The University of Jordan, Mathematics Department
Calculus II, Midterm Exam

Student's Name: ______ Student's Number: ______

Instructor's Name: Feras Yousef . Lecture Time: Tuesday / 8:30 => 9:30

$\frac{30}{30}$ Good job!

1) (3 points) Determine whether the integral is convergent or divergent (show your work) $\int_1^2 \frac{dx}{(x-1)^{\frac{4}{3}}}$

$x - 1 = w$

$dx = dw$

$x = 1 \Rightarrow w = 0$

$x = 2 \Rightarrow w = 1$

$$\int_0^1 \frac{dw}{w^{\frac{4}{3}}}$$

$$\lim_{R \to 0^-} \int_R^1 \frac{dw}{w^{\frac{4}{3}}} = \lim_{R \to 0^-} -3w^{-\frac{1}{3}} \Big|_R^1$$

$$= \lim_{R \to 0^-} -3 - \left(\frac{-3}{\sqrt[3]{R}}\right)$$

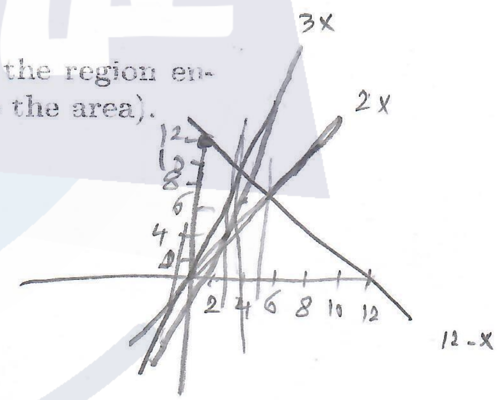
$$\lim_{R \to 0^-} \frac{3}{\sqrt[3]{R}} - 3 = \infty - 3 \text{ (divergent)}$$

$$= \infty$$

3

2) (4 points) Set up the integral that finds the area of the region enclosed by $y = 2x$, $y = 3x$ ~~about~~ and $y + x = 12$. (Do not evaluate the area).

$y = 12 - x$



$2x = 3x$

$\underline{x = 0}$

$2x = 12 - x$

$3x = 12$

$x = 4$

$3x = 12 - x$

$4x = 12$

$x = 3$

$$\int_0^3 (3x) - (2x)\, dx + \int_3^4 (12 - x) - (2x)\, dx$$

4

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3) (4 points) Show that $\int \cot^9(x)\,dx = \dfrac{-\cot^8(x)}{8} - \int \cot^7(x)\,dx$

$$\int \cot^7 x \cot^2 x \, dx$$

$$\frac{\tan^2 x + 1 = \sec^2 x}{\tan^2 x \qquad \tan^2 x}$$

$$1 + \cot^2 x = \csc^2 x$$

$$\int \cot^7 x (\csc^2 x - 1)\, dx$$

$$\int \cot^7 x \csc^2 x \, dx - \int \cot^7 x \, dx$$

$\cot x = w$

$dw = -\csc^2 x \, dx$

$$\int -w^7 \, dw - \int \cot^7 x \, dx$$

$$\frac{-w^8}{8} - \int \cot^7 x \, dx = -\frac{\cot^8 x}{8} - \int \cot^7 x \, dx$$

4

4) (4 points) Use disk's (washer's) method to set up the integral that finds the volume of the solid obtained by rotating the region bounded by $y = 2 + \sec(x)$ and $y = 4$ about $y = -2$. (Do not evaluate the volume).

$2 + \sec x = 4$

$-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\sec x = 2$

$x = \frac{\pi}{3}, -\frac{\pi}{3}$

$$\pi \int_{-\pi/3}^{\pi/3} (4 - -2)^2 - (\sec x + 2 - -2)^2 \, dx$$

$$= \pi \int_{-\pi/3}^{\pi/3} 36 - (\sec x + 4)^2 \, dx$$

4

5) (5 points) Find $\int \frac{4}{(x^2+4x+5)^{\frac{3}{2}}}\,dx$

$$= \int \frac{4}{(\sqrt{x^2+4x+4-4+5})^3}\,dx$$

$$\int \frac{4}{[\sqrt{(x+2)^2+1}]^3}\,dx$$

$x+2 = \tan\phi$

$dx = \sec^2\phi\,d\phi$

$\phi = \tan^{-1}(x+2)$

$\sqrt{(x+2)^2+1} = \sec\phi$

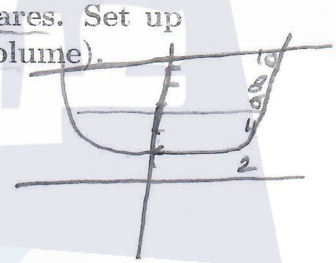
$$\int \frac{4\sec^2\phi\,d\phi}{\sec^3\phi} = \int 4\cos\phi\,d\phi$$

$$= 4\sin\phi + C$$

$$= \frac{4(x+2)}{\sqrt{(x+2)^2+1}} + C$$

5

6) (4 points) The base of the solid S is the region enclosed by $y = x^2 + 2$ and $y = 11$. Cross-sections perpendicular to the y-axis are squares. Set up the integral that gives the volume of S. (Do not evaluate the volume).



$x^2 = y-2$

$x = \mp\sqrt{y-2}$

$+\sqrt{y-2} = -\sqrt{y-2}$

$2\sqrt{y-2} = 0$

$y = 2$

4

$$\int_2^{14} A_{(y)}\,dy$$

$A = L^2$

$$\int_2^{11} L^2\,dy$$

$L = \sqrt{y-2} - -\sqrt{y-2} = 2\sqrt{y-2}$

$$= \int_2^{11} (2\sqrt{y-2})^2\,dy$$

7) (6 points) Find $\int \frac{-2\tan^{-1}(x)}{(x-1)^2}\,dx$

$$= \int -2(x-1)^{-2}\tan^{-1}(x)\,dx$$

$$\tan^{-1}x = u \qquad dv = -2(x-1)^{-2}\,dx$$

$$du = \frac{1}{x^2+1}\,dx \qquad V = 2(x-1)^{-1}$$

$$= \frac{2\tan^{-1}x}{x-1} - 2\int \underbrace{\frac{1}{(x^2+1)(x-1)}}_{\Downarrow}\,dx$$

$$\frac{Ax+b}{x^2+1} + \frac{C}{x-1}$$

$$= \frac{(Ax+b)(x-1) + C(x^2+1)}{(x^2+1)(x-1)}$$

6

$$Ax^2 - Ax + bx - b + Cx^2 + C = 1$$

(1) $A + C = 0$

(2) $-A + b = 0 \quad \Rightarrow A = b$

(3) $-b + C = 1$

(1+2) $C + b = 0$

(1+2+3) $2C = 1 \Rightarrow C = \frac{1}{2}$

$A + C = 0 \Rightarrow A = -\frac{1}{2}$

$A = b \Rightarrow b = -\frac{1}{2}$

$$= \frac{2\tan^{-1}x}{x-1} - 2\int\left(\frac{-\frac{1}{2}x - \frac{1}{2}}{x^2+1} + \frac{\frac{1}{2}}{x-1}\right)dx$$

$$= \frac{2\tan^{-1}x}{x-1} - \ln|x-1| + \frac{1}{2}\int \frac{2x}{x^2+1} - \frac{2}{x^2+1}\,dx$$