Linear – second – First semester – 2019

Q1) Let $p_1 = x^2 - 2x + 1$, $p_2 = 3x - 4$, $p_3 = 5x^2 + 2$.If s $=\{p_1, p_2, p_3\}$ is a basis for $p_2(R)$ *and* $q \in p_2(R)$ such that $(q)_s = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$, *find* q .

Q2) Let B= $\{ \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 3 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \}$ Determine whether B a basis for M_{22}(R) or not.

Q3) Determine whether the following statements always true or sometimes false. (Justify your answer)

a) The set {(1,2,5,0),(4,3,0,5),(5,3,9,2)} is a basis for R^4.

b) If {u,v} is an orthonormal set , then || u + v||=$\sqrt{2}$.

Q4) Let W ={(x,y,z): y=x+3z}

a) Show that W is a subspace of R^3 .

b) Find a basis for W.

c) Find a basis for W^t

Q5) If || 2a +b ||=3, ||a||=2, ||b||=3, find <a,b>.

Q6) If A is 7 x 4 matrix and rank(A)=3 , Find dim(row space),nullity(A),nullity(A^t), $rank(A^t)$.(Justify your answer).

Q7) Let M = {A ∈ M_{nn}:tr(A) =0 }.Find dim(M).

Q1) Let $W = \{(x, y, z) \in R^3 : x + y + z = 0\}$. Show that W is a subspace of R^3 and find dim(W).

$u = (u_1, u_2, u_3)$ $\quad v = (v_1, v_2, v_3)$ $\quad$ u, v ∈ W

$u_1 + u_2 + u_3 = 0$ $\quad v_1 + v_2 + v_3 = 0$

$(u+v) = (u_1+v_1) + (u_2+v_2) + (v_3+u_3) = 0$ $\quad (u+v) \in W$ ①

$ku = k(u_1 + u_2 + u_3) = 0$ $\quad ku \in W$ ②

$ku = (ku_1, ku_2, ku_3)$

$w = \{(x, y, z) \in R^3 : -x - y = z\}$

$w = \{(x, y, -x-y) \in R^3 : z = -x-y\}$ $\quad w(x(1, 0, -1), y(0, 1, -1))$

the basis has 2 vectors

dim(W) = 2

Q2) Let $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\}$. Show that S is a basis for M_{22}. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find $(A)_S$.

$S = a\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + c\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + d\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$a + b + c + d = 0$, $a = 0$
$b + c + d = 0$, $b = 0$
$c + d = 0$ → $c = 0$
$d = 0$

it has just the trivial solution

its independent ①

$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

det = 1 $\quad$ it spans M_{22}

* we can write b as a linear compenation of the vectors in (S)

$a + b + c + d = b_1$
$b + c + d = b_2$
$c + d = b_3$
$d = b_4$

Q3) Let $v_1 = (1, -1, 0)$, $v_2 = (2, 0, 1)$. Find all vectors $v_3 = (a, b, c)$ such that the set $\{v_1, v_2, v_3\}$ is a basis for R^3.

- independent det
- spans R_3 det ≠ 0

$$S = \begin{bmatrix} 1 & 2 & a \\ -1 & 0 & b \\ 0 & 1 & c \end{bmatrix}$$

det(S) = 0 + 0 − a − b ...

$-a - b + 2c \neq 0$

$c \neq \frac{a+b}{2}$

$v_3 \{ c, a, b \in R, c \neq \frac{a+b}{2} \}$

Q4) Show that there is no 4 × 6 matrix A such that nullity(A) = 1.

(1) max is the rank = min(4, 6) = 4

(2) 0 ≤ rank ≤ 4

0 ≤ 6 − nullity ≤ 4

−6 ≤ −nullity ≤ −2

6 ≥ nullity ≥ 2

can't be less than 2

nullity(A) ≥ 2 so ≠ 1 can't be = 1

Question 1

Correct

Mark 2.00 out of 2.00

Flag question

Let $A = \begin{bmatrix} x & -1 & 3 \\ 1 & 2 & 1 \\ 4 & -2 & 5 \end{bmatrix}$, and $C = \begin{bmatrix} 12 & -1 & -10 \\ -1 & -2 & y \\ -7 & 1 & 5 \end{bmatrix}$.

If C is the cofactor matrix of A, then $x + y =$

A) 2

B) −5

C) 6

D) 8

- A ✔
- B
- C
- D

The correct answer is:
A

Question **2**

Correct

Mark 2.00 out of 2.00

Flag question

One of the following is always true

A) If $det(A) = det(A^{-1})$**, then** $det(A) = \mp 1$

B) If $B = adj(A)$**, then** $AB = I$

C) $adj(AB) = adj(A)adj(B)$

D) If $det(A) = 3$**, then the system** $Ax = b$ **has infinitely many solutions**

- A ✔
- B
- C
- D

The correct answer is:

