# 1

# UNITS, PHYSICAL QUANTITIES, AND VECTORS

**1.1.** **IDENTIFY:** Convert units from mi to km and from km to ft.

**SET UP:** 1 in. = 2.54 cm, 1 km = 1000 m, 12 in. = 1 ft, 1 mi = 5280 ft.

**EXECUTE:** **(a)** $1.00 \text{ mi} = (1.00 \text{ mi})\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right)\left(\frac{2.54 \text{ cm}}{1 \text{ in.}}\right)\left(\frac{1 \text{ m}}{10^2 \text{ cm}}\right)\left(\frac{1 \text{ km}}{10^3 \text{ m}}\right) = 1.61 \text{ km}$

**(b)** $1.00 \text{ km} = (1.00 \text{ km})\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)\left(\frac{10^2 \text{ cm}}{1 \text{ m}}\right)\left(\frac{1 \text{ in.}}{2.54 \text{ cm}}\right)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right) = 3.28\times10^3 \text{ ft}$

**EVALUATE:** A mile is a greater distance than a kilometer. There are 5280 ft in a mile but only 3280 ft in a km.

**1.2.** **IDENTIFY:** Convert volume units from L to $\text{in.}^3$.

**SET UP:** $1 \text{ L} = 1000 \text{ cm}^3$. 1 in. = 2.54 cm

**EXECUTE:** $0.473 \text{ L}\times\left(\frac{1000 \text{ cm}^3}{1 \text{ L}}\right)\times\left(\frac{1 \text{ in.}}{2.54 \text{ cm}}\right)^3 = 28.9 \text{ in.}^3$.

**EVALUATE:** $1 \text{ in.}^3$ is greater than $1 \text{ cm}^3$, so the volume in $\text{in.}^3$ is a smaller number than the volume in $\text{cm}^3$, which is $473 \text{ cm}^3$.

**1.3.** **IDENTIFY:** We know the speed of light in m/s. $t = d/v$. Convert 1.00 ft to m and $t$ from s to ns.

**SET UP:** The speed of light is $v = 3.00\times10^8$ m/s. 1 ft = 0.3048 m. 1 s = $10^9$ ns.

**EXECUTE:** $t = \frac{0.3048 \text{ m}}{3.00\times10^8 \text{ m/s}} = 1.02\times10^{-9} \text{ s} = 1.02 \text{ ns}$

**EVALUATE:** In 1.00 s light travels $3.00\times10^8 \text{ m} = 3.00\times10^5 \text{ km} = 1.86\times10^5 \text{ mi}$.

**1.4.** **IDENTIFY:** Convert the units from g to kg and from $\text{cm}^3$ to $\text{m}^3$.

**SET UP:** 1 kg = 1000 g. 1 m = 100 cm.

**EXECUTE:** $19.3 \frac{\text{g}}{\text{cm}^3}\times\left(\frac{1 \text{ kg}}{1000 \text{ g}}\right)\times\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 = 1.93\times10^4 \frac{\text{kg}}{\text{m}^3}$

**EVALUATE:** The ratio that converts cm to m is cubed, because we need to convert $\text{cm}^3$ to $\text{m}^3$.

**1.5.** **IDENTIFY:** Convert volume units from $\text{in.}^3$ to L.

**SET UP:** $1 \text{ L} = 1000 \text{ cm}^3$. 1 in. = 2.54 cm.

**EXECUTE:** $(327 \text{ in.}^3)\times(2.54 \text{ cm/in.})^3\times(1 \text{ L}/1000 \text{ cm}^3) = 5.36 \text{ L}$

**EVALUATE:** The volume is $5360 \text{ cm}^3$. $1 \text{ cm}^3$ is less than $1 \text{ in.}^3$, so the volume in $\text{cm}^3$ is a larger number than the volume in $\text{in.}^3$.

**1.6.** **IDENTIFY:** Convert $\text{ft}^2$ to $\text{m}^2$ and then to hectares.

**SET UP:** $1.00 \text{ hectare} = 1.00\times10^4 \text{ m}^2$. $1 \text{ ft} = 0.3048 \text{ m}$.

**EXECUTE:** The area is $(12.0 \text{ acres})\left(\dfrac{43{,}600 \text{ ft}^2}{1 \text{ acre}}\right)\left(\dfrac{0.3048 \text{ m}}{1.00 \text{ ft}}\right)^2\left(\dfrac{1.00 \text{ hectare}}{1.00\times10^4 \text{ m}^2}\right) = 4.86 \text{ hectares}.$

**EVALUATE:** Since $1 \text{ ft} = 0.3048 \text{ m}$, $1 \text{ ft}^2 = (0.3048)^2 \text{ m}^2$.

**1.7.** **IDENTIFY:** Convert seconds to years. 1 gigasecond is a billion seconds.

**SET UP:** $1 \text{ gigasecond} = 1\times10^9 \text{ s}$. $1 \text{ day} = 24 \text{ h}$. $1 \text{ h} = 3600 \text{ s}$.

**EXECUTE:** $1.00 \text{ gigasecond} = (1.00\times10^9 \text{ s})\left(\dfrac{1 \text{ h}}{3600 \text{ s}}\right)\left(\dfrac{1 \text{ day}}{24 \text{ h}}\right)\left(\dfrac{1 \text{ y}}{365 \text{ days}}\right) = 31.7 \text{ y}.$

**EVALUATE:** The conversion $1 \text{ y} = 3.156\times10^7 \text{ s}$ assumes $1 \text{ y} = 365.24 \text{ d}$, which is the average for one extra day every four years, in leap years. The problem says instead to assume a 365-day year.

**1.8.** **IDENTIFY:** Apply the given conversion factors.

**SET UP:** $1 \text{ furlong} = 0.1250 \text{ mi}$ and $1 \text{ fortnight} = 14 \text{ days}$. $1 \text{ day} = 24 \text{ h}$.

**EXECUTE:** $(180{,}000 \text{ furlongs/fortnight})\left(\dfrac{0.125 \text{ mi}}{1 \text{ furlong}}\right)\left(\dfrac{1 \text{ fortnight}}{14 \text{ days}}\right)\left(\dfrac{1 \text{ day}}{24 \text{ h}}\right) = 67 \text{ mi/h}$

**EVALUATE:** A furlong is less than a mile and a fortnight is many hours, so the speed limit in mph is a much smaller number.

**1.9.** **IDENTIFY:** Convert miles/gallon to km/L.

**SET UP:** $1 \text{ mi} = 1.609 \text{ km}$. $1 \text{ gallon} = 3.788 \text{ L}$.

**EXECUTE:** **(a)** $55.0 \text{ miles/gallon} = (55.0 \text{ miles/gallon})\left(\dfrac{1.609 \text{ km}}{1 \text{ mi}}\right)\left(\dfrac{1 \text{ gallon}}{3.788 \text{ L}}\right) = 23.4 \text{ km/L}.$

**(b)** The volume of gas required is $\dfrac{1500 \text{ km}}{23.4 \text{ km/L}} = 64.1 \text{ L}$. $\dfrac{64.1 \text{ L}}{45 \text{ L/tank}} = 1.4 \text{ tanks}.$

**EVALUATE:** $1 \text{ mi/gal} = 0.425 \text{ km/L}$. A km is very roughly half a mile and there are roughly 4 liters in a gallon, so $1 \text{ mi/gal} \sim \frac{2}{4} \text{ km/L}$, which is roughly our result.

**1.10.** **IDENTIFY:** Convert units.

**SET UP:** Use the unit conversions given in the problem. Also, $100 \text{ cm} = 1 \text{ m}$ and $1000 \text{ g} = 1 \text{ kg}$.

**EXECUTE:** **(a)** $\left(60\dfrac{\text{mi}}{\text{h}}\right)\left(\dfrac{1 \text{ h}}{3600 \text{ s}}\right)\left(\dfrac{5280 \text{ ft}}{1 \text{ mi}}\right) = 88\dfrac{\text{ft}}{\text{s}}$

**(b)** $\left(32\dfrac{\text{ft}}{\text{s}^2}\right)\left(\dfrac{30.48 \text{ cm}}{1 \text{ ft}}\right)\left(\dfrac{1 \text{ m}}{100 \text{ cm}}\right) = 9.8\dfrac{\text{m}}{\text{s}^2}$

**(c)** $\left(1.0\dfrac{\text{g}}{\text{cm}^3}\right)\left(\dfrac{100 \text{ cm}}{1 \text{ m}}\right)^3\left(\dfrac{1 \text{ kg}}{1000 \text{ g}}\right) = 10^3\dfrac{\text{kg}}{\text{m}^3}$

**EVALUATE:** The relations $60 \text{ mi/h} = 88 \text{ ft/s}$ and $1 \text{ g/cm}^3 = 10^3 \text{ kg/m}^3$ are exact. The relation $32 \text{ ft/s}^2 = 9.8 \text{ m/s}^2$ is accurate to only two significant figures.

**1.11.** **IDENTIFY:** We know the density and mass; thus we can find the volume using the relation $\text{density} = \text{mass/volume} = m/V$. The radius is then found from the volume equation for a sphere and the result for the volume.

**SET UP:** $\text{Density} = 19.5 \text{ g/cm}^3$ and $m_{\text{critical}} = 60.0 \text{ kg}$. For a sphere $V = \frac{4}{3}\pi r^3$.

**EXECUTE:** $V = m_{\text{critical}}/\text{density} = \left(\dfrac{60.0 \text{ kg}}{19.5 \text{ g/cm}^3}\right)\left(\dfrac{1000 \text{ g}}{1.0 \text{ kg}}\right) = 3080 \text{ cm}^3.$

$$r = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3}{4\pi}(3080\text{ cm}^3)} = 9.0\text{ cm}.$$

**EVALUATE:** The density is very large, so the 130-pound sphere is small in size.

**1.12.** **IDENTIFY:** Convert units.

**SET UP:** We know the equalities $1\text{ mg} = 10^{-3}\text{ g}$, $1\ \mu\text{g}\ 10^{-6}\text{ g}$, and $1\text{ kg} = 10^{3}\text{ g}$.

**EXECUTE:** **(a)** $(410\text{ mg/day})\left(\frac{10^{-3}\text{ g}}{1\text{ mg}}\right)\left(\frac{1\ \mu\text{g}}{10^{-6}\text{ g}}\right) = 4.10\times 10^{5}\ \mu\text{g/day}.$

**(b)** $(12\text{ mg/kg})(75\text{ kg}) = (900\text{ mg})\left(\frac{10^{-3}\text{ g}}{1\text{ mg}}\right) = 0.900\text{ g}.$

**(c)** The mass of each tablet is $(2.0\text{ mg})\left(\frac{10^{-3}\text{ g}}{1\text{ mg}}\right) = 2.0\times 10^{-3}\text{ g}$. The number of tablets required each day is the number of grams recommended per day divided by the number of grams per tablet:

$\frac{0.0030\text{ g/day}}{2.0\times 10^{-3}\text{ g/tablet}} = 1.5$ tablet/day. Take 2 tablets each day.

**(d)** $(0.000070\text{ g/day})\left(\frac{1\text{ mg}}{10^{-3}\text{ g}}\right) = 0.070\text{ mg/day}.$

**EVALUATE:** Quantities in medicine and nutrition are frequently expressed in a wide variety of units.

**1.13.** **IDENTIFY:** Model the bacteria as spheres. Use the diameter to find the radius, then find the volume and surface area using the radius.

**SET UP:** From Appendix B, the volume $V$ of a sphere in terms of its radius is $V = \frac{4}{3}\pi r^3$ while its surface area $A$ is $A = 4\pi r^2$. The radius is one-half the diameter or $r = d/2 = 1.0\ \mu\text{m}$. Finally, the necessary equalities for this problem are: $1\ \mu\text{m} = 10^{-6}\text{ m}$; $1\text{ cm} = 10^{-2}\text{ m}$; and $1\text{ mm} = 10^{-3}\text{ m}$.

**EXECUTE:** $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.0\ \mu\text{m})^3\left(\frac{10^{-6}\text{ m}}{1\ \mu\text{m}}\right)^3\left(\frac{1\text{ cm}}{10^{-2}\text{ m}}\right)^3 = 4.2\times 10^{-12}\text{ cm}^3$ and

$$A = 4\pi r^2 = 4\pi(1.0\ \mu\text{m})^2\left(\frac{10^{-6}\text{ m}}{1\ \mu\text{m}}\right)^2\left(\frac{1\text{ mm}}{10^{-3}\text{ m}}\right)^2 = 1.3\times 10^{-5}\text{ mm}^2$$

**EVALUATE:** On a human scale, the results are extremely small. This is reasonable because bacteria are not visible without a microscope.

**1.14.** **IDENTIFY:** When numbers are multiplied or divided, the number of significant figures in the result can be no greater than in the factor with the fewest significant figures. When we add or subtract numbers it is the location of the decimal that matters.

**SET UP:** 12 mm has two significant figures and 5.98 mm has three significant figures.

**EXECUTE:** **(a)** $(12\text{ mm})\times(5.98\text{ mm}) = 72\text{ mm}^2$ (two significant figures)

**(b)** $\frac{5.98\text{ mm}}{12\text{ mm}} = 0.50$ (also two significant figures)

**(c)** 36 mm (to the nearest millimeter)

**(d)** 6 mm

**(e)** 2.0 (two significant figures)

**EVALUATE:** The length of the rectangle is known only to the nearest mm, so the answers in parts **(c)** and (d) are known only to the nearest mm.

**1.15.** **IDENTIFY:** Use your calculator to display $\pi\times 10^7$. Compare that number to the number of seconds in a year.

**SET UP:** $1\text{ yr} = 365.24\text{ days}$, $1\text{ day} = 24\text{ h}$, and $1\text{ h} = 3600\text{ s}$.

**EXECUTE:** $(365.24 \text{ days/1 yr})\left(\dfrac{24 \text{ h}}{1 \text{ day}}\right)\left(\dfrac{3600 \text{ s}}{1 \text{ h}}\right) = 3.15567\ldots\times10^7 \text{ s}; \ \pi\times10^7 \text{ s} = 3.14159\ldots\times10^7 \text{ s}$

The approximate expression is accurate to two significant figures. The percent error is 0.45%.

**EVALUATE:** The close agreement is a numerical accident.

**1.16.** **IDENTIFY:** To asses the accuracy of the approximations, we must convert them to decimals.

**SET UP:** Use a calculator to calculate the decimal equivalent of each fraction and then round the numeral to the specified number of significant figures. Compare to π rounded to the same number of significant figures.

**EXECUTE:** **(a)** 22/7 = 3.14286 **(b)** 355/113 = 3.14159 **(c)** The exact value of π rounded to six significant figures is 3.14159.

**EVALUATE:** We see that 355/113 is a much better approximation to π than is 22/7.

**1.17.** **IDENTIFY:** Express 200 kg in pounds. Express each of 200 m, 200 cm and 200 mm in inches. Express 200 months in years.

**SET UP:** A mass of 1 kg is equivalent to a weight of about 2.2 lbs. 1 in. = 2.54 cm. 1 y = 12 months.

**EXECUTE:** **(a)** 200 kg is a weight of 440 lb. This is much larger than the typical weight of a man.

**(b)** $200 \text{ m} = (2.00\times10^4 \text{ cm})\left(\dfrac{1 \text{ in.}}{2.54 \text{ cm}}\right) = 7.9\times10^3$ inches. This is much greater than the height of a person.

**(c)** 200 cm = 2.00 m = 79 inches = 6.6 ft. Some people are this tall, but not an ordinary man.

**(d)** 200 mm = 0.200 m = 7.9 inches. This is much too short.

**(e)** $200 \text{ months} = (200 \text{ mon})\left(\dfrac{1 \text{ y}}{12 \text{ mon}}\right) = 17 \text{ y}$. This is the age of a teenager; a middle-aged man is much older than this.

**EVALUATE:** None are plausible. When specifying the value of a measured quantity it is essential to give the units in which it is being expressed.

**1.18.** **IDENTIFY:** Estimate the number of people and then use the estimates given in the problem to calculate the number of gallons.

**SET UP:** Estimate $3\times10^8$ people, so $2\times10^8$ cars.

**EXECUTE:** (Number of cars × miles/car day)/(mi/gal) = gallons/day

$$(2\times10^8 \text{ cars}\times10000 \text{ mi/yr/car}\times1 \text{ yr/365 days})/(20 \text{ mi/gal}) = 3\times10^8 \text{ gal/day}$$

**EVALUATE:** The number of gallons of gas used each day approximately equals the population of the U.S.

**1.19.** **IDENTIFY:** Estimate the number of blinks per minute. Convert minutes to years. Estimate the typical lifetime in years.

**SET UP:** Estimate that we blink 10 times per minute. 1 y = 365 days. 1 day = 24 h, 1 h = 60 min. Use 80 years for the lifetime.

**EXECUTE:** The number of blinks is $(10 \text{ per min})\left(\dfrac{60 \text{ min}}{1 \text{ h}}\right)\left(\dfrac{24 \text{ h}}{1 \text{ day}}\right)\left(\dfrac{365 \text{ days}}{1 \text{ y}}\right)(80 \text{ y/lifetime}) = 4\times10^8$

**EVALUATE:** Our estimate of the number of blinks per minute can be off by a factor of two but our calculation is surely accurate to a power of 10.

**1.20.** **IDENTIFY:** Approximate the number of breaths per minute. Convert minutes to years and $\text{cm}^3$ to $\text{m}^3$ to find the volume in $\text{m}^3$ breathed in a year.

**SET UP:** Assume 10 breaths/min. $1 \text{ y} = (365 \text{ d})\left(\dfrac{24 \text{ h}}{1 \text{ d}}\right)\left(\dfrac{60 \text{ min}}{1 \text{ h}}\right) = 5.3\times10^5 \text{ min}$. $10^2 \text{ cm} = 1 \text{ m}$ so $10^6 \text{ cm}^3 = 1 \text{ m}^3$. The volume of a sphere is $V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$, where $r$ is the radius and $d$ is the diameter. Don't forget to account for four astronauts.

**EXECUTE:** **(a)** The volume is $(4)(10 \text{ breaths/min})(500\times10^{-6} \text{ m}^3)\left(\dfrac{5.3\times10^5 \text{ min}}{1 \text{ y}}\right) = 1\times10^4 \text{ m}^3\text{/yr}$.

**(b)** $d = \left(\frac{6V}{\pi}\right)^{1/3} = \left(\frac{6[1\times10^4\ \text{m}^3]}{\pi}\right)^{1/3} = 27\ \text{m}$

**EVALUATE:** Our estimate assumes that each $\text{cm}^3$ of air is breathed in only once, where in reality not all the oxygen is absorbed from the air in each breath. Therefore, a somewhat smaller volume would actually be required.

**1.21.** **IDENTIFY:** Estimation problem.

**SET UP:** Estimate that the pile is $18\ \text{in.} \times 18\ \text{in.} \times 5\ \text{ft}\ 8\ \text{in.}$. Use the density of gold to calculate the mass of gold in the pile and from this calculate the dollar value.

**EXECUTE:** The volume of gold in the pile is $V = 18\ \text{in.} \times 18\ \text{in.} \times 68\ \text{in.} = 22{,}000\ \text{in.}^3$. Convert to $\text{cm}^3$:

$$V = 22{,}000\ \text{in.}^3(1000\ \text{cm}^3/61.02\ \text{in.}^3) = 3.6\times10^5\ \text{cm}^3.$$

The density of gold is $19.3\ \text{g/cm}^3$, so the mass of this volume of gold is

$$m = (19.3\ \text{g/cm}^3)(3.6\times10^5\ \text{cm}^3) = 7\times10^6\ \text{g}.$$

The monetary value of one gram is \$10, so the gold has a value of $(\$10/\text{gram})(7\times10^6\ \text{grams}) = \$7\times10^7$, or about $\$100\times10^6$ (one hundred million dollars).

**EVALUATE:** This is quite a large pile of gold, so such a large monetary value is reasonable.

**1.22.** **IDENTIFY:** Estimate the number of beats per minute and the duration of a lifetime. The volume of blood pumped during this interval is then the volume per beat multiplied by the total beats.

**SET UP:** An average middle-aged (40 year-old) adult at rest has a heart rate of roughly 75 beats per minute. To calculate the number of beats in a lifetime, use the current average lifespan of 80 years.

**EXECUTE:** $N_{\text{beats}} = (75\ \text{beats/min})\left(\frac{60\ \text{min}}{1\ \text{h}}\right)\left(\frac{24\ \text{h}}{1\ \text{day}}\right)\left(\frac{365\ \text{days}}{\text{yr}}\right)\left(\frac{80\ \text{yr}}{\text{lifespan}}\right) = 3\times10^9\ \text{beats/lifespan}$

$V_{\text{blood}} = (50\ \text{cm}^3/\text{beat})\left(\frac{1\ \text{L}}{1000\ \text{cm}^3}\right)\left(\frac{1\ \text{gal}}{3.788\ \text{L}}\right)\left(\frac{3\times10^9\ \text{beats}}{\text{lifespan}}\right) = 4\times10^7\ \text{gal/lifespan}$

**EVALUATE:** This is a very large volume.

**1.23.** **IDENTIFY:** Estimate the diameter of a drop and from that calculate the volume of a drop, in $\text{m}^3$. Convert $\text{m}^3$ to L.

**SET UP:** Estimate the diameter of a drop to be $d = 2\ \text{mm}$. The volume of a spherical drop is $V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$. $10^3\ \text{cm}^3 = 1\ \text{L}$.

**EXECUTE:** $V = \frac{1}{6}\pi(0.2\ \text{cm})^3 = 4\times10^{-3}\ \text{cm}^3$. The number of drops in 1.0 L is $\frac{1000\ \text{cm}^3}{4\times10^{-3}\ \text{cm}^3} = 2\times10^5$

**EVALUATE:** Since $V \sim d^3$, if our estimate of the diameter of a drop is off by a factor of 2 then our estimate of the number of drops is off by a factor of 8.

**1.24.** **IDENTIFY:** Draw the vector addition diagram to scale.

**SET UP:** The two vectors $\vec{A}$ and $\vec{B}$ are specified in the figure that accompanies the problem.

**EXECUTE:** **(a)** The diagram for $\vec{R} = \vec{A} + \vec{B}$ is given in Figure 1.24a. Measuring the length and angle of $\vec{R}$ gives $R = 9.0$ m and an angle of $\theta = 34°$.

**(b)** The diagram for $\vec{E} = \vec{A} - \vec{B}$ is given in Figure 1.24b. Measuring the length and angle of $\vec{E}$ gives $D = 22$ m and an angle of $\theta = 250°$.

**(c)** $-\vec{A} - \vec{B} = -(\boldsymbol{A} + \boldsymbol{B})$, so $-\vec{A} - \vec{B}$ has a magnitude of 9.0 m (the same as $\vec{A} + \vec{B}$) and an angle with the $+x$ axis of $214°$ (opposite to the direction of $\vec{A} + \vec{B}$).

**(d)** $\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$, so $\vec{B} - \vec{A}$ has a magnitude of 22 m and an angle with the $+x$ axis of $70°$ (opposite to the direction of $\vec{A} - \vec{B}$).

**EVALUATE:** The vector $-\vec{A}$ is equal in magnitude and opposite in direction to the vector $\vec{A}$.

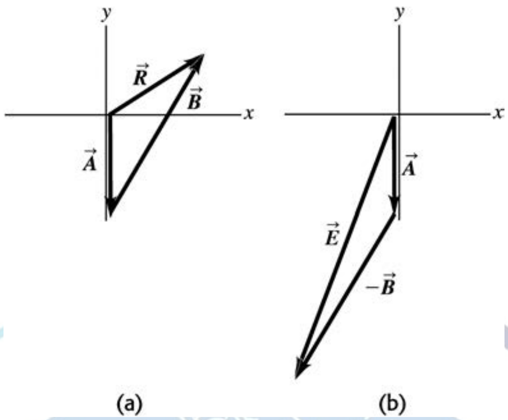



**Figure 1.24**

**1.25.** **IDENTIFY:** Draw each subsequent displacement tail to head with the previous displacement. The resultant displacement is the single vector that points from the starting point to the stopping point.

**SET UP:** Call the three displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$. The resultant displacement $\vec{R}$ is given by $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.

**EXECUTE:** The vector addition diagram is given in Figure 1.25. Careful measurement gives that $\vec{R}$ is 7.8 km, $38^\circ$ north of east.

**EVALUATE:** The magnitude of the resultant displacement, 7.8 km, is less than the sum of the magnitudes of the individual displacements, $2.6\text{ km} + 4.0\text{ km} + 3.1\text{ km}$.

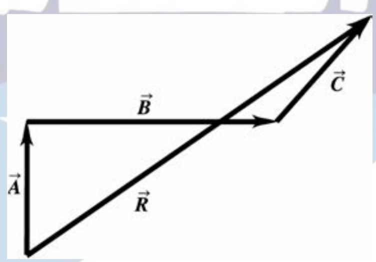



**Figure 1.25**

**1.26.** **IDENTIFY:** Since she returns to the starting point, the vector sum of the four displacements must be zero.

**SET UP:** Call the three given displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$, and call the fourth displacement $\vec{D}$. $\vec{A} + \vec{B} + \vec{C} + \vec{D} = 0$.

**EXECUTE:** The vector addition diagram is sketched in Figure 1.26. Careful measurement gives that $\vec{D}$ is 144 m, $41^\circ$ south of west.

**EVALUATE:** $\vec{D}$ is equal in magnitude and opposite in direction to the sum $\vec{A} + \vec{B} + \vec{C}$.

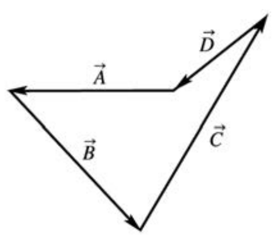



**Figure 1.26**

**1.27.** **IDENTIFY:** For each vector $\vec{V}$, use that $V_x = V\cos\theta$ and $V_y = V\sin\theta$, when $\theta$ is the angle $\vec{V}$ makes with the $+x$ axis, measured counterclockwise from the axis.

**SET UP:** For $\vec{A}$, $\theta = 270.0°$. For $\vec{B}$, $\theta = 60.0°$. For $\vec{C}$, $\theta = 205.0°$. For $\vec{D}$, $\theta = 143.0°$.

**EXECUTE:** $A_x = 0$, $A_y = -8.00$ m. $B_x = 7.50$ m, $B_y = 13.0$ m. $C_x = -10.9$ m, $C_y = -5.07$ m. $D_x = -7.99$ m, $D_y = 6.02$ m.

**EVALUATE:** The signs of the components correspond to the quadrant in which the vector lies.

**1.28.** **IDENTIFY:** $\tan\theta = \dfrac{A_y}{A_x}$, for $\theta$ measured counterclockwise from the $+x$-axis.

**SET UP:** A sketch of $A_x$, $A_y$ and $\vec{A}$ tells us the quadrant in which $\vec{A}$ lies.

**EXECUTE:**

**(a)** $\tan\,\theta = \dfrac{A_y}{A_x} = \dfrac{-1.00\text{ m}}{2.00\text{ m}} = -0.500.$ $\theta = \tan^{-1}(-0.500) = 360° - 26.6° = 333°.$

**(b)** $\tan\,\theta = \dfrac{A_y}{A_x} = \dfrac{1.00\text{ m}}{2.00\text{ m}} = 0.500.$ $\theta = \tan^{-1}(0.500) = 26.6°.$

**(c)** $\tan\,\theta = \dfrac{A_y}{A_x} = \dfrac{1.00\text{ m}}{-2.00\text{ m}} = -0.500.$ $\theta = \tan^{-1}(-0.500) = 180° - 26.6° = 153°.$

**(d)** $\tan\,\theta = \dfrac{A_y}{A_x} = \dfrac{-1.00\text{ m}}{-2.00\text{ m}} = 0.500.$ $\theta = \tan^{-1}(0.500) = 180° + 26.6° = 207°$

**EVALUATE:** The angles $26.6°$ and $207°$ have the same tangent. Our sketch tells us which is the correct value of $\theta$.

**1.29.** **IDENTIFY:** Given the direction and one component of a vector, find the other component and the magnitude.

**SET UP:** Use the tangent of the given angle and the definition of vector magnitude.

**EXECUTE:** **(a)** $\tan 32.0° = \dfrac{|A_x|}{|A_y|}$

$|A_x| = (9.60\text{ m})\tan 32.0° = 6.00\text{ m}.$ $A_x = -6.00$ m.

**(b)** $A = \sqrt{A_x^2 + A_y^2} = 11.3$ m.

**EVALUATE:** The magnitude is greater than either of the components.

**1.30.** **IDENTIFY:** Given the direction and one component of a vector, find the other component and the magnitude.

**SET UP:** Use the tangent of the given angle and the definition of vector magnitude.

**EXECUTE:** **(a)** $\tan 34.0° = \dfrac{|A_x|}{|A_y|}$

$$|A_y| = \frac{|A_x|}{\tan 34.0°} = \frac{16.0\text{ m}}{\tan 34.0°} = 23.72\text{ m}$$

$A_y = -23.7$ m.

**(b)** $A = \sqrt{A_x^2 + A_y^2} = 28.6$ m.

**EVALUATE:** The magnitude is greater than either of the components.

**1.31.** **IDENTIFY:** If $\vec{C} = \vec{A} + \vec{B}$, then $C_x = A_x + B_x$ and $C_y = A_y + B_y$. Use $C_x$ and $C_y$ to find the magnitude and direction of $\vec{C}$.

**SET UP:** From Figure E1.24 in the textbook, $A_x = 0$, $A_y = -8.00$ m and $B_x = +B\sin 30.0° = 7.50$ m, $B_y = +B\cos 30.0° = 13.0$ m.

**EXECUTE:** **(a)** $\vec{C} = \vec{A} + \vec{B}$ so $C_x = A_x + B_x = 7.50$ m and $C_y = A_y + B_y = +5.00$ m. $C = 9.01$ m.

$\tan\theta = \dfrac{C_y}{C_x} = \dfrac{5.00 \text{ m}}{7.50 \text{ m}}$ and $\theta = 33.7°$.

**(b)** $\vec{B} + \vec{A} = \vec{A} + \vec{B}$, so $\vec{B} + \vec{A}$ has magnitude 9.01 m and direction specified by $33.7°$.

**(c)** $\vec{D} = \vec{A} - \vec{B}$ so $D_x = A_x - B_x = -7.50$ m and $D_y = A_y - B_y = -21.0$ m. $D = 22.3$ m.

$\tan\phi = \dfrac{D_y}{D_x} = \dfrac{-21.0 \text{ m}}{-7.50 \text{ m}}$ and $\phi = 70.3°$. $\vec{D}$ is in the 3rd quadrant and the angle $\theta$ counterclockwise from the $+x$ axis is $180° + 70.3° = 250.3°$.

**(d)** $\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$, so $\vec{B} - \vec{A}$ has magnitude 22.3 m and direction specified by $\theta = 70.3°$.

**EVALUATE:** These results agree with those calculated from a scale drawing in Problem 1.24.

**1.32.** **IDENTIFY:** Find the vector sum of the three given displacements.

**SET UP:** Use coordinates for which $+x$ is east and $+y$ is north. The driver's vector displacements are: $\vec{A} = 2.6$ km, 0° of north; $\vec{B} = 4.0$ km, 0° of east; $\vec{C} = 3.1$ km, 45° north of east.

**EXECUTE:** $R_x = A_x + B_x + C_x = 0 + 4.0 \text{ km} + (3.1 \text{ km})\cos(45°) = 6.2$ km; $R_y = A_y + B_y + C_y = 2.6 \text{ km} + 0 + (3.1 \text{ km})(\sin 45°) = 4.8$ km; $R = \sqrt{R_x^2 + R_y^2} = 7.8$ km; $\theta = \tan^{-1}[(4.8 \text{ km})/(6.2 \text{ km})] = 38°$; $\vec{R} = 7.8$ km, 38° north of east. This result is confirmed by the sketch in Figure 1.32.

**EVALUATE:** Both $R_x$ and $R_y$ are positive and $\vec{R}$ is in the first quadrant.

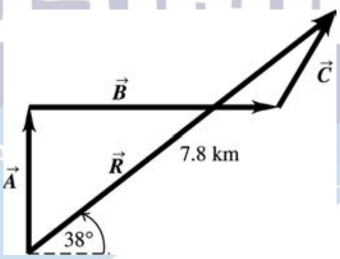
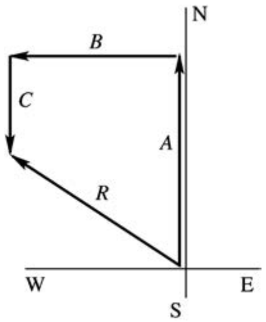



**Figure 1.32**

**1.33.** **IDENTIFY:** Vector addition problem. We are given the magnitude and direction of three vectors and are asked to find their sum.

**SET UP:**




$A = 3.25$ km

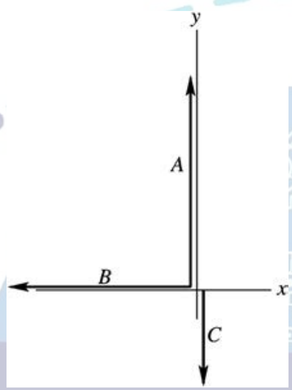
$B = 2.20$ km

$C = 1.50$ km

**Figure 1.33a**

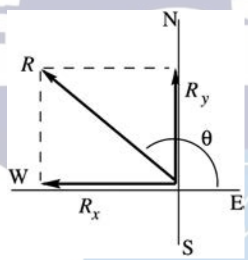
Select a coordinate system where $+x$ is east and $+y$ is north. Let $\vec{A}$, $\vec{B}$, and $\vec{C}$ be the three displacements of the professor. Then the resultant displacement $\vec{R}$ is given by $\vec{R} = \vec{A} + \vec{B} + \vec{C}$. By the method of components, $R_x = A_x + B_x + C_x$ and $R_y = A_y + B_y + C_y$. Find the $x$ and $y$ components of each vector; add them to find the components of the resultant. Then the magnitude and direction of the resultant can be found from its $x$ and $y$ components that we have calculated. As always it is essential to draw a sketch.

**EXECUTE:**




$$A_x = 0,\ A_y = +3.25 \text{ km}$$
$$B_x = -2.20 \text{ km},\ B_y = 0$$
$$C_x = 0,\ C_y = -1.50 \text{ km}$$
$$R_x = A_x + B_x + C_x$$
$$R_x = 0 - 2.20 \text{ km} + 0 = -2.20 \text{ km}$$
$$R_y = A_y + B_y + C_y$$
$$R_y = 3.25 \text{ km} + 0 - 1.50 \text{ km} = 1.75 \text{ km}$$

**Figure 1.33b**




$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-2.20 \text{ km})^2 + (1.75 \text{ km})^2}$$
$$R = 2.81 \text{ km}$$
$$\tan\theta = \frac{R_y}{R_x} = \frac{1.75 \text{ km}}{-2.20 \text{ km}} = -0.800$$
$$\theta = 141.5°$$

**Figure 1.33c**

The angle $\theta$ measured counterclockwise from the $+x$-axis. In terms of compass directions, the resultant displacement is $38.5°$ N of W.

**EVALUATE:** $R_x < 0$ and $R_y > 0$, so $\vec{R}$ is in the 2nd quadrant. This agrees with the vector addition diagram.

**1.34. IDENTIFY:** Use $A = \sqrt{A_x^2 + A_y^2}$ and $\tan\theta = \dfrac{A_y}{A_x}$ to calculate the magnitude and direction of each of the given vectors.

**SET UP:** A sketch of $A_x$, $A_y$ and $\vec{A}$ tells us the quadrant in which $\vec{A}$ lies.

**EXECUTE:** **(a)** $\sqrt{(-8.60 \text{ cm})^2 + (5.20 \text{ cm})^2} = 10.0 \text{ cm},\ \arctan\left(\dfrac{5.20}{-8.60}\right) = 148.8°$ (which is $180° - 31.2°$).

**(b)** $\sqrt{(-9.7 \text{ m})^2 + (-2.45 \text{ m})^2} = 10.0 \text{ m},\ \arctan\left(\dfrac{-2.45}{-9.7}\right) = 14° + 180° = 194°.$

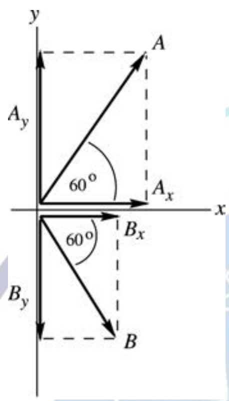
**(c)** $\sqrt{(7.75 \text{ km})^2 + (-2.70 \text{ km})^2} = 8.21 \text{ km},\ \arctan\left(\dfrac{-2.7}{7.75}\right) = 340.8°$ (which is $360° - 19.2°$).

**EVALUATE:** In each case the angle is measured counterclockwise from the $+x$ axis. Our results for $\theta$ agree with our sketches.

**1.35.** **IDENTIFY:** Vector addition problem. $\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$.

**SET UP:** Find the $x$- and $y$-components of $\vec{A}$ and $\vec{B}$. Then the $x$- and $y$-components of the vector sum are calculated from the $x$- and $y$-components of $\vec{A}$ and $\vec{B}$.

**EXECUTE:**




$A_x = A\cos(60.0°)$

$A_x = (2.80\text{ cm})\cos(60.0°) = +1.40\text{ cm}$

$A_y = A\sin(60.0°)$

$A_y = (2.80\text{ cm})\sin(60.0°) = +2.425\text{ cm}$

$B_x = B\cos(-60.0°)$

$B_x = (1.90\text{ cm})\cos(-60.0°) = +0.95\text{ cm}$

$B_y = B\sin(-60.0°)$

$B_y = (1.90\text{ cm})\sin(-60.0°) = -1.645\text{ cm}$

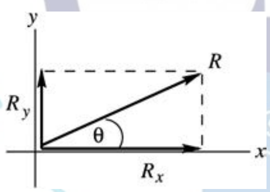
Note that the signs of the components correspond to the directions of the component vectors.

**Figure 1.35a**

**(a)** Now let $\vec{R}=\vec{A}+\vec{B}$.

$R_x = A_x + B_x = +1.40\text{ cm} + 0.95\text{ cm} = +2.35\text{ cm}.$

$R_y = A_y + B_y = +2.425\text{ cm} - 1.645\text{ cm} = +0.78\text{ cm}.$




$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(2.35\text{ cm})^2 + (0.78\text{ cm})^2}$

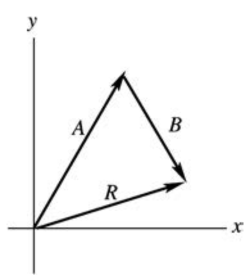
$R = 2.48\text{ cm}$

$\tan\theta = \dfrac{R_y}{R_x} = \dfrac{+0.78\text{ cm}}{+2.35\text{ cm}} = +0.3319$

$\theta = 18.4°$

**Figure 1.35b**

**EVALUATE:** The vector addition diagram for $\vec{R}=\vec{A}+\vec{B}$ is



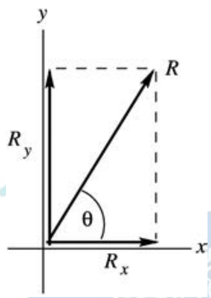

$\vec{R}$ is in the 1st quadrant, with $|R_y| < |R_x|$, in agreement with our calculation.

**Figure 1.35c**

**(b) EXECUTE:** Now let $\vec{R} = \vec{A} - \vec{B}$.

$R_x = A_x - B_x = +1.40 \text{ cm} - 0.95 \text{ cm} = +0.45 \text{ cm}.$

$R_y = A_y - B_y = +2.425 \text{ cm} + 1.645 \text{ cm} = +4.070 \text{ cm}.$




$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(0.45 \text{ cm})^2 + (4.070 \text{ cm})^2}$$

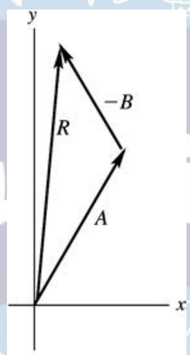
$R = 4.09$ cm

$$\tan\theta = \frac{R_y}{R_x} = \frac{4.070 \text{ cm}}{0.45 \text{ cm}} = +9.044$$

$\theta = 83.7°$

**Figure 1.35d**

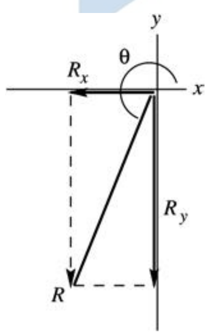
**EVALUATE:** The vector addition diagram for $\vec{R} = \vec{A} + (-\vec{B})$ is




$\vec{R}$ is in the 1st quadrant, with $|R_x| < |R_y|$, in agreement with our calculation.

**Figure 1.35e**

**(c) EXECUTE:**



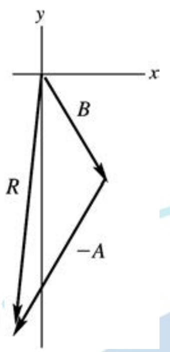

$\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$

$\vec{B} - \vec{A}$ and $\vec{A} - \vec{B}$ are equal in magnitude and opposite in direction.

$R = 4.09$ cm and $\theta = 83.7° + 180° = 264°$

**Figure 1.35f**

**EVALUATE:** The vector addition diagram for $\vec{R} = \vec{B} + (-\vec{A})$ is




$\vec{R}$ is in the 3rd quadrant, with $|R_x| < |R_y|$, in agreement with our calculation.

**Figure 1.35g**

**1.36.** **IDENTIFY:** The general expression for a vector written in terms of components and unit vectors is $\vec{A} = A_x\hat{i} + A_y\hat{j}$.

**SET UP:** $5.0\vec{B} = 5.0(4\hat{i} - 6\hat{j}) = 20\hat{i} - 30\hat{j}$

**EXECUTE:** **(a)** $A_x = 5.0,\ A_y = -6.3$ **(b)** $A_x = 11.2,\ A_y = -9.91$ **(c)** $A_x = -15.0,\ A_y = 22.4$ **(d)** $A_x = 20,\ A_y = -30$

**EVALUATE:** The components are signed scalars.

**1.37.** **IDENTIFY:** Find the components of each vector and then use the general equation $\vec{A} = A_x\hat{i} + A_y\hat{j}$ for a vector in terms of its components and unit vectors.

**SET UP:** $A_x = 0,\ A_y = -8.00$ m. $B_x = 7.50$ m, $B_y = 13.0$ m. $C_x = -10.9$ m, $C_y = -5.07$ m. $D_x = -7.99$ m, $D_y = 6.02$ m.

**EXECUTE:** $\vec{A} = (-8.00\text{ m})\hat{j}$; $\vec{B} = (7.50\text{ m})\hat{i} + (13.0\text{ m})\hat{j}$; $\vec{C} = (-10.9\text{ m})\hat{i} + (-5.07\text{ m})\hat{j}$; $\vec{D} = (-7.99\text{ m})\hat{i} + (6.02\text{ m})\hat{j}$.

**EVALUATE:** All these vectors lie in the *xy*-plane and have no *z*-component.

**1.38.** **IDENTIFY:** Find $A$ and $B$. Find the vector difference using components.

**SET UP:** Identify the *x*- and *y*-components and use $A = \sqrt{A_x^2 + A_y^2}$.

**EXECUTE:** **(a)** $\vec{A} = 4.00\hat{i} + 7.00\hat{j}$; $A_x = +4.00$; $A_y = +7.00$.

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(4.00)^2 + (7.00)^2} = 8.06$. $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$; $B_x = +5.00$; $B_y = -2.00$;

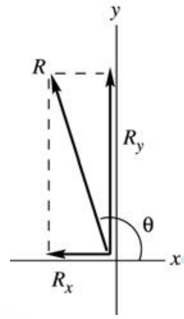
$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(5.00)^2 + (-2.00)^2} = 5.39$.

**EVALUATE:** Note that the magnitudes of $\vec{A}$ and $\vec{B}$ are each larger than either of their components.

**EXECUTE:** **(b)** $\vec{A} - \vec{B} = 4.00\hat{i} + 7.00\hat{j} - (5.00\hat{i} - 2.00\hat{j}) = (4.00 - 5.00)\hat{i} + (7.00 + 2.00)\hat{j}$.

$\vec{A} - \vec{B} = -1.00\hat{i} + 9.00\hat{j}$

**(c)** Let $\vec{R} = \vec{A} - \vec{B} = -1.00\hat{i} + 9.00\hat{j}$. Then $R_x = -1.00$, $R_y = 9.00$.




$$R = \sqrt{R_x^2 + R_y^2}$$
$$R = \sqrt{(-1.00)^2 + (9.00)^2} = 9.06.$$
$$\tan\theta = \frac{R_y}{R_x} = \frac{9.00}{-1.00} = -9.00$$
$$\theta = -83.6° + 180° = 96.3°.$$

**Figure 1.38**

**EVALUATE:** $R_x < 0$ and $R_y > 0$, so $\vec{R}$ is in the 2nd quadrant.

**1.39.** **IDENTIFY:** Use trigonometry to find the components of each vector. Use $R_x = A_x + B_x + \cdots$ and $R_y = A_y + B_y + \cdots$ to find the components of the vector sum. The equation $\vec{A} = A_x\hat{i} + A_y\hat{j}$ expresses a vector in terms of its components.

**SET UP:** Use the coordinates in the figure that accompanies the problem.

**EXECUTE:** **(a)** $\vec{A} = (3.60 \text{ m})\cos 70.0°\hat{i} + (3.60 \text{ m})\sin 70.0°\hat{j} = (1.23 \text{ m})\hat{i} + (3.38 \text{ m})\hat{j}$

$\vec{B} = -(2.40 \text{ m})\cos 30.0°\hat{i} - (2.40 \text{ m})\sin 30.0°\hat{j} = (-2.08 \text{ m})\hat{i} + (-1.20 \text{ m})\hat{j}$

**(b)** $\vec{C} = (3.00)\vec{A} - (4.00)\vec{B} = (3.00)(1.23 \text{ m})\hat{i} + (3.00)(3.38 \text{ m})\hat{j} - (4.00)(-2.08 \text{ m})\hat{i} - (4.00)(-1.20 \text{ m})\hat{j}$

$\vec{C} = (12.01 \text{ m})\hat{i} + (14.94 \text{ m})\hat{j}$

**(c)** From $A = \sqrt{A_x^2 + A_y^2}$ and $\tan\theta = \dfrac{A_y}{A_x}$,

$$C = \sqrt{(12.01 \text{ m})^2 + (14.94 \text{ m})^2} = 19.17 \text{ m}, \arctan\left(\frac{14.94 \text{ m}}{12.01 \text{ m}}\right) = 51.2°$$

**EVALUATE:** $C_x$ and $C_y$ are both positive, so $\theta$ is in the first quadrant.

**1.40.** **IDENTIFY:** We use the vector components and trigonometry to find the angles.

**SET UP:** Use the fact that $\tan\theta = A_y / A_x$.

**EXECUTE:** **(a)** $\tan\theta = A_y / A_x = \dfrac{6.00}{-3.00}$. $\theta = 117°$ with the +$x$-axis.

**(b)** $\tan\theta = B_y / B_x = \dfrac{2.00}{7.00}$. $\theta = 15.9°$.

**(c)** First find the components of $\vec{C}$. $C_x = A_x + B_x = -3.00 + 7.00 = 4.00$,
$C_y = A_y + B_y = 6.00 + 2.00 = 8.00$

$\tan\theta = C_y / C_x = \dfrac{8.00}{4.00} = 2.00$. $\theta = 63.4°$

**EVALUATE:** Sketching each of the three vectors to scale will show that the answers are reasonable.

**1.41.** **IDENTIFY:** $\vec{A}$ and $\vec{B}$ are given in unit vector form. Find $A$, $B$ and the vector difference $\vec{A} - \vec{B}$.

**SET UP:** $\vec{A} = -2.00\vec{i} + 3.00\vec{j} + 4.00\vec{k}$, $\vec{B} = 3.00\vec{i} + 1.00\vec{j} - 3.00\vec{k}$

Use $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$ to find the magnitudes of the vectors.

**EXECUTE:** **(a)** $A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(-2.00)^2 + (3.00)^2 + (4.00)^2} = 5.38$

$B = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(3.00)^2 + (1.00)^2 + (-3.00)^2} = 4.36$

**(b)** $\vec{A} - \vec{B} = (-2.00\hat{i} + 3.00\hat{j} + 4.00\hat{k}) - (3.00\hat{i} + 1.00\hat{j} - 3.00\hat{k})$

$\vec{A} - \vec{B} = (-2.00 - 3.00)\hat{i} + (3.00 - 1.00)\hat{j} + (4.00 - (-3.00))\hat{k} = -5.00\hat{i} + 2.00\hat{j} + 7.00\hat{k}.$

**(c)** Let $\vec{C} = \vec{A} - \vec{B}$, so $C_x = -5.00$, $C_y = +2.00$, $C_z = +7.00$

$$C = \sqrt{C_x^2 + C_y^2 + C_z^2} = \sqrt{(-5.00)^2 + (2.00)^2 + (7.00)^2} = 8.83$$

$\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$, so $\vec{A} - \vec{B}$ and $\vec{B} - \vec{A}$ have the same magnitude but opposite directions.

**EVALUATE:** $A$, $B$, and $C$ are each larger than any of their components.

**1.42.** **IDENTIFY:** Target variables are $\vec{A} \cdot \vec{B}$ and the angle $\phi$ between the two vectors.

**SET UP:** We are given $\vec{A}$ and $\vec{B}$ in unit vector form and can take the scalar product using $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$. The angle $\phi$ can then be found from $\vec{A} \cdot \vec{B} = AB\cos\phi$.

**EXECUTE:** **(a)** $\vec{A} = 4.00\hat{i} + 7.00\hat{j}$, $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$; $A = 8.06$, $B = 5.39$.

$\vec{A} \cdot \vec{B} = (4.00\hat{i} + 7.00\hat{j}) \cdot (5.00\hat{i} - 2.00\hat{j}) = (4.00)(5.00) + (7.00)(-2.00) = 20.0 - 14.0 = +6.00.$

**(b)** $\cos\phi = \dfrac{\vec{A} \cdot \vec{B}}{AB} = \dfrac{6.00}{(8.06)(5.39)} = 0.1382$; $\phi = 82.1°$.

**EVALUATE:** The component of $\vec{B}$ along $\vec{A}$ is in the same direction as $\vec{A}$, so the scalar product is positive and the angle $\phi$ is less than $90°$.

**1.43.** **IDENTIFY:** $\vec{A} \cdot \vec{B} = AB\cos\phi$

**SET UP:** For $\vec{A}$ and $\vec{B}$, $\phi = 150.0°$. For $\vec{B}$ and $\vec{C}$, $\phi = 145.0°$. For $\vec{A}$ and $\vec{C}$, $\phi = 65.0°$.

**EXECUTE:** **(a)** $\vec{A} \cdot \vec{B} = (8.00 \text{ m})(15.0 \text{ m})\cos 150.0° = -104 \text{ m}^2$

**(b)** $\vec{B} \cdot \vec{C} = (15.0 \text{ m})(12.0 \text{ m})\cos 145.0° = -148 \text{ m}^2$

**(c)** $\vec{A} \cdot \vec{C} = (8.00 \text{ m})(12.0 \text{ m})\cos 65.0° = 40.6 \text{ m}^2$

**EVALUATE:** When $\phi < 90°$ the scalar product is positive and when $\phi > 90°$ the scalar product is negative.

**1.44.** **IDENTIFY:** Target variable is the vector $\vec{A} \times \vec{B}$ expressed in terms of unit vectors.

**SET UP:** We are given $\vec{A}$ and $\vec{B}$ in unit vector form and can take the vector product using $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = 0$, $\hat{i} \times \hat{j} = \hat{k}$, and $\hat{j} \times \hat{i} = -\hat{k}$.

**EXECUTE:** $\vec{A} = 4.00\hat{i} + 7.00\hat{j}$, $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$.

$\vec{A} \times \vec{B} = (4.00\hat{i} + 7.00\hat{j}) \times (5.00\hat{i} - 2.00\hat{j}) = 20.0\hat{i} \times \hat{i} - 8.00\hat{i} \times \hat{j} + 35.0\hat{j} \times \hat{i} - 14.0\hat{j} \times \hat{j}$. But $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = 0$ and $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{i} = -\hat{k}$, so $\vec{A} \times \vec{B} = -8.00\hat{k} + 35.0(-\hat{k}) = -43.0\hat{k}$. The magnitude of $\vec{A} \times \vec{B}$ is 43.0.

**EVALUATE:** Sketch the vectors $\vec{A}$ and $\vec{B}$ in a coordinate system where the $xy$-plane is in the plane of the paper and the $z$-axis is directed out toward you. By the right-hand rule $\vec{A} \times \vec{B}$ is directed into the plane of the paper, in the $-z$-direction. This agrees with the above calculation that used unit vectors.

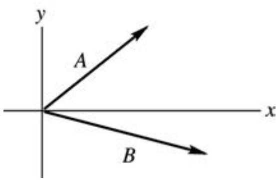



**Figure 1.44**

**1.45. IDENTIFY:** For all of these pairs of vectors, the angle is found from combining $\vec{A}\cdot\vec{B}=AB\cos\phi$ and $\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$, to give the angle $\phi$ as $\phi=\arccos\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)=\arccos\left(\frac{A_xB_x+A_yB_y}{AB}\right)$.

**SET UP:** $\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$ shows how to obtain the components for a vector written in terms of unit vectors.

**EXECUTE:** **(a)** $\vec{A}\cdot\vec{B}=-22,\ A=\sqrt{40},\ B=\sqrt{13},$ and so $\phi=\arccos\left(\frac{-22}{\sqrt{40}\sqrt{13}}\right)=165°.$

**(b)** $\vec{A}\cdot\vec{B}=60,\ A=\sqrt{34},\ B=\sqrt{136},\ \ \phi=\arccos\left(\frac{60}{\sqrt{34}\sqrt{136}}\right)=28°.$

**(c)** $\vec{A}\cdot\vec{B}=0$ and $\phi=90°.$

**EVALUATE:** If $\vec{A}\cdot\vec{B}>0,\ 0\le\phi<90°$. If $\vec{A}\cdot\vec{B}<0,\ 90°<\phi\le180°$. If $\vec{A}\cdot\vec{B}=0,\ \phi=90°$ and the two vectors are perpendicular.

**1.46. IDENTIFY:** The right-hand rule gives the direction and $|\vec{A}\times\vec{B}|=AB\sin\phi$ gives the magnitude.

**SET UP:** $\phi=120.0°$.

**EXECUTE:** **(a)** The direction of $\vec{A}\times\vec{B}$ is into the page (the $-z$-direction ). The magnitude of the vector product is $AB\sin\phi=(2.80\text{ cm})(1.90\text{ cm})\sin120°=4.61\text{ cm}^2$.

**(b)** Rather than repeat the calculations, $\vec{B}\times\vec{A}=-\vec{A}\times\vec{B}$ may be used to see that $\vec{B}\times\vec{A}$ has magnitude $4.61\text{ cm}^2$ and is in the $+z$-direction (out of the page).

**EVALUATE:** For part (a) we could use the components of the cross product and note that the only non-vanishing component is $C_z=A_xB_y-A_yB_x=(2.80\text{ cm})\cos60.0°(-1.90\text{ cm})\sin60°$

$$-(2.80\text{ cm})\sin60.0°(1.90\text{ cm})\cos60.0°=-4.61\text{ cm}^2.$$

This gives the same result.

**1.47. IDENTIFY:** $\vec{A}\times\vec{D}$ has magnitude $AD\sin\phi$. Its direction is given by the right-hand rule.

**SET UP:** $\phi=180°-53°=127°$

**EXECUTE:** **(a)** $|\vec{A}\times\vec{D}|=(8.00\text{ m})(10.0\text{ m})\sin127°=63.9\text{ m}^2$. The right-hand rule says $\vec{A}\times\vec{D}$ is in the $-z$-direction (into the page).

**(b)** $\vec{D}\times\vec{A}$ has the same magnitude as $\vec{A}\times\vec{D}$ and is in the opposite direction.

**EVALUATE:** The component of $\vec{D}$ perpendicular to $\vec{A}$ is $D_\perp=D\sin53.0°=7.99$ m.

$|\vec{A}\times\vec{D}|=AD_\perp=63.9\text{ m}^2$, which agrees with our previous result.

**1.48. IDENTIFY:** Apply Eqs. (1.16) and (1.20).

**SET UP:** The angle between the vectors is $20°+90°+30°=140°$.

**EXECUTE:** **(a)** $\vec{A}\cdot\vec{B}=AB\cos\phi$ gives $\vec{A}\cdot\vec{B}=(3.60\text{ m})(2.40\text{ m})\cos140°=-6.62\text{ m}^2$.

**(b)** From $|\vec{A}\times\vec{B}|=AB\sin\phi$, the magnitude of the cross product is $(3.60\text{ m})(2.40\text{ m})\sin140°=5.55\text{ m}^2$ and the direction, from the right-hand rule, is out of the page (the $+z$-direction ).

**EVALUATE:** We could also use $\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$ and the cross product, with the components of $\vec{A}$ and $\vec{B}$.

**1.49. IDENTIFY:** We model the earth, white dwarf, and neutron star as spheres. Density is mass divided by volume.

**SET UP:** We know that density = mass/volume = $m/V$ where $V=\frac{4}{3}\pi r^3$ for a sphere. From Appendix B, the earth has mass of $m=5.97\times10^{24}$ kg and a radius of $r=6.37\times10^6$ m whereas for the sun at the end of its lifetime, $m=1.99\times10^{30}$ kg and $r$ = 7500 km = $7.5\times10^6$ m. The star possesses a radius of $r$ = 10 km = $1.0\times10^4$ m and a mass of $m=1.99\times10^{30}$ kg.

**EXECUTE:** **(a)** The earth has volume $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6.37\times10^6\ \text{m})^3 = 1.0827\times10^{21}\ \text{m}^3$. Its density is

$$\text{density} = \frac{m}{V} = \frac{5.97\times10^{24}\ \text{kg}}{1.0827\times10^{21}\ \text{m}^3} = (5.51\times10^3\ \text{kg/m}^3)\left(\frac{10^3\ \text{g}}{1\ \text{kg}}\right)\left(\frac{1\ \text{m}}{10^2\ \text{cm}}\right)^3 = 5.51\ \text{g/cm}^3$$

**(b)** $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(7.5\times10^6\ \text{m})^3 = 1.77\times10^{21}\ \text{m}^3$

$$\text{density} = \frac{m}{V} = \frac{1.99\times10^{30}\ \text{kg}}{1.77\times10^{21}\ \text{m}^3} = (1.1\times10^9\ \text{kg/m}^3)\left(\frac{1\ \text{g/cm}^3}{1000\ \text{kg/m}^3}\right) = 1.1\times10^6\ \text{g/cm}^3$$

**(c)** $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.0\times10^4\ \text{m})^3 = 4.19\times10^{12}\ \text{m}^3$

$$\text{density} = \frac{m}{V} = \frac{1.99\times10^{30}\ \text{kg}}{4.19\times10^{12}\ \text{m}^3} = (4.7\times10^{17}\ \text{kg/m}^3)\left(\frac{1\ \text{g/cm}^3}{1000\ \text{kg/m}^3}\right) = 4.7\times10^{14}\ \text{g/cm}^3$$

**EVALUATE:** For a fixed mass, the density scales as $1/r^3$. Thus, the answer to (c) can also be obtained from (b) as

$$(1.1\times10^6\ \text{g/cm}^3)\left(\frac{7.50\times10^6\ \text{m}}{1.0\times10^4\ \text{m}}\right)^3 = 4.7\times10^{14}\ \text{g/cm}^3.$$

**1.50.** **IDENTIFY:** Area is length times width. Do unit conversions.

**SET UP:** $1\ \text{mi} = 5280\ \text{ft}$. $1\ \text{ft}^3 = 7.477\ \text{gal}$.

**EXECUTE:** **(a)** The area of one acre is $\frac{1}{8}\ \text{mi}\times\frac{1}{80}\ \text{mi} = \frac{1}{640}\ \text{mi}^2$, so there are 640 acres to a square mile.

**(b)** $(1\ \text{acre})\times\left(\frac{1\ \text{mi}^2}{640\ \text{acre}}\right)\times\left(\frac{5280\ \text{ft}}{1\ \text{mi}}\right)^2 = 43{,}560\ \text{ft}^2$

(all of the above conversions are exact).

**(c)** $(1\ \text{acre-foot}) = (43{,}560\ \text{ft}^3)\times\left(\frac{7.477\ \text{gal}}{1\ \text{ft}^3}\right) = 3.26\times10^5$ gal, which is rounded to three significant figures.

**EVALUATE:** An acre is much larger than a square foot but less than a square mile. A volume of 1 acre-foot is much larger than a gallon.

**1.51.** **IDENTIFY:** The density relates mass and volume. Use the given mass and density to find the volume and from this the radius.

**SET UP:** The earth has mass $m_E = 5.97\times10^{24}$ kg and radius $r_E = 6.37\times10^6$ m. The volume of a sphere is $V = \frac{4}{3}\pi r^3$. $\rho = 1.76\ \text{g/cm}^3 = 1760\ \text{km/m}^3$.

**EXECUTE:** **(a)** The planet has mass $m = 5.5m_E = 3.28\times10^{25}$ kg. $V = \frac{m}{\rho} = \frac{3.28\times10^{25}\ \text{kg}}{1760\ \text{kg/m}^3} = 1.86\times10^{22}\ \text{m}^3$.

$$r = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(\frac{3[1.86\times10^{22}\ \text{m}^3]}{4\pi}\right)^{1/3} = 1.64\times10^7\ \text{m} = 1.64\times10^4\ \text{km}$$

**(b)** $r = 2.57r_E$

**EVALUATE:** Volume $V$ is proportional to mass and radius $r$ is proportional to $V^{1/3}$, so $r$ is proportional to $m^{1/3}$. If the planet and earth had the same density its radius would be $(5.5)^{1/3}r_E = 1.8r_E$. The radius of the planet is greater than this, so its density must be less than that of the earth.

**1.52.** **IDENTIFY** and **SET UP:** Unit conversion.

**EXECUTE:** **(a)** $f = 1.420\times10^9$ cycles/s, so $\frac{1}{1.420\times10^9}\ \text{s} = 7.04\times10^{-10}\ \text{s}$ for one cycle.

**(b)** $\frac{3600\ \text{s/h}}{7.04\times10^{-10}\ \text{s/cycle}} = 5.11\times10^{12}\ \text{cycles/h}$

**(c)** Calculate the number of seconds in 4600 million years $= 4.6\times10^9$ y and divide by the time for 1 cycle:

$$\frac{(4.6\times10^9\ \text{y})(3.156\times10^7\ \text{s/y})}{7.04\times10^{-10}\ \text{s/cycle}} = 2.1\times10^{26}\ \text{cycles}$$

**(d)** The clock is off by 1 s in $100{,}000\ \text{y} = 1\times10^5\ \text{y}$, so in $4.60\times10^9$ y it is off by

$(1\ \text{s})\left(\dfrac{4.60\times10^9}{1\times10^5}\right) = 4.6\times10^4\ \text{s}$ (about 13 h).

**EVALUATE:** In each case the units in the calculation combine algebraically to give the correct units for the answer.

**1.53.** **IDENTIFY:** Using the density of the oxygen and volume of a breath, we want the mass of oxygen (the target variable in part (a)) breathed in per day and the dimensions of the tank in which it is stored.

**SET UP:** The mass is the density times the volume. Estimate 12 breaths per minute. We know 1 day = 24 h, 1 h = 60 min and 1000 L = 1 m$^3$. The volume of a cube having faces of length $l$ is $V = l^3$.

**EXECUTE:** **(a)** $(12\ \text{breaths/min})\left(\dfrac{60\ \text{min}}{1\ \text{h}}\right)\left(\dfrac{24\ \text{h}}{1\ \text{day}}\right) = 17{,}280\ \text{breaths/day}.$ The volume of air breathed in one day is $(\frac{1}{2}\ \text{L/breath})(17{,}280\ \text{breaths/day}) = 8640\ \text{L} = 8.64\ \text{m}^3$. The mass of air breathed in one day is the density of air times the volume of air breathed: $m = (1.29\ \text{kg/m}^3)(8.64\ \text{m}^3) = 11.1\ \text{kg}$. As 20% of this quantity is oxygen, the mass of oxygen breathed in 1 day is $(0.20)(11.1\ \text{kg}) = 2.2\ \text{kg} = 2200\ \text{g}$.

**(b)** $V = 8.64\ \text{m}^3$ and $V = l^3$, so $l = V^{1/3} = 2.1\ \text{m}$.

**EVALUATE:** A person could not survive one day in a closed tank of this size because the exhaled air is breathed back into the tank and thus reduces the percent of oxygen in the air in the tank. That is, a person cannot extract all of the oxygen from the air in an enclosed space.

**1.54.** **IDENTIFY:** Use the extreme values in the piece's length and width to find the uncertainty in the area.

**SET UP:** The length could be as large as 7.61 cm and the width could be as large as 1.91 cm.

**EXECUTE:** **(a)** The area is $14.44 \pm 0.095\ \text{cm}^2$.

**(b)** The fractional uncertainty in the area is $\dfrac{0.095\ \text{cm}^2}{14.44\ \text{cm}^2} = 0.66\%$, and the fractional uncertainties in the length and width are $\dfrac{0.01\ \text{cm}}{7.61\ \text{cm}} = 0.13\%$ and $\dfrac{0.01\ \text{cm}}{1.9\ \text{cm}} = 0.53\%$. The sum of these fractional uncertainties is $0.13\% + 0.53\% = 0.66\%$, in agreement with the fractional uncertainty in the area.

**EVALUATE:** The fractional uncertainty in a product of numbers is greater than the fractional uncertainty in any of the individual numbers.

**1.55.** **IDENTIFY:** Calculate the average volume and diameter and the uncertainty in these quantities.

**SET UP:** Using the extreme values of the input data gives us the largest and smallest values of the target variables and from these we get the uncertainty.

**EXECUTE:** **(a)** The volume of a disk of diameter $d$ and thickness $t$ is $V = \pi(d/2)^2 t$.

The average volume is $V = \pi(8.50\ \text{cm}/2)^2(0.050\ \text{cm}) = 2.837\ \text{cm}^3$. But $t$ is given to only two significant figures so the answer should be expressed to two significant figures: $V = 2.8\ \text{cm}^3$.
We can find the uncertainty in the volume as follows. The volume could be as large as $V = \pi(8.52\ \text{cm}/2)^2(0.055\ \text{cm}) = 3.1\ \text{cm}^3$, which is $0.3\ \text{cm}^3$ larger than the average value. The volume could be as small as $V = \pi(8.48\ \text{cm}/2)^2(0.045\ \text{cm}) = 2.5\ \text{cm}^3$, which is $0.3\ \text{cm}^3$ smaller than the average value. The uncertainty is $\pm0.3\ \text{cm}^3$, and we express the volume as $V = 2.8 \pm 0.3\ \text{cm}^3$.

**(b)** The ratio of the average diameter to the average thickness is $8.50 \text{ cm}/0.050 \text{ cm} = 170$. By taking the largest possible value of the diameter and the smallest possible thickness we get the largest possible value for this ratio: $8.52 \text{ cm}/0.045 \text{ cm} = 190$. The smallest possible value of the ratio is $8.48/0.055 = 150$. Thus the uncertainty is $\pm 20$ and we write the ratio as $170 \pm 20$.

**EVALUATE:** The thickness is uncertain by 10% and the percentage uncertainty in the diameter is much less, so the percentage uncertainty in the volume and in the ratio should be about 10%.

**1.56.** **IDENTIFY:** Estimate the volume of each object. The mass $m$ is the density times the volume.

**SET UP:** The volume of a sphere of radius $r$ is $V = \frac{4}{3}\pi r^3$. The volume of a cylinder of radius $r$ and length $l$ is $V = \pi r^2 l$. The density of water is $1000 \text{ kg/m}^3$.

**EXECUTE:** **(a)** Estimate the volume as that of a sphere of diameter 10 cm: $V = 5.2 \times 10^{-4} \text{ m}^3$.

$m = (0.98)(1000 \text{ kg / m}^3)(5.2 \times 10^{-4} \text{ m}^3) = 0.5 \text{ kg}$.

**(b)** Approximate as a sphere of radius $r = 0.25 \mu\text{m}$ (probably an overestimate): $V = 6.5 \times 10^{-20} \text{ m}^3$.

$m = (0.98)(1000 \text{ kg/m}^3)(6.5 \times 10^{-20} \text{ m}^3) = 6 \times 10^{-17} \text{ kg} = 6 \times 10^{-14} \text{ g}$.

**(c)** Estimate the volume as that of a cylinder of length 1 cm and radius 3 mm: $V = \pi r^2 l = 2.8 \times 10^{-7} \text{ m}^3$.

$m = (0.98)(1000 \text{ kg/m}^3)(2.8 \times 10^{-7} \text{ m}^3) = 3 \times 10^{-4} \text{ kg} = 0.3 \text{ g}$.

**EVALUATE:** The mass is directly proportional to the volume.

**1.57.** **IDENTIFY:** The number of atoms is your mass divided by the mass of one atom.

**SET UP:** Assume a 70-kg person and that the human body is mostly water. Use Appendix D to find the mass of one $H_2O$ molecule: $18.015 \text{ u} \times 1.661 \times 10^{-27} \text{ kg/u} = 2.992 \times 10^{-26} \text{ kg/molecule}$.

**EXECUTE:** $(70 \text{ kg})/(2.992 \times 10^{-26} \text{ kg/molecule}) = 2.34 \times 10^{27}$ molecules. Each $H_2O$ molecule has 3 atoms, so there are about $6 \times 10^{27}$ atoms.

**EVALUATE:** Assuming carbon to be the most common atom gives $3 \times 10^{27}$ molecules, which is a result of the same order of magnitude.

**1.58.** **IDENTIFY:** We know the vector sum and want to find the magnitude of the vectors. Use the method of components.

**SET UP:** The two vectors $\vec{A}$ and $\vec{B}$ and their resultant $\vec{C}$ are shown in Figure 1.58. Let $+y$ be in the direction of the resultant. $A = B$.

**EXECUTE:** $C_y = A_y + B_y$. $372 \text{ N} = 2A\cos 36.0°$ gives $A = 230 \text{ N}$.

**EVALUATE:** The sum of the magnitudes of the two forces exceeds the magnitude of the resultant force because only a component of each force is upward.

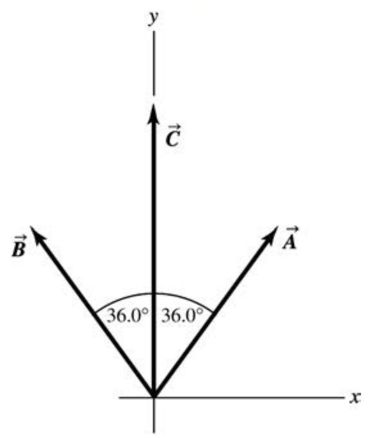



**Figure 1.58**

**1.59.** **IDENTIFY:** We know the magnitude and direction of the sum of the two vector pulls and the direction of one pull. We also know that one pull has twice the magnitude of the other. There are two unknowns, the magnitude of the smaller pull and its direction. $A_x + B_x = C_x$ and $A_y + B_y = C_y$ give two equations for these two unknowns.

**SET UP:** Let the smaller pull be $\vec{A}$ and the larger pull be $\vec{B}$. $B = 2A$. $\vec{C} = \vec{A} + \vec{B}$ has magnitude 460.0 N and is northward. Let $+x$ be east and $+y$ be north. $B_x = -B\sin 21.0°$ and $B_y = B\cos 21.0°$. $C_x = 0$, $C_y = 460.0$ N. $\vec{A}$ must have an eastward component to cancel the westward component of $\vec{B}$. There are then two possibilities, as sketched in Figures 1.59 a and b. $\vec{A}$ can have a northward component or $\vec{A}$ can have a southward component.

**EXECUTE:** In either Figure 1.59 a or b, $A_x + B_x = C_x$ and $B = 2A$ gives $(2A)\sin 21.0° = A\sin\phi$ and $\phi = 45.79°$. In Figure 1.59a, $A_y + B_y = C_y$ gives $2A\cos 21.0° + A\cos 45.79° = 460.0$ N, so $A = 179.4$ N. In Figure 1.59b, $2A\cos 21.0° - A\cos 45.79° = 460.0$ N and $A = 393$ N. One solution is for the smaller pull to be $45.8°$ east of north. In this case, the smaller pull is 179 N and the larger pull is 358 N. The other solution is for the smaller pull to be 45.8° south of east. In this case the smaller pull is 393 N and the larger pull is 786 N.

**EVALUATE:** For the first solution, with $\vec{A}$ east of north, each worker has to exert less force to produce the given resultant force and this is the sensible direction for the worker to pull.

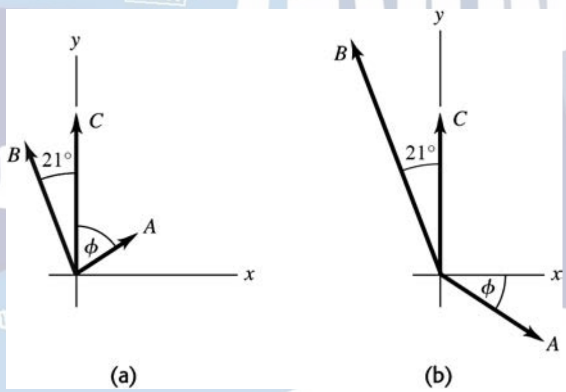



**Figure 1.59**

**1.60.** **IDENTIFY:** Let $\vec{D}$ be the fourth force. Find $\vec{D}$ such that $\vec{A} + \vec{B} + \vec{C} + \vec{D} = 0$, so $\vec{D} = -(\vec{A} + \vec{B} + \vec{C})$.

**SET UP:** Use components and solve for the components $D_x$ and $D_y$ of $\vec{D}$.

**EXECUTE:** $A_x = +A\cos 30.0° = +86.6\,\text{N}$, $A_y = +A\sin 30.0° = +50.00\,\text{N}$.

$B_x = -B\sin 30.0° = -40.00\,\text{N}$, $B_y = +B\cos 30.0° = +69.28\,\text{N}$.

$C_x = -C\cos 53.0° = -24.07\,\text{N}$, $C_y = -C\sin 53.0° = -31.90\,\text{N}$.

Then $D_x = -22.53$ N, $D_y = -87.34\,\text{N}$ and $D = \sqrt{D_x^2 + D_y^2} = 90.2$ N. $\tan\alpha = |D_y/D_x| = 87.34/22.53$. $\alpha = 75.54°$. $\phi = 180° + \alpha = 256°$, counterclockwise from the $+x$-axis.

**EVALUATE:** As shown in Figure 1.60, since $D_x$ and $D_y$ are both negative, $\vec{D}$ must lie in the third quadrant.

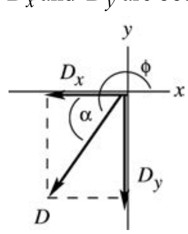



**Figure 1.60**

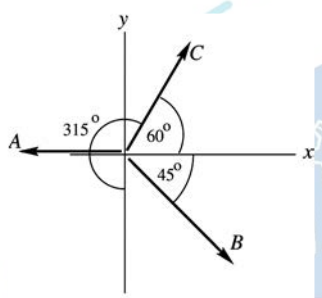
**1.61.** **IDENTIFY:** Vector addition. Target variable is the 4th displacement.

**SET UP:** Use a coordinate system where east is in the $+x$-direction and north is in the $+y$-direction. Let $\vec{A}$, $\vec{B}$, and $\vec{C}$ be the three displacements that are given and let $\vec{D}$ be the fourth unmeasured displacement. Then the resultant displacement is $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$. And since she ends up back where she started, $\vec{R} = 0$.

$0 = \vec{A} + \vec{B} + \vec{C} + \vec{D}$, so $\vec{D} = -(\vec{A} + \vec{B} + \vec{C})$

$D_x = -(A_x + B_x + C_x)$ and $D_y = -(A_y + B_y + C_y)$

**EXECUTE:**




$A_x = -180 \text{ m}, \quad A_y = 0$

$B_x = B\cos 315° = (210 \text{ m})\cos 315° = +148.5 \text{ m}$

$B_y = B\sin 315° = (210 \text{ m})\sin 315° = -148.5 \text{ m}$

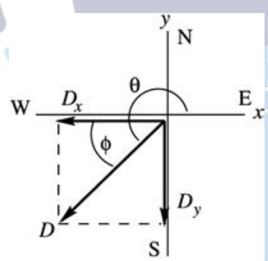
$C_x = C\cos 60° = (280 \text{ m})\cos 60° = +140 \text{ m}$

$C_y = C\sin 60° = (280 \text{ m})\sin 60° = +242.5 \text{ m}$

**Figure 1.61a**

$D_x = -(A_x + B_x + C_x) = -(-180 \text{ m} + 148.5 \text{ m} + 140 \text{ m}) = -108.5 \text{ m}$

$D_y = -(A_y + B_y + C_y) = -(0 - 148.5 \text{ m} + 242.5 \text{ m}) = -94.0 \text{ m}$




$D = \sqrt{D_x^2 + D_y^2}$

$D = \sqrt{(-108.5 \text{ m})^2 + (-94.0 \text{ m})^2} = 144 \text{ m}$

$\tan\theta = \dfrac{D_y}{D_x} = \dfrac{-94.0 \text{ m}}{-108.5 \text{ m}} = 0.8664$

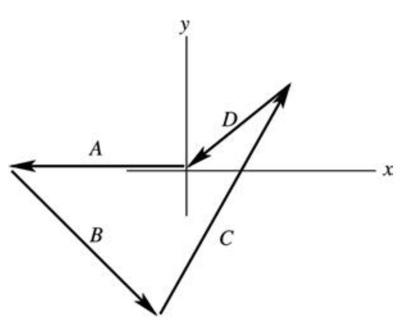
$\theta = 180° + 40.9° = 220.9°$

($\vec{D}$ is in the third quadrant since both $D_x$ and $D_y$ are negative.)

**Figure 1.61b**

The direction of $\vec{D}$ can also be specified in terms of $\phi = \theta - 180° = 40.9°$; $\vec{D}$ is $41°$ south of west.

**EVALUATE:** The vector addition diagram, approximately to scale, is




Vector $\vec{D}$ in this diagram agrees qualitatively with our calculation using components.

**Figure 1.61c**

**1.62.** **IDENTIFY:** Find the vector sum of the two displacements.

**SET UP:** Call the two displacements $\vec{A}$ and $\vec{B}$, where $A = 170$ km and $B = 230$ km. $\vec{A} + \vec{B} = \vec{R}$. $\vec{A}$ and $\vec{B}$ are as shown in Figure 1.62.

**EXECUTE:** $R_x = A_x + B_x = (170 \text{ km})\sin 68° + (230 \text{ km})\cos 36° = 343.7$ km.

$R_y = A_y + B_y = (170 \text{ km})\cos 68° - (230 \text{ km})\sin 36° = -71.5$ km.

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(343.7 \text{ km})^2 + (-71.5 \text{ km})^2} = 351$ km. $\tan\theta_R = |\dfrac{R_y}{R_x}| = \dfrac{71.5 \text{ km}}{343.7 \text{ km}} = 0.208$.

$\theta_R = 11.8°$ south of east.

**EVALUATE:** Our calculation using components agrees with $\vec{R}$ shown in the vector addition diagram, Figure 1.62.

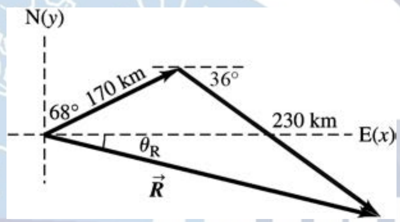



**Figure 1.62**

**1.63.** **IDENTIFY:** We know the resultant of two forces of known equal magnitudes and want to find that magnitude (the target variable).

**SET UP:** Use coordinates having a horizontal $+x$ axis and an upward $+y$ axis. Then $A_x + B_x = R_x$ and $R_x = 12.8$ N.

**SOLVE:** $A_x + B_x = R_x$ and $A\cos 32° + B\sin 32° = R_x$. Since $A = B$,

$2A\cos 32° = R_x$, so $A = \dfrac{R_x}{(2)(\cos 32°)} = 7.55$ N.

**EVALUATE:** The magnitude of the $x$ component of each pull is 6.40 N, so the magnitude of each pull (7.55 N) is greater than its $x$ component, as it should be.

**1.64.** **IDENTIFY:** Solve for one of the vectors in the vector sum. Use components.

**SET UP:** Use coordinates for which $+x$ is east and $+y$ is north. The vector displacements are:

$\vec{A} = 2.00$ km, 0°of east; $\vec{B} = 3.50$ m, 45° south of east; and $\vec{R} = 5.80$ m, 0° east

**EXECUTE:** $C_x = R_x - A_x - B_x = 5.80 \text{ km} - (2.00 \text{ km}) - (3.50 \text{ km})(\cos 45°) = 1.33$ km; $C_y = R_y - A_y - B_y$ $= 0 \text{ km} - 0 \text{ km} - (-3.50 \text{ km})(\sin 45°) = 2.47$ km; $C = \sqrt{(1.33 \text{ km})^2 + (2.47 \text{ km})^2} = 2.81$ km;

$\theta = \tan^{-1}[(2.47 \text{ km})/(1.33 \text{ km})] = 61.7°$ north of east. The vector addition diagram in Figure 1.64 shows good qualitative agreement with these values.

**EVALUATE:** The third leg lies in the first quadrant since its $x$ and $y$ components are both positive.

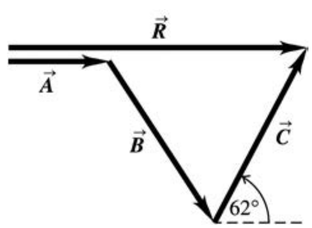



**Figure 1.64**

**1.65.** **IDENTIFY:** We have two known vectors and a third unknown vector, and we know the resultant of these three vectors.

**SET UP:** Use coordinates for which $+x$ is east and $+y$ is north. The vector displacements are:

$\vec{A} = 23.0$ km at 34.0° south of east; $\vec{B} = 46.0$ km due north; $\vec{R} = 32.0$ km due west ; $\vec{C}$ is unknown.

**EXECUTE:** $C_x = R_x - A_x - B_x = -32.0 \text{ km} - (23.0 \text{ km})\cos 34.0° - 0 = -51.07 \text{ km};$

$C_y = R_y - A_y - B_y = 0 - (-23.0 \text{ km})\sin 34.0° - 46.0 \text{ km} = -33.14 \text{ km};$

$C = \sqrt{C_x^2 + C_y^2} = 60.9 \text{ km}$

Calling $\theta$ the angle that $\vec{C}$ makes with the $-x$-axis (the westward direction), we have

$\tan\theta = C_y / C_x = \dfrac{33.14}{51.07}$; $\theta = 33.0°$ south of west.

**EVALUATE:** A graphical vector sum will confirm this result.

**1.66.** **IDENTIFY:** The four displacements return her to her starting point, so $\vec{D} = -(\vec{A} + \vec{B} + \vec{C})$, where $\vec{A}$, $\vec{B}$, and $\vec{C}$ are in the three given displacements and $\vec{D}$ is the displacement for her return.

**SET UP:** Let $+x$ be east and $+y$ be north.

**EXECUTE:** **(a)** $D_x = -[(147 \text{ km})\sin 85° + (106 \text{ km})\sin 167° + (166 \text{ km})\sin 235°] = -34.3 \text{ km}.$

$D_y = -[(147 \text{ km})\cos 85° + (106 \text{ km})\cos 167° + (166 \text{ km})\cos 235°] = +185.7 \text{ km}.$

$D = \sqrt{(-34.3 \text{ km})^2 + (185.7 \text{ km})^2} = 189 \text{ km}.$

**(b)** The direction relative to north is $\phi = \arctan\left(\dfrac{34.3 \text{ km}}{185.7 \text{ km}}\right) = 10.5°.$ Since $D_x < 0$ and $D_y > 0$, the direction of $\vec{D}$ is $10.5°$ west of north.

**EVALUATE:** The four displacements add to zero.

**1.67.** **IDENTIFY:** We want to find the resultant of three known displacement vectors: $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.

**SET UP:** Let $+x$ be east and $+y$ be north and find the components of the vectors.

**EXECUTE:** The magnitudes are $A = 20.8$ m, $B = 38.0$ m, $C = 18.0$ m. The components are
$A_x = 0$, $A_y = 28.0$ m, $B_x = 38.0$ m, $B_y = 0$,
$C_x = -(18.0 \text{ m})(\sin 33.0°) = -9.804$ m, $C_y = -(18.0 \text{ m})(\cos 33.0°) = -15.10$ m
$R_x = A_x + B_x + C_x = 0 + 38.0 \text{ m} + (-9.80 \text{ m}) = 28.2$ m
$R_y = A_y + B_y + C_y = 20.8 \text{ m} + 0 + (-15.10 \text{ m}) = 5.70$ m
$R = \sqrt{R_x^2 + R_y^2} = 28.8$ m is the distance you must run. Calling $\theta_R$ the angle the resultant makes with the $+x$-axis (the easterly direction), we have
$\tan\theta_R = R_y/R_x = (5.70 \text{ km})/(28.2 \text{ km})$; $\theta_R = 11.4°$ north of east.

**EVALUATE:** A graphical sketch will confirm this result.

**1.68.** **IDENTIFY:** Let the three given displacements be $\vec{A}$, $\vec{B}$ and $\vec{C}$, where $A = 40$ steps, $B = 80$ steps and $C = 50$ steps. $\vec{R} = \vec{A} + \vec{B} + \vec{C}$. The displacement $\vec{C}$ that will return him to his hut is $-\vec{R}$.

**SET UP:** Let the east direction be the $+x$-direction and the north direction be the $+y$-direction.

**EXECUTE:** **(a)** The three displacements and their resultant are sketched in Figure 1.68.

**(b)** $R_x = (40)\cos 45° - (80)\cos 60° = -11.7$ and $R_y = (40)\sin 45° + (80)\sin 60° - 50 = 47.6.$

The magnitude and direction of the resultant are $\sqrt{(-11.7)^2 + (47.6)^2} = 49$, acrtan$\left(\dfrac{47.6}{11.7}\right) = 76°$, north of west. We know that $\vec{R}$ is in the second quadrant because $R_x < 0$, $R_y > 0$. To return to the hut, the explorer must take 49 steps in a direction $76°$ south of east, which is $14°$ east of south.

**EVALUATE:** It is useful to show $R_x$, $R_y$, and $\vec{\boldsymbol{R}}$ on a sketch, so we can specify what angle we are computing.

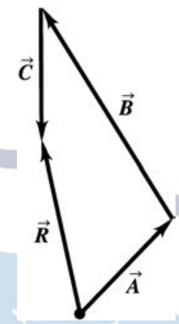



**Figure 1.68**

**1.69.** **IDENTIFY:** We know the resultant of two vectors and one of the vectors, and we want to find the second vector.

**SET UP:** Let the westerly direction be the $+x$-direction and the northerly direction be the $+y$-direction. We also know that $\vec{\boldsymbol{R}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$ where $\vec{\boldsymbol{R}}$ is the vector from you to the truck. Your GPS tells you that you are 122.0 m from the truck in a direction of 58.0° east of south, so a vector from the truck to you is 122.0 m at 58.0° east of south. Therefore the vector from you to the truck is 122.0 m at 58.0° west of north. Thus $\vec{\boldsymbol{R}}$ = 122.0 m at 58.0° west of north and $\vec{\boldsymbol{A}}$ is 72.0 m due west. We want to find the magnitude and direction of vector $\vec{\boldsymbol{B}}$.

**EXECUTE:** $B_x = R_x - A_x = (122.0\text{ m})(\sin 58.0°) - 72.0\text{ m} = 31.462\text{ m}$

$B_y = R_y - A_y = (122.0\text{ m})(\cos 58.0°) - 0 = 64.450\text{ m}$; $B = \sqrt{B_x^2 + B_y^2} = 71.9\text{ m}$.

$\tan\theta_B = B_y / B_x = \dfrac{64.650\text{ m}}{31.462\text{ m}} = 2.05486$; $\theta_B = 64.1°$ north of west.

**EVALUATE:** A graphical sum will show that the results are reasonable.

**1.70.** **IDENTIFY:** We use vector addition. One vector and the sum are given; find the magnitude and direction of the second vector.

**SET UP:** Let $+x$ be east and $+y$ be north. Let $\vec{\boldsymbol{A}}$ be the displacement 285 km at 62.0° north of west and let $\vec{\boldsymbol{B}}$ be the unknown displacement.

$\vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} = \vec{\boldsymbol{R}}$ where $\vec{\boldsymbol{R}} = 115\text{ km, east}$

$\vec{\boldsymbol{B}} = \vec{\boldsymbol{R}} - \vec{\boldsymbol{A}}$

$B_x = R_x - A_x$, $B_y = R_y - A_y$

**EXECUTE:** $A_x = -A\cos 62.0° = -133.8\text{ km}$, $A_y = +A\sin 62.0° = +251.6\text{ km}$

$R_x = 115\text{ km}, R_y = 0$

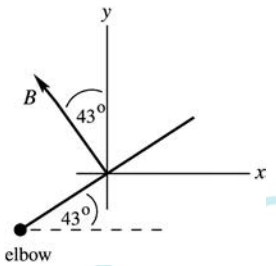
$B_x = R_x - A_x = 115\text{ km} - (-133.8\text{ km}) = 248.8\text{ km}$

$B_y = R_y - A_y = 0 - 251.6\text{ km} = -251.6\text{ km}$

$B = \sqrt{B_x^2 + B_y^2} = 354\text{ km}$. Since $\vec{\boldsymbol{B}}$ has a positive $x$ component and a negative $y$ component, it must lie in the fourth quadrant. Its angle with the $+x$-axis is given by $\tan\alpha = |B_y/B_x| = (251.6\text{ km})/(248.8\text{ km})$, so $\alpha = 45.3°$ south of east.

**EVALUATE:** A graphical vector sum will confirm these results.

**1.71.** **IDENTIFY:** Vector addition. One force and the vector sum are given; find the second force.
**SET UP:** Use components. Let $+y$ be upward.




$\vec{B}$ is the force the biceps exerts.

**Figure 1.71a**

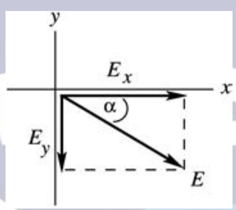
$\vec{E}$ is the force the elbow exerts. $\vec{E}+\vec{B}=\vec{R}$, where $R=132.5$ N and is upward.

$E_x = R_x - B_x,\; E_y = R_y - B_y$

**EXECUTE:** $B_x = -B\sin 43° = -158.2$ N, $B_y = +B\cos 43° = +169.7$ N, $R_x = 0$, $R_y = +132.5$ N

Then $E_x = +158.2$ N, $E_y = -37.2$ N.

$E=\sqrt{E_x^2+E_y^2}=160$ N;




$\tan\alpha = |E_y/E_x| = 37.2/158.2$

$\alpha = 13°$, below horizontal

**Figure 1.71b**

**EVALUATE:** The $x$-component of $\vec{E}$ cancels the $x$-component of $\vec{B}$. The resultant upward force is less than the upward component of $\vec{B}$, so $E_y$ must be downward.

**1.72.** **IDENTIFY:** Find the vector sum of the four displacements.
**SET UP:** Take the beginning of the journey as the origin, with north being the $y$-direction, east the $x$-direction, and the $z$-axis vertical. The first displacement is then $(-30\text{ m})\hat{k}$, the second is $(-15\text{ m})\hat{j}$, the third is $(200\text{ m})\hat{i}$, and the fourth is $(100\text{ m})\hat{j}$.
**EXECUTE:** **(a)** Adding the four displacements gives
$(-30\text{ m})\hat{k}+(-15\text{ m})\hat{j}+(200\text{ m})\hat{i}+(100\text{ m})\hat{j}=(200\text{ m})\hat{i}+(85\text{ m})\hat{j}-(30\text{ m})\hat{k}$.
**(b)** The total distance traveled is the sum of the distances of the individual segments:
$30\text{ m}+15\text{ m}+200\text{ m}+100\text{ m}=345\text{ m}$. The magnitude of the total displacement is:

$$D=\sqrt{D_x^2+D_y^2+D_z^2}=\sqrt{(200\text{ m})^2+(85\text{ m})^2+(-30\text{ m})^2}=219\text{ m}.$$

**EVALUATE:** The magnitude of the displacement is much less than the distance traveled along the path.

**1.73.** **IDENTIFY:** The sum of the four displacements must be zero. Use components.
**SET UP:** Call the displacements $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$, where $\vec{D}$ is the final unknown displacement for the return from the treasure to the oak tree. Vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are sketched in Figure 1.73a.
$\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$ says $A_x+B_x+C_x+D_x=0$ and $A_y+B_y+C_y+D_y=0$. $A=825$ m, $B=1250$ m, and $C=1000$ m. Let $+x$ be eastward and $+y$ be north.

**EXECUTE:** **(a)** $A_x + B_x + C_x + D_x = 0$ gives

$D_x = -(A_x + B_x + C_x) = -[0 - (1250\text{ m})\sin 30.0° + (1000\text{ m})\cos 32.0°] = -223.0\text{ m}$. $A_y + B_y + C_y + D_y = 0$ gives $D_y = -(A_y + B_y + C_y) = -[-825\text{ m} + (1250\text{ m})\cos 30.0° + (1000\text{ m})\sin 32.0°] = -787.4\text{ m}$. The fourth displacement $\vec{D}$ and its components are sketched in Figure 1.73b. $D = \sqrt{D_x^2 + D_y^2} = 818.4\text{ m}$.

$\tan\phi = \dfrac{|D_x|}{|D_y|} = \dfrac{223.0\text{ m}}{787.4\text{ m}}$ and $\phi = 15.8°$. You should head 15.8° west of south and must walk 818 m.

**(b)** The vector diagram is sketched in Figure 1.73c. The final displacement $\vec{D}$ from this diagram agrees with the vector $\vec{D}$ calculated in part **(a)** using components.

**EVALUATE:** Note that $\vec{D}$ is the negative of the sum of $\vec{A}$, $\vec{B}$, and $\vec{C}$, as it should be.

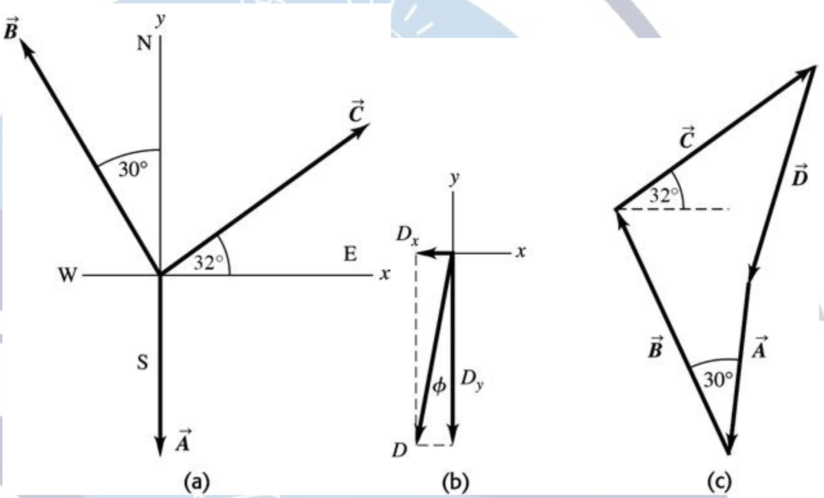



**Figure 1.73**

**1.74.** **IDENTIFY:** The displacements are vectors in which we want to find the magnitude of the resultant and know the other vectors.

**SET UP:** Calling $\vec{A}$ the vector from you to the first post, $\vec{B}$ the vector from you to the second post, and $\vec{C}$ the vector from the first post to the second post, we have $\vec{A} + \vec{C} = \vec{B}$. We want to find the magnitude of vector $\vec{B}$. We use components and the magnitude of $\vec{C}$. Let $+x$ be toward the east and $+y$ be toward the north.

**EXECUTE:** $B_x = 0$ and $B_y$ is unknown. $C_x = -A_x = -(52.0\text{ m})(\cos 37.0°) = -41.529\text{ m}$ $A_x = 41.53\text{ m}$

$C = 68.0\text{ m}$, so $C_y = \pm\sqrt{C^2 - C_x^2} = -53.8455\text{ m}$. We use the minus sign because the second post is south of the first post.

$B_y = A_y + C_y = (52.0\text{ m})(\sin 37°) + (-53.8455\text{ m}) = -22.551\text{ m}$.

Therefore you are 22.6 m from the second post.

**EVALUATE:** $B_y$ is negative since post is south of you (in the negative $y$ direction), but the distance to you is positive.

**1.75.** **IDENTIFY:** We are given the resultant of three vectors, two of which we know, and want to find the magnitude and direction of the third vector.

**SET UP:** Calling $\vec{C}$ the unknown vector and $\vec{A}$ and $\vec{B}$ the known vectors, we have $\vec{A} + \vec{B} + \vec{C} = \vec{R}$. The components are $A_x + B_x + C_x = R_x$ and $A_y + B_y + C_y = R_y$.

**EXECUTE:** The components of the known vectors are $A_x = 12.0\text{ m}$, $A_y = 0$, $B_x = -B\sin 50.0° = -21.45\text{ m}$, $B_y = B\cos 50.0° = +18.00\text{ m}$, $R_x = 0$, and $R_y = -10.0\text{ m}$. Therefore the components of $\vec{C}$ are $C_x = R_x - A_x - B_x = 0 - 12.0\text{ m} - (-21.45\text{ m}) = 9.45\text{ m}$ and $C_y = R_y - A_y - B_y = -10.0\text{ m} - 0 - 18.0\text{ m} = -28.0\text{ m}$.

Using these components to find the magnitude and direction of $\vec{C}$ gives $C = 29.6\text{ m}$ and $\tan\theta = \dfrac{9.45}{28.0}$ and $\theta = 18.6°$ east of south.

**EVALUATE:** A graphical sketch shows that this answer is reasonable.

**1.76.** **IDENTIFY:** The displacements are vectors in which we know the magnitude of the resultant and want to find the magnitude of one of the other vectors.

**SET UP:** Calling $\vec{A}$ the vector of Ricardo's displacement from the tree, $\vec{B}$ the vector of Jane's displacement from the tree, and $\vec{C}$ the vector from Ricardo to Jane, we have $\vec{A} + \vec{C} = \vec{B}$. Let the $+x$-axis be to the east and the $+y$-axis be to the north. Solving using components we have $A_x + C_x = B_x$ and $A_y + C_y = B_y$.

**EXECUTE:** **(a)** The components of $\vec{A}$ and $\vec{B}$ are $A_x = -(26.0\text{ m})\sin 60.0° = -22.52\text{ m}$, $A_y = (26.0\text{ m})\cos 60.0° = +13.0\text{ m}$, $B_x = -(16.0\text{ m})\cos 30.0° = -13.86\text{ m}$, $B_y = -(16.0\text{ m})\sin 30.0° = -8.00\text{ m}$, $C_x = B_x - A_x = -13.86\text{ m} - (-22.52\text{ m}) = +8.66\text{ m}$, $C_y = B_y - A_y = -8.00\text{ m} - (13.0\text{ m}) = -21.0\text{ m}$

Finding the magnitude from the components gives $C = 22.7\text{ m}$.

(b) Finding the direction from the components gives $\tan\theta = \dfrac{8.66}{21.0}$ and $\theta = 22.4°$, east of south.

**EVALUATE:** A graphical sketch confirms that this answer is reasonable.

**1.77.** **IDENTIFY:** If the vector from your tent to Joe's is $\vec{A}$ and from your tent to Karl's is $\vec{B}$, then the vector from Karl's tent to Joe's tent is $\vec{A} - \vec{B}$.

**SET UP:** Take your tent's position as the origin. Let $+x$ be east and $+y$ be north.

**EXECUTE:** The position vector for Joe's tent is $([21.0\text{ m}]\cos 23°)\hat{i} - ([21.0\text{ m}]\sin 23°)\hat{j} = (19.33\text{ m})\hat{i} - (8.205\text{ m})\hat{j}$.

The position vector for Karl's tent is $([32.0\text{ m}]\cos 37°)\hat{i} + ([32.0\text{ m}]\sin 37°)\hat{j} = (25.56\text{ m})\hat{i} + (19.26\text{ m})\hat{j}$.

The difference between the two positions is $(19.33\text{ m} - 25.56\text{ m})\hat{i} + (-8.205\text{ m} - 19.25\text{ m})\hat{j} = -(6.23\text{ m})\hat{i} - (27.46\text{ m})\hat{j}$. The magnitude of this vector is the distance between the two tents: $D = \sqrt{(-6.23\text{ m})^2 + (-27.46\text{ m})^2} = 28.2\text{ m}$

**EVALUATE:** If both tents were due east of yours, the distance between them would be $32.0\text{ m} - 21.0\text{ m} = 11.0\text{ m}$. If Joe's was due north of yours and Karl's was due south of yours, then the distance between them would be $32.0\text{ m} + 21.0\text{ m} = 53.0\text{ m}$. The actual distance between them lies between these limiting values.

**1.78.** **IDENTIFY:** Calculate the scalar product and use Eq. (1.16) to determine $\phi$.

**SET UP:** The unit vectors are perpendicular to each other.

**EXECUTE:** The direction vectors each have magnitude $\sqrt{3}$, and their scalar product is $(1)(1) + (1)(-1) + (1)(-1) = -1$, so from Eq. (1.16) the angle between the bonds is

$$\arccos\left(\frac{-1}{\sqrt{3}\sqrt{3}}\right) = \arccos\left(-\frac{1}{3}\right) = 109°.$$

**EVALUATE:** The angle between the two vectors in the bond directions is greater than $90°$.

**1.79.** **IDENTIFY:** We know the scalar product and the magnitude of the vector product of two vectors and want to know the angle between them.

**SET UP:** The scalar product is $\vec{A}\cdot\vec{B} = AB\cos\theta$ and the vector product is $\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$.

**EXECUTE:** $\vec{A}\cdot\vec{B} = AB\cos\theta = -6.00$ and $\left|\vec{A}\times\vec{B}\right| = AB\sin\theta = +9.00$. Taking the ratio gives $\tan\theta = \dfrac{9.00}{-6.00}$, so $\theta = 124°$.

**EVALUATE:** Since the scalar product is negative, the angle must be between 90° and 180°.

**1.80.** **IDENTIFY:** Find the angle between specified pairs of vectors.

**SET UP:** Use $\cos\phi = \dfrac{\vec{A}\cdot\vec{B}}{AB}$

**EXECUTE:** **(a)** $\vec{A} = \hat{k}$ (along line *ab*)

$\vec{B} = \hat{i} + \hat{j} + \hat{k}$ (along line *ad*)

$A = 1,\ B = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$\vec{A}\cdot\vec{B} = \hat{k}\cdot(\hat{i}+\hat{j}+\hat{k}) = 1$

So $\cos\phi = \dfrac{\vec{A}\cdot\vec{B}}{AB} = 1/\sqrt{3};\ \phi = 54.7°$

**(b)** $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ (along line *ad*)

$\vec{B} = \hat{j} + \hat{k}$ (along line *ac*)

$A = \sqrt{1^2+1^2+1^2} = \sqrt{3};\ \ B = \sqrt{1^2+1^2} = \sqrt{2}$

$\vec{A}\cdot\vec{B} = (\hat{i}+\hat{j}+\hat{k})\cdot(\hat{i}+\hat{j}) = 1+1 = 2$

So $\cos\phi = \dfrac{\vec{A}\cdot\vec{B}}{AB} = \dfrac{2}{\sqrt{3}\sqrt{2}} = \dfrac{2}{\sqrt{6}};\ \ \phi = 35.3°$

**EVALUATE:** Each angle is computed to be less than $90°$, in agreement with what is deduced from the figure shown with this problem in the textbook.

**1.81.** **IDENTIFY:** We know the magnitude of two vectors and their scalar product and want to find the magnitude of their vector product.

**SET UP:** The scalar product is $\vec{A}\cdot\vec{B} = AB\cos\phi$ and the vector product is $|\vec{A}\times\vec{B}| = AB\sin\phi$.

**EXECUTE:** $\vec{A}\cdot\vec{B} = AB\cos\phi = 90.0\ \text{m}^2$, which gives $\cos\phi = \dfrac{112.0\ \text{m}^2}{AB} = \dfrac{112.0\ \text{m}^2}{(12.0\ \text{m})(16.0\ \text{m})} = 0.5833$, so $\phi = 54.31°$. Therefore $\left|\vec{A}\times\vec{B}\right| = AB\sin\phi = (12.0\ \text{m})(16.0\ \text{m})(\sin 54.31°) = 156\ \text{m}^2$.

**EVALUATE:** The magnitude of the vector product is greater than the scalar product because the angle between the vectors is greater than 45º.

**1.82.** **IDENTIFY:** The cross product $\vec{A}\times\vec{B}$ is perpendicular to both $\vec{A}$ and $\vec{B}$.

**SET UP:** Use Eq. (1.23) to calculate the components of $\vec{A}\times\vec{B}$.

**EXECUTE:** The cross product is

$(-13.00)\hat{i} + (6.00)\hat{j} + (-11.00)\hat{k} = 13\left[-(1.00)\hat{i} + \left(\dfrac{6.00}{13.00}\right)\hat{j} - \dfrac{11.00}{13.00}\hat{k}\right]$. The magnitude of the vector in square brackets is $\sqrt{1.93}$, and so a unit vector in this direction is

$$\left[\frac{-(1.00)\hat{i} + (6.00/13.00)\hat{j} - (11.00/13.00)\hat{k}}{\sqrt{1.93}}\right].$$

The negative of this vector,

$$\left[\frac{(1.00)\hat{\boldsymbol{i}}-(6.00/13.00)\hat{\boldsymbol{j}}+(11.00/13.00)\hat{\boldsymbol{k}}}{\sqrt{1.93}}\right],$$

is also a unit vector perpendicular to $\vec{A}$ and $\vec{B}$.

**EVALUATE:** Any two vectors that are not parallel or antiparallel form a plane and a vector perpendicular to both vectors is perpendicular to this plane.

**1.83.** **IDENTIFY:** We know the scalar product of two vectors, both their directions, and the magnitude of one of them, and we want to find the magnitude of the other vector.

**SET UP:** $\vec{A}\cdot\vec{B} = AB\cos\phi$. Since we know the direction of each vector, we can find the angle between them.

**EXECUTE:** The angle between the vectors is $\theta = 79.0°$. Since $\vec{A}\cdot\vec{B} = AB\cos\phi$, we have

$$B = \frac{\vec{A}\cdot\vec{B}}{A\cos\phi} = \frac{48.0\text{ m}^2}{(9.00\text{ m})\cos 79.0°} = 28.0\text{ m}.$$

**EVALUATE:** Vector $\vec{B}$ has the same units as vector $\vec{A}$.

**1.84.** **IDENTIFY:** Calculate the magnitude of the vector product and then use $|\vec{A}\times\vec{B}| = AB\sin\theta$.

**SET UP:** The magnitude of a vector is related to its components by Eq. (1.11).

**EXECUTE:** $|\vec{A}\times\vec{B}| = AB\sin\theta$. $\sin\theta = \dfrac{|\vec{A}\times\vec{B}|}{AB} = \dfrac{\sqrt{(-5.00)^2+(2.00)^2}}{(3.00)(3.00)} = 0.5984$ and

$\theta = \sin^{-1}(0.5984) = 36.8°$.

**EVALUATE:** We haven't found $\vec{A}$ and $\vec{B}$, just the angle between them.

**1.85.** **IDENTIFY** and **SET UP:** The target variables are the components of $\vec{C}$. We are given $\vec{A}$ and $\vec{B}$. We also know $\vec{A}\cdot\vec{C}$ and $\vec{B}\cdot\vec{C}$, and this gives us two equations in the two unknowns $C_x$ and $C_y$.

**EXECUTE:** $\vec{A}$ and $\vec{C}$ are perpendicular, so $\vec{A}\cdot\vec{C} = 0$. $A_xC_x + A_yC_y = 0$, which gives $5.0C_x - 6.5C_y = 0$.

$\vec{B}\cdot\vec{C} = 15.0$, so $3.5C_x - 7.0C_y = 15.0$

We have two equations in two unknowns $C_x$ and $C_y$. Solving gives $C_x = -8.0$ and $C_y = -6.1$.

**EVALUATE:** We can check that our result does give us a vector $\vec{C}$ that satisfies the two equations $\vec{A}\cdot\vec{C} = 0$ and $\vec{B}\cdot\vec{C} = 15.0$.

**1.86.** **(a) IDENTIFY:** Prove that $\vec{A}\cdot(\vec{B}\times\vec{C}) = (\vec{A}\times\vec{B})\cdot\vec{C}$.

**SET UP:** Express the scalar and vector products in terms of components.

**EXECUTE:**

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = A_x(\vec{B}\times\vec{C})_x + A_y(\vec{B}\times\vec{C})_y + A_z(\vec{B}\times\vec{C})_z$$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = A_x(B_yC_z - B_zC_y) + A_y(B_zC_x - B_xC_z) + A_z(B_xC_y - B_yC_x)$$

$$(\vec{A}\times\vec{B})\cdot\vec{C} = (\vec{A}\times\vec{B})_xC_x + (\vec{A}\times\vec{B})_yC_y + (\vec{A}\times\vec{B})_zC_z$$

$$(\vec{A}\times\vec{B})\cdot\vec{C} = (A_yB_z - A_zB_y)C_x + (A_zB_x - A_xB_z)C_y + (A_xB_y - A_yB_x)C_z$$

Comparison of the expressions for $\vec{A}\cdot(\vec{B}\times\vec{C})$ and $(\vec{A}\times\vec{B})\cdot\vec{C}$ shows they contain the same terms, so $\vec{A}\cdot(\vec{B}\times\vec{C}) = (\vec{A}\times\vec{B})\cdot\vec{C}$.

**(b) IDENTIFY:** Calculate $(\vec{A}\times\vec{B})\cdot\vec{C}$, given the magnitude and direction of $\vec{A}$, $\vec{B}$, and $\vec{C}$.

**SET UP:** Use $|\vec{A}\times\vec{B}| = AB\sin\phi$ to find the magnitude and direction of $\vec{A}\times\vec{B}$. Then we know the components of $\vec{A}\times\vec{B}$ and of $\vec{C}$ and can use an expression like $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$ to find the scalar product in terms of components.

**EXECUTE:** $A = 5.00;\ \theta_A = 26.0°;\ B = 4.00,\ \theta_B = 63.0°$

$|\vec{A}\times\vec{B}| = AB\sin\phi.$

The angle $\phi$ between $\vec{A}$ and $\vec{B}$ is equal to $\phi = \theta_B - \theta_A = 63.0° - 26.0° = 37.0°$. So $|\vec{A}\times\vec{B}| = (5.00)(4.00)\sin 37.0° = 12.04$, and by the right hand-rule $\vec{A}\times\vec{B}$ is in the $+z$-direction. Thus $(\vec{A}\times\vec{B})\cdot\vec{C} = (12.04)(6.00) = 72.2$

**EVALUATE:** $\vec{A}\times\vec{B}$ is a vector, so taking its scalar product with $\vec{C}$ is a legitimate vector operation. $(\vec{A}\times\vec{B})\cdot\vec{C}$ is a scalar product between two vectors so the result is a scalar.

**1.87.** **IDENTIFY:** Express all the densities in the same units to make a comparison.

**SET UP:** Density $\rho$ is mass divided by volume. Use the numbers given in the table in the problem and convert all the densities to kg/m$^3$.

**EXECUTE:** Sample A: $\rho_A = \dfrac{8.00\text{ g}\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)}{1.67\times 10^{-6}\text{ m}^3} = 4790\text{ kg/m}^3$

Sample B: $\rho_B = \dfrac{6.00\times 10^{-6}\text{ g}\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)}{9.38\times 10^{6}\ \mu\text{m}^3\left(\dfrac{10^{-6}\text{ m}}{1\ \mu\text{m}}\right)^3} = 640\text{ kg/m}^3$

Sample C: $\rho_C = \dfrac{8.00\times 10^{-3}\text{ g}\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)}{2.50\times 10^{-3}\text{ cm}^3\left(\dfrac{1\text{ m}}{100\text{ cm}}\right)^3} = 3200\text{ kg/m}^3$

Sample D: $\rho_D = \dfrac{9.00\times 10^{-4}\text{ kg}}{2.81\times 10^{3}\text{ mm}^3\left(\dfrac{1\text{ m}}{1000\text{ mm}}\right)^3} = 320\text{ kg/m}^3$

Sample E: $\rho_E = \dfrac{9.00\times 10^{4}\text{ ng}\left(\dfrac{1\text{ g}}{10^9\text{ ng}}\right)\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)}{1.41\times 10^{-2}\text{ mm}^3\left(\dfrac{1\text{ m}}{1000\text{ mm}}\right)^3} = 6380\text{ kg/m}^3$

Sample F: $\rho_F = \dfrac{6.00\times 10^{-5}\text{ g}\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)}{1.25\times 10^{8}\ \mu\text{m}^3\left(\dfrac{1\text{ m}}{10^6\ \mu\text{m}}\right)^3} = 480\text{ kg/m}^3$

**EVALUATE:** In order of increasing density, the samples are D, F, B, C, A, E.

**1.88.** **IDENTIFY:** We know the magnitude of the resultant of two vectors at four known angles between them, and we want to find out the magnitude of each of these two vectors.

**SET UP:** Use the information in the table in the problem for $\theta = 0.0°$ and $90.0°$. Call $A$ and $B$ the magnitudes of the vectors.

**EXECUTE:** **(a)** At 0°: The vectors point in the same direction, so $A + B = 8.00$ N.

At 90.0°: The vectors are perpendicular to each other, so $A^2 + B^2 = R^2 = (5.83\text{ N})^2 = 33.99\text{ N}^2$.

Solving these two equations simultaneously gives

$B = 8.00\text{ N} - A$

$A^2 + (8.00\text{ N} - A)^2 = 33.99\text{ N}^2$

$A^2 + 64.00\ \text{N}^2 - 16.00\ \text{N}\ A + A^2 = 33.99\ \text{N}^2$

The quadratic formula gives two solutions: $A = 5.00$ N and $B = 3.00$ N or $A = 3.00$ N and $B = 5.00$ N. In either case, the larger force has magnitude 5.00 N.

**(b)** Let $A = 5.00$ N and $B = 3.00$ N, with the larger vector along the $x$-axis and the smaller one making an angle of $+30.0°$ with the $+x$-axis in the first quadrant. The components of the resultant are

$R_x = A_x + B_x = 5.00\ \text{N} + (3.00\ \text{N})(\cos 30.0°) = 7.598\ \text{N}$

$R_y = A_y + B_y = 0 + (3.00\ \text{N})(\sin 30.0°) = 1.500\ \text{N}$

$$R = \sqrt{R_x^2 + R_y^2} = 7.74\ \text{N}$$

**EVALUATE:** To check our answer, we could use the other resultants and angles given in the table with the problem.

**1.89.** **IDENTIFY:** Use the $x$ and $y$ coordinates for each object to find the vector from one object to the other; the distance between two objects is the magnitude of this vector. Use the scalar product to find the angle between two vectors.

**SET UP:** If object $A$ has coordinates $(x_A, y_A)$ and object $B$ has coordinates $(x_B, y_B)$, the vector $\vec{r}_{AB}$ from $A$ to $B$ has $x$-component $x_B - x_A$ and $y$-component $y_B - y_A$.

**EXECUTE:** **(a)** The diagram is sketched in Figure 1.89.

**(b)** (i) In AU, $\sqrt{(0.3182)^2 + (0.9329)^2} = 0.9857.$

(ii) In AU, $\sqrt{(1.3087)^2 + (-0.4423)^2 + (-0.0414)^2} = 1.3820.$

(iii) In AU, $\sqrt{(0.3182 - 1.3087)^2 + (0.9329 - (-0.4423))^2 + (0.0414)^2} = 1.695.$

**(c)** The angle between the directions from the Earth to the Sun and to Mars is obtained from the dot product. Combining Eqs. (1.16) and (1.19),

$$\phi = \arccos\left(\frac{(-0.3182)(1.3087 - 0.3182) + (-0.9329)(-0.4423 - 0.9329) + (0)}{(0.9857)(1.695)}\right) = 54.6°.$$

**(d)** Mars could not have been visible at midnight, because the Sun-Mars angle is less than $90°$.

**EVALUATE:** Our calculations correctly give that Mars is farther from the Sun than the earth is. Note that on this date Mars was farther from the earth than it is from the Sun.

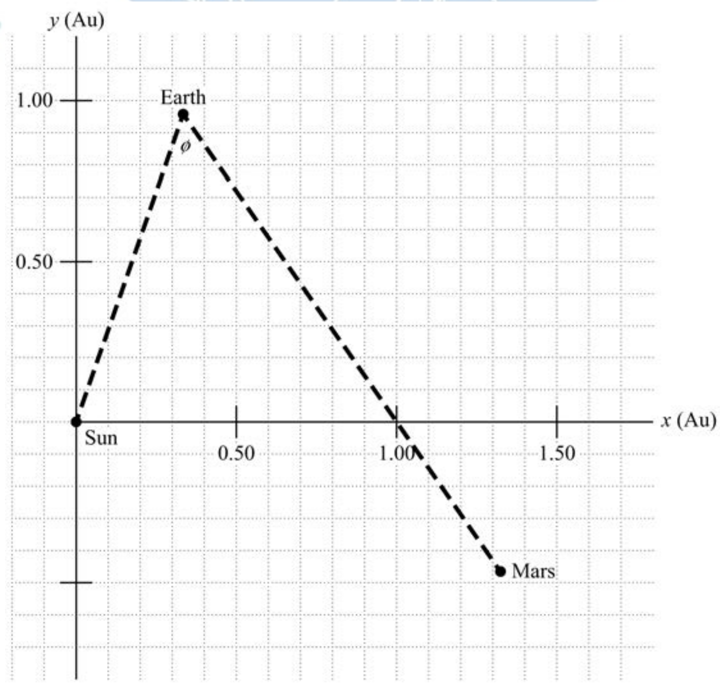



**Figure 1.89**

**1.90.** **IDENTIFY:** Add the vector displacements of the receiver and then find the vector from the quarterback to the receiver.

**SET UP:** Add the $x$-components and the $y$-components.

**EXECUTE:** The receiver's position is

$[(+1.0+9.0-6.0+12.0)\text{yd}]\hat{\boldsymbol{i}}+[(-5.0+11.0+4.0+18.0)\text{ yd}]\hat{\boldsymbol{j}}=(16.0\text{ yd})\hat{\boldsymbol{i}}+(28.0\text{ yd})\hat{\boldsymbol{j}}$.

The vector from the quarterback to the receiver is the receiver's position minus the quarterback's position, or $(16.0\text{ yd})\hat{\boldsymbol{i}}+(35.0\text{ yd})\hat{\boldsymbol{j}}$, a vector with magnitude $\sqrt{(16.0\text{ yd})^2+(35.0\text{ yd})^2}=38.5\text{ yd}$. The angle is $\arctan\left(\dfrac{16.0}{35.0}\right)=24.6°$ to the right of downfield.

**EVALUATE:** The vector from the quarterback to receiver has positive $x$-component and positive $y$-component.

**1.91.** **IDENTIFY:** Draw the vector addition diagram for the position vectors.

**SET UP:** Use coordinates in which the Sun to Merak line lies along the $x$-axis. Let $\vec{A}$ be the position vector of Alkaid relative to the Sun, $\vec{M}$ is the position vector of Merak relative to the Sun, and $\vec{R}$ is the position vector for Alkaid relative to Merak. $A=138$ ly and $M=77$ ly.

**EXECUTE:** The relative positions are shown in Figure 1.91. $\vec{M}+\vec{R}=\vec{A}$. $A_x=M_x+R_x$ so $R_x=A_x-M_x=(138\text{ ly})\cos 25.6°-77\text{ ly}=47.5\text{ ly}$. $R_y=A_y-M_y=(138\text{ ly})\sin 25.6°-0=59.6\text{ ly}$. $R=76.2$ ly is the distance between Alkaid and Merak.

**(b)** The angle is angle $\phi$ in Figure 1.91. $\cos\theta=\dfrac{R_x}{R}=\dfrac{47.5\text{ ly}}{76.2\text{ ly}}$ and $\theta=51.4°$. Then $\phi=180°-\theta=129°$.

**EVALUATE:** The concepts of vector addition and components make these calculations very simple.

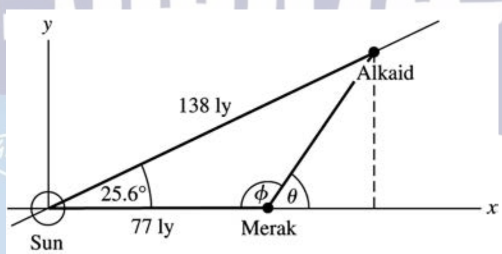



**Figure 1.91**

**1.92.** **IDENTIFY:** The total volume of the gas-exchanging region of the lungs must be at least as great as the total volume of all the alveoli, which is the product of the volume per alveoli times the number of alveoli.

**SET UP:** $V = NV_{alv}$, and we use the numbers given in the introduction to the problem.

**EXECUTE:** $V = NV_{alv} = (480\times 10^6)(4.2\times 10^6\ \mu\text{m}^3) = 2.02\times 10^{15}\ \mu\text{m}^3$. Converting to liters gives

$$V = 2.02\times10^{15}\ \text{m}^3\left(\frac{1\text{ m}}{10^6\ \mu\text{m}}\right)^3 = 2.02\text{ L}\approx 2.0\text{ L}.$$ Therefore choice (c) is correct.

**EVALUATE:** A volume of 2 L is reasonable for the lungs.

**1.93.** **IDENTIFY:** We know the volume and want to find the diameter of a typical alveolus, assuming it to be a sphere.

**SET UP:** The volume of a sphere of radius $r$ is $V = 4/3\ \pi r^3$ and its diameter is $D = 2r$.

**EXECUTE:** Solving for the radius in terms of the volume gives $r = (3V/4\pi)^{1/3}$, so the diameter is

$$D = 2r = 2(3V/4\pi)^{1/3} = 2\left[\frac{3\left(4.2\times10^6\ \mu\text{m}^3\right)}{4\pi}\right]^{1/3} = 200\ \mu\text{m}.$$ Converting to mm gives

$D = (200\ \mu\text{m})[(1\text{ mm})/(1000\ \mu\text{m})] = 0.20$ mm, so choice (a) is correct.

**EVALUATE:** A sphere that is 0.20 mm in diameter should be visible to the naked eye for someone with good eyesight.

**1.94.** **IDENTIFY:** Draw conclusions from a given graph.

**SET UP:** The dots lie more-or-less along a horizontal line, which means that the average alveolar volume does not vary significantly as the lung volume increases.

**EXECUTE:** The volume of individual alveoli does not vary (as stated in the introduction). The graph shows that the volume occupied by alveoli stays constant for higher and higher lung volumes, so there must be more of them, which makes choice (c) the correct one.

**EVALUATE:** It is reasonable that a large lung would need more alveoli than a small lung because a large lung probably belongs to a larger person than a small lung.

# 2

# MOTION ALONG A STRAIGHT LINE

**2.1. IDENTIFY:** $\Delta x = v_{\text{av-}x}\Delta t$

**SET UP:** We know the average velocity is 6.25 m/s.

**EXECUTE:** $\Delta x = v_{\text{av-}x}\Delta t = 25.0\text{ m}$

**EVALUATE:** In round numbers, 6 m/s × 4 s = 24 m ≈ 25 m, so the answer is reasonable.

**2.2. IDENTIFY:** $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$

**SET UP:** $13.5\text{ days} = 1.166\times10^6\text{ s}$. At the release point, $x = +5.150\times10^6\text{ m}$.

**EXECUTE:** **(a)** $v_{\text{av-}x} = \dfrac{x_2 - x_1}{\Delta t} = \dfrac{-5.150\times10^6\text{ m}}{1.166\times10^6\text{ s}} = -4.42\text{ m/s}$.

**(b)** For the round trip, $x_2 = x_1$ and $\Delta x = 0$. The average velocity is zero.

**EVALUATE:** The average velocity for the trip from the nest to the release point is positive.

**2.3. IDENTIFY:** Target variable is the time $\Delta t$ it takes to make the trip in heavy traffic. Use Eq. (2.2) that relates the average velocity to the displacement and average time.

**SET UP:** $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$ so $\Delta x = v_{\text{av-}x}\Delta t$ and $\Delta t = \dfrac{\Delta x}{v_{\text{av-}x}}$.

**EXECUTE:** Use the information given for normal driving conditions to calculate the distance between the two cities, where the time is 1 h and 50 min, which is 110 min:

$$\Delta x = v_{\text{av-}x}\Delta t = (105\text{ km/h})(1\text{ h}/60\text{ min})(110\text{ min}) = 192.5\text{ km}.$$

Now use $v_{\text{av-}x}$ for heavy traffic to calculate $\Delta t$; $\Delta x$ is the same as before:

$$\Delta t = \frac{\Delta x}{v_{\text{av-}x}} = \frac{192.5\text{ km}}{70\text{ km/h}} = 2.75\text{ h} = 2\text{ h and 45 min}.$$

The additional time is (2 h and 45 min) – (1 h and 50 min) = (1 h and 105 min) – (1 h and 50 min) = 55 min.

**EVALUATE:** At the normal speed of 105 km/s the trip takes 110 min, but at the reduced speed of 70 km/h it takes 165 min. So decreasing your average speed by about 30% adds 55 min to the time, which is 50% of 110 min. Thus a 30% reduction in speed leads to a 50% increase in travel time. This result (perhaps surprising) occurs because the time interval is inversely proportional to the average speed, not directly proportional to it.

**2.4. IDENTIFY:** The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$. Use the average speed for each segment to find the time traveled in that segment. The average speed is the distance traveled divided by the time.

**SET UP:** The post is 80 m west of the pillar. The total distance traveled is $200\text{ m} + 280\text{ m} = 480\text{ m}$.

**EXECUTE:** **(a)** The eastward run takes time $\dfrac{200\text{ m}}{5.0\text{ m/s}} = 40.0\text{ s}$ and the westward run takes $\dfrac{280\text{ m}}{4.0\text{ m/s}} = 70.0\text{ s}$. The average speed for the entire trip is $\dfrac{480\text{ m}}{110.0\text{ s}} = 4.4\text{ m/s}$.

**(b)** $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{-80\text{ m}}{110.0\text{ s}} = -0.73\text{ m/s}$. The average velocity is directed westward.

**EVALUATE:** The displacement is much less than the distance traveled, and the magnitude of the average velocity is much less than the average speed. The average speed for the entire trip has a value that lies between the average speed for the two segments.

**2.5.** **IDENTIFY:** Given two displacements, we want the average velocity and the average speed.

**SET UP:** The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$ and the average speed is just the total distance walked divided by the total time to walk this distance.

**EXECUTE:** **(a)** Let $+x$ be east. $\Delta x = 60.0\text{ m} - 40.0\text{ m} = 20.0\text{ m}$ and $\Delta t = 28.0\text{ s} + 36.0\text{ s} = 64.0\text{ s}$. So $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{20.0\text{ m}}{64.0\text{ s}} = 0.312\text{ m/s}$.

**(b)** average speed $= \dfrac{60.0\text{ m} + 40.0\text{ m}}{64.0\text{ s}} = 1.56\text{ m/s}$

**EVALUATE:** The average speed is much greater than the average velocity because the total distance walked is much greater than the magnitude of the displacement vector.

**2.6.** **IDENTIFY:** The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$. Use $x(t)$ to find $x$ for each $t$.

**SET UP:** $x(0) = 0$, $x(2.00\text{ s}) = 5.60\text{ m}$, and $x(4.00\text{ s}) = 20.8\text{ m}$

**EXECUTE:** **(a)** $v_{\text{av-}x} = \dfrac{5.60\text{ m} - 0}{2.00\text{ s}} = +2.80\text{ m/s}$

**(b)** $v_{\text{av-}x} = \dfrac{20.8\text{ m} - 0}{4.00\text{ s}} = +5.20\text{ m/s}$

**(c)** $v_{\text{av-}x} = \dfrac{20.8\text{ m} - 5.60\text{ m}}{2.00\text{ s}} = +7.60\text{ m/s}$

**EVALUATE:** The average velocity depends on the time interval being considered.

**2.7.** **(a) IDENTIFY:** Calculate the average velocity using $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$.

**SET UP:** $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$ so use $x(t)$ to find the displacement $\Delta x$ for this time interval.

**EXECUTE:** $t = 0$: $x = 0$

$t = 10.0\text{ s}$: $x = (2.40\text{ m/s}^2)(10.0\text{ s})^2 - (0.120\text{ m/s}^3)(10.0\text{ s})^3 = 240\text{ m} - 120\text{ m} = 120\text{ m}$.

Then $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{120\text{ m}}{10.0\text{ s}} = 12.0\text{ m/s}$.

**(b) IDENTIFY:** Use $v_x = \dfrac{dx}{dt}$ to calculate $v_x(t)$ and evaluate this expression at each specified $t$.

**SET UP:** $v_x = \dfrac{dx}{dt} = 2bt - 3ct^2$.

**EXECUTE:** (i) $t = 0$: $v_x = 0$

(ii) $t = 5.0\text{ s}$: $v_x = 2(2.40\text{ m/s}^2)(5.0\text{ s}) - 3(0.120\text{ m/s}^3)(5.0\text{ s})^2 = 24.0\text{ m/s} - 9.0\text{ m/s} = 15.0\text{ m/s}$.

(iii) $t = 10.0\text{ s}$: $v_x = 2(2.40\text{ m/s}^2)(10.0\text{ s}) - 3(0.120\text{ m/s}^3)(10.0\text{ s})^2 = 48.0\text{ m/s} - 36.0\text{ m/s} = 12.0\text{ m/s}$.

**(c) IDENTIFY:** Find the value of $t$ when $v_x(t)$ from part (b) is zero.

**SET UP:** $v_x = 2bt - 3ct^2$

$v_x = 0$ at $t = 0$.

$v_x = 0$ next when $2bt - 3ct^2 = 0$

**EXECUTE:** $2b = 3ct$ so $t = \dfrac{2b}{3c} = \dfrac{2(2.40\text{ m/s}^2)}{3(0.120\text{ m/s}^3)} = 13.3\text{ s}$

**EVALUATE:** $v_x(t)$ for this motion says the car starts from rest, speeds up, and then slows down again.

**2.8.** **IDENTIFY:** We know the position $x(t)$ of the bird as a function of time and want to find its instantaneous velocity at a particular time.

**SET UP:** The instantaneous velocity is $v_x(t) = \dfrac{dx}{dt} = \dfrac{d\left[28.0\text{ m} + (12.4\text{ m/s})t - (0.0450\text{ m/s}^3)t^3\right]}{dt}$.

**EXECUTE:** $v_x(t) = \dfrac{dx}{dt} = 12.4\text{ m/s} - (0.135\text{ m/s}^3)t^2$. Evaluating this at $t = 8.0\text{ s}$ gives $v_x = 3.76\text{ m/s}$.

**EVALUATE:** The acceleration is not constant in this case.

**2.9.** **IDENTIFY:** The average velocity is given by $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$. We can find the displacement $\Delta t$ for each constant velocity time interval. The average speed is the distance traveled divided by the time.

**SET UP:** For $t = 0$ to $t = 2.0\text{ s}$, $v_x = 2.0\text{ m/s}$. For $t = 2.0\text{ s}$ to $t = 3.0\text{ s}$, $v_x = 3.0\text{ m/s}$. In part (b), $v_x = -3.0\text{ m/s}$ for $t = 2.0\text{ s}$ to $t = 3.0\text{ s}$. When the velocity is constant, $\Delta x = v_x \Delta t$.

**EXECUTE:** **(a)** For $t = 0$ to $t = 2.0\text{ s}$, $\Delta x = (2.0\text{ m/s})(2.0\text{ s}) = 4.0\text{ m}$. For $t = 2.0\text{ s}$ to $t = 3.0\text{ s}$, $\Delta x = (3.0\text{ m/s})(1.0\text{ s}) = 3.0\text{ m}$. For the first 3.0 s, $\Delta x = 4.0\text{ m} + 3.0\text{ m} = 7.0\text{ m}$. The distance traveled is also 7.0 m. The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{7.0\text{ m}}{3.0\text{ s}} = 2.33\text{ m/s}$. The average speed is also 2.33 m/s.

**(b)** For $t = 2.0$ s to 3.0 s, $\Delta x = (-3.0\text{ m/s})(1.0\text{ s}) = -3.0\text{ m}$. For the first 3.0 s, $\Delta x = 4.0\text{ m} + (-3.0\text{ m}) = +1.0\text{ m}$. The ball travels 4.0 m in the $+x$-direction and then 3.0 m in the $-x$-direction, so the distance traveled is still 7.0 m. $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{1.0\text{ m}}{3.0\text{ s}} = 0.33\text{ m/s}$. The average speed is $\dfrac{7.00\text{ m}}{3.00\text{ s}} = 2.33\text{ m/s}$.

**EVALUATE:** When the motion is always in the same direction, the displacement and the distance traveled are equal and the average velocity has the same magnitude as the average speed. When the motion changes direction during the time interval, those quantities are different.

**2.10.** **IDENTIFY** and **SET UP:** The instantaneous velocity is the slope of the tangent to the $x$ versus $t$ graph.

**EXECUTE:** **(a)** The velocity is zero where the graph is horizontal; point IV.

**(b)** The velocity is constant and positive where the graph is a straight line with positive slope; point I.

**(c)** The velocity is constant and negative where the graph is a straight line with negative slope; point V.

**(d)** The slope is positive and increasing at point II.

**(e)** The slope is positive and decreasing at point III.

**EVALUATE:** The sign of the velocity indicates its direction.

**2.11.** **IDENTIFY:** Find the instantaneous velocity of a car using a graph of its position as a function of time.

**SET UP:** The instantaneous velocity at any point is the slope of the $x$ versus $t$ graph at that point. Estimate the slope from the graph.

**EXECUTE:** $A$: $v_x = 6.7\text{ m/s}$; $B$: $v_x = 6.7\text{ m/s}$; $C$: $v_x = 0$; $D$: $v_x = -40.0\text{ m/s}$; $E$: $v_x = -40.0\text{ m/s}$; $F$: $v_x = -40.0\text{ m/s}$; $G$: $v_x = 0$.

**EVALUATE:** The sign of $v_x$ shows the direction the car is moving. $v_x$ is constant when $x$ versus $t$ is a straight line.

**2.12.** **IDENTIFY:** $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$. $a_x(t)$ is the slope of the $v_x$ versus $t$ graph.

**SET UP:** $60\text{ km/h} = 16.7\text{ m/s}$

**EXECUTE:** **(a)** (i) $a_{\text{av-}x} = \dfrac{16.7\text{ m/s} - 0}{10\text{ s}} = 1.7\text{ m/s}^2$. (ii) $a_{\text{av-}x} = \dfrac{0 - 16.7\text{ m/s}}{10\text{ s}} = -1.7\text{ m/s}^2$.

(iii) $\Delta v_x = 0$ and $a_{\text{av-}x} = 0$. (iv) $\Delta v_x = 0$ and $a_{\text{av-}x} = 0$.

**(b)** At $t = 20\text{ s}$, $v_x$ is constant and $a_x = 0$. At $t = 35\text{ s}$, the graph of $v_x$ versus $t$ is a straight line and $a_x = a_{\text{av-}x} = -1.7\text{ m/s}^2$.

**EVALUATE:** When $a_{\text{av-}x}$ and $v_x$ have the same sign the speed is increasing. When they have opposite signs, the speed is decreasing.

**2.13. IDENTIFY:** The average acceleration for a time interval $\Delta t$ is given by $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$.

**SET UP:** Assume the car is moving in the $+x$ direction. $1 \text{ mi/h} = 0.447 \text{ m/s}$, so $60 \text{ mi/h} = 26.82 \text{ m/s}$, $200 \text{ mi/h} = 89.40 \text{ m/s}$ and $253 \text{ mi/h} = 113.1 \text{ m/s}$.

**EXECUTE:** **(a)** The graph of $v_x$ versus $t$ is sketched in Figure 2.13. The graph is not a straight line, so the acceleration is not constant.

**(b)** (i) $a_{\text{av-}x} = \dfrac{26.82 \text{ m/s} - 0}{2.1 \text{ s}} = 12.8 \text{ m/s}^2$ (ii) $a_{\text{av-}x} = \dfrac{89.40 \text{ m/s} - 26.82 \text{ m/s}}{20.0 \text{ s} - 2.1 \text{ s}} = 3.50 \text{ m/s}^2$

(iii) $a_{\text{av-}x} = \dfrac{113.1 \text{ m/s} - 89.40 \text{ m/s}}{53 \text{ s} - 20.0 \text{ s}} = 0.718 \text{ m/s}^2$. The slope of the graph of $v_x$ versus $t$ decreases as $t$ increases. This is consistent with an average acceleration that decreases in magnitude during each successive time interval.

**EVALUATE:** The average acceleration depends on the chosen time interval. For the interval between 0 and 53 s, $a_{\text{av-}x} = \dfrac{113.1 \text{ m/s} - 0}{53 \text{ s}} = 2.13 \text{ m/s}^2$.

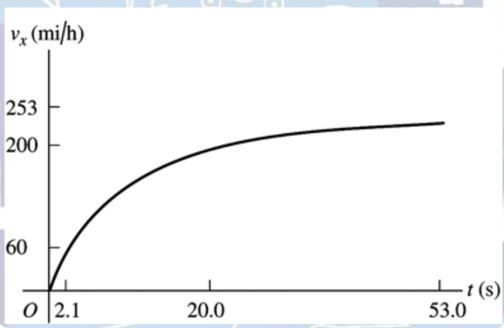



**Figure 2.13**

**2.14. IDENTIFY:** We know the velocity $v(t)$ of the car as a function of time and want to find its acceleration at the instant that its velocity is 12.0 m/s.

**SET UP:** We know that $v_x(t) = (0.860 \text{ m/s}^3)t^2$ and that $a_x(t) = \dfrac{dv_x}{dt} = \dfrac{d\left[(0.860 \text{ m/s}^3)t^2\right]}{dt}$.

**EXECUTE:** $a_x(t) = \dfrac{dv_x}{dt} = (1.72 \text{ m/s}^3)t$. When $v_x = 12.0 \text{ m/s}$, $(0.860 \text{ m/s}^3)t^2 = 12.0 \text{ m/s}$, which gives $t = 3.735 \text{ s}$. At this time, $a_x = 6.42 \text{ m/s}^2$.

**EVALUATE:** The acceleration of this car is not constant.

**2.15. IDENTIFY and SET UP:** Use $v_x = \dfrac{dx}{dt}$ and $a_x = \dfrac{dv_x}{dt}$ to calculate $v_x(t)$ and $a_x(t)$.

**EXECUTE:** $v_x = \dfrac{dx}{dt} = 2.00 \text{ cm/s} - (0.125 \text{ cm/s}^2)t$

$a_x = \dfrac{dv_x}{dt} = -0.125 \text{ cm/s}^2$

**(a)** At $t = 0$, $x = 50.0 \text{ cm}$, $v_x = 2.00 \text{ cm/s}$, $a_x = -0.125 \text{ cm/s}^2$.

**(b)** Set $v_x = 0$ and solve for $t$: $t = 16.0 \text{ s}$.

**(c)** Set $x = 50.0 \text{ cm}$ and solve for $t$. This gives $t = 0$ and $t = 32.0 \text{ s}$. The turtle returns to the starting point after 32.0 s.

**(d)** The turtle is 10.0 cm from starting point when $x = 60.0 \text{ cm}$ or $x = 40.0 \text{ cm}$.

Set $x = 60.0$ cm and solve for $t$: $t = 6.20$ s and $t = 25.8$ s.
At $t = 6.20$ s, $v_x = +1.23$ cm/s.
At $t = 25.8$ s, $v_x = -1.23$ cm/s.
Set $x = 40.0$ cm and solve for $t$: $t = 36.4$ s (other root to the quadratic equation is negative and hence nonphysical).
At $t = 36.4$ s, $v_x = -2.55$ cm/s.
**(e)** The graphs are sketched in Figure 2.15.

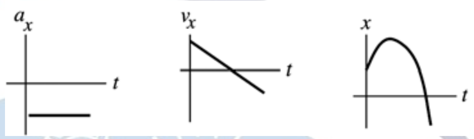



**Figure 2.15**

**EVALUATE:** The acceleration is constant and negative. $v_x$ is linear in time. It is initially positive, decreases to zero, and then becomes negative with increasing magnitude. The turtle initially moves farther away from the origin but then stops and moves in the $-x$-direction.

**2.16.** **IDENTIFY:** Use $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$, with $\Delta t = 10$ s in all cases.

**SET UP:** $v_x$ is negative if the motion is to the left.

**EXECUTE:** **(a)** $[(5.0 \text{ m/s}) - (15.0 \text{ m/s})]/(10 \text{ s}) = -1.0 \text{ m/s}^2$

**(b)** $[(-15.0 \text{ m/s}) - (-5.0 \text{ m/s})]/(10 \text{ s}) = -1.0 \text{ m/s}^2$

**(c)** $[(-15.0 \text{ m/s}) - (+15.0 \text{ m/s})]/(10 \text{ s}) = -3.0 \text{ m/s}^2$

**EVALUATE:** In all cases, the negative acceleration indicates an acceleration to the left.

**2.17.** **IDENTIFY:** The average acceleration is $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$. Use $v_x(t)$ to find $v_x$ at each $t$. The instantaneous acceleration is $a_x = \dfrac{dv_x}{dt}$.

**SET UP:** $v_x(0) = 3.00$ m/s and $v_x(5.00 \text{ s}) = 5.50$ m/s.

**EXECUTE:** **(a)** $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t} = \dfrac{5.50 \text{ m/s} - 3.00 \text{ m/s}}{5.00 \text{ s}} = 0.500 \text{ m/s}^2$

**(b)** $a_x = \dfrac{dv_x}{dt} = (0.100 \text{ m/s}^3)(2t) = (0.200 \text{ m/s}^3)t$. At $t = 0$, $a_x = 0$. At $t = 5.00$ s, $a_x = 1.00 \text{ m/s}^2$.

**(c)** Graphs of $v_x(t)$ and $a_x(t)$ are given in Figure 2.17 (next page).

**EVALUATE:** $a_x(t)$ is the slope of $v_x(t)$ and increases as $t$ increases. The average acceleration for $t = 0$ to $t = 5.00$ s equals the instantaneous acceleration at the midpoint of the time interval, $t = 2.50$ s, since $a_x(t)$ is a linear function of $t$.

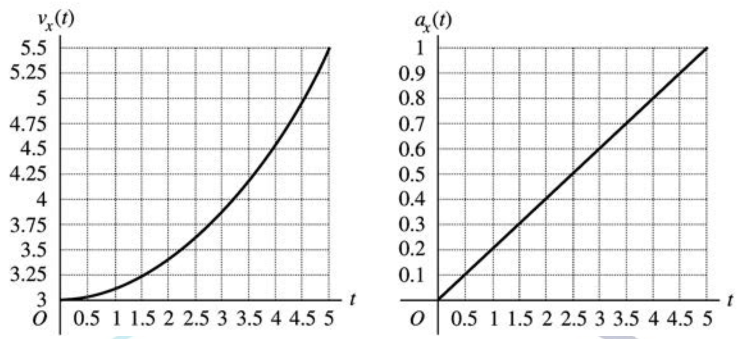



**Figure 2.17**

**2.18.** **IDENTIFY:** $v_x(t) = \dfrac{dx}{dt}$ and $a_x(t) = \dfrac{dv_x}{dt}$

**SET UP:** $\dfrac{d}{dt}(t^n) = nt^{n-1}$ for $n \geq 1$.

**EXECUTE:** **(a)** $v_x(t) = (9.60\ \text{m/s}^2)t - (0.600\ \text{m/s}^6)t^5$ and $a_x(t) = 9.60\ \text{m/s}^2 - (3.00\ \text{m/s}^6)t^4$. Setting $v_x = 0$ gives $t = 0$ and $t = 2.00$ s. At $t = 0$, $x = 2.17$ m and $a_x = 9.60\ \text{m/s}^2$. At $t = 2.00$ s, $x = 15.0$ m and $a_x = -38.4\ \text{m/s}^2$.

**(b)** The graphs are given in Figure 2.18.

**EVALUATE:** For the entire time interval from $t = 0$ to $t = 2.00$ s, the velocity $v_x$ is positive and $x$ increases. While $a_x$ is also positive the speed increases and while $a_x$ is negative the speed decreases.

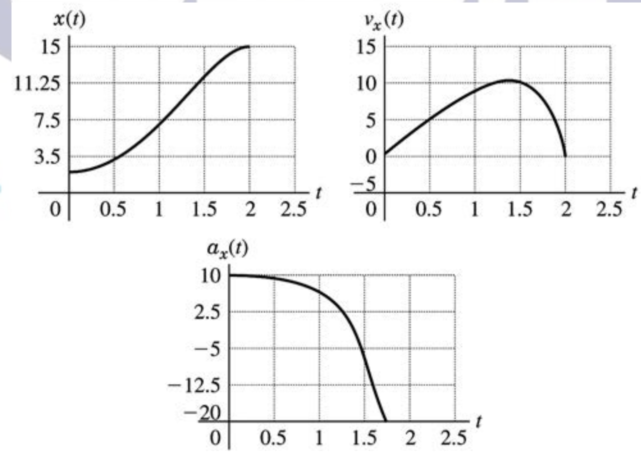
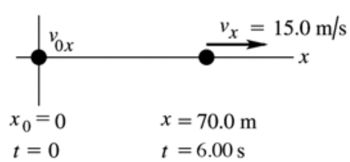



**Figure 2.18**

**2.19.** **IDENTIFY:** Use the constant acceleration equations to find $v_{0x}$ and $a_x$.

**(a) SET UP:** The situation is sketched in Figure 2.19.




$x - x_0 = 70.0$ m
$t = 6.00$ s
$v_x = 15.0$ m/s
$v_{0x} = ?$

**Figure 2.19**

**EXECUTE:** Use $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t$, so $v_{0x} = \frac{2(x - x_0)}{t} - v_x = \frac{2(70.0\text{ m})}{6.00\text{ s}} - 15.0\text{ m/s} = 8.33\text{ m/s}.$

**(b)** Use $v_x = v_{0x} + a_x t$, so $a_x = \frac{v_x - v_{0x}}{t} = \frac{15.0\text{ m/s} - 5.0\text{ m/s}}{6.00\text{ s}} = 1.11\text{ m/s}^2.$

**EVALUATE:** The average velocity is $(70.0\text{ m})/(6.00\text{ s}) = 11.7\text{ m/s}.$ The final velocity is larger than this, so the antelope must be speeding up during the time interval; $v_{0x} < v_x$ and $a_x > 0.$

**2.20.** **IDENTIFY:** In (a) find the time to reach the speed of sound with an acceleration of 5*g*, and in (b) find his speed at the end of 5.0 s if he has an acceleration of 5*g*.

**SET UP:** Let $+x$ be in his direction of motion and assume constant acceleration of 5*g* so the standard kinematics equations apply so $v_x = v_{0x} + a_x t.$ **(a)** $v_x = 3(331\text{ m/s}) = 993\text{ m/s},$ $v_{0x} = 0,$ and $a_x = 5g = 49.0\text{ m/s}^2.$ **(b)** $t = 5.0\text{ s}$

**EXECUTE:** **(a)** $v_x = v_{0x} + a_x t$ and $t = \frac{v_x - v_{0x}}{a_x} = \frac{993\text{ m/s} - 0}{49.0\text{ m/s}^2} = 20.3\text{ s}.$ Yes, the time required is larger than 5.0 s.

**(b)** $v_x = v_{0x} + a_x t = 0 + (49.0\text{ m/s}^2)(5.0\text{ s}) = 245\text{ m/s}.$

**EVALUATE:** In 5.0 s he can only reach about 2/3 the speed of sound without blacking out.

**2.21.** **IDENTIFY:** For constant acceleration, the standard kinematics equations apply.

**SET UP:** Assume the ball starts from rest and moves in the $+x$-direction.

**EXECUTE:** **(a)** $x - x_0 = 1.50\text{ m},$ $v_x = 45.0\text{ m/s}$ and $v_{0x} = 0.$ $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(45.0\text{ m/s})^2}{2(1.50\text{ m})} = 675\text{ m/s}^2.$$

**(b)** $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t$ gives $t = \frac{2(x - x_0)}{v_{0x} + v_x} = \frac{2(1.50\text{ m})}{45.0\text{ m/s}} = 0.0667\text{ s}$

**EVALUATE:** We could also use $v_x = v_{0x} + a_x t$ to find $t = \frac{v_x}{a_x} = \frac{45.0\text{ m/s}}{675\text{ m/s}^2} = 0.0667\text{ s}$ which agrees with our previous result. The acceleration of the ball is very large.

**2.22.** **IDENTIFY:** For constant acceleration, the standard kinematics equations apply.

**SET UP:** Assume the ball moves in the $+x$ direction.

**EXECUTE:** **(a)** $v_x = 73.14\text{ m/s},$ $v_{0x} = 0$ and $t = 30.0\text{ ms}.$ $v_x = v_{0x} + a_x t$ gives

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{73.14\text{ m/s} - 0}{30.0\times 10^{-3}\text{ s}} = 2440\text{ m/s}^2.$$

**(b)** $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t = \left(\frac{0 + 73.14\text{ m/s}}{2}\right)(30.0\times 10^{-3}\text{ s}) = 1.10\text{ m}.$

**EVALUATE:** We could also use $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ to calculate $x - x_0$:

$x - x_0 = \frac{1}{2}(2440\text{ m/s}^2)(30.0\times 10^{-3}\text{ s})^2 = 1.10\text{ m},$ which agrees with our previous result. The acceleration of the ball is very large.

**2.23.** **IDENTIFY:** Assume that the acceleration is constant and apply the constant acceleration kinematic equations. Set $|a_x|$ equal to its maximum allowed value.

**SET UP:** Let $+x$ be the direction of the initial velocity of the car. $a_x = -250\text{ m/s}^2.$

$105\text{ km/h} = 29.17\text{ m/s}.$

**EXECUTE:** $v_{0x} = 29.17\text{ m/s}.$ $v_x = 0.$ $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$x - x_0 = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0 - (29.17\text{ m/s})^2}{2(-250\text{ m/s}^2)} = 1.70\text{ m}.$$

**EVALUATE:** The car frame stops over a shorter distance and has a larger magnitude of acceleration. Part of your 1.70 m stopping distance is the stopping distance of the car and part is how far you move relative to the car while stopping.

**2.24.** **IDENTIFY:** In (a) we want the time to reach Mach 4 with an acceleration of $4g$, and in (b) we want to know how far he can travel if he maintains this acceleration during this time.

**SET UP:** Let $+x$ be the direction the jet travels and take $x_0 = 0$. With constant acceleration, the equations $v_x = v_{0x} + a_x t$ and $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ both apply. $a_x = 4g = 39.2\ \text{m/s}^2$, $v_x = 4(331\ \text{m/s}) = 1324\ \text{m/s}$, and $v_{0x} = 0$.

**EXECUTE:** **(a)** Solving $v_x = v_{0x} + a_x t$ for $t$ gives $t = \dfrac{v_x - v_{0x}}{a_x} = \dfrac{1324\ \text{m/s} - 0}{39.2\ \text{m/s}^2} = 33.8\ \text{s}$.

**(b)** $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = \frac{1}{2}(39.2\ \text{m/s}^2)(33.8\ \text{s})^2 = 2.24\times10^4\ \text{m} = 22.4\ \text{km}$.

**EVALUATE:** The answer in (a) is about ½ min, so if he wanted to reach Mach 4 any sooner than that, he would be in danger of blacking out.

**2.25.** **IDENTIFY:** If a person comes to a stop in 36 ms while slowing down with an acceleration of $60g$, how far does he travel during this time?

**SET UP:** Let $+x$ be the direction the person travels. $v_x = 0$ (he stops), $a_x$ is negative since it is opposite to the direction of the motion, and $t = 36\ \text{ms} = 3.6\times10^{-2}\ \text{s}$. The equations $v_x = v_{0x} + a_x t$ and $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ both apply since the acceleration is constant.

**EXECUTE:** Solving $v_x = v_{0x} + a_x t$ for $v_{0x}$ gives $v_{0x} = -a_x t$. Then $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ gives $x = -\frac{1}{2}a_x t^2 = -\frac{1}{2}(-588\ \text{m/s}^2)(3.6\times10^{-2}\ \text{s})^2 = 38\ \text{cm}$.

**EVALUATE:** Notice that we were not given the initial speed, but we could find it:

$$v_{0x} = -a_x t = -(-588\ \text{m/s}^2)(36\times10^{-3}\ \text{s}) = 21\ \text{m/s} = 47\ \text{mph}.$$

**2.26.** **IDENTIFY:** In (a) the hip pad must reduce the person's speed from 2.0 m/s to 1.3 m/s over a distance of 2.0 cm, and we want the acceleration over this distance, assuming constant acceleration. In (b) we want to find out how long the acceleration in (a) lasts.

**SET UP:** Let $+y$ be downward. $v_{0y} = 2.0\ \text{m/s}$, $v_y = 1.3\ \text{m/s}$, and $y - y_0 = 0.020\ \text{m}$. The equations $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ and $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ apply for constant acceleration.

**EXECUTE:** **(a)** Solving $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ for $a_y$ gives

$$a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \frac{(1.3\ \text{m/s})^2 - (2.0\ \text{m/s})^2}{2(0.020\ \text{m})} = -58\ \text{m/s}^2 = -5.9g.$$

**(b)** $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $t = \dfrac{2(y - y_0)}{v_{0y} + v_y} = \dfrac{2(0.020\ \text{m})}{2.0\ \text{m/s} + 1.3\ \text{m/s}} = 12\ \text{ms}$.

**EVALUATE:** The acceleration is very large, but it only lasts for 12 ms so it produces a small velocity change.

**2.27.** **IDENTIFY:** We know the initial and final velocities of the object, and the distance over which the velocity change occurs. From this we want to find the magnitude and duration of the acceleration of the object.

**SET UP:** The constant-acceleration kinematics formulas apply. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, where $v_{0x} = 0$, $v_x = 5.0\times10^3\ \text{m/s}$, and $x - x_0 = 4.0\ \text{m}$.

**EXECUTE:** **(a)** $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $a_x = \dfrac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \dfrac{(5.0\times10^3\ \text{m/s})^2}{2(4.0\ \text{m})} = 3.1\times10^6\ \text{m/s}^2 = 3.2\times10^5\ g$.

**(b)** $v_x = v_{0x} + a_x t$ gives $t = \dfrac{v_x - v_{0x}}{a_x} = \dfrac{5.0\times10^3\ \text{m/s}}{3.1\times10^6\ \text{m/s}^2} = 1.6\ \text{ms}$.

**EVALUATE:** **(c)** The calculated $a$ is less than 450,000 $g$ so the acceleration required doesn't rule out this hypothesis.

**2.28.** **IDENTIFY:** Apply constant acceleration equations to the motion of the car.

**SET UP:** Let $+x$ be the direction the car is moving.

**EXECUTE:** **(a)** From $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, with $v_{0x} = 0$, $a_x = \dfrac{v_x^2}{2(x - x_0)} = \dfrac{(20\ \text{m/s})^2}{2(120\ \text{m})} = 1.67\ \text{m/s}^2$.

**(b)** Using Eq. (2.14), $t = 2(x - x_0)/v_x = 2(120\text{ m})/(20\text{ m/s}) = 12\text{ s}$.

**(c)** $(12\text{ s})(20\text{ m/s}) = 240\text{ m}$.

**EVALUATE:** The average velocity of the car is half the constant speed of the traffic, so the traffic travels twice as far.

**2.29.** **IDENTIFY:** The average acceleration is $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$. For constant acceleration, the standard kinematics equations apply.

**SET UP:** Assume the rocket ship travels in the $+x$ direction. $161\text{ km/h} = 44.72\text{ m/s}$ and $1610\text{ km/h} = 447.2\text{ m/s}$. $1.00\text{ min} = 60.0\text{ s}$

**EXECUTE:** **(a)** (i) $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t} = \dfrac{44.72\text{ m/s} - 0}{8.00\text{ s}} = 5.59\text{ m/s}^2$

(ii) $a_{\text{av-}x} = \dfrac{447.2\text{ m/s} - 44.72\text{ m/s}}{60.0\text{ s} - 8.00\text{ s}} = 7.74\text{ m/s}^2$

**(b)** (i) $t = 8.00\text{ s}$, $v_{0x} = 0$, and $v_x = 44.72\text{ m/s}$. $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t = \left(\dfrac{0 + 44.72\text{ m/s}}{2}\right)(8.00\text{ s}) = 179\text{ m}$.

(ii) $\Delta t = 60.0\text{ s} - 8.00\text{ s} = 52.0\text{ s}$, $v_{0x} = 44.72\text{ m/s}$, and $v_x = 447.2\text{ m/s}$.

$$x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t = \left(\frac{44.72\text{ m/s} + 447.2\text{ m/s}}{2}\right)(52.0\text{ s}) = 1.28\times10^4\text{ m}.$$

**EVALUATE:** When the acceleration is constant the instantaneous acceleration throughout the time interval equals the average acceleration for that time interval. We could have calculated the distance in part (a) as $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2 = \tfrac{1}{2}(5.59\text{ m/s}^2)(8.00\text{ s})^2 = 179\text{ m}$, which agrees with our previous calculation.

**2.30.** **IDENTIFY:** The acceleration $a_x$ is the slope of the graph of $v_x$ versus $t$.

**SET UP:** The signs of $v_x$ and of $a_x$ indicate their directions.

**EXECUTE:** **(a)** Reading from the graph, at $t = 4.0\text{ s}$, $v_x = 2.7\text{ cm/s}$, to the right and at $t = 7.0\text{ s}$, $v_x = 1.3\text{ cm/s}$, to the left.

**(b)** $v_x$ versus $t$ is a straight line with slope $-\dfrac{8.0\text{ cm/s}}{6.0\text{ s}} = -1.3\text{ cm/s}^2$. The acceleration is constant and equal to $1.3\text{ cm/s}^2$, to the left. It has this value at all times.

**(c)** Since the acceleration is constant, $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$. For $t = 0$ to 4.5 s, $x - x_0 = (8.0\text{ cm/s})(4.5\text{ s}) + \tfrac{1}{2}(-1.3\text{ cm/s}^2)(4.5\text{ s})^2 = 22.8\text{ cm}$. For $t = 0$ to 7.5 s, $x - x_0 = (8.0\text{ cm/s})(7.5\text{ s}) + \tfrac{1}{2}(-1.3\text{ cm/s}^2)(7.5\text{ s})^2 = 23.4\text{ cm}$

**(d)** The graphs of $a_x$ and $x$ versus $t$ are given in Figure 2.30.

**EVALUATE:** In part (c) we could have instead used $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t$.

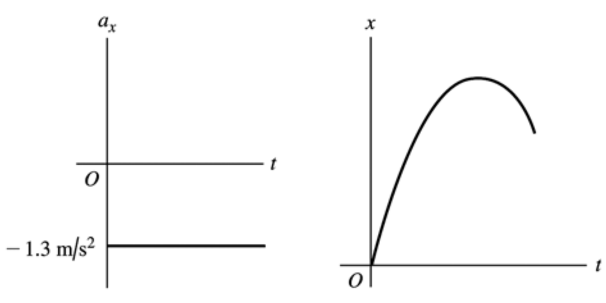



**Figure 2.30**

**2.31.** **(a) IDENTIFY and SET UP:** The acceleration $a_x$ at time *t* is the slope of the tangent to the $v_x$ versus *t* curve at time *t*.

**EXECUTE:** At $t = 3$ s, the $v_x$ versus *t* curve is a horizontal straight line, with zero slope. Thus $a_x = 0$.

At $t = 7$ s, the $v_x$ versus *t* curve is a straight-line segment with slope $\dfrac{45\text{ m/s} - 20\text{ m/s}}{9\text{ s} - 5\text{ s}} = 6.3\text{ m/s}^2$.

Thus $a_x = 6.3\text{ m/s}^2$.

At $t = 11$ s the curve is again a straight-line segment, now with slope $\dfrac{-0 - 45\text{ m/s}}{13\text{ s} - 9\text{ s}} = -11.2\text{ m/s}^2$.

Thus $a_x = -11.2\text{ m/s}^2$.

**EVALUATE:** $a_x = 0$ when $v_x$ is constant, $a_x > 0$ when $v_x$ is positive and the speed is increasing, and $a_x < 0$ when $v_x$ is positive and the speed is decreasing.

**(b) IDENTIFY:** Calculate the displacement during the specified time interval.

**SET UP:** We can use the constant acceleration equations only for time intervals during which the acceleration is constant. If necessary, break the motion up into constant acceleration segments and apply the constant acceleration equations for each segment. For the time interval $t = 0$ to $t = 5$ s the acceleration is constant and equal to zero. For the time interval $t = 5$ s to $t = 9$ s the acceleration is constant and equal to $6.25\text{ m/s}^2$. For the interval $t = 9$ s to $t = 13$ s the acceleration is constant and equal to $-11.2\text{ m/s}^2$.

**EXECUTE:** During the first 5 seconds the acceleration is constant, so the constant acceleration kinematic formulas can be used.

$v_{0x} = 20\text{ m/s}$ $a_x = 0$ $t = 5\text{ s}$ $x - x_0 = ?$

$x - x_0 = v_{0x}t$ ($a_x = 0$ so no $\frac{1}{2}a_x t^2$ term)

$x - x_0 = (20\text{ m/s})(5\text{ s}) = 100\text{ m}$; this is the distance the officer travels in the first 5 seconds.

During the interval $t = 5$ s to 9 s the acceleration is again constant. The constant acceleration formulas can be applied to this 4-second interval. It is convenient to restart our clock so the interval starts at time $t = 0$ and ends at time $t = 4$ s. (Note that the acceleration is *not* constant over the entire $t = 0$ to $t = 9$ s interval.)

$v_{0x} = 20\text{ m/s}$ $a_x = 6.25\text{ m/s}^2$ $t = 4\text{ s}$ $x_0 = 100\text{ m}$ $x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

$x - x_0 = (20\text{ m/s})(4\text{ s}) + \frac{1}{2}(6.25\text{ m/s}^2)(4\text{ s})^2 = 80\text{ m} + 50\text{ m} = 130\text{ m}$.

