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الاختبار الثاني: 2022/8/22	0301201 تفاضل وتكامل 3	الجامعة الأردنية
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الشعبة: 2		الرقم الجامعي:

يتكون الامتحان من 5 أسئلة في 3 ورقات

[1] (4 marks) Let $f(t) = \bar{\boldsymbol{u}}(t) \cdot \bar{\boldsymbol{v}}(t)$, where $\bar{\boldsymbol{u}}(2) = \langle 1,2,-1\rangle$, $\bar{\boldsymbol{u}}'(2) = \langle 3,0,4\rangle$, and $\bar{\boldsymbol{v}}(t) = \langle t, t^2, t^3\rangle$. Find $f'(2)$

$$f'(t) = \bar{u}(t) \cdot \bar{v}'(t) + \bar{u}'(t) \cdot \bar{v}(t)$$

$$\bar{v}'(t) = \langle 1, 2t, 3t^2 \rangle \quad \begin{cases} \bar{v}(2) = \langle 2, 4, 8 \rangle \\ \bar{v}'(2) = \langle 1, 4, 12 \rangle \end{cases}$$

$$f'(2) = 1 + 8 - 12 + 6 + 0 + 32$$
$$= 35$$

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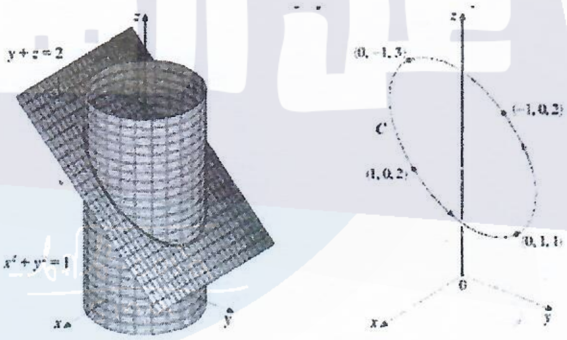
[2] (4 marks) Find parametric equations of the tangent line to the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane $y + z = 2$ at the point (0,1,1).



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intersection curve : $x = \cos t$, $y = \sin t$

$z = 2 - \sin t$

$\vec{r}(t) = \langle \cos t, \sin t, 2 - \sin t \rangle$ at $P = (0,1,1)$, $t = \frac{\pi}{2}$

$$T = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \frac{\langle -\sin t, \cos t, -\cos t \rangle}{\sqrt{1 + \cos^2 t}}$$

T

T parametric equations at (0,1,1) $t = \frac{\pi}{2}$

$x = 0 - t$

$y = 1$

$z = 1$

$x = \cos\frac{\pi}{2} (\sin\frac{\pi}{2}) t$

$y = \sin\frac{\pi}{2} + (\cos\frac{\pi}{2}) t$

$z = 2 - \sin\frac{\pi}{2} - (\cos\frac{\pi}{2}) t$

[3] (6 marks) Consider the vector valued function $\bar{r}(t) = \langle e^t \cos t\,, e^t\,, e^t \sin t \rangle$

(a) Sketch the graph of $\bar{r}(t)$.

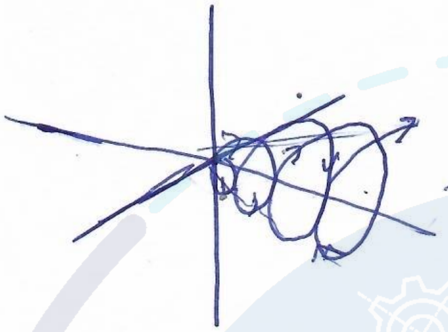
$x = e^t \cos t$

$y = e^t$

$z = e^t \sin t$

	$t=0$	$t=1$
x	1	0
y	1	$e^{\frac{\pi}{2}}$
z	0	$e^{\frac{\pi}{2}}$

$x^2 + z^2 = y^2 \quad , \; y > 0$



$\rightarrow$ one-sheet elliptic cone

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(b) Find the arc length for $\bar{r}(t)$ for $0 \le t \le 1$

$$L = \int_0^1 |\bar{r}'(t)|\, dt$$

$$r'(t) = \langle -e^t \sin t + e^t \cos t\,,\; e^t,\; e^t \cos t + e^t \sin t \rangle$$

$$|r'(t)| = \sqrt{e^{2t}(\cos t - \sin t)^2 + e^{2t} + e^{2t}(\cos t + \sin t)^2}$$

$$= \sqrt{e^{2t}\left((1 - \cancel{\sin 2t}) + 1 + (1 + \cancel{\sin 2t})\right)}$$

$$= \sqrt{3}\, e^t$$

$$L = \int_0^1 \sqrt{3}\, e^t\, dt$$

$$= \sqrt{3}\,(e^1 - e^0)$$

$$= \underline{\sqrt{3}(e - 1)}$$

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[4] (4 marks) Assume

$$f(x,y) = \begin{cases} \dfrac{3x^2y}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Is $f(x,y)$ continuous at $(0,0)$? Explain your answer.

$$\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+y^2} \overset{??}{=} 0$$

$$\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+y^2} \quad \left[\begin{array}{l} x = r\cos\phi \\ y = r\sin\phi \end{array}\right.$$

$$\lim_{(r,\phi)\to(0,\phi)} \frac{3\cancel{r^2}\cos^2\phi \; r\sin\phi}{\cancel{r^2}} = 0 \qquad \underline{\cos^2\phi\sin\phi} \text{ : bounded function}$$

$$1 \geq \cos^2\phi\sin\phi \geq -1$$

then $f(x,y)$ is continuous at $(0,0)$

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[5] (2 marks) Let $u(x,y) = \sqrt{x^2+y^2}$. Find u_{xy}

$$u_x = \frac{x}{\sqrt{x^2+y^2}}$$

$$u_{xy} = \frac{xy}{\sqrt{x^2+y^2}}$$

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