Linear – First exam – Second semester – 2019

Q1: Circle the correct answer :

1) If A=$\begin{bmatrix} -3 & 1 \\ 0 & 2 \end{bmatrix}$ and p(x)= $x^2 - 2x$-1, then p(A)=

   a) $\begin{bmatrix} 14 & -4 \\ 0 & -1 \end{bmatrix}$

   b) $\begin{bmatrix} 14 & -3 \\ 0 & -1 \end{bmatrix}$

   c) $\begin{bmatrix} 2 & -3 \\ 0 & -1 \end{bmatrix}$

   d) $\begin{bmatrix} 14 & 0 \\ 0 & -1 \end{bmatrix}$

2) For which values of the constant a does the system

   (a+3)x + 8y =0

   ax + (a-3)y =0

   have only the trivial solution ?

   a) a ∈ **R – {-9,1}**

   b) a ∈ **{-9,1}**

   c) a ∈ **R – {9,-1}**

   d) a ∈ **{9,-1}**

3) det $\left(tr\left(\begin{bmatrix} 4 & -3 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} -2 & 1 \\ -2 & 4 \end{bmatrix}\right)$=

a) 24

b) -24

c) 18

d) -18

4) Which one of the following matrices is elementary:

a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

5) If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -7 & -2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}, then \begin{bmatrix} a & b \\ c & d \end{bmatrix} =$

a) $\begin{bmatrix} -1 & 0 \\ -7 & -2 \end{bmatrix}$

b) $\begin{bmatrix} -11 & -5 \\ 7 & 3 \end{bmatrix}$

c) $\begin{bmatrix} -4 & 1 \\ 14 & -4 \end{bmatrix}$

d) $\begin{bmatrix} 6 & -2 \\ -14 & 7 \end{bmatrix}$

6) What a condition on b_1, b_2 and $b_3$in order for the system:

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ -2 & -4 & 2 \end{bmatrix}\begin{bmatrix} X \\ y \\ Z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$ to be consistent ?

a) b_3= -2b_1

b) b_1= 2b_2

c) b_1= -2b_3

d) b_2= 2b_1

7) If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ = -3 , then $\begin{vmatrix} a & 3d & -2g \\ b & 3e & -2h \\ c & 3f & -2i \end{vmatrix}$ =

a) -12

b) 18

c) -18

d) -81

8) Which one of the following statements is false for any n x n matrices A and B .

a) det(AB)$^{-1}$=det (A^{-1}) det (B^{-1})

b) A (A+B) =$A^2 + AB$

c) $(A + B)^2 = A^2 + 2AB + B^2$

d) $(A^{-1} + B^{-1})^t$= $(A^t)^{-1}$+$(B^t)^{-1}$

9) The solution of the system

x + 2y + z =2

-x -y + 2z =-9

2x + y -z =7

is:

a) {(1,2,-3)}

b) {(-1,2,-3)}

c) {(2,-1,1)}

d) {(2,-9,7)}

Q2)

a) Use Cramer's Rule to solve the following system

X + 2z =2

-3x + 4y +6z =30

-x -2x +3z =8

Q3) Let A = $\begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -1 \\ -2 & 3 & -2 \end{bmatrix}$

1) Find the adjoint of A.

2) Find the inverse of A.

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Q1: let B = $\begin{bmatrix} 3 & 4 \\ 2 & 2 \end{bmatrix}$

a. Find B^{-1}

b. If $(2A - 3I)^{-1}$=B, find A.

c. If BX=$\begin{bmatrix} 5 & 1 & 4 \\ -3 & -1 & -2 \end{bmatrix}$ .Find X.

Q2: If A and B are 4 x 4 matrices, |A |=2 and |B|=5. Find:

a. | A^tB|.

b. |$3B^{-2}$|.

Q3: Determine whether the following statements always true or sometimes false. (Justify your answer)

a. If B is invertible and skew symmetric, then $B^{-1}$is skew symmetric.

b. If the matrix A is invertible , then $\mathrm{adj}(A^{-1})=(\mathrm{adj}(\mathrm{A}))^{-1}$

Q4: Determine all values of (b) that make the following system has NO solution.

x-3y+z=4

-x-7y+z=10

2x-y+z=b

Q5: Let A = $\begin{bmatrix} 2 & 1 & 3 \\ -1 & 4 & 2 \\ 2 & 1 & 5 \end{bmatrix}$

Use adjoint method to find A^{-1}.

Q6:

a. For any matrix A, prove that the matrix C=A A^t– A^tA is symmetric .

b. If B is invertible matrix and AB^{-1}= B^{-1}A. Prove that AB=BA.

Q1) Find the value of k for which the system

$$x_1 + x_2 - x_3 = 1$$
$$2x_1 - x_2 + x_3 = -1$$
$$x_1 - 2x_2 + 2x_3 = k$$

is consistent.

$\rightarrow \begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & -1 & 1 & -1 \\ 1 & -2 & 2 & k \end{bmatrix} \xrightarrow[-R_1+R_3]{-2R_1+R_2} \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -3 & 3 & -3 \\ 0 & -3 & 3 & k-1 \end{bmatrix}$

$\xrightarrow{R_2} \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & -3 & 3 & (k-1) \end{bmatrix} \xrightarrow[3R_2+R_3]{-R_2+R_1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & k+2 \end{bmatrix}$

$k+2=0 \quad \boxed{k=-2}$

$A=A^t \quad B=B^t$

Q2) a) If A and B are symmetric matrices of the same size, prove that A + B is symmetric.