Thus $x - x_0 + 130\text{ m} = 100\text{ m} + 130\text{ m} = 230\text{ m}$.

At $t = 9$ s the officer is at $x = 230$ m, so she has traveled 230 m in the first 9 seconds.

During the interval $t = 9$ s to $t = 13$ s the acceleration is again constant. The constant acceleration formulas can be applied for this 4-second interval but *not* for the whole $t = 0$ to $t = 13$ s interval. To use the equations restart our clock so this interval begins at time $t = 0$ and ends at time $t = 4$ s.

$v_{0x} = 45\text{ m/s}$ (at the start of this time interval)

$a_x = -11.2\text{ m/s}^2$ $t = 4\text{ s}$ $x_0 = 230\text{ m}$ $x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

$x - x_0 = (45\text{ m/s})(4\text{ s}) + \frac{1}{2}(-11.2\text{ m/s}^2)(4\text{ s})^2 = 180\text{ m} - 89.6\text{ m} = 90.4\text{ m}$.

Thus $x = x_0 + 90.4\text{ m} = 230\text{ m} + 90.4\text{ m} = 320\text{ m}$.

At $t = 13$ s the officer is at $x = 320$ m, so she has traveled 320 m in the first 13 seconds.

**EVALUATE:** The velocity $v_x$ is always positive so the displacement is always positive and displacement and distance traveled are the same. The average velocity for time interval $\Delta t$ is $v_{\text{av-}x} = \Delta x/\Delta t$. For $t = 0$ to 5 s, $v_{\text{av-}x} = 20\text{ m/s}$. For $t = 0$ to 9 s, $v_{\text{av-}x} = 26\text{ m/s}$. For $t = 0$ to 13 s, $v_{\text{av-}x} = 25\text{ m/s}$. These results are consistent with the figure in the textbook.

**2.32.** **IDENTIFY:** $v_x(t)$ is the slope of the *x* versus *t* graph. Car *B* moves with constant speed and zero acceleration. Car *A* moves with positive acceleration; assume the acceleration is constant.

**SET UP:** For car $B$, $v_x$ is positive and $a_x = 0$. For car $A$, $a_x$ is positive and $v_x$ increases with $t$.

**EXECUTE:** **(a)** The motion diagrams for the cars are given in Figure 2.32a.

**(b)** The two cars have the same position at times when their $x$-$t$ graphs cross. The figure in the problem shows this occurs at approximately $t = 1$ s and $t = 3$ s.

**(c)** The graphs of $v_x$ versus $t$ for each car are sketched in Figure 2.32b.

**(d)** The cars have the same velocity when their $x$-$t$ graphs have the same slope. This occurs at approximately $t = 2$ s.

**(e)** Car $A$ passes car $B$ when $x_A$ moves above $x_B$ in the $x$-$t$ graph. This happens at $t = 3$ s.

**(f)** Car $B$ passes car $A$ when $x_B$ moves above $x_A$ in the $x$-$t$ graph. This happens at $t = 1$ s.

**EVALUATE:** When $a_x = 0$, the graph of $v_x$ versus $t$ is a horizontal line. When $a_x$ is positive, the graph of $v_x$ versus $t$ is a straight line with positive slope.

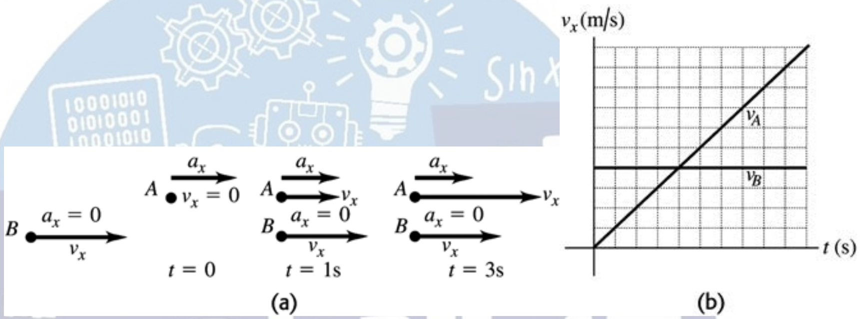



**Figure 2.32**

**2.33.** **IDENTIFY:** For constant acceleration, the kinematics formulas apply. We can use the total displacement ande final velocity to calculate the acceleration and then use the acceleration and shorter distance to find the speed.

**SET UP:** Take $+x$ to be down the incline, so the motion is in the $+x$ direction. The formula $v_x^2 = v_{0x}^2 + 2a(x - x_0)$ applies.

**EXECUTE:** First look at the motion over 6.80 m. We use the following numbers: $v_{0x} = 0$, $x - x_0 = 6.80$ m, and $v_x = 3.80$ /s. Solving the above equation for $a_x$ gives $a_x = 1.062$ m/s$^2$. Now look at the motion over the 3.40 m using $v_{0x} = 0$, $a_x = 1.062$ m/s$^2$ and $x - x_0 = 3.40$ m. Solving the same equation, but this time for $v_x$, gives $v_x = 2.69$ m/s.

**EVALUATE:** Even though the block has traveled half way down the incline, its speed is not half of its speed at the bottom.

**2.34.** **IDENTIFY:** Apply the constant acceleration equations to the motion of each vehicle. The truck passes the car when they are at the same $x$ at the same $t > 0$.

**SET UP:** The truck has $a_x = 0$. The car has $v_{0x} = 0$. Let $+x$ be in the direction of motion of the vehicles. Both vehicles start at $x_0 = 0$. The car has $a_C = 2.80\ \text{m/s}^2$. The truck has $v_x = 20.0$ m/s.

**EXECUTE:** **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x_T = v_{0T}t$ and $x_C = \frac{1}{2}a_C t^2$. Setting $x_T = x_C$ gives $t = 0$ and $v_{0T} = \frac{1}{2}a_C t$, so $t = \dfrac{2v_{0T}}{a_C} = \dfrac{2(20.0\ \text{m/s})}{2.80\ \text{m/s}^2} = 14.29$ s. At this $t$, $x_T = (20.0\ \text{m/s})(14.29\ \text{s}) = 286$ m and $x = \frac{1}{2}(3.20\ \text{m/s}^2)(14.29\ \text{s})^2 = 286$ m. The car and truck have each traveled 286 m.

**(b)** At $t = 14.29$ s, the car has $v_x = v_{0x} + a_x t = (2.80\ \text{m/s}^2)(14.29\ \text{s}) = 40$ m/s.

**(c)** $x_T = v_{0T}t$ and $x_C = \frac{1}{2}a_C t^2$. The $x$-$t$ graph of the motion for each vehicle is sketched in Figure 2.34a.

**(d)** $v_T = v_{0T}$. $v_C = a_C t$. The $v_x$-$t$ graph for each vehicle is sketched in Figure 2.34b (next page).

**EVALUATE:** When the car overtakes the truck its speed is twice that of the truck.

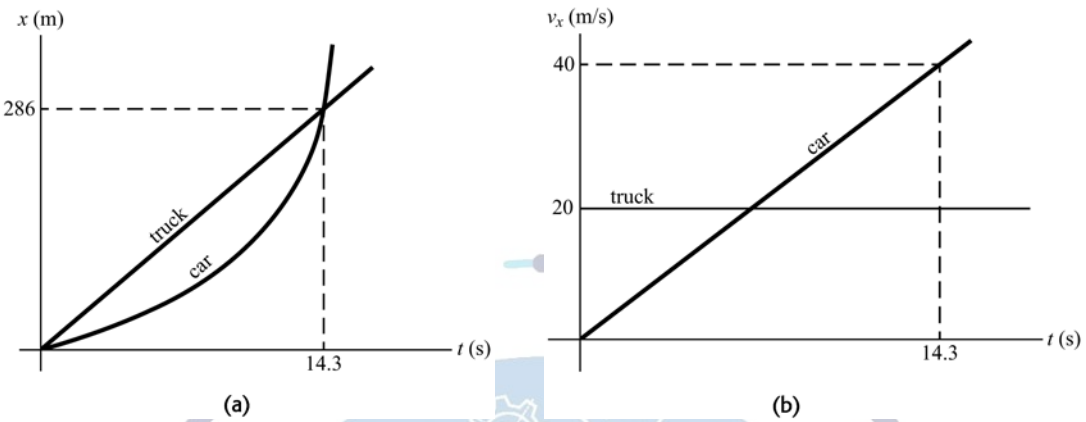



**Figure 2.34**

**2.35.** **IDENTIFY:** Apply the constant acceleration equations to the motion of the flea. After the flea leaves the ground, $a_y = g,$ downward. Take the origin at the ground and the positive direction to be upward.

**(a) SET UP:** At the maximum height $v_y = 0.$

$v_y = 0 \quad y - y_0 = 0.440 \text{ m} \quad a_y = -9.80 \text{ m/s}^2 \quad v_{0y} = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

**EXECUTE:** $v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80 \text{ m/s}^2)(0.440 \text{ m})} = 2.94 \text{ m/s}$

**(b) SET UP:** When the flea has returned to the ground $y - y_0 = 0.$

$y - y_0 = 0 \quad v_{0y} = +2.94 \text{ m/s} \quad a_y = -9.80 \text{ m/s}^2 \quad t = ?$

$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$

**EXECUTE:** With $y - y_0 = 0$ this gives $t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(2.94 \text{ m/s})}{-9.80 \text{ m/s}^2} = 0.600 \text{ s}.$

**EVALUATE:** We can use $v_y = v_{0y} + a_y t$ to show that with $v_{0y} = 2.94 \text{ m/s},$ $v_y = 0$ after 0.300 s.

**2.36.** **IDENTIFY:** The rock has a constant downward acceleration of 9.80 m/s$^2$. We know its initial velocity and position and its final position.

**SET UP:** We can use the kinematics formulas for constant acceleration.

**EXECUTE:** **(a)** $y - y_0 = -30 \text{ m},$ $v_{0y} = 22.0 \text{ m/s},$ $a_y = -9.80 \text{ m/s}^2.$ The kinematics formulas give

$v_y = -\sqrt{v_{0y}^2 + 2a_y(y - y_0)} = -\sqrt{(22.0 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(-30 \text{ m})} = -32.74 \text{ m/s},$ so the speed is 32.7 m/s.

**(b)** $v_y = v_{0y} + a_y t$ and $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-32.74 \text{ m/s} - 22.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = 5.59 \text{ s}.$

**EVALUATE:** The vertical velocity in part (a) is negative because the rock is moving downward, but the speed is always positive. The 5.59 s is the total time in the air.

**2.37.** **IDENTIFY:** The pin has a constant downward acceleration of 9.80 m/s$^2$ and returns to its initial position.

**SET UP:** We can use the kinematics formulas for constant acceleration.

**EXECUTE:** The kinematics formulas give $y - y_0 = v_{0y}t + \dfrac{1}{2}a_y t^2.$ We know that $y - y_0 = 0,$ so

$$t = -\frac{2v_{0y}}{a_y} = -\frac{2(8.20 \text{ m/s})}{-9.80 \text{ m/s}^2} = +1.67 \text{ s}.$$

**EVALUATE:** It takes the pin half this time to reach its highest point and the remainder of the time to return.

**2.38.** **IDENTIFY:** The putty has a constant downward acceleration of 9.80 m/s$^2$. We know the initial velocity of the putty and the distance it travels.

**SET UP:** We can use the kinematics formulas for constant acceleration.

**EXECUTE:** **(a)** $v_{0y} = 9.50$ m/s and $y - y_0 = 3.60$ m, which gives

$$v_y = \sqrt{v_{0y}^2 + 2a_y(y - y_0)} = \sqrt{(9.50 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(3.60 \text{ m})} = 4.44 \text{ m/s}$$

**(b)** $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{4.44 \text{ m/s} - 9.50 \text{ m/s}}{-9.8 \text{ m/s}^2} = 0.517 \text{ s}$

**EVALUATE:** The putty is stopped by the ceiling, not by gravity.

**2.39.** **IDENTIFY:** A ball on Mars that is hit directly upward returns to the same level in 8.5 s with a constant downward acceleration of $0.379g$. How high did it go and how fast was it initially traveling upward?

**SET UP:** Take $+y$ upward. $v_y = 0$ at the maximum height. $a_y = -0.379g = -3.71 \text{ m/s}^2$. The constant-acceleration formulas $v_y = v_{0y} + a_y t$ and $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ both apply.

**EXECUTE:** Consider the motion from the maximum height back to the initial level. For this motion $v_{0y} = 0$ and $t = 4.25$ s. $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(-3.71 \text{ m/s}^2)(4.25 \text{ s})^2 = -33.5 \text{ m}$. The ball went 33.5 m above its original position.

**(b)** Consider the motion from just after it was hit to the maximum height. For this motion $v_y = 0$ and $t = 4.25$ s. $v_y = v_{0y} + a_y t$ gives $v_{0y} = -a_y t = -(-3.71 \text{ m/s}^2)(4.25 \text{ s}) = 15.8 \text{ m/s}$.

**(c)** The graphs are sketched in Figure 2.39.

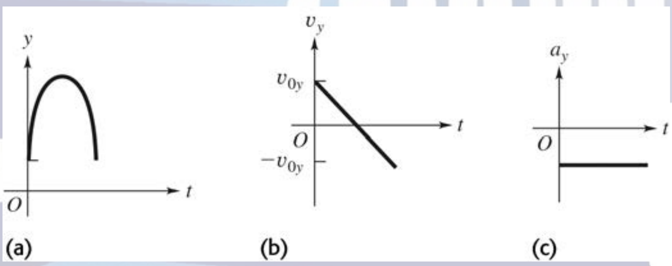



**Figure 2.39**

**EVALUATE:** The answers can be checked several ways. For example, $v_y = 0$, $v_{0y} = 15.8$ m/s, and $a_y = -3.71 \text{ m/s}^2$ in $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (15.8 \text{ m/s})^2}{2(-3.71 \text{ m/s}^2)} = 33.6 \text{ m}$, which agrees with the height calculated in (a).

**2.40.** **IDENTIFY:** Apply constant acceleration equations to the motion of the lander.

**SET UP:** Let $+y$ be downward. Since the lander is in free-fall, $a_y = +1.6 \text{ m/s}^2$.

**EXECUTE:** $v_{0y} = 0.8$ m/s, $y - y_0 = 5.0$ m, $a_y = +1.6 \text{ m/s}^2$ in $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$v_y = \sqrt{v_{0y}^2 + 2a_y(y - y_0)} = \sqrt{(0.8 \text{ m/s})^2 + 2(1.6 \text{ m/s}^2)(5.0 \text{ m})} = 4.1 \text{ m/s}.$$

**EVALUATE:** The same descent on earth would result in a final speed of 9.9 m/s, since the acceleration due to gravity on earth is much larger than on the moon.

**2.41.** **IDENTIFY:** Apply constant acceleration equations to the motion of the meterstick. The time the meterstick falls is your reaction time.

**SET UP:** Let $+y$ be downward. The meter stick has $v_{0y} = 0$ and $a_y = 9.80 \text{ m/s}^2$. Let $d$ be the distance the meterstick falls.

**EXECUTE:** **(a)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $d = (4.90 \text{ m/s}^2)t^2$ and $t = \sqrt{\dfrac{d}{4.90 \text{ m/s}^2}}$.

**(b)** $t = \sqrt{\dfrac{0.176 \text{ m}}{4.90 \text{ m/s}^2}} = 0.190 \text{ s}$

**EVALUATE:** The reaction time is proportional to the square of the distance the stick falls.

**2.42.** **IDENTIFY:** Apply constant acceleration equations to the vertical motion of the brick.

**SET UP:** Let $+y$ be downward. $a_y = 9.80\ \text{m/s}^2$

**EXECUTE:** **(a)** $v_{0y} = 0, t = 1.90\ \text{s}, a_y = 9.80\ \text{m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(9.80\ \text{m/s}^2)(1.90\ \text{s})^2 = 17.7\ \text{m}$. The building is 17.7 m tall.

**(b)** $v_y = v_{0y} + a_y t = 0 + (9.80\ \text{m/s}^2)(1.90\ \text{s}) = 18.6\ \text{m/s}$

**(c)** The graphs of $a_y$, $v_y$ and $y$ versus $t$ are given in Figure 2.42. Take $y = 0$ at the ground.

**EVALUATE:** We could use either $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ or $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to check our results.

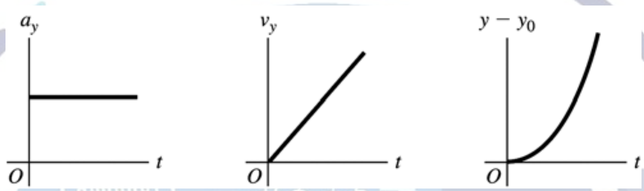



**Figure 2.42**

**2.43.** **IDENTIFY:** When the only force is gravity the acceleration is $9.80\ \text{m/s}^2$, downward. There are two intervals of constant acceleration and the constant acceleration equations apply during each of these intervals.

**SET UP:** Let $+y$ be upward. Let $y = 0$ at the launch pad. The final velocity for the first phase of the motion is the initial velocity for the free-fall phase.

**EXECUTE:** **(a)** Find the velocity when the engines cut off. $y - y_0 = 525\ \text{m}$, $a_y = 2.25\ \text{m/s}^2$, $v_{0y} = 0$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(2.25\ \text{m/s}^2)(525\ \text{m})} = 48.6\ \text{m/s}$.

Now consider the motion from engine cut-off to maximum height: $y_0 = 525\ \text{m}$, $v_{0y} = +48.6\ \text{m/s}$, $v_y = 0$ (at the maximum height), $a_y = -9.80\ \text{m/s}^2$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (48.6\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 121\ \text{m} \text{ and } y = 121\ \text{m} + 525\ \text{m} = 646\ \text{m}.$$

**(b)** Consider the motion from engine failure until just before the rocket strikes the ground: $y - y_0 = -525\ \text{m}$, $a_y = -9.80\ \text{m/s}^2$, $v_{0y} = +48.6\ \text{m/s}$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_y = -\sqrt{(48.6\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(-525\ \text{m})} = -112\ \text{m/s}$. Then $v_y = v_{0y} + a_y t$ gives

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-112\ \text{m/s} - 48.6\ \text{m/s}}{-9.80\ \text{m/s}^2} = 16.4\ \text{s}.$$

**(c)** Find the time from blast-off until engine failure: $y - y_0 = 525\ \text{m}$, $v_{0y} = 0$, $a_y = +2.25\ \text{m/s}^2$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(525\ \text{m})}{2.25\ \text{m/s}^2}} = 21.6\ \text{s}$. The rocket strikes the launch pad $21.6\ \text{s} + 16.4\ \text{s} = 38.0\ \text{s}$ after blast-off. The acceleration $a_y$ is $+2.25\ \text{m/s}^2$ from $t = 0$ to $t = 21.6\ \text{s}$. It is $-9.80\ \text{m/s}^2$ from $t = 21.6\ \text{s}$ to $38.0\ \text{s}$. $v_y = v_{0y} + a_y t$ applies during each constant acceleration segment, so the graph of $v_y$ versus $t$ is a straight line with positive slope of $2.25\ \text{m/s}^2$ during the blast-off phase and with negative slope of $-9.80\ \text{m/s}^2$ after engine failure. During each phase $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. The sign of $a_y$ determines the curvature of $y(t)$. At $t = 38.0\ \text{s}$ the rocket has returned to $y = 0$. The graphs are sketched in Figure 2.43.

**EVALUATE:** In part (b) we could have found the time from $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$, finding $v_y$ first allows us to avoid solving for $t$ from a quadratic equation.

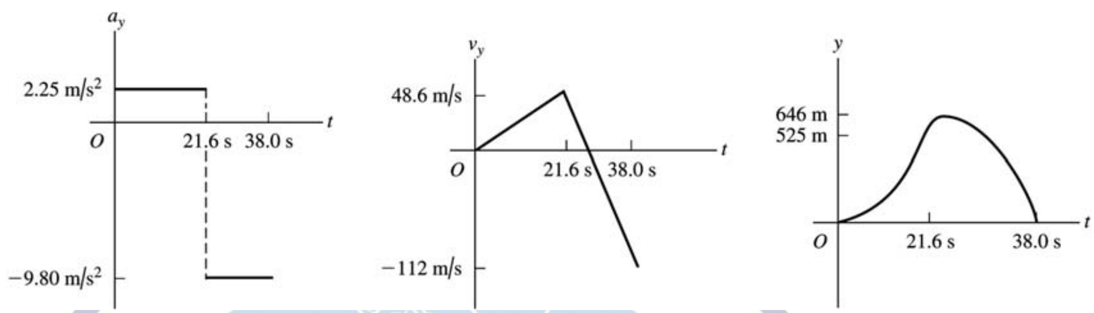



**Figure 2.43**

**2.44.** **IDENTIFY:** Apply constant acceleration equations to the vertical motion of the sandbag.

**SET UP:** Take $+y$ upward. $a_y = -9.80\ \text{m/s}^2$. The initial velocity of the sandbag equals the velocity of the balloon, so $v_{0y} = +5.00\ \text{m/s}$. When the balloon reaches the ground, $y - y_0 = -40.0\ \text{m}$. At its maximum height the sandbag has $v_y = 0$.

**EXECUTE:** **(a)**

$t = 0.250\ \text{s}$: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (5.00\ \text{m/s})(0.250\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(0.250\ \text{s})^2 = 0.94\ \text{m}$. The sandbag is 40.9 m above the ground. $v_y = v_{0y} + a_y t = +5.00\ \text{m/s} + (-9.80\ \text{m/s}^2)(0.250\ \text{s}) = 2.55\ \text{m/s}$.

$t = 1.00\ \text{s}$: $y - y_0 = (5.00\ \text{m/s})(1.00\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(1.00\ \text{s})^2 = 0.10\ \text{m}$. The sandbag is 40.1 m above the ground. $v_y = v_{0y} + a_y t = +5.00\ \text{m/s} + (-9.80\ \text{m/s}^2)(1.00\ \text{s}) = -4.80\ \text{m/s}$.

**(b)** $y - y_0 = -40.0\ \text{m}$, $v_{0y} = 5.00\ \text{m/s}$, $a_y = -9.80\ \text{m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$-40.0\ \text{m} = (5.00\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2$. $(4.90\ \text{m/s}^2)t^2 - (5.00\ \text{m/s})t - 40.0\ \text{m} = 0$ and

$t = \dfrac{1}{9.80}\left(5.00 \pm \sqrt{(-5.00)^2 - 4(4.90)(-40.0)}\right)\text{s} = (0.51 \pm 2.90)\ \text{s}$. $t$ must be positive, so $t = 3.41\ \text{s}$.

**(c)** $v_y = v_{0y} + a_y t = +5.00\ \text{m/s} + (-9.80\ \text{m/s}^2)(3.41\ \text{s}) = -28.4\ \text{m/s}$

**(d)** $v_{0y} = 5.00\ \text{m/s}$, $a_y = -9.80\ \text{m/s}^2$, $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (5.00\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 1.28\ \text{m}$. The maximum height is 41.3 m above the ground.

**(e)** The graphs of $a_y$, $v_y$, and $y$ versus $t$ are given in Figure 2.44. Take $y = 0$ at the ground.

**EVALUATE:** The sandbag initially travels upward with decreasing velocity and then moves downward with increasing speed.

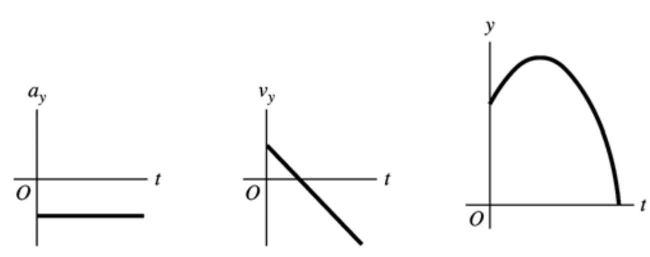



**Figure 2.44**

**2.45.** **IDENTIFY:** Use the constant acceleration equations to calculate $a_x$ and $x - x_0$.

**(a) SET UP:** $v_x = 224\text{ m/s},\ v_{0x} = 0,\ t = 0.900\text{ s},\ a_x = ?$

$v_x = v_{0x} + a_x t$

**EXECUTE:** $a_x = \dfrac{v_x - v_{0x}}{t} = \dfrac{224\text{ m/s} - 0}{0.900\text{ s}} = 249\text{ m/s}^2$

**(b)** $a_x/g = (249\text{ m/s}^2)/(9.80\text{ m/s}^2) = 25.4$

**(c)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(249\text{ m/s}^2)(0.900\text{ s})^2 = 101\text{ m}$

**(d) SET UP:** Calculate the acceleration, assuming it is constant:

$t = 1.40\text{ s},\ v_{0x} = 283\text{ m/s},\ v_x = 0\ \text{(stops)},\ a_x = ?$

$v_x = v_{0x} + a_x t$

**EXECUTE:** $a_x = \dfrac{v_x - v_{0x}}{t} = \dfrac{0 - 283\text{ m/s}}{1.40\text{ s}} = -202\text{ m/s}^2$

$a_x/g = (-202\text{ m/s}^2)/(9.80\text{ m/s}^2) = -20.6;\ a_x = -20.6g$

If the acceleration while the sled is stopping is constant then the magnitude of the acceleration is only 20.6*g*. But if the acceleration is not constant it is certainly possible that at some point the instantaneous acceleration could be as large as 40*g*.

**EVALUATE:** It is reasonable that for this motion the acceleration is much larger than *g*.

**2.46.** **IDENTIFY:** Since air resistance is ignored, the egg is in free-fall and has a constant downward acceleration of magnitude $9.80\text{ m/s}^2$. Apply the constant acceleration equations to the motion of the egg.

**SET UP:** Take $+y$ to be upward. At the maximum height, $v_y = 0$.

**EXECUTE:** **(a)** $y - y_0 = -30.0\text{ m},\ t = 5.00\text{ s},\ a_y = -9.80\text{ m/s}^2.\ y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$$v_{0y} = \frac{y - y_0}{t} - \tfrac{1}{2}a_y t = \frac{-30.0\text{ m}}{5.00\text{ s}} - \tfrac{1}{2}(-9.80\text{ m/s}^2)(5.00\text{ s}) = +18.5\text{ m/s}.$$

**(b)** $v_{0y} = +18.5\text{ m/s},\ v_y = 0$ (at the maximum height), $a_y = -9.80\text{ m/s}^2.\ v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (18.5\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 17.5\text{ m}.$$

**(c)** At the maximum height $v_y = 0$.

**(d)** The acceleration is constant and equal to $9.80\text{ m/s}^2$, downward, at all points in the motion, including at the maximum height.

**(e)** The graphs are sketched in Figure 2.46.

**EVALUATE:** The time for the egg to reach its maximum height is $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-18.5\text{ m/s}}{-9.8\text{ m/s}^2} = 1.89\text{ s}$. The egg has returned to the level of the cornice after 3.78 s and after 5.00 s it has traveled downward from the cornice for 1.22 s.

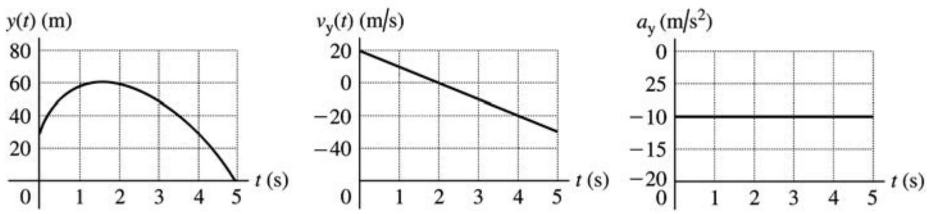



**Figure 2.46**

**2.47.** **IDENTIFY:** We can avoid solving for the common height by considering the relation between height, time of fall, and acceleration due to gravity, and setting up a ratio involving time of fall and acceleration due to gravity.

**SET UP:** Let $g_{\text{En}}$ be the acceleration due to gravity on Enceladus and let $g$ be this quantity on earth. Let $h$ be the common height from which the object is dropped. Let $+y$ be downward, so $y-y_0=h$. $v_{0y}=0$

**EXECUTE:** $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ gives $h=\frac{1}{2}gt_{\text{E}}^2$ and $h=\frac{1}{2}g_{\text{En}}t_{\text{En}}^2$. Combining these two equations gives

$$gt_{\text{E}}^2=g_{\text{En}}t_{\text{En}}^2 \text{ and } g_{\text{En}}=g\left(\frac{t_{\text{E}}}{t_{\text{En}}}\right)^2=(9.80\text{ m/s}^2)\left(\frac{1.75\text{ s}}{18.6\text{ s}}\right)^2=0.0868\text{ m/s}^2.$$

**EVALUATE:** The acceleration due to gravity is inversely proportional to the square of the time of fall.

**2.48.** **IDENTIFY:** Since air resistance is ignored, the boulder is in free-fall and has a constant downward acceleration of magnitude $9.80\text{ m/s}^2$. Apply the constant acceleration equations to the motion of the boulder.

**SET UP:** Take $+y$ to be upward.

**EXECUTE:** **(a)** $v_{0y}=+40.0\text{ m/s}$, $v_y=+20.0\text{ m/s}$, $a_y=-9.80\text{ m/s}^2$. $v_y=v_{0y}+a_yt$ gives

$$t=\frac{v_y-v_{0y}}{a_y}=\frac{20.0\text{ m/s}-40.0\text{ m/s}}{-9.80\text{ m/s}^2}=+2.04\text{ s}.$$

**(b)** $v_y=-20.0\text{ m/s}$. $t=\dfrac{v_y-v_{0y}}{a_y}=\dfrac{-20.0\text{ m/s}-40.0\text{ m/s}}{-9.80\text{ m/s}^2}=+6.12\text{ s}.$

**(c)** $y-y_0=0$, $v_{0y}=+40.0\text{ m/s}$, $a_y=-9.80\text{ m/s}^2$. $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ gives $t=0$ and

$$t=-\frac{2v_{0y}}{a_y}=-\frac{2(40.0\text{ m/s})}{-9.80\text{ m/s}^2}=+8.16\text{ s}.$$

**(d)** $v_y=0$, $v_{0y}=+40.0\text{ m/s}$, $a_y=-9.80\text{ m/s}^2$. $v_y=v_{0y}+a_yt$ gives $t=\dfrac{v_y-v_{0y}}{a_y}=\dfrac{0-40.0\text{ m/s}}{-9.80\text{ m/s}^2}=4.08\text{ s}.$

**(e)** The acceleration is $9.80\text{ m/s}^2$, downward, at all points in the motion.

**(f)** The graphs are sketched in Figure 2.48.

**EVALUATE:** $v_y=0$ at the maximum height. The time to reach the maximum height is half the total time in the air, so the answer in part (d) is half the answer in part (c). Also note that $2.04\text{ s}<4.08\text{ s}<6.12\text{ s}$. The boulder is going upward until it reaches its maximum height and after the maximum height it is traveling downward.

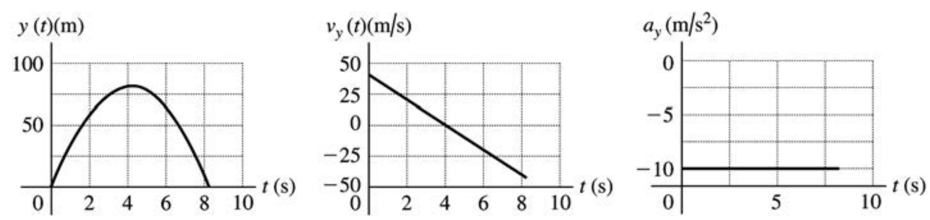



**Figure 2.48**

**2.49.** **IDENTIFY:** The rock has a constant downward acceleration of $9.80\text{ m/s}^2$. The constant-acceleration kinematics formulas apply.

**SET UP:** The formulas $y=y_0+v_{0y}t+\frac{1}{2}a_yt^2$ and $v_y^2=v_{0y}^2+2a_y(y-y_0)$ both apply. Call $+y$ upward. First find the initial velocity and then the final speed.

**EXECUTE:** **(a)** 6.00 s after it is thrown, the rock is back at its original height, so $y=y_0$ at that instant. Using $a_y=-9.80\text{ m/s}^2$ and $t=6.00$ s, the equation $y=y_0+v_{0y}t+\frac{1}{2}a_yt^2$ gives $v_{0y}=29.4$ m/s. When the rock reaches the water, $y - y_0 = -28.0$ m. The equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = -37.6$ m/s, so its speed is 37.6 m/s.

**EVALUATE:** The final speed is greater than the initial speed because the rock accelerated on its way down below the bridge.

**2.50.** **IDENTIFY:** The acceleration is not constant, so we must use calculus instead of the standard kinematics formulas.

**SET UP:** The general calculus formulas are $v_x = v_{0x} + \int_0^t a_x dt$ and $x = x_0 + \int_0^t v_x dt$. First integrate $a_x$ to find $v(t)$, and then integrate that to find $x(t)$.

**EXECUTE:** Find $v(t)$: $v_x(t) = v_{0x} + \int_0^t a_x dt = v_{0x} + \int_0^t -(0.0320 \text{ m/s}^3)(15.0 \text{ s} - t)dt$. Carrying out the integral and putting in the numbers gives $v_x(t) = 8.00 \text{ m/s} - (0.0320 \text{ m/s}^3)[(15.0 \text{ s})t - t^2/2]$. Now use this result to find $x(t)$.

$x = x_0 + \int_0^t v_x dt = x_0 + \int_0^t \left[8.00 \text{ m/s} - (0.0320 \text{ m/s}^3)((15.0 \text{ s})t - \frac{t^2}{2})\right]dt$, which gives

$x = x_0 + (8.00 \text{ m/s})t - (0.0320 \text{ m/s}^3)[(7.50 \text{ s})t^2 - t^3/6)]$. Using $x_0 = -14.0$ m and $t = 10.0$ s, we get $x = 47.3$ m.

**EVALUATE:** The standard kinematics formulas apply only when the acceleration is constant.

**2.51.** **IDENTIFY:** The acceleration is not constant, but we know how it varies with time. We can use the definitions of instantaneous velocity and position to find the rocket's position and speed.

**SET UP:** The basic definitions of velocity and position are $v_y(t) = v_{0y} + \int_0^t a_y dt$ and $y - y_0 = \int_0^t v_y dt$.

**EXECUTE:** **(a)** $v_y(t) = \int_0^t a_y dt = \int_0^t (2.80 \text{ m/s}^3) t dt = (1.40 \text{ m/s}^3)t^2$

$y - y_0 = \int_0^t v_y dt = \int_0^t (1.40 \text{ m/s}^3)t^2 dt = (0.4667 \text{ m/s}^3)t^3$. For $t = 10.0$ s, $y - y_0 = 467$ m.

**(b)** $y - y_0 = 325$ m so $(0.4667 \text{ m/s}^3)t^3 = 325$ m and $t = 8.864$ s. At this time $v_y = (1.40 \text{ m/s}^3)(8.864 \text{ s})^2 = 110$ m/s.

**EVALUATE:** The time in part (b) is less than 10.0 s, so the given formulas are valid.

**2.52.** **IDENTIFY:** The acceleration is not constant so the constant acceleration equations cannot be used. Instead, use $v_x = v_{0x} + \int_0^t a_x dt$ and $x = x_0 + \int_0^t v_x dt$. Use the values of $v_x$ and of $x$ at $t = 1.0$ s to evaluate $v_{0x}$ and $x_0$.

**SET UP:** $\int t^n dt = \frac{1}{n+1} t^{n+1}$, for $n \ge 0$.

**EXECUTE:** **(a)** $v_x = v_{0x} + \int_0^t \alpha t dt = v_{0x} + \frac{1}{2}\alpha t^2 = v_{0x} + (0.60 \text{ m/s}^3)t^2$. $v_x = 5.0$ m/s when $t = 1.0$ s gives $v_{0x} = 4.4$ m/s. Then, at $t = 2.0$ s, $v_x = 4.4 \text{ m/s} + (0.60 \text{ m/s}^3)(2.0 \text{ s})^2 = 6.8$ m/s.

**(b)** $x = x_0 + \int_0^t (v_{0x} + \frac{1}{2}\alpha t^2)dt = x_0 + v_{0x}t + \frac{1}{6}\alpha t^3$. $x = 6.0$ m at $t = 1.0$ s gives $x_0 = 1.4$ m. Then, at $t = 2.0$ s, $x = 1.4 \text{ m} + (4.4 \text{ m/s})(2.0 \text{ s}) + \frac{1}{6}(1.2 \text{ m/s}^3)(2.0 \text{ s})^3 = 11.8$ m.

**(c)** $x(t) = 1.4 \text{ m} + (4.4 \text{ m/s})t + (0.20 \text{ m/s}^3)t^3$. $v_x(t) = 4.4 \text{ m/s} + (0.60 \text{ m/s}^3)t^2$. $a_x(t) = (1.20 \text{ m/s}^3)t$. The graphs are sketched in Figure 2.52.

**EVALUATE:** We can verify that $a_x = \frac{dv_x}{dt}$ and $v_x = \frac{dx}{dt}$.

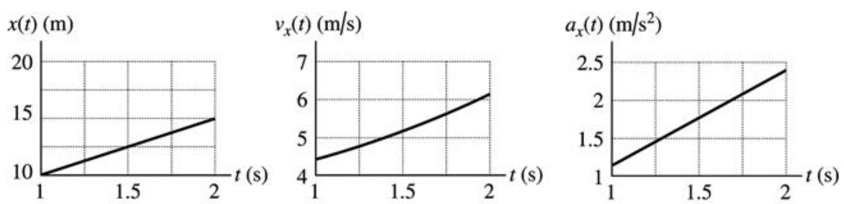



**Figure 2.52**

**2.53.** **(a) IDENTIFY:** Integrate $a_x(t)$ to find $v_x(t)$ and then integrate $v_x(t)$ to find $x(t)$.

**SET UP:** $v_x = v_{0x} + \int_0^t a_x\,dt$, $a_x = At - Bt^2$ with $A = 1.50\text{ m/s}^3$ and $B = 0.120\text{ m/s}^4$.

**EXECUTE:** $v_x = v_{0x} + \int_0^t (At - Bt^2)\,dt = v_{0x} + \frac{1}{2}At^2 - \frac{1}{3}Bt^3$

At rest at $t = 0$ says that $v_{0x} = 0$, so

$v_x = \frac{1}{2}At^2 - \frac{1}{3}Bt^3 = \frac{1}{2}(1.50\text{ m/s}^3)t^2 - \frac{1}{3}(0.120\text{ m/s}^4)t^3$

$v_x = (0.75\text{ m/s}^3)t^2 - (0.040\text{ m/s}^4)t^3$

**SET UP:** $x - x_0 + \int_0^t v_x\,dt$

**EXECUTE:** $x = x_0 + \int_0^t (\frac{1}{2}At^2 - \frac{1}{3}Bt^3)\,dt = x_0 + \frac{1}{6}At^3 - \frac{1}{12}Bt^4$

At the origin at $t = 0$ says that $x_0 = 0$, so

$x = \frac{1}{6}At^3 - \frac{1}{12}Bt^4 = \frac{1}{6}(1.50\text{ m/s}^3)t^3 - \frac{1}{12}(0.120\text{ m/s}^4)t^4$

$x = (0.25\text{ m/s}^3)t^3 - (0.010\text{ m/s}^4)t^4$

**EVALUATE:** We can check our results by using them to verify that $v_x(t) = \dfrac{dx}{dt}$ and $a_x(t) = \dfrac{dv_x}{dt}$.

**(b) IDENTIFY and SET UP:** At time *t*, when $v_x$ is a maximum, $\dfrac{dv_x}{dt} = 0$. (Since $a_x = \dfrac{dv_x}{dt}$, the maximum velocity is when $a_x = 0$. For earlier times $a_x$ is positive so $v_x$ is still increasing. For later times $a_x$ is negative and $v_x$ is decreasing.)

**EXECUTE:** $a_x = \dfrac{dv_x}{dt} = 0$ so $At - Bt^2 = 0$

One root is $t = 0$, but at this time $v_x = 0$ and not a maximum.

The other root is $t = \dfrac{A}{B} = \dfrac{1.50\text{ m/s}^3}{0.120\text{ m/s}^4} = 12.5\text{ s}$

At this time $v_x = (0.75\text{ m/s}^3)t^2 - (0.040\text{ m/s}^4)t^3$ gives

$v_x = (0.75\text{ m/s}^3)(12.5\text{ s})^2 - (0.040\text{ m/s}^4)(12.5\text{ s})^3 = 117.2\text{ m/s} - 78.1\text{ m/s} = 39.1\text{ m/s}.$

**EVALUATE:** For $t < 12.5$ s, $a_x > 0$ and $v_x$ is increasing. For $t > 12.5$ s, $a_x < 0$ and $v_x$ is decreasing.

**2.54.** **IDENTIFY:** $a(t)$ is the slope of the *v* versus *t* graph and the distance traveled is the area under the *v* versus *t* graph.

**SET UP:** The *v* versus *t* graph can be approximated by the graph sketched in Figure 2.54 (next page).

**EXECUTE:** **(a)** Slope $= a = 0$ for $t \ge 1.3$ ms.

**(b)** $h_{\text{max}} = \text{Area under } v\text{-}t \text{ graph} \approx A_{\text{Triangle}} + A_{\text{Rectangle}} \approx \dfrac{1}{2}(1.3\text{ ms})(133\text{ cm/s}) + (2.5\text{ ms} - 1.3\text{ ms})(133\text{ cm/s}) \approx 0.25\text{ cm}$

**(c)** $a = \text{slope of } v\text{-}t \text{ graph. } a(0.5\text{ ms}) \approx a(1.0\text{ ms}) \approx \dfrac{133\text{ cm/s}}{1.3\text{ ms}} = 1.0 \times 10^5\text{ cm/s}^2.$

$a(1.5\text{ ms}) = 0$ because the slope is zero.

**(d)** $h = \text{area under } v\text{-}t \text{ graph. } h(0.5\text{ ms}) \approx A_{\text{Triangle}} = \dfrac{1}{2}(0.5\text{ ms})(33\text{ cm/s}) = 8.3 \times 10^{-3}\text{ cm}.$

$h(1.0\text{ ms}) \approx A_{\text{Triangle}} = \dfrac{1}{2}(1.0\text{ ms})(100\text{ cm/s}) = 5.0 \times 10^{-2}\text{ cm}.$

$h(1.5\text{ ms}) \approx A_{\text{Triangle}} + A_{\text{Rectangle}} = \dfrac{1}{2}(1.3\text{ ms})(133\text{ cm/s}) + (0.2\text{ ms})(133\text{ cm/s}) = 0.11\text{ cm}.$

**EVALUATE:** The acceleration is constant until $t = 1.3$ ms, and then it is zero. $g = 980\ \text{cm/s}^2$. The acceleration during the first 1.3 ms is much larger than this and gravity can be neglected for the portion of the jump that we are considering.

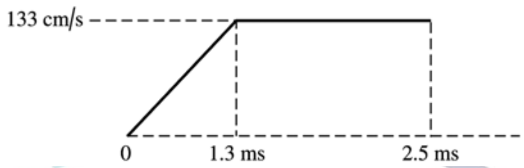



**Figure 2.54**

**2.55.** **IDENTIFY:** The sprinter's acceleration is constant for the first 2.0 s but zero after that, so it is not constant over the entire race. We need to break up the race into segments.

**SET UP:** When the acceleration is constant, the formula $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t$ applies. The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$.

**EXECUTE:** **(a)** $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t = \left(\dfrac{0 + 10.0\ \text{m/s}}{2}\right)(2.0\ \text{s}) = 10.0\ \text{m}.$

**(b)** (i) 40.0 m at 10.0 m/s so time at constant speed is 4.0 s. The total time is 6.0 s, so

$$v_{\text{av-}x} = \frac{\Delta x}{\Delta t} = \frac{50.0\ \text{m}}{6.0\ \text{s}} = 8.33\ \text{m/s}.$$

(ii) He runs 90.0 m at 10.0 m/s so the time at constant speed is 9.0 s. The total time is 11.0 s, so

$$v_{\text{av-}x} = \frac{100\ \text{m}}{11.0\ \text{s}} = 9.09\ \text{m/s}.$$

(iii) He runs 190 m at 10.0 m/s so time at constant speed is 19.0 s. His total time is 21.0 s, so

$$v_{\text{av-}x} = \frac{200\ \text{m}}{21.0\ \text{s}} = 9.52\ \text{m/s}.$$

**EVALUATE:** His average velocity keeps increasing because he is running more and more of the race at his top speed.

**2.56.** **IDENTIFY:** We know the vertical position of the lander as a function of time and want to use this to find its velocity initially and just before it hits the lunar surface.

**SET UP:** By definition, $v_y(t) = \dfrac{dy}{dt}$, so we can find $v_y$ as a function of time and then evaluate it for the desired cases.

**EXECUTE:** **(a)** $v_y(t) = \dfrac{dy}{dt} = -c + 2dt$. At $t = 0$, $v_y(t) = -c = -60.0\ \text{m/s}$. The initial velocity is 60.0 m/s downward.

**(b)** $y(t) = 0$ says $b - ct + dt^2 = 0$. The quadratic formula says $t = 28.57\ \text{s} \pm 7.38\ \text{s}$. It reaches the surface at $t = 21.19\ \text{s}$. At this time, $v_y = -60.0\ \text{m/s} + 2(1.05\ \text{m/s}^2)(21.19\ \text{s}) = -15.5\ \text{m/s}$.

**EVALUATE:** The given formula for $y(t)$ is of the form $y = y_0 + v_{0y}t + \frac{1}{2}at^2$. For part (a), $v_{0y} = -c = -60$ m/s.

**2.57.** **IDENTIFY:** In time $t_S$ the S-waves travel a distance $d = v_S t_S$ and in time $t_P$ the P-waves travel a distance $d = v_P t_P$.

**SET UP:** $t_S = t_P + 33\ \text{s}$

**EXECUTE:** $\dfrac{d}{v_S} = \dfrac{d}{v_P} + 33\ \text{s}$. $d\left(\dfrac{1}{3.5\ \text{km/s}} - \dfrac{1}{6.5\ \text{km/s}}\right) = 33\ \text{s}$ and $d = 250$ km.

**EVALUATE:** The times of travel for each wave are $t_S = 71\text{ s}$ and $t_P = 38\text{ s}$.

**2.58.** **IDENTIFY:** The brick has a constant downward acceleration, so we can use the usual kinematics formulas. We know that it falls 40.0 m in 1.00 s, but we do not know which second that is. We want to find out how far it falls in the next 1.00-s interval.

**SET UP:** Let the $+y$ direction be downward. The final velocity at the end of the first 1.00-s interval will be the initial velocity for the second 1.00-s interval. $a_y = 9.80\text{ m/s}^2$ and the formula $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ applies.

**EXECUTE:** **(a)** First find the initial speed at the beginning of the first 1.00-s interval. Applying the above formula with $a_y = 9.80\text{ m/s}^2$, $t = 1.00$ s, and $y - y_0 = 40.0$ m, we get $v_{0y} = 35.1$ m/s. At the end of this 1.00-s interval, the velocity is $v_y = 35.1\text{ m/s} + (9.80\text{ m/s}^2)(1.00\text{ s}) = 44.9$ m/s. This is $v_{0y}$ for the next 1.00-s interval. Using $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with this initial velocity gives $y - y_0 = 49.8$ m.

**EVALUATE:** The distance the brick falls during the second 1.00-s interval is greater than during the first 1.00-s interval, which it must be since the brick is accelerating downward.

**2.59.** **IDENTIFY:** The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$.

**SET UP:** Let $+x$ be upward.

**EXECUTE:** **(a)** $v_{\text{av-}x} = \dfrac{1000\text{ m} - 63\text{ m}}{4.75\text{ s}} = 197\text{ m/s}$

**(b)** $v_{\text{av-}x} = \dfrac{1000\text{ m} - 0}{5.90\text{ s}} = 169\text{ m/s}$

**EVALUATE:** For the first 1.15 s of the flight, $v_{\text{av-}x} = \dfrac{63\text{ m} - 0}{1.15\text{ s}} = 54.8\text{ m/s}.$ When the velocity isn't constant the average velocity depends on the time interval chosen. In this motion the velocity is increasing.

**2.60.** **IDENTIFY:** Use constant acceleration equations to find $x - x_0$ for each segment of the motion.

**SET UP:** Let $+x$ be the direction the train is traveling.

**EXECUTE:** $t = 0$ to 14.0 s: $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = \frac{1}{2}(1.60\text{ m/s}^2)(14.0\text{ s})^2 = 157\text{ m}.$

At $t = 14.0$ s, the speed is $v_x = v_{0x} + a_x t = (1.60\text{ m/s}^2)(14.0\text{ s}) = 22.4\text{ m/s}.$ In the next 70.0 s, $a_x = 0$ and $x - x_0 = v_{0x}t = (22.4\text{ m/s})(70.0\text{ s}) = 1568\text{ m}.$

For the interval during which the train is slowing down, $v_{0x} = 22.4$ m/s, $a_x = -3.50\text{ m/s}^2$ and $v_x = 0$.

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $x - x_0 = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = \dfrac{0 - (22.4\text{ m/s})^2}{2(-3.50\text{ m/s}^2)} = 72\text{ m}.$

The total distance traveled is $157\text{ m} + 1568\text{ m} + 72\text{ m} = 1800\text{ m}.$

**EVALUATE:** The acceleration is not constant for the entire motion, but it does consist of constant acceleration segments, and we can use constant acceleration equations for each segment.

**2.61.** **IDENTIFY:** When the graph of $v_x$ versus $t$ is a straight line the acceleration is constant, so this motion consists of two constant acceleration segments and the constant acceleration equations can be used for each segment. Since $v_x$ is always positive the motion is always in the $+x$ direction and the total distance moved equals the magnitude of the displacement. The acceleration $a_x$ is the slope of the $v_x$ versus $t$ graph.

**SET UP:** For the $t = 0$ to $t = 10.0$ s segment, $v_{0x} = 4.00$ m/s and $v_x = 12.0$ m/s. For the $t = 10.0$ s to 12.0 s segment, $v_{0x} = 12.0$ m/s and $v_x = 0$.

**EXECUTE:** **(a)** For $t = 0$ to $t = 10.0$ s, $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t = \left(\dfrac{4.00\text{ m/s} + 12.0\text{ m/s}}{2}\right)(10.0\text{ s}) = 80.0\text{ m}.$

For $t = 10.0$ s to $t = 12.0$ s, $x - x_0 = \left(\dfrac{12.0\text{ m/s} + 0}{2}\right)(2.00\text{ s}) = 12.0\text{ m}.$ The total distance traveled is 92.0 m.

**(b)** $x - x_0 = 80.0\text{ m} + 12.0\text{ m} = 92.0\text{ m}$

**(c)** For $t=0$ to 10.0 s, $a_x = \dfrac{12.0\text{ m/s} - 4.00\text{ m/s}}{10.0\text{ s}} = 0.800\text{ m/s}^2$. For $t = 10.0$ s to 12.0 s, $a_x = \dfrac{0 - 12.0\text{ m/s}}{2.00\text{ s}} = -6.00\text{ m/s}^2$. The graph of $a_x$ versus $t$ is given in Figure 2.61.

**EVALUATE:** When $v_x$ and $a_x$ are both positive, the speed increases. When $v_x$ is positive and $a_x$ is negative, the speed decreases.

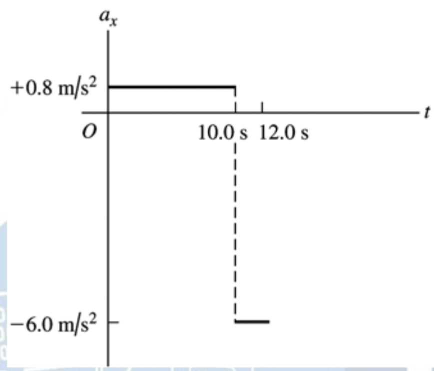



**Figure 2.61**

**2.62.** **IDENTIFY:** Apply $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ to the motion of each train. A collision means the front of the passenger train is at the same location as the caboose of the freight train at some common time.

**SET UP:** Let P be the passenger train and F be the freight train. For the front of the passenger train $x_0 = 0$ and for the caboose of the freight train $x_0 = 200$ m. For the freight train $v_F = 15.0$ m/s and $a_F = 0$. For the passenger train $v_P = 25.0$ m/s and $a_P = -0.100\text{ m/s}^2$.

**EXECUTE:** **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ for each object gives $x_P = v_P t + \frac{1}{2}a_P t^2$ and $x_F = 200\text{ m} + v_F t$. Setting $x_P = x_F$ gives $v_P t + \frac{1}{2}a_P t^2 = 200\text{ m} + v_F t$. $(0.0500\text{ m/s}^2)t^2 - (10.0\text{ m/s})t + 200\text{ m} = 0$. The quadratic formula gives $t = \dfrac{1}{0.100}\left(+10.0 \pm \sqrt{(10.0)^2 - 4(0.0500)(200)}\right)\text{ s} = (100 \pm 77.5)\text{ s}$. The collision occurs at $t = 100\text{ s} - 77.5\text{ s} = 22.5$ s. The equations that specify a collision have a physical solution (real, positive $t$), so a collision does occur.

**(b)** $x_P = (25.0\text{ m/s})(22.5\text{ s}) + \frac{1}{2}(-0.100\text{ m/s}^2)(22.5\text{ s})^2 = 537$ m. The passenger train moves 537 m before the collision. The freight train moves $(15.0\text{ m/s})(22.5\text{ s}) = 337$ m.

**(c)** The graphs of $x_F$ and $x_P$ versus $t$ are sketched in Figure 2.62.

**EVALUATE:** The second root for the equation for $t$, $t = 177.5$ s is the time the trains would meet again if they were on parallel tracks and continued their motion after the first meeting.

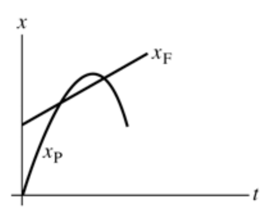



**Figure 2.62**

**2.63.** **IDENTIFY** and **SET UP:** Apply constant acceleration kinematics equations.
Find the velocity at the start of the second 5.0 s; this is the velocity at the end of the first 5.0 s. Then find $x - x_0$ for the first 5.0 s.

**EXECUTE:** For the first 5.0 s of the motion, $v_{0x} = 0,\ t = 5.0\text{ s}.$

$v_x = v_{0x} + a_x t$ gives $v_x = a_x(5.0\text{ s}).$

This is the initial speed for the second 5.0 s of the motion. For the second 5.0 s:

$v_{0x} = a_x(5.0\text{ s}),\ t = 5.0\text{ s},\ x - x_0 = 200\text{ m}.$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $200\text{ m} = (25\text{ s}^2)a_x + (12.5\text{ s}^2)a_x$ so $a_x = 5.333\text{ m/s}^2$.

Use this $a_x$ and consider the first 5.0 s of the motion:

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(5.333\text{ m/s}^2)(5.0\text{ s})^2 = 67\text{ m}.$

**EVALUATE:** The ball is speeding up so it travels farther in the second 5.0 s interval than in the first.

**2.64.** **IDENTIFY:** The insect has constant speed 15 m/s during the time it takes the cars to come together.

**SET UP:** Each car has moved 100 m when they hit.

**EXECUTE:** The time until the cars hit is $\dfrac{100\text{ m}}{10\text{ m/s}} = 10\text{ s}.$ During this time the grasshopper travels a distance of $(15\text{ m/s})(10\text{ s}) = 150\text{ m}.$

**EVALUATE:** The grasshopper ends up 100 m from where it started, so the magnitude of his final displacement is 100 m. This is less than the total distance he travels since he spends part of the time moving in the opposite direction.

**2.65.** **IDENTIFY:** Apply constant acceleration equations to each object.

Take the origin of coordinates to be at the initial position of the truck, as shown in Figure 2.65a. Let *d* be the distance that the car initially is behind the truck, so $x_0(\text{car}) = -d$ and $x_0(\text{truck}) = 0.$ Let *T* be the time it takes the car to catch the truck. Thus at time *T* the truck has undergone a displacement $x - x_0 = 60.0\text{ m},$ so is at $x = x_0 + 60.0\text{ m} = 60.0\text{ m}.$ The car has caught the truck so at time *T* is also at $x = 60.0\text{ m}.$

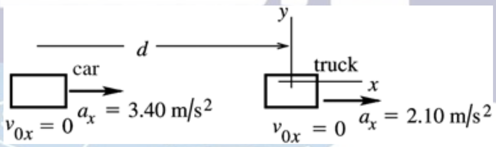



**Figure 2.65a**

**(a) SET UP:** Use the motion of the truck to calculate *T*:

$x - x_0 = 60.0\text{ m},\ v_{0x} = 0$ (starts from rest), $a_x = 2.10\text{ m/s}^2,\ t = T$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

Since $v_{0x} = 0,$ this gives $t = \sqrt{\dfrac{2(x - x_0)}{a_x}}$

**EXECUTE:** $T = \sqrt{\dfrac{2(60.0\text{ m})}{2.10\text{ m/s}^2}} = 7.56\text{ s}$

**(b) SET UP:** Use the motion of the car to calculate *d*:

$x - x_0 = 60.0\text{ m} + d,\ v_{0x} = 0,\ a_x = 3.40\text{ m/s}^2,\ t = 7.56\text{ s}$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

**EXECUTE:** $d + 60.0\text{ m} = \frac{1}{2}(3.40\text{ m/s}^2)(7.56\text{ s})^2$

$d = 97.16\text{ m} - 60.0\text{ m} = 37.2\text{ m}$ .

**(c)** car: $v_x = v_{0x} + a_x t = 0 + (3.40\text{ m/s}^2)(7.56\text{ s}) = 25.7\text{ m/s}$

truck: $v_x = v_{0x} + a_x t = 0 + (2.10\text{ m/s}^2)(7.56\text{ s}) = 15.9\text{ m/s}$

**(d)** The graph is sketched in Figure 2.65b.

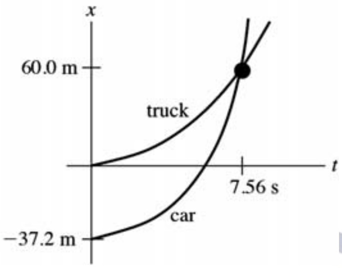



**Figure 2.65b**

**EVALUATE:** In part (c) we found that the auto was traveling faster than the truck when they came abreast. The graph in part (d) agrees with this: at the intersection of the two curves the slope of the $x$-$t$ curve for the auto is greater than that of the truck. The auto must have an average velocity greater than that of the truck since it must travel farther in the same time interval.

**2.66.** **IDENTIFY:** The bus has a constant velocity but you have a constant acceleration, starting from rest.

**SET UP:** When you catch the bus, you and the bus have been traveling for the same time, but you have traveled an extra 12.0 m during that time interval. The constant-acceleration kinematics formula $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ applies.

**EXECUTE:** Call $d$ the distance the bus travels after you start running and $t$ the time until you catch the bus. For the bus we have $d = (5.00\text{ m/s})t$, and for you we have $d + 12.0\text{ m} = (1/2)(0.960\text{ m/s}^2)t^2$. Solving these two equations simultaneously, and using the positive root, gives $t = 12.43$ s and $d = 62.14$ m. The distance you must run is 12.0 m + 62.14 m = 74.1 m. Your final speed just as you reach the bus is $v_x = (0.960\text{ m/s}^2)(12.43\text{s}) = 11.9$ m/s. This might be possible for a college runner for a brief time, but it would be highly demanding!

**EVALUATE:** Note that when you catch the bus, you are moving much faster than it is.

**2.67.** **IDENTIFY:** Apply constant acceleration equations to each vehicle.

**SET UP:** **(a)** It is very convenient to work in coordinates attached to the truck.
Note that these coordinates move at constant velocity relative to the earth. In these coordinates the truck is at rest, and the initial velocity of the car is $v_{0x} = 0$. Also, the car's acceleration in these coordinates is the same as in coordinates fixed to the earth.

**EXECUTE:** First, let's calculate how far the car must travel relative to the truck: The situation is sketched in Figure 2.67.

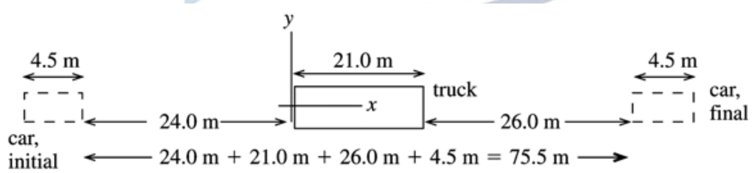



**Figure 2.67**

The car goes from $x_0 = -24.0$ m to $x = 51.5$ m. So $x - x_0 = 75.5$ m for the car.

Calculate the time it takes the car to travel this distance:

$a_x = 0.600\text{ m/s}^2,\ v_{0x} = 0,\ x - x_0 = 75.5\text{ m},\ t = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

$$t = \sqrt{\frac{2(x - x_0)}{a_x}} = \sqrt{\frac{2(75.5\text{ m})}{0.600\text{ m/s}^2}} = 15.86\text{ s}$$

It takes the car 15.9 s to pass the truck.

**(b)** Need how far the car travels relative to the earth, so go now to coordinates fixed to the earth. In these coordinates $v_{0x} = 20.0$ m/s for the car. Take the origin to be at the initial position of the car.

$v_{0x} = 20.0 \text{ m/s},\ a_x = 0.600 \text{ m/s}^2,\ t = 15.86 \text{ s},\ x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (20.0 \text{ m/s})(15.86 \text{ s}) + \frac{1}{2}(0.600 \text{ m/s}^2)(15.86 \text{ s})^2$

$x - x_0 = 317.2 \text{ m} + 75.5 \text{ m} = 393 \text{ m}.$

**(c)** In coordinates fixed to the earth:

$v_x = v_{0x} + a_x t = 20.0 \text{ m/s} + (0.600 \text{ m/s}^2)(15.86 \text{ s}) = 29.5 \text{ m/s}$

**EVALUATE:** In 15.86 s the truck travels $x - x_0 = (20.0 \text{ m/s})(15.86 \text{ s}) = 317.2 \text{ m}.$ The car travels $392.7 \text{ m} - 317.2 \text{ m} = 75 \text{ m}$ farther than the truck, which checks with part (a). In coordinates attached to the truck, for the car $v_{0x} = 0,\ v_x = 9.5 \text{ m/s}$ and in 15.86 s the car travels $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t = 75 \text{ m},$ which checks with part (a).

**2.68.** **IDENTIFY:** The acceleration is not constant so the constant acceleration equations cannot be used. Instead, use $a_x(t) = \frac{dv_x}{dt}$ and $x = x_0 + \int_0^t v_x(t)dt.$

**SET UP:** $\int t^n dt = \frac{1}{n+1}t^{n+1}$ for $n \geq 0.$

**EXECUTE:** **(a)** $x(t) = x_0 + \int_0^t [\alpha - \beta t^2]dt = x_0 + \alpha t - \frac{1}{3}\beta t^3.$ $x = 0$ at $t = 0$ gives $x_0 = 0$ and

$x(t) = \alpha t - \frac{1}{3}\beta t^3 = (4.00 \text{ m/s})t - (0.667 \text{ m/s}^3)t^3.$ $a_x(t) = \frac{dv_x}{dt} = -2\beta t = -(4.00 \text{ m/s}^3)t.$

**(b)** The maximum positive $x$ is when $v_x = 0$ and $a_x < 0.$ $v_x = 0$ gives $\alpha - \beta t^2 = 0$ and

$t = \sqrt{\frac{\alpha}{\beta}} = \sqrt{\frac{4.00 \text{ m/s}}{2.00 \text{ m/s}^3}} = 1.41 \text{ s}.$ At this $t,$ $a_x$ is negative. For $t = 1.41$ s,

$x = (4.00 \text{ m/s})(1.41 \text{ s}) - (0.667 \text{ m/s}^3)(1.41 \text{ s})^3 = 3.77 \text{ m}.$

**EVALUATE:** After $t = 1.41$ s the object starts to move in the $-x$ direction and goes to $x = -\infty$ as $t \to \infty.$

**2.69.** **(a) IDENTIFY and SET UP:** Integrate $a_x(t)$ to find $v_x(t)$ and then integrate $v_x(t)$ to find $x(t).$ We know $a_x(t) = \alpha + \beta t,$ with $\alpha = -2.00 \text{ m/s}^2$ and $\beta = 3.00 \text{ m/s}^3.$

**EXECUTE:** $v_x = v_{0x} + \int_0^t a_x\, dt = v_{0x} + \int_0^t (\alpha + \beta t)\, dt = v_{0x} + \alpha t + \frac{1}{2}\beta t^2$

$x = x_0 + \int_0^t v_x\, dt = x_0 + \int_0^t (v_{0x} + \alpha t + \frac{1}{2}\beta t^2)\, dt = x_0 + v_{0x}t + \frac{1}{2}\alpha t^2 + \frac{1}{6}\beta t^3$

At $t = 0,$ $x = x_0.$

To have $x = x_0$ at $t_1 = 4.00$ s requires that $v_{0x}t_1 + \frac{1}{2}\alpha t_1^2 + \frac{1}{6}\beta t_1^3 = 0.$

Thus $v_{0x} = -\frac{1}{6}\beta t_1^2 - \frac{1}{2}\alpha t_1 = -\frac{1}{6}(3.00 \text{ m/s}^3)(4.00 \text{ s})^2 - \frac{1}{2}(-2.00 \text{ m/s}^2)(4.00 \text{ s}) = -4.00 \text{ m/s}.$

**(b)** With $v_{0x}$ as calculated in part (a) and $t = 4.00$ s,

$v_x = v_{0x} + \alpha t + \frac{1}{2}\beta t^2 = -4.00 \text{ m/s} + (-2.00 \text{ m/s}^2)(4.00 \text{ s}) + \frac{1}{2}(3.00 \text{ m/s}^3)(4.00 \text{ s})^2 = +12.0 \text{ m/s}.$

**EVALUATE:** $a_x = 0$ at $t = 0.67$ s. For $t > 0.67$ s, $a_x > 0.$ At $t = 0,$ the particle is moving in the $-x$-direction and is speeding up. After $t = 0.67$ s, when the acceleration is positive, the object slows down and then starts to move in the $+x$-direction with increasing speed.

**2.70.** **IDENTIFY:** Find the distance the professor walks during the time $t$ it takes the egg to fall to the height of his head.

**SET UP:** Let $+y$ be downward. The egg has $v_{0y} = 0$ and $a_y = 9.80 \text{ m/s}^2.$ At the height of the professor's head, the egg has $y - y_0 = 44.2$ m.

**EXECUTE:** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(44.2 \text{ m})}{9.80 \text{ m/s}^2}} = 3.00 \text{ s}.$ The professor walks a distance $x - x_0 = v_{0x}t = (1.20 \text{ m/s})(3.00 \text{ s}) = 3.60 \text{ m}.$ Release the egg when your professor is 3.60 m from the point directly below you.

**EVALUATE:** Just before the egg lands its speed is $(9.80\ \text{m/s}^2)(3.00\ \text{s}) = 29.4\ \text{m/s}$. It is traveling much faster than the professor.

**2.71.** **IDENTIFY:** Use the constant acceleration equations to establish a relationship between maximum height and acceleration due to gravity and between time in the air and acceleration due to gravity.

**SET UP:** Let $+y$ be upward. At the maximum height, $v_y = 0$. When the rock returns to the surface, $y - y_0 = 0$.

**EXECUTE:** **(a)** $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $a_y H = -\frac{1}{2}v_{0y}^2$, which is constant, so $a_\text{E} H_\text{E} = a_\text{M} H_\text{M}$.

$$H_\text{M} = H_\text{E}\left(\frac{a_\text{E}}{a_\text{M}}\right) = H\left(\frac{9.80\ \text{m/s}^2}{3.71\ \text{m/s}^2}\right) = 2.64H.$$

**(b)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = 0$ gives $a_y t = -2v_{0y}$, which is constant, so $a_\text{E} T_\text{E} = a_\text{M} T_\text{M}$.

$$T_\text{M} = T_\text{E}\left[\frac{a_\text{E}}{a_\text{M}}\right] = 2.64T.$$

**EVALUATE:** On Mars, where the acceleration due to gravity is smaller, the rocks reach a greater height and are in the air for a longer time.

**2.72.** **IDENTIFY:** Calculate the time it takes her to run to the table and return. This is the time in the air for the thrown ball. The thrown ball is in free-fall after it is thrown. Assume air resistance can be neglected.

**SET UP:** For the thrown ball, let $+y$ be upward. $a_y = -9.80\ \text{m/s}^2$. $y - y_0 = 0$ when the ball returns to its original position. The constant-acceleration kinematics formulas apply.

**EXECUTE:** **(a)** It takes her $\dfrac{5.50\ \text{m}}{3.00\ \text{m/s}} = 1.833\ \text{s}$ to reach the table and an equal time to return, so the total time ball is in the air is 3.667 s. For the ball, $y - y_0 = 0$, $t = 3.667\ \text{s}$ and $a_y = -9.80\ \text{m/s}^2$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $v_{0y} = -\frac{1}{2}a_y t = -\frac{1}{2}(-9.80\ \text{m/s}^2)(3.667\ \text{s}) = 18.0\ \text{m/s}$.

**(b)** Find $y - y_0$ when $t = 1.833\ \text{s}$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (18.0\ \text{m/s})(1.833\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(1.833\ \text{s})^2 = 16.5\ \text{m}$.

**EVALUATE:** It takes the ball the same amount of time to reach its maximum height as to return from its maximum height, so when she is at the table the ball is at its maximum height. Note that this large maximum height requires that the act either be done outdoors, or in a building with a very high ceiling.

**2.73.** **(a) IDENTIFY:** Consider the motion from when he applies the acceleration to when the shot leaves his hand.

**SET UP:** Take positive $y$ to be upward. $v_{0y} = 0$, $v_y = ?$, $a_y = 35.0\ \text{m/s}^2$, $y - y_0 = 0.640\ \text{m}$, $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

**EXECUTE:** $v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(35.0\ \text{m/s}^2)(0.640\ \text{m})} = 6.69\ \text{m/s}$

**(b) IDENTIFY:** Consider the motion of the shot from the point where he releases it to its maximum height, where $v = 0$. Take $y = 0$ at the ground.

**SET UP:** $y_0 = 2.20\ \text{m}$, $y = ?$, $a_y = -9.80\ \text{m/s}^2$ (free fall), $v_{0y} = 6.69\ \text{m/s}$ (from part (a), $v_y = 0$ at maximum height), $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

**EXECUTE:** $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (6.69\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 2.29\ \text{m}$, $y = 2.20\ \text{m} + 2.29\ \text{m} = 4.49\ \text{m}$.

**(c) IDENTIFY:** Consider the motion of the shot from the point where he releases it to when it returns to the height of his head. Take $y = 0$ at the ground.

**SET UP:** $y_0 = 2.20\ \text{m}$, $y = 1.83\ \text{m}$, $a_y = -9.80\ \text{m/s}^2$, $v_{0y} = +6.69\ \text{m/s}$, $t = ?$ $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

**EXECUTE:** $1.83\ \text{m} - 2.20\ \text{m} = (6.69\ \text{m/s})t + \frac{1}{2}(-9.80\ \text{m/s}^2)t^2 = (6.69\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2$,

$4.90t^2 - 6.69t - 0.37 = 0$, with $t$ in seconds. Use the quadratic formula to solve for $t$:

$t = \dfrac{1}{9.80}\left(6.69 \pm \sqrt{(6.69)^2 - 4(4.90)(-0.37)}\right) = 0.6830 \pm 0.7362$. Since $t$ must be positive,
$t = 0.6830\text{ s} + 0.7362\text{ s} = 1.42\text{ s}$.

**EVALUATE:** Calculate the time to the maximum height: $v_y = v_{0y} + a_y t$, so $t = (v_y - v_{0y})/a_y =$ $-(6.69\text{ m/s})/(-9.80\text{ m/s}^2) = 0.68\text{ s}$. It also takes 0.68 s to return to 2.2 m above the ground, for a total time of 1.36 s. His head is a little lower than 2.20 m, so it is reasonable for the shot to reach the level of his head a little later than 1.36 s after being thrown; the answer of 1.42 s in part (c) makes sense.

**2.74.** **IDENTIFY:** The flowerpot is in free-fall. Apply the constant acceleration equations. Use the motion past the window to find the speed of the flowerpot as it reaches the top of the window. Then consider the motion from the windowsill to the top of the window.

**SET UP:** Let $+y$ be downward. Throughout the motion $a_y = +9.80\text{ m/s}^2$. The constant-acceleration kinematics formulas all apply.

**EXECUTE:** Motion past the window: $y - y_0 = 1.90\text{ m}$, $t = 0.380\text{ s}$, $a_y = +9.80\text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $v_{0y} = \dfrac{y - y_0}{t} - \frac{1}{2}a_y t = \dfrac{1.90\text{ m}}{0.380\text{ s}} - \frac{1}{2}(9.80\text{ m/s}^2)(0.380\text{ s}) = 3.138\text{ m/s}$. This is the velocity of the flowerpot when it is at the top of the window.

Motion from the windowsill to the top of the window: $v_{0y} = 0$, $v_y = 2.466\text{ m/s}$, $a_y = +9.80\text{ m/s}^2$.
$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{(3.138\text{ m/s})^2 - 0}{2(9.80\text{ m/s}^2)} = 0.502\text{ m}$. The top of the window is 0.502 m below the windowsill.

**EVALUATE:** It takes the flowerpot $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{3.138\text{ m/s}}{9.80\text{ m/s}^2} = 0.320\text{ s}$ to fall from the sill to the top of the window. Our result says that from the windowsill the pot falls $0.502\text{ m} + 1.90\text{ m} = 2.4\text{ m}$ in $0.320\text{ s} + 0.380\text{ s} = 0.700\text{ s}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(9.80\text{ m/s}^2)(0.700\text{ s})^2 = 2.4\text{ m}$, which checks.

**2.75.** **IDENTIFY:** Two stones are thrown up with different speeds. (a) Knowing how soon the faster one returns to the ground, how long it will take the slow one to return? (b) Knowing how high the slower stone went, how high did the faster stone go?

**SET UP:** Use subscripts f and s to refer to the faster and slower stones, respectively. Take $+y$ to be upward and $y_0 = 0$ for both stones. $v_{0\text{f}} = 3v_{0\text{s}}$. When a stone reaches the ground, $y = 0$. The constant-acceleration formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ both apply.

**EXECUTE:** **(a)** $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives $a_y = -\dfrac{2v_{0y}}{t}$. Since both stones have the same $a_y$, $\dfrac{v_{0\text{f}}}{t_\text{f}} = \dfrac{v_{0\text{s}}}{t_\text{s}}$ and

$$t_\text{s} = t_\text{f}\left(\frac{v_{0\text{s}}}{v_{0\text{f}}}\right) = \left(\tfrac{1}{3}\right)(10\text{ s}) = 3.3\text{ s}.$$

**(b)** Since $v_y = 0$ at the maximum height, then $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $a_y = -\dfrac{v_{0y}^2}{2y}$. Since both have the same $a_y$, $\dfrac{v_{0\text{f}}^2}{y_\text{f}} = \dfrac{v_{0\text{s}}^2}{y_\text{s}}$ and $y_\text{f} = y_\text{s}\left(\dfrac{v_{0\text{f}}}{v_{0\text{s}}}\right)^2 = 9H$.

**EVALUATE:** The faster stone reaches a greater height so it travels a greater distance than the slower stone and takes more time to return to the ground.

**2.76.** **IDENTIFY:** The motion of the rocket can be broken into 3 stages, each of which has constant acceleration, so in each stage we can use the standard kinematics formulas for constant acceleration. But the acceleration is not the same throughout all 3 stages.

**SET UP:** The formulas $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$, $y - y_0 = v_{0y}t + \dfrac{1}{2}a_y t^2$, and $v_y = v_{0y} + a_y t$ apply.

**EXECUTE:** **(a)** Let $+y$ be upward. At $t = 25.0\text{ s}$, $y - y_0 = 1094\text{ m}$ and $v_y = 87.5\text{ m/s}$. During the next 10.0 s the rocket travels upward an additional distance $y - y_0 = \left(\frac{v_{0y} + v_y}{2}\right)t = \left(\frac{87.5\text{ m/s} + 132.5\text{ m/s}}{2}\right)(10.0\text{ s}) =$ $1100\text{ m}$. The height above the launch pad when the second stage quits therefore is $1094\text{ m} + 1100\text{ m} =$ $2194\text{ m}$. For the free-fall motion after the second stage quits: $y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (132.5\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} = 896\text{ m}$.

The maximum height above the launch pad that the rocket reaches is $2194\text{ m} + 896\text{ m} = 3090\text{ m}$.

**(b)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $-2194\text{ m} = (132.5\text{ m/s})t - (4.9\text{ m/s}^2)t^2$. From the quadratic formula the positive root is $t = 38.6\text{ s}$.

**(c)** $v_y = v_{0y} + a_y t = 132.5\text{ m/s} + (-9.8\text{ m/s}^2)(38.6\text{ s}) = -246\text{ m/s}$. The rocket's speed will be 246 m/s just before it hits the ground.

**EVALUATE:** We cannot solve this problem in a single step because the acceleration, while constant in each stage, is not constant over the entire motion. The standard kinematics equations apply to each stage but not to the motion as a whole.

**2.77.** **IDENTIFY:** The rocket accelerates uniformly upward at 16.0 m/s$^2$ with the engines on. After the engines are off, it moves upward but accelerates downward at 9.80 m/s$^2$.

**SET UP:** The formulas $y - y_0 = v_{0_y}t = \frac{1}{2}a_y t^2$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ both apply to both parts of the motion since the accelerations are both constant, but the accelerations are different in both cases. Let $+y$ be upward.

**EXECUTE:** With the engines on, $v_{oy} = 0$, $a_y = 16.0$ m/s$^2$ upward, and $t = T$ at the instant the engines just shut off. Using these quantities, we get

$y - y_0 = v_{0_y}t + \frac{1}{2}a_y t^2 = (8.00\text{ m/s}^2)T^2$ and $v_y = v_{0y} + a_y t = (16.0\text{ m/s}^2)T$.

With the engines off (free fall), the formula $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ for the highest point gives $y - y_0 = (13.06\text{ m/s}^2)T^2$, using $v_{0y} = (16.0\text{ m/s}^2)T$, $v_y = 0$, and $a_y = -9.80\text{ m/s}^2$.

The total height reached is 960 m, so (distance in free-fall) + (distance with engines on) = 960 m. Therefore $(13.06\text{ m/s}^2)\,T^2 + (8.00\text{ m/s}^2)\,T^2 = 960\text{ m}$, which gives $T = 6.75$ s.

**EVALUATE:** It we put in 6.75 s for $T$, we see that the rocket travels considerably farther during free fall than with the engines on.

**2.78.** **IDENTIFY:** The teacher is in free-fall and falls with constant acceleration $9.80\text{ m/s}^2$, downward. The sound from her shout travels at constant speed. The sound travels from the top of the cliff, reflects from the ground and then travels upward to her present location. If the height of the cliff is $h$ and she falls a distance $y$ in 3.0 s, the sound must travel a distance $h + (h - y)$ in 3.0 s.

**SET UP:** Let $+y$ be downward, so for the teacher $a_y = 9.80\text{ m/s}^2$ and $v_{0y} = 0$. Let $y = 0$ at the top of the cliff.

**EXECUTE:** **(a)** For the teacher, $y = \frac{1}{2}(9.80\text{ m/s}^2)(3.0\text{ s})^2 = 44.1\text{ m}$. For the sound, $h + (h - y) = v_s t$.

$h = \frac{1}{2}(v_s t + y) = \frac{1}{2}\left([340\text{ m/s}][3.0\text{ s}] + 44.1\text{ m}\right) = 532\text{ m}$, which rounds to 530 m.

**(b)** $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(9.80\text{ m/s}^2)(532\text{ m})} = 102\text{ m/s}$.

**EVALUATE:** She is in the air for $t = \frac{v_y - v_{0y}}{a_y} = \frac{102\text{ m/s}}{9.80\text{ m/s}^2} = 10.4\text{ s}$ and strikes the ground at high speed.

**2.79.** **IDENTIFY:** The helicopter has two segments of motion with constant acceleration: upward acceleration for 10.0 s and then free-fall until it returns to the ground. Powers has three segments of motion with constant acceleration: upward acceleration for 10.0 s, free-fall for 7.0 s and then downward acceleration of $2.0\text{ m/s}^2$.

**SET UP:** Let $+y$ be upward. Let $y = 0$ at the ground.

**EXECUTE:** **(a)** When the engine shuts off both objects have upward velocity $v_y = v_{0y} + a_y t =$ $(5.0\text{ m/s}^2)(10.0\text{ s}) = 50.0\text{ m/s}$ and are at $y = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(5.0\text{ m/s}^2)(10.0\text{ s})^2 = 250\text{ m}$.

For the helicopter, $v_y = 0$ (at the maximum height), $v_{0y} = +50.0\text{ m/s}$, $y_0 = 250\text{ m}$, and $a_y = -9.80\text{ m/s}^2$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y = \dfrac{v_y^2 - v_{0y}^2}{2a_y} + y_0 = \dfrac{0 - (50.0\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} + 250\text{ m} = 378\text{ m}$, which rounds to 380 m.

**(b)** The time for the helicopter to crash from the height of 250 m where the engines shut off can be found using $v_{0y} = +50.0\text{ m/s}$, $a_y = -9.80\text{ m/s}^2$, and $y - y_0 = -250\text{ m}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $-250\text{ m} = (50.0\text{ m/s})t - (4.90\text{ m/s}^2)t^2$. $(4.90\text{ m/s}^2)t^2 - (50.0\text{ m/s})t - 250\text{ m} = 0$. The quadratic formula gives $t = \dfrac{1}{9.80}\left(50.0 \pm \sqrt{(50.0)^2 + 4(4.90)(250)}\right)$ s. Only the positive solution is physical, so $t = 13.9\text{ s}$.

Powers therefore has free-fall for 7.0 s and then downward acceleration of $2.0\text{ m/s}^2$ for $13.9\text{ s} - 7.0\text{ s} = 6.9\text{ s}$. After 7.0 s of free-fall he is at $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 250\text{ m} + (50.0\text{ m/s})(7.0\text{ s}) +$ $\frac{1}{2}(-9.80\text{ m/s}^2)(7.0\text{ s})^2 = 360\text{ m}$ and has velocity $v_x = v_{0x} + a_x t = 50.0\text{ m/s} + (-9.80\text{ m/s}^2)(7.0\text{ s}) =$ $-18.6\text{ m/s}$. After the next 6.9 s he is at $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 360\text{ m} + (-18.6\text{ m/s})(6.9\text{ s}) +$ $\frac{1}{2}(-2.00\text{ m/s}^2)(6.9\text{ s})^2 = 184\text{ m}$. Powers is 184 m above the ground when the helicopter crashes.

**EVALUATE:** When Powers steps out of the helicopter he retains the initial velocity he had in the helicopter but his acceleration changes abruptly from $5.0\text{ m/s}^2$ upward to $9.80\text{ m/s}^2$ downward. Without the jet pack he would have crashed into the ground at the same time as the helicopter. The jet pack slows his descent so he is above the ground when the helicopter crashes.

**2.80.** **IDENTIFY:** Apply constant acceleration equations to the motion of the rock. Sound travels at constant speed.

**SET UP:** Let $t_f$ be the time for the rock to fall to the ground and let $t_s$ be the time it takes the sound to travel from the impact point back to you. $t_f + t_s = 8.00\text{ s}$. Both the rock and sound travel a distance $h$ that is equal to the height of the cliff. Take $+y$ downward for the motion of the rock. The rock has $v_{0y} = 0$ and $a_y = g = 9.80\text{ m/s}^2$.

**EXECUTE:** **(a)** For the falling rock, $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $h = \frac{1}{2}g t_f^2$. For the sound, $h = v_s t_s$. Equating these two equations for $h$ and using the fact that $t_f + t_s = 8.00\text{ s}$, we get $\frac{1}{2}g t_f^2 = v_s t_s = v_s(8.00\text{ s} - t_f)$. Using $v_s = 330\text{ m/s}$ and $g = 9.80\text{ m/s}^2$, we get a quadratic equation. Solving it using the quadratic formula and using the positive square root, we get $t_f = 7.225\text{ s}$. Therefore $h = \frac{1}{2}g t_f^2 = (1/2)(9.80\text{ m/s}^2)(7.225\text{ s})^2 = 256\text{ m}$.

**(b)** Ignoring sound you would calculate $d = \frac{1}{2}(9.80\text{ m/s}^2)(8.00\text{ s})^2 = 314\text{ m}$, which is greater than the actual distance. So you would have overestimated the height of the cliff. It actually takes the rock less time than 8.00 s to fall to the ground.

**EVALUATE:** Once we know $h$ we can calculate that $t_f = 7.225\text{ s}$ and $t_s = 0.775\text{ s}$. The time for the sound of impact to travel back to you is 6% of the total time and should not be neglected for best precision.

**2.81.** **(a) IDENTIFY:** We have nonconstant acceleration, so we must use calculus instead of the standard kinematics formulas.

**SET UP:** We know the acceleration as a function of time is $a_x(t) = -Ct$, so we can integrate to find the velocity and then the $x$-coordinate of the object. We know that $v_x(t) = v_{0x} + \int_0^t a_x dt$ and $x(t) = x_0 + \int_0^t v_x(t)dt$.

**EXECUTE:** **(a)** We have information about the velocity, so we need to find that by integrating the acceleration. $v_x(t) = v_{0x} + \int_0^t a_x dt = v_{0x} + \int_0^t -Ctdt = v_{0x} - \frac{1}{2}Ct^2$. Using the facts that the initial velocity is 20.0 m/s and $v_x = 0$ when $t = 8.00$ s, we have $0 = 20.0\text{ m/s} - C(8.00\text{ s})^2/2$, which gives $C = 0.625\text{ m/s}^3$.

**(b)** We need the change in position during the first 8.00 s. Using $x(t) = x_0 + \int_0^t v_x(t)dt$ gives

$$x - x_0 = \int_0^t \left(-\tfrac{1}{2}Ct^2 + (20.0\text{ m/s})\right)dt = -Ct^3/6 + (20.0\text{ m/s})t$$

Putting in $C = 0.625$ m/s$^3$ and $t = 8.00$ s gives an answer of 107 m.

**EVALUATE:** The standard kinematics formulas are of no use in this problem since the acceleration varies with time.

**2.82.** **IDENTIFY:** Both objects are in free-fall and move with constant acceleration $9.80\text{ m/s}^2$, downward. The two balls collide when they are at the same height at the same time.

**SET UP:** Let $+y$ be upward, so $a_y = -9.80\text{ m/s}^2$ for each ball. Let $y = 0$ at the ground. Let ball $A$ be the one thrown straight up and ball $B$ be the one dropped from rest at height $H$. $y_{0A} = 0$, $y_{0B} = H$.

**EXECUTE:** **(a)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ applied to each ball gives $y_A = v_0 t - \frac{1}{2}gt^2$ and $y_B = H - \frac{1}{2}gt^2$.

$y_A = y_B$ gives $v_0 t - \frac{1}{2}gt^2 = H - \frac{1}{2}gt^2$ and $t = \dfrac{H}{v_0}$.

**(b)** For ball $A$ at its highest point, $v_{yA} = 0$ and $v_y = v_{0y} + a_y t$ gives $t = \dfrac{v_0}{g}$. Setting this equal to the time in part (a) gives $\dfrac{H}{v_0} = \dfrac{v_0}{g}$ and $H = \dfrac{v_0^2}{g}$.

**EVALUATE:** In part (a), using $t = \dfrac{H}{v_0}$ in the expressions for $y_A$ and $y_B$ gives $y_A = y_B = H\left(1 - \dfrac{gH}{2v_0^2}\right)$.

$H$ must be less than $\dfrac{2v_0^2}{g}$ in order for the balls to collide before ball $A$ returns to the ground. This is because it takes ball $A$ time $t = \dfrac{2v_0}{g}$ to return to the ground and ball $B$ falls a distance $\frac{1}{2}gt^2 = \dfrac{2v_0^2}{g}$ during this time. When $H = \dfrac{2v_0^2}{g}$ the two balls collide just as ball $A$ reaches the ground and for $H$ greater than this ball $A$ reaches the ground before they collide.

**2.83.** **IDENTIFY** and **SET UP:** Use $v_x = dx/dt$ and $a_x = dv_x/dt$ to calculate $v_x(t)$ and $a_x(t)$ for each car. Use these equations to answer the questions about the motion.

**EXECUTE:** $x_A = \alpha t + \beta t^2$, $v_{Ax} = \dfrac{dx_A}{dt} = \alpha + 2\beta t$, $a_{Ax} = \dfrac{dv_{Ax}}{dt} = 2\beta$

$x_B = \gamma t^2 - \delta t^3$, $v_{Bx} = \dfrac{dx_B}{dt} = 2\gamma t - 3\delta t^2$, $a_{Bx} = \dfrac{dv_{Bx}}{dt} = 2\gamma - 6\delta t$

**(a) IDENTIFY** and **SET UP:** The car that initially moves ahead is the one that has the larger $v_{0x}$.

**EXECUTE:** At $t = 0$, $v_{Ax} = \alpha$ and $v_{Bx} = 0$. So initially car $A$ moves ahead.

**(b) IDENTIFY** and **SET UP:** Cars at the same point implies $x_A = x_B$.

$\alpha t + \beta t^2 = \gamma t^2 - \delta t^3$

**EXECUTE:** One solution is $t = 0$, which says that they start from the same point. To find the other solutions, divide by $t$: $\alpha + \beta t = \gamma t - \delta t^2$

$\delta t^2 + (\beta - \gamma)t + \alpha = 0$

$$t = \frac{1}{2\delta}\left(-(\beta - \gamma) \pm \sqrt{(\beta - \gamma)^2 - 4\delta\alpha}\right) = \frac{1}{0.40}\left(+1.60 \pm \sqrt{(1.60)^2 - 4(0.20)(2.60)}\right) = 4.00\text{ s} \pm 1.73\text{ s}$$

So $x_A = x_B$ for $t = 0$, $t = 2.27$ s and $t = 5.73$ s.

**EVALUATE:** Car $A$ has constant, positive $a_x$. Its $v_x$ is positive and increasing. Car $B$ has $v_{0x} = 0$ and $a_x$ that is initially positive but then becomes negative. Car $B$ initially moves in the $+x$-direction but then slows down and finally reverses direction. At $t = 2.27$ s car $B$ has overtaken car $A$ and then passes it. At $t = 5.73$ s, car $B$ is moving in the $-x$-direction as it passes car $A$ again.

**(c) IDENTIFY:** The distance from $A$ to $B$ is $x_B - x_A$. The rate of change of this distance is $\frac{d(x_B - x_A)}{dt}$. If this distance is not changing, $\frac{d(x_B - x_A)}{dt} = 0$. But this says $v_{Bx} - v_{Ax} = 0$. (The distance between $A$ and $B$ is neither decreasing nor increasing at the instant when they have the same velocity.)

**SET UP:** $v_{Ax} = v_{Bx}$ requires $\alpha + 2\beta t = 2\gamma t - 3\delta t^2$

**EXECUTE:** $3\delta t^2 + 2(\beta - \gamma)t + \alpha = 0$

$$t = \frac{1}{6\delta}\left(-2(\beta-\gamma) \pm \sqrt{4(\beta-\gamma)^2 - 12\delta\alpha}\right) = \frac{1}{1.20}\left(3.20 \pm \sqrt{4(-1.60)^2 - 12(0.20)(2.60)}\right)$$

$t = 2.667\text{ s} \pm 1.667\ s$, so $v_{Ax} = v_{Bx}$ for $t = 1.00\text{ s}$ and $t = 4.33\text{ s}$.

**EVALUATE:** At $t = 1.00\text{ s}$, $v_{Ax} = v_{Bx} = 5.00\text{ m/s}$. At $t = 4.33\text{ s}$, $v_{Ax} = v_{Bx} = 13.0\text{ m/s}$. Now car $B$ is slowing down while $A$ continues to speed up, so their velocities aren't ever equal again.

**(d) IDENTIFY and SET UP:** $a_{Ax} = a_{Bx}$ requires $2\beta = 2\gamma - 6\delta t$

**EXECUTE:** $t = \frac{\gamma - \beta}{3\delta} = \frac{2.80\text{ m/s}^2 - 1.20\text{ m/s}^2}{3(0.20\text{ m/s}^3)} = 2.67\text{ s}.$

**EVALUATE:** At $t = 0$, $a_{Bx} > a_{Ax}$, but $a_{Bx}$ is decreasing while $a_{Ax}$ is constant. They are equal at $t = 2.67\text{ s}$ but for all times after that $a_{Bx} < a_{Ax}$.

**2.84.** **IDENTIFY:** Interpret the data on a graph to draw conclusions about the motion of a glider having constant acceleration down a frictionless air track, starting from rest at the top.

**SET UP:** The constant-acceleration kinematics formulas apply. Take the +$x$-axis along the surface of the track pointing downward.

**EXECUTE:** **(a)** For constant acceleration starting from rest, we have $x = \frac{1}{2}a_x t^2$. Therefore a plot of $x$ versus $t^2$ should be a straight line, and the slope of that line should be $a_x/2$.

**(b)** To construct the graph of $x$ versus $t^2$, we can use readings from the graph given in the text to construct a table of values for $x$ and $t^2$, or we could use graphing software if available. The result is a graph similar to the one shown in Figure 2.84, which was obtained using software. A graph done by hand could vary slightly from this one, depending on how one reads the values on the graph in the text. The graph shown is clearly a straight line having slope 3.77 m/s$^2$ and $x$-intercept 0.0092 m. Using the slope $y$-intercept form of the equation of a straight line, the equation of this line is $x = 3.77t^2 + 0.0092$, where $x$ is in meters and $t$ in seconds.

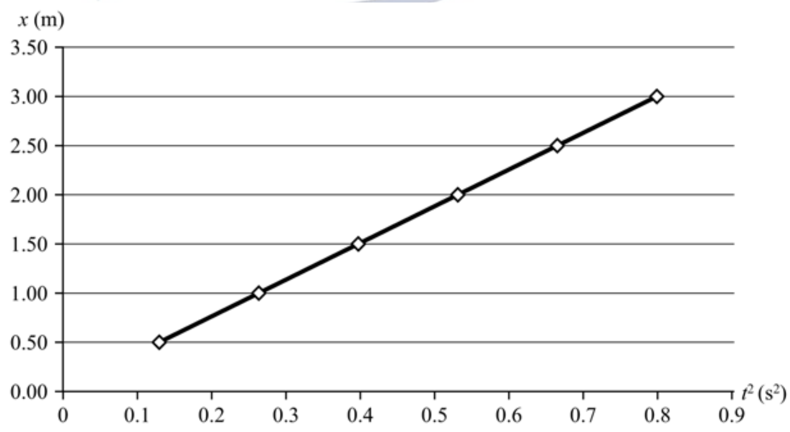



**Figure 2.84**

**(c)** The slope of the straight line in the graph is $a_x/2$, so $a_x = 2(3.77\ \text{m/s}^2) = 7.55\ \text{m/s}^2$.
**(d)** We know the distance traveled is 1.35 m, the acceleration is 7.55 m/s$^2$, and the initial velocity is zero, so we use the equation $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ and solve for $v_x$, giving $v_x = 4.51$ m/s.

**EVALUATE:** For constant acceleration in part (d), the average velocity is (4.51 m/s)/2 = 2.25 m/s. With this average velocity, the time for the glider to travel 1.35 m is $x/v_{\text{av}}$ = (1.35 m)/(2.25 m) = 0.6 s, which is approximately the value of $t$ read from the graph in the text for $x = 1.35$ m.

**2.85.** **IDENTIFY:** A ball is dropped from rest and falls from various heights with constant acceleration. Interpret a graph of the square of its velocity just as it reaches the floor as a function of its release height.
**SET UP:** Let $+y$ be downward since all motion is downward. The constant-acceleration kinematics formulas apply for the ball.
**EXECUTE:** **(a)** The equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ applies to the falling ball. Solving for $y - y_0$ and using $v_{0y} = 0$ and $a_y = g$, we get $y - y_0 = \dfrac{v_y^2}{2g}$. A graph of $y - y_0$ versus $v_y^2$ will be a straight line with slope $1/2g$ = $1/(19.6\ \text{m/s}^2) = 0.0510\ \text{s}^2/\text{m}$.
**(b)** With air resistance the acceleration is less than 9.80 m/s$^2$, so the final speed will be smaller.
**(c)** The graph will not be a straight line because the acceleration will vary with the speed of the ball. For a given release height, $v_y$ with air resistance is less than without it. Alternatively, with air resistance the ball will have to fall a greater distance to achieve a given velocity than without air resistance. The graph is sketched in Figure 2.85.

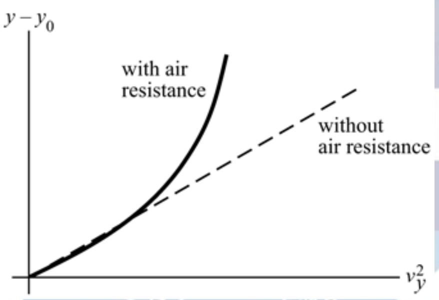



**Figure 2.85**

**EVALUATE:** Graphing $y - y_0$ versus $v_y^2$ for a set of data will tell us if the acceleration is constant. If the graph is a straight line, the acceleration is constant; if not, the acceleration is not constant.

**2.86.** **IDENTIFY:** Use data of acceleration and time for a model car to find information about its velocity and position.
**SET UP:** From the table of data in the text, we can see that the acceleration is not constant, so the constant-acceleration kinematics formlas do not apply. Therefore we must use calculus. The equations $v_x(t) = v_{0x} + \int_0^t a_x dt$ and $x(t) = x_0 + \int_0^t v_x dt$ apply.
**EXECUTE:** **(a)** Figure 2.86a shows the graph of $a_x$ versus $t$. From the graph, we find that the slope of the line is –0.5131 m/s$^3$ and the $a$-intercept is 6.026 m/s$^2$. Using the slope $y$-intercept equation of a straight line, the equation is $a(t) = -0.513\ \text{m/s}^3\ t + 6.026\ \text{m/s}^2$, where $t$ is in seconds and $a$ is in m/s$^2$.

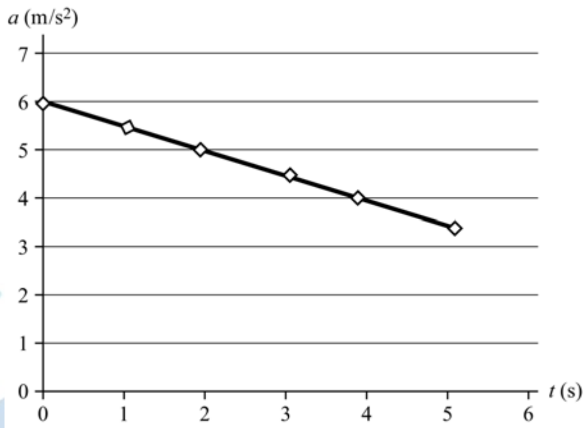



**Figure 2.86a**

**(b)** Integrate the acceleration to find the velocity, with the initial velocity equal to zero.

$v_x(t) = v_{0x} + \int_0^t a_x dt = v_{0x} + \int_0^t (6.026 \text{ m/s}^2 - 0.513 \text{ m/s}^3\ t)dt = 6.026 \text{ m/s}^2\ t - 0.2565 \text{ m/s}^3\ t^2$.

Figure 2.86b shows a sketch of the graph of $v_x$ versus $t$.

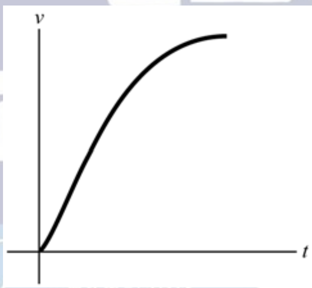



**Figure 2.86b**

**(c)** Putting $t = 5.00$ s into the equation we found in (b) gives $v_x = 23.7$ m/s.
**(d)** Integrate the velocity to find the change in position of the car.

$x - x_0 = \int_0^t v_x dt = \int_0^t [(6.026 \text{ m/s}^2)t - (0.2565 \text{ m/s}^3)t^2]dt = 3.013 \text{ m/s}^2\ t^2 - 0.0855 \text{ m/s}^3\ t^3$

At $t = 5.00$ s, this gives $x - x_0 = 64.6$ m.

**EVALUATE:** Since the acceleration is not constant, the standard kinematics formulas do not apply, so we must go back to basic definitions involving calculus.

**2.87. IDENTIFY:** Apply $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ to the motion from the maximum height, where $v_{0y} = 0$. The time spent above $y_{\text{max}}/2$ on the way down equals the time spent above $y_{\text{max}}/2$ on the way up.

**SET UP:** Let $+y$ be downward. $a_y = g$. $y - y_0 = y_{\text{max}}/2$ when he is a distance $y_{\text{max}}/2$ above the floor.

**EXECUTE:** The time from the maximum height to $y_{\text{max}}/2$ above the floor is given by $y_{\text{max}}/2 = \frac{1}{2}gt_1^2$. The time from the maximum height to the floor is given by $y_{\text{max}} = \frac{1}{2}gt_{\text{tot}}^2$ and the time from a height of $y_{\text{max}}/2$ to the floor is $t_2 = t_{\text{tot}} - t_1$.

$$\frac{2t_1}{t_2} = \frac{2\sqrt{y_{\text{max}}/2}}{\sqrt{y_{\text{max}}} - \sqrt{y_{\text{max}}/2}} = \frac{2}{\sqrt{2} - 1} = 4.8.$$

**EVALUATE:** The person spends over twice as long above $y_{\text{max}}/2$ as below $y_{\text{max}}/2$. His average speed is less above $y_{\text{max}}/2$ than it is when he is below this height.

**2.88.** **IDENTIFY:** Apply constant acceleration equations to the motion of the two objects, the student and the bus.

**SET UP:** For convenience, let the student's (constant) speed be $v_0$ and the bus's initial position be $x_0$. Note that these quantities are for separate objects, the student and the bus. The initial position of the student is taken to be zero, and the initial velocity of the bus is taken to be zero. The positions of the student $x_1$ and the bus $x_2$ as functions of time are then $x_1 = v_0 t$ and $x_2 = x_0 + (1/2)at^2$.

**EXECUTE:** **(a)** Setting $x_1 = x_2$ and solving for the times $t$ gives $t = \frac{1}{a}\left(v_0 \pm \sqrt{v_0^2 - 2ax_0}\right)$.

$$t = \frac{1}{0.170\ \text{m/s}^2}\left(5.0\ \text{m/s} \pm \sqrt{(5.0\ \text{m/s})^2 - 2(0.170\ \text{m/s}^2)(40.0\ \text{m})}\right) = 9.55\ \text{s and } 49.3\ \text{s}.$$

The student will be likely to hop on the bus the first time she passes it (see part (d) for a discussion of the later time). During this time, the student has run a distance $v_0 t = (5\ \text{m/s})(9.55\ \text{s}) = 47.8\ \text{m}$.

**(b)** The speed of the bus is $(0.170\ \text{m/s}^2)(9.55\ \text{s}) = 1.62\ \text{m/s}$.

**(c)** The results can be verified by noting that the $x$ lines for the student and the bus intersect at two points, as shown in Figure 2.88a.

**(d)** At the later time, the student has passed the bus, maintaining her constant speed, but the accelerating bus then catches up to her. At this later time the bus's velocity is $(0.170\ \text{m/s}^2)(49.3\ \text{s}) = 8.38\ \text{m/s}$.

**(e)** No; $v_0^2 < 2ax_0$, and the roots of the quadratic are imaginary. When the student runs at $3.5\ \text{m/s}$, Figure 2.88b shows that the two lines do *not* intersect.

**(f)** For the student to catch the bus, $v_0^2 > 2ax_0$. And so the minimum speed is $\sqrt{2(0.170\ \text{m/s}^2)(40\ \text{m/s})} =$ $3.688\ \text{m/s}$. She would be running for a time $\frac{3.69\ \text{m/s}}{0.170\ \text{m/s}^2} = 21.7\ \text{s}$, and covers a distance $(3.688\ \text{m/s})(21.7\ \text{s}) =$ $80.0\ \text{m}$. However, when the student runs at $3.688\ \text{m/s}$, the lines intersect at *one* point, at $x = 80\ \text{m}$, as shown in Figure 2.88c.

**EVALUATE:** The graph in part (c) shows that the student is traveling faster than the bus the first time they meet but at the second time they meet the bus is traveling faster.

$t_2 = t_{\text{tot}} - t_1$

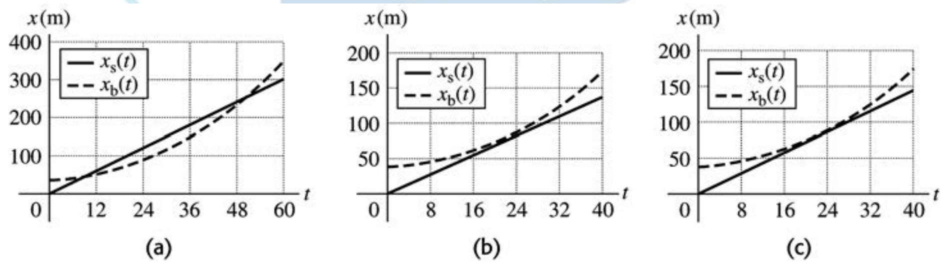



**Figure 2.88**

**2.89.** **IDENTIFY:** Apply constant acceleration equations to both objects.

**SET UP:** Let $+y$ be upward, so each ball has $a_y = -g$. For the purpose of doing all four parts with the least repetition of algebra, quantities will be denoted symbolically. That is, let $y_1 = h + v_0 t - \frac{1}{2}gt^2$, $y_2 = h - \frac{1}{2}g(t - t_0)^2$. In this case, $t_0 = 1.00\ \text{s}$.

**EXECUTE:** **(a)** Setting $y_1 = y_2 = 0,$ expanding the binomial $(t - t_0)^2$ and eliminating the common term $\frac{1}{2}gt^2$ yields $v_0 t = gt_0 t - \frac{1}{2}gt_0^2.$ Solving for $t$: $t = \dfrac{\frac{1}{2}gt_0^2}{gt_0 - v_0} = \dfrac{t_0}{2}\left(\dfrac{1}{1 - v_0/(gt_0)}\right).$

Substitution of this into the expression for $y_1$ and setting $y_1 = 0$ and solving for $h$ as a function of $v_0$ yields, after some algebra, $h = \frac{1}{2}gt_0^2 \dfrac{\left(\frac{1}{2}gt_0 - v_0\right)^2}{(gt_0 - v_0)^2}.$ Using the given value $t_0 = 1.00$ s and $g = 9.80\ \text{m/s}^2,$

$h = 20.0\ \text{m} = (4.9\ \text{m})\left(\dfrac{4.9\ \text{m/s} - v_0}{9.8\ \text{m/s} - v_0}\right)^2.$

This has two solutions, one of which is unphysical (the first ball is still going up when the second is released; see part (c)). The physical solution involves taking the negative square root before solving for $v_0,$ and yields $8.2\ \text{m/s}.$ The graph of $y$ versus $t$ for each ball is given in Figure 2.89.

**(b)** The above expression gives for (i) 0.411 m and for (ii) 1.15 km.

**(c)** As $v_0$ approaches $9.8\ \text{m/s},$ the height $h$ becomes infinite, corresponding to a relative velocity at the time the second ball is thrown that approaches zero. If $v_0 > 9.8\ \text{m/s},$ the first ball can never catch the second ball.

**(d)** As $v_0$ approaches 4.9 m/s, the height approaches zero. This corresponds to the first ball being closer and closer (on its way down) to the top of the roof when the second ball is released. If $v_0 < 4.9\ \text{m/s},$ the first ball will already have passed the roof on the way down before the second ball is released, and the second ball can never catch up.

**EVALUATE:** Note that the values of $v_0$ in parts (a) and (b) are all greater than $v_{\text{min}}$ and less than $v_{\text{max}}.$

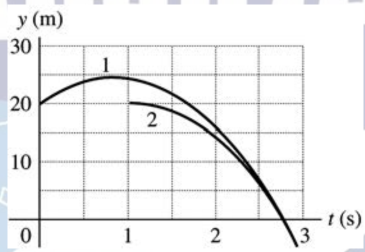



**Figure 2.89**

**2.90.** **IDENTIFY:** We know the change in velocity and the time for that change. We can use these quantities to find the average acceleration.

**SET UP:** The average acceleration is the change in velocity divided by the time for that change.

**EXECUTE:** $a_{\text{av}} = (v - v_0)/t = (0.80\ \text{m/s} - 0)/(250 \times 10^{-3}\ \text{s}) = 32\ \text{m/s}^2,$ which is choice (c).

**EVALUATE:** This is about 1/3 the acceleration due to gravity, which is a reasonable acceleration for an organ.

**2.91.** **IDENTIFY:** The original area is divided into two equal areas. We want the diameter of these two areas, assuming the original and final areas are circular.

**SET UP:** The area $A$ of a circle or radius $r$ is $A = \pi r^2$ and the diameter $d$ is $d = 2r$. $A_\text{i} = 2A_\text{f}$, and $r = d/2$, where $A_\text{f}$ is the area of each of the two arteries.

**EXECUTE:** Call $d$ the diameter of each artery. $A_\text{i} = \pi(d_\text{a}/2)^2 = 2[\pi(d/2)^2]$, which gives $d = d_\text{a}/\sqrt{2},$ which is choice (b).

**EVALUATE:** The area of each artery is half the area of the aorta, but the diameters of the arteries are not half the diameter of the aorta.

**2.92.** **IDENTIFY:** We must interpret a graph of blood velocity during a heartbeat as a function of time.

**SET UP:** The instantaneous acceleration of a blood molecule is the slope of the velocity-versus-time graph.

**EXECUTE:** The magnitude of the acceleration is greatest when the slope of the $v$-$t$ graph is steepest. That occurs at the upward sloping part of the graph, around $t = 0.10$ s, which makes choice (d) the correct one.
**EVALUATE:** The slope of the given graph is positive during the first 0.25 s and negative after that. Yet the velocity is positive throughout. Therefore the blood is always flowing forward, but it is increasing in speed during the first 0.25 s and slowing down after that.

# 3

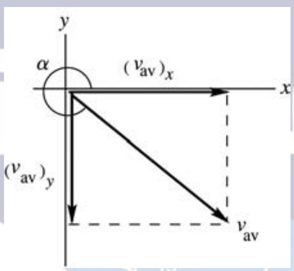
# MOTION IN TWO OR THREE DIMENSIONS

**3.1.** **IDENTIFY** and **SET UP:** Use $\vec{v}_{av} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$ in component form.

**EXECUTE:** **(a)** $v_{av\text{-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{5.3\text{ m} - 1.1\text{ m}}{3.0\text{ s} - 0} = 1.4\text{ m/s}$

$v_{av\text{-}y} = \dfrac{\Delta y}{\Delta t} = \dfrac{y_2 - y_1}{t_2 - t_1} = \dfrac{-0.5\text{ m} - 3.4\text{ m}}{3.0\text{ s} - 0} = -1.3\text{ m/s}$

**(b)**




$\tan\alpha = \dfrac{(v_{av})_y}{(v_{av})_x} = \dfrac{-1.3\text{ m/s}}{1.4\text{ m/s}} = -0.9286$

$\alpha = 360° - 42.9° = 317°$

$v_{av} = \sqrt{(v_{av})_x^2 + (v_{av})_y^2}$

$v_{av} = \sqrt{(1.4\text{ m/s})^2 + (-1.3\text{ m/s})^2} = 1.9\text{ m/s}$

**Figure 3.1**

**EVALUATE:** Our calculation gives that $\vec{v}_{av}$ is in the 4th quadrant. This corresponds to increasing *x* and decreasing *y*.

**3.2.** **IDENTIFY:** Use $\vec{v}_{av} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$ in component form. The distance from the origin is the magnitude of $\vec{r}$.

**SET UP:** At time $t_1$, $x_1 = y_1 = 0$.

**EXECUTE:** **(a)** $x = (v_{av\text{-}x})\Delta t = (-3.8\text{ m/s})(12.0\text{ s}) = -45.6\text{ m}$ and $y = (v_{av\text{-}y})\Delta t = (4.9\text{ m/s})(12.0\text{ s}) = 58.8\text{ m}$.

**(b)** $r = \sqrt{x^2 + y^2} = \sqrt{(-45.6\text{ m})^2 + (58.8\text{ m})^2} = 74.4\text{ m}$.

**EVALUATE:** $\Delta\vec{r}$ is in the direction of $\vec{v}_{av}$. Therefore, $\Delta x$ is negative since $v_{av\text{-}x}$ is negative and $\Delta y$ is positive since $v_{av\text{-}y}$ is positive.

**3.3.** **(a) IDENTIFY** and **SET UP:** From $\vec{r}$ we can calculate *x* and *y* for any *t*.

Then use $\vec{v}_{av} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$ in component form.

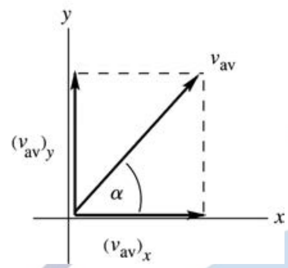
**EXECUTE:** $\vec{r} = [4.0\text{ cm} + (2.5\text{ cm/s}^2)t^2]\hat{i} + (5.0\text{ cm/s})t\hat{j}$

At $t = 0$, $\vec{r} = (4.0\text{ cm})\,\hat{i}$.

At $t = 2.0\text{ s}$, $\vec{r} = (14.0\text{ cm})\,\hat{i} + (10.0\text{ cm})\,\hat{j}$.

$$v_{\text{av-}x} = \frac{\Delta x}{\Delta t} = \frac{10.0 \text{ cm}}{2.0 \text{ s}} = 5.0 \text{ cm/s}.$$

$$v_{\text{av-}y} = \frac{\Delta y}{\Delta t} = \frac{10.0 \text{ cm}}{2.0 \text{ s}} = 5.0 \text{ cm/s}.$$




$$v_{\text{av}} = \sqrt{(v_{\text{av}})_x^2 + (v_{\text{av}})_y^2} = 7.1 \text{ cm/s}$$

$$\tan\alpha = \frac{(v_{\text{av}})_y}{(v_{\text{av}})_x} = 1.00$$

$$\theta = 45°.$$

**Figure 3.3a**

**EVALUATE:** Both $x$ and $y$ increase, so $\vec{v}_{\text{av}}$ is in the 1st quadrant.

**(b) IDENTIFY** and **SET UP:** Calculate $\vec{r}$ by taking the time derivative of $\vec{r}(t)$.

**EXECUTE:** $\vec{v} = \dfrac{d\vec{r}}{dt} = ([5.0 \text{ cm/s}^2]t)\hat{i} + (5.0 \text{ cm/s})\hat{j}$

$\underline{t = 0}$: $v_x = 0$, $v_y = 5.0$ cm/s; $v = 5.0$ cm/s and $\theta = 90°$

$\underline{t = 1.0 \text{ s}}$: $v_x = 5.0$ cm/s, $v_y = 5.0$ cm/s; $v = 7.1$ cm/s and $\theta = 45°$

$\underline{t = 2.0 \text{ s}}$: $v_x = 10.0$ cm/s, $v_y = 5.0$ cm/s; $v = 11$ cm/s and $\theta = 27°$

**(c)** The trajectory is a graph of $y$ versus $x$.

$x = 4.0 \text{ cm} + (2.5 \text{ cm/s}^2)t^2$, $y = (5.0 \text{ cm/s})t$

For values of $t$ between 0 and 2.0 s, calculate $x$ and $y$ and plot $y$ versus $x$.



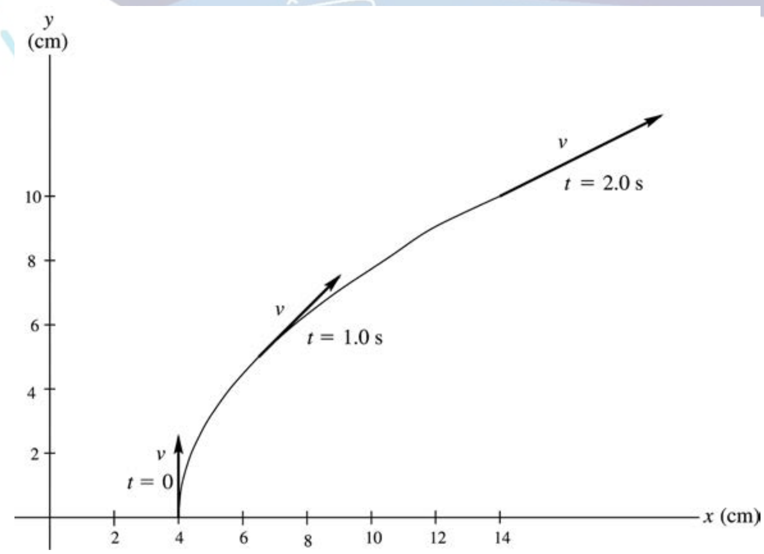



**Figure 3.3b**

**EVALUATE:** The sketch shows that the instantaneous velocity at any $t$ is tangent to the trajectory.

**3.4.** **IDENTIFY:** Given the position vector of a squirrel, find its velocity components in general, and at a specific time find its velocity components and the magnitude and direction of its position vector and velocity.

**SET UP:** $v_x = dx/dt$ and $v_y = dy/dt$; the magnitude of a vector is $A = \sqrt{(A_x^2 + A_y^2)}$.

**EXECUTE:** **(a)** Taking the derivatives gives $v_x(t) = 0.280\text{ m/s} + (0.0720\text{ m/s}^2)t$ and $v_y(t) = (0.0570\text{ m/s}^3)t^2$.

**(b)** Evaluating the position vector at $t = 5.00\text{ s}$ gives $x = 2.30\text{ m}$ and $y = 2.375\text{ m}$, which gives $r = 3.31\text{ m}$.

**(c)** At $t = 5.00\text{ s}$, $v_x = +0.64\text{ m/s}$, $v_y = 1.425\text{ m/s}$, which gives $v = 1.56\text{ m/s}$ and $\tan\theta = \dfrac{1.425}{0.64}$ so the direction is $\theta = 65.8^\text{o}$ (counterclockwise from $+x$-axis)

**EVALUATE:** The acceleration is not constant, so we cannot use the standard kinematics formulas.

**3.5.** **IDENTIFY** and **SET UP:** Use Eq. $\vec{a}_{\text{av}} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$ in component form to calculate $a_{\text{av-}x}$ and $a_{\text{av-}y}$.

**EXECUTE:** **(a)** The velocity vectors at $t_1 = 0$ and $t_2 = 30.0\text{ s}$ are shown in Figure 3.5a.

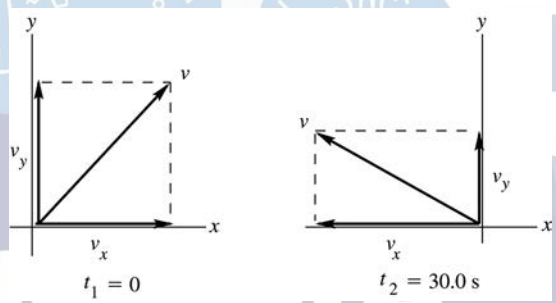
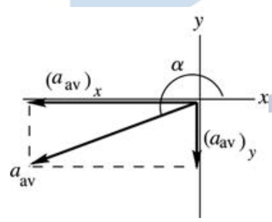



**Figure 3.5a**

**(b)** $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t} = \dfrac{v_{2x} - v_{1x}}{t_2 - t_1} = \dfrac{-170\text{ m/s} - 90\text{ m/s}}{30.0\text{ s}} = -8.67\text{ m/s}^2$

$a_{\text{av-}y} = \dfrac{\Delta v_y}{\Delta t} = \dfrac{v_{2y} - v_{1y}}{t_2 - t_1} = \dfrac{40\text{ m/s} - 110\text{ m/s}}{30.0\text{ s}} = -2.33\text{ m/s}^2$

**(c)**




$a = \sqrt{(a_{\text{av-}x})^2 + (a_{\text{av-}y})^2} = 8.98\text{ m/s}^2$

$\tan\alpha = \dfrac{a_{\text{av-}y}}{a_{\text{av-}x}} = \dfrac{-2.33\text{ m/s}^2}{-8.67\text{ m/s}^2} = 0.269$

$\alpha = 15° + 180° = 195°$

**Figure 3.5b**

**EVALUATE:** The changes in $v_x$ and $v_y$ are both in the negative $x$ or $y$ direction, so both components of $\vec{a}_{\text{av}}$ are in the 3rd quadrant.

**3.6.** **IDENTIFY:** Use $\vec{a}_{\text{av}} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$ in component form.

**SET UP:** $a_x = (0.45\text{ m/s}^2)\cos 31.0° = 0.39\text{ m/s}^2$, $a_y = (0.45\text{ m/s}^2)\sin 31.0° = 0.23\text{ m/s}^2$

**EXECUTE:** **(a)** $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$ and $v_x = 2.6\text{ m/s} + (0.39\text{ m/s}^2)(10.0\text{ s}) = 6.5\text{ m/s}$. $a_{\text{av-}y} = \dfrac{\Delta v_y}{\Delta t}$ and $v_y = -1.8\text{ m/s} + (0.23\text{ m/s}^2)(10.0\text{ s}) = 0.52\text{ m/s}$.

**(b)** $v=\sqrt{(6.5\text{ m/s})^2+(0.52\text{ m/s})^2}=6.52\text{ m/s},$ at an angle of $\arctan\left(\dfrac{0.52}{6.5}\right)=4.6°$ counterclockwise from the $+x$-axis.

**(c)** The velocity vectors $\vec{v}_1$ and $\vec{v}_2$ are sketched in Figure 3.6. The two velocity vectors differ in magnitude and direction.

**EVALUATE:** $\vec{v}_1$ is at an angle of $35°$ below the $+x$-axis and has magnitude $v_1=3.2$ m/s, so $v_2>v_1$ and the direction of $\vec{v}_2$ is rotated counterclockwise from the direction of $\vec{v}_1$.

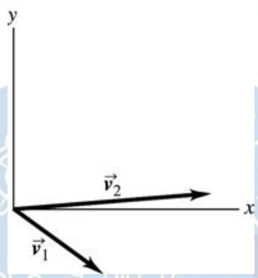



**Figure 3.6**

**3.7.** **IDENTIFY** and **SET UP:** Use $\vec{v}=\dfrac{d\vec{r}}{dt}$ and $\vec{a}=\dfrac{d\vec{v}}{dt}$ to find $v_x$, $v_y$, $a_x$, and $a_y$ as functions of time. The magnitude and direction of $\vec{r}$ and $\vec{a}$ can be found once we know their components.

**EXECUTE:** **(a)** Calculate $x$ and $y$ for $t$ values in the range 0 to 2.0 s and plot $y$ versus $x$. The results are given in Figure 3.7a.

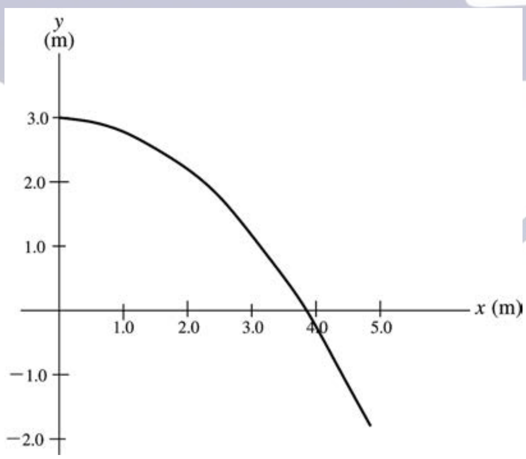


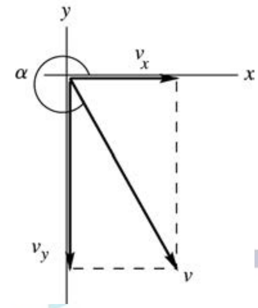

**Figure 3.7a**

**(b)** $v_x=\dfrac{dx}{dt}=\alpha \quad v_y=\dfrac{dy}{dt}=-2\beta t$

$a_x=\dfrac{dv_x}{dt}=0 \quad a_y=\dfrac{dv_y}{dt}=-2\beta$

Thus $\vec{v}=\alpha\hat{i}-2\beta t\hat{j}$, $\vec{a}=-2\beta\hat{j}$

**(c)** velocity: At $t=2.0$ s, $v_x=2.4$ m/s, $v_y=-2(1.2\text{ m/s}^2)(2.0\text{ s})=-4.8$ m/s



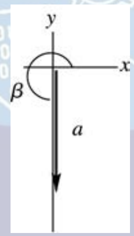

$$v = \sqrt{v_x^2 + v_y^2} = 5.4 \text{ m/s}$$

$$\tan\alpha = \frac{v_y}{v_x} = \frac{-4.8 \text{ m/s}}{2.4 \text{ m/s}} = -2.00$$

$$\alpha = -63.4° + 360° = 297°$$

**Figure 3.7b**

<u>acceleration:</u> At $t = 2.0$ s, $a_x = 0$, $a_y = -2\,(1.2 \text{ m/s}^2) = -2.4 \text{ m/s}^2$




$$a = \sqrt{a_x^2 + a_y^2} = 2.4 \text{ m/s}^2$$

$$\tan\beta = \frac{a_y}{a_x} = \frac{-2.4 \text{ m/s}^2}{0} = -\infty$$

$$\beta = 270°$$

**Figure 3.7c**

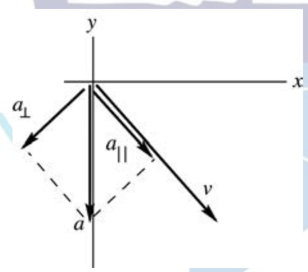



**Figure 3.7d**

**EVALUATE:** **(d)** $\vec{a}$ has a component $a_{||}$ in the same direction as $\vec{v}$, so we know that $v$ is increasing (the bird is speeding up). $\vec{a}$ also has a component $a_\perp$ perpendicular to $\vec{v}$, so that the direction of $\vec{v}$ is changing; the bird is turning toward the $-y$-direction (toward the right)

$\vec{v}$ is always tangent to the path; $\vec{v}$ at $t = 2.0$ s shown in part (c) is tangent to the path at this $t$, conforming to this general rule. $\vec{a}$ is constant and in the $-y$-direction; the direction of $\vec{v}$ is turning toward the $-y$-direction.

**3.8.** **IDENTIFY:** Use the velocity components of a car (given as a function of time) to find the acceleration of the car as a function of time and to find the magnitude and direction of the car's velocity and acceleration at a specific time.

**SET UP:** $a_x = dv_x/dt$ and $a_y = dv_y/dt$; the magnitude of a vector is $A = \sqrt{(A_x^2 + A_y^2)}$.

**EXECUTE:** **(a)** Taking the derivatives gives $a_x(t) = (-0.0360 \text{ m/s}^3)t$ and $a_y(t) = 0.550 \text{ m/s}^2$.

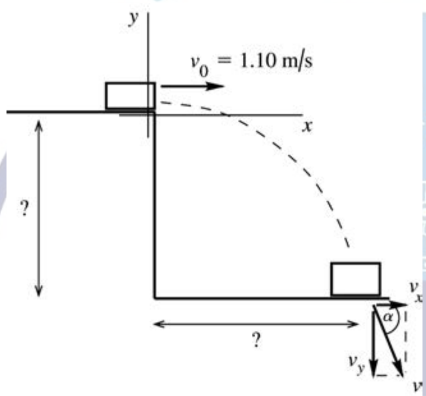
**(b)** Evaluating the velocity components at $t = 8.00$ s gives $v_x = 3.848$ m/s and $v_y = 6.40$ m/s, which gives $v = 7.47$ m/s. The direction is $\tan\theta = \dfrac{6.40}{3.848}$ so $\theta = 59.0^\circ$ (counterclockwise from +$x$-axis).

**(c)** Evaluating the acceleration components at $t = 8.00$ s gives $a_x = -0.288 \text{ m/s}^2$ and $a_y = 0.550 \text{ m/s}^2$, which gives $a = 0.621 \text{ m/s}^2$. The angle with the $+y$ axis is given by $\tan\theta = \dfrac{0.288}{0.550}$, so $\theta = 27.6^\circ$. The direction is therefore $118^\circ$ counterclockwise from $+x$-axis.

**EVALUATE:** The acceleration is not constant, so we cannot use the standard kinematics formulas.

**3.9.** **IDENTIFY:** The book moves in projectile motion once it leaves the tabletop. Its initial velocity is horizontal.

**SET UP:** Take the positive $y$-direction to be upward. Take the origin of coordinates at the initial position of the book, at the point where it leaves the table top.




<u>x-component</u>:
$a_x = 0$, $v_{0x} = 1.10$ m/s,
$t = 0.480$ s
<u>y-component</u>:
$a_y = -9.80 \text{ m/s}^2$,
$v_{0y} = 0$,
$t = 0.480$ s

**Figure 3.9a**

Use constant acceleration equations for the $x$ and $y$ components of the motion, with $a_x = 0$ and $a_y = -g$.

**EXECUTE:** **(a)** $y - y_0 = ?$

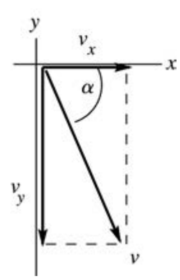
$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.480 \text{ s})^2 = -1.129$ m. The tabletop is therefore 1.13 m above the floor.

**(b)** $x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (1.10 \text{ m/s})(0.480 \text{ s}) + 0 = 0.528$ m.

**(c)** $v_x = v_{0x} + a_x t = 1.10$ m/s (The $x$-component of the velocity is constant, since $a_x = 0$.)

$v_y = v_{0y} + a_y t = 0 + (-9.80 \text{ m/s}^2)(0.480 \text{ s}) = -4.704$ m/s




$v = \sqrt{v_x^2 + v_y^2} = 4.83$ m/s

$\tan\alpha = \dfrac{v_y}{v_x} = \dfrac{-4.704 \text{ m/s}}{1.10 \text{ m/s}} = -4.2764$

$\alpha = -76.8^\circ$

Direction of $\vec{v}$ is $76.8^\circ$ below the horizontal

**Figure 3.9b**

**(d)** The graphs are given in Figure 3.9c.

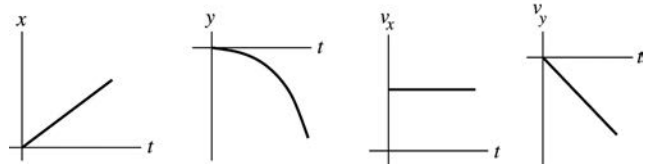



**Figure 3.9c**

**EVALUATE:** In the *x*-direction, $a_x = 0$ and $v_x$ is constant. In the *y*-direction, $a_y = -9.80\ \text{m/s}^2$ and $v_y$ is downward and increasing in magnitude since $a_y$ and $v_y$ are in the same directions. The *x* and *y* motions occur independently, connected only by the time. The time it takes the book to fall 1.13 m is the time it travels horizontally.

**3.10.** **IDENTIFY:** The person moves in projectile motion. She must travel 1.75 m horizontally during the time she falls 9.00 m vertically.

**SET UP:** Take $+y$ downward. $a_x = 0$, $a_y = +9.80\ \text{m/s}^2$. $v_{0x} = v_0$, $v_{0y} = 0$.

**EXECUTE:** Time to fall 9.00 m: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(9.00\ \text{m})}{9.80\ \text{m/s}^2}} = 1.36\ \text{s}.$

Speed needed to travel 1.75 m horizontally during this time: $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives

$$v_0 = v_{0x} = \frac{x - x_0}{t} = \frac{1.75\ \text{m}}{1.36\ \text{s}} = 1.29\ \text{m/s}.$$

**EVALUATE:** If she increases her initial speed she still takes 1.36 s to reach the level of the ledge, but has traveled horizontally farther than 1.75 m.

**3.11.** **IDENTIFY:** Each object moves in projectile motion.

**SET UP:** Take $+y$ to be downward. For each cricket, $a_x = 0$ and $a_y = +9.80\ \text{m/s}^2$. For Chirpy, $v_{0x} = v_{0y} = 0$. For Milada, $v_{0x} = 0.950\ \text{m/s}$, $v_{0y} = 0$.

**EXECUTE:** Milada's horizontal component of velocity has no effect on her vertical motion. She also reaches the ground in 2.70 s. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (0.950\ \text{m/s})(2.70\ \text{s}) = 2.57\ \text{m}.$

**EVALUATE:** The *x* and *y* components of motion are totally separate and are connected only by the fact that the time is the same for both.

**3.12.** **IDENTIFY:** The football moves in projectile motion.

**SET UP:** Let $+y$ be upward. $a_x = 0$, $a_y = -g$. At the highest point in the trajectory, $v_y = 0$.

**EXECUTE:** **(a)** $v_y = v_{0y} + a_y t$. The time *t* is $\dfrac{v_{0y}}{g} = \dfrac{12.0\ \text{m/s}}{9.80\ \text{m/s}^2} = 1.224\ \text{s}$, which we round to 1.22 s.

**(b)** Different constant acceleration equations give different expressions but the same numerical result:

$$\tfrac{1}{2}gt^2 = \tfrac{1}{2}v_{y0}t = \frac{v_{0y}^2}{2g} = 7.35\ \text{m}.$$

**(c)** Regardless of how the algebra is done, the time will be twice that found in part (a), which is $2(1.224\ \text{s}) = 2.45\ \text{s}$.

**(d)** $a_x = 0$, so $x - x_0 = v_{0x}t = (20.0\ \text{m/s})(2.45\ \text{s}) = 49.0\ \text{m}$.

**(e)** The graphs are sketched in Figure 3.12 (next page).

**EVALUATE:** When the football returns to its original level, $v_x = 20.0$ m/s and $v_y = -12.0$ m/s.

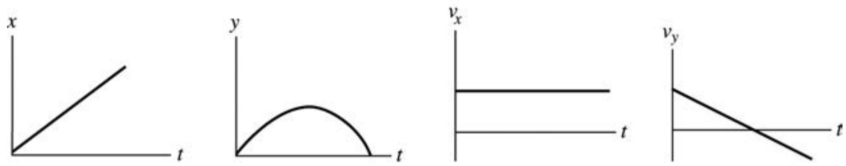



**Figure 3.12**

**3.13.** **IDENTIFY:** The car moves in projectile motion. The car travels $21.3\text{ m} - 1.80\text{ m} = 19.5$ m downward during the time it travels 48.0 m horizontally.

**SET UP:** Take $+y$ to be downward. $a_x = 0$, $a_y = +9.80\text{ m/s}^2$. $v_{0x} = v_0$, $v_{0y} = 0$.

**EXECUTE:** **(a)** Use the vertical motion to find the time in the air:

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(19.5\text{ m})}{9.80\text{ m/s}^2}} = 1.995$ s

Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $v_0 = v_{0x} = \dfrac{x - x_0}{t} = \dfrac{48.0\text{ m}}{1.995\text{ s}} = 24.1$ m/s.

**(b)** $v_x = 24.06$ m/s since $a_x = 0$. $v_y = v_{0y} + a_y t = -19.55$ m/s. $v = \sqrt{v_x^2 + v_y^2} = 31.0$ m/s.

**EVALUATE:** Note that the speed is considerably less than the algebraic sum of the $x$- and $y$-components of the velocity.

**3.14.** **IDENTIFY:** Knowing the maximum reached by the froghopper and its angle of takeoff, we want to find its takeoff speed and the horizontal distance it travels while in the air.

**SET UP:** Use coordinates with the origin at the ground and $+y$ upward. $a_x = 0$, $a_y = -9.80\text{ m/s}^2$. At the maximum height $v_y = 0$. The constant-acceleration formulas $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ and $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ apply.

**EXECUTE:** **(a)** $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80\text{ m/s}^2)(0.587\text{ m})} = 3.39$ m/s. $v_{0y} = v_0 \sin\theta_0$ so

$v_0 = \dfrac{v_{0y}}{\sin\theta_0} = \dfrac{3.39\text{ m/s}}{\sin 58.0°} = 4.00$ m/s.

**(b)** Use the vertical motion to find the time in the air. When the froghopper has returned to the ground, $y - y_0 = 0$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(3.39\text{ m/s})}{-9.80\text{ m/s}^2} = 0.692$ s.

Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (v_0 \cos\theta_0)t = (4.00\text{ m/s})(\cos 58.0°)(0.692\text{ s}) = 1.47$ m.

**EVALUATE:** $v_y = 0$ when $t = -\dfrac{v_{0y}}{a_y} = -\dfrac{3.39\text{ m/s}}{-9.80\text{ m/s}^2} = 0.346$ s. The total time in the air is twice this.

**3.15.** **IDENTIFY:** The ball moves with projectile motion with an initial velocity that is horizontal and has magnitude $v_0$. The height $h$ of the table and $v_0$ are the same; the acceleration due to gravity changes from $g_E = 9.80\text{ m/s}^2$ on earth to $g_X$ on planet X.

**SET UP:** Let $+x$ be horizontal and in the direction of the initial velocity of the marble and let $+y$ be upward. $v_{0x} = v_0$, $v_{0y} = 0$, $a_x = 0$, $a_y = -g$, where $g$ is either $g_E$ or $g_X$.

**EXECUTE:** Use the vertical motion to find the time in the air: $y - y_0 = -h$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2h}{g}}$. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x - x_0 = v_{0x}t = v_0\sqrt{\dfrac{2h}{g}}$. $x - x_0 = D$ on earth and $2.76D$ on

Planet X. $(x-x_0)\sqrt{g}=v_0\sqrt{2h}$, which is constant, so $D\sqrt{g_E}=2.76D\sqrt{g_X}$.

$$g_X=\frac{g_E}{(2.76)^2}=0.131g_E=1.28\text{ m/s}^2.$$

**EVALUATE:** On Planet X the acceleration due to gravity is less, it takes the ball longer to reach the floor and it travels farther horizontally.

**3.16.** **IDENTIFY:** The shell moves in projectile motion.

**SET UP:** Let $+x$ be horizontal, along the direction of the shell's motion, and let $+y$ be upward. $a_x=0$, $a_y=-9.80\text{ m/s}^2$.

**EXECUTE:** **(a)** $v_{0x}=v_0\cos\alpha_0=(40.0\text{ m/s})\cos 60.0°=20.0\text{ m/s}$,
$v_{0y}=v_0\sin\alpha_0=(40.0\text{ m/s})\sin 60.0°=34.6\text{ m/s}$.

**(b)** At the maximum height $v_y=0$. $v_y=v_{0y}+a_yt$ gives $t=\dfrac{v_y-v_{0y}}{a_y}=\dfrac{0-34.6\text{ m/s}}{-9.80\text{ m/s}^2}=3.53\text{ s}$.

**(c)** $v_y^2=v_{0y}^2+2a_y(y-y_0)$ gives $y-y_0=\dfrac{v_y^2-v_{0y}^2}{2a_y}=\dfrac{0-(34.6\text{ m/s})^2}{2(-9.80\text{ m/s}^2)}=61.2\text{ m}$.

**(d)** The total time in the air is twice the time to the maximum height, so
$x-x_0=v_{0x}t+\frac{1}{2}a_xt^2=(20.0\text{ m/s})(2)(3.53\text{ s})=141\text{ m}$.

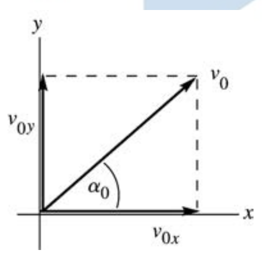
**(e)** At the maximum height, $v_x=v_{0x}=20.0\text{ m/s}$ and $v_y=0$. At all points in the motion, $a_x=0$ and $a_y=-9.80\text{ m/s}^2$.

**EVALUATE:** The equation for the horizontal range $R$ derived in the text is $R=\dfrac{v_0^2\sin 2\alpha_0}{g}$. This gives

$$R=\frac{(40.0\text{ m/s})^2\sin(120.0°)}{9.80\text{ m/s}^2}=141\text{ m},$$ which agrees with our result in part (d).

**3.17.** **IDENTIFY:** The baseball moves in projectile motion. In part (c) first calculate the components of the velocity at this point and then get the resultant velocity from its components.

**SET UP:** First find the $x$- and $y$-components of the initial velocity. Use coordinates where the $+y$-direction is upward, the $+x$-direction is to the right and the origin is at the point where the baseball leaves the bat.




$v_{0x}=v_0\cos\alpha_0=(30.0\text{ m/s})\cos 36.9°=24.0\text{ m/s}$

$v_{0y}=v_0\sin\alpha_0=(30.0\text{ m/s})\sin 36.9°=18.0\text{ m/s}$

**Figure 3.17a**

Use constant acceleration equations for the $x$ and $y$ motions, with $a_x=0$ and $a_y=-g$.

**EXECUTE:** **(a)** $y$-component (vertical motion):

$y-y_0=+10.0\text{ m}$, $v_{0y}=18.0\text{ m/s}$, $a_y=-9.80\text{ m/s}^2$, $t=?$

$y-y_0=v_{0y}+\frac{1}{2}a_yt^2$

$10.0\text{ m}=(18.0\text{ m/s})t-(4.90\text{ m/s}^2)t^2$

$(4.90\text{ m/s}^2)t^2-(18.0\text{ m/s})t+10.0\text{ m}=0$

Apply the quadratic formula: $t = \frac{1}{9.80}\left[18.0 \pm \sqrt{(-18.0)^2 - 4\,(4.90)(10.0)}\right]\ \text{s} = (1.837 \pm 1.154)\ \text{s}$

The ball is at a height of 10.0 above the point where it left the bat at $t_1 = 0.683\ \text{s}$ and at $t_2 = 2.99\ \text{s}$. At the earlier time the ball passes through a height of 10.0 m as its way up and at the later time it passes through 10.0 m on its way down.

**(b)** $v_x = v_{0x} = +24.0\ \text{m/s}$, at all times since $a_x = 0$.

$v_y = v_{0y} + a_y t$

$\underline{t_1 = 0.683\ \text{s}}$: $v_y = +18.0\ \text{m/s} + (-9.80\ \text{m/s}^2)(0.683\ \text{s}) = +11.3\ \text{m/s}$. ($v_y$ is positive means that the ball is traveling upward at this point.)

$\underline{t_2 = 2.99\ \text{s}}$: $v_y = +18.0\ \text{m/s} + (-9.80\ \text{m/s}^2)(2.99\ \text{s}) = -11.3\ \text{m/s}$. ($v_y$ is negative means that the ball is traveling downward at this point.)

**(c)** $v_x = v_{0x} = 24.0\ \text{m/s}$

Solve for $v_y$:

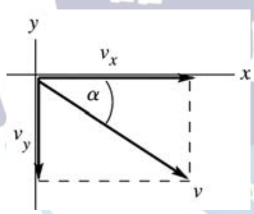
$v_y = ?$, $y - y_0 = 0$ (when ball returns to height where motion started),

$a_y = -9.80\ \text{m/s}^2$, $v_{0y} = +18.0\ \text{m/s}$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$v_y = -v_{0y} = -18.0\ \text{m/s}$ (negative, since the baseball must be traveling downward at this point)

Now solve for the magnitude and direction of $\vec{v}$.




$v = \sqrt{v_x^2 + v_y^2}$

$v = \sqrt{(24.0\ \text{m/s})^2 + (-18.0\ \text{m/s})^2} = 30.0\ \text{m/s}$

$\tan\alpha = \dfrac{v_y}{v_x} = \dfrac{-18.0\ \text{m/s}}{24.0\ \text{m/s}}$

$\alpha = -36.9°$, $36.9°$ below the horizontal

**Figure 3.17b**

The velocity of the ball when it returns to the level where it left the bat has magnitude 30.0 m/s and is directed at an angle of $36.9°$ below the horizontal.

**EVALUATE:** The discussion in parts (a) and (b) explains the significance of two values of $t$ for which $y - y_0 = +10.0\ \text{m}$. When the ball returns to its initial height, our results give that its speed is the same as its initial speed and the angle of its velocity below the horizontal is equal to the angle of its initial velocity above the horizontal; both of these are general results.

**3.18.** **IDENTIFY:** The shot moves in projectile motion.

**SET UP:** Let $+y$ be upward.

**EXECUTE:** **(a)** If air resistance is to be ignored, the components of acceleration are 0 horizontally and $-g = -9.80\ \text{m/s}^2$ vertically downward.

**(b)** The $x$-component of velocity is constant at $v_x = (12.0\ \text{m/s})\cos 51.0° = 7.55\ \text{m/s}$. The $y$-component is $v_{0y} = (12.0\ \text{m/s})\sin 51.0° = 9.32\ \text{m/s}$ at release and

$v_y = v_{0y} - gt = (9.32\ \text{m/s}) - (9.80\ \text{m/s})(2.08\ \text{s}) = -11.06\ \text{m/s}$ when the shot hits.

**(c)** $x - x_0 = v_{0x}t = (7.55\ \text{m/s})(2.08\ \text{s}) = 15.7\ \text{m}$.

**(d)** The initial and final heights are not the same.

**(e)** With $y = 0$ and $v_{0y}$ as found above, the equation for $y - y_0$ as a function of time gives $y_0 = 1.81\text{m}$.

**(f)** The graphs are sketched in Figure 3.18.

**EVALUATE:** When the shot returns to its initial height, $v_y = -9.32$ m/s. The shot continues to accelerate downward as it travels downward 1.81 m to the ground and the magnitude of $v_y$ at the ground is larger than 9.32 m/s.

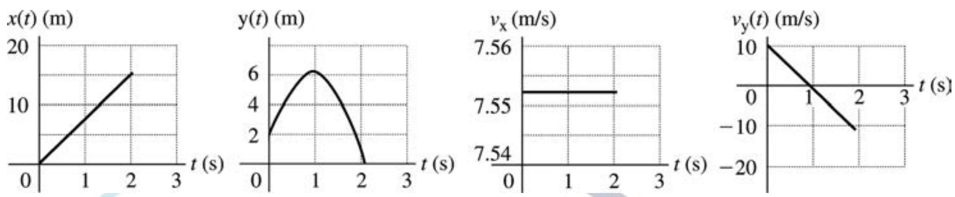
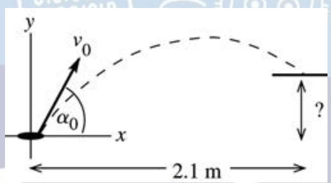



**Figure 3.18**

**3.19.** **IDENTIFY:** Take the origin of coordinates at the point where the quarter leaves your hand and take positive $y$ to be upward. The quarter moves in projectile motion, with $a_x = 0$, and $a_y = -g$. It travels vertically for the time it takes it to travel horizontally 2.1 m.




$v_{0x} = v_0 \cos\alpha_0 = (6.4 \text{ m/s}) \cos 60°$

$v_{0x} = 3.20$ m/s

$v_{0y} = v_0 \sin\alpha_0 = (6.4 \text{ m/s}) \sin 60°$

$v_{0y} = 5.54$ m/s

**Figure 3.19**

**(a) SET UP:** Use the horizontal ($x$-component) of motion to solve for $t$, the time the quarter travels through the air:

$t = ?,\ \ x - x_0 = 2.1 \text{ m},\ \ v_{0x} = 3.2 \text{ m/s},\ \ a_x = 0$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = v_{0x}t,$ since $a_x = 0$

**EXECUTE:** $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{2.1 \text{ m}}{3.2 \text{ m/s}} = 0.656 \text{ s}$

**SET UP:** Now find the vertical displacement of the quarter after this time:

$y - y_0 = ?,\ \ a_y = -9.80 \text{ m/s}^2,\ \ v_{0y} = +5.54 \text{ m/s},\ \ t = 0.656 \text{ s}$

$y - y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

**EXECUTE:** $y - y_0 = (5.54 \text{ m/s})(0.656 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.656 \text{ s})^2 = 3.63 \text{ m} - 2.11 \text{ m} = 1.5 \text{ m}.$

**(b) SET UP:** $v_y = ?,\ \ t = 0.656 \text{ s},\ \ a_y = -9.80 \text{ m/s}^2,\ \ v_{0y} = +5.54 \text{ m/s}\ \ \ v_y = v_{0y} + a_y t$

**EXECUTE:** $v_y = 5.54 \text{ m/s} + (-9.80 \text{ m/s}^2)(0.656 \text{ s}) = -0.89 \text{ m/s}.$

**EVALUATE:** The minus sign for $v_y$ indicates that the $y$-component of $\vec{v}$ is downward. At this point the quarter has passed through the highest point in its path and is on its way down. The horizontal range if it returned to its original height (it doesn't!) would be 3.6 m. It reaches its maximum height after traveling horizontally 1.8 m, so at $x - x_0 = 2.1$ m it is on its way down.

**3.20.** **IDENTIFY:** Consider the horizontal and vertical components of the projectile motion. The water travels 45.0 m horizontally in 3.00 s.

**SET UP:** Let $+y$ be upward. $a_x = 0,\ \ a_y = -9.80 \text{ m/s}^2.\ \ v_{0x} = v_0 \cos\theta_0,\ \ v_{0y} = v_0 \sin\theta_0.$

**EXECUTE:** **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x - x_0 = v_0(\cos\theta_0)t$ and $\cos\theta_0 = \dfrac{45.0 \text{ m}}{(25.0 \text{ m/s})(3.00 \text{ s})} = 0.600;$

$\theta_0 = 53.1°$

**(b)** At the highest point $v_x = v_{0x} = (25.0\text{ m/s})\cos 53.1° = 15.0\text{ m/s}$, $v_y = 0$ and $v = \sqrt{v_x^2 + v_y^2} = 15.0\text{ m/s}$. At all points in the motion, $a = 9.80\text{ m/s}^2$ downward.

**(c)** Find $y - y_0$ when $t = 3.00\text{s}$:

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2 = (25.0\text{ m/s})(\sin 53.1°)(3.00\text{ s}) + \tfrac{1}{2}(-9.80\text{ m/s}^2)(3.00\text{ s})^2 = 15.9\text{ m}$$

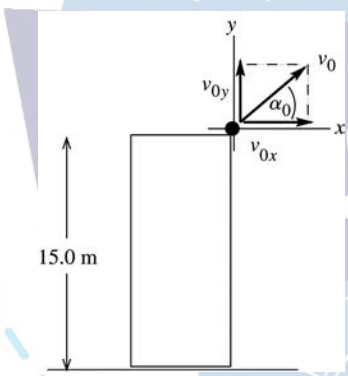
$v_x = v_{0x} = 15.0\text{ m/s}$, $v_y = v_{0y} + a_y t = (25.0\text{ m/s})(\sin 53.1°) - (9.80\text{m/s}^2)(3.00\text{ s}) = -9.41\text{ m/s}$, and

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.0\text{ m/s})^2 + (-9.41\text{ m/s})^2} = 17.7\text{ m/s}$$

**EVALUATE:** The acceleration is the same at all points of the motion. It takes the water $t = -\dfrac{v_{0y}}{a_y} = -\dfrac{20.0\text{ m/s}}{-9.80\text{ m/s}^2} = 2.04\text{ s}$ to reach its maximum height. When the water reaches the building it has passed its maximum height and its vertical component of velocity is downward.

**3.21.** **IDENTIFY:** Take the origin of coordinates at the roof and let the $+y$-direction be upward. The rock moves in projectile motion, with $a_x = 0$ and $a_y = -g$. Apply constant acceleration equations for the $x$ and $y$ components of the motion.

**SET UP:**




$v_{0x} = v_0 \cos\alpha_0 = 25.2\text{ m/s}$

$v_{0y} = v_0 \sin\alpha_0 = 16.3\text{ m/s}$

**Figure 3.21a**

**(a)** At the maximum height $v_y = 0$.

$a_y = -9.80\text{ m/s}^2$, $v_y = 0$, $v_{0y} = +16.3\text{ m/s}$, $y - y_0 = ?$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

**EXECUTE:** $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (16.3\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = +13.6\text{ m}$

**(b) SET UP:** Find the velocity by solving for its $x$ and $y$ components.

$v_x = v_{0x} = 25.2\text{ m/s}$ (since $a_x = 0$)

$v_y = ?$, $a_y = -9.80\text{ m/s}^2$, $y - y_0 = -15.0\text{ m}$ (negative because at the ground the rock is below its initial position), $v_{0y} = 16.3\text{ m/s}$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$v_y = -\sqrt{v_{0y}^2 + 2a_y(y - y_0)}$ ($v_y$ is negative because at the ground the rock is traveling downward.)

**EXECUTE:** $v_y = -\sqrt{(16.3\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(-15.0\text{ m})} = -23.7\text{ m/s}$

Then $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(25.2\text{ m/s})^2 + (-23.7\text{ m/s})^2} = 34.6\text{ m/s}$.

**(c) SET UP:** Use the vertical motion ($y$-component) to find the time the rock is in the air:
$t = ?$, $v_y = -23.7$ m/s (from part (b)), $a_y = -9.80$ m/s$^2$, $v_{0y} = +16.3$ m/s

**EXECUTE:** $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-23.7 \text{ m/s} - 16.3 \text{ m/s}}{-9.80 \text{ m/s}^2} = +4.08 \text{ s}$

**SET UP:** Can use this $t$ to calculate the horizontal range:
$t = 4.08$ s, $v_{0x} = 25.2$ m/s, $a_x = 0$, $x - x_0 = ?$

**EXECUTE:** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (25.2 \text{ m/s})(4.08 \text{ s}) + 0 = 103 \text{ m}$

**(d)** Graphs of $x$ versus $t$, $y$ versus $t$, $v_x$ versus $t$ and $v_y$ versus $t$:

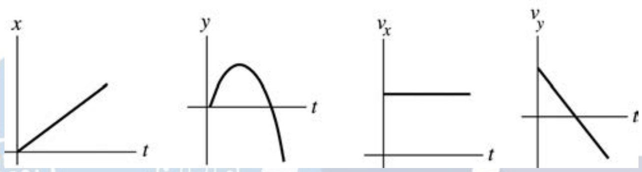



**Figure 3.21b**

**EVALUATE:** The time it takes the rock to travel vertically to the ground is the time it has to travel horizontally. With $v_{0y} = +16.3$ m/s the time it takes the rock to return to the level of the roof $(y = 0)$ is $t = 2v_{0y}/g = 3.33$ s. The time in the air is greater than this because the rock travels an additional 15.0 m to the ground.

**3.22. IDENTIFY and SET UP:** The stone moves in projectile motion. Its initial velocity is the same as that of the balloon. Use constant acceleration equations for the $x$ and $y$ components of its motion. Take $+y$ to be downward.

**EXECUTE:** **(a)** Use the vertical motion of the rock to find the initial height.
$t = 5.00$ s, $v_{0y} = +20.0$ m/s, $a_y = +9.80$ m/s$^2$, $y - y_0 = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $y - y_0 = 223$ m.

**(b)** In 5.00 s the balloon travels downward a distance $y - y_0 = (20.0 \text{ m/s})(5.00 \text{ s}) = 100 \text{ m}$. So, its height above ground when the rock hits is $223 \text{ m} - 100 \text{ m} = 123 \text{ m}$.

**(c)** The horizontal distance the rock travels in 5.00 s is (15.0 m/s)(5.00 s) = 75.0 m. The vertical component of the distance between the rock and the basket is 123 m, so the rock is $\sqrt{(75 \text{ m})^2 + (123 \text{ m})^2} = 144 \text{ m}$ from the basket when it hits the ground.

**(d)** (i) The basket has no horizontal velocity, so the rock has horizontal velocity 15.0 m/s relative to the basket. Just before the rock hits the ground, its vertical component of velocity is $v_y = v_{0y} + a_y t = 20.0 \text{ m/s} + (9.80 \text{ m/s}^2)(5.00 \text{ s}) = 69.0 \text{ m/s}$, downward, relative to the ground. The basket is moving downward at 20.0 m/s, so relative to the basket the rock has a downward component of velocity 49.0 m/s.
(ii) horizontal: 15.0 m/s; vertical: 69.0 m/s

**EVALUATE:** The rock has a constant horizontal velocity and accelerates downward.

**3.23. IDENTIFY:** Circular motion.

**SET UP:** Apply the equation $a_{\text{rad}} = 4\pi^2 R/T^2$, where $T = 24$ h.

**EXECUTE:** **(a)** $a_{\text{rad}} = \dfrac{4\pi^2(6.38\times10^6 \text{ m})}{[(24 \text{ h})(3600 \text{ s/h})]^2} = 0.034 \text{ m/s}^2 = 3.4\times10^{-3} g.$

**(b)** Solving the equation $a_{\text{rad}} = 4\pi^2 R/T^2$ for the period $T$ with $a_{\text{rad}} = g$,

$$T = \sqrt{\frac{4\pi^2(6.38\times10^6 \text{m})}{9.80 \text{ m/s}^2}} = 5070 \text{ s } = 1.4 \text{ h}.$$

**EVALUATE:** $a_{\text{rad}}$ is proportional to $1/T^2$, so to increase $a_{\text{rad}}$ by a factor of $\dfrac{1}{3.4\times10^{-3}} = 294$ requires that $T$ be multiplied by a factor of $\dfrac{1}{\sqrt{294}}$. $\dfrac{24\text{ h}}{\sqrt{294}} = 1.4\text{ h}.$

**3.24.** **IDENTIFY:** We want to find the acceleration of the inner ear of a dancer, knowing the rate at which she spins.

**SET UP:** $R = 0.070$ m. For 3.0 rev/s, the period $T$ (time for one revolution) is $T = \dfrac{1.0\text{ s}}{3.0\text{ rev}} = 0.333$ s. The speed is $v = d/T = (2\pi R)/T$, and $a_{\text{rad}} = v^2/R$.

**EXECUTE:** $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(2\pi R/T)^2}{R} = \dfrac{4\pi^2 R}{T^2} = \dfrac{4\pi^2(0.070\text{ m})}{(0.333\text{ s})^2} = 25\text{ m/s}^2 = 2.5g.$

**EVALUATE:** The acceleration is large and the force on the fluid must be 2.5 times its weight.

**3.25.** **IDENTIFY:** For the curved lowest part of the dive, the pilot's motion is approximately circular. We know the pilot's acceleration and the radius of curvature, and from this we want to find the pilot's speed.

**SET UP:** $a_{\text{rad}} = 5.5g = 53.9\text{ m/s}^2$. 1 mph = 0.4470 m/s. $a_{\text{rad}} = \dfrac{v^2}{R}$.

**EXECUTE:** $a_{\text{rad}} = \dfrac{v^2}{R}$, so $v = \sqrt{Ra_{\text{rad}}} = \sqrt{(280\text{ m})(53.9\text{ m/s}^2)} = 122.8\text{ m/s} = 274.8\text{ mph}$. Rounding these answers to 2 significant figures (because of $5.5g$), gives $v = 120$ m/s = 270 mph.

**EVALUATE:** This speed is reasonable for the type of plane flown by a test pilot.

**3.26.** **IDENTIFY:** Each blade tip moves in a circle of radius $R = 3.40$ m and therefore has radial acceleration $a_{\text{rad}} = v^2/R$.

**SET UP:** 550 rev/min = 9.17 rev/s, corresponding to a period of $T = \dfrac{1}{9.17\text{ rev/s}} = 0.109$ s.

**EXECUTE:** **(a)** $v = \dfrac{2\pi R}{T} = 196\text{ m/s}.$

**(b)** $a_{\text{rad}} = \dfrac{v^2}{R} = 1.13\times10^4\text{ m/s}^2 = 1.15\times10^3 g.$

**EVALUATE:** $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$ gives the same results for $a_{\text{rad}}$ as in part (b).

**3.27.** **IDENTIFY:** Uniform circular motion.

**SET UP:** Since the magnitude of $\vec{v}$ is constant, $v_{\tan} = \dfrac{d|\vec{v}|}{dt} = 0$ and the resultant acceleration is equal to the radial component. At each point in the motion the radial component of the acceleration is directed in toward the center of the circular path and its magnitude is given by $v^2/R$.

**EXECUTE:** **(a)** $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(6.00\text{ m/s})^2}{14.0\text{ m}} = 2.57\text{ m/s}^2$, upward.

**(b)** The radial acceleration has the same magnitude as in part (a), but now the direction toward the center of the circle is downward. The acceleration at this point in the motion is $2.57\text{ m/s}^2$, downward.

**(c) SET UP:** The time to make one rotation is the period $T$, and the speed $v$ is the distance for one revolution divided by $T$.

**EXECUTE:** $v = \dfrac{2\pi R}{T}$ so $T = \dfrac{2\pi R}{v} = \dfrac{2\pi\,(14.0\text{ m})}{6.00\text{ m/s}} = 14.7\text{ s}.$

**EVALUATE:** The radial acceleration is constant in magnitude since $v$ is constant and is at every point in the motion directed toward the center of the circular path. The acceleration is perpendicular to $\vec{v}$ and is nonzero because the direction of $\vec{v}$ changes.

**3.28.** **IDENTIFY:** Each planet moves in a circular orbit and therefore has acceleration $a_{\text{rad}} = v^2/R$.

**SET UP:** The radius of the earth's orbit is $r = 1.50\times10^{11}$ m and its orbital period is $T = 365$ days $= 3.16\times10^{7}$ s. For Mercury, $r = 5.79\times10^{10}$ m and $T = 88.0$ days $= 7.60\times10^{6}$ s.

**EXECUTE:** **(a)** $v = \dfrac{2\pi r}{T} = 2.98\times10^{4}$ m/s

**(b)** $a_{\text{rad}} = \dfrac{v^2}{r} = 5.91\times10^{-3}$ m/s$^2$.

**(c)** $v = 4.79\times10^{4}$ m/s, and $a_{\text{rad}} = 3.96\times10^{-2}$ m/s$^2$.

**EVALUATE:** Mercury has a larger orbital velocity and a larger radial acceleration than earth.

**3.29.** **IDENTIFY:** Each part of his body moves in uniform circular motion, with $a_{\text{rad}} = \dfrac{v^2}{R}$. The speed in rev/s is $1/T$, where $T$ is the period in seconds (time for 1 revolution). The speed $v$ increases with $R$ along the length of his body but all of him rotates with the same period $T$.

**SET UP:** For his head $R = 8.84$ m and for his feet $R = 6.84$ m.

**EXECUTE:** **(a)** $v = \sqrt{Ra_{\text{rad}}} = \sqrt{(8.84\text{ m})(12.5)(9.80\text{ m/s}^2)} = 32.9$ m/s

**(b)** Use $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$. Since his head has $a_{\text{rad}} = 12.5g$ and $R = 8.84$ m,

$T = 2\pi\sqrt{\dfrac{R}{a_{\text{rad}}}} = 2\pi\sqrt{\dfrac{8.84\,\text{m}}{12.5(9.80\,\text{m/s}^2)}} = 1.688\,\text{s}$. Then his feet have $a_{\text{rad}} = \dfrac{R}{T^2} = \dfrac{4\pi^2(6.84\,\text{m})}{(1.688\,\text{s})^2} = 94.8\,\text{m/s}^2 = 9.67$ g.

