The University of Jordan
Department of Mathematics
Calculus III Final Exam

Student's Name

Instructor's Name

Student's Num

Lecture's time 9:45-11

Q1)(3 points) Find the curvature of the curve $\vec{r(t)} = < e^{-2t}, 3\cos 2t, 4\sin t >$ at the point $1, 3, 0 \longrightarrow t = 0$

$K = \frac{|T'(t)|}{|r'(t)|} = \frac{|r'(t) \times r''(t)|}{|r'|^3}$

$r'(t) = \langle -2e^{-2t}, -6\sin 2t, 4\cos t\rangle$

$r''(t) = \langle -4e^{-2t}, -12\cos 2t, -4\sin t\rangle$

$\langle 24\sin t \sin 2t, -(8e^{-2t}\sin t + 16\cos t e^{-2t}), 24e^{-2t}(\cos 2t - \sin 2t)\rangle$

$K = \frac{\sqrt{(24)^2 + 16^2 + 24^2}}{\left(\sqrt{(-2)^2+0^2+4^2}\right)^3}$

$\langle 24(\sin t \sin 2t + \cos t \cos 2t), -(8\sin t e^{-2t} + 16\cos t e^{-2t}), 24e^{-2t}(\cos 2t - \sin 2t)\rangle$

Q2)(4 points) Find the point on the curve $\vec{r(t)} = < e^t, 2\cos t, 2\sin t >, (0 \le t \le \pi)$, where the tangent line is parallel to the plane $y + \sqrt{3}z = 1$.

$r'(t) = \langle e^t, -2\sin t, 2\cos t\rangle \cdot \langle 0, 1, \sqrt{3}\rangle = 0$

$0 \times e^t + (-2\sin t \times 1) + (2\sqrt{3}\cos t) = 0$

$\sin t = \sqrt{3}\cos t$

$\tan t = \sqrt{3}$

$t = \frac{\pi}{3}$

$P_0 = (e^{\frac{\pi}{3}}, 1, \sqrt{3})$

Q3)(4 points) Let $f(x,y) = x^2y^3 + by^2 + x^3$ and let $u = \langle 3,4 \rangle$. Find b such that $D_u f(1,1) = -1$.

$\vec{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$ $\sqrt{9+16} = 5$

$f_x = 2xy^3 + 3x^2 \to f_x(1,1) = 5$

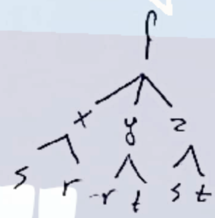
$f_y = 3y^2x^2 + 2by \to f_y(1,1) = 3 + 2b$

$\frac{3}{5} \times 5 + \frac{4}{5}(3+2b) = -1$

$3 + \frac{12}{5} + \frac{8b}{5} = -1 \Rightarrow \frac{8b}{5} = -\frac{32}{5}$ $\boxed{b = -4}$

Q4)(4 points) If $W = f(x,y,z)$, $x = sr$, $y = 2rt$ and $z = 3st$, show that

$$s\frac{\partial f}{\partial s} + r\frac{\partial f}{\partial r} - t\frac{\partial f}{\partial t} = 2srf_x.$$



$\frac{df}{ds} = \frac{df}{dx} r + \frac{df}{dz} 3t$

$\frac{df}{dr} = \frac{df}{dx} s + \frac{df}{dy} 2t$

$\frac{df}{dt} = \frac{df}{dy} 2r + \frac{df}{dz} 3s$

$\left(\frac{df}{dx}(sr) + \cancel{\frac{df}{dz} 3st}\right) + \left(\frac{df}{dx} sr + \cancel{\frac{df}{dy} 2rt}\right) - \cancel{\frac{df}{dy} 2rt} - \cancel{\frac{df}{dz} 3st}$

$= 2\frac{df}{dx} sr$

Q5)(4 points) Find the equation of the tangent plane for the surface $8e^{zy} = x^3z^2 + x^2 \ln z$ at the point (2,0, 1).

$x^3z^2 + x^2 \ln z - 8e^{zy} = 0$

$f_x = 3x^2z^2 + 2x \ln z \Rightarrow 12 + 0 = f_x(2,0,1)$

$f_y = -8z e^{zy} \Rightarrow -8 = f_y(2,0,1)$

$f_z = 2x^3z + \frac{x^2}{z} - 8ye^{zy} \Rightarrow 16 + 4 = f_z(2,0,1)$

$T: 12(x-2) + (-8)y + 20(z-1) = 0$

Q6) (4 points) Find and classify The critical point(s) of $f(x, y) = e^x + 2e^{2y} - xe^{2y}$.

$f_x = e^x - e^{2y} = 0$

$f_y = 4e^{2y} - 2xe^{2y} = 0$

$x = 2y$

$4 = 2x$

$x = 2 \quad y = 1 \Rightarrow$ Critical Point

$f_{xx} = e^x - 0$

$f_{xy} = -2e^{2y}$

$f_{yy} = 8e^{2y} - 4xe^{2y}$

هنا يوجد خطا في القانون

*يوجد تربيع على fxy *

وبالتالي خطا في النتيجة

$D = f_{xx}(2,1)\, f_{yy}(2,1) - f_{xy}(2,1)$

$= e^2 \times 0 - -2e^2 = 2e^2 > 0$

$f_{xx}(2,1) = e > 0$

*Saddle point --> $P_0(2,1)$ local minimum

Q7) (5 points) Find the point on the plane $2x - 4y + 2z = 14$ that is closest to the point $(0, 0, 1)$.

