln(n))^6}\,dn$$

$y = \ln(n)$

$\frac{dy}{dn} = \frac{1}{n}$

$\boxed{dn = n\,dy}$

$$= \lim_{t\to\infty} \int_2 \frac{1}{\cancel{n}(y)^6} \cdot \cancel{n}\,dy = \lim_{t\to\infty} \int y^{-6}\,dy$$

$$= \lim_{t\to\infty} \frac{y^{-5}}{-5} = \lim_{t\to\infty} -\frac{1}{5(\ln(n))^5}\Bigg]_2^t = \lim_{t\to\infty} \left(-\frac{1}{5(\ln t)^5}\right) - \left(-\frac{1}{5(\ln 2)^5}\right)$$

$$= \frac{-1}{\infty} - \left(-\frac{1}{5(\ln 2)^5}\right)$$

$$= 0 + \frac{1}{5(\ln 2)^5} = \frac{1}{5(\ln 2)^5}$$

It is absolutely Convergent by (Integral Test).

$\ln \frac{\infty}{\infty}$

st the following series for convergence:

a) (3 points) $\sum_{n=5}^{\infty} \ln(2n-5) - \ln(5n+7)$ = $\sum_{n=5}^{\infty} \ln\left(\frac{2n-5}{5n+7}\right)$

L'Hospital Rule

$\lim_{n\to\infty} \ln\left(\frac{2n-5}{5n+7}\right) = \ln\left(\lim_{n\to\infty}\left(\frac{2n-5}{5n+7}\right)\right) = \ln\left(\lim_{n\to\infty} \frac{2n}{5n}\right)$

$= \ln\left(\frac{2}{5}\right) \neq 0$ So It is divergent by Divergance Test

3

b) (3 points) $\sum_{n=1}^{\infty} \left(1 - \frac{3}{n}\right)^{n^2}$ = $\sum_{n=1}^{\infty} \left(\left(1 + \frac{-3}{n}\right)^n\right)^n$

$\left(1 + \frac{-3}{n}\right)^n = e^{-3}$

$\sum_{n=1}^{\infty} e^{-3n}$ By root test

$\lim_{n\to\infty} \left(e^{-3n}\right)^{1/n} = \lim_{n\to\infty} e^{-3} = e^{-3} = \frac{1}{e^3} < 1$

3

So the Series is Convergant by root test.

3

c) (3 points) $\sum_{k=1}^{\infty} \tan\left(\frac{1}{7^k}\right)$ By Limit Comparison Test

$\lim_{n\to\infty} \left|\frac{a_n}{b_n}\right| = \lim_{n\to\infty} \frac{\tan\left(\frac{1}{7^k}\right)}{\left(\frac{1}{7^k}\right)}$

$b_n = \frac{1}{7^K} = \left(\frac{1}{7}\right)^K$

Since $\frac{1}{7} < 1$ So (b_n) is Convergant by geometric Series.

$\lim_{n\to\infty} 1 = 1$

$\boxed{\text{THEORY} = \frac{\tan(x)}{x} = 1}$

Since $1 > 0$

* So a_n has the Same attitude as b_n.

b_n is Conv $\longrightarrow$ a_n is Convergent by Limit Comparison test.