The difference between the acceleration of his head and his feet is $12.5g - 9.67g = 2.83g = 27.7$ m/s$^2$.

**(c)** $\dfrac{1}{T} = \dfrac{1}{1.69\text{ s}} = 0.592$ rev/s $= 35.5$ rpm

**EVALUATE:** His feet have speed $v = \sqrt{Ra_{\text{rad}}} = \sqrt{(6.84\text{ m})(94.8\text{ m/s}^2)} = 25.5$ m/s.

**3.30.** **IDENTIFY:** The relative velocities are $\vec{v}_{\text{S/F}}$, the velocity of the scooter relative to the flatcar, $\vec{v}_{\text{S/G}}$, the scooter relative to the ground and $\vec{v}_{\text{F/G}}$, the flatcar relative to the ground. $\vec{v}_{\text{S/G}} = \vec{v}_{\text{S/F}} + \vec{v}_{\text{F/G}}$. Carry out the vector addition by drawing a vector addition diagram.

**SET UP:** $\vec{v}_{\text{S/F}} = \vec{v}_{\text{S/G}} - \vec{v}_{\text{F/G}}$. $\vec{v}_{\text{F/G}}$ is to the right, so $-\vec{v}_{\text{F/G}}$ is to the left.

**EXECUTE:** In each case the vector addition diagram gives

**(a)** 5.0 m/s to the right

**(b)** 16.0 m/s to the left

**(c)** 13.0 m/s to the left.

**EVALUATE:** The scooter has the largest speed relative to the ground when it is moving to the right relative to the flatcar, since in that case the two velocities $\vec{v}_{\text{S/F}}$ and $\vec{v}_{\text{F/G}}$ are in the same direction and their magnitudes add.

**3.31.** **IDENTIFY:** Relative velocity problem. The time to walk the length of the moving sidewalk is the length divided by the velocity of the woman relative to the ground.

**SET UP:** Let W stand for the woman, G for the ground and S for the sidewalk. Take the positive direction to be the direction in which the sidewalk is moving.

The velocities are $v_{\text{W/G}}$ (woman relative to the ground), $v_{\text{W/S}}$ (woman relative to the sidewalk), and $v_{\text{S/G}}$ (sidewalk relative to the ground).

The equation for relative velocity becomes $v_{\text{W/G}} = v_{\text{W/S}} + v_{\text{S/G}}$.

The time to reach the other end is given by $t = \dfrac{\text{distance traveled relative to ground}}{v_{\text{W/G}}}$

**EXECUTE:** **(a)** $v_{\text{S/G}} = 1.0$ m/s

$v_{\text{W/S}} = +1.5$ m/s

$v_{\text{W/G}} = v_{\text{W/S}} + v_{\text{S/G}} = 1.5\text{ m/s} + 1.0\text{ m/s} = 2.5$ m/s.

$t = \dfrac{35.0\text{ m}}{v_{\text{W/G}}} = \dfrac{35.0\text{ m}}{2.5\text{ m/s}} = 14\text{ s.}$

**(b)** $v_{\text{S/G}} = 1.0\text{ m/s}$

$v_{\text{W/S}} = -1.5\text{ m/s}$

$v_{\text{W/G}} = v_{\text{W/S}} + v_{\text{S/G}} = -1.5\text{ m/s} + 1.0\text{ m/s} = -0.5\text{ m/s}$. (Since $v_{\text{W/G}}$ now is negative, she must get on the moving sidewalk at the opposite end from in part (a).)

$t = \dfrac{-35.0\text{ m}}{v_{\text{W/G}}} = \dfrac{-35.0\text{ m}}{-0.5\text{ m/s}} = 70\text{ s.}$

**EVALUATE:** Her speed relative to the ground is much greater in part (a) when she walks with the motion of the sidewalk.

**3.32.** **IDENTIFY:** Calculate the rower's speed relative to the shore for each segment of the round trip.
**SET UP:** The boat's speed relative to the shore is 6.8 km/h downstream and 1.2 km/h upstream.
**EXECUTE:** The walker moves a total distance of 3.0 km at a speed of 4.0 km/h, and takes a time of three fourths of an hour (45.0 min).

The total time the rower takes is $\dfrac{1.5\text{ km}}{6.8\text{ km/h}} + \dfrac{1.5\text{ km}}{1.2\text{ km/h}} = 1.47\text{ h} = 88.2\text{ min.}$

**EVALUATE:** It takes the rower longer, even though for half the distance his speed is greater than 4.0 km/h. The rower spends more time at the slower speed.

**3.33.** **IDENTIFY:** Apply the relative velocity relation.
**SET UP:** The relative velocities are $\vec{v}_{\text{C/E}}$, the canoe relative to the earth, $\vec{v}_{\text{R/E}}$, the velocity of the river relative to the earth and $\vec{v}_{\text{C/R}}$, the velocity of the canoe relative to the river.
**EXECUTE:** $\vec{v}_{\text{C/E}} = \vec{v}_{\text{C/R}} + \vec{v}_{\text{R/E}}$ and therefore $\vec{v}_{\text{C/R}} = \vec{v}_{\text{C/E}} - \vec{v}_{\text{R/E}}$. The velocity components of $\vec{v}_{\text{C/R}}$ are $-0.50\text{ m/s} + (0.40\text{ m/s})/\sqrt{2}$, east and $(0.40\text{ m/s})/\sqrt{2}$, south, for a velocity relative to the river of 0.36 m/s, at $52.5°$ south of west.
**EVALUATE:** The velocity of the canoe relative to the river has a smaller magnitude than the velocity of the canoe relative to the earth.

**3.34.** **IDENTIFY:** Relative velocity problem in two dimensions.
**(a) SET UP:** $\vec{v}_{\text{P/A}}$ is the velocity of the plane relative to the air. The problem states that $\vec{v}_{\text{P/A}}$ has magnitude 35 m/s and direction south.
$\vec{v}_{\text{A/E}}$ is the velocity of the air relative to the earth. The problem states that $\vec{v}_{\text{A/E}}$ is to the southwest ($45°$ S of W) and has magnitude 10 m/s.
The relative velocity equation is $\vec{v}_{\text{P/E}} = \vec{v}_{\text{P/A}} + \vec{v}_{\text{A/E}}$.

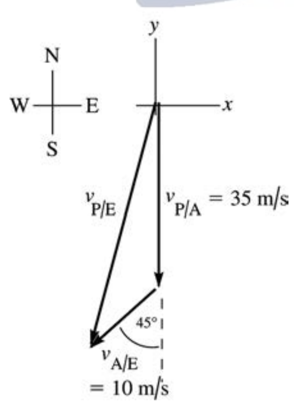



**Figure 3.34a**

**EXECUTE:** **(b)** $(v_{\text{P/A}})_x = 0$, $(v_{\text{P/A}})_y = -35\text{ m/s}$

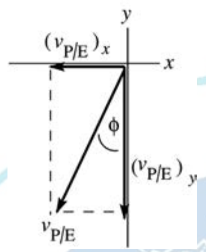
$(v_{\text{A/E}})_x = -(10 \text{ m/s})\cos 45° = -7.07 \text{ m/s},$

$(v_{\text{A/E}})_y = -(10 \text{ m/s})\sin 45° = -7.07 \text{ m/s}$

$(v_{\text{P/E}})_x = (v_{\text{P/A}})_x + (v_{\text{A/E}})_x = 0 - 7.07 \text{ m/s} = -7.1 \text{ m/s}$

$(v_{\text{P/E}})_y = (v_{\text{P/A}})_y + (v_{\text{A/E}})_y = -35 \text{ m/s} - 7.07 \text{ m/s} = -42 \text{ m/s}$

**(c)**




$$v_{\text{P/E}} = \sqrt{(v_{\text{P/E}})_x^2 + (v_{\text{P/E}})_y^2}$$

$$v_{\text{P/E}} = \sqrt{(-7.1 \text{ m/s})^2 + (-42 \text{ m/s})^2} = 43 \text{ m/s}$$

$$\tan\phi = \frac{(v_{\text{P/E}})_x}{(v_{\text{P/E}})_y} = \frac{-7.1}{-42} = 0.169$$

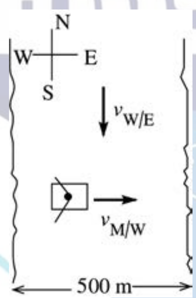
$\phi = 9.6°$; ($9.6°$ west of south)

**Figure 3.34b**

**EVALUATE:** The relative velocity addition diagram does not form a right triangle so the vector addition must be done using components. The wind adds both southward and westward components to the velocity of the plane relative to the ground.

**3.35.** **IDENTIFY:** Relative velocity problem in two dimensions. His motion relative to the earth (time displacement) depends on his velocity relative to the earth so we must solve for this velocity.

**(a) SET UP:** View the motion from above.




The velocity vectors in the problem are:

$\vec{v}_{\text{M/E}}$, the velocity of the man relative to the earth

$\vec{v}_{\text{W/E}}$, the velocity of the water relative to the earth

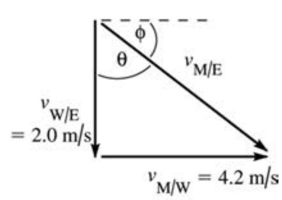
$\vec{v}_{\text{M/W}}$, the velocity of the man relative to the water

The rule for adding these velocities is

$\vec{v}_{\text{M/E}} = \vec{v}_{\text{M/W}} + \vec{v}_{\text{W/E}}$

**Figure 3.35a**

The problem tells us that $\vec{v}_{\text{W/E}}$ has magnitude 2.0 m/s and direction due south. It also tells us that $\vec{v}_{\text{M/W}}$ has magnitude 4.2 m/s and direction due east. The vector addition diagram is then as shown in Figure 3.35b.




This diagram shows the vector addition

$\vec{v}_{\text{M/E}} = \vec{v}_{\text{M/W}} + \vec{v}_{\text{W/E}}$

and also has $\vec{v}_{\text{M/W}}$ and $\vec{v}_{\text{W/E}}$ in their specified directions. Note that the vector diagram forms a right triangle.

**Figure 3.35b**

The Pythagorean theorem applied to the vector addition diagram gives $v_{\text{M/E}}^2 = v_{\text{M/W}}^2 + v_{\text{W/E}}^2$.

**EXECUTE:** $v_{\mathrm{M/E}} = \sqrt{v_{\mathrm{M/W}}^2 + v_{\mathrm{W/E}}^2} = \sqrt{(4.2\ \mathrm{m/s})^2 + (2.0\ \mathrm{m/s})^2} = 4.7\ \mathrm{m/s};\ \tan\theta = \dfrac{v_{\mathrm{M/W}}}{v_{\mathrm{W/E}}} = \dfrac{4.2\ \mathrm{m/s}}{2.0\ \mathrm{m/s}} = 2.10;$

$\theta = 65°$; or $\phi = 90° - \theta = 25°$. The velocity of the man relative to the earth has magnitude 4.7 m/s and direction 25° S of E.

**(b)** This requires careful thought. To cross the river the man must travel 500 m due east relative to the earth. The man's velocity relative to the earth is $\vec{v}_{\mathrm{M/E}}$. But, from the vector addition diagram the eastward component of $v_{\mathrm{M/E}}$ equals $v_{\mathrm{M/W}} = 4.2$ m/s.

Thus $t = \dfrac{x - x_0}{v_x} = \dfrac{500\ \mathrm{m}}{4.2\ \mathrm{m/s}} = 119$ s, which we round to 120 s.

**(c)** The southward component of $\vec{v}_{\mathrm{M/E}}$ equals $v_{\mathrm{W/E}} = 2.0$ m/s. Therefore, in the 120 s it takes him to cross the river, the distance south the man travels relative to the earth is

$$y - y_0 = v_y t = (2.0\ \mathrm{m/s})(119\ \mathrm{s}) = 240\ \mathrm{m}.$$

**EVALUATE:** If there were no current he would cross in the same time, $(500\ \mathrm{m})/(4.2\ \mathrm{m/s}) = 120$ s. The current carries him downstream but doesn't affect his motion in the perpendicular direction, from bank to bank.

**3.36.** **IDENTIFY:** Use the relation that relates the relative velocities.

**SET UP:** The relative velocities are the water relative to the earth, $\vec{v}_{\mathrm{W/E}}$, the boat relative to the water, $\vec{v}_{\mathrm{B/W}}$, and the boat relative to the earth, $\vec{v}_{\mathrm{B/E}}$. $\vec{v}_{\mathrm{B/E}}$ is due east, $\vec{v}_{\mathrm{W/E}}$ is due south and has magnitude 2.0 m/s. $v_{\mathrm{B/W}} = 4.2$ m/s. $\vec{v}_{\mathrm{B/E}} = \vec{v}_{\mathrm{B/W}} + \vec{v}_{\mathrm{W/E}}$. The velocity addition diagram is given in Figure 3.36.

**EXECUTE:** **(a)** Find the direction of $\vec{v}_{\mathrm{B/W}}$. $\sin\theta = \dfrac{v_{\mathrm{W/E}}}{v_{\mathrm{B/W}}} = \dfrac{2.0\ \mathrm{m/s}}{4.2\ \mathrm{m/s}}$. $\theta = 28.4°$, north of east.

**(b)** $v_{\mathrm{B/E}} = \sqrt{v_{\mathrm{B/W}}^2 - v_{\mathrm{W/E}}^2} = \sqrt{(4.2\ \mathrm{m/s})^2 - (2.0\ \mathrm{m/s})^2} = 3.7\ \mathrm{m/s}$

**(c)** $t = \dfrac{800\ \mathrm{m}}{v_{\mathrm{B/E}}} = \dfrac{800\ \mathrm{m}}{3.7\ \mathrm{m/s}} = 216$ s.

**EVALUATE:** It takes longer to cross the river in this problem than it did in Problem 3.35. In the direction straight across the river (east) the component of his velocity relative to the earth is lass than 4.2 m/s.

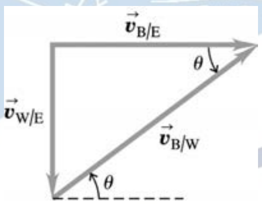



**Figure 3.36**

**3.37.** **IDENTIFY:** The resultant velocity, relative to the ground, is directly southward. This velocity is the sum of the velocity of the bird relative to the air and the velocity of the air relative to the ground.

**SET UP:** $v_{\mathrm{B/A}} = 100$ km/h. $\vec{v}_{\mathrm{A/G}} = 40$ km/h, east. $\vec{v}_{\mathrm{B/G}} = \vec{v}_{\mathrm{B/A}} + \vec{v}_{\mathrm{A/G}}$.

**EXECUTE:** We want $\vec{v}_{\mathrm{B/G}}$ to be due south. The relative velocity addition diagram is shown in Figure 3.37.

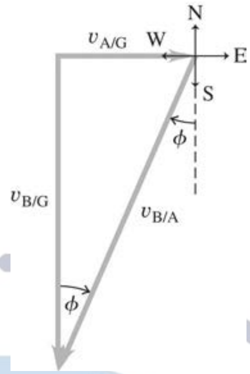



**Figure 3.37**

**(a)** $\sin\phi = \dfrac{v_{A/G}}{v_{B/A}} = \dfrac{40 \text{ km/h}}{100 \text{ km/h}}$, $\phi = 24°$, west of south.

**(b)** $v_{B/G} = \sqrt{v_{B/A}{}^2 - v_{A/G}{}^2} = 91.7$ km/h. $t = \dfrac{d}{v_{B/G}} = \dfrac{500 \text{ km}}{91.7 \text{ km/h}} = 5.5$ h.

**EVALUATE:** The speed of the bird relative to the ground is less than its speed relative to the air. Part of its velocity relative to the air is directed to oppose the effect of the wind.

**3.38.** **IDENTIFY:** Use the relation that relates the relative velocities.

**SET UP:** The relative velocities are the velocity of the plane relative to the ground, $\vec{v}_{P/G}$, the velocity of the plane relative to the air, $\vec{v}_{P/A}$, and the velocity of the air relative to the ground, $\vec{v}_{A/G}$. $\vec{v}_{P/G}$ must be due west and $\vec{v}_{A/G}$ must be south. $v_{A/G} = 80$ km/h and $v_{P/A} = 320$ km/h. $\vec{v}_{P/G} = \vec{v}_{P/A} + \vec{v}_{A/G}$. The relative velocity addition diagram is given in Figure 3.38.

**EXECUTE:** **(a)** $\sin\theta = \dfrac{v_{A/G}}{v_{P/A}} = \dfrac{80 \text{ km/h}}{320 \text{ km/h}}$ and $\theta = 14°$, north of west.

**(b)** $v_{P/G} = \sqrt{v_{P/A}^2 - v_{A/G}^2} = \sqrt{(320 \text{ km/h})^2 - (80.0 \text{ km/h})^2} = 310$ km/h.

**EVALUATE:** To travel due west the velocity of the plane relative to the air must have a westward component and also a component that is northward, opposite to the wind direction.

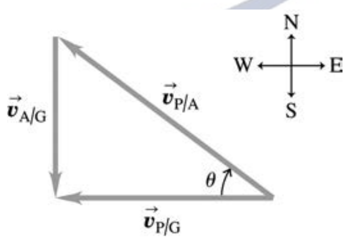



**Figure 3.38**

**3.39.** **IDENTIFY:** $\vec{v} = \dfrac{d\vec{r}}{dt}$ and $\vec{a} = \dfrac{d\vec{v}}{dt}$

**SET UP:** $\dfrac{d}{dt}(t^n) = nt^{n-1}$. At $t = 1.00$ s, $a_x = 4.00 \text{ m/s}^2$ and $a_y = 3.00 \text{ m/s}^2$. At $t = 0$, $x = 0$ and $y = 50.0$ m.

**EXECUTE:** **(a)** $v_x = \dfrac{dx}{dt} = 2Bt$. $a_x = \dfrac{dv_x}{dt} = 2B$, which is independent of $t$. $a_x = 4.00\ \text{m/s}^2$ gives $B = 2.00\ \text{m/s}^2$. $v_y = \dfrac{dy}{dt} = 3Dt^2$. $a_y = \dfrac{dv_y}{dt} = 6Dt$. $a_y = 3.00\ \text{m/s}^2$ gives $D = 0.500\ \text{m/s}^3$. $x = 0$ at $t = 0$ gives $A = 0$. $y = 50.0$ m at $t = 0$ gives $C = 50.0$ m.

**(b)** At $t = 0$, $v_x = 0$ and $v_y = 0$, so $\vec{v} = 0$. At $t = 0$, $a_x = 2B = 4.00\ \text{m/s}^2$ and $a_y = 0$, so $\vec{a} = (4.00\ \text{m/s}^2)\hat{i}$.

**(c)** At $t = 10.0$ s, $v_x = 2\,(2.00\ \text{m/s}^2)(10.0\ \text{s}) = 40.0\ \text{m/s}$ and $v_y = 3(0.500\ \text{m/s}^3)(10.0\ \text{s})^2 = 150\ \text{m/s}$.
$v = \sqrt{v_x^2 + v_y^2} = 155\ \text{m/s}$.

**(d)** $x = (2.00\ \text{m/s}^2)(10.0\ \text{s})^2 = 200\ \text{m}$, $y = 50.0\ \text{m} + (0.500\ \text{m/s}^3)(10.0\ \text{s})^3 = 550\ \text{m}$.
$\vec{r} = (200\ \text{m})\hat{i} + (550\ \text{m})\hat{j}$.

**EVALUATE:** The velocity and acceleration vectors as functions of time are
$\vec{v}(t) = (2Bt)\hat{i} + (3Dt^2)\hat{j}$ and $\vec{a}(t) = (2B)\hat{i} + (6Dt)\hat{j}$. The acceleration is not constant.

**3.40.** **IDENTIFY:** The acceleration is not constant but is known as a function of time.
**SET UP:** Integrate the acceleration to get the velocity and the velocity to get the position. At the maximum height $v_y = 0$.

**EXECUTE:** **(a)** $v_x = v_{0x} + \dfrac{\alpha}{3}t^3$, $v_y = v_{0y} + \beta t - \dfrac{\gamma}{2}t^2$, and $x = v_{0x}t + \dfrac{\alpha}{12}t^4$, $y = v_{0y}t + \dfrac{\beta}{2}t^2 - \dfrac{\gamma}{6}t^3$.

**(b)** Setting $v_y = 0$ yields a quadratic in $t$, $0 = v_{0y} + \beta t - \dfrac{\gamma}{2}t^2$. Using the numerical values given in the problem, this equation has as the positive solution $t = \dfrac{1}{\gamma}\left[\beta + \sqrt{\beta^2 + 2v_{0y}\gamma}\right] = 13.59$ s. Using this time in the expression for $y(t)$ gives a maximum height of 341 m.

**(c)** $y = 0$ gives $0 = v_{0y}t + \dfrac{\beta}{2}t^2 - \dfrac{\gamma}{6}t^3$ and $\dfrac{\gamma}{6}t^2 - \dfrac{\beta}{2}t - v_{0y} = 0$. Using the numbers given in the problem, the positive solution is $t = 20.73$ s. For this $t$, $x = 3.85 \times 10^4$ m.
**EVALUATE:** We cannot use the constant-acceleration kinematics formulas, but calculus provides the solution.

**3.41.** **IDENTIFY:** $\vec{v} = d\vec{r}/dt$. This vector will make a $45°$ angle with both axes when its $x$- and $y$-components are equal.

**SET UP:** $\dfrac{d(t^n)}{dt} = nt^{n-1}$.

**EXECUTE:** $\vec{v} = 2bt\hat{i} + 3ct^2\hat{j}$. $v_x = v_y$ gives $t = 2b/3c$.

**EVALUATE:** Both components of $\vec{v}$ change with $t$.

**3.42.** **IDENTIFY:** Use the position vector of a dragonfly to determine information about its velocity vector and acceleration vector.
**SET UP:** Use the definitions $v_x = dx/dt$, $v_y = dy/dt$, $a_x = dv_x/dt$, and $a_y = dv_y/dt$.

**EXECUTE:** **(a)** Taking derivatives of the position vector gives the components of the velocity vector:
$v_x(t) = (0.180\ \text{m/s}^2)t$, $v_y(t) = (-0.0450\ \text{m/s}^3)t^2$. Use these components and the given direction:

$\tan 30.0^\circ = \dfrac{(0.0450\ \text{m/s}^3)t^2}{(0.180\ \text{m/s}^2)t}$, which gives $t = 2.31$ s.

**(b)** Taking derivatives of the velocity components gives the acceleration components: $a_x = 0.180\ \text{m/s}^2$, $a_y(t) = -(0.0900\ \text{m/s}^3)t$. At $t = 2.31$ s, $a_x = 0.180\ \text{m/s}^2$ and $a_y = -0.208\ \text{m/s}^2$, giving $a = 0.275\ \text{m/s}^2$. The direction is $\tan\theta = \dfrac{0.208}{0.180}$, so $\theta = 49.1^\circ$ clockwise from $+x$-axis.

**EVALUATE:** The acceleration is not constant, so we cannot use the standard kinematics formulas.

**3.43.** **IDENTIFY:** Once the rocket leaves the incline it moves in projectile motion. The acceleration along the incline determines the initial velocity and initial position for the projectile motion.

**SET UP:** For motion along the incline let $+x$ be directed up the incline. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = \sqrt{2(1.90\ \text{m/s}^2)(200\ \text{m})} = 27.57\ \text{m/s}$. When the projectile motion begins the rocket has $v_0 = 27.57$ m/s at $35.0^\circ$ above the horizontal and is at a vertical height of $(200.0\ \text{m})\sin 35.0^\circ = 114.7$ m. For the projectile motion let $+x$ be horizontal to the right and let $+y$ be upward. Let $y = 0$ at the ground. Then $y_0 = 114.7$ m, $v_{0x} = v_0 \cos 35.0^\circ = 22.57$ m/s, $v_{0y} = v_0 \sin 35.0^\circ = 15.81$ m/s, $a_x = 0$, $a_y = -9.80\ \text{m/s}^2$. Let $x = 0$ at point $A$, so $x_0 = (200.0\ \text{m})\cos 35.0^\circ = 163.8$ m.

**EXECUTE:** **(a)** At the maximum height $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (15.81\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 12.77$ m and $y = 114.7\ \text{m} + 12.77\ \text{m} = 128$ m. The maximum height above ground is 128 m.

**(b)** The time in the air can be calculated from the vertical component of the projectile motion: $y - y_0 = -114.7$ m, $v_{0y} = 15.81$ m/s, $a_y = -9.80\ \text{m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $(4.90\ \text{m/s}^2)t^2 - (15.81\ \text{m/s})t - 114.7$ m. The quadratic formula gives $t = 6.713$ s for the positive root. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (22.57\ \text{m/s})(6.713\ \text{s}) = 151.6$ m and $x = 163.8\ \text{m} + 151.6\ \text{m} = 315$ m. The horizontal range of the rocket is 315 m.

**EVALUATE:** The expressions for $h$ and $R$ derived in the range formula do not apply here. They are only for a projectile fired on level ground.

**3.44.** **IDENTIFY:** $\vec{r} = \vec{r}_0 + \int_0^t \vec{v}(t)dt$ and $\boldsymbol{a} = \dfrac{d\vec{v}}{dt}$.

**SET UP:** At $t = 0$, $x_0 = 0$ and $y_0 = 0$.

**EXECUTE:** **(a)** Integrating, $\vec{r} = \left(\alpha t - \dfrac{\beta}{3}t^3\right)\hat{\boldsymbol{i}} + \left(\dfrac{\gamma}{2}t^2\right)\hat{\boldsymbol{j}}$. Differentiating, $\vec{a} = (-2\beta t)\hat{\boldsymbol{i}} + \gamma\hat{\boldsymbol{j}}$.

**(b)** The positive time at which $x = 0$ is given by $t^2 = 3\alpha/\beta$. At this time, the $y$-coordinate is

$$y = \frac{\gamma}{2}t^2 = \frac{3\alpha\gamma}{2\beta} = \frac{3(2.4\ \text{m/s})(4.0\ \text{m/s}^2)}{2(1.6\ \text{m/s}^3)} = 9.0\ \text{m}.$$

**EVALUATE:** The acceleration is not constant.

**3.45.** **IDENTIFY:** Take $+y$ to be downward. Both objects have the same vertical motion, with $v_{0y}$ and $a_y = +g$. Use constant acceleration equations for the $x$ and $y$ components of the motion.

**SET UP:** Use the vertical motion to find the time in the air:

$v_{0y} = 0$, $a_y = 9.80\ \text{m/s}^2$, $y - y_0 = 25$ m, $t = ?$

**EXECUTE:** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 2.259$ s.

During this time the dart must travel 90 m, so the horizontal component of its velocity must be

$v_{0x} = \dfrac{x - x_0}{t} = \dfrac{70\ \text{m}}{2.259\ \text{s}} = 31\ \text{m/s}.$

**EVALUATE:** Both objects hit the ground at the same time. The dart hits the monkey for any muzzle velocity greater than 31 m/s.

**3.46. IDENTIFY:** The velocity has a horizontal tangential component and a vertical component. The vertical component of acceleration is zero and the horizontal component is $a_{\text{rad}} = \dfrac{v_x^2}{R}$.

**SET UP:** Let $+y$ be upward and $+x$ be in the direction of the tangential velocity at the instant we are considering.

**EXECUTE:** **(a)** The bird's tangential velocity can be found from

$$v_x = \frac{\text{circumference}}{\text{time of rotation}} = \frac{2\pi(6.00\text{ m})}{5.00\text{ s}} = 7.54\text{ m/s}.$$

Thus its velocity consists of the components $v_x = 7.54$ m/s and $v_y = 3.00$ m/s. The speed relative to the ground is then $v = \sqrt{v_x^2 + v_y^2} = 8.11$ m/s.

**(b)** The bird's speed is constant, so its acceleration is strictly centripetal—entirely in the horizontal direction, toward the center of its spiral path—and has magnitude $a_{\text{rad}} = \dfrac{v_x^2}{r} = \dfrac{(7.54\text{ m/s})^2}{6.00\text{ m}} = 9.48\text{ m/s}^2$.

**(c)** Using the vertical and horizontal velocity components $\theta = \tan^{-1}\dfrac{3.00\text{ m/s}}{7.54\text{ m/s}} = 21.7°$.

**EVALUATE:** The angle between the bird's velocity and the horizontal remains constant as the bird rises.

**3.47. IDENTIFY:** The cannister moves in projectile motion. Its initial velocity is horizontal. Apply constant acceleration equations for the *x* and *y* components of motion.

**SET UP:**

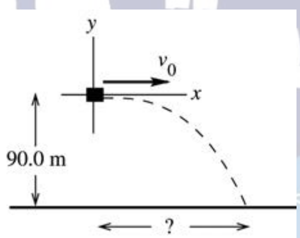



**Figure 3.47**

Take the origin of coordinates at the point where the cannister is released. Take $+y$ to be upward. The initial velocity of the cannister is the velocity of the plane, 64.0 m/s in the $+x$-direction.

Use the vertical motion to find the time of fall:

$t = ?$, $v_{0y} = 0$, $a_y = -9.80\text{ m/s}^2$, $y - y_0 = -90.0$ m (When the cannister reaches the ground it is 90.0 m below the origin.)

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$$

**EXECUTE:** Since $v_{0y} = 0$, $t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(-90.0\text{ m})}{-9.80\text{ m/s}^2}} = 4.286$ s.

**SET UP:** Then use the horizontal component of the motion to calculate how far the cannister falls in this time:

$x - x_0 = ?$, $a_x - 0$, $v_{0x} = 64.0$ m/s

**EXECUTE:** $x - x_0 = v_0 t + \frac{1}{2}at^2 = (64.0\text{ m/s})(4.286\text{ s}) + 0 = 274$ m.

**EVALUATE:** The time it takes the cannister to fall 90.0 m, starting from rest, is the time it travels horizontally at constant speed.

**3.48. IDENTIFY:** The person moves in projectile motion. Her vertical motion determines her time in the air.

**SET UP:** Take $+y$ upward. $v_{0x} = 15.0$ m/s, $v_{0y} = +10.0$ m/s, $a_x = 0$, $a_y = -9.80\text{ m/s}^2$.

**EXECUTE:** **(a)** Use the vertical motion to find the time in the air: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = -30.0$ m gives $-30.0\text{ m} = (10.0\text{ m/s})t - (4.90\text{ m/s}^2)t^2$. The quadratic formula gives

$t = \frac{1}{2(4.9)}\left(+10.0 \pm \sqrt{(-10.0)^2 - 4(4.9)(-30)}\right)$ s. The positive solution is $t = 3.70$ s. During this time she travels a horizontal distance $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (15.0 \text{ m/s})(3.70 \text{ s}) = 55.5$ m. She will land 55.5 m south of the point where she drops from the helicopter and this is where the mats should have been placed.

**(b)** The *x-t*, *y-t*, $v_x$ *-t* and $v_y$ *-t* graphs are sketched in Figure 3.48.

**EVALUATE:** If she had dropped from rest at a height of 30.0 m it would have taken her $t = \sqrt{\frac{2(30.0 \text{ m})}{9.80 \text{ m/s}^2}} = 2.47$ s. She is in the air longer than this because she has an initial vertical component of velocity that is upward.

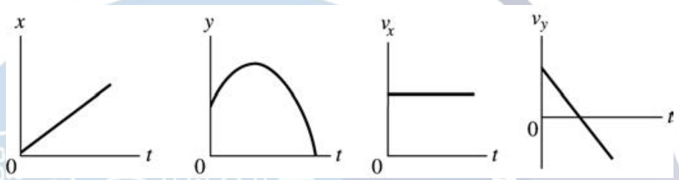



**Figure 3.48**

**3.49.** **IDENTIFY:** The suitcase moves in projectile motion. The initial velocity of the suitcase equals the velocity of the airplane.

**SET UP:** Take $+y$ to be upward. $a_x = 0$, $a_y = -g$.

**EXECUTE:** Use the vertical motion to find the time it takes the suitcase to reach the ground: $v_{0y} = v_0 \sin 23°$, $a_y = -9.80 \text{ m/s}^2$, $y - y_0 = -114$ m, $t = ?$ $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 9.60$ s.

The distance the suitcase travels horizontally is $x - x_0 = v_{0x} = (v_0 \cos 23.0°)t = 795$ m.

**EVALUATE:** An object released from rest at a height of 114 m strikes the ground at $t = \sqrt{\frac{2(y - y_0)}{-g}} = 4.82$ s. The suitcase is in the air much longer than this since it initially has an upward component of velocity.

**3.50.** **IDENTIFY:** The shell moves as a projectile. To just clear the top of the cliff, the shell must have $y - y_0 = 25.0$ m when it has $x - x_0 = 60.0$ m.

**SET UP:** Let $+y$ be upward. $a_x = 0$, $a_y = -g$. $v_{0x} = v_0 \cos 43°$, $v_{0y} = v_0 \sin 43°$.

**EXECUTE:** **(a)** horizontal motion: $x - x_0 = v_{0x}t$ so $t = \frac{60.0 \text{ m}}{(v_0 \cos 43°)}$.

vertical motion: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $25.0 \text{ m} = (v_0 \sin 43.0°)\, t + \frac{1}{2}(-9.80 \text{ m/s}^2)\, t^2$.

Solving these two simultaneous equations for $v_0$ and $t$ gives $v_0 = 32.6$ m/s and $t = 2.51$ s.

**(b)** $v_y$ when shell reaches cliff:

$$v_y = v_{0y} + a_y t = (32.6 \text{ m/s}) \sin 43.0° - (9.80 \text{ m/s}^2)(2.51 \text{ s}) = -2.4 \text{ m/s}$$

The shell is traveling downward when it reaches the cliff, so it lands right at the edge of the cliff.

**EVALUATE:** The shell reaches its maximum height at $t = -\frac{v_{0y}}{a_y} = 2.27$ s, which confirms that at $t = 2.51$ s it has passed its maximum height and is on its way down when it strikes the edge of the cliff.

**3.51.** **IDENTIFY:** Find the horizontal distance a rocket moves if it has a non-constant horizontal acceleration but a constant vertical acceleration of *g* downward.

**SET UP:** The vertical motion is *g* downward, so we can use the constant acceleration formulas for that component of the motion. We must use integration for the horizontal motion because the acceleration is not constant. Solving for $t$ in the kinematics formula for $y$ gives $t=\sqrt{\dfrac{2(y-y_0)}{a_y}}$. In the horizontal direction we must use $v_x(t)=v_{0x}+\int_0^t a_x(t')dt'$ and $x-x_0=\int_0^t v_x(t')dt'$.

**EXECUTE:** Use vertical motion to find $t$. $t=\sqrt{\dfrac{2(y-y_0)}{a_y}}=\sqrt{\dfrac{2(30.0\text{ m})}{9.80\text{ m/s}^2}}=2.474\text{ s}$.

In the horizontal direction we have

$v_x(t)=v_{0x}+\int_0^t a_x(t')dt'=v_{0x}+(0.800\text{ m/s}^3)t^2=12.0\text{ m/s}+(0.800\text{ m/s}^3)t^2$. Integrating $v_x(t)$ gives $x-x_0=(12.0\text{ m/s})t+(0.2667\text{ m/s}^3)t^3$. At $t=2.474\text{ s}$, $x-x_0=29.69\text{ m}+4.04\text{ m}=33.7\text{ m}$.

**EVALUATE:** The vertical part of the motion is familiar projectile motion, but the horizontal part is not.

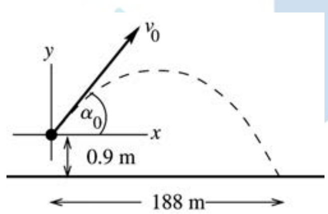
**3.52.** **IDENTIFY:** The equipment moves in projectile motion. The distance $D$ is the horizontal range of the equipment plus the distance the ship moves while the equipment is in the air.

**SET UP:** For the motion of the equipment take $+x$ to be to the right and $+y$ to be upward. Then $a_x=0$, $a_y=-9.80\text{ m/s}^2$, $v_{0x}=v_0\cos\alpha_0=7.50\text{ m/s}$ and $v_{0y}=v_0\sin\alpha_0=13.0\text{ m/s}$. When the equipment lands in the front of the ship, $y-y_0=-8.75\text{ m}$.

**EXECUTE:** Use the vertical motion of the equipment to find its time in the air: $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ gives $t=\dfrac{1}{9.80}\left(13.0\pm\sqrt{(-13.0)^2+4(4.90)(8.75)}\right)\text{ s}$. The positive root is $t=3.21\text{ s}$. The horizontal range of the equipment is $x-x_0=v_{0x}t+\frac{1}{2}a_xt^2=(7.50\text{ m/s})(3.21\text{ s})=24.1\text{ m}$. In 3.21 s the ship moves a horizontal distance $(0.450\text{ m/s})(3.21\text{ s})=1.44\text{ m}$, so $D=24.1\text{ m}+1.44\text{ m}=25.5\text{ m}$.

**EVALUATE:** The range equation $R=\dfrac{v_0^2\sin 2\alpha_0}{g}$ cannot be used here because the starting and ending points of the projectile motion are at different heights.

**3.53.** **IDENTIFY:** Projectile motion problem.




Take the origin of coordinates at the point where the ball leaves the bat, and take $+y$ to be upward.

$v_{0x}=v_0\cos\alpha_0$

$v_{0y}=v_0\sin\alpha_0$,

but we don't know $v_0$.

**Figure 3.53**

Write down the equation for the horizontal displacement when the ball hits the ground and the corresponding equation for the vertical displacement. The time $t$ is the same for both components, so this will give us two equations in two unknowns ($v_0$ and $t$).

**(a) SET UP:** <u>$y$-component:</u>

$a_y=-9.80\text{ m/s}^2$, $y-y_0=-0.9\text{ m}$, $v_{0y}=v_0\sin 45°$

$y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$

**EXECUTE:** $-0.9\text{ m}=(v_0\sin 45°)t+\frac{1}{2}(-9.80\text{ m/s}^2)t^2$

**SET UP:** <u>$x$-component:</u>

$a_x=0$, $x-x_0=188\text{ m}$, $v_{0x}=v_0\cos 45°$

$x-x_0=v_{0x}t+\frac{1}{2}a_xt^2$

**EXECUTE:** $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{188\text{ m}}{v_0 \cos 45°}$

Put the expression for $t$ from the $x$-component motion into the $y$-component equation and solve for $v_0$. (Note that $\sin 45° = \cos 45°$.)

$$-0.9\text{ m} = (v_0 \sin 45°)\left(\frac{188\text{ m}}{v_0 \cos 45°}\right) - (4.90\text{ m/s}^2)\left(\frac{188\text{ m}}{v_0 \cos 45°}\right)^2$$

$$4.90\text{ m/s}^2\left(\frac{188\text{ m}}{v_0 \cos 45°}\right)^2 = 188\text{ m} + 0.9\text{ m} = 188.9\text{ m}$$

$$\left(\frac{v_0 \cos 45°}{188\text{ m}}\right)^2 = \frac{4.90\text{ m/s}^2}{188.9\text{ m}},\quad v_0 = \left(\frac{188\text{ m}}{\cos 45°}\right)\sqrt{\frac{4.90\text{ m/s}^2}{188.9\text{ m}}} = 42.8\text{ m/s}$$

**(b)** Use the horizontal motion to find the time it takes the ball to reach the fence:
**SET UP:** <u>$x$-component:</u>
$x - x_0 = 116\text{ m},\ a_x = 0,\ v_{0x} = v_0 \cos 45° = (42.8\text{ m/s})\cos 45° = 30.3\text{ m/s},\ t = ?$
$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

**EXECUTE:** $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{116\text{ m}}{30.3\text{ m/s}} = 3.83\text{ s}$

**SET UP:** Find the vertical displacement of the ball at this $t$:
<u>$y$-component:</u>
$y - y_0 = ?,\ a_y = -9.80\text{ m/s}^2,\ v_{0y} = v_0 \sin 45° = 30.3\text{ m/s},\ t = 3.83\text{ s}$
$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

**EXECUTE:** $y - y_0 = (30.3\text{ s})(3.83\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(3.83\text{ s})^2$

$y - y_0 = 116.0\text{ m} - 71.9\text{ m} = +44.1\text{ m},$ above the point where the ball was hit. The height of the ball above the ground is $44.1\text{ m} + 0.90\text{ m} = 45.0\text{ m}.$ Its height then above the top of the fence is $45.0\text{ m} - 3.0\text{ m} = 42.0\text{ m}.$

**EVALUATE:** With $v_0 = 42.8\text{ m/s},\ v_{0y} = 30.3\text{ m/s}$ and it takes the ball 6.18 s to return to the height where it was hit and only slightly longer to reach a point 0.9 m below this height. $t = (188\text{ m})/(v_0 \cos 45°)$ gives $t = 6.21\text{ s},$ which agrees with this estimate. The ball reaches its maximum height approximately $(188\text{ m})/2 = 94\text{ m}$ from home plate, so at the fence the ball is not far past its maximum height of 47.6 m, so a height of 45.0 m at the fence is reasonable.

**3.54. IDENTIFY:** While the hay falls 150 m with an initial upward velocity and with a downward acceleration of $g$, it must travel a horizontal distance (the target variable) with constant horizontal velocity.
**SET UP:** Use coordinates with $+y$ upward and $+x$ horizontal. The bale has initial velocity components $v_{0x} = v_0 \cos\alpha_0 = (75\text{ m/s})\cos 55° = 43.0\text{ m/s}$ and $v_{0y} = v_0 \sin\alpha_0 = (75\text{ m/s})\sin 55° = 61.4\text{ m/s}.$ $y_0 = 150\text{ m}$ and $y = 0.$ The equation $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ applies to the vertical motion and a similar equation to the horizontal motion.
**EXECUTE:** Use the vertical motion to find $t$: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $-150\text{ m} = (61.4\text{ m/s})t - (4.90\text{ m/s}^2)t^2.$ The quadratic formula gives $t = 6.27 \pm 8.36\text{ s}.$ The physical value is the positive one, and $t = 14.6\text{ s}.$ Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (43.0\text{ m/s})(14.6\text{ s}) = 630\text{ m}.$
**EVALUATE:** If the airplane maintains constant velocity after it releases the bales, it will also travel horizontally 630 m during the time it takes the bales to fall to the ground, so the airplane will be directly over the impact spot when the bales land.

**3.55. IDENTIFY:** Two-dimensional projectile motion.

**SET UP:** Let $+y$ be upward. $a_x = 0$, $a_y = -9.80\ \text{m/s}^2$. With $x_0 = y_0 = 0$, algebraic manipulation of the equations for the horizontal and vertical motion shows that $x$ and $y$ are related by

$$y = (\tan\theta_0)x - \frac{g}{2\upsilon_0{}^2 \cos^2\theta_0}x^2.$$

$\theta_0 = 60.0°$. $y = 8.00$ m when $x = 18.0$ m.

**EXECUTE:** **(a)** Solving for $v_0$ gives $\upsilon_0 = \sqrt{\dfrac{gx^2}{2(\cos^2\theta_0)(x\tan\theta_0 - y)}} = 16.6$ m/s.

**(b)** We find the horizontal and vertical velocity components:

$\upsilon_x = \upsilon_{0x} = \upsilon_0 \cos\theta_0 = 8.3$ m/s.

$\upsilon_y{}^2 = \upsilon_{0y}{}^2 + 2a_y(y - y_0)$ gives

$\upsilon_y = -\sqrt{(\upsilon_0 \sin\theta_0)^2 + 2a_y(y - y_0)} = -\sqrt{(14.4\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(8.00\ \text{m})} = -7.1$ m/s

$\upsilon = \sqrt{\upsilon_x{}^2 + \upsilon_y{}^2} = 10.9$ m/s. $\tan\theta = \dfrac{|\upsilon_y|}{|\upsilon_x|} = \dfrac{7.1}{8.3}$ and $\theta = 40.5°$, below the horizontal.

**EVALUATE:** We can check our calculated $v_0$.

$$t = \frac{x - x_0}{\upsilon_{0x}} = \frac{18.0\ \text{m}}{8.3\ \text{m/s}} = 2.17\ \text{s}.$$

Then $y - y_0 = \upsilon_{0y}t + \frac{1}{2}a_y t^2 = (14.4\ \text{m/s})(2.17\ \text{s}) - (4.9\ \text{m/s}^2)(2.17\ \text{s})^2 = 8$ m, which checks.

**3.56.** **IDENTIFY:** The water moves in projectile motion.

**SET UP:** Let $x_0 = y_0 = 0$ and take $+y$ to be positive. $a_x = 0$, $a_y = -g$.

**EXECUTE:** The equations of motions are $y = (v_0 \sin\alpha)t - \frac{1}{2}gt^2$ and $x = (v_0 \cos\alpha)t$. When the water goes in the tank for the *minimum* velocity, $y = 2D$ and $x = 6D$. When the water goes in the tank for the *maximum* velocity, $y = 2D$ and $x = 7D$. In both cases, $\sin\alpha = \cos\alpha = \sqrt{2}/2$.

To reach the *minimum* distance: $6D = \dfrac{\sqrt{2}}{2}v_0 t$, and $2D = \dfrac{\sqrt{2}}{2}v_0 t - \frac{1}{2}gt^2$. Solving the first equation for $t$ gives $t = \dfrac{6D\sqrt{2}}{v_0}$. Substituting this into the second equation gives $2D = 6D - \frac{1}{2}g\left(\dfrac{6D\sqrt{2}}{v_0}\right)^2$. Solving this for $v_0$ gives $v_0 = 3\sqrt{gD}$.

To reach the *maximum* distance: $7D = \dfrac{\sqrt{2}}{2}v_0 t$, and $2D = \dfrac{\sqrt{2}}{2}v_0 t - \frac{1}{2}gt^2$. Solving the first equation for $t$ gives $t = \dfrac{7D\sqrt{2}}{v_0}$. Substituting this into the second equation gives $2D = 7D - \frac{1}{2}g\left(\dfrac{7D\sqrt{2}}{v_0}\right)^2$. Solving this for $v_0$ gives $v_0 = \sqrt{49gD/5} = 3.13\sqrt{gD}$, which, as expected, is larger than the previous result.

**EVALUATE:** A launch speed of $v_0 = \sqrt{6}\sqrt{gD} = 2.45\sqrt{gD}$ is required for a horizontal range of $6D$. The minimum speed required is greater than this, because the water must be at a height of at least $2D$ when it reaches the front of the tank.

**3.57.** **IDENTIFY:** From the figure in the text, we can read off the maximum height and maximum horizontal distance reached by the grasshopper. Knowing its acceleration is $g$ downward, we can find its initial speed and the height of the cliff (the target variables).

**SET UP:** Use coordinates with the origin at the ground and $+y$ upward. $a_x = 0$, $a_y = -9.80\ \text{m/s}^2$. The constant-acceleration kinematics formulas $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ and $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ apply.

**EXECUTE:** **(a)** $v_y = 0$ when $y - y_0 = 0.0674$ m. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80 \text{ m/s}^2)(0.0674 \text{ m})} = 1.15$ m/s. $v_{0y} = v_0 \sin\alpha_0$ so

$v_0 = \dfrac{v_{0y}}{\sin\alpha_0} = \dfrac{1.15 \text{ m/s}}{\sin 50.0°} = 1.50$ m/s.

**(b)** Use the horizontal motion to find the time in the air. The grasshopper travels horizontally $x - x_0 = 1.06$ m. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{x - x_0}{v_0 \cos 50.0°} = 1.10$ s. Find the vertical displacement of the grasshopper at $t = 1.10$ s:

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (1.15 \text{ m/s})(1.10 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(1.10 \text{ s})^2 = -4.66$ m. The height of the cliff is 4.66 m.

**EVALUATE:** The grasshopper's maximum height (6.74 cm) is physically reasonable, so its takeoff speed of 1.50 m/s must also be reasonable. Note that the equation $R = \dfrac{v_0^2 \sin 2\alpha_0}{g}$ does *not* apply here since the launch point is not at the same level as the landing point.

**3.58.** **IDENTIFY:** To clear the bar the ball must have a height of 10.0 ft when it has a horizontal displacement of 36.0 ft. The ball moves as a projectile. When $v_0$ is very large, the ball reaches the goal posts in a very short time and the acceleration due to gravity causes negligible downward displacement.

**SET UP:** 36.0 ft = 10.97 m; 10.0 ft = 3.048 m. Let $+x$ be to the right and $+y$ be upward, so $a_x = 0$, $a_y = -g$, $v_{0x} = v_0 \cos\alpha_0$ and $v_{0y} = v_0 \sin\alpha_0$.

**EXECUTE:** **(a)** The ball cannot be aimed lower than directly at the bar. $\tan\alpha_0 = \dfrac{10.0 \text{ ft}}{36.0 \text{ ft}}$ and $\alpha_0 = 15.5°$.

**(b)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{x - x_0}{v_0 \cos\alpha_0}$. Then $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$$y - y_0 = (v_0 \sin\alpha_0)\left(\frac{x - x_0}{v_0 \cos\alpha_0}\right) - \frac{1}{2}g\frac{(x - x_0)^2}{v_0^2 \cos^2\alpha_0} = (x - x_0)\tan\alpha_0 - \frac{1}{2}g\frac{(x - x_0)^2}{v_0^2 \cos^2\alpha_0}.$$

$$v_0 = \frac{(x - x_0)}{\cos\alpha_0}\sqrt{\frac{g}{2[(x - x_0)\tan\alpha_0 - (y - y_0)]}} = \frac{10.97 \text{ m}}{\cos 45.0°}\sqrt{\frac{9.80 \text{ m/s}^2}{2[10.97 \text{ m} - 3.048 \text{ m}]}} = 12.2 \text{ m/s} = 43.9 \text{ km/h}.$$

**EVALUATE:** With the $v_0$ and 45° launch angle in part (b), the horizontal range of the ball is $R = \dfrac{v_0^2 \sin 2\alpha_0}{g}$ = 15.2 m = 49.9 ft. The ball reaches the highest point in its trajectory when $x - x_0 = R/2$, which is 25 ft, so when it reaches the goal posts it is on its way down.

**3.59.** **IDENTIFY:** The snowball moves in projectile motion. In part (a) the vertical motion determines the time in the air. In part (c), find the height of the snowball above the ground after it has traveled horizontally 4.0 m.

**SET UP:** Let $+y$ be downward. $a_x = 0$, $a_y = +9.80$ m/s$^2$. $v_{0x} = v_0 \cos\theta_0 = 5.36$ m/s, $v_{0y} = v_0 \sin\theta_0 = 4.50$ m/s.

**EXECUTE:** **(a)** Use the vertical motion to find the time in the air: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = 14.0$ m gives $14.0 \text{ m} = (4.50 \text{ m/s})\,t + (4.9 \text{ m/s}^2)\,t^2$. The quadratic formula gives

$t = \dfrac{1}{2(4.9)}\left(-4.50 \pm \sqrt{(4.50)^2 - 4(4.9)(-14.0)}\right)$ s. The positive root is $t = 1.29$ s. Then

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (5.36 \text{ m/s})(1.29 \text{ s}) = 6.91$ m.

**(b)** The $x$-$t$, $y$-$t$, $v_x$-$t$ and $v_y$-$t$ graphs are sketched in Figure 3.59.

**(c)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{4.0\text{ m}}{5.36\text{ m/s}} = 0.746\text{ s}$. In this time the snowball travels downward a distance $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 6.08\text{ m}$ and is therefore $14.0\text{ m} - 6.08\text{ m} = 7.9\text{ m}$ above the ground. The snowball passes well above the man and doesn't hit him.

**EVALUATE:** If the snowball had been released from rest at a height of 14.0 m it would have reached the ground in $t = \sqrt{\dfrac{2(14.0\text{ m})}{9.80\text{ m/s}^2}} = 1.69\text{ s}$. The snowball reaches the ground in a shorter time than this because of its initial downward component of velocity.

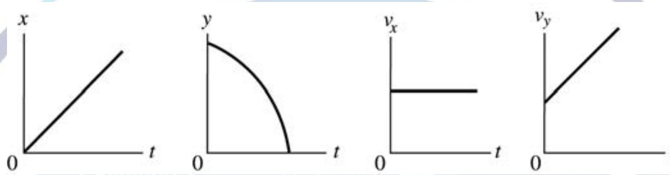



**Figure 3.59**

**3.60.** **IDENTIFY:** The dog runs horizontally at constant velocity, and the ball is in two-dimensional projectile motion. The ball starts out traveling only horizontally.

**SET UP:** Use coordinates with the origin at the boy and with $+y$ downward. For the ball $v_{0y} = 0$, $v_{0x} = 8.50\text{ m/s}$, $a_x = 0$ and $a_y = 9.80\text{ m/s}^2$.

**EXECUTE:** **(a)** The dog must travel horizontally the same distance the ball travels horizontally, so the dog must have speed 8.50 m/s.

**(b)** Use the vertical motion of the ball to find its time in the air. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(12.0\text{ m})}{9.80\text{ m/s}^2}} = 1.56\text{ s}$. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (8.50\text{ m/s})(1.56\text{ s}) = 13.3\text{ m}$

**EVALUATE:** The dog is about 40 ft from the tree, which is not unreasonable since the tree is nearly 40 ft high.

**3.61.** **IDENTIFY:** The dog runs horizontally at constant velocity, and the ball is in two-dimensional projectile motion. But this time the ball has an upward component to its initial velocity.

**SET UP:** Use coordinates with the origin at the boy and with $+y$ upward. The ball has $v_{0x} = v_0\cos\theta_0 = (8.50\text{ m/s})\cos 60.0^\circ = 4.25\text{ m/s}$, $v_{0y} = v_0\sin\theta_0 = (8.50\text{ m/s})\sin 60.0^\circ = 7.36\text{ m/s}$, $a_x = 0$ and $a_y = -9.80\text{ m/s}^2$.

**EXECUTE:** **(a)** The dog must travel horizontally the same distance the ball travels horizontally, so the dog must have speed 4.25 m/s.

**(b)** Use the vertical motion of the ball to find its time in the air. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$-12.0\text{ m} = (7.36\text{ m/s})t - (4.90\text{ m/s}^2)t^2$. The quadratic formula gives $t = 0.751 \pm 1.74\text{ s}$. The negative value is not physical, so $t = 2.49\text{ s}$. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (4.25\text{ m/s})(2.49\text{ s}) = 10.6\text{ m}$.

**EVALUATE:** The ball is in the air longer than when it is thrown horizontally (as we saw in the previous problem), but it doesn't travel as far horizontally. The dog doesn't have to run as far or as fast as when the ball is thrown horizontally.

**3.62.** **IDENTIFY:** The rock moves in projectile motion.

**SET UP:** Let $+y$ be upward. $a_x = 0$, $a_y = -g$. Eqs. (3.21) and (3.22) give $v_x$ and $v_y$.

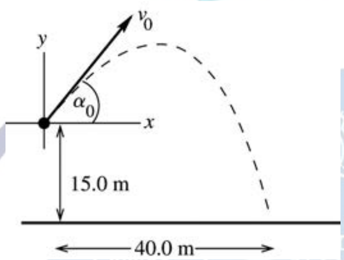
**EXECUTE:** Combining Eqs. 3.24, 3.21 and 3.22 gives

$v^2 = v_0^2\cos^2\alpha_0 + (v_0\sin\alpha_0 - gt)^2 = v_0^2(\sin^2\alpha_0 + \cos^2\alpha_0) - 2v_0\sin\alpha_0 gt + (gt)^2$.

$v^2 = v_0^2 - 2g\left(v_0 \sin\alpha_0 t - \frac{1}{2}gt^2\right) = v_0^2 - 2gy$, where Eq. (3.20) has been used to eliminate $t$ in favor of $y$. For the case of a rock thrown from the roof of a building of height $h$, the speed at the ground is found by substituting $y = -h$ into the above expression, yielding $v = \sqrt{v_0^2 + 2gh}$, which is independent of $\alpha_0$.

**EVALUATE:** This result, as will be seen in the chapter dealing with conservation of energy (Chapter 7), is valid for any $y$, positive, negative or zero, as long as $v_0^2 - 2gy > 0$.

**3.63.** **(a) IDENTIFY:** Projectile motion.




Take the origin of coordinates at the top of the ramp and take $+y$ to be upward.

The problem specifies that the object is displaced 40.0 m to the right when it is 15.0 m below the origin.

**Figure 3.63**

We don't know $t$, the time in the air, and we don't know $v_0$. Write down the equations for the horizontal and vertical displacements. Combine these two equations to eliminate one unknown.

**SET UP:** <u>$y$-component:</u>

$y - y_0 = -15.0\text{ m},\ \ a_y = -9.80\text{ m/s}^2,\ \ v_{0y} = v_0 \sin 53.0°$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

**EXECUTE:** $-15.0\text{ m} = (v_0 \sin 53.0°)\,t - (4.90\text{ m/s}^2)\,t^2$

**SET UP:** <u>$x$-component:</u>

$x - x_0 = 40.0\text{ m},\ \ a_x = 0,\ \ v_{0x} = v_0 \cos 53.0°$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

**EXECUTE:** $40.0\text{ m} = (v_0 t)\cos 53.0°$

The second equation says $v_0 t = \dfrac{40.0\text{ m}}{\cos 53.0°} = 66.47\text{ m}$.

Use this to replace $v_0 t$ in the first equation:

$-15.0\text{ m} = (66.47\text{ m})\sin 53° - (4.90\text{ m/s}^2)\,t^2$

$$t = \sqrt{\frac{(66.47\text{ m})\sin 53° + 15.0\text{ m}}{4.90\text{ m/s}^2}} = \sqrt{\frac{68.08\text{ m}}{4.90\text{ m/s}^2}} = 3.727\text{ s}.$$

Now that we have $t$ we can use the $x$-component equation to solve for $v_0$:

$$v_0 = \frac{40.0\text{ m}}{t\cos 53.0°} = \frac{40.0\text{ m}}{(3.727\text{ s})\cos 53.0°} = 17.8\text{ m/s}.$$

**EVALUATE:** Using these values of $v_0$ and $t$ in the $y = y_0 = v_{0y} + \frac{1}{2}a_y t^2$ equation verifies that $y - y_0 = -15.0\text{ m}$.

**(b) IDENTIFY:** $v_0 = (17.8\text{ m/s})/2 = 8.9\text{ m/s}$

This is less than the speed required to make it to the other side, so he lands in the river.
Use the vertical motion to find the time it takes him to reach the water:

**SET UP:** $y - y_0 = -100\text{ m};\ \ v_{0y} = +v_0 \sin 53.0° = 7.11\text{ m/s};\ \ a_y = -9.80\text{ m/s}^2$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $-100 = 7.11t - 4.90t^2$

**EXECUTE:** $4.90t^2 - 7.11t - 100 = 0$ and $t = \frac{1}{9.80}\left(7.11 \pm \sqrt{(7.11)^2 - 4(4.90)(-100)}\right)$

$t = 0.726 \text{ s} \pm 4.57 \text{ s}$ so $t = 5.30$ s.

The horizontal distance he travels in this time is

$$x - x_0 = v_{0x}t = (v_0 \cos 53.0°)\, t = (5.36 \text{ m/s})(5.30 \text{ s}) = 28.4 \text{ m}.$$

He lands in the river a horizontal distance of 28.4 m from his launch point.

**EVALUATE:** He has half the minimum speed and makes it only about halfway across.

**3.64.** **IDENTIFY:** The ball moves in projectile motion.

**SET UP:** The woman and ball travel for the same time and must travel the same horizontal distance, so for the ball $v_{0x} = 6.00$ m/s.

**EXECUTE:** **(a)** $v_{0x} = v_0 \cos\theta_0$. $\cos\theta_0 = \dfrac{v_{0x}}{v_0} = \dfrac{6.00 \text{ m/s}}{20.0 \text{ m/s}}$ and $\theta_0 = 72.5°$. The ball is in the air for 5.55s and she runs a distance of $(6.00 \text{ m/s})(5.55 \text{ s}) = 33.3$ m.

**(b)** Relative to the ground the ball moves in a parabola. The ball and the runner have the same horizontal component of velocity, so relative to the runner the ball has only vertical motion. The trajectories as seen by each observer are sketched in Figure 3.64.

**EVALUATE:** The ball could be thrown with a different speed, so long as the angle at which it was thrown was adjusted to keep $v_{0x} = 6.00$ m/s.

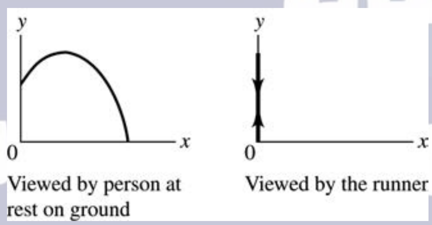



**Figure 3.64**

**3.65.** **IDENTIFY:** The boulder moves in projectile motion.

**SET UP:** Take $+y$ downward. $v_{0x} = v_0$, $a_x = 0$, $a_x = 0$, $a_y = +9.80 \text{ m/s}^2$.

**EXECUTE:** **(a)** Use the vertical motion to find the time for the boulder to reach the level of the lake: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = +20$ m gives $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(20 \text{ m})}{9.80 \text{ m/s}^2}} = 2.02$ s. The rock must travel horizontally 100 m during this time. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $v_0 = v_{0x} = \dfrac{x - x_0}{t} = \dfrac{100 \text{ m}}{2.02 \text{ s}} = 49.5$ m/s

**(b)** In going from the edge of the cliff to the plain, the boulder travels downward a distance of $y - y_0 = 45$ m. $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(45 \text{ m})}{9.80 \text{ m/s}^2}} = 3.03$ s and $x - x_0 = v_{0x}t = (49.5 \text{ m/s})(3.03 \text{ s}) = 150$ m.

The rock lands $150 \text{ m} - 100 \text{ m} = 50$ m beyond the foot of the dam.

**EVALUATE:** The boulder passes over the dam 2.02 s after it leaves the cliff and then travels an additional 1.01 s before landing on the plain. If the boulder has an initial speed that is less than 49 m/s, then it lands in the lake.

**3.66.** **IDENTIFY:** The bagels move in projectile motion. Find Henrietta's location when the bagels reach the ground, and require the bagels to have this horizontal range.

**SET UP:** Let $+y$ be downward and let $x_0 = y_0 = 0$. $a_x = 0$, $a_y = +g$. When the bagels reach the ground, $y = 38.0$ m.

**EXECUTE:** **(a)** When she catches the bagels, Henrietta has been jogging for 9.00 s plus the time for the bagels to fall 38.0 m from rest. Get the time to fall: $y = \dfrac{1}{2}gt^2$, $38.0 \text{ m} = \dfrac{1}{2}(9.80 \text{ m/s}^2)\, t^2$ and $t = 2.78$ s.

So, she has been jogging for $9.00 \text{ s} + 2.78 \text{ s} = 11.78$ s. During this time she has gone

$x = vt = (3.05\text{ m/s})(11.78\text{ s}) = 35.9\text{ m}$. Bruce must throw the bagels so they travel 35.9 m horizontally in 2.78 s. This gives $x = vt$. $35.9\text{ m} = v\,(2.78\text{ s})$ and $v = 12.9\text{ m/s}$.

**(b)** 35.9 m from the building.

**EVALUATE:** If $v > 12.9\text{ m/s}$ the bagels land in front of her and if $v < 12.9\text{ m/s}$ they land behind her. There is a range of velocities greater than 12.9 m/s for which she would catch the bagels in the air, at some height above the sidewalk.

**3.67. IDENTIFY:** The cart has a constant horizontal velocity, but the missile has horizontal and vertical motion once it has left the cart and is in free fall.

**SET UP:** Let +*y* be upward and +*x* be to the right. The missile has $\upsilon_{0x} = 30.0\text{ m/s}$, $\upsilon_{0y} = 40.0\text{ m/s}$, $a_x = 0$ and $a_y = -9.80\text{ m/s}^2$. The cart has $a_x = 0$ and $\upsilon_{0x} = 30.0\text{ m/s}$.

**EXECUTE:** **(a)** At the missile's maximum height, $\upsilon_y = 0$.

$$\upsilon_y{}^2 = \upsilon_{0y}{}^2 + 2a_y(y - y_0) \text{ gives } y - y_0 = \frac{\upsilon_y{}^2 - \upsilon_{0y}{}^2}{2a_y} = \frac{0 - (40.0\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 81.6\text{ m}$$

**(b)** Find *t* for $y - y_0 = 0$ (missile returns to initial level).

$$y - y_0 = \upsilon_{0y}t + \tfrac{1}{2}a_y t^2 \text{ gives } t = -\frac{2\upsilon_{0y}}{a_y} = -\frac{2(40.0\text{ m/s})}{-9.80\text{ m/s}^2} = 8.16\text{ s}$$

Then $x - x_0 = \upsilon_{0x}t + \tfrac{1}{2}a_x t^2 = (30.0\text{ m/s})(8.16\text{ s}) = 245\text{ m}$.

**(c)** The missile also travels horizontally 245 m so the missile lands in the cart.

**EVALUATE:** The vertical motion of the missile does not affect its horizontal motion, which is the same as that of the cart, so the missile is always directly above the cart throughout its motion.

**3.68. IDENTIFY:** The water follows a parabolic trajectory since it is affected only by gravity, so we apply the principles of projectile motion to it.

**SET UP:** Use coordinates with +*y* upward. Once the water leaves the cannon it is in free-fall and has $a_x = 0$ and $a_y = -9.80\text{ m/s}^2$. The water has $\upsilon_{0x} = \upsilon_0\cos\theta_0 = 15.0\text{ m/s}$ and $\upsilon_{0y} = \upsilon_0\sin\theta_0 = 20.0\text{ m/s}$.

**EXECUTE:** Use the vertical motion to find *t* that gives $y - y_0 = 10.0\text{ m}$: $y - y_0 = \upsilon_{0y}t + \tfrac{1}{2}a_y t^2$ gives

$$10.0\text{ m} = (20.0\text{ m/s})t - (4.90\text{ m/s}^2)t^2.$$

The quadratic formula gives $t = 2.04 \pm 1.45\text{ s}$, and $t = 0.59\text{ s}$ or $t = 3.49\text{ s}$. Both answers are physical.

$$\text{For } t = 0.59\text{ s},\quad x - x_0 = \upsilon_{0x}t = (15.0\text{ m/s})(0.59\text{ s}) = 8.8\text{ m}.$$

$$\text{For } t = 3.49\text{ s},\quad x - x_0 = \upsilon_{0x}t = (15.0\text{ m/s})(3.49\text{ s}) = 52.4\text{ m}.$$

When the cannon is 8.8 m from the building, the water hits this spot on the wall on its way up to its maximum height. When is it 52.4 m from the building it hits this spot after it has passed through its maximum height.

**EVALUATE:** The fact that we have two possible answers means that the firefighters have some choice on where to stand. If the fire is extremely fierce, they would no doubt prefer to stand at the more distant location.

**3.69. IDENTIFY:** The rock is in free fall once it is in the air, so it has only a downward acceleration of $9.80\text{ m/s}^2$, and we apply the principles of two-dimensional projectile motion to it. The constant-acceleration kinematics formulas apply.

**SET UP:** The vertical displacement must be $\Delta y = y - y_0 = 5.00\text{ m} - 1.60\text{ m} = 3.40\text{ m}$ at the instant that the horizontal displacement $\Delta x = x - x_0 = 14.0\text{ m}$, and $a_y = -9.80\text{ m/s}^2$ with +*y* upward.

**EXECUTE:** **(a)** There is no horizontal acceleration, so $14.0\text{ m} = v_0\cos(56.0°)t$, which gives $t = \dfrac{14.0\text{ m}}{v_0\cos 56.0°}$. Putting this quantity, along with the numerical quantities, into the equation $y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ and solving for $v_0$ we get $v_0 = 13.3\text{ m/s}$.

**(b)** The initial horizontal velocity of the rock is (13.3 m/s)(cos 56.0°), and when it lands on the ground, $y - y_0 = -1.60$ m. Putting these quantities into the equation $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ leads to a quadratic equation in $t$. Using the positive square root, we get $t = 2.388$ s when the rock lands. The horizontal position at that instant is $x - x_0 = (13.3\text{ m/s})(\cos 56.0°)(2.388\text{ s}) = 17.8$ m from the launch point. So the distance beyond the fence is 17.8 m – 14.0 m = 3.8 m.

**EVALUATE:** We cannot use the range formula to find the distance in (b) because the rock's motion does not start and end at the same height.

**3.70.** **IDENTIFY:** The object moves with constant acceleration in both the horizontal and vertical directions.

**SET UP:** Let $+y$ be downward and let $+x$ be the direction in which the firecracker is thrown.

**EXECUTE:** The firecracker's falling time can be found from the vertical motion: $t = \sqrt{\dfrac{2h}{g}}$.

The firecracker's horizontal position at any time $t$ (taking the student's position as $x = 0$) is $x = vt - \frac{1}{2}at^2$. $x = 0$ when cracker hits the ground, so $t = 2v/a$. Combining this with the expression for the falling time gives $\dfrac{2v}{a} = \sqrt{\dfrac{2h}{g}}$ and $h = \dfrac{2v^2 g}{a^2}$.

**EVALUATE:** When $h$ is smaller, the time in the air is smaller and either $v$ must be smaller or $a$ must be larger.

**3.71.** **IDENTIFY:** Relative velocity problem. The plane's motion relative to the earth is determined by its velocity relative to the earth.

**SET UP:** Select a coordinate system where $+y$ is north and $+x$ is east.

The velocity vectors in the problem are:

$\vec{v}_{P/E}$, the velocity of the plane relative to the earth.

$\vec{v}_{P/A}$, the velocity of the plane relative to the air (the magnitude $v_{P/A}$ is the airspeed of the plane and the direction of $\vec{v}_{P/A}$ is the compass course set by the pilot).

$\vec{v}_{A/E}$, the velocity of the air relative to the earth (the wind velocity).

The rule for combining relative velocities gives $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$.

**(a)** We are given the following information about the relative velocities:

$\vec{v}_{P/A}$ has magnitude 220 km/h and its direction is west. In our coordinates it has components $(v_{P/A})_x = -220$ km/h and $(v_{P/A})_y = 0$.

From the displacement of the plane relative to the earth after 0.500 h, we find that $\vec{v}_{P/E}$ has components in our coordinate system of

$$(v_{P/E})_x = -\frac{120\text{ km}}{0.500\text{ h}} = -240\text{ km/h (west)}$$

$$(v_{P/E})_y = -\frac{20\text{ km}}{0.500\text{ h}} = -40\text{ km/h (south)}$$

With this information the diagram corresponding to the velocity addition equation is shown in Figure 3.71a.

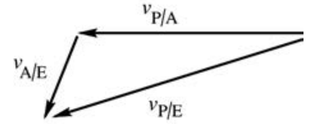



**Figure 3.71a**

We are asked to find $\vec{v}_{A/E}$, so solve for this vector:

$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$ gives $\vec{v}_{A/E} = \vec{v}_{P/E} - \vec{v}_{P/A}$.

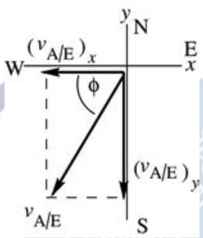
**EXECUTE:** The $x$-component of this equation gives

$(v_{A/E})_x = (v_{P/E})_x - (v_{P/A})_x = -240 \text{ km/h} - (-220 \text{ km/h}) = -20 \text{ km/h}$.

The $y$-component of this equation gives

$(v_{A/E})_y = (v_{P/E})_y - (v_{P/A})_y = -40 \text{ km/h}$.

Now that we have the components of $\vec{v}_{A/E}$ we can find its magnitude and direction.




$$v_{A/E} = \sqrt{(v_{A/E})_x^2 + (v_{A/E})_y^2}$$

$$v_{A/E} = \sqrt{(-20 \text{ km/h})^2 + (-40 \text{ km/h})^2} = 44.7 \text{ km/h}$$

$$\tan\phi = \frac{40 \text{ km/h}}{20 \text{ km/h}} = 2.00; \quad \phi = 63.4^\circ$$

The direction of the wind velocity is $63.4^\circ$ S of W, or $26.6^\circ$ W of S.

**Figure 3.71b**

**EVALUATE:** The plane heads west. It goes farther west than it would without wind and also travels south, so the wind velocity has components west and south.

**(b) SET UP:** The rule for combining the relative velocities is still $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$, but some of these velocities have different values than in part (a).

$\vec{v}_{P/A}$ has magnitude 220 km/h but its direction is to be found.

$\vec{v}_{A/E}$ has magnitude 40 km/h and its direction is due south.

The direction of $\vec{v}_{P/E}$ is west; its magnitude is not given.

The vector diagram for $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$ and the specified directions for the vectors is shown in Figure 3.71c.

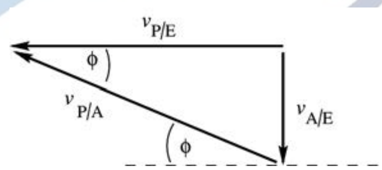



**Figure 3.71c**

The vector addition diagram forms a right triangle.

**EXECUTE:** $\sin\phi = \dfrac{v_{A/E}}{v_{P/A}} = \dfrac{40 \text{ km/h}}{220 \text{ km/h}} = 0.1818; \quad \phi = 10.5^\circ$.

The pilot should set her course $10.5^\circ$ north of west.

**EVALUATE:** The velocity of the plane relative to the air must have a northward component to counteract the wind and a westward component in order to travel west.

**3.72.** **IDENTIFY:** Use the relation that relates the relative velocities.

**SET UP:** The relative velocities are the raindrop relative to the earth, $\vec{v}_{\text{R/E}}$, the raindrop relative to the train, $\vec{v}_{\text{R/T}}$, and the train relative to the earth, $\vec{v}_{\text{T/E}}$. $\vec{v}_{\text{R/E}} = \vec{v}_{\text{R/T}} + \vec{v}_{\text{T/E}}$. $\vec{v}_{\text{T/E}}$ is due east and has magnitude 12.0 m/s. $\vec{v}_{\text{R/T}}$ is $30.0°$ west of vertical. $\vec{v}_{\text{R/E}}$ is vertical. The relative velocity addition diagram is given in Figure 3.72.

**EXECUTE:** **(a)** $\vec{v}_{\text{R/E}}$ is vertical and has zero horizontal component. The horizontal component of $\vec{v}_{\text{R/T}}$ is $-\vec{v}_{\text{T/E}}$, so is 12.0 m/s westward.

**(b)** $v_{\text{R/E}} = \dfrac{v_{\text{T/E}}}{\tan 30.0°} = \dfrac{12.0\text{ m/s}}{\tan 30.0°} = 20.8\text{ m/s}$. $v_{\text{R/T}} = \dfrac{v_{\text{T/E}}}{\sin 30.0°} = \dfrac{12.0\text{ m/s}}{\sin 30.0°} = 24.0\text{ m/s}$.

**EVALUATE:** The speed of the raindrop relative to the train is greater than its speed relative to the earth, because of the motion of the train.

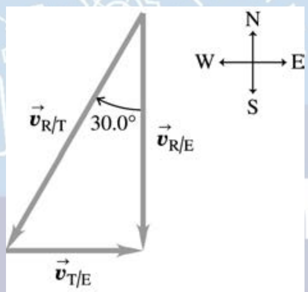



**Figure 3.72**

**3.73.** **IDENTIFY:** Relative velocity problem.

**SET UP:** The three relative velocities are:

$\vec{v}_{\text{J/G}}$, Juan relative to the ground. This velocity is due north and has magnitude $v_{\text{J/G}} = 8.00\text{ m/s}$.

$\vec{v}_{\text{B/G}}$, the ball relative to the ground. This vector is $37.0°$ east of north and has magnitude $v_{\text{B/G}} = 12.00\text{ m/s}$.

$\vec{v}_{\text{B/J}}$, the ball relative to Juan. We are asked to find the magnitude and direction of this vector.

The relative velocity addition equation is $\vec{v}_{\text{B/G}} = \vec{v}_{\text{B/J}} + \vec{v}_{\text{J/G}}$, so $\vec{v}_{\text{B/J}} = \vec{v}_{\text{B/G}} - \vec{v}_{\text{J/G}}$.

The relative velocity addition diagram does not form a right triangle so we must do the vector addition using components.

Take $+y$ to be north and $+x$ to be east.

**EXECUTE:** $v_{\text{B/J}x} = +v_{\text{B/G}} \sin 37.0° = 7.222\text{ m/s}$

$v_{\text{B/J}y} = +v_{\text{B/G}} \cos 37.0° - v_{\text{J/G}} = 1.584\text{ m/s}$

These two components give $v_{\text{B/J}} = 7.39\text{ m/s}$ at $12.4°$ north of east.

**EVALUATE:** Since Juan is running due north, the ball's eastward component of velocity relative to him is the same as its eastward component relative to the earth. The northward component of velocity for Juan and the ball are in the same direction, so the component for the ball relative to Juan is the difference in their components of velocity relative to the ground.

**3.74.** **IDENTIFY:** Both the bolt and the elevator move vertically with constant acceleration.

**SET UP:** Let $+y$ be upward and let $y = 0$ at the initial position of the floor of the elevator, so $y_0$ for the bolt is 3.00 m.

**EXECUTE:** **(a)** The position of the bolt is $3.00\text{ m} + (2.50\text{ m/s})\,t - (1/2)(9.80\text{ m/s}^2)\,t^2$ and the position of the floor is (2.50 m/s)$t$. Equating the two, $3.00\text{ m} = (4.90\text{ m/s}^2)\,t^2$. Therefore, $t = 0.782\text{ s}$.

**(b)** The velocity of the bolt is $2.50\text{ m/s} - (9.80\text{ m/s}^2)(0.782\text{ s}) = -5.17\text{ m/s}$ relative to earth, therefore, relative to an observer in the elevator $v = -5.17\text{ m/s} - 2.50\text{ m/s} = -7.67\text{ m/s}$.

**(c)** As calculated in part (b), the speed relative to earth is 5.17 m/s.
**(d)** Relative to earth, the distance the bolt traveled is
$(2.50\text{ m/s})\,t-(1/2)(9.80\text{ m/s}^2)\,t^2=(2.50\text{ m/s})(0.782\text{ s})-(4.90\text{ m/s}^2)(0.782\text{ s})^2=-1.04\text{ m}.$

**EVALUATE:** As viewed by an observer in the elevator, the bolt has $v_{0y}=0$ and $a_y=-9.80\text{ m/s}^2$, so in 0.782 s it falls $-\frac{1}{2}(9.80\text{ m/s}^2)(0.782\text{ s})^2=-3.00\text{ m}.$

**3.75.** **IDENTIFY:** We need to use relative velocities.
**SET UP:** If B is moving relative to M and M is moving relative to E, the velocity of B relative to E is $\vec{v}_{\text{B/E}}=\vec{v}_{\text{B/M}}+\vec{v}_{\text{M/E}}$.
**EXECUTE:** Let $+x$ be east and $+y$ be north. We have $v_{\text{B/M},x}=2.50\text{ m/s}$, $v_{\text{B/M,y}}=-4.33\text{ m/s}$, $v_{\text{M/E},x}=0$, and $v_{\text{M/E,y}}=6.00\text{ m/s}$. Therefore $v_{\text{B/E},x}=v_{\text{B/M},x}+v_{\text{M/E},x}=2.50\text{ m/s}$ and $v_{\text{B/E,y}}=v_{\text{B/M,y}}+v_{\text{M/E,y}}=-4.33\text{ m/s}+6.00\text{ m/s}=+1.67\text{ m/s}$. The magnitude is $v_{\text{B/E}}=\sqrt{(2.50\text{ m/s})^2+(1.67\text{ m/s})^2}=3.01\text{ m/s}$, and the direction is $\tan\theta=\dfrac{1.67}{2.50}$, which gives $\theta=33.7^\circ$ north of east.
**EVALUATE:** Since Mia is moving, the velocity of the ball relative to her is different from its velocity relative to the ground or relative to Alice.

**3.76.** **IDENTIFY:** You have a graph showing the horizontal range of the rock as a function of the angle at which it was launched and want to find its initial velocity. Because air resistance is negligible, the rock is in free fall. The range formula applies since the rock rock was launced from the ground and lands at the ground.

**SET UP:** **(a)** The range formula is $R=\dfrac{v_0^2\sin(2\theta)}{g}$, so a plot of $R$ versus $\sin(2\theta_0)$ will give a straight line having slope equal to $v_0^2/g$. We can use that data in the graph in the problem to construct our graph by hand, or we can use graphing software. The resulting graph is shown in Figure 3.76.

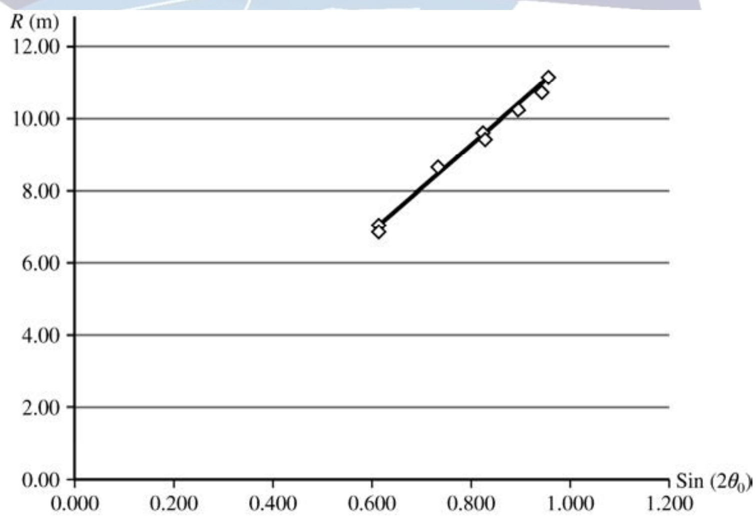



**Figure 3.76**

**(b)** The slope of the graph is 10.95 m, so 10.95 m = $v_0^2/g$. Solving for $v_0$ we get $v_0$ = 10.4 m/s.

**(c)** Solving the formula $v_y^2=v_{0y}^2+2a_y(y-y_0)$ for $y-y_0$ with $v_y=0$ at the highest point, we get $y-y_0$ = 1.99 m.

**EVALUATE:** This approach to finding the launch speed $v_0$ requires only simple measurements: the range and the launch angle. It would be difficult and would require special equipment to measure $v_0$ directly.

**3.77.** **IDENTIFY:** The table gives data showing the horizontal range of the potato for various launch heights. You want to use this information to determine the launch speed of the potato, assuming negligible air resistance.

**SET UP:** The potatoes are launched horizontally, so $v_{0y} = 0$, and they are in free fall, so $a_y = 9.80\text{ m/s}^2$ downward and $a_x = 0$. The time a potato is in the air is just the time it takes for it to fall vertically from the launch point to the ground, a distance $h$.

**EXECUTE:** **(a)** For the vertical motion of a potato, we have $h = ½ gt^2$, so $t = \sqrt{2h/g}$. The horizontal range $R$ is given by $R = v_0 t = v_0\sqrt{2h/g}$. Squaring gives $R^2 = \left(\dfrac{2v_0^2}{g}\right)h$. Graphing $R^2$ versus $h$ will give a straight line with slope $2v_0^2/g$. We can graph the data from the table in the text by hand, or we could use graphing software. The result is shown in Figure 3.77.

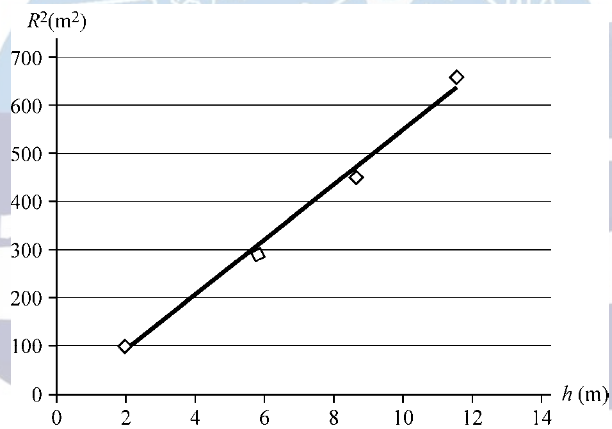



**Figure 3.77**

**(b)** The slope of the graph is 55.2 m, so $v_0 = \sqrt{\dfrac{(9.80\text{ m/s}^2)(55.2\text{ m})}{2}} = 16.4\text{ m/s}.$

**(c)** In this case, the potatoes are launched and land at ground level, so we can use the range formula with $\theta$ = 30.0° and $v_0 = 16.4$ m/s. The result is $R = \dfrac{v_0^2 \sin(2\theta)}{g} = 23.8\text{ m}.$

**EVALUATE:** This approach to finding the launch speed $v_0$ requires only simple measurements: the range and the launch height. It would be difficult and would require special equipment to measure $v_0$ directly.

**3.78.** **IDENTIFY:** This is a vector addition problem. The boat moves relative to the water and the water moves relative to the earth. We know the speed of the boat relative to the water and the times for the boat to go directly across the river, and from these things we want to find out how fast the water is moving and the width of the river.

**SET UP:** For both trips of the boat, $\vec{v}_{B/E} = \vec{v}_{B/W} + \vec{v}_{W/E}$, where the subscripts refer to the boat, earth, and water. The speed of the boat relative to the earth is $v_{B/E} = d/t$, where $d$ is the width of the river and $t$ is the time to cross the river, which is different in the two crossings.

**EXECUTE:** Figure 3.78 shows a vector sum for the first trip and for the return trip.

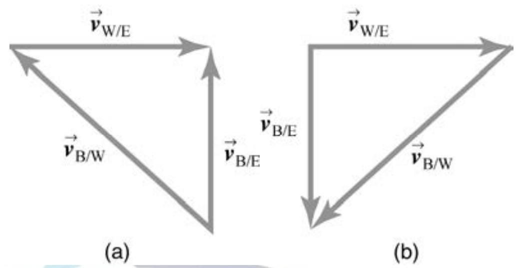



**Figure 3.78a-b**

**(a)** For both trips, the vectors in Figures 3.78 a & b form right triangles, so we can apply the Pythagorean theorem. $v_{B/E}^2 = v_{B/W}^2 - v_{W/E}^2$ and $v_{B/E} = d/t$. For the first trip, $v_{B/W} = 6.00$ m/s and $t = 20.1$ s, giving $d^2/(20.1\text{ s})^2 = (60.00\text{ m/s}^2) - (v_{W/E})^2$. For the return trip, $v_{B/W} = 9.0$ m/s and $t = 11.2$ s, which gives $d^2/(11.2\text{ s})^2 = (9.00\text{ m/s}^2) - (v_{W/E})^2$. Solving these two equations together gives $d = 90.48$ m, which rounds to 90.5 m (the width of the river) and $v_{W/E} = 3.967$ m/s which rounds to 3.97 m/s (the speed of the current).

**(b)** The shortest time is when the boat heads perpendicular to the current, which is due north. Figure 3.78c illustrates this situation. The time to cross is $t = d/v_{B/W} = (90.48\text{ m})/(6.00\text{ m/s}) = 15.1$ s. The distance $x$ east (down river) that you travel is $x = v_{W/E}t = (3.967\text{ m/s})(15.1\text{ s}) = 59.9$ m east of your starting point.

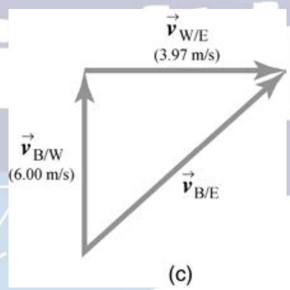



**Figure 3.78c**

**EVALUATE:** In part (a), the boat must have a velocity component up river to cancel out the current velocity. In part (b), velocity of the current has no effect on the crossing time, but it does affect the landing position of the boat.

**3.79. IDENTIFY:** Write an expression for the square of the distance $(D^2)$ from the origin to the particle, expressed as a function of time. Then take the derivative of $D^2$ with respect to $t$, and solve for the value of $t$ when this derivative is zero. If the discriminant is zero or negative, the distance $D$ will never decrease.

**SET UP:** $D^2 = x^2 + y^2$, with $x(t)$ and $y(t)$ given by Eqs. (3.19) and (3.20).

**EXECUTE:** Following this process, $\sin^{-1}\sqrt{8/9} = 70.5°$.

**EVALUATE:** We know that if the object is thrown straight up it moves away from $P$ and then returns, so we are not surprised that the projectile angle must be less than some maximum value for the distance to always increase with time.

**3.80. IDENTIFY:** Apply the relative velocity relation.

**SET UP:** Let $v_{C/W}$ be the speed of the canoe relative to water and $v_{W/G}$ be the speed of the water relative to the ground.

**EXECUTE:** **(a)** Taking all units to be in km and h, we have three equations. We know that heading upstream $v_{C/W} - v_{W/G} = 2.$ We know that heading downstream for a time $t$, $(v_{C/W} + v_{W/G})t = 5.$ We also know that for the bottle $v_{W/G}(t+1)=3$. Solving these three equations for $v_{W/G}=x$, $v_{C/W}=2+x$, therefore $(2+x+x)t=5$ or $(2+2x)t=5$. Also $t=3/x-1$, so $(2+2x)\left(\dfrac{3}{x}-1\right)=5$ or $2x^2+x-6=0$.

The positive solution is $x=v_{W/G}=1.5$ km/h.

**(b)** $v_{C/W}=2\text{ km/h}+v_{W/G}=3.5\text{ km/h}$.

**EVALUATE:** When they head upstream, their speed relative to the ground is $3.5\text{ km/h}-1.5\text{ km/h}=2.0\text{ km/h}$. When they head downstream, their speed relative to the ground is $3.5\text{ km/h}+1.5\text{ km/h}=5.0\text{ km/h}$. The bottle is moving downstream at 1.5 km/s relative to the earth, so they are able to overtake it.

**3.81.** **IDENTIFY:** The rocket has two periods of constant acceleration motion.

**SET UP:** Let $+y$ be upward. During the free-fall phase, $a_x=0$ and $a_y=-g$. After the engines turn on, $a_x=(3.00g)\cos 30.0°$ and $a_y=(3.00g)\sin 30.0°$. Let *t* be the total time since the rocket was dropped and let *T* be the time the rocket falls before the engine starts.

**EXECUTE:** (i) The diagram is given in Figure 3.81 a.

(ii) The *x*-position of the plane is $(236\text{ m/s})t$ and the *x*-position of the rocket is $(236\text{ m/s})t+(1/2)(3.00)(9.80\text{ m/s}^2)\cos 30°(t-T)^2$. The graphs of these two equations are sketched in Figure 3.81 b.

(iii) If we take $y=0$ to be the altitude of the airliner, then $y(t)=-1/2gT^2-gT(t-T)+1/2(3.00)(9.80\text{ m/s}^2)(\sin 30°)(t-T)^2$ for the rocket. The airliner has constant *y*. The graphs are sketched in Figure 3.81b.

In each of the Figures 3.81a–c, the rocket is dropped at $t=0$ and the time *T* when the motor is turned on is indicated.

By setting $y=0$ for the rocket, we can solve for *t* in terms of *T*:

$0=-(4.90\text{ m/s}^2)T^2-(9.80\text{ m/s}^2)T(t-T)+(7.35\text{ m/s}^2)(t-T)^2$. Using the quadratic formula for the variable $x=t-T$ we find $x=t-T=\dfrac{(9.80\text{ m/s}^2)T+\sqrt{(9.80\text{ m/s}^2T)^2+(4)(7.35\text{ m/s}^2)(4.9)T^2}}{2(7.35\text{ m/s}^2)}$, or

$t=2.72\,T$. Now, using the condition that $x_{\text{rocket}}-x_{\text{plane}}=1000\text{ m}$, we find

$(236\text{ m/s})t+(12.7\text{ m/s}^2)(t-T)^2-(236\text{ m/s})t=1000\text{ m}$, or $(1.72T)^2=78.6\text{ s}^2$. Therefore $T=5.15$ s.

**EVALUATE:** During the free-fall phase the rocket and airliner have the same *x* coordinate but the rocket moves downward from the airliner. After the engines fire, the rocket starts to move upward and its horizontal component of velocity starts to exceed that of the airliner.

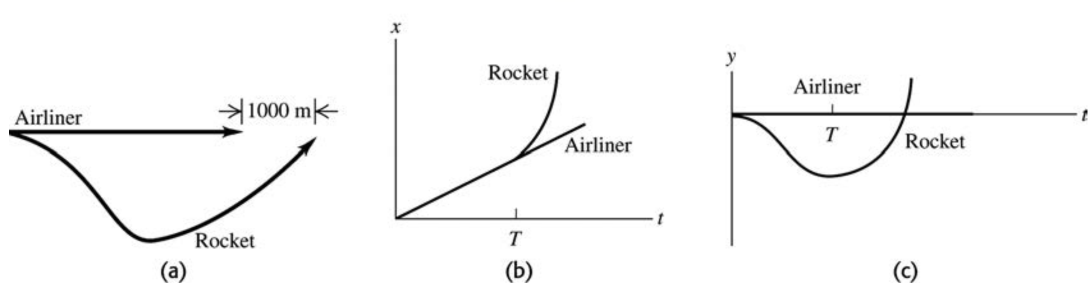



**Figure 3. 81**

**3.82.** **IDENTIFY:** We know the speed of the seeds and the distance they travel.

**SET UP:** We can treat the speed as constant over a very short distance, so $v = d/t$. The minimum frame rate is determined by the maximum speed of the seeds, so we use $v = 4.6$ m/s.

**EXECUTE:** Solving for $t$ gives $t = d/v = (0.20 \times 10^{-3}\ \text{s})/(4.6\ \text{m/s}) = 4.3 \times 10^{-5}$ s per frame. The frame rate is $1/(4.3 \times 10^{-5}$ s per frame) = 23,000 frames/seconde. Choice (c) 25,000 frames per second is closest to this result, so choice (c) is the best one.

**EVALUATE:** This experiment would clearly require high-speed photography.

**3.83.** **IDENTIFY:** A seed launched at 90° goes straight up. Since we are ignoring air resistance, its acceleration is 9.80 m/s$^2$ downward.

**SET UP:** For the highest possible speed $v_{0y} = 4.6$ m/s, and $v_y = 0$ at the highest point.

**EXECUTE:** $v_y = v_{0y} - gt$ gives $t = v_{0y}/g = (4.6\ \text{m/s})/(9.80\ \text{m/s}^2) = 0.47$ s, which is choice (b).

**EVALUATE:** Seeds are rather light and 4.6 m/s is fairly fast, so it might not be such a good idea to ignore air resistance. But doing so is acceptable to get a first approximation to the time.

**3.84.** **IDENTIFY:** A seed launched at 0° starts out traveling horizontally from a height of 20 cm above the ground. Since we are ignoring air resistance, its acceleration is 9.80 m/s$^2$ downward.

**SET UP:** Its horizontal distance is determined by the time it takes the seed to fall 20 cm, starting from rest vertically.

**EXECUTE:** The time to fall 20 cm is $0.20\ \text{m} = \frac{1}{2}gt^2$, which gives $t = 0.202$ s. The horizontal distance traveled during this time is $x = (4.6\ \text{m/s})(0.202\ \text{s}) = 0.93\ \text{m} = 93$ cm, which is choice (b).

**EVALUATE:** In reality the seed would travel a bit less distance due to air resistance.

**3.85.** **IDENTIFY:** About 2/3 of the seeds are launched between 6° and 56° above the horizontal, and the average for all the seeds is 31°. So clearly most of the seeds are launched above the horizontal.

**SET UP and EXECUTE:** For choice (a) to be correct, the seeds would need to cluster around 90°, which they do not. For choice (b), most seeds would need to launch below the horizontal, which is not the case. For choice (c), the launch angle should be around +45°. Since 31° is not far from 45°, this is the best choice. For choice (d), the seeds should go straight downward. This would require a launch angle of –90°, which is not the case.

**EVALUATE:** Evolutionarily it would be an advantage for the seeds to get as far from the parent plant as possible so the young plants would not compete with the parent for water and soil nutrients, so 45° is a biologically plausible result. Natural selection would tend to favor plants that launched their seeds at this angle over those that did not.

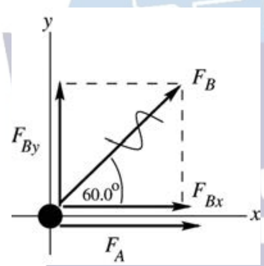
4

# NEWTON'S LAWS OF MOTION

**4.1.** **IDENTIFY:** Vector addition.

**SET UP:** Use a coordinate system where the $+x$-axis is in the direction of $\vec{F}_A$, the force applied by dog A. The forces are sketched in Figure 4.1.

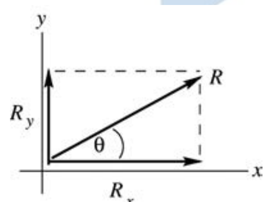
**EXECUTE:**




$$F_{Ax} = +270 \text{ N}, \quad F_{Ay} = 0$$
$$F_{Bx} = F_B \cos 60.0° = (300 \text{ N}) \cos 60.0° = +150 \text{ N}$$
$$F_{By} = F_B \sin 60.0° = (300 \text{ N}) \sin 60.0° = +260 \text{ N}$$

**Figure 4.1a**

$$\vec{R} = \vec{F}_A + \vec{F}_B$$
$$R_x = F_{Ax} + F_{Bx} = +270 \text{ N} + 150 \text{ N} = +420 \text{ N}$$
$$R_y = F_{Ay} + F_{By} = 0 + 260 \text{ N} = +260 \text{ N}$$




$$R = \sqrt{R_x^2 + R_y^2}$$
$$R = \sqrt{(420 \text{ N})^2 + (260 \text{ N})^2} = 494 \text{ N}$$
$$\tan\theta = \frac{R_y}{R_x} = 0.619$$
$$\theta = 31.8°$$

**Figure 4.1b**

**EVALUATE:** The forces must be added as vectors. The magnitude of the resultant force is less than the sum of the magnitudes of the two forces and depends on the angle between the two forces.

**4.2.** **IDENTIFY:** We know the magnitudes and directions of three vectors and want to use them to find their components, and then to use the components to find the magnitude and direction of the resultant vector.

**SET UP:** Let $F_1 = 985$ N, $F_2 = 788$ N, and $F_3 = 411$ N. The angles $\theta$ that each force makes with the $+x$ axis are $\theta_1 = 31°$, $\theta_2 = 122°$, and $\theta_3 = 233°$. The components of a force vector are $F_x = F\cos\theta$ and $F_y = F\sin\theta$, and $R = \sqrt{R_x^2 + R_y^2}$ and $\tan\theta = \dfrac{R_y}{R_x}$.

**EXECUTE:** **(a)** $F_{1x} = F_1 \cos\theta_1 = 844\ \text{N},\ \ F_{1y} = F_1 \sin\theta_1 = 507\ \text{N},\ \ F_{2x} = F_2 \cos\theta_2 = -418\ \text{N},$
$F_{2y} = F_2 \sin\theta_2 = 668\ \text{N},\ \ F_{3x} = F_3 \cos\theta_3 = -247\ \text{N},$ and $F_{3y} = F_3 \sin\theta_3 = -328\ \text{N}.$

**(b)** $R_x = F_{1x} + F_{2x} + F_{3x} = 179\ \text{N};\ \ R_y = F_{1y} + F_{2y} + F_{3y} = 847\ \text{N}.\ \ R = \sqrt{R_x^2 + R_y^2} = 886\ \text{N};\ \ \tan\theta = \dfrac{R_y}{R_x}$ so

$\theta = 78.1°.$ $\vec{\boldsymbol{R}}$ and its components are shown in Figure 4.2.

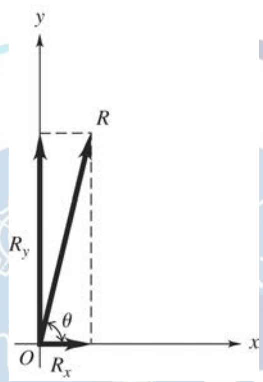



**Figure 4.2**

**EVALUATE:** A graphical sketch of the vector sum should agree with the results found in (b). Adding the forces as vectors gives a very different result from adding their magnitudes.

**4.3.** **IDENTIFY:** We know the resultant of two vectors of equal magnitude and want to find their magnitudes. They make the same angle with the vertical.

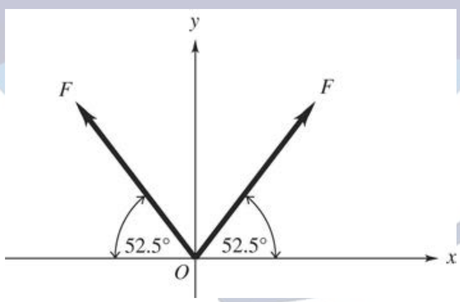



**Figure 4.3**

**SET UP:** Take $+y$ to be upward, so $\sum F_y = 5.00\ \text{N}.$ The strap on each side of the jaw exerts a force $F$ directed at an angle of 52.5° above the horizontal, as shown in Figure 4.3.

**EXECUTE:** $\sum F_y = 2F \sin 52.5° = 5.00\ \text{N},$ so $F = 3.15\ \text{N}.$

**EVALUATE:** The resultant force has magnitude 5.00 N which is *not* the same as the sum of the magnitudes of the two vectors, which would be 6.30 N.

**4.4.** **IDENTIFY:** $F_x = F\cos\theta,\ \ F_y = F\sin\theta.$

**SET UP:** Let $+x$ be parallel to the ramp and directed up the ramp. Let $+y$ be perpendicular to the ramp and directed away from it. Then $\theta = 30.0°.$

**EXECUTE:** **(a)** $F = \dfrac{F_x}{\cos\theta} = \dfrac{90.0\ \text{N}}{\cos 30°} = 104\ \text{N}.$

**(b)** $F_y = F\sin\theta = F_x \tan\theta = (90\ \text{N})(\tan 30°) = 52.0\ \text{N}.$

**EVALUATE:** We can verify that $F_x^2 + F_y^2 = F^2.$ The signs of $F_x$ and $F_y$ show their direction.

**4.5.** **IDENTIFY:** Add the two forces using components.

**SET UP:** $F_x = F\cos\theta$, $F_y = F\sin\theta$, where $\theta$ is the angle $\vec{F}$ makes with the $+x$ axis.

**EXECUTE:** **(a)** $F_{1x} + F_{2x} = (9.00\text{ N})\cos 120° + (6.00\text{ N})\cos(233.1°) = -8.10\text{ N}$

$F_{1y} + F_{2y} = (9.00\text{ N})\sin 120° + (6.00\text{ N})\sin(233.1°) = +3.00\text{ N}.$

**(b)** $R = \sqrt{R_x^2 + R_y^2} = \sqrt{(8.10\text{ N})^2 + (3.00\text{ N})^2} = 8.64\text{ N}.$

**EVALUATE:** Since $F_x < 0$ and $F_y > 0$, $\vec{F}$ is in the second quadrant.

**4.6.** **IDENTIFY:** Use constant acceleration equations to calculate $a_x$ and $t$. Then use $\sum\vec{F} = m\vec{a}$ to calculate the net force.

**SET UP:** Let $+x$ be in the direction of motion of the electron.

**EXECUTE:** **(a)** $v_{0x} = 0$, $(x - x_0) = 1.80\times10^{-2}\text{ m}$, $v_x = 3.00\times10^6\text{ m/s}$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(3.00\times10^6\text{ m/s})^2 - 0}{2(1.80\times10^{-2}\text{ m})} = 2.50\times10^{14}\text{ m/s}^2$$

**(b)** $v_x = v_{0x} + a_x t$ gives $t = \dfrac{v_x - v_{0x}}{a_x} = \dfrac{3.00\times10^6\text{ m/s} - 0}{2.50\times10^{14}\text{ m/s}^2} = 1.2\times10^{-8}\text{ s}$

**(c)** $\sum F_x = ma_x = (9.11\times10^{-31}\text{ kg})(2.50\times10^{14}\text{ m/s}^2) = 2.28\times10^{-16}\text{ N}.$

**EVALUATE:** The acceleration is in the direction of motion since the speed is increasing, and the net force is in the direction of the acceleration.

**4.7.** **IDENTIFY:** Friction is the only horizontal force acting on the skater, so it must be the one causing the acceleration. Newton's second law applies.

**SET UP:** Take $+x$ to be the direction in which the skater is moving initially. The final velocity is $v_x = 0$, since the skater comes to rest. First use the kinematics formula $v_x = v_{0x} + a_x t$ to find the acceleration, then apply $\sum\vec{F} = m\vec{a}$ to the skater.

**EXECUTE:** $v_x = v_{0x} + a_x t$ so $a_x = \dfrac{v_x - v_{0x}}{t} = \dfrac{0 - 2.40\text{ m/s}}{3.52\text{ s}} = -0.682\text{ m/s}^2$. The only horizontal force on the skater is the friction force, so $f_x = ma_x = (68.5\text{ kg})(-0.682\text{ m/s}^2) = -46.7\text{ N}$. The force is 46.7 N, directed opposite to the motion of the skater.

**EVALUATE:** Although other forces are acting on the skater (gravity and the upward force of the ice), they are vertical and therefore do not affect the horizontal motion.

**4.8.** **IDENTIFY:** The elevator and everything in it are accelerating upward, so we apply Newton's second law in the vertical direction.

**SET UP:** Your mass is $m = w/g = 63.8\text{ kg}$. Both you and the package have the same acceleration as the elevator. Take $+y$ to be upward, in the direction of the acceleration of the elevator, and apply $\sum F_y = ma_y$.

**EXECUTE:** **(a)** Your free-body diagram is shown in Figure 4.8a, where $n$ is the scale reading. $\sum F_y = ma_y$ gives $n - w = ma$. Solving for $n$ gives $n = w + ma = 625\text{ N} + (63.8\text{ kg})(2.50\text{ m/s}^2) = 784\text{ N}.$

**(b)** The free-body diagram for the package is given in Figure 4.8b. $\sum F_y = ma_y$ gives $T - w = ma$, so $T = w + ma = (3.85\text{ kg})(9.80\text{ m/s}^2 + 2.50\text{ m/s}^2) = 47.4\text{ N}.$

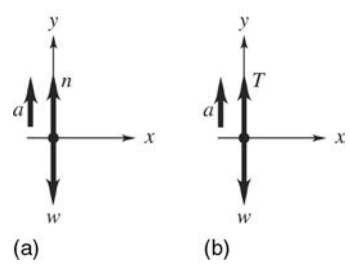



**Figure 4.8**

**EVALUATE:** The objects accelerate upward so for each of them the upward force is greater than the downward force.

**4.9.** **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the box.

**SET UP:** Let $+x$ be the direction of the force and acceleration. $\sum F_x = 48.0\text{ N}$.

**EXECUTE:** $\sum F_x = ma_x$ gives $m = \dfrac{\Sigma F_x}{a_x} = \dfrac{48.0\text{ N}}{2.20\text{ m/s}^2} = 21.8\text{ kg}$.

**EVALUATE:** The vertical forces sum to zero and there is no motion in that direction.

**4.10.** **IDENTIFY:** Use the information about the motion to find the acceleration and then use $\sum F_x = ma_x$ to calculate $m$.

**SET UP:** Let $+x$ be the direction of the force. $\sum F_x = 80.0\text{ N}$.

**EXECUTE:** **(a)** $x - x_0 = 11.0\text{ m}$, $t = 5.00\text{ s}$, $v_{0x} = 0$. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives

$a_x = \dfrac{2(x - x_0)}{t^2} = \dfrac{2(11.0\text{ m})}{(5.00\text{ s})^2} = 0.880\text{ m/s}^2$. $m = \dfrac{\Sigma F_x}{a_x} = \dfrac{80.0\text{ N}}{0.880\text{ m/s}^2} = 90.9\text{ kg}$.

**(b)** $a_x = 0$ and $v_x$ is constant. After the first 5.0 s, $v_x = v_{0x} + a_x t =$ $(0.880\text{ m/s}^2)(5.00\text{ s}) = 4.40\text{ m/s}$.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 =$ $(4.40\text{ m/s})(5.00\text{ s}) = 22.0\text{ m}$.

**EVALUATE:** The mass determines the amount of acceleration produced by a given force. The block moves farther in the second 5.00 s than in the first 5.00 s.

**4.11.** **IDENTIFY and SET UP:** Use Newton's second law in component form to calculate the acceleration produced by the force. Use constant acceleration equations to calculate the effect of the acceleration on the motion.

**EXECUTE:** **(a)** During this time interval the acceleration is constant and equal to

$$a_x = \frac{F_x}{m} = \frac{0.250\text{ N}}{0.160\text{ kg}} = 1.562\text{ m/s}^2$$

We can use the constant acceleration kinematic equations from Chapter 2.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(1.562\text{ m/s}^2)(2.00\text{ s})^2 = 3.12\text{ m}$, so the puck is at $x = 3.12\text{ m}$.

$v_x = v_{0x} + a_x t = 0 + (1.562\text{ m/s}^2)(2.00\text{ s}) = 3.12\text{ m/s}$.

**(b)** In the time interval from $t = 2.00\text{ s}$ to 5.00 s the force has been removed so the acceleration is zero. The speed stays constant at $v_x = 3.12\text{ m/s}$. The distance the puck travels is

$x - x_0 = v_{0x}t = (3.12\text{ m/s})(5.00\text{ s} - 2.00\text{ s}) = 9.36\text{ m}$. At the end of the interval it is at

$x = x_0 + 9.36\text{ m} = 12.5\text{ m}$.

In the time interval from $t = 5.00\text{ s}$ to 7.00 s the acceleration is again $a_x = 1.562\text{ m/s}^2$. At the start of this interval $v_{0x} = 3.12\text{ m/s}$ and $x_0 = 12.5\text{ m}$.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (3.12\text{ m/s})(2.00\text{ s}) + \frac{1}{2}(1.562\text{ m/s}^2)(2.00\text{ s})^2$.

$x - x_0 = 6.24\text{ m} + 3.12\text{ m} = 9.36\text{ m}$.

Therefore, at $t = 7.00\text{ s}$ the puck is at $x = x_0 + 9.36\text{ m} = 12.5\text{ m} + 9.36\text{ m} = 21.9\text{ m}$.

$$v_x = v_{0x} + a_x t = 3.12\text{ m/s} + (1.562\text{ m/s}^2)(2.00\text{ s}) = 6.24\text{ m/s}.$$

**EVALUATE:** The acceleration says the puck gains 1.56 m/s of velocity for every second the force acts. The force acts a total of 4.00 s so the final velocity is $(1.56\text{ m/s})(4.0\text{ s}) = 6.24\text{ m/s}$.

**4.12.** **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$. Then use a constant acceleration equation to relate the kinematic quantities.

**SET UP:** Let $+x$ be in the direction of the force.

**EXECUTE:** **(a)** $a_x = F_x / m$ = (14.0 N)/(32.5 kg) = 0.4308 $\text{m/s}^2$, which rounds to 0.431 $\text{m/s}^2$ for the final answer.

**(b)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$. With $v_{0x} = 0$, $x = \frac{1}{2}a_x t^2 = \frac{1}{2}(0.4308\text{ m/s}^2)(10.0\text{ s})^2 = 21.5\text{ m}$.

**(c)** $v_x = v_{0x} + a_x t$. With $v_{0x} = 0$, $v_x = a_x t$ = (0.4308 $\text{m/s}^2$)(10.0 s) = 4.31 m/s.

**EVALUATE:** The acceleration connects the motion to the forces.

**4.13.** **IDENTIFY:** The force and acceleration are related by Newton's second law.

**SET UP:** $\sum F_x = ma_x$, where $\sum F_x$ is the net force. $m = 4.50\text{ kg}$.

**EXECUTE:** **(a)** The maximum net force occurs when the acceleration has its maximum value. $\sum F_x = ma_x = (4.50\text{ kg})(10.0\text{ m/s}^2) = 45.0\text{ N}$. This maximum force occurs between 2.0 s and 4.0 s.

**(b)** The net force is constant when the acceleration is constant. This is between 2.0 s and 4.0 s.

**(c)** The net force is zero when the acceleration is zero. This is the case at $t = 0$ and $t = 6.0\text{ s}$.

**EVALUATE:** A graph of $\sum F_x$ versus *t* would have the same shape as the graph of $a_x$ versus *t*.

**4.14.** **IDENTIFY:** The force and acceleration are related by Newton's second law. $a_x = \dfrac{dv_x}{dt}$, so $a_x$ is the slope of the graph of $v_x$ versus *t*.

**SET UP:** The graph of $v_x$ versus *t* consists of straight-line segments. For $t = 0$ to $t = 2.00\text{ s}$, $a_x = 4.00\text{ m/s}^2$. For $t = 2.00\text{ s}$ to 6.00 s, $a_x = 0$. For $t = 6.00\text{ s}$ to 10.0 s, $a_x = 1.00\text{ m/s}^2$. $\sum F_x = ma_x$, with $m = 2.75\text{ kg}$. $\sum F_x$ is the net force.

**EXECUTE:** **(a)** The maximum net force occurs when the acceleration has its maximum value. $\sum F_x = ma_x = (2.75\text{ kg})(4.00\text{ m/s}^2) = 11.0\text{ N}$. This maximum occurs in the interval $t = 0$ to $t = 2.00\text{ s}$.

**(b)** The net force is zero when the acceleration is zero. This is between 2.00 s and 6.00 s.

**(c)** Between 6.00 s and 10.0 s, $a_x = 1.00\text{ m/s}^2$, so $\sum F_x = (2.75\text{ kg})(1.00\text{ m/s}^2) = 2.75\text{ N}$.

**EVALUATE:** The net force is largest when the velocity is changing most rapidly.

**4.15.** **IDENTIFY:** The net force and the acceleration are related by Newton's second law. When the rocket is near the surface of the earth the forces on it are the upward force $\vec{F}$ exerted on it because of the burning fuel and the downward force $\vec{F}_{\text{grav}}$ of gravity. $F_{\text{grav}} = mg$.

**SET UP:** Let $+y$ be upward. The weight of the rocket is $F_{\text{grav}} = (8.00\text{ kg})(9.80\text{ m/s}^2) = 78.4\text{ N}$.

**EXECUTE:** **(a)** At $t = 0$, $F = A = 100.0\text{ N}$. At $t = 2.00\text{ s}$, $F = A + (4.00\text{ s}^2)B = 150.0\text{ N}$ and

$$B = \frac{150.0\text{ N} - 100.0\text{ N}}{4.00\text{ s}^2} = 12.5\text{ N/s}^2.$$

**(b)** (i) At $t = 0$, $F = A = 100.0\text{ N}$. The net force is $\sum F_y = F - F_{\text{grav}} = 100.0\text{ N} - 78.4\text{ N} = 21.6\text{ N}$.

$a_y = \dfrac{\sum F_y}{m} = \dfrac{21.6\text{ N}}{8.00\text{ kg}} = 2.70\text{ m/s}^2$. (ii) At $t = 3.00\text{ s}$, $F = A + B(3.00\text{ s})^2 = 212.5\text{ N}$.

$\sum F_y = 212.5\text{ N} - 78.4\text{ N} = 134.1\text{ N}$. $a_y = \dfrac{\sum F_y}{m} = \dfrac{134.1\text{ N}}{8.00\text{ kg}} = 16.8\text{ m/s}^2$.

**(c)** Now $F_{\text{grav}} = 0$ and $\sum F_y = F = 212.5\text{ N}$. $a_y = \dfrac{212.5\text{ N}}{8.00\text{ kg}} = 26.6\text{ m/s}^2$.

**EVALUATE:** The acceleration increases as *F* increases.

**4.16.** **IDENTIFY:** Weight and mass are related by $w = mg$. The mass is constant but *g* and *w* depend on location.

**SET UP:** On Earth, $g = 9.80\text{ m/s}^2$.

**EXECUTE:** **(a)** $\dfrac{w}{g} = m$, which is constant, so $\dfrac{w_{\text{E}}}{g_{\text{E}}} = \dfrac{w_{\text{A}}}{g_{\text{A}}}$. $w_{\text{E}} = 17.5\text{ N}$, $g_{\text{E}} = 9.80\text{ m/s}^2$, and

$$w_{\text{M}} = 3.24\text{ N}.\quad g_{\text{M}} = \left(\frac{w_{\text{A}}}{w_{\text{E}}}\right)g_{\text{E}} = \left(\frac{3.24\text{ N}}{17.5\text{ N}}\right)(9.80\text{ m/s}^2) = 1.81\text{ m/s}^2.$$

**(b)** $m = \dfrac{w_{\text{E}}}{g_{\text{E}}} = \dfrac{17.5\text{ N}}{9.80\text{ m/s}^2} = 1.79\text{ kg}$.

**EVALUATE:** The weight at a location and the acceleration due to gravity at that location are directly proportional.

**4.17.** **IDENTIFY and SET UP:** $F = ma$. We must use $w = mg$ to find the mass of the boulder.

**EXECUTE:** $m = \frac{w}{g} = \frac{2400\ \text{N}}{9.80\ \text{m/s}^2} = 244.9\ \text{kg}$

Then $F = ma = (244.9\ \text{kg})(12.0\ \text{m/s}^2) = 2940\ \text{N}$.

**EVALUATE:** We must use mass in Newton's second law. Mass and weight are proportional.

**4.18.** **IDENTIFY:** Find weight from mass and vice versa.

**SET UP:** Equivalencies we'll need are: $1\ \mu\text{g} = 10^{-6}\ \text{g} = 10^{-9}\ \text{kg}$, $1\ \text{mg} = 10^{-3}\ \text{g} = 10^{-6}\ \text{kg}$, $1\ \text{N} = 0.2248\ \text{lb}$, and $g = 9.80\ \text{m/s}^2 = 32.2\ \text{ft/s}^2$.

**EXECUTE:** **(a)** $m = 210\ \mu\text{g} = 2.10\times10^{-7}\ \text{kg}$. $w = mg = (2.10\times10^{-7}\ \text{kg})(9.80\ \text{m/s}^2) = 2.06\times10^{-6}\ \text{N}$.

**(b)** $m = 12.3\ \text{mg} = 1.23\times10^{-5}\ \text{kg}$. $w = mg = (1.23\times10^{-5}\ \text{kg})(9.80\ \text{m/s}^2) = 1.21\times10^{-4}\ \text{N}$.

**(c)** $(45\ \text{N})\left(\frac{0.2248\ \text{lb}}{1\ \text{N}}\right) = 10.1\ \text{lb}$. $m = \frac{w}{g} = \frac{45\ \text{N}}{9.80\ \text{m/s}^2} = 4.6\ \text{kg}$.

**EVALUATE:** We are not converting mass to weight (or vice versa) since they are different types of quantities. We are finding what a given mass will weigh and how much mass a given weight contains.

**4.19.** **IDENTIFY and SET UP:** $w = mg$. The mass of the watermelon is constant, independent of its location. Its weight differs on earth and Jupiter's moon. Use the information about the watermelon's weight on earth to calculate its mass:

**EXECUTE:** **(a)** $w = mg$ gives that $m = \frac{w}{g} = \frac{44.0\ \text{N}}{9.80\ \text{m/s}^2} = 4.49\ \text{kg}$.

**(b)** On Jupiter's moon, $m = 4.49\ \text{kg}$, the same as on earth. Thus the weight on Jupiter's moon is $w = mg = (4.49\ \text{kg})(1.81\ \text{m/s}^2) = 8.13\ \text{N}$.

**EVALUATE:** The weight of the watermelon is less on Io, since g is smaller there.

**4.20.** **IDENTIFY:** Newton's third law applies.

**SET UP:** The car exerts a force on the truck and the truck exerts a force on the car.

**EXECUTE:** The force and the reaction force are always exactly the same in magnitude, so the force that the truck exerts on the car is 1600 N, by Newton's third law.

**EVALUATE:** Even though the truck is much larger and more massive than the car, it cannot exert a larger force on the car than the car exerts on it.

**4.21.** **IDENTIFY:** Apply $\sum F_x = ma_x$ to find the resultant horizontal force.

**SET UP:** Let the acceleration be in the $+x$ direction.

**EXECUTE:** $\sum F_x = ma_x = (55\ \text{kg})(15\ \text{m/s}^2) = 825\ \text{N}$. The force is exerted by the blocks. The blocks push on the sprinter because the sprinter pushes on the blocks.

**EVALUATE:** The force the blocks exert on the sprinter has the same magnitude as the force the sprinter exerts on the blocks. The harder the sprinter pushes, the greater the force on her.

**4.22.** **IDENTIFY:** The reaction forces in Newton's third law are always between a pair of objects. In Newton's second law all the forces act on a single object.

**SET UP:** Let $+y$ be downward. $m = w/g$.

**EXECUTE:** The reaction to the upward normal force on the passenger is the downward normal force, also of magnitude 620 N, that the passenger exerts on the floor. The reaction to the passenger's weight is the gravitational force that the passenger exerts on the earth, upward and also of magnitude 650 N. $\frac{\sum F_y}{m} = a_y$ gives $a_y = \frac{650\ \text{N} - 620\ \text{N}}{(650\ \text{N})/(9.80\ \text{m/s}^2)} = 0.452\ \text{m/s}^2$. The passenger's acceleration is $0.452\ \text{m/s}^2$, downward.

**EVALUATE:** There is a net downward force on the passenger, and the passenger has a downward acceleration.

**4.23.** **IDENTIFY:** The system is accelerating so we use Newton's second law.

**SET UP:** The acceleration of the entire system is due to the 250-N force, but the acceleration of box $B$ is due to the force that box $A$ exerts on it. $\sum F = ma$ applies to the two-box system and to each box individually.

**EXECUTE:** For the two-box system: $a_x = \dfrac{250\text{ N}}{25.0\text{ kg}} = 10.0\text{ m/s}^2$. Then for box *B*, where $F_A$ is the force exerted on *B* by *A*, $F_A = m_B a = (5.0\text{ kg})(10.0\text{ m/s}^2) = 50\text{ N}$.

**EVALUATE:** The force on *B* is less than the force on *A*.

**4.24.** **IDENTIFY:** Apply Newton's second law to the earth.

**SET UP:** The force of gravity that the earth exerts on her is her weight, $w = mg = (45\text{ kg})(9.8\text{ m/s}^2) = 441\text{ N}$. By Newton's third law, she exerts an equal and opposite force on the earth.

Apply $\sum\vec{F} = m\vec{a}$ to the earth, with $\left|\sum\vec{F}\right| = w = 441\text{ N}$, but must use the mass of the earth for *m*.

**EXECUTE:** $a = \dfrac{w}{m} = \dfrac{441\text{ N}}{6.0\times10^{24}\text{ kg}} = 7.4\times10^{-23}\text{ m/s}^2$.

**EVALUATE:** This is *much* smaller than her acceleration of $9.8\text{ m/s}^2$. The force she exerts on the earth equals in magnitude the force the earth exerts on her, but the acceleration the force produces depends on the mass of the object and her mass is *much* less than the mass of the earth.

**4.25.** **IDENTIFY:** Identify the forces on each object.

**SET UP:** In each case the forces are the noncontact force of gravity (the weight) and the forces applied by objects that are in contact with each crate. Each crate touches the floor and the other crate, and some object applies $\vec{F}$ to crate *A*.

**EXECUTE:** **(a)** The free-body diagrams for each crate are given in Figure 4.25.

$F_{AB}$ (the force on $m_A$ due to $m_B$) and $F_{BA}$ (the force on $m_B$ due to $m_A$) form an action-reaction pair.

**(b)** Since there is no horizontal force opposing *F*, any value of *F*, no matter how small, will cause the crates to accelerate to the right. The weight of the two crates acts at a right angle to the horizontal, and is in any case balanced by the upward force of the surface on them.

**EVALUATE:** Crate *B* is accelerated by $F_{BA}$ and crate *A* is accelerated by the net force $F - F_{AB}$. The greater the total weight of the two crates, the greater their total mass and the smaller will be their acceleration.

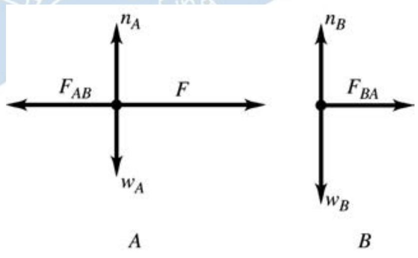



**Figure 4.25**

**4.26.** **IDENTIFY:** The surface of block *B* can exert both a friction force and a normal force on block *A*. The friction force is directed so as to oppose relative motion between blocks *B* and *A*. Gravity exerts a downward force *w* on block *A*.

**SET UP:** The pull is a force on *B* not on *A*.

**EXECUTE:** **(a)** If the table is frictionless there is a net horizontal force on the combined object of the two blocks, and block *B* accelerates in the direction of the pull. The friction force that *B* exerts on *A* is to the right, to try to prevent *A* from slipping relative to *B* as *B* accelerates to the right. The free-body diagram is sketched in Figure 4.26a (next page). *f* is the friction force that *B* exerts on *A* and *n* is the normal force that *B* exerts on *A*.

**(b)** The pull and the friction force exerted on *B* by the table cancel and the net force on the system of two blocks is zero. The blocks move with the same constant speed and *B* exerts no friction force on *A*. The free-body diagram is sketched in Figure 4.26b (next page).

**EVALUATE:** If in part (b) the pull force is decreased, block $B$ will slow down, with an acceleration directed to the left. In this case the friction force on $A$ would be to the left, to prevent relative motion between the two blocks by giving $A$ an acceleration equal to that of $B$.

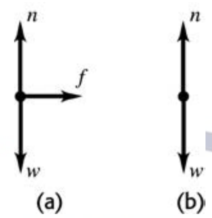



**Figure 4.26**

**4.27.** **IDENTIFY:** Since the observer in the train sees the ball hang motionless, the ball must have the same acceleration as the train car. By Newton's second law, there must be a net force on the ball in the same direction as its acceleration.

**SET UP:** The forces on the ball are gravity, which is $w$, downward, and the tension $\vec{T}$ in the string, which is directed along the string.

**EXECUTE:** **(a)** The acceleration of the train is zero, so the acceleration of the ball is zero. There is no net horizontal force on the ball and the string must hang vertically. The free-body diagram is sketched in Figure 4.27a.

**(b)** The train has a constant acceleration directed east so the ball must have a constant eastward acceleration. There must be a net horizontal force on the ball, directed to the east. This net force must come from an eastward component of $\vec{T}$ and the ball hangs with the string displaced west of vertical. The free-body diagram is sketched in Figure 4.27b.

**EVALUATE:** When the motion of an object is described in an inertial frame, there must be a net force in the direction of the acceleration.

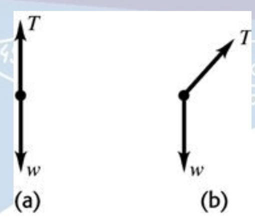



**Figure 4.27**

**4.28.** **IDENTIFY:** Use a constant acceleration equation to find the stopping time and acceleration. Then use $\sum \vec{F} = m\vec{a}$ to calculate the force.

**SET UP:** Let $+x$ be in the direction the bullet is traveling. $\vec{F}$ is the force the wood exerts on the bullet.

**EXECUTE:** **(a)** $v_{0x} = 350\text{ m/s}$, $v_x = 0$ and $(x - x_0) = 0.130\text{ m}$. $(x - x_0) = \left(\dfrac{v_{0x} + v_x}{2}\right)t$ gives

$$t = \frac{2(x - x_0)}{v_{0x} + v_x} = \frac{2(0.130\text{ m})}{350\text{ m/s}} = 7.43\times10^{-4}\text{ s}.$$

**(b)** $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $a_x = \dfrac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \dfrac{0 - (350\text{ m/s})^2}{2(0.130\text{ m})} = -4.71\times10^5\text{ m/s}^2$

$\sum F_x = ma_x$ gives $-F = ma_x$ and $F = -ma_x = -(1.80\times10^{-3}\text{ kg})(-4.71\times10^5\text{ m/s}^2) = 848\text{ N}$.

**EVALUATE:** The acceleration and net force are opposite to the direction of motion of the bullet.

**4.29.** **IDENTIFY:** Identify the forces on the chair. The floor exerts a normal force and a friction force.

**SET UP:** Let $+y$ be upward and let $+x$ be in the direction of the motion of the chair.

**Execute:** **(a)** The free-body diagram for the chair is given in Figure 4.29.
**(b)** For the chair, $a_y = 0$ so $\sum F_y = ma_y$ gives $n - mg - F\sin 37° = 0$ and $n = 142$ N.

**Evaluate:** $n$ is larger than the weight because $\vec{F}$ has a downward component.

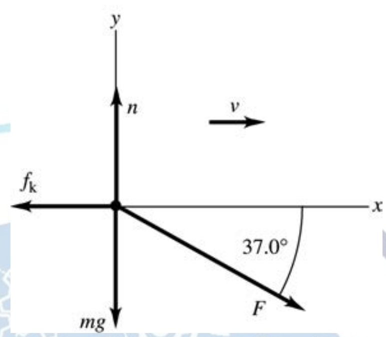



**Figure 4.29**

**4.30.** **Identify:** Identify the forces for each object. Action-reaction pairs of forces act between two objects.
**Set Up:** Friction is parallel to the surfaces and is directly opposite to the relative motion between the surfaces.
**Execute:** The free-body diagram for the box is given in Figure 4.30a. The free-body diagram for the truck is given in Figure 4.30b. The box's friction force on the truck bed and the truck bed's friction force on the box form an action-reaction pair. There would also be some small air-resistance force action to the left, presumably negligible at this speed.
**Evaluate:** The friction force on the box, exerted by the bed of the truck, is in the direction of the truck's acceleration. This friction force can't be large enough to give the box the same acceleration that the truck has and the truck acquires a greater speed than the box.

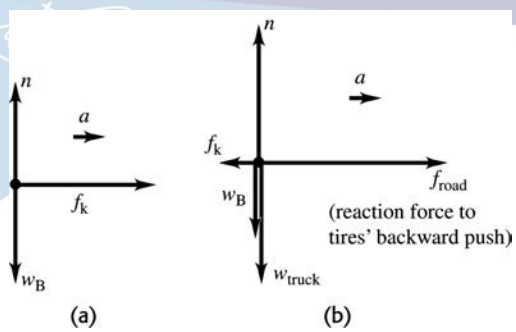



**Figure 4.30**

**4.31.** **Identify:** Apply Newton's second law to the bucket and constant-acceleration kinematics.
**Set Up:** The minimum time to raise the bucket will be when the tension in the cord is a maximum since this will produce the greatest acceleration of the bucket.
**Execute:** Apply Newton's second law to the bucket: $T - mg = ma$. For the maximum acceleration, the tension is greatest, so $a = \dfrac{T - mg}{m} = \dfrac{75.0\text{ N} - (5.60\text{ kg})(9.8\text{ m/s}^2)}{5.60\text{ kg}} = 3.593\text{ m/s}^2$.

The kinematics equation for $y(t)$ gives $t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(12.0\text{ m})}{3.593\text{ m/s}^2}} = 2.58\text{ s}$.

**Evaluate:** A shorter time would require a greater acceleration and hence a stronger pull, which would break the cord.

**4.32.** **IDENTIFY:** Use the motion of the ball to calculate $g$, the acceleration of gravity on the planet. Then $w = mg$.

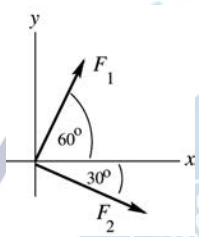
**SET UP:** Let $+y$ be downward and take $y_0 = 0$. $v_{0y} = 0$ since the ball is released from rest.

**EXECUTE:** Get $g$ on X: $y = \frac{1}{2}gt^2$ gives $10.0\text{ m} = \frac{1}{2}g(3.40\text{ s})^2$. $g = 1.73\text{ m/s}^2$ and then

$w_X = mg_X = (0.100\text{ kg})(1.73\text{ m/s}^2) = 0.173\text{ N}$.

**EVALUATE:** $g$ on Planet X is smaller than on earth and the object weighs less than it would on earth.

**4.33.** **IDENTIFY:** If the box moves in the $+x$-direction it must have $a_y = 0$, so $\sum F_y = 0$.




The smallest force the child can exert and still produce such motion is a force that makes the $y$-components of all three forces sum to zero, but that doesn't have any $x$-component.

**Figure 4.33**

**SET UP:** $\vec{F}_1$ and $\vec{F}_2$ are sketched in Figure 4.33. Let $\vec{F}_3$ be the force exerted by the child.

$\sum F_y = ma_y$ implies $F_{1y} + F_{2y} + F_{3y} = 0$, so $F_{3y} = -(F_{1y} + F_{2y})$.

**EXECUTE:** $F_{1y} = +F_1\sin 60° = (100\text{ N})\sin 60° = 86.6\text{ N}$

$F_{2y} = +F_2\sin(-30°) = -F_2\sin 30° = -(140\text{ N})\sin 30° = -70.0\text{ N}$

Then $F_{3y} = -(F_{1y} + F_{2y}) = -(86.6\text{ N} - 70.0\text{ N}) = -16.6\text{ N}$; $F_{3x} = 0$

The smallest force the child can exert has magnitude 17 N and is directed at $90°$ clockwise from the $+x$-axis shown in the figure.

**(b) IDENTIFY** and **SET UP:** Apply $\sum F_x = ma_x$. We know the forces and $a_x$ so can solve for $m$. The force exerted by the child is in the $-y$-direction and has no $x$-component.

**EXECUTE:** $F_{1x} = F_1\cos 60° = 50\text{ N}$

$F_{2x} = F_2\cos 30° = 121.2\text{ N}$

$\sum F_x = F_{1x} + F_{2x} = 50\text{ N} + 121.2\text{ N} = 171.2\text{ N}$

$$m = \frac{\sum F_x}{a_x} = \frac{171.2\text{ N}}{2.00\text{ m/s}^2} = 85.6\text{ kg}$$

Then $w = mg = 840\text{ N}$.

**EVALUATE:** In part (b) we don't need to consider the $y$-component of Newton's second law. $a_y = 0$ so the mass doesn't appear in the $\sum F_y = ma_y$ equation.

**4.34.** **IDENTIFY:** Use $\sum \vec{F} = m\vec{a}$ to calculate the acceleration of the tanker and then use constant acceleration kinematic equations.

**SET UP:** Let $+x$ be the direction the tanker is moving initially. Then $a_x = -F/m$.

**EXECUTE:** $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ says that if the reef weren't there the ship would stop in a distance of

$$x - x_0 = -\frac{v_{0x}^2}{2a_x} = \frac{v_0^2}{2(F/m)} = \frac{mv_0^2}{2F} = \frac{(3.6\times 10^7\text{ kg})(1.5\text{ m/s})^2}{2(8.0\times 10^4\text{ N})} = 506\text{ m},$$

so the ship would hit the reef. The speed when the tanker hits the reef is found from $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, so it is

$$v=\sqrt{v_0^2-(2Fx/m)}=\sqrt{(1.5\text{ m/s})^2-\frac{2(8.0\times10^4\text{ N})(500\text{ m})}{(3.6\times10^7\text{ kg})}}=0.17\text{ m/s},$$

and the oil should be safe.

**EVALUATE:** The force and acceleration are directed opposite to the initial motion of the tanker and the speed decreases.

**4.35.** **IDENTIFY:** We can apply constant acceleration equations to relate the kinematic variables and we can use Newton's second law to relate the forces and acceleration.

**(a) SET UP:** First use the information given about the height of the jump to calculate the speed he has at the instant his feet leave the ground. Use a coordinate system with the $+y$-axis upward and the origin at the position when his feet leave the ground.

$v_y=0$ (at the maximum height), $v_{0y}=?$, $a_y=-9.80\text{ m/s}^2$, $y-y_0=+1.2\text{ m}$

$v_y^2=v_{0y}^2+2a_y(y-y_0)$

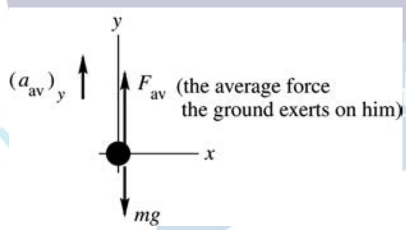
**EXECUTE:** $v_{0y}=\sqrt{-2a_y(y-y_0)}=\sqrt{-2(-9.80\text{ m/s}^2)(1.2\text{ m})}=4.85\text{ m/s}$

**(b) SET UP:** Now consider the acceleration phase, from when he starts to jump until when his feet leave the ground. Use a coordinate system where the $+y$-axis is upward and the origin is at his position when he starts his jump.

**EXECUTE:** Calculate the average acceleration:

$$(a_{\text{av}})_y=\frac{v_y-v_{0y}}{t}=\frac{4.85\text{ m/s}-0}{0.300\text{ s}}=16.2\text{ m/s}^2$$

**(c) SET UP:** Finally, find the average upward force that the ground must exert on him to produce this average upward acceleration. (Don't forget about the downward force of gravity.) The forces are sketched in Figure 4.35.




**EXECUTE:**

$$m=w/g=\frac{890\text{ N}}{9.80\text{ m/s}^2}=90.8\text{ kg}$$

$$\sum F_y=ma_y$$

$$F_{\text{av}}-mg=m(a_{\text{av}})_y$$

$$F_{\text{av}}=m(g+(a_{\text{av}})_y)$$

$$F_{\text{av}}=90.8\text{ kg}(9.80\text{ m/s}^2+16.2\text{ m/s}^2)$$

$$F_{\text{av}}=2360\text{ N}$$

**Figure 4.35**

This is the average force exerted on him by the ground. But by Newton's third law, the average force he exerts on the ground is equal and opposite, so is 2360 N, downward. The net force on him is equal to $ma$, so $F_{\text{net}}=ma=(90.8\text{ kg})(16.2\text{ m/s}^2)=1470\text{ N}$ upward.

**EVALUATE:** In order for him to accelerate upward, the ground must exert an upward force greater than his weight.

**4.36.** **IDENTIFY:** Use constant acceleration equations to calculate the acceleration $a_x$ that would be required. Then use $\sum F_x=ma_x$ to find the necessary force.

**SET UP:** Let $+x$ be the direction of the initial motion of the auto.

**EXECUTE:** $v_x^2=v_{0x}^2+2a_x(x-x_0)$ with $v_x=0$ gives $a_x=-\dfrac{v_{0x}^2}{2(x-x_0)}$. The force $F$ is directed opposite to the motion and $a_x = -\dfrac{F}{m}$. Equating these two expressions for $a_x$ gives

$$F = m\frac{v_{0x}^2}{2(x - x_0)} = (850\text{ kg})\frac{(12.5\text{ m/s})^2}{2(1.8\times10^{-2}\text{ m})} = 3.7\times10^6\text{ N}.$$

**EVALUATE:** A very large force is required to stop such a massive object in such a short distance.

**4.37.** **IDENTIFY:** Using constant-acceleration kinematics, we can find the acceleration of the ball. Then we can apply Newton's second law to find the force causing that acceleration.

**SET UP:** Use coordinates where $+x$ is in the direction the ball is thrown. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ and $\sum F_x = ma_x$.

**EXECUTE:** **(a)** Solve for $a_x$: $x - x_0 = 1.0$ m, $v_{0x} = 0$, $v_x = 46$ m/s. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x)} = \frac{(46\text{ m/s})^2 - 0}{2(1.0\text{ m})} = 1058\text{ m/s}^2.$$

The free-body diagram for the ball during the pitch is shown in Figure 4.37a. The force $\vec{F}$ is applied to the ball by the pitcher's hand. $\sum F_x = ma_x$ gives $F = (0.145\text{ kg})(1058\text{ m/s}^2) = 153$ N.

**(b)** The free-body diagram after the ball leaves the hand is given in Figure 4.37b. The only force on the ball is the downward force of gravity.

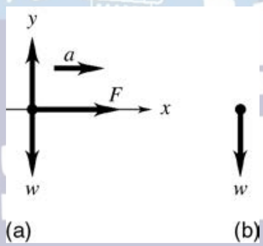



**Figure 4.37**

**EVALUATE:** The force is much greater than the weight of the ball because it gives it an acceleration much greater than $g$.

**4.38.** **IDENTIFY:** Kinematics will give us the ball's acceleration, and Newton's second law will give us the horizontal force acting on it.

**SET UP:** Use coordinates with $+x$ horizontal and in the direction of the motion of the ball and with $+y$ upward. $\sum F_x = ma_x$ and for constant acceleration, $v_x = v_{0x} + a_x t$.

**SOLVE:** **(a)** $v_{0x} = 0$, $v_x = 73.14$ m/s, $t = 3.00\times10^{-2}$ s. $v_x = v_{0x} + a_x t$ gives

$a_x = \dfrac{v_x - v_{0x}}{t} = \dfrac{73.14\text{ m/s} - 0}{3.00\times10^{-2}\text{ s}} = 2.44\times10^3\text{ m/s}^2$. $\sum F_x = ma_x$ gives

$F = ma_x = (57\times10^{-3}\text{ kg})(2.44\times10^3\text{ m/s}^2) = 140$ N.

**(b)** The free-body diagram while the ball is in contact with the racket is given in Figure 4.38a. $\vec{F}$ is the force exerted on the ball by the racket. After the ball leaves the racket, $\vec{F}$ ceases to act, as shown in Figure 4.38b.

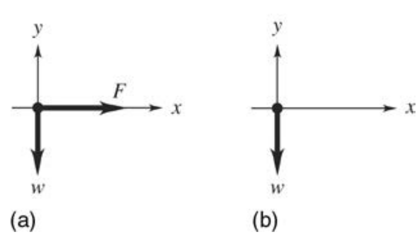


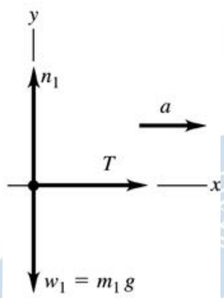

**Figure 4.38**

**EVALUATE:** The force is around 30 lb, which is quite large for a light-weight object like a tennis ball, but is reasonable because it acts for only 30 ms yet during that time gives the ball an acceleration of about 250*g*.

**4.39.** **IDENTIFY:** Use Newton's second law to relate the acceleration and forces for each crate.

**(a) SET UP:** Since the crates are connected by a rope, they both have the same acceleration, $2.50\ \text{m/s}^2$.

**(b)** The forces on the 4.00 kg crate are shown in Figure 4.39a.



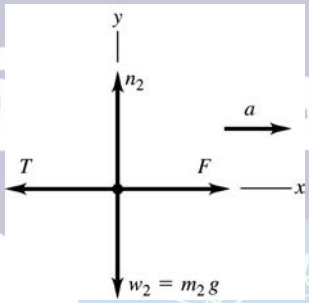

**EXECUTE:**

$\sum F_x = ma_x$

$T = m_1 a = (4.00\ \text{kg})(2.50\ \text{m/s}^2) = 10.0\ \text{N}.$

**Figure 4.39a**

**(c) SET UP:** Forces on the 6.00 kg crate are shown in Figure 4.39b.



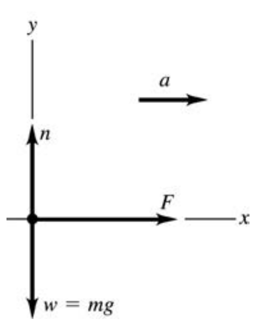

The crate accelerates to the right, so the net force is to the right. $F$ must be larger than $T$.

**Figure 4.39b**

**(d) EXECUTE:** $\sum F_x = ma_x$ gives $F - T = m_2 a$

$$F = T + m_2 a = 10.0\ \text{N} + (6.00\ \text{kg})(2.50\ \text{m/s}^2) = 10.0\ \text{N} + 15.0\ \text{N} = 25.0\ \text{N}$$

**EVALUATE:** We can also consider the two crates and the rope connecting them as a single object of mass $m = m_1 + m_2 = 10.0\ \text{kg}$. The free-body diagram is sketched in Figure 4.39c.




$\sum F_x = ma_x$

$F = ma = (10.0\ \text{kg})(2.50\ \text{m/s}^2) = 25.0\ \text{N}$

This agrees with our answer in part (d).

**Figure 4.39c**

**4.40. IDENTIFY:** Use kinematics to find the acceleration and then apply Newton's second law.
**SET UP:** The 60.0-N force accelerates both blocks, but only the tension in the rope accelerates block $B$. The force $F$ is constant, so the acceleration is constant, which means that the standard kinematics formulas apply. There is no friction.
**EXECUTE:** **(a)** First use kinematics to find the acceleration of the system. Using $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ with $x - x_0 = 18.0$ m, $v_{0x} = 0$, and $t = 5.00$ s, we get $a_x = 1.44\ \text{m/s}^2$. Now apply Newton's second law to the horizontal motion of block $A$, which gives $F - T = m_A a$. $T = 60.0\ \text{N} - (15.0\ \text{kg})(1.44\ \text{m/s}^2) = 38.4$ N.
**(b)** Apply Newton's second law to block $B$, giving $T = m_B a$. $m_B = T/a = (38.4\ \text{N})/(1.44\ \text{m/s}^2) = 26.7$ kg.
**EVALUATE:** As an alternative approach, consider the two blocks as a single system, which makes the tension an internal force. Newton's second law gives $F = (m_A + m_B)a$. Putting in numbers gives 60.0 N = $(15.0\ \text{kg} + m_B)(1.44\ \text{m/s}^2)$, and solving for $m_B$ gives 26.7 kg. Now apply Newton's second law to either block $A$ or block $B$ and find the tension.

**4.41. IDENTIFY and SET UP:** Take derivatives of $x(t)$ to find $v_x$ and $a_x$. Use Newton's second law to relate the acceleration to the net force on the object.
**EXECUTE:**
**(a)** $x = (9.0\times10^3\ \text{m/s}^2)t^2 - (8.0\times10^4\ \text{m/s}^3)t^3$
$x = 0$ at $t = 0$
When $t = 0.025$ s, $x = (9.0\times10^3\ \text{m/s}^2)(0.025\ \text{s})^2 - (8.0\times10^4\ \text{m/s}^3)(0.025\ \text{s})^3 = 4.4$ m.
The length of the barrel must be 4.4 m.
**(b)** $v_x = \dfrac{dx}{dt} = (18.0\times10^3\ \text{m/s}^2)t - (24.0\times10^4\ \text{m/s}^3)t^2$
At $t = 0$, $v_x = 0$ (object starts from rest).
At $t = 0.025$ s, when the object reaches the end of the barrel,
$v_x = (18.0\times10^3\ \text{m/s}^2)(0.025\ \text{s}) - (24.0\times10^4\ \text{m/s}^3)(0.025\ \text{s})^2 = 300$ m/s
**(c)** $\sum F_x = ma_x$, so must find $a_x$.
$a_x = \dfrac{dv_x}{dt} = 18.0\times10^3\ \text{m/s}^2 - (48.0\times10^4\ \text{m/s}^3)t$
(i) At $t = 0$, $a_x = 18.0\times10^3\ \text{m/s}^2$ and $\sum F_x = (1.50\ \text{kg})(18.0\times10^3\ \text{m/s}^2) = 2.7\times10^4$ N.
(ii) At $t = 0.025$ s, $a_x = 18\times10^3\ \text{m/s}^2 - (48.0\times10^4\ \text{m/s}^3)(0.025\ \text{s}) = 6.0\times10^3\ \text{m/s}^2$ and $\sum F_x = (1.50\ \text{kg})(6.0\times10^3\ \text{m/s}^2) = 9.0\times10^3$ N.
**EVALUATE:** The acceleration and net force decrease as the object moves along the barrel.

**4.42. IDENTIFY:** The ship and instrument have the same acceleration. The forces and acceleration are related by Newton's second law. We can use a constant acceleration equation to calculate the acceleration from the information given about the motion.
**SET UP:** Let $+y$ be upward. The forces on the instrument are the upward tension $\vec{T}$ exerted by the wire and the downward force $\vec{w}$ of gravity. $w = mg = (6.50\ \text{kg})(9.80\ \text{m/s}^2) = 63.7$ N
**EXECUTE:** **(a)** The free-body diagram is sketched in Figure 4.42. The acceleration is upward, so $T > w$.
**(b)** $y - y_0 = 276$ m, $t = 15.0$ s, $v_{0y} = 0$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $a_y = \dfrac{2(y - y_0)}{t^2} = \dfrac{2(276\ \text{m})}{(15.0\ \text{s})^2} =$ $2.45\ \text{m/s}^2$. $\sum F_y = ma_y$ gives $T - w = ma$ and $T = w + ma = 63.7\ \text{N} + (6.50\ \text{kg})(2.45\ \text{m/s}^2) = 79.6$ N.
**EVALUATE:** There must be a net force in the direction of the acceleration.

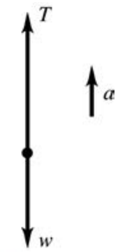



**Figure 4.42**

**4.43.** **IDENTIFY:** Using kinematics we can find the acceleration of the froghopper and then apply Newton's second law to find the force on it from the ground.

**SET UP:** Take $+y$ to be upward. $\sum F_y = ma_y$ and for constant acceleration, $v_y = v_{0y} + a_y t$.

**EXECUTE:** **(a)** The free-body diagram for the froghopper while it is still pushing against the ground is given in Figure 4.43.

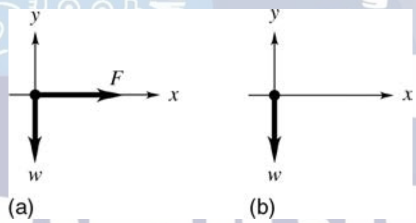



**Figure 4.43**

**(b)** $v_{0y} = 0$, $v_y = 4.0\text{ m/s}$, $t = 1.0\times 10^{-3}\text{ s}$. $v_y = v_{0y} + a_y t$ gives

$$a_y = \frac{v_y - v_{0y}}{t} = \frac{4.0\text{ m/s} - 0}{1.0\times 10^{-3}\text{ s}} = 4.0\times 10^3\text{ m/s}^2.\ \ \sum F_y = ma_y \text{ gives } n - w = ma, \text{ so}$$

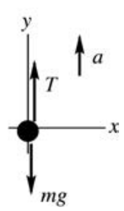
$$n = w + ma = m(g + a) = (12.3\times 10^{-6}\text{ kg})(9.8\text{ m/s}^2 + 4.0\times 10^3\text{ m/s}^2) = 0.049\text{ N}.$$

**(c)** $$\frac{F}{w} = \frac{0.049\text{ N}}{(12.3\times 10^{-6}\text{ kg})(9.8\text{ m/s}^2)} = 410;\ \ F = 410w.$$

**EVALUATE:** Because the force from the ground is huge compared to the weight of the froghopper, it produces an acceleration of around 400$g$!

**4.44.** **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the elevator to relate the forces on it to the acceleration.

**(a) SET UP:** The free-body diagram for the elevator is sketched in Figure 4.44.




The net force is $T - mg$ (upward).

**Figure 4.44**

Take the +$y$-direction to be upward since that is the direction of the acceleration. The maximum upward acceleration is obtained from the maximum possible tension in the cables.

**EXECUTE:** $\sum F_y = ma_y$ gives $T - mg = ma$

$$a = \frac{T - mg}{m} = \frac{28{,}000\ \text{N} - (2200\ \text{kg})(9.80\ \text{m/s}^2)}{2200\ \text{kg}} = 2.93\ \text{m/s}^2.$$

**(b)** What changes is the weight *mg* of the elevator.

$$a = \frac{T - mg}{m} = \frac{28{,}000\ \text{N} - (2200\ \text{kg})(1.62\ \text{m/s}^2)}{2200\ \text{kg}} = 11.1\ \text{m/s}^2.$$

**EVALUATE:** The cables can give the elevator a greater acceleration on the moon since the downward force of gravity is less there and the same $T$ then gives a greater net force.

**4.45.** **IDENTIFY:** You observe that your weight is different from your normal in an elevator, so you must have acceleration. Apply $\sum \vec{F} = m\vec{a}$ to your body inside the elevator.

**SET UP:** The quantity $w = 683\ \text{N}$ is the force of gravity exerted on you, independent of your motion. Your mass is $m = w/g = 69.7\ \text{kg}$. Use coordinates with $+y$ upward. Your free-body diagram is shown in Figure 4.45, where $n$ is the scale reading, which is the force the scale exerts on you. You and the elevator have the same acceleration.

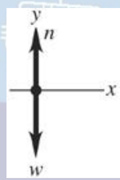



**Figure 4.45**

**EXECUTE:** $\sum F_y = ma_y$ gives $n - w = ma_y$ so $a_y = \dfrac{n - w}{m}$.

**(a)** $n = 725\ \text{N}$, so $a_y = \dfrac{725\ \text{N} - 683\ \text{N}}{69.7\ \text{kg}} = 0.603\ \text{m/s}^2$. $a_y$ is positive so the acceleration is upward.

**(b)** $n = 595\ \text{N}$, so $a_y = \dfrac{595\ \text{N} - 683\ \text{N}}{69.7\ \text{kg}} = -1.26\ \text{m/s}^2$. $a_y$ is negative so the acceleration is downward.

**EVALUATE:** If you appear to weigh less than your normal weight, you must be accelerating downward, but not necessarily *moving* downward. Likewise if you appear to weigh more than your normal weight, you must be acceleration upward, but you could be *moving* downward.

**4.46.** **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the hammer head. Use a constant acceleration equation to relate the motion to the acceleration.

**SET UP:** Let $+y$ be upward.

**EXECUTE:** **(a)** The free-body diagram for the hammer head is sketched in Figure 4.46.

**(b)** The acceleration of the hammer head is given by $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ with $v_y = 0$, $v_{0y} = -3.2\ \text{m/s}$ and $y - y_0 = -0.0045\ \text{m}$. $a_y = v_{0y}^2/2(y - y_0) = (3.2\ \text{m/s})^2/2(0.0045\ \text{m}) = 1.138 \times 10^3\ \text{m/s}^2$. The mass of the hammer head is its weight divided by $g$, $(4.9\ \text{N})/(9.80\ \text{m/s}^2) = 0.50\ \text{kg}$, and so the net force on the hammer head is $(0.50\ \text{kg})(1.138 \times 10^3\ \text{m/s}^2) = 570\ \text{N}$. This is the sum of the forces on the hammer head: the upward force that the nail exerts, the downward weight and the downward 15-N force. The force that the nail exerts is then 590 N, and this must be the magnitude of the force that the hammer head exerts on the nail.

**(c)** The distance the nail moves is 0.12 cm, so the acceleration will be $4267\ \text{m/s}^2$, and the net force on the hammer head will be 2133 N. The magnitude of the force that the nail exerts on the hammer head, and hence the magnitude of the force that the hammer head exerts on the nail, is 2153 N, or about 2200 N.

**EVALUATE:** For the shorter stopping distance the acceleration has a larger magnitude and the force between the nail and hammer head is larger.

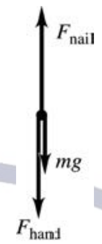



**Figure 4.46**

**4.47.** **IDENTIFY:** He is in free-fall until he contacts the ground. Use the constant acceleration equations and apply $\sum \vec{F} = m\vec{a}$.

**SET UP:** Take $+y$ downward. While he is in the air, before he touches the ground, his acceleration is $a_y = 9.80 \text{ m/s}^2$.

**EXECUTE:** **(a)** $v_{0y} = 0$, $y - y_0 = 3.10$ m, and $a_y = 9.80 \text{ m/s}^2$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(9.80 \text{ m/s}^2)(3.10 \text{ m})} = 7.79 \text{ m/s}$

**(b)** $v_{0y} = 7.79$ m/s, $v_y = 0$, $y - y_0 = 0.60$ m. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$a_y = \dfrac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \dfrac{0 - (7.79 \text{ m/s})^2}{2(0.60 \text{ m})} = -50.6 \text{ m/s}^2$. The acceleration is upward.

**(c)** The free-body diagram is given in Fig. 4.47. $\vec{F}$ is the force the ground exerts on him.

$\sum F_y = ma_y$ gives $mg - F = -ma$. $F = m(g + a) = (75.0 \text{ kg})(9.80 \text{ m/s}^2 + 50.6 \text{ m/s}^2) = 4.53 \times 10^3$ N, upward.

$$\frac{F}{w} = \frac{4.53 \times 10^3 \text{ N}}{(75.0 \text{ kg})(9.80 \text{ m/s}^2)} \text{ so, } F = 6.16\, w = 6.16\, mg.$$

By Newton's third law, the force his feet exert on the ground is $-\vec{F}$.

**EVALUATE:** The force the ground exerts on him is about six times his weight.

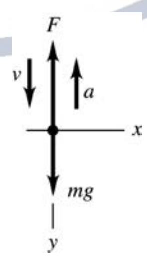



**Figure 4.47**

**4.48.** **IDENTIFY:** Note that in this problem the mass of the rope is given, and that it is not negligible compared to the other masses. Apply $\sum \vec{F} = m\vec{a}$ to each object to relate the forces to the acceleration.

**(a) SET UP:** The free-body diagrams for each block and for the rope are given in Figure 4.48a.

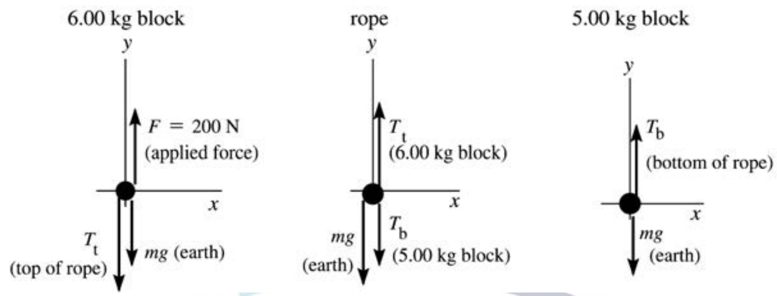
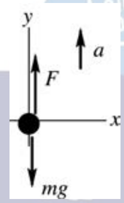



**Figure 4.48a**

$T_\text{t}$ is the tension at the top of the rope and $T_\text{b}$ is the tension at the bottom of the rope.

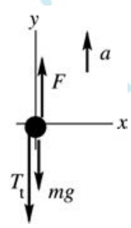
**EXECUTE:** **(b)** Treat the rope and the two blocks together as a single object, with mass $m = 6.00\text{ kg} + 4.00\text{ kg} + 5.00\text{ kg} = 15.0\text{ kg}$. Take $+y$ upward, since the acceleration is upward. The free-body diagram is given in Figure 4.48b.




$$\sum F_y = ma_y$$
$$F - mg = ma$$
$$a = \frac{F - mg}{m}$$
$$a = \frac{200\text{ N} - (15.0\text{ kg})(9.80\text{ m/s}^2)}{15.0\text{ kg}} = 3.53\text{ m/s}^2$$

**Figure 4.48b**

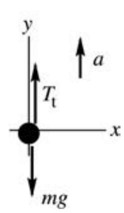
**(c)** Consider the forces on the top block $(m = 6.00\text{ kg})$, since the tension at the top of the rope $(T_\text{t})$ will be one of these forces.




$$\sum F_y = ma_y$$
$$F - mg - T_\text{t} = ma$$
$$T_\text{t} = F - m(g + a)$$
$$T_\text{t} = 200\text{ N} - (6.00\text{ kg})(9.80\text{ m/s}^2 + 3.53\text{ m/s}^2) = 120\text{ N}.$$

**Figure 4.48c**

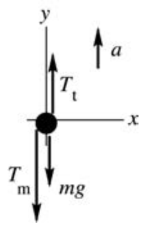
Alternatively, can consider the forces on the combined object rope plus bottom block $(m = 9.00\text{ kg})$:




$$\sum F_y = ma_y$$
$$T_\text{t} - mg = ma$$
$$T_\text{t} = m(g + a) = 9.00\text{ kg}(9.80\text{ m/s}^2 + 3.53\text{ m/s}^2) = 120\text{ N},$$
which checks

**Figure 4.48d**

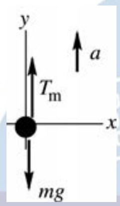
(d) One way to do this is to consider the forces on the top half of the rope $(m = 2.00\text{ kg})$. Let $T_m$ be the tension at the midpoint of the rope.




$$\sum F_y = ma_y$$
$$T_t - T_m - mg = ma$$
$$T_m = T_t - m(g+a) = 120\ \text{N} - 2.00\ \text{kg}(9.80\ \text{m/s}^2 + 3.53\ \text{m/s}^2) = 93.3\ \text{N}$$

**Figure 4.48e**

To check this answer we can alternatively consider the forces on the bottom half of the rope plus the lower block taken together as a combined object $(m = 2.00\ \text{kg} + 5.00\ \text{kg} = 7.00\ \text{kg})$:




$$\sum F_y = ma_y$$
$$T_m - mg = ma$$
$$T_m = m(g+a) = 7.00\ \text{kg}(9.80\ \text{m/s}^2 + 3.53\ \text{m/s}^2) = 93.3\ \text{N},$$
which checks

**Figure 4.48f**

**EVALUATE:** The tension in the rope is not constant but increases from the bottom of the rope to the top. The tension at the top of the rope must accelerate the rope as well the 5.00-kg block. The tension at the top of the rope is less than *F*; there must be a net upward force on the 6.00-kg block.

**4.49.** **IDENTIFY:** The system is accelerating, so we apply Newton's second law to each box and can use the constant acceleration kinematics for formulas to find the acceleration.

**SET UP:** First use the constant acceleration kinematics for formulas to find the acceleration of the system. Then apply $\sum F = ma$ to each box.

**EXECUTE:** **(a)** The kinematics formula $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$a_y = \dfrac{2(y-y_0)}{t^2} = \dfrac{2(12.0\ \text{m})}{(4.0\ \text{s})^2} = 1.5\ \text{m/s}^2$. For box *B*, $mg - T = ma$ and

$m = \dfrac{T}{g-a} = \dfrac{36.0\ \text{N}}{9.8\ \text{m/s}^2 - 1.5\ \text{m/s}^2} = 4.34\ \text{kg}.$

**(b)** For box *A*, $T + mg - F = ma$ and $m = \dfrac{F-T}{g-a} = \dfrac{80.0\ \text{N} - 36.0\ \text{N}}{9.8\ \text{m/s}^2 - 1.5\ \text{m/s}^2} = 5.30\ \text{kg}.$

**EVALUATE:** The boxes have the same acceleration but experience different forces because they have different masses.

**4.50.** **IDENTIFY:** On the planet Newtonia, you make measurements on a tool by pushing on it and by dropping it. You want to use those results to find the weight of the object on that planet and on Earth.