A

Incorrect

Mark 0.00 out of 2.00

Flag question

One of the following is Not true

A) $W = \{(a, b, c) : a < b\}$ **is a subspace of** R^3

B) $W = \{(a, b, c) : b = 2c + a\}$ **is a subspace of** R^3

C) $W = \{A \in M_{nn} : tr(A) = 0\}$ **is a subspace of** M_{nn}

D) $W = \{A \in M_{nn} : A \text{ is lower}\}$ **is a subspace of** M_{nn}

- A
- B ✖
- C
- D

The correct answer is:
A

Question **4**

Correct

Mark 3.00 out of 3.00

Flag question

Let $s = \{(x^2-3x+6, 2x-5, x^2+1\}$ be a basis for P_2, let $q \in P_2$. If $(q)_s = [2, -3, 2]$, then $q =$

A) $3x^2 - 12x + 28$

B) $3x^2 - 12x + 28$

C) $3x^2 - 12x + 28$

D) $4x^2 - 12x + 29$

- A
- B
- C
- D ✔

The correct answer is:

D

Question 5

Incorrect

Mark 0.00 out of 3.00

Flag question

Let A and B be 3×3 matrices, such that $det(A) = 2$, $det(B) = -4$, then $det(2A^T B^{-2}) =$

A) 1

B) −1

C) $\frac{1}{4}$

D) $\frac{-1}{4}$

- A
- B
- C
- D

The correct answer is:
A

Question 6

Correct

Mark 2.00 out of 2.00

Flag question

Let $s = \{(1,3,5), (3,3,2), (-1,1,8)\}$ be a basis for R^3. If $u = (6, 10, 17)$, then $(u)_s =$

A) [1, 2, 1]

B) [2, 2, 1]

C) [1, 1, 1]

D) [2, 1, 1]

A ✔

B

C

D

The correct answer is:
A

Question **7**

Correct

Mark 2.00 out of 2.00

Flag question

Let $s = \{(\lambda, 2, 5), (3, \lambda, 4), (-1, 3, 6)\}$, then one of the following is true

A) If $\lambda = \frac{1}{6}$, then s is linearly dependent

B) If $\lambda = 1$, then s is linearly independent

C) If $\lambda = \frac{1}{6}$, then s is linearly independent

D) If $\lambda = 2$, then s is linearly dependent

- A ✔
- B
- C
- D

The correct answer is:
A

Question 8

Correct

Mark 3.00 out of 3.00

Flag question

Let $A = \begin{bmatrix} 2 & 4 & -1 \\ 5 & 3 & 2 \\ 6 & 7 & -1 \end{bmatrix}$.

If $B = adj(A)$, then $b_{21} + b_{23} =$

A) 7

B) −8

C) 8

D) −7

A

B

C ✔

D

The correct answer is:
C

Question 9

Correct

Mark 3.00 out of 3.00

Flag question

If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 5$, then $\begin{vmatrix} -d & -e & -f \\ a & b & c \\ a-4g & b-4h & c-4i \end{vmatrix} =$

A) 5

B) −5

C) −20

D) 20

A

B

C ✓

D

The correct answer is:
C

Question **10**

Incorrect

Mark 0.00 out of 2.00

Flag question

Let $W = \{A \in M_{33} : A \text{ is lower and } tr(A) = 0\}$, **then** $dim(W) =$

A) 5

B) 9

C) 6

D) 3

A

B

C

D ✖

The correct answer is:
A

Question 11

Correct

Mark 3.00 out of 3.00

Flag question

Consider the following system:

$x + 3y + az = -2,$

$x - y + bz = 1,$

$2x + 4y + cz = -3.$

If $\begin{vmatrix} 1 & 3 & a \\ 1 & -1 & b \\ 2 & 4 & c \end{vmatrix} = 6$, and $x = s_1$, $y = s_2$, $z = s_3$ is the solution of the system, then $s_3 =$

A) $\frac{-1}{3}$

B) $\frac{2}{3}$

C) $\frac{-2}{3}$

D) $\frac{1}{3}$

- A
- B
- C
- D ✔

The correct answer is:

D

Question 12

Incorrect

Mark 0.00 out of 3.00

Flag question

Let $W = \{(a, b, c, d) : a + 2b = d,\ c = 3a\}$ be a subspace of R^4, then one of the following is a basis for W

A) $\{(1, 0, 3, 1), (0, 1, 0, 2)\}$

B) $\{(1, 0, 3, 2), (0, 1, 0, 1)\}$

C) $\{(1, 0, 0, 1), (0, 1, 3, 2)\}$

D) $\{(1, 0, 1, 3), (0, 1, 2, 0)\}$

- A
- B
- C
- D ✖

The correct answer is:
A

Jordan University, Mathematics Department

Linear Algebra, Second Exam II

Answer all Questions. Final answer without supporting work will not receive any credit.

1) a) (4 points) Let $A = \begin{bmatrix} 2 & 2 & 1 & 1 \\ -2 & 3 & 1 & 2 \\ -2 & -2 & 1 & 3 \\ 4 & 4 & 2 & 1 \end{bmatrix}$. Find $\det(A)$.

b) (2 points) Suppose that B is a 2×2 matrix with $\det(B) = 3$. Find $\det(4B^{-1}B^T)$.

