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الجامعة الأردنية (قسم الرياضيات)   الاختبار الثاني (تفاضل وتكامل2)   18/5/2023

اسم الطالب :   مدرس المادة: د. مها سكحها

الرقم الجامعي :   وقت المحاضرة : 9:30

Q1: (3 pts) Find the area of the surface of revolution obtained by rotating the curve of $x=\sqrt{9-y^2}$, $0 \le y \le 1$ **about** $\boxed{\text{y-axis}}$ $r = x = \sqrt{9-y^2}$

$$S = 2\pi \int_a^b r\sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy \rightarrow S = 2\pi \int_0^1 r + \sqrt{1+\left(\frac{dx}{dy}\right)^2} \cdot dy$$

$$\left(\frac{dx}{dx}\right)^2 = \frac{-2y}{2\sqrt{9-y^2}} = \left(\frac{-y}{\sqrt{9-y^2}}\right)^2 = \frac{y^2}{9-y^2} + 1 = \frac{\cancel{y^2}+9-\cancel{y^2}}{9-y^2}$$

$$= \frac{9}{9-y^2}$$

$$S = 2\pi \int_0^1 \cancel{\sqrt{9-y^2}} \cdot \frac{\sqrt{9}}{\cancel{\sqrt{9-y^2}}} = 2\pi \int_0^1 3$$

$$= 2\pi \left(3y\Big]_0^1\right) = 2\pi(3-0)$$

$$= 6\pi$$

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Q2:(2pts) Find $\lim_{n\to\infty}\left(\frac{5n+3}{5n}\right)^{2n} = \lim_{n\to\infty}\left(\frac{\cancel{5n}}{\cancel{5n}} + \frac{3}{5n}\right)^{2n}$

$$= \lim_{n\to\infty}\left(1+\frac{3}{5n}\right)^{2n} = \lim_{n\to\infty}\left(1+\frac{3*2}{5*2n}\right)^{2n} = \lim_{n\to\infty}\left(1+\frac{6}{10n}\right)^{2n}$$

$$\lim_{n\to\infty}\left(1+\frac{\frac{6}{5}}{\frac{10n}{5}}\right)^{2n} = \lim_{n\to\infty}\left(1+\frac{\frac{6}{5}}{2n}\right)^{2n} = e^{\frac{6}{5}}$$

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Q3:(3pts) Find the sum of the series $\sum_{n=3}^{\infty}\left(e^{\frac{1}{n}}-e^{\frac{1}{n+1}}\right)$

$S_n = \left(\boxed{e^{\frac{1}{3}}} - e^{\frac{1}{4}}\right) + \left(e^{\frac{1}{4}} - e^{\frac{1}{5}}\right) + \left(e^{\frac{1}{5}} - e^{\frac{1}{6}}\right) \dots + \left(e^{\frac{1}{n-1}} - e^{\frac{1}{n}}\right) + \left(e^{\frac{1}{n}} - \boxed{e^{\frac{1}{n+1}}}\right)$

$\boxed{S_n = \frac{1}{e^3} - e^{\frac{1}{n+1}}}$ $\lim_{n\to\infty} \frac{1}{e^3} - e^{\frac{1}{n+1}} =$

$\lim_{n\to\infty} \frac{1}{e^3} - \lim_{n\to\infty} e^{\frac{1}{n+1}} = \frac{1}{e^3} - 1$

$\Downarrow$

$e^{\frac{1}{\infty}} = e^0 = \boxed{1}$

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Q4: (3pts each) Test the series for convergence :

(a) $\sum_{n=1}^{\infty}\frac{1}{n+n^3}$

$\frac{1}{n+n^3} < \frac{1}{n^3} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n+n^3} < \sum_{n=1}^{\infty} \frac{1}{n^3}$

$\Downarrow$

$p = 3 > 1$ conv by p. series

$\therefore \sum_{n=1}^{\infty} \frac{1}{n+n^3}$ is conv by Comprision test

$\boxed{(\text{small})\ \frac{1}{n+n^3} \quad (\text{bigger})\ \frac{1}{n^3}}$

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(b) $\sum_{n=1}^{\infty}\frac{5^n}{4^n+3^n}$

$b_n = \frac{5^n}{4^n} = \left(\frac{5}{4}\right)^n$ div by geometric Series

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{\cancel{5^n}}{4^n+3^n}}{\frac{\cancel{5^n}}{4^n}} = \lim_{n\to\infty} \frac{4^n}{4^n+3^n} = \frac{\infty}{\infty}$

$\xrightarrow{C.R} \frac{\cancel{4^n}}{\cancel{4^n}} = \boxed{1}$ a_n تسلك نفس سلوك b_n

$\boxed{1} = \lim \frac{a_n}{b_n}$ عدد

$\therefore a_n = \sum_{n=1}^{\infty} \frac{5^n}{4^n+3^n}$ is div by Limit Comprison test

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(c) $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^2}$ $\lim_{n\to\infty}\left(\left(\frac{n}{n+1}\right)^{n^2}\right)^{\frac{1}{n}} = \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^{n}$

$= \lim_{n\to\infty}\left(\frac{\cancel{n}}{\cancel{n}(1+\frac{1}{n})}\right)^{n} = \lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right)^{n} = \lim_{n\to\infty}\frac{(1)^n}{(1+\frac{1}{n})^n} =$

$\frac{1}{\lim_{n\to\infty}(1+\frac{1}{n})^n} = \frac{1}{e^1} = e^{-1} < 1$ Conv by Root test.

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(d) $\sum_{n=1}^{\infty}(-1)^n\frac{\ln n}{\sqrt{n}}$ $\lim_{n\to\infty}\left|\frac{(-1)^n \ln n}{\sqrt{n}}\right| = \lim_{n\to\infty}\frac{\text{Ln}(n)}{\sqrt{n}} = \frac{\infty}{\infty}$ x

$\xrightarrow{C.R}$ $\frac{\frac{1}{n}}{\frac{1}{2\sqrt{n}}} = \frac{2\sqrt{n}}{n} = \frac{2}{\sqrt{n}} = \frac{2}{\infty} = 0$ faild by divergent test ✓

dicresing $\Rightarrow$ $\left(\frac{\text{Ln}(n)}{\sqrt{n}}\right)' \rightarrow \frac{\frac{1}{n}*\sqrt{n} - \ln(n)\cdot\frac{1}{2\sqrt{n}}}{(\sqrt{n})^2} = \frac{\frac{1}{\sqrt{n}} - \frac{\ln(n)}{2\sqrt{n}}}{n}$

$= \frac{\frac{2-(\text{Ln}(n))}{2\sqrt{n}}}{n} = \frac{2-(\text{Ln}(n))}{2n^{\frac{3}{2}}} < 0$ is dicresing.

$\frac{2-\text{Ln}(1)}{2}$

The Series Convergent by Altraning Series test.

or $a_1 = \text{Ln}1 > a_2 = \frac{\text{Ln}2}{\sqrt{2}}$ $\therefore$ dicrering.

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