**SET UP:** Using $w = mg$, you could find the weight if you could calculate the mass of the tool and the acceleration due to gravity on Newtonia. Newton's laws of motion are applicable on Newtonia, as is your knowledge of falling objects. Let *m* be the mass of the tool. There is no appreciable friction. Use coordinates where $+x$ is horizontal, in the direction of the 12.0 N force, and let $+y$ be downward.

**EXECUTE:** First find the mass *m*: $x - x_0 = 16.0\ \text{m}$, $t = 2.00\ \text{s}$, $v_{0x} = 0$. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives

$a_x = \dfrac{2(x-x_0)}{t^2} = \dfrac{2(16.0\ \text{m})}{(2.00\ \text{s})^2} = 8.00\ \text{m/s}^2$. Now apply Newton's second law to the tool. $\sum F_x = ma_x$ gives

$F = ma_x$ and $m = \dfrac{F}{a_x} = \dfrac{12.0\ \text{N}}{8.00\ \text{m/s}^2} = 1.50\ \text{kg}$. Find $g_N$, the acceleration due to gravity on Newtonia.

$y - y_0 = 10.0\ \text{m}$, $v_{0y} = 0$, $t = 2.58\ \text{s}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$a_y = \dfrac{2(y-y_0)}{t^2} = \dfrac{2(10.0\text{ m})}{(2.58\text{ s})^2} = 3.00\text{ m/s}^2$; $g_N = 3.00\text{ m/s}^2$. The weight on Newtonia is

$w_N = mg_N = (1.50\text{ kg})(3.00\text{ m/s}^2) = 4.50\text{ N}$. The weight on Earth is

$w_E = mg_E = (1.50\text{ kg})(9.80\text{ m/s}^2) = 14.7\text{ N}$.

**EVALUATE:** The tool weighs about 1/3 on Newtonia of what it weighs on Earth since the acceleration due to gravity on Newtonia is about 1/3 what it is on Earth.

**4.51.** **IDENTIFY:** The rocket accelerates due to a variable force, so we apply Newton's second law. But the acceleration will not be constant because the force is not constant.

**SET UP:** We can use $a_x = F_x/m$ to find the acceleration, but must integrate to find the velocity and then the distance the rocket travels.

**EXECUTE:** Using $a_x = F_x/m$ gives $a_x(t) = \dfrac{(16.8\text{ N/s})t}{45.0\text{ kg}} = (0.3733\text{ m/s}^3)t$. Now integrate the acceleration to get the velocity, and then integrate the velocity to get the distance moved.

$v_x(t) = v_{0x} + \int_0^t a_x(t')dt' = (0.1867\text{ m/s}^3)t^2$ and $x - x_0 = \int_0^t v_x(t')dt' = (0.06222\text{ m/s}^3)t^3$. At $t = 5.00$ s, $x - x_0 = 7.78$ m.

**EVALUATE:** The distance moved during the next 5.0 s would be considerably greater because the acceleration is increasing with time.

**4.52.** **IDENTIFY:** Calculate $\vec{a}$ from $\vec{a} = d^2\vec{r}/dt^2$. Then $\vec{F}_{\text{net}} = m\vec{a}$.

**SET UP:** $w = mg$

**EXECUTE:** Differentiating twice, the acceleration of the helicopter as a function of time is $\vec{a} = (0.120\text{ m/s}^3)t\hat{i} - (0.12\text{ m/s}^2)\hat{k}$ and at $t = 5.0$ s, the acceleration is $\vec{a} = (0.60\text{ m/s}^2)\hat{i} - (0.12\text{ m/s}^2)\hat{k}$. The force is then

$$\vec{F} = m\vec{a} = \frac{w}{g}\vec{a} = \frac{(2.75\times10^5\text{ N})}{(9.80\text{ m/s}^2)}\left[(0.60\text{ m/s}^2)\hat{i} - (0.12\text{ m/s}^2)\hat{k}\right] = (1.7\times10^4\text{ N})\hat{i} - (3.4\times10^3\text{ N})\hat{k}$$

**EVALUATE:** The force and acceleration are in the same direction. They are both time dependent.

**4.53.** **IDENTIFY:** Kinematics will give us the average acceleration of each car, and Newton's second law will give us the average force that is accelerating each car.

**SET UP:** The cars start from rest and all reach a final velocity of 60 mph (26.8 m/s). We first use kinematics to find the average acceleration of each car, and then use Newton's second law to find the average force on each car.

**EXECUTE:** **(a)** We know the initial and final velocities of each car and the time during which this change in velocity occurs. The definition of average acceleration gives $a_{\text{av}} = \dfrac{\Delta v}{\Delta t}$. Then $F = ma$ gives the force on each car. For the Alpha Romeo, the calculations are $a_{\text{av}} = (26.8\text{ m/s})/(4.4\text{ s}) = 6.09\text{ m/s}^2$. The force is $F = ma = (895\text{ kg})(6.09\text{ m/s}^2) = 5.451\times10^3\text{ N} = 5.451$ kN, which we should round to 5.5 kN for 2 significant figures. Repeating this calculation for the other cars and rounding the force to 2 significant figures gives:

Alpha Romeo: $a = 6.09\text{ m/s}^2$, $F = 5.5$ kN
Honda Civic: $a = 4.19\text{ m/s}^2$, $F = 5.5$ kN
Ferrari: $a = 6.88\text{ m/s}^2$, $F = 9.9$ kN
Ford Focus: $a = 4.97\text{ m/s}^2$, $F = 7.3$ kN
Volvo: $a = 3.72\text{ m/s}^2$, $F = 6.1$ kN

The smallest net force is on the Alpha Romeo and Honda Civic, to two-figure accuracy. The largest net force is on the Ferrari.

**(b)** The largest force would occur for the largest acceleration, which would be in the Ferrari. The smallest force would occur for the smallest acceleration, which would be in the Volvo.

**(c)** We use the same approach as in part (a), but now the final velocity is 100 mph (44.7 m/s). $a_{\text{av}} = (44.7\text{ m/s})/(8.6\text{ s}) = 5.20\text{ m/s}^2$, and $F = ma = (1435\text{ kg})(5.20\text{ m/s}^2) = 7.5$ kN. The average force is considerably smaller in this case. This is because air resistance increases with speed.

**(d)** As the speed increases, so does the air resistance. Eventually the air resistance will be equal to the force from the roadway, so the new force will be zero and the acceleration will also be zero, so the speed will remain constant.

**EVALUATE:** The actual forces and accelerations involved with auto dynamics can be quite complicated because the forces (and hence the accelerations) are not constant but depend on the speed of the car.

**4.54.** **IDENTIFY:** The box comes to a stop, so it must have acceleration, so Newton's second law applies. For constant acceleration, the standard kinematics formulas apply.

**SET UP:** For constant acceleration, $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ and $v_x = v_{0x} + a_x t$ apply. For any motion, $\vec{F}_{\text{net}} = m\vec{a}$.

**EXECUTE:** **(a)** If the box comes to rest with constant acceleration, its final velocity is zero so $v_{0x} = -a_x t$. And if during this time it travels a distance $x - x_0 = d$, the distance formula above can be put into the form $d = (-a_x t) + ½\, a_x t^2 = -½\, a_x t^2$. This gives $a_x = -2d/t^2$. For the first push on the box, this gives $a_x = -2(8.22\text{ m})/(2.8\text{ s})^2 = -2.1\text{ m/s}^2$. If the acceleration is constant, the distance the box should travel after the second push is $d = -½\, a_x t^2 = -(½)(-2.1\text{ m/s}^2)(2.0\text{ s})^2 = 4.2\text{ m}$, which is in fact the distance the box did travel. Therefore the acceleration was constant.

**(b)** The total mass $m_T$ of the box is the initial mass (8.00 kg) plus the added mass. Since $v_x = 0$ and $a_x = 2d/t^2$ as shown in part (a), the magnitude of the initial speed $v_{0x}$ is $v_{0x} = a_x t = (2d/t^2)t = 2d/t$. For no added mass, this calculation gives $v_{0x} = 2(8.22\text{ m})/(2.8\text{ s}) = 5.87\text{ m/s}$. Similar calculations with added mass give

$m_T = 8.00\text{ kg}$, $v_{0x} = 5.87\text{ m/s} \approx 5.9\text{ m/s}$
$m_T = 11.00\text{ kg}$, $v_{0x} = 6.72\text{ m/s} \approx 6.7\text{ m/s}$
$m_T = 15.00\text{ kg}$, $v_{0x} = 6.30\text{ m/s} \approx 6.3\text{ m/s}$
$m_T = 20.00\text{ kg}$, $v_{0x} = 5.46\text{ m/s} \approx 5.5\text{ m/s}$

where all answers have been rounded to 2 significant figures. It is obvious that the initial speed was *not* the same in each case. The ratio of maximum speed to minimum speed is
$v_{0,\max}/v_{0,\min} = (6.72\text{ m/s})/(5.46\text{ m/s}) = 1.2$

**(c)** We calculate the magnitude of the force $f$ using $f = ma$, getting $a$ using $a = -2d/t^2$, as we showed in part (a). In each case the acceleration is $2.1\text{ m/s}^2$. So for example, when $m = 11.00$ kg, the force is $f = (11.00\text{ kg})(2.1\text{ m/s}^2) = 23\text{ N}$. Similar calculations produce a set of values for $f$ and $m$. These can be graphed by hand or using graphing software. The resulting graph is shown in Figure 4.54. The slope of this straight-line graph is $2.1\text{ m/s}^2$ and it passes through the origin, so the slope-$y$ intercept equation of the line is $f = (2.1\text{ m/s}^2)m$.

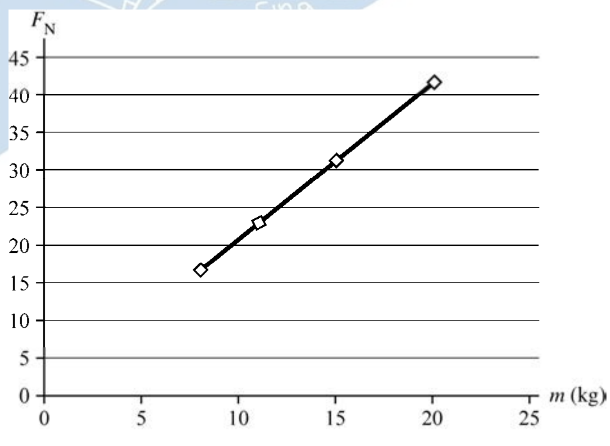



**Figure 4.54**

**EVALUATE:** The results of the graph certainly agree with Newton's second law. A graph of $F$ versus $m$ should have slope equal to the acceleration $a$. This is in fact just what we get, since the acceleration is $2.1\text{ m/s}^2$ which is the same as the slope of the graph.

**4.55.** **IDENTIFY:** A block is accelerated upward by a force of magnitude $F$. For various forces, we know the time for the block to move upward a distance of 8.00 m starting from rest. Since the upward force is constant, so is the acceleration. Newton's second law applies to the accelerating block.

**SET UP:** The acceleration is constant, so $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ applies, and $\sum F_y=ma_y$ also applies to the block.

**EXECUTE:** **(a)** Using the above formula with $v_{0y}=0$ and $y-y_0=8.00$ m, we get $a_y=(16.0\text{ m})/t^2$. We use this formula to calculate the acceleration for each value of the force $F$. For example, when $F$= 250 N, we have $a=(16.0\text{ m})/(3.3\text{ s})^2=1.47\text{ m/s}^2$. We make similar calculations for all six values of $F$ and then graph $F$ versus $a$. We can do this graph by hand or using graphing software. The result is shown in Figure 4.55.

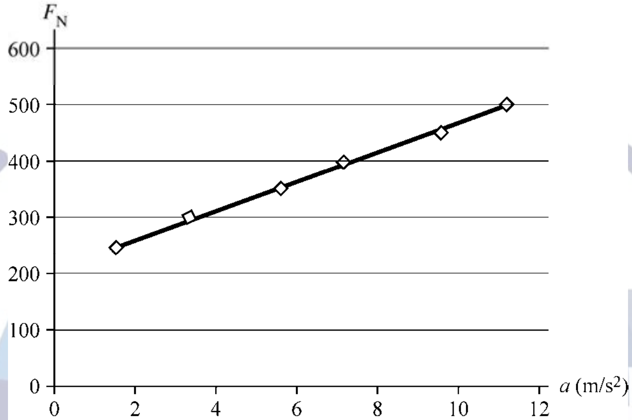



**Figure 4.55**

**(b)** Applying Newton's second law to the block gives $F-mg=ma$, so $F=mg+ma$. The equation of our best-fit graph in part (a) is $F=(25.58\text{ kg})a+213.0$ N. The slope of the graph is the mass $m$, so the mass of the block is $m=26$ kg. The $y$ intercept is $mg$, so $mg=213$ N, which gives $g=(213\text{ N})/(25.58\text{ kg})=8.3\text{ m/s}^2$ on the distant planet.

**EVALUATE:** The acceleration due to gravity on this planet is not too different from what it is on Earth.

**4.56.** **IDENTIFY:** $x=\int_0^t v_x dt$ and $v_x=\int_0^t a_x dt$, and similar equations apply to the $y$-component.

**SET UP:** In this situation, the $x$-component of force depends explicitly on the $y$-component of position. As the $y$-component of force is given as an explicit function of time, $v_y$ and $y$ can be found as functions of time and used in the expression for $a_x(t)$.

**EXECUTE:** $a_y=(k_3/m)t$, so $v_y=(k_3/2m)t^2$ and $y=(k_3/6m)t^3$, where the initial conditions $v_{0y}=0, y_0=0$ have been used. Then, the expressions for $a_x, v_x$, and $x$ are obtained as functions of time: $a_x=\frac{k_1}{m}+\frac{k_2k_3}{6m^2}t^3$, $v_x=\frac{k_1}{m}t+\frac{k_2k_3}{24m^2}t^4$ and $x=\frac{k_1}{2m}t^2+\frac{k_2k_3}{120m^2}t^5$.

In vector form, $\vec{r}=\left(\frac{k_1}{2m}t^2+\frac{k_2k_3}{120m^2}t^5\right)\hat{i}+\left(\frac{k_3}{6m}t^3\right)\hat{j}$ and $\vec{v}=\left(\frac{k_1}{m}t+\frac{k_2k_3}{24m^2}t^4\right)\hat{i}+\left(\frac{k_3}{2m}t^2\right)\hat{j}$.

**EVALUATE:** $a_x$ depends on time because it depends on $y$, and $y$ is a function of time.

**4.57.** **IDENTIFY:** Newton's second law applies to the dancer's head.

**SET UP:** We use $a_{av}=\frac{\Delta v}{\Delta t}$ and $\vec{F}_{net}=m\vec{a}$.

**EXECUTE:** First find the average acceleration: $a_{av}=(4.0\text{ m/s})/(0.20\text{ s})=20\text{ m/s}^2$. Now apply Newton's second law to the dancer's head. Two vertical force act on the head: $F_{neck}-mg=ma$, so $F_{neck}=m(g+a)$, which gives $F_{neck}=(0.094)(65\text{ kg})(9.80\text{ m/s}^2+20\text{ m/s}^2)=180$ N, which is choice (d).

**EVALUATE:** The neck force is not simply $ma$ because the neck must balance her head against gravity, even if the head were not accelerating. That error would lead one to incorrectly select choice (c).

**4.58.** **IDENTIFY:** Newton's third law of motion applies.

**SET UP:** The force the neck exerts on her head is the same as the force the head exerts on the neck.

**EXECUTE:** Choice (a) is correct.

**EVALUATE:** These two forces form an action-reaction pair.

**4.59.** **IDENTIFY:** The dancer is in the air and holding a pose, so she is in free fall.

**SET UP:** The dancer, including all parts of her body, are in free fall, so they all have the same downward acceleration of $9.80 \text{ m/s}^2$.

**EXECUTE:** Since her head and her neck have the same downward acceleration, and that is produced by gravity, her neck does not exert any force on her head, so choice (a) 0 N is correct.

**EVALUATE:** During falling motion such as this, a person (including her head) is often described as being "weightless."

**4.60.** **IDENTIFY:** The graph shows the vertical force that a force plate exerts on her body.

**SET UP** and **EXECUTE:** When the dancer is not moving, the force that the force plate exerts on her will be her weight, which appears to be about 650 N. Between 0.0 s and 0.4 s, the force on her is less than her weight and is decreasing, so she must be accelerating downward. At 0.4 s, the graph reaches a relative minimum of around 300 N and then begins to increase after that. Only choice (a) is consistent with this part of the graph.

**EVALUATE:** At the high points in the graph, the force on her is over twice her weight.

# 5

# APPLYING NEWTON'S LAWS

**5.1.** **IDENTIFY:** $a = 0$ for each object. Apply $\Sigma F_y = ma_y$ to each weight and to the pulley.

**SET UP:** Take $+y$ upward. The pulley has negligible mass. Let $T_r$ be the tension in the rope and let $T_c$ be the tension in the chain.

**EXECUTE:** **(a)** The free-body diagram for each weight is the same and is given in Figure 5.1a. $\Sigma F_y = ma_y$ gives $T_r = w = 25.0$ N.

**(b)** The free-body diagram for the pulley is given in Figure 5.1b. $T_c = 2T_r = 50.0$ N.

**EVALUATE:** The tension is the same at all points along the rope.

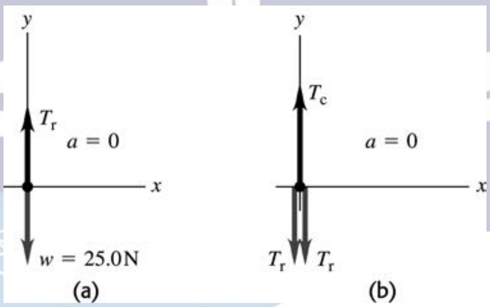



**Figure 5.1**

**5.2.** **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to each weight.

**SET UP:** Two forces act on each mass: $w$ down and $T(= w)$ up.

**EXECUTE:** In all cases, each string is supporting a weight $w$ against gravity, and the tension in each string is $w$.

**EVALUATE:** The tension is the same in all three cases.

**5.3.** **IDENTIFY:** Both objects are at rest and $a = 0$. Apply Newton's first law to the appropriate object. The maximum tension $T_{max}$ is at the top of the chain and the minimum tension is at the bottom of the chain.

**SET UP:** Let $+y$ be upward. For the maximum tension take the object to be the chain plus the ball. For the minimum tension take the object to be the ball. For the tension $T$ three-fourths of the way up from the bottom of the chain, take the chain below this point plus the ball to be the object. The free-body diagrams in each of these three cases are sketched in Figure 5.3. $m_{b+c} = 75.0 \text{ kg} + 26.0 \text{ kg} = 101.0 \text{ kg}$. $m_b = 75.0$ kg. $m$ is the mass of three-fourths of the chain: $m = \frac{3}{4}(26.0 \text{ kg}) = 19.5$ kg.

**EXECUTE:** **(a)** From Figure 5.3a, $\Sigma F_y = 0$ gives $T_{max} - m_{b+c}g = 0$ and $T_{max} = (101.0 \text{ kg})(9.80 \text{ m/s}^2) = 990$ N. From Figure 5.3b, $\Sigma F_y = 0$ gives $T_{min} - m_b g = 0$ and $T_{min} = (75.0 \text{ kg})(9.80 \text{ m/s}^2) = 735$ N.

**(b)** From Figure 5.3c, $\Sigma F_y = 0$ gives $T - (m + m_b)g = 0$ and $T = (19.5 \text{ kg} + 75.0 \text{ kg})(9.80 \text{ m/s}^2) = 926$ N.

**Evaluate:** The tension in the chain increases linearly from the bottom to the top of the chain.

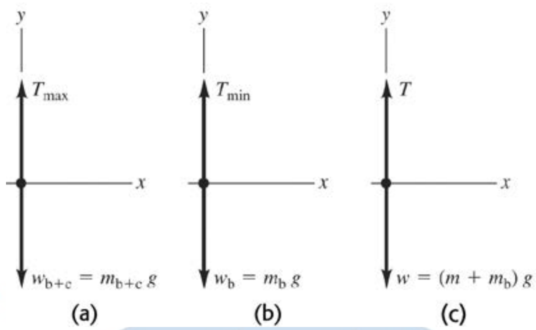



**Figure 5.3**

**5.4.** **Identify:** For the maximum tension, the patient is just ready to slide so static friction is at its maximum and the forces on him add to zero.

**Set Up:** **(a)** The free-body diagram for the person is given in Figure 5.4a. $F$ is magnitude of the traction force along the spinal column and $w = mg$ is the person's weight. At maximum static friction, $f_s = \mu_s n$.

**(b)** The free-body diagram for the collar where the cables are attached is given in Figure 5.4b. The tension in each cable has been resolved into its $x$- and $y$-components.

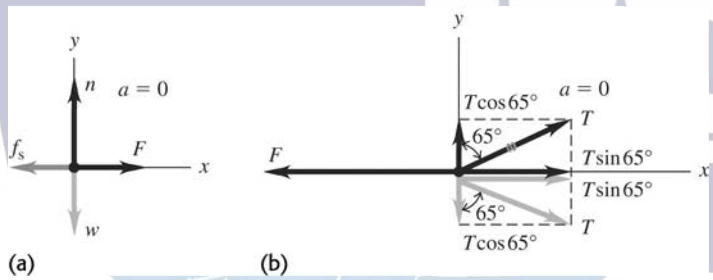



**Figure 5.4**

**Execute:** **(a)** $n = w$ and $F = f_s = \mu_s n = 0.75w = 0.75(9.80\ \text{m/s}^2)(78.5\ \text{kg}) = 577\ \text{N}$.

**(b)** $2T\sin 65° - F = 0$ so $T = \dfrac{F}{2\sin 65°} = \dfrac{0.75w}{2\sin 65°} = 0.41w = (0.41)(9.80\ \text{m/s}^2)(78.5\ \text{kg}) = 315\ \text{N}$.

**Evaluate:** The two tensions add up to 630 N, which is more than the traction force, because the cables do not pull directly along the spinal column.

**5.5.** **Identify:** Apply $\Sigma\vec{F} = m\vec{a}$ to the frame.

**Set Up:** Let $w$ be the weight of the frame. Since the two wires make the same angle with the vertical, the tension is the same in each wire. $T = 0.75w$.

**Execute:** The vertical component of the force due to the tension in each wire must be half of the weight, and this in turn is the tension multiplied by the cosine of the angle each wire makes with the vertical.

$\dfrac{w}{2} = \dfrac{3w}{4}\cos\theta$ and $\theta = \text{arc}\cos\frac{2}{3} = 48°$.

**Evaluate:** If $\theta = 0°$, $T = w/2$ and $T \to \infty$ as $\theta \to 90°$. Therefore, there must be an angle where $T = 3w/4$.

**5.6.** **Identify:** Apply Newton's first law to the wrecking ball. Each cable exerts a force on the ball, directed along the cable.

**Set Up:** The force diagram for the wrecking ball is sketched in Figure 5.6.

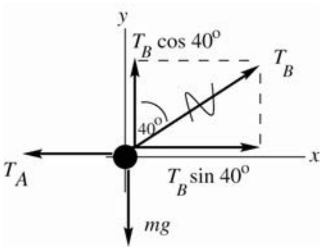



**Figure 5.6**

**EXECUTE:** **(a)** $\Sigma F_y = ma_y$

$T_B \cos 40° - mg = 0$

$$T_B = \frac{mg}{\cos 40°} = \frac{(3620\text{ kg})(9.80\text{ m/s}^2)}{\cos 40°} = 4.63\times 10^4\text{ N} = 46.3\text{ kN}$$

**(b)** $\Sigma F_x = ma_x$

$T_B \sin 40° - T_A = 0$

$T_A = T_B \sin 40° = 2.98\times 10^4\text{ N} = 29.8\text{ kN}$

**EVALUATE:** If the angle 40° is replaced by 0° (cable $B$ is vertical), then $T_B = mg$ and $T_A = 0$.

**5.7.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the object and to the knot where the cords are joined.

**SET UP:** Let $+y$ be upward and $+x$ be to the right.

**EXECUTE:** **(a)** $T_C = w$, $T_A \sin 30° + T_B \sin 45° = T_C = w$, and $T_A \cos 30° - T_B \cos 45° = 0$. Since $\sin 45° = \cos 45°$, adding the last two equations gives $T_A(\cos 30° + \sin 30°) = w$, and so

$$T_A = \frac{w}{1.366} = 0.732w.\ \text{Then, } T_B = T_A \frac{\cos 30°}{\cos 45°} = 0.897w.$$

**(b)** Similar to part (a), $T_C = w$, $-T_A \cos 60° + T_B \sin 45° = w$, and $T_A \sin 60° - T_B \cos 45° = 0$.

Adding these two equations, $T_A = \dfrac{w}{(\sin 60° - \cos 60°)} = 2.73w$, and $T_B = T_A \dfrac{\sin 60°}{\cos 45°} = 3.35w$.

**EVALUATE:** In part (a), $T_A + T_B > w$ since only the vertical components of $T_A$ and $T_B$ hold the object against gravity. In part (b), since $T_A$ has a downward component $T_B$ is greater than $w$.

**5.8.** **IDENTIFY:** Apply Newton's first law to the car.

**SET UP:** Use $x$- and $y$-coordinates that are parallel and perpendicular to the ramp.

**EXECUTE:** **(a)** The free-body diagram for the car is given in Figure 5.8 (next page). The vertical weight $w$ and the tension $T$ in the cable have each been replaced by their $x$- and $y$-components.

**(b)** $\Sigma F_x = 0$ gives $T\cos 31.0° - w\sin 25.0° = 0$ and

$$T = w\frac{\sin 25.0°}{\cos 31.0°} = (1130\text{ kg})(9.80\text{ m/s}^2)\frac{\sin 25.0°}{\cos 31.0°} = 5460\text{ N}.$$

**(c)** $\Sigma F_y = 0$ gives $n + T\sin 31.0° - w\cos 25.0° = 0$ and

$$n = w\cos 25.0° - T\sin 31.0° = (1130\text{ kg})(9.80\text{ m/s}^2)\cos 25.0° - (5460\text{ N})\sin 31.0° = 7220\text{ N}$$

**EVALUATE:** We could also use coordinates that are horizontal and vertical and would obtain the same values of $n$ and $T$.

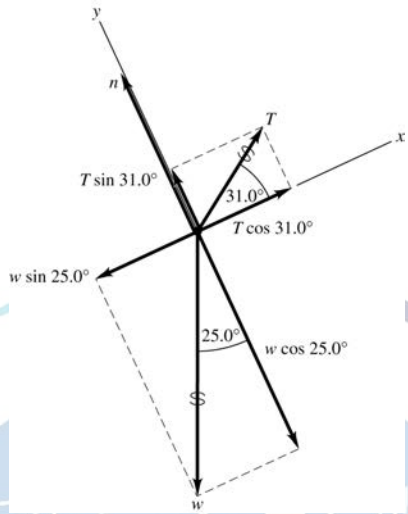



**Figure 5.8**

**5.9.** **IDENTIFY:** Since the velocity is constant, apply Newton's first law to the piano. The push applied by the man must oppose the component of gravity down the incline.

**SET UP:** The free-body diagrams for the two cases are shown in Figure 5.9. $\vec{F}$ is the force applied by the man. Use the coordinates shown in the figure.

**EXECUTE:** **(a)** $\Sigma F_x = 0$ gives $F - w\sin 19.0° = 0$ and $F = (180\text{ kg})(9.80\text{ m/s}^2)\sin 19.0° = 574\text{ N}$.

**(b)** $\Sigma F_y = 0$ gives $n\cos 19.0° - w = 0$ and $n = \dfrac{w}{\cos 19.0°}$. $\Sigma F_x = 0$ gives $F - n\sin 19.0° = 0$ and

$$F = \left(\frac{w}{\cos 19.0°}\right)\sin 19.0° = w\tan 19.0° = 607\text{ N}.$$

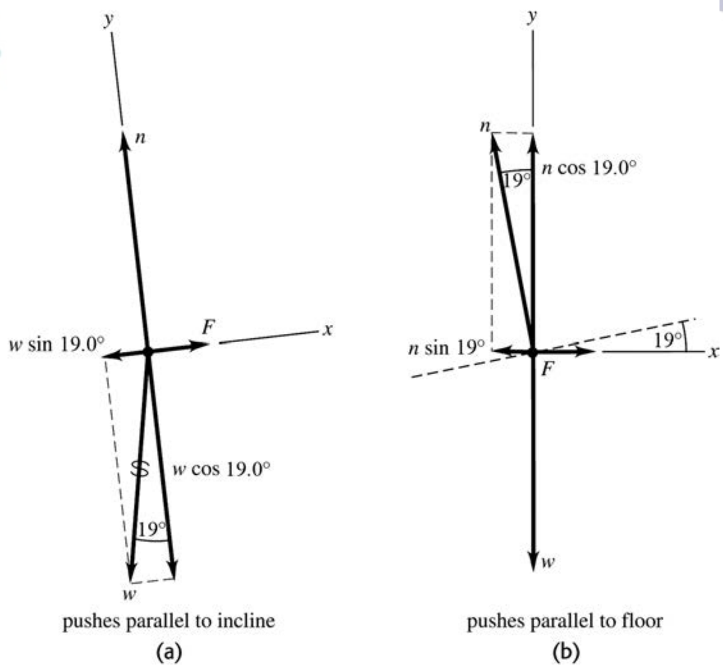



**Figure 5.9**

**EVALUATE:** When pushing parallel to the floor only part of the push is up the ramp to balance the weight of the piano, so you need a larger push in this case than if you push parallel to the ramp.

**5.10.** **IDENTIFY:** Apply Newton's first law to the hanging weight and to each knot. The tension force at each end of a string is the same.

**(a)** Let the tensions in the three strings be $T$, $T'$, and $T''$, as shown in Figure 5.10a.

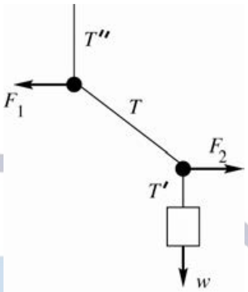
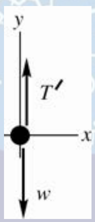



**Figure 5.10a**

**SET UP:** The free-body diagram for the block is given in Figure 5.10b.




**EXECUTE:**

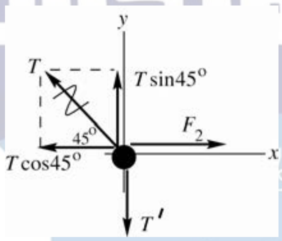
$\Sigma F_y = 0$

$T' - w = 0$

$T' = w = 60.0\text{ N}$

**Figure 5.10b**

**SET UP:** The free-body diagram for the lower knot is given in Figure 5.10c.




**EXECUTE:**

$\Sigma F_y = 0$

$T\sin 45° - T' = 0$

$$T = \frac{T'}{\sin 45°} = \frac{60.0\text{ N}}{\sin 45°} = 84.9\text{ N}$$

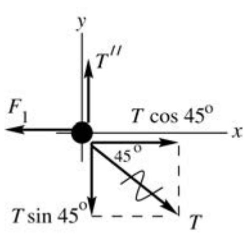
**Figure 5.10c**

**(b)** Apply $\Sigma F_x = 0$ to the force diagram for the lower knot:

$\Sigma F_x = 0$

$F_2 = T\cos 45° = (84.9\text{ N})\cos 45° = 60.0\text{ N}$

**SET UP:** The free-body diagram for the upper knot is given in Figure 5.10d.




**EXECUTE:**

$\Sigma F_x = 0$

$T\cos 45° - F_1 = 0$

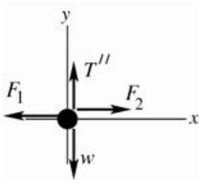
$F_1 = (84.9\text{ N})\cos 45°$

$F_1 = 60.0\text{ N}$

**Figure 5.10d**

Note that $F_1 = F_2$.

**EVALUATE:** Applying $\Sigma F_y = 0$ to the upper knot gives $T'' = T\sin 45° = 60.0\text{ N} = w$. If we treat the whole system as a single object, the force diagram is given in Figure 5.10e (next page).




$\Sigma F_x = 0$ gives $F_2 = F_1$, which checks
$\Sigma F_y = 0$ gives $T'' = w$, which checks

**Figure 5.10e**

**5.11.** **IDENTIFY:** We apply Newton's second law to the rocket and the astronaut in the rocket. A constant force means we have constant acceleration, so we can use the standard kinematics equations.

**SET UP:** The free-body diagrams for the rocket (weight $w_r$) and astronaut (weight $w$) are given in Figure 5.11. $F_T$ is the thrust and $n$ is the normal force the rocket exerts on the astronaut. The speed of sound is 331 m/s. We use $\Sigma F_y = ma_y$ and $v = v_0 + at$.

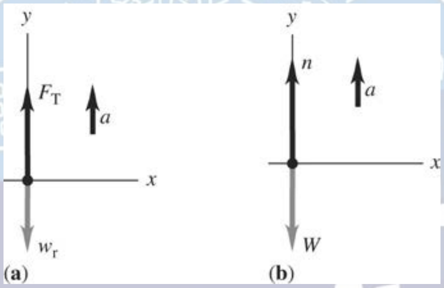



**Figure 5.11**

**EXECUTE:** **(a)** Apply $\Sigma F_y = ma_y$ to the rocket: $F_T - w_r = ma$. $a = 4g$ and $w_r = mg$, so
$F = m\,(5g) = (2.25\times10^6\text{ kg})\,(5)\,(9.80\text{ m/s}^2) = 1.10\times10^8\text{ N}$.

**(b)** Apply $\Sigma F_y = ma_y$ to the astronaut: $n - w = ma$. $a = 4g$ and $m = \dfrac{w}{g}$, so $n = w + \left(\dfrac{w}{g}\right)(4g) = 5w$.

**(c)** $v_0 = 0$, $v = 331$ m/s and $a = 4g = 39.2\text{ m/s}^2$. $v = v_0 + at$ gives $t = \dfrac{v - v_0}{a} = \dfrac{331\text{ m/s}}{39.2\text{ m/s}^2} = 8.4$ s.

**EVALUATE:** The 8.4 s is probably an unrealistically short time to reach the speed of sound because you would not want your astronauts at the brink of blackout during a launch.

**5.12.** **IDENTIFY:** Apply Newton's second law to the rocket plus its contents and to the power supply. Both the rocket and the power supply have the same acceleration.

**SET UP:** The free-body diagrams for the rocket and for the power supply are given in Figure 5.12. Since the highest altitude of the rocket is 120 m, it is near to the surface of the earth and there is a downward gravity force on each object. Let $+y$ be upward, since that is the direction of the acceleration. The power supply has mass $m_{ps} = (15.5\text{ N})/(9.80\text{ m/s}^2) = 1.58$ kg.

**EXECUTE:** **(a)** $\Sigma F_y = ma_y$ applied to the rocket gives $F - m_r g = m_r a$.

$$a = \frac{F - m_r g}{m_r} = \frac{1720\text{ N} - (125\text{ kg})(9.80\text{ m/s}^2)}{125\text{ kg}} = 3.96\text{ m/s}^2.$$

**(b)** $\Sigma F_y = ma_y$ applied to the power supply gives $n - m_{ps} g = m_{ps} a$.

$n = m_{ps}(g + a) = (1.58\text{ kg})(9.80\text{ m/s}^2 + 3.96\text{ m/s}^2) = 21.7$ N.

**EVALUATE:** The acceleration is constant while the thrust is constant, and the normal force is constant while the acceleration is constant. The altitude of 120 m is not used in the calculation.

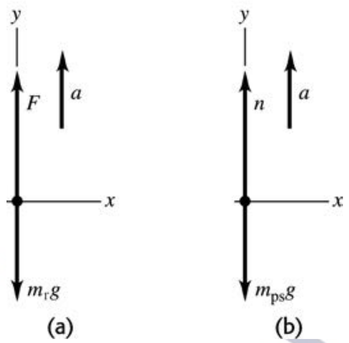



**Figure 5.12**

**5.13.** **IDENTIFY:** Use the kinematic information to find the acceleration of the capsule and the stopping time. Use Newton's second law to find the force $F$ that the ground exerted on the capsule during the crash.

**SET UP:** Let $+y$ be upward. 311 km/h = 86.4 m/s. The free-body diagram for the capsule is given in Figure 5.13.

**EXECUTE:** $y - y_0 = -0.810$ m, $v_{0y} = -86.4$ m/s, $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \frac{0 - (-86.4\text{ m/s})^2}{2(-0.810)\text{ m}} = 4610\text{ m/s}^2 = 470g.$$

**(b)** $\Sigma F_y = ma_y$ applied to the capsule gives $F - mg = ma$ and

$F = m(g + a) = (210\text{ kg})(9.80\text{ m/s}^2 + 4610\text{ m/s}^2) = 9.70\times10^5\text{ N} = 471w.$

**(c)** $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $t = \dfrac{2(y - y_0)}{v_{0y} + v_y} = \dfrac{2(-0.810\text{ m})}{-86.4\text{ m/s} + 0} = 0.0187\text{ s}$

**EVALUATE:** The upward force exerted by the ground is much larger than the weight of the capsule and stops the capsule in a short amount of time. After the capsule has come to rest, the ground still exerts a force $mg$ on the capsule, but the large $9.70\times10^5$ N force is exerted only for 0.0187 s.

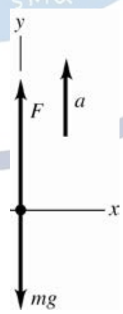



**Figure 5.13**

**5.14.** **IDENTIFY:** Apply Newton's second law to the three sleds taken together as a composite object and to each individual sled. All three sleds have the same horizontal acceleration $a$.

**SET UP:** The free-body diagram for the three sleds taken as a composite object is given in Figure 5.14a and for each individual sled in Figures 5.14b–d. Let $+x$ be to the right, in the direction of the acceleration. $m_{tot} = 60.0$ kg.

**EXECUTE:** **(a)** $\Sigma F_x = ma_x$ for the three sleds as a composite object gives $P = m_{tot}a$ and

$$a = \frac{P}{m_{tot}} = \frac{190\text{ N}}{60.0\text{ kg}} = 3.17\text{ m/s}^2.$$

**(b)** $\Sigma F_x = ma_x$ applied to the 10.0 kg sled gives $P - T_A = m_{10}a$ and

$T_A = P - m_{10}a = 190\text{ N} - (10.0\text{ kg})(3.17\text{ m/s}^2) = 158\text{ N}$. $\Sigma F_x = ma_x$ applied to the 30.0 kg sled gives

$T_B = m_{30}a = (30.0\text{ kg})(3.17\text{ m/s}^2) = 95.1\text{ N}$.

**EVALUATE:** If we apply $\Sigma F_x = ma_x$ to the 20.0 kg sled and calculate $a$ from $T_A$ and $T_B$ found in part (b), we get $T_A - T_B = m_{20}a$. $a = \dfrac{T_A - T_B}{m_{20}} = \dfrac{158\text{ N} - 95.1\text{ N}}{20.0\text{ kg}} = 3.15\text{ m/s}^2$, which agrees closely with the value we calculated in part (a), the difference being due to rounding.

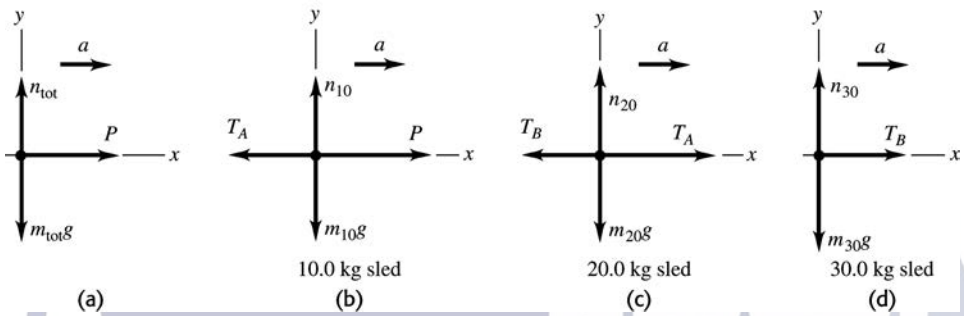



**Figure 5.14**

**5.15. IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the load of bricks and to the counterweight. The tension is the same at each end of the rope. The rope pulls up with the same force $(T)$ on the bricks and on the counterweight. The counterweight accelerates downward and the bricks accelerate upward; these accelerations have the same magnitude.

**(a) SET UP:** The free-body diagrams for the bricks and counterweight are given in Figure 5.15.

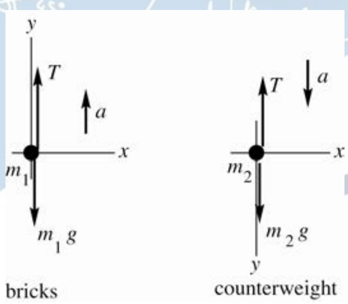



**Figure 5.15**

**(b) EXECUTE:** Apply $\Sigma F_y = ma_y$ to each object. The acceleration magnitude is the same for the two objects. For the bricks take $+y$ to be upward since $\vec{a}$ for the bricks is upward. For the counterweight take $+y$ to be downward since $\vec{a}$ is downward.

bricks: $\Sigma F_y = ma_y$

$T - m_1 g = m_1 a$

counterweight: $\Sigma F_y = ma_y$

$m_2 g - T = m_2 a$

Add these two equations to eliminate $T$:

$$(m_2 - m_1)g = (m_1 + m_2)a$$

$$a=\left(\frac{m_2-m_1}{m_1+m_2}\right)g=\left(\frac{28.0\text{ kg}-15.0\text{ kg}}{15.0\text{ kg}+28.0\text{ kg}}\right)(9.80\text{ m/s}^2)=2.96\text{ m/s}^2$$

**(c)** $T-m_1g=m_1a$ gives $T=m_1(a+g)=(15.0\text{ kg})(2.96\text{ m/s}^2+9.80\text{ m/s}^2)=191\text{ N}$

As a check, calculate $T$ using the other equation.

$m_2g-T=m_2a$ gives $T=m_2(g-a)=28.0\text{ kg}(9.80\text{ m/s}^2-2.96\text{ m/s}^2)=191\text{ N}$, which checks.

**EVALUATE:** The tension is 1.30 times the weight of the bricks; this causes the bricks to accelerate upward. The tension is 0.696 times the weight of the counterweight; this causes the counterweight to accelerate downward. If $m_1=m_2$, $a=0$ and $T=m_1g=m_2g$. In this special case the objects don't move. If $m_1=0$, $a=g$ and $T=0$; in this special case the counterweight is in free fall. Our general result is correct in these two special cases.

**5.16.** **IDENTIFY:** In part (a) use the kinematic information and the constant acceleration equations to calculate the acceleration of the ice. Then apply $\Sigma\vec{F}=m\vec{a}$. In part (b) use $\Sigma\vec{F}=m\vec{a}$ to find the acceleration and use this in the constant acceleration equations to find the final speed.

**SET UP:** Figure 5.16 gives the free-body diagrams for the ice both with and without friction. Let $+x$ be directed down the ramp, so $+y$ is perpendicular to the ramp surface. Let $\phi$ be the angle between the ramp and the horizontal. The gravity force has been replaced by its $x$- and $y$-components.

**EXECUTE:** **(a)** $x-x_0=1.50\text{ m}$, $v_{0x}=0$. $v_x=2.50\text{ m/s}$. $v_x^2=v_{0x}^2+2a_x(x-x_0)$ gives

$$a_x=\frac{v_x^2-v_{0x}^2}{2(x-x_0)}=\frac{(2.50\text{ m/s})^2-0}{2(1.50\text{ m})}=2.08\text{ m/s}^2.$$ $\Sigma F_x=ma_x$ gives $mg\sin\phi=ma$ and $\sin\phi=\frac{a}{g}=\frac{2.08\text{ m/s}^2}{9.80\text{ m/s}^2}$.

$\phi=12.3°$.

**(b)** $\Sigma F_x=ma_x$ gives $mg\sin\phi-f=ma$ and

$$a=\frac{mg\sin\phi-f}{m}=\frac{(8.00\text{ kg})(9.80\text{ m/s}^2)\sin 12.3°-10.0\text{ N}}{8.00\text{ kg}}=0.838\text{ m/s}^2.$$

Then $x-x_0=1.50\text{ m}$, $v_{0x}=0$. $a_x=0.838\text{ m/s}^2$ and $v_x^2=v_{0x}^2+2a_x(x-x_0)$ gives

$$v_x=\sqrt{2a_x(x-x_0)}=\sqrt{2(0.838\text{ m/s}^2)(1.50\text{ m})}=1.59\text{ m/s}$$

**EVALUATE:** With friction present the speed at the bottom of the ramp is less.

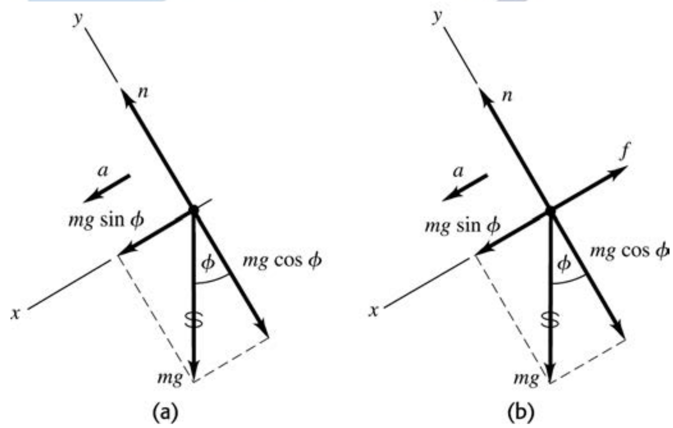



**Figure 5.16**

**5.17.** **IDENTIFY:** Apply $\Sigma\vec{F}=m\vec{a}$ to each block. Each block has the same magnitude of acceleration $a$.

**SET UP:** Assume the pulley is to the right of the 4.00 kg block. There is no friction force on the 4.00 kg block; the only force on it is the tension in the rope. The 4.00 kg block therefore accelerates to the right and the suspended block accelerates downward. Let $+x$ be to the right for the 4.00 kg block, so for it $a_x = a$, and let $+y$ be downward for the suspended block, so for it $a_y = a$.

**EXECUTE:** **(a)** The free-body diagrams for each block are given in Figures 5.17a and b.

**(b)** $\Sigma F_x = ma_x$ applied to the 4.00 kg block gives $T = (4.00\text{ kg})a$ and $a = \dfrac{T}{4.00\text{ kg}} = \dfrac{15.0\text{ N}}{4.00\text{ kg}} = 3.75\text{ m/s}^2$.

**(c)** $\Sigma F_y = ma_y$ applied to the suspended block gives $mg - T = ma$ and

$$m = \frac{T}{g-a} = \frac{15.0\text{ N}}{9.80\text{ m/s}^2 - 3.75\text{ m/s}^2} = 2.48\text{ kg}.$$

**(d)** The weight of the hanging block is $mg = (2.48\text{ kg})(9.80\text{ m/s}^2) = 24.3\text{ N}$. This is greater than the tension in the rope; $T = 0.617mg$.

**EVALUATE:** Since the hanging block accelerates downward, the net force on this block must be downward and the weight of the hanging block must be greater than the tension in the rope. Note that the blocks accelerate no matter how small $m$ is. It is not necessary to have $m > 4.00$ kg, and in fact in this problem $m$ is less than 4.00 kg.

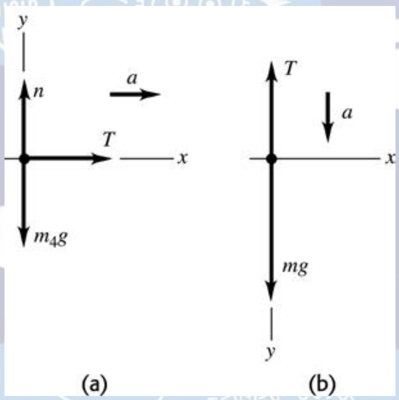



**Figure 5.17**

**5.18.** **IDENTIFY:** **(a)** Consider both gliders together as a single object, apply $\Sigma\vec{F} = m\vec{a}$, and solve for $a$. Use $a$ in a constant acceleration equation to find the required runway length.

**(b)** Apply $\Sigma\vec{F} = m\vec{a}$ to the second glider and solve for the tension $T_g$ in the towrope that connects the two gliders.

**SET UP:** In part (a), set the tension $T_t$ in the towrope between the plane and the first glider equal to its maximum value, $T_t = 12{,}000$ N.

**EXECUTE:** **(a)** The free-body diagram for both gliders as a single object of mass $2m = 1400$ kg is given in Figure 5.18a. $\Sigma F_x = ma_x$ gives $T_t - 2f = (2m)a$ and $a = \dfrac{T_t - 2f}{2m} = \dfrac{12{,}000\text{ N} - 5000\text{ N}}{1400\text{ kg}} = 5.00\text{ m/s}^2$. Then

$a_x = 5.00\text{ m/s}^2$, $v_{0x} = 0$ and $v_x = 40$ m/s in $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $(x - x_0) = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = 160$ m.

**(b)** The free-body diagram for the second glider is given in Figure 5.18b.

$\Sigma F_x = ma_x$ gives $T_g - f = ma$ and $T_g = f + ma = 2500\text{ N} + (700\text{ kg})(5.00\text{ m/s}^2) = 6000$ N.

**EVALUATE:** We can verify that $\Sigma F_x = ma_x$ is also satisfied for the first glider.

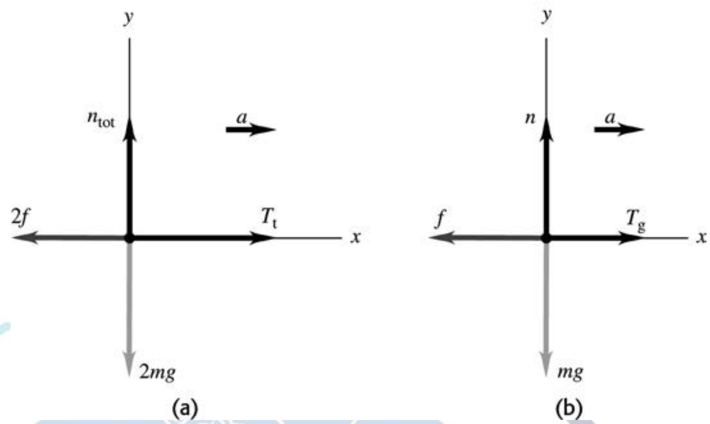



**Figure 5.18**

**5.19.** **IDENTIFY:** The maximum tension in the chain is at the top of the chain. Apply $\Sigma\vec{F} = m\vec{a}$ to the composite object of chain and boulder. Use the constant acceleration kinematic equations to relate the acceleration to the time.

**SET UP:** Let $+y$ be upward. The free-body diagram for the composite object is given in Figure 5.19. $T = 2.50w_{chain}$. $m_{tot} = m_{chain} + m_{boulder} = 1325$ kg.

**EXECUTE:** **(a)** $\Sigma F_y = ma_y$ gives $T - m_{tot}g = m_{tot}a$.

$$a = \frac{T - m_{tot}g}{m_{tot}} = \frac{2.50m_{chain}g - m_{tot}g}{m_{tot}} = \left(\frac{2.50m_{chain}}{m_{tot}} - 1\right)g$$

$$a = \left(\frac{2.50(575\text{ kg})}{1325\text{ kg}} - 1\right)(9.80\text{ m/s}^2) = 0.832\text{ m/s}^2$$

**(b)** Assume the acceleration has its maximum value: $a_y = 0.832\text{ m/s}^2$, $y - y_0 = 125$ m and $v_{0y} = 0$.

$$y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2 \text{ gives } t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(125\text{ m})}{0.832\text{ m/s}^2}} = 17.3\text{ s}$$

**EVALUATE:** The tension in the chain is $T = 1.41 \times 10^4$ N and the total weight is $1.30 \times 10^4$ N. The upward force exceeds the downward force and the acceleration is upward.

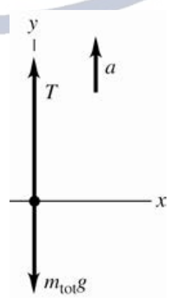



**Figure 5.19**

**5.20.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the composite object of elevator plus student $(m_{tot} = 850\text{ kg})$ and also to the student $(w = 550\text{ N})$. The elevator and the student have the same acceleration.

**SET UP:** Let $+y$ be upward. The free-body diagrams for the composite object and for the student are given in Figure 5.20. $T$ is the tension in the cable and $n$ is the scale reading, the normal force the scale exerts on the student. The mass of the student is $m = w/g = 56.1$ kg.

**EXECUTE:** **(a)** $\Sigma F_y = ma_y$ applied to the student gives $n - mg = ma_y$.

$a_y = \dfrac{n - mg}{m} = \dfrac{450\text{ N} - 550\text{ N}}{56.1\text{ kg}} = -1.78\text{ m/s}^2$. The elevator has a downward acceleration of $1.78\text{ m/s}^2$.

**(b)** $a_y = \dfrac{670\text{ N} - 550\text{ N}}{56.1\text{ kg}} = 2.14\text{ m/s}^2$.

**(c)** $n = 0$ means $a_y = -g$. The student should worry; the elevator is in free fall.

**(d)** $\Sigma F_y = ma_y$ applied to the composite object gives $T - m_{\text{tot}}g = m_{\text{tot}}a_y$. $T = m_{\text{tot}}(a_y + g)$. In part (a), $T = (850\text{ kg})(-1.78\text{ m/s}^2 + 9.80\text{ m/s}^2) = 6820\text{ N}$. In part (c), $a_y = -g$ and $T = 0$.

**EVALUATE:** In part (b), $T = (850\text{ kg})(2.14\text{ m/s}^2 + 9.80\text{ m/s}^2) = 10{,}150\text{ N}$. The weight of the composite object is 8330 N. When the acceleration is upward the tension is greater than the weight and when the acceleration is downward the tension is less than the weight.

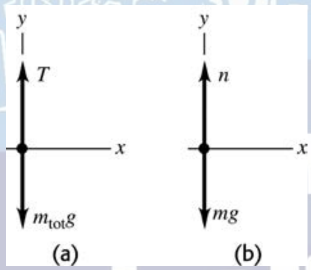



**Figure 5.20**

**5.21.** **IDENTIFY:** While the person is in contact with the ground, he is accelerating upward and experiences two forces: gravity downward and the upward force of the ground. Once he is in the air, only gravity acts on him so he accelerates downward. Newton's second law applies during the jump (and at all other times).

**SET UP:** Take $+y$ to be upward. After he leaves the ground the person travels upward 60 cm and his acceleration is $g = 9.80\text{ m/s}^2$, downward. His weight is $w$ so his mass is $w/g$. $\Sigma F_y = ma_y$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply to the jumper.

**EXECUTE:** **(a)** $v_y = 0$ (at the maximum height), $y - y_0 = 0.60\text{ m}$, $a_y = -9.80\text{ m/s}^2$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80\text{ m/s}^2)(0.60\text{ m})} = 3.4\text{ m/s}$.

**(b)** The free-body diagram for the person while he is pushing up against the ground is given in Figure 5.21 (next page).

**(c)** For the jump, $v_{0y} = 0$, $v_y = 3.4\text{ m/s}$ (from part (a)), and $y - y_0 = 0.50\text{ m}$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $a_y = \dfrac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \dfrac{(3.4\text{ m/s})^2 - 0}{2(0.50\text{ m})} = 11.6\text{ m/s}^2$. $\Sigma F_y = ma_y$ gives $n - w = ma$.

$n = w + ma = w\left(1 + \dfrac{a}{g}\right) = 2.2w$.

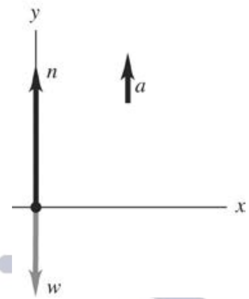



**Figure 5.21**

**EVALUATE:** To accelerate the person upward during the jump, the upward force from the ground must exceed the downward pull of gravity. The ground pushes up on him because he pushes down on the ground.

**5.22.** **IDENTIFY:** Acceleration and velocity are related by $a_y = \frac{dv_y}{dt}$. Apply $\Sigma\vec{F} = m\vec{a}$ to the rocket.

**SET UP:** Let $+y$ be upward. The free-body diagram for the rocket is sketched in Figure 5.22. $\vec{F}$ is the thrust force.

**EXECUTE:** **(a)** $v_y = At + Bt^2$. $a_y = A + 2Bt$. At $t = 0$, $a_y = 1.50\ \text{m/s}^2$ so $A = 1.50\ \text{m/s}^2$. Then $v_y = 2.00\ \text{m/s}$ at $t = 1.00\ \text{s}$ gives $2.00\ \text{m/s} = (1.50\ \text{m/s}^2)(1.00\ \text{s}) + B(1.00\ \text{s})^2$ and $B = 0.50\ \text{m/s}^3$.

**(b)** At $t = 4.00\ \text{s}$, $a_y = 1.50\ \text{m/s}^2 + 2(0.50\ \text{m/s}^3)(4.00\ \text{s}) = 5.50\ \text{m/s}^2$.

**(c)** $\Sigma F_y = ma_y$ applied to the rocket gives $T - mg = ma$ and

$T = m(a + g) = (2540\ \text{kg})(9.80\ \text{m/s}^2 + 5.50\ \text{m/s}^2) = 3.89\times10^4\ \text{N}$. $T = 1.56w$.

**(d)** When $a = 1.50\ \text{m/s}^2$, $T = (2540\ \text{kg})(9.80\ \text{m/s}^2 + 1.50\ \text{m/s}^2) = 2.87\times10^4\ \text{N}$.

**EVALUATE:** During the time interval when $v(t) = At + Bt^2$ applies the magnitude of the acceleration is increasing, and the thrust is increasing.

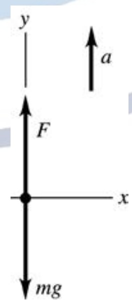



**Figure 5.22**

**5.23.** **IDENTIFY:** We know the external forces on the box and want to find the distance it moves and its speed. The force is not constant, so the acceleration will not be constant, so we cannot use the standard constant-acceleration kinematics formulas. But Newton's second law will apply.

**SET UP:** First use Newton's second law to find the acceleration as a function of time: $a_x(t) = \frac{F_x}{m}$. Then integrate the acceleration to find the velocity as a function of time, and next integrate the velocity to find the position as a function of time.

**EXECUTE:** Let $+x$ be to the right. $a_x(t) = \dfrac{F_x}{m} = \dfrac{(-6.00\ \text{N/s}^2)t^2}{2.00\ \text{kg}} = -(3.00\ \text{m/s}^4)t^2$. Integrate the acceleration to find the velocity as a function of time: $v_x(t) = -(1.00\ \text{m/s}^4)t^3 + 9.00\ \text{m/s}$. Next integrate the velocity to find the position as a function of time: $x(t) = -(0.250\ \text{m/s}^4)t^4 + (9.00\ \text{m/s})t$. Now use the given values of time.

**(a)** $v_x = 0$ when $(1.00\ \text{m/s}^4)t^3 = 9.00\ \text{m/s}$. This gives $t = 2.08$ s. At $t = 2.08$ s,
$x = (9.00\ \text{m/s})(2.08\ \text{s}) - (0.250\ \text{m/s}^4)(2.08\ \text{s})^4 = 18.72\ \text{m} - 4.68\ \text{m} = 14.0\ \text{m}$.

**(b)** At $t = 3.00$ s, $v_x(t) = -(1.00\ \text{m/s}^4)(3.00\ \text{s})^3 + 9.00\ \text{m/s} = -18.0\ \text{m/s}$, so the speed is $18.0\ \text{m/s}$.

**EVALUATE:** The box starts out moving to the right. But because the acceleration is to the left, it reverses direction and $v_x$ is negative in part (b).

**5.24. IDENTIFY:** We know the position of the crate as a function of time, so we can differentiate to find its acceleration. Then we can apply Newton's second law to find the upward force.

**SET UP:** $v_y(t) = dy/dt$, $a_y(t) = dv_y/dt$, and $\Sigma F_y = ma_y$.

**EXECUTE:** Let $+y$ be upward. $dy/dt = v_y(t) = 2.80\ \text{m/s} + (1.83\ \text{m/s}^3)t^2$ and $dv_y/dt = a_y(t) = (3.66\ \text{m/s}^3)t$. At $t = 4.00$ s, $a_y = 14.64\ \text{m/s}^2$. Newton's second law in the $y$ direction gives $F - mg = ma$. Solving for $F$ gives $F = 49\ \text{N} + (5.00\ \text{kg})(14.64\ \text{m/s}^2) = 122\ \text{N}$.

**EVALUATE:** The force is greater than the weight since it is accelerating the crate upwards.

**5.25. IDENTIFY:** At the maximum tilt angle, the patient is just ready to slide down, so static friction is at its maximum and the forces on the patient balance.

**SET UP:** Take $+x$ to be down the incline. At the maximum angle $f_s = \mu_s n$ and $\Sigma F_x = ma_x = 0$.

**EXECUTE:** The free-body diagram for the patient is given in Figure 5.25. $\Sigma F_y = ma_y$ gives $n = mg\cos\theta$. $\Sigma F_x = 0$ gives $mg\sin\theta - \mu_s n = 0$. $mg\sin\theta - \mu_s mg\cos\theta = 0$. $\tan\theta = \mu_s$ so $\theta = 50°$.

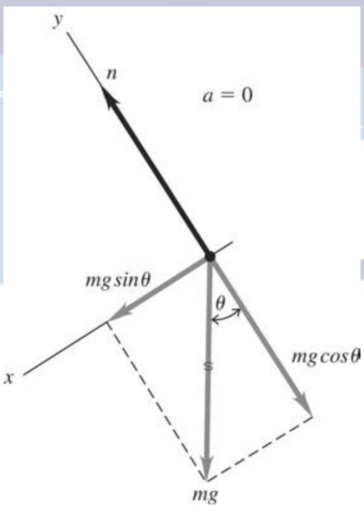



**Figure 5.25**

**EVALUATE:** A larger angle of tilt would cause more blood to flow to the brain, but it would also cause the patient to slide down the bed.

**5.26. IDENTIFY:** $f_s \le \mu_s n$ and $f_k = \mu_k n$. The normal force $n$ is determined by applying $\Sigma\vec{F} = m\vec{a}$ to the block. Normally, $\mu_k \le \mu_s$. $f_s$ is only as large as it needs to be to prevent relative motion between the two surfaces.

**SET UP:** Since the table is horizontal, with only the block present $n = 135$ N. With the brick on the block, $n = 270$ N.

**EXECUTE:** **(a)** The friction is static for $P = 0$ to $P = 75.0$ N. The friction is kinetic for $P > 75.0$ N.

**(b)** The maximum value of $f_s$ is $\mu_s n$. From the graph the maximum $f_s$ is $f_s = 75.0\text{ N}$, so $\mu_s = \dfrac{\max f_s}{n} = \dfrac{75.0\text{ N}}{135\text{ N}} = 0.556$. $f_k = \mu_k n$. From the graph, $f_k = 50.0\text{ N}$ and $\mu_k = \dfrac{f_k}{n} = \dfrac{50.0\text{ N}}{135\text{ N}} = 0.370$.

**(c)** When the block is moving the friction is kinetic and has the constant value $f_k = \mu_k n$, independent of $P$. This is why the graph is horizontal for $P > 75.0\text{ N}$. When the block is at rest, $f_s = P$ since this prevents relative motion. This is why the graph for $P < 75.0\text{ N}$ has slope $+1$.

**(d)** max $f_s$ and $f_k$ would double. The values of *f* on the vertical axis would double but the shape of the graph would be unchanged.

**EVALUATE:** The coefficients of friction are independent of the normal force.

**5.27.** **(a) IDENTIFY:** Constant speed implies $a = 0$. Apply Newton's first law to the box. The friction force is directed opposite to the motion of the box.

**SET UP:** Consider the free-body diagram for the box, given in Figure 5.27a. Let $\vec{F}$ be the horizontal force applied by the worker. The friction is kinetic friction since the box is sliding along the surface.

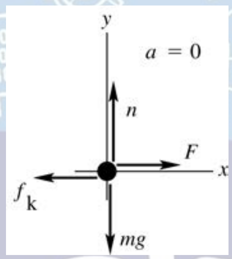



Figure 5.27a

**EXECUTE:**

$$\Sigma F_y = ma_y$$
$$n - mg = 0$$
$$n = mg$$
so $f_k = \mu_k n = \mu_k mg$

$$\Sigma F_x = ma_x$$
$$F - f_k = 0$$
$$F = f_k = \mu_k mg = (0.20)(16.8\text{ kg})(9.80\text{ m/s}^2) = 33\text{ N}$$

**(b) IDENTIFY:** Now the only horizontal force on the box is the kinetic friction force. Apply Newton's second law to the box to calculate its acceleration. Once we have the acceleration, we can find the distance using a constant acceleration equation. The friction force is $f_k = \mu_k mg$, just as in part (a).

**SET UP:** The free-body diagram is sketched in Figure 5.27b.

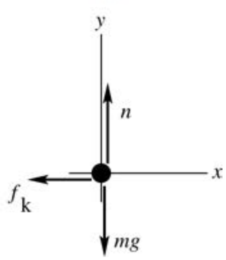



Figure 5.27b

**EXECUTE:**

$$\Sigma F_x = ma_x$$
$$-f_k = ma_x$$
$$-\mu_k mg = ma_x$$
$$a_x = -\mu_k g = -(0.20)(9.80\text{ m/s}^2) = -1.96\text{ m/s}^2$$

Use the constant acceleration equations to find the distance the box travels:

$$v_x = 0,\ \ v_{0x} = 3.50\text{ m/s},\ \ a_x = -1.96\text{ m/s}^2,\ \ x - x_0 = ?$$
$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$
$$x - x_0 = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0 - (3.50\text{ m/s})^2}{2(-1.96\text{ m/s}^2)} = 3.1\text{ m}$$

**Evaluate:** The normal force is the component of force exerted by a surface perpendicular to the surface. Its magnitude is determined by $\Sigma\vec{F} = m\vec{a}$. In this case $n$ and $mg$ are the only vertical forces and $a_y = 0$, so $n = mg$. Also note that $f_k$ and $n$ are proportional in magnitude but perpendicular in direction.

**5.28. Identify:** Apply $\Sigma\vec{F} = m\vec{a}$ to the box.

**Set Up:** Since the only vertical forces are $n$ and $w$, the normal force on the box equals its weight. Static friction is as large as it needs to be to prevent relative motion between the box and the surface, up to its maximum possible value of $f_s^{max} = \mu_s n$. If the box is sliding then the friction force is $f_k = \mu_k n$.

**Execute:** **(a)** If there is no applied force, no friction force is needed to keep the box at rest.

**(b)** $f_s^{max} = \mu_s n = (0.40)(40.0\ \text{N}) = 16.0\ \text{N}$. If a horizontal force of 6.0 N is applied to the box, then $f_s = 6.0\ \text{N}$ in the opposite direction.

**(c)** The monkey must apply a force equal to $f_s^{max}$, 16.0 N.

**(d)** Once the box has started moving, a force equal to $f_k = \mu_k n = 8.0\ \text{N}$ is required to keep it moving at constant velocity.

**(e)** $f_k = 8.0\ \text{N}$. $a = (18.0\ \text{N} - 8.0\ \text{N})/(40.0\ \text{N}/9.80\ \text{m/s}^2) = 2.45\ \text{m/s}^2$

**Evaluate:** $\mu_k < \mu_s$ and less force must be applied to the box to maintain its motion than to start it moving.

**5.29. Identify:** Apply $\Sigma\vec{F} = m\vec{a}$ to the crate. $f_s \le \mu_s n$ and $f_k = \mu_k n$.

**Set Up:** Let $+y$ be upward and let $+x$ be in the direction of the push. Since the floor is horizontal and the push is horizontal, the normal force equals the weight of the crate: $n = mg = 441\ \text{N}$. The force it takes to start the crate moving equals max $f_s$ and the force required to keep it moving equals $f_k$.

**Execute:** **(a)** max $f_s = 313\ \text{N}$, so $\mu_s = \dfrac{313\ \text{N}}{441\ \text{N}} = 0.710$. $f_k = 208\ \text{N}$, so $\mu_k = \dfrac{208\ \text{N}}{441\ \text{N}} = 0.472$.

**(b)** The friction is kinetic. $\Sigma F_x = ma_x$ gives $F - f_k = ma$ and

$F = f_k + ma = 208\ \text{N} + (45.0\ \text{kg})(1.10\ \text{m/s}^2) = 258\ \text{N}$.

**(c)** (i) The normal force now is $mg = 72.9\ \text{N}$. To cause it to move,

$F = \max f_s = \mu_s n = (0.710)(72.9\ \text{N}) = 51.8\ \text{N}$.

(ii) $F = f_k + ma$ and $a = \dfrac{F - f_k}{m} = \dfrac{258\ \text{N} - (0.472)(72.9\ \text{N})}{45.0\ \text{kg}} = 4.97\ \text{m/s}^2$

**Evaluate:** The kinetic friction force is independent of the speed of the object. On the moon, the mass of the crate is the same as on earth, but the weight and normal force are less.

**5.30. Identify:** Newton's second law applies to the rocks on the hill. When they are moving, kinetic friction acts on them, but when they are at rest, static friction acts.

**Set Up:** Use coordinates with axes parallel and perpendicular to the incline, with $+x$ in the direction of the acceleration. $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y = 0$.

**Execute:** With the rock sliding up the hill, the friction force is down the hill. The free-body diagram is given in Figure 5.30a.

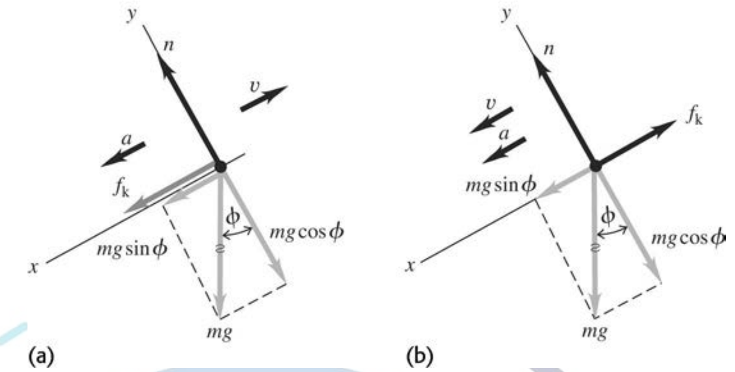



**Figure 5.30**

$\Sigma F_y = ma_y = 0$ gives $n = mg\cos\phi$ and $f_k = \mu_k n = \mu_k mg\cos\phi$. $\Sigma F_x = ma_x$ gives $mg\sin\phi + \mu_k mg\cos\phi = ma$.

$a = g(\sin\phi + \mu_k\cos\phi) = (9.80\text{ m/s}^2)[\sin 36° + (0.45)\cos 36°]$. $a = 9.33\text{ m/s}^2$, down the incline.

**(b)** The component of gravity down the incline is $mg\sin\phi = 0.588mg$. The maximum possible static friction force is $f_s = \mu_s n = \mu_s mg\cos\phi = 0.526mg$. $f_s$ can't be as large as $mg\sin\phi$ and the rock slides back down. As the rock slides down, $f_k$ is up the incline. The free-body diagram is given in Figure 5.30b. $\Sigma F_y = ma_y = 0$ gives $n = mg\cos\phi$ and $f_k = \mu_k n = \mu_k mg\cos\phi$. $\Sigma F_x = ma_x$ gives $mg\sin\phi - \mu_k mg\cos\phi = ma$, so $a = g(\sin\phi - \mu_k\cos\phi) = 2.19\text{ m/s}^2$, down the incline.

**EVALUATE:** The acceleration down the incline in (a) is greater than that in (b) because in (a) the static friction and gravity are both acting down the incline, whereas in (b) friction is up the incline, opposing gravity which still acts down the incline.

**5.31.** **IDENTIFY:** A 10.0-kg box is pushed on a ramp, causing it to accelerate. Newton's second law applies.

**SET UP:** Choose the $x$-axis along the surface of the ramp and the $y$-axis perpendicular to the surface. The only acceleration of the box is in the $x$-direction, so $\Sigma F_x = ma_x$ and $\Sigma F_y = 0$. The external forces acting on the box are the push $P$ along the surface of the ramp, friction $f_k$, gravity $mg$, and the normal force $n$. The ramp rises at 55.0° above the horizontal, and $f_k = \mu_k n$. The friction force opposes the sliding, so it is directed up the ramp in part (a) and down the ramp in part (b).

**EXECUTE:** **(a)** Applying $\Sigma F_y = 0$ gives $n = mg\cos(55.0°)$, so the force of kinetic friction is $f_k = \mu_k n = (0.300)(10.0\text{ kg})(9.80\text{ m/s}^2)(\cos 55.0°) = 16.86\text{ N}$. Call the $+x$-direction down the ramp since that is the direction of the acceleration of the box. Applying $\Sigma F_x = ma_x$ gives $P + mg\sin(55.0°) - f_k = ma$. Putting in the numbers gives $(10.0\text{ kg})a = 120\text{ N} + (98.0\text{ N})(\sin 55.0°) - 16.86\text{ N}$; $a = 18.3\text{ m/s}^2$.

**(b)** Now $P$ is up the up the ramp and $f_k$ is down the ramp, but the other force components are unchanged, so $f_k = 16.86\text{ N}$ as before. We now choose $+x$ to be up the ramp, so $\Sigma F_x = ma_x$ gives $P - mg\sin(55.0°) - f_k = ma$. Putting in the same numbers as before gives $a = 2.29\text{ m/s}^2$.

**EVALUATE:** Pushing up the ramp produces a much smaller acceleration than pushing down the ramp because gravity helps the downward push but opposes the upward push.

**5.32.** **IDENTIFY:** For the shortest time, the acceleration is a maximum, so the toolbox is just ready to slide relative to the bed of the truck. The box is at rest relative to the truck, but it is accelerating relative to the ground because the truck is accelerating. Therefore Newton's second law will be useful.

**SET UP:** If the truck accelerates to the right the static friction force on the box is to the right, to try to prevent the box from sliding relative to the truck. The free-body diagram for the box is given in

Figure 5.32. The maximum acceleration of the box occurs when $f_s$ has its maximum value, so $f_s = \mu_s n$. If the box doesn't slide, its acceleration equals the acceleration of the truck. The constant-acceleration equation $v_x = v_{0x} + a_x t$ applies.

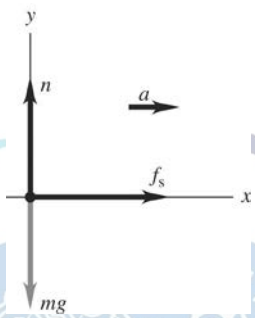



**Figure 5.32**

**EXECUTE:** $n = mg$. $\Sigma F_x = ma_x$ gives $f_s = ma$ so $\mu_s mg = ma$ and $a = \mu_s g = 6.37\ \text{m/s}^2$. $v_{0x} = 0$, $v_x = 30.0$ m/s. $v_x = v_{0x} + a_x t$ gives $t = \dfrac{v_x - v_{0x}}{a_x} = \dfrac{30.0\ \text{m/s} - 0}{6.37\ \text{m/s}^2} = 4.71$ s.

**EVALUATE:** If the truck has a smaller acceleration it is still true that $f_s = ma$, but now $f_s < \mu_s n$.

**5.33.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the composite object consisting of the two boxes and to the top box. The friction the ramp exerts on the lower box is kinetic friction. The upper box doesn't slip relative to the lower box, so the friction between the two boxes is static. Since the speed is constant the acceleration is zero.

**SET UP:** Let $+x$ be up the incline. The free-body diagrams for the composite object and for the upper box are given in Figure 5.33. The slope angle $\phi$ of the ramp is given by $\tan\phi = \dfrac{2.50\ \text{m}}{4.75\ \text{m}}$, so $\phi = 27.76°$. Since the boxes move down the ramp, the kinetic friction force exerted on the lower box by the ramp is directed up the incline. To prevent slipping relative to the lower box the static friction force on the upper box is directed up the incline. $m_{\text{tot}} = 32.0\ \text{kg} + 48.0\ \text{kg} = 80.0\ \text{kg}$.

**EXECUTE:** **(a)** $\Sigma F_y = ma_y$ applied to the composite object gives $n_{\text{tot}} = m_{\text{tot}} g\cos\phi$ and $f_k = \mu_k m_{\text{tot}} g\cos\phi$. $\Sigma F_x = ma_x$ gives $f_k + T - m_{\text{tot}} g\sin\phi = 0$ and

$T = (\sin\phi - \mu_k\cos\phi)m_{\text{tot}} g = (\sin 27.76° - [0.444]\cos 27.76°)(80.0\ \text{kg})(9.80\ \text{m/s}^2) = 57.1\ \text{N}.$

The person must apply a force of 57.1 N, directed up the ramp.

**(b)** $\Sigma F_x = ma_x$ applied to the upper box gives $f_s = mg\sin\phi = (32.0\ \text{kg})(9.80\ \text{m/s}^2)\sin 27.76° = 146$ N, directed up the ramp.

**EVALUATE:** For each object the net force is zero.

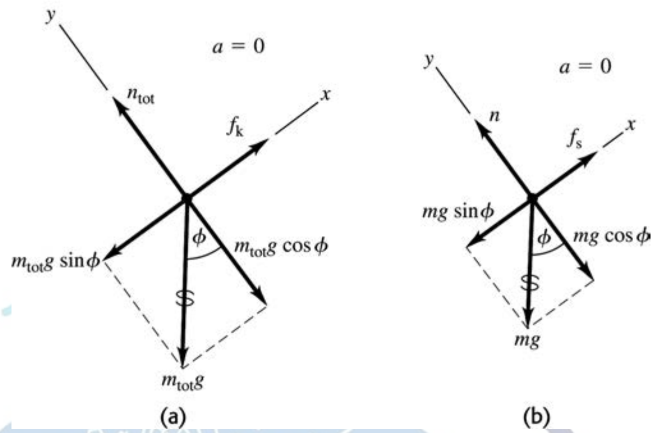



**Figure 5.33**

**5.34.** **IDENTIFY:** Constant speed means zero acceleration for each block. If the block is moving, the friction force the tabletop exerts on it is kinetic friction. Apply $\Sigma\vec{F} = m\vec{a}$ to each block.

**SET UP:** The free-body diagrams and choice of coordinates for each block are given by Figure 5.34. $m_A = 4.59$ kg and $m_B = 2.55$ kg.

**EXECUTE:** **(a)** $\Sigma F_y = ma_y$ with $a_y = 0$ applied to block $B$ gives $m_B g - T = 0$ and $T = 25.0$ N. $\Sigma F_x = ma_x$ with $a_x = 0$ applied to block $A$ gives $T - f_k = 0$ and $f_k = 25.0$ N. $n_A = m_A g = 45.0$ N and $\mu_k = \dfrac{f_k}{n_A} = \dfrac{25.0\text{ N}}{45.0\text{ N}} = 0.556.$

**(b)** Now let $A$ be block $A$ plus the cat, so $m_A = 9.18$ kg. $n_A = 90.0$ N and $f_k = \mu_k n = (0.556)(90.0\text{ N}) = 50.0$ N. $\sum F_x = ma_x$ for $A$ gives $T - f_k = m_A a_x$. $\sum F_y = ma_y$ for block $B$ gives $m_B g - T = m_B a_y$. $a_x$ for $A$ equals $a_y$ for $B$, so adding the two equations gives

$m_B g - f_k = (m_A + m_B)a_y$ and $a_y = \dfrac{m_B g - f_k}{m_A + m_B} = \dfrac{25.0\text{ N} - 50.0\text{ N}}{9.18\text{ kg} + 2.55\text{ kg}} = -2.13\text{ m/s}^2$. The acceleration is upward and block $B$ slows down.

**EVALUATE:** The equation $m_B g - f_k = (m_A + m_B)a_y$ has a simple interpretation. If both blocks are considered together then there are two external forces: $m_B g$ that acts to move the system one way and $f_k$ that acts oppositely. The net force of $m_B g - f_k$ must accelerate a total mass of $m_A + m_B$.

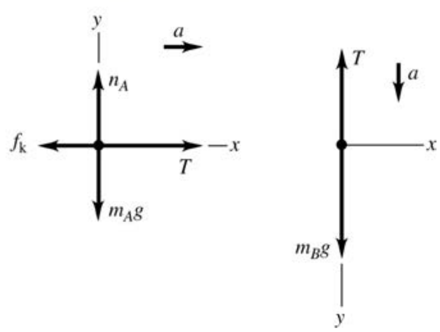



**Figure 5.34**

**5.35.** **IDENTIFY:** Use $\Sigma\vec{F} = m\vec{a}$ to find the acceleration that can be given to the car by the kinetic friction force. Then use a constant acceleration equation.

**SET UP:** Take $+x$ in the direction the car is moving.

**EXECUTE:** **(a)** The free-body diagram for the car is shown in Figure 5.35. $\Sigma F_y = ma_y$ gives $n = mg$. $\Sigma F_x = ma_x$ gives $-\mu_k n = ma_x$. $-\mu_k mg = ma_x$ and $a_x = -\mu_k g$. Then $v_x = 0$ and $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $(x - x_0) = -\dfrac{v_{0x}^2}{2a_x} = +\dfrac{v_{0x}^2}{2\mu_k g} = \dfrac{(28.7\text{ m/s})^2}{2(0.80)(9.80\text{ m/s}^2)} = 52.5\text{ m}.$

**(b)** $v_{0x} = \sqrt{2\mu_k g(x - x_0)} = \sqrt{2(0.25)(9.80\text{ m/s}^2)52.5\text{ m}} = 16.0\text{ m/s}$

**EVALUATE:** For constant stopping distance $\dfrac{v_{0x}^2}{\mu_k}$ is constant and $v_{0x}$ is proportional to $\sqrt{\mu_k}$. The answer to part (b) can be calculated as $(28.7\text{ m/s})\sqrt{0.25/0.80} = 16.0\text{ m/s}$.

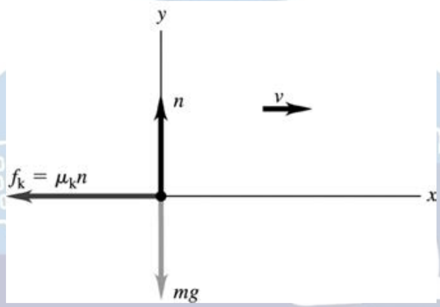



**Figure 5.35**

**5.36.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the box. When the box is ready to slip the static friction force has its maximum possible value, $f_s = \mu_s n$.

**SET UP:** Use coordinates parallel and perpendicular to the ramp.

**EXECUTE:** **(a)** The normal force will be $w\cos\alpha$ and the component of the gravitational force along the ramp is $w\sin\alpha$. The box begins to slip when $w\sin\alpha > \mu_s w\cos\alpha$, or $\tan\alpha > \mu_s = 0.35$, so slipping occurs at $\alpha = \arctan(0.35) = 19.3°$.

**(b)** When moving, the friction force along the ramp is $\mu_k w\cos\alpha$, the component of the gravitational force along the ramp is $w\sin\alpha$, so the acceleration is

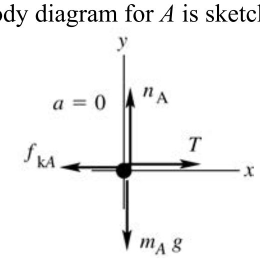
$$(w\sin\alpha - w\mu_k\cos\alpha)/m = g(\sin\alpha - \mu_k\cos\alpha) = 0.92\text{ m/s}^2.$$

**(c)** Since $v_{0x} = 0$, $2ax = v^2$, so $v = (2ax)^{1/2}$, or $v = [(2)(0.92\text{ m/s}^2)(5\text{ m})]^{1/2} = 3\text{ m/s}$.

**EVALUATE:** When the box starts to move, friction changes from static to kinetic and the friction force becomes smaller.

**5.37.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each crate. The rope exerts force $T$ to the right on crate $A$ and force $T$ to the left on crate $B$. The target variables are the forces $T$ and $F$. Constant $v$ implies $a = 0$.

**SET UP:** The free-body diagram for $A$ is sketched in Figure 5.37a.




**EXECUTE:**

$\Sigma F_y = ma_y$

$n_A - m_A g = 0$

$n_A = m_A g$

$f_{kA} = \mu_k n_A = \mu_k m_A g$

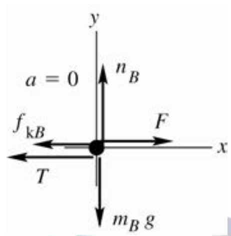
**Figure 5.37a**

$\Sigma F_x = ma_x$

$T - f_{kA} = 0$

$T = \mu_k m_A g$

**SET UP:** The free-body diagram for $B$ is sketched in Figure 5.37b.




**EXECUTE:**

$\Sigma F_y = ma_y$

$n_B - m_B g = 0$

$n_B = m_B g$

$f_{kB} = \mu_k n_B = \mu_k m_B g$

**Figure 5.37b**

$\Sigma F_x = ma_x$

$F - T - f_{kB} = 0$

$F = T + \mu_k m_B g$

Use the first equation to replace $T$ in the second:

$F = \mu_k m_A g + \mu_k m_B g.$

**(a)** $F = \mu_k (m_A + m_B) g$

**(b)** $T = \mu_k m_A g$

**EVALUATE:** We can also consider both crates together as a single object of mass $(m_A + m_B)$. $\Sigma F_x = ma_x$ for this combined object gives $F = f_k = \mu_k (m_A + m_B) g$, in agreement with our answer in part (a).

**5.38.** **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to the box.

**SET UP:** Let $+y$ be upward and $+x$ be horizontal, in the direction of the acceleration. Constant speed means $a = 0$.

**EXECUTE:** **(a)** There is no net force in the vertical direction, so $n + F\sin\theta - w = 0$, or $n = w - F\sin\theta = mg - F\sin\theta$. The friction force is $f_k = \mu_k n = \mu_k (mg - F\sin\theta)$. The net horizontal force is $F\cos\theta - f_k = F\cos\theta - \mu_k (mg - F\sin\theta)$, and so at constant speed,

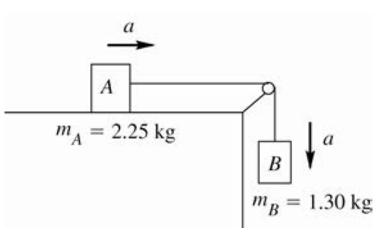
$$F = \frac{\mu_k mg}{\cos\theta + \mu_k \sin\theta}$$

**(b)** Using the given values, $F = \dfrac{(0.35)(90\text{ kg})(9.80\text{ m/s}^2)}{(\cos 25° + (0.35)\sin 25°)} = 290\text{ N}.$

**EVALUATE:** If $\theta = 0°$, $F = \mu_k mg$.

**5.39.** **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to each block. The target variables are the tension $T$ in the cord and the acceleration $a$ of the blocks. Then $a$ can be used in a constant acceleration equation to find the speed of each block. The magnitude of the acceleration is the same for both blocks.

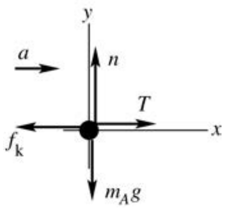
**SET UP:** The system is sketched in Figure 5.39a.




For each block take a positive coordinate direction to be the direction of the block's acceleration.

**Figure 5.39a**

<u>block on the table</u>: The free-body is sketched in Figure 5.39b (next page).




**Execute:**

$\Sigma F_y = ma_y$

$n - m_A g = 0$

$n = m_A g$

$f_k = \mu_k n = \mu_k m_A g$

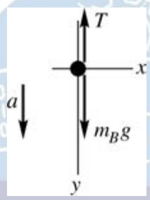
**Figure 5.39b**

$\Sigma F_x = ma_x$

$T - f_k = m_A a$

$T - \mu_k m_A g = m_A a$

**Set Up:** <u>hanging block</u>: The free-body is sketched in Figure 5.39c.




**Execute:**

$\Sigma F_y = ma_y$

$m_B g - T = m_B a$

$T = m_B g - m_B a$

**Figure 5.39c**

**(a)** Use the second equation in the first

$$m_B g - m_B a - \mu_k m_A g = m_A a$$

$$(m_A + m_B)a = (m_B - \mu_k m_A)g$$

$$a = \frac{(m_B - \mu_k m_A)g}{m_A + m_B} = \frac{(1.30\text{ kg} - (0.45)(2.25\text{ kg}))(9.80\text{ m/s}^2)}{2.25\text{ kg} + 1.30\text{ kg}} = 0.7937\text{ m/s}^2$$

**Set Up:** Now use the constant acceleration equations to find the final speed. Note that the blocks have the same speeds. $x - x_0 = 0.0300$ m, $a_x = 0.7937\text{ m/s}^2$, $v_{0x} = 0$, $v_x = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

**Execute:** $v_x = \sqrt{2a_x(x - x_0)} = \sqrt{2(0.7937\text{ m/s}^2)(0.0300\text{ m})} = 0.218\text{ m/s} = 21.8\text{ cm/s}$.

**(b)** $T = m_B g - m_B a = m_B(g - a) = 1.30\text{ kg}(9.80\text{ m/s}^2 - 0.7937\text{ m/s}^2) = 11.7\text{ N}$

Or, to check, $T - \mu_k m_A g = m_A a$.

$T = m_A(a + \mu_k g) = 2.25\text{ kg}(0.7937\text{ m/s}^2 + (0.45)(9.80\text{ m/s}^2)) = 11.7\text{ N}$, which checks.

**Evaluate:** The force $T$ exerted by the cord has the same value for each block. $T < m_B g$ since the hanging block accelerates downward. Also, $f_k = \mu_k m_A g = 9.92$ N. $T > f_k$ and the block on the table accelerates in the direction of $T$.

**5.40.** **Identify:** Apply $\Sigma \vec{F} = m\vec{a}$ to the ball. At the terminal speed, $f = mg$.

**Set Up:** The fluid resistance is directed opposite to the velocity of the object. At half the terminal speed, the magnitude of the frictional force is one-fourth the weight.

**Execute:** **(a)** If the ball is moving up, the frictional force is down, so the magnitude of the net force is $(5/4)w$ and the acceleration is $(5/4)g$, down.

**(b)** While moving down, the frictional force is up, and the magnitude of the net force is $(3/4)w$ and the acceleration is $(3/4)g$, down.

**Evaluate:** The frictional force is less than $mg$ in each case and in each case the net force is downward and the acceleration is downward.

**5.41.** **(a) IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the crate. Constant $v$ implies $a = 0$. Crate moving says that the friction is kinetic friction. The target variable is the magnitude of the force applied by the woman.
**SET UP:** The free-body diagram for the crate is sketched in Figure 5.41.

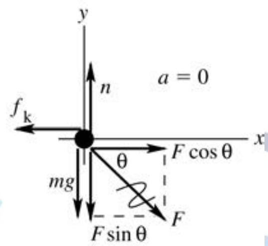



Figure 5.41

**EXECUTE:**

$\Sigma F_y = ma_y$

$n - mg - F\sin\theta = 0$

$n = mg + F\sin\theta$

$f_k = \mu_k n = \mu_k mg + \mu_k F\sin\theta$

$\Sigma F_x = ma_x$

$F\cos\theta - f_k = 0$

$F\cos\theta - \mu_k mg - \mu_k F\sin\theta = 0$

$F(\cos\theta - \mu_k \sin\theta) = \mu_k mg$

$$F = \frac{\mu_k mg}{\cos\theta - \mu_k \sin\theta}$$

**(b) IDENTIFY and SET UP:** "Start the crate moving" means the same force diagram as in part (a), except that $\mu_k$ is replaced by $\mu_s$. Thus $F = \dfrac{\mu_s mg}{\cos\theta - \mu_s \sin\theta}$.

**EXECUTE:** $F \to \infty$ if $\cos\theta - \mu_s \sin\theta = 0$. This gives $\mu_s = \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$.

**EVALUATE:** $\vec{F}$ has a downward component so $n > mg$. If $\theta = 0$ (woman pushes horizontally), $n = mg$ and $F = f_k = \mu_k mg$.

**5.42.** **IDENTIFY and SET UP:** Apply $v_t = \sqrt{\dfrac{mg}{D}}$.

**EXECUTE:** **(a)** Solving for $D$ in terms of $v_t$, $D = \dfrac{mg}{v_t^2} = \dfrac{(80\text{ kg})(9.80\text{ m/s}^2)}{(42\text{ m/s})^2} = 0.44\text{ kg/m}$.

**(b)** $v_t = \sqrt{\dfrac{mg}{D}} = \sqrt{\dfrac{(45\text{ kg})(9.80\text{ m/s}^2)}{(0.25\text{ kg/m})}} = 42\text{ m/s}$.

**EVALUATE:** "Terminal speed depends on the mass of the falling object."

**5.43.** **IDENTIFY:** Since the stone travels in a circular path, its acceleration is $a_{rad} = v^2/R$, directed toward the center of the circle. The only horizontal force on the stone is the tension of the string. Set the tension in the string equal to its maximum value.

**SET UP:** $\sum F_x = ma_x$ gives $T = m\dfrac{v^2}{R}$.

**EXECUTE:** **(a)** The free-body diagram for the stone is given in Figure 5.43 (next page). In the diagram the stone is at a point to the right of the center of the path.

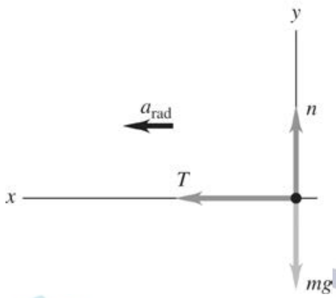



**Figure 5.43**

**(b)** Solving for $v$ gives $v = \sqrt{\frac{TR}{m}} = \sqrt{\frac{(60.0\text{ N})(0.90\text{ m})}{0.80\text{ kg}}} = 8.2\text{ m/s}.$

**EVALUATE:** The tension is directed toward the center of the circular path of the stone. Gravity plays no role in this case because it is a vertical force and the acceleration is horizontal.

**5.44.** **IDENTIFY:** The wrist exerts a force on the hand causing the hand to move in a horizontal circle. Newton's second law applies to the hand.

**SET UP:** Each hand travels in a circle of radius 0.750 m and has mass (0.0125)(52 kg) = 0.65 kg and weight 6.4 N. The period for each hand is $T = (1.0\text{ s})/(2.0) = 0.50\text{ s}.$ Let $+x$ be toward the center of the circular path. The speed of the hand is $v = 2\pi R/T$, the radial acceleration is $a_{\text{rad}} = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}$, and $\sum F_x = ma_x = ma_{\text{rad}}.$

**EXECUTE:** **(a)** The free-body diagram for one hand is given in Figure 5.44. $\vec{F}$ is the force exerted on the hand by the wrist. This force has both horizontal and vertical components.

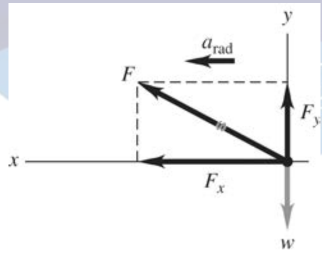



**Figure 5.44**

**(b)** $a_{\text{rad}} = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2(0.750\text{ m})}{(0.50\text{ s})^2} = 118\text{ m/s}^2$, so $F_x = ma_{\text{rad}} = (0.65\text{ kg})(118\text{ m/s}^2) = 77\text{ N}.$

**(c)** $\frac{F}{w} = \frac{77\text{ N}}{6.4\text{ N}} = 12,$ so the horizontal force from the wrist is 12 times the weight of the hand.

**EVALUATE:** The wrist must also exert a vertical force on the hand equal to the weight of the hand.

**5.45.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the car. It has acceleration $\vec{a}_{\text{rad}}$, directed toward the center of the circular path.

**SET UP:** The analysis is the same as in Example 5.23.

**EXECUTE:** **(a)** $F_A = m\left(g + \frac{v^2}{R}\right) = (1.60\text{ kg})\left(9.80\text{ m/s}^2 + \frac{(12.0\text{ m/s})^2}{5.00\text{ m}}\right) = 61.8\text{ N}.$

**(b)** $F_B = m\left(g - \frac{v^2}{R}\right) = (1.60\text{ kg})\left(9.80\text{ m/s}^2 - \frac{(12.0\text{ m/s})^2}{5.00\text{ m}}\right) = -30.4\text{ N},$ where the minus sign indicates that the track pushes down on the car. The magnitude of this force is 30.4 N.

**EVALUATE:** $|F_A| > |F_B|$. $|F_A| - 2mg = |F_B|$.

**5.46.** **IDENTIFY:** The acceleration of the car at the top and bottom is toward the center of the circle, and Newton's second law applies to it.

**SET UP:** Two forces are acting on the car, gravity and the normal force. At point $B$ (the top), both forces are toward the center of the circle, so Newton's second law gives $mg + n_B = ma$. At point $A$ (the bottom), gravity is downward but the normal force is upward, so $n_A - mg = ma$.

**EXECUTE:** Solving the equation at $B$ for the acceleration gives

$a = \frac{mg + n_B}{m} = \frac{(0.800\text{ kg})(9.8\text{ m/s}^2) + 6.00\text{ N}}{0.800\text{ kg}} = 17.3\text{ m/s}^2$. Solving the equation at $A$ for the normal force gives $n_A = m(g + a) = (0.800\text{ kg})(9.8\text{ m/s}^2 + 17.3\text{ m/s}^2) = 21.7\text{ N}$.

**EVALUATE:** The normal force at the bottom is greater than at the top because it must balance the weight in addition to accelerate the car toward the center of its track.

**5.47.** **IDENTIFY:** A model car travels in a circle so it has radial acceleration, and Newton's second law applies to it.

**SET UP:** We use $\Sigma\vec{F} = m\vec{a}$, where the acceleration is $a_{\text{rad}} = \frac{v^2}{R}$ and the time $T$ for one revolution is $T = 2\pi R/v$.

**EXECUTE:** At the bottom of the track, taking $+y$ upward, $\Sigma\vec{F} = m\vec{a}$ gives $n - mg = ma$, where $n$ is the normal force. This gives $2.50mg - mg = ma$, so $a = 1.50g$. The acceleration is $a_{\text{rad}} = \frac{v^2}{R}$, so

$v = \sqrt{aR} = \sqrt{(1.50)(9.80\text{ m/s}^2)(5.00\text{ m})} = 8.573\text{ m/s}$, so $T = 2\pi R/v = 2\pi(5.00\text{ m})/(8.573\text{ m}) = 3.66\text{ s}$.

**EVALUATE:** We never need the mass of the car because we know the acceleration as a fraction of $g$ and the force as a fraction of $mg$.

**5.48.** **IDENTIFY:** Since the car travels in an arc of a circle, it has acceleration $a_{\text{rad}} = v^2/R$, directed toward the center of the arc. The only horizontal force on the car is the static friction force exerted by the roadway. To calculate the minimum coefficient of friction that is required, set the static friction force equal to its maximum value, $f_s = \mu_s n$. Friction is static friction because the car is not sliding in the radial direction.

**SET UP:** The free-body diagram for the car is given in Figure 5.48 (next page). The diagram assumes the center of the curve is to the left of the car.

**EXECUTE:** **(a)** $\Sigma F_y = ma_y$ gives $n = mg$. $\Sigma F_x = ma_x$ gives $\mu_s n = m\frac{v^2}{R}$. $\mu_s mg = m\frac{v^2}{R}$ and

$$\mu_s = \frac{v^2}{gR} = \frac{(25.0\text{ m/s})^2}{(9.80\text{ m/s}^2)(170\text{ m})} = 0.375$$

**(b)** $\frac{v^2}{\mu_s} = Rg = \text{constant}$, so $\frac{v_1^2}{\mu_{s1}} = \frac{v_2^2}{\mu_{s2}}$. $v_2 = v_1\sqrt{\frac{\mu_{s2}}{\mu_{s1}}} = (25.0\text{ m/s})\sqrt{\frac{\mu_{s1}/3}{\mu_{s1}}} = 14.4\text{ m/s}$.

**EVALUATE:** A smaller coefficient of friction means a smaller maximum friction force, a smaller possible acceleration and therefore a smaller speed.

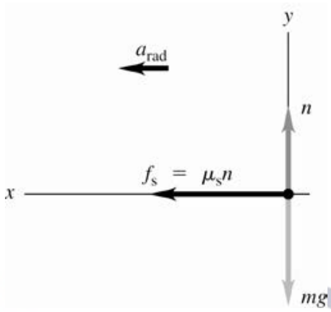



**Figure 5.48**

**5.49.** **IDENTIFY:** Apply Newton's second law to the car in circular motion, assume friction is negligible.

**SET UP:** The acceleration of the car is $a_{rad} = v^2/R$. As shown in the text, the banking angle $\beta$ is given by $\tan\beta = \frac{v^2}{gR}$. Also, $n = mg/\cos\beta$. $65.0\text{ mi/h} = 29.1\text{ m/s}$.

**EXECUTE:** **(a)** $\tan\beta = \frac{(29.1\text{ m/s})^2}{(9.80\text{ m/s}^2)(225\text{ m})}$ and $\beta = 21.0°$. The expression for $\tan\beta$ does not involve the mass of the vehicle, so the truck and car should travel at the same speed.

**(b)** For the car, $n_{car} = \frac{(1125\text{ kg})(9.80\text{ m/s}^2)}{\cos 21.0°} = 1.18\times10^4\text{ N}$ and $n_{truck} = 2n_{car} = 2.36\times10^4\text{ N}$, since $m_{truck} = 2m_{car}$.

**EVALUATE:** The vertical component of the normal force must equal the weight of the vehicle, so the normal force is proportional to $m$.

**5.50.** **IDENTIFY:** The acceleration of the person is $a_{rad} = v^2/R$, directed horizontally to the left in the figure in the problem. The time for one revolution is the period $T = \frac{2\pi R}{v}$. Apply $\Sigma\vec{F} = m\vec{a}$ to the person.

**SET UP:** The person moves in a circle of radius $R = 3.00\text{ m} + (5.00\text{ m})\sin 30.0° = 5.50\text{ m}$. The free-body diagram is given in Figure 5.50. $\vec{F}$ is the force applied to the seat by the rod.

**EXECUTE:** **(a)** $\Sigma F_y = ma_y$ gives $F\cos 30.0° = mg$ and $F = \frac{mg}{\cos 30.0°}$. $\Sigma F_x = ma_x$ gives $F\sin 30.0° = m\frac{v^2}{R}$. Combining these two equations gives

$v = \sqrt{Rg\tan\theta} = \sqrt{(5.50\text{ m})(9.80\text{ m/s}^2)\tan 30.0°} = 5.58\text{ m/s}$. Then the period is

$T = \frac{2\pi R}{v} = \frac{2\pi(5.50\text{ m})}{5.58\text{ m/s}} = 6.19\text{ s}$.

**(b)** The net force is proportional to $m$ so in $\Sigma\vec{F} = m\vec{a}$ the mass divides out and the angle for a given rate of rotation is independent of the mass of the passengers.

**EVALUATE:** The person moves in a horizontal circle so the acceleration is horizontal. The net inward force required for circular motion is produced by a component of the force exerted on the seat by the rod.

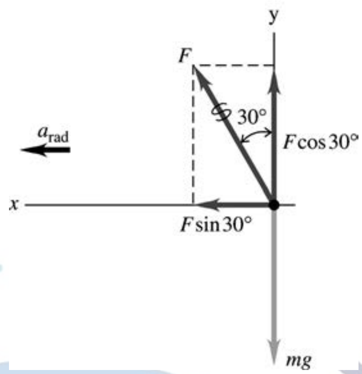



**Figure 5.50**

**5.51.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the composite object of the person plus seat. This object moves in a horizontal circle and has acceleration $a_{\text{rad}}$, directed toward the center of the circle.

**SET UP:** The free-body diagram for the composite object is given in Figure 5.51. Let $+x$ be to the right, in the direction of $\vec{a}_{\text{rad}}$. Let $+y$ be upward. The radius of the circular path is $R = 7.50$ m. The total mass is $(255\text{ N} + 825\text{ N})/(9.80\text{ m/s}^2) = 110.2$ kg. Since the rotation rate is $28.0\text{ rev/min} = 0.4667\text{ rev/s}$, the period $T$ is $\dfrac{1}{0.4667\text{ rev/s}} = 2.143$ s.

**EXECUTE:** $\Sigma F_y = ma_y$ gives $T_A\cos 40.0° - mg = 0$ and $T_A = \dfrac{mg}{\cos 40.0°} = \dfrac{255\text{ N} + 825\text{ N}}{\cos 40.0°} = 1410$ N.

$\Sigma F_x = ma_x$ gives $T_A\sin 40.0° + T_B = ma_{\text{rad}}$ and

$$T_B = m\frac{4\pi^2 R}{T^2} - T_A\sin 40.0° = (110.2\text{ kg})\frac{4\pi^2(7.50\text{ m})}{(2.143\text{ s})^2} - (1410\text{ N})\sin 40.0° = 6200\text{ N}$$

The tension in the horizontal cable is 6200 N and the tension in the other cable is 1410 N.

**EVALUATE:** The weight of the composite object is 1080 N. The tension in cable $A$ is larger than this since its vertical component must equal the weight. The tension in cable $B$ is less than $ma_{\text{rad}}$ because part of the required inward force comes from a component of the tension in cable $A$.

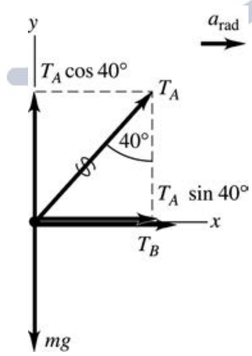



**Figure 5.51**

**5.52.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the button. The button moves in a circle, so it has acceleration $a_{\text{rad}}$.

**SET UP:** We apply Newton's second law to the horizontal and vertical motion. Vertically we get $n = w$, and horizontally we get $\mu_s mg = mv^2/R$. Combining these equations gives $\mu_s = \dfrac{v^2}{Rg}$. Also, $v = 2\pi R/T$.

**Execute:** **(a)** $\mu_s = \dfrac{v^2}{Rg}$. Expressing $v$ in terms of the period $T$, $v = \dfrac{2\pi R}{T}$ so $\mu_s = \dfrac{4\pi^2 R}{T^2 g}$. A platform speed of 40.0 rev/min corresponds to a period of 1.50 s, so $\mu_s = \dfrac{4\pi^2 (0.220\text{ m})}{(1.50\text{ s})^2 (9.80\text{ m/s}^2)} = 0.394.$

**(b)** For the same coefficient of static friction, the maximum radius is proportional to the square of the period (longer periods mean slower speeds, so the button may be moved farther out) and so is inversely proportional to the square of the speed. Thus, at the higher speed, the maximum radius is $(0.220\text{ m})\left(\dfrac{40.0}{60.0}\right)^2 = 0.0978\text{ m}.$

**Evaluate:** $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$. The maximum radial acceleration that friction can give is $\mu_s mg$. At the faster rotation rate $T$ is smaller so $R$ must be smaller to keep $a_{\text{rad}}$ the same.

**5.53.** **Identify:** The acceleration due to circular motion is $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$.

**Set Up:** $R = 400$ m. $1/T$ is the number of revolutions per second.

**Execute:** **(a)** Setting $a_{\text{rad}} = g$ and solving for the period $T$ gives

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{400\text{ m}}{9.80\text{ m/s}^2}} = 40.1\text{ s},$$

so the number of revolutions per minute is $(60\text{ s/min})/(40.1\text{ s}) = 1.5$ rev/min.

**(b)** The lower acceleration corresponds to a longer period, and hence a lower rotation rate, by a factor of the square root of the ratio of the accelerations, $T' = (1.5\text{ rev/min}) \times \sqrt{3.70/9.8} = 0.92$ rev/min.

**Evaluate:** In part (a) the tangential speed of a point at the rim is given by $a_{\text{rad}} = \dfrac{v^2}{R}$, so $v = \sqrt{Ra_{\text{rad}}} = \sqrt{Rg} = 62.6$ m/s; the space station is rotating rapidly.

**5.54.** **Identify:** $T = \dfrac{2\pi R}{v}$. The apparent weight of a person is the normal force exerted on him by the seat he is sitting on. His acceleration is $a_{\text{rad}} = v^2/R$, directed toward the center of the circle.

**Set Up:** The period is $T = 60.0$ s. The passenger has mass $m = w/g = 90.0$ kg.

**Execute:** **(a)** $v = \dfrac{2\pi R}{T} = \dfrac{2\pi(50.0\text{ m})}{60.0\text{ s}} = 5.24\text{ m/s}$. Note that $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(5.24\text{ m/s})^2}{50.0\text{ m}} = 0.549\text{ m/s}^2$.

**(b)** The free-body diagram for the person at the top of his path is given in Figure 5.54a. The acceleration is downward, so take $+y$ downward. $\Sigma F_y = ma_y$ gives $mg - n = ma_{\text{rad}}$.

$n = m(g - a_{\text{rad}}) = (90.0\text{ kg})(9.80\text{ m/s}^2 - 0.549\text{ m/s}^2) = 833\text{ N}.$

The free-body diagram for the person at the bottom of his path is given in Figure 5.54b. The acceleration is upward, so take $+y$ upward. $\Sigma F_y = ma_y$ gives $n - mg = ma_{\text{rad}}$ and $n = m(g + a_{\text{rad}}) = 931\text{ N}$.

**(c)** Apparent weight $= 0$ means $n = 0$ and $mg = ma_{\text{rad}}$. $g = \dfrac{v^2}{R}$ and $v = \sqrt{gR} = 22.1$ m/s. The time for one revolution would be $T = \dfrac{2\pi R}{v} = \dfrac{2\pi(50.0\text{ m})}{22.1\text{ m/s}} = 14.2$ s. Note that $a_{\text{rad}} = g$.

**(d)** $n = m(g + a_{\text{rad}}) = 2mg = 2(882\text{ N}) = 1760\text{ N}$, twice his true weight.

**Evaluate:** At the top of his path his apparent weight is less than his true weight and at the bottom of his path his apparent weight is greater than his true weight.

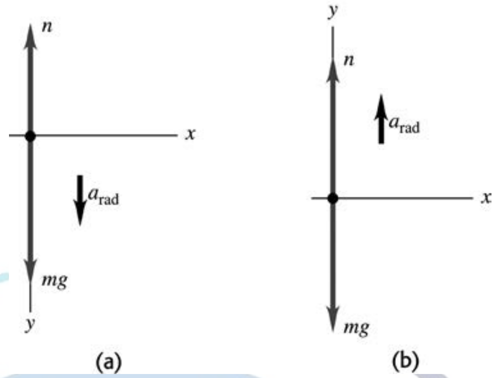
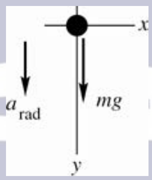



**Figure 5.54**

**5.55.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the motion of the pilot. The pilot moves in a vertical circle. The apparent weight is the normal force exerted on him. At each point $\vec{a}_{\text{rad}}$ is directed toward the center of the circular path.

**(a) SET UP:** "the pilot feels weightless" means that the vertical normal force $n$ exerted on the pilot by the chair on which the pilot sits is zero. The force diagram for the pilot at the top of the path is given in Figure 5.55a.




**EXECUTE:**

$$\Sigma F_y = ma_y$$

$$mg = ma_{\text{rad}}$$

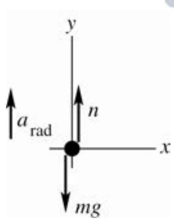
$$g = \frac{v^2}{R}$$

**Figure 5.55a**

Thus $v = \sqrt{gR} = \sqrt{(9.80\ \text{m/s}^2)(150\ \text{m})} = 38.34\ \text{m/s}$

$$v = (38.34\ \text{m/s})\left(\frac{1\ \text{km}}{10^3\ \text{m}}\right)\left(\frac{3600\ \text{s}}{1\ \text{h}}\right) = 138\ \text{km/h}$$

**(b) SET UP:** The force diagram for the pilot at the bottom of the path is given in Figure 5.55b. Note that the vertical normal force exerted on the pilot by the chair on which the pilot sits is now upward.




**EXECUTE:**

$$\Sigma F_y = ma_y$$

$$n - mg = m\frac{v^2}{R}$$

$$n = mg + m\frac{v^2}{R}$$

This normal force is the pilot's apparent weight.

**Figure 5.55b**

$w = 700\ \text{N}$, so $m = \dfrac{w}{g} = 71.43\ \text{kg}$

$$v = (280\ \text{km/h})\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right) = 77.78\ \text{m/s}$$

Thus $n = 700\ \text{N} + 71.43\ \text{kg}\dfrac{(77.78\ \text{m/s})^2}{150\ \text{m}} = 3580\ \text{N}.$

**EVALUATE:** In part (b), $n > mg$ since the acceleration is upward. The pilot feels he is much heavier than when at rest. The speed is not constant, but it is still true that $a_{\text{rad}} = v^2/R$ at each point of the motion.

**5.56. IDENTIFY:** $a_{\text{rad}} = v^2/R,$ directed toward the center of the circular path. At the bottom of the dive, $\vec{a}_{\text{rad}}$ is upward. The apparent weight of the pilot is the normal force exerted on her by the seat on which she is sitting.

**SET UP:** The free-body diagram for the pilot is given in Figure 5.56.

**EXECUTE:** **(a)** $a_{\text{rad}} = \dfrac{v^2}{R}$ gives $R = \dfrac{v^2}{a_{\text{rad}}} = \dfrac{(95.0\ \text{m/s})^2}{4.00(9.80\ \text{m/s}^2)} = 230\ \text{m}.$

**(b)** $\Sigma F_y = ma_y$ gives $n - mg = ma_{\text{rad}}.$

$n = m(g + a_{\text{rad}}) = m(g + 4.00g) = 5.00mg = (5.00)(50.0\ \text{kg})(9.80\ \text{m/s}^2) = 2450\ \text{N}$

**EVALUATE:** Her apparent weight is five times her true weight, the force of gravity the earth exerts on her.

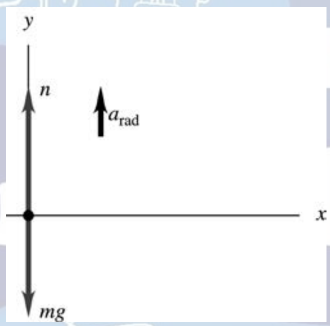
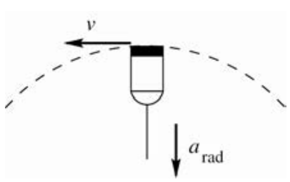



**Figure 5.56**

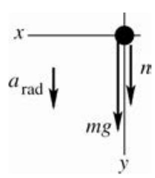
**5.57. IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the water. The water moves in a vertical circle. The target variable is the speed $v$; we will calculate $a_{\text{rad}}$ and then get $v$ from $a_{\text{rad}} = v^2/R.$

**SET UP:** Consider the free-body diagram for the water when the pail is at the top of its circular path, as shown in Figures 5.57a and b.




The radial acceleration is in toward the center of the circle so at this point is downward. $n$ is the downward normal force exerted on the water by the bottom of the pail.

**Figure 5.57a**




**EXECUTE:**

$\Sigma F_y = ma_y$

$n + mg = m\dfrac{v^2}{R}$

**Figure 5.57b**

At the minimum speed the water is just ready to lose contact with the bottom of the pail, so at this speed, $n \to 0.$ (Note that the force $n$ cannot be upward.)

With $n \to 0$ the equation becomes $mg = m\dfrac{v^2}{R}$. $v = \sqrt{gR} = \sqrt{(9.80\text{ m/s}^2)(0.600\text{ m})} = 2.42\text{ m/s}$.

**EVALUATE:** At the minimum speed $a_{\text{rad}} = g$. If $v$ is less than this minimum speed, gravity pulls the water (and bucket) out of the circular path.

**5.58.** **IDENTIFY:** The ball has acceleration $a_{\text{rad}} = v^2/R$, directed toward the center of the circular path. When the ball is at the bottom of the swing, its acceleration is upward.

**SET UP:** Take $+y$ upward, in the direction of the acceleration. The bowling ball has mass $m = w/g = 7.27\text{ kg}$.

**EXECUTE:** **(a)** $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(4.20\text{ m/s})^2}{3.80\text{ m}} = 4.64\text{ m/s}$, upward.

**(b)** The free-body diagram is given in Figure 5.58. $\Sigma F_y = ma_y$ gives $T - mg = ma_{\text{rad}}$.

$T = m(g + a_{\text{rad}}) = (7.27\text{ kg})(9.80\text{ m/s}^2 + 4.64\text{ m/s}^2) = 105\text{ N}$

**EVALUATE:** The acceleration is upward, so the net force is upward and the tension is greater than the weight.

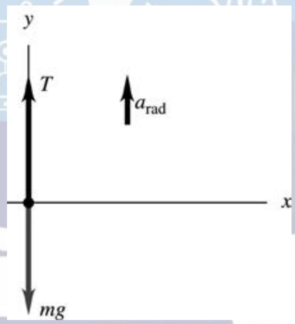



**Figure 5.58**

**5.59.** **IDENTIFY:** Since the arm is swinging in a circle, objects in it are accelerated toward the center of the circle, and Newton's second law applies to them.

**SET UP:** $R = 0.700\text{ m}$. A 45° angle is $\frac{1}{8}$ of a full rotation, so in $\frac{1}{2}$ s a hand travels through a distance of $\frac{1}{8}(2\pi R)$. In (c) use coordinates where $+y$ is upward, in the direction of $\vec{a}_{\text{rad}}$ at the bottom of the swing.

The acceleration is $a_{\text{rad}} = \dfrac{v^2}{R}$.

**EXECUTE:** **(a)** $v = \dfrac{1}{8}\left(\dfrac{2\pi R}{0.50\text{ s}}\right) = 1.10\text{ m/s}$ and $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(1.10\text{ m/s})^2}{0.700\text{ m}} = 1.73\text{ m/s}^2$.

**(b)** The free-body diagram is shown in Figure 5.59. $F$ is the force exerted by the blood vessel.

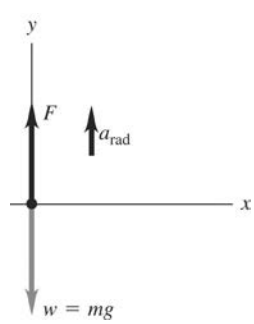



**Figure 5.59**

**(c)** $\Sigma F_y = ma_y$ gives $F - w = ma_{\text{rad}}$ and

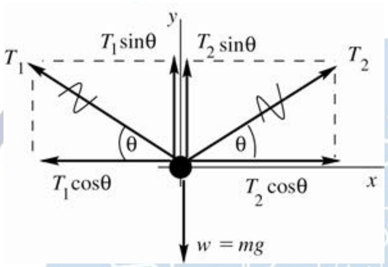
$F = m(g + a_{\text{rad}}) = (1.00\times 10^{-3}\text{ kg})(9.80\text{ m/s}^2 + 1.73\text{ m/s}^2) = 1.15\times 10^{-2}\text{ N},$ upward.

**(d)** When the arm hangs vertically and is at rest, $a_{\text{rad}} = 0$ so $F = w = mg = 9.8\times 10^{-3}\text{ N}.$

**EVALUATE:** The acceleration of the hand is only about 20% of $g$, so the increase in the force on the blood drop when the arm swings is about 20%.

**5.60.** **IDENTIFY:** Apply Newton's first law to the person. Each half of the rope exerts a force on him, directed along the rope and equal to the tension $T$ in the rope.

**SET UP:** **(a)** The force diagram for the person is given in Figure 5.60.




$T_1$ and $T_2$ are the tensions in each half of the rope.

**Figure 5.60**

**EXECUTE:** $\Sigma F_x = 0$

$T_2\cos\theta - T_1\cos\theta = 0$

This says that $T_1 = T_2 = T$ (The tension is the same on both sides of the person.)

$\Sigma F_y = 0$

$T_1\sin\theta + T_2\sin\theta - mg = 0$

But $T_1 = T_2 = T,$ so $2T\sin\theta = mg$

$$T = \frac{mg}{2\sin\theta} = \frac{(90.0\text{ kg})(9.80\text{ m/s}^2)}{2\sin 10.0^\circ} = 2540\text{ N}$$

**(b)** The relation $2T\sin\theta = mg$ still applies but now we are given that $T = 2.50\times 10^4\text{ N}$ (the breaking strength) and are asked to find $\theta$.

$$\sin\theta = \frac{mg}{2T} = \frac{(90.0\text{ kg})(9.80\text{ m/s}^2)}{2(2.50\times 10^4\text{ N})} = 0.01764, \quad \theta = 1.01^\circ.$$

**EVALUATE:** $T = mg/(2\sin\theta)$ says that $T = mg/2$ when $\theta = 90^\circ$ (rope is vertical).

$T \to \infty$ when $\theta \to 0$ since the upward component of the tension becomes a smaller fraction of the tension.

**5.61.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the knot.

**SET UP:** $a = 0.$ Use coordinates with axes that are horizontal and vertical.

**EXECUTE:** **(a)** The free-body diagram for the knot is sketched in Figure 5.61.

$T_1$ is more vertical so supports more of the weight and is larger. You can also see this from $\Sigma F_x = ma_x$:

$T_2\cos 40^\circ - T_1\cos 60^\circ = 0.$ $T_2\cos 40^\circ - T_1\cos 60^\circ = 0.$

**(b)** $T_1$ is larger so set $T_1 = 5000\text{ N}.$ Then $T_2 = T_1/1.532 = 3263.5\text{ N}.$ $\Sigma F_y = ma_y$ gives

$T_1\sin 60^\circ + T_2\sin 40^\circ = w$ and $w = 6400\text{ N}.$

**EVALUATE:** The sum of the vertical components of the two tensions equals the weight of the suspended object. The sum of the tensions is greater than the weight.

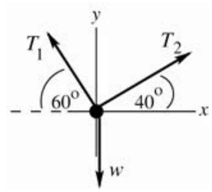



**Figure 5.61**

**5.62.** **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to each object. Constant speed means $a = 0$.

**SET UP:** The free-body diagrams are sketched in Figure 5.62. $T_1$ is the tension in the lower chain, $T_2$ is the tension in the upper chain and $T = F$ is the tension in the rope.

**EXECUTE:** The tension in the lower chain balances the weight and so is equal to *w*. The lower pulley must have no net force on it, so twice the tension in the rope must be equal to *w* and the tension in the rope, which equals *F*, is $w/2$. Then, the downward force on the upper pulley due to the rope is also *w*, and so the upper chain exerts a force *w* on the upper pulley, and the tension in the upper chain is also *w*.

**EVALUATE:** The pulley combination allows the worker to lift a weight *w* by applying a force of only $w/2$.

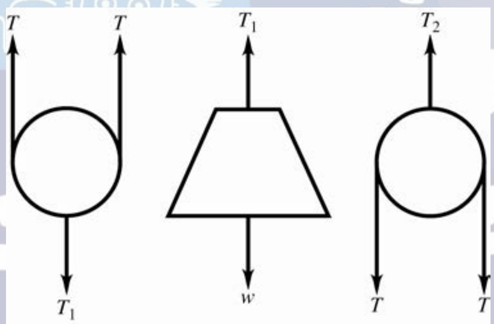



**Figure 5.62**

**5.63.** **IDENTIFY:** The engine is hanging at rest, so its acceleration is zero which means that the forces on it must balance. We balance horizontal components and vertical components.

**SET UP:** In addition to the tensions in the four cables shown in the text, gravity also acts on the engine. Call $+x$ horizontally to the right and $+y$ vertically upward, and call $\theta$ the angle that cable *C* makes with cable *D*. The mass of the engine is 409 kg and the tension $T_A$ in cable *A* is 722 N.

**EXECUTE:** The tension in cable *D* is the only force balancing gravity on the engine, so $T_D = mg$. In the *x*-direction, we have $T_A = T_C \sin\theta$, which gives $T_C = T_A/\sin\theta = (722\text{ N})/(\sin 37.1°) = 1197\text{ N}$. In the *y*-direction, we have $T_B - T_D - T_C \cos\theta = 0$, which gives $T_B = (409\text{ kg})(9.80\text{ m/s}^2) + (1197\text{ N})\cos(37.1°) = 4963\text{ N}$. Rounding to 3 significant figures gives $T_B = 4960$ N and $T_C = 1200$ N.

**EVALUATE:** The tension in cable *B* is greater than the weight of the engine because cable *C* has a downward component that *B* must also balance.

**5.64.** **IDENTIFY:** Apply Newton's first law to the ball. Treat the ball as a particle.

**SET UP:** The forces on the ball are gravity, the tension in the wire and the normal force exerted by the surface. The normal force is perpendicular to the surface of the ramp. Use *x*- and *y*-axes that are horizontal and vertical.

**EXECUTE:** **(a)** The free-body diagram for the ball is given in Figure 5.64 (next page). The normal force has been replaced by its *x* and *y* components.

**(b)** $\Sigma F_y = 0$ gives $n\cos 35.0° - w = 0$ and $n = \dfrac{mg}{\cos 35.0°} = 1.22mg$.

**(c)** $\Sigma F_x = 0$ gives $T - n\sin 35.0° = 0$ and $T = (1.22mg)\sin 35.0° = 0.700mg$.

**Evaluate:** Note that the normal force is greater than the weight, and increases without limit as the angle of the ramp increases toward $90°$. The tension in the wire is $w\tan\phi$, where $\phi$ is the angle of the ramp and $T$ also increases without limit as $\phi \to 90°$.

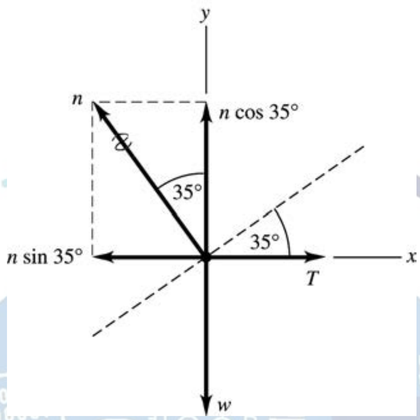



**Figure 5.64**

**5.65.** **Identify:** Apply Newton's first law to the ball. The force of the wall on the ball and the force of the ball on the wall are related by Newton's third law.

**Set Up:** The forces on the ball are its weight, the tension in the wire, and the normal force applied by the wall. To calculate the angle $\phi$ that the wire makes with the wall, use Figure 5.65a: $\sin\phi = \dfrac{16.0 \text{ cm}}{46.0 \text{ cm}}$ and $\phi = 20.35°$

**Execute:** **(a)** The free-body diagram is shown in Figure 5.65b. Use the $x$ and $y$ coordinates shown in the figure. $\Sigma F_y = 0$ gives $T\cos\phi - w = 0$ and $T = \dfrac{w}{\cos\phi} = \dfrac{(45.0 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 20.35°} = 470 \text{ N}$

**(b)** $\Sigma F_x = 0$ gives $T\sin\phi - n = 0$. $n = (470 \text{ N})\sin 20.35° = 163 \text{ N}$. By Newton's third law, the force the ball exerts on the wall is 163 N, directed to the right.

**Evaluate:** $n = \left(\dfrac{w}{\cos\phi}\right)\sin\phi = w\tan\phi$. As the angle $\phi$ decreases (by increasing the length of the wire), $T$ decreases and $n$ decreases.

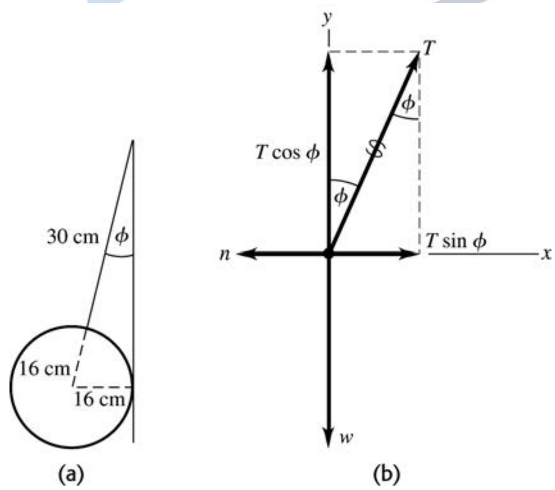



**Figure 5.65**

**5.66.** **IDENTIFY:** In each rough patch, the kinetic friction (and hence the acceleration) is constant, but the constants are different in the two patches. Newton's second law applies, as well as the constant-acceleration kinematics formulas in each patch.

**SET UP:** Choose the $+y$-axis upward and the $+x$-axis in the direction of the velocity.

**EXECUTE:** **(a)** Find the velocity and time when the box is at $x = 2.00$ m. Newton's second law tells us that $n = mg$ and $-f_k = ma_x$ which gives $-\mu_k mg = ma_x$; $a_x = -\mu_k g = -(0.200)(9.80\text{ m/s}^2) = -1.96\text{ m/s}^2$. Now use the kinematics equations involving $v_x$. Using $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ we get

$v_x = \sqrt{(4.00\text{ m/s})^2 + 2(-1.96\text{ m/s}^2)(2.00\text{ m})} = 2.857$ m/s. Now solve the equation $v_x = v_{0x} + a_x t$ for $t$ to get $t = (2.857\text{ m/s} - 4.00\text{ m/s})/(-1.96\text{ m/s}^2) = 0.5834$ s.

Now look at the motion in the section for which $\mu_k = 0.400$: $a_x = -(0.400)(9.80\text{ m/s}^2) = -3.92\text{ m/s}^2$, $v_x = 0$, $v_{0x} = 2.857$ m/s. Solving $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ for $x - x_0$ gives $x - x_0 = -(2.857\text{ m/s})^2/[2(-3.92\text{ m/s}^2)] = 1.041$ m. The box is at the point $x = 2.00\text{ m} + 1.041\text{ m} = 3.04$ m.

Solving $v_x = v_{0x} + a_x t$ for $t$ gives $t = (-2.857\text{ m/s})/(-3.92\text{ m/s}^2) = 0.7288$ s. The total time is $0.5834\text{ s} + 0.7288\text{ s} = 1.31$ s.

**EVALUATE:** We cannot do this problem in a single process because the acceleration, although constant in each patch, is different in the two patches.

**5.67.** **IDENTIFY:** Kinematics will give us the acceleration of the person, and Newton's second law will give us the force (the target variable) that his arms exert on the rest of his body.

**SET UP:** Let the person's weight be $W$, so $W = 680$ N. Assume constant acceleration during the speeding up motion and assume that the body moves upward 15 cm in 0.50 s while speeding up. The constant-acceleration kinematics formula $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ and $\Sigma F_y = ma_y$ apply. The free-body diagram for the person is given in Figure 5.67. $F$ is the force exerted on him by his arms.

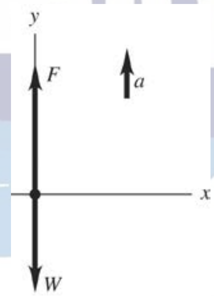



**Figure 5.67**

**EXECUTE:** $v_{0y} = 0$, $y - y_0 = 0.15$ m, $t = 0.50$ s. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$$a_y = \frac{2(y - y_0)}{t^2} = \frac{2(0.15\text{ m})}{(0.50\text{ s})^2} = 1.2\text{ m/s}^2.\ \ \Sigma F_y = ma_y \text{ gives } F - W = ma.\ \ m = \frac{W}{g}, \text{ so}$$

$$F = W\left(1 + \frac{a}{g}\right) = 1.12W = 762\text{ N}.$$

**EVALUATE:** The force is greater than his weight, which it must be if he is to accelerate upward.

**5.68.** **IDENTIFY:** The force is time-dependent, so the acceleration is not constant. Therefore we must use calculus instead of the standard kinematics formulas. Newton's second law applies.

**SET UP:** The acceleration is the time derivative of the velocity and $\Sigma F_y = ma_y$.

**EXECUTE:** Differentiating the velocity gives $a_y = dv_y/dt = 2.00\text{ m/s}^2 + (1.20\text{ m/s}^3)t$. Find the time when $v_y = 9.00$ m/s: $9.00\text{ m/s} = (2.00\text{ m/s}^2)t + (0.600\text{ m/s}^3)t^2$. Solving this quadratic for $t$ and taking the positive value gives $t = 2.549$ s. At this time the acceleration is $a = 2.00\text{ m/s}^2 + (1.20\text{ m/s}^3)(2.549\text{ s}) = 5.059\text{ m/s}^2$. Now apply Newton's second law to the box, calling $T$ the tension in the rope: $T - mg = ma$, which gives $T = m(g + a) = (2.00\text{ kg})(9.80\text{ m/s}^2 + 5.059\text{ m/s}^2) = 29.7$ N.

**EVALUATE:** The tension is greater than the weight of the box, which it must be to accelerate the box upward. As time goes on, the acceleration, and hence the tension, would increase.

**5.69.** **IDENTIFY:** We know the forces on the box and want to find information about its position and velocity. Newton's second law will give us the box's acceleration.

**SET UP:** $a_y(t) = \dfrac{\Sigma F_y}{m}$. We can integrate the acceleration to find the velocity and the velocity to find the position. At an altitude of several hundred meters, the acceleration due to gravity is essentially the same as it is at the earth's surface.

**EXECUTE:** Let $+y$ be upward. Newton's second law gives $T - mg = ma_y$, so $a_y(t) = (12.0\text{ m/s}^3)t - 9.8\text{ m/s}^2$. Integrating the acceleration gives $v_y(t) = (6.00\text{ m/s}^3)t^2 - (9.8\text{ m/s}^2)t$.

**(a)** (i) At $t = 1.00$ s, $v_y = -3.80$ m/s. (ii) At $t = 3.00$ s, $v_y = 24.6$ m/s.

**(b)** Integrating the velocity gives $y - y_0 = (2.00\text{ m/s}^3)t^3 - (4.9\text{ m/s}^2)t^2$. $v_y = 0$ at $t = 1.63$ s. At $t = 1.63$ s, $y - y_0 = 8.71\text{ m} - 13.07\text{ m} = -4.36\text{ m}$.

**(c)** Setting $y - y_0 = 0$ and solving for $t$ gives $t = 2.45$ s.

**EVALUATE:** The box accelerates and initially moves downward until the tension exceeds the weight of the box. Once the tension exceeds the weight, the box will begin to accelerate upward and will eventually move upward, as we saw in part (b).

**5.70.** **IDENTIFY:** We can use the standard kinematics formulas because the force (and hence the acceleration) is constant, and we can use Newton's second law to find the force needed to cause that acceleration. Kinetic friction, not static friction, is acting.

**SET UP:** From kinematics, we have $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ and $\Sigma F_x = ma_x$ applies. Forces perpendicular to the ramp balance. The force of kinetic friction is $f_k = \mu_k mg\cos\theta$.

**EXECUTE:** Call $+x$ upward along the surface of the ramp. Kinematics gives $a_x = \dfrac{2(x - x_0)}{t^2} = \dfrac{2(8.00\text{ m})}{(6.00\text{ s})^2} = 0.4444\text{ m/s}^2$. $\Sigma F_x = ma_x$ gives $F - mg\sin\theta - \mu_k mg\cos\theta = ma_x$. Solving for $F$ and putting in the numbers for this problem gives

$$F = m(a_x + g\sin\theta + \mu_k mg\cos\theta) = (5.00\text{ kg})(0.4444\text{ m/s}^2 + 4.9\text{ m/s}^2 + 3.395\text{ m/s}^2) = 43.7\text{ N}.$$

**EVALUTE:** As long as the box is moving, only kinetic friction, not static friction, acts on it. The force is less than the weight of the box because only part of the box's weight acts down the ramp. We should also investigate if the force is great enough to start the box moving in the first place. In that case, static friction would have it maximum value, so $f_s = \mu_s n$. The force $F$ in this would be $F = \mu_s mg\cos(30°) + mg\sin(30°) = mg(\mu_s\cos30° + \sin30°) = (5.00\text{ kg})(9.80\text{ m/s}^2)[(0.43)(\cos30°) + \sin30°] = 42.7$ N. Since the force we found is 43.7 N, it is great enough to overcome static friction and cause the box to move.

**5.71.** **IDENTIFY:** The system of boxes is accelerating, so we apply Newton's second law to each box. The friction is kinetic friction. We can use the known acceleration to find the tension and the mass of the second box.

**SET UP:** The force of friction is $f_k = \mu_k n$, $\Sigma F_x = ma_x$ applies to each box, and the forces perpendicular to the surface balance.

**EXECUTE:** **(a)** Call the $+x$-axis along the surface. For the 5 kg block, the vertical forces balance, so $n + F\sin 53.1° - mg = 0$, which gives $n = 49.0\text{ N} - 31.99\text{ N} = 17.01\text{ N}$. The force of kinetic friction is $f_k = \mu_k n = 5.104$ N. Applying Newton's second law along the surface gives $F\cos 53.1° - T - f_k = ma$. Solving for $T$ gives $T = F\cos 53.1° - f_k - ma = 24.02\text{ N} - 5.10\text{ N} - 7.50\text{ N} = 11.4\text{ N}$.

**(b)** For the second box, $T - f_k = ma$. $T - \mu_k mg = ma$. Solving for $m$ gives

$$m = \frac{T}{\mu_k g + a} = \frac{11.42\text{ N}}{(0.3)(9.8\text{ m/s}^2) + 1.5\text{ m/s}^2} = 2.57\text{ kg}.$$

**EVALUATE:** The normal force for box $B$ is less than its weight due to the upward pull, but the normal force for box $A$ is equal to its weight because the rope pulls horizontally on $A$.

**5.72. IDENTIFY:** The horizontal force has a component up the ramp and a component perpendicular to the surface of the ramp. The upward component causes the upward acceleration and the perpendicular component affects the normal force on the box. Newton's second law applies. The forces perpendicular to the surface balance.

**SET UP:** Balance forces perpendicular to the ramp: $n - mg\cos\theta - F\sin\theta = 0$. Applying Newton's second law parallel to the ramp surface gives $F\cos\theta - f_k - mg\sin\theta = ma$.

**EXECUTE:** Using the above equations gives $n = mg\cos\theta + F\sin\theta$. The force of friction is $f_k = \mu_k n$, so $f_k = \mu_k(mg\cos\theta + F\sin\theta)$. $F\cos\theta - \mu_k mg\cos\theta - \mu_k F\sin\theta - mg\sin\theta = ma$. Solving for $F$ gives

$$F = \frac{m(a + \mu_k g\cos\theta + g\sin\theta)}{\cos\theta - \mu_k\sin\theta}.$$ Putting in the numbers, we get

$$F = \frac{(6.00\text{ kg})[3.60\text{ m/s}^2 + (0.30)(9.80\text{ m/s}^2)\cos 37.0° + (9.80\text{ m/s}^2)\sin 37.0°]}{\cos 37.0° - (0.30)\sin 37.0°} = 115\text{ N}$$

**EVALUATE:** Even though the push is horizontal, it can cause a vertical acceleration because it causes the normal force to have a vertical component greater than the vertical component of the box's weight.

**5.73. IDENTIFY:** Newton's second law applies to the box.

**SET UP:** $f_k = \mu_k n$, $\Sigma F_x = ma_x$, and $\Sigma F_y = ma_y$ apply to the box. Take the $+x$-axis down the surface of the ramp and the $+y$-axis perpendicular to the surface upward.

**EXECUTE:** $\Sigma F_y = ma_y$ gives $n + F\sin(33.0°) - mg\cos(33.0°) = 0$, which gives $n = 51.59$ N. The friction force is $f_k = \mu_k n = (0.300)(51.59\text{ N}) = 15.48$ N. Parallel to the surface we have $\Sigma F_x = ma_x$ which gives $F\cos(33.0°) + mg\sin(33.0°) - f_k = ma$, which gives $a = 6.129\text{ m/s}^2$. Finally the velocity formula gives us $v_x = v_{0x} + a_x t = 0 + (6.129\text{ m/s}^2)(2.00\text{ s}) = 12.3$ m/s.

**EVALUATE:** Even though $F$ is horizontal and $mg$ is vertical, it is best to choose the axes as we have done, rather than horizontal-vertical, because the acceleration is then in the $x$-direction. Taking $x$ and $y$ to be horizontal-vertical would give the acceleration $x$- and $y$-components, which would complicate the solution.

**5.74. IDENTIFY:** This is a system having constant acceleration, so we can use the standard kinematics formulas as well as Newton's second law to find the unknown mass $m_2$.

**SET UP:** Newton's second law applies to each block. The standard kinematics formulas can be used to find the acceleration because the acceleration is constant. The normal force on $m_1$ is $m_1 g\cos\alpha$, so the force of friction on it is $f_k = \mu_k m_1 g\cos\alpha$.

**EXECUTE:** Standard kinematics gives the acceleration of the system to be

$a_y = \dfrac{2(y - y_0)}{t^2} = \dfrac{2(12.0\text{ m})}{(3.00\text{ s})^2} = 2.667\text{ m/s}^2$. For $m_1$, $n = m_1 g\cos\alpha = 117.7$ N, so the friction force on $m_1$ is $f_k = (0.40)(117.7\text{ N}) = 47.08$ N. Applying Newton's second law to $m_1$ gives $T - f_k - m_1 g\sin\alpha = m_1 a$, where $T$ is the tension in the cord. Solving for $T$ gives

$T = f_k + m_1 g\sin\alpha + m_1 a = 47.08\text{ N} + 156.7\text{ N} + 53.34\text{ N} = 257.1$ N. Newton's second law for $m_2$ gives

$m_2 g - T = m_2 a$, so $m_2 = \dfrac{T}{g - a} = \dfrac{257.1\text{ N}}{9.8\text{ m/s}^2 - 2.667\text{ m/s}^2} = 36.0$ kg.

**EVALUATE:** We could treat these blocks as a two-block system. Newton's second law would then give $m_2 g - m_1 g\sin\alpha - \mu_k m_1 g\cos\alpha = (m_1 + m_2)a$, which gives the same result as above.

**5.75. IDENTIFY:** Newton's second law applies, as do the constant-acceleration kinematics equations.

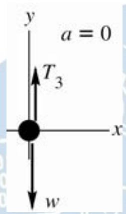
**SET UP:** Call the $+x$-axis horizontal and to the right and the $+y$-axis vertically upward. $\Sigma F_y = ma_y$ and $\Sigma F_x = ma_x$ both apply to the book.

**EXECUTE:** The book has no horizontal motion, so $\Sigma F_x = ma_x = 0$, which gives us the normal force $n$: $n = F\cos(60.0°)$. The kinetic friction force is $f_k = \mu_k n = (0.300)(96.0\text{ N})(\cos 60.0°) = 14.4\text{ N}$. In the vertical direction, we have $\Sigma F_y = ma_y$, which gives $F\sin(60.0°) - mg - f_k = ma$. Solving for $a$ gives us $a = [(96.0\text{ N})(\sin 60.0°) - 49.0\text{ N} - 14.4\text{ N}]/(5.00\text{ kg}) = 3.948\text{ m/s}^2$ upward. Now the velocity formula $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(3.948\text{ m/s}^2)(0.400\text{ m})} = 1.78\text{ m/s}$.

**EVALUATE:** Only the upward component of the force $F$ makes the book accelerate upward, while the horizontal component of $T$ is the magnitude of the normal force.

**5.76.** **IDENTIFY:** The system is in equilibrium. Apply Newton's first law to block $A$, to the hanging weight and to the knot where the cords meet. Target variables are the two forces.

**(a) SET UP:** The free-body diagram for the hanging block is given in Figure 5.76a.

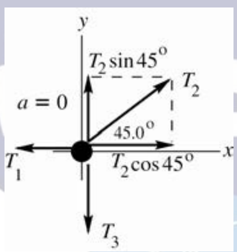



**EXECUTE:**

$$\Sigma F_y = ma_y$$
$$T_3 - w = 0$$
$$T_3 = 12.0\text{ N}$$

Figure 5.76a

**SET UP:** The free-body diagram for the knot is given in Figure 5.76b.



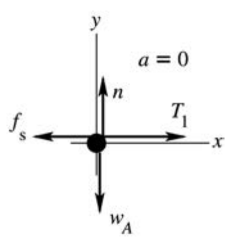

**EXECUTE:**

$$\Sigma F_y = ma_y$$
$$T_2 \sin 45.0° - T_3 = 0$$
$$T_2 = \frac{T_3}{\sin 45.0°} = \frac{12.0\text{ N}}{\sin 45.0°}$$
$$T_2 = 17.0\text{ N}$$

Figure 5.76b

$$\Sigma F_x = ma_x$$
$$T_2 \cos 45.0° - T_1 = 0$$
$$T_1 = T_2 \cos 45.0° = 12.0\text{ N}$$

**SET UP:** The free-body diagram for block $A$ is given in Figure 5.76c.




**EXECUTE:**

$$\Sigma F_x = ma_x$$
$$T_1 - f_s = 0$$
$$f_s = T_1 = 12.0\text{ N}$$

Figure 5.76c

**EVALUATE:** Also can apply $\Sigma F_y = ma_y$ to this block:

$$n - w_A = 0$$
$$n = w_A = 60.0\text{ N}$$

Then $\mu_s n = (0.25)(60.0\text{ N}) = 15.0\text{ N}$; this is the maximum possible value for the static friction force. We see that $f_s < \mu_s n$; for this value of $w$ the static friction force can hold the blocks in place.

**(b) SET UP:** We have all the same free-body diagrams and force equations as in part (a) but now the static friction force has its largest possible value, $f_s = \mu_s n = 15.0\text{ N}$. Then $T_1 = f_s = 15.0\text{ N}$.

**EXECUTE:** From the equations for the forces on the knot

$T_2 \cos 45.0° - T_1 = 0$ implies $T_2 = T_1/\cos 45.0° = \dfrac{15.0\text{ N}}{\cos 45.0°} = 21.2\text{ N}$

$T_2 \sin 45.0° - T_3 = 0$ implies $T_3 = T_2 \sin 45.0° = (21.2\text{ N})\sin 45.0° = 15.0\text{ N}$

And finally $T_3 - w = 0$ implies $w = T_3 = 15.0\text{ N}$.

**EVALUATE:** Compared to part (a), the friction is larger in part (b) by a factor of $(15.0/12.0)$ and $w$ is larger by this same ratio.

**5.77.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block.

**SET UP:** Constant speed means $a = 0$. When the blocks are moving, the friction force is $f_k$ and when they are at rest, the friction force is $f_s$.

**EXECUTE:** **(a)** The tension in the cord must be $m_2 g$ in order that the hanging block move at constant speed. This tension must overcome friction and the component of the gravitational force along the incline, so $m_2 g = (m_1 g \sin\alpha + \mu_k m_1 g \cos\alpha)$ and $m_2 = m_1(\sin\alpha + \mu_k \cos\alpha)$.

**(b)** In this case, the friction force acts in the same direction as the tension on the block of mass $m_1$, so $m_2 g = (m_1 g \sin\alpha - \mu_k m_1 g \cos\alpha)$, or $m_2 = m_1(\sin\alpha - \mu_k \cos\alpha)$.

**(c)** Similar to the analysis of parts (a) and (b), the largest $m_2$ could be is $m_1(\sin\alpha + \mu_s \cos\alpha)$ and the smallest $m_2$ could be is $m_1(\sin\alpha - \mu_s \cos\alpha)$.

**EVALUATE:** In parts (a) and (b) the friction force changes direction when the direction of the motion of $m_1$ changes. In part (c), for the largest $m_2$ the static friction force on $m_1$ is directed down the incline and for the smallest $m_2$ the static friction force on $m_1$ is directed up the incline.

**5.78.** **IDENTIFY:** The net force at any time is $F_{net} = ma$.

**SET UP:** At $t = 0$, $a = 62g$. The maximum acceleration is 140*g* at $t = 1.2$ ms.

**EXECUTE:** **(a)** $F_{net} = ma = 62mg = 62(210\times10^{-9}\text{ kg})(9.80\text{ m/s}^2) = 1.3\times10^{-4}\text{ N}$. This force is 62 times the flea's weight.

**(b)** $F_{net} = 140mg = 2.9\times10^{-4}\text{ N}$, at $t = 1.2$ ms.

**(c)** Since the initial speed is zero, the maximum speed is the area under the $a_x - t$ graph. This gives 1.2 m/s.

**EVALUATE:** *a* is much larger than *g* and the net external force is much larger than the flea's weight.

**5.79.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block. Use Newton's third law to relate forces on *A* and on *B*.

**SET UP:** Constant speed means $a = 0$.

**EXECUTE:** **(a)** Treat *A* and *B* as a single object of weight $w = w_A + w_B = 1.20\text{ N} + 3.60\text{ N} = 4.80\text{ N}$. The free-body diagram for this combined object is given in Figure 5.79a. $\Sigma F_y = ma_y$ gives

$n = w = 4.80\text{ N}$. $f_k = \mu_k n = (0.300)(4.80\text{ N}) = 1.44\text{ N}$. $\Sigma F_x = ma_x$ gives $F = f_k = 1.44\text{ N}$.

**(b)** The free-body force diagrams for blocks *A* and *B* are given in Figure 5.79b. *n* and $f_k$ are the normal and friction forces applied to block *B* by the tabletop and are the same as in part (a). $f_{kB}$ is the friction force that *A* applies to *B*. It is to the right because the force from *A* opposes the motion of *B*. $n_B$ is the downward force that *A* exerts on *B*. $f_{kA}$ is the friction force that *B* applies to *A*. It is to the left because block *B* wants *A* to move with it. $n_A$ is the normal force that block *B* exerts on *A*. By Newton's third law, $f_{kB} = f_{kA}$ and these forces are in opposite directions. Also, $n_A = n_B$ and these forces are in opposite directions.

$\Sigma F_y = ma_y$ for block *A* gives $n_A = w_A = 1.20\text{ N}$, so $n_B = 1.20\text{ N}$.

$f_{kA} = \mu_k n_A = (0.300)(1.20\text{ N}) = 0.360\text{ N}$, and $f_{kB} = 0.360\text{ N}$.

$\Sigma F_x = ma_x$ for block $A$ gives $T = f_{\mathrm{k}A} = 0.360\ \mathrm{N}$.

$\Sigma F_x = ma_x$ for block $B$ gives $F = f_{\mathrm{k}B} + f_{\mathrm{k}} = 0.360\ \mathrm{N} + 1.44\ \mathrm{N} = 1.80\ \mathrm{N}$.

**EVALUATE:** In part (a) block $A$ is at rest with respect to $B$ and it has zero acceleration. There is no horizontal force on $A$ besides friction, and the friction force on $A$ is zero. A larger force $F$ is needed in part (b), because of the friction force between the two blocks.

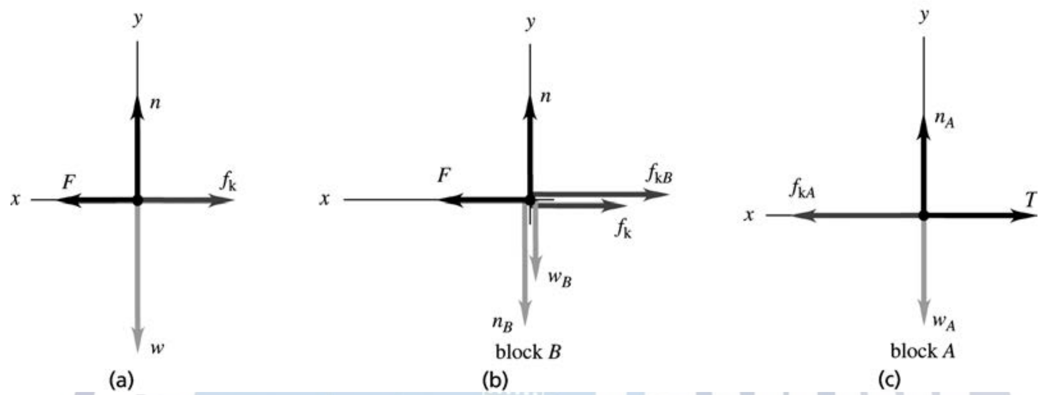



**Figure 5.79**

**5.80.** **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to the passenger to find the maximum allowed acceleration. Then use a constant acceleration equation to find the maximum speed.

**SET UP:** The free-body diagram for the passenger is given in Figure 5.80.

**EXECUTE:** $\Sigma F_y = ma_y$ gives $n - mg = ma$. $n = 1.6mg$, so $a = 0.60g = 5.88\ \mathrm{m/s^2}$.

$y - y_0 = 3.0\ \mathrm{m}$, $a_y = 5.88\ \mathrm{m/s^2}$, $v_{0y} = 0$ so $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = 5.9\ \mathrm{m/s}$.

**EVALUATE:** A larger final speed would require a larger value of $a_y$, which would mean a larger normal force on the person.

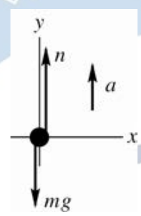



**Figure 5.80**

**5.81.** **IDENTIFY:** $a = dv/dt$. Apply $\Sigma \vec{F} = m\vec{a}$ to yourself.

**SET UP:** The reading of the scale is equal to the normal force the scale applies to you.

**EXECUTE:** The elevator's acceleration is $a = \dfrac{dv(t)}{dt} = 3.0\ \mathrm{m/s^2} + 2(0.20\ \mathrm{m/s^3})t = 3.0\ \mathrm{m/s^2} + (0.40\ \mathrm{m/s^3})t$.

At $t = 4.0\ \mathrm{s}$, $a = 3.0\ \mathrm{m/s^2} + (0.40\ \mathrm{m/s^3})(4.0\ \mathrm{s}) = 4.6\ \mathrm{m/s^2}$. From Newton's second law, the net force on you is $F_{\mathrm{net}} = F_{\mathrm{scale}} - w = ma$ and $F_{\mathrm{scale}} = w + ma = (64\ \mathrm{kg})(9.8\ \mathrm{m/s^2}) + (64\ \mathrm{kg})(4.6\ \mathrm{m/s^2}) = 920\ \mathrm{N}$.

**EVALUATE:** $a$ increases with time, so the scale reading is increasing.

**5.82.** **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to the hammer. Since the hammer is at rest relative to the bus, its acceleration equals that of the bus.

**SET UP:** The free-body diagram for the hammer is given in Figure 5.82.

**EXECUTE:** $\Sigma F_y = ma_y$ gives $T\sin 56.0° - mg = 0$ so $T\sin 56.0° = mg$. $\Sigma F_x = ma_x$ gives $T\cos 56.0° = ma$.

Divide the second equation by the first: $\dfrac{a}{g} = \dfrac{1}{\tan 56.0°}$ and $a = 6.61\ \text{m/s}^2$.

**EVALUATE:** When the acceleration increases, the angle between the rope and the ceiling of the bus decreases, and the angle the rope makes with the vertical increases.

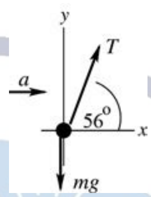
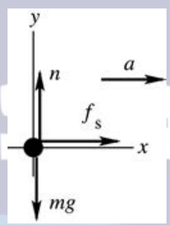



**Figure 5.82**

**5.83.** **IDENTIFY:** First calculate the maximum acceleration that the static friction force can give to the case. Apply $\Sigma\vec{F} = m\vec{a}$ to the case.

**(a) SET UP:** The static friction force is to the right in Figure 5.83a (northward) since it tries to make the case move with the truck. The maximum value it can have is $f_s = \mu_s N$.




**EXECUTE:**

$\Sigma F_y = ma_y$

$n - mg = 0$

$n = mg$

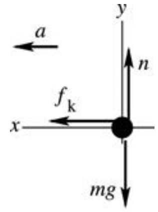
$f_s = \mu_s n = \mu_s mg$

**Figure 5.83a**

$\Sigma F_x = ma_x$. $f_s = ma$. $\mu_s mg = ma$. $a = \mu_s g = (0.30)(9.80\ \text{m/s}^2) = 2.94\ \text{m/s}^2$. The truck's acceleration is less than this so the case doesn't slip relative to the truck; the case's acceleration is $a = 2.20\ \text{m/s}^2$ (northward). Then $f_s = ma = (40.0\ \text{kg})(2.20\ \text{m/s}^2) = 88.0\ \text{N}$, northward.

**(b) IDENTIFY:** Now the acceleration of the truck is greater than the acceleration that static friction can give the case. Therefore, the case slips relative to the truck and the friction is kinetic friction. The friction force still tries to keep the case moving with the truck, so the acceleration of the case and the friction force are both southward. The free-body diagram is sketched in Figure 5.83b.

**SET UP:**




**EXECUTE:**

$\Sigma F_y = ma_y$

$n - mg = 0$

$n = mg$

$f_k = \mu_k mg = (0.20)(40.0\ \text{kg})(9.80\ \text{m/s}^2)$

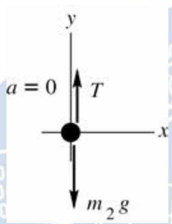
$f_k = 78\ \text{N}$, southward

**Figure 5.83b**

**EVALUATE:** $f_k = ma$ implies $a = \dfrac{f_k}{m} = \dfrac{78\text{ N}}{40.0\text{ kg}} = 2.0\text{ m/s}^2$. The magnitude of the acceleration of the case is less than that of the truck and the case slides toward the front of the truck. In both parts (a) and (b) the friction is in the direction of the motion and accelerates the case. Friction opposes *relative* motion between two surfaces in contact.

**5.84.** **IDENTIFY:** Apply Newton's first law to the rope. Let $m_1$ be the mass of that part of the rope that is on the table, and let $m_2$ be the mass of that part of the rope that is hanging over the edge. ($m_1 + m_2 = m$, the total mass of the rope). Since the mass of the rope is not being neglected, the tension in the rope varies along the length of the rope. Let $T$ be the tension in the rope at that point that is at the edge of the table.

**SET UP:** The free-body diagram for the hanging section of the rope is given in Figure 5.84a.




**EXECUTE:**

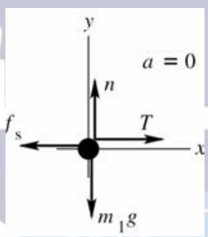
$\Sigma F_y = ma_y$

$T - m_2 g = 0$

$T = m_2 g$

**Figure 5.84a**

**SET UP:** The free-body diagram for that part of the rope that is on the table is given in Figure 5.84b.




**EXECUTE:**

$\Sigma F_y = ma_y$

$n - m_1 g = 0$

$n = m_1 g$

**Figure 5.84b**

When the maximum amount of rope hangs over the edge the static friction has its maximum value:

$f_s = \mu_s n = \mu_s m_1 g$

$\Sigma F_x = ma_x$

$T - f_s = 0$

$T = \mu_s m_1 g$

Use the first equation to replace $T$:

$m_2 g = \mu_s m_1 g$

$m_2 = \mu_s m_1$

The fraction that hangs over is $\dfrac{m_2}{m} = \dfrac{\mu_s m_1}{m_1 + \mu_s m_1} = \dfrac{\mu_s}{1 + \mu_s}$.

**EVALUATE:** As $\mu_s \to 0$, the fraction goes to zero and as $\mu_s \to \infty$, the fraction goes to unity.

**5.85.** **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to the point where the three wires join and also to one of the balls. By symmetry the tension in each of the 35.0 cm wires is the same.

**SET UP:** The geometry of the situation is sketched in Figure 5.85a. The angle $\phi$ that each wire makes with the vertical is given by $\sin\phi = \dfrac{12.5\text{ cm}}{47.5\text{ cm}}$ and $\phi = 15.26°$. Let $T_A$ be the tension in the vertical wire and let $T_B$ be the tension in each of the other two wires. Neglect the weight of the wires. The free-body diagram for the left-hand ball is given in Figure 5.85b and for the point where the wires join in Figure 5.85c. $n$ is the force one ball exerts on the other.

**EXECUTE:** **(a)** $\Sigma F_y = ma_y$ applied to the ball gives $T_B \cos\phi - mg = 0$.

$T_B = \dfrac{mg}{\cos\phi} = \dfrac{(15.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 15.26°} = 152\text{ N}$. Then $\Sigma F_y = ma_y$ applied in Figure 5.85c gives

$T_A - 2T_B \cos\phi = 0$ and $T_A = 2(152\text{ N})\cos\phi = 294\text{ N}$.

**(b)** $\Sigma F_x = ma_x$ applied to the ball gives $n - T_B \sin\phi = 0$ and $n = (152\text{ N})\sin 15.26° = 40.0\text{ N}$.

**EVALUATE:** $T_A$ equals the total weight of the two balls.

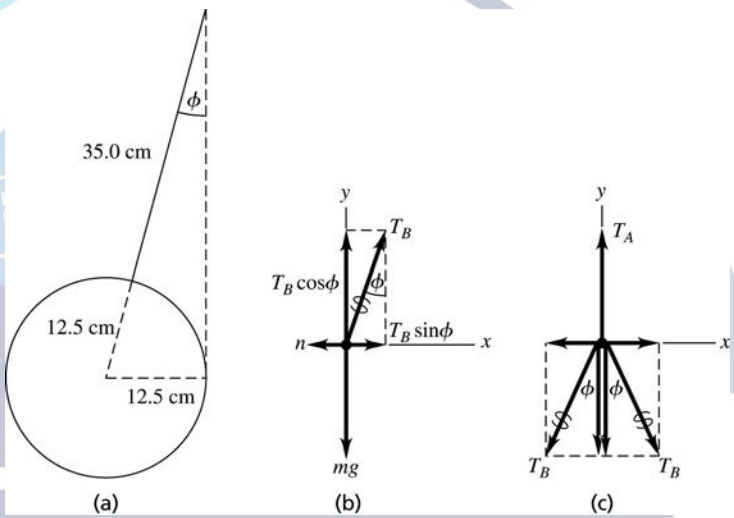



**Figure 5.85**

**5.86.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the car to calculate its acceleration. Then use a constant acceleration equation to find the initial speed.

**SET UP:** Let $+x$ be in the direction of the car's initial velocity. The friction force $f_k$ is then in the $-x$-direction. $192\text{ ft} = 58.52\text{ m}$.

**EXECUTE:** $n = mg$ and $f_k = \mu_k mg$. $\Sigma F_x = ma_x$ gives $-\mu_k mg = ma_x$ and

$a_x = -\mu_k g = -(0.750)(9.80\text{ m/s}^2) = -7.35\text{ m/s}^2$. $v_x = 0$ (stops), $x - x_0 = 58.52\text{ m}$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

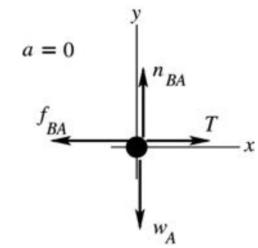
gives $v_{0x} = \sqrt{-2a_x(x - x_0)} = \sqrt{-2(-7.35\text{ m/s}^2)(58.52\text{ m})} = 29.3\text{ m/s} = 65.5\text{ mi/h}$. He was guilty.

