Linear – second – First semester – 2019

Q1) Let  $p_1 = x^2 - 2x + 1$ ,  $p_2 = 3x - 4$ ,  $p_3 = 5x^2 + 2$ . If  $s = \{p_1, p_2, p_3\}$  is a basis for  $p_2(R)$  and  $q \in p_2(R)$  such that  $(q)_s = \begin{bmatrix} 3\\ 2\\ -1 \end{bmatrix}$ , find q.

Q2) Let B= {  $\begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$ ,  $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ ,  $\begin{bmatrix} 0 & 3 \\ 0 & 1 \end{bmatrix}$ ,  $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ } Determine whether B a basis for  $M_{22}(R)$  or not.

Q3) Determine whether the following statements always true or sometimes false. (Justify your answer)

- a) The set {(1,2,5,0), (4,3,0,5), (5,3,9,2)} is a basis for  $\mathbb{R}^4$ .
- b) If {u,v} is an orthonormal set , then  $|| u + v || = \sqrt{2}$ .

Q4) Let  $W = \{(x,y,z): y=x+3z\}$ 

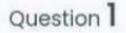
- a) Show that W is a subspace of  $R^3$ .
- b) Find a basis for W.
- c) Find a basis for  $W^t$

Q5) If || 2a +b ||=3, ||a||=2, ||b||=3, find <a,b>.

# Q6) If A is 7 x 4 matrix and rank(A)=3, Find dim(row space), nullity(A), nullity( $A^t$ ), rank( $A^t$ ). (Justify your answer).

Q7) Let  $M = \{A \in M_{nn}: tr(A) = 0\}$ . Find dim(M).

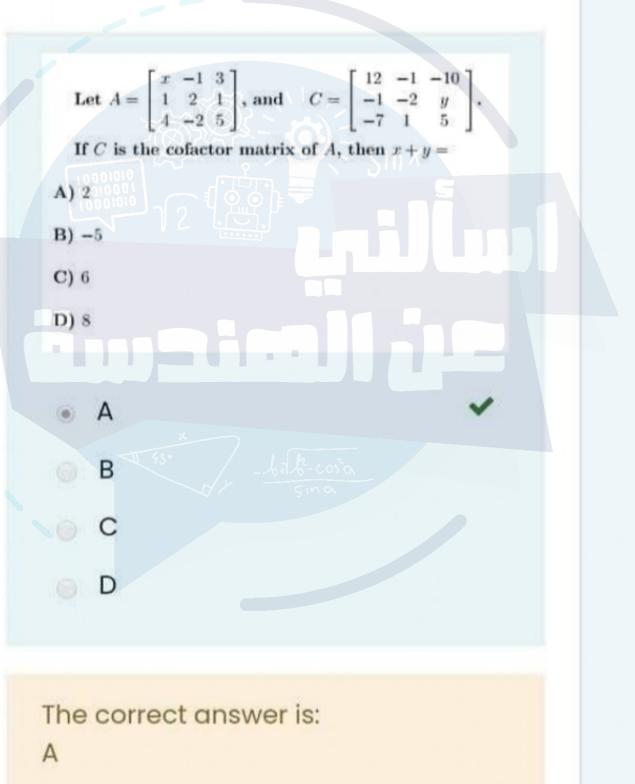
(3) Let 
$$v_1 = (1, -1, 0)$$
,  $v_2 = (2, 0, 1)$ . Find all vectors  $v_1 = (a, b, c)$   
such that the set  $(v_1, v_2, v_1)$  is a basis for  $B^3$ . Involve feedden 1 def  
spans  $R_3$  def 2 def  
 $(3, -1)$   $(3, -1)$ 