2) a) (4 points) Let $W = \{(x, y) : x + 4y = 0 \text{ and } x, y \in \mathbb{R}\}$. Show that W is a subspace of $\mathbb{R}^2$

b) (3 points) Let S be the set of all 2×2 matrices of real entries of the form $\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$ such that $a_2 \geq 0$ with usual addition and scaler multiplication. Is S a vector space? Explain your answer.

3) a) (3 points) Show that $(3, -3, 2)$ in span$\{(1, 0, 0),\ (1, 1, -1),\ (0, 2, -1)\}$.

b) (4 points) Determine whether the matrices $\begin{bmatrix} 4 & -4 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 2 & -1 \\ 2 & -2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$ are linearly independent or dependent in M_{22}. Show your work.

Q1 a) $A = \begin{bmatrix} 2 & 2 & 1 & 1 \\ -1 & 3 & 1 & 2 \\ -2 & -2 & 1 & 3 \\ 4 & 4 & 2 & 1 \end{bmatrix} \xrightarrow{\text{Row operations}} \begin{bmatrix} 2 & 2 & 1 & 1 \\ 0 & 5 & 2 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

$\rightarrow \det(A) = 2(5)(2)(-1) = -20$

b) $\det(4B^{-1}B^T) = 4^2 \det(B^{-1}B^T) = 4^2 \det(B^{-1}) \det(B^T)$

$= 4^2 \frac{1}{\det B} \det B = 16$.

Q2 a) First $(0,0) \in W$ since $0 + 4(0) = 0$ -

Suppose (x_1, y_1) & $(x_2, y_2) \in W$. Then $x_1 + 4y_1 = 0$

and $x_2 + 4y_2 = 0 \rightarrow (x_1 + 4y_1) + (x_2 + 4y_2) = 0$

$\rightarrow (x_1 + x_2) + 4(y_1 + y_2) = 0$

$\rightarrow (x_1 + x_2, y_1 + y_2) \in W$.

Now, let $a \in \mathbb{R}$, then ~~$a(x_1 + 4y_1) \Rightarrow ax_1 + 4(ay_1) = 0$~~

$a(x_1 + 4y_1) = 0 \rightarrow (ax_1) + 4(ay_1) = 0$

$\rightarrow (ax_1, ay_1) \in W$.

Thus W is a subspace of $\mathbb{R}^2$.

b) No, Take $u = \begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix} \in S$ $(2 > 0)$

but $-2u = \begin{bmatrix} -2 & -4 \\ -10 & -6 \end{bmatrix} \notin S$ since $-4 < 0$.

Thus S is not a vector space -

3)a) Find x_1, x_2, x_3 such that $x_1(1,0,0) + x_2(1,1,-1) + x_3(0,2,-1) = (3,-3,2)$

→ Solve
$$x_1 + x_2 + 0x_3 = 3$$
$$0x_1 + x_2 + 2x_3 = -3$$
$$0x_1 - x_2 - x_3 = 2$$

The coeficient matrix $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & -1 & -1 \end{bmatrix} \xrightarrow{\text{Row operations}}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \det(\ ) = 1 \rightarrow$ The coefficient invertible → System is Consistent ∴

$(3,-3,2) \in \text{span}\{(1,0,0), (1,1,-1), (0,2,-1)\}$.

b) Look at the hom. system

$$c_1\begin{bmatrix} 4 & -4 \\ 0 & 0 \end{bmatrix} + c_2\begin{bmatrix} 2 & -1 \\ 2 & -2 \end{bmatrix} + c_3\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

→
$$4c_1 + 2c_2 + c_3 = 0$$
$$-4c_1 - c_2 + c_3 = 0$$
$$0c_1 + 2c_2 - c_3 = 0$$
$$0c_1 - 2c_2 + 2c_3 = 0$$
$\longrightarrow \left[\begin{array}{ccc|c} 4 & 2 & 1 & 0 \\ -4 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & -2 & 2 & 0 \end{array}\right]$

Row operations → $\left[\begin{array}{ccc|c} 4 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \rightarrow c_3 = 0$, $2c_2 - c_3 = 0 \xrightarrow{c_3=0} c_2 = 0$

$4c_1 + 2c_2 + c_3 = 0 \xrightarrow[c_2=0]{c_3=0} c_1 = 0$

The System has only trivial solution
→ ~~Trivial solution~~.

→ Matrices are linearly independent.

Linear– second exam – First semester – 2022

(Ch4 without Ch4.6)

Q1) Determine which of the following is a subspace of M_{nn} and justify your answer.

a) W ={A ∈ M_{nn} : $A = A^t$}

b) W ={A ∈ M_{nn} : $A\ is\ invertible$ }

Q2) If S={(1,k,2),(k,1,1),(-1,3,1)} , find k such that S is linearly independent .

Q3) If S = {1 , 1+x , 1+x+x^2}. Show that S is a basis for P_2.

Q4) If A = $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 3 & 2 & 1 & 2 \\ -1 & 2 & -3 & 6 \end{bmatrix}$, find rank(A) , nullity(A) and nullity(A^t).