**EVALUATE:** $x - x_0 = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = -\dfrac{v_{0x}^2}{2a_x}$. If his initial speed had been 45 mi/h he would have stopped in

$\left(\dfrac{45\text{ mi/h}}{65.5\text{ mi/h}}\right)^2 (192\text{ ft}) = 91\text{ ft}$.

**5.87.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block. Forces between the blocks are related by Newton's third law. The target variable is the force $F$. Block $B$ is pulled to the left at constant speed, so block $A$ moves to the right at constant speed and $a = 0$ for each block.

**SET UP:** The free-body diagram for block $A$ is given in Figure 5.87a. $n_{BA}$ is the normal force that $B$ exerts on $A$. $f_{BA} = \mu_k n_{BA}$ is the kinetic friction force that $B$ exerts on $A$. Block $A$ moves to the right relative to $B$, and $f_{BA}$ opposes this motion, so $f_{BA}$ is to the left. Note also that $F$ acts just on $B$, not on $A$.




**Execute:**

$\Sigma F_y = ma_y$

$n_{BA} - w_A = 0$

$n_{BA} = 1.90\ \text{N}$

$f_{BA} = \mu_k n_{BA} = (0.30)(1.90\ \text{N}) = 0.57\ \text{N}$

**Figure 5.87a**

$\Sigma F_x = ma_x$. $T - f_{BA} = 0$. $T = f_{BA} = 0.57$ N.

**Set Up:** The free-body diagram for block $B$ is given in Figure 5.87b.

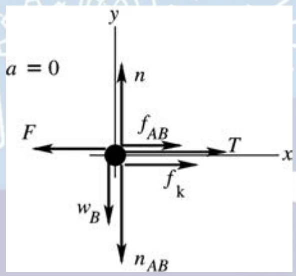



**Figure 5.87b**

**Execute:** $n_{AB}$ is the normal force that block $A$ exerts on block $B$. By Newton's third law $n_{AB}$ and $n_{BA}$ are equal in magnitude and opposite in direction, so $n_{AB} = 1.90$ N. $f_{AB}$ is the kinetic friction force that $A$ exerts on $B$. Block $B$ moves to the left relative to $A$ and $f_{AB}$ opposes this motion, so $f_{AB}$ is to the right. $f_{AB} = \mu_k n_{AB} = (0.30)(1.90\ \text{N}) = 0.57$ N. $n$ and $f_k$ are the normal and friction force exerted by the floor on block $B$; $f_k = \mu_k n$. Note that block $B$ moves to the left relative to the floor and $f_k$ opposes this motion, so $f_k$ is to the right.

$\Sigma F_y = ma_y$: $n - w_B - n_{AB} = 0$. $n = w_B + n_{AB} = 4.20\ \text{N} + 1.90\ \text{N} = 6.10$ N. Then $f_k = \mu_k n = (0.30)(6.10\ \text{N}) = 1.83$ N. $\Sigma F_x = ma_x$: $f_{AB} + T + f_k - F = 0$.
$F = T + f_{AB} + f_k = 0.57\ \text{N} + 0.57\ \text{N} + 1.83\ \text{N} = 3.0$ N.

**Evaluate:** Note that $f_{AB}$ and $f_{BA}$ are a third law action-reaction pair, so they must be equal in magnitude and opposite in direction and this is indeed what our calculation gives.

**5.88.** **Identify:** Apply $\Sigma \vec{F} = m\vec{a}$ to the box. Compare the acceleration of the box to the acceleration of the truck and use constant acceleration equations to describe the motion.

**Set Up:** Both objects have acceleration in the same direction; take this to be the $+x$-direction.

**Execute:** If the box were to remain at rest relative to the truck, the friction force would need to cause an acceleration of $2.20\ \text{m/s}^2$; however, the maximum acceleration possible due to static friction is $(0.19)(9.80\ \text{m/s}^2) = 1.86\ \text{m/s}^2$, and so the box will move relative to the truck; the acceleration of the box would be $\mu_k g = (0.15)(9.80\ \text{m/s}^2) = 1.47\ \text{m/s}^2$. The difference between the distance the truck moves and the distance the box moves (i.e., the distance the box moves relative to the truck) will be 1.80 m after a time

$$t = \sqrt{\frac{2\Delta x}{a_{\text{truck}} - a_{\text{box}}}} = \sqrt{\frac{2(1.80\ \text{m})}{(2.20\ \text{m/s}^2 - 1.47\ \text{m/s}^2)}} = 2.221\ \text{s}.$$

In this time, the truck moves $\frac{1}{2}a_{\text{truck}}t^2 = \frac{1}{2}(2.20\ \text{m/s}^2)(2.221\ \text{s})^2 = 5.43$ m.

**EVALUATE:** To prevent the box from sliding off the truck the coefficient of static friction would have to be $\mu_s = (2.20 \text{ m/s}^2)/g = 0.224$.

**5.89.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block. Parts (a) and (b) will be done together.

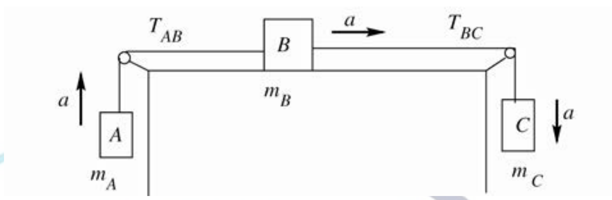
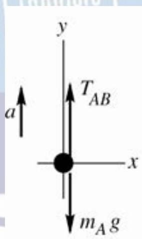



**Figure 5.89a**

Note that each block has the same magnitude of acceleration, but in different directions. For each block let the direction of $\vec{a}$ be a positive coordinate direction.

**SET UP:** The free-body diagram for block $A$ is given in Figure 5.89b.




**EXECUTE:**

$$\Sigma F_y = ma_y$$

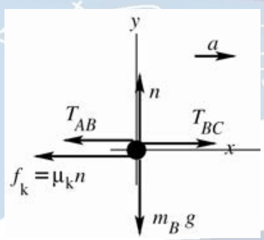
$$T_{AB} - m_A g = m_A a$$

$$T_{AB} = m_A(a+g)$$

$$T_{AB} = 4.00 \text{ kg}(2.00 \text{ m/s}^2 + 9.80 \text{ m/s}^2) = 47.2 \text{ N}$$

**Figure 5.89b**

**SET UP:** The free-body diagram for block $B$ is given in Figure 5.89c.




**EXECUTE:**

$$\Sigma F_y = ma_y$$

$$n - m_B g = 0$$

$$n = m_B g$$

**Figure 5.89c**

$$f_k = \mu_k n = \mu_k m_B g = (0.25)(12.0 \text{ kg})(9.80 \text{ m/s}^2) = 29.4 \text{ N}$$

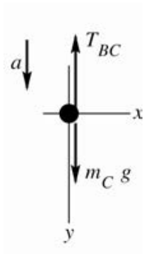
$$\Sigma F_x = ma_x$$

$$T_{BC} - T_{AB} - f_k = m_B a$$

$$T_{BC} = T_{AB} + f_k + m_B a = 47.2 \text{ N} + 29.4 \text{ N} + (12.0 \text{ kg})(2.00 \text{ m/s}^2)$$

$$T_{BC} = 100.6 \text{ N}$$

**SET UP:** The free-body diagram for block $C$ is sketched in Figure 5.89d (next page).




**EXECUTE:**

$$\Sigma F_y = ma_y$$

$$m_C g - T_{BC} = m_C a$$

$$m_C(g-a) = T_{BC}$$

$$m_C = \frac{T_{BC}}{g-a} = \frac{100.6\text{ N}}{9.80\text{ m/s}^2 - 2.00\text{ m/s}^2} = 12.9\text{ kg}$$

**Figure 5.89d**

**EVALUATE:** If all three blocks are considered together as a single object and $\Sigma\vec{F} = m\vec{a}$ is applied to this combined object, $m_C g - m_A g - \mu_k m_B g = (m_A + m_B + m_C)a$. Using the values for $\mu_k$, $m_A$ and $m_B$ given in the problem and the mass $m_C$ we calculated, this equation gives $a = 2.00\text{ m/s}^2$, which checks.

**5.90.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block. They have the same magnitude of acceleration, $a$.

**SET UP:** Consider positive accelerations to be to the right (up and to the right for the left-hand block, down and to the right for the right-hand block).

**EXECUTE:** **(a)** The forces along the inclines and the accelerations are related by $T - (100\text{ kg})g\sin 30.0° = (100\text{ kg})a$ and $(50\text{ kg})g\sin 53.1° - T = (50\text{ kg})a$, where $T$ is the tension in the cord and $a$ the mutual magnitude of acceleration. Adding these relations, $(50\text{ kg}\sin 53.1° - 100\text{ kg}\sin 30.0°)g = (50\text{ kg} + 100\text{ kg})a$, or $a = -0.067g$. Since $a$ comes out negative, the blocks will slide to the left; the 100-kg block will slide down. Of course, if coordinates had been chosen so that positive accelerations were to the left, $a$ would be $+0.067g$.

**(b)** $a = 0.067(9.80\text{ m/s}^2) = 0.658\text{ m/s}^2$.

**(c)** Substituting the value of $a$ (including the proper sign, depending on choice of coordinates) into either of the above relations involving $T$ yields 424 N.

**EVALUATE:** For part (a) we could have compared $mg\sin\theta$ for each block to determine which direction the system would move.

**5.91.** **IDENTIFY:** Let the tensions in the ropes be $T_1$ and $T_2$.

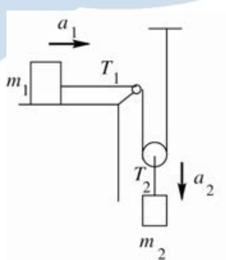
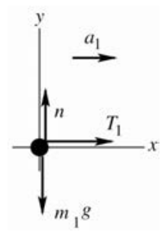



**Figure 5.91a**

Consider the forces on each block. In each case take a positive coordinate direction in the direction of the acceleration of that block.

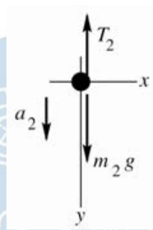
**SET UP:** The free-body diagram for $m_1$ is given in Figure 5.91b.




**EXECUTE:**
$\Sigma F_x = ma_x$
$T_1 = m_1 a_1$

Figure 5.91b

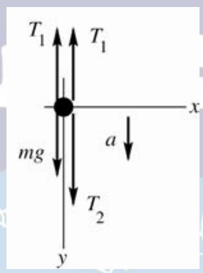
**SET UP:** The free-body diagram for $m_2$ is given in Figure 5.91c.




**EXECUTE:**
$\Sigma F_y = ma_y$
$m_2 g - T_2 = m_2 a_2$

Figure 5.91c

This gives us two equations, but there are four unknowns ($T_1$, $T_2$, $a_1$ and $a_2$) so two more equations are required.
**SET UP:** The free-body diagram for the moveable pulley (mass $m$) is given in Figure 5.91d.




**EXECUTE:**
$\Sigma F_y = ma_y$
$mg + T_2 - 2T_1 = ma$

Figure 5.91d

But our pulleys have negligible mass, so $mg = ma = 0$ and $T_2 = 2T_1$. Combine these three equations to eliminate $T_1$ and $T_2$: $m_2 g - T_2 = m_2 a_2$ gives $m_2 g - 2T_1 = m_2 a_2$. And then with $T_1 = m_1 a_1$ we have $m_2 g - 2m_1 a_1 = m_2 a_2$.
**SET UP:** There are still two unknowns, $a_1$ and $a_2$. But the accelerations $a_1$ and $a_2$ are related. In any time interval, if $m_1$ moves to the right a distance $d$, then in the same time $m_2$ moves downward a distance $d/2$. One of the constant acceleration kinematic equations says $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$, so if $m_2$ moves half the distance it must have half the acceleration of $m_1$: $a_2 = a_1/2$, or $a_1 = 2a_2$.
**EXECUTE:** This is the additional equation we need. Use it in the previous equation and get
$m_2 g - 2m_1(2a_2) = m_2 a_2$.
$a_2(4m_1 + m_2) = m_2 g$
$$a_2 = \frac{m_2 g}{4m_1 + m_2} \text{ and } a_1 = 2a_2 = \frac{2m_2 g}{4m_1 + m_2}.$$
**EVALUATE:** If $m_2 \to 0$ or $m_1 \to \infty$, $a_1 = a_2 = 0$. If $m_2 \gg m_1$, $a_2 = g$ and $a_1 = 2g$.

**5.92.** **IDENTIFY:** Apply $\Sigma \vec{F} = m\vec{a}$ to block $B$, to block $A$ and $B$ as a composite object, and to block $C$. If $A$ and $B$ slide together all three blocks have the same magnitude of acceleration.
**SET UP:** If $A$ and $B$ don't slip, the friction between them is static. The free-body diagrams for block $B$, for blocks $A$ and $B$, and for $C$ are given in Figure 5.92. Block $C$ accelerates downward and $A$ and $B$ accelerate

to the right. In each case take a positive coordinate direction to be in the direction of the acceleration. Since block $A$ moves to the right, the friction force $f_s$ on block $B$ is to the right, to prevent relative motion between the two blocks. When $C$ has its largest mass, $f_s$ has its largest value: $f_s = \mu_s n$.

**EXECUTE:** $\Sigma F_x = ma_x$ applied to the block $B$ gives $f_s = m_B a$. $n = m_B g$ and $f_s = \mu_s m_B g$. $\mu_s m_B g = m_B a$ and $a = \mu_s g$. $\Sigma F_x = ma_x$ applied to blocks $A + B$ gives $T = m_{AB} a = m_{AB}\mu_s g$. $\Sigma F_y = ma_y$ applied to block $C$ gives $m_C g - T = m_C a$. $m_C g - m_{AB}\mu_s g = m_C \mu_s g$. $m_C = \dfrac{m_{AB}\mu_s}{1-\mu_s} = (5.00\text{ kg} + 8.00\text{ kg})\left(\dfrac{0.750}{1-0.750}\right) = 39.0\text{ kg}.$

**EVALUATE:** With no friction from the tabletop, the system accelerates no matter how small the mass of $C$ is. If $m_C$ is less than 39.0 kg, the friction force that $A$ exerts on $B$ is less than $\mu_s n$. If $m_C$ is greater than 39.0 kg, blocks $C$ and $A$ have a larger acceleration than friction can give to block $B$, and $A$ accelerates out from under $B$.

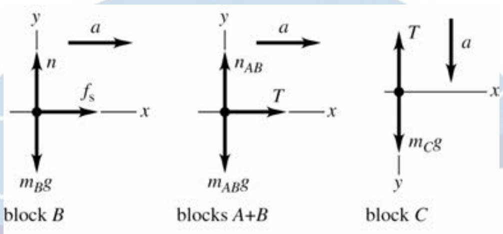



**Figure 5.92**

**5.93. IDENTIFY:** Apply the method of Exercise 5.15 to calculate the acceleration of each object. Then apply constant acceleration equations to the motion of the 2.00 kg object.

**SET UP:** After the 5.00 kg object reaches the floor, the 2.00 kg object is in free fall, with downward acceleration $g$.

**EXECUTE:** The 2.00-kg object will accelerate upward at $g\dfrac{5.00\text{ kg} - 2.00\text{ kg}}{5.00\text{ kg} + 2.00\text{ kg}} = 3g/7$, and the 5.00-kg object will accelerate downward at $3g/7$. Let the initial height above the ground be $h_0$. When the large object hits the ground, the small object will be at a height $2h_0$, and moving upward with a speed given by $v_0^2 = 2ah_0 = 6gh_0/7$. The small object will continue to rise a distance $v_0^2/2g = 3h_0/7$, and so the maximum height reached will be $2h_0 + 3h_0/7 = 17h_0/7 = 1.46$ m above the floor , which is 0.860 m above its initial height.

**EVALUATE:** The small object is 1.20 m above the floor when the large object strikes the floor, and it rises an additional 0.26 m after that.

**5.94. IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the box.

**SET UP:** The box has an upward acceleration of $a = 1.90\text{ m/s}^2$.

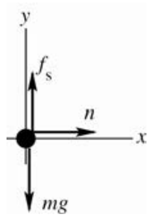
**EXECUTE:** The floor exerts an upward force $n$ on the box, obtained from $n - mg = ma$, or $n = m(a+g)$. The friction force that needs to be balanced is

$$\mu_k n = \mu_k m(a+g) = (0.32)(36.0\text{ kg})(1.90\text{ m/s}^2 + 9.80\text{ m/s}^2) = 135\text{ N}.$$

**EVALUATE:** If the elevator were not accelerating the normal force would be $n = mg$ and the friction force that would have to be overcome would be 113 N. The upward acceleration increases the normal force and that increases the friction force.

**5.95. IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the block. The cart and the block have the same acceleration. The normal force exerted by the cart on the block is perpendicular to the front of the cart, so is horizontal and to the right. The friction force on the block is directed so as to hold the block up against the downward pull of gravity. We want to calculate the minimum $a$ required, so take static friction to have its maximum value, $f_s = \mu_s n$.

**SET UP:** The free-body diagram for the block is given in Figure 5.95.




**EXECUTE:**
$\Sigma F_x = ma_x$
$n = ma$
$f_s = \mu_s n = \mu_s ma$

**Figure 5.95**

$\Sigma F_y = ma_y$: $f_s - mg = 0$

$\mu_s ma = mg$, so $a = g/\mu_s$.

**EVALUATE:** An observer on the cart sees the block pinned there, with no reason for a horizontal force on it because the block is at rest relative to the cart. Therefore, such an observer concludes that $n = 0$ and thus $f_s = 0$, and he doesn't understand what holds the block up against the downward force of gravity. The reason for this difficulty is that $\Sigma\vec{F} = m\vec{a}$ does not apply in a coordinate frame attached to the cart. This reference frame is accelerated, and hence not inertial. The smaller $\mu_s$ is, the larger $a$ must be to keep the block pinned against the front of the cart.

**5.96.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block.

**SET UP:** Use coordinates where $+x$ is directed down the incline.

**EXECUTE:** **(a)** Since the larger block (the trailing block) has the larger coefficient of friction, it will need to be pulled down the plane; i.e., the larger block will not move faster than the smaller block, and the blocks will have the same acceleration. For the smaller block, $(4.00\text{ kg})g(\sin 30° - (0.25)\cos 30°) - T = (4.00\text{ kg})a$, or $11.11\text{ N} - T = (4.00\text{ kg})a$, and similarly for the larger, $15.44\text{ N} + T = (8.00\text{ kg})a$. Adding these two relations, $26.55\text{ N} = (12.00\text{ kg})a$, $a = 2.21\text{ m/s}^2$.

**(b)** Substitution into either of the above relations gives $T = 2.27\text{ N}$.

**(c)** The string will be slack. The 4.00-kg block will have $a = 2.78\text{ m/s}^2$ and the 8.00-kg block will have $a = 1.93\text{ m/s}^2$, until the 4.00-kg block overtakes the 8.00-kg block and collides with it.

**EVALUATE:** If the string is cut the acceleration of each block will be independent of the mass of that block and will depend only on the slope angle and the coefficient of kinetic friction. The 8.00-kg block would have a smaller acceleration even though it has a larger mass, since it has a larger $\mu_k$.

**5.97.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the block and to the plank.

**SET UP:** Both objects have $a = 0$.

**EXECUTE:** Let $n_B$ be the normal force between the plank and the block and $n_A$ be the normal force between the block and the incline. Then, $n_B = w\cos\theta$ and $n_A = n_B + 3w\cos\theta = 4w\cos\theta$. The net frictional force on the block is $\mu_k(n_A + n_B) = \mu_k 5w\cos\theta$. To move at constant speed, this must balance the component of the block's weight along the incline, so $3w\sin\theta = \mu_k 5w\cos\theta$, and

$\mu_k = \frac{3}{5}\tan\theta = \frac{3}{5}\tan 37° = 0.452$.

**EVALUATE:** In the absence of the plank the block slides down at constant speed when the slope angle and coefficient of friction are related by $\tan\theta = \mu_k$. For $\theta = 36.9°$, $\mu_k = 0.75$. A smaller $\mu_k$ is needed when the plank is present because the plank provides an additional friction force.

**5.98.** **IDENTIFY:** Apply Newton's second law to Jack in the Ferris wheel.

**SET UP:** $\Sigma\vec{F} = m\vec{a}$ and Jack's acceleration is $a_{rad} = v^2/R$, and $v = 2\pi R/T$. At the highest point, the normal force that the chair exerts on Jack is ¼ of his weight, or $0.25mg$. Take $+y$ downward.

**EXECUTE:** $\Sigma F_y = ma_y$ gives $mg - n = mv^2/R$. $mg - 0.25mg = mv^2/R$, so $v^2/R = 0.75g$. Using $T = 2\pi R/T$, we get $v^2/R = 4\pi^2 R/T^2$. Therefore $4\pi^2 R/T^2 = 0.750g$. $T = 1/(0.100\text{ rev/s}) = 10.0\text{ s/rev}$, so
$R = (0.750g)T^2/(4\pi^2) = (0.750)(9.80\text{ m/s}^2)[(10.0\text{ s})/(2\pi)]^2 = 18.6\text{ m}$.

**EVALUATE:** This Ferris wheel would be about 120 ft in diameter, which is certainly large but not impossible.

**5.99.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the automobile.

**SET UP:** The "correct" banking angle is for zero friction and is given by $\tan\beta = \frac{v_0^2}{gR}$, as derived in the text. Use coordinates that are vertical and horizontal, since the acceleration is horizontal.

**EXECUTE:** For speeds larger than $v_0$, a frictional force is needed to keep the car from skidding. In this case, the inward force will consist of a part due to the normal force $n$ and the friction force $f$; $n\sin\beta + f\cos\beta = ma_{\text{rad}}$. The normal and friction forces both have vertical components; since there is no vertical acceleration, $n\cos\beta - f\sin\beta = mg$. Using $f = \mu_s n$ and $a_{\text{rad}} = \frac{v^2}{R} = \frac{(1.5v_0)^2}{R} = 2.25\, g\tan\beta$, these two relations become $n\sin\beta + \mu_s n\cos\beta = 2.25\, mg\tan\beta$ and $n\cos\beta - \mu_s n\sin\beta = mg$. Dividing to cancel $n$ gives $\frac{\sin\beta + \mu_s\cos\beta}{\cos\beta - \mu_s\sin\beta} = 2.25\tan\beta$. Solving for $\mu_s$ and simplifying yields $\mu_s = \frac{1.25\sin\beta\cos\beta}{1 + 1.25\sin^2\beta}$.

Using $\beta = \arctan\left(\frac{(20\text{ m/s})^2}{(9.80\text{ m/s}^2)(120\text{ m})}\right) = 18.79°$ gives $\mu_s = 0.34$.

**EVALUATE:** If $\mu_s$ is insufficient, the car skids away from the center of curvature of the roadway, so the friction is inward.

**5.100.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the car. The car moves in the arc of a horizontal circle, so $\vec{a} = \vec{a}_{\text{rad}}$, directed toward the center of curvature of the roadway. The target variable is the speed of the car. $a_{\text{rad}}$ will be calculated from the forces and then $v$ will be calculated from $a_{\text{rad}} = v^2/R$.

**(a)** To keep the car from sliding up the banking the static friction force is directed down the incline. At maximum speed the static friction force has its maximum value $f_s = \mu_s n$.

**SET UP:** The free-body diagram for the car is sketched in Figure 5.100a.

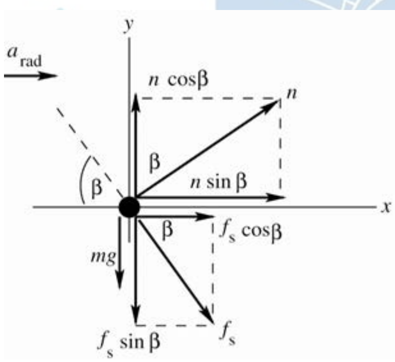



**Figure 5.100a**

**EXECUTE:**

$\Sigma F_y = ma_y$

$n\cos\beta - f_s\sin\beta - mg = 0$

But $f_s = \mu_s n$, so

$n\cos\beta - \mu_s n\sin\beta - mg = 0$

$$n = \frac{mg}{\cos\beta - \mu_s\sin\beta}$$

$\Sigma F_x = ma_x$

$n\sin\beta + \mu_s n\cos\beta = ma_{\text{rad}}$

$n(\sin\beta + \mu_s\cos\beta) = ma_{\text{rad}}$

Use the $\Sigma F_y$ equation to replace $n$:

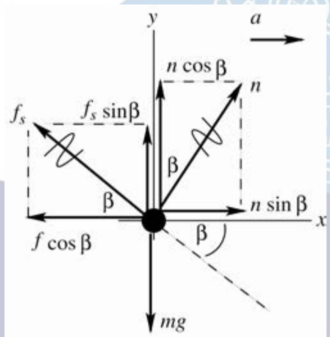
$$\left(\frac{mg}{\cos\beta - \mu_s \sin\beta}\right)(\sin\beta + \mu_s \cos\beta) = ma_{rad}$$

$$a_{rad} = \left(\frac{\sin\beta + \mu_s \cos\beta}{\cos\beta - \mu_s \sin\beta}\right)g = \left(\frac{\sin 25° + (0.30)\cos 25°}{\cos 25° - (0.30)\sin 25°}\right)(9.80\ \text{m/s}^2) = 8.73\ \text{m/s}^2$$

$a_{rad} = v^2/R$ implies $v = \sqrt{a_{rad}R} = \sqrt{(8.73\ \text{m/s}^2)(50\ \text{m})} = 21\ \text{m/s}.$

**(b) IDENTIFY:** To keep the car from sliding *down* the banking the static friction force is directed up the incline. At the minimum speed the static friction force has its maximum value $f_s = \mu_s n$.

**SET UP:** The free-body diagram for the car is sketched in Figure 5.100b.




The free-body diagram is identical to that in part (a) except that now the components of $f_s$ have opposite directions. The force equations are all the same except for the opposite sign for terms containing $\mu_s$.

**Figure 5.100b**

**EXECUTE:** $a_{rad} = \left(\frac{\sin\beta - \mu_s \cos\beta}{\cos\beta + \mu_s \sin\beta}\right)g = \left(\frac{\sin 25° - (0.30)\cos 25°}{\cos 25° + (0.30)\sin 25°}\right)(9.80\ \text{m/s}^2) = 1.43\ \text{m/s}^2$

$v = \sqrt{a_{rad}R} = \sqrt{(1.43\ \text{m/s}^2)(50\ \text{m})} = 8.5\ \text{m/s}.$

**EVALUATE:** For $v$ between these maximum and minimum values, the car is held on the road at a constant height by a static friction force that is less than $\mu_s n$. When $\mu_s \to 0$, $a_{rad} = g\tan\beta$. Our analysis agrees with the result of the banking derived in the text for this special case.

**5.101.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block.

**SET UP:** For block $B$ use coordinates parallel and perpendicular to the incline. Since they are connected by ropes, blocks $A$ and $B$ also move with constant speed.

**EXECUTE:** **(a)** The free-body diagrams are sketched in Figure 5.101 (next page).

**(b)** The blocks move with constant speed, so there is no net force on block $A$; the tension in the rope connecting $A$ and $B$ must be equal to the frictional force on block $A$, $T_1 = (0.35)(25.0\ \text{N}) = 8.8\ \text{N}$.

**(c)** The weight of block $C$ will be the tension in the rope connecting $B$ and $C$; this is found by considering the forces on block $B$. The components of force along the ramp are the tension in the first rope (8.8 N, from part (b)), the component of the weight along the ramp, the friction on block $B$ and the tension in the second rope. Thus, the weight of block $C$ is

$$w_C = 8.8\ \text{N} + w_B(\sin 36.9° + \mu_k \cos 36.9°) = 8.8\ \text{N} + (25.0\ \text{N})(\sin 36.9° + (0.35)\cos 36.9°) = 30.8\ \text{N}$$

The intermediate calculation of the first tension may be avoided to obtain the answer in terms of the common weight $w$ of blocks $A$ and $B$, $w_C = w(\mu_k + (\sin\theta + \mu_k \cos\theta))$, giving the same result.

**(d)** Applying Newton's second law to the remaining masses ($B$ and $C$) gives:

$$a = g(w_C - \mu_k w_B \cos\theta - w_B \sin\theta)/(w_B + w_C) = 1.54\ \text{m/s}^2.$$

**EVALUATE:** Before the rope between $A$ and $B$ is cut the net external force on the system is zero. When the rope is cut the friction force on $A$ is removed from the system and there is a net force on the system of blocks $B$ and $C$.

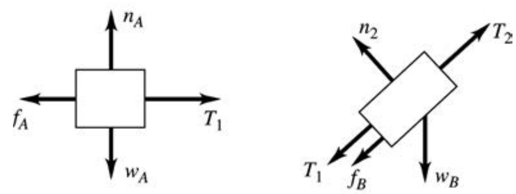



**Figure 5.101**

**5.102.** **IDENTIFY:** The analysis of this problem is similar to that of the conical pendulum in the text.

**SET UP:** As shown in the text for a conical pendulum, $\tan\beta = \dfrac{a_{\text{rad}}}{g} = \dfrac{v^2}{Rg}$.

**EXECUTE:** Solving for $v$ in terms of $\beta$ and $R$,

$v = \sqrt{gR\tan\beta} = \sqrt{(9.80\ \text{m/s}^2)(50.0\ \text{m})\tan 30.0°} = 16.8\ \text{m/s}$, about $60.6\ \text{km/h}$.

**EVALUATE:** The greater the speed of the bus the larger will be the angle $\beta$, so $T$ will have a larger horizontal, inward component.

**5.103.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$, with $f = kv$.

**SET UP:** Follow the analysis that leads to the equation $v_y = v_t[1 - e^{-(k/m)t}]$, except now the initial speed is $v_{0y} = 3mg/k = 3v_t$ rather than zero.

**EXECUTE:** The separated equation of motion has a lower limit of $3v_t$ instead of zero; specifically,

$$\int_{3v_t}^{v} \frac{dv}{v - v_t} = \ln\frac{v_t - v}{-2v_t} = \ln\left(\frac{v}{2v_t} - \frac{1}{2}\right) = -\frac{k}{m}t, \text{ or } v = 2v_t\left[\frac{1}{2} + e^{-(k/m)t}\right]$$

where $v_t = mg/k$.

**EVALUATE:** As $t \to \infty$ the speed approaches $v_t$. The speed is always greater than $v_t$ and this limit is approached from above.

**5.104.** **IDENTIFY:** The block has acceleration $a_{\text{rad}} = v^2/r$, directed to the left in the figure in the problem. Apply $\Sigma\vec{F} = m\vec{a}$ to the block.

**SET UP:** The block moves in a horizontal circle of radius $r = \sqrt{(1.25\ \text{m})^2 - (1.00\ \text{m})^2} = 0.75\ \text{m}$. Each string makes an angle $\theta$ with the vertical. $\cos\theta = \dfrac{1.00\ \text{m}}{1.25\ \text{m}}$, so $\theta = 36.9°$. The free-body diagram for the block is given in Figure 5.104. Let $+x$ be to the left and let $+y$ be upward.

**EXECUTE:** **(a)** $\Sigma F_y = ma_y$ gives $T_u\cos\theta - T_l\cos\theta - mg = 0$.

$$T_l = T_u - \frac{mg}{\cos\theta} = 80.0\ \text{N} - \frac{(4.00\ \text{kg})(9.80\ \text{m/s}^2)}{\cos 36.9°} = 31.0\ \text{N}.$$

**(b)** $\Sigma F_x = ma_x$ gives $(T_u + T_l)\sin\theta = m\dfrac{v^2}{r}$.

$v = \sqrt{\dfrac{r(T_u + T_l)\sin\theta}{m}} = \sqrt{\dfrac{(0.75\ \text{m})(80.0\ \text{N} + 31.0\ \text{N})\sin 36.9°}{4.00\ \text{kg}}} = 3.53\ \text{m/s}$. The number of revolutions per second is $\dfrac{v}{2\pi r} = \dfrac{3.53\ \text{m/s}}{2\pi(0.75\ \text{m})} = 0.749\ \text{rev/s} = 44.9\ \text{rev/min}$.

**(c)** If $T_l \to 0$, $T_u \cos\theta = mg$ and $T_u = \dfrac{mg}{\cos\theta} = \dfrac{(4.00\text{ kg})(9.80\text{ m/s}^2)}{\cos 36.9°} = 49.0\text{ N}$. $T_u \sin\theta = m\dfrac{v^2}{r}$.

$v = \sqrt{\dfrac{rT_u \sin\theta}{m}} = \sqrt{\dfrac{(0.75\text{ m})(49.0\text{ N})\sin 36.9°}{4.00\text{ kg}}} = 2.35\text{ m/s}$. The number of revolutions per minute is

$(44.9\text{ rev/min})\left(\dfrac{2.35\text{ m/s}}{3.53\text{ m/s}}\right) = 29.9\text{ rev/min}$.

**EVALUATE:** The tension in the upper string must be greater than the tension in the lower string so that together they produce an upward component of force that balances the weight of the block.

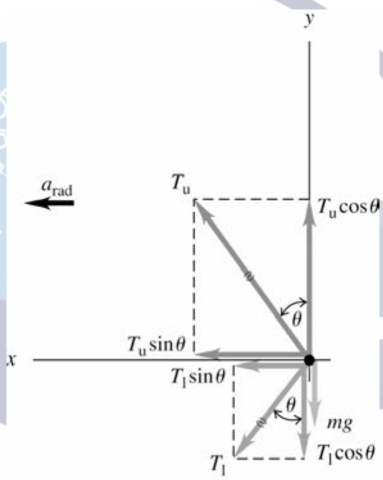



**Figure 5.104**

**5.105.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the person. The person moves in a horizontal circle so his acceleration is $a_{rad} = v^2/R$, directed toward the center of the circle. The target variable is the coefficient of static friction between the person and the surface of the cylinder.

$v = (0.60\text{ rev/s})\left(\dfrac{2\pi R}{1\text{ rev}}\right) = (0.60\text{ rev/s})\left(\dfrac{2\pi(2.5\text{ m})}{1\text{ rev}}\right) = 9.425\text{ m/s}$

**(a) SET UP:** The problem situation is sketched in Figure 5.105a.

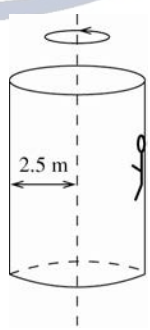
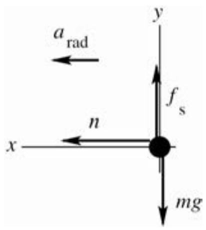



**Figure 5.105a**




The free-body diagram for the person is sketched in Figure 5.105b.
The person is held up against gravity by the static friction force exerted on him by the wall.
The acceleration of the person is $a_{\text{rad}}$, directed in toward the axis of rotation.

**Figure 5.105b**

**(b) EXECUTE:** To calculate the minimum $\mu_s$ required, take $f_s$ to have its maximum value, $f_s = \mu_s n$.

$\Sigma F_y = ma_y$: $f_s - mg = 0$

$\mu_s n = mg$

$\Sigma F_x = ma_x$: $n = mv^2/R$

Combine these two equations to eliminate $n$: $\mu_s mv^2/R = mg$

$$\mu_s = \frac{Rg}{v^2} = \frac{(2.5\text{ m})(9.80\text{ m/s}^2)}{(9.425\text{ m/s})^2} = 0.28$$

**(c) EVALUATE:** No, the mass of the person divided out of the equation for $\mu_s$. Also, the smaller $\mu_s$ is, the larger $v$ must be to keep the person from sliding down. For smaller $\mu_s$ the cylinder must rotate faster to make $n$ large enough.

**5.106.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the person and to the cart.

**SET UP:** The apparent weight, $w_{\text{app}}$ is the same as the upward force on the person exerted by the car seat.

**EXECUTE:** (a) The apparent weight is the actual weight of the person minus the centripetal force needed to keep him moving in his circular path:

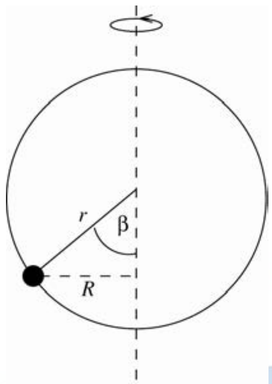
$$w_{\text{app}} = mg - \frac{mv^2}{R} = (70\text{ kg})\left[(9.8\text{ m/s}^2) - \frac{(12\text{ m/s})^2}{40\text{ m}}\right] = 434\text{ N}.$$

(b) The cart will lose contact with the surface when its apparent weight is zero; i.e., when the road no longer has to exert any upward force on it: $mg - \dfrac{mv^2}{R} = 0$. $v = \sqrt{Rg} = \sqrt{(40\text{ m})(9.8\text{ m/s}^2)} = 19.8\text{ m/s}$. The answer doesn't depend on the cart's mass, because the centripetal force needed to hold it on the road is proportional to its mass and so to its weight, which provides the centripetal force in this situation.

**EVALUATE:** At the speed calculated in part (b), the downward force needed for circular motion is provided by gravity. For speeds greater than this, more downward force is needed and there is no source for it and the cart leaves the circular path. For speeds less than this, less downward force than gravity is needed, so the roadway must exert an upward vertical force.

**5.107.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the circular motion of the bead. Also use $a_{\text{rad}} = 4\pi^2 R/T^2$ to relate $a_{\text{rad}}$ to the period of rotation $T$.

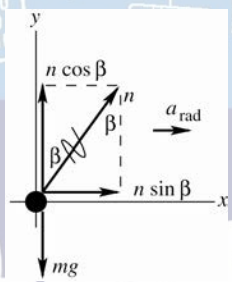
**SET UP:** The bead and hoop are sketched in Figure 5.107a.




The bead moves in a circle of radius $R = r\sin\beta$. The normal force exerted on the bead by the hoop is radially inward.

**Figure 5.107a**

The free-body diagram for the bead is sketched in Figure 5.107b.




**EXECUTE:**
$\Sigma F_y = ma_y$
$n\cos\beta - mg = 0$
$n = mg/\cos\beta$
$\Sigma F_x = ma_x$
$n\sin\beta = ma_{\text{rad}}$

**Figure 5.107b**

Combine these two equations to eliminate $n$:

$$\left(\frac{mg}{\cos\beta}\right)\sin\beta = ma_{\text{rad}}$$

$$\frac{\sin\beta}{\cos\beta} = \frac{a_{\text{rad}}}{g}$$

$a_{\text{rad}} = v^2/R$ and $v = 2\pi R/T$, so $a_{\text{rad}} = 4\pi^2 R/T^2$, where $T$ is the time for one revolution.

$R = r\sin\beta$, so $a_{\text{rad}} = \dfrac{4\pi^2 r\sin\beta}{T^2}$

Use this in the above equation: $\dfrac{\sin\beta}{\cos\beta} = \dfrac{4\pi^2 r\sin\beta}{T^2 g}$

This equation is satisfied by $\sin\beta = 0$, so $\beta = 0$, or by $\dfrac{1}{\cos\beta} = \dfrac{4\pi^2 r}{T^2 g}$, which gives $\cos\beta = \dfrac{T^2 g}{4\pi^2 r}$.

**(a)** 4.00 rev/s implies $T = (1/4.00)\text{ s} = 0.250\text{ s}$

Then $\cos\beta = \dfrac{(0.250\text{ s})^2(9.80\text{ m/s}^2)}{4\pi^2(0.100\text{ m})}$ and $\beta = 81.1°$.

**(b)** This would mean $\beta = 90°$. But $\cos 90° = 0$, so this requires $T \to 0$. So $\beta$ approaches $90°$ as the hoop rotates very fast, but $\beta = 90°$ is not possible.

**(c)** 1.00 rev/s implies $T = 1.00\text{ s}$

The $\cos\beta = \dfrac{T^2 g}{4\pi^2 r}$ equation then says $\cos\beta = \dfrac{(1.00\text{ s})^2(9.80\text{ m/s}^2)}{4\pi^2(0.100\text{ m})} = 2.48,$ which is not possible. The only way to have the $\Sigma\vec{F} = m\vec{a}$ equations satisfied is for $\sin\beta = 0.$ This means $\beta = 0;$ the bead sits at the bottom of the hoop.

**EVALUATE:** $\beta \to 90°$ as $T \to 0$ (hoop moves faster). The largest value $T$ can have is given by $T^2 g/(4\pi^2 r) = 1$ so $T = 2\pi\sqrt{r/g} = 0.635$ s. This corresponds to a rotation rate of $(1/0.635)$ rev/s $= 1.58$ rev/s. For a rotation rate less than 1.58 rev/s, $\beta = 0$ is the only solution and the bead sits at the bottom of the hoop. Part (c) is an example of this.

**5.108.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the combined object of motorcycle plus rider.

**SET UP:** The object has acceleration $a_{\text{rad}} = v^2/r,$ directed toward the center of the circular path.

**EXECUTE:** **(a)** For the tires not to lose contact, there must be a downward force on the tires. Thus, the (downward) acceleration at the top of the sphere must exceed $mg$, so $m\dfrac{v^2}{R} > mg,$ and

$v > \sqrt{gR} = \sqrt{(9.80\text{ m/s}^2)(13.0\text{ m})} = 11.3\text{ m/s}.$

**(b)** The (upward) acceleration will then be $4g$, so the upward normal force must be $5mg = 5(110\text{ kg})(9.80\text{ m/s}^2) = 5390\text{ N}.$

**EVALUATE:** At any nonzero speed the normal force at the bottom of the path exceeds the weight of the object.

**5.109.** **IDENTIFY:** The block begins to move when static friction has reached its maximum value. After that, kinetic friction acts and the block accelerates, obeying Newton's second law.

**SET UP:** $\Sigma F_x = ma_x$ and $f_{s,max} = \mu_s n$, where $n$ is the normal force (the weight of the block in this case).

**EXECUTE:** **(a) & (b)** $\Sigma F_x = ma_x$ gives $T - \mu_k mg = ma$. The graph with the problem shows the acceleration $a$ of the block versus the tension $T$ in the cord. So we solve the equation from Newton's second law for $a$ versus $T$, giving $a = (1/m)T - \mu_k g$. Therefore the slope of the graph will be $1/m$ and the intercept with the vertical axis will be $-\mu_k g$. Using the information given in the problem for the best-fit equation, we have $1/m = 0.182\text{ kg}^{-1}$, so $m = 5.4945$ kg and $-\mu_k g = -2.842\text{ m/s}^2,$ so $\mu_k = 0.290$.

When the block is just ready to slip, we have $f_{s,max} = \mu_s n$, which gives $\mu_s = (20.0\text{ N})/[(5.4945\text{ kg})(9.80\text{ m/s}^2)] = 0.371$.

**(c)** On the Moon, $g$ is less than on earth, but the mass $m$ of the block would be the same as would $\mu_k$. Therefore the slope $(1/m)$ would be the same, but the intercept $(-\mu_k g)$ would be less negative.

**EVALUATE:** Both coefficients of friction are reasonable or ordinary materials, so our results are believable.

**5.110.** **IDENTIFY:** Near the top of the hill the car is traveling in a circular arc, so it has radial acceleration and Newton's second law applies. We have measurements for the force the car exerts on the road at various speeds.

**SET UP:** The acceleration of the car is $a_{\text{rad}} = v^2/R$ and $\Sigma F_y = ma_y$ applies to the car. Let the $+y$-axis be downward, since that is the direction of the acceleration of the car.

**EXECUTE:** **(a)** Apply $\Sigma F_y = ma_y$ to the car at the top of the hill: $mg - n = mv^2/R$, where $n$ is the force the road exerts on the car (which is the same as the force the car exerts on the road). Solving for $n$ gives $n = mg - (m/R)v^2$. So if we plot $n$ versus $v^2$, we should get a straight line having slope equal to $-m/R$ and intercept with the vertical axis at $mg$. We could make a table of $v^2$ and $n$ using the given numbers given with the problem, or we could use graphing software. The resulting graph is shown in Figure 5.110.

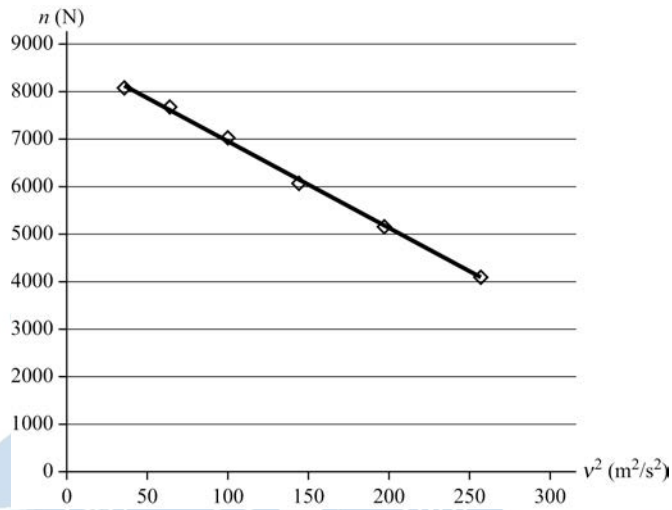



**Figure 5.110**

**(b)** The best-fit equation for the graph in Figure 5.110 is $n = [-18.12\text{ N/(m/s)}^2]v^2 + 8794$ N. Therefore $mg = 8794$ N, which gives $m = (8794\text{ N})/(9.80\text{ m/s}^2) = 897$ kg.
The slope is equal to $-m/R$, so $R = -m/\text{slope} = -(897\text{ kg})/[-18.12\text{ N/(m/s)}^2] = 49.5$ m.

**(c)** At the maximum speed, $n = 0$. Using $mg - n = mv^2/R$, this gives $v = \sqrt{gR} = \sqrt{(9.80\text{ m/s}^2)(49.5\text{ m})} = 22.0$ m/s.

**EVALUATE:** We can double check (c) using our graph. Putting $n = 0$ into the best-fit equation, we get $v = \sqrt{(8794\text{ N})(18.14\text{ N}\cdot\text{s}^2/\text{m}^2)} = 22.0$ m/s, which checks. Also 22 m/s is about 49 mph, which is not an unreasonabled speed on a hill.

**5.111.** **IDENTIFY:** A cable pulling parallel to the surface of a ramp accelerates 2170-kg metal blocks up a ramp that rises at 40.0° above the horizontal. Newton's second law applies to the blocks, and the constant-acceleration kinematics formulas can be used.

**SET UP:** Call the +$x$-axis parallel to the ramp surface pointing upward because that is the direction of the acceleration of the blocks, and let the $y$-axis be perpendicular to the surface. There is no acceleration in the $y$-direction. $\Sigma F_x = ma_x$, $f_k = \mu_k n$, and $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$.

**EXECUTE:** **(a)** First use $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ to find the acceleration of a block. Since $v_{0x} = 0$, we have $a_x = 2(x - x_0)/t^2 = 2(8.00\text{ m})/(4.20\text{ s})^2 = 0.9070\text{ m/s}^2$. The forces in the $y$-direction balance, so $n = mg\cos(40.0°)$, so $f_k = (0.350)(2170\text{ kg})(9.80\text{ m/s}^2)\cos(40.0°) = 5207$ N. Using $\Sigma F_x = ma_x$, we have $T - mg\sin(40.0°) - f_k = ma$. Solving for $T$ gives
$T = (2170\text{ kg})(9.80\text{ m/s}^2)\sin(40.0°) + 5207\text{ N} + (2170\text{ kg})(0.9070\text{ m/s}^2) = 2.13\times 10^4\text{ N} = 21.3$ kN.
From the table shown with the problem, this tension is greater than the safe load of a ½ inch diameter cable (which is 19.0 kN), so we need to use a 5/8-inch cable.

**(b)** We assume that the safe load (SL) is proportional to the cross-sectional area of the cable, which means that SL $\propto \pi(D/2)^2 \propto (\pi/4)D^2$, where $D$ is the diameter of the cable. Therefore a graph of SL versus $D^2$ should give a straight line. We could use the data given in the table with the problem to make the graph by hand, or we could use graphing software. The resulting graph is shown in Figure 5.111 (next page). The best-fit line has a slope of 74.09 kN/in.$^2$ and a $y$-intercept of 0.499 kN. For a cable of diameter $D = 9/16$ in., this equation gives SL $= (74.09\text{ kN/in.}^2)(9/16\text{ in.})^2 + 0.499\text{ kN} = 23.9$ kN.

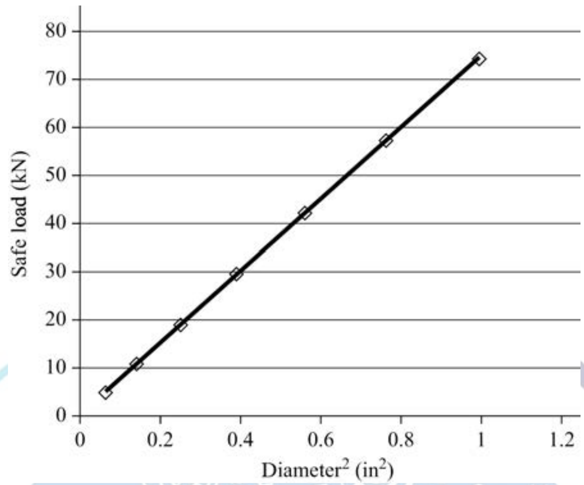



**Figure 5.111**

**(c)** The acceleration is now zero, so the forces along the surface balance, giving $T + f_s = mg\sin(40.0°)$. Using the numbers we get $T = 3.57$ kN.

**(d)** The tension at the top of the cable must accelerate the block and the cable below it, so the tension at the top would be larger. For a 5/8-inch cable, the mass per meter is 0.98 kg/m, so the 9.00-m long cable would have a mass of (0.98 kg/m)(9.00 m) = 8.8 kg. This is only 0.4% of the mass of the block, so neglecting the cable weight has little effect on accuracy.

**EVALUATE:** It is reasonable that the safe load of a cable is proportional to its cross-sectional area. If we think of the cable as consisting of many tiny strings each pulling, doubling the area would double the number of strings.

**5.112.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the block and to the wedge.

**SET UP:** For both parts, take the $x$-direction to be horizontal and positive to the right, and the $y$-direction to be vertical and positive upward. The normal force between the block and the wedge is $n$; the normal force between the wedge and the horizontal surface will not enter, as the wedge is presumed to have zero vertical acceleration. The horizontal acceleration of the wedge is $A$, and the components of acceleration of the block are $a_x$ and $a_y$.

**EXECUTE:** **(a)** The equations of motion are then $MA = -n\sin\alpha$, $ma_x = n\sin\alpha$ and $ma_y = n\cos\alpha - mg$. Note that the normal force gives the wedge a negative acceleration; the wedge is expected to move to the left. These are three equations in four unknowns, $A$, $a_x, a_y$ and $n$. Solution is possible with the imposition of the relation between $A$, $a_x$ and $a_y$. An observer on the wedge is not in an inertial frame, and should not apply Newton's laws, but the kinematic relation between the components of acceleration are not so restricted. To such an observer, the vertical acceleration of the block is $a_y$, but the horizontal acceleration of the block is $a_x - A$. To this observer, the block descends at an angle $\alpha$, so the relation needed is $\dfrac{a_y}{a_x - A} = -\tan\alpha$. At this point, algebra is unavoidable. A possible approach is to eliminate $a_x$ by noting that $a_x = -\dfrac{M}{m}A$, using this in the kinematic constraint to eliminate $a_y$ and then eliminating $n$. The results are:

$$A = \frac{-gm}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

$$a_x = \frac{gM}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

$$a_y = \frac{-g(M+m)\ \tan\alpha}{(M+m)\ \tan\alpha + (M/\tan\alpha)}$$

**(b)** When $M \gg m,\ A \to 0,$ as expected (the large block won't move). Also,

$a_x \to \dfrac{g}{\tan\alpha + (1/\tan\alpha)} = g\dfrac{\tan\alpha}{\tan^2\alpha+1} = g\sin\alpha\cos\alpha$ which is the acceleration of the block ($g\sin\alpha$ in this case), with the factor of $\cos\alpha$ giving the horizontal component. Similarly, $a_y \to -g\sin^2\alpha.$

**(c)** The trajectory is a straight line with slope $-\left(\dfrac{M+m}{M}\right)\tan\alpha.$

**EVALUATE:** If $m \gg M,$ our general results give $a_x = 0$ and $a_y = -g.$ The massive block accelerates straight downward, as if it were in free fall.

**5.113.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the block and to the wedge.

**SET UP:** From Problem 5.112, $ma_x = n\sin\alpha$ and $ma_y = n\cos\alpha - mg$ for the block. $a_y = 0$ gives $a_x = g\tan\alpha.$

**EXECUTE:** If the block is not to move vertically, both the block and the wedge have this horizontal acceleration and the applied force must be $F = (M+m)a = (M+m)g\tan\alpha.$

**EVALUATE:** $F \to 0$ as $\alpha \to 0$ and $F \to \infty$ as $\alpha \to 90°.$

**5.114.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each of the three masses and to the pulley $B$.

**SET UP:** Take all accelerations to be positive downward. The equations of motion are straightforward, but the kinematic relations between the accelerations, and the resultant algebra, are not immediately obvious. If the acceleration of pulley $B$ is $a_B$, then $a_B = -a_3$, and $a_B$ is the average of the accelerations of masses 1 and 2, or $a_1 + a_2 = 2a_B = -2a_3.$

**EXECUTE:** **(a)** There can be no net force on the massless pulley $B$, so $T_C = 2T_A.$ The five equations to be solved are then $m_1g - T_A = m_1a_1,\ \ m_2g - T_A = m_2a_2,\ \ m_3g - T_C = m_3a_3,\ \ a_1 + a_2 + 2a_3 = 0$ and $2T_A - T_C = 0.$ These are five equations in five unknowns, and may be solved by standard means.

The accelerations $a_1$ and $a_2$ may be eliminated using $2a_3 = -(a_1+a_2) = -[2g - T_A((1/m_1)+(1/m_2))].$

The tension $T_A$ may be eliminated by using $T_A = (1/2)T_C = (1/2)m_3(g - a_3).$

Combining and solving for $a_3$ gives $a_3 = g\dfrac{-4m_1m_2 + m_2m_3 + m_1m_3}{4m_1m_2 + m_2m_3 + m_1m_3}.$

**(b)** The acceleration of the pulley $B$ has the same magnitude as $a_3$ and is in the opposite direction.

**(c)** $a_1 = g - \dfrac{T_A}{m_1} = g - \dfrac{T_C}{2m_1} = g - \dfrac{m_3}{2m_1}(g - a_3).$ Substituting the above expression for $a_3$ gives

$a_1 = g\dfrac{4m_1m_2 - 3m_2m_3 + m_1m_3}{4m_1m_2 + m_2m_3 + m_1m_3}.$

**(d)** A similar analysis (or, interchanging the labels 1 and 2) gives $a_2 = g\dfrac{4m_1m_2 - 3m_1m_3 + m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}.$

**(e), (f)** Once the accelerations are known, the tensions may be found by substitution into the appropriate equation of motion, giving $T_A = g\dfrac{4m_1m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3},\ T_C = g\dfrac{8m_1m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}.$

**(g)** If $m_1 = m_2 = m$ and $m_3 = 2m,$ all of the accelerations are zero, $T_C = 2mg$ and $T_A = mg.$ All masses and pulleys are in equilibrium, and the tensions are equal to the weights they support, which is what is expected.

**EVALUATE:** It is useful to consider special cases. For example, when $m_1 = m_2 \gg m_3$ our general result gives $a_1 = a_2 = +g$ and $a_3 = g.$

**5.115.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the ball at each position.

**SET UP:** When the ball is at rest, $a = 0$. When the ball is swinging in an arc it has acceleration component $a_{\text{rad}} = \dfrac{v^2}{R}$, directed inward.

**EXECUTE:** Before the horizontal string is cut, the ball is in equilibrium, and the vertical component of the tension force must balance the weight, so $T_A \cos\beta = w$ or $T_A = w/\cos\beta$. At point $B$, the ball is not in equilibrium; its speed is instantaneously 0, so there is no radial acceleration, and the tension force must balance the radial component of the weight, so $T_B = w\cos\beta$ and the ratio $(T_B/T_A) = \cos^2\beta$.

**EVALUATE:** At point $B$ the net force on the ball is not zero; the ball has a tangential acceleration.

**5.116.** **IDENTIFY:** The forces must balance for the person not to slip.

**SET UP and EXECUTE:** As was done in earlier problems, balancing forces parallel to and perpendicular to the surface of the rock leads to the equation $\mu_s = \tan\theta = 1.2$, so $\theta = 50°$, which is choice (b).

**EVALUATE:** The condition $\mu_s = \tan\theta$ applies only when the person is just ready to slip, which would be the case at the maximum angle.

**5.117.** **IDENTIFY:** Friction changes from static friction to kinetic friction.

**SET UP and EXECUTE:** When she slipped, static friction must have been at its maximum value, and that was enough to support her weight just before she slipped. But the kinetic friction will be less than the maximum static friction, so the kinetic friction force will not be enough to balance her weight down the incline. Therefore she will slide down the surface and continue to accelerate downward, making (b) the correct choice.

**EVALUATE:** Shoes with a greater coefficient of static friction would enable her to walk more safely.

**5.118.** **IDENTIFY:** The person pushes off horizontally and acclerates herself, so Newton's second law applies.

**SET UP and EXECUTE:** She runs horizontally, so her vertical acceleration is zero, which makes the normal force $n$ due to the ground equal to her weight $mg$. In the horizontal direction, static friction accelerates her forward, and it must be its maximum value to achieve her maximum acceleration. Therefore $f_s = ma = \mu_s n = \mu_s mg$, which gives $a = \mu_s g = 1.2g$, making (d) the correct choice.

**EVALUATE:** Shoes with more friction would allow her to accelerate even faster.

# 6

## WORK AND KINETIC ENERGY

**6.1.** **IDENTIFY** and **SET UP:** For parts (a) through (d), identify the appropriate value of $\phi$ and use the relation $W = F_{\text{P}}s = (F\cos\phi)s$. In part (e), apply the relation $W_{\text{net}} = W_{\text{student}} + W_{\text{grav}} + W_n + W_f$.

**EXECUTE:** **(a)** Since you are applying a horizontal force, $\phi = 0°$. Thus,
$W_{\text{student}} = (2.40\text{ N})(\cos 0°)(1.50\text{ m}) = 3.60\text{ J}$.

**(b)** The friction force acts in the horizontal direction, opposite to the motion, so $\phi = 180°$.
$W_f = (F_f \cos\phi)s = (0.600\text{ N})(\cos 180°)(1.50\text{ m}) = -0.900\text{ J}$.

**(c)** Since the normal force acts upward and perpendicular to the tabletop, $\phi = 90°$.
$W_n = (n\cos\phi)s = (ns)(\cos 90°) = 0.0\text{ J}$.

**(d)** Since gravity acts downward and perpendicular to the tabletop, $\phi = 270°$.
$W_{\text{grav}} = (mg\cos\phi)s = (mgs)(\cos 270°) = 0.0\text{ J}$.

**(e)** $W_{\text{net}} = W_{\text{student}} + W_{\text{grav}} + W_n + W_f = 3.60\text{ J} + 0.0\text{ J} + 0.0\text{ J} - 0.900\text{ J} = 2.70\text{ J}$.

**EVALUATE:** Whenever a force acts perpendicular to the direction of motion, its contribution to the net work is zero.

**6.2.** **IDENTIFY:** In each case the forces are constant and the displacement is along a straight line, so $W = Fs\cos\phi$.

**SET UP:** In part **(a)**, when the cable pulls horizontally $\phi = 0°$ and when it pulls at $35.0°$ above the horizontal $\phi = 35.0°$. In part **(b)**, if the cable pulls horizontally $\phi = 180°$. If the cable pulls on the car at $35.0°$ above the horizontal it pulls on the truck at $35.0°$ below the horizontal and $\phi$ $145.0°$. For the gravity force $\phi = 90°$, since the force is vertical and the displacement is horizontal.

**EXECUTE:** **(a)** When the cable is horizontal, $W = (1350\text{ N})(5.00\times10^3\text{ m})\cos 0° = 6.75\times10^6\text{ J}$. When the cable is $35.0°$ above the horizontal, $W = (1350\text{ N})(5.00\times10^3\text{ m})\cos 35.0° = 5.53\times10^6\text{ J}$.

**(b)** $\cos 180° = -\cos 0°$ and $\cos 145.0° = -\cos 35.0°$, so the answers are $-6.75\times10^6\text{ J}$ and $-5.53\times10^6\text{ J}$.

**(c)** Since $\cos\phi = \cos 90° = 0$, $W = 0$ in both cases.

**EVALUATE:** If the car and truck are taken together as the system, the tension in the cable does no net work.

**6.3.** **IDENTIFY:** Each force can be used in the relation $W = F_{\|}s = (F\cos\phi)s$ for parts (b) through (d). For part (e), apply the net work relation as $W_{\text{net}} = W_{\text{worker}} + W_{\text{grav}} + W_n + W_f$.

**SET UP:** In order to move the crate at constant velocity, the worker must apply a force that equals the force of friction, $F_{\text{worker}} = f_{\text{k}} = \mu_{\text{k}} n$.

**EXECUTE:** **(a)** The magnitude of the force the worker must apply is:

$$F_{\text{worker}} = f_{\text{k}} = \mu_{\text{k}} n = \mu_{\text{k}} mg = (0.25)(30.0\text{ kg})(9.80\text{ m/s}^2) = 74\text{ N}$$

**(b)** Since the force applied by the worker is horizontal and in the direction of the displacement, $\phi = 0°$ and the work is:

$$W_{\text{worker}} = (F_{\text{worker}}\cos\phi)s = [(74\text{ N})(\cos 0°)](4.5\text{ m}) = +333\text{ J}$$

**(c)** Friction acts in the direction opposite of motion, thus $\phi = 180°$ and the work of friction is:

$$W_f = (f_k \cos\phi)s = [(74 \text{ N})(\cos 180°)](4.5 \text{ m}) = -333 \text{ J}$$

**(d)** Both gravity and the normal force act perpendicular to the direction of displacement. Thus, neither force does any work on the crate and $W_{\text{grav}} = W_n = 0.0$ J.

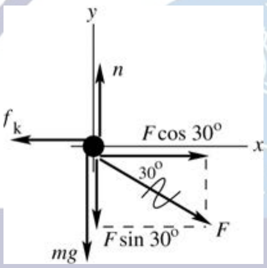
**(e)** Substituting into the net work relation, the net work done on the crate is:

$$W_{\text{net}} = W_{\text{worker}} + W_{\text{grav}} + W_n + W_f = +333 \text{ J} + 0.0 \text{ J} + 0.0 \text{ J} - 333 \text{ J} = 0.0 \text{ J}$$

**EVALUATE:** The net work done on the crate is zero because the two contributing forces, $F_{\text{worker}}$ and $F_f$, are equal in magnitude and opposite in direction.

**6.4.** **IDENTIFY:** The forces are constant so Eq. (6.2) can be used to calculate the work. Constant speed implies $a = 0$. We must use $\Sigma\vec{F} = m\vec{a}$ applied to the crate to find the forces acting on it.

**(a) SET UP:** The free-body diagram for the crate is given in Figure 6.4.




**EXECUTE:** $\Sigma F_y = ma_y$

$n - mg - F\sin 30° = 0$

$n = mg + F\sin 30°$

$f_k = \mu_k n = \mu_k mg + F\mu_k \sin 30°$

**Figure 6.4**

$\Sigma F_x = ma_x$

$F\cos 30° - f_k = 0$

$F\cos 30° - \mu_k mg - \mu_k \sin 30° F = 0$

$$F = \frac{\mu_k mg}{\cos 30° - \mu_k \sin 30°} = \frac{0.25(30.0 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 30° - (0.25)\sin 30°} = 99.2 \text{ N}$$

**(b)** $W_F = (F\cos\phi)s = (99.2 \text{ N})(\cos 30°)(4.5 \text{ m}) = 387 \text{ J}$

($F\cos 30°$ is the horizontal component of $\vec{F}$; the work done by $\vec{F}$ is the displacement times the component of $\vec{F}$ in the direction of the displacement.)

**(c)** We have an expression for $f_k$ from part (a):

$$f_k = \mu_k(mg + F\sin 30°) = (0.250)[(30.0 \text{ kg})(9.80 \text{ m/s}^2) + (99.2 \text{ N})(\sin 30°)] = 85.9 \text{ N}$$

$\phi = 180°$ since $f_k$ is opposite to the displacement. Thus $W_f = (f_k \cos\phi)s = (85.9 \text{ N})(\cos 180°)(4.5 \text{ m}) = -387$ J.

**(d)** The normal force is perpendicular to the displacement so $\phi = 90°$ and $W_n = 0$. The gravity force (the weight) is perpendicular to the displacement so $\phi = 90°$ and $W_w = 0$.

**(e)** $W_{\text{tot}} = W_F + W_f + W_n + W_w = +387 \text{ J} + (-387 \text{ J}) = 0$

**EVALUATE:** Forces with a component in the direction of the displacement do positive work, forces opposite to the displacement do negative work, and forces perpendicular to the displacement do zero work. The total work, obtained as the sum of the work done by each force, equals the work done by the net force. In this problem, $F_{\text{net}} = 0$ since $a = 0$ and $W_{\text{tot}} = 0$, which agrees with the sum calculated in part (e).

**6.5.** **IDENTIFY:** The gravity force is constant and the displacement is along a straight line, so $W = Fs\cos\phi$.

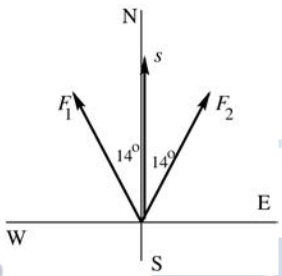
**SET UP:** The displacement is upward along the ladder and the gravity force is downward, so $\phi = 180.0° - 30.0° = 150.0°$. $w = mg = 735$ N.

**EXECUTE:** **(a)** $W = (735 \text{ N})(2.75 \text{ m})\cos 150.0° = -1750$ J.

**(b)** No, the gravity force is independent of the motion of the painter.

**EVALUATE:** Gravity is downward and the vertical component of the displacement is upward, so the gravity force does negative work.

**6.6.** **IDENTIFY** and **SET UP:** $W_F = (F\cos\phi)s,$ since the forces are constant. We can calculate the total work by summing the work done by each force. The forces are sketched in Figure 6.6.




**EXECUTE:** $W_1 = F_1 s\cos\phi_1$

$W_1 = (1.80\times10^6\ \text{N})(0.75\times10^3\ \text{m})\cos14°$

$W_1 = 1.31\times10^9\ \text{J}$

$W_2 = F_2 s\cos\phi_2 = W_1$

**Figure 6.6**

$W_{tot} = W_1 + W_2 = 2(1.31\times10^9\ \text{J}) = 2.62\times10^9\ \text{J}$

**EVALUATE:** Only the component $F\cos\phi$ of force in the direction of the displacement does work. These components are in the direction of $\vec{s}$ so the forces do positive work.

**6.7.** **IDENTIFY:** All forces are constant and each block moves in a straight line, so $W = Fs\cos\phi.$ The only direction the system can move at constant speed is for the 12.0 N block to descend and the 20.0 N block to move to the right.

**SET UP:** Since the 12.0 N block moves at constant speed, $a = 0$ for it and the tension $T$ in the string is $T = 12.0$ N. Since the 20.0 N block moves to the right at constant speed, the friction force $f_k$ on it is to the left and $f_k = T = 12.0$ N.

**EXECUTE:** **(a)** (i) $\phi = 0°$ and $W = (12.0\ \text{N})(0.750\ \text{m})\cos0° = 9.00\ \text{J}.$ (ii) $\phi = 180°$ and $W = (12.0\ \text{N})(0.750\ \text{m})\cos180° = -9.00\ \text{J}.$

**(b)** (i) $\phi = 90°$ and $W = 0.$ (ii) $\phi = 0°$ and $W = (12.0\ \text{N})(0.750\ \text{m})\cos0° = 9.00\ \text{J}.$ (iii) $\phi = 180°$ and $W = (12.0\ \text{N})(0.750\ \text{m})\cos180° = -9.00\ \text{J}.$ (iv) $\phi = 90°$ and $W = 0.$

**(c)** $W_{tot} = 0$ for each block.

**EVALUATE:** For each block there are two forces that do work, and for each block the two forces do work of equal magnitude and opposite sign. When the force and displacement are in opposite directions, the work done is negative.

**6.8.** **IDENTIFY:** Apply Eq. (6.5).

**SET UP:** $\hat{\boldsymbol{i}}\cdot\hat{\boldsymbol{i}} = \hat{\boldsymbol{j}}\cdot\hat{\boldsymbol{j}} = 1$ and $\hat{\boldsymbol{i}}\cdot\hat{\boldsymbol{j}} = \hat{\boldsymbol{j}}\cdot\hat{\boldsymbol{i}} = 0$

**EXECUTE:** The work you do is $\vec{F}\cdot\vec{s} = \left[(30\ \text{N})\hat{\boldsymbol{i}} - (40\ \text{N})\hat{\boldsymbol{j}}\right]\cdot\left[(-9.0\ \text{m})\hat{\boldsymbol{i}} - (3.0\ \text{m})\hat{\boldsymbol{j}}\right]$

$\vec{F}\cdot\vec{s} = (30\ \text{N})(-9.0\ \text{m}) + (-40\ \text{N})(-3.0\ \text{m}) = -270\ \text{N}\cdot\text{m} + 120\ \text{N}\cdot\text{m} = -150\ \text{J}.$

**EVALUATE:** The $x$-component of $\vec{F}$ does negative work and the $y$-component of $\vec{F}$ does positive work. The total work done by $\vec{F}$ is the sum of the work done by each of its components.

**6.9.** **IDENTIFY:** Apply Eq. (6.2) or (6.3).

**SET UP:** The gravity force is in the $-y$-direction, so $\vec{F}_{mg}\cdot\vec{s} = -mg(y_2 - y_1)$

**EXECUTE:** **(a)** (i) Tension force is always perpendicular to the displacement and does no work.
(ii) Work done by gravity is $-mg(y_2 - y_1).$ When $y_1 = y_2,$ $W_{mg} = 0.$

**(b)** (i) Tension does no work. (ii) Let $l$ be the length of the string. $W_{mg} = -mg(y_2 - y_1) = -mg(2l) = -25.1\ \text{J}$

**EVALUATE:** In part (b) the displacement is upward and the gravity force is downward, so the gravity force does negative work.

**6.10.** **IDENTIFY** and **SET UP:** Use $W = F_p s = (F\cos\phi)s$ to calculate the work done in each of parts (a) through (c). In part (d), the net work consists of the contributions due to all three forces, or $w_{net} = w_{grav} + w_n + w_f$.

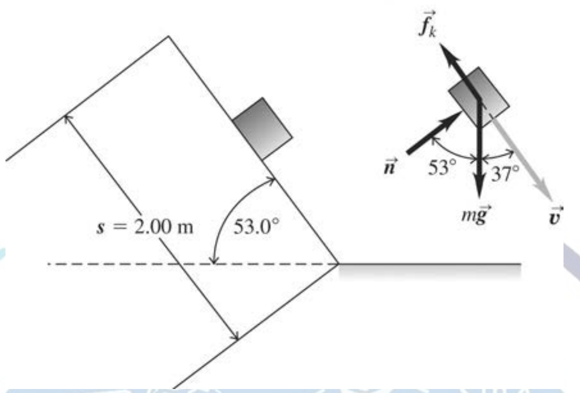



**Figure 6.10**

**EXECUTE:** **(a)** As the package slides, work is done by the frictional force which acts at $\phi = 180°$ to the displacement. The normal force is $mg\cos 53.0°$. Thus for $\mu_k = 0.40$,

$W_f = F_p s = (f_k \cos\phi)s = (\mu_k n \cos\phi)s = [\mu_k(mg\cos 53.0°)](\cos 180°)s.$

$W_f = (0.40)[(12.0\text{ kg})(9.80\text{ m/s}^2)(\cos 53.0°)](\cos 180°)(2.00\text{ m}) = -57\text{ J}.$

**(b)** Work is done by the component of the gravitational force parallel to the displacement. $\phi = 90° - 53° = 37°$ and the work of gravity is

$W_{grav} = (mg\cos\phi)s = [(12.0\text{ kg})(9.80\text{ m/s}^2)(\cos 37.0°)](2.00\text{ m}) = +188\text{ J}.$

**(c)** $W_n = 0$ since the normal force is perpendicular to the displacement.

**(d)** The net work done on the package is $W_{net} = W_{grav} + W_n + W_f = 188\text{ J} + 0.0\text{ J} - 57\text{ J} = 131\text{ J}.$

**EVALUATE:** The net work is positive because gravity does more positive work than the magnitude of the negative work done by friction.

**6.11.** **IDENTIFY:** As the carton is pulled up the ramp, the forces acting on it are gravity, the tension in the rope, and the normal force. Each of these forces may do work on the carton.

**SET UP:** Use $W = F_{\parallel}s = (F\cos\phi)s$. Calculate the work done by each force. In each case, identify the angle $\phi$. In part (d), the net work is the algebraic sum of the work done by each force.

**EXECUTE:** **(a)** Since the force exerted by the rope and the displacement are in the same direction, $\phi = 0°$ and $W_{rope} = (72.0\text{ N})(\cos 0°)(5.20\text{ m}) = +374\text{ J}.$

**(b)** Gravity is downward and the displacement is at $30.0°$ above the horizontal, so $\phi = 90.0° + 30.0° = 120.0°$. $W_{grav} = (128.0\text{ N})(\cos 120°)(5.20\text{ m}) = -333\text{ J}.$

**(c)** The normal force $n$ is perpendicular to the surface of the ramp while the displacement is parallel to the surface of the ramp, so $\phi = 90°$ and $W_n = 0.$

**(d)** $W_{net} = W_{rope} + W_{grav} + W_n = +374\text{ J} - 333\text{ J} + 0 = +41\text{ J}$

**(e)** Now $\phi = 50.0° - 30.0° = 20.0°$ and $W_{rope} = (72.0\text{ N})(\cos 20.0°)(5.20\text{ m}) = +352\text{ J}$

**EVALUATE:** In part (b), gravity does negative work since the gravity force acts downward and the carton moves upward. Less work is done by the rope in part (e), but the net work is still positive.

**6.12.** **IDENTIFY:** Since the speed is constant, the acceleration and the net force on the monitor are zero.

**SET UP:** Use the fact that the net force on the monitor is zero to develop expressions for the friction force, $f_k$, and the normal force, $n$. Then use $W = F_P s = (F\cos\phi)s$ to calculate $W$.

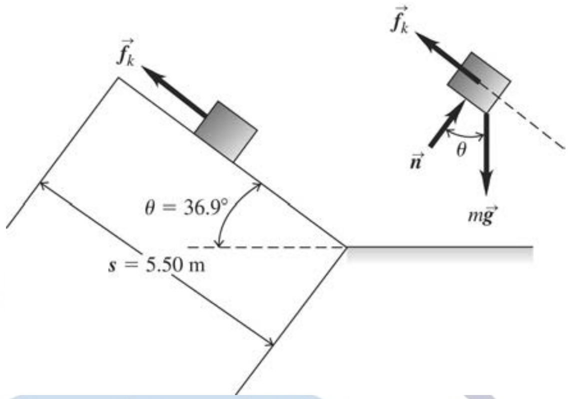



**Figure 6.12**

**EXECUTE:** **(a)** Summing forces along the incline, $\Sigma F = ma = 0 = f_k - mg\sin\theta$, giving $f_k = mg\sin\theta$, directed up the incline. Substituting gives $W_f = (f_k \cos\phi)s = [(mg\sin\theta)\cos\phi]s$.

$W_f = [(10.0 \text{ kg})(9.80 \text{ m/s}^2)(\sin 36.9°)](\cos 0°)(5.50 \text{ m}) = +324 \text{ J}$.

**(b)** The gravity force is downward and the displacement is directed up the incline so $\phi = 126.9°$.

$W_{grav} = (10.0 \text{ kg})(9.80 \text{ m/s}^2)(\cos 126.9°)(5.50 \text{ m}) = -324 \text{ J}$.

**(c)** The normal force, $n$, is perpendicular to the displacement and thus does zero work.

**EVALUATE:** Friction does positive work and gravity does negative work. The net work done is zero.

**6.13.** **IDENTIFY:** We want the work done by a known force acting through a known displacement.

**SET UP:** $W = Fs\cos\phi$

**EXECUTE:** $W = (48.0 \text{ N})(12.0 \text{ m})\cos(173°) = -572 \text{ J}$.

**EVALUATE:** The force has a component opposite to the displacement, so it does negative work.

**6.14.** **IDENTIFY:** We want to find the work done by a known force acting through a known displacement.

**SET UP:** $W = \vec{F}\cdot\vec{s} = F_x s_x + F_y s_y$. We know the components of $\vec{F}$ but need to find the components of the displacement $\vec{s}$.

**EXECUTE:** Using the magnitude and direction of $\vec{s}$, its components are $x = (48.0 \text{ m})\cos 240.0^{\text{o}} = -24.0$ m and $y = (48.0 \text{ m})\sin 240.0^{\text{o}} = -41.57$ m. Therefore, $\vec{s} = (-24.0 \text{ m})\hat{i} + (-41.57 \text{ m})\hat{j}$. The definition of work gives $W = \vec{F}\cdot\vec{s} = (-68.0 \text{ N})(-24.0 \text{ m}) + (36.0 \text{ N})(-41.57 \text{ m}) = +1632 \text{ J} - 1497 \text{ J} = +135 \text{ J}$.

**EVALUATE:** The mass of the car is not needed since it is the given force that is doing the work.

**6.15.** **IDENTIFY:** We want the work done by the force, and we know the force and the displacement in terms of their components.

**SET UP:** We can use either $W = \vec{F}\cdot\vec{s} = F_x s_x + F_y s_y$ or $W = Fs\cos\phi$, depending on what we know.

**EXECUTE:** (a) We know the magnitudes of the two given vectors and the angle between them, so $W = Fs\cos\phi = (30.0 \text{ N})(5.00 \text{ m})(\cos 37°) = 120 \text{ J}$.

(b) As in (a), we have $W = Fs\cos\phi = (30.0 \text{ N})(6.00 \text{ m})(\cos 127°) = -108 \text{ J}$.

(c) We know the components of both vectors, so we use $W = \vec{F}\cdot\vec{s} = F_x s_x + F_y s_y$.

$W = \vec{F}\cdot\vec{s} = F_x s_x + F_y s_y = (30.0 \text{ N})(\cos 37°)(-2.00 \text{ m}) + (30.00 \text{ N})(\sin 37°)(4.00 \text{ m}) = 24.3 \text{ J}$.

**EVALUATE:** We could check parts (a) and (b) using the method from part (c).

**6.16.** **IDENTIFY:** The book changes its speed and hence its kinetic energy, so work must have been done on it.

**SET UP:** Use the work-kinetic energy theorem $W_{net} = K_f - K_i$, with $K = \frac{1}{2}mv^2$. In part (a) use $K_i$ and $K_f$ to calculate $W$. In parts (b) and (c) use $K_i$ and $W$ to calculate $K_f$.

**EXECUTE:** **(a)** Substituting the notation $i = A$ and $f = B$,

$W_{net} = K_B - K_A = \frac{1}{2}(1.50 \text{ kg})[(1.25 \text{ m/s})^2 - (3.21 \text{ m/s})^2] = -6.56 \text{ J}$.

**(b)** Noting $\text{i} = B$ and $\text{f} = C$, $K_C = K_B + W_{\text{net}} = \frac{1}{2}(1.50\text{ kg})(1.25\text{ m/s})^2 - 0.750\text{ J} = +0.422\text{ J}$. $K_C = \frac{1}{2}mv_C^2$ so $v_C = \sqrt{2K_C/m} = 0.750\text{ m/s}$.

**(c)** Similarly, $K_C = \frac{1}{2}(1.50\text{ kg})(1.25\text{ m/s})^2 + 0.750\text{ J} = 1.922\text{ J}$ and $v_C = 1.60\text{ m/s}$.

**EVALUATE:** Negative $W_{\text{net}}$ corresponds to a decrease in kinetic energy (slowing down) and positive $W_{\text{net}}$ corresponds to an increase in kinetic energy (speeding up).

**6.17.** **IDENTIFY:** Find the kinetic energy of the cheetah knowing its mass and speed.

**SET UP:** Use $K = \frac{1}{2}mv^2$ to relate $v$ and $K$.

**EXECUTE:** **(a)** $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(70\text{ kg})(32\text{ m/s})^2 = 3.6\times10^4\text{ J}$.

**(b)** $K$ is proportional to $v^2$, so $K$ increases by a factor of 4 when $v$ doubles.

**EVALUATE:** A running person, even with a mass of 70 kg, would have only 1/100 of the cheetah's kinetic energy since a person's top speed is only about 1/10 that of the cheetah.

**6.18.** **IDENTIFY:** Use the equations for free-fall to find the speed of the weight when it reaches the ground and use the formula for kinetic energy.

**SET UP:** Kinetic energy is $K = \frac{1}{2}mv^2$. The mass of an electron is $9.11\times10^{-31}$ kg. In part (b) take $+y$ downward, so $a_y = +9.80\text{ m/s}^2$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$.

**EXECUTE:** **(a)** $K = \frac{1}{2}(9.11\times10^{-31}\text{ kg})(2.19\times10^6\text{ m/s})^2 = 2.18\times10^{-18}\text{ J}$.

**(b)** $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(9.80\text{ m/s}^2)(1.0\text{ m})} = 4.43\text{ m/s}$. $K = \frac{1}{2}(1.0\text{ kg})(4.43\text{ m/s})^2 = 9.8\text{ J}$.

**(c)** Solving $K = \frac{1}{2}mv^2$ for $v$ gives $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(100\text{ J})}{30\text{ kg}}} = 2.6\text{ m/s}$. Yes, this is reasonable.

**EVALUATE:** A running speed of 6 m/s corresponds to running a 100-m dash in about 17 s, so 2.6 m/s is reasonable for a running child.

**6.19.** **IDENTIFY:** $K = \frac{1}{2}mv^2$. Since the meteor comes to rest the energy it delivers to the ground equals its original kinetic energy.

**SET UP:** $v = 12\text{ km/s} = 1.2\times10^4\text{ m/s}$. A 1.0 megaton bomb releases $4.184\times10^{15}$ J of energy.

**EXECUTE:** **(a)** $K = \frac{1}{2}(1.4\times10^8\text{ kg})(1.2\times10^4\text{ m/s})^2 = 1.0\times10^{16}\text{ J}$.

**(b)** $\dfrac{1.0\times10^{16}\text{ J}}{4.184\times10^{15}\text{ J}} = 2.4$. The energy is equivalent to 2.4 one-megaton bombs.

**EVALUATE:** Part of the energy transferred to the ground lifts soil and rocks into the air and creates a large crater.

**6.20.** **IDENTIFY:** Only gravity does work on the watermelon, so $W_{\text{tot}} = W_{\text{grav}}$. $W_{\text{tot}} = \Delta K$ and $K = \frac{1}{2}mv^2$.

**SET UP:** Since the watermelon is dropped from rest, $K_1 = 0$.

**EXECUTE:** **(a)** $W_{\text{grav}} = mgs = (4.80\text{ kg})(9.80\text{ m/s}^2)(18.0\text{ m}) = 847\text{ J}$.

**(b) (i)** $W_{\text{tot}} = K_2 - K_1$ so $K_2 = 847\text{ J}$. **(ii)** $v = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(847\text{ J})}{4.80\text{ kg}}} = 18.8\text{ m/s}$.

**(c)** The work done by gravity would be the same. Air resistance would do negative work and $W_{\text{tot}}$ would be less than $W_{\text{grav}}$. The answer in (a) would be unchanged and both answers in (b) would decrease.

**EVALUATE:** The gravity force is downward and the displacement is downward, so gravity does positive work.

**6.21.** **IDENTIFY:** $W_{\text{tot}} = K_2 - K_1$. In each case calculate $W_{\text{tot}}$ from what we know about the force and the displacement.

**SET UP:** The gravity force is $mg$, downward. The mass of the object isn't given, so we expect that it will divide out in the calculation.

**EXECUTE:** **(a)** $K_1 = 0$. $W_{tot} = W_{grav} = mgs$. $mgs = \frac{1}{2}mv_2^2$ and

$v_2 = \sqrt{2gs} = \sqrt{2(9.80\text{ m/s}^2)(95.0\text{ m})} = 43.2\text{ m/s}$.

**(b)** $K_2 = 0$ (at the maximum height). $W_{tot} = W_{grav} = -mgs$. $-mgs = -\frac{1}{2}mv_1^2$ and

$v_1 = \sqrt{2gs} = \sqrt{2(9.80\text{ m/s}^2)(525\text{ m})} = 101\text{ m/s}$.

**EVALUATE:** In part (a), gravity does positive work and the speed increases. In part (b), gravity does negative work and the speed decreases.

**6.22.** **IDENTIFY:** $W_{tot} = K_2 - K_1$. In each case calculate $W_{tot}$ from what we know about the force and the displacement.

**SET UP:** The gravity force is $mg$, downward. The friction force is $f_k = \mu_k n = \mu_k mg$ and is directed opposite to the displacement. The mass of the object isn't given, so we expect that it will divide out in the calculation.

**EXECUTE:** **(a)** $K_1 = \frac{1}{2}mv_1^2$. $K_2 = 0$. $W_{tot} = W_f = -\mu_k mgs$. $-\mu_k mgs = -\frac{1}{2}mv_1^2$.

$$s = \frac{v_1^2}{2\mu_k g} = \frac{(5.00\text{ m/s})^2}{2(0.220)(9.80\text{ m/s}^2)} = 5.80\text{ m}.$$

**(b)** $K_1 = \frac{1}{2}mv_1^2$. $K_2 = \frac{1}{2}mv_2^2$. $W_{tot} = W_f = -\mu_k mgs$. $K_2 = W_{tot} + K_1$. $\frac{1}{2}mv_2^2 = -\mu_k mgs + \frac{1}{2}mv_1^2$.

$v_2 = \sqrt{v_1^2 - 2\mu_k gs} = \sqrt{(5.00\text{ m/s})^2 - 2(0.220)(9.80\text{ m/s}^2)(2.90\text{ m})} = 3.53\text{ m/s}$.

**(c)** $K_1 = \frac{1}{2}mv_1^2$. $K_2 = 0$. $W_{grav} = -mgy_2$, where $y_2$ is the vertical height. $-mgy_2 = -\frac{1}{2}mv_1^2$ and

$$y_2 = \frac{v_1^2}{2g} = \frac{(12.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 7.35\text{ m}.$$

**EVALUATE:** In parts (a) and (b), friction does negative work and the kinetic energy is reduced. In part (c), gravity does negative work and the speed decreases. The vertical height in part (c) is independent of the slope angle of the hill.

**6.23.** **IDENTIFY** and **SET UP:** Apply Eq. (6.6) to the box. Let point 1 be at the bottom of the incline and let point 2 be at the skier. Work is done by gravity and by friction. Solve for $K_1$ and from that obtain the required initial speed.

**EXECUTE:** $W_{tot} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2$, $K_2 = 0$

Work is done by gravity and friction, so $W_{tot} = W_{mg} + W_f$.

$W_{mg} = -mg(y_2 - y_1) = -mgh$

$W_f = -fs$. The normal force is $n = mg\cos\alpha$ and $s = h/\sin\alpha$, where $s$ is the distance the box travels along the incline.

$W_f = -(\mu_k mg\cos\alpha)(h/\sin\alpha) = -\mu_k mgh/\tan\alpha$

Substituting these expressions into the work-energy theorem gives $-mgh - \mu_k mgh/\tan\alpha = -\frac{1}{2}mv_0^2$.

Solving for $v_0$ then gives $v_0 = \sqrt{2gh(1 + \mu_k/\tan\alpha)}$.

**EVALUATE:** The result is independent of the mass of the box. As $\alpha \to 90°$, $h = s$ and $v_0 = \sqrt{2gh}$, the same as throwing the box straight up into the air. For $\alpha = 90°$ the normal force is zero so there is no friction.

**6.24.** **IDENTIFY:** From the work-energy relation, $W = W_{grav} = \Delta K_{rock}$.

**SET UP:** As the rock rises, the gravitational force, $F = mg$, does work on the rock. Since this force acts in the direction opposite to the motion and displacement, $s$, the work is negative. Let $h$ be the vertical distance the rock travels.

**EXECUTE:** **(a)** Applying $W_{\text{grav}} = K_2 - K_1$ we obtain $-mgh = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$. Dividing by $m$ and solving for $v_1$, $v_1 = \sqrt{v_2^2 + 2gh}$. Substituting $h = 15.0$ m and $v_2 = 25.0$ m/s,

$$v_1 = \sqrt{(25.0 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(15.0 \text{ m})} = 30.3 \text{ m/s}$$

**(b)** Solve the same work-energy relation for $h$. At the maximum height $v_2 = 0$.

$$-mgh = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \text{ and } h = \frac{v_1^2 - v_2^2}{2g} = \frac{(30.3 \text{ m/s})^2 - (0.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 46.8 \text{ m}.$$

**EVALUATE:** Note that the weight of the rock was never used in the calculations because both gravitational potential and kinetic energy are proportional to mass, $m$. Thus any object, that attains 25.0 m/s at a height of 15.0 m, must have an initial velocity of 30.3 m/s. As the rock moves upward gravity does negative work and this reduces the kinetic energy of the rock.

**6.25.** **IDENTIFY:** Apply $W = Fs\cos\phi$ and $W_{\text{tot}} = \Delta K$.

**SET UP:** $\phi = 0°$

**EXECUTE:** From Eqs. (6.1), (6.5) and (6.6), and solving for $F$,

$$F = \frac{\Delta K}{s} = \frac{\frac{1}{2}m(v_2^2 - v_1^2)}{s} = \frac{\frac{1}{2}(12.0 \text{ kg})\left[(6.00 \text{ m/s})^2 - (4.00 \text{ m/s})^2\right]}{(2.50 \text{ m})} = 48.0 \text{ N}$$

**EVALUATE:** The force is in the direction of the displacement, so the force does positive work and the kinetic energy of the object increases.

**6.26.** **IDENTIFY:** Apply $W = Fs\cos\phi$ and $W_{\text{tot}} = \Delta K$.

**SET UP:** Parallel to incline: force component $W_{\parallel} = mg\sin\alpha$, down incline; displacement $s = h/\sin\alpha$, down incline. Perpendicular to the incline: $s = 0$.

**EXECUTE:** **(a)** $W_{\parallel} = (mg\sin\alpha)(h/\sin\alpha) = mgh$. $W_{\perp} = 0$, since there is no displacement in this direction.

$W_{mg} = W_{\parallel} + W_{\perp} = mgh$, same as falling height $h$.

**(b)** $W_{\text{tot}} = K_2 - K_1$ gives $mgh = \frac{1}{2}mv^2$ and $v = \sqrt{2gh}$, same as if had been dropped from height $h$. The work done by gravity depends only on the vertical displacement of the object. When the slope angle is small, there is a small force component in the direction of the displacement but a large displacement in this direction. When the slope angle is large, the force component in the direction of the displacement along the incline is larger but the displacement in this direction is smaller.

**(c)** $h = 15.0\text{ m}$, so $v = \sqrt{2gh} = 17.1\text{ s}$.

**EVALUATE:** The acceleration and time of travel are different for an object sliding down an incline and an object in free-fall, but the final velocity is the same in these two cases.

**6.27.** **IDENTIFY:** Apply $W_{\text{tot}} = \Delta K$.

**SET UP:** $v_1 = 0$, $v_2 = v$. $f_k = \mu_k mg$ and $f_k$ does negative work. The force $F = 36.0$ N is in the direction of the motion and does positive work.

**EXECUTE:** **(a)** If there is no work done by friction, the final kinetic energy is the work done by the applied force, and solving for the speed,

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2Fs}{m}} = \sqrt{\frac{2(36.0 \text{ N})(1.20 \text{ m})}{(4.30 \text{ kg})}} = 4.48 \text{ m/s}.$$

**(b)** The net work is $Fs - f_k s = (F - \mu_k mg)s$, so

$$v = \sqrt{\frac{2(F - \mu_k mg)s}{m}} = \sqrt{\frac{2(36.0 \text{ N} - (0.30)(4.30 \text{ kg})(9.80 \text{ m/s}^2)(1.20 \text{ m})}{(4.30 \text{ kg})}} = 3.61 \text{ m/s}$$

**EVALUATE:** The total work done is larger in the absence of friction and the final speed is larger in that case.

**6.28.** **IDENTIFY** and **SET UP:** Use Eq. (6.6) to calculate the work done by the foot on the ball. Then use Eq. (6.2) to find the distance over which this force acts.

**EXECUTE:** $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.420\text{ kg})(2.00\text{ m/s})^2 = 0.84\text{ J}$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.420\text{ kg})(6.00\text{ m/s})^2 = 7.56\text{ J}$

$W_{\text{tot}} = K_2 - K_1 = 7.56\text{ J} - 0.84\text{ J} = 6.72\text{ J}$

The 40.0 N force is the only force doing work on the ball, so it must do 6.72 J of work. $W_F = (F\cos\phi)s$ gives that $s = \dfrac{W}{F\cos\phi} = \dfrac{6.72\text{ J}}{(40.0\text{ N})(\cos 0)} = 0.168\text{ m.}$

**EVALUATE:** The force is in the direction of the motion so positive work is done and this is consistent with an increase in kinetic energy.

**6.29.** **(a) IDENTIFY** and **SET UP:** Use $W_F = (F\cos\phi)s$ to find the work done by the force. Then use $W_{\text{tot}} = K_2 - K_1$ to find the final kinetic energy, and then $K_2 = \frac{1}{2}mv_2^2$ gives the final speed.

**EXECUTE:** $W_{\text{tot}} = K_2 - K_1$, so $K_2 = W_{\text{tot}} + K_1$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(7.00\text{ kg})(4.00\text{ m/s})^2 = 56.0\text{ J}$

The only force that does work on the wagon is the 10.0 N force. This force is in the direction of the displacement so $\phi = 0°$ and the force does positive work:

$$W_F = (F\cos\phi)s = (10.0\text{ N})(\cos 0)(3.0\text{ m}) = 30.0\text{ J}$$

Then $K_2 = W_{\text{tot}} + K_1 = 30.0\text{ J} + 56.0\text{ J} = 86.0\text{ J.}$

$$K_2 = \tfrac{1}{2}mv_2^2;\quad v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(86.0\text{ J})}{7.00\text{ kg}}} = 4.96\text{ m/s}$$

**(b) IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the wagon to calculate $a$. Then use a constant acceleration equation to calculate the final speed. The free-body diagram is given in Figure 6.29.

**SET UP:**

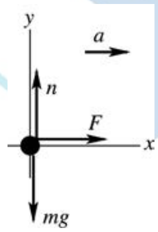



Figure 6.29

**EXECUTE:** $\Sigma F_x = ma_x$

$F = ma_x$

$a_x = \dfrac{F}{m} = \dfrac{10.0\text{ N}}{7.00\text{ kg}} = 1.43\text{ m/s}^2$

$v_{2x}^2 = v_{1x}^2 + 2a_2(x - x_0)$

$v_{2x} = \sqrt{v_{1x}^2 + 2a_x(x - x_0)} = \sqrt{(4.00\text{ m/s})^2 + 2(1.43\text{ m/s}^2)(3.0\text{ m})} = 4.96\text{ m/s}$

**EVALUATE:** This agrees with the result calculated in part (a). The force in the direction of the motion does positive work and the kinetic energy and speed increase. In part (b), the equivalent statement is that the force produces an acceleration in the direction of the velocity and this causes the magnitude of the velocity to increase.

**6.30.** **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$.

**SET UP:** $K_1 = 0$. The normal force does no work. The work $W$ done by gravity is $W = mgh$, where $h = L\sin\theta$ is the vertical distance the block has dropped when it has traveled a distance $L$ down the incline and $\theta$ is the angle the plane makes with the horizontal.

**EXECUTE:** The work-energy theorem gives $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2W}{m}} = \sqrt{2gh} = \sqrt{2gL\sin\theta}$. Using the given numbers, $v = \sqrt{2(9.80\text{ m/s}^2)(1.35\text{ m})\sin 36.9^\circ} = 3.99\text{ m/s}$.

**EVALUATE:** The final speed of the block is the same as if it had been dropped from a height $h$.

**6.31.** **IDENTIFY:** $W_{\text{tot}} = K_2 - K_1$. Only friction does work.

**SET UP:** $W_{\text{tot}} = W_{f_k} = -\mu_k mgs$. $K_2 = 0$ (car stops). $K_1 = \frac{1}{2}mv_0^2$.

**EXECUTE:** **(a)** $W_{\text{tot}} = K_2 - K_1$ gives $-\mu_k mgs = -\frac{1}{2}mv_0^2$. $s = \dfrac{v_0^2}{2\mu_k g}$.

**(b)** (i) $\mu_{kb} = 2\mu_{ka}$. $s\mu_k = \dfrac{v_0^2}{2g} = \text{constant}$ so $s_a\mu_{ka} = s_b\mu_{kb}$. $s_b = \left(\dfrac{\mu_{ka}}{\mu_{kb}}\right)s_a = s_a/2$. The minimum stopping distance would be halved. (ii) $v_{0b} = 2v_{0a}$. $\dfrac{s}{v_0^2} = \dfrac{1}{2\mu_k g} = \text{constant}$, so $\dfrac{s_a}{v_{0a}^2} = \dfrac{s_b}{v_{0b}^2}$. $s_b = s_a\left(\dfrac{v_{0b}}{v_{0a}}\right)^2 = 4s_a$. The stopping distance would become 4 times as great. (iii) $v_{0b} = 2v_{0a}$, $\mu_{kb} = 2\mu_{ka}$. $\dfrac{s\mu_k}{v_0^2} = \dfrac{1}{2g} = \text{constant}$, so $\dfrac{s_a\mu_{ka}}{v_{0a}^2} = \dfrac{s_b\mu_{kb}}{v_{0b}^2}$. $s_b = s_a\left(\dfrac{\mu_{ka}}{\mu_{kb}}\right)\left(\dfrac{v_{0b}}{v_{0a}}\right)^2 = s_a\left(\dfrac{1}{2}\right)(2)^2 = 2s_a$. The stopping distance would double.

**EVALUATE:** The stopping distance is directly proportional to the square of the initial speed and indirectly proportional to the coefficient of kinetic friction.

**6.32.** **IDENTIFY:** We know (or can calculate) the change in the kinetic energy of the crate and want to find the work needed to cause this change, so the work-energy theorem applies.

**SET UP:** $W_{\text{tot}} = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$.

**EXECUTE:** $W_{\text{tot}} = K_f - K_i = \frac{1}{2}(30.0\text{ kg})(5.62\text{ m/s})^2 - \frac{1}{2}(30.0\text{ kg})(3.90\text{ m/s})^2$.

$W_{\text{tot}} = 473.8\text{ J} - 228.2\text{ J} = 246\text{ J}$.

**EVALUATE:** Kinetic energy is a scalar and does not depend on direction, so only the initial and final speeds are relevant.

**6.33.** **IDENTIFY:** The elastic aortal material behaves like a spring, so we can apply Hooke's law to it.

**SET UP:** $|F_{\text{spr}}| = F$, where $F$ is the pull on the strip or the force the strip exerts, and $F = kx$.

**EXECUTE:** **(a)** Solving $F = kx$ for $k$ gives $k = \dfrac{F}{x} = \dfrac{1.50\text{ N}}{0.0375\text{ m}} = 40.0\text{ N/m}$.

**(b)** $F = kx = (40.0\text{ N/m})(0.0114\text{ m}) = 0.456\text{ N}$.

**EVALUATE:** It takes 0.40 N to stretch this material by 1.0 cm, so it is not as stiff as many laboratory springs.

**6.34.** **IDENTIFY:** The work that must be done to move the end of a spring from $x_1$ to $x_2$ is $W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$. The force required to hold the end of the spring at displacement $x$ is $F_x = kx$.

**SET UP:** When the spring is at its unstretched length, $x = 0$. When the spring is stretched, $x > 0$, and when the spring is compressed, $x < 0$.

**EXECUTE:** **(a)** $x_1 = 0$ and $W = \frac{1}{2}kx_2^2$. $k = \dfrac{2W}{x_2^2} = \dfrac{2(12.0\text{ J})}{(0.0300\text{ m})^2} = 2.67\times 10^4\text{ N/m}$.

**(b)** $F_x = kx = (2.67\times 10^4\text{ N/m})(0.0300\text{ m}) = 801\text{ N}$.

**(c)** $x_1 = 0$, $x_2 = -0.0400\text{ m}$. $W = \frac{1}{2}(2.67\times 10^4\text{ N/m})(-0.0400\text{ m})^2 = 21.4\text{ J}$.

$F_x = kx = (2.67\times 10^4\text{ N/m})(0.0400\text{ m}) = 1070\text{ N}$.

**EVALUATE:** When a spring, initially unstretched, is either compressed or stretched, positive work is done by the force that moves the end of the spring.

**6.35.** **IDENTIFY:** The springs obey Hooke's law and balance the downward force of gravity.

**SET UP:** Use coordinates with $+y$ upward. Label the masses 1, 2, and 3, with 1 the top mass and 3 the bottom mass, and call the amounts the springs are stretched $x_1$, $x_2$, and $x_3$. Each spring force is $kx$.

**EXECUTE:** **(a)** The three free-body diagrams are shown in Figure 6.35.

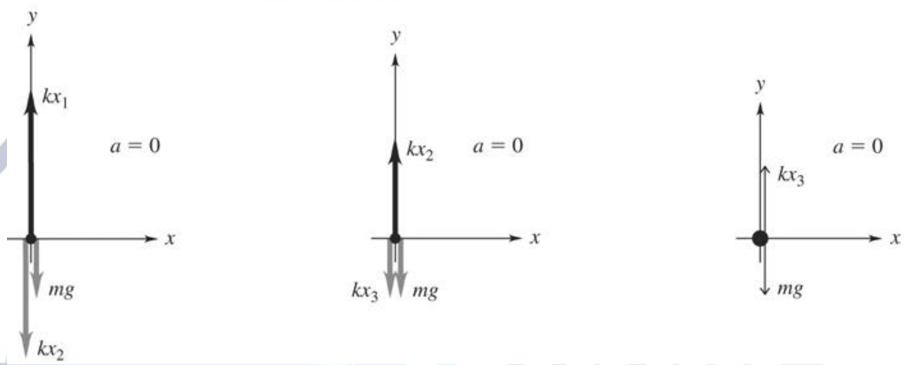



**Figure 6.35**

**(b)** Balancing forces on each of the masses and using $F = kx$ gives $kx_3 = mg$ so

$$x_3 = \frac{mg}{k} = \frac{(8.50\text{ kg})(9.80\text{ m/s}^2)}{7.80\times 10^3\text{ N/m}} = 1.068\text{ cm}. \quad kx_2 = mg + kx_3 = 2mg \text{ so } x_2 = 2\left(\frac{mg}{k}\right) = 2.136\text{ cm}.$$

$$kx_1 = mg + kx_2 = 3mg \text{ so } x_3 = 3\left(\frac{mg}{k}\right) = 3.204\text{ cm}.$$ Adding the original lengths to the distance stretched, the lengths of the springs, starting from the bottom one, are 13.1 cm, 14.1 cm, and 15.2 cm.

**EVALUATE:** The top spring stretches most because it supports the most weight, while the bottom spring stretches least because it supports the least weight.

**6.36.** **IDENTIFY:** The magnitude of the work can be found by finding the area under the graph.

**SET UP:** The area under each triangle is $1/2 \text{ base} \times \text{height}$. $F_x > 0$, so the work done is positive when $x$ increases during the displacement.

**EXECUTE:** **(a)** $1/2\ (8\text{ m})(10\text{ N}) = 40\text{ J}$.

**(b)** $1/2\ (4\text{ m})(10\text{ N}) = 20\text{ J}$.

**(c)** $1/2\ (12\text{ m})(10\text{ N}) = 60\text{ J}$.

**EVALUATE:** The sum of the answers to parts (a) and (b) equals the answer to part (c).

**6.37.** **IDENTIFY:** Use the work-energy theorem and the results of Problem 6.36.

**SET UP:** For $x = 0$ to $x = 8.0$ m, $W_{\text{tot}} = 40$ J. For $x = 0$ to $x = 12.0$ m, $W_{\text{tot}} = 60$ J.

**EXECUTE:** **(a)** $v = \sqrt{\dfrac{(2)(40\text{ J})}{10\text{ kg}}} = 2.83\text{ m/s}$

**(b)** $v = \sqrt{\dfrac{(2)(60\text{ J})}{10\text{ kg}}} = 3.46\text{ m/s}$.

**EVALUATE:** $\vec{F}$ is always in the $+x$-direction. For this motion $\vec{F}$ does positive work and the speed continually increases during the motion.

**6.38.** **IDENTIFY:** The spring obeys Hooke's law.

**SET UP:** Solve $F = kx$ for $x$ to determine the length of stretch and use $W = +\frac{1}{2}kx^2$ to assess the corresponding work.

**EXECUTE:** $x = \dfrac{F}{k} = \dfrac{15.0\text{ N}}{300.0\text{ N/m}} = 0.0500\text{ m}.$ The new length will be $0.240\text{ m} + 0.0500\text{ m} = 0.290\text{ m}.$

The corresponding work done is $W = \dfrac{1}{2}(300.0\text{ N/m})(0.0500\text{ m})^2 = 0.375\text{ J}.$

**EVALUATE:** In $F = kx$, $F$ is always the force applied to one end of the spring, thus we did not need to double the 15.0 N force. Consider a free-body diagram of a spring at rest; forces of equal magnitude and opposite direction are always applied to both ends of every section of the spring examined.

**6.39.** **IDENTIFY:** Apply Eq. (6.6) to the box.

**SET UP:** Let point 1 be just before the box reaches the end of the spring and let point 2 be where the spring has maximum compression and the box has momentarily come to rest.

**EXECUTE:** $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2,\ \ K_2 = 0$

Work is done by the spring force. $W_{\text{tot}} = -\frac{1}{2}kx_2^2,$ where $x_2$ is the amount the spring is compressed.

$-\frac{1}{2}kx_2^2 = -\frac{1}{2}mv_0^2$ and $x_2 = v_0\sqrt{m/k} = (3.0\text{ m/s})\sqrt{(6.0\text{ kg})/(7500\text{ N/m})} = 8.5\text{ cm}$

**EVALUATE:** The compression of the spring increases when either $v_0$ or $m$ increases and decreases when $k$ increases (stiffer spring).

**6.40.** **IDENTIFY:** The force applied to the springs is $F_x = kx.$ The work done on a spring to move its end from $x_1$ to $x_2$ is $W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2.$ Use the information that is given to calculate $k$.

**SET UP:** When the springs are compressed 0.200 m from their uncompressed length, $x_1 = 0$ and $x_2 = -0.200\text{ m}.$ When the platform is moved 0.200 m farther, $x_2$ becomes $-0.400\text{ m}.$

**EXECUTE:** **(a)** $k = \dfrac{2W}{x_2^2 - x_1^2} = \dfrac{2(80.0\text{ J})}{(0.200\text{ m})^2 - 0} = 4000\text{ N/m}.$ $F_x = kx = (4000\text{ N/m})(-0.200\text{ m}) = -800\text{ N}.$

The magnitude of force that is required is 800 N.

**(b)** To compress the springs from $x_1 = 0$ to $x_2 = -0.400\text{ m},$ the work required is

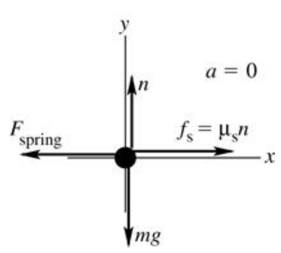
$W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 = \frac{1}{2}(4000\text{ N/m})(-0.400\text{ m})^2 = 320\text{ J}.$ The additional work required is

$320\text{ J} - 80\text{ J} = 240\text{ J}.$ For $x = -0.400\text{ m},$ $F_x = kx = -1600\text{ N}.$ The magnitude of force required is 1600 N.

**EVALUATE:** More work is required to move the end of the spring from $x = -0.200\text{ m}$ to $x = -0.400\text{ m}$ than to move it from $x = 0$ to $x = -0.200\text{ m},$ even though the displacement of the platform is the same in each case. The magnitude of the force increases as the compression of the spring increases.

**6.41.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to calculate the $\mu_s$ required for the static friction force to equal the spring force.

**SET UP:** **(a)** The free-body diagram for the glider is given in Figure 6.41.




**EXECUTE:** $\Sigma F_y = ma_y$

$n - mg = 0$

$n = mg$

$f_s = \mu_s mg$

**Figure 6.41**

$\Sigma F_x = ma_x$

$f_s - F_{spring} = 0$

$\mu_s mg - kd = 0$

$$\mu_s = \frac{kd}{mg} = \frac{(20.0 \text{ N/m})(0.086 \text{ m})}{(0.100 \text{ kg})(9.80 \text{ m/s}^2)} = 1.76$$

**(b) IDENTIFY and SET UP:** Apply $\Sigma\vec{F} = m\vec{a}$ to find the maximum amount the spring can be compressed and still have the spring force balanced by friction. Then use $W_{tot} = K_2 - K_1$ to find the initial speed that results in this compression of the spring when the glider stops.

**EXECUTE:** $\mu_s mg = kd$

$$d = \frac{\mu_s mg}{k} = \frac{(0.60)(0.100 \text{ kg})(9.80 \text{ m/s}^2)}{20.0 \text{ N/m}} = 0.0294 \text{ m}$$

Now apply the work-energy theorem to the motion of the glider:

$W_{tot} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_1^2$, $K_2 = 0$ (instantaneously stops)

$W_{tot} = W_{spring} + W_{fric} = -\frac{1}{2}kd^2 - \mu_k mgd$ (as in Example 6.7)

$W_{tot} = -\frac{1}{2}(20.0 \text{ N/m})(0.0294 \text{ m})^2 - 0.47(0.100 \text{ kg})(9.80 \text{ m/s}^2)(0.0294 \text{ m}) = -0.02218 \text{ J}$

Then $W_{tot} = K_2 - K_1$ gives $-0.02218 \text{ J} = -\frac{1}{2}mv_1^2$.

$$v_1 = \sqrt{\frac{2(0.02218 \text{ J})}{0.100 \text{ kg}}} = 0.67 \text{ m/s}.$$

**EVALUATE:** In Example 6.7 an initial speed of 1.50 m/s compresses the spring 0.086 m and in part (a) of this problem we found that the glider doesn't stay at rest. In part (b) we found that a smaller displacement of 0.0294 m when the glider stops is required if it is to stay at rest. And we calculate a smaller initial speed (0.67 m/s) to produce this smaller displacement.

**6.42. IDENTIFY:** For the spring, $W = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$. Apply $W_{tot} = K_2 - K_1$.

**SET UP:** $x_1 = -0.025$ m and $x_2 = 0$.

**EXECUTE:** **(a)** $W = \frac{1}{2}kx_1^2 = \frac{1}{2}(200 \text{ N/m})(-0.025 \text{ m})^2 = 0.0625 \text{ J}$, which rounds to 0.063 J.

**(b)** The work-energy theorem gives $v_2 = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(0.0625 \text{ J})}{(4.0 \text{ kg})}} = 0.18 \text{ m/s}$.

**EVALUATE:** The block moves in the direction of the spring force, the spring does positive work and the kinetic energy of the block increases.

**6.43. IDENTIFY and SET UP:** The magnitude of the work done by $F_x$ equals the area under the $F_x$ versus $x$ curve. The work is positive when $F_x$ and the displacement are in the same direction; it is negative when they are in opposite directions.

**EXECUTE:** **(a)** $F_x$ is positive and the displacement $\Delta x$ is positive, so $W > 0$.

$W = \frac{1}{2}(2.0 \text{ N})(2.0 \text{ m}) + (2.0 \text{ N})(1.0 \text{ m}) = +4.0 \text{ J}$

**(b)** During this displacement $F_x = 0$, so $W = 0$.

**(c)** $F_x$ is negative, $\Delta x$ is positive, so $W < 0$. $W = -\frac{1}{2}(1.0 \text{ N})(2.0 \text{ m}) = -1.0 \text{ J}$

**(d)** The work is the sum of the answers to parts (a), (b), and (c), so $W = 4.0 \text{ J} + 0 - 1.0 \text{ J} = +3.0 \text{ J}$.

**(e)** The work done for $x = 7.0$ m to $x = 3.0$ m is $+1.0$ J. This work is positive since the displacement and the force are both in the $-x$-direction. The magnitude of the work done for $x = 3.0$ m to $x = 2.0$ m is 2.0 J, the area under $F_x$ versus $x$. This work is negative since the displacement is in the $-x$-direction and the force is in the $+x$-direction. Thus $W = +1.0 \text{ J} - 2.0 \text{ J} = -1.0 \text{ J}$.

**EVALUATE:** The work done when the car moves from $x = 2.0$ m to $x = 0$ is $-\frac{1}{2}(2.0\text{ N})(2.0\text{ m}) = -2.0$ J. Adding this to the work for $x = 7.0$ m to $x = 2.0$ m gives a total of $W = -3.0$ J for $x = 7.0$ m to $x = 0$. The work for $x = 7.0$ m to $x = 0$ is the negative of the work for $x = 0$ to $x = 7.0$ m.

**6.44.** **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$.

**SET UP:** $K_1 = 0$. From Exercise 6.43, the work for $x = 0$ to $x = 3.0$ m is 4.0 J. $W$ for $x = 0$ to $x = 4.0$ m is also 4.0 J. For $x = 0$ to $x = 7.0$ m, $W = 3.0$ J.

**EXECUTE:** **(a)** $K = 4.0$ J, so $v = \sqrt{2K/m} = \sqrt{2(4.0\text{ J})/(2.0\text{ kg})} = 2.00$ m/s.

**(b)** No work is done between $x = 3.0$ m and $x = 4.0$ m, so the speed is the same, 2.00 m/s.

**(c)** $K = 3.0$ J, so $v = \sqrt{2K/m} = \sqrt{2(3.0\text{ J})/(2.0\text{ kg})} = 1.73$ m/s.

**EVALUATE:** In each case the work done by $F$ is positive and the car gains kinetic energy.

**6.45.** **IDENTIFY** and **SET UP:** Apply Eq. (6.6). Let point 1 be where the sled is released and point 2 be at $x = 0$ for part (a) and at $x = -0.200$ m for part (b). Use Eq. (6.10) for the work done by the spring and calculate $K_2$. Then $K_2 = \frac{1}{2}mv_2^2$ gives $v_2$.

**EXECUTE:** **(a)** $W_{\text{tot}} = K_2 - K_1$ so $K_2 = K_1 + W_{\text{tot}}$

$K_1 = 0$ (released with no initial velocity), $K_2 = \frac{1}{2}mv_2^2$

The only force doing work is the spring force. Eq. (6.10) gives the work done *on* the spring to move its end from $x_1$ to $x_2$. The force the spring exerts on an object attached to it is $F = -kx$, so the work the spring does is

$W_{\text{spr}} = -\left(\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2\right) = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$. Here $x_1 = -0.375$ m and $x_2 = 0$. Thus

$W_{\text{spr}} = \frac{1}{2}(4000\text{ N/m})(-0.375\text{ m})^2 - 0 = 281$ J.

$K_2 = K_1 + W_{\text{tot}} = 0 + 281\text{ J} = 281$ J.

Then $K_2 = \frac{1}{2}mv_2^2$ implies $v_2 = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(281\text{ J})}{70.0\text{ kg}}} = 2.83$ m/s.

**(b)** $K_2 = K_1 + W_{\text{tot}}$

$K_1 = 0$

$W_{\text{tot}} = W_{\text{spr}} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$. Now $x_2 = -0.200$ m, so

$W_{\text{spr}} = \frac{1}{2}(4000\text{ N/m})(-0.375\text{ m})^2 - \frac{1}{2}(4000\text{ N/m})(-0.200\text{ m})^2 = 281\text{ J} - 80\text{ J} = 201$ J

Thus $K_2 = 0 + 201\text{ J} = 201$ J and $K_2 = \frac{1}{2}mv_2^2$ gives $v_2 = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(201\text{ J})}{70.0\text{ kg}}} = 2.40$ m/s.

**EVALUATE:** The spring does positive work and the sled gains speed as it returns to $x = 0$. More work is done during the larger displacement in part (a), so the speed there is larger than in part (b).

**6.46.** **IDENTIFY:** $F_x = kx$

**SET UP:** When the spring is in equilibrium, the same force is applied to both ends of any segment of the spring.

**EXECUTE:** **(a)** When a force $F$ is applied to each end of the original spring, the end of the spring is displaced a distance $x$. Each half of the spring elongates a distance $x_{\text{h}}$, where $x_{\text{h}} = x/2$. Since $F$ is also the force applied to each half of the spring, $F = kx$ and $F = k_{\text{h}}x_{\text{h}}$. $kx = k_{\text{h}}x_{\text{h}}$ and $k_{\text{h}} = k\left(\dfrac{x}{x_{\text{h}}}\right) = 2k$.

**(b)** The same reasoning as in part (a) gives $k_{\text{seg}} = 3k$, where $k_{\text{seg}}$ is the force constant of each segment.

**EVALUATE:** For half of the spring the same force produces less displacement than for the original spring. Since $k = F/x$, smaller $x$ for the same $F$ means larger $k$.

**6.47.** **IDENTIFY** and **SET UP:** Apply Eq. (6.6) to the glider. Work is done by the spring and by gravity. Take point 1 to be where the glider is released. In part (a) point 2 is where the glider has traveled 1.80 m and $K_2 = 0.$ There are two points shown in Figure 6.47a. In part (b) point 2 is where the glider has traveled 0.80 m.

**EXECUTE:** (a) $W_{\text{tot}} = K_2 - K_1 = 0.$ Solve for $x_1,$ the amount the spring is initially compressed.

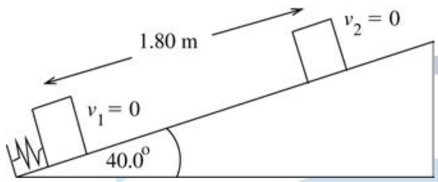



**Figure 6.47a**

$W_{\text{tot}} = W_{\text{spr}} + W_w = 0$

So $W_{\text{spr}} = -W_w$

(The spring does positive work on the glider since the spring force is directed up the incline, the same as the direction of the displacement.)

The directions of the displacement and of the gravity force are shown in Figure 6.47b.

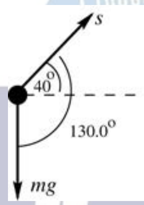



**Figure 6.47b**

$W_w = (w\cos\phi)s = (mg\cos 130.0°)s$

$W_w = (0.0900 \text{ kg})(9.80 \text{ m/s}^2)(\cos 130.0°)(1.80 \text{ m}) = -1.020 \text{ J}$

(The component of $w$ parallel to the incline is directed down the incline, opposite to the displacement, so gravity does negative work.)

$W_{\text{spr}} = -W_w = +1.020 \text{ J}$

$W_{\text{spr}} = \tfrac{1}{2}kx_1^2$ so $x_1 = \sqrt{\dfrac{2W_{\text{spr}}}{k}} = \sqrt{\dfrac{2(1.020 \text{ J})}{640 \text{ N/m}}} = 0.0565 \text{ m}$

**(b)** The spring was compressed only 0.0565 m so at this point in the motion the glider is no longer in contact with the spring. Points 1 and 2 are shown in Figure 6.47c.

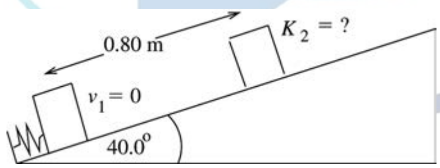



**Figure 6.47c**

$W_{\text{tot}} = K_2 - K_1$

$K_2 = K_1 + W_{\text{tot}}$

$K_1 = 0$

$W_{\text{tot}} = W_{\text{spr}} + W_w$

From part (a), $W_{\text{spr}} = 1.020 \text{ J}$ and

$W_w = (mg\cos 130.0°)s = (0.0900 \text{ kg})(9.80 \text{ m/s}^2)(\cos 130.0°)(0.80 \text{ m}) = -0.454 \text{ J}$

Then $K_2 = W_{\text{spr}} + W_w = +1.020 \text{ J} - 0.454 \text{ J} = +0.57 \text{ J}.$

**EVALUATE:** The kinetic energy in part (b) is positive, as it must be. In part (a), $x_2 = 0$ since the spring force is no longer applied past this point. In computing the work done by gravity we use the full 0.80 m the glider moves.

**6.48. IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$ to the brick. Work is done by the spring force and by gravity.

**SET UP:** At the maximum height, $v = 0$. Gravity does negative work, $W_{\text{grav}} = -mgh$. The work done by the spring is $\frac{1}{2}kd^2$, where $d$ is the distance the spring is compressed initially.

**EXECUTE:** The initial and final kinetic energies of the brick are both zero, so the net work done on the brick by the spring and gravity is zero, so $(1/2)kd^2 - mgh = 0$, or $d = \sqrt{2mgh/k} = \sqrt{2(1.80\text{ kg})(9.80\text{ m/s}^2)(3.6\text{ m})/(450\text{ N/m})} = 0.53\text{ m}$. The spring will provide an upward force while the spring and the brick are in contact. When this force goes to zero, the spring is at its uncompressed length. But when the spring reaches its uncompressed length the brick has an upward velocity and leaves the spring.

**EVALUATE:** Gravity does negative work because the gravity force is downward and the brick moves upward. The spring force does positive work on the brick because the spring force is upward and the brick moves upward.

**6.49. IDENTIFY:** The force does work on the box, which gives it kinetic energy, so the work-energy theorem applies. The force is variable so we must integrate to calculate the work it does on the box.

**SET UP:** $W_{\text{tot}} = \Delta K = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2$ and $W_{\text{tot}} = \int_{x_1}^{x_2} F(x)dx$.

**EXECUTE:** $W_{\text{tot}} = \int_{x_1}^{x_2} F(x)dx = \int_0^{14.0\text{m}} [18.0\text{ N} - (0.530\text{ N/m})x]dx$

$W_{\text{tot}} = (18.0\text{ N})(14.0\text{ m}) - (0.265\text{ N/m})(14.0\text{ m})^2 = 252.0\text{ J} - 51.94\text{ J} = 200.1\text{ J}$. The initial kinetic energy is zero, so $W_{\text{tot}} = \Delta K = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}mv_{\text{f}}^2$. Solving for $v_{\text{f}}$ gives $v_{\text{f}} = \sqrt{\frac{2W_{\text{tot}}}{m}} = \sqrt{\frac{2(200.1\text{ J})}{6.00\text{ kg}}} = 8.17\text{ m/s}$.

**EVALUATE:** We could not readily do this problem by integrating the acceleration over time because we know the force as a function of *x*, not of *t*. The work-energy theorem provides a much simpler method.

**6.50. IDENTIFY:** The force acts through a distance over time, so it does work on the crate and hence supplies power to it. The force exerted by the worker is variable but the acceleration of the cart is constant.

**SET UP:** Use $P = Fv$ to find the power, and we can use $v = v_0 + at$ to find the instantaneous velocity.

**EXECUTE:** First find the instantaneous force and velocity: $F = (5.40\text{ N/s})(5.00\text{ s}) = 27.0\text{ N}$ and $v = v_0 + at = (2.80\text{ m/s}^2)(5.00\text{ s}) = 14.0\text{ m/s}$. Now find the power: $P = (27.0\text{ N})(14.0\text{ m/s}) = 378\text{ W}$.

**EVALUATE:** The instantaneous power will increase as the worker pushes harder and harder.

**6.51. IDENTIFY:** Apply the relation between energy and power.

**SET UP:** Use $P = \frac{W}{\Delta t}$ to solve for *W*, the energy the bulb uses. Then set this value equal to $\frac{1}{2}mv^2$ and solve for the speed.

**EXECUTE:** $W = P\Delta t = (100\text{ W})(3600\text{ s}) = 3.6\times 10^5\text{ J}$

$K = 3.6\times 10^5\text{ J}$ so $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(3.6\times 10^5\text{ J})}{70\text{ kg}}} = 100\text{ m/s}$

**EVALUATE:** Olympic runners achieve speeds up to approximately 10 m/s, or roughly one-tenth the result calculated.

**6.52. IDENTIFY:** Knowing the rate at which energy is consumed, we want to find out the total energy used.

**SET UP:** Find the elapsed time $\Delta t$ in each case by dividing the distance by the speed, $\Delta t = d/v$. Then calculate the energy as $W = P\Delta t$.

**EXECUTE:** *Running:* $\Delta t = (5.0\text{ km})/(10\text{ km/h}) = 0.50\text{ h} = 1.8\times 10^3\text{ s}$. The energy used is $W = (700\text{ W})(1.8\times 10^3\text{ s}) = 1.3\times 10^6\text{ J}$.

*Walking:* $\Delta t = \frac{5.0\text{ km}}{3.0\text{ km/h}}\left(\frac{3600\text{ s}}{\text{h}}\right) = 6.0\times 10^3\text{ s}$. The energy used is

$W = (290\text{ W})(6.0\times 10^3\text{ s}) = 1.7\times 10^6\text{ J}$.

**EVALUATE:** The less intense exercise lasts longer and therefore burns up more energy than the intense exercise.

**6.53.** **IDENTIFY:** $P_{av} = \dfrac{\Delta W}{\Delta t}$. $\Delta W$ is the energy released.

**SET UP:** $\Delta W$ is to be the same. $1\text{ y} = 3.156\times 10^7\text{ s}$.

**EXECUTE:** $P_{av}\Delta t = \Delta W = \text{constant}$, so $P_{\text{av-sun}}\Delta t_{\text{sun}} = P_{\text{av-m}}\Delta t_{\text{m}}$.

$$P_{\text{av-m}} = P_{\text{av-sun}}\left(\frac{\Delta t_{\text{sun}}}{\Delta t_{\text{m}}}\right) = P\left(\frac{(2.5\times 10^5\text{ y})(3.156\times 10^7\text{ s/y})}{0.20\text{ s}}\right) = 3.9\times 10^{13}P.$$

**EVALUATE:** Since the power output of the magnetar is so much larger than that of our sun, the mechanism by which it radiates energy must be quite different.

**6.54.** **IDENTIFY:** The thermal energy is produced as a result of the force of friction, $F = \mu_k mg$. The average thermal power is thus the average rate of work done by friction or $P = F_{\|}v_{av}$.

**SET UP:** $v_{av} = \dfrac{v_2 + v_1}{2} = \left(\dfrac{8.00\text{ m/s} + 0}{2}\right) = 4.00\text{ m/s}$

**EXECUTE:** $P = Fv_{av} = [(0.200)(20.0\text{ kg})(9.80\text{ m/s}^2)](4.00\text{ m/s}) = 157\text{ W}$

**EVALUATE:** The power could also be determined as the rate of change of kinetic energy, $\Delta K/t$, where the time is calculated from $v_f = v_i + at$ and $a$ is calculated from a force balance, $\Sigma F = ma = \mu_k mg$.

**6.55.** **IDENTIFY:** Use the relation $P = F_{\|}v$ to relate the given force and velocity to the total power developed.

**SET UP:** $1\text{ hp} = 746\text{ W}$

**EXECUTE:** The total power is $P = F_{\|}v = (165\text{ N})(9.00\text{ m/s}) = 1.49\times 10^3\text{ W}$. Each rider therefore contributes $P_{\text{each rider}} = (1.49\times 10^3\text{ W})/2 = 745\text{ W} \approx 1\text{ hp}$.

**EVALUATE:** The result of one horsepower is very large; a rider could not sustain this output for long periods of time.

**6.56.** **IDENTIFY and SET UP:** Calculate the power used to make the plane climb against gravity. Consider the vertical motion since gravity is vertical.

**EXECUTE:** The rate at which work is being done against gravity is

$P = Fv = mgv = (700\text{ kg})(9.80\text{ m/s}^2)(2.5\text{ m/s}) = 17.15\text{ kW}$.

This is the part of the engine power that is being used to make the airplane climb. The fraction this is of the total is $17.15\text{ kW}/75\text{ kW} = 0.23$.

**EVALUATE:** The power we calculate for making the airplane climb is considerably less than the power output of the engine.

**6.57.** **IDENTIFY:** $P_{av} = \dfrac{\Delta W}{\Delta t}$. The work you do in lifting mass *m* a height *h* is *mgh*.

**SET UP:** $1\text{ hp} = 746\text{ W}$

**EXECUTE:** **(a)** The number per minute would be the average power divided by the work (*mgh*) required to lift one box, $\dfrac{(0.50\text{ hp})(746\text{ W/hp})}{(30\text{ kg})(9.80\text{ m/s}^2)(0.90\text{ m})} = 1.41/\text{s}$, or $84.6/\text{min}$.

**(b)** Similarly, $\dfrac{(100\text{ W})}{(30\text{ kg})(9.80\text{ m/s}^2)(0.90\text{ m})} = 0.378/\text{s}$, or $22.7/\text{min}$.

**EVALUATE:** A 30-kg crate weighs about 66 lbs. It is not possible for a person to perform work at this rate.

**6.58.** **IDENTIFY and SET UP:** Use Eq. (6.15) to relate the power provided and the amount of work done against gravity in 16.0 s. The work done against gravity depends on the total weight which depends on the number of passengers.

**EXECUTE:** Find the total mass that can be lifted:

$$P_{av} = \frac{\Delta W}{\Delta t} = \frac{mgh}{t},\ \text{so}\ m = \frac{P_{av}t}{gh}$$

$$P_{av} = (40\text{ hp})\left(\frac{746\text{ W}}{1\text{ hp}}\right) = 2.984\times 10^4\text{ W}$$

$$m = \frac{P_{\text{av}}t}{gh} = \frac{(2.984\times 10^4\ \text{W})(16.0\ \text{s})}{(9.80\ \text{m/s}^2)(20.0\ \text{m})} = 2.436\times 10^3\ \text{kg}$$

This is the total mass of elevator plus passengers. The mass of the passengers is

$2.436\times 10^3\ \text{kg} - 600\ \text{kg} = 1.836\times 10^3\ \text{kg}$. The number of passengers is $\dfrac{1.836\times 10^3\ \text{kg}}{65.0\ \text{kg}} = 28.2$.

28 passengers can ride.

**Evaluate:** Typical elevator capacities are about half this, in order to have a margin of safety.

**6.59.** **Identify:** To lift the skiers, the rope must do positive work to counteract the negative work developed by the component of the gravitational force acting on the total number of skiers, $F_{\text{rope}} = Nmg\sin\alpha$.

**Set Up:** $P = F_{\parallel}v = F_{\text{rope}}v$

**Execute:** $P_{\text{rope}} = F_{\text{rope}}v = [+Nmg(\cos\phi)]v$.

$$P_{\text{rope}} = [(50\ \text{riders})(70.0\ \text{kg})(9.80\ \text{m/s}^2)(\cos 75.0)]\left[(12.0\ \text{km/h})\left(\frac{1\ \text{m/s}}{3.60\ \text{km/h}}\right)\right].$$

$P_{\text{rope}} = 2.96\times 10^4\ \text{W} = 29.6\ \text{kW}$.

**Evaluate:** Some additional power would be needed to give the riders kinetic energy as they are accelerated from rest.

**6.60.** **Identify:** We want to find the power supplied by a known force acting on a crate at a known velocity.

**Set Up:** We know the vector components, so we use $P = \vec{F}\cdot\vec{v} = F_xv_x + F_yv_y$

**Execute:** $P = F_xv_x + F_yv_y = (-8.00\ \text{N})(3.20\ \text{m/s}) + (3.00\ \text{N})(2.20\ \text{m/s}) = -19.0\ \text{W}$.

**Evaluate:** The power is negative because the $x$-component of the force is opposite to the $x$-component of the velocity and hence opposes the motion of the crate.

**6.61.** **Identify:** Relate power, work, and time.

**Set Up:** Work done in each stroke is $W = Fs$ and $P_{\text{av}} = W/t$.

**Execute:** 100 strokes per second means $P_{\text{av}} = 100Fs/t$ with $t = 1.00$ s, $F = 2mg$ and $s = 0.010$ m. $P_{\text{av}} = 0.20$ W.

**Evaluate:** For a 70-kg person to apply a force of twice his weight through a distance of 0.50 m for 100 times per second, the average power output would be $7.0\times 10^4$ W. This power output is very far beyond the capability of a person.

**6.62.** **Identify:** The force has only an $x$-component and the motion is along the $x$-direction, so $W = \int_{x_1}^{x_2} F_x dx$.

**Set Up:** $x_1 = 0$ and $x_2 = 6.9$ m.

**Execute:** The work you do with your changing force is

$$W = \int_{x_1}^{x_2} F(x)dx = \int_{x_1}^{x_2}(-20.0\ \text{N})dx - \int_{x_1}^{x_2}(3.0\ \text{N/m})x dx = (-20.0\ \text{N})x\Big|_{x_1}^{x_2} - (3.0\ \text{N/m})(x^2/2)\Big|_{x_1}^{x_2}$$

$W = -138\ \text{N}\cdot\text{m} - 71.4\ \text{N}\cdot\text{m} = -209\ \text{J}$.

**Evaluate:** The work is negative because the cow continues to move forward (in the $+x$-direction) as you vainly attempt to push her backward.

**6.63.** **Identify and Set Up:** Since the forces are constant, Eq. (6.2) can be used to calculate the work done by each force. The forces on the suitcase are shown in Figure 6.63a.

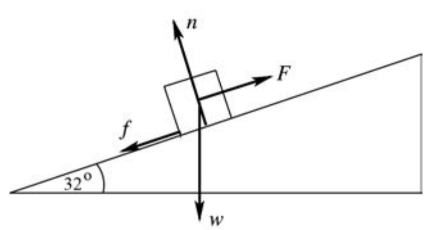


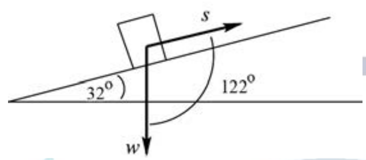

**Figure 6.63a**

In part (f), Eq. (6.6) is used to relate the total work to the initial and final kinetic energy.

**EXECUTE:** **(a)** $W_F = (F\cos\phi)s$

Both $\vec{F}$ and $\vec{s}$ are parallel to the incline and in the same direction, so $\phi = 90°$ and $W_F = Fs = (160\ \text{N})(3.80\ \text{m}) = 608\ \text{J}.$

**(b)** The directions of the displacement and of the gravity force are shown in Figure 6.63b.




$W_w = (w\cos\phi)s$

$\phi = 122°,$ so

$W_w = (196\ \text{N})(\cos 122°)(3.80\ \text{m})$

$W_w = -395\ \text{J}$

**Figure 6.63b**

Alternatively, the component of $w$ parallel to the incline is $w\sin 32°$. This component is down the incline so its angle with $\vec{s}$ is $\phi = 180°$. $W_{w\sin 32°} = (196\ \text{N}\sin 32°)(\cos 180°)(3.80\ \text{m}) = -395\ \text{J}.$ The other component of $w$, $w\cos 32°$, is perpendicular to $\vec{s}$ and hence does no work. Thus $W_w = W_{w\sin 25°} = -315\ \text{J},$ which agrees with the above.

**(c)** The normal force is perpendicular to the displacement $(\phi = 90°)$, so $W_n = 0.$

**(d)** $n = w\cos 32°$ so $f_\text{k} = \mu_\text{k} n = \mu_\text{k} w\cos 32° = (0.30)(196\ \text{N})\cos 32° = 49.87\ \text{N}$

$W_f = (f_\text{k}\cos\phi)x = (49.87\ \text{N})(\cos 180°)(3.80\ \text{m}) = -189\ \text{J}.$

**(e)** $W_\text{tot} = W_F + W_w + W_n + W_f = +608\ \text{J} - 395\ \text{J} + 0 - 189\ \text{J} = 24\ \text{J}.$

**(f)** $W_\text{tot} = K_2 - K_1,\ \ K_1 = 0,$ so $K_2 = W_\text{tot}$

$\frac{1}{2}mv_2^2 = W_\text{tot}$ so $v_2 = \sqrt{\dfrac{2W_\text{tot}}{m}} = \sqrt{\dfrac{2(24\ \text{J})}{20.0\ \text{kg}}} = 1.5\ \text{m/s}.$

**EVALUATE:** The total work done is positive and the kinetic energy of the suitcase increases as it moves up the incline.

**6.64.** **IDENTIFY:** The work he does to lift his body a distance $h$ is $W = mgh$. The work per unit mass is $(W/m) = gh$.

**SET UP:** The quantity $gh$ has units of N/kg.

**EXECUTE:** **(a)** The man does work, $(9.8\ \text{N/kg})(0.4\ \text{m}) = 3.92\ \text{J/kg}.$

**(b)** $(3.92\ \text{J/kg})/(70\ \text{J/kg}) \times 100 = 5.6\%.$

**(c)** The child does work $(9.8\ \text{N/kg})(0.2\ \text{m}) = 1.96\ \text{J/kg}.$ $(1.96\ \text{J/kg})/(70\ \text{J/kg}) \times 100 = 2.8\%.$

**(d)** If both the man and the child can do work at the rate of $70\ \text{J/kg}$, and if the child only needs to use $1.96\ \text{J/kg}$ instead of $3.92\ \text{J/kg}$, the child should be able to do more chin-ups.

**EVALUATE:** Since the child has arms half the length of his father's arms, the child must lift his body only 0.20 m to do a chin-up.

**6.65.** **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block to find the tension in the string. Each force is constant and $W = Fs\cos\phi.$

**SET UP:** The free-body diagram for each block is given in Figure 6.65 (next page).

$m_A = \dfrac{20.0\ \text{N}}{g} = 2.04\ \text{kg}$ and $m_B = \dfrac{12.0\ \text{N}}{g} = 1.22\ \text{kg}.$

**EXECUTE:** $T - f_\text{k} = m_A a.\ w_B - T = m_B a.\ \ w_B - f_\text{k} = (m_A + m_B)a.$

**(a)** $f_\text{k} = 0.$ $a = \left(\dfrac{w_B}{m_A + m_B}\right)$ and $T = w_B\left(\dfrac{m_A}{m_A + m_B}\right) = w_B\left(\dfrac{w_A}{w_A + w_B}\right) = 7.50\ \text{N}.$

20.0 N block: $W_\text{tot} = Ts = (7.50\ \text{N})(0.750\ \text{m}) = 5.62\ \text{J}.$

12.0 N block: $W_\text{tot} = (w_B - T)s = (12.0\ \text{N} - 7.50\ \text{N})(0.750\ \text{m}) = 3.38\ \text{J}.$

**(b)** $f_k = \mu_k w_A = 6.50$ N. $a = \dfrac{w_B - \mu_k w_A}{m_A + m_B}$.

$T = f_k + (w_B - \mu_k w_A)\left(\dfrac{m_A}{m_A + m_B}\right) = \mu_k w_A + (w_B - \mu_k w_A)\left(\dfrac{w_A}{w_A + w_B}\right)$. $T = 6.50\ \text{N} + (5.50\ \text{N})(0.625) = 9.94\ \text{N}$.

20.0 N block: $W_{\text{tot}} = (T - f_k)s = (9.94\ \text{N} - 6.50\ \text{N})(0.750\ \text{m}) = 2.58\ \text{J}$.

12.0 N block: $W_{\text{tot}} = (w_B - T)s = (12.0\ \text{N} - 9.94\ \text{N})(0.750\ \text{m}) = 1.54\ \text{J}$.

**EVALUATE:** Since the two blocks move with equal speeds, for each block $W_{\text{tot}} = K_2 - K_1$ is proportional to the mass (or weight) of that block. With friction the gain in kinetic energy is less, so the total work on each block is less.

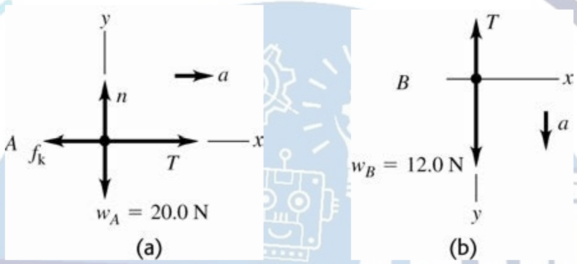



**Figure 6.65**

**6.66.** **IDENTIFY:** $W = Fs\cos\phi$ and $W_{\text{tot}} = K_2 - K_1$.

**SET UP:** $f_k = \mu_k n$. The normal force is $n = mg\cos\theta$, with $\theta = 24.0°$. The component of the weight parallel to the incline is $mg\sin\theta$.

**EXECUTE:** **(a)** $\phi = 180°$ and

$W_f = -f_k s = -(\mu_k mg\cos\theta)s = -(0.31)(5.00\ \text{kg})(9.80\ \text{m/s}^2)(\cos 24.0°)(2.80\ \text{m}) = -38.9\ \text{J}$.

**(b)** $(5.00\ \text{kg})(9.80\ \text{m/s}^2)(\sin 24.0°)(2.80\ \text{m}) = 55.8\ \text{J}$.

**(c)** The normal force does no work.

**(d)** $W_{\text{tot}} = 55.8\ \text{J} - 38.9\ \text{J} = +16.9\ \text{J}$.

**(e)** $K_2 = K_1 + W_{\text{tot}} = (1/2)(5.00\ \text{kg})(2.20\ \text{m/s})^2 + 16.9\ \text{J} = 29.0\ \text{J}$, and so

$v_2 = \sqrt{2(29.0\ \text{J})/(5.00\ \text{kg})} = 3.41\ \text{m/s}$.

**EVALUATE:** Friction does negative work and gravity does positive work. The net work is positive and the kinetic energy of the object increases.

**6.67.** **IDENTIFY:** The initial kinetic energy of the head is absorbed by the neck bones during a sudden stop. Newton's second law applies to the passengers as well as to their heads.

**SET UP:** In part (a), the initial kinetic energy of the head is absorbed by the neck bones, so $\frac{1}{2}mv_{\text{max}}^2 = 8.0$ J. For part (b), assume constant acceleration and use $v_f = v_i + at$ with $v_i = 0$, to calculate $a$; then apply $F_{\text{net}} = ma$ to find the net accelerating force.

**Solve: (a)** $v_{\text{max}} = \sqrt{\dfrac{2(8.0\ \text{J})}{5.0\ \text{kg}}} = 1.8\ \text{m/s} = 4.0\ \text{mph}$.

**(b)** $a = \dfrac{v_f - v_i}{t} = \dfrac{1.8\ \text{m/s} - 0}{10.0 \times 10^{-3}\ \text{s}} = 180\ \text{m/s}^2 \approx 18g$, and $F_{\text{net}} = ma = (5.0\ \text{kg})(180\ \text{m/s}^2) = 900\ \text{N}$.

**EVALUATE:** The acceleration is very large, but if it lasts for only 10 ms it does not do much damage.

**6.68.** **IDENTIFY:** The force does work on the object, which changes its kinetic energy, so the work-energy theorem applies. The force is variable so we must integrate to calculate the work it does on the object.

**SET UP:** $W_{\text{tot}} = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ and $W_{\text{tot}} = \int_{x_1}^{x_2} F(x)dx$.

**EXECUTE:** $W_{\text{tot}} = \int_{x_1}^{x_2} F(x)dx = \int_0^{5.00\text{ m}} [-12.0\text{ N} + (0.300\text{ N/m}^2)x^2]dx.$

$W_{\text{tot}} = -(12.0\text{ N})(5.00\text{ m}) + (0.100\text{ N/m}^2)(5.00\text{ m})^3 = -60.0\text{ J} + 12.5\text{ J} = -47.5\text{ J}.$

$W_{\text{tot}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -47.5\text{ J},$ so the final velocity is

$$v_f = \sqrt{v_i^2 - \frac{2(47.5\text{ J})}{m}} = \sqrt{(6.00\text{ m/s})^2 - \frac{2(47.5\text{ J})}{5.00\text{ kg}}} = 4.12\text{ m/s}.$$

**EVALUATE:** We could not readily do this problem by integrating the acceleration over time because we know the force as a function of $x$, not of $t$. The work-energy theorem provides a much simpler method.

**6.69. IDENTIFY:** Calculate the work done by friction and apply $W_{\text{tot}} = K_2 - K_1$. Since the friction force is not constant, use Eq. (6.7) to calculate the work.

**SET UP:** Let $x$ be the distance past $P$. Since $\mu_k$ increases linearly with $x$, $\mu_k = 0.100 + Ax$. When $x = 12.5\text{ m}$, $\mu_k = 0.600$, so $A = 0.500/(12.5\text{ m}) = 0.0400\text{/m}$.

**EXECUTE:** **(a)** $W_{\text{tot}} = \Delta K = K_2 - K_1$ gives $-\int \mu_k mg dx = 0 - \frac{1}{2}mv_1^2$. Using the above expression for $\mu_k$,

$g\int_0^{x_2}(0.100 + Ax)dx = \frac{1}{2}v_1^2$ and $g\left[(0.100)x_2 + A\frac{x_2^2}{2}\right] = \frac{1}{2}v_1^2.$

$(9.80\text{ m/s}^2)\left[(0.100)x_2 + (0.0400\text{/m})\frac{x_2^2}{2}\right] = \frac{1}{2}(4.50\text{ m/s})^2$. Solving for $x_2$ gives $x_2 = 5.11\text{ m}$.

**(b)** $\mu_k = 0.100 + (0.0400\text{/m})(5.11\text{ m}) = 0.304$

**(c)** $W_{\text{tot}} = K_2 - K_1$ gives $-\mu_k mgx_2 = 0 - \frac{1}{2}mv_1^2$. $x_2 = \frac{v_1^2}{2\mu_k g} = \frac{(4.50\text{ m/s})^2}{2(0.100)(9.80\text{ m/s}^2)} = 10.3\text{ m}.$

**EVALUATE:** The box goes farther when the friction coefficient doesn't increase.

**6.70. IDENTIFY:** Use Eq. (6.7) to calculate $W$.

**SET UP:** $x_1 = 0$. In part (a), $x_2 = 0.050\text{ m}$. In part (b), $x_2 = -0.050\text{ m}$.

**EXECUTE:** **(a)** $W = \int_0^{x_2} Fdx = \int_0^{x_2}(kx - bx^2 + cx^3)\,dx = \frac{k}{2}x_2^2 - \frac{b}{3}x_2^3 + \frac{c}{4}x_2^4.$

$W = (50.0\text{ N/m})\,x_2^2 - (233\text{ N/m}^2)\,x_2^3 + (3000\text{ N/m}^3)\,x_2^4$. When $x_2 = 0.050\text{ m}$, $W = 0.12\text{ J}$.

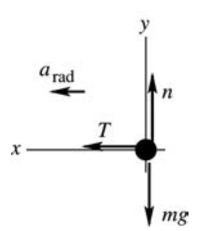
**(b)** When $x_2 = -0.050\text{ m}$, $W = 0.17\text{ J}$.

**(c)** It's easier to stretch the spring; the quadratic $-bx^2$ term is always in the $-x$-direction, and so the needed force, and hence the needed work, will be less when $x_2 > 0$.

**EVALUATE:** When $x = 0.050\text{ m}$, $F_x = 4.75\text{ N}$. When $x = -0.050\text{ m}$, $F_x = -8.25\text{ N}$.

**6.71. IDENTIFY and SET UP:** Use $\Sigma\vec{F} = m\vec{a}$ to find the tension force $T$. The block moves in uniform circular motion and $\vec{a} = \vec{a}_{\text{rad}}$.

**(a)** The free-body diagram for the block is given in Figure 6.71.




**EXECUTE:** $\Sigma F_x = ma_x$

$T = m\frac{v^2}{R}$

$T = (0.0600\text{ kg})\frac{(0.70\text{ m/s})^2}{0.40\text{ m}} = 0.074\text{ N}.$

**Figure 6.71**

**(b)** $T = m\dfrac{v^2}{R} = (0.0600 \text{ kg})\dfrac{(2.80 \text{ m/s})^2}{0.10 \text{ m}} = 4.7 \text{ N}.$

**(c) SET UP:** The tension changes as the distance of the block from the hole changes. We could use $W = \int_{x_1}^{x_2} F_x\, dx$ to calculate the work. But a much simpler approach is to use $W_{\text{tot}} = K_2 - K_1$.

**EXECUTE:** The only force doing work on the block is the tension in the cord, so $W_{\text{tot}} = W_T$.

$K_1 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(0.0600 \text{ kg})(0.70 \text{ m/s})^2 = 0.01470 \text{ J}$, $K_2 = \tfrac{1}{2}mv_2^2 = \tfrac{1}{2}(0.0600 \text{ kg})(2.80 \text{ m/s})^2 = 0.2352 \text{ J}$, so $W_{\text{tot}} = K_2 - K_1 = 0.2352 \text{ J} - 0.01470 \text{ J} = 0.22 \text{ J}$. This is the amount of work done by the person who pulled the cord.

**EVALUATE:** The block moves inward, in the direction of the tension, so $T$ does positive work and the kinetic energy increases.

**6.72.** **IDENTIFY:** Use Eq. (6.7) to find the work done by $F$. Then apply $W_{\text{tot}} = K_2 - K_1$.

**SET UP:** $\displaystyle\int \frac{dx}{x^2} = -\frac{1}{x}.$

**EXECUTE:** $W = \displaystyle\int_{x_1}^{x_2} \frac{\alpha}{x^2}\, dx = \alpha\left(\frac{1}{x_1} - \frac{1}{x_2}\right).$

$W = (2.12 \times 10^{-26} \text{ N}\cdot\text{m}^2)\left[(0.200 \text{ m}^{-1}) - (1.25 \times 10^{9} \text{ m}^{-1})\right] = -2.65 \times 10^{-17} \text{ J}.$

Note that $x_1$ is so large compared to $x_2$ that the term $1/x_1$ is negligible. Then, using Eq. (6.13) and solving for $v_2$,

$$v_2 = \sqrt{v_1^2 + \frac{2W}{m}} = \sqrt{(3.00 \times 10^5 \text{ m/s})^2 + \frac{2(-2.65 \times 10^{-17} \text{ J})}{(1.67 \times 10^{-27} \text{ kg})}} = 2.41 \times 10^5 \text{ m/s}.$$

**(b)** With $K_2 = 0, W = -K_1$. Using $W = -\dfrac{\alpha}{x_2}$,

$$x_2 = \frac{\alpha}{K_1} = \frac{2\alpha}{mv_1^2} = \frac{2(2.12 \times 10^{-26} \text{ N}\cdot\text{m}^2)}{(1.67 \times 10^{-27} \text{ kg})(3.00 \times 10^5 \text{ m/s})^2} = 2.82 \times 10^{-10} \text{ m}.$$

**(c)** The repulsive force has done no net work, so the kinetic energy and hence the speed of the proton have their original values, and the speed is $3.00 \times 10^5$ m/s.

**EVALUATE:** As the proton moves toward the uranium nucleus the repulsive force does negative work and the kinetic energy of the proton decreases. As the proton moves away from the uranium nucleus the repulsive force does positive work and the kinetic energy of the proton increases.

**6.73.** **IDENTIFY:** The negative work done by the spring equals the change in kinetic energy of the car.

**SET UP:** The work done by a spring when it is compressed a distance $x$ from equilibrium is $-\dfrac{1}{2}kx^2$. $K_2 = 0$.

**EXECUTE:** $-\tfrac{1}{2}kx^2 = K_2 - K_1$ gives $\tfrac{1}{2}kx^2 = \tfrac{1}{2}mv_1^2$ and

$k = (mv_1^2)/x^2 = [(1200 \text{ kg})(0.65 \text{ m/s})^2]/(0.090 \text{ m})^2 = 6.3 \times 10^4 \text{ N/m}.$

**EVALUATE:** When the spring is compressed, the spring force is directed opposite to the displacement of the object and the work done by the spring is negative.

**6.74.** **IDENTIFY and SET UP:** Use Eq. (6.6). You do positive work and gravity does negative work. Let point 1 be at the base of the bridge and point 2 be at the top of the bridge.

**EXECUTE:** **(a)** $W_{\text{tot}} = K_2 - K_1$

$K_1 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(80.0 \text{ kg})(5.00 \text{ m/s})^2 = 1000 \text{ J}$

$K_2 = \tfrac{1}{2}mv_2^2 = \tfrac{1}{2}(80.0 \text{ kg})(1.50 \text{ m/s})^2 = 90 \text{ J}$

$W_{\text{tot}} = 90 \text{ J} - 1000 \text{ J} = -910 \text{ J}$

**(b)** Neglecting friction, work is done by you (with the force you apply to the pedals) and by gravity: $W_{\text{tot}} = W_{\text{you}} + W_{\text{gravity}}$. The gravity force is $w = mg = (80.0\text{ kg})(9.80\text{ m/s}^2) = 784\text{ N}$, downward. The displacement is 5.20 m, upward. Thus $\phi = 180°$ and

$$W_{\text{gravity}} = (F\cos\phi)s = (784\text{ N})(5.20\text{ m})\cos 180° = -4077\text{ J}$$

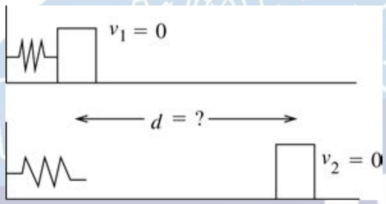
Then $W_{\text{tot}} = W_{\text{you}} + W_{\text{gravity}}$ gives

$$W_{\text{you}} = W_{\text{tot}} - W_{\text{gravity}} = -910\text{ J} - (-4077\text{ J}) = +3170\text{ J}$$

**EVALUATE:** The total work done is negative and you lose kinetic energy.

**6.75.** **IDENTIFY** and **SET UP:** Use Eq. (6.6). Work is done by the spring and by gravity. Let point 1 be where the textbook is released and point 2 be where it stops sliding. $x_2 = 0$ since at point 2 the spring is neither stretched nor compressed. The situation is sketched in Figure 6.75.

**EXECUTE:**




$W_{\text{tot}} = K_2 - K_1$

$K_1 = 0,\ K_2 = 0$

$W_{\text{tot}} = W_{\text{fric}} + W_{\text{spr}}$

**Figure 6.75**

$W_{\text{spr}} = \frac{1}{2}kx_1^2$, where $x_1 = 0.250\text{ m}$ (Spring force is in direction of motion of block so it does positive work.)

$W_{\text{fric}} = -\mu_k mgd$

Then $W_{\text{tot}} = K_2 - K_1$ gives $\frac{1}{2}kx_1^2 - \mu_k mgd = 0$

$$d = \frac{kx_1^2}{2\mu_k mg} = \frac{(250\text{ N/m})\,(0.250\text{ m})^2}{2(0.30)\,(2.50\text{ kg})\,(9.80\text{ m/s}^2)} = 1.1\text{ m},$$ measured from the point where the block was released.

**EVALUATE:** The positive work done by the spring equals the magnitude of the negative work done by friction. The total work done during the motion between points 1 and 2 is zero, and the textbook starts and ends with zero kinetic energy.

**6.76.** **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$.

**SET UP:** Let $x_0$ be the initial distance the spring is compressed. The work done by the spring is $\frac{1}{2}kx_0^2 - \frac{1}{2}kx^2$, where $x$ is the final distance the spring is compressed.

**EXECUTE:** **(a)** Equating the work done by the spring to the gain in kinetic energy, $\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2$, so

$$v = \sqrt{\frac{k}{m}}x_0 = \sqrt{\frac{400\text{ N/m}}{0.0300\text{ kg}}}(0.060\text{ m}) = 6.93\text{ m/s}.$$

**(b)** $W_{\text{tot}}$ must now include friction, so $\frac{1}{2}mv^2 = W_{\text{tot}} = \frac{1}{2}kx_0^2 - fx_0$, where $f$ is the magnitude of the friction force. Then,

$$v = \sqrt{\frac{k}{m}x_0^2 - \frac{2f}{m}x_0} = \sqrt{\frac{400\text{ N/m}}{0.0300\text{ kg}}(0.060\text{ m})^2 - \frac{2(6.00\text{ N})}{(0.0300\text{ kg})}(0.060\text{ m})} = 4.90\text{ m/s}.$$

**(c)** The greatest speed occurs when the acceleration (and the net force) are zero. Let $x$ be the amount the spring is still compressed, so the distance the ball has moved is $x_0 - x$. $kx = f$, $x = \frac{f}{k} = \frac{6.00\text{ N}}{400\text{ N/m}} =$ 0.0150 m.

The ball is 0.0150 m from the end of the barrel, or 0.0450 m from its initial position.

To find the speed, the net work is $W_{tot} = \frac{1}{2}k(x_0^2 - x^2) - f(x_0 - x)$, so the maximum speed is

$$v_{max} = \sqrt{\frac{k}{m}(x_0^2 - x^2) - \frac{2f}{m}(x_0 - x)}.$$

$$v_{max} = \sqrt{\frac{400\ \text{N/m}}{(0.0300\ \text{kg})}\left[(0.060\ \text{m})^2 - (0.0150\ \text{m})^2\right] - \frac{2(6.00\ \text{N})}{(0.0300\ \text{kg})}(0.060\ \text{m} - 0.0150\ \text{m})} = 5.20\ \text{m/s}$$

**EVALUATE:** The maximum speed with friction present (part (c)) is larger than the result of part (b) but smaller than the result of part (a).

**6.77.** **IDENTIFY:** A constant horizontal force pushes a block against a spring on a rough floor. The work-energy theorem and Newton's second law both apply.

**SET UP:** In part (a), we apply the work-energy theorem $W_{tot} = K_2 - K_1$ to the block. $f_k = \mu_k n$ and $W_{spring} = -½\, kx^2$. In part (b), we apply Newton's second law to the block.

**EXECUTE:** **(a)** $W_F + W_{spring} + W_f = K_2 - K_1$. $Fx - ½\, kx^2 - \mu_k mgx = ½\, mv^2 - 0$. Putting in the numbers from the problem gives (82.0 N)(0.800 m) – (130.0 N/m)$(0.800\ \text{m})^2/2$ – (0.400)(4.00 kg)(9.80 m/s$^2$)(0.800 m) = (4.00 kg)$v^2/2$, $v$ = 2.39 m/s.

**(b)** Looking at quantities parallel to the floor, with the positive direction toward the wall, Newton's second law gives $F - f_k - F_{spring} = ma$.

$F - \mu_k mg - kx = ma$: 82.0 N – (0.400)(4.00 kg)(9.80 m/s$^2$) – (130.0 N/m)(0.800 m) = (4.00 kg)$a$

$a$ = –9.42 m/s$^2$. The minus sign means that the acceleration is away from the wall.

**EVALUATE:** The force you apply is toward the wall but the block is accelerating away from the wall.

**6.78.** **IDENTIFY:** A constant horizontal force pushes a frictionless block of ice against a spring on the floor. The work-energy theorem and Newton's second law both apply.

**SET UP:** In part (a), we apply the work-energy theorem $W_{tot} = K_2 - K_1$ to the ice. $W_{spring} = -½\, kx^2$. In part (b), we apply Newton's second law to the ice.

**EXECUTE:** **(a)** $W_F + W_{spring} = K_2 - K_1$. $Fx - ½\, kx^2 = ½\, mv^2 - 0$. Putting in the numbers from the problem gives (54.0 N)(0.400 m) – (76.0 N/m)$(0.400\ \text{m})^2/2$ = (2.00 kg)$v^2/2$, $v$ = 3.94 m/s.

**(b)** Looking at quantities parallel to the floor, with the positive direction away from the post, Newton's second law gives $F - F_{spring} = ma$, so $F - kx = ma$.

54.0 N – (76.0 N/m)(0.400 m) = (2.00 kg)$a$, which gives $a$ = 11.8 m/s$^2$. The acceleration is positive, so the block is accelerating away from the post.

**EVALUATE:** The given force must be greater than the spring force since the ice is accelerating away from the post.

**6.79.** **IDENTIFY:** Apply $W_{tot} = K_2 - K_1$ to the blocks.

**SET UP:** If $X$ is the distance the spring is compressed, the work done by the spring is $-\frac{1}{2}kX^2$. At maximum compression, the spring (and hence the block) is not moving, so the block has no kinetic energy.

**EXECUTE:** **(a)** The work done *by* the block is equal to its initial kinetic energy, and the maximum compression is found from $\frac{1}{2}kX^2 = \frac{1}{2}mv_0^2$ and $X = \sqrt{\frac{m}{k}}v_0 = \sqrt{\frac{5.00\ \text{kg}}{500\ \text{N/m}}}(6.00\ \text{m/s}) = 0.600\ \text{m}$.

**(b)** Solving for $v_0$ in terms of a known $X$, $v_0 = \sqrt{\frac{k}{m}}X = \sqrt{\frac{500\ \text{N/m}}{5.00\ \text{kg}}}(0.150\ \text{m}) = 1.50\ \text{m/s}$.

**EVALUATE:** The negative work done by the spring removes the kinetic energy of the block.

**6.80.** **IDENTIFY:** Apply $W_{tot} = K_2 - K_1$. $W = Fs\cos\phi$.

**SET UP:** The students do positive work, and the force that they exert makes an angle of $30.0°$ with the direction of motion. Gravity does negative work, and is at an angle of $120.0°$ with the chair's motion.

**EXECUTE:** The total work done is $W_{tot} = ((600\ \text{N})\cos 30.0° + (85.0\ \text{kg})(9.80\ \text{m/s}^2)\cos 120.0°)(2.50\ \text{m}) =$ $257.8\ \text{J}$, and so the speed at the top of the ramp is $v_2 = \sqrt{v_1^2 + \frac{2W_{tot}}{m}} = \sqrt{(2.00\ \text{m/s})^2 + \frac{2(257.8\ \text{J})}{(85.0\ \text{kg})}} = 3.17\ \text{m/s}$.

**EVALUATE:** The component of gravity down the incline is $mg\sin 30° = 417\text{ N}$ and the component of the push up the incline is $(600\text{ N})\cos 30° = 520\text{ N}$. The force component up the incline is greater than the force component down the incline; the net work done is positive and the speed increases.

**6.81.** **IDENTIFY** and **SET UP:** Apply $W_{\text{tot}} = K_2 - K_1$ to the system consisting of both blocks. Since they are connected by the cord, both blocks have the same speed at every point in the motion. Also, when the 6.00-kg block has moved downward 1.50 m, the 8.00-kg block has moved 1.50 m to the right. The target variable, $\mu_k$, will be a factor in the work done by friction. The forces on each block are shown in Figure 6.81.

