The University of Jordan
Department of Mathematics
Calculus III
First Exam

Student's Name: Student's Number:

29.5

Good

Instructor's Name: Dr. Manal Ghanem Section's Number: 2

Q1) (5 points) If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $\vec{a}.\vec{b} = -2$. Find the angle between $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$.

$\theta = \cos^{-1}\left(\frac{(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})}{|\vec{a}+\vec{b}||\vec{a}-\vec{b}|}\right)$

$|\vec{a}+\vec{b}|^2 = (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = |\vec{a}|^2 + 2(\vec{a}\cdot\vec{b}) + |\vec{b}|^2$

$= 9 + (2 \times -2) + 16 = 21$

$|a+b| = \sqrt{21}$

$|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$

$= 9 + 4 + 16 = 29$

$|\vec{a}-\vec{b}| = \sqrt{29}$

$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 9 + \cancel{2} - \cancel{2} - 16$

$= -7$

$\theta = \cos^{-1}\left(\frac{-7}{\sqrt{21}\sqrt{29}}\right)$

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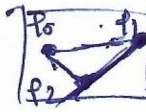
Q2) (5 points) Find the equation of the plane that contains the point (3, −1, 4) and the line x = 2 − y = 2z.

$P_0 = (3,-1,4)$ $x = t$, $y = 2-t$, $z = \frac{t}{2}$

$t=0 \to P_1 = (0,2,0)$

$t=2 \to P_2 = (2,0,1)$



$\overrightarrow{P_0P_1} = \langle 3, -3, 4\rangle$

$\overrightarrow{P_0P_2} = \langle 1, -1, 3\rangle$

$\vec{n} = \overrightarrow{P_0P_1} \times \overrightarrow{P_0P_2}$

$\vec{n} = \begin{vmatrix} i & j & k \\ 3 & -3 & 4 \\ 1 & -1 & 3 \end{vmatrix}$

$= -5i - 5j + (-3+3)k$

$= -5i - 5j$

5

$\vec{n} = \langle -5, -5, 0\rangle$

equation of: $-5(x-3) - 5(y+1) + 0(z-4)$

$-5x + 15 - 5y - 5 = 0$

$-5x - 5y + 8 = 0$

$-5x - 5y = -\cancel{8}$ -5

Q3) (6 points)

a) Find the parametric equations of the line passes through the point $(2, -3, 5)$ and perpendicular to the plane $3x - y + 4z = 0$.

$\vec{V} \perp$ Plane $\Rightarrow \vec{n} \parallel \vec{V}$

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$\vec{V} = \langle 3, -1, 4 \rangle \quad , \quad P = (2, -3, 5)$

$x = 2 + 3t \quad , \quad y = -3 - t \quad , \quad z = 5 + 4t$

* b) Find the intersection between the line $x = 4 - 2t$, $y = 3 + t$, $z = 5 - 3t$ and the $yz -$ plane.

plane equation $\Rightarrow \quad P = (0,0,0) \quad \vec{V} = \langle 1, 0, 0 \rangle$

$\boxed{\therefore x = 0}$

x - intersection $\Rightarrow 0 = 4 - 2t \Rightarrow t = 2$

y - intersection $\Rightarrow y = 3 + 2 = 5$

z - intersection $\Rightarrow z = 5 - 6 = -1$

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intersection point $P = (0, 5, -1)$

Q4) (5 points) Find the equation of the sphere with center $(-2, 1, 3)$ and touch the plane $2x - y + 2z = -5$.

radius $= D$

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$$D = \frac{|(2 \times -2) - (1) + (2 \times 3) + 5|}{\sqrt{9}} = \frac{6}{3} = 2$$

$-4 - 1 + 6 + 5$

$S \Rightarrow (x+2)^2 + (y-1)^2 + (z-3)^2 = 4$

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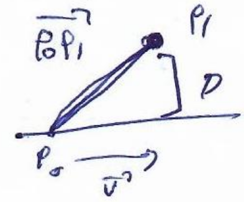
Q5) **(5 points) Find the distance between the point $(4, 2, -1)$ and the line** $x = 2 + 3t,\ y = 3 + 2t,\ z = -4t$.



$\vec{v} = \langle 3, 2, -4 \rangle$

$P_0 = (4, 2, -1)$ $\quad$ $P_1\,(2, 3, 0)$

$t = 0$

5

$\overrightarrow{P_0P_1} = \langle -2, 1, 1 \rangle$

$$D = \frac{|\overrightarrow{P_0P_1} \times \vec{v}|}{|\vec{v}|} = \frac{\sqrt{36+25+49}}{\sqrt{9+4+16}} = \frac{\sqrt{115}}{\sqrt{29}}$$

$$\overrightarrow{P_0P_1} \times \vec{v} = \begin{vmatrix} i & j & k \\ -2 & 1 & 1 \\ 3 & 2 & -4 \end{vmatrix}$$

$(-6)i - (5)j + (-7)k$

Q6) **(4 points) Identify and sketch**

1) $z = 2 - y^2$ $\qquad$ $(z - 2) = -y^2$

Parabolic cylinder whose axis is parallel to x-axis $(x, 0, 2)$ and opening downward

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2) $2x = x^2 - \frac{y^2}{9} + \frac{z^2}{4}$ $\qquad$ $0 = x^2 - 2x + \frac{z^2}{4} - \frac{y^2}{9} \implies 1 = (x-1)^2 + \frac{z^2}{4} - \frac{y^2}{9}$

hyperboloid of one sheet whose axis is parallel to y-axis $(1, y, 0)$