$f = d^2 = x^2 + y^2 + (z-1)^2$

$f_x = \lambda g_x$

$f_y = \lambda g_y$

$f_z = \lambda g_z$

$2x - 4y + 2z = 14$

$2x = \lambda 2$

$2y = \lambda(-4)$

$2(z-1) = \lambda 2$

$\frac{x}{y} = \frac{-1}{2} \Big/ \frac{y}{z-1} = -2 \Big/ \frac{x}{z-1} = 1$

$2x = -y \quad y = -2z + 2 \quad x = z - 1$

$2x - 4(-2x) + 2(x+1) = 14$

$2x + 8x + 2x = 12$

$12x = 12$

~~$x = 2$~~

~~$y = 6$~~

~~$P_0(-2, 6, 2)$~~

$x = 1$

$y = -2$

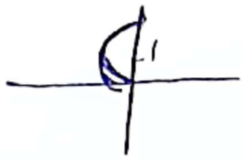
$z = 2$

$P_0 = (1, -2, 2)$

Q8) (5 points) Evaluate the integral $\int_D\int \sqrt{x^2+y^2}\, dA$, where D is the region inside the circle $x^2+y^2=2y$ and $x \le 0$.

$x^2+y^2-2y+1=1$

$x^2+(y-1)^2=1 \Rightarrow r^2=2r\sin\phi$



$$\int_{\frac{\pi}{2}}^{\pi}\int_0^{2\sin\theta} r^2\,dr\,d\phi$$

$$\int_{\frac{\pi}{2}}^{\pi} \frac{r^3}{3}\Big|_0^{2\sin\phi} d\phi = \int_{\frac{\pi}{2}}^{\pi} \frac{8}{3}\sin^3\theta\, d\phi$$

$\cos\phi = u$

$du = -\sin\phi$

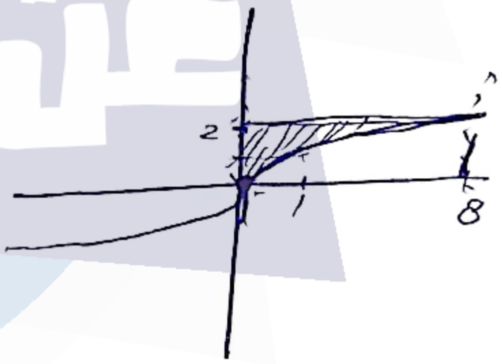
$$-\frac{8}{3}\int_0^{-1}(1-u^2)\,du$$

$$-\frac{8}{3}\left(u-\frac{u^3}{3}\Big|_0^{-1}\right)$$

$$-\frac{8}{3}\left(-1+\frac{1}{3}\right) = \frac{8}{3}\times\frac{2}{3} = \frac{16}{9}$$

Q9) (4 points) Sketch the region and reverse the order of the integration

$$\int_0^8\int_{x^{\frac{1}{3}}}^2 f(x,y)\,dy\,dx.$$



$$\int_0^2\int_0^{y^3} f(x,y)\,dx\,dy$$

Q10) (2 points) If the rectangular coordinate of the point p is $(1, -1, -\sqrt{2})$, find the spherical coordinate of p.

$$P = \left(2, \frac{7\pi}{4}, \frac{3\pi}{4}\right)$$

$z = -\sqrt{2} = \rho\cos\phi \qquad \phi = \frac{3\pi}{4}$

$\rho^2 = 1+1+2 = 4$

$\rho = 2$

$y = -1 = \rho\sin\phi\sin\theta = \frac{2}{\sqrt{2}}\sin\theta \qquad \sin\theta = -\frac{1}{\sqrt{2}} \quad \theta = \frac{7\pi}{4}$

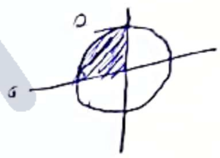
Q11) (4 points) Convert the following integral to cylindrical coordinate (Don't evaluate the integral).

$$\int_0^2 \int_{-\sqrt{4-y^2}}^{0} \int_{\sqrt{3x^2+3y^2}}^{\sqrt{16-x^2-y^2}} x \, dz dx dy.$$

$3r^2 = 16 - r^2$

$r^2 = 4$

$r = 2$



$$\int_{\frac{\pi}{2}}^{\pi} \int_0^2 \int_{\sqrt{3}r}^{\sqrt{16-r^2}} r^2\cos\theta \, dz dr d\theta$$

Q12) (5 points) Use the spherical coordinate to set up the integral that represents the volume of the solid bounded by $z = \sqrt{\frac{x^2+y^2}{3}}$, $z = 2$ and $y \geq 0$.

$\rho\cos\phi = 2$

$\rho = 2\sec\phi$

$$\int_0^{\pi} \int_0^{\frac{\pi}{3}} \int_0^{2\sec\phi} \rho^2 \sin\phi \, d\rho d\phi d\theta$$

$\rho\cos^2\phi = \frac{\rho^2\sin^2\phi}{3}$

$\frac{\sin\phi}{\cos\phi} = \sqrt{3}$

$\tan\phi = \sqrt{3}$

$\phi = \frac{\pi}{3}$

$\rho\cos\phi = \frac{\rho\sin\phi}{\sqrt{3}}$