The University of Jordan, Mathematics Department
Calculus II, Second Exam, 19-5-2022

$\frac{19}{20}$

Student's Name: .... Student's Number: ....

Instructor's Name: Feras yousef .. Lecture Time: 8:30-9:30 (Tues) الثلاثاء

1) (4 points) Find the area of the surface obtained by rotating the curve $y = \sqrt{7-x}$, $5 \le x \le 6$ about the x-axis.

$f'(x) = \frac{-1}{2\sqrt{7-x}} = \frac{-1}{2}(7-x)^{-\frac{1}{2}}$

$(f'(x))^2 + 1 = \frac{1}{4}(7-x)^{-1} + 1$

$S = 2\pi \int_5^6 \sqrt{7-x} * \sqrt{1 + \frac{1}{4(7-x)}}\, dx$

$= 2\pi \int_5^6 \cancel{\sqrt{7-x}} * \frac{\sqrt{28-4x+1}}{2\cancel{\sqrt{7-x}}}\, dx$

$= \pi \int_5^6 (29-4x)^{\frac{1}{2}}\, dx \quad = \frac{2\pi}{-4\cdot 3}(29-4x)^{\frac{3}{2}} \Big|_5^6$

$= \frac{-\pi}{6}\left((29-24)^{\frac{3}{2}} - (29-20)^{\frac{3}{2}}\right)$

$= \frac{-\pi}{6}(8-27) = \frac{19\pi}{6}$

4

2) (2 points) Test the sequence $\left\{ \sqrt[n]{4^{1+2n}} \right\}_{n=1}^{\infty}$ for convergence.

$a_n = \sqrt[n]{4^{1+2n}}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} 4^{\frac{1+2n}{n}}$

$= \lim_{n\to\infty} 4^{\frac{1}{n}} * 16 \quad = 16 \times 1$ (convergent)

$= 16$

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3) (3 points) Find the value of the sum $\frac{9}{25} - \frac{27}{125} + \frac{81}{625} - \frac{243}{3125} + \cdots$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} * \left(\frac{3}{5}\right)^n$$

$$= -\sum_{n=1}^{\infty} \left(\frac{-3}{5}\right)^n = -1 * \frac{\left(\frac{-3}{5}\right)}{1 - -\frac{3}{5}} \quad (r = -\tfrac{3}{5} > -1)$$

$$= \frac{\frac{3}{5}}{\frac{8}{5}} = \frac{3}{8}$$

1

4) Test the following series for convergence:

a) (4 points) $\sum_{n=2}^{\infty} \frac{1}{n \sqrt[3]{\ln n}}$

The function $f(x) = \frac{1}{x \sqrt[3]{\ln x}}$ is positive, decreasing and continuous for $x \geq 2$, Then:

$$\int_2^{\infty} \frac{1}{x (\ln x)^{\frac{1}{3}}} dx = \lim_{R \to \infty} \int_2^R \frac{1}{x (\ln x)^{\frac{1}{3}}} dx$$

$\ln x = w$
$dw = \frac{1}{x} dx$
$R \to \ln R$
$2 \to \ln 2$

$$= \lim_{R \to \infty} \int_{\ln 2}^{\ln R} w^{-\frac{1}{3}} dw$$

$$= \lim_{R \to \infty} \frac{3 w^{\frac{2}{3}}}{2} \Big|_{\ln 2}^{\ln R}$$

$$= \lim_{R \to \infty} \frac{3}{2} \left((\ln R)^{\frac{2}{3}} - (\ln 2)^{\frac{2}{3}} \right) = \underline{\infty}$$

divergent by I.T

4

b) (2 points) $\sum_{n=1}^{\infty} \ln\left(\frac{2n^3+4}{n^3+5}\right)$

$a_n = \ln\left(\frac{2n^3+4}{n^3+5}\right)$

2

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \ln\left(\frac{2n^3+4}{n^3+5}\right) = \ln\left(\lim_{n\to\infty} \frac{2n^3+4}{n^3+5}\right)$

$= \ln\left(\lim_{n\to\infty} \frac{2\cancel{n^3}}{\cancel{n^3}}\right)$

$= \ln 2 \neq 0$

divergent by D.T

c) (3 points) $\sum_{k=1}^{\infty} \tan\left(\frac{1}{k^4}\right)$

$a_k = \tan\left(\frac{1}{k^4}\right)$, $b_k = \frac{1}{k^4}$

$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{\tan\left(\frac{1}{k^4}\right)}{\frac{1}{k^4}} = 1 > 0$

3

since $\frac{1}{k^4}$ convergent $(P = 4 > 1)$

$\sum_{k=1}^{\infty} a_n = \sum_{k=1}^{\infty} \tan\left(\frac{1}{k^4}\right)$ converge by L.C.T

d) (3 points) $\sum_{n=1}^{\infty} \frac{4+\sin(7n)+\cos(3n)}{3^n}$

since $a_n = \frac{4+\sin(7n)+\cos(3n)}{3^n} \leq \frac{6}{3^n} = b_n$

and b_n is converge $(r = \frac{1}{3} < 1)$

3

then $\sum_{n=1}^{\infty} a_n$ converge by C.T