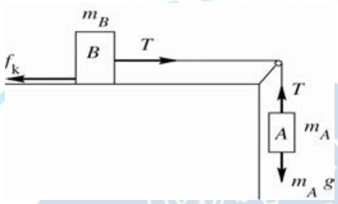



Figure 6.81

**EXECUTE:**

$K_1 = \frac{1}{2}m_A v_1^2 + \frac{1}{2}m_B v_1^2 = \frac{1}{2}(m_A + m_B)v_1^2$

$K_2 = 0$

The tension $T$ in the rope does positive work on block $B$ and the same magnitude of negative work on block $A$, so $T$ does no net work on the system. Gravity does work $W_{mg} = m_A g d$ on block $A$, where $d = 2.00\text{ m}$. (Block $B$ moves horizontally, so no work is done on it by gravity.) Friction does work $W_{\text{fric}} = -\mu_k m_B g d$ on block $B$. Thus $W_{\text{tot}} = W_{mg} + W_{\text{fric}} = m_A g d - \mu_k m_B g d$. Then $W_{\text{tot}} = K_2 - K_1$ gives $m_A g d - \mu_k m_B g d = -\frac{1}{2}(m_A + m_B)v_1^2$ and

$$\mu_k = \frac{m_A}{m_B} + \frac{\frac{1}{2}(m_A + m_B)v_1^2}{m_B g d} = \frac{6.00\text{ kg}}{8.00\text{ kg}} + \frac{(6.00\text{ kg} + 8.00\text{ kg})\,(0.900\text{ m/s})^2}{2(8.00\text{ kg})\,(9.80\text{ m/s}^2)\,(2.00\text{ m})} = 0.786$$

**EVALUATE:** The weight of block $A$ does positive work and the friction force on block $B$ does negative work, so the net work is positive and the kinetic energy of the blocks increases as block $A$ descends. Note that $K_1$ includes the kinetic energy of both blocks. We could have applied the work-energy theorem to block $A$ alone, but then $W_{\text{tot}}$ includes the work done on block $A$ by the tension force.

**6.82.** **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$ to the system of the two blocks. The total work done is the sum of that done by gravity (on the hanging block) and that done by friction (on the block on the table).

**SET UP:** Let $h$ be the distance the 6.00 kg block descends. The work done by gravity is (6.00 kg)$gh$ and the work done by friction is $-\mu_k(8.00\text{ kg})gh$.

**EXECUTE:** $W_{\text{tot}} = (6.00\text{ kg} - (0.25)(8.00\text{ kg}))(9.80\text{ m/s}^2)(1.50\text{ m}) = 58.8\text{ J}$. This work increases the kinetic energy of both blocks: $W_{\text{tot}} = \frac{1}{2}(m_1 + m_2)v^2$, so $v = \sqrt{\dfrac{2(58.8\text{ J})}{(14.00\text{ kg})}} = 2.90\text{ m/s}$.

**EVALUATE:** Since the two blocks are connected by the rope, they move the same distance $h$ and have the same speed $v$.

**6.83.** **IDENTIFY:** Apply Eq. (6.6) to the skater.

**SET UP:** Let point 1 be just before she reaches the rough patch and let point 2 be where she exits from the patch. Work is done by friction. We don't know the skater's mass so can't calculate either friction or the initial kinetic energy. Leave her mass $m$ as a variable and expect that it will divide out of the final equation.

**EXECUTE:** $f_k = 0.25mg$ so $W_f = W_{\text{tot}} = -(0.25mg)s$, where $s$ is the length of the rough patch.

$W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2,\ \ K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}m(0.55v_0)^2 = 0.3025\left(\frac{1}{2}mv_0^2\right)$

The work-energy relation gives $-(0.25mg)s = (0.3025 - 1)\frac{1}{2}mv_0^2$.

The mass divides out, and solving gives $s = 1.3\text{ m}$.

**EVALUATE:** Friction does negative work and this reduces her kinetic energy.

**6.84.** **IDENTIFY and SET UP:** $W = Pt$

**EXECUTE:** **(a)** The hummingbird produces energy at a rate of 0.7 J/s to 1.75 J/s. At 10 beats/s, the bird must expend between 0.07 J/beat and 0.175 J/beat.

**(b)** The steady output of the athlete is $(500 \text{ W})/(70 \text{ kg}) = 7 \text{ W/kg}$, which is below the 10 W/kg necessary to stay aloft. Though the athlete can expend $1400 \text{ W}/70 \text{ kg} = 20 \text{ W/kg}$ for short periods of time, no human-powered aircraft could stay aloft for very long.

**EVALUATE:** Movies of early attempts at human-powered flight bear out our results.

**6.85.** **IDENTIFY:** To lift a mass $m$ a height $h$ requires work $W = mgh$. To accelerate mass $m$ from rest to speed $v$ requires $W = K_2 - K_1 = \frac{1}{2}mv^2$. $P_{\text{av}} = \dfrac{\Delta W}{\Delta t}$.

**SET UP:** $t = 60 \text{ s}$

**EXECUTE:** **(a)** $(800 \text{ kg})(9.80 \text{ m/s}^2)(14.0 \text{ m}) = 1.10\times10^5 \text{ J}$.

**(b)** $(1/2)(800 \text{ kg})(18.0 \text{ m/s}^2) = 1.30\times10^5 \text{ J}$.

**(c)** $\dfrac{1.10\times10^5 \text{ J} + 1.30\times10^5 \text{ J}}{60 \text{ s}} = 3.99 \text{ kW}$.

**EVALUATE:** Approximately the same amount of work is required to lift the water against gravity as to accelerate it to its final speed.

**6.86.** **IDENTIFY and SET UP:** Use Eq. (6.15). The work done on the water by gravity is $mgh$, where $h = 170 \text{ m}$. Solve for the mass $m$ of water for 1.00 s and then calculate the volume of water that has this mass.

**EXECUTE:** The power output is $P_{\text{av}} = 2000 \text{ MW} = 2.00\times10^9 \text{ W}$. $P_{\text{av}} = \dfrac{\Delta W}{\Delta t}$ and 92% of the work done on the water by gravity is converted to electrical power output, so in 1.00 s the amount of work done on the water by gravity is

$$W = \frac{P_{\text{av}}\Delta t}{0.92} = \frac{(2.00\times10^9 \text{ W})(1.00 \text{ s})}{0.92} = 2.174\times10^9 \text{ J}.$$

$W = mgh$, so the mass of water flowing over the dam in 1.00 s must be

$$m = \frac{W}{gh} = \frac{2.174\times10^9 \text{ J}}{(9.80 \text{ m/s}^2)(170 \text{ m})} = 1.30\times10^6 \text{ kg}.$$

$$\text{density} = \frac{m}{V} \text{ so } V = \frac{m}{\text{density}} = \frac{1.30\times10^6 \text{ kg}}{1.00\times10^3 \text{ kg/m}^3} = 1.30\times10^3 \text{ m}^3.$$

**EVALUATE:** The dam is 1270 m long, so this volume corresponds to about a $\text{m}^3$ flowing over each 1 m length of the dam, a reasonable amount.

**6.87.** **IDENTIFY and SET UP:** Energy is $P_{\text{av}}t$. The total energy expended in one day is the sum of the energy expended in each type of activity.

**EXECUTE:** $1 \text{ day} = 8.64\times10^4 \text{ s}$

Let $t_{\text{walk}}$ be the time she spends walking and $t_{\text{other}}$ be the time she spends in other activities;

$t_{\text{other}} = 8.64\times10^4 \text{ s} - t_{\text{walk}}$.

The energy expended in each activity is the power output times the time, so

$$E = Pt = (280 \text{ W})t_{\text{walk}} + (100 \text{ W})t_{\text{other}} = 1.1\times10^7 \text{ J}$$

$$(280 \text{ W})t_{\text{walk}} + (100 \text{ W})(8.64\times10^4 \text{ s} - t_{\text{walk}}) = 1.1\times10^7 \text{ J}$$

$$(180 \text{ W})t_{\text{walk}} = 2.36\times10^6 \text{ J}$$

$$t_{\text{walk}} = 1.31\times10^4 \text{ s} = 218 \text{ min} = 3.6 \text{ h}.$$

**EVALUATE:** Her average power for one day is $(1.1\times10^7 \text{ J})/[(24)(3600 \text{ s})] = 127 \text{ W}$. This is much closer to her 100 W rate than to her 280 W rate, so most of her day is spent at the 100 W rate.

**6.88.** **IDENTIFY:** $W = \int_{x_1}^{x_2} F_x dx,$ and $F_x$ depends on both $x$ and $y$.

**SET UP:** In each case, use the value of $y$ that applies to the specified path. $\int x dx = \frac{1}{2}x^2$. $\int x^2 dx = \frac{1}{3}x^3$.

**EXECUTE:** **(a)** Along this path, $y$ is constant, with the value $y = 3.00$ m.

$$W = \alpha y \int_{x_1}^{x_2} x dx = (2.50\ \text{N/m}^2)(3.00\ \text{m})\frac{(2.00\ \text{m})^2}{2} = 15.0\ \text{J},\ \text{since}\ x_1 = 0\ \text{and}\ x_2 = 2.00\ \text{m}.$$

**(b)** Since the force has no $y$-component, no work is done moving in the $y$-direction.

**(c)** Along this path, $y$ varies with position along the path, given by $y = 1.5x$, so $F_x = \alpha(1.5x)x = 1.5\alpha x^2$, and

$$W = \int_{x_1}^{x_2} F dx = 1.5\alpha \int_{x_1}^{x_2} x^2 dx = 1.5(2.50\ \text{N/m}^2)\frac{(2.00\ \text{m})^3}{3} = 10.0\ \text{J}.$$

**EVALUATE:** The force depends on the position of the object along its path.

**6.89.** **IDENTIFY** and **SET UP:** For part (a) calculate $m$ from the volume of blood pumped by the heart in one day. For part (b) use $W$ calculated in part (a) in Eq. (6.15).

**EXECUTE:** (a) $W = mgh$, as in Example 6.10. We need the mass of blood lifted; we are given the volume

$$V = (7500\ \text{L})\left(\frac{1\times 10^{-3}\ \text{m}^3}{1\ \text{L}}\right) = 7.50\ \text{m}^3.$$

$$m = \text{density} \times \text{volume} = (1.05\times 10^3\ \text{kg/m}^3)(7.50\ \text{m}^3) = 7.875\times 10^3\ \text{kg}$$

Then $W = mgh = (7.875\times 10^3\ \text{kg})(9.80\ \text{m/s}^2)(1.63\ \text{m}) = 1.26\times 10^5\ \text{J}.$

**(b)** $P_{\text{av}} = \dfrac{\Delta W}{\Delta t} = \dfrac{1.26\times 10^5\ \text{J}}{(24\ \text{h})(3600\ \text{s/h})} = 1.46\ \text{W}.$

**EVALUATE:** Compared to light bulbs or common electrical devices, the power output of the heart is rather small.

**6.90.** **IDENTIFY:** We know information about the force exerted by a stretched rubber band and want to know if it obeys Hooke's law.

**SET UP:** Hooke's law is $F = kx$. The graph fits the equation $F = 33.55x^{0.4871}$, with $F$ in newtons and $x$ in meters.

**EXECUTE:** **(a)** For Hooke's law, a graph of $F$ versus $x$ is a straight line through the origin. This graph is not a straight line, so the rubber band does not obey Hooke's law.

**(b)** $k_{\text{eff}} = \dfrac{dF}{dx} = \dfrac{d}{dx}(33.55x^{0.4871}) = 16.34x^{-0.5129}$. Because of the negative exponent for $x$, as $x$ increases, $k_{\text{eff}}$ decreases.

**(c)** The definition of work gives $W = \int_a^b F_x dx = \int_0^{0.0400\ \text{m}} 0.3355x^{0.4871} dx = (33.55/1.4871)\ 0.0400^{1.4871}$

$W = 0.188$ J. From 0.0400 m to 0.0800 m, we follow the same procedure but with different limits of integration. The result is $W = (33.55/1.4871)\ (0.0800^{1.4871} - 0.0400^{1.4871}) = 0.339$ J.

**(d)** $W = K_2 - K_1 = ½\ mv^2 - 0$, which gives $0.339\ \text{J} = (0.300\ \text{kg})v^2/2$, $v = 1.50$ m/s.

**EVALUATE:** The rubber band does not obey Hooke's law, but it does obey the work-energy theorem.

**6.91.** **IDENTIFY:** We know a spring obeys Hooke's law, and we want to use observations of the motion of a block attached to this spring to determine its force constant and the coefficient of friction between the block and the surface on which it is sliding. The work-energy theorem applies.

**SET UP:** $W_{\text{tot}} = K_2 - K_1$, $W_{\text{spring}} = ½\ kx^2$.

**EXECUTE:** **(a)** The spring force is initially greater than friction, so the block accelerates forward. But eventually the spring force decreases enough so that it is less than the force of friction, and the block then slows down (decelerates).

**(b)** The spring is initially compressed a distance $x_0$, and after the block has moved a distance $d$, the spring is compressed a distance $x = x_0 - d$. Therefore the work done by the spring is

$W_{\text{spring}} = \dfrac{1}{2}kx_0^2 - \dfrac{1}{2}k(x_0 - d)^2$. The work done by friction is $W_f = -\mu_k mgd$.

The work-energy theorem gives $W_{\text{spring}} + W_f = K_2 - K_1 = ½\, mv^2$. Using our previous results, we get $\frac{1}{2}kx_0^2 - \frac{1}{2}k(x_0 - d)^2 - \mu_k mgd = \frac{1}{2}mv^2$. Solving for $v^2$ gives $v^2 = -\frac{k}{m}d^2 + 2d\left(\frac{k}{m}x_0 - \mu_k g\right)$, where $x_0 = 0.400$ m.

**(c)** Figure 6.91 shows the resulting graph of $v^2$ versus $d$. Using a graphing program and a quadratic fit gives $v^2 = -39.96d^2 + 16.31d$. The maximum speed occurs when $dv^2/dd = 0$, which gives $(-39.96)(2d) + 16.31 = 0$, so $d = 0.204$ m. For this value of $d$, we have $v^2 = (-39.96)(0.204\text{ m})^2 + (16.31)(0.204\text{ m})$, giving $v = 1.29$ m/s.

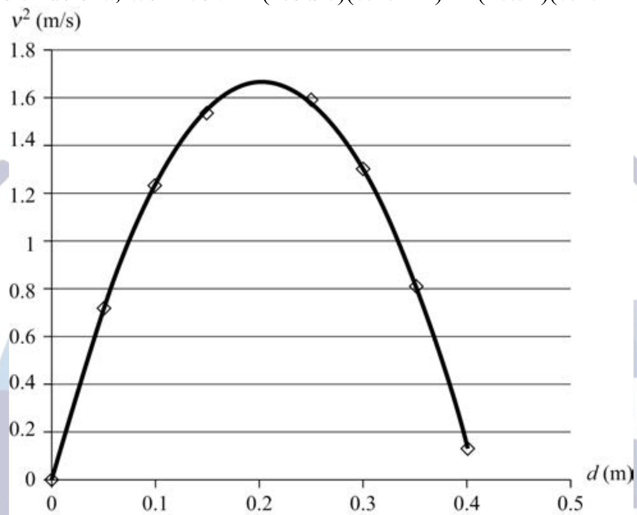



**Figure 6.91**

**(d)** From our work in (b) and (c), we know that $-k/m$ is the coefficient of $d^2$, so $-k/m = -39.96$, which gives $k = (39.96)(0.300\text{ kg}) = 12.0$ N/m. We also know that $2(kx_0/m - \mu_k g)$ is the coefficient of $d$. Solving for $\mu_k$ and putting in the numbers gives $\mu_k = 0.800$.

**EVALUATE:** The graphing program makes analysis of complicated behavior relatively easy.

**6.92.** **IDENTIFY:** The power output of the runners is the work they do in running from the basement to the top floor divided by the time it takes to make this run.

**SET UP:** $P = W/t$ and $W = mgh$.

**EXECUTE:** **(a)** For each runner, $P = mgh/t$. We must read the time of each runner from the figure shown with the problem. For example, for Tatiana we have $P = (50.2\text{ kg})(9.80\text{ m/s}^2)(16.0\text{ m})/32\text{ s} = 246.0$ W, which we must round to 2 significant figures because we cannot read the times any more accurate than that using the figure in the text. Carrying out these calculations for all the runners, we get the following results. Tatiana: 250 W, Bill: 210 W, Ricardo: 290 W, Melanie: 170 W. Ricardo had the greatest power output, and Melanie had the least.

**(b)** Solving $P = mgh/t$ for $t$ gives $t = mgh/P = (62.3\text{ kg})(9.80\text{ m/s}^2)(16.0\text{ m})/(746\text{ W}) = 13.1$ s, where we have used the fact that 1 hp = 746 W.

**EVALUATE:** Even though Tatiana had the shortest time, her power output was less than Ricardo's because she weighs less than he does.

**6.93.** **IDENTIFY:** In part (a) follow the steps outlined in the problem. For parts (b), (c), and (d) apply the work-energy theorem.

**SET UP:** $\int x^2 dx = \frac{1}{3}x^3$

**EXECUTE:** **(a)** Denote the position of a piece of the spring by $l$; $l = 0$ is the fixed point and $l = L$ is the moving end of the spring. Then the velocity of the point corresponding to $l$, denoted $u$, is $u(l) = v(l/L)$ (when the spring is moving, $l$ will be a function of time, and so $u$ is an implicit function of time). The mass of a piece of length $dl$ is $dm = (M/L)dl$, and so $dK = \frac{1}{2}(dm)u^2 = \frac{1}{2}\frac{Mv^2}{L^3}l^2 dl$, and $K = \int dK = \frac{Mv^2}{2L^3}\int_0^L l^2 dl = \frac{Mv^2}{6}$.

**(b)** $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, so $v = \sqrt{(k/m)x} = \sqrt{(3200\text{ N/m})/(0.053\text{ kg})}\,(2.50\times10^{-2}\text{ m}) = 6.1$ m/s.

**(c)** With the mass of the spring included, the work that the spring does goes into the kinetic energies of both the ball and the spring, so $\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{6}Mv^2$. Solving for $v$,

$$v = \sqrt{\frac{k}{m + M/3}}\,x = \sqrt{\frac{(3200\ \text{N/m})}{(0.053\ \text{kg}) + (0.243\ \text{kg})/3}}(2.50 \times 10^{-2}\ \text{m}) = 3.9\ \text{m/s}.$$

**(d)** Algebraically, $\frac{1}{2}mv^2 = \frac{(1/2)kx^2}{(1 + M/3m)} = 0.40\ \text{J}$ and $\frac{1}{6}Mv^2 = \frac{(1/2)kx^2}{(1 + 3m/M)} = 0.60\ \text{J}$.

**EVALUATE:** For this ball and spring, $\frac{K_{\text{ball}}}{K_{\text{spring}}} = \frac{3m}{M} = 3\left(\frac{0.053\ \text{kg}}{0.243\ \text{kg}}\right) = 0.65$. The percentage of the final kinetic energy that ends up with each object depends on the ratio of the masses of the two objects. As expected, when the mass of the spring is a small fraction of the mass of the ball, the fraction of the kinetic energy that ends up in the spring is small.

**6.94.** **IDENTIFY:** In both cases, a given amount of fuel represents a given amount of work $W_0$ that the engine does in moving the plane forward against the resisting force. Write $W_0$ in terms of the range $R$ and speed $v$ and in terms of the time of flight $T$ and $v$.

**SET UP:** In both cases assume $v$ is constant, so $W_0 = RF$ and $R = vT$.

**EXECUTE:** In terms of the range $R$ and the constant speed $v$, $W_0 = RF = R\left(\alpha v^2 + \frac{\beta}{v^2}\right)$.

In terms of the time of flight $T, R = vt$, so $W_0 = vTF = T\left(\alpha v^3 + \frac{\beta}{v}\right)$.

**(a)** Rather than solve for $R$ as a function of $v$, differentiate the first of these relations with respect to $v$, setting $\frac{dW_0}{dv} = 0$ to obtain $\frac{dR}{dv}F + R\frac{dF}{dv} = 0$. For the maximum range, $\frac{dR}{dv} = 0$, so $\frac{dF}{dv} = 0$. Performing the differentiation, $\frac{dF}{dv} = 2\alpha v - 2\beta/v^3 = 0$, which is solved for

$$v = \left(\frac{\beta}{\alpha}\right)^{1/4} = \left(\frac{3.5 \times 10^5\ \text{N} \cdot \text{m}^2/\text{s}^2}{0.30\ \text{N} \cdot \text{s}^2/\text{m}^2}\right)^{1/4} = 32.9\ \text{m/s} = 118\ \text{km/h}.$$

**(b)** Similarly, the maximum time is found by setting $\frac{d}{dv}(Fv) = 0$; performing the differentiation,

$$3\alpha v^2 - \beta/v^2 = 0.\quad v = \left(\frac{\beta}{3\alpha}\right)^{1/4} = \left(\frac{3.5 \times 10^5\ \text{N} \cdot \text{m}^2/\text{s}^2}{3(0.30\ \text{N} \cdot \text{s}^2/\text{m}^2)}\right)^{1/4} = 25\ \text{m/s} = 90\ \text{km/h}.$$

**EVALUATE:** When $v = (\beta/\alpha)^{1/4}$, $F_{\text{air}}$ has its minimum value $F_{\text{air}} = 2\sqrt{\alpha\beta}$. For this $v$, $R_1 = (0.50)\frac{W_0}{\sqrt{\alpha\beta}}$ and $T_1 = (0.50)\alpha^{-1/4}\beta^{-3/4}$. When $v = (\beta/3\alpha)^{1/4}$, $F_{\text{air}} = 2.3\sqrt{\alpha\beta}$. For this $v$, $R_2 = (0.43)\frac{W_0}{\sqrt{\alpha\beta}}$ and $T_2 = (0.57)\alpha^{-1/4}\beta^{-3/4}$. $R_1 > R_2$ and $T_2 > T_1$, as they should be.

**6.95.** **IDENTIFY:** Using 300 W of metabolic power, the person travels 3 times as fast when biking than when walking.

**SET UP:** $P = W/t$, so $W = Pt$.

**EXECUTE:** When biking, the person travels 3 times as fast as when walking, so the bike trip takes 1/3 the time. Since $W = Pt$ and the power is the same, the energy when biking will be 1/3 of the energy when walking, which makes choice (a) the correct one.

**EVALUATE:** Walking is obviously a better way to burn calories than biking.

**6.96.** **IDENTIFY:** When walking on a grade, metabolic power is required for walking horizontally as well as the vertical climb.

**SET UP:** $P = W/t$, $W = mgh$.

**EXECUTE:** $P_{tot} = P_{horiz} + P_{vert} = P_{horiz} + mgh/t = P_{horiz} + mg(v_{vert})$. The slope is a 5% grade, so $v_{vert} = 0.05v_{horiz}$. Therefore $P_{tot} = 300\text{ W} + (70\text{ kg})(9.80\text{ m/s}^2)(0.05)(1.4\text{ m/s}) = 348\text{ W} \approx 350\text{ W}$, which makes choice (c) correct.

**EVALUATE:** Even a small grade of only 5% makes a difference of about 17% in power output.

**6.97.** **IDENTIFY:** Using 300 W of metabolic power, the person travels 3 times as fast when biking than when walking.

**SET UP:** $K = \frac{1}{2} mv^2$.

**EXECUTE:** The speed when biking is 3 times the speed when walking. Since the kinetic energy is proportional to the square of the speed, the kinetic energy will be $3^2 = 9$ times as great when biking, making choice (d) correct.

**EVALUATE:** Even a small increase in speed gives a considerable increase in kinetic energy due to the $v^2$.

# 7

## POTENTIAL ENERGY AND ENERGY CONSERVATION

**7.1.** **IDENTIFY:** $U_{\text{grav}} = mgy$ so $\Delta U_{\text{grav}} = mg(y_2 - y_1)$

**SET UP:** $+y$ is upward.

**EXECUTE:** **(a)** $\Delta U = (75\text{ kg})(9.80\text{ m/s}^2)(2400\text{ m} - 1500\text{ m}) = +6.6\times10^5\text{ J}$

**(b)** $\Delta U = (75\text{ kg})(9.80\text{ m/s}^2)(1350\text{ m} - 2400\text{ m}) = -7.7\times10^5\text{ J}$

**EVALUATE:** $U_{\text{grav}}$ increases when the altitude of the object increases.

**7.2.** **IDENTIFY:** The change in height of a jumper causes a change in their potential energy.

**SET UP:** Use $\Delta U_{\text{grav}} = mg(y_2 - y_1)$.

**EXECUTE:** $\Delta U_{\text{grav}} = (72\text{ kg})(9.80\text{ m/s}^2)(0.60\text{ m}) = 420\text{ J}$.

**EVALUATE:** This gravitational potential energy comes from elastic potential energy stored in the jumper's tensed muscles.

**7.3.** **IDENTIFY:** Use the free-body diagram for the bag and Newton's first law to find the force the worker applies. Since the bag starts and ends at rest, $K_2 - K_1 = 0$ and $W_{\text{tot}} = 0$.

**SET UP:** A sketch showing the initial and final positions of the bag is given in Figure 7.3a. $\sin\phi = \dfrac{2.0\text{ m}}{3.5\text{ m}}$ and $\phi = 34.85°$. The free-body diagram is given in Figure 7.3b. $\vec{F}$ is the horizontal force applied by the worker. In the calculation of $U_{\text{grav}}$ take $+y$ upward and $y = 0$ at the initial position of the bag.

**EXECUTE:** **(a)** $\Sigma F_y = 0$ gives $T\cos\phi = mg$ and $\Sigma F_x = 0$ gives $F = T\sin\phi$. Combining these equations to eliminate $T$ gives $F = mg\tan\phi = (90.0\text{ kg})(9.80\text{ m/s}^2)\tan 34.85° = 610\text{ N}$.

**(b)** (i) The tension in the rope is radial and the displacement is tangential so there is no component of $T$ in the direction of the displacement during the motion and the tension in the rope does no work.

(ii) $W_{\text{tot}} = 0$ so

$$W_{\text{worker}} = -W_{\text{grav}} = U_{\text{grav},2} - U_{\text{grav},1} = mg(y_2 - y_1) = (90.0\text{ kg})(9.80\text{ m/s}^2)(0.6277\text{ m}) = 550\text{ J}.$$

**EVALUATE:** The force applied by the worker varies during the motion of the bag and it would be difficult to calculate $W_{\text{worker}}$ directly.

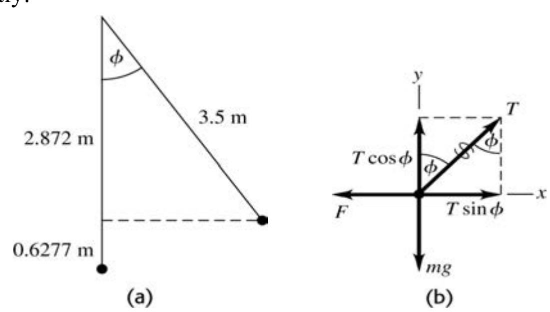



**Figure 7.3**

**7.4.** **IDENTIFY:** The energy from the food goes into the increased gravitational potential energy of the hiker. We must convert food calories to joules.

**SET UP:** The change in gravitational potential energy is $\Delta U_{grav} = mg\ (y_f - y_i)$, while the increase in kinetic energy is negligible. Set the food energy, expressed in joules, equal to the mechanical energy developed.

**EXECUTE:** **(a)** The food energy equals $mg(y_2 - y_1)$, so

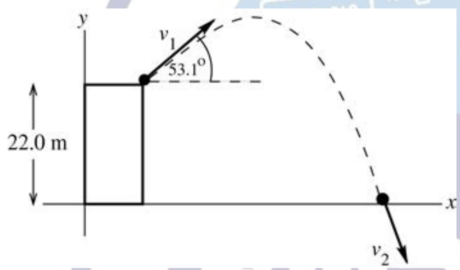
$$y_2 - y_1 = \frac{(140\text{ food calories})(4186\text{ J/1 food calorie})}{(65\text{ kg})(9.80\text{ m/s}^2)} = 920\text{ m}.$$

**(b)** The mechanical energy would be 20% of the results of part (a), so $\Delta y = (0.20)(920\text{ m}) = 180\text{ m}$.

**EVALUATE:** Since only 20% of the food calories go into mechanical energy, the hiker needs much less of climb to turn off the calories in the bar.

**7.5.** **IDENTIFY** and **SET UP:** Use $K_1 + U_1 + W_{other} = K_2 + U_2$. Points 1 and 2 are shown in Figure 7.5.

**(a)** $K_1 + U_1 + W_{other} = K_2 + U_2$. Solve for $K_2$ and then use $K_2 = \frac{1}{2}mv_2^2$ to obtain $v_2$.




$W_{other} = 0$ (The only force on the ball while it is in the air is gravity.)
$K_1 = \frac{1}{2}mv_1^2$; $K_2 = \frac{1}{2}mv_2^2$
$U_1 = mgy_1$, $y_1 = 22.0\text{ m}$
$U_2 = mgy_2 = 0$, since $y_2 = 0$ for our choice of coordinates.

**Figure 7.5**

**EXECUTE:** $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2$

$$v_2 = \sqrt{v_1^2 + 2gy_1} = \sqrt{(12.0\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(22.0\text{ m})} = 24.0\text{ m/s}$$

**EVALUATE:** The projection angle of $53.1°$ doesn't enter into the calculation. The kinetic energy depends only on the magnitude of the velocity; it is independent of the direction of the velocity.

**(b)** Nothing changes in the calculation. The expression derived in part (a) for $v_2$ is independent of the angle, so $v_2 = 24.0\text{ m/s}$, the same as in part (a).

**(c)** The ball travels a shorter distance in part (b), so in that case air resistance will have less effect.

**7.6.** **IDENTIFY:** The normal force does no work, so only gravity does work and $K_1 + U_1 = K_2 + U_2$ applies.

**SET UP:** $K_1 = 0$. The crate's initial point is at a vertical height of $d\sin\alpha$ above the bottom of the ramp.

**EXECUTE:** **(a)** $y_2 = 0$, $y_1 = d\sin\alpha$. $K_1 + U_{grav,1} = K_2 + U_{grav,2}$ gives $U_{grav,1} = K_2$. $mgd\sin\alpha = \frac{1}{2}mv_2^2$ and $v_2 = \sqrt{2gd\sin\alpha}$.

**(b)** $y_1 = 0$, $y_2 = -d\sin\alpha$. $K_1 + U_{grav,1} = K_2 + U_{grav,2}$ gives $0 = K_2 + U_{grav,2}$. $0 = \frac{1}{2}mv_2^2 + (-mgd\sin\alpha)$ and $v_2 = \sqrt{2gd\sin\alpha}$, the same as in part (a).

**(c)** The normal force is perpendicular to the displacement and does no work.

**EVALUATE:** When we use $U_{grav} = mgy$ we can take any point as $y = 0$ but we must take $+y$ to be upward.

**7.7.** **IDENTIFY:** The take-off kinetic energy of the flea goes into gravitational potential energy.

**SET UP:** Use $K_1 + U_1 = K_2 + U_2$. Let $y_1 = 0$ and $y_2 = h$ and note that $U_1 = 0$ while $K_2 = 0$ at the maximum height. Consequently, conservation of energy becomes $mgh = \frac{1}{2}mv_1^2$.

**EXECUTE:** **(a)** $v_1 = \sqrt{2gh} = \sqrt{2(9.80 \text{ m/s}^2)(0.20 \text{ m})} = 2.0 \text{ m/s}.$

**(b)** $K_1 = mgh = (0.50 \times 10^{-6} \text{ kg})(9.80 \text{ m/s}^2)(0.20 \text{ m}) = 9.8 \times 10^{-7} \text{ J}.$ The kinetic energy per kilogram is $\dfrac{K_1}{m} = \dfrac{9.8 \times 10^{-7} \text{ J}}{0.50 \times 10^{-6} \text{ kg}} = 2.0 \text{ J/kg}.$

**(c)** The human can jump to a height of $h_\text{h} = h_\text{f}\left(\dfrac{l_\text{h}}{l_\text{f}}\right) = (0.20 \text{ m})\left(\dfrac{2.0 \text{ m}}{2.0 \times 10^{-3} \text{ m}}\right) = 200 \text{ m}.$ To attain this height, he would require a takeoff speed of: $v_1 = \sqrt{2gh} = \sqrt{2(9.80 \text{ m/s}^2)(200 \text{ m})} = 63 \text{ m/s}.$

**(d)** The human's kinetic energy per kilogram is $\dfrac{K_1}{m} = gh = (9.80 \text{ m/s}^2)(0.60 \text{ m}) = 5.9 \text{ J/kg}.$

**(e) EVALUATE:** The flea stores the energy in its tensed legs.

**7.8.** **IDENTIFY:** The potential energy is transformed into kinetic energy which is then imparted to the bone.

**SET UP:** The initial gravitational potential energy must be absorbed by the leg bones. $U_1 = mgh.$

**EXECUTE:** **(a)** $mgh = 2(200 \text{ J}),$ so $h = \dfrac{400 \text{ J}}{(60 \text{ kg})(9.80 \text{ m/s}^2)} = 0.68 \text{ m} = 68 \text{ cm}.$

**(b) EVALUATE:** They flex when they land and their joints and muscles absorb most of the energy.

**(c) EVALUATE:** Their bones are more fragile so can absorb less energy without breaking and their muscles and joints are weaker and less flexible and therefore less able to absorb energy.

**7.9.** **IDENTIFY:** $W_\text{tot} = K_B - K_A.$ The forces on the rock are gravity, the normal force and friction.

**SET UP:** Let $y = 0$ at point $B$ and let $+y$ be upward. $y_A = R = 0.50 \text{ m}.$ The work done by friction is negative; $W_f = -0.22 \text{ J}.$ $K_A = 0.$ The free-body diagram for the rock at point $B$ is given in Figure 7.9. The acceleration of the rock at this point is $a_\text{rad} = v^2/R,$ upward.

**EXECUTE:** **(a)** (i) The normal force is perpendicular to the displacement and does zero work.

(ii) $W_\text{grav} = U_{\text{grav},A} - U_{\text{grav},B} = mgy_A = (0.20 \text{ kg})(9.80 \text{ m/s}^2)(0.50 \text{ m}) = 0.98 \text{ J}.$

**(b)** $W_\text{tot} = W_n + W_f + W_\text{grav} = 0 + (-0.22 \text{ J}) + 0.98 \text{ J} = 0.76 \text{ J}.$ $W_\text{tot} = K_B - K_A$ gives $\frac{1}{2}mv_B^2 = W_\text{tot}.$

$$v_B = \sqrt{\frac{2W_\text{tot}}{m}} = \sqrt{\frac{2(0.76 \text{ J})}{0.20 \text{ kg}}} = 2.8 \text{ m/s}.$$

**(c)** Gravity is constant and equal to $mg$. $n$ is not constant; it is zero at $A$ and not zero at $B$. Therefore, $f_\text{k} = \mu_\text{k} n$ is also not constant.

**(d)** $\Sigma F_y = ma_y$ applied to Figure 7.9 gives $n - mg = ma_\text{rad}.$

$$n = m\left(g + \frac{v^2}{R}\right) = (0.20 \text{ kg})\left(9.80 \text{ m/s}^2 + \frac{[2.8 \text{ m/s}]^2}{0.50 \text{ m}}\right) = 5.1 \text{ N}.$$

**EVALUATE:** In the absence of friction, the speed of the rock at point $B$ would be $\sqrt{2gR} = 3.1 \text{ m/s}.$ As the rock slides through point $B$, the normal force is greater than the weight $mg = 2.0 \text{ N}$ of the rock.

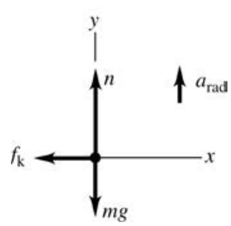



**Figure 7.9**

**7.10.** **IDENTIFY:** The child's energy is transformed from gravitational potential energy to kinetic energy as she swings downward.

**SET UP:** Let $y_2 = 0$. For part (a), $U_1 = mgy_1$. For part (b) use $K_2 + U_2 = K_1 + U_1$ with $U_2 = K_1 = 0$ and $K_2 = \frac{1}{2}mv_2{}^2$; the result is $\frac{1}{2}mv_2{}^2 = mgy_1$.

**EXECUTE:** **(a)** Figure 7.10 shows that the difference in potential energy at the top of the swing is proportional to the height difference, $y_1 = (2.20\text{ m})(1 - \cos 42°) = 0.56$ m. The difference in potential energy is thus $U_1 = mgy_1 = (25\text{ kg})(9.80\text{ m/s}^2)(0.56\text{ m}) = 140$ J.

**(b)** $v_2 = \sqrt{2gy_1} = \sqrt{2(9.80\text{ m/s}^2)(0.56\text{ m})} = 3.3\text{ m/s}$.

**EVALUATE:** **(c)** The tension is radial while the displacement is tangent to the circular path; thus there is no component of the tension along the direction of the displacement and the tension in the ropes does no work on the child.

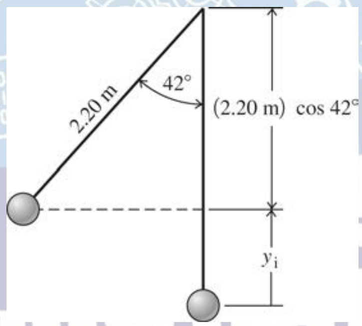



**Figure 7.10**

**7.11.** **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the car.

**SET UP:** Take $y = 0$ at point $A$. Let point 1 be $A$ and point 2 be $B$.

**EXECUTE:** $U_1 = 0$, $U_2 = mg(2R) = 28{,}224$ J, $W_{\text{other}} = W_f$

$K_1 = \frac{1}{2}mv_1^2 = 37{,}500$ J, $K_2 = \frac{1}{2}mv_2^2 = 3840$ J

The work-energy relation then gives $W_f = K_2 + U_2 - K_1 = -5400$ J.

**EVALUATE:** Friction does negative work. The final mechanical energy $(K_2 + U_2 = 32{,}064\text{ J})$ is less than the initial mechanical energy $(K_1 + U_1 = 37{,}500\text{ J})$ because of the energy removed by friction work.

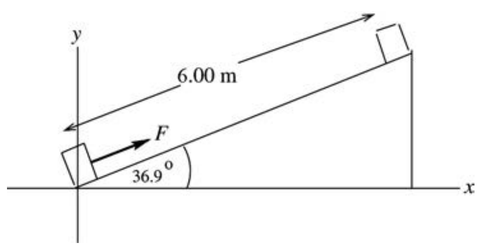
**7.12.** **IDENTIFY:** Only gravity does work, so apply $K_1 + U_1 = K_2 + U_2$.

**SET UP:** $v_1 = 0$, so $\frac{1}{2}mv_2^2 = mg(y_1 - y_2)$.

**EXECUTE:** Tarzan is lower than his original height by a distance $y_1 - y_2 = l(\cos 30° - \cos 45°)$ so his speed is $v = \sqrt{2gl(\cos 30° - \cos 45°)} = 7.9$ m/s, a bit quick for conversation.

**EVALUATE:** The result is independent of Tarzan's mass.

**7.13.** **(a) IDENTIFY and SET UP:** $\vec{F}$ is constant so Eq. (6.2) can be used. The situation is sketched in Figure 7.13a.




$y_1 = 0$

$y_2 = (6.00\text{ m})\sin 36.9°$

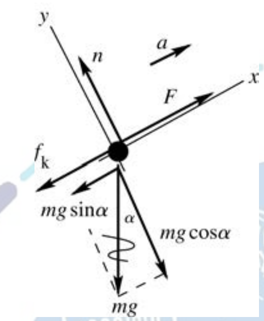
$y_2 = 3.60$ m

**Figure 7.13a**

**Execute:** $W_F = (F\cos\phi)s = (110\text{ N})(\cos 0°)(6.00\text{ m}) = 660\text{ J}.$

**Evaluate:** $\vec{F}$ is in the direction of the displacement and does positive work.

**(b) Identify** and **Set Up**: Calculate $W$ using but first we must calculate the friction force. Use the free-body diagram for the oven sketched in Figure 7.13b to calculate the normal force $n$; then the friction force can be calculated from $f_k = \mu_k n$. For this calculation use coordinates parallel and perpendicular to the incline.




**Execute:** $\Sigma F_y = ma_y$

$n - mg\cos 36.9° = 0$

$n = mg\cos 36.9°$

$f_k = \mu_k n = \mu_k mg\cos 36.9°$

$f_k = (0.25)(10.0\text{ kg})(9.80\text{ m/s}^2)\cos 36.9° = 19.6\text{ N}$

**Figure 7.13b**

$W_f = (f_k\cos\phi)s = (19.6\text{ N})(\cos 180°)(6.00\text{ m}) = -117.6\text{ J}$, which rounds to $-118$ J.

**Evaluate:** Friction does negative work.

**(c) Identify** and **Set Up:** $U = mgy$; take $y = 0$ at the bottom of the ramp.

**Execute:** $\Delta U = U_2 - U_1 = mg(y_2 - y_1) = (10.0\text{ kg})(9.80\text{ m/s}^2)(3.60\text{ m} - 0) = 352.8\text{ J}$, which rounds to 353 J.

**Evaluate:** The object moves upward and $U$ increases.

**(d) Identify** and **Set Up:** Use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ and solve for $\Delta K$.

**Execute:** $\Delta K = K_2 - K_1 = U_1 - U_2 + W_{\text{other}}$

$\Delta K = W_{\text{other}} - \Delta U$

$W_{\text{other}} = W_F + W_f = 660\text{ J} - 117.6\text{ J} = 542.4\text{ J}$

$\Delta U = 352.8\text{ J}$

Thus $\Delta K = 542.4\text{ J} - 352.8\text{ J} = 189.6\text{ J}$, which rounds to 190 J.

**Evaluate:** $W_{\text{other}}$ is positive. Some of $W_{\text{other}}$ goes to increasing $U$ and the rest goes to increasing $K$.

**(e) Identify:** Apply $\Sigma\vec{F} = m\vec{a}$ to the oven. Solve for $\vec{a}$ and then use a constant acceleration equation to calculate $v_2$.

**Set Up:** We can use the free-body diagram that is in part (b):

$\Sigma F_x = ma_x$

$F - f_k - mg\sin 36.9° = ma$

**Execute:** $a = \dfrac{F - f_k - mg\sin 36.9°}{m} = \dfrac{110\text{ N} - 19.6\text{ N} - (10\text{ kg})(9.80\text{ m/s}^2)\sin 36.9°}{10.0\text{ kg}} = 3.16\text{ m/s}^2$

**Set Up:** $v_{1x} = 0$, $a_x = 3.16\text{ m/s}^2$, $x - x_0 = 6.00\text{ m}$, $v_{2x} = ?$

$v_{2x}^2 = v_{1x}^2 + 2a_x(x - x_0)$

**Execute:** $v_{2x} = \sqrt{2a_x(x - x_0)} = \sqrt{2(3.16\text{ m/s}^2)(6.00\text{ m})} = 6.158\text{ m/s}$. Then

$\Delta K = K_2 - K_1 = \frac{1}{2}mv_2^2 = \frac{1}{2}(10.0\text{ kg})(6.158\text{ m/s})^2 = 189.6\text{ J}$, which rounds to 190 J.

**Evaluate:** The result in (e) using Newton's second law agrees with the result calculated in part (d) using energy methods.

**7.14. IDENTIFY:** Use the information given in the problem with $F = kx$ to find $k$. Then $U_{el} = \frac{1}{2}kx^2$.

**SET UP:** $x$ is the amount the spring is stretched. When the weight is hung from the spring, $F = mg$.

**EXECUTE:** $k = \frac{F}{x} = \frac{mg}{x} = \frac{(3.15\text{ kg})(9.80\text{ m/s}^2)}{0.1340\text{ m} - 0.1200\text{ m}} = 2205\text{ N/m}.$

$x = \pm\sqrt{\frac{2U_{el}}{k}} = \pm\sqrt{\frac{2(10.0\text{ J})}{2205\text{ N/m}}} = \pm 0.0952\text{ m} = \pm 9.52\text{ cm}.$ The spring could be either stretched 9.52 cm or compressed 9.52 cm. If it were stretched, the total length of the spring would be $12.00\text{ cm} + 9.52\text{ cm} = 21.52\text{ cm}.$ If it were compressed, the total length of the spring would be $12.00\text{ cm} - 9.52\text{ cm} = 2.48\text{ cm}.$

**EVALUATE:** To stretch or compress the spring 9.52 cm requires a force $F = kx = 210\text{ N}.$

**7.15. IDENTIFY:** Apply $U_{el} = \frac{1}{2}kx^2$.

**SET UP:** $kx = F$, so $U_{el} = \frac{1}{2}Fx$, where $F$ is the magnitude of force required to stretch or compress the spring a distance $x$.

**EXECUTE:** **(a)** (1/2)(520 N)(0.200 m) = 52.0 J.

**(b)** The potential energy is proportional to the square of the compression or extension;
$(52.0\text{ J})(0.050\text{ m}/0.200\text{ m})^2 = 3.25\text{ J}.$

**EVALUATE:** We could have calculated $k = \frac{F}{x} = \frac{520\text{ N}}{0.200\text{ m}} = 2600\text{ N/m}$ and then used $U_{el} = \frac{1}{2}kx^2$ directly.

**7.16. IDENTIFY:** We treat the tendon like a spring and apply Hooke's law to it. Knowing the force stretching the tendon and how much it stretched, we can find its force constant.

**SET UP:** Use $F_{\text{on tendon}} = kx$. In part (a), $F_{\text{on tendon}}$ equals $mg$, the weight of the object suspended from it. In part (b), also apply $U_{el} = \frac{1}{2}kx^2$ to calculate the stored energy.

**EXECUTE:** **(a)** $k = \frac{F_{\text{on tendon}}}{x} = \frac{(0.250\text{ kg})(9.80\text{ m/s}^2)}{0.0123\text{ m}} = 199\text{ N/m}.$

**(b)** $x = \frac{F_{\text{on tendon}}}{k} = \frac{138\text{ N}}{199\text{ N/m}} = 0.693\text{m} = 69.3\text{ cm};$ $U_{el} = \frac{1}{2}(199\text{ N/m})(0.693\text{ m})^2 = 47.8\text{ J}.$

**EVALUATE:** The 250 g object has a weight of 2.45 N. The 138 N force is much larger than this and stretches the tendon a much greater distance.

**7.17. IDENTIFY:** Apply $U_{el} = \frac{1}{2}kx^2$.

**SET UP:** $U_0 = \frac{1}{2}kx_0^2$. $x$ is the distance the spring is stretched or compressed.

**EXECUTE:** **(a)** (i) $x = 2x_0$ gives $U_{el} = \frac{1}{2}k(2x_0)^2 = 4(\frac{1}{2}kx_0^2) = 4U_0$. (ii) $x = x_0/2$ gives $U_{el} = \frac{1}{2}k(x_0/2)^2 = \frac{1}{4}(\frac{1}{2}kx_0^2) = U_0/4.$

(b) (i) $U = 2U_0$ gives $\frac{1}{2}kx^2 = 2(\frac{1}{2}kx_0^2)$ and $x = x_0\sqrt{2}$. (ii) $U = U_0/2$ gives $\frac{1}{2}kx^2 = \frac{1}{2}(\frac{1}{2}kx_0^2)$ and $x = x_0/\sqrt{2}$.

**EVALUATE:** $U$ is proportional to $x^2$ and $x$ is proportional to $\sqrt{U}$.

**7.18. IDENTIFY:** Apply energy conservation, $K_1 + U_1 = K_2 + U_2$.

**SET UP:** Initially and at the highest point, $v = 0$, so $K_1 = K_2 = 0$. $W_{\text{other}} = 0$.

**EXECUTE:** **(a)** In going from rest in the slingshot's pocket to rest at the maximum height, the potential energy stored in the rubber band is converted to gravitational potential energy.
$U = mgy = (10\times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)(22.0\text{ m}) = 2.16\text{ J}.$

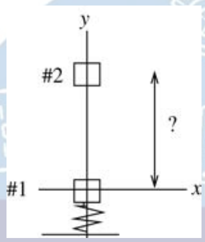
**(b)** Because gravitational potential energy is proportional to mass, the larger pebble rises only 8.8 m.
**(c)** The lack of air resistance and no deformation of the rubber band are two possible assumptions.
**EVALUATE:** The potential energy stored in the rubber band depends on $k$ for the rubber band and the maximum distance it is stretched.

**7.19.** **IDENTIFY** and **SET UP:** Use energy methods. There are changes in both elastic and gravitational potential energy; elastic; $U = \frac{1}{2}kx^2$, gravitational: $U = mgy$.

**EXECUTE:** **(a)** $U_{\text{el}} = \frac{1}{2}kx^2$ so $x = \sqrt{\dfrac{2U_{\text{el}}}{k}} = \sqrt{\dfrac{2(1.20\text{ J})}{800\text{ N/m}}} = 0.0548\text{ m} = 5.48\text{ cm}.$

**(b)** The work done by gravity is equal to the gain in elastic potential energy: $W_{\text{grav}} = U_{\text{el}}$.
$mgx = ½\, kx^2$, so $x = 2mg/k = 2(1.60\text{ kg})(9.80\text{ m/s}^2)/(800\text{ N/m}) = 0.0392\text{ m} = 3.92\text{ cm}$.
**EVALUATE:** When the spring is compressed 3.92 cm, it exerts an upward force of 31.4 N on the book, which is greater than the weight of the book (15.6 N). The book will be accelerated upward from this position.

**7.20.** **IDENTIFY:** Use energy methods. There are changes in both elastic and gravitational potential energy.
**SET UP:** $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Points 1 and 2 in the motion are sketched in Figure 7.20.




The spring force and gravity are the only forces doing work on the cheese, so $W_{\text{other}} = 0$ and $U = U_{\text{grav}} + U_{\text{el}}$.

**Figure 7.20**

**EXECUTE:** Cheese released from rest implies $K_1 = 0$.
At the maximum height $v_2 = 0$ so $K_2 = 0$. $U_1 = U_{1,\text{el}} + U_{1,\text{grav}}$
$y_1 = 0$ implies $U_{1,\text{grav}} = 0$
$U_{1,\text{el}} = \frac{1}{2}kx_1^2 = \frac{1}{2}(1800\text{ N/m})(0.15\text{ m})^2 = 20.25\text{ J}$
(Here $x_1$ refers to the amount the spring is stretched or compressed when the cheese is at position 1; it is *not* the *x*-coordinate of the cheese in the coordinate system shown in the sketch.)
$U_2 = U_{2,\text{el}} + U_{2,\text{grav}}$ $U_{2,\text{grav}} = mgy_2$, where $y_2$ is the height we are solving for. $U_{2,\text{el}} = 0$ since now the spring is no longer compressed. Putting all this into $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives $U_{1,\text{el}} = U_{2,\text{grav}}$

$$y_2 = \frac{20.25\text{ J}}{mg} = \frac{20.25\text{ J}}{(1.20\text{ kg})(9.80\text{ m/s}^2)} = 1.72\text{ m}$$

**EVALUATE:** The description in terms of energy is very simple; the elastic potential energy originally stored in the spring is converted into gravitational potential energy of the system.

**7.21.** **IDENTIFY:** The energy of the book-spring system is conserved. There are changes in both elastic and gravitational potential energy.
**SET UP:** $U_{\text{el}} = \frac{1}{2}kx^2$, $U_{\text{grav}} = mgy$, $W_{\text{other}} = 0$.

**EXECUTE:** **(a)** $U = \frac{1}{2}kx^2$ so $x = \sqrt{\dfrac{2U}{k}} = \sqrt{\dfrac{2(3.20\text{ J})}{1600\text{ N/m}}} = 0.0632\text{ m} = 6.32\text{ cm}$

**(b)** Points 1 and 2 in the motion are sketched in Figure 7.21. We have $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, where $W_{\text{other}} = 0$ (only work is that done by gravity and spring force), $K_1 = 0$, $K_2 = 0$, and $y = 0$ at final position of book. Using $U_1 = mg(h+d)$ and $U_2 = \frac{1}{2}kd^2$ we obtain $0 + mg(h+d) + 0 = \frac{1}{2}kd^2$. The original gravitational potential energy of the system is converted into potential energy of the compressed spring.

Finally, we use the quadratic formula to solve for $d$: $\frac{1}{2}kd^2 - mgd - mgh = 0$, which gives

$d = \dfrac{1}{k}\left(mg \pm \sqrt{(mg)^2 + 4\left(\dfrac{1}{2}k\right)(mgh)}\right)$. In our analysis we have assumed that $d$ is positive, so we get

$$d = \frac{(1.20\text{ kg})(9.80\text{ m/s}^2) + \sqrt{\left[(1.20\text{ kg})(9.80\text{ m/s}^2)\right]^2 + 2(1600\text{ N/m})(1.20\text{ kg})(9.80\text{ m/s}^2)(0.80\text{ m})}}{1600\text{ N/m}},$$

which gives $d = 0.12$ m = 12 cm.

**EVALUATE:** It was important to recognize that the total displacement was $h + d$; gravity continues to do work as the book moves against the spring. Also note that with the spring compressed 0.12 m it exerts an upward force (192 N) greater than the weight of the book (11.8 N). The book will be accelerated upward from this position.

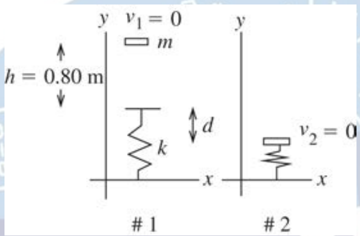



**Figure 7.21**

**7.22.** **(a) IDENTIFY** and **SET UP:** Use energy methods. Both elastic and gravitational potential energy changes. Work is done by friction.

Choose point 1 and let that be the origin, so $y_1 = 0$. Let point 2 be 1.00 m below point 1, so $y_2 = -1.00$ m.

**EXECUTE:** $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(2000\text{ kg})(4.0\text{ m/s})^2 = 16{,}000\text{ J},\ \ U_1 = 0$

$W_{\text{other}} = -f|y_2| = -(17{,}000\text{ N})(1.00\text{ m}) = -17{,}000\text{ J}$

$K_2 = \frac{1}{2}mv_2^2$

$U_2 = U_{2,\text{grav}} + U_{2,\text{el}} = mgy_2 + \frac{1}{2}ky_2^2$

$U_2 = (2000\text{ kg})(9.80\text{ m/s}^2)(-1.00\text{ m}) + \frac{1}{2}(1.06\times 10^4\text{ N/m})(1.00\text{ m})^2$

$U_2 = -19{,}600\text{ J} + 5300\text{ J} = -14{,}300\text{ J}$

Thus $16{,}000\text{ J} - 17{,}000\text{ J} = \frac{1}{2}mv_2^2 - 14{,}300\text{ J}$

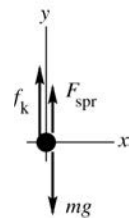
$\frac{1}{2}mv_2^2 = 13{,}300\text{ J}$

$v_2 = \sqrt{\dfrac{2(13{,}300\text{ J})}{2000\text{ kg}}} = 3.65\text{ m/s}$

**EVALUATE:** The elevator stops after descending 3.00 m. After descending 1.00 m it is still moving but has slowed down.

**(b) IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the elevator. We know the forces and can solve for $\vec{a}$.

**SET UP:** The free-body diagram for the elevator is given in Figure 7.22.




**EXECUTE:** $F_{spr} = kd$, where $d$ is the distance the spring is compressed

$\Sigma F_y = ma_y$

$f_k + F_{spr} - mg = ma$

$f_k + kd - mg = ma$

**Figure 7.22**

$$a = \frac{f_k + kd - mg}{m} = \frac{17{,}000\text{ N} + (1.06\times10^4\text{ N/m})(1.00\text{ m}) - (2000\text{ kg})(9.80\text{ m/s}^2)}{2000\text{ kg}} = 4.00\text{ m/s}^2$$

We calculate that $a$ is positive, so the acceleration is upward.

**EVALUATE:** The velocity is downward and the acceleration is upward, so the elevator is slowing down at this point.

**7.23.** **IDENTIFY:** Only the spring does work and $K_1 + U_1 = K_2 + U_2$ applies. $a = \frac{F}{m} = \frac{-kx}{m}$, where $F$ is the force the spring exerts on the mass.

**SET UP:** Let point 1 be the initial position of the mass against the compressed spring, so $K_1 = 0$ and $U_1 = 11.5$ J. Let point 2 be where the mass leaves the spring, so $U_{el,2} = 0$.

**EXECUTE:** **(a)** $K_1 + U_{el,1} = K_2 + U_{el,2}$ gives $U_{el,1} = K_2$. $\frac{1}{2}mv_2^2 = U_{el,1}$ and

$$v_2 = \sqrt{\frac{2U_{el,1}}{m}} = \sqrt{\frac{2(11.5\text{ J})}{2.50\text{ kg}}} = 3.03\text{ m/s}.$$

$K$ is largest when $U_{el}$ is least and this is when the mass leaves the spring. The mass achieves its maximum speed of 3.03 m/s as it leaves the spring and then slides along the surface with constant speed.

**(b)** The acceleration is greatest when the force on the mass is the greatest, and this is when the spring has its maximum compression. $U_{el} = \frac{1}{2}kx^2$ so $x = -\sqrt{\frac{2U_{el}}{k}} = -\sqrt{\frac{2(11.5\text{ J})}{2500\text{ N/m}}} = -0.0959$ m. The minus sign indicates compression. $F = -kx = ma_x$ and $a_x = -\frac{kx}{m} = -\frac{(2500\text{ N/m})(-0.0959\text{ m})}{2.50\text{ kg}} = 95.9\text{ m/s}^2$.

**EVALUATE:** If the end of the spring is displaced to the left when the spring is compressed, then $a_x$ in part (b) is to the right, and vice versa.

**7.24.** **IDENTIFY:** The spring force is conservative but the force of friction is nonconservative. Energy is conserved during the process. Initially all the energy is stored in the spring, but part of this goes to kinetic energy, part remains as elastic potential energy, and the rest does work against friction.

**SET UP:** Energy conservation: $K_1 + U_1 + W_{other} = K_2 + U_2$, the elastic energy in the spring is $U = \frac{1}{2}kx^2$, and the work done by friction is $W_f = -f_k s = -\mu_k mgs$.

**EXECUTE:** The initial and final elastic potential energies are

$U_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(840\text{ N/m})(0.0300\text{ m})^2 = 0.378$ J and $U_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(840\text{ N/m})(0.0100\text{ m})^2 = 0.0420$ J.

The initial and final kinetic energies are $K_1 = 0$ and $K_2 = \frac{1}{2}mv_2^2$. The work done by friction is

$W_{other} = W_{f_k} = -f_k s = -\mu_k mgs = -(0.40)(2.50\text{ kg})(9.8\text{ m/s}^2)(0.0200\text{ m}) = -0.196$ J. Energy conservation gives $K_2 = \frac{1}{2}mv_2^2 = K_1 + U_1 + W_{other} - U_2 = 0.378\text{ J} + (-0.196\text{ J}) - 0.0420\text{ J} = 0.140$ J. Solving for $v_2$ gives

$$v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(0.140\text{ J})}{2.50\text{ kg}}} = 0.335\text{ m/s}.$$

**EVALUATE:** Mechanical energy is not conserved due to friction.

**7.25. IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ and $F = ma$.

**SET UP:** $W_{\text{other}} = 0$. There is no change in $U_{\text{grav}}$. $K_1 = 0$, $U_2 = 0$.

**EXECUTE:** $\frac{1}{2}kx^2 = \frac{1}{2}mv_x^2$. The relations for $m$, $v_x$, $k$ and $x$ are $kx^2 = mv_x^2$ and $kx = 5mg$.

Dividing the first equation by the second gives $x = \dfrac{v_x^2}{5g}$, and substituting this into the second gives

$k = 25\dfrac{mg^2}{v_x^2}$.

**(a)** $k = 25\dfrac{(1160\text{ kg})(9.80\text{ m/s}^2)^2}{(2.50\text{ m/s})^2} = 4.46\times10^5\text{ N/m}$

**(b)** $x = \dfrac{(2.50\text{ m/s})^2}{5(9.80\text{ m/s}^2)} = 0.128\text{ m}$

**EVALUATE:** Our results for $k$ and $x$ do give the required values for $a_x$ and $v_x$:

$$a_x = \frac{kx}{m} = \frac{(4.46\times10^5\text{ N/m})(0.128\text{ m})}{1160\text{ kg}} = 49.2\text{ m/s}^2 = 5.0g \text{ and } v_x = x\sqrt{\frac{k}{m}} = 2.5\text{ m/s}.$$

**7.26. IDENTIFY:** $W_{\text{grav}} = mg\cos\phi$.

**SET UP:** When he moves upward, $\phi = 180°$ and when he moves downward, $\phi = 0°$. When he moves parallel to the ground, $\phi = 90°$.

**EXECUTE:** (a) $W_{\text{grav}} = (75\text{ kg})(9.80\text{ m/s}^2)(7.0\text{ m})\cos180° = -5100\text{ J}$.

**(b)** $W_{\text{grav}} = (75\text{ kg})(9.80\text{ m/s}^2)(7.0\text{ m})\cos0° = +5100\text{ J}$.

**(c)** $\phi = 90°$ in each case and $W_{\text{grav}} = 0$ in each case.

**(d)** The total work done on him by gravity during the round trip is $-5100\text{ J} + 5100\text{ J} = 0$.

**(e)** Gravity is a conservative force since the total work done for a round trip is zero.

**EVALUATE:** The gravity force is independent of the position and motion of the object. When the object moves upward gravity does negative work and when the object moves downward gravity does positive work.

**7.27. IDENTIFY:** Since the force is constant, use $W = Fs\cos\phi$.

**SET UP:** For both displacements, the direction of the friction force is opposite to the displacement and $\phi = 180°$.

**EXECUTE:** **(a)** When the book moves to the left, the friction force is to the right, and the work is $-(1.8\text{ N})(3.0\text{ m}) = -5.4\text{ J}$.

**(b)** The friction force is now to the left, and the work is again $-5.4\text{ J}$.

**(c)** The total work is sum of the work in both directions, which is –10.8 J.

**(d)** The net work done by friction for the round trip is not zero, so friction is not a conservative force.

**EVALUATE:** The direction of the friction force depends on the motion of the object. For the gravity force, which is conservative, the force does not depend on the motion of the object.

**7.28. IDENTIFY and SET UP:** The force is not constant so we must integrate to calculate the work.

$W = \int_1^2 \vec{F}\cdot d\vec{l}$, $\vec{F} = -\alpha x^2\hat{i}$ .

**EXECUTE:** **(a)** $d\vec{l} = dy\hat{j}$ ($x$ is constant; the displacement is in the $+y$-direction)

$\vec{F}\cdot d\vec{l} = 0$ (since $\hat{i}\cdot\hat{j} = 0$) and thus $W = 0$.

**(b)** $d\vec{l} = dx\hat{i}$

$\vec{F}\cdot d\vec{l} = (-\alpha x^2\hat{i})\cdot(dx\hat{i}) = -\alpha x^2\,dx$

$$W = \int_{x_1}^{x_2}(-\alpha x^2)\,dx = -\tfrac{1}{3}ax^3\Big|_{x_1}^{x_2} = -\tfrac{1}{3}\alpha\,(x_2^3 - x_1^3) = -\frac{12\text{ N/m}^2}{3}\left[(0.300\text{ m})^3 - (0.10\text{ m})^3\right] = -0.10\text{ J}$$

**(c)** $d\vec{l} = dx\hat{i}$ as in part (b), but now $x_1 = 0.30$ m and $x_2 = 0.10$ m, so $W = -\frac{1}{3}\alpha(x_2^3 - x_1^3) = +0.10$ J.

**(d) EVALUATE:** The total work for the displacement along the $x$-axis from 0.10 m to 0.30 m and then back to 0.10 m is the sum of the results of parts (b) and (c), which is zero. The total work is zero when the starting and ending points are the same, so the force is conservative.

**EXECUTE:** $W_{x_1 \to x_2} = -\frac{1}{3}\alpha(x_2^3 - x_1^3) = \frac{1}{3}\alpha x_1^3 - \frac{1}{3}\alpha x_2^3$

The definition of the potential energy function is $W_{x_1 \to x_2} = U_1 - U_2$. Comparison of the two expressions for $W$ gives $U = \frac{1}{3}\alpha x^3$. This does correspond to $U = 0$ when $x = 0$.

**EVALUATE:** In part (a) the work done is zero because the force and displacement are perpendicular. In part (b) the force is directed opposite to the displacement and the work done is negative. In part (c) the force and displacement are in the same direction and the work done is positive.

**7.29.** **IDENTIFY:** Some of the mechanical energy of the skier is converted to internal energy by the nonconservative force of friction on the rough patch. Use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

**SET UP:** For part (a) use $E_{\text{mech, 2}} = E_{\text{mech, 1}} - f_k s$ where $f_k = \mu_k mg$. Let $y_2 = 0$ at the bottom of the hill; then $y_1 = 2.50$ m along the rough patch. The energy equation is $\frac{1}{2}mv_2{}^2 = \frac{1}{2}mv_1{}^2 + mgy_1 - \mu_k mgs$. Solving for her final speed gives $v_2 = \sqrt{v_1{}^2 + 2gy_1 - 2\mu_k gs}$. For part (b), the internal energy is calculated as the negative of the work done by friction: $-W_f = +f_k s = +\mu_k mgs$.

**EXECUTE:** **(a)** $v_2 = \sqrt{(6.50 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(2.50 \text{ m}) - 2(0.300)(9.80 \text{ m/s}^2)(4.20 \text{ m})} = 8.16$ m/s.

**(b)** Internal energy $= \mu_k mgs = (0.300)(62.0 \text{ kg})(9.80 \text{ m/s}^2)(4.20 \text{ m}) = 766$ J.

**EVALUATE:** Without friction the skier would be moving faster at the bottom of the hill than at the top, but in this case she is moving *slower* because friction converted some of her initial kinetic energy into internal energy.

**7.30.** **IDENTIFY:** Some of the initial gravitational potential energy is converted to kinetic energy, but some of it is lost due to work by the nonconservative friction force.

**SET UP:** The energy of the box at the edge of the roof is given by: $E_{\text{mech, f}} = E_{\text{mech, i}} - f_k s$. Setting $y_f = 0$ at this point, $y_i = (4.25 \text{ m}) \sin 36° = 2.50$ m. Furthermore, by substituting $K_i = 0$ and $K_f = \frac{1}{2}mv_f{}^2$ into the conservation equation, $\frac{1}{2}mv_f{}^2 = mgy_i - f_k s$ or $v_f = \sqrt{2gy_i - 2f_k sg/w} = \sqrt{2g(y_i - f_k s/w)}$.

**EXECUTE:** $v_f = \sqrt{2(9.80 \text{ m/s}^2)[(2.50 \text{ m}) - (22.0 \text{ N})(4.25 \text{ m})/(85.0 \text{ N})]} = 5.24$ m/s.

**EVALUATE:** Friction does negative work and removes mechanical energy from the system. In the absence of friction the final speed of the toolbox would be 7.00 m/s.

**7.31.** **IDENTIFY:** We know the potential energy function and want to find the force causing this energy.

**SET UP:** $F_x = -\dfrac{dU}{dx}$. The sign of $F_x$ indicates its direction.

**EXECUTE:** $F_x = -\dfrac{dU}{dx} = -4\alpha x^3 = -4(0.630 \text{ J/m}^4)x^3$. $F_x(-0.800 \text{ m}) = -4(0.630 \text{ J/m}^4)(-0.80 \text{ m})^3 = 1.29$ N. The force is in the $+x$-direction.

**EVALUATE:** $F_x > 0$ when $x < 0$ and $F_x < 0$ when $x > 0$, so the force is always directed towards the origin.

**7.32.** **IDENTIFY and SET UP:** Use $F_x = -\dfrac{dU}{dx}$ to calculate the force from $U(x)$. Use coordinates where the origin is at one atom. The other atom then has coordinate $x$.

**EXECUTE:**

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left(-\frac{C_6}{x^6}\right) = +C_6\frac{d}{dx}\left(\frac{1}{x^6}\right) = -\frac{6C_6}{x^7}$$

The minus sign mean that $F_x$ is directed in the $-x$-direction, toward the origin. The force has magnitude $6C_6/x^7$ and is attractive.

**Evaluate:** $U$ depends only on $x$ so $\vec{F}$ is along the $x$-axis; it has no $y$- or $z$-components.

**7.33.** **Identify:** From the potential energy function of the block, we can find the force on it, and from the force we can use Newton's second law to find its acceleration.

**Set Up:** The force components are $F_x = -\dfrac{\partial U}{\partial x}$ and $F_y = -\dfrac{\partial U}{\partial y}$. The acceleration components are $a_x = F_x/m$ and $a_y = F_y/m$. The magnitude of the acceleration is $a = \sqrt{a_x^2 + a_y^2}$ and we can find its angle with the $+x$ axis using $\tan\theta = a_y/a_x$.

**Execute:** $F_x = -\dfrac{\partial U}{\partial x} = -(11.6\ \text{J/m}^2)x$ and $F_y = -\dfrac{\partial U}{\partial y} = (10.8\ \text{J/m}^3)y^2$. At the point ($x = 0.300$ m, $y = 0.600$ m), $F_x = -(11.6\ \text{J/m}^2)(0.300\ \text{m}) = -3.48$ N and $F_y = (10.8\ \text{J/m}^3)(0.600\ \text{m})^2 = 3.89$ N. Therefore $a_x = \dfrac{F_x}{m} = -87.0\ \text{m/s}^2$ and $a_y = \dfrac{F_y}{m} = 97.2\ \text{m/s}^2$, giving $a = \sqrt{a_x^2 + a_y^2} = 130\ \text{m/s}^2$ and $\tan\theta = \dfrac{97.2}{87.0}$, so $\theta = 48.2°$. The direction is $132^\circ$ counterclockwise from the $+x$-axis.

**Evaluate:** The force is not constant, so the acceleration will not be the same at other points.

**7.34.** **Identify:** Apply $\vec{F} = -\dfrac{\partial U}{\partial x}\hat{i} - \dfrac{\partial U}{\partial y}\hat{j}$.

**Set Up:** $\dfrac{d}{dx}\left(\dfrac{1}{x^2}\right) = -\dfrac{2}{x^3}$ and $\dfrac{d}{dy}\left(\dfrac{1}{y^2}\right) = -\dfrac{2}{y^3}$.

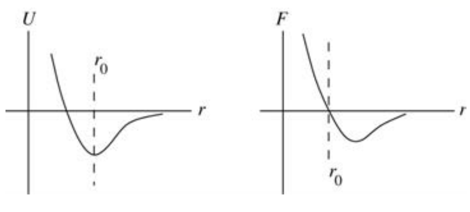
**Execute:** $\vec{F} = -\dfrac{\partial U}{\partial x}\hat{i} - \dfrac{\partial U}{\partial y}\hat{j}$ since $U$ has no $z$-dependence. $\dfrac{\partial U}{\partial x} = \dfrac{-2\alpha}{x^3}$ and $\dfrac{\partial U}{\partial y} = \dfrac{-2\alpha}{y^3}$, so

$$\vec{F} = -\alpha\left(\frac{-2}{x^3}\hat{i} + \frac{-2}{y^3}\hat{j}\right) = 2\alpha\left(\frac{\hat{i}}{x^3} + \frac{\hat{j}}{y^3}\right).$$

**Evaluate:** $F_x$ and $x$ have the same sign and $F_y$ and $y$ have the same sign. When $x > 0$, $F_x$ is in the $+x$-direction, and so forth.

**7.35.** **Identify** and **Set Up:** Use $F = -dU/dr$ to calculate the force from $U$. At equilibrium $F = 0$.

**(a) Execute:** The graphs are sketched in Figure 7.35.




$$U = \frac{a}{r^{12}} - \frac{b}{r^6}$$

$$F = -\frac{dU}{dr} = +\frac{12a}{r^{13}} - \frac{6b}{r^7}$$

**Figure 7.35**

**(b)** At equilibrium $F = 0$, so $\dfrac{dU}{dr} = 0$

$$F = 0 \text{ implies } \frac{+12a}{r^{13}} - \frac{6b}{r^7} = 0$$

$6br^6 = 12a$; solution is the equilibrium distance $r_0 = (2a/b)^{1/6}$

$U$ is a minimum at this $r$; the equilibrium is stable.

**(c)** At $r=(2a/b)^{1/6}$, $U=a/r^{12}-b/r^{6}=a(b/2a)^{2}-b(b/2a)=-b^{2}/4a$.

At $r\to\infty$, $U=0$. The energy that must be added is $-\Delta U=b^{2}/4a$.

**(d)** $r_0=(2a/b)^{1/6}=1.13\times10^{-10}\ \text{m}$ gives that

$2a/b=2.082\times10^{-60}\ \text{m}^6$ and $b/4a=2.402\times10^{59}\ \text{m}^{-6}$

$b^2/4a=b(b/4a)=1.54\times10^{-18}\ \text{J}$

$b(2.402\times10^{59}\ \text{m}^{-6})=1.54\times10^{-18}\ \text{J}$ and $b=6.41\times10^{-78}\ \text{J}\cdot\text{m}^6$.

Then $2a/b=2.082\times10^{-60}\ \text{m}^6$ gives $a=(b/2)(2.082\times10^{-60}\ \text{m}^6)=$

$\frac{1}{2}(6.41\times10^{-78}\ \text{J}\cdot\text{m}^6)\ (2.082\times10^{-60}\ \text{m}^6)\ =6.67\times10^{-138}\ \text{J}\cdot\text{m}^{12}$

**EVALUATE:** As the graphs in part (a) show, $F(r)$ is the slope of $U(r)$ at each $r$. $U(r)$ has a minimum where $F=0$.

**7.36.** **IDENTIFY:** Apply $F_x=-\dfrac{dU}{dx}$.

**SET UP:** $\dfrac{dU}{dx}$ is the slope of the $U$ versus $x$ graph.

**EXECUTE:** **(a)** Considering only forces in the $x$-direction, $F_x=-\dfrac{dU}{dx}$ and so the force is zero when the slope of the $U$ *vs* $x$ graph is zero, at points $b$ and $d$.

**(b)** Point $b$ is at a potential minimum; to move it away from $b$ would require an input of energy, so this point is stable.

**(c)** Moving away from point $d$ involves a decrease of potential energy, hence an increase in kinetic energy, and the marble tends to move further away, and so $d$ is an unstable point.

**EVALUATE:** At point $b$, $F_x$ is negative when the marble is displaced slightly to the right and $F_x$ is positive when the marble is displaced slightly to the left, the force is a restoring force, and the equilibrium is stable. At point $d$, a small displacement in either direction produces a force directed away from $d$ and the equilibrium is unstable.

**7.37.** **IDENTIFY:** Apply $\Sigma\vec{F}=m\vec{a}$ to the bag and to the box. Apply $K_1+U_1+W_{\text{other}}=K_2+U_2$ to the motion of the system of the box and bucket after the bag is removed.

**SET UP:** Let $y=0$ at the final height of the bucket, so $y_1=2.00\ \text{m}$ and $y_2=0$. $K_1=0$. The box and the bucket move with the same speed $v$, so $K_2=\frac{1}{2}(m_{\text{box}}+m_{\text{bucket}})v^2$. $W_{\text{other}}=-f_{\text{k}}d$, with $d=2.00\ \text{m}$ and $f_{\text{k}}=\mu_{\text{k}}m_{\text{box}}g$. Before the bag is removed, the maximum possible friction force the roof can exert on the box is $(0.700)(80.0\ \text{kg}+50.0\ \text{kg})(9.80\ \text{m/s}^2)=892\ \text{N}$. This is larger than the weight of the bucket (637 N), so before the bag is removed the system is at rest.

**EXECUTE:** **(a)** The friction force on the bag of gravel is zero, since there is no other horizontal force on the bag for friction to oppose. The static friction force on the box equals the weight of the bucket, 637 N.

**(b)** Applying $K_1+U_1+W_{\text{other}}=K_2+U_2$ gives $m_{\text{bucket}}gy_1-f_{\text{k}}d=\frac{1}{2}m_{\text{tot}}v^2$, with $m_{\text{tot}}=145.0\ \text{kg}$.

$$v=\sqrt{\frac{2}{m_{\text{tot}}}(m_{\text{bucket}}gy_1-\mu_{\text{k}}m_{\text{box}}gd)}.$$

$$v=\sqrt{\frac{2}{145.0\ \text{kg}}\left[(65.0\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m})-(0.400)(80.0\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m})\right]}=2.99\ \text{m/s}.$$

**EVALUATE:** If we apply $\Sigma\vec{F}=m\vec{a}$ to the box and to the bucket we can calculate their common acceleration $a$. Then a constant acceleration equation applied to either object gives $v=2.99\ \text{m/s}$, in agreement with our result obtained using energy methods.

**7.38. IDENTIFY:** For the system of two blocks, only gravity does work. Apply $K_1 + U_1 = K_2 + U_2$.

**SET UP:** Call the blocks $A$ and $B$, where $A$ is the more massive one. $v_{A1} = v_{B1} = 0$. Let $y = 0$ for each block to be at the initial height of that block, so $y_{A1} = y_{B1} = 0$. $y_{A2} = -1.20$ m and $y_{B2} = +1.20$ m. $v_{A2} = v_{B2} = v_2 = 3.00$ m/s.

**EXECUTE:** $K_1 + U_1 = K_2 + U_2$ gives $0 = \frac{1}{2}(m_A + m_B)v_2^2 + g(1.20\text{ m})(m_B - m_A)$, with $m_A + m_B = 22.0$ kg.

Therefore $\frac{1}{2}(22.0\text{ kg})(3.00\text{ m/s})^2 + (9.80\text{ m/s}^2)(1.20\text{ m})(22.0\text{ kg} - 2m_A)$. Solving for $m_A$ gives

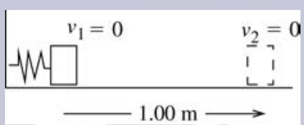
$m_A = 15.2$ kg. And then $m_B = 6.79$ kg.

**EVALUATE:** The final kinetic energy of the two blocks is 99 J. The potential energy of block $A$ decreases by 179 J. The potential energy of block $B$ increases by 80 J. The total decrease in potential energy is $179\text{ J} - 80\text{ J} = 99\text{ J}$, which equals the increase in kinetic energy of the system.

**7.39. IDENTIFY:** Use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. The target variable $\mu_k$ will be a factor in the work done by friction.

**SET UP:** Let point 1 be where the block is released and let point 2 be where the block stops, as shown in Figure 7.39.

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$




Work is done on the block by the spring and by friction, so $W_{\text{other}} = W_f$ and $U = U_{\text{el}}$.

**Figure 7.39**

**EXECUTE:** $K_1 = K_2 = 0$

$U_1 = U_{1,\text{el}} = \frac{1}{2}kx_1^2 = \frac{1}{2}(100\text{ N/m})(0.200\text{ m})^2 = 2.00\text{ J}$

$U_2 = U_{2,\text{el}} = 0$, since after the block leaves the spring has given up all its stored energy

$W_{\text{other}} = W_f = (f_k \cos\phi)s = \mu_k mg(\cos\phi)s = -\mu_k mgs$, since $\phi = 180°$ (The friction force is directed opposite to the displacement and does negative work.)

Putting all this into $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$U_{1,\text{el}} + W_f = 0$

$\mu_k mgs = U_{1,\text{el}}$

$$\mu_k = \frac{U_{1,\text{el}}}{mgs} = \frac{2.00\text{ J}}{(0.50\text{ kg})(9.80\text{ m/s}^2)(1.00\text{ m})} = 0.41.$$

**EVALUATE:** $U_{1,\text{el}} + W_f = 0$ says that the potential energy originally stored in the spring is taken out of the system by the negative work done by friction.

**7.40. IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

**SET UP:** Only the spring force and gravity do work, so $W_{\text{other}} = 0$. Let $y = 0$ at the horizontal surface.

**EXECUTE:** **(a)** Equating the potential energy stored in the spring to the block's kinetic energy,

$$\tfrac{1}{2}kx^2 = \tfrac{1}{2}mv^2,\text{ or } v = \sqrt{\frac{k}{m}}x = \sqrt{\frac{400\text{ N/m}}{2.00\text{ kg}}}(0.220\text{ m}) = 3.11\text{ m/s}.$$

**(b)** Using energy methods directly, the initial potential energy of the spring equals the final gravitational potential energy, $\frac{1}{2}kx^2 = mgL\sin\theta$, or $L = \dfrac{\frac{1}{2}kx^2}{mg\sin\theta} = \dfrac{\frac{1}{2}(400\text{ N/m})(0.220\text{ m})^2}{(2.00\text{ kg})(9.80\text{ m/s}^2)\sin 37.0°} = 0.821\text{ m}.$

**EVALUATE:** The total energy of the system is constant. Initially it is all elastic potential energy stored in the spring, then it is all kinetic energy and finally it is all gravitational potential energy.

**7.41.** **IDENTIFY:** The mechanical energy of the roller coaster is conserved since there is no friction with the track. We must also apply Newton's second law for the circular motion.

**SET UP:** For part (a), apply conservation of energy to the motion from point $A$ to point $B$: $K_B + U_{\text{grav},B} = K_A + U_{\text{grav},A}$ with $K_A = 0$. Defining $y_B = 0$ and $y_A = 13.0$ m, conservation of energy becomes $\frac{1}{2}mv_B{}^2 = mgy_A$ or $v_B = \sqrt{2gy_A}$. In part (b), the free-body diagram for the roller coaster car at point $B$ is shown in Figure 7.41. $\Sigma F_y = ma_y$ gives $mg + n = ma_{\text{rad}}$, where $a_{\text{rad}} = v^2/r$. Solving for the normal force gives $n = m\left(\dfrac{v^2}{r} - g\right)$.

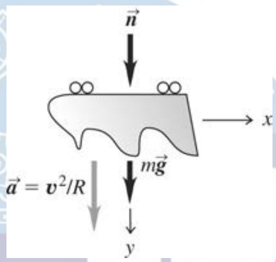



**Figure 7.41**

**EXECUTE:** **(a)** $v_B = \sqrt{2(9.80\text{ m/s}^2)(13.0\text{ m})} = 16.0\text{ m/s}$.

**(b)** $n = (350\text{ kg})\left[\dfrac{(16.0\text{ m/s})^2}{6.0\text{ m}} - 9.80\text{ m/s}^2\right] = 1.15 \times 10^4\text{ N}$.

**EVALUATE:** The normal force $n$ is the force that the tracks exert on the roller coaster car. The car exerts a force of equal magnitude and opposite direction on the tracks.

**7.42.** **IDENTIFY:** Mechanical energy is conserved since no nonconservative forces do work on the system. Newton's second law also applies.

**SET UP:** Relate $h$ and $v_B$. Apply $\Sigma\vec{F} = m\vec{a}$ at point $B$ to find the minimum speed required at $B$ for the car not to fall off the track. At $B$, $a = v_B^2/R$, downward. The minimum speed is when $n \to 0$ and $mg = mv_B^2/R$. The minimum speed required is $v_B = \sqrt{gR}$. $K_1 = 0$ and $W_{\text{other}} = 0$.

**EXECUTE:** **(a)** Conservation of mechanical energy applied to points $A$ and $B$ gives $U_A - U_B = \frac{1}{2}mv_B^2$. The speed at the top must be at least $\sqrt{gR}$. Thus, $mg(h - 2R) > \dfrac{1}{2}mgR$, or $h > \dfrac{5}{2}R$.

**(b)** Conservation of mechanical energy applied to points $A$ and $C$ gives $U_A - U_C = (2.50)Rmg = K_C$, so $v_C = \sqrt{(5.00)gR} = \sqrt{(5.00)(9.80\text{ m/s}^2)(14.0\text{ m})} = 26.2\text{ m/s}$. The radial acceleration is $a_{\text{rad}} = \dfrac{v_C^2}{R} = 49.0\text{ m/s}^2$. The tangential direction is down, the normal force at point $C$ is horizontal, there is no friction, so the only downward force is gravity, and $a_{\tan} = g = 9.80\text{ m/s}^2$.

**EVALUATE:** If $h > \frac{5}{2}R$, then the downward acceleration at $B$ due to the circular motion is greater than $g$ and the track must exert a downward normal force $n$. $n$ increases as $h$ increases and hence $v_B$ increases.

**7.43.** **(a) IDENTIFY:** Use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to find the kinetic energy of the wood as it enters the rough bottom.

**SET UP:** Let point 1 be where the piece of wood is released and point 2 be just before it enters the rough bottom. Let $y = 0$ be at point 2.

**EXECUTE:** $U_1 = K_2$ gives $K_2 = mgy_1 = 78.4$ J.

**IDENTIFY:** Now apply $K_1 + U_1 + W_{other} = K_2 + U_2$ to the motion along the rough bottom.

**SET UP:** Let point 1 be where it enters the rough bottom and point 2 be where it stops.
$K_1 + U_1 + W_{other} = K_2 + U_2$.

**EXECUTE:** $W_{other} = W_f = -\mu_k mgs$, $K_2 = U_1 = U_2 = 0$; $K_1 = 78.4$ J

$78.4\text{ J} - \mu_k mgs = 0$; solving for $s$ gives $s = 20.0$ m.

The wood stops after traveling 20.0 m along the rough bottom.
**(b)** Friction does $-78.4$ J of work.

**EVALUATE:** The piece of wood stops before it makes one trip across the rough bottom. The final mechanical energy is zero. The negative friction work takes away all the mechanical energy initially in the system.

**7.44.** **IDENTIFY:** Apply $K_1 + U_1 + W_{other} = K_2 + U_2$ to the rock. $W_{other} = W_{f_k}$.

**SET UP:** Let $y = 0$ at the foot of the hill, so $U_1 = 0$ and $U_2 = mgh$, where $h$ is the vertical height of the rock above the foot of the hill when it stops.

**EXECUTE:** **(a)** At the maximum height, $K_2 = 0$. $K_1 + U_1 + W_{other} = K_2 + U_2$ gives

$K_{Bottom} + W_{f_k} = U_{Top}$. $\frac{1}{2}mv_0^2 - \mu_k mg\cos\theta d = mgh$. $d = h/\sin\theta$, so $\frac{1}{2}v_0^2 - \mu_k g\cos\theta\frac{h}{\sin\theta} = gh$.

$\frac{1}{2}(15\text{ m/s})^2 - (0.20)(9.8\text{ m/s}^2)\frac{\cos 40°}{\sin 40°}h = (9.8\text{ m/s}^2)h$ which gives $h = 9.3$ m.

**(b)** Compare maximum static friction force to the weight component down the plane.

$f_s = \mu_s mg\cos\theta = (0.75)(28\text{ kg})(9.8\text{ m/s}^2)\cos 40° = 158$ N.

$mg\sin\theta = (28\text{ kg})(9.8\text{ m/s}^2)(\sin 40°) = 176\text{ N} > f_s$, so the rock will slide down.

**(c)** Use same procedure as in part (a), with $h = 9.3$ m and $v_B$ being the speed at the bottom of the hill.

$U_{Top} + W_{f_k} = K_B$. $mgh - \mu_k mg\cos\theta\frac{h}{\sin\theta} = \frac{1}{2}mv_B^2$ and $v_B = \sqrt{2gh - 2\mu_k gh\cos\theta/\sin\theta} = 11.8$ m/s.

**EVALUATE:** For the round trip up the hill and back down, there is negative work done by friction and the speed of the rock when it returns to the bottom of the hill is less than the speed it had when it started up the hill.

**7.45.** **IDENTIFY:** Apply $K_1 + U_1 + W_{other} = K_2 + U_2$ to the motion of the stone.

**SET UP:** $K_1 + U_1 + W_{other} = K_2 + U_2$. Let point 1 be point $A$ and point 2 be point $B$. Take $y = 0$ at $B$.

**EXECUTE:** $mgy_1 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2$, with $h = 20.0$ m and $v_1 = 10.0$ m/s, so $v_2 = \sqrt{v_1^2 + 2gh} = 22.2$ m/s.

**EVALUATE:** The loss of gravitational potential energy equals the gain of kinetic energy.

**(b) IDENTIFY:** Apply $K_1 + U_1 + W_{other} = K_2 + U_2$ to the motion of the stone from point $B$ to where it comes to rest against the spring.

**SET UP:** Use $K_1 + U_1 + W_{other} = K_2 + U_2$, with point 1 at $B$ and point 2 where the spring has its maximum compression $x$.

**EXECUTE:** $U_1 = U_2 = K_2 = 0$; $K_1 = \frac{1}{2}mv_1^2$ with $v_1 = 22.2$ m/s. $W_{other} = W_f + W_{el} = -\mu_k mgs - \frac{1}{2}kx^2$, with $s = 100\text{ m} + x$. The work-energy relation gives $K_1 + W_{other} = 0$. $\frac{1}{2}mv_1^2 - \mu_k mgs - \frac{1}{2}kx^2 = 0$.

Putting in the numerical values gives $x^2 + 29.4x - 750 = 0$. The positive root to this equation is $x = 16.4$ m.

**EVALUATE:** Part of the initial mechanical (kinetic) energy is removed by friction work and the rest goes into the potential energy stored in the spring.

**(c) IDENTIFY** and **SET UP:** Consider the forces.

**EXECUTE:** When the spring is compressed $x = 16.4$ m the force it exerts on the stone is $F_{el} = kx = 32.8$ N. The maximum possible static friction force is

$\max f_s = \mu_s mg = (0.80)(15.0\text{ kg})(9.80\text{ m/s}^2) = 118$ N.

**EVALUATE:** The spring force is less than the maximum possible static friction force so the stone remains at rest.

**7.46.** **IDENTIFY:** Once the block leaves the top of the hill it moves in projectile motion. Use $K_1+U_1=K_2+U_2$ to relate the speed $v_B$ at the bottom of the hill to the speed $v_{Top}$ at the top and the 70 m height of the hill.

**SET UP:** For the projectile motion, take $+y$ to be downward. $a_x=0$, $a_y=g$. $v_{0x}=v_{Top}$, $v_{0y}=0$. For the motion up the hill only gravity does work. Take $y=0$ at the base of the hill.

**EXECUTE:** First get speed at the top of the hill for the block to clear the pit. $y=\frac{1}{2}gt^2$.

$20\text{ m}=\frac{1}{2}(9.8\text{ m/s}^2)t^2$. $t=2.0$ s. Then $v_{Top}t=40$ m gives $v_{Top}=\frac{40\text{ m}}{2.0\text{ s}}=20$ m/s.

Energy conservation applied to the motion up the hill: $K_{Bottom}=U_{Top}+K_{Top}$ gives

$\frac{1}{2}mv_B^2=mgh+\frac{1}{2}mv_{Top}^2$. $v_B=\sqrt{v_{Top}^2+2gh}=\sqrt{(20\text{ m/s})^2+2(9.8\text{ m/s}^2)(70\text{ m})}=42$ m/s.

**EVALUATE:** The result does not depend on the mass of the block.

**7.47.** **IDENTIFY:** Apply $K_1+U_1+W_{other}=K_2+U_2$ to the motion of the person.

**SET UP:** Point 1 is where he steps off the platform and point 2 is where he is stopped by the cord. Let $y=0$ at point 2. $y_1=41.0$ m. $W_{other}=-\frac{1}{2}kx^2$, where $x=11.0$ m is the amount the cord is stretched at point 2. The cord does negative work.

**EXECUTE:** $K_1=K_2=U_2=0$, so $mgy_1-\frac{1}{2}kx^2=0$ and $k=631$ N/m.

Now apply $F=kx$ to the test pulls:

$F=kx$ so $x=F/k=0.602$ m.

**EVALUATE:** All his initial gravitational potential energy is taken away by the negative work done by the force exerted by the cord, and this amount of energy is stored as elastic potential energy in the stretched cord.

**7.48.** **IDENTIFY:** To be at equilibrium at the bottom, with the spring compressed a distance $x_0$, the spring force must balance the component of the weight down the ramp plus the largest value of the static friction, or $kx_0=w\sin\theta+f$. Apply energy conservation to the motion down the ramp.

**SET UP:** $K_2=0$, $K_1=\frac{1}{2}mv^2$, where $v$ is the speed at the top of the ramp. Let $U_2=0$, so $U_1=wL\sin\theta$, where $L$ is the total length traveled down the ramp.

**EXECUTE:** Energy conservation gives $\frac{1}{2}kx_0^2=(w\sin\theta-f)L+\frac{1}{2}mv^2$. With the given parameters, $\frac{1}{2}kx_0^2=421$ J and $kx_0=1.066\times10^3$ N. Solving for $k$ gives $k=1350$ N/m.

**EVALUATE:** $x_0=0.790$ m. $w\sin\theta=551$ N. The decrease in gravitational potential energy is larger than the amount of mechanical energy removed by the negative work done by friction. $\frac{1}{2}mv^2=243$ J. The energy stored in the spring is larger than the initial kinetic energy of the crate at the top of the ramp.

**7.49.** **IDENTIFY:** Use $K_1+U_1+W_{other}=K_2+U_2$. Solve for $K_2$ and then for $v_2$.

**SET UP:** Let point 1 be at his initial position against the compressed spring and let point 2 be at the end of the barrel, as shown in Figure 7.49. Use $F=kx$ to find the amount the spring is initially compressed by the 4400 N force.

$K_1+U_1+W_{other}=K_2+U_2$

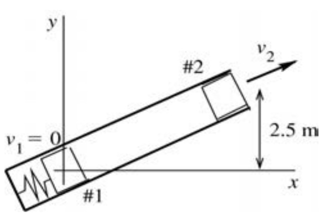



**Figure 7.49**

Take $y=0$ at his initial position.

**EXECUTE:** $K_1=0$, $K_2=\frac{1}{2}mv_2^2$

$W_{other}=W_{fric}=-fs$

$W_{other}=-(40\text{ N})(4.0\text{ m})=-160$ J

$U_{1,\text{grav}} = 0$, $U_{1,\text{el}} = \frac{1}{2}kd^2$, where $d$ is the distance the spring is initially compressed.

$F = kd$ so $d = \dfrac{F}{k} = \dfrac{4400\ \text{N}}{1100\ \text{N/m}} = 4.00\ \text{m}$

and $U_{1,\text{el}} = \frac{1}{2}(1100\ \text{N/m})(4.00\ \text{m})^2 = 8800\ \text{J}$

$U_{2,\text{grav}} = mgy_2 = (60\ \text{kg})(9.80\ \text{m/s}^2)(2.5\ \text{m}) = 1470\ \text{J}$, $U_{2,\text{el}} = 0$

Then $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$8800\ \text{J} - 160\ \text{J} = \frac{1}{2}mv_2^2 + 1470\ \text{J}$

$\frac{1}{2}mv_2^2 = 7170\ \text{J}$ and $v_2 = \sqrt{\dfrac{2(7170\ \text{J})}{60\ \text{kg}}} = 15.5\ \text{m/s}.$

**EVALUATE:** Some of the potential energy stored in the compressed spring is taken away by the work done by friction. The rest goes partly into gravitational potential energy and partly into kinetic energy.

**7.50.** **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the rocket from the starting point to the base of the ramp. $W_{\text{other}}$ is the work done by the thrust and by friction.

**SET UP:** Let point 1 be at the starting point and let point 2 be at the base of the ramp. $v_1 = 0$, $v_2 = 50.0\ \text{m/s}$. Let $y = 0$ at the base and take $+y$ upward. Then $y_2 = 0$ and $y_1 = d\sin 53°$, where $d$ is the distance along the ramp from the base to the starting point. Friction does negative work.

**EXECUTE:** $K_1 = 0$, $U_2 = 0$. $U_1 + W_{\text{other}} = K_2$. $W_{\text{other}} = (2000\ \text{N})d - (500\ \text{N})d = (1500\ \text{N})d$.

$mgd\sin 53° + (1500\ \text{N})d = \frac{1}{2}mv_2^2.$

$$d = \frac{mv_2^2}{2[mg\sin 53° + 1500\ \text{N}]} = \frac{(1500\ \text{kg})(50.0\ \text{m/s})^2}{2[(1500\ \text{kg})(9.80\ \text{m/s}^2)\sin 53° + 1500\ \text{N}]} = 142\ \text{m}.$$

**EVALUATE:** The initial height is $y_1 = (142\ \text{m})\sin 53° = 113\ \text{m}$. An object free-falling from this distance attains a speed $v = \sqrt{2gy_1} = 47.1\ \text{m/s}$. The rocket attains a greater speed than this because the forward thrust is greater than the friction force.

**7.51.** **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the system consisting of the two buckets. If we ignore the inertia of the pulley we ignore the kinetic energy it has.

**SET UP:** $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Points 1 and 2 in the motion are sketched in Figure 7.51.

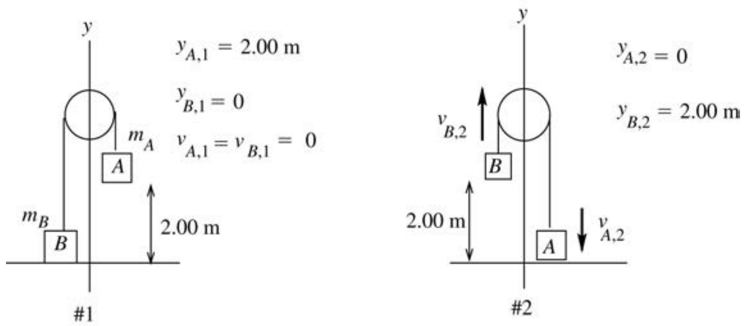



**Figure 7.51**

The tension force does positive work on the 4.0 kg bucket and an equal amount of negative work on the 12.0 kg bucket, so the net work done by the tension is zero.
Work is done on the system only by gravity, so $W_{\text{other}} = 0$ and $U = U_{\text{grav}}$.

**EXECUTE:** $K_1 = 0$, $K_2 = \frac{1}{2}m_Av_{A,2}^2 + \frac{1}{2}m_Bv_{B,2}^2$. But since the two buckets are connected by a rope they move together and have the same speed: $v_{A,2} = v_{B,2} = v_2$. Thus $K_2 = \frac{1}{2}(m_A + m_B)v_2^2 = (8.00\ \text{kg})v_2^2$.

$U_1 = m_A g y_{A,1} = (12.0 \text{ kg})(9.80 \text{ m/s}^2)(2.00 \text{ m}) = 235.2 \text{ J}.$

$U_2 = m_B g y_{B,2} = (4.0 \text{ kg})(9.80 \text{ m/s}^2)(2.00 \text{ m}) = 78.4 \text{ J}.$

Putting all this into $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives $U_1 = K_2 + U_2$.

$235.2 \text{ J} = (8.00 \text{ kg})v_2^2 + 78.4 \text{ J}.\ \ v_2 = \sqrt{\dfrac{235.2 \text{ J} - 78.4 \text{ J}}{8.00 \text{ kg}}} = 4.4 \text{ m/s}$

**EVALUATE:** The gravitational potential energy decreases and the kinetic energy increases by the same amount. We could apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to one bucket, but then we would have to include in $W_{\text{other}}$ the work done on the bucket by the tension $T$.

**7.52.** **IDENTIFY:** $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ says $W_{\text{other}} = K_2 + U_2 - (K_1 + U_1)$. $W_{\text{other}}$ is the work done on the baseball by the force exerted by the air.

**SET UP:** $U = mgy$. $K = \frac{1}{2}mv^2$, where $v^2 = v_x^2 + v_y^2$.

**EXECUTE:** **(a)** The change in total energy is the work done by the air,

$W_{\text{other}} = (K_2 + U_2) - (K_1 + U_1) = m\left(\frac{1}{2}(v_2^2 - v_1^2) + gy_2\right).$

$W_{\text{other}} = (0.145 \text{ kg})((1/2[(18.6 \text{ m/s})^2 - (30.0 \text{ m/s})^2 - (40.0 \text{ m/s})^2] + (9.80 \text{ m/s}^2)(53.6 \text{ m})).$

$W_{\text{other}} = -80.0 \text{ J}.$

**(b)** Similarly, $W_{\text{other}} = (K_3 + U_3) - (K_2 + U_2)$.

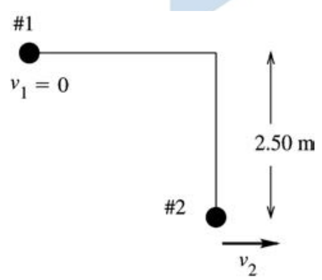
$W_{\text{other}} = (0.145 \text{ kg})((1/2)[(11.9 \text{ m/s})^2 + (-28.7 \text{ m/s})^2 - (18.6 \text{ m/s})^2] - (9.80 \text{ m/s}^2)(53.6 \text{ m})).$

$W_{\text{other}} = -31.3 \text{ J}.$

**(c)** The ball is moving slower on the way down, and does not go as far (in the $x$-direction), and so the work done by the air is smaller in magnitude.

**EVALUATE:** The initial kinetic energy of the baseball is $\frac{1}{2}(0.145 \text{ kg})(50.0 \text{ m/s})^2 = 181 \text{ J}$. For the total motion from the ground, up to the maximum height, and back down the total work done by the air is 111 J. The ball returns to the ground with $181 \text{ J} - 111 \text{ J} = 70 \text{ J}$ of kinetic energy and a speed of 31 m/s, less than its initial speed of 50 m/s.

**7.53.** **(a) IDENTIFY** and **SET UP:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the potato. Let point 1 be where the potato is released and point 2 be at the lowest point in its motion, as shown in Figure 7.53a.




$y_1 = 2.50 \text{ m}$

$y_2 = 0$

The tension in the string is at all points in the motion perpendicular to the displacement, so $W_r = 0$

The only force that does work on the potato is gravity, so $W_{\text{other}} = 0$.

**Figure 7.53a**

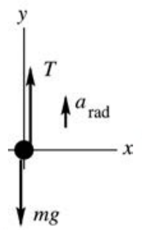
**EXECUTE:** $K_1 = 0$, $K_2 = \frac{1}{2}mv_2^2$, $U_1 = mgy_1$, $U_2 = 0$. Thus $U_1 = K_2$. $mgy_1 = \frac{1}{2}mv_2^2$, which gives

$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80 \text{ m/s}^2)(2.50 \text{ m})} = 7.00 \text{ m/s}.$

**EVALUATE:** The speed $v_2$ is the same as if the potato fell through 2.50 m.

**(b) IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the potato. The potato moves in an arc of a circle so its acceleration is $\vec{a}_{rad}$, where $a_{rad} = v^2/R$ and is directed toward the center of the circle. Solve for one of the forces, the tension $T$ in the string.

**SET UP:** The free-body diagram for the potato as it swings through its lowest point is given in Figure 7.53b.




The acceleration $\vec{a}_{rad}$ is directed in toward the center of the circular path, so at this point it is upward.

**Figure 7.53b**

**EXECUTE:** $\Sigma F_y = ma_y$ gives $T - mg = ma_{rad}$. Solving for $T$ gives $T = m(g + a_{rad}) = m\left(g + \dfrac{v_2^2}{R}\right)$, where the radius $R$ for the circular motion is the length $L$ of the string. It is instructive to use the algebraic expression for $v_2$ from part (a) rather than just putting in the numerical value: $v_2 = \sqrt{2gy_1} = \sqrt{2gL}$, so $v_2^2 = 2gL$. Then $T = m\left(g + \dfrac{v_2^2}{L}\right) = m\left(g + \dfrac{2gL}{L}\right) = 3mg$. The tension at this point is three times the weight of the potato, so $T = 3mg = 3(0.300\text{ kg})(9.80\text{ m/s}^2) = 8.82\text{ N}$.

**EVALUATE:** The tension is greater than the weight; the acceleration is upward so the net force must be upward.

**7.54. IDENTIFY:** Apply $K_1 + U_1 + W_{other} = K_2 + U_2$ to each stage of the motion.

**SET UP:** Let $y = 0$ at the bottom of the slope. In part (a), $W_{other}$ is the work done by friction. In part (b), $W_{other}$ is the work done by friction and the air resistance force. In part (c), $W_{other}$ is the work done by the force exerted by the snowdrift.

**EXECUTE:** **(a)** The skier's kinetic energy at the bottom can be found from the potential energy at the top minus the work done by friction, $K_1 = mgh - W_f = (60.0\text{ kg})(9.8\text{ N/kg})(65.0\text{ m}) - 10{,}500\text{ J}$, or

$K_1 = 38{,}200\text{ J} - 10{,}500\text{ J} = 27{,}720\text{ J}$. Then $v_1 = \sqrt{\dfrac{2K_1}{m}} = \sqrt{\dfrac{2(27{,}720\text{ J})}{60\text{ kg}}} = 30.4\text{ m/s}$.

**(b)** $K_2 = K_1 - (W_f + W_{air}) = 27{,}720\text{ J} - (\mu_k mgd + f_{air}d)$.

$K_2 = 27{,}720\text{ J} - [(0.2)(588\text{ N})(82\text{ m}) + (160\text{ N})(82\text{ m})]$ or $K_2 = 27{,}720\text{ J} - 22{,}763\text{ J} = 4957\text{ J}$. Then,

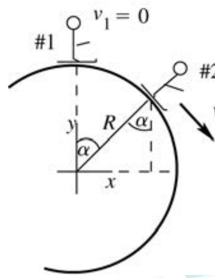
$v_2 = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(4957\text{ J})}{60\text{ kg}}} = 12.9\text{ m/s}$.

**(c)** Use the work-energy theorem to find the force. $W = \Delta K$, $F = K/d = (4957\text{ J})/(2.5\text{ m}) = 2000\text{ N}$.

**EVALUATE:** In each case, $W_{other}$ is negative and removes mechanical energy from the system.

**7.55. IDENTIFY and SET UP:** First apply $\Sigma\vec{F} = m\vec{a}$ to the skier.

Find the angle $\alpha$ where the normal force becomes zero, in terms of the speed $v_2$ at this point. Then apply the work-energy theorem to the motion of the skier to obtain another equation that relates $v_2$ and $\alpha$. Solve these two equations for $\alpha$.

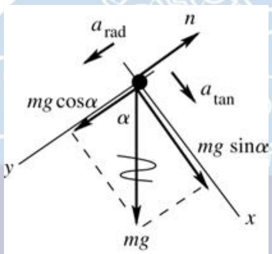



Let point 2 be where the skier loses contact with the snowball, as sketched in Figure 7.55a Loses contact implies $n \to 0$.

$y_1 = R, \quad y_2 = R\cos\alpha$

**Figure 7.55a**

First, analyze the forces on the skier when she is at point 2. The free-body diagram is given in Figure 7.55b. For this use coordinates that are in the tangential and radial directions. The skier moves in an arc of a circle, so her acceleration is $a_{\text{rad}} = v^2/R$, directed in towards the center of the snowball.




**EXECUTE:** $\Sigma F_y = ma_y$

$mg\cos\alpha - n = mv_2^2/R$

But $n = 0$ so $mg\cos\alpha = mv_2^2/R$

$v_2^2 = Rg\cos\alpha$

**Figure 7.55b**

Now use conservation of energy to get another equation relating $v_2$ to $\alpha$:

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

The only force that does work on the skier is gravity, so $W_{\text{other}} = 0$.

$K_1 = 0, \quad K_2 = \frac{1}{2}mv_2^2$

$U_1 = mgy_1 = mgR, \quad U_2 = mgy_2 = mgR\cos\alpha$

Then $mgR = \frac{1}{2}mv_2^2 + mgR\cos\alpha$

$v_2^2 = 2gR(1-\cos\alpha)$

Combine this with the $\Sigma F_y = ma_y$ equation:

$Rg\cos\alpha = 2gR(1-\cos\alpha)$

$\cos\alpha = 2 - 2\cos\alpha$

$3\cos\alpha = 2$ so $\cos\alpha = 2/3$ and $\alpha = 48.2°$

**EVALUATE:** She speeds up and her $a_{\text{rad}}$ increases as she loses gravitational potential energy. She loses contact when she is going so fast that the radially inward component of her weight isn't large enough to keep her in the circular path. Note that $\alpha$ where she loses contact does not depend on her mass or on the radius of the snowball.

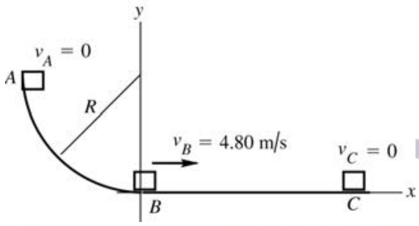
**7.56.** **IDENTIFY:** Initially the ball has all kinetic energy, but at its highest point it has kinetic energy and potential energy. Since it is thrown upward at an angle, its kinetic energy is *not* zero at its highest point.

**SET UP:** Apply conservation of energy: $K_f + U_f = K_i + U_i$. Let $y_i = 0$, so $y_f = h$, the maximum height. At this maximum height, $v_{f,y} = 0$ and $v_{f,x} = v_{i,x}$, so $v_f = v_{i,x} = (15 \text{ m/s})(\cos 60.0°) = 7.5 \text{ m/s}$. Substituting into conservation of energy equation gives $\frac{1}{2}mv_i^2 = mgh + \frac{1}{2}m(7.5 \text{ m/s})^2$.

**EXECUTE:** Solve for $h$: $h = \dfrac{v_i^2 - (7.5 \text{ m/s})^2}{2g} = \dfrac{(15 \text{ m/s})^2 - (7.5 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 8.6 \text{ m}$.

**EVALUATE:** If the ball were thrown straight up, its maximum height would be 11.5 m, since all of its kinetic energy would be converted to potential energy. But in this case it reaches a lower height because it still retains some kinetic energy at its highest point.

**7.57.** **IDENTIFY** and **SET UP:**




$y_A = R$

$y_B = y_C = 0$

**Figure 7.57**

**(a)** Apply conservation of energy to the motion from $B$ to $C$:
$K_B + U_B + W_{\text{other}} = K_C + U_C$. The motion is described in Figure 7.57.

**EXECUTE:** The only force that does work on the package during this part of the motion is friction, so
$W_{\text{other}} = W_f = f_{\text{k}}(\cos\phi)s = \mu_{\text{k}} mg(\cos 180°)s = -\mu_{\text{k}} mgs$

$K_B = \frac{1}{2}mv_B^2, \quad K_C = 0$

$U_B = 0, \quad U_C = 0$

Thus $K_B + W_f = 0$

$\frac{1}{2}mv_B^2 - \mu_{\text{k}} mgs = 0$

$$\mu_{\text{k}} = \frac{v_B^2}{2gs} = \frac{(4.80 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(3.00 \text{ m})} = 0.392.$$

**EVALUATE:** The negative friction work takes away all the kinetic energy.

**(b) IDENTIFY** and **SET UP:** Apply conservation of energy to the motion from $A$ to $B$:

$$K_A + U_A + W_{\text{other}} = K_B + U_B$$

**EXECUTE:** Work is done by gravity and by friction, so $W_{\text{other}} = W_f$.

$K_A = 0, \quad K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.200 \text{ kg})(4.80 \text{ m/s})^2 = 2.304 \text{ J}$

$U_A = mgy_A = mgR = (0.200 \text{ kg})(9.80 \text{ m/s}^2)(1.60 \text{ m}) = 3.136 \text{ J}, \quad U_B = 0$

Thus $U_A + W_f = K_B$

$W_f = K_B - U_A = 2.304 \text{ J} - 3.136 \text{ J} = -0.83 \text{ J}$

**EVALUATE:** $W_f$ is negative as expected; the friction force does negative work since it is directed opposite to the displacement.

**7.58.** **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the initial and final positions of the truck.

**SET UP:** Let $y = 0$ at the lowest point of the path of the truck. $W_{\text{other}}$ is the work done by friction. $f_{\text{r}} = \mu_{\text{r}} n = \mu_{\text{r}} mg\cos\beta$.

**EXECUTE:** Denote the distance the truck moves up the ramp by $x$. $K_1 = \frac{1}{2}mv_0^2$, $U_1 = mgL\sin\alpha$, $K_2 = 0$, $U_2 = mgx\sin\beta$ and $W_{\text{other}} = -\mu_{\text{r}} mgx\cos\beta$. From $W_{\text{other}} = (K_2 + U_2) - (K_1 + U_1)$, and solving for $x$, we get $x = \dfrac{K_1 + mgL\sin\alpha}{mg(\sin\beta + \mu_{\text{r}}\cos\beta)} = \dfrac{(v_0^2/2g) + L\sin\alpha}{\sin\beta + \mu_{\text{r}}\cos\beta}$.

**EVALUATE:** $x$ increases when $v_0$ increases and decreases when $\mu_{\text{r}}$ increases.

**7.59.** **(a) IDENTIFY:** We are given that $F_x = -\alpha x - \beta x^2$, $\alpha = 60.0\text{ N/m}$ and $\beta = 18.0\text{ N/m}^2$. Use $W_{F_x} = \int_{x_1}^{x_2} F_x(x)dx$ to calculate $W$ and then use $W = -\Delta U$ to identify the potential energy function $U(x)$.

**SET UP:** $W_{F_x} = U_1 - U_2 = \int_{x_1}^{x_2} F_x(x)\,dx$

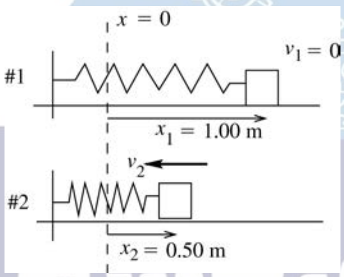
Let $x_1 = 0$ and $U_1 = 0$. Let $x_2$ be some arbitrary point $x$, so $U_2 = U(x)$.

**EXECUTE:** $U(x) = -\int_0^x F_x(x)dx = -\int_0^x(-\alpha x - \beta x^2)dx = \int_0^x(\alpha x + \beta x^2)dx = \frac{1}{2}\alpha x^2 + \frac{1}{3}\beta x^3.$

**EVALUATE:** If $\beta = 0$, the spring does obey Hooke's law, with $k = \alpha$, and our result reduces to $\frac{1}{2}kx^2$.

**(b) IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the object.

**SET UP:** The system at points 1 and 2 is sketched in Figure 7.59.




$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

The only force that does work on the object is the spring force, so $W_{\text{other}} = 0$.

**Figure 7.59**

**EXECUTE:** $K_1 = 0$, $K_2 = \frac{1}{2}mv_2^2$

$U_1 = U(x_1) = \frac{1}{2}\alpha x_1^2 + \frac{1}{3}\beta x_1^3 = \frac{1}{2}(60.0\text{ N/m})(1.00\text{ m})^2 + \frac{1}{3}(18.0\text{ N/m}^2)(1.00\text{ m})^3 = 36.0\text{ J}$

$U_2 = U(x_2) = \frac{1}{2}\alpha x_2^2 + \frac{1}{3}\beta x_2^3 = \frac{1}{2}(60.0\text{ N/m})(0.500\text{ m})^2 + \frac{1}{3}(18.0\text{ N/m}^2)(0.500\text{ m})^3 = 8.25\text{ J}$

Thus $36.0\text{ J} = \frac{1}{2}mv_2^2 + 8.25\text{ J}$, which gives $v_2 = \sqrt{\dfrac{2(36.0\text{ J} - 8.25\text{ J})}{0.900\text{ kg}}} = 7.85\text{ m/s}.$

**EVALUATE:** The elastic potential energy stored in the spring decreases and the kinetic energy of the object increases.

**7.60.** **IDENTIFY:** Mechanical energy is conserved on the hill, which gives us the speed of the sled at the top. After it leaves the cliff, we must use projectile motion.

**SET UP:** Use conservation of energy to find the speed of the sled at the edge of the cliff. Let $y_\text{i} = 0$ so $y_\text{f} = h = 11.0$ m. $K_\text{f} + U_\text{f} = K_\text{i} + U_\text{i}$ gives $\frac{1}{2}mv_\text{f}^{\,2} + mgh = \frac{1}{2}mv_\text{i}^{\,2}$ or $v_\text{f} = \sqrt{v_\text{i}^{\,2} - 2gh}$. Then analyze the projectile motion of the sled: use the vertical component of motion to find the time $t$ that the sled is in the air; then use the horizontal component of the motion with $a_x = 0$ to find the horizontal displacement.

**EXECUTE:** $v_\text{f} = \sqrt{(22.5\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(11.0\text{ m})} = 17.1\text{ m/s}$. $y_\text{f} = v_{\text{i},y}t + \frac{1}{2}a_y t^2$ gives

$t = \sqrt{\dfrac{2y_\text{f}}{a_y}} = \sqrt{\dfrac{2(-11.0\text{ m})}{-9.80\text{ m/s}^2}} = 1.50\text{ s}$. $x_\text{f} = v_{\text{i},x}t + \frac{1}{2}a_x t^2$ gives $x_\text{f} = v_{\text{i},x}t = (17.1\text{ m/s})(1.50\text{ s}) = 25.6\text{ m}$.

**EVALUATE:** Conservation of energy can be used to find the speed of the sled at any point of the motion but does not specify how far the sled travels while it is in the air.

**7.61. IDENTIFY:** We have a conservative force, so we can relate the force and the potential energy function. Energy conservation applies.

**SET UP:** $F_x = -dU/dx$, $U$ goes to 0 as $x$ goes to infinity, and $F(x) = \dfrac{\alpha}{(x+x_0)^2}$.

**EXECUTE:** **(a)** Using $dU = -F_x dx$, we get $U_x - U_\infty = -\displaystyle\int_\infty^x \frac{\alpha}{(x+x_0)^2}dx = \frac{\alpha}{x+x_0}$.

**(b)** Energy conservation tells us that $U_1 = K_2 + U_2$. Therefore $\dfrac{\alpha}{x_1+x_0} = \dfrac{1}{2}mv_x^2 + \dfrac{\alpha}{x_2+x_0}$. Putting in $m$ = 0.500 kg, $\alpha = 0.800\ \text{N}\cdot\text{m}$, $x_0$ = 0.200 m, $x_1$ = 0, and $x_2$ = 0.400 m, solving for $v$ gives $v$ = 3.27 m/s.

**EVALUATE:** The potential energy is not infinite even though the integral in (a) is taken over an infinite distance because the force rapidly gets smaller with increasing distance $x$.

**7.62. IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. $U$ is the total elastic potential energy of the two springs.

**SET UP:** Call the two points in the motion where $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ is applied $A$ and $B$ to avoid confusion with springs 1 and 2, that have force constants $k_1$ and $k_2$. At any point in the motion the distance one spring is stretched equals the distance the other spring is compressed. Let $+x$ be to the right. Let point $A$ be the initial position of the block, where it is released from rest, so $x_{1A} = +0.150$ m and $x_{2A} = -0.150$ m.

**EXECUTE:** **(a)** With no friction, $W_{\text{other}} = 0$. $K_A = 0$ and $U_A = K_B + U_B$. The maximum speed is when $U_B = 0$ and this is at $x_{1B} = x_{2B} = 0$, when both springs are at their natural length.

$\frac{1}{2}k_1x_{1A}^2 + \frac{1}{2}k_2x_{2A}^2 = \frac{1}{2}mv_B^2$. $x_{1A}^2 = x_{2A}^2 = (0.150\text{ m})^2$, so

$$v_B = \sqrt{\frac{k_1+k_2}{m}}(0.150\text{ m}) = \sqrt{\frac{2500\text{ N/m} + 2000\text{ N/m}}{3.00\text{ kg}}}(0.150\text{ m}) = 6.00\text{ m/s}.$$

**(b)** At maximum compression of spring 1, spring 2 has its maximum extension and $v_B = 0$. Therefore, at this point $U_A = U_B$. The distance spring 1 is compressed equals the distance spring 2 is stretched, and vice versa: $x_{1A} = -x_{2A}$ and $x_{1B} = -x_{2B}$. Then $U_A = U_B$ gives $\frac{1}{2}(k_1+k_2)x_{1A}^2 = \frac{1}{2}(k_1+k_2)x_{1B}^2$ and $x_{1B} = -x_{1A} = -0.150$ m. The maximum compression of spring 1 is 15.0 cm.

**EVALUATE:** When friction is not present mechanical energy is conserved and is continually transformed between kinetic energy of the block and potential energy in the springs. If friction is present, its work removes mechanical energy from the system.

**7.63. IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the block.

**SET UP:** Let $y = 0$ at the floor. Let point 1 be the initial position of the block against the compressed spring and let point 2 be just before the block strikes the floor.

**EXECUTE:** With $U_2 = 0, K_1 = 0,$ $K_2 = U_1$. $\frac{1}{2}mv_2^2 = \frac{1}{2}kx^2 + mgh$. Solving for $v_2$,

$$v_2 = \sqrt{\frac{kx^2}{m} + 2gh} = \sqrt{\frac{(1900\text{ N/m})(0.045\text{ m})^2}{(0.150\text{ kg})} + 2(9.80\text{ m/s}^2)(1.20\text{ m})} = 7.01\text{ m/s}.$$

**EVALUATE:** The potential energy stored in the spring and the initial gravitational potential energy all go into the final kinetic energy of the block.

**7.64. IDENTIFY:** At equilibrium the upward spring force equals the weight $mg$ of the object. Apply conservation of energy to the motion of the fish.

**SET UP:** The distance that the mass descends equals the distance the spring is stretched. $K_1 = K_2 = 0$, so $U_1(\text{gravitational}) = U_2(\text{spring})$

**EXECUTE:** Following the hint, the force constant $k$ is found from $mg = kd$, or $k = mg/d$. When the fish falls from rest, its gravitational potential energy decreases by $mgy$; this becomes the potential energy of the spring, which is $\frac{1}{2}ky^2 = \frac{1}{2}(mg/d)y^2$. Equating these, $\frac{1}{2}\frac{mg}{d}y^2 = mgy$, or $y = 2d$.

**EVALUATE:** At its lowest point the fish is not in equilibrium. The upward spring force at this point is $ky = 2kd$, and this is equal to twice the weight. At this point the net force is $mg$, upward, and the fish has an upward acceleration equal to $g$.

**7.65. IDENTIFY:** The spring does positive work on the box but friction does negative work.

**SET UP:** $U_{el} = \frac{1}{2}kx^2$ and $W_{other} = W_f = -\mu_k mgx$.

**EXECUTE:** **(a)** $U_{el} + W_{other} = K$ gives ½ $kx^2 + (-\mu_k mgx) =$ ½ $mv^2$. Using the numbers for the problem, $k = 45.0$ N/m, $x = 0.280$ m, $\mu_k = 0.300$, and $m = 1.60$ kg, solving for $v$ gives $v = 0.747$ m/s.

**(b)** Call $x$ the distance the spring is compressed when the speed of the box is a maximum and $x_0$ the initial compression distance of the spring. Using an approach similar to that in part (a) gives ½ $kx_0^2 - \mu_k mg(x_0 - x) =$ ½ $mv^2 +$ ½ $kx^2$. Rearranging gives $mv^2 = kx_0^2 - kx^2 - 2\mu_k mg(x_0 - x)$. For the maximum speed, $d(v^2)/dx = 0$, which gives $-2kx + 2\mu_k mg = 0$. Solving for $x_{max}$, the compression distance at maximum speed, gives $x_{max} = \mu_k mg/k$. Now substitute this result into the expression above for $mv^2$, put in the numbers, and solve for $v$, giving $v = 0.931$ m/s.

**EVALUATE:** Another way to find the result in (b) is to realize that the spring force decreases as $x$ decreases, but the friction force remains constant. Eventually these two forces will be equal in magnitude. After that the friction force will be greater than the spring force, and friction will begin to slow down the box. So the maximum box speed occurs when the spring force is equal to the friction force. At that instant, $kx = f_k$, which gives $x = 0.105$ m. Then energy conservation can be used to find $v$ with this value of $x$.

**7.66. IDENTIFY:** The spring obeys Hooke's law. Gravity and the spring provide the vertical forces on the brick. The mechanical energy of the system is conserved.

**SET UP:** Use $K_f + U_f = K_i + U_i$. In part (a), setting $y_f = 0$, we have $y_i = x$, the amount the spring will stretch. Also, since $K_i = K_f = 0$, $\frac{1}{2}kx^2 = mgx$. In part (b), $y_i = h + x$, where $h = 1.0$ m.

**EXECUTE:** **(a)** $x = \dfrac{2mg}{k} = \dfrac{2(3.0\text{ kg})(9.80\text{ m/s}^2)}{1500\text{ N/m}} = 0.039\text{ m} = 3.9\text{ cm}.$

**(b)** $\frac{1}{2}kx^2 = mg(h+x)$, $kx^2 - 2mgx - 2mgh = 0$ and $x = \dfrac{mg}{k}\left(1 \pm \sqrt{1 + \dfrac{2hk}{mg}}\right)$. Since $x$ must be positive, we have $x = \dfrac{mg}{k}\left(1 + \sqrt{1 + \dfrac{2hk}{mg}}\right) = \dfrac{(3.0\text{ kg})(9.80\text{ m/s}^2)}{1500\text{ N/m}}\left(1 + \sqrt{1 + \dfrac{2(1.0\text{ m})(1500\text{ N/m})}{3.0\text{ kg}(9.80\text{ m/s}^2)}}\right) = 0.22\text{ m} = 22\text{ cm}.$

**EVALUATE:** In part (b) there is additional initial energy (from gravity), so the spring is stretched more.

**7.67. IDENTIFY:** Only conservative forces (gravity and the spring) act on the fish, so its mechanical energy is conserved.

**SET UP:** Energy conservation tells us $K_1 + U_1 + W_{other} = K_2 + U_2$, where $W_{other} = 0$. $U_g = mgy$, $K = \frac{1}{2}mv^2$, and $U_{spring} = \frac{1}{2}ky^2$.

**EXECUTE:** **(a)** $K_1 + U_1 + W_{other} = K_2 + U_2$. Let $y$ be the distance the fish has descended, so $y = 0.0500$ m. $K_1 = 0$, $W_{other} = 0$, $U_1 = mgy$, $K_2 = \frac{1}{2}mv_2^2$, and $U_2 = \frac{1}{2}ky^2$. Solving for $K_2$ gives

$$K_2 = U_1 - U_2 = mgy - \frac{1}{2}ky^2 = (3.00\text{ kg})(9.8\text{ m/s}^2)(0.0500\text{ m}) - \frac{1}{2}(900\text{ N/m})(0.0500\text{ m})^2$$

$K_2 = 1.47\text{ J} - 1.125\text{ J} = 0.345\text{ J}$. Solving for $v_2$ gives $v_2 = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(0.345\text{ J})}{3.00\text{ kg}}} = 0.480\text{ m/s}.$

**(b)** The maximum speed is when $K_2$ is maximum, which is when $dK_2/dy = 0$. Using $K_2 = mgy - \frac{1}{2}ky^2$ gives $\frac{dK_2}{dy} = mg - ky = 0$. Solving for $y$ gives $y = \frac{mg}{k} = \frac{(3.00\text{ kg})(9.8\text{ m/s}^2)}{900\text{ N/m}} = 0.03267\text{ m}$. At this $y$,

$K_2 = (3.00\text{ kg})(9.8\text{ m/s}^2)(0.03267\text{ m}) - \frac{1}{2}(900\text{ N/m})(0.03267\text{ m})^2$. $K_2 = 0.9604\text{ J} - 0.4803\text{ J} = 0.4801\text{ J}$,

so $v_2 = \sqrt{\frac{2K_2}{m}} = 0.566\text{ m/s}$.

**EVALUATE:** The speed in part (b) is greater than the speed in part (a), as it should be since it is the maximum speed.

**7.68.** **IDENTIFY:** The mechanical energy is conserved and Newton's second law applies. The kinetic energy of the cart (with riders) is transformed into elastic potential energy at maximum compression of the spring, and the acceleration is greatest at that instant.

**SET UP:** $F = ma$, $K_1 = U_{el}$, $a_{max} = 3.00g$.

**EXECUTE:** **(a)** and **(b)** $a_{max} = kx_{max}/m$ and $\frac{1}{2}mv^2 = \frac{1}{2}kx_{max}^2$, where $m = 300$ kg, $v = 6.00$ m/s, and $a_{max} = 3.00g$. Solving these two equations simultaneously gives $k = 7210$ N/m and $x_{max} = 1.22$ m.

**EVALUATE:** The force constant is 72 N/cm, so this is a rather stiff spring, as it would have to be to stop a 300-kg cart with an acceleration of $3g$.

**7.69.** **(a) IDENTIFY** and **SET UP:** Apply $K_A + U_A + W_{other} = K_B + U_B$ to the motion from $A$ to $B$.

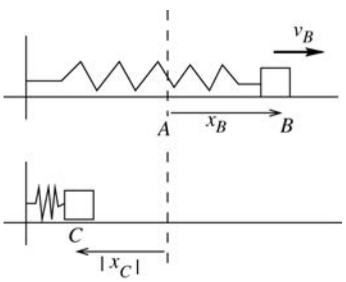
**EXECUTE:** $K_A = 0$, $K_B = \frac{1}{2}mv_B^2$, $U_A = 0$, $U_B = U_{el,B} = \frac{1}{2}kx_B^2$, where $x_B = 0.25$ m, and $W_{other} = W_F = Fx_B$. Thus $Fx_B = \frac{1}{2}mv_B^2 + \frac{1}{2}kx_B^2$. (The work done by $F$ goes partly to the potential energy of the stretched spring and partly to the kinetic energy of the block.)

$Fx_B = (20.0\text{ N})(0.25\text{ m}) = 5.0\text{ J}$ and $\frac{1}{2}kx_B^2 = \frac{1}{2}(40.0\text{ N/m})(0.25\text{ m})^2 = 1.25\text{ J}$

Thus $5.0\text{ J} = \frac{1}{2}mv_B^2 + 1.25\text{ J}$ and $v_B = \sqrt{\frac{2(3.75\text{ J})}{0.500\text{ kg}}} = 3.87\text{ m/s}$.

**(b) IDENTIFY:** Apply $K_1 + U_1 + W_{other} = K_2 + U_2$ to the motion of the block. Let point $C$ be where the block is closest to the wall. When the block is at point $C$ the spring is compressed an amount $|x_C|$, so the block is $0.60\text{ m} - |x_C|$ from the wall, and the distance between $B$ and $C$ is $x_B + |x_C|$.

**SET UP:** The motion from $A$ to $B$ to $C$ is described in Figure 7.69.




$K_B + U_B + W_{other} = K_C + U_C$

**EXECUTE:** $W_{other} = 0$

$K_B = \frac{1}{2}mv_B^2 = 5.0\text{ J} - 1.25\text{ J} = 3.75\text{ J}$ (from part (a))

$U_B = \frac{1}{2}kx_B^2 = 1.25\text{ J}$

$K_C = 0$ (instantaneously at rest at point closest to wall)

$U_C = \frac{1}{2}k|x_C|^2$

**Figure 7.69**

Thus $3.75\text{ J} + 1.25\text{ J} = \frac{1}{2}k|x_C|^2$, giving $|x_C| = \sqrt{\frac{2(5.0\text{ J})}{40.0\text{ N/m}}} = 0.50\text{ m}$. The distance of the block from the wall is $0.60\text{ m} - 0.50\text{ m} = 0.10\text{ m}$.

**EVALUATE:** The work $(20.0\text{ N})(0.25\text{ m}) = 5.0\text{ J}$ done by $F$ puts 5.0 J of mechanical energy into the system. No mechanical energy is taken away by friction, so the total energy at points $B$ and $C$ is 5.0 J.

**7.70.** **IDENTIFY:** Applying Newton's second law, we can use the known normal forces to find the speeds of the block at the top and bottom of the circle. We can then use energy conservation to find the work done by friction, which is the target variable.

**SET UP:** For circular motion $\Sigma F = m\dfrac{v^2}{R}$. Energy conservation tells us that $K_A + U_A + W_{\text{other}} = K_B + U_B$, where $W_{\text{other}}$ is the work done by friction. $U_g = mgy$ and $K = \frac{1}{2}mv^2$.

**EXECUTE:** Use the given values for the normal force to find the block's speed at points $A$ and $B$. At point $A$, Newton's second law gives $n_A - mg = m\dfrac{v_A^2}{R}$. So

$v_A = \sqrt{\dfrac{R}{m}(n_A - mg)} = \sqrt{\dfrac{0.500\text{ m}}{0.0400\text{ kg}}(3.95\text{ N} - 0.392\text{ N})} = 6.669\text{ m/s}$. Similarly at point $B$, $n_B + mg = m\dfrac{v_B^2}{R}$.

Solving for $v_B$ gives $v_B = \sqrt{\dfrac{R}{m}(n_B + mg)} = \sqrt{\dfrac{0.500\text{ m}}{0.0400\text{ kg}}(0.680\text{ N} + 0.392\text{ N})} = 3.660\text{ m/s}$. Now apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to find the work done by friction. $K_A + U_A + W_{\text{other}} = K_B + U_B$.

$W_{\text{other}} = K_B + U_B - K_A$.

$W_{\text{other}} = \dfrac{1}{2}(0.040\text{ kg})(3.66\text{ m/s})^2 + (0.04\text{ kg})(9.8\text{ m/s}^2)(1.0\text{ m}) - \dfrac{1}{2}(0.04\text{ kg})(6.669\text{ m/s})^2$.

$W_{\text{other}} = 0.2679\text{ J} + 0.392\text{ J} - 0.8895\text{ J} = -0.230\text{ J}$.

**EVALUATE:** The work done by friction is negative, as it should be. This work is equal to the loss of mechanical energy between the top and bottom of the circle.

**7.71.** **IDENTIFY:** We can apply Newton's second law to the block. The only forces acting on the block are gravity downward and the normal force from the track pointing toward the center of the circle. The mechanical energy of the block is conserved since only gravity does work on it. The normal force does no work since it is perpendicular to the displacement of the block. The target variable is the normal force at the top of the track.

**SET UP:** For circular motion $\Sigma F = m\dfrac{v^2}{R}$. Energy conservation tells us that $K_A + U_A + W_{\text{other}} = K_B + U_B$, where $W_{\text{other}} = 0$. $U_g = mgy$ and $K = \frac{1}{2}mv^2$.

**EXECUTE:** Let point $A$ be at the bottom of the path and point $B$ be at the top of the path. At the bottom of the path, $n_A - mg = m\dfrac{v^2}{R}$ (from Newton's second law).

$v_A = \sqrt{\dfrac{R}{m}(n_A - mg)} = \sqrt{\dfrac{0.800\text{ m}}{0.0500\text{ kg}}(3.40\text{ N} - 0.49\text{ N})} = 6.82\text{ m/s}$. Use energy conservation to find the speed at point $B$. $K_A + U_A + W_{\text{other}} = K_B + U_B$, giving $\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + mg(2R)$. Solving for $v_B$ gives $v_B = \sqrt{v_A^2 - 4Rg} = \sqrt{(6.82\text{ m/s})^2 - 4(0.800\text{ m})(9.8\text{ m/s}^2)} = 3.89\text{ m/s}$. Then at point $B$, Newton's second law gives $n_B + mg = m\dfrac{v_B^2}{R}$. Solving for $n_B$ gives $n_B = m\dfrac{v_B^2}{R} - mg =$

$$(0.0500\text{ kg})\left(\frac{(3.89\text{ m/s})^2}{0.800\text{ m}} - 9.8\text{ m/s}^2\right) = 0.456\text{ N}.$$

**EVALUATE:** The normal force at the top is considerably less than it is at the bottom for two reasons: the block is moving slower at the top and the downward force of gravity at the top aids the normal force in keeping the block moving in a circle.

**7.72.** **IDENTIFY:** Only gravity does work, so apply $K_1 + U_1 = K_2 + U_2$. Use $\Sigma \vec{F} = m\vec{a}$ to calculate the tension.

**SET UP:** Let $y = 0$ at the bottom of the arc. Let point 1 be when the string makes a $45°$ angle with the vertical and point 2 be where the string is vertical. The rock moves in an arc of a circle, so it has radial acceleration $a_{\text{rad}} = v^2/r$.

**EXECUTE:** **(a)** At the top of the swing, when the kinetic energy is zero, the potential energy (with respect to the bottom of the circular arc) is $mgl(1-\cos\theta)$, where $l$ is the length of the string and $\theta$ is the angle the string makes with the vertical. At the bottom of the swing, this potential energy has become kinetic energy, so $mgl(1-\cos\theta) = \frac{1}{2}mv^2$, which gives $v = \sqrt{2gl(1-\cos\theta)} = \sqrt{2(9.80\text{ m/s}^2)(0.80\text{ m})(1-\cos 45°)} = 2.1\text{ m/s}$.

**(b)** At $45°$ from the vertical, the speed is zero, and there is no radial acceleration; the tension is equal to the radial component of the weight, or $mg\cos\theta = (0.12\text{ kg})(9.80\text{ m/s}^2)\cos 45° = 0.83\text{ N}$.

**(c)** At the bottom of the circle, the tension is the sum of the weight and the mass times the radial acceleration, $mg + mv_2^2/l = mg(1 + 2(1-\cos 45°)) = 1.9\text{ N}$.

**EVALUATE:** When the string passes through the vertical, the tension is greater than the weight because the acceleration is upward.

**7.73.** **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the block.

**SET UP:** The motion from $A$ to $B$ is described in Figure 7.73.

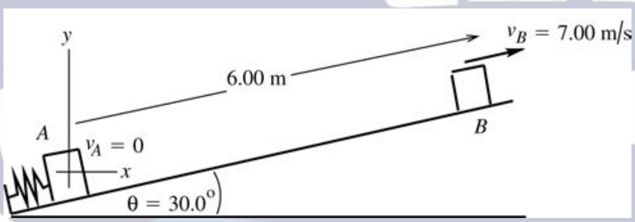



**Figure 7.73**

The normal force is $n = mg\cos\theta$, so $f_k = \mu_k n = \mu_k mg\cos\theta$. $y_A = 0$; $y_B = (6.00\text{ m})\sin 30.0° = 3.00\text{ m}$.

$K_A + U_A + W_{\text{other}} = K_B + U_B$

**EXECUTE:** Work is done by gravity, by the spring force, and by friction, so $W_{\text{other}} = W_f$ and $U = U_{\text{el}} + U_{\text{grav}}$

$K_A = 0$, $K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(1.50\text{ kg})(7.00\text{ m/s})^2 = 36.75\text{ J}$

$U_A = U_{\text{el},A} + U_{\text{grav},A} = U_{\text{el},A}$, since $U_{\text{grav},A} = 0$

$U_B = U_{\text{el},B} + U_{\text{grav},B} = 0 + mgy_B = (1.50\text{ kg})(9.80\text{ m/s}^2)(3.00\text{ m}) = 44.1\text{ J}$

$W_{\text{other}} = W_f = (f_k\cos\phi)s = \mu_k mg\cos\theta(\cos 180°)s = -\mu_k mg\cos\theta s$

$W_{\text{other}} = -(0.50)(1.50\text{ kg})(9.80\text{ m/s}^2)(\cos 30.0°)(6.00\text{ m}) = -38.19\text{ J}$

Thus $U_{\text{el},A} - 38.19\text{ J} = 36.75\text{ J} + 44.10\text{ J}$, giving $U_{\text{el},A} = 38.19\text{ J} + 36.75\text{ J} + 44.10\text{ J} = 119\text{ J}$.

**EVALUATE:** $U_{\text{el}}$ must always be positive. Part of the energy initially stored in the spring was taken away by friction work; the rest went partly into kinetic energy and partly into an increase in gravitational potential energy.

**7.74.** **IDENTIFY:** We know the potential energy function for a conservative force. Mechanical energy is conserved.

**SET UP:** $F_x = -dU/dx$ and $U(x) = -\alpha x^2 + \beta x^3$.

**EXECUTE:** **(a)** $U_1 + K_1 = U_2 + K_2$ gives $0 + 0 = U_2 + K_2$, so $K_2 = -U_2 = -(-\alpha x_2^2 + \beta x_2^3) = \frac{1}{2}mv^2$. Using $m = 0.0900$ kg, $x = 4.00$ m, $\alpha = 2.00\ \text{J/m}^2$, and $\beta = 0.300\ \text{J/m}^3$, solving for $v$ gives $v = 16.9$ m/s.

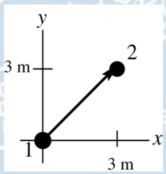
**(b)** $F_x = -dU/dx = -(-2\alpha x + 3\beta x^2)$. In addition, $F_x = ma_x$, so $a_x = F_x/m$. Using the numbers from (a), gives $a = 17.8\ \text{m/s}^2$.

**(c)** The maximum $x$ will occur when $U = 0$ since the total energy is zero. Therefore $-\alpha x^2 + \beta x^3 = 0$, so $x_{max} = \alpha/\beta = (2.00\ \text{J/m}^2)/(0.300\ \text{J/m}^3) = 6.67$ m.

**EVALUATE:** The object is released from rest but at a small (but not zero) $x$. Therefore $F_x$ is small but not zero initially, so it will start the object moving.

**7.75.** **IDENTIFY:** We are given that $\vec{F} = -\alpha xy^2\hat{j}$, $\alpha = 2.50\ \text{N/m}^3$. $\vec{F}$ is not constant so use $W = \int_1^2 \vec{F}\cdot d\vec{l}$ to calculate the work. $\vec{F}$ must be evaluated along the path.

**(a) SET UP:** The path is sketched in Figure 7.75a.




$d\vec{l} = dx\hat{i} + dy\hat{j}$

$\vec{F}\cdot d\vec{l} = -\alpha xy^2\,dy$

On the path, $x = y$ so $\vec{F}\cdot d\vec{l} = -\alpha y^3\,dy$

**Figure 7.75a**

**EXECUTE:** $W = \int_1^2 \vec{F}\cdot d\vec{l} = \int_{y_1}^{y_2} (-\alpha y^3)\,dy = -(\alpha/4)(y_2^4 - y_1^4)$

$y_1 = 0$, $y_2 = 3.00$ m, so $W = -\frac{1}{4}(2.50\ \text{N/m}^3)(3.00\ \text{m})^4 = -50.6$ J

**(b) SET UP:** The path is sketched in Figure 7.75b.

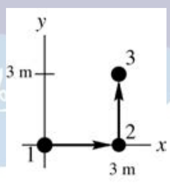



**Figure 7.75b**

For the displacement from point 1 to point 2, $d\vec{l} = dx\hat{i}$, so $\vec{F}\cdot d\vec{l} = 0$ and $W = 0$. (The force is perpendicular to the displacement at each point along the path, so $W = 0$.)

For the displacement from point 2 to point 3, $d\vec{l} = dy\hat{j}$, so $\vec{F}\cdot d\vec{l} = -\alpha xy^2\,dy$. On this path, $x = 3.00$ m, so

$$\vec{F}\cdot d\vec{l} = -(2.50\ \text{N/m}^3)(3.00\ \text{m})y^2\,dy = -(7.50\ \text{N/m}^2)y^2\,dy.$$

**EXECUTE:** $W = \int_2^3 \vec{F}\cdot d\vec{l} = -(7.50\ \text{N/m}^2)\int_{y_2}^{y_3} y^2\,dy = -(7.50\ \text{N/m}^2)\frac{1}{3}(y_3^3 - y_2^3)$

$W = -(7.50\ \text{N/m}^2)\left(\frac{1}{3}\right)(3.00\ \text{m})^3 = -67.5$ J.

**(c) EVALUATE:** For these two paths between the same starting and ending points the work is different, so the force is nonconservative.

**7.76.** **IDENTIFY:** Use $F_x = -\dfrac{dU}{dx}$ to relate $F_x$ and $U(x)$. The equilibrium is stable where $U(x)$ is a local minimum and the equilibrium is unstable where $U(x)$ is a local maximum.

**SET UP:** $dU/dx$ is the slope of the graph of $U$ versus $x$. $K = E - U$, so $K$ is a maximum when $U$ is a minimum. The maximum $x$ is where $E = U$.

**EXECUTE:** **(a)** The slope of the *U vs. x* curve is negative at point *A*, so $F_x$ is positive because $F_x = -dU/dx$.

**(b)** The slope of the curve at point *B* is positive, so the force is negative.

**(c)** The kinetic energy is a maximum when the potential energy is a minimum, and that figures to be at around 0.75 m.

**(d)** The curve at point *C* looks pretty close to flat, so the force is zero.

**(e)** The object had zero kinetic energy at point *A*, and in order to reach a point with more potential energy than $U(A)$, the kinetic energy would need to be negative. Kinetic energy is never negative, so the object can never be at any point where the potential energy is larger than $U(A)$. On the graph, that looks to be at about 2.2 m.

**(f)** The point of minimum potential (found in part (c)) is a stable point, as is the relative minimum near 1.9 m.

**(g)** The only potential maximum, and hence the only point of unstable equilibrium, is at point *C*.

**EVALUATE:** If *E* is less than *U* at point *C*, the particle is trapped in one or the other of the potential "wells" and cannot move from one allowed region of *x* to the other.

**7.77.** **IDENTIFY:** The mechanical energy of the system is conserved, and Newton's second law applies. As the pendulum swings, gravitational potential energy gets transformed to kinetic energy.

**SET UP:** For circular motion, $F = mv^2/r$. $U_{grav} = mgh$.

**EXECUTE:** **(a)** Conservation of mechanical energy gives $mgh = \frac{1}{2}mv^2 + mgh_0$, where $h_0 = 0.800$ m. Applying Newton's second law at the bottom of the swing gives $T = mv^2/L + mg$. Combining these two equations and solving for *T* as a function of *h* gives $T = (2mg/L)h + mg(1 - 2h_0/L)$. In a graph of *T* versus *h*, the slope is $2mg/L$. Graphing the data given in the problem, we get the graph shown in Figure 7.77. Using the best-fit equation, we get $T = (9.293\text{ N/m})h + 257.3$ N. Therefore $2mg/L = 9.293$ N/m. Using $mg = 265$ N and solving for *L*, we get $L = 2(265\text{ N})/(9.293\text{ N/m}) = 57.0$ m.

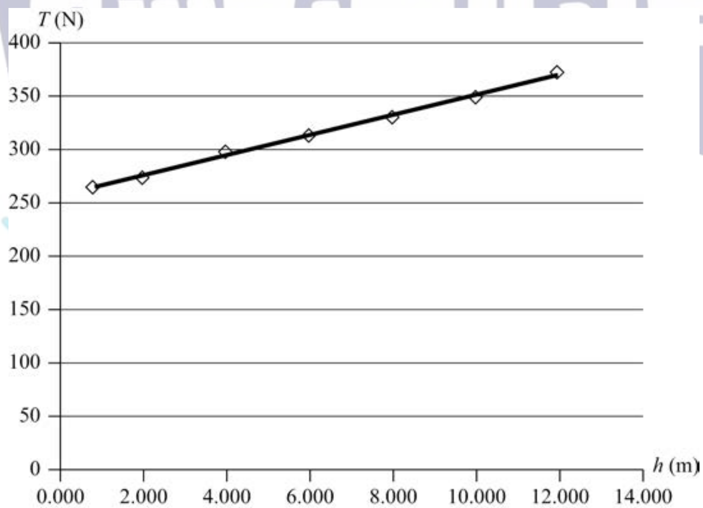



**Figure 7.77**

**(b)** $T_{max} = 822$ N, so $T = T_{max}/2 = 411$ N. We use the equation for the graph with $T = 411$ N and solve for *h*. $411\text{ N} = (9.293\text{ N/m})h + 257.3$ N, which gives $h = 16.5$ m.

**(c)** The pendulum is losing energy because negative work is being done on it by friction with the air and at the point of contact where it swings.

**EVALUATE:** The length of this pendulum may seem extremely large, but it is not unreasonable for a museum exhibit, which can cover a height of several floor levels.

**7.78.** **IDENTIFY:** Friction does negative work, and we can use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

**SET UP:** $U_1 + W_{\text{other}} = K_2$

**EXECUTE:** **(a)** Using $K_2 = U_1 + W_{\text{other}}$ gives $\frac{1}{2}mv^2 = mgh - (\mu_k mg\cos\theta)s$ and geometry gives $s = \frac{h}{\sin\theta}$.

Combining these equations and solving for $h$ gives $h = \frac{v^2}{2g\left(1 - \frac{\mu_k}{\tan\theta}\right)}$. For each material, $\theta$ = 52.0° and $v$ = 4.00 m/s. Using the coefficients of sliding friction from the table in the problem, this formula gives the following results for $h$. (i) 0.92 m (ii) 1.1 m (iii) 2.4 m.

**(b)** The mass divides out, so $h$ is unchanged and remains at 1.1 m.

**(c)** In the formula for $h$ in part (a), we solve for $v^2$ giving $v^2 = 2gh\left(1 - \frac{\mu_k}{\tan\theta}\right)$. As $\theta$ increases (but $h$ remains the same), $\tan\theta$ increases, so the quantity in parentheses increases since $\tan\theta$ is in the denominator. Therefore $v$ increases.

**EVALUATE:** The answer in (c) makes physical sense because with $h$ constant, a larger value for $\theta$ means that the normal force decreases so the magnitude of the friction force also decreases, and therefore friction is less able to oppose the motion of the block as it slides down the slope.

**7.79.** **IDENTIFY:** For a conservative force, mechanical energy is conserved and we can relate the force to its potential energy function.

**SET UP:** $F_x = -dU/dx$.

**EXECUTE:** **(a)** $U + K = E$ = constant. If two points have the same kinetic energy, they must have the same potential energy since the sum of $U$ and $K$ is constant. Since the kinetic energy curve symmetric, the potential energy curve must also be symmetric.

**(b)** At $x = 0$ we can see from the graph with the problem that $E = K + 0 = 0.14$ J. Since $E$ is constant, if $K = 0$ at $x = -1.5$ m, then $U$ must be equal to 0.14 J at that point.

**(c)** $U(x) = E - K(x) = 0.14\text{ J} - K(x)$, so the graph of $U(x)$ is like the sketch in Figure 7.79.

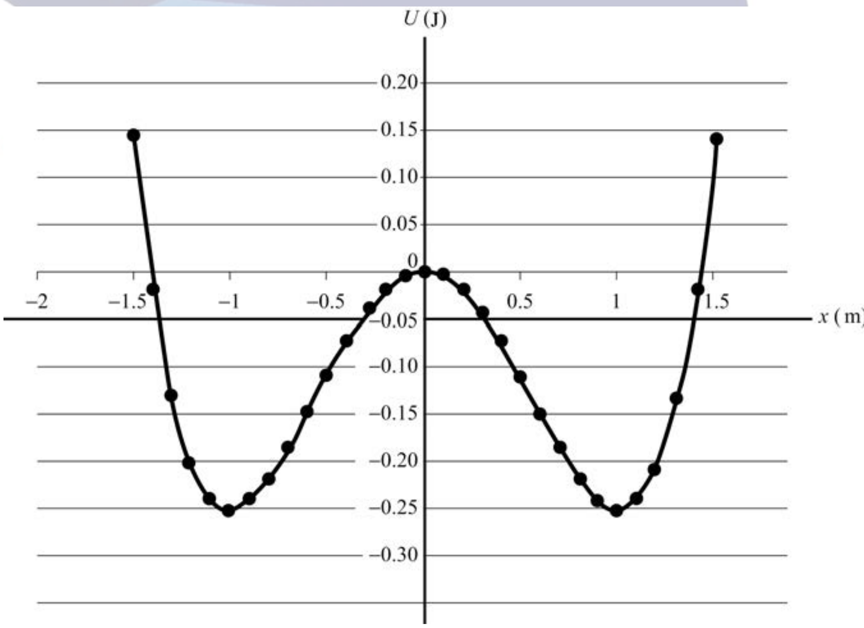



**Figure 7.79**

**(d)** Since $F_x = -dU/dx$, $F(x) = 0$ at $x$ = 0, +1.0 m, and –1.0 m.

**(e)** $F(x)$ is positive when the slope of the $U(x)$ curve is negative, and $F(x)$ is negative when the slope of the $U(x)$ curve is positive. Therefore $F(x)$ is positive between $x = -1.5$ m and $x = -1.0$ m and between $x = 0$ and $x = 1.0$ m. $F(x)$ is negative between $x = -1.0$ m and 0 and between $x = 1.0$ m and $x = 1.5$ m.

**(f)** When released from $x = -1.30$ m, the sphere will move to the right until it reaches $x = -0.55$ m, at which point it has 0.12 J of potential energy, the same as at is original point of release.

**EVALUATE:** Even though we do not have the equation of the kinetic energy function, we can still learn much about the behavior of the system by studying its graph.

**7.80.** **IDENTIFY:** $K = E - U$ determines $v(x)$.

**SET UP:** $v$ is a maximum when $U$ is a minimum and $v$ is a minimum when $U$ is a maximum. $F_x = -dU/dx$. The extreme values of $x$ are where $E = U(x)$.

**EXECUTE:** **(a)** Eliminating $\beta$ in favor of $\alpha$ and $x_0(\beta = \alpha/x_0)$,

$$U(x) = \frac{\alpha}{x^2} - \frac{\beta}{x} = \frac{\alpha}{x_0^2}\frac{x_0^2}{x^2} - \frac{\alpha}{x_0 x} = \frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right)^2 - \left(\frac{x_0}{x}\right)\right].$$

$U(x_0) = \left(\dfrac{\alpha}{x_0^2}\right)(1-1) = 0.$ $U(x)$ is positive for $x < x_0$ and negative for $x > x_0$ ($\alpha$ and $\beta$ must be taken as positive). The graph of $U(x)$ is sketched in Figure 7.80a.

**(b)** $v(x) = \sqrt{-\dfrac{2}{m}U} = \sqrt{\left(\dfrac{2\alpha}{mx_0^2}\right)\left(\left(\dfrac{x_0}{x}\right) - \left(\dfrac{x_0}{x}\right)^2\right)}$. The proton moves in the positive $x$-direction, speeding up until it reaches a maximum speed (see part (c)), and then slows down, although it never stops. The minus sign in the square root in the expression for $v(x)$ indicates that the particle will be found only in the region where $U < 0$, that is, $x > x_0$. The graph of $v(x)$ is sketched in Figure 7.80b.

**(c)** The maximum speed corresponds to the maximum kinetic energy, and hence the minimum potential energy. This minimum occurs when $\dfrac{dU}{dx} = 0$, or $\dfrac{dU}{dx} = \dfrac{\alpha}{x_0}\left[-2\left(\dfrac{x_0}{x}\right)^3 + \left(\dfrac{x_0}{x}\right)^2\right] = 0,$

which has the solution $x = 2x_0$. $U(2x_0) = -\dfrac{\alpha}{4x_0^2}$, so $v = \sqrt{\dfrac{\alpha}{2mx_0^2}}$.

**(d)** The maximum speed occurs at a point where $\dfrac{dU}{dx} = 0$, and since $F_x = -\dfrac{dU}{dx}$, the force at this point is zero.

**(e)** $x_1 = 3x_0$, and $U(3x_0) = -\dfrac{2\alpha}{9x_0^2}$.

$$v(x) = \sqrt{\frac{2}{m}(U(x_1) - U(x))} = \sqrt{\frac{2}{m}\left[\left(\frac{-2}{9}\frac{\alpha}{x_0^2}\right) - \frac{\alpha}{x_0^2}\left(\left(\frac{x_0}{x}\right)^2 - \frac{x_0}{x}\right)\right]} = \sqrt{\frac{2\alpha}{mx_0^2}\left(\left(\frac{x_0}{x}\right) - \left(\frac{x_0}{x}\right)^2 - \frac{2}{9}\right)}.$$

The particle is confined to the region where $U(x) < U(x_1)$. The maximum speed still occurs at $x = 2x_0$, but now the particle will oscillate between $x_1$ and some minimum value (see part (f)).

**(f)** Note that $U(x) - U(x_1)$ can be written as

$$\frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right)^2 - \left(\frac{x_0}{x}\right) + \left(\frac{2}{9}\right)\right] = \frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right) - \frac{1}{3}\right]\left[\left(\frac{x_0}{x}\right) - \frac{2}{3}\right],$$

which is zero (and hence the kinetic energy is zero) at $x = 3x_0 = x_1$ and $x = \frac{3}{2}x_0$. Thus, when the particle is released from $x_0$, it goes on to infinity, and doesn't reach any maximum distance. When released from $x_1$, it oscillates between $\frac{3}{2}x_0$ and $3x_0$.

**EVALUATE:** In each case the proton is released from rest and $E = U(x_i)$, where $x_i$ is the point where it is released. When $x_i = x_0$ the total energy is zero. When $x_i = x_1$ the total energy is negative. $U(x) \to 0$ as $x \to \infty$, so for this case the proton can't reach $x \to \infty$ and the maximum $x$ it can have is limited.

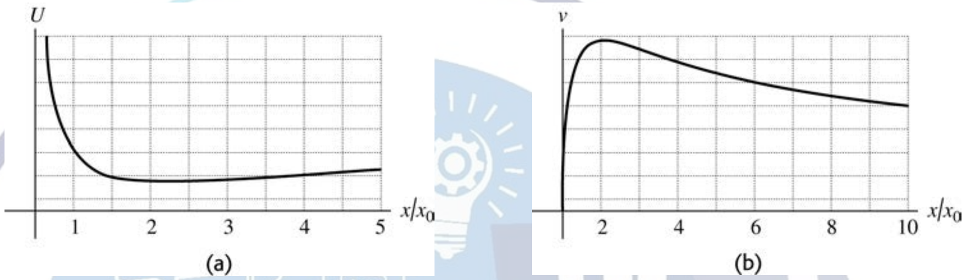



**Figure 7.80**

**7.81.** **IDENTIFY:** We model the DNA molecule as an ideal spring.
**SET UP:** Hooke's law is $F = kx$.
**EXECUTE:** Since $F$ is proportional to $x$, if a 3.0-pN force causes a 0.10-nm deflection, a 6.0-pN force, which is twice as great, should use twice as much deflection, or 0.2 nm. This makes choice (c) correct.
**EVALUATE:** A simple model can give rough but often meaningful insight into the behavior of a complicated system.

**7.82.** **IDENTIFY** and **SET UP:** If a system obeys Hooke's law, a graph of force versus displacement will be a straight line through the origin having positive slope equal to the force constant.
**EXECUTE:** The graph is a straight line. Reading its slope from the graph gives (2.0 pN)/(20 nm) = 0.1 pN/nm, which makes choice (b) correct.
**EVALUATE:** The molecule would obey Hooke's law only over a restricted range of displacements.

**7.83.** **IDENTIFY** and **SET UP:** The energy is the area under the force-displacement curve.
**EXECUTE:** Using the area under the triangular section from 0 to 50 nm, we have
$A = \frac{1}{2}(5.0 \text{ pN})(50 \text{ nm}) = 1.25 \times 10^{-19} \text{ J} \approx 1.2 \times 10^{-19}$ J, which makes choice (b) correct.
**EVALUATE:** This amount of energy is quite small, but recall that this is the energy of a microscopic molecule.

**7.84.** **IDENTIFY** and **SET UP:** $P = Fv$ and at constant speed $x = vt$. The DNA follows Hooke's law, so $F = kx$.
**EXECUTE:** $P = Fv = kxv = k(vt)v = kv^2t$. Since $k$ and $v$ are constant, the power is proportional to the time, so the graph of power versus time should be a straight line through the origin, which fits choice (a).
**EVALUATE:** The power increases with time because the force increases with $x$ and $x$ increases with $t$.

# 8

## MOMENTUM, IMPULSE, AND COLLISIONS

**8.1.** **IDENTIFY** and **SET UP:** $p = mv$. $K = \frac{1}{2}mv^2$.

**EXECUTE:** (a) $p = (10{,}000 \text{ kg})(12.0 \text{ m/s}) = 1.20 \times 10^5 \text{ kg} \cdot \text{m/s}$

**(b)** (i) $v = \dfrac{p}{m} = \dfrac{1.20 \times 10^5 \text{ kg} \cdot \text{m/s}}{2000 \text{ kg}} = 60.0 \text{ m/s}$. (ii) $\frac{1}{2}m_\text{T}v_\text{T}^2 = \frac{1}{2}m_\text{SUV}v_\text{SUV}^2$, so

$$v_\text{SUV} = \sqrt{\frac{m_\text{T}}{m_\text{SUV}}}v_\text{T} = \sqrt{\frac{10{,}000 \text{ kg}}{2000 \text{ kg}}}(12.0 \text{ m/s}) = 26.8 \text{ m/s}$$

**EVALUATE:** The SUV must have less speed to have the same kinetic energy as the truck than to have the same momentum as the truck.

**8.2.** **IDENTIFY:** Each momentum component is the mass times the corresponding velocity component.

**SET UP:** Let $+x$ be along the horizontal motion of the shotput. Let $+y$ be vertically upward. $v_x = v\cos\theta$, $v_y = v\sin\theta$.

**EXECUTE:** The horizontal component of the initial momentum is

$$p_x = mv_x = mv\cos\theta = (7.30 \text{ kg})(15.0 \text{ m/s})\cos 40.0° = 83.9 \text{ kg} \cdot \text{m/s}.$$

The vertical component of the initial momentum is

$$p_y = mv_y = mv\sin\theta = (7.30 \text{ kg})(15.0 \text{ m/s})\sin 40.0° = 70.4 \text{ kg} \cdot \text{m/s}.$$

**EVALUATE:** The initial momentum is directed at $40.0°$ above the horizontal.

**8.3.** **IDENTIFY** and **SET UP:** We use $p = mv$ and add the respective components.

**EXECUTE:** **(a)** $P_x = p_{Ax} + p_{Cx} = 0 + (10.0 \text{ kg})(-3.0 \text{ m/s}) = -30 \text{ kg} \cdot \text{m/s}$

$P_y = p_{Ay} + p_{Cy} = (5.0 \text{ kg})(-11.0 \text{ m/s}) + 0 = -55 \text{ kg} \cdot \text{m/s}$

**(b)** $P_x = p_{Bx} + p_{Cx} = (6.0 \text{ kg})(10.0 \text{ m/s} \cos 60°) + (10.0 \text{ kg})(-3.0 \text{ m/s}) = 0$

$P_y = p_{By} + p_{Cy} = (6.0 \text{ kg})(10.0 \text{ m/s} \sin 60°) + 0 = 52 \text{ kg} \cdot \text{m/s}$

**(c)** $P_x = p_{Ax} + p_{Bx} + p_{Cx} = 0 + (6.0 \text{ kg})(10.0 \text{ m/s} \cos 60°) + (10.0 \text{ kg})(-3.0 \text{ m/s}) = 0$

$P_y = p_{Ay} + p_{By} + p_{Cy} = (5.0 \text{ kg})(-11.0 \text{ m/s}) + (6.0 \text{ kg})(10.0 \text{ m/s} \sin 60°) + 0 = -3.0 \text{ kg} \cdot \text{m/s}$

**EVALUATE:** $A$ has no $x$-component of momentum so $P_x$ is the same in (b) and (c). $C$ has no $y$-component of momentum so $P_y$ in (c) is the sum of $P_y$ in (a) and (b).

**8.4.** **IDENTIFY:** For each object $\vec{p} = m\vec{v}$ and the net momentum of the system is $\vec{P} = \vec{p}_A + \vec{p}_B$. The momentum vectors are added by adding components. The magnitude and direction of the net momentum is calculated from its $x$- and $y$-components.