$(A+B)^t \rightarrow A^t + B^t = A+B \quad \therefore (A+B)^t = A+B$ so

$(A+B)$ is symmetric

$A^t + B^t \xrightarrow[B=B^t]{A=A^t} (A+B) \quad \therefore (A+B)^t = A+B$

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b) Give an example of 2×2 symmetric matrices A and B such that AB is not symmetric.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \quad AB = \begin{bmatrix} 3 & 1 \\ 5 & 10 \end{bmatrix}$$

Q3) a) Let $A = \begin{bmatrix} a & 0 \\ b & 1 \end{bmatrix}$. Find the values of a and b for which $A^tA = I$.

$$\overset{A}{\begin{bmatrix} a & 0 \\ b & 1 \end{bmatrix}} \overset{A^t}{\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}} = \begin{bmatrix} a^2 & ab \\ ab & b^2+1 \end{bmatrix}$$

$\to ab = 0$

(a) or $b = 0$

$a^2 = 1 \therefore b = 0$

$a^2 = 1$

$a = \pm 1$

$\therefore a = \pm 1 \quad b = 0$

$a = 1 / a = -1$

$$\begin{bmatrix} a^2 & ab \\ ab & b^2+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \therefore$$

$b^2 + 1 = 1 \to b = 0$

$a^2 = 1 \to a = -1 / a = 1$

$a^2 = b^2 + 1 = 1$

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b) Let A be a square matrix such that $A^2 = 0$. Prove that A is not invertible.

$\det(0) = 0 \to$

$\det(A^2) = 0 \to (\det(A))^2 = 0$

$\to \det(A) = 0$

$\therefore \det(A) = 0$ A is not invertable

Q4) Let A and B be 3 × 3 matrices such that $\det(A) = 2$ and $\det(B) = -3$. Find

a) $\det(A^t B^{-1})$ b) $\det(\frac{1}{2}A)$ c) $\det(\text{adj}(B))$.

$\det(A^t)\det(B^{-1})$
$= \det(A)\frac{1}{\det(B)}$
$= 2 \times \frac{-1}{3}$
$= \frac{-2}{3}$

$= (\frac{1}{2})^3 \det(A)$
$= \frac{1}{8} \times 2$
$= \frac{1}{4}$

$(\det(B))^{n-1}$
$(-3)^2 = 9$

$\det(\text{adj}(B)) =$
$\det(B^{-1}\det(B)) =$
$(\det(B))^3 \det(B^{-1})$
$(\det B)^3 \frac{1}{\det B}$
$(\det(B))^{n-1}$
$(-3)^2 = 9$

Q5) If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -5$, find $\begin{vmatrix} -2a & -2c & -2b \\ d+3g & f+3i & e+3h \\ g & i & h \end{vmatrix}$.

$-2\begin{vmatrix} a & c & b \\ d+3g & f+3i & e+3h \\ g & i & h \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ d+3g & e+3h & f+3i \\ g & h & i \end{vmatrix}$

$\xrightarrow{-3R_3+R_2} 2\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2 \times -5 = -10$

$-2\begin{vmatrix} a & c & b \\ d+3g & f+3i & e+3h \\ g & i & h \end{vmatrix} = -(-2)\begin{vmatrix} a & b & c \\ d+3g & e+3h & f+3i \\ g & h & i \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} =$

Linear – First exam – First semester – 2022

Q1) Find the condition on a,b,c that the system is consistent :

$x-y+2z=a$

$-2x+y+z=b$

$-x+3z=c$

Q2) Let A be any square matrix .show that AA^t-A^tA is a symmetric matrix.

Q3) Let A = $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ find A^{-1}.

Q4) Let A and B be 3x3 matrices such that det(A)=-2 and det(B)=4 . Find :

a) det($A^{-1}B^2$)

b) det($2A^t$)

c) det(adj(B))

Q5) If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ =-5, find $\begin{vmatrix} -4a & -4b & -4c \\ g & h & i \\ d+2g & e+2h & f+2i \end{vmatrix}$.

Q6) Use Cramer's rule to find the value of x_1 in the system of linear equations.

x_1-2x_2+x_3=-1

3x_1+$x_2 - x_3$=-12

$2x_1 + 3x_2 + 4x_3$=1

Linear-first-2022

Q1) (6 points)

a) Let A= $\begin{bmatrix} 1 & k \\ 0 & -1 \end{bmatrix}$ find the value of k for which $AA^t = A^tA$

b) Let A be a square matrix such that $A^2 - 2A + 3I = 0$. Show that A is invertible.

Q2: (6 points) For which value(s) of λ does the system

$(\lambda-1)x + y = 0$

$X + (\lambda-1)y = 0$

Have nontrivial solutions?

Q3)(6 points)

a) Let A be an invertible symmetric matrix . prove that A^{-1} is symmetric.

b) Let A and B be square matrices of the same size such that AB =0.

If A is invertible show that B =0.

Q4) (6 points) find the value of a such that the system

$X + 2y + 9z = 10$

$2x + 3y + 17z = 16$

$X + y + (a^2-1)z = 2a$

Is inconsistent.

Q5) (6 points) let A=$\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix}$. if det(A)= -5, find

a) $\det(2A^{-1})$

b) $\det((2A)^{-1})$

c) $\det\begin{matrix} a & g & d \\ b & h & e \\ c & i & f \end{matrix}$