Correct

Mark 2.00 out of 2.00

₱ Flag question



Correct

Mark 2.00 out of 2.00

𝒫 Flag question

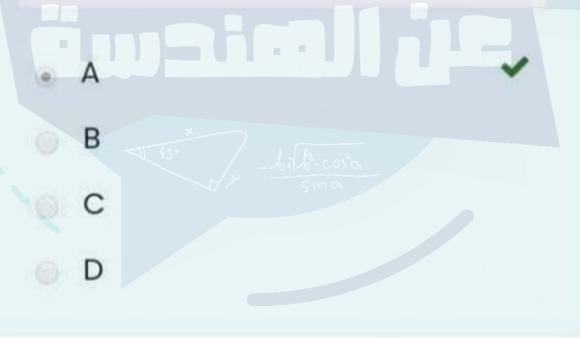
One of the following is always true

A) If 
$$det(A) = det(A^{-1})$$
, then  $det(A) = \mp 1$ 

**B)** If 
$$B = adj(A)$$
, then  $AB = I$ 

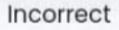
C) 
$$adj(AB) = adj(A)adj(B)$$

D) If det(A) = 3, then the system Ax = b has infinitely many solutions

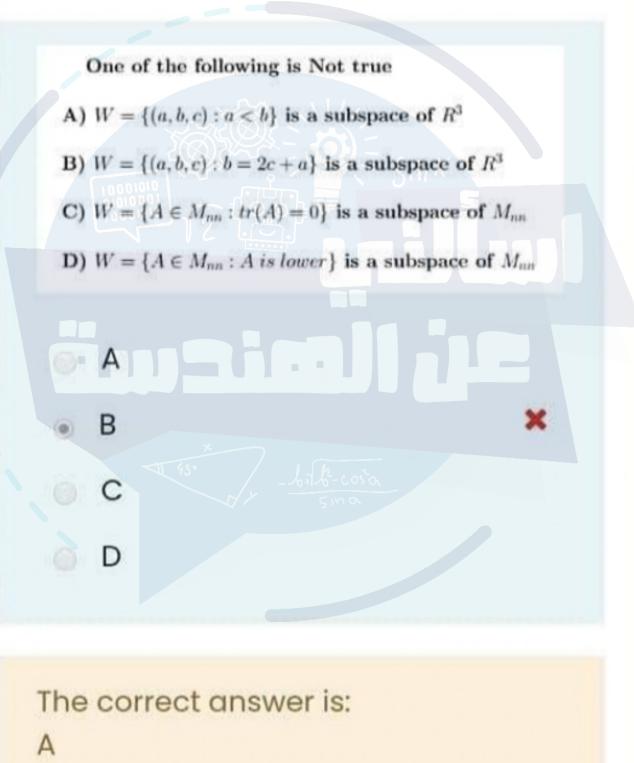


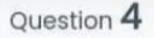
#### The correct answer is:

A



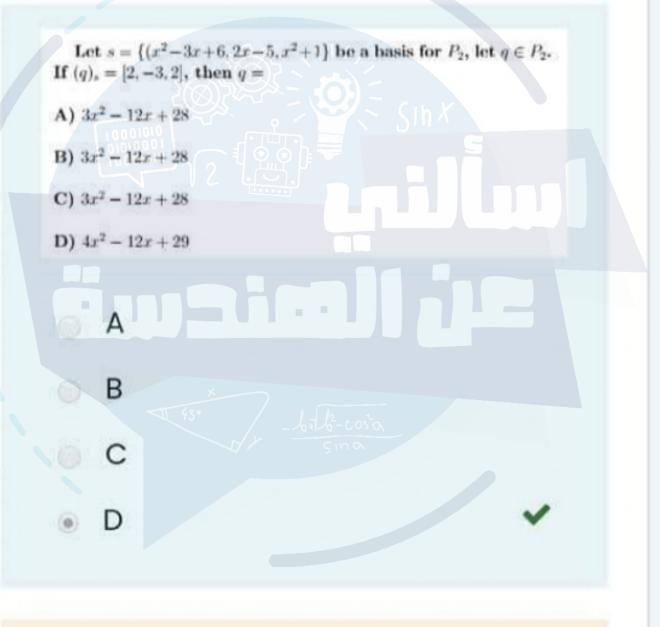
Mark 0.00 out of 2.00



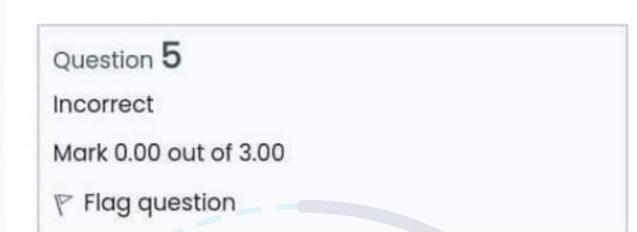


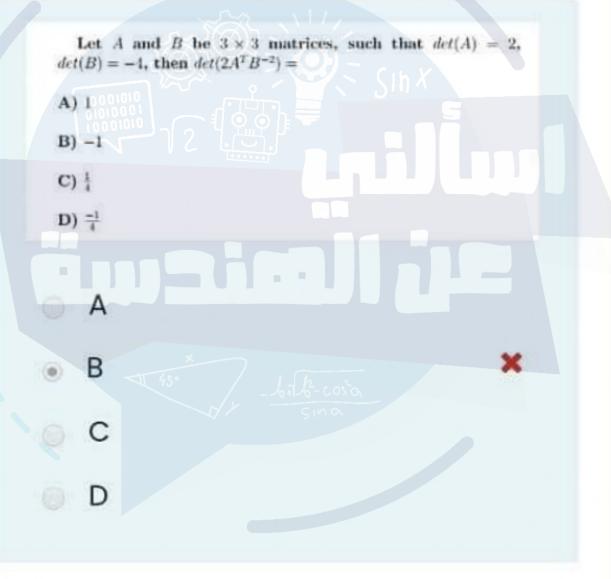
Correct

Mark 3.00 out of 3.00



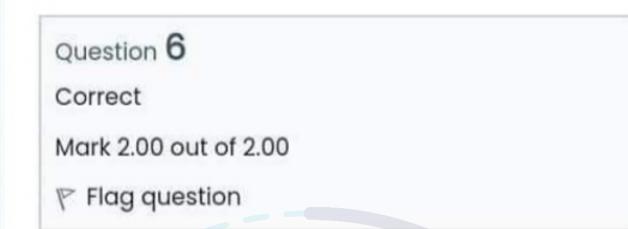
The correct answer is:

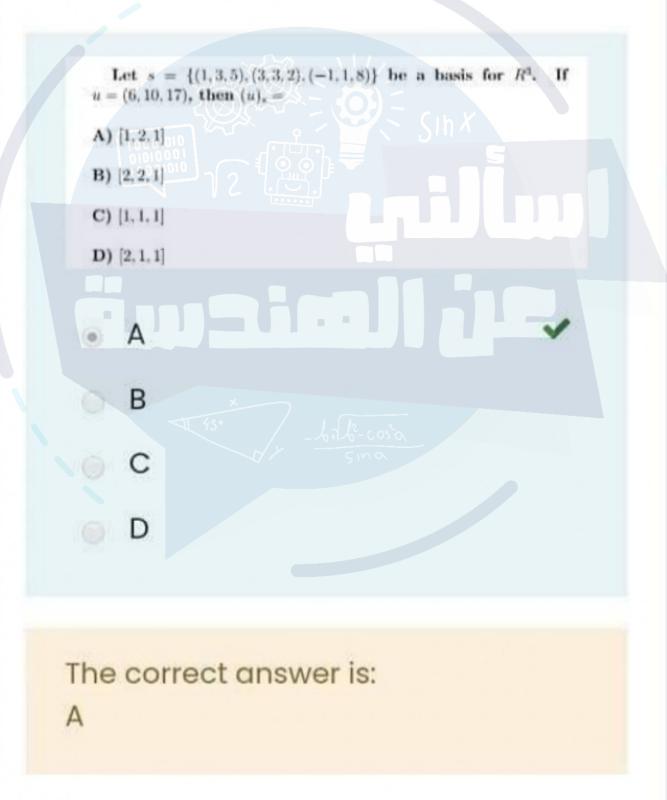




### The correct answer is:

A





Correct

Mark 2.00 out of 2.00

𝒫 Flag question

A

В

C

D

A

Let  $s = \{(\lambda, 2, 5), (3, \lambda, 4), (-1, 3, 6)\}$ , then one of the following is true