**SET UP:** Let object $A$ be the pickup and object $B$ be the sedan. $v_{Ax} = -14.0 \text{ m/s}$, $v_{Ay} = 0$. $v_{Bx} = 0$, $v_{By} = +23.0 \text{ m/s}$.

**EXECUTE:** **(a)** $P_x = p_{Ax} + p_{Bx} = m_A v_{Ax} + m_B v_{Bx} = (2500 \text{ kg})(-14.0 \text{ m/s}) + 0 = -3.50 \times 10^4 \text{ kg} \cdot \text{m/s}$

$$P_y = p_{Ay} + p_{By} = m_A v_{Ay} + m_B v_{By} = (1500 \text{ kg})(+23.0 \text{ m/s}) = +3.45 \times 10^4 \text{ kg} \cdot \text{m/s}$$

**(b)** $P = \sqrt{P_x^2 + P_y^2} = 4.91 \times 10^4 \text{ kg} \cdot \text{m/s}$. From Figure 8.4, $\tan\theta = \left|\dfrac{P_x}{P_y}\right| = \dfrac{3.50 \times 10^4 \text{ kg} \cdot \text{m/s}}{3.45 \times 10^4 \text{ kg} \cdot \text{m/s}}$ and $\theta = 45.4°$.

The net momentum has magnitude $4.91 \times 10^4 \text{ kg} \cdot \text{m/s}$ and is directed at $45.4°$ west of north.

**EVALUATE:** The momenta of the two objects must be added as vectors. The momentum of one object is west and the other is north. The momenta of the two objects are nearly equal in magnitude, so the net momentum is directed approximately midway between west and north.

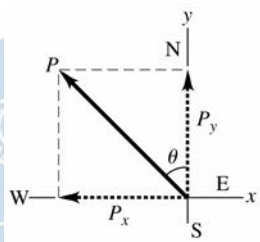



**Figure 8.4**

**8.5.** **IDENTIFY:** For each object, $\vec{p} = m\vec{v}$ and $K = \frac{1}{2}mv^2$. The total momentum is the vector sum of the momenta of each object. The total kinetic energy is the scalar sum of the kinetic energies of each object.

**SET UP:** Let object $A$ be the 110 kg lineman and object $B$ the 125 kg lineman. Let $+x$ be to the right, so $v_{Ax} = +2.75 \text{ m/s}$ and $v_{Bx} = -2.60 \text{ m/s}$.

**EXECUTE:** **(a)** $P_x = m_A v_{Ax} + m_B v_{Bx} = (110 \text{ kg})(2.75 \text{ m/s}) + (125 \text{ kg})(-2.60 \text{ m/s}) = -22.5 \text{ kg} \cdot \text{m/s}$. The net momentum has magnitude $22.5 \text{ kg} \cdot \text{m/s}$ and is directed to the left.

**(b)** $K = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}(110 \text{ kg})(2.75 \text{ m/s})^2 + \frac{1}{2}(125 \text{ kg})(2.60 \text{ m/s})^2 = 838 \text{ J}$

**EVALUATE:** The kinetic energy of an object is a scalar and is never negative. It depends only on the magnitude of the velocity of the object, not on its direction. The momentum of an object is a vector and has both magnitude and direction. When two objects are in motion, their total kinetic energy is greater than the kinetic energy of either one. But if they are moving in opposite directions, the net momentum of the system has a smaller magnitude than the magnitude of the momentum of either object.

**8.6.** **IDENTIFY:** We know the contact time of the ball with the racket, the change in velocity of the ball, and the mass of the ball. From this information we can use the fact that the impulse is equal to the change in momentum to find the force exerted on the ball by the racket.

**SET UP:** $J_x = \Delta p_x$ and $J_x = F_x \Delta t$. In part (a), take the $+x$-direction to be along the final direction of motion of the ball. The initial speed of the ball is zero. In part (b), take the $+x$-direction to be in the direction the ball is traveling before it is hit by the opponent's racket.

**EXECUTE:** **(a)** $J_x = mv_{2x} - mv_{1x} = (57 \times 10^{-3} \text{ kg})(73 \text{ m/s} - 0) = 4.16 \text{ kg} \cdot \text{m/s}$. Using $J_x = F_x \Delta t$ gives

$$F_x = \frac{J_x}{\Delta t} = \frac{4.16 \text{ kg} \cdot \text{m/s}}{30.0 \times 10^{-3} \text{ s}} = 140 \text{ N}.$$

**(b)** $J_x = mv_{2x} - mv_{1x} = (57 \times 10^{-3} \text{ kg})(-55 \text{ m/s} - 73 \text{ m/s}) = -7.30 \text{ kg} \cdot \text{m/s}$.

$$F_x = \frac{J_x}{\Delta t} = \frac{-7.30 \text{ kg} \cdot \text{m/s}}{30.0 \times 10^{-3} \text{ s}} = -240 \text{ N}.$$

**EVALUATE:** The signs of $J_x$ and $F_x$ show their direction. $140 \text{ N} = 31 \text{ lb}$. This very attainable force has a large effect on the light ball. 140 N is 250 times the weight of the ball.

**8.7.** **IDENTIFY:** The average force on an object and the object's change in momentum are related by $(F_{\text{av}})_x = \dfrac{J_x}{\Delta t}$. The weight of the ball is $w = mg$.

**SET UP:** Let $+x$ be in the direction of the final velocity of the ball, so $v_{1x} = 0$ and $v_{2x} = 25.0$ m/s.

**EXECUTE:** $(F_{av})_x(t_2 - t_1) = mv_{2x} - mv_{1x}$ gives $(F_{av})_x = \dfrac{mv_{2x} - mv_{1x}}{t_2 - t_1} = \dfrac{(0.0450 \text{ kg})(25.0 \text{ m/s})}{2.00\times10^{-3} \text{ s}} = 562 \text{ N}.$

$w = (0.0450 \text{ kg})(9.80 \text{ m/s}^2) = 0.441$ N. The force exerted by the club is much greater than the weight of the ball, so the effect of the weight of the ball during the time of contact is not significant.

**EVALUATE:** Forces exerted during collisions typically are very large but act for a short time.

**8.8.** **IDENTIFY:** The change in momentum, the impulse, and the average force are related by $J_x = \Delta p_x$ and $(F_{av})_x = \dfrac{J_x}{\Delta t}.$

**SET UP:** Let the direction in which the batted ball is traveling be the $+x$-direction, so $v_{1x} = -45.0$ m/s and $v_{2x} = 55.0$ m/s.

**EXECUTE:** **(a)** $\Delta p_x = p_{2x} - p_{1x} = m(v_{2x} - v_{1x}) = (0.145 \text{ kg})[55.0 \text{ m/s} - (-45.0 \text{ m/s})] = 14.5 \text{ kg}\cdot\text{m/s}.$

$J_x = \Delta p_x$, so $J_x = 14.5 \text{ kg}\cdot\text{m/s}$. Both the change in momentum and the impulse have magnitude $14.5 \text{ kg}\cdot\text{m/s}$.

**(b)** $(F_{av})_x = \dfrac{J_x}{\Delta t} = \dfrac{14.5 \text{ kg}\cdot\text{m/s}}{2.00\times10^{-3} \text{ s}} = 7250 \text{ N}.$

**EVALUATE:** The force is in the direction of the momentum change.

**8.9.** **IDENTIFY:** Use $J_x = p_{2x} - p_{1x}$. We know the initial momentum and the impulse so can solve for the final momentum and then the final velocity.

**SET UP:** Take the $x$-axis to be toward the right, so $v_{1x} = +3.00$ m/s. Use $J_x = F_x\Delta t$ to calculate the impulse, since the force is constant.

**EXECUTE:** **(a)** $J_x = p_{2x} - p_{1x}$

$$J_x = F_x(t_2 - t_1) = (+25.0 \text{ N})(0.050 \text{ s}) = +1.25 \text{ kg}\cdot\text{m/s}$$

Thus $p_{2x} = J_x + p_{1x} = +1.25 \text{ kg}\cdot\text{m/s} + (0.160 \text{ kg})(+3.00 \text{ m/s}) = +1.73 \text{ kg}\cdot\text{m/s}$

$$v_{2x} = \frac{p_{2x}}{m} = \frac{1.73 \text{ kg}\cdot\text{m/s}}{0.160 \text{ kg}} = +10.8 \text{ m/s (to the right)}$$

**(b)** $J_x = F_x(t_2 - t_1) = (-12.0 \text{ N})(0.050 \text{ s}) = -0.600 \text{ kg}\cdot\text{m/s}$ (negative since force is to left)

$$p_{2x} = J_x + p_{1x} = -0.600 \text{ kg}\cdot\text{m/s} + (0.160 \text{ kg})(+3.00 \text{ m/s}) = -0.120 \text{ kg}\cdot\text{m/s}$$

$$v_{2x} = \frac{p_{2x}}{m} = \frac{-0.120 \text{ kg}\cdot\text{m/s}}{0.160 \text{ kg}} = -0.75 \text{ m/s (to the left)}$$

**EVALUATE:** In part (a) the impulse and initial momentum are in the same direction and $v_x$ increases. In part (b) the impulse and initial momentum are in opposite directions and the velocity decreases.

**8.10.** **IDENTIFY:** Apply $J_x = \Delta p_x = mv_{2x} - mv_{1x}$ and $J_y = \Delta p_y = mv_{2y} - mv_{1y}$ to relate the change in momentum to the components of the average force on it.

**SET UP:** Let $+x$ be to the right and $+y$ be upward.

**EXECUTE:** $J_x = \Delta p_x = mv_{2x} - mv_{1x} = (0.145 \text{ kg})[-(52.0 \text{ m/s})\cos 30° - 40.0 \text{ m/s}] = -12.33 \text{ kg}\cdot\text{m/s}.$

$J_y = \Delta p_y = mv_{2y} - mv_{1y} = (0.145 \text{ kg})[(52.0 \text{ m/s})\sin 30° - 0] = 3.770 \text{ kg}\cdot\text{m/s}.$

The horizontal component is $12.33 \text{ kg}\cdot\text{m/s}$, to the left and the vertical component is $3.770 \text{ kg}\cdot\text{m/s}$, upward.

$F_{\text{av-}x} = \dfrac{J_x}{\Delta t} = \dfrac{-12.33 \text{ kg}\cdot\text{m/s}}{1.75\times10^{-3} \text{ s}} = -7050 \text{ N}.$ $F_{\text{av-}y} = \dfrac{J_y}{\Delta t} = \dfrac{3.770 \text{ kg}\cdot\text{m/s}}{1.75\times10^{-3} \text{ s}} = 2150 \text{ N}.$

The horizontal component is 7050 N, to the left, and the vertical component is 2150 N, upward.

**EVALUATE:** The ball gains momentum to the left and upward and the force components are in these directions.

**8.11.** **IDENTIFY:** The force is not constant so $\vec{J} = \int_{t_1}^{t_2} \vec{F} dt$. The impulse is related to the change in velocity by $J_x = m(v_{2x} - v_{1x})$.

**SET UP:** Only the $x$-component of the force is nonzero, so $J_x = \int_{t_1}^{t_2} F_x dt$ is the only nonzero component of $\vec{J}$. $J_x = m(v_{2x} - v_{1x})$. $t_1 = 2.00$ s, $t_2 = 3.50$ s.

**EXECUTE:** **(a)** $A = \frac{F_x}{t^2} = \frac{781.25\text{ N}}{(1.25\text{ s})^2} = 500\text{ N/s}^2$.

**(b)** $J_x = \int_{t_1}^{t_2} At^2 dt = \frac{1}{3}A(t_2^3 - t_1^3) = \frac{1}{3}(500\text{ N/s}^2)([3.50\text{ s}]^3 - [2.00\text{ s}]^3) = 5.81\times 10^3\text{ N}\cdot\text{s}$.

**(c)** $\Delta v_x = v_{2x} - v_{1x} = \frac{J_x}{m} = \frac{5.81\times 10^3\text{ N}\cdot\text{s}}{2150\text{ kg}} = 2.70\text{ m/s}$. The $x$-component of the velocity of the rocket increases by 2.70 m/s.

**EVALUATE:** The change in velocity is in the same direction as the impulse, which in turn is in the direction of the net force. In this problem the net force equals the force applied by the engine, since that is the only force on the rocket.

**8.12.** **IDENTIFY:** The force imparts an impulse to the forehead, which changes the momentum of the skater.

**SET UP:** $J_x = \Delta p_x$ and $J_x = F_x \Delta t$. With $A = 1.5\times 10^{-4}\text{ m}^2$, the maximum force without breaking the bone is $(1.5\times 10^{-4}\text{ m}^2)(1.03\times 10^8\text{ N/m}^2) = 1.5\times 10^4\text{ N}$. Set the magnitude of the average force $F_{\text{av}}$ during the collision equal to this value. Use coordinates where $+x$ is in his initial direction of motion. $F_x$ is opposite to this direction, so $F_x = -1.5\times 10^4\text{ N}$.

**EXECUTE:** $J_x = F_x\,\Delta t = (-1.5\times 10^4\text{ N})(10.0\times 10^{-3}\text{ s}) = -150.0\text{ N}\cdot\text{s}$. $J_x = mx_{2x} - mx_{1x}$ and $v_{2x} = 0$. $v_{1x} = -\frac{J_x}{m} = -\frac{-150\text{ N}\cdot\text{s}}{70\text{ kg}} = 2.1\text{ m/s}$.

**EVALUATE:** This speed is about the same as a jog. However, in most cases the skater would not be completely stopped, so in that case a greater speed would not result in injury.

**8.13.** **IDENTIFY:** The force is constant during the 1.0 ms interval that it acts, so $\vec{J} = \vec{F}\Delta t$. $\vec{J} = \vec{p}_2 - \vec{p}_1 = m(\vec{v}_2 - \vec{v}_1)$.

**SET UP:** Let $+x$ be to the right, so $v_{1x} = +5.00\text{ m/s}$. Only the $x$-component of $\vec{J}$ is nonzero, and $J_x = m(v_{2x} - v_{1x})$.

**EXECUTE:** **(a)** The magnitude of the impulse is $J = F\Delta t = (2.50\times 10^3\text{ N})(1.00\times 10^{-3}\text{ s}) = 2.50\text{ N}\cdot\text{s}$. The direction of the impulse is the direction of the force.

**(b)** (i) $v_{2x} = \frac{J_x}{m} + v_{1x}$. $J_x = +2.50\text{ N}\cdot\text{s}$. $v_{2x} = \frac{+2.50\text{ N}\cdot\text{s}}{2.00\text{ kg}} + 5.00\text{ m/s} = 6.25\text{ m/s}$. The stone's velocity has magnitude 6.25 m/s and is directed to the right. (ii) Now $J_x = -2.50\text{ N}\cdot\text{s}$ and $v_{2x} = \frac{-2.50\text{ N}\cdot\text{s}}{2.00\text{ kg}} + 5.00\text{ m/s} = 3.75\text{ m/s}$. The stone's velocity has magnitude 3.75 m/s and is directed to the right.

**EVALUATE:** When the force and initial velocity are in the same direction the speed increases, and when they are in opposite directions the speed decreases.

**8.14.** **IDENTIFY:** We know the force acting on a box as a function of time and its initial momentum and want to find its momentum at a later time. The target variable is the final momentum.

**SET UP:** Use $\int_{t_1}^{t_2} \vec{F}(t)dt = \vec{p}_2 - \vec{p}_1$ to find $\vec{p}_2$ since we know $\vec{p}_1$ and $\vec{F}(t)$.

**EXECUTE:** $\vec{p}_1 = (-3.00\ \text{kg}\cdot\text{m/s})\hat{i} + (4.00\ \text{kg}\cdot\text{m/s})\hat{j}$ at $t_1 = 0$, and $t_2 = 2.00$ s. Work with the components of the force and momentum. $\int_{t_1}^{t_2} F_x(t)dt = (0.280\ \text{N/s})\int_{t_1}^{t_2} t\,dt = (0.140\ \text{N/s})t_2^2 = 0.560\ \text{N}\cdot\text{s}$

$p_{2x} = p_{1x} + 0.560\ \text{N}\cdot\text{s} = -3.00\ \text{kg}\cdot\text{m/s} + 0.560\ \text{N}\cdot\text{s} = -2.44\ \text{kg}\cdot\text{m/s}$.

$\int_{t_1}^{t_2} F_y(t)dt = (-0.450\ \text{N/s}^2)\int_{t_1}^{t_2} t^2 dt = (-0.150\ \text{N/s}^2)t_2^3 = -1.20\ \text{N}\cdot\text{s}$.

$p_{2y} = p_{1y} + (-1.20\ \text{N}\cdot\text{s}) = 4.00\ \text{kg}\cdot\text{m/s} + (-1.20\ \text{N}\cdot\text{s}) = +2.80\ \text{kg}\cdot\text{m/s}$. So

$\vec{p}_2 = (-2.44\ \text{kg}\cdot\text{m/s})\hat{i} + (2.80\ \text{kg}\cdot\text{m/s})\hat{j}$

**EVALUATE:** Since the given force has $x$- and $y$-components, it changes both components of the box's momentum.

**8.15.** **IDENTIFY:** The player imparts an impulse to the ball which gives it momentum, causing it to go upward.

**SET UP:** Take $+y$ to be upward. Use the motion of the ball after it leaves the racket to find its speed just after it is hit. After it leaves the racket $a_y = -g$. At the maximum height $v_y = 0$. Use $J_y = \Delta p_y$ and the kinematics equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ for constant acceleration.

**EXECUTE:** $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80\ \text{m/s}^2)(5.50\ \text{m})} = 10.4\ \text{m/s}$.

For the interaction with the racket $v_{1y} = 0$ and $v_{2y} = 10.4$ m/s.

$J_y = mv_{2y} - mv_{1y} = (57\times10^{-3}\ \text{kg})(10.4\ \text{m/s} - 0) = 0.593\ \text{kg}\cdot\text{m/s}$.

**EVALUATE:** We could have found the initial velocity using energy conservation instead of free-fall kinematics.

**8.16.** **IDENTIFY:** Apply conservation of momentum to the system of the astronaut and tool.

**SET UP:** Let $A$ be the astronaut and $B$ be the tool. Let $+x$ be the direction in which she throws the tool, so $v_{B2x} = +3.20$ m/s. Assume she is initially at rest, so $v_{A1x} = v_{B1x} = 0$. Solve for $v_{A2x}$.

**EXECUTE:** $P_{1x} = P_{2x}$. $P_{1x} = m_A v_{A1x} + m_B v_{B1x} = 0$. $P_{2x} = m_A v_{A2x} + m_B v_{B2x} = 0$ and

$v_{A2x} = -\dfrac{m_B v_{A2x}}{m_A} = -\dfrac{(2.25\ \text{kg})(3.20\ \text{m/s})}{68.5\ \text{kg}} = -0.105$ m/s. Her speed is 0.105 m/s and she moves opposite to the direction in which she throws the tool.

**EVALUATE:** Her mass is much larger than that of the tool, so to have the same magnitude of momentum as the tool her speed is much less.

**8.17.** **IDENTIFY:** Since the rifle is loosely held there is no net external force on the system consisting of the rifle, bullet, and propellant gases and the momentum of this system is conserved. Before the rifle is fired everything in the system is at rest and the initial momentum of the system is zero.

**SET UP:** Let $+x$ be in the direction of the bullet's motion. The bullet has speed $601\ \text{m/s} - 1.85\ \text{m/s} = 599\ \text{m/s}$ relative to the earth. $P_{2x} = p_{\text{rx}} + p_{\text{bx}} + p_{\text{gx}}$, the momenta of the rifle, bullet, and gases. $v_{\text{rx}} = -1.85$ m/s and $v_{\text{bx}} = +599$ m/s.

**EXECUTE:** $P_{2x} = P_{1x} = 0$. $p_{\text{rx}} + p_{\text{bx}} + p_{\text{gx}} = 0$.

$p_{\text{gx}} = -p_{\text{rx}} - p_{\text{bx}} = -(2.80\ \text{kg})(-1.85\ \text{m/s}) - (0.00720\ \text{kg})(599\ \text{m/s})$ and

$p_{\text{gx}} = +5.18\ \text{kg}\cdot\text{m/s} - 4.31\ \text{kg}\cdot\text{m/s} = 0.87\ \text{kg}\cdot\text{m/s}$. The propellant gases have momentum $0.87\ \text{kg}\cdot\text{m/s}$, in the same direction as the bullet is traveling.

**EVALUATE:** The magnitude of the momentum of the recoiling rifle equals the magnitude of the momentum of the bullet plus that of the gases as both exit the muzzle.

**8.18.** **IDENTIFY:** The total momentum of the two skaters is conserved, but not their kinetic energy.

**SET UP:** There is no horizontal external force so, $P_{\text{i},x} = P_{\text{f},x}$, $p = mv$, $K = ½\, mv^2$.

**EXECUTE:** **(a)** $P_{\text{i},x} = P_{\text{f},x}$. The skaters are initially at rest so $P_{\text{i},x} = 0$. $0 = m_A(v_{A,\text{f}})_x + m_B(v_{B,\text{f}})_x$

$$(v_{A,\text{f}})_x = -\frac{m_B(v_{B,\text{f}})_x}{m_A} = -\frac{(74.0\text{ kg})(1.50\text{ m/s})}{63.8\text{ kg}} = -1.74\text{ m/s}.$$ The lighter skater travels to the left at 1.74 m/s.

**(b)** $K_\text{i} = 0$. $K_\text{f} = \frac{1}{2}m_A v_{A,\text{f}}{}^2 + \frac{1}{2}m_B v_{B,\text{f}}{}^2 = \frac{1}{2}(63.8\text{ kg})(1.74\text{ m/s})^2 + \frac{1}{2}(74.0\text{ kg})(1.50\text{ m/s})^2 = 180\text{ J}$.

**EVALUATE:** The kinetic energy of the system was produced by the work the two skaters do on each other.

**8.19.** **IDENTIFY:** Since drag effects are neglected, there is no net external force on the system of squid plus expelled water, and the total momentum of the system is conserved. Since the squid is initially at rest, with the water in its cavity, the initial momentum of the system is zero. For each object, $K = \frac{1}{2}mv^2$.

**SET UP:** Let $A$ be the squid and $B$ be the water it expels, so $m_A = 6.50\text{ kg} - 1.75\text{ kg} = 4.75\text{ kg}$. Let $+x$ be the direction in which the water is expelled. $v_{A2x} = -2.50\text{ m/s}$. Solve for $v_{B2x}$.

**EXECUTE:** **(a)** $P_{1x} = 0$. $P_{2x} = P_{1x}$, so $0 = m_A v_{A2x} + m_B v_{B2x}$.

$$v_{B2x} = -\frac{m_A v_{A2x}}{m_B} = -\frac{(4.75\text{ kg})(-2.50\text{ m/s})}{1.75\text{ kg}} = +6.79\text{ m/s}.$$

**(b)** $K_2 = K_{A2} + K_{B2} = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2 = \frac{1}{2}(4.75\text{ kg})(2.50\text{ m/s})^2 + \frac{1}{2}(1.75\text{ kg})(6.79\text{ m/s})^2 = 55.2\text{ J}$. The initial kinetic energy is zero, so the kinetic energy produced is $K_2 = 55.2\text{ J}$.

**EVALUATE:** The two objects end up with momenta that are equal in magnitude and opposite in direction, so the total momentum of the system remains zero. The kinetic energy is created by the work done by the squid as it expels the water.

**8.20.** **IDENTIFY:** Apply conservation of momentum to the system of you and the ball. In part (a) both objects have the same final velocity.

**SET UP:** Let $+x$ be in the direction the ball is traveling initially. $m_A = 0.600\text{ kg}$ (ball). $m_B = 70.0\text{ kg}$ (you).

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$ gives $(0.600\text{ kg})(10.0\text{ m/s}) = (0.600\text{ kg} + 70.0\text{ kg})v_2$ so $v_2 = 0.0850\text{ m/s}$.

**(b)** $P_{1x} = P_{2x}$ gives $(0.600\text{ kg})(10.0\text{ m/s}) = (0.600\text{ kg})(-8.00\text{ m/s}) + (70.0\text{ kg})v_{B2}$ so $v_{B2} = 0.154\text{ m/s}$.

**EVALUATE:** When the ball bounces off it has a greater change in momentum and you acquire a greater final speed.

**8.21.** **IDENTIFY:** Apply conservation of momentum to the system of the two pucks.

**SET UP:** Let $+x$ be to the right.

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$ says $(0.250\text{ kg})v_{A1} = (0.250\text{ kg})(-0.120\text{ m/s}) + (0.350\text{ kg})(0.650\text{ m/s})$ and $v_{A1} = 0.790\text{ m/s}$.

**(b)** $K_1 = \frac{1}{2}(0.250\text{ kg})(0.790\text{ m/s})^2 = 0.0780\text{ J}$.

$K_2 = \frac{1}{2}(0.250\text{ kg})(0.120\text{ m/s})^2 + \frac{1}{2}(0.350\text{ kg})(0.650\text{ m/s})^2 = 0.0757\text{ J}$ and $\Delta K = K_2 - K_1 = -0.0023\text{ J}$.

**EVALUATE:** The total momentum of the system is conserved but the total kinetic energy decreases.

**8.22.** **IDENTIFY:** Since road friction is neglected, there is no net external force on the system of the two cars and the total momentum of the system is conserved. For each object, $K = \frac{1}{2}mv^2$.

**SET UP:** Let $A$ be the 1750 kg car and $B$ be the 1450 kg car. Let $+x$ be to the right, so $v_{A1x} = +1.50\text{ m/s}$, $v_{B1x} = -1.10\text{ m/s}$, and $v_{A2x} = +0.250\text{ m/s}$. Solve for $v_{B2x}$.

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$. $v_{B2x} = \dfrac{m_A v_{A1x} + m_B v_{B1x} - m_A v_{A2x}}{m_B}$.

$$v_{B2x} = \frac{(1750\text{ kg})(1.50\text{ m/s}) + (1450\text{ kg})(-1.10\text{ m/s}) - (1750\text{ kg})(0.250\text{ m/s})}{1450\text{ kg}} = 0.409\text{ m/s}.$$

After the collision the lighter car is moving to the right with a speed of 0.409 m/s.

**(b)** $K_1 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}(1750\text{ kg})(1.50\text{ m/s})^2 + \frac{1}{2}(1450\text{ kg})(1.10\text{ m/s})^2 = 2846\text{ J}$.

$$K_2 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2 = \frac{1}{2}(1750\text{ kg})(0.250\text{ m/s})^2 + \frac{1}{2}(1450\text{ kg})(0.409\text{ m/s})^2 = 176\text{ J}.$$

The change in kinetic energy is $\Delta K = K_2 - K_1 = 176\text{ J} - 2846\text{ J} = -2670\text{ J}$.

**EVALUATE:** The total momentum of the system is constant because there is no net external force during the collision. The kinetic energy of the system decreases because of negative work done by the forces the cars exert on each other during the collision.

**8.23.** **IDENTIFY:** The momentum and the mechanical energy of the system are both conserved. The mechanical energy consists of the kinetic energy of the masses and the elastic potential energy of the spring. The potential energy stored in the spring is transformed into the kinetic energy of the two masses.

**SET UP:** Let the system be the two masses and the spring. The system is sketched in Figure 8.23, in its initial and final situations. Use coordinates where $+x$ is to the right. Call the masses *A* and *B*.

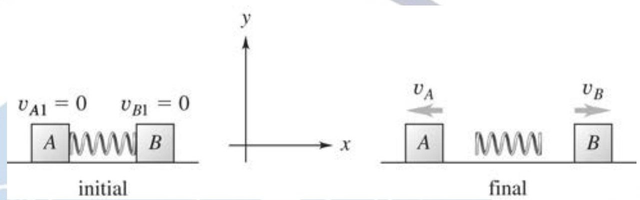



**Figure 8.23**

**EXECUTE:** $P_{1x} = P_{2x}$ so $0 = (0.900\text{ kg})(-v_A) + (0.900\text{ kg})(v_B)$ and, since the masses are equal, $v_A = v_B$. Energy conservation says the potential energy originally stored in the spring is all converted into kinetic energy of the masses, so $\frac{1}{2}kx_1^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2$. Since $v_A = v_B$, this equation gives

$$v_A = x_1\sqrt{\frac{k}{2m}} = (0.200\text{ m})\sqrt{\frac{175\text{ N/m}}{2(0.900\text{ kg})}} = 1.97\text{ m/s}.$$

**EVALUATE:** If the objects have different masses they will end up with different speeds. The lighter one will have the greater speed, since they end up with equal magnitudes of momentum.

**8.24.** **IDENTIFY:** In part (a) no horizontal force implies $P_x$ is constant. In part (b) use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to find the potential energy initially in the spring.

**SET UP:** Initially both blocks are at rest.

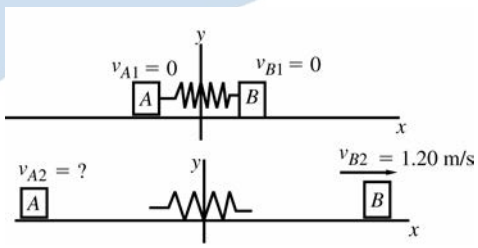



**Figure 8.24**

**EXECUTE:** **(a)** $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$

$$0 = m_A v_{A2x} + m_B v_{B2x}$$

$$v_{A2x} = -\left(\frac{m_B}{m_A}\right)v_{B2x} = -\left(\frac{3.00\text{ kg}}{1.00\text{ kg}}\right)(+1.20\text{ m/s}) = -3.60\text{ m/s}$$

Block *A* has a final speed of 3.60 m/s, and moves off in the opposite direction to *B*.

**(b)** Use energy conservation: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

Only the spring force does work so $W_{\text{other}} = 0$ and $U = U_{\text{el}}$.

$K_1 = 0$ (the blocks initially are at rest)

$U_2 = 0$ (no potential energy is left in the spring)

$$K_2 = \tfrac{1}{2}m_A v_{A2}^2 + \tfrac{1}{2}m_B v_{B2}^2 = \tfrac{1}{2}(1.00\text{ kg})(3.60\text{ m/s})^2 + \tfrac{1}{2}(3.00\text{ kg})(1.20\text{ m/s})^2 = 8.64\text{ J}$$

$U_1 = U_{1,\text{el}}$ the potential energy stored in the compressed spring.

Thus $U_{1,\text{el}} = K_2 = 8.64$ J.

**EVALUATE:** The blocks have equal and opposite momenta as they move apart, since the total momentum is zero. The kinetic energy of each block is positive and doesn't depend on the direction of the block's velocity, just on its magnitude.

**8.25.** **IDENTIFY:** Since friction at the pond surface is neglected, there is no net external horizontal force, and the horizontal component of the momentum of the system of hunter plus bullet is conserved. Both objects are initially at rest, so the initial momentum of the system is zero. Gravity and the normal force exerted by the ice together produce a net vertical force while the rifle is firing, so the vertical component of momentum is not conserved.

**SET UP:** Let object $A$ be the hunter and object $B$ be the bullet. Let $+x$ be the direction of the horizontal component of velocity of the bullet. Solve for $v_{A2x}$.

**EXECUTE:** **(a)** $v_{B2x} = +965$ m/s. $P_{1x} = P_{2x} = 0$. $0 = m_A v_{A2x} + m_B v_{B2x}$ and

$$v_{A2x} = -\frac{m_B}{m_A}v_{B2x} = -\left(\frac{4.20\times10^{-3}\text{ kg}}{72.5\text{ kg}}\right)(965\text{ m/s}) = -0.0559\text{ m/s}.$$

**(b)** $v_{B2x} = v_{B2}\cos\theta = (965\text{ m/s})\cos 56.0° = 540$ m/s. $v_{A2x} = -\left(\dfrac{4.20\times10^{-3}\text{ kg}}{72.5\text{ kg}}\right)(540\text{ m/s}) = -0.0313$ m/s.

**EVALUATE:** The mass of the bullet is much less than the mass of the hunter, so the final mass of the hunter plus gun is still 72.5 kg, to three significant figures. Since the hunter has much larger mass, his final speed is much less than the speed of the bullet.

**8.26.** **IDENTIFY:** Assume the nucleus is initially at rest. $K = \tfrac{1}{2}mv^2$.

**SET UP:** Let $+x$ be to the right. $v_{A2x} = -v_A$ and $v_{B2x} = +v_B$.

**EXECUTE:** **(a)** $P_{2x} = P_{1x} = 0$ gives $m_A v_{A2x} + m_B v_{B2x} = 0$. $v_B = \left(\dfrac{m_A}{m_B}\right)v_A$.

**(b)** $\dfrac{K_A}{K_B} = \dfrac{\tfrac{1}{2}m_A v_A^2}{\tfrac{1}{2}m_B v_B^2} = \dfrac{m_A v_A^2}{m_B(m_A v_A/m_B)^2} = \dfrac{m_B}{m_A}$.

**EVALUATE:** The lighter fragment has the greater kinetic energy.

**8.27.** **IDENTIFY:** Each horizontal component of momentum is conserved. $K = \tfrac{1}{2}mv^2$.

**SET UP:** Let $+x$ be the direction of Rebecca's initial velocity and let the $+y$ axis make an angle of 36.9° with respect to the direction of her final velocity. $v_{\text{D}1x} = v_{\text{D}1y} = 0$. $v_{\text{R}1x} = 13.0$ m/s; $v_{\text{R}1y} = 0$. $v_{\text{R}2x} = (8.00\text{ m/s})\cos 53.1° = 4.80$ m/s; $v_{\text{R}2y} = (8.00\text{ m/s})\sin 53.1° = 6.40$ m/s. Solve for $v_{\text{D}2x}$ and $v_{\text{D}2y}$.

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$ gives $m_\text{R} v_{\text{R}1x} = m_\text{R} v_{\text{R}2x} + m_\text{D} v_{\text{D}2x}$.

$$v_{\text{D}2x} = \frac{m_\text{R}(v_{\text{R}1x} - v_{\text{R}2x})}{m_\text{D}} = \frac{(45.0\text{ kg})(13.0\text{ m/s} - 4.80\text{ m/s})}{65.0\text{ kg}} = 5.68\text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $0 = m_\text{R} v_{\text{R}2y} + m_\text{D} v_{\text{D}2y}$. $v_{\text{D}2y} = -\dfrac{m_\text{R}}{m_\text{D}}v_{\text{R}2y} = -\left(\dfrac{45.0\text{ kg}}{65.0\text{ kg}}\right)(6.40\text{ m/s}) = -4.43$ m/s.

The directions of $\vec{v}_{\text{R}1}$, $\vec{v}_{\text{R}2}$ and $\vec{v}_{\text{D}2}$ are sketched in Figure 8.27. $\tan\theta = \left|\dfrac{v_{\text{D}2y}}{v_{\text{D}2x}}\right| = \dfrac{4.43\text{ m/s}}{5.68\text{ m/s}}$ and

$\theta = 38.0°$. $v_\text{D} = \sqrt{v_{\text{D}2x}^2 + v_{\text{D}2y}^2} = 7.20$ m/s.

**(b)** $K_1 = \tfrac{1}{2}m_\text{R} v_{\text{R}1}^2 = \tfrac{1}{2}(45.0\text{ kg})(13.0\text{ m/s})^2 = 3.80\times10^3$ J.

$K_2 = \tfrac{1}{2}m_\text{R} v_{\text{R}2}^2 + \tfrac{1}{2}m_\text{D} v_{\text{D}2}^2 = \tfrac{1}{2}(45.0\text{ kg})(8.00\text{ m/s})^2 + \tfrac{1}{2}(65.0\text{ kg})(7.20\text{ m/s})^2 = 3.12\times10^3$ J.

$\Delta K = K_2 - K_1 = -680\ \text{J}.$

**EVALUATE:** Each component of momentum is separately conserved. The kinetic energy of the system decreases.

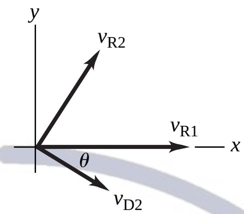



**Figure 8.27**

**8.28.** **IDENTIFY** and **SET UP:** Let the $+x$-direction be horizontal, along the direction the rock is thrown. There is no net horizontal force, so $P_x$ is constant. Let object $A$ be you and object $B$ be the rock.

**EXECUTE:** $0 = -m_A v_A + m_B v_B \cos 35.0°$ gives $v_A = \dfrac{m_B v_B \cos 35.0°}{m_A} = 0.421\ \text{m/s}.$

**EVALUATE:** $P_y$ is not conserved because there is a net external force in the vertical direction; as you throw the rock the normal force exerted on you by the ice is larger than the total weight of the system.

**8.29.** **IDENTIFY:** In the absence of a horizontal force, we know that momentum is conserved.

**SET UP:** $p = mv$. Let $+x$ be the direction you are moving. Before you catch it, the flour sack has no momentum along the $x$-axis. The total mass of you and your skateboard is 60 kg. You, the skateboard, and the flour sack are all moving with the same velocity, after the catch.

**EXECUTE:** **(a)** Since $P_{i,x} = P_{f,x}$, we have $(60\ \text{kg})(4.5\ \text{m/s}) = (62.5\ \text{kg})\upsilon_{f,x}$. Solving for the final velocity we obtain $\upsilon_{f,x} = 4.3\ \text{m/s}$.

**(b)** To bring the flour sack up to your speed, you must exert a horizontal force on it. Consequently, it exerts an equal and opposite force on you, which slows you down.

**(c)** Since you exert a vertical force on the flour sack, your horizontal speed does not change and remains at 4.3 m/s. Since the flour sack is only accelerated in the vertical direction, its horizontal velocity-component remains at 4.3 m/s as well.

**EVALUATE:** Unless you or the flour sack are deflected by an outside force, you will need to be ready to catch the flour sack as it returns to your arms!

**8.30.** **IDENTIFY:** There is no net external force on the system of astronaut plus canister, so the momentum of the system is conserved.

**SET UP:** Let object $A$ be the astronaut and object $B$ be the canister. Assume the astronaut is initially at rest. After the collision she must be moving in the same direction as the canister. Let $+x$ be the direction in which the canister is traveling initially, so $v_{A1x} = 0$, $v_{A2x} = +2.40\ \text{m/s}$, $v_{B1x} = +3.50\ \text{m/s}$, and $v_{B2x} = +1.20\ \text{m/s}$. Solve for $m_B$.

**EXECUTE:** $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$.

$$m_B = \frac{m_A(v_{A2x} - v_{A1x})}{v_{B1x} - v_{B2x}} = \frac{(78.4\ \text{kg})(2.40\ \text{m/s} - 0)}{3.50\ \text{m/s} - 1.20\ \text{m/s}} = 81.8\ \text{kg}.$$

**EVALUATE:** She must exert a force on the canister in the $-x$-direction to reduce its velocity component in the $+x$-direction. By Newton's third law, the canister exerts a force on her that is in the $+x$-direction and she gains velocity in that direction.

**8.31.** **IDENTIFY:** The $x$- and $y$-components of the momentum of the system of the two asteroids are separately conserved.

**SET UP:** The before and after diagrams are given in Figure 8.31 (next page) and the choice of coordinates is indicated. Each asteroid has mass $m$.

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$ gives $mv_{A1} = mv_{A2}\cos 30.0° + mv_{B2}\cos 45.0°$. $40.0\ \text{m/s} = 0.866 v_{A2} + 0.707 v_{B2}$ and $0.707 v_{B2} = 40.0\ \text{m/s} - 0.866 v_{A2}$.

$$P_{2y} = P_{2y} \text{ gives } 0 = mv_{A2}\sin 30.0° - mv_{B2}\sin 45.0° \text{ and } 0.500v_{A2} = 0.707v_{B2}.$$

Combining these two equations gives $0.500v_{A2} = 40.0\text{ m/s} - 0.866v_{A2}$ and $v_{A2} = 29.3$ m/s. Then

$$v_{B2} = \left(\frac{0.500}{0.707}\right)(29.3\text{ m/s}) = 20.7\text{ m/s}.$$

**(b)** $K_1 = \frac{1}{2}mv_{A1}^2$. $K_2 = \frac{1}{2}mv_{A2}^2 + \frac{1}{2}mv_{B2}^2$. $\dfrac{K_2}{K_1} = \dfrac{v_{A2}^2 + v_{B2}^2}{v_{A1}^2} = \dfrac{(29.3\text{ m/s})^2 + (20.7\text{ m/s})^2}{(40.0\text{ m/s})^2} = 0.804.$

$$\frac{\Delta K}{K_1} = \frac{K_2 - K_1}{K_1} = \frac{K_2}{K_1} - 1 = -0.196.$$

19.6% of the original kinetic energy is dissipated during the collision.

**EVALUATE:** We could use any directions we wish for the $x$- and $y$-coordinate directions, but the particular choice we have made is especially convenient.

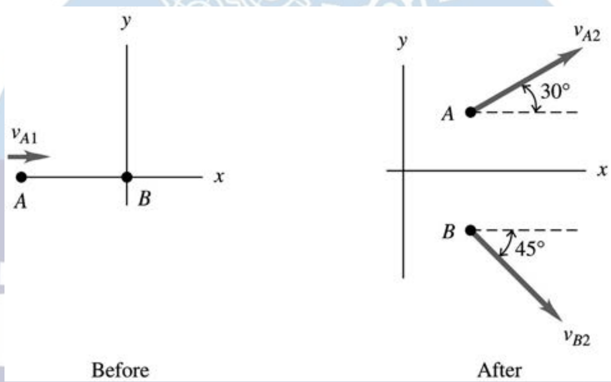



**Figure 8.31**

**8.32.** **IDENTIFY:** There is no net external force on the system of the two skaters and the momentum of the system is conserved.

**SET UP:** Let object $A$ be the skater with mass 70.0 kg and object $B$ be the skater with mass 65.0 kg. Let $+x$ be to the right, so $v_{A1x} = +4.00$ m/s and $v_{B1x} = -2.50$ m/s. After the collision, the two objects are combined and move with velocity $\vec{v}_2$. Solve for $v_{2x}$.

**EXECUTE:** $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

$v_{2x} = \dfrac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \dfrac{(70.0\text{ kg})(4.00\text{ m/s}) + (65.0\text{ kg})(-2.50\text{ m/s})}{70.0\text{ kg} + 65.0\text{ kg}} = 0.870$ m/s. The two skaters move to the right at 0.870 m/s.

**EVALUATE:** There is a large decrease in kinetic energy.

**8.33.** **IDENTIFY:** Since drag effects are neglected there is no net external force on the system of two fish and the momentum of the system is conserved. The mechanical energy equals the kinetic energy, which is $K = \frac{1}{2}mv^2$ for each object.

**SET UP:** Let object $A$ be the 15.0 kg fish and $B$ be the 4.50 kg fish. Let $+x$ be the direction the large fish is moving initially, so $v_{A1x} = 1.10$ m/s and $v_{B1x} = 0$. After the collision the two objects are combined and move with velocity $\vec{v}_2$. Solve for $v_{2x}$.

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \frac{(15.0\text{ kg})(1.10\text{ m/s}) + 0}{15.0\text{ kg} + 4.50\text{ kg}} = 0.846\text{ m/s}.$$

**(b)** $K_1 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}(15.0\text{ kg})(1.10\text{ m/s})^2 = 9.08\text{ J}.$

$K_2 = \frac{1}{2}(m_A + m_B)v_2^2 = \frac{1}{2}(19.5\text{ kg})(0.846\text{ m/s})^2 = 6.98\text{ J}.$

$\Delta K = K_2 - K_1 = -2.10\text{ J}.$ 2.10 J of mechanical energy is dissipated.

**EVALUATE:** The total kinetic energy always decreases in a collision where the two objects become combined.

**8.34.** **IDENTIFY:** There is no net external force on the system of the two otters and the momentum of the system is conserved. The mechanical energy equals the kinetic energy, which is $K = \frac{1}{2}mv^2$ for each object.

**SET UP:** Let $A$ be the 7.50 kg otter and $B$ be the 5.75 kg otter. After the collision their combined velocity is $\vec{v}_2$. Let $+x$ be to the right, so $v_{A1x} = -5.00\text{ m/s}$ and $v_{B1x} = +6.00\text{ m/s}$. Solve for $v_{2x}$.

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \frac{(7.50\text{ kg})(-5.00\text{ m/s}) + (5.75\text{ kg})(+6.00\text{ m/s})}{7.50\text{ kg} + 5.75\text{ kg}} = -0.226\text{ m/s}.$$

**(b)** $K_1 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}(7.50\text{ kg})(5.00\text{ m/s})^2 + \frac{1}{2}(5.75\text{ kg})(6.00\text{ m/s})^2 = 197.2\text{ J}.$

$K_2 = \frac{1}{2}(m_A + m_B)v_2^2 = \frac{1}{2}(13.25\text{ kg})(0.226\text{ m/s})^2 = 0.338\text{ J}.$

$\Delta K = K_2 - K_1 = -197\text{ J}.$ 197 J of mechanical energy is dissipated.

**EVALUATE:** The total kinetic energy always decreases in a collision where the two objects become combined.

**8.35.** **IDENTIFY:** Treat the comet and probe as an isolated system for which momentum is conserved.

**SET UP:** In part (a) let object $A$ be the probe and object $B$ be the comet. Let $-x$ be the direction the probe is traveling just before the collision. After the collision the combined object moves with speed $v_2$. The change in velocity is $\Delta v = v_{2x} - v_{B1x}$. In part (a) the impact speed of 37,000 km/h is the speed of the probe relative to the comet just before impact: $v_{A1x} - v_{B1x} = -37{,}000\text{ km/h}.$ In part (b) let object $A$ be the comet and object $B$ be the earth. Let $-x$ be the direction the comet is traveling just before the collision. The impact speed is 40,000 km/h, so $v_{A1x} - v_{B1x} = -40{,}000\text{ km/h}.$

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$. $v_{2x} = \dfrac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B}$.

$$\Delta v = v_{2x} - v_{B1x} = \left(\frac{m_A}{m_A + m_B}\right)v_{A1x} + \left(\frac{m_B - m_A - m_B}{m_A + m_B}\right)v_{B1x} = \left(\frac{m_A}{m_A + m_B}\right)(v_{A1x} - v_{B1x}).$$

$$\Delta v = \left(\frac{372\text{ kg}}{372\text{ kg} + 0.10\times 10^{14}\text{ kg}}\right)(-37{,}000\text{ km/h}) = -1.4\times 10^{-6}\text{ km/h}.$$

The speed of the comet decreased by $1.4\times 10^{-6}$ km/h. This change is not noticeable.

**(b)** $\Delta v = \left(\dfrac{0.10\times 10^{14}\text{ kg}}{0.10\times 10^{14}\text{ kg} + 5.97\times 10^{24}\text{ kg}}\right)(-40{,}000\text{ km/h}) = -6.7\times 10^{-8}\text{ km/h}.$ The speed of the earth would change by $6.7\times 10^{-8}$ km/h. This change is not noticeable.

**EVALUATE:** $v_{A1x} - v_{B1x}$ is the velocity of the projectile (probe or comet) relative to the target (comet or earth). The expression for $\Delta v$ can be derived directly by applying momentum conservation in coordinates in which the target is initially at rest.

**8.36.** **IDENTIFY:** The forces the two vehicles exert on each other during the collision are much larger than the horizontal forces exerted by the road, and it is a good approximation to assume momentum conservation.

**SET UP:** Let $+x$ be eastward. After the collision two vehicles move with a common velocity $\vec{v}_2$.

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$ gives $m_{SC}v_{SCx} + m_T v_{Tx} = (m_{SC} + m_T)v_{2x}$.

$$v_{2x} = \frac{m_{SC}v_{SCx} + m_T v_{Tx}}{m_{SC} + m_T} = \frac{(1050\text{ kg})(-15.0\text{ m/s}) + (6320\text{ kg})(+10.0\text{ m/s})}{1050\text{ kg} + 6320\text{ kg}} = 6.44\text{ m/s}.$$

The final velocity is 6.44 m/s, eastward.

**(b)** $P_{1x} = P_{2x} = 0$ gives $m_{SC}v_{SCx} + m_T v_{Tx} = 0$. $v_{Tx} = -\left(\frac{m_{SC}}{m_T}\right)v_{SCx} = -\left(\frac{1050\text{ kg}}{6320\text{ kg}}\right)(-15.0\text{ m/s}) = 2.50\text{ m/s}.$

The truck would need to have initial speed 2.50 m/s.

**(c)** part (a): $\Delta K = \frac{1}{2}(7370\text{ kg})(6.44\text{ m/s})^2 - \frac{1}{2}(1050\text{ kg})(15.0\text{ m/s})^2 - \frac{1}{2}(6320\text{ kg})(10.0\text{ m/s})^2 = -2.81\times10^5\text{ J}$

part (b): $\Delta K = 0 - \frac{1}{2}(1050\text{ kg})(15.0\text{ m/s})^2 - \frac{1}{2}(6320\text{ kg})(2.50\text{ m/s})^2 = -1.38\times10^5\text{ J}$. The change in kinetic energy has the greater magnitude in part (a).

**EVALUATE:** In part (a) the eastward momentum of the truck has a greater magnitude than the westward momentum of the car and the wreckage moves eastward after the collision. In part (b) the two vehicles have equal magnitudes of momentum, the total momentum of the system is zero and the wreckage is at rest after the collision.

**8.37.** **IDENTIFY:** The forces the two players exert on each other during the collision are much larger than the horizontal forces exerted by the slippery ground and it is a good approximation to assume momentum conservation. Each component of momentum is separately conserved.

**SET UP:** Let $+x$ be east and $+y$ be north. After the collision the two players have velocity $\vec{v}_2$. Let the linebacker be object $A$ and the halfback be object $B$, so $v_{A1x} = 0$, $v_{A1y} = 8.8$ m/s, $v_{B1x} = 7.2$ m/s and $v_{B1y} = 0$. Solve for $v_{2x}$ and $v_{2y}$.

**EXECUTE:** $P_{1x} = P_{2x}$ gives $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \frac{(85\text{ kg})(7.2\text{ m/s})}{110\text{ kg} + 85\text{ kg}} = 3.14\text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $m_A v_{A1y} + m_B v_{B1y} = (m_A + m_B)v_{2y}$.

$$v_{2y} = \frac{m_A v_{A1y} + m_B v_{B1y}}{m_A + m_B} = \frac{(110\text{ kg})(8.8\text{ m/s})}{110\text{ kg} + 85\text{ kg}} = 4.96\text{ m/s}.$$

$v = \sqrt{v_{2x}^2 + v_{2y}^2} = 5.9$ m/s.

$$\tan\theta = \frac{v_{2y}}{v_{2x}} = \frac{4.96\text{ m/s}}{3.14\text{ m/s}} \text{ and } \theta = 58°.$$

The players move with a speed of 5.9 m/s and in a direction 58° north of east.

**EVALUATE:** Each component of momentum is separately conserved.

**8.38.** **IDENTIFY:** The momentum is conserved during the collision. Since the motions involved are in two dimensions, we must consider the components separately.

**SET UP:** Use coordinates where $+x$ is east and $+y$ is south. The system of two cars before and after the collision is sketched in Figure 8.38. Neglect friction from the road during the collision. The enmeshed cars have a total mass of $2000\text{ kg} + 1500\text{ kg} = 3500\text{ kg}$. Momentum conservation tells us that $P_{1x} = P_{2x}$ and $P_{1y} = P_{2y}$.

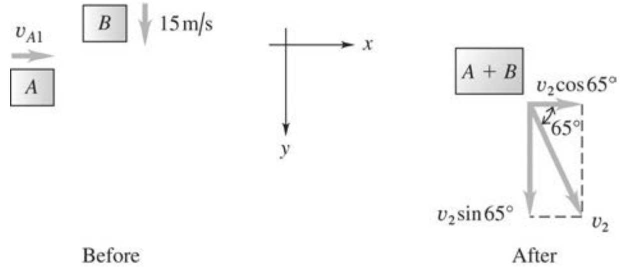



**Figure 8.38**

**EXECUTE:** There are no external horizontal forces during the collision, so $P_{1x} = P_{2x}$ and $P_{1y} = P_{2y}$.

**(a)** $P_{1x} = P_{2x}$ gives $(1500\text{ kg})(15\text{ m/s}) = (3500\text{ kg})v_2\sin 65°$ and $v_2 = 7.1$ m/s.

**(b)** $P_{1y} = P_{2y}$ gives $(2000\text{ kg})v_{A1} = (3500\text{ kg})v_2\cos 65°$. And then using $v_2 = 7.1$ m/s, we have $v_{A1} = 5.2$ m/s.

**EVALUATE:** Momentum is a vector so we must treat each component separately.

**8.39.** **IDENTIFY:** The collision generates only internal forces to the Jack-Jill system, so momentum is conserved.

**SET UP:** Call the $x$-axis Jack's initial direction (eastward), and the $y$-axis perpendicular to that (northward). The initial $y$-component of the momentum is zero. Call $v$ Jill's speed just after the collision and call $\theta$ the angle her velocity makes with the $+x$-axis.

**EXECUTE:** In the $x$-direction: (55.0 kg)(8.00 m/s) = (55.0 kg)(5.00 m/s)(cos34.0°) + (48.0 kg)$v \cos\theta$.
In the $y$-direction: (55.0 kg)(5.00 m/s)(sin34.0°) = (48.0 kg)$v \sin\theta$.
Separating $v \sin\theta$ and $v \cos\theta$ and dividing gives
$\tan\theta$ = (5.00 m/s)(sin34.0°)/[8.00 m/s – (5.00 m/s)(cos34.0°)] = 0.72532, so $\theta$ = 36.0° south of east.
Using the $y$-direction momentum equation gives
$v$ = (55.0 kg)(5.00 m/s)(sin34.0°)/[(48.0 kg)(sin36.0°) = 5.46 m/s.

**EVALUATE:** Jill has a bit less mass than Jack, so the angle her momentum makes with the $+x$-axis (36.0°) has to be a bit larger than Jack's (34.0°) for their $y$-component momenta to be equal in magnitude.

**8.40.** **IDENTIFY:** The collision forces are large so gravity can be neglected during the collision. Therefore, the horizontal and vertical components of the momentum of the system of the two birds are conserved.

**SET UP:** The system before and after the collision is sketched in Figure 8.40. Use the coordinates shown.

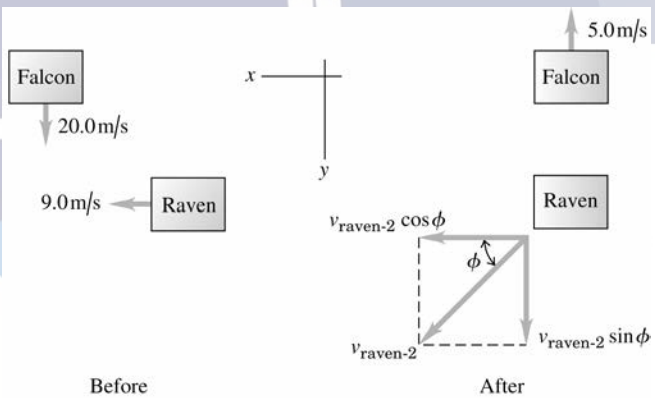



**Figure 8.40**

**EXECUTE:** **(a)** There is no external force on the system so $P_{1x} = P_{2x}$ and $P_{1y} = P_{2y}$.

$P_{1x} = P_{2x}$ gives $(1.5\text{ kg})(9.0\text{ m/s}) = (1.5\text{ kg})v_{\text{raven-2}}\cos\phi$ and $v_{\text{raven-2}}\cos\phi = 9.0$ m/s.

$P_{1y} = P_{2y}$ gives $(0.600\text{ kg})(20.0\text{ m/s}) = (0.600\text{ kg})(-5.0\text{ m/s}) + (1.5\text{ kg})v_{\text{raven-2}}\sin\phi$ and $v_{\text{raven-2}}\sin\phi = 10.0$ m/s.

Combining these two equations gives $\tan\phi = \dfrac{10.0\text{ m/s}}{9.0\text{ m/s}}$ and $\phi = 48°$.

**(b)** $v_{\text{raven-2}} = 13.5$ m/s

**EVALUATE:** Due to its large initial speed the lighter falcon was able to produce a large change in the raven's direction of motion.

**8.41.** **IDENTIFY:** Since friction forces from the road are ignored, the $x$- and $y$-components of momentum are conserved.

**SET UP:** Let object $A$ be the subcompact and object $B$ be the truck. After the collision the two objects move together with velocity $\vec{v}_2$. Use the $x$- and $y$-coordinates given in the problem. $v_{A1y} = v_{B1x} = 0$.

$v_{2x} = (16.0\text{ m/s})\sin 24.0° = 6.5$ m/s; $v_{2y} = (16.0\text{ m/s})\cos 24.0° = 14.6$ m/s.

**EXECUTE:** $P_{1x} = P_{2x}$ gives $m_A v_{A1x} = (m_A + m_B)v_{2x}$.

$$v_{A1x} = \left(\frac{m_A + m_B}{m_A}\right)v_{2x} = \left(\frac{950\text{ kg} + 1900\text{ kg}}{950\text{ kg}}\right)(6.5\text{ m/s}) = 19.5\text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $m_B v_{B1y} = (m_A + m_B)v_{2y}$.

$$v_{B1y} = \left(\frac{m_A + m_B}{m_B}\right)v_{2y} = \left(\frac{950\text{ kg} + 1900\text{ kg}}{1900\text{ kg}}\right)(14.6\text{ m/s}) = 21.9\text{ m/s}.$$

Before the collision the subcompact car has speed 19.5 m/s and the truck has speed 21.9 m/s.

**EVALUATE:** Each component of momentum is independently conserved.

**8.42.** **IDENTIFY:** Apply conservation of momentum to the collision. Apply conservation of energy to the motion of the block after the collision.

**SET UP:** Conservation of momentum applied to the collision between the bullet and the block: Let object *A* be the bullet and object *B* be the block. Let $v_A$ be the speed of the bullet before the collision and let *V* be the speed of the block with the bullet inside just after the collision.

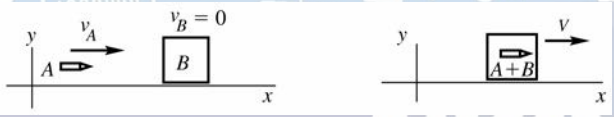



**Figure 8.42a**

$P_x$ is constant gives $m_A v_A = (m_A + m_B)V$.

Conservation of energy applied to the motion of the block after the collision:

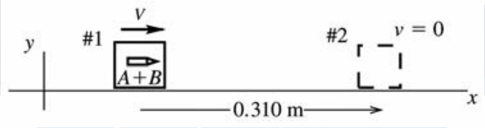



**Figure 8.42b**

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

**EXECUTE:** Work is done by friction so $W_{\text{other}} = W_f = (f_k \cos\phi)s = -f_k s = -\mu_k mgs$

$U_1 = U_2 = 0$ (no work done by gravity)

$K_1 = \frac{1}{2}mV^2$; $K_2 = 0$ (block has come to rest)

Thus $\frac{1}{2}mV^2 - \mu_k mgs = 0$

$V = \sqrt{2\mu_k gs} = \sqrt{2(0.20)(9.80\text{ m/s}^2)(0.310\text{ m})} = 1.1\text{ m/s}$

Use this result in the conservation of momentum equation

$$v_A = \left(\frac{m_A + m_B}{m_A}\right)V = \left(\frac{5.00\times10^{-3}\text{ kg} + 1.20\text{ kg}}{5.00\times10^{-3}\text{ kg}}\right)(1.1\text{ m/s}) = 266\text{ m/s},\ \text{which rounds to 270 m/s.}$$

**EVALUATE:** When we apply conservation of momentum to the collision we are ignoring the impulse of the friction force exerted by the surface during the collision. This is reasonable since this force is much smaller than the forces the bullet and block exert on each other during the collision. This force does work as the block moves after the collision, and takes away all the kinetic energy.

**8.43.** **IDENTIFY:** Apply conservation of momentum to the collision and conservation of energy to the motion after the collision. After the collision the kinetic energy of the combined object is converted to gravitational potential energy.

**SET UP:** Immediately after the collision the combined object has speed $V$. Let $h$ be the vertical height through which the pendulum rises.

**EXECUTE:** **(a)** Conservation of momentum applied to the collision gives

$(12.0\times10^{-3}\ \text{kg})(380\ \text{m/s}) = (6.00\ \text{kg} + 12.0\times10^{-3}\ \text{kg})V$ and $V = 0.758$ m/s.

Conservation of energy applied to the motion after the collision gives $\frac{1}{2}m_{\text{tot}}V^2 = m_{\text{tot}}gh$ and

$$h = \frac{V^2}{2g} = \frac{(0.758\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = 0.0293\ \text{m} = 2.93\ \text{cm}.$$

**(b)** $K = \frac{1}{2}m_{\text{b}}v_{\text{b}}^2 = \frac{1}{2}(12.0\times10^{-3}\ \text{kg})(380\ \text{m/s})^2 = 866\ \text{J}.$

**(c)** $K = \frac{1}{2}m_{\text{tot}}V^2 = \frac{1}{2}(6.00\ \text{kg} + 12.0\times10^{-3}\ \text{kg})(0.758\ \text{m/s})^2 = 1.73\ \text{J}.$

**EVALUATE:** Most of the initial kinetic energy of the bullet is dissipated in the collision.

**8.44.** **IDENTIFY:** During the collision, momentum is conserved. After the collision, mechanical energy is conserved.

**SET UP:** The collision occurs over a short time interval and the block moves very little during the collision, so the spring force during the collision can be neglected. Use coordinates where $+x$ is to the right. During the collision, momentum conservation gives $P_{1x} = P_{2x}$. After the collision, $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$.

**EXECUTE:** *Collision*: There is no external horizontal force during the collision and $P_{1x} = P_{2x}$, so

$(3.00\ \text{kg})(8.00\ \text{m/s}) = (15.0\ \text{kg})v_{\text{block, 2}} - (3.00\ \text{kg})(2.00\ \text{m/s})$ and $v_{\text{block, 2}} = 2.00$ m/s.

*Motion after the collision*: When the spring has been compressed the maximum amount, all the initial kinetic energy of the block has been converted into potential energy $\frac{1}{2}kx^2$ that is stored in the compressed spring. Conservation of energy gives $\frac{1}{2}(15.0\ \text{kg})(2.00\ \text{m/s})^2 = \frac{1}{2}(500.0\ \text{kg})x^2$, so $x = 0.346$ m.

**EVALUATE:** We cannot say that the momentum was converted to potential energy, because momentum and energy are different types of quantities.

**8.45.** **IDENTIFY:** The missile gives momentum to the ornament causing it to swing in a circular arc and thereby be accelerated toward the center of the circle.

**SET UP:** After the collision the ornament moves in an arc of a circle and has acceleration $a_{\text{rad}} = \dfrac{v^2}{r}$.

During the collision, momentum is conserved, so $P_{1x} = P_{2x}$. The free-body diagram for the ornament plus missile is given in Figure 8.45. Take $+y$ to be upward, since that is the direction of the acceleration. Take the $+x$-direction to be the initial direction of motion of the missile.

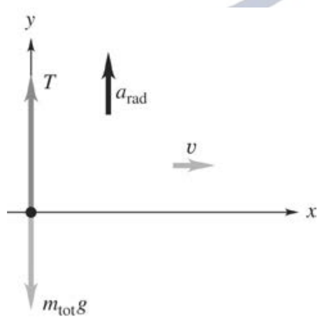



**Figure 8.45**

**EXECUTE:** Apply conservation of momentum to the collision. Using $P_{1x} = P_{2x}$, we get

$(0.200\ \text{kg})(12.0\ \text{m/s}) = (1.00\ \text{kg})V$, which gives $V = 2.40$ m/s, the speed of the ornament immediately after the collision. Then $\Sigma F_y = ma_y$ gives $T - m_{\text{tot}}g = m_{\text{tot}}\dfrac{v^2}{r}$. Solving for $T$ gives

$$T = m_{\text{tot}}\left(g + \frac{v^2}{r}\right) = (1.00\ \text{kg})\left(9.80\ \text{m/s}^2 + \frac{(2.40\ \text{m/s})^2}{1.50\ \text{m}}\right) = 13.6\ \text{N}.$$

**EVALUATE:** We cannot use energy conservation during the collision because it is an inelastic collision (the objects stick together).

**8.46.** **IDENTIFY:** No net external horizontal force so $P_x$ is conserved. Elastic collision so $K_1 = K_2$ and can use $v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$.

**SET UP:**

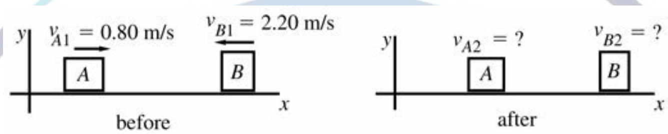



**Figure 8.46**

**EXECUTE:** From conservation of $x$-component of momentum:

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$

$$m_A v_{A1} - m_B v_{B1} = m_A v_{A2x} + m_B v_{B2x}$$

$$(0.150\ \text{kg})(0.80\ \text{m/s}) - (0.300\ \text{kg})(2.20\ \text{m/s}) = (0.150\ \text{kg})v_{A2x} + (0.300\ \text{kg})v_{B2x}$$

$$-3.60\ \text{m/s} = v_{A2x} + 2v_{B2x}$$

From the relative velocity equation for an elastic collision Eq. 8.27:

$$v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x}) = -(-2.20\ \text{m/s} - 0.80\ \text{m/s}) = +3.00\ \text{m/s}$$

$$3.00\ \text{m/s} = -v_{A2x} + v_{B2x}$$

Adding the two equations gives $-0.60\ \text{m/s} = 3v_{B2x}$ and $v_{B2x} = -0.20$ m/s. Then $v_{A2x} = v_{B2x} - 3.00\ \text{m/s} = -3.20$ m/s.

The 0.150 kg glider ($A$) is moving to the left at 3.20 m/s and the 0.300 kg glider ($B$) is moving to the left at 0.20 m/s.

**EVALUATE:** We can use our $v_{A2x}$ and $v_{B2x}$ to show that $P_x$ is constant and $K_1 = K_2$.

**8.47.** **IDENTIFY:** When the spring is compressed the maximum amount the two blocks aren't moving relative to each other and have the same velocity $\vec{V}$ relative to the surface. Apply conservation of momentum to find $V$ and conservation of energy to find the energy stored in the spring. Since the collision is elastic, $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_{A1x}$ and $v_{B2x} = \left(\dfrac{2m_A}{m_A + m_B}\right)v_{A1x}$ give the final velocity of each block after the collision.

**SET UP:** Let $+x$ be the direction of the initial motion of $A$.

**EXECUTE:** **(a)** Momentum conservation gives $(2.00\ \text{kg})(2.00\ \text{m/s}) = (8.00\ \text{kg})V$ so $V = 0.500$ m/s. Both blocks are moving at 0.500 m/s, in the direction of the initial motion of block $A$. Conservation of energy says the initial kinetic energy of $A$ equals the total kinetic energy at maximum compression plus the potential energy $U_{\text{b}}$ stored in the bumpers: $\frac{1}{2}(2.00\ \text{kg})(2.00\ \text{m/s})^2 = U_{\text{b}} + \frac{1}{2}(8.00\ \text{kg})(0.500\ \text{m/s})^2$ so $U_{\text{b}} = 3.00$ J.

**(b)** $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_{A1x} = \left(\dfrac{2.00\ \text{kg} - 6.0\ \text{kg}}{8.00\ \text{kg}}\right)(2.00\ \text{m/s}) = -1.00$ m/s. Block $A$ is moving in the $-x$-direction at 1.00 m/s.

$v_{B2x} = \left(\dfrac{2m_A}{m_A + m_B}\right)v_{A1x} = \dfrac{2(2.00\text{ kg})}{8.00\text{ kg}}(2.00\text{ m/s}) = +1.00\text{ m/s}.$ Block $B$ is moving in the $+x$-direction at 1.00 m/s.

**EVALUATE:** When the spring is compressed the maximum amount, the system must still be moving in order to conserve momentum.

**8.48.** **IDENTIFY:** Since the collision is elastic, both momentum conservation and equation $v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$ apply.

**SET UP:** Let object $A$ be the 30.0 g marble and let object $B$ be the 10.0 g marble. Let $+x$ be to the right.

**EXECUTE:** **(a)** Conservation of momentum gives
$(0.0300\text{ kg})(0.200\text{ m/s}) + (0.0100\text{ kg})(-0.400\text{ m/s}) = (0.0300\text{ kg})v_{A2x} + (0.0100\text{ kg})v_{B2x}.$
$3v_{A2x} + v_{B2x} = 0.200\text{ m/s}.$ $v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$ says
$v_{B2x} - v_{A2x} = -(-0.400\text{ m/s} - 0.200\text{ m/s}) = +0.600\text{ m/s}.$ Solving this pair of equations gives
$v_{A2x} = -0.100\text{ m/s}$ and $v_{B2x} = +0.500\text{ m/s}.$ The 30.0 g marble is moving to the left at 0.100 m/s and the 10.0 g marble is moving to the right at 0.500 m/s.

**(b)** For marble $A$, $\Delta P_{Ax} = m_A v_{A2x} - m_A v_{A1x} = (0.0300\text{ kg})(-0.100\text{ m/s} - 0.200\text{ m/s}) = -0.00900\text{ kg}\cdot\text{m/s}.$
For marble $B$, $\Delta P_{Bx} = m_B v_{B2x} - m_B v_{B1x} = (0.0100\text{ kg})(0.500\text{ m/s} - [-0.400\text{ m/s}]) = +0.00900\text{ kg}\cdot\text{m/s}.$
The changes in momentum have the same magnitude and opposite sign.

**(c)** For marble $A$, $\Delta K_A = \frac{1}{2}m_A v_{A2}^2 - \frac{1}{2}m_A v_{A1}^2 = \frac{1}{2}(0.0300\text{ kg})([0.100\text{ m/s}]^2 - [0.200\text{ m/s}]^2) = -4.5\times 10^{-4}\text{ J}.$
For marble $B$, $\Delta K_B = \frac{1}{2}m_B v_{B2}^2 - \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}(0.0100\text{ kg})([0.500\text{ m/s}]^2 - [0.400\text{ m/s}]^2) = +4.5\times 10^{-4}\text{ J}.$
The changes in kinetic energy have the same magnitude and opposite sign.

**EVALUATE:** The results of parts (b) and (c) show that momentum and kinetic energy are conserved in the collision.

**8.49.** **IDENTIFY:** Equation $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_{A1x}$ applyies, with object $A$ being the neutron.

**SET UP:** Let $+x$ be the direction of the initial momentum of the neutron. The mass of a neutron is $m_n = 1.0\text{ u}.$

**EXECUTE:** **(a)** $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_{A1x} = \dfrac{1.0\text{ u} - 2.0\text{ u}}{1.0\text{ u} + 2.0\text{ u}}v_{A1x} = -v_{A1x}/3.0.$ The speed of the neutron after the collision is one-third its initial speed.

**(b)** $K_2 = \frac{1}{2}m_n v_n^2 = \frac{1}{2}m_n (v_{A1}/3.0)^2 = \dfrac{1}{9.0}K_1.$

**(c)** After $n$ collisions, $v_{A2} = \left(\dfrac{1}{3.0}\right)^n v_{A1}.$ $\left(\dfrac{1}{3.0}\right)^n = \dfrac{1}{59{,}000},$ so $3.0^n = 59{,}000.$ $n\log 3.0 = \log 59{,}000$ and $n = 10.$

**EVALUATE:** Since the collision is elastic, in each collision the kinetic energy lost by the neutron equals the kinetic energy gained by the deuteron.

**8.50.** **IDENTIFY:** Elastic collision. Solve for mass and speed of target nucleus.

**SET UP:** **(a)** Let $A$ be the proton and $B$ be the target nucleus. The collision is elastic, all velocities lie along a line, and $B$ is at rest before the collision. Hence the results of equations $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_{A1x}$ and $v_{B2x} = \left(\dfrac{2m_A}{m_A + m_B}\right)v_{A1x}$ apply.

**EXECUTE:** $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_{A1x}$: $m_B(v_x + v_{Ax}) = m_A(v_x - v_{Ax}),$ where $v_x$ is the velocity component of $A$ before the collision and $v_{Ax}$ is the velocity component of $A$ after the collision. Here, $v_x = 1.50\times 10^7$ m/s (take direction of incident beam to be positive) and $v_{Ax} = -1.20\times10^7$ m/s (negative since traveling in direction opposite to incident beam).

$$m_B = m_A\left(\frac{v_x - v_{Ax}}{v_x + v_{Ax}}\right) = m\left(\frac{1.50\times10^7 \text{ m/s} + 1.20\times10^7 \text{ m/s}}{1.50\times10^7 \text{ m/s} - 1.20\times10^7 \text{ m/s}}\right) = m\left(\frac{2.70}{0.30}\right) = 9.00m.$$

**(b)** $v_{B2x} = \left(\dfrac{2m_A}{m_A + m_B}\right)v_{A1x}$: $v_{Bx} = \left(\dfrac{2m_A}{m_A + m_B}\right)v_x = \left(\dfrac{2m}{m + 9.00m}\right)(1.50\times10^7 \text{ m/s}) = 3.00\times10^6 \text{ m/s}.$

**EVALUATE:** Can use our calculated $v_{Bx}$ and $m_B$ to show that $P_x$ is constant and that $K_1 = K_2$.

**8.51.** **IDENTIFY:** Apply $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$.

**SET UP:** $m_A = 0.300$ kg, $m_B = 0.400$ kg, $m_C = 0.200$ kg.

**EXECUTE:** $x_{cm} = \dfrac{m_Ax_A + m_Bx_B + m_Cx_C}{m_A + m_B + m_C}$.

$$x_{cm} = \frac{(0.300 \text{ kg})(0.200 \text{ m}) + (0.400 \text{ kg})(0.100 \text{ m}) + (0.200 \text{ kg})(-0.300 \text{ m})}{0.300 \text{ kg} + 0.400 \text{ kg} + 0.200 \text{ kg}} = 0.0444 \text{ m}.$$

$$y_{cm} = \frac{m_Ay_A + m_By_B + m_Cy_C}{m_A + m_B + m_C}.$$

$$y_{cm} = \frac{(0.300 \text{ kg})(0.300 \text{ m}) + (0.400 \text{ kg})(-0.400 \text{ m}) + (0.200 \text{ kg})(0.600 \text{ m})}{0.300 \text{ kg} + 0.400 \text{ kg} + 0.200 \text{ kg}} = 0.0556 \text{ m}.$$

**EVALUATE:** There is mass at both positive and negative $x$ and at positive and negative $y$, and therefore the center of mass is close to the origin.

**8.52.** **IDENTIFY:** Calculate $x_{cm}$.

**SET UP:** Apply $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$ with the sun as mass 1 and Jupiter as mass 2. Take the origin at the sun and let Jupiter lie on the positive $x$-axis.

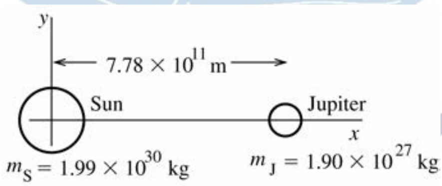



**Figure 8.52**

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

**EXECUTE:** $x_1 = 0$ and $x_2 = 7.78\times10^{11}$ m

$$x_{cm} = \frac{(1.90\times10^{27} \text{ kg})(7.78\times10^{11} \text{ m})}{1.99\times10^{30} \text{ kg} + 1.90\times10^{27} \text{ kg}} = 7.42\times10^8 \text{ m}$$

The center of mass is $7.42\times10^8$ m from the center of the sun and is on the line connecting the centers of the sun and Jupiter. The sun's radius is $6.96\times10^8$ m so the center of mass lies just outside the sun.

**EVALUATE:** The mass of the sun is much greater than the mass of Jupiter, so the center of mass is much closer to the sun. For each object we have considered all the mass as being at the center of mass (geometrical center) of the object.

**8.53.** **IDENTIFY:** The location of the center of mass is given by $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$. The mass can be expressed in terms of the diameter. Each object can be replaced by a point mass at its center.
**SET UP:** Use coordinates with the origin at the center of Pluto and the $+x$-direction toward Charon, so $x_P = 0$, $x_C = 19{,}700$ km. $m = \rho V = \rho\frac{4}{3}\pi r^3 = \frac{1}{6}\rho\pi d^3$.

**EXECUTE:** $x_{cm} = \dfrac{m_Px_P + m_Cx_C}{m_P + m_C} = \left(\dfrac{m_C}{m_P + m_C}\right)x_C = \left(\dfrac{\frac{1}{6}\rho\pi d_C^3}{\frac{1}{6}\rho\pi d_P^3 + \frac{1}{6}\rho\pi d_C^3}\right)x_C = \left(\dfrac{d_C^3}{d_P^3 + d_C^3}\right)x_C$.

$$x_{cm} = \left(\frac{[1250\text{ km}]^3}{[2370\text{ km}]^3 + [1250\text{ km}]^3}\right)(19{,}700\text{ km}) = 2.52\times10^3\text{ km}.$$

The center of mass of the system is $2.52\times10^3$ km from the center of Pluto.
**EVALUATE:** The center of mass is closer to Pluto because Pluto has more mass than Charon.

**8.54.** **IDENTIFY:** Apply $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$, $v_{cm,x} = \dfrac{m_Av_{A,x} + m_Bv_{B,x}}{m_A + m_B}$, and $P_x = Mv_{cm-x}$. There is only one component of position and velocity.
**SET UP:** $m_A = 1200$ kg, $m_B = 1800$ kg. $M = m_A + m_B = 3000$ kg. Let $+x$ be to the right and let the origin be at the center of mass of the station wagon.

**EXECUTE:** **(a)** $x_{cm} = \dfrac{m_Ax_A + m_Bx_B}{m_A + m_B} = \dfrac{0 + (1800\text{ kg})(40.0\text{ m})}{1200\text{ kg} + 1800\text{ kg}} = 24.0$ m.

The center of mass is between the two cars, 24.0 m to the right of the station wagon and 16.0 m behind the lead car.

**(b)** $P_x = m_Av_{A,x} + m_Bv_{B,x} = (1200\text{ kg})(12.0\text{ m/s}) + (1800\text{ kg})(20.0\text{ m/s}) = 5.04\times10^4\text{ kg}\cdot\text{m/s}$.

**(c)** $v_{cm,x} = \dfrac{m_Av_{A,x} + m_Bv_{B,x}}{m_A + m_B} = \dfrac{(1200\text{ kg})(12.0\text{ m/s}) + (1800\text{ kg})(20.0\text{ m/s})}{1200\text{ kg} + 1800\text{ kg}} = 16.8$ m/s.

**(d)** $P_x = Mv_{cm-x} = (3000\text{ kg})(16.8\text{ m/s}) = 5.04\times10^4\text{ kg}\cdot\text{m/s}$, the same as in part (b).

**EVALUATE:** The total momentum can be calculated either as the vector sum of the momenta of the individual objects in the system, or as the total mass of the system times the velocity of the center of mass.

**8.55.** **IDENTIFY:** Use $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$ and $y_{cm} = \dfrac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$ to find the $x$- and $y$-coordinates of the center of mass of the machine part for each configuration of the part. In calculating the center of mass of the machine part, each uniform bar can be represented by a point mass at its geometrical center.
**SET UP:** Use coordinates with the axis at the hinge and the $+x$- and $+y$-axes along the horizontal and vertical bars in the figure in the problem. Let $(x_i, y_i)$ and $(x_f, y_f)$ be the coordinates of the bar before and after the vertical bar is pivoted. Let object 1 be the horizontal bar, object 2 be the vertical bar and 3 be the ball.

**EXECUTE:** $x_i = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \dfrac{(4.00\text{ kg})(0.750\text{ m}) + 0 + 0}{4.00\text{ kg} + 3.00\text{ kg} + 2.00\text{ kg}} = 0.333$ m.

$$y_i = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{0 + (3.00\text{ kg})(0.900\text{ m}) + (2.00\text{ kg})(1.80\text{ m})}{9.00\text{ kg}} = 0.700\text{ m}.$$

$$x_f = \frac{(4.00\text{ kg})(0.750\text{ m}) + (3.00\text{ kg})(-0.900\text{ m}) + (2.00\text{ kg})(-1.80\text{ m})}{9.00\text{ kg}} = -0.366\text{ m}.$$

$y_f = 0$. $x_f - x_i = -0.700$ m and $y_f - y_i = -0.700$ m. The center of mass moves 0.700 m to the right and 0.700 m upward.
**EVALUATE:** The vertical bar moves upward and to the right, so it is sensible for the center of mass of the machine part to move in these directions.

**8.56.** **IDENTIFY:** Use $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$.

**SET UP:** The target variable is $m_1$.

**EXECUTE:** $x_{cm} = 2.0 \text{ m}, \quad y_{cm} = 0$

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{m_1(0) + (0.10 \text{ kg})(8.0 \text{ m})}{m_1 + (0.10 \text{ kg})} = \frac{0.80 \text{ kg}\cdot\text{m}}{m_1 + 0.10 \text{ kg}}.$$

$$x_{cm} = 2.0 \text{ m gives } 2.0 \text{ m} = \frac{0.80 \text{ kg}\cdot\text{m}}{m_1 + 0.10 \text{ kg}}.$$

$$m_1 + 0.10 \text{ kg} = \frac{0.80 \text{ kg}\cdot\text{m}}{2.0 \text{ m}} = 0.40 \text{ kg}.$$

$$m_1 = 0.30 \text{ kg}.$$

**EVALUATE:** The cm is closer to $m_1$ so its mass is larger then $m_2$.

**(b) IDENTIFY:** Use $\vec{P} = M\vec{v}_{cm}$ to calculate $\vec{P}$.

**SET UP:** $\vec{v}_{cm} = (5.0 \text{ m/s})\,\hat{i}$.

$$\vec{P} = M\vec{v}_{cm} = (0.10 \text{ kg} + 0.30 \text{ kg})(5.0 \text{ m/s})\,\hat{i} = (2.0 \text{ kg}\cdot\text{m/s})\hat{i}.$$

**(c) IDENTIFY:** Use $\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$.

**SET UP:** $\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$. The target variable is $\vec{v}_1$. Particle 2 at rest says $v_2 = 0$.

**EXECUTE:** $\vec{v}_1 = \left(\dfrac{m_1 + m_2}{m_1}\right)\vec{v}_{cm} = \left(\dfrac{0.30 \text{ kg} + 0.10 \text{ kg}}{0.30 \text{ kg}}\right)(5.00 \text{ m/s})\hat{i} = (6.7 \text{ m/s})\hat{i}$.

**EVALUATE:** Using the result of part (c) we can calculate $\vec{p}_1$ and $\vec{p}_2$ and show that $\vec{P}$ as calculated in part (b) does equal $\vec{p}_1 + \vec{p}_2$.

**8.57.** **IDENTIFY:** There is no net external force on the system of James, Ramon, and the rope; the momentum of the system is conserved, and the velocity of its center of mass is constant. Initially there is no motion, and the velocity of the center of mass remains zero after Ramon has started to move.

**SET UP:** Let $+x$ be in the direction of Ramon's motion. Ramon has mass $m_R = 60.0 \text{ kg}$ and James has mass $m_J = 90.0 \text{ kg}$.

**EXECUTE:** $v_{cm\text{-}x} = \dfrac{m_R v_{Rx} + m_J v_{Jx}}{m_R + m_J} = 0$.

$$v_{Jx} = -\left(\frac{m_R}{m_J}\right)v_{Rx} = -\left(\frac{60.0 \text{ kg}}{90.0 \text{ kg}}\right)(1.10 \text{ m/s}) = -0.733 \text{ m/s. James' speed is } 0.733 \text{ m/s.}$$

**EVALUATE:** As they move, the two men have momenta that are equal in magnitude and opposite in direction, and the total momentum of the system is zero. Also, Example 8.14 shows that Ramon moves farther than James in the same time interval. This is consistent with Ramon having a greater speed.

**8.58.** **(a) IDENTIFY and SET UP:** Apply $y_{cm} = \dfrac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$ and solve for $m_1$ and $m_2$.

**EXECUTE:** $y_{cm} = \dfrac{m_1y_1 + m_2y_2}{m_1 + m_2}$

$$m_1 + m_2 = \frac{m_1y_1 + m_2y_2}{y_{cm}} = \frac{m_1(0) + (0.50 \text{ kg})(6.0 \text{ m})}{2.4 \text{ m}} = 1.25 \text{ kg and } m_1 = 0.75 \text{ kg}.$$

**EVALUATE:** $y_{cm}$ is closer to $m_1$ since $m_1 > m_2$.

**(b) IDENTIFY and SET UP:** Apply $\vec{a} = d\vec{v}/dt$ for the cm motion.

**EXECUTE:** $\vec{a}_{cm} = \dfrac{d\vec{v}_{cm}}{dt} = (1.5 \text{ m/s}^3)t\hat{i}$.

**(c) IDENTIFY and SET UP:** Apply $\sum \vec{F}_{ext} = M\vec{a}_{cm}$.

**EXECUTE:** $\sum \vec{F}_{ext} = M\vec{a}_{cm} = (1.25 \text{ kg})(1.5 \text{ m/s}^3)t\hat{i}$.

At $t = 3.0$ s, $\sum \vec{F}_{ext} = (1.25 \text{ kg})(1.5 \text{ m/s}^3)(3.0 \text{ s})\hat{i} = (5.6 \text{ N})\hat{i}$.

**EVALUATE:** $v_{cm\text{-}x}$ is positive and increasing so $a_{cm\text{-}x}$ is positive and $\vec{F}_{ext}$ is in the $+x$-direction. There is no motion and no force component in the $y$-direction.

**8.59.** **IDENTIFY:** Apply $\sum \vec{F} = \dfrac{d\vec{P}}{dt}$ to the airplane.

**SET UP:** $\dfrac{d}{dt}(t^n) = nt^{n-1}$. $1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$

**EXECUTE:** $\dfrac{d\vec{P}}{dt} = [-(1.50 \text{ kg} \cdot \text{m/s}^3)t]\,\hat{i} + (0.25 \text{ kg} \cdot \text{m/s}^2)\,\hat{j}$. $F_x = -(1.50 \text{ N/s})t$, $F_y = 0.25$ N, $F_z = 0$.

**EVALUATE:** There is no momentum or change in momentum in the $z$-direction and there is no force component in this direction.

**8.60.** **IDENTIFY:** Raising your leg changes the location of its center of mass and hence the location of your body's center of mass.

**SET UP:** The leg in each position is sketched in Figure 8.60. Use the coordinates shown. The mass of each part of the leg may be taken as concentrated at the center of that part. The location of the $x$-coordinate of the center of mass of two particles is $x_{cm} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$. and likewise for the $y$-coordinate.

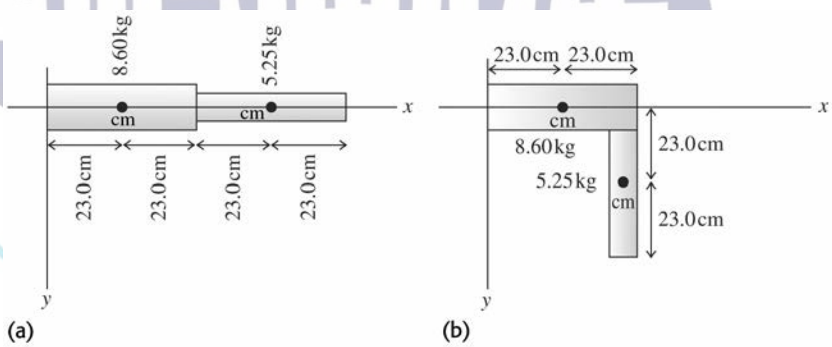



**Figure 8.60**

**EXECUTE:** **(a)** $y_{cm} = 0$, $x_{cm} = \dfrac{(23.0 \text{ cm})(8.60 \text{ kg}) + (69.0 \text{ cm})(5.25 \text{ kg})}{8.60 \text{ kg} + 5.25 \text{ kg}} = 40.4$ cm. The center of mass of the leg is a horizontal distance of 40.4 cm from the hip.

**(b)** $x_{cm} = \dfrac{(23.0 \text{ cm})(8.60 \text{ kg}) + (46.0 \text{ cm})(5.25 \text{ kg})}{8.60 \text{ kg} + 5.25 \text{ kg}} = 31.7 \text{ cm}$ and $y_{cm} = \dfrac{0 + (23.0 \text{ cm})(5.25 \text{ kg})}{8.60 \text{ kg} + 5.25 \text{ kg}} = 8.7$ cm.

The center of mass is a vertical distance of 8.7 cm below the hip and a horizontal distance of 31.7 cm from the hip.

**EVALUATE:** Since the body is not a rigid object, the location of its center of mass is not fixed.

**8.61.** **IDENTIFY:** $a = -\dfrac{v_{ex}}{m}\dfrac{dm}{dt}$. Assume that $dm/dt$ is constant over the 5.0 s interval, since $m$ doesn't change much during that interval. The thrust is $F = -v_{ex}\dfrac{dm}{dt}$.

**Set Up:** Take $m$ to have the constant value $110\text{ kg} + 70\text{ kg} = 180\text{ kg}$. $dm/dt$ is negative since the mass of the MMU decreases as gas is ejected.

**Execute:** **(a)** $\frac{dm}{dt} = -\frac{m}{v_{\text{ex}}}a = -\left(\frac{180\text{ kg}}{490\text{ m/s}}\right)(0.029\text{ m/s}^2) = -0.0106\text{ kg/s}$. In 5.0 s the mass that is ejected is $(0.0106\text{ kg/s})(5.0\text{ s}) = 0.053\text{ kg}$.

**(b)** $F = -v_{\text{ex}}\frac{dm}{dt} = -(490\text{ m/s})(-0.0106\text{ kg/s}) = 5.19\text{ N}$.

**Evaluate:** The mass change in the 5.0 s is a very small fraction of the total mass $m$, so it is accurate to take $m$ to be constant.

**8.62.** **Identify:** Use $F = -v_{\text{ex}}\frac{\Delta m}{\Delta t}$, applied to a finite time interval.

**Set Up:** $v_{\text{ex}} = 1600\text{ m/s}$

**Execute:** **(a)** $F = -v_{\text{ex}}\frac{\Delta m}{\Delta t} = -(1600\text{ m/s})\frac{-0.0500\text{ kg}}{1.00\text{ s}} = +80.0\text{ N}$.

**(b)** The absence of atmosphere would not prevent the rocket from operating. The rocket could be steered by ejecting the gas in a direction with a component perpendicular to the rocket's velocity and braked by ejecting it in a direction parallel (as opposed to antiparallel) to the rocket's velocity.

**Evaluate:** The thrust depends on the speed of the ejected gas relative to the rocket and on the mass of gas ejected per second.

**8.63.** **Identify** and **Set Up:** Use $v - v_0 = v_{\text{ex}}\ln(m_0/m)$.

$v_0 = 0$ ("fired from rest"), so $v/v_{\text{ex}} = \ln(m_0/m)$.

Thus $m_0/m = e^{v/v_{\text{ex}}}$, or $m/m_0 = e^{-v/v_{\text{ex}}}$.

If $v$ is the final speed then $m$ is the mass left when all the fuel has been expended; $m/m_0$ is the fraction of the initial mass that is not fuel.

**(a) Execute:** $v = 1.00\times10^{-3}c = 3.00\times10^5\text{ m/s}$ gives

$$m/m_0 = e^{-(3.00\times10^5\text{ m/s})/(2000\text{ m/s})} = 7.2\times10^{-66}.$$

**Evaluate:** This is clearly not feasible, for so little of the initial mass to not be fuel.

**(b) Execute:** $v = 3000\text{ m/s}$ gives $m/m_0 = e^{-(3000\text{ m/s})/(2000\text{ m/s})} = 0.223$.

**Evaluate:** 22.3% of the total initial mass not fuel, so 77.7% is fuel; this is possible.

**8.64.** **Identify:** Use the heights to find $v_{1y}$ and $v_{2y}$, the velocity of the ball just before and just after it strikes the slab. Then apply $J_y = F_y\Delta t = \Delta p_y$.

**Set Up:** Let $+y$ be downward.

**Execute:** **(a)** $\frac{1}{2}mv^2 = mgh$ so $v = \pm\sqrt{2gh}$.

$v_{1y} = +\sqrt{2(9.80\text{ m/s}^2)(2.00\text{ m})} = 6.26\text{ m/s}$. $v_{2y} = -\sqrt{2(9.80\text{ m/s}^2)(1.60\text{ m})} = -5.60\text{ m/s}$.

$$J_y = \Delta p_y = m(v_{2y} - v_{1y}) = (40.0\times10^{-3}\text{ kg})(-5.60\text{ m/s} - 6.26\text{ m/s}) = -0.474\text{ kg}\cdot\text{m/s}.$$

The impulse is $0.474\text{ kg}\cdot\text{m/s}$, upward.

**(b)** $F_y = \frac{J_y}{\Delta t} = \frac{-0.474\text{ kg}\cdot\text{m/s}}{2.00\times10^{-3}\text{ s}} = -237\text{ N}$. The average force on the ball is 237 N, upward.

**Evaluate:** The upward force, on the ball changes the direction of its momentum.

**8.65.** **Identify:** The impulse, force, and change in velocity are related by $J_x = F_x\Delta t$.

**Set Up:** $m = w/g = 0.0571\text{ kg}$. Since the force is constant, $\vec{F} = \vec{F}_{\text{av}}$.

**Execute:** **(a)** $J_x = F_x\Delta t = (-380\text{ N})(3.00\times10^{-3}\text{ s}) = -1.14\text{ N}\cdot\text{s}$.

$J_y = F_y\Delta t = (110\text{ N})(3.00\times10^{-3}\text{ s}) = 0.330\text{ N}\cdot\text{s}$.

**(b)** $v_{2x} = \dfrac{J_x}{m} + v_{1x} = \dfrac{-1.14\ \text{N}\cdot\text{s}}{0.0571\ \text{kg}} + 20.0\ \text{m/s} = 0.04\ \text{m/s}.$

$v_{2y} = \dfrac{J_y}{m} + v_{1y} = \dfrac{0.330\ \text{N}\cdot\text{s}}{0.0571\ \text{kg}} + (-4.0\ \text{m/s}) = +1.8\ \text{m/s}.$

**EVALUATE:** The change in velocity $\Delta\vec{v}$ is in the same direction as the force, so $\Delta\vec{v}$ has a negative $x$-component and a positive $y$-component.

**8.66.** **IDENTIFY:** The total momentum of the system is conserved and is equal to zero, since the pucks are released from rest.

**SET UP:** Each puck has the same mass $m$. Let $+x$ be east and $+y$ be north. Let object $A$ be the puck that moves west. All three pucks have the same speed $v$.

**EXECUTE:** $P_{1x} = P_{2x}$ gives $0 = -mv + mv_{Bx} + mv_{Cx}$ and $v = v_{Bx} + v_{Cx}$. $P_{1y} = P_{2y}$ gives $0 = mv_{By} + mv_{Cy}$ and $v_{By} = -v_{Cy}$. Since $v_B = v_C$ and the $y$-components are equal in magnitude, the $x$-components must also be equal: $v_{Bx} = v_{Cx}$ and $v = v_{Bx} + v_{Cx}$ says $v_{Bx} = v_{Cx} = v/2$. If $v_{By}$ is positive then $v_{Cy}$ is negative. The angle $\theta$ that puck $B$ makes with the $x$-axis is given by $\cos\theta = \dfrac{v/2}{v}$ and $\theta = 60°$. One puck moves in a direction $60°$ north of east and the other puck moves in a direction $60°$ south of east.

**EVALUATE:** Each component of momentum is separately conserved.

**8.67.** **IDENTIFY** and **SET UP:** When the spring is compressed the maximum amount the two blocks aren't moving relative to each other and have the same velocity $V$ relative to the surface. Apply conservation of momentum to find $V$ and conservation of energy to find the energy stored in the spring. Let $+x$ be the direction of the initial motion of $A$. The collision is elastic.

**SET UP:** $p = mv$, $K = ½\, mv^2$, $v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$ for an elastic collision.

**EXECUTE:** **(a)** The maximum energy stored in the spring is at maximum compression, at which time the blocks have the same velocity. Momentum conservation gives $m_A v_{A1} + m_B v_{B1} = (m_A + m_B)V$. Putting in the numbers we have (2.00 kg)(2.00 m/s) + (10.0 kg)(–0.500 m/s) = (12.0 kg)$V$, giving $V$ = –0.08333 m/s. The energy $U_{\text{spring}}$ stored in the spring is the loss of kinetic of the system. Therefore $U_{\text{spring}} = K_1 - K_2 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{V1}^2 - \frac{1}{2}(m_A + m_B)V^2$. Putting in the same set of numbers as above, and using $V$ = –0.08333 m/s, we get $U_{\text{spring}}$ = 5.21 J. At this time, the blocks are both moving to the left, so their velocities are each –0.0833 m/s.

**(b)** Momentum conservation gives $m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2}$. Putting in the numbers gives –1 m/s = $2v_{A2} + 10v_{B2}$. Using $v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$ we get $v_{B2x} - v_{A2x}$ = –(–0.500 m/s – 2.00 m/s) = +2.50 m/s. Solving this equation and the momentum equation simultaneously gives $v_{A2x}$ = 2.17 m/s and $v_{B2x}$ = 0.333 m/s.

**EVALUATE:** The total kinetic energy before the collision is 5.25 J, and it is the same after, which is consistent with an elastic collision.

**8.68.** **IDENTIFY:** Use a coordinate system attached to the ground. Take the $x$-axis to be east (along the tracks) and the $y$-axis to be north (parallel to the ground and perpendicular to the tracks). Then $P_x$ is conserved and $P_y$ is *not* conserved, due to the sideways force exerted by the tracks, the force that keeps the handcar on the tracks.

**(a) SET UP:** Let $A$ be the 25.0 kg mass and $B$ be the car (mass 175 kg). After the mass is thrown sideways relative to the car it still has the same eastward component of velocity, 5.00 m/s as it had before it was thrown.

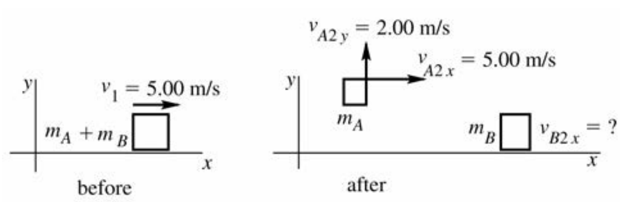



**Figure 8.68a**

$P_x$ is conserved so $(m_A + m_B)v_1 = m_A v_{A2x} + m_B v_{B2x}$

**EXECUTE:** $(200\text{ kg})(5.00\text{ m/s}) = (25.0\text{ kg})(5.00\text{ m/s}) + (175\text{ kg})v_{B2x}$.

$$v_{B2x} = \frac{1000\text{ kg}\cdot\text{m/s} - 125\text{ kg}\cdot\text{m/s}}{175\text{ kg}} = 5.00\text{ m/s}.$$

The final velocity of the car is 5.00 m/s, east (unchanged).

**EVALUATE:** The thrower exerts a force on the mass in the $y$-direction and by Newton's third law the mass exerts an equal and opposite force in the $-y$-direction on the thrower and car.

**(b) SET UP:** We are applying $P_x = \text{constant}$ in coordinates attached to the ground, so we need the final velocity of $A$ relative to the ground. Use the relative velocity addition equation. Then use $P_x = \text{constant}$ to find the final velocity of the car.

**EXECUTE:** $\vec{v}_{A/E} = \vec{v}_{A/B} + \vec{v}_{B/E}$

$v_{B/E} = +5.00$ m/s

$v_{A/B} = -5.00$ m/s (minus since the mass is moving west relative to the car). This gives $v_{A/E} = 0$; the mass is at rest relative to the earth after it is thrown backwards from the car.

As in part (a) $(m_A + m_B)v_1 = m_A v_{A2x} + m_B v_{B2x}$.

Now $v_{A2x} = 0$, so $(m_A + m_B)v_1 = m_B v_{B2x}$.

$$v_{B2x} = \left(\frac{m_A + m_B}{m_B}\right)v_1 = \left(\frac{200\text{ kg}}{175\text{ kg}}\right)(5.00\text{ m/s}) = 5.71\text{ m/s}.$$

The final velocity of the car is 5.71 m/s, east.

**EVALUATE:** The thrower exerts a force in the $-x$-direction so the mass exerts a force on him in the $+x$-direction, and he and the car speed up.

**(c) SET UP:** Let $A$ be the 25.0 kg mass and $B$ be the car (mass $m_B = 200$ kg).

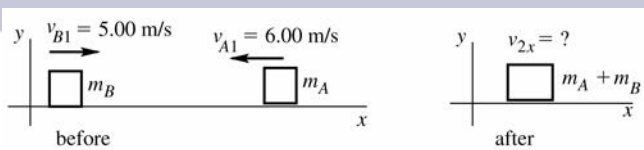



**Figure 8.68b**

$P_x$ is conserved so $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

**EXECUTE:** $-m_A v_{A1} + m_B v_{B1} = (m_A + m_B)v_{2x}$.

$$v_{2x} = \frac{m_B v_{B1} - m_A v_{A1}}{m_A + m_B} = \frac{(200\text{ kg})(5.00\text{ m/s}) - (25.0\text{ kg})(6.00\text{ m/s})}{200\text{ kg} + 25.0\text{ kg}} = 3.78\text{ m/s}.$$

The final velocity of the car is 3.78 m/s, east.

**EVALUATE:** The mass has negative $p_x$ so reduces the total $P_x$ of the system and the car slows down.

**8.69. IDENTIFY:** The $x$- and $y$-components of the momentum of the system are conserved.

**SET UP:** After the collision the combined object with mass $m_{\text{tot}} = 0.100$ kg moves with velocity $\vec{v}_2$. Solve for $v_{Cx}$ and $v_{Cy}$.

**EXECUTE:** **(a)** $P_{1x} = P_{2x}$ gives $m_A v_{Ax} + m_B v_{Bx} + m_C v_{Cx} = m_{\text{tot}} v_{2x}$.

$$v_{Cx} = -\frac{m_A v_{Ax} + m_B v_{Bx} - m_{\text{tot}} v_{2x}}{m_C}$$

$$v_{Cx} = -\frac{(0.020\text{ kg})(-1.50\text{ m/s}) + (0.030\text{ kg})(-0.50\text{ m/s})\cos 60° - (0.100\text{ kg})(0.50\text{ m/s})}{0.050\text{ kg}}.$$

$$v_{Cx} = 1.75\text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $m_A v_{Ay} + m_B v_{By} + m_C v_{Cy} = m_{\text{tot}} v_{2y}$.

$$v_{Cy} = -\frac{m_A v_{Ay} + m_B v_{By} - m_{\text{tot}} v_{2y}}{m_C} = -\frac{(0.030\text{ kg})(-0.50\text{ m/s})\sin 60°}{0.050\text{ kg}} = +0.260\text{ m/s}.$$

**(b)** $v_C = \sqrt{v_{Cx}^2 + v_{Cy}^2} = 1.77\text{ m/s}$. $\Delta K = K_2 - K_1$.

$\Delta K = \frac{1}{2}(0.100\text{ kg})(0.50\text{ m/s})^2 - [\frac{1}{2}(0.020\text{ kg})(1.50\text{ m/s})^2 + \frac{1}{2}(0.030\text{ kg})(0.50\text{ m/s})^2 + \frac{1}{2}(0.050\text{ kg})(1.77\text{ m/s})^2]$

$\Delta K = -0.092\text{ J}$.

**EVALUATE:** Since there is no horizontal external force the vector momentum of the system is conserved. The forces the spheres exert on each other do negative work during the collision and this reduces the kinetic energy of the system.

**8.70.** **IDENTIFY:** Each component of horizontal momentum is conserved.

**SET UP:** Let $+x$ be east and $+y$ be north. $v_{S1y} = v_{A1x} = 0$. $v_{S2x} = (6.00\text{ m/s})\cos 37.0° = 4.79\text{ m/s}$, $v_{S2y} = (6.00\text{ m/s})\sin 37.0° = 3.61\text{ m/s}$, $v_{A2x} = (9.00\text{ m/s})\cos 23.0° = 8.28\text{ m/s}$ and $v_{A2y} = -(9.00\text{ m/s})\sin 23.0° = -3.52\text{ m/s}$.

**EXECUTE:** $P_{1x} = P_{2x}$ gives $m_S v_{S1x} = m_S v_{S2x} + m_A v_{A2x}$.

$$v_{S1x} = \frac{m_S v_{S2x} + m_A v_{A2x}}{m_S} = \frac{(80.0\text{ kg})(4.79\text{ m/s}) + (50.0\text{ kg})(8.28\text{ m/s})}{80.0\text{ kg}} = 9.97\text{ m/s}.$$

Sam's speed before the collision was 9.97 m/s.

$P_{1y} = P_{2y}$ gives $m_A v_{A1y} = m_S v_{S2y} + m_A v_{A2y}$.

$$v_{A1y} = \frac{m_S v_{S2y} + m_A v_{A2y}}{m_S} = \frac{(80.0\text{ kg})(3.61\text{ m/s}) + (50.0\text{ kg})(-3.52\text{ m/s})}{50.0\text{ kg}} = 2.26\text{ m/s}.$$

Abigail's speed before the collision was 2.26 m/s.

**(b)** $\Delta K = \frac{1}{2}(80.0\text{ kg})(6.00\text{ m/s})^2 + \frac{1}{2}(50.0\text{ kg})(9.00\text{ m/s})^2 - \frac{1}{2}(80.0\text{ kg})(9.97\text{ m/s})^2 - \frac{1}{2}(50.0\text{ kg})(2.26\text{ m/s})^2$.

$\Delta K = -639\text{ J}$.

**EVALUATE:** The total momentum is conserved because there is no net external horizontal force. The kinetic energy decreases because the forces between the objects do negative work during the collision.

**8.71.** **IDENTIFY:** Momentum is conserved during the collision, and the wood (with the clay attached) is in free fall as it falls since only gravity acts on it.

**SET UP:** Apply conservation of momentum to the collision to find the velocity $V$ of the combined object just after the collision. After the collision, the wood's downward acceleration is $g$ and it has no horizontal acceleration, so we can use the standard kinematics equations: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ and $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$.

**EXECUTE:** Momentum conservation gives $(0.500\text{ kg})(24.0\text{ m/s}) = (8.50\text{ kg})V$, so $V = 1.412\text{ m/s}$. Consider the projectile motion after the collision: $a_y = +9.8\text{ m/s}^2$, $v_{0y} = 0$, $y - y_0 = +2.20\text{ m}$, and $t$ is unknown.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(2.20\text{ m})}{9.8\text{ m/s}^2}} = 0.6701\text{ s}$. The horizontal acceleration is zero so $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (1.412\text{ m/s})(0.6701\text{ s}) = 0.946\text{ m}$.

**EVALUATE:** The momentum is *not* conserved after the collision because an external force (gravity) acts on the system. Mechanical energy is *not* conserved during the collision because the clay and block stick together, making it an inelastic collision.

**8.72.** **IDENTIFY:** An inelastic collision (the objects stick together) occurs during which momentum is conserved, followed by a swing during which mechanical energy is conserved. The target variable is the initial speed of the bullet.

**Set Up:** Newton's second law, $\Sigma\vec{F} = m\vec{a}$, will relate the tension in the cord to the speed of the block during the swing. Mechanical energy is conserved after the collision, and momentum is conserved during the collision.

**Execute:** First find the speed $v$ of the block, at a height of 0.800 m. The mass of the combined object is 0.812 kg. $\cos\theta = \frac{0.8\text{ m}}{1.6\text{ m}} = 0.50$ so $\theta = 60.0^\circ$ is the angle the cord makes with the vertical. At this position, Newton's second law gives $T - mg\cos\theta = m\frac{v^2}{R}$, where we have taken force components toward the center of the circle. Solving for $v$ gives $v = \sqrt{\frac{R}{m}(T - mg\cos\theta)} = \sqrt{\frac{1.6\text{ m}}{0.812\text{ kg}}(4.80\text{ N} - 3.979\text{ N})} = 1.272\text{ m/s}.$ Now apply conservation of energy to find the velocity $V$ of the combined object just after the collision: $\frac{1}{2}mV^2 = mgh + \frac{1}{2}mv^2$. Solving for $V$ gives

$V = \sqrt{2gh + v^2} = \sqrt{2(9.8\text{ m/s}^2)(0.8\text{ m}) + (1.272\text{ m/s})^2} = 4.159\text{ m/s}.$ Now apply conservation of momentum to the collision: $(0.012\text{ kg})v_0 = (0.812\text{ kg})(4.159\text{ m/s})$, which gives $v_0 = 281\text{ m/s}.$

**Evaluate:** We cannot solve this problem in a single step because different conservation laws apply to the collision and the swing.

**8.73.** **Identify:** During the collision, momentum is conserved, but after the collision mechanical energy is conserved. We cannot solve this problem in a single step because the collision and the motion after the collision involve different conservation laws.

**Set Up:** Use coordinates where $+x$ is to the right and $+y$ is upward. Momentum is conserved during the collision, so $P_{1x} = P_{2x}$. Energy is conserved after the collision, so $K_1 = U_2$, where $K = \frac{1}{2}mv^2$ and $U = mgh$.

**Execute:** *Collision*: There is no external horizontal force during the collision so $P_{1x} = P_{2x}$. This gives $(5.00\text{ kg})(12.0\text{ m/s}) = (10.0\text{ kg})v_2$ and $v_2 = 6.0\text{ m/s}$.

*Motion after the collision*: Only gravity does work and the initial kinetic energy of the combined chunks is converted entirely to gravitational potential energy when the chunk reaches its maximum height $h$ above the valley floor. Conservation of energy gives $\frac{1}{2}m_{\text{tot}}v^2 = m_{\text{tot}}gh$ and $h = \frac{v^2}{2g} = \frac{(6.0\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 1.8\text{ m}.$

**Evaluate:** After the collision the energy of the system is $\frac{1}{2}m_{\text{tot}}v^2 = \frac{1}{2}(10.0\text{ kg})(6.0\text{ m/s})^2 = 180\text{ J}$ when it is all kinetic energy and the energy is $m_{\text{tot}}gh = (10.0\text{ kg})(9.8\text{ m/s}^2)(1.8\text{ m}) = 180\text{ J}$ when it is all gravitational potential energy. Mechanical energy is conserved during the motion after the collision. But before the collision the total energy of the system is $\frac{1}{2}(5.0\text{ kg})(12.0\text{ m/s})^2 = 360\text{ J}$; 50% of the mechanical energy is dissipated during the inelastic collision of the two chunks.

**8.74.** **Identify:** Momentum is conserved during the collision. After that we use energy conservation for $B$.

**Set Up:** $P_1 = P_2$ during the collision. For $B$, $K_1 + U_1 = K_2 + U_2$ after the collision.

**Execute:** For the collision, $P_1 = P_2$: (2.00 kg)(8.00 m/s) = (2.00 kg)(–2.00 m/s) + (4.00 kg)$v_B$, which gives $v_B$ = 5.00 m/s. Now look at $B$ after the collision and apply $K_1 + U_1 = K_2 + U_2$.

$K_1 + U_1 = K_2 + 0$: $\frac{1}{2} m v_B^2 + mgh = \frac{1}{2} mv^2$

$v^2 = (5.00\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(2.60\text{ m})$, which gives $v$ = 8.72 m/s.

**Evaluate:** We cannot do this problem in a single step because we have two different conservation laws involved: momentum during the collision and energy after the collision. The energy is not conserved during the collision, and the momentum of $B$ is not conserved after the collision.

**8.75.** **Identify:** The system initially has elastic potential energy in the spring. This will eventually be converted to kinetic energy by the spring. The spring produces only internal forces on the two-block system, so momentum is conserved. The spring force is conservative, so mechanical energy is conserved. Newton's second law applies.

**SET UP:** $K_1 + U_1 = K_2 + U_2$, $P_1 = P_2$, $p = mv$, $U_{el} = ½\, kx^2$, $F = kx$, $\Sigma\vec{F} = m\vec{a}$.

**EXECUTE:** **(a)** The spring exerts the same magnitude force on each block, so $F = kx = ma$, which gives $a = kx/m$. $a_A$ = (720 N/m)(0.225 m)/(1.00 kg) = 162 m/s$^2$. $a_B = kx/m$ = (720 N/m)?(0.225 m)/(3.00 kg) = 54.0 m/s$^2$.

**(b)** The initial momentum and kinetic energy are zero. After the blocks have separated from the spring, momentum conservation tells us that $0 = p_A - p_B$, which gives (1.00 kg)$v_A$ = (3.00 kg)$v_B$, so $v_A = 3v_B$.

Energy conservation gives $K_1 + U_1 = K_2 + U_2$, so $0 + ½\, kx^2 = K_A + K_B = \frac{1}{2}kx^2 = \frac{1}{2}m_Av_A^2 + \frac{1}{2}m_Bv_B^2$.

Substituting $v_A = 3v_B$ into this last equation and solving for $v_B$ gives $v_B$ = 1.74 m/s and $v_A$ = 5.23 m/s.

**EVALUATE:** The kinetic energy of $A$ is $\frac{1}{2}$ (1.00 kg)(5.23 m/s)$^2$ = 13.7 J, and the kinetic energy of $B$ is $\frac{1}{2}$ (3.00 kg)(1.74 m/s)$^2$ = 4.56 J. The two blocks do not share the energy equally, but they do have the same magnitude momentum.

**8.76.** **IDENTIFY:** During the inelastic collision, momentum is conserved but not mechanical energy. After the collision, momentum is not conserved and the kinetic energy of the cars is dissipated by nonconservative friction.

**SET UP:** Treat the collision and motion after the collision as separate events. Apply conservation of momentum to the collision and conservation of energy to the motion after the collision. The friction force on the combined cars is $\mu_k(m_A + m_B)g$.

**EXECUTE:** *Motion after the collision*: The kinetic energy of the combined cars immediately after the collision is taken away by the negative work done by friction: $\frac{1}{2}(m_A + m_B)V^2 = \mu_k(m_A + m_B)gd$, where $d = 7.15$ m. This gives $V = \sqrt{2\mu_k gd} = 9.54$ m/s.

*Collision*: Momentum conservation gives $m_Av_A = (m_A + m_B)V$, which gives

$$v_A = \left(\frac{m_A + m_B}{m_A}\right)V = \left(\frac{1500\text{ kg} + 1900\text{ kg}}{1500\text{ kg}}\right)(9.54\text{ m/s}) = 21.6\text{ m/s}.$$

**(b)** $v_A = 21.6$ m/s = 48 mph, which is 13 mph greater than the speed limit.

**EVALUATE:** We cannot solve this problem in a single step because the collision and the motion after the collision involve different principles (momentum conservation and energy conservation).

**8.77.** **IDENTIFY:** During the inelastic collision, momentum is conserved (in two dimensions), but after the collision we must use energy principles.

**SET UP:** The friction force is $\mu_k m_{tot} g$. Use energy considerations to find the velocity of the combined object immediately after the collision. Apply conservation of momentum to the collision. Use coordinates where $+x$ is west and $+y$ is south. For momentum conservation, we have $P_{1x} = P_{2x}$ and $P_{1y} = P_{2y}$.

**EXECUTE:** *Motion after collision*: The negative work done by friction takes away all the kinetic energy that the combined object has just after the collision. Calling $\phi$ the angle south of west at which the enmeshed cars slid, we have $\tan\phi = \frac{6.43\text{ m}}{5.39\text{ m}}$ and $\phi = 50.0°$. The wreckage slides 8.39 m in a direction $50.0°$ south of west. Energy conservation gives $\frac{1}{2}m_{tot}V^2 = \mu_k m_{tot} gd$, so

$V = \sqrt{2\mu_k gd} = \sqrt{2(0.75)(9.80\text{ m/s}^2)(8.39\text{ m})} = 11.1$ m/s. The velocity components are

$V_x = V\cos\phi = 7.13$ m/s; $V_y = V\sin\phi = 8.50$ m/s.

*Collision*: $P_{1x} = P_{2x}$ gives (2200 kg)$v_{SUV}$ = (1500 kg + 2200 kg)$V_x$ and $v_{SUV} = 12$ m/s. $P_{1y} = P_{2y}$ gives (1500 kg)$v_{sedan}$ = (1500 kg + 2200 kg)$V_y$ and $v_{sedan} = 21$ m/s.

**EVALUATE:** We cannot solve this problem in a single step because the collision and the motion after the collision involve different principles (momentum conservation and energy conservation).

**8.78.** **IDENTIFY:** Find $k$ for the spring from the forces when the frame hangs at rest, use constant acceleration equations to find the speed of the putty just before it strikes the frame, apply conservation of momentum to the collision between the putty and the frame, and then apply conservation of energy to the motion of the frame after the collision.

**SET UP:** Use the free-body diagram in Figure 8.78a for the frame when it hangs at rest on the end of the spring to find the force constant $k$ of the spring. Let $s$ be the amount the spring is stretched.

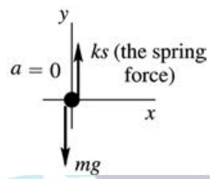



**Figure 8.78a**

**EXECUTE:** $\Sigma F_y = ma_y$ gives $-mg + ks = 0$. $k = \dfrac{mg}{s} = \dfrac{(0.150\text{ kg})(9.80\text{ m/s}^2)}{0.0400\text{ m}} = 36.75\text{ N/m}$.

**SET UP:** Next find the speed of the putty when it reaches the frame. The putty falls with acceleration $a = g$, downward (see Figure 8.78b).

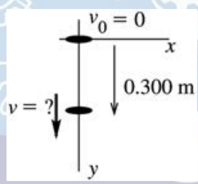



**Figure 8.78b**

$v_0 = 0$, $y - y_0 = 0.300$ m, $a = +9.80\text{ m/s}^2$, and we want to find $v$. The constant-acceleration $v^2 = v_0^2 + 2a(y - y_0)$ applies to this motion.

**EXECUTE:** $v = \sqrt{2a(y-y_0)} = \sqrt{2(9.80\text{ m/s}^2)(0.300\text{ m})} = 2.425\text{ m/s}$.

**SET UP:** Apply conservation of momentum to the collision between the putty ($A$) and the frame ($B$). See Figure 8.78c.

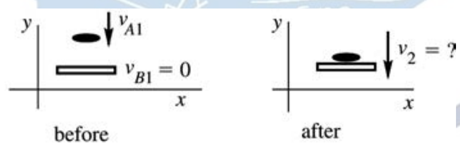



**Figure 8.78c**

$P_y$ is conserved, so $-m_A v_{A1} = -(m_A + m_B)v_2$.

**EXECUTE:** $v_2 = \left(\dfrac{m_A}{m_A + m_B}\right) v_{A1} = \left(\dfrac{0.200\text{ kg}}{0.350\text{ kg}}\right)(2.425\text{ m/s}) = 1.386\text{ m/s}$.

**SET UP:** Apply conservation of energy to the motion of the frame on the end of the spring after the collision. Let point 1 be just after the putty strikes and point 2 be when the frame has its maximum downward displacement. Let $d$ be the amount the frame moves downward (see Figure 8.78d).

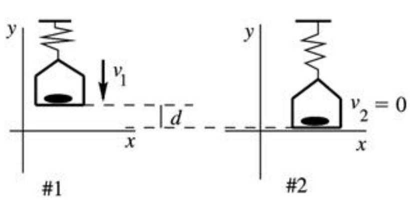



**Figure 8.78d**

When the frame is at position 1 the spring is stretched a distance $x_1 = 0.0400$ m. When the frame is at position 2 the spring is stretched a distance $x_2 = 0.040 \text{ m} + d$. Use coordinates with the $y$-direction upward and $y = 0$ at the lowest point reached by the frame, so that $y_1 = d$ and $y_2 = 0$. Work is done on the frame by gravity and by the spring force, so $W_{\text{other}} = 0$, and $U = U_{\text{el}} + U_{\text{gravity}}$.

**EXECUTE:** $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. $W_{\text{other}} = 0$.

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.350 \text{ kg})(1.386 \text{ m/s})^2 = 0.3362$ J.

$U_1 = U_{1,\text{el}} + U_{1,\text{grav}} = \frac{1}{2}kx_1^2 + mgy_1 = \frac{1}{2}(36.75 \text{ N/m})(0.0400 \text{ m})^2 + (0.350 \text{ kg})(9.80 \text{ m/s}^2)d$.

$U_1 = 0.02940 \text{ J} + (3.43 \text{ N})d$. $U_2 = U_{2,\text{el}} + U_{2,\text{grav}} = \frac{1}{2}kx_2^2 + mgy_2 = \frac{1}{2}(36.75 \text{ N/m})(0.0400 \text{ m} + d)^2$.

$U_2 = 0.02940 \text{ J} + (1.47 \text{ N})d + (18.375 \text{ N/m})d^2$. Thus

$0.3362 \text{ J} + 0.02940 \text{ J} + (3.43 \text{ N})d = 0.02940 \text{ J} + (1.47 \text{ N})d + (18.375 \text{ N/m})d^2$.

$(18.375 \text{ N/m})d^2 - (1.96 \text{ N})d - 0.3362 \text{ J} = 0$. Using the quadratic formula, with the positive solution, we get $d = 0.199$ m.

**EVALUATE:** The collision is inelastic and mechanical energy is lost. Thus the decrease in gravitational potential energy is *not* equal to the increase in potential energy stored in the spring.

**8.79.** **IDENTIFY:** Apply conservation of momentum to the collision and conservation of energy to the motion after the collision.

**SET UP:** Let $+x$ be to the right. The total mass is $m = m_{\text{bullet}} + m_{\text{block}} = 1.00$ kg. The spring has force constant $k = \dfrac{|F|}{|x|} = \dfrac{0.750 \text{ N}}{0.250 \times 10^{-2} \text{ m}} = 300$ N/m. Let $V$ be the velocity of the block just after impact.

**EXECUTE:** **(a)** Conservation of energy for the motion after the collision gives $K_1 = U_{\text{el}2}$. $\frac{1}{2}mV^2 = \frac{1}{2}kx^2$ and

$$V = x\sqrt{\frac{k}{m}} = (0.150 \text{ m})\sqrt{\frac{300 \text{ N/m}}{1.00 \text{ kg}}} = 2.60 \text{ m/s}.$$

**(b)** Conservation of momentum applied to the collision gives $m_{\text{bullet}}v_1 = mV$.

$$v_1 = \frac{mV}{m_{\text{bullet}}} = \frac{(1.00 \text{ kg})(2.60 \text{ m/s})}{8.00 \times 10^{-3} \text{ kg}} = 325 \text{ m/s}.$$

**EVALUATE:** The initial kinetic energy of the bullet is 422 J. The energy stored in the spring at maximum compression is 3.38 J. Most of the initial mechanical energy of the bullet is dissipated in the collision.

**8.80.** **IDENTIFY:** The horizontal components of momentum of the system of bullet plus stone are conserved. The collision is elastic if $K_1 = K_2$.

**SET UP:** Let $A$ be the bullet and $B$ be the stone.

**(a)**

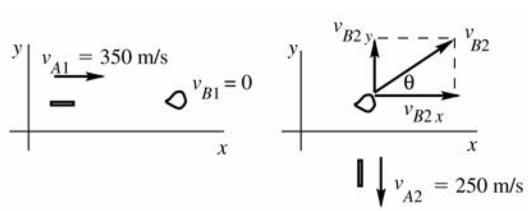



**Figure 8.80**

**EXECUTE:** $P_x$ is conserved so $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$.

$m_A v_{A1} = m_B v_{B2x}$.

$$v_{B2x} = \left(\frac{m_A}{m_B}\right)v_{A1} = \left(\frac{6.00 \times 10^{-3} \text{ kg}}{0.100 \text{ kg}}\right)(350 \text{ m/s}) = 21.0 \text{ m/s}$$

$P_y$ is conserved so $m_A v_{A1y} + m_B v_{B1y} = m_A v_{A2y} + m_B v_{B2y}$.

$0 = -m_A v_{A2} + m_B v_{B2y}$.

$$v_{B2y} = \left(\frac{m_A}{m_B}\right) v_{A2} = \left(\frac{6.00\times10^{-3}\text{ kg}}{0.100\text{ kg}}\right)(250\text{ m/s}) = 15.0\text{ m/s}.$$

$$v_{B2} = \sqrt{v_{B2x}^2 + v_{B2y}^2} = \sqrt{(21.0\text{ m/s})^2 + (15.0\text{ m/s})^2} = 25.8\text{ m/s}.$$

$\tan\theta = \dfrac{v_{B2y}}{v_{B2x}} = \dfrac{15.0\text{ m/s}}{21.0\text{ m/s}} = 0.7143;\quad \theta = 35.5°$ (defined in the sketch).

**(b)** To answer this question compare $K_1$ and $K_2$ for the system:

$$K_1 = \tfrac{1}{2}m_A v_{A1}^2 + \tfrac{1}{2}m_B v_{B1}^2 = \tfrac{1}{2}(6.00\times10^{-3}\text{ kg})(350\text{ m/s})^2 = 368\text{ J}.$$

$$K_2 = \tfrac{1}{2}m_A v_{A2}^2 + \tfrac{1}{2}m_B v_{B2}^2 = \tfrac{1}{2}(6.00\times10^{-3}\text{ kg})(250\text{ m/s})^2 + \tfrac{1}{2}(0.100\text{ kg})(25.8\text{ m/s})^2 = 221\text{ J}.$$

$$\Delta K = K_2 - K_1 = 221\text{ J} - 368\text{ J} = -147\text{ J}.$$

**EVALUATE:** The kinetic energy of the system decreases by 147 J as a result of the collision; the collision is *not* elastic. Momentum is conserved because $\Sigma \boldsymbol{F}_{\text{ext},x} = 0$ and $\Sigma \boldsymbol{F}_{\text{ext},y} = 0$. But there are internal forces between the bullet and the stone. These forces do negative work that reduces $K$.

**8.81.** **IDENTIFY:** Apply conservation of momentum to the collision between the two people. Apply conservation of energy to the motion of the stuntman before the collision and to the entwined people after the collision.

**SET UP:** For the motion of the stuntman, $y_1 - y_2 = 5.0$ m. Let $v_S$ be the magnitude of his horizontal velocity just before the collision. Let $V$ be the speed of the entwined people just after the collision. Let $d$ be the distance they slide along the floor.

**EXECUTE:** **(a)** Motion before the collision: $K_1 + U_1 = K_2 + U_2$. $K_1 = 0$ and $\tfrac{1}{2}mv_S^2 = mg(y_1 - y_2)$.

$v_S = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.80\text{ m/s}^2)(5.0\text{ m})} = 9.90\text{ m/s}.$

Collision: $m_S v_S = m_{\text{tot}} V$. $V = \dfrac{m_S}{m_{\text{tot}}} v_S = \left(\dfrac{80.0\text{ kg}}{150.0\text{ kg}}\right)(9.90\text{ m/s}) = 5.28\text{ m/s}.$

**(b)** Motion after the collision: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives $\tfrac{1}{2}m_{\text{tot}}V^2 - \mu_k m_{\text{tot}} g d = 0$.

$d = \dfrac{V^2}{2\mu_k g} = \dfrac{(5.28\text{ m/s})^2}{2(0.250)(9.80\text{ m/s}^2)} = 5.7\text{ m}.$

**EVALUATE:** Mechanical energy is dissipated in the inelastic collision, so the kinetic energy just after the collision is less than the initial potential energy of the stuntman.

**8.82.** **IDENTIFY:** Apply conservation of energy to the motion before and after the collision and apply conservation of momentum to the collision.

**SET UP:** Let $v$ be the speed of the mass released at the rim just before it strikes the second mass. Let each object have mass $m$.

**EXECUTE:** Conservation of energy says $\tfrac{1}{2}mv^2 = mgR$; $v = \sqrt{2gR}$.

**SET UP:** This is speed $v_1$ for the collision. Let $v_2$ be the speed of the combined object just after the collision.

**EXECUTE:** Conservation of momentum applied to the collision gives $mv_1 = 2mv_2$ so $v_2 = v_1/2 = \sqrt{gR/2}$.

**SET UP:** Apply conservation of energy to the motion of the combined object after the collision. Let $y_3$ be the final height above the bottom of the bowl.

**EXECUTE:** $\tfrac{1}{2}(2m)v_2^2 = (2m)gy_3$.

$y_3 = \dfrac{v_2^2}{2g} = \dfrac{1}{2g}\left(\dfrac{gR}{2}\right) = R/4.$

**EVALUATE:** Mechanical energy is lost in the collision, so the final gravitational potential energy is less than the initial gravitational potential energy.

**8.83.** **IDENTIFY:** Eqs. $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right) v_{A1x}$ and $v_{B2x} = \left(\dfrac{2m_A}{m_A + m_B}\right) v_{A1x}$ give the outcome of the elastic collision. Apply conservation of energy to the motion of the block after the collision.

**SET UP:** Object $B$ is the block, initially at rest. If $L$ is the length of the wire and $\theta$ is the angle it makes with the vertical, the height of the block is $y = L(1-\cos\theta)$. Initially, $y_1 = 0$.

**EXECUTE:** Eq. $v_{B2x} = \left(\dfrac{2m_A}{m_A + m_B}\right) v_{A1x}$ gives $v_B = \left(\dfrac{2m_A}{m_A + m_B}\right) v_A = \left(\dfrac{2M}{M + 3M}\right)(4.00 \text{ m/s}) = 2.00 \text{ m/s}$.

Conservation of energy gives $\frac{1}{2} m_B v_B^2 = m_B g L(1-\cos\theta)$.

$$\cos\theta = 1 - \frac{v_B^2}{2gL} = 1 - \frac{(2.00 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(0.500 \text{ m})} = 0.5918, \text{ which gives } \theta = 53.7°.$$

**EVALUATE:** Only a portion of the initial kinetic energy of the ball is transferred to the block in the collision.

**8.84.** **IDENTIFY:** Apply conservation of energy to the motion before and after the collision. Apply conservation of momentum to the collision.

**SET UP:** First consider the motion after the collision. The combined object has mass $m_{\text{tot}} = 25.0$ kg. Apply $\Sigma\vec{F} = m\vec{a}$ to the object at the top of the circular loop, where the object has speed $v_3$. The acceleration is $a_{\text{rad}} = v_3^2/R$, downward.

**EXECUTE:** $T + mg = m\dfrac{v_3^2}{R}$.

The minimum speed $v_3$ for the object not to fall out of the circle is given by setting $T = 0$. This gives $v_3 = \sqrt{Rg}$, where $R = 2.80$ m.

**SET UP:** Next, use conservation of energy with point 2 at the bottom of the loop and point 3 at the top of the loop. Take $y = 0$ at point 2. Only gravity does work, so $K_2 + U_2 = K_3 + U_3$

**EXECUTE:** $\frac{1}{2} m_{\text{tot}} v_2^2 = \frac{1}{2} m_{\text{tot}} v_3^2 + m_{\text{tot}} g(2R)$.

Use $v_3 = \sqrt{Rg}$ and solve for $v_2$: $v_2 = \sqrt{5gR} = 11.71$ m/s.

**SET UP:** Now apply conservation of momentum to the collision between the dart and the sphere. Let $v_1$ be the speed of the dart before the collision.

**EXECUTE:** $(5.00 \text{ kg})v_1 = (25.0 \text{ kg})(11.71 \text{ m/s})$, which gives $v_1 = 58.6$ m/s.

**EVALUATE:** The collision is inelastic and mechanical energy is removed from the system by the negative work done by the forces between the dart and the sphere.

**8.85.** **IDENTIFY:** Apply conservation of momentum to the collision between the bullet and the block and apply conservation of energy to the motion of the block after the collision.

**(a) SET UP:** For the collision between the bullet and the block, let object $A$ be the bullet and object $B$ be the block. Apply momentum conservation to find the speed $v_{B2}$ of the block just after the collision (see Figure 8.85a).

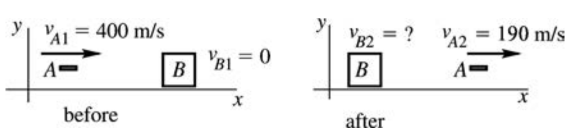



**Figure 8.85a**

**EXECUTE:** $P_x$ is conserved so $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$. $m_A v_{A1} = m_A v_{A2} + m_B v_{B2x}$.

$$v_{B2x} = \frac{m_A(v_{A1} - v_{A2})}{m_B} = \frac{4.00\times10^{-3} \text{ kg}(400 \text{ m/s} - 190 \text{ m/s})}{0.800 \text{ kg}} = 1.05 \text{ m/s}.$$

**SET UP:** For the motion of the block after the collision, let point 1 in the motion be just after the collision, where the block has the speed 1.05 m/s calculated above, and let point 2 be where the block has come to rest (see Figure 8.85b).

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2.$

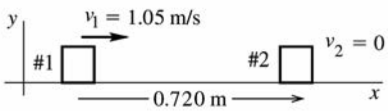



**Figure 8.85b**

**EXECUTE:** Work is done on the block by friction, so $W_{\text{other}} = W_f$.

$W_{\text{other}} = W_f = (f_k \cos\phi)s = -f_k s = -\mu_k mgs$, where $s = 0.720$ m. $U_1 = 0$, $U_2 = 0$, $K_1 = \frac{1}{2}mv_1^2$, $K_2 = 0$ (the block has come to rest). Thus $\frac{1}{2}mv_1^2 - \mu_k mgs = 0$. Therefore $\mu_k = \dfrac{v_1^2}{2gs} = \dfrac{(1.05\text{ m/s})^2}{2(9.80\text{ m/s}^2)(0.720\text{ m})} = 0.0781.$

**(b)** For the bullet, $K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(4.00\times10^{-3}\text{ kg})(400\text{ m/s})^2 = 320\text{ J}$ and

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(4.00\times10^{-3}\text{ kg})(190\text{ m/s})^2 = 72.2\text{ J}$. $\Delta K = K_2 - K_1 = 72.2\text{ J} - 320\text{ J} = -248\text{ J}$. The kinetic energy of the bullet decreases by 248 J.

**(c)** Immediately after the collision the speed of the block is 1.05 m/s, so its kinetic energy is

$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.800\text{ kg})(1.05\text{ m/s})^2 = 0.441\text{ J}.$

**EVALUATE:** The collision is highly inelastic. The bullet loses 248 J of kinetic energy but only 0.441 J is gained by the block. But momentum is conserved in the collision. All the momentum lost by the bullet is gained by the block.

**8.86.** **IDENTIFY:** Apply conservation of momentum to the collision and conservation of energy to the motion of the block after the collision.

**SET UP:** Let $+x$ be to the right. Let the bullet be $A$ and the block be $B$. Let $V$ be the velocity of the block just after the collision.

**EXECUTE:** *Motion of block after the collision*: $K_1 = U_{\text{grav2}}$. $\frac{1}{2}m_B V^2 = m_B gh$.

$$V = \sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(0.38\times10^{-2}\text{ m})} = 0.273\text{ m/s}.$$

*Collision*: $v_{B2} = 0.273$ m/s. $P_{1x} = P_{2x}$ gives $m_A v_{A1} = m_A v_{A2} + m_B v_{B2}$.

$$v_{A2} = \frac{m_A v_{A1} - m_B v_{B2}}{m_A} = \frac{(5.00\times10^{-3}\text{ kg})(450\text{ m/s}) - (1.00\text{ kg})(0.273\text{ m/s})}{5.00\times10^{-3}\text{ kg}} = 395\text{ m/s}.$$

**EVALUATE:** We assume the block moves very little during the time it takes the bullet to pass through it.

**8.87.** **IDENTIFY:** Apply conservation of energy to the motion of the package before the collision and apply conservation of the horizontal component of momentum to the collision.

**(a) SET UP:** Apply conservation of energy to the motion of the package from point 1 as it leaves the chute to point 2 just before it lands in the cart. Take $y = 0$ at point 2, so $y_1 = 4.00$ m. Only gravity does work, so

$$K_1 + U_1 = K_2 + U_2.$$

**EXECUTE:** $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2$.

$v_2 = \sqrt{v_1^2 + 2gy_1} = 9.35$ m/s.

**(b) SET UP:** In the collision between the package and the cart, momentum is conserved in the horizontal direction. (But not in the vertical direction, due to the vertical force the floor exerts on the cart.) Take $+x$ to be to the right. Let $A$ be the package and $B$ be the cart.

**EXECUTE:** $P_x$ is constant gives $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

$v_{B1x} = -5.00$ m/s.

$v_{A1x} = (3.00 \text{ m/s})\cos 37.0°$. (The horizontal velocity of the package is constant during its free fall.)

Solving for $v_{2x}$ gives $v_{2x} = -3.29$ m/s. The cart is moving to the left at 3.29 m/s after the package lands in it.

**EVALUATE:** The cart is slowed by its collision with the package, whose horizontal component of momentum is in the opposite direction to the motion of the cart.

**8.88.** **IDENTIFY:** Apply conservation of momentum to the system of the neutron and its decay products.

**SET UP:** Let the proton be moving in the $+x$-direction with speed $v_p$ after the decay. The initial momentum of the neutron is zero, so to conserve momentum the electron must be moving in the $-x$-direction after the decay. Let the speed of the electron be $v_e$.

**EXECUTE:** $P_{1x} = P_{2x}$ gives $0 = m_p v_p - m_e v_e$ and $v_e = \left(\dfrac{m_p}{m_e}\right)v_p$. The total kinetic energy after the decay is

$$K_{tot} = \tfrac{1}{2}m_e v_e^2 + \tfrac{1}{2}m_p v_p^2 = \tfrac{1}{2}m_e\left(\frac{m_p}{m_e}\right)^2 v_p^2 + \tfrac{1}{2}m_p v_p^2 = \tfrac{1}{2}m_p v_p^2\left(1+\frac{m_p}{m_e}\right).$$

Thus, $\dfrac{K_p}{K_{tot}} = \dfrac{1}{1+m_p/m_e} = \dfrac{1}{1+1836} = 5.44\times10^{-4} = 0.0544\%$.

**EVALUATE:** Most of the released energy goes to the electron, since it is much lighter than the proton.

**8.89.** **IDENTIFY:** The momentum of the system is conserved.

**SET UP:** Let $+x$ be to the right. $P_{1x} = 0$. $p_{ex}$, $p_{nx}$ and $p_{anx}$ are the momenta of the electron, polonium nucleus, and antineutrino, respectively.

**EXECUTE:** $P_{1x} = P_{2x}$ gives $p_{ex} + p_{nx} + p_{anx} = 0$. $p_{anx} = -(p_{ex} + p_{nx})$.

$$p_{anx} = -(5.60\times10^{-22} \text{ kg}\cdot\text{m/s} + [3.50\times10^{-25} \text{ kg}][-1.14\times10^{3} \text{ m/s}]) = -1.61\times10^{-22} \text{ kg}\cdot\text{m/s}.$$

The antineutrino has momentum to the left with magnitude $1.61\times10^{-22}$ kg·m/s.

**EVALUATE:** The antineutrino interacts very weakly with matter and most easily shows its presence by the momentum it carries away.

**8.90.** **IDENTIFY:** Since there is no friction, the horizontal component of momentum of the system of Jonathan, Jane, and the sleigh is conserved.

**SET UP:** Let $+x$ be to the right. $w_A = 800$ N, $w_B = 600$ N and $w_C = 1000$ N.

**EXECUTE:** $P_{1x} = P_{2x}$ gives $0 = m_A v_{A2x} + m_B v_{B2x} + m_C v_{C2x}$.

$$v_{C2x} = -\frac{m_A v_{A2x} + m_B v_{B2x}}{m_C} = -\frac{w_A v_{A2x} + w_B v_{B2x}}{w_C}.$$

$$v_{C2x} = -\frac{(800 \text{ N})[-(5.00 \text{ m/s})\cos 30.0°] + (600 \text{ N})[+(7.00 \text{ m/s})\cos 36.9°]}{1000 \text{ N}} = 0.105 \text{ m/s}.$$

The sleigh's velocity is 0.105 m/s, to the right.

**EVALUATE:** The vertical component of the momentum of the system consisting of the two people and the sleigh is not conserved, because of the net force exerted on the sleigh by the ice while they jump.

**8.91.** **IDENTIFY:** No net external force acts on the Burt-Ernie-log system, so the center of mass of the system does not move.

**SET UP:** $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1+m_2+m_3}$.

**EXECUTE:** Use coordinates where the origin is at Burt's end of the log and where $+x$ is toward Ernie, which makes $x_1 = 0$ for Burt initially. The initial coordinate of the center of mass is $x_{cm,1} = \dfrac{(20.0 \text{ kg})(1.5 \text{ m}) + (40.0 \text{ kg})(3.0 \text{ m})}{90.0 \text{ kg}}$. Let $d$ be the distance the log moves toward Ernie's original position. The final location of the center of mass is $x_{cm,2} = \dfrac{(30.0 \text{ kg})d + (1.5 \text{ kg} + d)(20.0 \text{ kg}) + (40.0 \text{ kg})d}{90.0 \text{ kg}}$.

The center of mass does not move, so $x_{cm,1} = x_{cm,2}$, which gives

$(20.0\text{ kg})(1.5\text{ m})+(40.0\text{ kg})(3.0\text{ m})=(30.0\text{ kg})d+(20.0\text{ kg})(1.5\text{ m}+d)+(40.0\text{ kg})d$. Solving for $d$ gives $d=1.33\text{ m}$.

**EVALUATE:** Burt, Ernie, and the log all move, but the center of mass of the system does not move.

**8.92.** **IDENTIFY:** There is no net horizontal external force so $v_{\text{cm}}$ is constant.

**SET UP:** Let $+x$ be to the right, with the origin at the initial position of the left-hand end of the canoe. $m_{\text{A}}=45.0\text{ kg}$, $m_B=60.0\text{ kg}$. The center of mass of the canoe is at its center.

**EXECUTE:** Initially, $v_{\text{cm}}=0$, so the center of mass doesn't move. Initially, $x_{\text{cm1}}=\dfrac{m_A x_{A1}+m_B x_{B1}}{m_A+m_B}$. After she walks, $x_{\text{cm2}}=\dfrac{m_A x_{A2}+m_B x_{B2}}{m_A+m_B}$. $x_{\text{cm1}}=x_{\text{cm2}}$ gives $m_A x_{A1}+m_B x_{B1}=m_A x_{A2}+m_B x_{B2}$. She walks to a point 1.00 m from the right-hand end of the canoe, so she is 1.50 m to the right of the center of mass of the canoe and $x_{A2}=x_{B2}+1.50\text{ m}$.

$$(45.0\text{ kg})(1.00\text{ m})+(60.0\text{ kg})(2.50\text{ m})=(45.0\text{ kg})(x_{B2}+1.50\text{ m})+(60.0\text{ kg})x_{B2}.$$

$(105.0\text{ kg})x_{B2}=127.5\text{ kg}\cdot\text{m}$ and $x_{B2}=1.21\text{ m}$. $x_{B2}-x_{B1}=1.21\text{ m}-2.50\text{ m}=-1.29\text{ m}$. The canoe moves 1.29 m to the left.

**EVALUATE:** When the woman walks to the right, the canoe moves to the left. The woman walks 3.00 m to the right relative to the canoe and the canoe moves 1.29 m to the left, so she moves $3.00\text{ m}-1.29\text{ m}=1.71\text{ m}$ to the right relative to the water. Note that this distance is $(60.0\text{ kg}/45.0\text{ kg})(1.29\text{ m})$.

**8.93.** **IDENTIFY:** Take as the system you and the slab. There is no horizontal force, so horizontal momentum is conserved. Since $\vec{P}=M\vec{v}_{\text{cm}}$, if $\vec{P}$ is constant, $\vec{v}_{\text{cm}}$ is constant (for a system of constant mass). Use coordinates fixed to the ice, with the direction you walk as the $x$-direction. $\vec{v}_{\text{cm}}$ is constant and initially $\vec{v}_{\text{cm}}=0$.

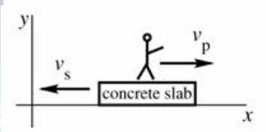



**Figure 8.93**

$$\vec{v}_{\text{cm}}=\frac{m_{\text{p}}\vec{v}_{\text{p}}+m_{\text{s}}\vec{v}_{\text{s}}}{m_{\text{p}}+m_{\text{s}}}=0.$$

$m_{\text{p}}\vec{v}_{\text{p}}+m_{\text{s}}\vec{v}_{\text{s}}=0.$

$m_{\text{p}}v_{\text{p}x}+m_{\text{s}}v_{\text{s}x}=0.$

$v_{\text{s}x}=-(m_{\text{p}}/m_{\text{s}})v_{\text{p}x}=-(m_{\text{p}}/5m_{\text{p}})2.00\text{ m/s}=-0.400\text{ m/s}.$

The slab moves at $0.400\text{ m/s}$, in the direction opposite to the direction you are walking.

**EVALUATE:** The initial momentum of the system is zero. You gain momentum in the $+x$-direction so the slab gains momentum in the $-x$-direction. The slab exerts a force on you in the $+x$-direction so you exert a force on the slab in the $-x$-direction.

**8.94.** **IDENTIFY:** The explosion produces only internal forces for the fragments, so the momentum of the two-fragment system is conserved. Therefore the explosion does not affect the motion of the center of mass of this system.

**SET UP:** The center of mass follows a parabolic path just as a single particle would. Its horizontal range is $R=\dfrac{v_0^2\sin(2\alpha)}{g}$. The center of mass of a two-particle system is $x_{\text{cm}}=\dfrac{m_1x_1+m_2x_2}{m_1+m_2}$.

**EXECUTE: (a)** The range formula gives $R=(18.0\text{ m/s})^2(\sin 102°)/(9.80\text{ m/s}^2)=32.34\text{ m}$, which rounds to 32.3 m.
**(b)** The center of mass is 32.3 m from the firing point and one fragment lands at $x_2=26.0\text{ m}$. Using the center of mass formula, with the origin at the firing point and calling $m$ the mass of each fragment, we have $32.34\text{ m}=[m(26.0\text{ m})+mx_2]/(2m)$, which gives $x_2=38.68\text{ m}$, which rounds to 38.7 m.

**EVALUATE:** Since the fragments have equal masses, their center of mass should be midway between them. So it should be at (26.0 m + 38.68 m)/2 = 32.3 m, which it is.

**8.95.** **IDENTIFY:** The explosion releases energy which goes into the kinetic energy of the two fragments. The explosive forces are internal to the two-fragment system, so momentum is conserved.

**SET UP:** Call the fragments $A$ and $B$, with $m_A = 2.0\text{ kg}$ and $m_B = 5.0\text{ kg}$. After the explosion fragment $A$ moves in the $+x$-direction with speed $\upsilon_A$ and fragment $B$ moves in the $-x$-direction with speed $\upsilon_B$.

**EXECUTE:** $P_{\text{i},x} = P_{\text{f},x}$ gives $0 = m_A v_A + m_B(-v_B)$ and $v_A = \left(\dfrac{m_B}{m_A}\right)v_B = \left(\dfrac{5.0\text{ kg}}{2.0\text{ kg}}\right)v_B = 2.5v_B$.

$\dfrac{K_A}{K_B} = \dfrac{\frac{1}{2}m_A v_A{}^2}{\frac{1}{2}m_B v_B{}^2} = \dfrac{\frac{1}{2}(2.0\text{ kg})(2.5v_B)^2}{\frac{1}{2}(5.0\text{ kg})v_B{}^2} = \dfrac{12.5}{5.0} = 2.5.$ $K_A = 100\text{ J}$ so $K_B = 250\text{ J}$.

**EVALUATE:** In an explosion the lighter fragment receives the most of the liberated energy, which agrees with our results here.

**8.96.** **IDENTIFY:** Conservation of $x$- and $y$-components of momentum applies to the collision. At the highest point of the trajectory the vertical component of the velocity of the projectile is zero.

**SET UP:** Let $+y$ be upward and $+x$ be horizontal and to the right. Let the two fragments be $A$ and $B$, each with mass $m$. For the projectile before the explosion and the fragments after the explosion. $a_x = 0$, $a_y = -9.80\text{ m/s}^2$.

**EXECUTE:** **(a)** $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ with $v_y = 0$ gives that the maximum height of the projectile is

$h = -\dfrac{v_{0y}^2}{2a_y} = -\dfrac{[(80.0\text{ m/s})\sin 60.0°]^2}{2(-9.80\text{ m/s}^2)} = 244.9\text{ m}.$ Just before the explosion the projectile is moving to the right with horizontal velocity $v_x = v_{0x} = v_0\cos 60.0° = 40.0\text{ m/s}$. After the explosion $v_{Ax} = 0$ since fragment $A$ falls vertically. Conservation of momentum applied to the explosion gives $(2m)(40.0\text{ m/s}) = mv_{Bx}$ and $v_{Bx} = 80.0\text{ m/s}$. Fragment $B$ has zero initial vertical velocity so $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives a time of fall of

$t = \sqrt{-\dfrac{2h}{a_y}} = \sqrt{-\dfrac{2(244.9\text{ m})}{-9.80\text{ m/s}^2}} = 7.07\text{ s}.$ During this time the fragment travels horizontally a distance $(80.0\text{ m/s})(7.07\text{ s}) = 566\text{ m}$. It also took the projectile 7.07 s to travel from launch to maximum height and during this time it travels a horizontal distance of $([80.0\text{ m/s}]\cos 60.0°)(7.07\text{ s}) = 283\text{ m}$. The second fragment lands $283\text{ m} + 566\text{ m} = 849\text{ m}$ from the firing point.

**(b)** For the explosion, $K_1 = \frac{1}{2}(20.0\text{ kg})(40.0\text{ m/s})^2 = 1.60\times10^4\text{ J}$.

$K_2 = \frac{1}{2}(10.0\text{ kg})(80.0\text{ m/s})^2 = 3.20\times10^4\text{ J}$. The energy released in the explosion is $1.60\times10^4\text{ J}$.

**EVALUATE:** The kinetic energy of the projectile just after it is launched is $6.40\times10^4\text{ J}$. We can calculate the speed of each fragment just before it strikes the ground and verify that the total kinetic energy of the fragments just before they strike the ground is $6.40\times10^4\text{ J} + 1.60\times10^4\text{ J} = 8.00\times10^4\text{ J}$. Fragment $A$ has speed 69.3 m/s just before it strikes the ground, and hence has kinetic energy $2.40\times10^4\text{ J}$. Fragment $B$ has speed $\sqrt{(80.0\text{ m/s})^2 + (69.3\text{ m/s})^2} = 105.8\text{ m/s}$ just before it strikes the ground, and hence has kinetic energy $5.60\times10^4\text{ J}$. Also, the center of mass of the system has the same horizontal range $R = \dfrac{v_0^2}{g}\sin(2\alpha_0) = 565\text{ m}$ that the projectile would have had if no explosion had occurred. One fragment lands at $R/2$ so the other, equal mass fragment lands at a distance $3R/2$ from the launch point.

**8.97.** **IDENTIFY:** The rocket moves in projectile motion before the explosion and its fragments move in projectile motion after the explosion. Apply conservation of energy and conservation of momentum to the explosion.

**(a) SET UP:** Apply conservation of energy to the explosion. Just before the explosion the rocket is at its maximum height and has zero kinetic energy. Let $A$ be the piece with mass 1.40 kg and $B$ be the piece with mass 0.28 kg. Let $v_A$ and $v_B$ be the speeds of the two pieces immediately after the collision.

**EXECUTE:** $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = 860\text{ J}$

**SET UP:** Since the two fragments reach the ground at the same time, their velocities just after the explosion must be horizontal. The initial momentum of the rocket before the explosion is zero, so after the explosion the pieces must be moving in opposite horizontal directions and have equal magnitude of momentum: $m_A v_A = m_B v_B$.

**EXECUTE:** Use this to eliminate $v_A$ in the first equation and solve for $v_B$:

$$\tfrac{1}{2}m_B v_B^2(1 + m_B/m_A) = 860\text{ J and } v_B = 71.6\text{ m/s}.$$

Then $v_A = (m_B/m_A)v_B = 14.3\text{ m/s}$.

**(b) SET UP:** Use the vertical motion from the maximum height to the ground to find the time it takes the pieces to fall to the ground after the explosion. Take $+y$ downward.

$$v_{0y} = 0,\quad a_y = +9.80\text{ m/s}^2,\quad y - y_0 = 80.0\text{ m},\quad t = ?$$

**EXECUTE:** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 4.04$ s.

During this time the horizontal distance each piece moves is $x_A = v_A t = 57.8\text{ m}$ and $x_B = v_B t = 289.1\text{ m}$. They move in opposite directions, so they are $x_A + x_B = 347\text{ m}$ apart when they land.

**EVALUATE:** Fragment $A$ has more mass so it is moving slower right after the collision, and it travels horizontally a smaller distance as it falls to the ground.

**8.98. IDENTIFY:** Apply conservation of momentum to the explosion. At the highest point of its trajectory the shell is moving horizontally. If one fragment received some upward momentum in the explosion, the other fragment would have had to receive a downward component. Since they each hit the ground at the same time, each must have zero vertical velocity immediately after the explosion.

**SET UP:** Let $+x$ be horizontal, along the initial direction of motion of the projectile and let $+y$ be upward. At its maximum height the projectile has $v_x = v_0\cos 55.0° = 86.0\text{ m/s}$. Let the heavier fragment be $A$ and the lighter fragment be $B$. $m_A = 9.00\text{ kg}$ and $m_B = 3.00\text{ kg}$.

**EXECUTE:** Since fragment $A$ returns to the launch point, immediately after the explosion it has $v_{Ax} = -86.0\text{ m/s}$. Conservation of momentum applied to the explosion gives $(12.0\text{ kg})(86.0\text{ m/s}) = (9.00\text{ kg})(-86.0\text{ m/s}) + (3.00\text{ kg})v_{Bx}$ and $v_{Bx} = 602\text{ m/s}$. The horizontal range of the projectile, if no explosion occurred, would be $R = \dfrac{v_0^2}{g}\sin(2\alpha_0) = 2157\text{ m}$. The horizontal distance each fragment travels is proportional to its initial speed and the heavier fragment travels a horizontal distance $R/2 = 1078\text{ m}$ after the explosion, so the lighter fragment travels a horizontal distance $\left(\dfrac{602\text{ m}}{86\text{ m}}\right)(1078\text{ m}) = 7546\text{ m}$ from the point of explosion and $1078\text{ m} + 7546\text{ m} = 8624\text{ m}$ from the launch point. The energy released in the explosion is

$K_2 - K_1 = \frac{1}{2}(9.00\text{ kg})(86.0\text{ m/s})^2 + \frac{1}{2}(3.00\text{ kg})(602\text{ m/s})^2 - \frac{1}{2}(12.0\text{ kg})(86.0\text{ m/s})^2 = 5.33\times 10^5\text{ J}$.

**EVALUATE:** The center of mass of the system has the same horizontal range $R = 2157$ m as if the explosion didn't occur. This gives $(12.0\text{ kg})(2157\text{ m}) = (9.00\text{ kg})(0) + (3.00\text{ kg})d$ and $d = 8630$ m, where $d$ is the distance from the launch point to where the lighter fragment lands. This agrees with our calculation.

**8.99. IDENTIFY:** Apply conservation of energy to the motion of the wagon before the collision. After the collision the combined object moves with constant speed on the level ground. In the collision the horizontal component of momentum is conserved.

**SET UP:** Let the wagon be object $A$ and treat the two people together as object $B$. Let $+x$ be horizontal and to the right. Let $V$ be the speed of the combined object after the collision.

**EXECUTE:** **(a)** The speed $v_{A1}$ of the wagon just before the collision is given by conservation of energy applied to the motion of the wagon prior to the collision. $U_1 = K_2$ says $m_A g([50\text{ m}][\sin 6.0°]) = \frac{1}{2} m_A v_{A1}^2$. $v_{A1} = 10.12$ m/s. $P_{1x} = P_{2x}$ for the collision says $m_A v_{A1} = (m_A + m_B)V$ and

$$V = \left(\frac{300\text{ kg}}{300\text{ kg} + 75.0\text{ kg} + 60.0\text{ kg}}\right)(10.12\text{ m/s}) = 6.98\text{ m/s}.$$ In 5.0 s the wagon travels $(6.98\text{ m/s})(5.0\text{ s}) = 34.9$ m, and the people will have time to jump out of the wagon before it reaches the edge of the cliff.

**(b)** For the wagon, $K_1 = \frac{1}{2}(300\text{ kg})(10.12\text{ m/s})^2 = 1.54 \times 10^4$ J. Assume that the two heroes drop from a small height, so their kinetic energy just before the wagon can be neglected compared to $K_1$ of the wagon. $K_2 = \frac{1}{2}(435\text{ kg})(6.98\text{ m/s})^2 = 1.06 \times 10^4$ J. The kinetic energy of the system decreases by $K_1 - K_2 = 4.8 \times 10^3$ J.

**EVALUATE:** The wagon slows down when the two heroes drop into it. The mass that is moving horizontally increases, so the speed decreases to maintain the same horizontal momentum. In the collision the vertical momentum is not conserved, because of the net external force due to the ground.

**8.100.** **IDENTIFY:** Impulse is equal to the area under the curve in a graph of force versus time.

**SET UP:** $J_x = \Delta p_x = F_x \Delta t$.

**EXECUTE:** **(a)** Impulse is the area under *F*-*t* curve
$J_x = [7500\text{ N} + ½\,(7500\text{ N} + 3500\text{ N}) + 3500\text{ N}](1.50\text{ s}) = 2.475 \times 10^4\ \text{N}\cdot\text{s}$.
**(b)** The total mass of the car and driver is $(3071\text{ lb})(4.448\text{ N/lb})/(9.80\text{ m/s}^2) = 1394$ kg.
$J_x = \Delta p_x = mv_x - 0$, so $v_x = J_x/m = (2.475 \times 10^4\ \text{N}\cdot\text{s})/(1394\text{ kg}) = 17.8$ m/s.
**(c)** The braking force must produce an impulse opposite to the one that accelerated the car, so $J_x = -2.475 \times 10^4\ \text{N}\cdot\text{s}$. Therefore $J_x = F_x \Delta t$ gives $\Delta t = J_x/F_x = (-24{,}750\ \text{N}\cdot\text{s})/(-5200\text{ N}) = 4.76$ s.
**(d)** $W_{\text{brake}} = \Delta K = -K = -½\, mv^2 = -½\,(1394\text{ kg})(17.76\text{ m/s})^2 = -2.20 \times 10^5$ J.
**(e)** $W_{\text{brake}} = -B_x s$, so $s = -W_{\text{brake}}/B_x = -(2.20 \times 10^5\text{ J})/(-5200\text{ N}) = 42.3$ m.

**EVALUATE:** The result in (e) could be checked by using kinematics with an average velocity of (17.8 m/s)/2 for 4.76 s.

**8.101.** **IDENTIFY:** As the bullet strikes and embeds itself in the block, momentum is conserved. After that, we use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, where $W_{\text{other}}$ is due to kinetic friction.

**SET UP:** Momentum conservation during the collision gives $m_b v_b = (m_b + m)V$, where $m$ is the mass of the block and $m_b$ is the mass of the bullet. After the collision, $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives $\frac{1}{2}MV^2 - \mu_k Mgd = \frac{1}{2}kd^2$, where $M$ is the mass of the block plus the bullet.

**EXECUTE:** **(a)** From the energy equation above, we can see that the greatest compression of the spring will occur for the greatest $V$ (since $M >> m_b$), and the greatest $V$ will occur for the bullet with the greatest initial momentum. Using the data in the table with the problem, we get the following momenta expressed in units of grain·ft/s.

A: $1.334 \times 10^5$ grain·ft/s   B: $1.181 \times 10^5$ grain·ft/s   C: $2.042 \times 10^5$ grain·ft/s

D: $1.638 \times 10^5$ grain·ft/s   E: $1.869 \times 10^5$ grain·ft/s

From these results, it is clear that bullet C will produce the maximum compression of the spring and bullet B will produce the least compression.

**(b)** For bullet C, we use $p_b = m_b v_b = (m_b + m)V$. Converting mass (in grains) and speed to SI units gives $m_b = 0.01555$ kg and $v_b = 259.38$ m/s, we have
$(0.01555\text{ kg})(259.38\text{ m/s}) = (0.01555\text{ kg} + 2.00\text{ kg})V$, so $V = 2.001$ m/s.

Now use $\frac{1}{2}MV^2 - \mu_k Mgd = \frac{1}{2}kd^2$ and solve for $k$, giving
$k = (2.016\text{ kg})[(2.001\text{ m/s})^2 - 2(0.38)(9.80\text{ m/s}^2)(0.25\text{ m})]/(0.25\text{ m})^2 = 69.1$ N/m, which rounds to 69 N/m.

**(c)** For bullet B, $m_b = 125$ grains $= 0.00810$ kg and $v_b = 945$ ft/s $= 288.0$ m/s. Momentum conservation gives $V = (0.00810\text{ kg})(288.0\text{ m/s})/(2.00810\text{ kg}) = 1.162$ m/s.

Using $\frac{1}{2}MV^2 - \mu_k Mgd = \frac{1}{2}kd^2$, the above numbers give $33.55d^2 + 7.478d - 1.356 = 0$. The quadratic formula, using the positive square root, gives $d = 0.118$ m, which rounds to 0.12 m.

**EVALUATE:** This method for measuring muzzle velocity involves a spring displacement of around 12 cm, which should be readily measurable.

**8.102.** **IDENTIFY** Momentum is conserved during the collision. After the collision, we can use energy methods.

**SET UP:** $p = mv$, $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, where $W_{\text{other}}$ is due to kinetic friction. We need to use components of momentum. Call $+x$ eastward and $+y$ northward.

**EXECUTE:** **(a)** Momentum conservation gives

$p_x = [(6500\text{ lb})/g]v_D = [(9542\text{ lb})/g]v_w\cos(39°)$

$p_y = [(3042\text{ lb})/g](50\text{ mph}) = [(9542\text{ lb})/g]v_w\sin(39°)$

Solving for $v_D$ gives $v_D = 28.9$ mph, which rounds to 29 mph.

**(b)** The above equations also give that the velocity of the wreckage just after impact is 25.3 mph = 37.1 ft/s.

Using $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, we have $\frac{1}{2}mv_1^2 - \mu_k mgd = \frac{1}{2}mv_2^2$. Solving for $v_2$ gives

$v_2 = \sqrt{v_1^2 - 2\mu_k gd}$. Using $v_1 = 37.1$ ft/s, $g = 32.2\text{ ft/s}^2$ and $d = 35$ ft, we get $v_2 = 19.1$ ft/s = 13 mph.

**EVALUATE:** We were able to minimize unit conversions by working in British units instead of SI units since the data was given in British units.