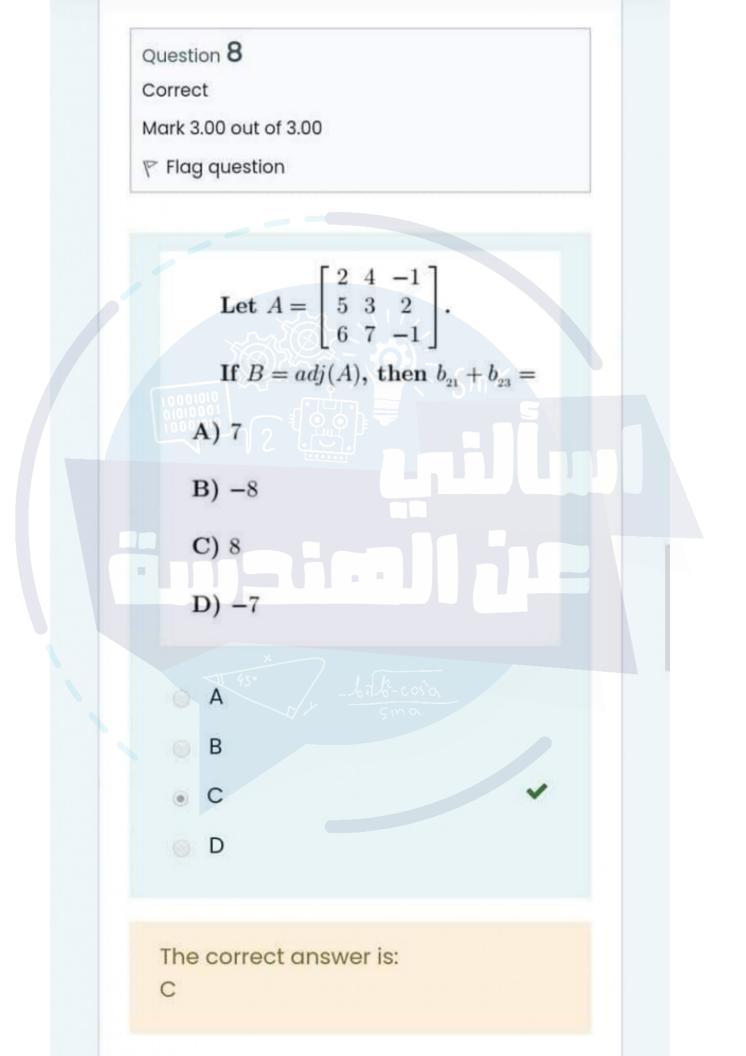
A) If  $\lambda = \frac{1}{6}$ , then s is linearly dependent

B) If  $\lambda = 1$ , then s is linearly independent

C) If  $\lambda = \frac{1}{6}$ , then s is linearly independent

D) If  $\lambda = 2$ , then s is linearly dependent

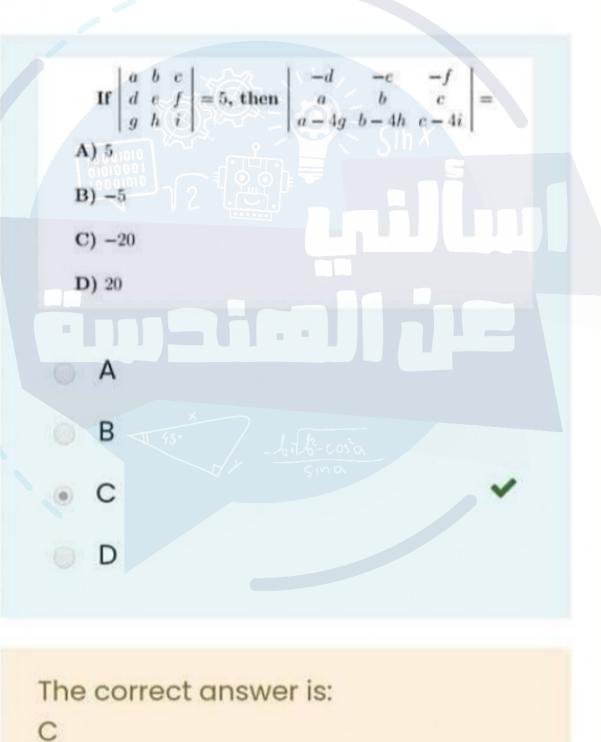
The correct answer is:





Mark 3.00 out of 3.00

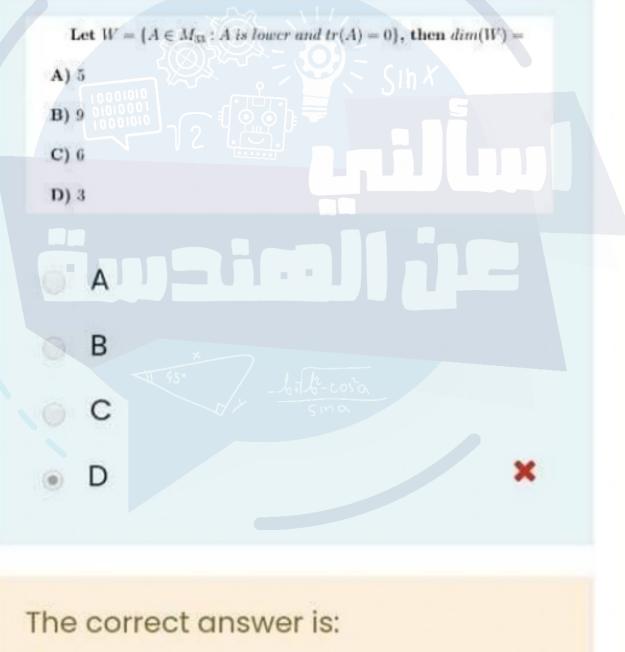
♥ Flag question



Incorrect

Mark 0.00 out of 2.00

𝒫 Flag question

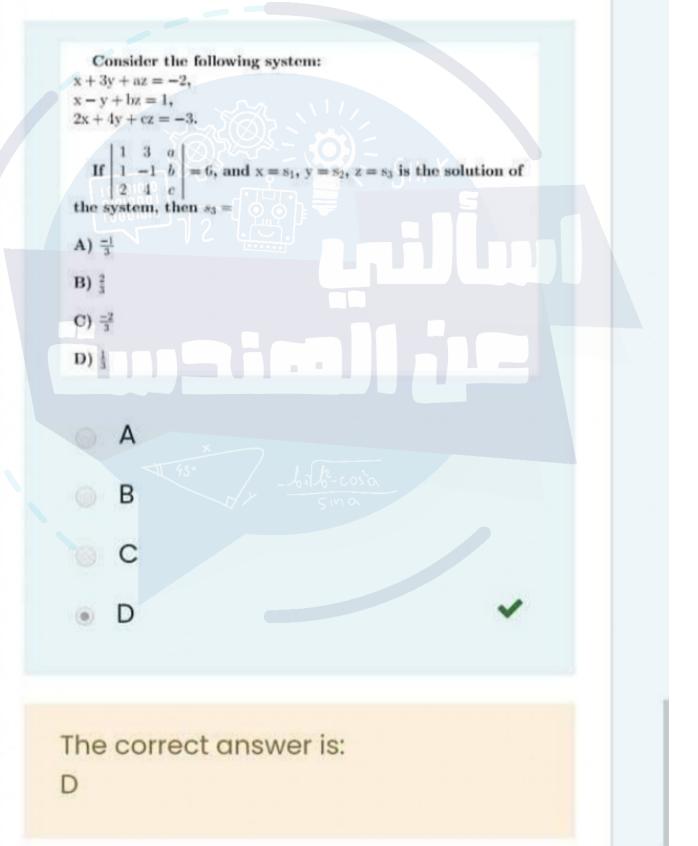


A

Correct

Mark 3.00 out of 3.00

♥ Flag question





Incorrect

Mark 0.00 out of 3.00

₱ Flag question

Let  $W = \{(a, b, c, d) : a + 2b = d, c = 3a\}$  be a subspace of  $R^4$ , then one of the following is a basis for W

A)  $\{(1,0,3,1), (0,1,0,2)\}$ 

**B)**  $\{(1,0,3,2), (0,1,0,1)\}$ 

C)  $\{(1,0,0,1), (0,1,3,2)\}$ 

**D)**  $\{(1,0,1,3), (0,1,2,0)\}$ 

A

В

C

D

The correct answer is: A

#### Jordan University, Mathematics Department Linear Algebra, Second Exam II

Answer all Questions. Final answer without supporting work will not receive any credit.

1) a) (4 points) Let 
$$A = \begin{bmatrix} 2 & 2 & 1 & 1 \\ -2 & 3 & 1 & 2 \\ -2 & -2 & 1 & 3 \\ 4 & 4 & 2 & 1 \end{bmatrix}$$
. Find det(A).

b) (2 points) Suppose that B is a  $2 \times 2$  matrix with det(B) = 3. Find  $\det(4B^{-1}B^T).$ 

- 2) a) (4 points) Let  $W = \{(x, y) : x + 4y = 0 \text{ and } x, y \in \mathbb{R}\}$ . Show that W is a subspace of  $\mathbb{R}^2$
- b) (3 points) Let S be the set of all  $2 \times 2$  matrices of real entries of the

form  $\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$  such that  $a_2 \ge 0$  with usual addition and scaler

multiplication. Is S a vector space? Explain your answer.

3) a) (3 points) Show that (3, -3, 2) in span $\{(1, 0, 0), (1, 1, -1), (0, 2, -1)\}$ .

b) (4 points) Determine whether the matrices  $\begin{bmatrix} 4 & -4 \\ 0 & 0 \end{bmatrix}$ ,  $\begin{bmatrix} 2 & -1 \\ 2 & -2 \end{bmatrix}$  and  $\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$  are linearly independent or dependent in  $M_{22}$ . Show your work.

$$\begin{array}{c} -1-t_{max} \\ (Q,\alpha) \ A_{2} \left[ \begin{array}{c} 2 & 2 & 1 & 1 \\ 1 & -2 & 1 & 3 \\ 1 & -2 & 2 & 1 \end{array} \right] \xrightarrow{R} ev quarkins \left[ \begin{array}{c} 2 & 2 & 1 & 1 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & -1 \end{array} \right] \\ \rightarrow dut(A) = 2(r)(1)(1) = -2i \\ (A) = u^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}) ut(B) \\ = u^{2} t_{0}t^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}) ut(B) \\ = u^{2} t_{0}t^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}) ut(B) \\ = u^{2} t_{0}t^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}) ut(B) \\ = u^{2} t_{0}t^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}) ut(B) \\ = u^{2} t_{0}t^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}) ut(B) \\ = u^{2} t_{0}t^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}) ut(B) \\ = u^{2} t_{0}t^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}) ut(B) \\ = u^{2} t_{0}t^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}) ut(B) \\ = u^{2} t_{0}t^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}) ut(B) \\ = u^{2} t_{0}t^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}) ut(B^{-1}B^{T}) \\ = u^{2} t_{0}t^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}) ut(B^{-1}B^{T}) \\ = u^{2} t_{0}t^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}B^{T}) \\ = u^{2} t_{0}t^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}B^{T}) \\ = u^{2} t_{0}t^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}B^{T}) \\ = u^{2} t_{0}t^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}B^{T}) \\ = u^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}B^{T}) = u^{2} dut(B^{-1}B^{T}) \\ = u^{2} dut(B^{-1}B^{T})$$

$$-2 - \operatorname{Term} 2$$
3)a) Find  $x_{1,1}x_{1,2}x_{3} \operatorname{such} + \operatorname{flat} x_{1}(1,q_{0}) + x_{3}(1,1_{1}-1) + x_{3}(0,1_{1}-1) = (2_{1},2_{1})$ 

$$\Rightarrow S.bec \begin{bmatrix} x_{1} + y_{2} + ex_{3} = 3 \\ ex_{1} + x_{2} + 2x_{3} = -3 \\ ex_{1} - x_{2} - x_{3} = 2 \end{bmatrix}$$
The coefficient matrix  $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{\circ} \operatorname{det} () = 1 \xrightarrow{\circ} \operatorname{The} \operatorname{for} \operatorname{firster} f$ 

$$= \left[ \begin{array}{c} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{array} \right] \xrightarrow{\circ} \operatorname{det} () = 1 \xrightarrow{\circ} \operatorname{The} \operatorname{for} \operatorname{firster} f$$

$$= \left[ \begin{array}{c} 3_{1}, 3_{1}, 2 \\ 0 & 0 & 1 \end{array} \right] \xrightarrow{\circ} \operatorname{det} () = 1 \xrightarrow{\circ} \operatorname{The} \operatorname{for} \operatorname{firster} f$$

$$= \left[ \begin{array}{c} 3_{1}, 3_{1}, 2 \\ 0 & 0 & 1 \end{array} \right] \xrightarrow{\circ} \operatorname{det} () = 1 \xrightarrow{\circ} \operatorname{The} \operatorname{for} \operatorname{firster} f$$

$$= \left[ \begin{array}{c} 3_{1}, 3_{1}, 2 \\ 0 & 0 & 1 \end{array} \right] \xrightarrow{\circ} \operatorname{ferm} \left\{ \operatorname{firster} f = 1 \\ \left[ \begin{array}{c} 3_{1}, 3_{1}, 2 \\ 0 & 0 \end{array} \right] + \operatorname{for} \left[ \begin{array}{c} 2 & -1 \\ 2 & -2 \end{array} \right] + \operatorname{for} \left[ \begin{array}{c} 1 & -1 \\ -4 & -1 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 & 0 \end{array} \right] \xrightarrow{\circ} \operatorname{ferm} \left\{ \operatorname{firster} f = 1 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ -4 & -1 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 2 \\ -4 & -1 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 2 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 2 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 2 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 & 0 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 & 2 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \end{array} \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 \end{array} \right] \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \end{array} \xrightarrow{\circ} \left[ \begin{array}{c} 0 & 0 \\ 0 \end{array} \end{array}$$

$$\xrightarrow{$$

Linear— second exam — First semester — 2022 (Ch4 without Ch4.6)

Q1) Determine which of the following is a subspace of  $M_{nn}$  and justify your answer.

- a) W ={A  $\in M_{nn}$  :  $A = A^t$ }
- b) W ={A  $\in M_{nn}$  : A is invertible }