**8.103.** **IDENTIFY:** From our analysis of motion with constant acceleration, if $v = at$ and $a$ is constant, then $x - x_0 = v_0t + \frac{1}{2}at^2$.

**SET UP:** Take $v_0 = 0$, $x_0 = 0$ and let $+x$ downward.

**EXECUTE:** **(a)** $\frac{dv}{dt} = a$, $v = at$ and $x = \frac{1}{2}at^2$. Substituting into $xg = x\frac{dv}{dt} + v^2$ gives $\frac{1}{2}at^2g = \frac{1}{2}at^2a + a^2t^2 = \frac{3}{2}a^2t^2$. The nonzero solution is $a = g/3$.

**(b)** $x = \frac{1}{2}at^2 = \frac{1}{6}gt^2 = \frac{1}{6}(9.80\text{ m/s}^2)(3.00\text{ s})^2 = 14.7\text{ m}$.

**(c)** $m = kx = (2.00\text{ g/m})(14.7\text{ m}) = 29.4\text{ g}$.

**EVALUATE:** The acceleration is less than $g$ because the small water droplets are initially at rest, before they adhere to the falling drop. The small droplets are suspended by buoyant forces that we ignore for the raindrops.

**8.104.** **IDENTIFY** and **SET UP:** $dm = \rho dV$. $dV = Adx$. Since the thin rod lies along the $x$-axis, $y_{\text{cm}} = 0$. The mass of the rod is given by $M = \int dm$.

**EXECUTE:** **(a)** $x_{\text{cm}} = \frac{1}{M}\int_0^L xdm = \frac{\rho}{M}A\int_0^L xdx = \frac{\rho A}{M}\frac{L^2}{2}$. The volume of the rod is $AL$ and $M = \rho AL$.

$x_{\text{cm}} = \frac{\rho AL^2}{2\rho AL} = \frac{L}{2}$. The center of mass of the uniform rod is at its geometrical center, midway between its ends.

**(b)** $x_{\text{cm}} = \frac{1}{M}\int_0^L xdm = \frac{1}{M}\int_0^L x\rho Adx = \frac{A\alpha}{M}\int_0^L x^2dx = \frac{A\alpha L^3}{3M}$. $M = \int dm = \int_0^L \rho Adx = \alpha A\int_0^L xdx = \frac{\alpha AL^2}{2}$.

Therefore, $x_{\text{cm}} = \left(\frac{A\alpha L^3}{3}\right)\left(\frac{2}{\alpha AL^2}\right) = \frac{2L}{3}$.

**EVALUATE:** When the density increases with $x$, the center of mass is to the right of the center of the rod.

**8.105.** **IDENTIFY:** $x_{\text{cm}} = \frac{1}{M}\int xdm$ and $y_{\text{cm}} = \frac{1}{M}\int ydm$. At the upper surface of the plate, $y^2 + x^2 = a^2$.

**SET UP:** To find $x_{\text{cm}}$, divide the plate into thin strips parallel to the $y$-axis, as shown in Figure 8.105a. To find $y_{\text{cm}}$, divide the plate into thin strips parallel to the $x$-axis as shown in Figure 8.105b. The plate has volume one-half that of a circular disk, so $V = \frac{1}{2}\pi a^2t$ and $M = \frac{1}{2}\rho\pi a^2t$.

**EXECUTE:** In Figure 8.105a each strip has length $y = \sqrt{a^2 - x^2}$. $x_{cm} = \frac{1}{M}\int x dm$, where $dm = \rho t y dx = \rho t\sqrt{a^2 - x^2}dx$. $x_{cm} = \frac{\rho t}{M}\int_{-a}^{a} x\sqrt{a^2 - x^2}dx = 0$, since the integrand is an odd function of $x$. $x_{cm} = 0$ because of symmetry. In Figure 8.105b each strip has length $2x = 2\sqrt{a^2 - y^2}$. $y_{cm} = \frac{1}{M}\int y dm$, where $dm = 2\rho t x dy = 2\rho t\sqrt{a^2 - y^2}dy$. $y_{cm} = \frac{2\rho t}{M}\int_{-a}^{a} y\sqrt{a^2 - y^2}dy$. The integral can be evaluated using $u = a^2 - y^2$, $du = -2y dy$. This substitution gives

$$y_{cm} = \frac{2\rho t}{M}\left(-\frac{1}{2}\right)\int_{a^2}^{0} u^{1/2}du = \frac{2\rho t a^3}{3M} = \left(\frac{2\rho t a^3}{3}\right)\left(\frac{2}{\rho\pi a^2 t}\right) = \frac{4a}{3\pi}.$$

**EVALUATE:** $\frac{4}{3\pi} = 0.424$. $y_{cm}$ is less than $a/2$, as expected, since the plate becomes wider as $y$ decreases.

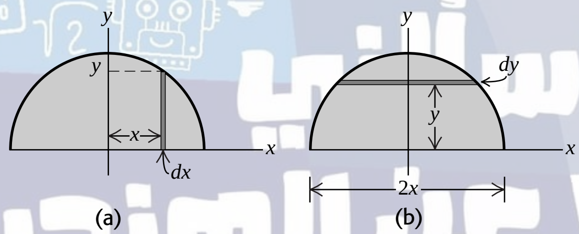



**Figure 8.105**

**8.106.** **IDENTIFY** and **SET UP:** $p = mv$.
**EXECUTE:** $p = mv = (0.30 \times 10^{-3}\text{ kg})(2.5\text{ m/s}) = 7.5 \times 10^{-4}\text{ kg}\cdot\text{m/s}$, which makes choice (a) correct.
**EVALUATE:** This is a small amount of momentum for a speed of 2.5 m/s, but the water drop is very light.

**8.107.** **IDENTIFY** and **SET UP:** Momentum is conserved, $p = mv$.
**EXECUTE:** $(65 \times 10^{-3}\text{ kg})v_{fish} = 7.5 \times 10^{-4}\text{ kg}\cdot\text{m/s}$, so $v_{fish} = 0.012$ m/s, which makes choice (b) correct.
**EVALUATE:** The fish is much ligher than the water drop and thus moves much slower.

**8.108.** **IDENTIFY** and **SET UP:** $J = F_{av}t = \Delta p$.
**EXECUTE:** $F_{av} = \Delta p/t = (7.5\times10^{-4}\text{ kg}\cdot\text{m/s})/(0.0050\text{ s}) = 0.15$ N, which is choice (d).
**EVALUATE:** This is a rather small force, but it acts on a very light-weight water drop, so it can give the water considerable speed.

**8.109.** **IDENTIFY** and **SET UP:** Momentum is conserved in the collision with the insect. $p = mv$.
**EXECUTE:** Using $P_1 = P_2$ gives $7.5 \times 10^{-4}\text{ kg}\cdot\text{m/s} = (m_{insect} + 3.0 \times 10^{-4}\text{ kg})(2.0\text{ m/s})$, which gives $m_{insect} = 0.075$ g, so choice (b) is correct.
**EVALUATE:** The insect has considerably less mass than the water drop.

# 9

## ROTATION OF RIGID BODIES

**9.1.** **IDENTIFY:** $s = r\theta$, with $\theta$ in radians.

**SET UP:** $\pi$ rad $= 180°$.

**EXECUTE:** **(a)** $\theta = \dfrac{s}{r} = \dfrac{1.50\text{ m}}{2.50\text{ m}} = 0.600\text{ rad} = 34.4°$

**(b)** $r = \dfrac{s}{\theta} = \dfrac{14.0\text{ cm}}{(128°)(\pi\text{ rad}/180°)} = 6.27\text{ cm}$

**(c)** $s = r\theta = (1.50\text{ m})(0.700\text{ rad}) = 1.05\text{ m}$

**EVALUATE:** An angle is the ratio of two lengths and is dimensionless. But, when $s = r\theta$ is used, $\theta$ must be in radians. Or, if $\theta = s/r$ is used to calculate $\theta$, the calculation gives $\theta$ in radians.

**9.2.** **IDENTIFY:** $\theta - \theta_0 = \omega t$, since the angular velocity is constant.

**SET UP:** 1 rpm $= (2\pi/60)$ rad/s.

**EXECUTE:** **(a)** $\omega = (1900)(2\pi\text{ rad}/60\text{ s}) = 199\text{ rad/s}$

**(b)** $35° = (35°)(\pi/180°) = 0.611\text{ rad}$. $t = \dfrac{\theta - \theta_0}{\omega} = \dfrac{0.611\text{ rad}}{199\text{ rad/s}} = 3.1\times 10^{-3}\text{ s}$

**EVALUATE:** In $t = \dfrac{\theta - \theta_0}{\omega}$ we must use the same angular measure (radians, degrees or revolutions) for both $\theta - \theta_0$ and $\omega$.

**9.3.** **IDENTIFY:** $\alpha_z(t) = \dfrac{d\omega_z}{dt}$. Using $\omega_z = d\theta/dt$ gives $\theta - \theta_0 = \int_{t_1}^{t_2} \omega_z dt$.

**SET UP:** $\dfrac{d}{dt}t^n = nt^{n-1}$ and $\int t^n dt = \dfrac{1}{n+1}t^{n+1}$

**EXECUTE:** **(a)** $A$ must have units of rad/s and $B$ must have units of $\text{rad/s}^3$.

**(b)** $\alpha_z(t) = 2Bt = (3.00\text{ rad/s}^3)t$. (i) For $t = 0$, $\alpha_z = 0$. (ii) For $t = 5.00$ s, $\alpha_z = 15.0\text{ rad/s}^2$.

**(c)** $\theta_2 - \theta_1 = \int_{t_1}^{t_2}(A + Bt^2)dt = A(t_2 - t_1) + \frac{1}{3}B(t_2^3 - t_1^3)$. For $t_1 = 0$ and $t_2 = 2.00$ s,

$\theta_2 - \theta_1 = (2.75\text{ rad/s})(2.00\text{ s}) + \frac{1}{3}(1.50\text{ rad/s}^3)(2.00\text{ s})^3 = 9.50\text{ rad}$.

**EVALUATE:** Both $\alpha_z$ and $\omega_z$ are positive and the angular speed is increasing.

**9.4.** **IDENTIFY:** $\alpha_z = d\omega_z/dt$. $\alpha_{\text{av-}z} = \dfrac{\Delta\omega_z}{\Delta t}$.

**SET UP:** $\dfrac{d}{dt}(t^2) = 2t$

**EXECUTE:** **(a)** $\alpha_z(t) = \dfrac{d\omega_z}{dt} = -2\beta t = (-1.60\text{ rad/s}^3)t$.

**(b)** $\alpha_z(3.0\text{ s}) = (-1.60\text{ rad/s}^3)(3.0\text{ s}) = -4.80\text{ rad/s}^2.$

$$\alpha_{\text{av-}z} = \frac{\omega_z(3.0\text{ s}) - \omega_z(0)}{3.0\text{ s}} = \frac{-2.20\text{ rad/s} - 5.00\text{ rad/s}}{3.0\text{ s}} = -2.40\text{ rad/s}^2,$$

which is half as large (in magnitude) as the acceleration at $t = 3.0$ s.

**EVALUATE:** $\alpha_z(t)$ increases linearly with time, so $\alpha_{\text{av-}z} = \dfrac{\alpha_z(0) + \alpha_z(3.0\text{ s})}{2}$. $\alpha_z(0) = 0$.

**9.5.** **IDENTIFY and SET UP:** Use $\omega_z = \dfrac{d\theta}{dt}$ to calculate the angular velocity and $\omega_{\text{av-}z} = \dfrac{\Delta\theta}{\Delta t} = \dfrac{\theta_2 - \theta_1}{t_2 - t_1}$ to calculate the average angular velocity for the specified time interval.

**EXECUTE:** $\theta = \gamma t + \beta t^3$; $\gamma = 0.400\text{ rad/s}$, $\beta = 0.0120\text{ rad/s}^3$

**(a)** $\omega_z = \dfrac{d\theta}{dt} = \gamma + 3\beta t^2$

**(b)** At $t = 0$, $\omega_z = \gamma = 0.400\text{ rad/s}$

**(c)** At $t = 5.00$ s, $\omega_z = 0.400\text{ rad/s} + 3(0.0120\text{ rad/s}^3)(5.00\text{ s})^2 = 1.30\text{ rad/s}$

$$\omega_{\text{av-}z} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

For $t_1 = 0$, $\theta_1 = 0$.

For $t_2 = 5.00$ s, $\theta_2 = (0.400\text{ rad/s})(5.00\text{ s}) + (0.012\text{ rad/s}^3)(5.00\text{ s})^3 = 3.50\text{ rad}$

So $\omega_{\text{av-}z} = \dfrac{3.50\text{ rad} - 0}{5.00\text{ s} - 0} = 0.700\text{ rad/s}.$

**EVALUATE:** The average of the instantaneous angular velocities at the beginning and end of the time interval is $\frac{1}{2}(0.400\text{ rad/s} + 1.30\text{ rad/s}) = 0.850\text{ rad/s}$. This is larger than $\omega_{\text{av-}z}$, because $\omega_z(t)$ is increasing faster than linearly.

**9.6.** **IDENTIFY:** $\omega_z(t) = \dfrac{d\theta}{dt}$. $\alpha_z(t) = \dfrac{d\omega_z}{dt}$. $\omega_{\text{av}-z} = \dfrac{\Delta\theta}{\Delta t}$.

**SET UP:** $\omega_z = (250\text{ rad/s}) - (40.0\text{ rad/s}^2)t - (4.50\text{ rad/s}^3)t^2$. $\alpha_z = -(40.0\text{ rad/s}^2) - (9.00\text{ rad/s}^3)t$.

**EXECUTE:** **(a)** Setting $\omega_z = 0$ results in a quadratic in *t*. The only positive root is $t = 4.23$ s.

**(b)** At $t = 4.23$ s, $\alpha_z = -78.1\text{ rad/s}^2$.

**(c)** At $t = 4.23$ s, $\theta = 586\text{ rad} = 93.3\text{ rev}$.

**(d)** At $t = 0$, $\omega_z = 250\text{ rad/s}$.

**(e)** $\omega_{\text{av-}z} = \dfrac{586\text{ rad}}{4.23\text{ s}} = 138\text{ rad/s}.$

**EVALUATE:** Between $t = 0$ and $t = 4.23$ s, $\omega_z$ decreases from 250 rad/s to zero. $\omega_z$ is not linear in *t*, so $\omega_{\text{av-}z}$ is not midway between the values of $\omega_z$ at the beginning and end of the interval.

**9.7.** **IDENTIFY:** $\omega_z(t) = \dfrac{d\theta}{dt}$. $\alpha_z(t) = \dfrac{d\omega_z}{dt}$. Use the values of $\theta$ and $\omega_z$ at $t = 0$ and $\alpha_z$ at 1.50 s to calculate *a*, *b*, and *c*.

**SET UP:** $\dfrac{d}{dt}t^n = nt^{n-1}$

**EXECUTE:** **(a)** $\omega_z(t) = b - 3ct^2$. $\alpha_z(t) = -6ct$. At $t = 0$, $\theta = a = \pi/4$ rad and $\omega_z = b = 2.00$ rad/s. At $t = 1.50$ s, $\alpha_z = -6c(1.50\text{ s}) = 1.25\text{ rad/s}^2$ and $c = -0.139\text{ rad/s}^3$.

**(b)** $\theta = \pi/4$ rad and $\alpha_z = 0$ at $t = 0$.

**(c)** $\alpha_z = 3.50\text{ rad/s}^2$ at $t = -\dfrac{\alpha_z}{6c} = -\dfrac{3.50\text{ rad/s}^2}{6(-0.139\text{ rad/s}^3)} = 4.20\text{ s}$. At $t = 4.20$ s,

$\theta = \frac{\pi}{4}\text{ rad} + (2.00\text{ rad/s})(4.20\text{ s}) - (-0.139\text{ rad/s}^3)(4.20\text{ s})^3 = 19.5\text{ rad}.$

$\omega_z = 2.00\text{ rad/s} - 3(-0.139\text{ rad/s}^3)(4.20\text{ s})^2 = 9.36\text{ rad/s}.$

**EVALUATE:** $\theta$, $\omega_z$, and $\alpha_z$ all increase as $t$ increases.

**9.8.** **IDENTIFY:** $\alpha_z = \frac{d\omega_z}{dt}$. $\theta - \theta_0 = \omega_{\text{av-}z}t$. When $\omega_z$ is linear in $t$, $\omega_{\text{av-}z}$ for the time interval $t_1$ to $t_2$ is $\omega_{\text{av-}z} = \frac{\omega_{z1} + \omega_{z2}}{t_2 - t_1}$.

**SET UP:** From the information given, $\alpha_z = \frac{\Delta\omega}{\Delta t} = \frac{4.00\text{ rad/s} - (-6.00\text{ rad/s})}{7.00\text{ s}} = 1.429\text{ rad/s}^2$.

$\omega_z(t) = -6.00\text{ rad/s} + (1.429\text{ rad/s}^2)t.$

**EXECUTE:** **(a)** The angular acceleration is positive, since the angular velocity increases steadily from a negative value to a positive value.

**(b)** It takes time $t = -\frac{\omega_{0z}}{\alpha_z} = -(-6.00\text{ rad/s})/(1.429\text{ rad/s}^2) = 4.20\text{ s}$ for the wheel to stop $(\omega_z = 0)$. During this time its speed is decreasing. For the next 2.80 s its speed is increasing from $0\text{ rad/s}$ to $+4.00\text{ rad/s}$.

**(c)** The average angular velocity is $\frac{-6.00\text{ rad/s} + 4.00\text{ rad/s}}{2} = -1.00\text{ rad/s}$. $\theta - \theta_0 = \omega_{\text{av-}z}t$ then leads to displacement of –7.00 rad after 7.00 s.

**EVALUATE:** When $\alpha_z$ and $\omega_z$ have the same sign, the angular speed is increasing; this is the case for $t = 4.20\text{ s}$ to $t = 7.00\text{ s}$. When $\alpha_z$ and $\omega_z$ have opposite signs, the angular speed is decreasing; this is the case between $t = 0$ and $t = 4.20\text{ s}$.

**9.9.** **IDENTIFY:** Apply the constant angular acceleration equations.

**SET UP:** Let the direction the wheel is rotating be positive.

**EXECUTE:** **(a)** $\omega_z = \omega_{0z} + \alpha_z t = 1.50\text{ rad/s} + (0.200\text{ rad/s}^2)(2.50\text{ s}) = 2.00\text{ rad/s}.$

**(b)** $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (1.50\text{ rad/s})(2.50\text{ s}) + \frac{1}{2}(0.200\text{ rad/s}^2)(2.50\text{ s})^2 = 4.38\text{ rad}.$

**EVALUATE:** $\theta - \theta_0 = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t = \left(\frac{1.50\text{ rad/s} + 2.00\text{ rad/s}}{2}\right)(2.50\text{ s}) = 4.38\text{ rad}$, the same as calculated with another equation in part (b).

**9.10.** **IDENTIFY:** Apply the constant angular acceleration equations to the motion of the fan.

**(a) SET UP:** $\omega_{0z} = (500\text{ rev/min})(1\text{ min}/60\text{ s}) = 8.333\text{ rev/s}$, $\omega_z = (200\text{ rev/min})(1\text{ min}/60\text{ s}) = 3.333\text{ rev/s}$, $t = 4.00\text{ s}$, $\alpha_z = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

**EXECUTE:** $\alpha_z = \frac{\omega_z - \omega_{0z}}{t} = \frac{3.333\text{ rev/s} - 8.333\text{ rev/s}}{4.00\text{ s}} = -1.25\text{ rev/s}^2$

$\theta - \theta_0 = ?$

$\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (8.333\text{ rev/s})(4.00\text{ s}) + \frac{1}{2}(-1.25\text{ rev/s}^2)(4.00\text{ s})^2 = 23.3\text{ rev}$

**(b) SET UP:** $\omega_z = 0$ (comes to rest); $\omega_{0z} = 3.333\text{ rev/s}$; $\alpha_z = -1.25\text{ rev/s}^2$; $t = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

**EXECUTE:** $t = \frac{\omega_z - \omega_{0z}}{\alpha_z} = \frac{0 - 3.333\text{ rev/s}}{-1.25\text{ rev/s}^2} = 2.67\text{ s}$

**EVALUATE:** The angular acceleration is negative because the angular velocity is decreasing. The average angular velocity during the 4.00 s time interval is 350 rev/min and $\theta - \theta_0 = \omega_{\text{av-}z}t$ gives $\theta - \theta_0 = 23.3\text{ rev}$, which checks.

**9.11.** **IDENTIFY:** Apply the constant angular acceleration equations to the motion. The target variables are $t$ and $\theta - \theta_0$.

**SET UP:** **(a)** $\alpha_z = 1.50\text{ rad/s}^2$; $\omega_{0z} = 0$ (starts from rest); $\omega_z = 36.0\text{ rad/s}$; $t = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

**EXECUTE:** $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{36.0\text{ rad/s} - 0}{1.50\text{ rad/s}^2} = 24.0\text{ s}$

**(b)** $\theta - \theta_0 = ?$

$\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = 0 + \frac{1}{2}(1.50\text{ rad/s}^2)(24.0\text{ s})^2 = 432\text{ rad}$

$\theta - \theta_0 = 432\text{ rad}(1\text{ rev}/2\pi\text{ rad}) = 68.8\text{ rev}$

**EVALUATE:** We could use $\theta - \theta_0 = \frac{1}{2}(\omega_z + \omega_{0z})t$ to calculate $\theta - \theta_0 = \frac{1}{2}(0 + 36.0\text{ rad/s})(24.0\text{ s}) = 432\text{ rad}$, which checks.

**9.12.** **IDENTIFY:** In part (b) apply the equation derived in part (a).

**SET UP:** Let the direction the propeller is rotating be positive.

**EXECUTE:** **(a)** Solving $\omega_z = \omega_{0z} + \alpha_z t$ for $t$ gives $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z}$. Rewriting $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$ as $\theta - \theta_0 = t(\omega_{0z} + \frac{1}{2}\alpha_z t)$ and substituting for $t$ gives

$$\theta - \theta_0 = \left(\frac{\omega_z - \omega_{0z}}{\alpha_z}\right)\left(\omega_{0z} + \tfrac{1}{2}(\omega_z - \omega_{0z})\right) = \frac{1}{\alpha_z}(\omega_z - \omega_{0z})\left(\frac{\omega_z + \omega_{0z}}{2}\right) = \frac{1}{2\alpha_z}(\omega_z^2 - \omega_{0z}^2),$$

which when rearranged gives $\omega_z^2 = \omega_{z0}^2 + 2\alpha_z(\theta - \theta_0)$.

**(b)** $\alpha_z = \frac{1}{2}\left(\dfrac{1}{\theta - \theta_0}\right)(\omega_z^2 - \omega_{0z}^2) = \frac{1}{2}\left(\dfrac{1}{7.00\text{ rad}}\right)((16.0\text{ rad/s})^2 - (12.0\text{ rad/s})^2) = 8.00\text{ rad/s}^2$

**EVALUATE:** We could also use $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ to calculate $t = 0.500\text{ s}$. Then $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = 8.00\text{ rad/s}^2$, which agrees with our results in part (b).

**9.13.** **IDENTIFY:** Use a constant angular acceleration equation and solve for $\omega_{0z}$.

**SET UP:** Let the direction of rotation of the flywheel be positive.

**EXECUTE:** $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$ gives

$$\omega_{0z} = \frac{\theta - \theta_0}{t} - \tfrac{1}{2}\alpha_z t = \frac{30.0\text{ rad}}{4.00\text{ s}} - \tfrac{1}{2}(2.25\text{ rad/s}^2)(4.00\text{ s}) = 3.00\text{ rad/s}.$$

**EVALUATE:** At the end of the 4.00 s interval, $\omega_z = \omega_{0z} + \alpha_z t = 12.0\text{ rad/s}$.

$$\theta - \theta_0 = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t = \left(\frac{3.00\text{ rad/s} + 12.0\text{ rad/s}}{2}\right)(4.00\text{ s}) = 30.0\text{ rad},$$ which checks.

**9.14.** **IDENTIFY:** Apply the constant angular acceleration equations.

**SET UP:** Let the direction of the rotation of the blade be positive. $\omega_{0z} = 0$.

**EXECUTE:** $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = \dfrac{\omega_z - \omega_{0z}}{t} = \dfrac{140\text{ rad/s} - 0}{6.00\text{ s}} = 23.3\text{ rad/s}^2$.

$$(\theta - \theta_0) = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t = \left(\frac{0 + 140\text{ rad/s}}{2}\right)(6.00\text{ s}) = 420\text{ rad}$$

**EVALUATE:** We could also use $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$. This equation gives

$\theta - \theta_0 = \frac{1}{2}(23.3\text{ rad/s}^2)(6.00\text{ s})^2 = 419\text{ rad}$, in agreement with the result obtained above.

**9.15.** **IDENTIFY:** Apply constant angular acceleration equations.

**SET UP:** Let the direction the flywheel is rotating be positive.

$\theta - \theta_0 = 200$ rev, $\omega_{0z} = 500$ rev/min $= 8.333$ rev/s, $t = 30.0$ s.

**EXECUTE:** **(a)** $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ gives $\omega_z = 5.00$ rev/s $= 300$ rpm

**(b)** Use the information in part (a) to find $\alpha_z$: $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = -0.1111\ \text{rev/s}^2$. Then $\omega_z = 0$, $\alpha_z = -0.1111\ \text{rev/s}^2$, $\omega_{0z} = 8.333$ rev/s in $\omega_z = \omega_{0z} + \alpha_z t$ gives $t = 75.0$ s and $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t$ gives $\theta - \theta_0 = 312$ rev.

**EVALUATE:** The mass and diameter of the flywheel are not used in the calculation.

**9.16.** **IDENTIFY:** Apply the constant angular acceleration equations separately to the time intervals 0 to 2.00 s and 2.00 s until the wheel stops.

**(a) SET UP:** Consider the motion from $t = 0$ to $t = 2.00$ s:

$\theta - \theta_0 = ?$; $\omega_{0z} = 24.0$ rad/s; $\alpha_z = 30.0\ \text{rad/s}^2$; $t = 2.00$ s

**EXECUTE:** $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (24.0\ \text{rad/s})(2.00\ \text{s}) + \frac{1}{2}(30.0\ \text{rad/s}^2)(2.00\ \text{s})^2$

$\theta - \theta_0 = 48.0\ \text{rad} + 60.0\ \text{rad} = 108\ \text{rad}$

Total angular displacement from $t = 0$ until stops: $108\ \text{rad} + 432\ \text{rad} = 540\ \text{rad}$

Note: At $t = 2.00$ s, $\omega_z = \omega_{0z} + \alpha_z t = 24.0\ \text{rad/s} + (30.0\ \text{rad/s}^2)(2.00\ \text{s}) = 84.0\ \text{rad/s}$; angular speed when breaker trips.

**(b) SET UP:** Consider the motion from when the circuit breaker trips until the wheel stops. For this calculation let $t = 0$ when the breaker trips.

$t = ?$; $\theta - \theta_0 = 432$ rad; $\omega_z = 0$; $\omega_{0z} = 84.0$ rad/s (from part (a))

$$\theta - \theta_0 = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t$$

**EXECUTE:** $t = \dfrac{2(\theta - \theta_0)}{\omega_{0z} + \omega_z} = \dfrac{2(432\ \text{rad})}{84.0\ \text{rad/s} + 0} = 10.3\ \text{s}$

The wheel stops 10.3 s after the breaker trips so $2.00\ \text{s} + 10.3\ \text{s} = 12.3\ \text{s}$ from the beginning.

**(c) SET UP:** $\alpha_z = ?$; consider the same motion as in part (b):

$\omega_z = \omega_{0z} + \alpha_z t$

**EXECUTE:** $\alpha_z = \dfrac{\omega_z - \omega_{0z}}{t} = \dfrac{0 - 84.0\ \text{rad/s}}{10.3\ \text{s}} = -8.16\ \text{rad/s}^2$

**EVALUATE:** The angular acceleration is positive while the wheel is speeding up and negative while it is slowing down. We could also use $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ to calculate

$\alpha_z = \dfrac{\omega_z^2 - \omega_{0z}^2}{2(\theta - \theta_0)} = \dfrac{0 - (84.0\ \text{rad/s})^2}{2(432\ \text{rad})} = -8.16\ \text{rad/s}^2$ for the acceleration after the breaker trips.

**9.17.** **IDENTIFY:** Apply Eq. (9.12) to relate $\omega_z$ to $\theta - \theta_0$.

**SET UP:** Establish a proportionality.

**EXECUTE:** From $\omega_z^2 = \omega_{z0}^2 + 2\alpha_z(\theta - \theta_0)$, with $\omega_{0z} = 0$, the number of revolutions is proportional to the square of the initial angular velocity, so tripling the initial angular velocity increases the number of revolutions by 9, to 9.00 rev.

**EVALUATE:** We don't have enough information to calculate $\alpha_z$; all we need to know is that it is constant.

**9.18.** **IDENTIFY:** The linear distance the elevator travels, its speed and the magnitude of its acceleration are equal to the tangential displacement, speed and acceleration of a point on the rim of the disk. $s = r\theta$, $v = r\omega$ and $a = r\alpha$. In these equations the angular quantities must be in radians.

**SET UP:** $1\ \text{rev} = 2\pi$ rad. $1\ \text{rpm} = 0.1047$ rad/s. $\pi\ \text{rad} = 180°$. For the disk, $r = 1.25$ m.

**EXECUTE:** **(a)** $v = 0.250$ m/s so $\omega = \frac{v}{r} = \frac{0.250 \text{ m/s}}{1.25 \text{ m}} = 0.200 \text{ rad/s} = 1.91 \text{ rpm}$.

**(b)** $a = \frac{1}{8}g = 1.225 \text{ m/s}^2$. $\alpha = \frac{a}{r} = \frac{1.225 \text{ m/s}^2}{1.25 \text{ m}} = 0.980 \text{ rad/s}^2$.

**(c)** $s = 3.25$ m. $\theta = \frac{s}{r} = \frac{3.25 \text{ m}}{1.25 \text{ m}} = 2.60 \text{ rad} = 149°$.

**EVALUATE:** When we use $s = r\theta$, $v = r\omega$ and $a_{\tan} = r\alpha$ to solve for $\theta$, $\omega$ and $\alpha$, the results are in rad, rad/s, and $\text{rad/s}^2$.

**9.19.** **IDENTIFY:** When the angular speed is constant, $\omega = \theta/t$. $v_{\tan} = r\omega$, $a_{\tan} = r\alpha$ and $a_{\text{rad}} = r\omega^2$. In these equations radians must be used for the angular quantities.

**SET UP:** The radius of the earth is $R_{\text{E}} = 6.37 \times 10^6$ m and the earth rotates once in 1 day = 86,400 s. The orbit radius of the earth is $1.50 \times 10^{11}$ m and the earth completes one orbit in 1 y $= 3.156 \times 10^7$ s. When $\omega$ is constant, $\omega = \theta/t$.

**EXECUTE:** **(a)** $\theta = 1 \text{ rev} = 2\pi$ rad in $t = 3.156 \times 10^7$ s. $\omega = \frac{2\pi \text{ rad}}{3.156 \times 10^7 \text{ s}} = 1.99 \times 10^{-7}$ rad/s.

**(b)** $\theta = 1 \text{ rev} = 2\pi$ rad in $t = 86{,}400$ s. $\omega = \frac{2\pi \text{ rad}}{86{,}400 \text{ s}} = 7.27 \times 10^{-5}$ rad/s

**(c)** $v = r\omega = (1.50 \times 10^{11} \text{ m})(1.99 \times 10^{-7} \text{ rad/s}) = 2.98 \times 10^4$ m/s.

**(d)** $v = r\omega = (6.37 \times 10^6 \text{ m})(7.27 \times 10^{-5} \text{ rad/s}) = 463$ m/s.

**(e)** $a_{\text{rad}} = r\omega^2 = (6.37 \times 10^6 \text{ m})(7.27 \times 10^{-5} \text{ rad/s})^2 = 0.0337 \text{ m/s}^2$. $a_{\tan} = r\alpha = 0$. $\alpha = 0$ since the angular velocity is constant.

**EVALUATE:** The tangential speeds associated with these motions are large even though the angular speeds are very small, because the radius for the circular path in each case is quite large.

**9.20.** **IDENTIFY:** Linear and angular velocities are related by $v = r\omega$. Use $\omega_z = \omega_{0z} + \alpha_z t$ to calculate $\alpha_z$.

**SET UP:** $\omega = v/r$ gives $\omega$ in rad/s.

**EXECUTE:** **(a)** $\frac{1.25 \text{ m/s}}{25.0 \times 10^{-3} \text{ m}} = 50.0$ rad/s, $\frac{1.25 \text{ m/s}}{58.0 \times 10^{-3} \text{ m}} = 21.6$ rad/s.

**(b)** $(1.25 \text{ m/s})(74.0 \text{ min})(60 \text{ s/min}) = 5.55$ km.

**(c)** $\alpha_z = \frac{21.55 \text{ rad/s} - 50.0 \text{ rad/s}}{(74.0 \text{ min})(60 \text{ s/min})} = -6.41 \times 10^{-3} \text{ rad/s}^2$.

**EVALUATE:** The width of the tracks is very small, so the total track length on the disc is huge.

**9.21.** **IDENTIFY:** Use constant acceleration equations to calculate the angular velocity at the end of two revolutions. $v = r\omega$.

**SET UP:** 2 rev $= 4\pi$ rad. $r = 0.200$ m.

**EXECUTE:** **(a)** $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$. $\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(3.00 \text{ rad/s}^2)(4\pi \text{ rad})} = 8.68$ rad/s.

$a_{\text{rad}} = r\omega^2 = (0.200 \text{ m})(8.68 \text{ rad/s})^2 = 15.1 \text{ m/s}^2$.

**(b)** $v = r\omega = (0.200 \text{ m})(8.68 \text{ rad/s}) = 1.74$ m/s. $a_{\text{rad}} = \frac{v^2}{r} = \frac{(1.74 \text{ m/s})^2}{0.200 \text{ m}} = 15.1 \text{ m/s}^2$.

**EVALUATE:** $r\omega^2$ and $v^2/r$ are completely equivalent expressions for $a_{\text{rad}}$.

**9.22.** **IDENTIFY:** $v = r\omega$ and $a_{\tan} = r\alpha$.

**SET UP:** The linear acceleration of the bucket equals $a_{\tan}$ for a point on the rim of the axle.

**EXECUTE:** **(a)** $v = R\omega$. 2.00 cm/s $= R\left(\frac{7.5 \text{ rev}}{\text{min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)$ gives $R = 2.55$ cm.

$D = 2R = 5.09$ cm.

**(b)** $a_{tan} = R\alpha$. $\alpha = \frac{a_{tan}}{R} = \frac{0.400\text{ m/s}^2}{0.0255\text{ m}} = 15.7\text{ rad/s}^2$.

**EVALUATE:** In $v = R\omega$ and $a_{tan} = R\alpha$, $\omega$ and $\alpha$ must be in radians.

**9.23.** **IDENTIFY** and **SET UP:** Use constant acceleration equations to find $\omega$ and $\alpha$ after each displacement. Use $a_{tan} = R\alpha$ and $a_{rad} = r\omega^2$ to find the components of the linear acceleration.

**EXECUTE:** **(a)** <u>at the start</u> $t = 0$

flywheel starts from rest so $\omega = \omega_{0z} = 0$

$a_{tan} = r\alpha = (0.300\text{ m})(0.600\text{ rad/s}^2) = 0.180\text{ m/s}^2$

$a_{rad} = r\omega^2 = 0$

$a = \sqrt{a_{rad}^2 + a_{tan}^2} = 0.180\text{ m/s}^2$

**(b)** $\underline{\theta - \theta_0 = 60°}$

$a_{tan} = r\alpha = 0.180\text{ m/s}^2$

Calculate $\omega$:

$\theta - \theta_0 = 60°(\pi\text{ rad}/180°) = 1.047\text{ rad}$; $\omega_{0z} = 0$; $\alpha_z = 0.600\text{ rad/s}^2$; $\omega_z = ?$

$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$

$\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(0.600\text{ rad/s}^2)(1.047\text{ rad})} = 1.121\text{ rad/s}$ and $\omega = \omega_z$.

Then $a_{rad} = r\omega^2 = (0.300\text{ m})(1.121\text{ rad/s})^2 = 0.377\text{ m/s}^2$.

$a = \sqrt{a_{rad}^2 + a_{tan}^2} = \sqrt{(0.377\text{ m/s}^2)^2 + (0.180\text{ m/s}^2)^2} = 0.418\text{ m/s}^2$

**(c)** $\underline{\theta - \theta_0 = 120°}$

$a_{tan} = r\alpha = 0.180\text{ m/s}^2$

Calculate $\omega$:

$\theta - \theta_0 = 120°(\pi\text{ rad}/180°) = 2.094\text{ rad}$; $\omega_{0z} = 0$; $\alpha_z = 0.600\text{ rad/s}^2$; $\omega_z = ?$

$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$

$\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(0.600\text{ rad/s}^2)(2.094\text{ rad})} = 1.585\text{ rad/s}$ and $\omega = \omega_z$.

Then $a_{rad} = r\omega^2 = (0.300\text{ m})(1.585\text{ rad/s})^2 = 0.754\text{ m/s}^2$.

$a = \sqrt{a_{rad}^2 + a_{tan}^2} = \sqrt{(0.754\text{ m/s}^2)^2 + (0.180\text{ m/s}^2)^2} = 0.775\text{ m/s}^2$.

**EVALUATE:** $\alpha$ is constant so $\alpha_{tan}$ is constant. $\omega$ increases so $a_{rad}$ increases.

**9.24.** **IDENTIFY:** Apply constant angular acceleration equations. $v = r\omega$. A point on the rim has both tangential and radial components of acceleration.

**SET UP:** $a_{tan} = r\alpha$ and $a_{rad} = r\omega^2$.

**EXECUTE:** **(a)** $\omega_z = \omega_{0z} + \alpha_z t = 0.250\text{ rev/s} + (0.900\text{ rev/s}^2)(0.200\text{ s}) = 0.430\text{ rev/s}$

(Note that since $\omega_{0z}$ and $\alpha_z$ are given in terms of revolutions, it's not necessary to convert to radians).

**(b)** $\omega_{av\text{-}z}\Delta t = (0.340\text{ rev/s})(0.2\text{ s}) = 0.068\text{ rev}$.

**(c)** Here, the conversion to radians must be made to use $v = r\omega$, and

$$v = r\omega = \left(\frac{0.750\text{ m}}{2}\right)(0.430\text{ rev/s})(2\pi\text{ rad/rev}) = 1.01\text{ m/s}.$$

**(d)** Combining $a_{\text{rad}} = r\omega^2$ and $a_{\tan} = R\alpha$,

$$a = \sqrt{a_{\text{rad}}^2 + a_{\tan}^2} = \sqrt{(\omega^2 r)^2 + (\alpha r)^2}.$$

$$a = \sqrt{\left[((0.430\text{ rev/s})(2\pi\text{ rad/rev}))^2(0.375\text{ m})\right]^2 + \left[(0.900\text{ rev/s}^2)(2\pi\text{ rad/rev})(0.375\text{ m})\right]^2}.$$

$a = 3.46\text{ m/s}^2$.

**EVALUATE:** If the angular acceleration is constant, $a_{\tan}$ is constant but $a_{\text{rad}}$ increases as $\omega$ increases.

**9.25. IDENTIFY:** Use $a_{\text{rad}} = r\omega^2$ and solve for $r$.

**SET UP:** $a_{\text{rad}} = r\omega^2$ so $r = a_{\text{rad}}/\omega^2$, where $\omega$ must be in rad/s

**EXECUTE:** $a_{\text{rad}} = 3000g = 3000(9.80\text{ m/s}^2) = 29{,}400\text{ m/s}^2$

$$\omega = (5000\text{ rev/min})\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 523.6\text{ rad/s}$$

Then $r = \dfrac{a_{\text{rad}}}{\omega^2} = \dfrac{29{,}400\text{ m/s}^2}{(523.6\text{ rad/s})^2} = 0.107\text{ m}.$

**EVALUATE:** The diameter is then 0.214 m, which is larger than 0.127 m, so the claim is *not* realistic.

**9.26. IDENTIFY:** $a_{\tan} = r\alpha$, $v = r\omega$ and $a_{\text{rad}} = v^2/r$. $\theta - \theta_0 = \omega_{\text{av-}z}t$.

**SET UP:** When $\alpha_z$ is constant, $\omega_{\text{av-}z} = \dfrac{\omega_{0z} + \omega_z}{2}$. Let the direction the wheel is rotating be positive.

**EXECUTE: (a)** $\alpha = \dfrac{a_{\tan}}{r} = \dfrac{-10.0\text{ m/s}^2}{0.200\text{ m}} = -50.0\text{ rad/s}^2$

**(b)** At $t = 3.00$ s, $v = 50.0$ m/s and $\omega = \dfrac{v}{r} = \dfrac{50.0\text{ m/s}}{0.200\text{ m}} = 250\text{ rad/s}$ and at $t = 0$,

$v = 50.0\text{ m/s} + (-10.0\text{ m/s}^2)(0 - 3.00\text{ s}) = 80.0\text{ m/s}$, $\omega = 400\text{ rad/s}$.

**(c)** $\omega_{\text{av-}z}t = (325\text{ rad/s})(3.00\text{ s}) = 975\text{ rad} = 155\text{ rev}$.

**(d)** $v = \sqrt{a_{\text{rad}}r} = \sqrt{(9.80\text{ m/s}^2)(0.200\text{ m})} = 1.40\text{ m/s}$. This speed will be reached at time $\dfrac{50.0\text{ m/s} - 1.40\text{ m/s}}{10.0\text{ m/s}^2} = 4.86\text{ s}$ after $t = 3.00$ s, or at $t = 7.86$ s. (There are many equivalent ways to do this calculation.)

**EVALUATE:** At $t = 0$, $a_{\text{rad}} = r\omega^2 = 3.20\times10^4\text{ m/s}^2$. At $t = 3.00$ s, $a_{\text{rad}} = 1.25\times10^4\text{ m/s}^2$. For $a_{\text{rad}} = g$ the wheel must be rotating more slowly than at 3.00 s so it occurs some time after 3.00 s.

**9.27. IDENTIFY:** $v = r\omega$ and $a_{\text{rad}} = r\omega^2 = v^2/r$.

**SET UP:** $2\pi$ rad = 1 rev, so $\pi$ rad/s = 30 rev/min.

**EXECUTE: (a)** $\omega r = (1250\text{ rev/min})\left(\dfrac{\pi\text{ rad/s}}{30\text{ rev/min}}\right)\left(\dfrac{12.7\times10^{-3}\text{ m}}{2}\right) = 0.831\text{ m/s}.$

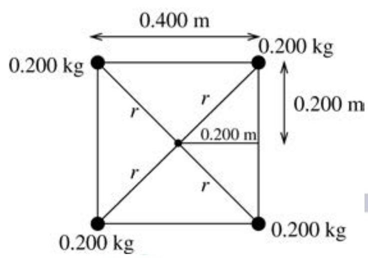
**(b)** $\dfrac{v^2}{r} = \dfrac{(0.831\text{ m/s})^2}{(12.7\times10^{-3}\text{ m})/2} = 109\text{ m/s}^2.$

**EVALUATE:** In $v = r\omega$, $\omega$ must be in rad/s.

**9.28. IDENTIFY and SET UP:** Use $I = \sum m_i r_i^2$. Treat the spheres as point masses and ignore $I$ of the light rods.

**EXECUTE:** The object is shown in Figure 9.28a.

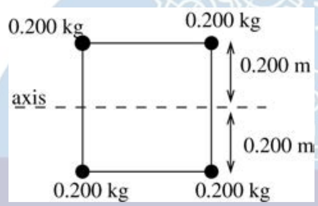
**(a)**




$$r=\sqrt{(0.200\text{ m})^2+(0.200\text{ m})^2}=0.2828\text{ m}$$
$$I=\sum m_i r_i^2=4(0.200\text{ kg})(0.2828\text{ m})^2$$
$$I=0.0640\text{ kg}\cdot\text{m}^2$$

**Figure 9.28a**

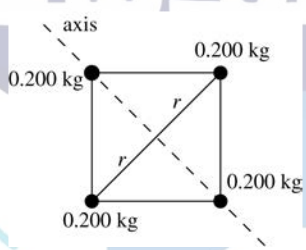
**(b)** The object is shown in Figure 9.28b.




$$r=0.200\text{ m}$$
$$I=\sum m_i r_i^2=4(0.200\text{ kg})(0.200\text{ m})^2$$
$$I=0.0320\text{ kg}\cdot\text{m}^2$$

**Figure 9.28b**

**(c)** The object is shown in Figure 9.28c.




$$r=0.2828\text{ m}$$
$$I=\sum m_i r_i^2=2(0.200\text{ kg})(0.2828\text{ m})^2$$
$$I=0.0320\text{ kg}\cdot\text{m}^2$$

**Figure 9.28c**

**EVALUATE:** In general $I$ depends on the axis and our answer for part (a) is larger than for parts (b) and (c). It just happens that $I$ is the same in parts (b) and (c).

**9.29.** **IDENTIFY:** Use Table 9.2. The correct expression to use in each case depends on the shape of the object and the location of the axis.

**SET UP:** In each case express the mass in kg and the length in m, so the moment of inertia will be in $\text{kg}\cdot\text{m}^2$.

**EXECUTE:** **(a)** (i) $I=\frac{1}{3}ML^2=\frac{1}{3}(2.50\text{ kg})(0.750\text{ m})^2=0.469\text{ kg}\cdot\text{m}^2$.

(ii) $I=\frac{1}{12}ML^2=\frac{1}{4}(0.469\text{ kg}\cdot\text{m}^2)=0.117\text{ kg}\cdot\text{m}^2$. (iii) For a very thin rod, all of the mass is at the axis and $I=0$.

**(b)** (i) $I=\frac{2}{5}MR^2=\frac{2}{5}(3.00\text{ kg})(0.190\text{ m})^2=0.0433\text{ kg}\cdot\text{m}^2$.

(ii) $I=\frac{2}{3}MR^2=\frac{5}{3}(0.0433\text{ kg}\cdot\text{m}^2)=0.0722\text{ kg}\cdot\text{m}^2$.

**(c)** (i) $I=MR^2=(8.00\text{ kg})(0.0600\text{ m})^2=0.0288\text{ kg}\cdot\text{m}^2$.

(ii) $I=\frac{1}{2}MR^2=\frac{1}{2}(8.00\text{ kg})(0.0600\text{ m})^2=0.0144\text{ kg}\cdot\text{m}^2$.

**EVALUATE:** $I$ depends on how the mass of the object is distributed relative to the axis.

**9.30.** **IDENTIFY:** Treat each block as a point mass, so for each block $I = mr^2$, where $r$ is the distance of the block from the axis. The total $I$ for the object is the sum of the $I$ for each of its pieces.

**SET UP:** In part (a) two blocks are a distance $L/2$ from the axis and the third block is on the axis. In part (b) two blocks are a distance $L/4$ from the axis and one is a distance $3L/4$ from the axis.

**EXECUTE:** **(a)** $I = 2m(L/2)^2 = \frac{1}{2}mL^2$.

**(b)** $I = 2m(L/4)^2 + m(3L/4)^2 = \dfrac{1}{16}mL^2(2+9) = \dfrac{11}{16}mL^2$.

**EVALUATE:** For the same object $I$ is in general different for different axes.

**9.31.** **IDENTIFY:** $I$ for the object is the sum of the values of $I$ for each part.

**SET UP:** For the bar, for an axis perpendicular to the bar, use the appropriate expression from Table 9.2. For a point mass, $I = mr^2$, where $r$ is the distance of the mass from the axis.

**EXECUTE:** **(a)** $I = I_{\text{bar}} + I_{\text{balls}} = \dfrac{1}{12}M_{\text{bar}}L^2 + 2m_{\text{balls}}\left(\dfrac{L}{2}\right)^2$.

$$I = \frac{1}{12}(4.00\text{ kg})(2.00\text{ m})^2 + 2(0.300\text{ kg})(1.00\text{ m})^2 = 1.93\text{ kg}\cdot\text{m}^2$$

**(b)** $I = \dfrac{1}{3}m_{\text{bar}}L^2 + m_{\text{ball}}L^2 = \dfrac{1}{3}(4.00\text{ kg})(2.00\text{ m})^2 + (0.300\text{ kg})(2.00\text{ m})^2 = 6.53\text{ kg}\cdot\text{m}^2$

**(c)** $I = 0$ because all masses are on the axis.

**(d)** All the mass is a distance $d = 0.500$ m from the axis and

$I = m_{\text{bar}}d^2 + 2m_{\text{ball}}d^2 = M_{\text{Total}}d^2 = (4.60\text{ kg})(0.500\text{ m})^2 = 1.15\text{ kg}\cdot\text{m}^2$.

**EVALUATE:** $I$ for an object depends on the location and direction of the axis.

**9.32.** **IDENTIFY:** Moment of inertia of a bar.

**SET UP:** $I_{\text{end}} = \dfrac{1}{3}ML^2$, $I_{\text{center}} = \dfrac{1}{12}ML^2$

**EXECUTE:** **(a)** $\dfrac{1}{12}ML^2 = (0.400\text{ kg})(0.600\text{ m})^2/12 = 0.0120\text{ kg}\cdot\text{m}^2$.

**(b)** Now we want the moment of inertia of two bars about their ends. Each has mass $M/2$ and length $L/2$.

$\dfrac{1}{3}ML^2 = \dfrac{1}{3}\left(\dfrac{M}{2}\right)\left(\dfrac{L}{2}\right)^2 + \dfrac{1}{3}\left(\dfrac{M}{2}\right)\left(\dfrac{L}{2}\right)^2 = \dfrac{1}{12}ML^2 = 0.0120\text{ kg}\cdot\text{m}^2$.

**EVALUATE:** Neither the bend nor the 60° angle affects the moment of inertia. In (a) and (b), we can think of the rod as two 0.200-kg rods, each 0.300 m long, with the moment of inertia calculated about one end.

**9.33.** **IDENTIFY** and **SET UP:** $I = \sum m_i r_i^2$ implies $I = I_{\text{rim}} + I_{\text{spokes}}$

**EXECUTE:** $I_{\text{rim}} = MR^2 = (1.40\text{ kg})(0.300\text{ m})^2 = 0.126\text{ kg}\cdot\text{m}^2$

Each spoke can be treated as a slender rod with the axis through one end, so

$I_{\text{spokes}} = 8(\frac{1}{3}ML^2) = \frac{8}{3}(0.280\text{ kg})(0.300\text{ m})^2 = 0.0672\text{ kg}\cdot\text{m}^2$

$I = I_{\text{rim}} + I_{\text{spokes}} = 0.126\text{ kg}\cdot\text{m}^2 + 0.0672\text{ kg}\cdot\text{m}^2 = 0.193\text{ kg}\cdot\text{m}^2$

**EVALUATE:** Our result is smaller than $m_{\text{tot}}R^2 = (3.64\text{ kg})(0.300\text{ m})^2 = 0.328\text{ kg}\cdot\text{m}^2$, since the mass of each spoke is distributed between $r = 0$ and $r = R$.

**9.34.** **IDENTIFY:** $K = \frac{1}{2}I\omega^2$. Use Table 9.2 to calculate $I$.

**SET UP:** $I = \frac{1}{12}ML^2$. $1\text{ rpm} = 0.1047\text{ rad/s}$

**EXECUTE:** **(a)** $I = \frac{1}{12}(117\text{ kg})(2.08\text{ m})^2 = 42.2\text{ kg}\cdot\text{m}^2$. $\omega = (2400\text{ rev/min})\left(\dfrac{0.1047\text{ rad/s}}{1\text{ rev/min}}\right) = 251\text{ rad/s}$.

$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(42.2\text{ kg}\cdot\text{m}^2)(251\text{ rad/s})^2 = 1.33\times10^6\text{ J}$.

**(b)** $K_1 = \frac{1}{12}M_1L_1^2\omega_1^2$, $K_2 = \frac{1}{12}M_2L_2^2\omega_2^2$. $L_1 = L_2$ and $K_1 = K_2$, so $M_1\omega_1^2 = M_2\omega_2^2$.

$$\omega_2 = \omega_1\sqrt{\frac{M_1}{M_2}} = (2400\text{ rpm})\sqrt{\frac{M_1}{0.750M_1}} = 2770\text{ rpm}$$

**EVALUATE:** The rotational kinetic energy is proportional to the square of the angular speed and directly proportional to the mass of the object.

**9.35.** **IDENTIFY:** $I$ for the compound disk is the sum of $I$ of the solid disk and of the ring.

**SET UP:** For the solid disk, $I = \frac{1}{2}m_\text{d}r_\text{d}^2$. For the ring, $I_\text{r} = \frac{1}{2}m_\text{r}(r_1^2 + r_2^2)$, where $r_1 = 50.0$ cm, $r_2 = 70.0$ cm. The mass of the disk and ring is their area times their area density.

**EXECUTE:** $I = I_\text{d} + I_\text{r}$.

Disk: $m_\text{d} = (3.00\text{ g/cm}^2)\pi r_\text{d}^2 = 23.56$ kg. $I_\text{d} = \frac{1}{2}m_\text{d}r_\text{d}^2 = 2.945\text{ kg}\cdot\text{m}^2$.

Ring: $m_\text{r} = (2.00\text{ g/cm}^2)\pi(r_2^2 - r_1^2) = 15.08$ kg. $I_\text{r} = \frac{1}{2}m_\text{r}(r_1^2 + r_2^2) = 5.580\text{ kg}\cdot\text{m}^2$.

$I = I_\text{d} + I_\text{r} = 8.52\text{ kg}\cdot\text{m}^2$.

**EVALUATE:** Even though $m_\text{r} < m_\text{d}$, $I_\text{r} > I_\text{d}$ since the mass of the ring is farther from the axis.

**9.36.** **IDENTIFY:** We can use angular kinematics (for constant angular acceleration) to find the angular velocity of the wheel. Then knowing its kinetic energy, we can find its moment of inertia, which is the target variable.

**SET UP:** $\theta - \theta_0 = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t$ and $K = \frac{1}{2}I\omega^2$.

**EXECUTE:** Converting the angle to radians gives $\theta - \theta_0 = (8.20\text{ rev})(2\pi\text{ rad/1 rev}) = 51.52$ rad.

$\theta - \theta_0 = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t$ gives $\omega_z = \frac{2(\theta - \theta_0)}{t} = \frac{2(51.52\text{ rad})}{12.0\text{ s}} = 8.587$ rad/s. Solving $K = \frac{1}{2}I\omega^2$ for $I$ gives

$$I = \frac{2K}{\omega^2} = \frac{2(36.0\text{ J})}{(8.587\text{ rad/s})^2} = 0.976\text{ kg}\cdot\text{m}^2.$$

**EVALUATE:** The angular velocity must be in radians to use the formula $K = \frac{1}{2}I\omega^2$.

**9.37.** **IDENTIFY:** Knowing the kinetic energy, mass and radius of the sphere, we can find its angular velocity. From this we can find the tangential velocity (the target variable) of a point on the rim.

**SET UP:** $K = \frac{1}{2}I\omega^2$ and $I = \frac{2}{5}MR^2$ for a solid uniform sphere. The tagential velocity is $v = r\omega$.

**EXECUTE:** $I = \frac{2}{5}MR^2 = \frac{2}{5}(28.0\text{ kg})(0.380\text{ m})^2 = 1.617\text{ kg}\cdot\text{m}^2$. $K = \frac{1}{2}I\omega^2$ so

$$\omega = \sqrt{\frac{2K}{I}} = \sqrt{\frac{2(236\text{ J})}{1.617\text{ kg}\cdot\text{m}^2}} = 17.085\text{ rad/s}.$$

$v = r\omega = (0.380\text{ m})(17.085\text{ rad/s}) = 6.49$ m/s.

**EVALUATE:** This is the speed of a point on the surface of the sphere that is farthest from the axis of rotation (the "equator" of the sphere). Points off the "equator" would have smaller tangential velocity but the same angular velocity.

**9.38.** **IDENTIFY:** Knowing the angular acceleration of the sphere, we can use angular kinematics (with constant angular acceleration) to find its angular velocity. Then using its mass and radius, we can find its kinetic energy, the target variable.

**SET UP:** $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$, $K = \frac{1}{2}I\omega^2$, and $I = \frac{2}{3}MR^2$ for a uniform hollow spherical shell.

**EXECUTE:** $I = \frac{2}{3}MR^2 = \frac{2}{3}(8.20\text{ kg})(0.220\text{ m})^2 = 0.2646\text{ kg}\cdot\text{m}^2$. Converting the angle to radians gives $\theta - \theta_0 = (6.00\text{ rev})(2\pi\text{ rad/1 rev}) = 37.70$ rad. The angular velocity is $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$, which gives

$$\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(0.890\text{ rad/s}^2)(37.70\text{ rad})} = 8.192\text{ rad/s}.$$

$K = \frac{1}{2}(0.2646\ \text{kg}\cdot\text{m}^2)(8.192\ \text{rad/s})^2 = 8.88\ \text{J}.$

**EVALUATE:** The angular velocity must be in radians to use the formula $K = \frac{1}{2}I\omega^2$.

**9.39.** **IDENTIFY:** $K = \frac{1}{2}I\omega^2$, with $\omega$ in rad/s. Solve for $I$.

**SET UP:** $1\ \text{rev/min} = (2\pi/60)\ \text{rad/s}$. $\Delta K = -500\ \text{J}$

**EXECUTE:** $\omega_\text{i} = 650\ \text{rev/min} = 68.1\ \text{rad/s}$. $\omega_\text{f} = 520\ \text{rev/min} = 54.5\ \text{rad/s}$. $\Delta K = K_\text{f} - K_\text{i} = \frac{1}{2}I(\omega_\text{f}^2 - \omega_\text{i}^2)$

and $I = \dfrac{2(\Delta K)}{\omega_\text{f}^2 - \omega_\text{i}^2} = \dfrac{2(-500\ \text{J})}{(54.5\ \text{rad/s})^2 - (68.1\ \text{rad/s})^2} = 0.600\ \text{kg}\cdot\text{m}^2.$

**EVALUATE:** In $K = \frac{1}{2}I\omega^2$, $\omega$ must be in rad/s.

**9.40.** **IDENTIFY:** $K = \frac{1}{2}I\omega^2$. Use Table 9.2 to relate $I$ to the mass $M$ of the disk.

**SET UP:** $45.0\ \text{rpm} = 4.71\ \text{rad/s}$. For a uniform solid disk, $I = \frac{1}{2}MR^2$.

**EXECUTE:** **(a)** $I = \dfrac{2K}{\omega^2} = \dfrac{2(0.250\ \text{J})}{(4.71\ \text{rad/s})^2} = 0.0225\ \text{kg}\cdot\text{m}^2.$

**(b)** $I = \frac{1}{2}MR^2$ and $M = \dfrac{2I}{R^2} = \dfrac{2(0.0225\ \text{kg}\cdot\text{m}^2)}{(0.300\ \text{m})^2} = 0.500\ \text{kg}.$

**EVALUATE:** No matter what the shape is, the rotational kinetic energy is proportional to the mass of the object.

**9.41.** **IDENTIFY** and **SET UP:** Combine $K = \frac{1}{2}I\omega^2$ and $a_\text{rad} = r\omega^2$ to solve for $K$. Use Table 9.2 to get $I$.

**EXECUTE:** $K = \frac{1}{2}I\omega^2$

$a_\text{rad} = R\omega^2$, so $\omega = \sqrt{a_\text{rad}/R} = \sqrt{(3500\ \text{m/s}^2)/1.20\ \text{m}} = 54.0\ \text{rad/s}$

For a disk, $I = \frac{1}{2}MR^2 = \frac{1}{2}(70.0\ \text{kg})(1.20\ \text{m})^2 = 50.4\ \text{kg}\cdot\text{m}^2$

Thus $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(50.4\ \text{kg}\cdot\text{m}^2)(54.0\ \text{rad/s})^2 = 7.35\times10^4\ \text{J}$

**EVALUATE:** The limit on $a_\text{rad}$ limits $\omega$ which in turn limits $K$.

**9.42.** **IDENTIFY:** The work done on the cylinder equals its gain in kinetic energy.

**SET UP:** The work done on the cylinder is $PL$, where $L$ is the length of the rope. $K_1 = 0$. $K_2 = \frac{1}{2}I\omega^2$.

$I = mr^2 = \left(\dfrac{w}{g}\right)r^2.$

**EXECUTE:** $PL = \dfrac{1}{2}\dfrac{w}{g}v^2$, or $P = \dfrac{1}{2}\dfrac{w}{g}\dfrac{v^2}{L} = \dfrac{(40.0\ \text{N})(6.00\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)(5.00\ \text{m})} = 14.7\ \text{N}.$

**EVALUATE:** The linear speed $v$ of the end of the rope equals the tangential speed of a point on the rim of the cylinder. When $K$ is expressed in terms of $v$, the radius $r$ of the cylinder doesn't appear.

**9.43.** **IDENTIFY:** Apply conservation of energy to the system of stone plus pulley. $v = r\omega$ relates the motion of the stone to the rotation of the pulley.

**SET UP:** For a uniform solid disk, $I = \frac{1}{2}MR^2$. Let point 1 be when the stone is at its initial position and point 2 be when it has descended the desired distance. Let $+y$ be upward and take $y = 0$ at the initial position of the stone, so $y_1 = 0$ and $y_2 = -h$, where $h$ is the distance the stone descends.

**EXECUTE:** **(a)** $K_\text{p} = \frac{1}{2}I_\text{p}\omega^2$. $I_\text{p} = \frac{1}{2}M_\text{p}R^2 = \frac{1}{2}(2.50\ \text{kg})(0.200\ \text{m})^2 = 0.0500\ \text{kg}\cdot\text{m}^2.$

$\omega = \sqrt{\dfrac{2K_\text{p}}{I_\text{p}}} = \sqrt{\dfrac{2(4.50\ \text{J})}{0.0500\ \text{kg}\cdot\text{m}^2}} = 13.4\ \text{rad/s}$. The stone has speed $v = R\omega = (0.200\ \text{m})(13.4\ \text{rad/s}) = 2.68\ \text{m/s}.$

The stone has kinetic energy $K_s = \frac{1}{2}mv^2 = \frac{1}{2}(1.50 \text{ kg})(2.68 \text{ m/s})^2 = 5.39 \text{ J}$. $K_1 + U_1 = K_2 + U_2$ gives $0 = K_2 + U_2$. $0 = 4.50 \text{ J} + 5.39 \text{ J} + mg(-h)$. $h = \dfrac{9.89 \text{ J}}{(1.50 \text{ kg})(9.80 \text{ m/s}^2)} = 0.673 \text{ m}$.

**(b)** $K_{tot} = K_p + K_s = 9.89 \text{ J}$. $\dfrac{K_p}{K_{tot}} = \dfrac{4.50 \text{ J}}{9.89 \text{ J}} = 45.5\%$.

**EVALUATE:** The gravitational potential energy of the pulley doesn't change as it rotates. The tension in the wire does positive work on the pulley and negative work of the same magnitude on the stone, so no net work on the system.

**9.44.** **IDENTIFY:** $K_p = \frac{1}{2}I\omega^2$ for the pulley and $K_b = \frac{1}{2}mv^2$ for the bucket. The speed of the bucket and the rotational speed of the pulley are related by $v = R\omega$.

**SET UP:** $K_p = \frac{1}{2}K_b$

**EXECUTE:** $\frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}mv^2) = \frac{1}{4}mR^2\omega^2$. $I = \frac{1}{2}mR^2$.

**EVALUATE:** The result is independent of the rotational speed of the pulley and the linear speed of the mass.

**9.45.** **IDENTIFY:** With constant acceleration, we can use kinematics to find the speed of the falling object. Then we can apply the work-energy expression to the entire system and find the moment of inertia of the wheel. Finally, using its radius we can find its mass, the target variable.

**SET UP:** With constant acceleration, $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$. The angular velocity of the wheel is related to the linear velocity of the falling mass by $\omega_z = \dfrac{v_y}{R}$. The work-energy theorem is $K_1 + U_1 + W_{other} = K_2 + U_2$, and the moment of inertia of a uniform disk is $I = \frac{1}{2}MR^2$.

**EXECUTE:** Find $v_y$, the velocity of the block after it has descended 3.00 m. $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $v_y = \dfrac{2(y - y_0)}{t} = \dfrac{2(3.00 \text{ m})}{2.00 \text{ s}} = 3.00 \text{ m/s}$. For the wheel, $\omega_z = \dfrac{v_y}{R} = \dfrac{3.00 \text{ m/s}}{0.280 \text{ m}} = 10.71 \text{ rad/s}$. Apply the work-energy expression: $K_1 + U_1 + W_{other} = K_2 + U_2$, giving $mg(3.00 \text{ m}) = \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2$. Solving for $I$ gives

$$I = \frac{2}{\omega^2}\left[mg(3.00 \text{ m}) - \frac{1}{2}mv^2\right].$$

$I = \dfrac{2}{(10.71 \text{ rad/s})^2}\left[(4.20 \text{ kg})(9.8 \text{ m/s}^2)(3.00 \text{ m}) - \dfrac{1}{2}(4.20 \text{ kg})(3.00 \text{ m/s})^2\right]$. $I = 1.824 \text{ kg}\cdot\text{m}^2$. For a solid disk, $I = \frac{1}{2}MR^2$ gives $M = \dfrac{2I}{R^2} = \dfrac{2(1.824 \text{ kg}\cdot\text{m}^2)}{(0.280 \text{ m})^2} = 46.5 \text{ kg}$.

**EVALUATE:** The gravitational potential of the falling object is converted into the kinetic energy of that object and the rotational kinetic energy of the wheel.

**9.46.** **IDENTIFY:** The work the person does is the negative of the work done by gravity. $W_{grav} = U_{grav,1} - U_{grav,2}$. $U_{grav} = Mgy_{cm}$.

**SET UP:** The center of mass of the ladder is at its center, 1.00 m from each end. $y_{cm,1} = (1.00 \text{ m})\sin 53.0° = 0.799 \text{ m}$. $y_{cm,2} = 1.00 \text{ m}$.

**EXECUTE:** $W_{grav} = (9.00 \text{ kg})(9.80 \text{ m/s}^2)(0.799 \text{ m} - 1.00 \text{ m}) = -17.7 \text{ J}$. The work done by the person is 17.7 J. The increase in gravitational potential energy of the ladder is $U_{grav,1} - U_{grav,2} = -W_{grav} = +17.7 \text{ J}$.

**EVALUATE:** The gravity force is downward and the center of mass of the ladder moves upward, so gravity does negative work. The person pushes upward and does positive work.

**9.47.** **IDENTIFY:** The general expression for $I$ is $I = \sum m_i r_i^2$. $K = \frac{1}{2} I\omega^2$.

**SET UP:** $R$ will be multiplied by $f$.

**EXECUTE:** **(a)** In the equation $I = \sum m_i r_i^2$, each term will have the mass multiplied by $f^3$ and the distance multiplied by $f$, and so the moment of inertia is multiplied by $f^3(f)^2 = f^5$.

**(b)** $(2.5\text{ J})(48)^5 = 6.37\times 10^8$ J.

**EVALUATE:** Mass and volume are proportional to each other so both scale by the same factor.

**9.48.** **IDENTIFY:** Apply the parallel-axis theorem.

**SET UP:** The center of mass of the hoop is at its geometrical center.

**EXECUTE:** In the parallel-axis theorem, $I_{\text{cm}} = MR^2$ and $d = R^2$, so $I_P = 2MR^2$.

**EVALUATE:** $I$ is larger for an axis at the edge than for an axis at the center. Some mass is closer than distance $R$ from the axis but some is also farther away. Since $I$ for each piece of the hoop is proportional to the square of the distance from the axis, the increase in distance has a larger effect.

**9.49.** **IDENTIFY:** Use the parallel-axis theorem to relate $I$ for the wood sphere about the desired axis to $I$ for an axis along a diameter.

**SET UP:** For a thin-walled hollow sphere, axis along a diameter, $I = \frac{2}{3}MR^2$.

For a solid sphere with mass $M$ and radius $R$, $I_{\text{cm}} = \frac{2}{5}MR^2$, for an axis along a diameter.

**EXECUTE:** Find $d$ such that $I_P = I_{\text{cm}} + Md^2$ with $I_P = \frac{2}{3}MR^2$:

$$\tfrac{2}{3}MR^2 = \tfrac{2}{5}MR^2 + Md^2$$

The factors of $M$ divide out and the equation becomes $(\frac{2}{3} - \frac{2}{5})R^2 = d^2$

$$d = \sqrt{(10-6)/15}R = 2R/\sqrt{15} = 0.516R.$$

The axis is parallel to a diameter and is $0.516R$ from the center.

**EVALUATE:** $I_{\text{cm}}(\text{lead}) > I_{\text{cm}}(\text{wood})$ even though $M$ and $R$ are the same since for a hollow sphere all the mass is a distance $R$ from the axis. The parallel-axis theorem says $I_P > I_{\text{cm}}$, so there must be a $d$ where $I_P(\text{wood}) = I_{\text{cm}}(\text{lead})$.

**9.50.** **IDENTIFY:** Consider the plate as made of slender rods placed side-by-side.

**SET UP:** The expression in Table 9.2 gives $I$ for a rod and an axis through the center of the rod.

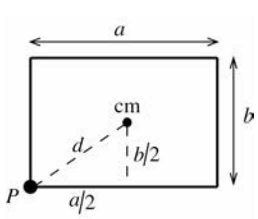
**EXECUTE:** **(a)** $I$ is the same as for a rod with length $a$: $I = \frac{1}{12}Ma^2$.

**(b)** $I$ is the same as for a rod with length $b$: $I = \frac{1}{12}Mb^2$.

**EVALUATE:** $I$ is smaller when the axis is through the center of the plate than when it is along one edge.

**9.51.** **IDENTIFY** and **SET UP:** Use the parallel-axis theorem. The cm of the sheet is at its geometrical center. The object is sketched in Figure 9.51.

**EXECUTE:** $I_P = I_{\text{cm}} + Md^2$.




From Table 9.2, $I_{\text{cm}} = \frac{1}{12}M(a^2 + b^2)$.

The distance $d$ of $P$ from the cm is $d = \sqrt{(a/2)^2 + (b/2)^2}$.

**Figure 9.51**

Thus $I_P = I_{\text{cm}} + Md^2 = \frac{1}{12}M(a^2+b^2) + M(\frac{1}{4}a^2 + \frac{1}{4}b^2) = (\frac{1}{12} + \frac{1}{4})M(a^2+b^2) = \frac{1}{3}M(a^2+b^2)$

**EVALUATE:** $I_P = 4I_{\text{cm}}$. For an axis through $P$ mass is farther from the axis.

**9.52.** **IDENTIFY:** Use the equations in Table 9.2. $I$ for the rod is the sum of $I$ for each segment. The parallel-axis theorem says $I_p = I_{cm} + Md^2$.

**SET UP:** The bent rod and axes $a$ and $b$ are shown in Figure 9.52. Each segment has length $L/2$ and mass $M/2$.

**EXECUTE:** **(a)** For each segment the moment of inertia is for a rod with mass $M/2$, length $L/2$ and the axis through one end. For one segment, $I_s = \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{24}ML^2$. For the rod, $I_a = 2I_s = \frac{1}{12}ML^2$.

**(b)** The center of mass of each segment is at the center of the segment, a distance of $L/4$ from each end. For each segment, $I_{cm} = \frac{1}{12}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{96}ML^2$. Axis $b$ is a distance $L/4$ from the cm of each segment, so for each segment the parallel axis theorem gives $I$ for axis $b$ to be $I_s = \frac{1}{96}ML^2 + \frac{M}{2}\left(\frac{L}{4}\right)^2 = \frac{1}{24}ML^2$ and $I_b = 2I_s = \frac{1}{12}ML^2$.

**EVALUATE:** $I$ for these two axes are the same.

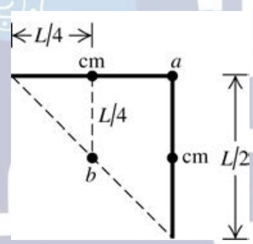



**Figure 9.52**

**9.53.** **IDENTIFY:** Apply $I = \int r^2\,dm$.

**SET UP:** $dm = \rho dV = \rho(2\pi rL\,dr)$, where $L$ is the thickness of the disk. $M = \pi L\rho R^2$.

**EXECUTE:** The analysis is identical to that of Example 9.10, with the lower limit in the integral being zero and the upper limit being $R$. The result is $I = \frac{1}{2}MR^2$.

**9.54.** **IDENTIFY:** Use $I = \int r^2\,dm$.

**SET UP:**

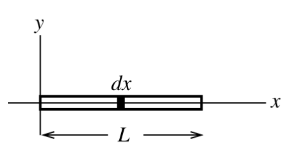



**Figure 9.54**

Take the $x$-axis to lie along the rod, with the origin at the left end. Consider a thin slice at coordinate $x$ and width $dx$, as shown in Figure 9.54. The mass per unit length for this rod is $M/L$, so the mass of this slice is $dm = (M/L)\,dx$.

**EXECUTE:** $I = \int_0^L x^2(M/L)\,dx = (M/L)\int_0^L x^2\,dx = (M/L)(L^3/3) = \frac{1}{3}ML^2$

**EVALUATE:** This result agrees with the table in the text.

**9.55.** **IDENTIFY:** Apply $I=\int r^2\,dm$ and $M=\int dm.$

**SET UP:** For this case, $dm=\gamma x\,dx.$

**EXECUTE:** **(a)** $M=\int dm=\int_0^L \gamma x\,dx=\gamma \frac{x^2}{2}\Big|_0^L=\frac{\gamma L^2}{2}$

**(b)** $I=\int_0^L x^2(\gamma x)dx=\gamma\frac{x^4}{4}\Big|_0^L=\frac{\gamma L^4}{4}=\frac{M}{2}L^2.$ This is larger than the moment of inertia of a uniform rod of the same mass and length, since the mass density is greater farther away from the axis than nearer the axis.

**(c)** $I=\int_0^L (L-x)^2\gamma x dx=\gamma\int_0^L (L^2x-2Lx^2+x^3)dx=\gamma\left(L^2\frac{x^2}{2}-2L\frac{x^3}{3}+\frac{x^4}{4}\right)\Big|_0^L=\gamma\frac{L^4}{12}=\frac{M}{6}L^2.$

This is a third of the result of part (b), reflecting the fact that more of the mass is concentrated at the right end.

**EVALUATE:** For a uniform rod with an axis at one end, $I=\frac{1}{3}ML^2.$ The result in (b) is larger than this and the result in (c) is smaller than this.

**9.56.** **IDENTIFY:** Using the equation for the angle as a function of time, we can find the angular acceleration of the disk at a given time and use this to find the linear acceleration of a point on the rim (the target variable).

**SET UP:** We can use the definitions of the angular velocity and the angular acceleration: $\omega_z(t)=\frac{d\theta}{dt}$ and $\alpha_z(t)=\frac{d\omega_z}{dt}.$ The acceleration components are $a_{\text{rad}}=R\omega^2$ and $a_{\tan}=R\alpha,$ and the magnitude of the acceleration is $a=\sqrt{a_{\text{rad}}^2+a_{\tan}^2}.$

**EXECUTE:** $\omega_z(t)=\frac{d\theta}{dt}=1.10\text{ rad/s}+(12.6\text{ rad/s}^2)t.$ $\alpha_z(t)=\frac{d\omega_z}{dt}=12.6\text{ rad/s}^2$ (constant).

$\theta=0.100\text{ rev}=0.6283\text{ rad}$ gives $6.30t^2+1.10t-0.6283=0,$ so $t=0.2403\text{ s},$ using the positive root. At this $t,$ $\omega_z(t)=4.1278\text{ rad/s}$ and $\alpha_z(t)=12.6\text{ rad/s}^2.$ For a point on the rim, $a_{\text{rad}}=R\omega^2=6.815\text{ m/s}^2$ and $a_{\tan}=R\alpha=5.04\text{ m/s}^2,$ so $a=\sqrt{a_{\text{rad}}^2+a_{\tan}^2}=8.48\text{ m/s}^2.$

**EVALUATE:** Since the angular acceleration is constant, we could use the constant acceleration formulas as a check. For example, the coefficient of $t^2$ is $\frac{1}{2}\alpha_z=6.30\text{ rad/s}^2$ gives $\alpha_z=12.6\text{ rad/s}^2.$

**9.57.** **IDENTIFY:** The target variable is the horizontal distance the piece travels before hitting the floor. Using the angular acceleration of the blade, we can find its angular velocity when the piece breaks off. This will give us the linear horizontal speed of the piece. It is then in free fall, so we can use the linear kinematics equations.

**SET UP:** $\omega_z^2=\omega_{0z}^2+2\alpha_z(\theta-\theta_0)$ for the blade, and $v=r\omega$ is the horizontal velocity of the piece.

$y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ for the falling piece.

**EXECUTE:** Find the initial horizontal velocity of the piece just after it breaks off.

$\theta-\theta_0=(155\text{ rev})(2\pi\text{ rad/1 rev})=973.9\text{ rad}.$

$\alpha_z=(2.00\text{ rev/s}^2)(2\pi\text{ rad/1 rev})=12.566\text{ rad/s}^2.$ $\omega_z^2=\omega_{0z}^2+2\alpha_z(\theta-\theta_0).$

$\omega_z=\sqrt{2\alpha_z(\theta-\theta_0)}=\sqrt{2(12.566\text{ rad/s}^2)(973.9\text{ rad})}=156.45\text{ rad/s}.$ The horizontal velocity of the piece is $v=r\omega=(0.120\text{ m})(156.45\text{ rad/s})=18.774\text{ m/s}.$ Now consider the projectile motion of the piece. Take

$+y$ downward and use the vertical motion to find $t$. Solving $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ for $t$ gives

$$t=\sqrt{\frac{2(y-y_0)}{a_y}}=\sqrt{\frac{2(0.820\text{ m})}{9.8\text{ m/s}^2}}=0.4091\text{ s. Then } x-x_0=v_{0x}t+\tfrac{1}{2}a_xt^2=(18.774\text{ m/s})(0.4091\text{ s})=7.68\text{ m.}$$

**EVALUATE:** Once the piece is free of the blade, the only force acting on it is gravity so its acceleration is $g$ downward.

**9.58.** **IDENTIFY** and **SET UP:** Use $\omega_z=\dfrac{d\theta}{dt}$ and $\alpha_z=\dfrac{d\omega_z}{dt}$. As long as $\alpha_z>0$, $\omega_z$ increases. At the $t$ when $\alpha_z=0$, $\omega_z$ is at its maximum positive value and then starts to decrease when $\alpha_z$ becomes negative.

$\theta(t)=\gamma t^2-\beta t^3$; $\gamma=3.20\text{ rad/s}^2$, $\beta=0.500\text{ rad/s}^3$

**EXECUTE:** **(a)** $\omega_z(t)=\dfrac{d\theta}{dt}=\dfrac{d(\gamma t^2-\beta t^3)}{dt}=2\gamma t-3\beta t^2$

**(b)** $\alpha_z(t)=\dfrac{d\omega_z}{dt}=\dfrac{d(2\gamma t-3\beta t^2)}{dt}=2\gamma-6\beta t$

**(c)** The maximum angular velocity occurs when $\alpha_z=0$.

$$2\gamma-6\beta t=0 \text{ implies } t=\frac{2\gamma}{6\beta}=\frac{\gamma}{3\beta}=\frac{3.20\text{ rad/s}^2}{3(0.500\text{ rad/s}^3)}=2.133\text{ s}$$

At this $t$, $\omega_z=2\gamma t-3\beta t^2=2(3.20\text{ rad/s}^2)(2.133\text{ s})-3(0.500\text{ rad/s}^3)(2.133\text{ s})^2=6.83\text{ rad/s}$

The maximum positive angular velocity is 6.83 rad/s and it occurs at 2.13 s.

**EVALUATE:** For large $t$ both $\omega_z$ and $\alpha_z$ are negative and $\omega_z$ increases in magnitude. In fact, $\omega_z\to-\infty$ at $t\to\infty$. So the answer in (c) is not the largest angular speed, just the largest positive angular velocity.

**9.59.** **IDENTIFY:** The angular acceleration $\alpha$ of the disk is related to the linear acceleration $a$ of the ball by $a=R\alpha$. Since the acceleration is not constant, use $\omega_z-\omega_{0z}=\int_0^t\alpha_z dt$ and $\theta-\theta_0=\int_0^t\omega_z dt$ to relate $\theta$, $\omega_z$, $\alpha_z$, and $t$ for the disk. $\omega_{0z}=0$.

**SET UP:** $\int t^n dt=\dfrac{1}{n+1}t^{n+1}$. In $a=R\alpha$, $\alpha$ is in rad/s$^2$.

**EXECUTE:** **(a)** $A=\dfrac{a}{t}=\dfrac{1.80\text{ m/s}^2}{3.00\text{ s}}=0.600\text{ m/s}^3$

**(b)** $\alpha=\dfrac{a}{R}=\dfrac{(0.600\text{ m/s}^3)t}{0.250\text{ m}}=(2.40\text{ rad/s}^3)t$

**(c)** $\omega_z=\int_0^t(2.40\text{ rad/s}^3)tdt=(1.20\text{ rad/s}^3)t^2$. $\omega_z=15.0$ rad/s for $t=\sqrt{\dfrac{15.0\text{ rad/s}}{1.20\text{ rad/s}^3}}=3.54$ s.

**(d)** $\theta-\theta_0=\int_0^t\omega_z dt=\int_0^t(1.20\text{ rad/s}^3)t^2dt=(0.400\text{ rad/s}^3)t^3$. For $t=3.54$ s, $\theta-\theta_0=17.7$ rad.

**EVALUATE:** If the disk had turned at a constant angular velocity of 15.0 rad/s for 3.54 s it would have turned through an angle of 53.1 rad in 3.54 s. It actually turns through less than half this because the angular velocity is increasing in time and is less than 15.0 rad/s at all but the end of the interval.

**9.60.** **IDENTIFY:** The flywheel gains rotational kinetic energy as it spins. This kinetic energy depends on the flywheel's rate of spin but also on its moment of inertia. The angular acceleration is constant.

**SET UP:** $K=\frac{1}{2}I\omega^2$, $I=\frac{1}{2}mR^2$, $\omega=\omega_0+\alpha t$, $m=\rho V=\rho\pi R^2h$.

**EXECUTE:** $K=\frac{1}{2}I\omega^2=\frac{1}{2}(\frac{1}{2}mR^2)(\omega_0+\alpha t)^2=\frac{1}{4}\left[(\rho\pi R^2h)R^2\right](0+\alpha t)^2$. Solving for $h$ gives

$$h=\frac{4K}{\rho\pi R^4(\alpha t)^2}=4(800\text{ J})/[\pi(8600\text{ kg/m}^3)(0.250\text{ m})^4(3.00\text{ rad/s}^2)^2(8.00\text{ s})^2]=0.0526\text{ m}=5.26\text{ cm.}$$

**EVALUATE:** If we could turn the disk into a thin-walled cylinder of the same mass and radius, the moment of inertia would be twice as great, so we could store twice as much energy as for the given disk.

**9.61.** **IDENTIFY:** As it turns, the wheel gives kinetic energy to the marble, and this energy is converted into gravitational potential energy as the marble reaches its highest point in the air.

**SET UP:** The marble starts from rest at point $A$ at the same level as the center of the wheel and after 20.0 revolutions it leaves the rim of the wheel at point $A$. $K_1+U_1=K_2+U_2$ applies once the marble has left the cup. While the marble is turning with the wheel, $\omega^2=\omega_0^2+2\alpha(\theta-\theta_0)$ applies.

**EXECUTE:** Applying $K_1+U_1=K_2+U_2$ gives $v_A=\sqrt{2gh}$. The marble is at the rim of the wheel, so $v_A=R\omega_A$. Using this formula in the angular velocity formula gives $(v_A/R)^2 = 0 + 2\alpha(\theta-\theta_0)$. The marble turns through 20.0 rev = $40.0\pi$ rad, $R$ = 0.260 m, and $h$ = 12.0 m. Solving the previous equation for $\alpha$ gives $\alpha = gh/40\pi R^2 = (9.80\ \text{m/s}^2)(12.0\ \text{m})/[40\pi(0.260\ \text{m})^2] = 13.8\ \text{rad/s}^2$.

**EVALUATE:** The marble has a tangential acceleration $a_{\text{tang}} = R\alpha = (0.260\ \text{m})(13.8\ \text{rad/s}^2) = 3.59\ \text{m/s}^2$ upward just before it leaves the cup. But this acceleration ends the instant the marble leaves the cup, and after that its acceleration is 9.80 m/s$^2$ downward due to gravity.

**9.62.** **IDENTIFY:** Apply conservation of energy to the system of drum plus falling mass, and compare the results for earth and for Mars.

**SET UP:** $K_{\text{drum}}=\frac{1}{2}I\omega^2$. $K_{\text{mass}}=\frac{1}{2}mv^2$. $v=R\omega$ so if $K_{\text{drum}}$ is the same, $\omega$ is the same and $v$ is the same on both planets. Therefore, $K_{\text{mass}}$ is the same. Let $y=0$ at the initial height of the mass and take $+y$ upward. Configuration 1 is when the mass is at its initial position and 2 is when the mass has descended 5.00 m, so $y_1=0$ and $y_2=-h$, where $h$ is the height the mass descends.

**EXECUTE:** **(a)** $K_1+U_1=K_2+U_2$ gives $0=K_{\text{drum}}+K_{\text{mass}}-mgh$. $K_{\text{drum}}+K_{\text{mass}}$ are the same on both planets, so $mg_{\text{E}}h_{\text{E}}=mg_{\text{M}}h_{\text{M}}$. $h_{\text{M}}=h_{\text{E}}\left(\dfrac{g_{\text{E}}}{g_{\text{M}}}\right)=(5.00\ \text{m})\left(\dfrac{9.80\ \text{m/s}^2}{3.71\ \text{m/s}^2}\right)=13.2\ \text{m}$.

**(b)** $mg_{\text{M}}h_{\text{M}}=K_{\text{drum}}+K_{\text{mass}}$. $\frac{1}{2}mv^2=mg_{\text{M}}h_{\text{M}}-K_{\text{drum}}$ and

$$v=\sqrt{2g_{\text{M}}h_{\text{M}}-\frac{2K_{\text{drum}}}{m}}=\sqrt{2(3.71\ \text{m/s}^2)(13.2\ \text{m})-\frac{2(250.0\ \text{J})}{15.0\ \text{kg}}}=8.04\ \text{m/s}$$

**EVALUATE:** We did the calculations without knowing the moment of inertia $I$ of the drum, or the mass and radius of the drum.

**9.63.** **IDENTIFY** and **SET UP:** All points on the belt move with the same speed. Since the belt doesn't slip, the speed of the belt is the same as the speed of a point on the rim of the shaft and on the rim of the wheel, and these speeds are related to the angular speed of each circular object by $v=r\omega$.

**EXECUTE:**

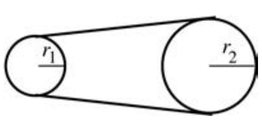



**Figure 9.63**

**(a)** $v_1=r_1\omega_1$

$\omega_1=(60.0\ \text{rev/s})(2\pi\ \text{rad/1 rev})=377\ \text{rad/s}$

$v_1=r_1\omega_1=(0.45\times10^{-2}\ \text{m})(377\ \text{rad/s})=1.70\ \text{m/s}$

**(b)** $v_1=v_2$

$r_1\omega_1=r_2\omega_2$

$\omega_2=(r_1/r_2)\omega_1=(0.45\ \text{cm}/1.80\ \text{cm})(377\ \text{rad/s})=94.2\ \text{rad/s}$

**EVALUATE:** The wheel has a larger radius than the shaft so turns slower to have the same tangential speed for points on the rim.

**9.64.** **IDENTIFY:** The speed of all points on the belt is the same, so $r_1\omega_1 = r_2\omega_2$ applies to the two pulleys.

**SET UP:** The second pulley, with half the diameter of the first, must have twice the angular velocity, and this is the angular velocity of the saw blade. $\pi$ rad/s $= 30$ rev/min.

**EXECUTE:** **(a)** $v_2 = (2(3450\text{ rev/min}))\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)\left(\dfrac{0.208\text{ m}}{2}\right) = 75.1\text{ m/s}.$

**(b)** $a_{\text{rad}} = \omega^2 r = \left(2(3450\text{ rev/min})\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)\right)^2\left(\dfrac{0.208\text{ m}}{2}\right) = 5.43\times10^4\text{ m/s}^2,$

so the force holding sawdust on the blade would have to be about 5500 times as strong as gravity.

**EVALUATE:** In $v = r\omega$ and $a_{\text{rad}} = r\omega^2$, $\omega$ must be in rad/s.

**9.65.** **IDENTIFY:** Apply $v = r\omega$.

**SET UP:** Points on the chain all move at the same speed, so $r_\text{r}\omega_\text{r} = r_\text{f}\omega_\text{f}$.

**EXECUTE:** The angular velocity of the rear wheel is $\omega_\text{r} = \dfrac{v_\text{r}}{r_\text{r}} = \dfrac{5.00\text{ m/s}}{0.330\text{ m}} = 15.15\text{ rad/s}.$

The angular velocity of the front wheel is $\omega_\text{f} = 0.600\text{ rev/s} = 3.77\text{ rad/s}$. $r_\text{r} = r_\text{f}(\omega_\text{f}/\omega_\text{r}) = 2.99\text{ cm}$.

**EVALUATE:** The rear sprocket and wheel have the same angular velocity and the front sprocket and wheel have the same angular velocity. $r\omega$ is the same for both, so the rear sprocket has a smaller radius since it has a larger angular velocity. The speed of a point on the chain is $v = r_\text{r}\omega_\text{r} = (2.99\times10^{-2}\text{ m})(15.15\text{ rad/s}) =$ $0.453\text{ m/s}$. The linear speed of the bicycle is 5.00 m/s.

**9.66.** **IDENTIFY:** Use the constant angular acceleration equations, applied to the first revolution and to the first two revolutions.

**SET UP:** Let the direction the disk is rotating be positive. 1 rev $= 2\pi$ rad. Let $t$ be the time for the first revolution. The time for the first two revolutions is $t + 0.0865$ s.

**EXECUTE:** **(a)** $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$ applied to the first revolution and then to the first two revolutions gives $2\pi\text{ rad} = \frac{1}{2}\alpha_z t^2$ and $4\pi\text{ rad} = \frac{1}{2}\alpha_z(t + 0.0865\text{ s})^2$. Eliminating $\alpha_z$ between these equations gives

$4\pi\text{ rad} = \dfrac{2\pi\text{ rad}}{t^2}(t+0.0865\text{ s})^2$. $2t^2 = (t+0.0865\text{ s})^2$. $\sqrt{2}t = \pm(t+0.0865\text{ s})$. The positive root is

$t = \dfrac{0.0865\text{ s}}{\sqrt{2}-1} = 0.209\text{ s}.$

**(b)** $2\pi\text{ rad} = \frac{1}{2}\alpha_z t^2$ and $t = 0.209$ s gives $\alpha_z = 288\text{ rad/s}^2$

**EVALUATE:** At the start of the second revolution, $\omega_{0z} = (288\text{ rad/s}^2)(0.209\text{ s}) = 60.19\text{ rad/s}$. The distance the disk rotates in the next 0.0865 s is

$\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (60.19\text{ rad/s})(0.0865\text{ s}) + \frac{1}{2}(288\text{ rad/s}^2)(0.0865\text{ s})^2 = 6.28\text{ rad}$, which is two revolutions.

**9.67.** **IDENTIFY:** $K = \frac{1}{2}I\omega^2$. $a_{\text{rad}} = r\omega^2$. $m = \rho V$.

**SET UP:** For a disk with the axis at the center, $I = \frac{1}{2}mR^2$. $V = t\pi R^2$, where $t = 0.100$ m is the thickness of the flywheel. $\rho = 7800\text{ kg/m}^3$ is the density of the iron.

**EXECUTE:** **(a)** $\omega = 90.0\text{ rpm} = 9.425\text{ rad/s}$. $I = \dfrac{2K}{\omega^2} = \dfrac{2(10.0\times10^6\text{ J})}{(9.425\text{ rad/s})^2} = 2.252\times10^5\text{ kg}\cdot\text{m}^2.$

$m = \rho V = \rho\pi R^2 t$. $I = \frac{1}{2}mR^2 = \frac{1}{2}\rho\pi t R^4$. This gives $R = (2I/\rho\pi t)^{1/4} = 3.68$ m and the diameter is 7.36 m.

**(b)** $a_{\text{rad}} = R\omega^2 = 327\text{ m/s}^2$

**EVALUATE:** In $K = \frac{1}{2}I\omega^2$, $\omega$ must be in rad/s. $a_{\text{rad}}$ is about 33$g$; the flywheel material must have large cohesive strength to prevent the flywheel from flying apart.

**9.68.** **IDENTIFY:** The moment of inertia of the section that is removed must be one-half the moment of inertia of the original disk.

**SET UP:** For a solid disk, $I = \frac{1}{2}mR^2$. Call $m$ the mass of the removed piece and $R$ its radius. $I_m = \dfrac{1}{2} I_{M_0}$.

**EXECUTE:** $I_m = \frac{1}{2} I_{M_0}$ gives $\frac{1}{2}mR^2 = \frac{1}{2}(\frac{1}{2}M_0 R_0^2)$. We need to find $m$. Since the disk is uniform, the mass of a given segment will be proportional to the area of that segment. In this case, the segment is the piece cut out of the center. So $\dfrac{m}{M_0} = \dfrac{A_R}{A_{R_0}} = \dfrac{\pi R^2}{\pi R_0^2} = \dfrac{R^2}{R_0^2}$, which gives $m = M_0\left(\dfrac{R^2}{R_0^2}\right)$. Combining the two results gives $\frac{1}{2}M_0\left(\dfrac{R^2}{R_0^2}\right)R^2 = \frac{1}{2}(\frac{1}{2}M_0 R_0^2)$, from which we get $R = \dfrac{R_0}{2^{1/4}} = 0.841 R_0$.

**EVALUATE:** Notice that the piece that is removed does not have one-half the mass of the original disk, nor it its radius one-half the original radius.

**9.69.** **IDENTIFY:** The falling wood accelerates downward as the wheel undergoes angular acceleration. Newton's second law applies to the wood and the wheel, and the linear kinematics formulas apply to the wood because it has constant acceleration.

**SET UP:** $\Sigma \vec{F} = m\vec{a}$, $\tau = I\alpha$, $a_{\text{tan}} = R\alpha$, $y - y_0 = v_{0y}t + \dfrac{1}{2}a_y t^2$.

**EXECUTE:** First use $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ to find the downward acceleration of the wood. With $v_0 = 0$, we have $a_y = 2(y - y_0)/t^2 = 2(12.0\text{ m})/(4.00\text{ s})^2 = 1.50\text{ m/s}^2$. Now apply Newton's second to the wood to find the tension in the rope. $\Sigma \vec{F} = m\vec{a}$ gives $mg - T = ma$, $T = m(g - a)$, which gives $T = (8.20\text{ kg})(9.80\text{ m/s}^2 - 1.50\text{ m/s}^2) = 68.06\text{ N}$. Now use $a_{\text{tan}} = R\alpha$ and apply Newton's second law (in its rotational form) to the wheel. $\tau = I\alpha$ gives $TR = I\alpha$, $I = TR/\alpha = TR/(a/R) = TR^2/a$

$I = (68.06\text{ N})(0.320\text{ m})^2/(1.50\text{ m/s}^2) = 4.65\ \text{kg}\cdot\text{m}^2$.

**EVALUATE:** The tension in the rope affects the acceleration of the wood and causes the angular acceleration of the wheel.

**9.70.** **IDENTIFY:** Using energy considerations, the system gains as kinetic energy the lost potential energy, $mgR$.

**SET UP:** The kinetic energy is $K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$, with $I = \frac{1}{2}mR^2$ for the disk. $v = R\omega$.

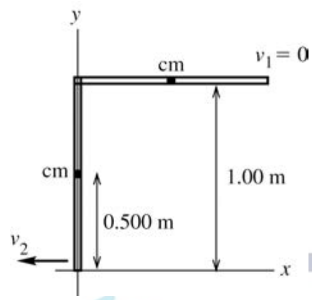
**EXECUTE:** $K = \frac{1}{2}I\omega^2 + \frac{1}{2}m(\omega R)^2 = \frac{1}{2}(I + mR^2)\omega^2$. Using $I = \frac{1}{2}mR^2$ and solving for $\omega$, $\omega^2 = \dfrac{4}{3}\dfrac{g}{R}$ and

$$\omega = \sqrt{\frac{4}{3}\frac{g}{R}}.$$

**EVALUATE:** The small object has speed $v = \sqrt{\dfrac{2}{3}}\sqrt{2gR}$. If it was not attached to the disk and was dropped from a height $h$, it would attain a speed $\sqrt{2gR}$. Being attached to the disk reduces its final speed by a factor of $\sqrt{\dfrac{2}{3}}$.

**9.71.** **IDENTIFY:** Use conservation of energy. The stick rotates about a fixed axis so $K = \frac{1}{2}I\omega^2$. Once we have $\omega$ use $v = r\omega$ to calculate $v$ for the end of the stick.

**SET UP:** The object is sketched in Figure 9.71.




Take the origin of coordinates at the lowest point reached by the stick and take the positive $y$-direction to be upward.

**Figure 9.71**

**EXECUTE:** **(a)** Use $U = Mgy_{cm}$. $\Delta U = U_2 - U_1 = Mg(y_{cm2} - y_{cm1})$. The center of mass of the meter stick is at its geometrical center, so $y_{cm1} = 1.00$ m and $y_{cm2} = 0.50$ m. Then

$\Delta U = (0.180 \text{ kg})(9.80 \text{ m/s}^2)(0.50 \text{ m} - 1.00 \text{ m}) = -0.882$ J.

**(b)** Use conservation of energy: $K_1 + U_1 + W_{other} = K_2 + U_2$. Gravity is the only force that does work on the meter stick, so $W_{other} = 0$. $K_1 = 0$. Thus $K_2 = U_1 - U_2 = -\Delta U$, where $\Delta U$ was calculated in part (a). $K_2 = \frac{1}{2}I\omega_2^2$ so $\frac{1}{2}I\omega_2^2 = -\Delta U$ and $\omega_2 = \sqrt{2(-\Delta U)/I}$. For stick pivoted about one end, $I = \frac{1}{3}ML^2$ where $L = 1.00$ m, so $\omega_2 = \sqrt{\dfrac{6(-\Delta U)}{ML^2}} = \sqrt{\dfrac{6(0.882 \text{ J})}{(0.180 \text{ kg})(1.00 \text{ m})^2}} = 5.42$ rad/s.

**(c)** $v = r\omega = (1.00 \text{ m})(5.42 \text{ rad/s}) = 5.42$ m/s.

**(d)** For a particle in free fall, with $+y$ upward, $v_{0y} = 0$; $y - y_0 = -1.00$ m; $a_y = -9.80$ m/s$^2$; and $v_y = ?$ Solving the equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ for $v_y$ gives

$v_y = -\sqrt{2a_y(y - y_0)} = -\sqrt{2(-9.80 \text{ m/s}^2)(-1.00 \text{ m})} = -4.43$ m/s.

**EVALUATE:** The magnitude of the answer in part (c) is larger. $U_{1,grav}$ is the same for the stick as for a particle falling from a height of 1.00 m. For the stick $K = \frac{1}{2}I\omega_2^2 = \frac{1}{2}(\frac{1}{3}ML^2)(v/L)^2 = \frac{1}{6}Mv^2$. For the stick and for the particle, $K_2$ is the same but the same $K$ gives a larger $v$ for the end of the stick than for the particle. The reason is that all the other points along the stick are moving slower than the end opposite the axis.

**9.72.** **IDENTIFY:** The student accelerates downward and causes the wheel to turn. Newton's second law applies to the student and to the wheel. The acceleration is constant so the kinematics formulas apply.

**SET UP:** $\Sigma\tau = I\alpha$, $\Sigma\vec{F} = m\vec{a}$, $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$, $v_y = v_{0y} + a_yt$.

**EXECUTE:** Apply $\Sigma\tau = I\alpha$ to the wheel: $TR = I\alpha = I(\alpha/R)$, so $T = I\alpha/R^2$.

Apply $\Sigma\vec{F} = m\vec{a}$ to the student: $mg - T = ma$, so $T = m(g - a)$.

Equating these two expressions for $T$ and solving for the acceleration gives $a = \dfrac{mg}{m + I/R^2}$. Now apply kinematics for $y - y_0$ to the student, using $v_{0y} = 0$, and solve for $t$. $t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(y - y_0)(m + I/R^2)}{mg}}$.

Putting in $y - y_0 = 12.0$ m, $m = 43.0$ kg, $I = 9.60$ kg $\cdot$ m$^2$, and $R = 0.300$ m, we get $t = 2.92$ s.

Now use $v_y = v_{0y} + a_yt$ to get $v_y$, where $a = \dfrac{mg}{m + I/R^2}$. Putting in the numbers listed above, the result is $v_y = 8.22$ m/s.

**EVALUATE:** If the wheel were massless, her speed would simply be $v = \sqrt{2gy} = 15.3$ m/s, so the effect of the massive wheel reduces her speed by nearly half.

**9.73.** **IDENTIFY:** Mechanical energy is conserved since there is no friction.

**SET UP:** $K_1 + U_1 = K_2 + U_2$, $K = \frac{1}{2}I\omega^2$ (for rotational motion), $K = \frac{1}{2}mv^2$ (for linear motion), $I = \frac{1}{12}ML^2$ for a slender rod.

**EXECUTE:** Take the initial position with the rod horizontal, and the final position with the rod vertical. The heavier sphere will be at the bottom and the lighter one at the top. Call the gravitational potential energy zero with the rod horizontal, which makes the initial potential energy zero. The initial kinetic energy is also zero. Applying $K_1 + U_1 = K_2 + U_2$ and calling A and B the spheres gives $0 = K_A + K_B + K_{rod} + U_A + U_B + U_{rod}$. $U_{rod} = 0$ in the final position since its center of mass has not moved. Therefore $0 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 + \frac{1}{2}I\omega^2 + m_A g\frac{L}{2} - m_B g\frac{L}{2}$. We also know that $v_A = v_B = (L/2)\,\omega$.

Calling $v$ the speed of the spheres, we get $0 = \frac{1}{2}m_A v^2 + \frac{1}{2}m_B v^2 + \frac{1}{2}(\frac{1}{12})(ML^2)(2v/L)^2 + m_A g\frac{L}{2} - m_B g\frac{L}{2}$

Putting in $m_A = 0.0200$ kg, $m_B = 0.0500$ kg, $M = 0.120$ kg, and $L = 800$ m, we get $v = 1.46$ m/s.

**EVALUATE:** As the rod turns, the heavier sphere loses potential energy but the lighter one gains potential energy.

**9.74.** **IDENTIFY:** Apply conservation of energy to the system of cylinder and rope.

**SET UP:** Taking the zero of gravitational potential energy to be at the axle, the initial potential energy is zero (the rope is wrapped in a circle with center on the axle).When the rope has unwound, its center of mass is a distance $\pi R$ below the axle, since the length of the rope is $2\pi R$ and half this distance is the position of the center of the mass. Initially, every part of the rope is moving with speed $\omega_0 R$, and when the rope has unwound, and the cylinder has angular speed $\omega$, the speed of the rope is $\omega R$ (the upper end of the rope has the same tangential speed at the edge of the cylinder). $I = (1/2)MR^2$ for a uniform cylinder.

**EXECUTE:** $K_1 = K_2 + U_2$. $\left(\frac{M}{4} + \frac{m}{2}\right)R^2\omega_0^2 = \left(\frac{M}{4} + \frac{m}{2}\right)R^2\omega^2 - mg\pi R$. Solving for $\omega$ gives

$\omega = \sqrt{\omega_0^2 + \frac{(4\pi mg/R)}{(M + 2m)}}$, and the speed of any part of the rope is $v = \omega R$.

**EVALUATE:** When $m \to 0$, $\omega \to \omega_0$, When $m >> M$, $\omega = \sqrt{\omega_0^2 + \frac{2\pi g}{R}}$ and $v = \sqrt{v_0^2 + 2\pi gR}$. This is the final speed when an object with initial speed $v_0$ descends a distance $\pi R$.

**9.75.** **IDENTIFY:** Apply conservation of energy to the system consisting of blocks $A$ and $B$ and the pulley.

**SET UP:** The system at points 1 and 2 of its motion is sketched in Figure 9.75.

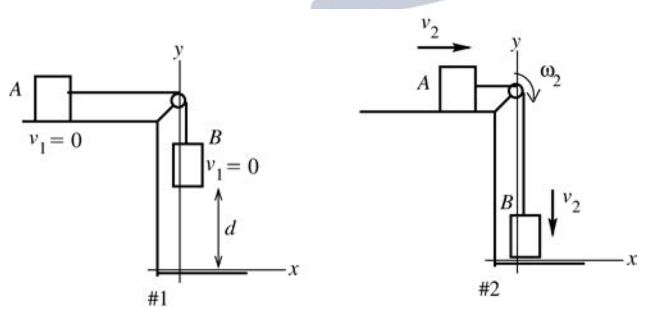



**Figure 9.75**

Use the work-energy relation $K_1 + U_1 + W_{other} = K_2 + U_2$. Use coordinates where $+y$ is upward and where the origin is at the position of block $B$ after it has descended. The tension in the rope does positive work on block $A$ and negative work of the same magnitude on block $B$, so the net work done by the tension in the rope is zero. Both blocks have the same speed.

**EXECUTE:** Gravity does work on block $B$ and kinetic friction does work on block $A$. Therefore $W_{\text{other}} = W_f = -\mu_k m_A g d$.

$K_1 = 0$ (system is released from rest)

$U_1 = m_B g y_{B1} = m_B g d;\ \ U_2 = m_B g y_{B2} = 0$

$K_2 = \frac{1}{2}m_A v_2^2 + \frac{1}{2}m_B v_2^2 + \frac{1}{2}I\omega_2^2.$

But $v(\text{blocks}) = R\omega(\text{pulley})$, so $\omega_2 = v_2/R$ and

$K_2 = \frac{1}{2}(m_A + m_B)v_2^2 + \frac{1}{2}I(v_2/R)^2 = \frac{1}{2}(m_A + m_B + I/R^2)v_2^2$

Putting all this into the work-energy relation gives

$m_B g d - \mu_k m_A g d = \frac{1}{2}(m_A + m_B + I/R^2)v_2^2$

$(m_A + m_B + I/R^2)v_2^2 = 2gd(m_B - \mu_k m_A)$

$$v_2 = \sqrt{\frac{2gd(m_B - \mu_k m_A)}{m_A + m_B + I/R^2}}$$

**EVALUATE:** If $m_B >> m_A$ and $I/R^2$, then $v_2 = \sqrt{2gd}$; block $B$ falls freely. If $I$ is very large, $v_2$ is very small. Must have $m_B > \mu_k m_A$ for motion, so the weight of $B$ will be larger than the friction force on $A$. $I/R^2$ has units of mass and is in a sense the "effective mass" of the pulley.

**9.76.** **IDENTIFY:** Apply conservation of energy to the system of two blocks and the pulley.

**SET UP:** Let the potential energy of each block be zero at its initial position. The kinetic energy of the system is the sum of the kinetic energies of each object. $v = R\omega$, where $v$ is the common speed of the blocks and $\omega$ is the angular velocity of the pulley.

**EXECUTE:** The amount of gravitational potential energy which has become kinetic energy is $K = (4.00\text{ kg} - 2.00\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m}) = 98.0\text{ J}$. In terms of the common speed $v$ of the blocks, the kinetic energy of the system is $K = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2$.

$K = v^2\frac{1}{2}\left(4.00\text{ kg} + 2.00\text{ kg} + \frac{(0.380\text{ kg}\cdot\text{m}^2)}{(0.160\text{ m})^2}\right) = v^2(10.422\text{ kg})$. Solving for $v$ gives

$$v = \sqrt{\frac{98.0\text{ J}}{10.422\text{ kg}}} = 3.07\text{ m/s}.$$

**EVALUATE:** If the pulley is massless, $98.0\text{ J} = \frac{1}{2}(4.00\text{ kg} + 2.00\text{ kg})v^2$ and $v = 5.72\text{ m/s}$. The moment of inertia of the pulley reduces the final speed of the blocks.

**9.77.** **IDENTIFY:** $I = I_1 + I_2$. Apply conservation of energy to the system. The calculation is similar to Example 9.8.

**SET UP:** $\omega = \dfrac{v}{R_1}$ for part (b) and $\omega = \dfrac{v}{R_2}$ for part (c).

**EXECUTE:** **(a)** $I = \frac{1}{2}M_1R_1^2 + \frac{1}{2}M_2R_2^2 = \frac{1}{2}((0.80\text{ kg})(2.50\times10^{-2}\text{ m})^2 + (1.60\text{ kg})(5.00\times10^{-2}\text{ m})^2)$

$I = 2.25\times10^{-3}\text{ kg}\cdot\text{m}^2.$

**(b)** The method of Example 9.8 yields $v = \sqrt{\dfrac{2gh}{1 + (I/mR_1^2)}}$.

$$v = \sqrt{\frac{2(9.80\text{ m/s}^2)(2.00\text{ m})}{(1 + ((2.25\times10^{-3}\text{ kg}\cdot\text{m}^2)/(1.50\text{ kg})(0.025\text{ m})^2))}} = 3.40\text{ m/s}.$$

**(c)** The same calculation, with $R_2$ instead of $R_1$ gives $v = 4.95\text{ m/s}$.

**EVALUATE:** The final speed of the block is greater when the string is wrapped around the larger disk. $v = R\omega$, so when $R = R_2$ the factor that relates $v$ to $\omega$ is larger. For $R = R_2$ a larger fraction of the total kinetic energy resides with the block. The total kinetic energy is the same in both cases (equal to $mgh$), so when $R = R_2$ the kinetic energy and speed of the block are greater.

**9.78.** **IDENTIFY:** The potential energy of the falling block is transformed into kinetic energy of the block and kinetic energy of the turning wheel, but some of it is lost to the work by friction. Energy conservation applies, with the target variable being the angular velocity of the wheel when the block has fallen a given distance.

**SET UP:** $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, where $K = \frac{1}{2}mv^2$, $U = mgh$, and $W_{\text{other}}$ is the work done by friction.

**EXECUTE:** Energy conservation gives $mgh + (-9.00\text{ J}) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. $v = R\omega$, so $\frac{1}{2}mv^2 = \frac{1}{2}mR^2\omega^2$ and $mgh + (-9.00\text{ J}) = \frac{1}{2}(mR^2 + I)\omega^2$. Solving for $\omega$ gives

$$\omega = \sqrt{\frac{2[mgh + (-9.00\text{ J})]}{mR^2 + I}} = \sqrt{\frac{2[(0.340\text{ kg})(9.8\text{ m/s}^2)(3.00\text{ m}) - 9.00\text{ J}]}{(0.340\text{ kg})(0.180\text{ m})^2 + 0.480\text{ kg}\cdot\text{m}^2}} = 2.01\text{ rad/s.}$$

**EVALUATE:** Friction does negative work because it opposes the turning of the wheel.

**9.79.** **IDENTIFY:** Apply conservation of energy to relate the height of the mass to the kinetic energy of the cylinder.

**SET UP:** First use $K(\text{cylinder}) = 480\text{ J}$ to find $\omega$ for the cylinder and $v$ for the mass.

**EXECUTE:** $I = \frac{1}{2}MR^2 = \frac{1}{2}(10.0\text{ kg})(0.150\text{ m})^2 = 0.1125\text{ kg}\cdot\text{m}^2$. $K = \frac{1}{2}I\omega^2$ so $\omega = \sqrt{2K/I} = 92.38\text{ rad/s}$. $v = R\omega = 13.86\text{ m/s}$.

**SET UP:** Use conservation of energy $K_1 + U_1 = K_2 + U_2$ to solve for the distance the mass descends. Take $y = 0$ at lowest point of the mass, so $y_2 = 0$ and $y_1 = h$, the distance the mass descends.

**EXECUTE:** $K_1 = U_2 = 0$ so $U_1 = K_2$. $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, where $m = 12.0\text{ kg}$. For the cylinder, $I = \frac{1}{2}MR^2$ and $\omega = v/R$, so $\frac{1}{2}I\omega^2 = \frac{1}{4}Mv^2$. Solving $mgh = \frac{1}{2}mv^2 + \frac{1}{4}Mv^2$ for $h$ gives

$$h = \frac{v^2}{2g}\left(1 + \frac{M}{2m}\right) = 13.9\text{ m.}$$

**EVALUATE:** For the cylinder $K_{\text{cyl}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}MR^2)(v/R)^2 = \frac{1}{4}Mv^2$. $K_{\text{mass}} = \frac{1}{2}mv^2$, so $K_{\text{mass}} = (2m/M)K_{\text{cyl}} = [2(12.0\text{ kg})/10.0\text{ kg}](480\text{ J}) = 1150\text{ J}$. The mass has 1150 J of kinetic energy when the cylinder has 480 J of kinetic energy and at this point the system has total energy 1630 J since $U_2 = 0$. Initially the total energy of the system is $U_1 = mgy_1 = mgh = 1630\text{ J}$, so the total energy is shown to be conserved.

**9.80.** **IDENTIFY:** Energy conservation: Loss of $U$ of box equals gain in $K$ of system. Both the cylinder and pulley have kinetic energy of the form $K = \frac{1}{2}I\omega^2$.

$$m_{\text{box}}gh = \frac{1}{2}m_{\text{box}}v_{\text{box}}^2 + \frac{1}{2}I_{\text{pulley}}\omega_{\text{pulley}}^2 + \frac{1}{2}I_{\text{cylinder}}\omega_{\text{cylinder}}^2.$$

**SET UP:** $\omega_{\text{pulley}} = \dfrac{v_{\text{box}}}{r_{\text{pulley}}}$ and $\omega_{\text{cylinder}} = \dfrac{v_{\text{box}}}{r_{\text{cylinder}}}$.

Let B = box, P = pulley, and C = cylinder.

**EXECUTE:** $m_B gh = \frac{1}{2}m_B v_B^2 + \frac{1}{2}\left(\frac{1}{2}m_P r_P^2\right)\left(\frac{v_B}{r_P}\right)^2 + \frac{1}{2}\left(\frac{1}{2}m_C r_C^2\right)\left(\frac{v_B}{r_C}\right)^2$. $m_B gh = \dfrac{1}{2}m_B v_B^2 + \dfrac{1}{4}m_P v_B^2 + \dfrac{1}{4}m_C v_B^2$

and $v_B = \sqrt{\dfrac{m_B gh}{\frac{1}{2}m_B + \frac{1}{4}m_P + \frac{1}{4}m_C}} = \sqrt{\dfrac{(3.00\text{ kg})(9.80\text{ m/s}^2)(2.50\text{ m})}{1.50\text{ kg} + \frac{1}{4}(7.00\text{ kg})}} = 4.76\text{ m/s}$.

**EVALUATE:** If the box was disconnected from the rope and dropped from rest, after falling 2.50 m its speed would be $v = \sqrt{2g(2.50\text{ m})} = 7.00\text{ m/s}$. Since in the problem some of the energy of the system goes into kinetic energy of the cylinder and of the pulley, the final speed of the box is less than this.

**9.81.** **IDENTIFY:** The total kinetic energy of a walker is the sum of his translational kinetic energy plus the rotational kinetic of his arms and legs. We can model these parts of the body as uniform bars.

**SET UP:** For a uniform bar pivoted about one end, $I = \frac{1}{3}mL^2$. $v = 5.0\text{ km/h} = 1.4\text{ m/s}$.

$K_{\text{tran}} = \frac{1}{2}mv^2$ and $K_{\text{rot}} = \frac{1}{2}I\omega^2$.

**EXECUTE:** **(a)** $60° = (\frac{1}{3})$ rad. The average angular speed of each arm and leg is $\frac{\frac{1}{3}\text{ rad}}{1\text{ s}} = 1.05\text{ rad/s}$.

**(b)** Adding the moments of inertia gives

$I = \frac{1}{3}m_{\text{arm}}L_{\text{arm}}^{\;2} + \frac{1}{3}m_{\text{leg}}L_{\text{leg}}^{\;2} = \frac{1}{3}[(0.13)(75\text{ kg})(0.70\text{ m})^2 + (0.37)(75\text{ kg})(0.90\text{ m})^2]$. $I = 9.08\text{ kg}\cdot\text{m}^2$.

$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(9.08\text{ kg}\cdot\text{m}^2)(1.05\text{ rad/s})^2 = 5.0\text{ J}$.

**(c)** $K_{\text{tran}} = \frac{1}{2}mv^2 = \frac{1}{2}(75\text{ kg})(1.4\text{ m/s})^2 = 73.5\text{ J}$ and $K_{\text{tot}} = K_{\text{tran}} + K_{\text{rot}} = 78.5\text{ J}$.

**(d)** $\frac{K_{\text{rot}}}{K_{\text{tran}}} = \frac{5.0\text{ J}}{78.5\text{ J}} = 6.4\%$.

**EVALUATE:** If you swing your arms more vigorously more of your energy input goes into the kinetic energy of walking and it is more effective exercise. Carrying weights in our hands would also be effective.

**9.82.** **IDENTIFY:** The total kinetic energy of a runner is the sum of his translational kinetic energy plus the rotational kinetic of his arms and legs. We can model these parts of the body as uniform bars.

**SET UP:** Now $v = 12\text{ km/h} = 3.33\text{ m/s}$. $I_{\text{tot}} = 9.08\text{ kg}\cdot\text{m}^2$ as in the previous problem.

**EXECUTE:** **(a)** $\omega_{\text{av}} = \frac{\pi/3\text{ rad}}{0.5\text{ s}} = 2.1\text{ rad/s}$.

**(b)** $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(9.08\text{ kg}\cdot\text{m}^2)(2.1\text{ rad/s})^2 = 20\text{ J}$.

**(c)** $K_{\text{tran}} = \frac{1}{2}mv^2 = \frac{1}{2}(75\text{ kg})(3.33\text{ m/s})^2 = 416\text{ J}$. Therefore

$K_{\text{tot}} = K_{\text{tran}} + K_{\text{rot}} = 416\text{ J} + 20\text{ J} = 436\text{ J}$.

**(d)** $\frac{K_{\text{rot}}}{K_{\text{tot}}} = \frac{20\text{ J}}{436\text{ J}} = 0.046$, so $K_{\text{rot}}$ is 4.6% of $K_{\text{tot}}$.

**EVALUATE:** The amount rotational energy depends on the geometry of the object.

**9.83.** **IDENTIFY:** We know (or can calculate) the masses and geometric measurements of the various parts of the body. We can model them as familiar objects, such as uniform spheres, rods, and cylinders, and calculate their moments of inertia and kinetic energies.

**SET UP:** My total mass is $m = 90$ kg. I model my head as a uniform sphere of radius 8 cm. I model my trunk and legs as a uniform solid cylinder of radius 12 cm. I model my arms as slender rods of length 60 cm. $\omega = 72\text{ rev/min} = 7.5\text{ rad/s}$. For a solid uniform sphere, $I = 2/5\,MR^2$, for a solid cylinder, $I = \frac{1}{2}MR^2$, and for a rod rotated about one end $I = 1/3\,ML^2$.

**EXECUTE:** **(a)** Using the formulas indicated above, we have $I_{\text{tot}} = I_{\text{head}} + I_{\text{trunk+legs}} + I_{\text{arms}}$, which gives $I_{\text{tot}} = \frac{2}{5}(0.070m)(0.080\text{ m})^2 + \frac{1}{2}(0.80m)(0.12\text{ m})^2 + 2\left(\frac{1}{3}\right)(0.13m)(0.60\text{ m})^2 = 3.3\text{ kg}\cdot\text{m}^2$ where we have used $m = 90$ kg.

**(b)** $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(3.3\text{ kg}\cdot\text{m}^2)(7.5\text{ rad/s})^2 = 93\text{ J}$.

**EVALUATE:** According to these estimates about 85% of the total $I$ is due to the outstretched arms. If the initial translational kinetic energy $\frac{1}{2}mv^2$ of the skater is converted to this rotational kinetic energy as he goes into a spin, his initial speed must be $1.4\text{ m/s}$.

**9.84.** **IDENTIFY:** Apply the parallel-axis theorem to each side of the square.

**SET UP:** Each side has length $a$ and mass $M/4$, and the moment of inertia of each side about an axis perpendicular to the side and through its center is $\frac{1}{12}\left(\frac{1}{4}Ma^2\right)=\frac{1}{48}Ma^2$.

**EXECUTE:** The moment of inertia of each side about the axis through the center of the square is, from the perpendicular axis theorem, $\dfrac{Ma^2}{48}+\dfrac{M}{4}\left(\dfrac{a}{2}\right)^2=\dfrac{Ma^2}{12}$. The total moment of inertia is the sum of the contributions from the four sides, or $4\times\dfrac{Ma^2}{12}=\dfrac{Ma^2}{3}$.

**EVALUATE:** If all the mass of a side were at its center, a distance $a/2$ from the axis, we would have $I=4\left(\dfrac{M}{4}\right)\left(\dfrac{a}{2}\right)^2=\dfrac{1}{4}Ma^2$. If all the mass was divided equally among the four corners of the square, a distance $a/\sqrt{2}$ from the axis, we would have $I=4\left(\dfrac{M}{4}\right)\left(\dfrac{a}{\sqrt{2}}\right)^2=\frac{1}{2}Ma^2$. The actual $I$ is between these two values.

**9.85.** **IDENTIFY:** The density depends on the distance from the center of the sphere, so it is a function of $r$. We need to integrate to find the mass and the moment of inertia.

**SET UP:** $M=\int dm=\int\rho dV$ and $I=\int dI$.

**EXECUTE:** **(a)** Divide the sphere into thin spherical shells of radius $r$ and thickness $dr$. The volume of each shell is $dV=4\pi r^2dr$. $\rho(r)=a-br$, with $a=3.00\times10^3\ \text{kg/m}^3$ and $b=9.00\times10^3\ \text{kg/m}^4$. Integrating gives $M=\int dm=\int\rho dV=\int_0^R(a-br)4\pi r^2dr=\dfrac{4}{3}\pi R^3\left(a-\dfrac{3}{4}bR\right)$.

$$M=\frac{4}{3}\pi(0.200\ \text{m})^3\left(3.00\times10^3\ \text{kg/m}^3-\frac{3}{4}(9.00\times10^3\ \text{kg/m}^4)(0.200\ \text{m})\right)=55.3\ \text{kg}.$$

**(b)** The moment of inertia of each thin spherical shell is

$$dI=\frac{2}{3}r^2dm=\frac{2}{3}r^2\rho dV=\frac{2}{3}r^2(a-br)4\pi r^2dr=\frac{8\pi}{3}r^4(a-br)dr.$$

$$I=\int_0^R dI=\frac{8\pi}{3}\int_0^R r^4(a-br)dr=\frac{8\pi}{15}R^5\left(a-\frac{5b}{6}R\right).$$

$$I=\frac{8\pi}{15}(0.200\ \text{m})^5\left(3.00\times10^3\ \text{kg/m}^3-\frac{5}{6}(9.00\times10^3\ \text{kg/m}^4)(0.200\ \text{m})\right)=0.804\ \text{kg}\cdot\text{m}^2.$$

**EVALUATE:** We cannot use the formulas $M=\rho V$ and $I=\frac{1}{2}MR^2$ because this sphere is not uniform throughout. Its density increases toward the surface. For a uniform sphere with density $3.00\times10^3\ \text{kg/m}^3$, the mass is $\dfrac{4}{3}\pi R^3\rho=100.5\ \text{kg}$. The mass of the sphere in this problem is less than this. For a uniform sphere with mass 55.3 kg and $R=0.200$ m, $I=\dfrac{2}{5}MR^2=0.885\ \text{kg}\cdot\text{m}^2$. The moment of inertia for the sphere in this problem is less than this, since the density decreases with distance from the center of the sphere.

**9.86.** **IDENTIFY:** Write $K$ in terms of the period $T$ and take derivatives of both sides of this equation to relate $dK/dt$ to $dT/dt$.

**SET UP:** $\omega=\dfrac{2\pi}{T}$ and $K=\frac{1}{2}I\omega^2$. The speed of light is $c=3.00\times10^8$ m/s.

**EXECUTE:** **(a)** $K=\dfrac{2\pi^2I}{T^2}$. $\dfrac{dK}{dt}=-\dfrac{4\pi^2I}{T^3}\dfrac{dT}{dt}$. The rate of energy loss is $\dfrac{4\pi^2I}{T^3}\dfrac{dT}{dt}$. Solving for the moment of inertia $I$ in terms of the power $P$,

$$I = \frac{PT^3}{4\pi^2}\frac{1}{dT/dt} = \frac{(5\times10^{31}\ \text{W})(0.0331\ \text{s})^3}{4\pi^2}\frac{1\ \text{s}}{4.22\times10^{-13}\ \text{s}} = 1.09\times10^{38}\ \text{kg}\cdot\text{m}^2$$

**(b)** $R = \sqrt{\frac{5I}{2M}} = \sqrt{\frac{5(1.08\times10^{38}\ \text{kg}\cdot\text{m}^2)}{2(1.4)(1.99\times10^{30}\ \text{kg})}} = 9.9\times10^3\ \text{m}$, about 10 km.

**(c)** $v = \frac{2\pi R}{T} = \frac{2\pi(9.9\times10^3\text{m})}{(0.0331\ \text{s})} = 1.9\times10^6\ \text{m/s} = 6.3\times10^{-3}c.$

**(d)** $\rho = \frac{M}{V} = \frac{M}{(4\pi/3)R^3} = 6.9\times10^{17}\,\text{kg/m}^3$, which is much higher than the density of ordinary rock by 14 orders of magnitude, and is comparable to nuclear mass densities.

**EVALUATE:** $I$ is huge because $M$ is huge. A small rate of change in the period corresponds to a large release of energy.

**9.87.** **IDENTIFY:** The graph with the problem in the text shows that the angular acceleration increases linearly with time and is therefore not constant.

**SET UP:** $\omega_z = d\theta/dt$, $\alpha_z = d\omega_z/dt$.

**EXECUTE:** **(a)** Since the angular acceleration is not constant, Eq. (9.11) cannot be used, so we must use $\alpha_z = d\omega_z/dt$ and $\omega_z = d\theta/dt$ and integrate to find the angle. The graph passes through the origin and has a constant positive slope of 6/5 rad/s$^3$, so the equation for $\alpha_z$ is $\alpha_z$ = (1.2 rad/s$^3$)$t$. Using $\alpha_z = d\omega_z/dt$ gives $\omega_z = \omega_{0z} + \int_0^t \alpha_z dt = 0 + \int_0^t (1.2\ \text{rad/s}^3)t\,dt = (0.60\ \text{rad/s}^3)t^2$. Now we must use $\omega_z = d\theta/dt$ and integrate again to get the angle.

$\theta_2 - \theta_1 = \int_0^t \omega_z dt = \int_0^t (0.60\ \text{rad/s}^3)t^2 dt = (0.20\ \text{rad/s}^3)t^3$ = (0.20 rad/s$^3$)(5.0 s)$^3$ = 25 rad.

**(b)** The result of our first integration gives $\omega_z$ = (0.60 rad/s$^3$)(5.0 s)$^2$ = 15 rad/s.

**(c)** The result of our second integration gives $4\pi$ rad = (0.20 rad/s$^3$)$t^3$, so $t$ = 3.98 s. Therefore $\omega_z$ = (0.60 rad/s$^3$)(3.98 s)$^2$ = 9.48 rad/s.

**EVALUATE:** When the constant-acceleration angular kinematics formulas do not apply, we must go back to basic definitions.

**9.88.** **IDENTIFY** and **SET UP:** The graph of $a^2$ versus $(\theta - \theta_0)^2$ is shown in Figure 9.88. It is a straight line with a positive slope. The angular acceleration is constant.

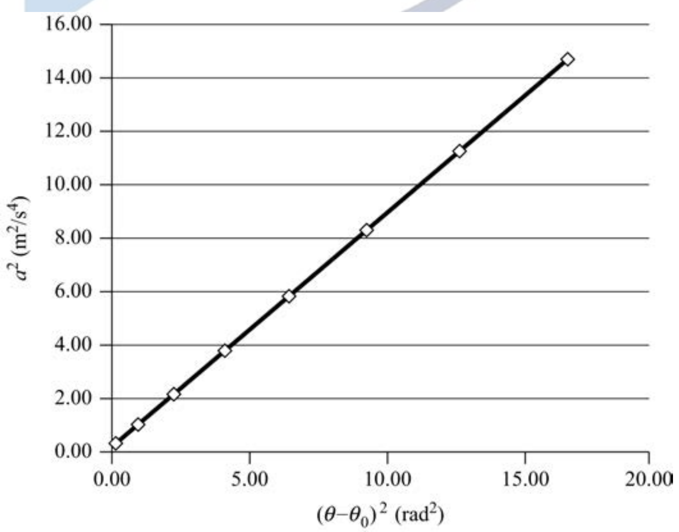



**Figure 9.88**

**EXECUTE:** **(a)** From graphing software, the slope is 0.921 $m^2/s^4$ and the $y$-intercept is 0.233 $m^2/s^4$.
**(b)** The resultant acceleration is $a^2 = a_{tan}^2 + a_{rad}^2$. $a_{tan} = r\alpha_z$ and $a_{rad} = r\omega_z^2$, where $\omega_z^2 = \omega_{z0}^2 + 2\alpha_z(\theta-\theta_0) = 0 + 2\alpha_z(\theta-\theta_0)$. Therefore the resultant acceleration is

$a^2 = (r\,\alpha_z)^2 + [2r\,\alpha_z(\theta-\theta_0)]^2$

$a^2 = 4r^2\alpha_z^2(\theta-\theta_0)^2 + (r\alpha_z)^2$.

From this result, we see that the slope of the graph is $4r^2\alpha_z^2$, so $4r^2\alpha_z^2 = 0.921\ \text{m}^2/\text{s}^4$. Solving for $\alpha_z$ gives $\alpha_z = \sqrt{\dfrac{0.921\ \text{m}^2/\text{s}^4}{4(0.800\ \text{m})^2}} = 0.600\ \text{rad/s}^2$.

**(c)** Using $\omega_z^2 = \omega_{z0}^2 + 2\alpha_z(\theta-\theta_0)$ gives $\omega_z^2 = 0 + 2(0.600\ \text{rad/s}^2)(3\pi/4\ \text{rad})$, $\omega_z = 1.6815$ rad/s. The speed is $v = r\,\omega_z = (0.800\ \text{m})(1.6815\ \text{rad/s}) = 1.35$ m/s.
**(d)** Call $\phi$ the angle between the linear velocity and the resultant acceleration. The resultant velocity is tangent to the circle, so $\tan\phi = \dfrac{a_{rad}}{a_{tan}} = \dfrac{r\omega_z^2}{r\alpha_z} = \dfrac{\omega_z^2}{\alpha_z}$. It is also the case that $\omega_z^2 = 2\alpha_z\Delta\theta$, so

$\tan\phi = \dfrac{2\alpha_z\Delta\theta}{\alpha_z} = 2\Delta\theta = 2(\pi/2) = \pi$. Thus $\phi = \arctan \pi = 72.3°$.

**EVALUATE:** According to the work in parts (a) and (b), the $y$-intercept of the graph is $(r\alpha_z)^2$ and is equal to 0.233 $m^2/s^4$. Solving for $\alpha_z$ gives $\alpha_z = \sqrt{\dfrac{0.233\ \text{m}^2/\text{s}^4}{(0.800\ \text{m})^2}} = 0.60\ \text{rad/s}^2$, as we found in part (b).

**9.89.** **IDENTIFY** and **SET UP:** The equation of the graph in the text is $d = (165\ \text{cm/s}^2)t^2$. For constant acceleration, the second time derivative of the position ($d$ in this case) is a constant.

**EXECUTE:** **(a)** $\dfrac{d(d)}{dt} = (330\ \text{cm/s}^2)t$ and $\dfrac{d^2(d)}{dt^2} = 330\ \text{cm/s}^2$, which is a constant. Therefore the acceleration of the metal block is a constant 330 $\text{cm/s}^2$ = 3.30 $\text{m/s}^2$.
**(b)** $v = \dfrac{d(d)}{dt} = (330\ \text{cm/s}^2)t$. When $d$ = 1.50 m = 150 cm, we have 150 cm = $(165\ \text{cm/s}^2)t^2$, which gives $t$ = 0.9535 s. Thus $v$ = 330 $\text{cm/s}^2$)(0.9535 s) = 315 cm/s = 3.15 m/s.
**(c)** Energy conservation $K_1 + U_1 = K_2 + U_2$ gives $mgd = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}mv^2$. Using $\omega = v/r$, solving for $I$ and putting in the numbers $m$ = 5.60 kg, $d$ = 1.50 m, $r$ = 0.178 m, $v$ = 3.15 m/s, we get $I = 0.348\ \text{kg}\cdot\text{m}^2$.
**(d)** Newton's second law gives $mg - T = ma$, $T = m(g - a) = (5.60\ \text{kg})(9.80\ \text{m/s}^2 - 3.30\ \text{m/s}^2) = 36.4$ N.
**EVALUATE:** When dealing with non-uniform objects, such as this flywheel, we cannot use the standard moment of inertia formulas and must resort to other ways.

**9.90.** **IDENTIFY:** Apply $I = \int r^2\,dm.$

**SET UP:** Let $z$ be the coordinate along the vertical axis. $r(z) = \dfrac{zR}{h}$. $dm = \pi\rho\dfrac{R^2z^2}{h^2}$ and $dI = \dfrac{\pi\rho}{2}\dfrac{R^4}{h^4}z^4\,dz.$

**EXECUTE:** $I = \int dI = \dfrac{\pi\rho}{2}\dfrac{R^4}{h^4}\int_0^h z^4\,dz = \dfrac{\pi\rho}{10}\dfrac{R^4}{h^4}\left[z^5\right]_0^h = \dfrac{1}{10}\pi\rho R^4 h$. The volume of a right circular cone is $V = \tfrac{1}{3}\pi R^2 h$, the mass is $\tfrac{1}{3}\pi\rho R^2 h$ and so $I = \dfrac{3}{10}\left(\dfrac{\pi\rho R^2 h}{3}\right)R^2 = \dfrac{3}{10}MR^2.$

**EVALUATE:** For a uniform cylinder of radius $R$ and for an axis through its center, $I = \tfrac{1}{2}MR^2$. $I$ for the cone is less, as expected, since the cone is constructed from a series of parallel discs whose radii decrease from $R$ to zero along the vertical axis of the cone.

**9.91.** **IDENTIFY:** Follow the steps outlined in the problem.

**SET UP:** $\omega_z = d\theta/dt$. $\alpha_z = d^2\omega_z/dt^2$.

**EXECUTE:** **(a)** $ds = r\,d\theta = r_0 d\theta + \beta\theta\,d\theta$ so $s(\theta) = r_0\theta + \frac{\beta}{2}\theta^2$. $\theta$ must be in radians.

**(b)** Setting $s = vt = r_0\theta + \frac{\beta}{2}\theta^2$ gives a quadratic in $\theta$. The positive solution is

$$\theta(t) = \frac{1}{\beta}\left[\sqrt{r_0{}^2 + 2\beta vt} - r_0\right].$$

(The negative solution would be going backwards, to values of $r$ smaller than $r_0$.)

**(c)** Differentiating, $\omega_z(t) = \dfrac{d\theta}{dt} = \dfrac{v}{\sqrt{r_0^2 + 2\beta vt}}$, $\alpha_z = \dfrac{d\omega_z}{dt} = -\dfrac{\beta v^2}{(r_0^2 + 2\beta vt)^{3/2}}$. The angular acceleration $\alpha_z$ is not constant.

**(d)** $r_0 = 25.0$ mm. $\theta$ must be measured in radians, so $\beta = (1.55\,\mu\text{m/rev})(1\text{ rev}/2\pi\text{ rad}) = 0.247\,\mu\text{m/rad}$. Using $\theta(t)$ from part (b), the total angle turned in $74.0\text{ min} = 4440\text{ s}$ is

$$\theta = \frac{1}{2.47\times10^{-7}\text{ m/rad}}\left(\sqrt{2(2.47\times10^{-7}\text{ m/rad})(1.25\text{ m/s})(4440\text{ s}) + (25.0\times10^{-3}\text{ m})^2} - 25.0\times10^{-3}\text{ m}\right)$$

$\theta = 1.337\times10^5$ rad, which is $2.13\times10^4$ rev.

**(e)** The graphs are sketched in Figure 9.91.

**EVALUATE:** $\omega_z$ must decrease as $r$ increases, to keep $v = r\omega$ constant. For $\omega_z$ to decrease in time, $\alpha_z$ must be negative.

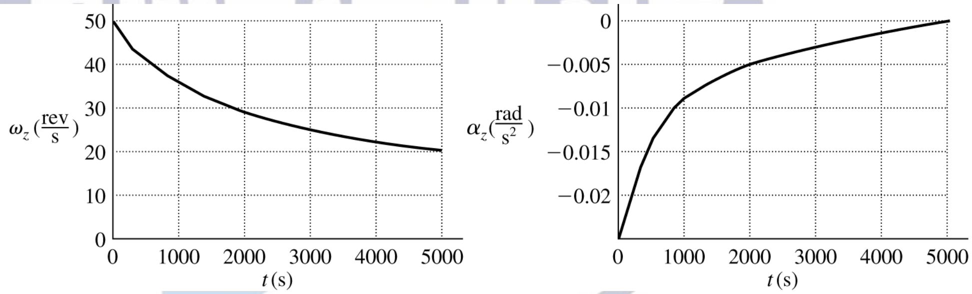



**Figure 9.91**

**9.92.** **IDENTIFY and SET UP:** For constant angular speed $\theta = \omega t$.

**EXECUTE:** **(a)** $\theta = \omega t$ = (14 rev/s)(2π rad/rev)(1/120 s) = 42°, which is choice (d).

**EVALUATE:** This is quite a large rotation in just one frame.

**9.93.** **IDENTIFY and SET UP:** The average angular acceleration is $\alpha_{\text{av}} = \dfrac{\omega - \omega_0}{t}$.

**EXECUTE:** **(a)** $\alpha_{\text{av}} = \dfrac{\omega - \omega_0}{t}$ = [8 rev/s – (–14 rev/s)]/(10 s) = (2.2 rev/s)(2π rad/rev) = 44π/10 rad/s$^2$ which is choice (d).

**EVALUATE:** This is nearly 14 rad/s$^2$.

**9.94.** **IDENTIFY and SET UP:** The rotational kinetic energy is $K = \frac{1}{2}I\omega^2$ and the kinetic energy due to running is $K = \frac{1}{2}mv^2$.

**EXECUTE:** Equating the two kinetic energies gives $\frac{1}{2}mv^2 = \frac{1}{2}I\omega^2$. Using $I = \frac{1}{2}mR^2$, we have $\frac{1}{2}(\frac{1}{2}mr^2)\omega^2 = \frac{1}{2}mv^2$, which gives $v = \frac{r\omega}{\sqrt{2}} = \frac{(0.05\text{ m})(14\text{ rev/s})(2\pi\text{ rad/rev})}{\sqrt{2}} = 3.11$ m/s, choice (c).

**EVALUATE:** This is about 3 times as fast as a human walks.

**9.95.** **IDENTIFY** and **SET UP:** $I = \frac{1}{2}mR^2$.

**EXECUTE:** **(a)** $I = \frac{1}{2}mR^2$, so if we double the radius but keep the mass fixed, the moment of inertia increases by a factor of 4, which is choice (d).

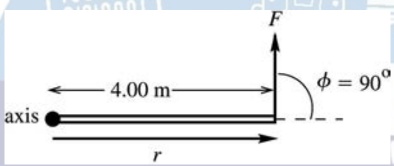
**EVALUATE:** The difference in length of the two eels plays no part in their moment of inertia if their mass is the same in both cases.

# DYNAMICS OF ROTATIONAL MOTION

# 10

**10.1. IDENTIFY:** Use $\tau = Fl$ to calculate the magnitude of the torque and use the right-hand rule illustrated in Section 10.1 in the textbook to calculate the torque direction.

**(a) SET UP:** Consider Figure 10.1a.




**EXECUTE:** $\tau = Fl$

$l = r\sin\phi = (4.00\text{ m})\sin 90°$

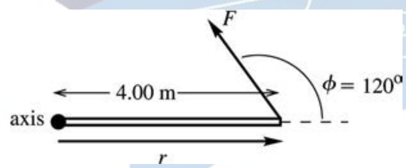
$l = 4.00\text{ m}$

$\tau = (10.0\text{ N})(4.00\text{ m}) = 40.0\text{ N}\cdot\text{m}$

**Figure 10.1a**

This force tends to produce a counterclockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed out of the plane of the figure.

**(b) SET UP:** Consider Figure 10.1b.




**EXECUTE:** $\tau = Fl$

$l = r\sin\phi = (4.00\text{ m})\sin 120°$

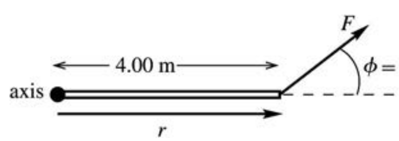
$l = 3.464\text{ m}$

$\tau = (10.0\text{ N})(3.464\text{ m}) = 34.6\text{ N}\cdot\text{m}$

**Figure 10.1b**

This force tends to produce a counterclockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed out of the plane of the figure.

**(c) SET UP:** Consider Figure 10.1c.




**EXECUTE:** $\tau = Fl$

$l = r\sin\phi = (4.00\text{ m})\sin 30°$

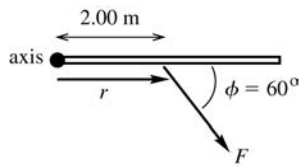
$l = 2.00\text{ m}$

$\tau = (10.0\text{ N})(2.00\text{ m}) = 20.0\text{ N}\cdot\text{m}$

**Figure 10.1c**

This force tends to produce a counterclockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed out of the plane of the figure.

**(d) SET UP:** Consider Figure 10.1d.




**EXECUTE:** $\tau = Fl$

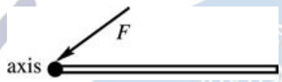
$l = r\sin\phi = (2.00\text{ m})\sin 60° = 1.732\text{ m}$

$\tau = (10.0\text{ N})(1.732\text{ m}) = 17.3\text{ N}\cdot\text{m}$

**Figure 10.1d**

This force tends to produce a clockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed into the plane of the figure.

**(e) SET UP:** Consider Figure 10.1e.

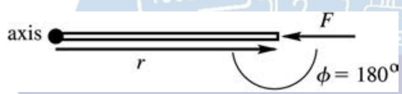



**EXECUTE:** $\tau = Fl$

$r = 0$ so $l = 0$ and $\tau = 0$

**Figure 10.1e**

**(f) SET UP:** Consider Figure 10.1f.




**EXECUTE:** $\tau = Fl$

$l = r\sin\phi$, $\phi = 180°$,

so $l = 0$ and $\tau = 0$

**Figure 10.1f**

**EVALUATE:** The torque is zero in parts (e) and (f) because the moment arm is zero; the line of action of the force passes through the axis.

**10.2. IDENTIFY:** $\tau = Fl$ with $l = r\sin\phi$. Add the two torques to calculate the net torque.

**SET UP:** Let counterclockwise torques be positive.

**EXECUTE:** $\tau_1 = -F_1 l_1 = -(8.00\text{ N})(5.00\text{ m}) = -40.0\text{ N}\cdot\text{m}$.

$\tau_2 = +F_2 l_2 = (12.0\text{ N})(2.00\text{ m})\sin 30.0° = +12.0\text{ N}\cdot\text{m}$. $\sum\tau = \tau_1 + \tau_2 = -28.0\text{ N}\cdot\text{m}$. The net torque is $28.0\text{ N}\cdot\text{m}$, clockwise.

**EVALUATE:** Even though $F_1 < F_2$, the magnitude of $\tau_1$ is greater than the magnitude of $\tau_2$, because $F_1$ has a larger moment arm.

**10.3. IDENTIFY and SET UP:** Use $\tau = Fl$ to calculate the magnitude of each torque and use the right-hand rule (Figure 10.4 in the textbook) to determine the direction. Consider Figure 10.3.

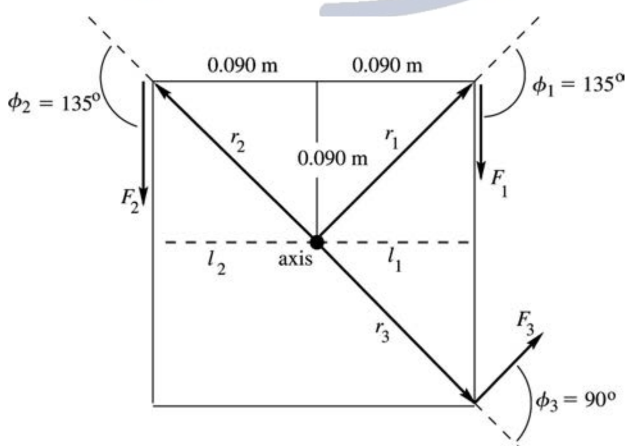



**Figure 10.3**

Let counterclockwise be the positive sense of rotation.

**EXECUTE:** $r_1 = r_2 = r_3 = \sqrt{(0.090\text{ m})^2 + (0.090\text{ m})^2} = 0.1273\text{ m}$

$\tau_1 = -F_1 l_1$

$l_1 = r_1 \sin\phi_1 = (0.1273\text{ m})\sin 135° = 0.0900\text{ m}$

$\tau_1 = -(18.0\text{ N})(0.0900\text{ m}) = -1.62\text{ N}\cdot\text{m}$

$\vec{\tau}_1$ is directed into paper

$\tau_2 = +F_2 l_2$

$l_2 = r_2 \sin\phi_2 = (0.1273\text{ m})\sin 135° = 0.0900\text{ m}$

$\tau_2 = +(26.0\text{ N})(0.0900\text{ m}) = +2.34\text{ N}\cdot\text{m}$

$\vec{\tau}_2$ is directed out of paper

$\tau_3 = +F_3 l_3$

$l_3 = r_3 \sin\phi_3 = (0.1273\text{ m})\sin 90° = 0.1273\text{ m}$

$\tau_3 = +(14.0\text{ N})(0.1273\text{ m}) = +1.78\text{ N}\cdot\text{m}$

$\vec{\tau}_3$ is directed out of paper

$\sum\tau = \tau_1 + \tau_2 + \tau_3 = -1.62\text{ N}\cdot\text{m} + 2.34\text{ N}\cdot\text{m} + 1.78\text{ N}\cdot\text{m} = 2.50\text{ N}\cdot\text{m}$

**EVALUATE:** The net torque is positive, which means it tends to produce a counterclockwise rotation; the vector torque is directed out of the plane of the paper. In summing the torques it is important to include $+$ or $-$ signs to show direction.

**10.4.** **IDENTIFY:** Use $\tau = Fl = rF\sin\phi$ to calculate the magnitude of each torque and use the right-hand rule to determine the direction of each torque. Add the torques to find the net torque.

**SET UP:** Let counterclockwise torques be positive. For the 11.9 N force $(F_1)$, $r = 0$. For the 14.6 N force $(F_2)$, $r = 0.350\text{ m}$ and $\phi = 40.0°$. For the 8.50 N force $(F_3)$, $r = 0.350\text{ m}$ and $\phi = 90.0°$.

**EXECUTE:** $\tau_1 = 0$. $\tau_2 = -(14.6\text{ N})(0.350\text{ m})\sin 40.0° = -3.285\text{ N}\cdot\text{m}$.

$\tau_3 = +(8.50\text{ N})(0.350\text{ m})\sin 90.0° = +2.975\text{ N}\cdot\text{m}$. $\sum\tau = -3.285\text{ N}\cdot\text{m} + 2.975\text{ N}\cdot\text{m} = -0.31\text{ N}\cdot\text{m}$. The net torque is $0.31\text{ N}\cdot\text{m}$ and is clockwise.

**EVALUATE:** If we treat the torques as vectors, $\vec{\tau}_2$ is into the page and $\vec{\tau}_3$ is out of the page.

**10.5.** **IDENTIFY and SET UP:** Calculate the torque using Eq. (10.3) and also determine the direction of the torque using the right-hand rule.

**(a)** $\vec{r} = (-0.450\text{ m})\hat{i} + (0.150\text{ m})\hat{j}$; $\vec{F} = (-5.00\text{ N})\hat{i} + (4.00\text{ N})\hat{j}$. The sketch is given in Figure 10.5.

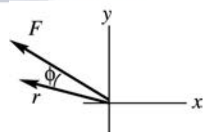



**Figure 10.5**

**EXECUTE:** **(b)** When the fingers of your right hand curl from the direction of $\vec{r}$ into the direction of $\vec{F}$ (through the smaller of the two angles, angle $\phi$) your thumb points into the page (the direction of $\vec{\tau}$, the $-z$-direction).

**(c)** $\vec{\tau} = \vec{r}\times\vec{F} = [(-0.450\text{ m})\hat{i} + (0.150\text{ m})\hat{j}]\times[(-5.00\text{ N})\hat{i} + (4.00\text{ N})\hat{j}]$

$\vec{\tau} = +(2.25\text{ N}\cdot\text{m})\hat{i}\times\hat{i} - (1.80\text{ N}\cdot\text{m})\hat{i}\times\hat{j} - (0.750\text{ N}\cdot\text{m})\hat{j}\times\hat{i} + (0.600\text{ N}\cdot\text{m})\hat{j}\times\hat{j}$

$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = 0$

$\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{i} = -\hat{k}$

Thus $\vec{\tau} = -(1.80\ \text{N}\cdot\text{m})\hat{k} - (0.750\ \text{N}\cdot\text{m})(-\hat{k}) = (-1.05\ \text{N}\cdot\text{m})\hat{k}$.

**EVALUATE:** The calculation gives that $\vec{\tau}$ is in the $-z$-direction. This agrees with what we got from the right-hand rule.

**10.6. IDENTIFY:** Knowing the force on a bar and the point where it acts, we want to find the position vector for the point where the force acts and the torque the force exerts on the bar.

**SET UP:** The position vector is $\vec{r} = x\hat{i} + y\hat{j}$ and the torque is $\vec{\tau} = \vec{r} \times \vec{F}$.

**EXECUTE:** **(a)** Using $x = 3.00$ m and $y = 4.00$ m, we have $\vec{r} = (3.00\ \text{m})\hat{i} + (4.00\ \text{m})\hat{j}$.

**(b)** $\vec{\tau} = \vec{r} \times \vec{F} = [(3.00\ \text{m})\hat{i} + (4.00\ \text{m})\hat{j}] \times [(7.00\ \text{N})\hat{i} + (-3.00\ \text{N})\hat{j}]$.

$\vec{\tau} = (-9.00\ \text{N}\cdot\text{m})\hat{k} + (-28.0\ \text{N}\cdot\text{m})(-\hat{k}) = (-37.0\ \text{N}\cdot\text{m})\hat{k}$. The torque has magnitude $37.0\ \text{N}\cdot\text{m}$ and is in the $-z$-direction.

**EVALUATE:** Applying the right-hand rule for the vector product to $\vec{r} \times \vec{F}$ shows that the torque must be in the $-z$-direction because it is perpendicular to both $\vec{r}$ and $\vec{F}$, which are both in the $x$-$y$ plane.

**10.7. IDENTIFY:** Use $\tau = Fl = rF\sin\phi$ for the magnitude of the torque and the right-hand rule for the direction.

**SET UP:** In part (a), $r = 0.250$ m and $\phi = 37°$.

**EXECUTE:** **(a)** $\tau = (17.0\ \text{N})(0.250\ \text{m})\sin 37° = 2.56\ \text{N}\cdot\text{m}$. The torque is counterclockwise.

**(b)** The torque is maximum when $\phi = 90°$ and the force is perpendicular to the wrench. This maximum torque is $(17.0\ \text{N})(0.250\ \text{m}) = 4.25\ \text{N}\cdot\text{m}$.

**EVALUATE:** If the force is directed along the handle then the torque is zero. The torque increases as the angle between the force and the handle increases.

**10.8. IDENTIFY:** The constant force produces a torque which gives a constant angular acceleration to the disk and a linear acceleration to points on the disk.

**SET UP:** $\sum\tau_z = I\alpha_z$ applies to the disk, $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ because the angular acceleration is constant. The acceleration components of the rim are $a_{\text{tan}} = r\alpha$ and $a_{\text{rad}} = r\omega^2$, and the magnitude of the acceleration is $a = \sqrt{a_{\text{tan}}^2 + a_{\text{rad}}^2}$.

**EXECUTE:** **(a)** $\sum\tau_z = I\alpha_z$ gives $Fr = I\alpha_z$. For a uniform disk,

$$I = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(40.0\ \text{kg})(0.200\ \text{m})^2 = 0.800\ \text{kg}\cdot\text{m}^2.\quad \alpha_z = \frac{Fr}{I} = \frac{(30.0\ \text{N})(0.200\ \text{m})}{0.800\ \text{kg}\cdot\text{m}^2} = 7.50\ \text{rad/s}^2.$$

$\theta - \theta_0 = 0.200\ \text{rev} = 1.257$ rad. $\omega_{0z} = 0$, so $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ gives

$\omega_z = \sqrt{2(7.50\ \text{rad/s}^2)(1.257\ \text{rad})} = 4.342$ rad/s. $v = r\omega = (0.200\ \text{m})(4.342\ \text{rad/s}) = 0.868$ m/s.

**(b)** $a_{\text{tan}} = r\alpha = (0.200\ \text{m})(7.50\ \text{rad/s}^2) = 1.50\ \text{m/s}^2$. $a_{\text{rad}} = r\omega^2 = (0.200\ \text{m})(4.342\ \text{rad/s})^2 = 3.771\ \text{m/s}^2$.

$a = \sqrt{a_{\text{tan}}^2 + a_{\text{rad}}^2} = 4.06\ \text{m/s}^2$.

**EVALUATE:** The net acceleration is neither toward the center nor tangent to the disk.

**10.9. IDENTIFY:** Apply $\sum\tau_z = I\alpha_z$.

**SET UP:** $\omega_{0z} = 0$. $\omega_z = (400\ \text{rev/min})\left(\dfrac{2\pi\ \text{rad/rev}}{60\ \text{s/min}}\right) = 41.9$ rad/s

**EXECUTE:** $\tau_z = I\alpha_z = I\dfrac{\omega_z - \omega_{0z}}{t} = (1.60\ \text{kg}\cdot\text{m}^2)\dfrac{41.9\ \text{rad/s}}{8.00\ \text{s}} = 8.38\ \text{N}\cdot\text{m}$.

**EVALUATE:** In $\tau_z = I\alpha_z$, $\alpha_z$ must be in $\text{rad/s}^2$.

**10.10. IDENTIFY:** Apply $\sum\tau_z = I\alpha_z$ to the wheel. The acceleration $a$ of a point on the cord and the angular acceleration $\alpha$ of the wheel are related by $a = R\alpha$.

**SET UP:** Let the direction of rotation of the wheel be positive. The wheel has the shape of a disk and $I = \tfrac{1}{2}MR^2$. The free-body diagram for the wheel is sketched in Figure 10.10a for a horizontal pull and in Figure 10.10b for a vertical pull. $P$ is the pull on the cord and $F$ is the force exerted on the wheel by the axle.

**EXECUTE:** **(a)** $\alpha_z = \dfrac{\tau_z}{I} = \dfrac{(40.0\ \text{N})(0.250\ \text{m})}{\frac{1}{2}(9.20\ \text{kg})(0.250\ \text{m})^2} = 34.8\ \text{rad/s}^2.$

$a = R\alpha = (0.250\ \text{m})(34.8\ \text{rad/s}^2) = 8.70\ \text{m/s}^2.$

**(b)** $F_x = -P,\ F_y = Mg.\ F = \sqrt{P^2 + (Mg)^2} = \sqrt{(40.0\ \text{N})^2 + ([9.20\ \text{kg}][9.80\ \text{m/s}^2])^2} = 98.6\ \text{N}.$

$\tan\phi = \dfrac{|F_y|}{|F_x|} = \dfrac{Mg}{P} = \dfrac{(9.20\ \text{kg})(9.80\ \text{m/s}^2)}{40.0\ \text{N}}$ and $\phi = 66.1°.$ The force exerted by the axle has magnitude 98.6 N and is directed at $66.1°$ above the horizontal, away from the direction of the pull on the cord.

**(c)** The pull exerts the same torque as in part (a), so the answers to part (a) don't change. In part (b), $F + P = Mg$ and $F = Mg - P = (9.20\ \text{kg})(9.80\ \text{m/s}^2) - 40.0\ \text{N} = 50.2\ \text{N}.$ The force exerted by the axle has magnitude 50.2 N and is upward.

**EVALUATE:** The weight of the wheel and the force exerted by the axle produce no torque because they act at the axle.

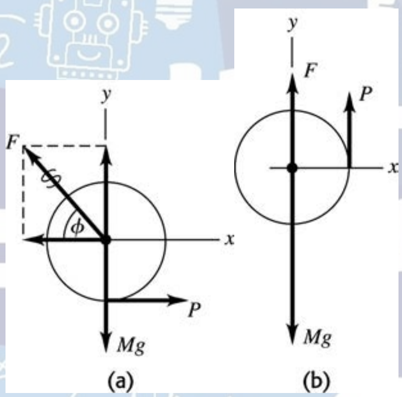



**Figure 10.10**

**10.11.** **IDENTIFY:** Use $\sum\tau_z = I\alpha_z$ to calculate $\alpha$. Use a constant angular acceleration kinematic equation to relate $\alpha_z$, $\omega_z$, and $t$.

**SET UP:** For a solid uniform sphere and an axis through its center, $I = \frac{2}{5}MR^2$. Let the direction the sphere is spinning be the positive sense of rotation. The moment arm for the friction force is $l = 0.0150\ \text{m}$ and the torque due to this force is negative.

**EXECUTE:** **(a)** $\alpha_z = \dfrac{\tau_z}{I} = \dfrac{-(0.0200\ \text{N})(0.0150\ \text{m})}{\frac{2}{5}(0.225\ \text{kg})(0.0150\ \text{m})^2} = -14.8\ \text{rad/s}^2$

**(b)** $\omega_z - \omega_{0z} = -22.5\ \text{rad/s}.$ $\omega_z = \omega_{0z} + \alpha_z t$ gives $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{-22.5\ \text{rad/s}}{-14.8\ \text{rad/s}^2} = 1.52\ \text{s}.$

**EVALUATE:** The fact that $\alpha_z$ is negative means its direction is opposite to the direction of spin. The negative $\alpha_z$ causes $\omega_z$ to decrease.

**10.12.** **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to the stone and $\sum\tau_z = I\alpha_z$ to the pulley. Use a constant acceleration equation to find $a$ for the stone.

**SET UP:** For the motion of the stone take $+y$ to be downward. The pulley has $I = \frac{1}{2}MR^2$. $a = R\alpha$.

**EXECUTE:** **(a)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $12.6\ \text{m} = \frac{1}{2}a_y(3.00\ \text{s})^2$ and $a_y = 2.80\ \text{m/s}^2.$

Then $\sum F_y = ma_y$ applied to the stone gives $mg - T = ma.$

$\sum\tau_z = I\alpha_z$ applied to the pulley gives $TR = \frac{1}{2}MR^2\alpha = \frac{1}{2}MR^2(a/R)$. $T = \frac{1}{2}Ma$.

Combining these two equations to eliminate $T$ gives

$$m = \frac{M}{2}\left(\frac{a}{g-a}\right) = \left(\frac{10.0\text{ kg}}{2}\right)\left(\frac{2.80\text{ m/s}^2}{9.80\text{ m/s}^2 - 2.80\text{ m/s}^2}\right) = 2.00\text{ kg}.$$

**(b)** $T = \frac{1}{2}Ma = \frac{1}{2}(10.0\text{ kg})(2.80\text{ m/s}^2) = 14.0\text{ N}$

**EVALUATE:** The tension in the wire is less than the weight $mg = 19.6\text{ N}$ of the stone, because the stone has a downward acceleration.

**10.13. IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to each book and apply $\sum\tau_z = I\alpha_z$ to the pulley. Use a constant acceleration equation to find the common acceleration of the books.

**SET UP:** $m_1 = 2.00\text{ kg}$, $m_2 = 3.00\text{ kg}$. Let $T_1$ be the tension in the part of the cord attached to $m_1$ and $T_2$ be the tension in the part of the cord attached to $m_2$. Let the $+x$-direction be in the direction of the acceleration of each book. $a = R\alpha$.

**EXECUTE:** **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2$ gives $a_x = \frac{2(x-x_0)}{t^2} = \frac{2(1.20\text{ m})}{(0.800\text{ s})^2} = 3.75\text{ m/s}^2$. $a_1 = 3.75\text{ m/s}^2$ so $T_1 = m_1a_1 = 7.50\text{ N}$ and $T_2 = m_2(g - a_1) = 18.2\text{ N}$.

**(b)** The torque on the pulley is $(T_2 - T_1)R = 0.803\text{ N}\cdot\text{m}$, and the angular acceleration is $\alpha = a_1/R = 50\text{ rad/s}^2$, so $I = \tau/\alpha = 0.016\text{ kg}\cdot\text{m}^2$.

**EVALUATE:** The tensions in the two parts of the cord must be different, so there will be a net torque on the pulley.

**10.14. IDENTIFY:** Apply $\sum F_y = ma_y$ to the bucket, with $+y$ downward. Apply $\sum\tau_z = I\alpha_z$ to the cylinder, with the direction the cylinder rotates positive.

**SET UP:** The free-body diagram for the bucket is given in Figure 10.14a and the free-body diagram for the cylinder is given in Figure 10.14b. $I = \frac{1}{2}MR^2$. $a(\text{bucket}) = R\alpha(\text{cylinder})$

**EXECUTE:** **(a)** For the bucket, $mg - T = ma$. For the cylinder, $\sum\tau_z = I\alpha_z$ gives $TR = \frac{1}{2}MR^2\alpha$. $\alpha = a/R$ then gives $T = \frac{1}{2}Ma$. Combining these two equations gives $mg - \frac{1}{2}Ma = ma$ and

$$a = \frac{mg}{m + M/2} = \left(\frac{15.0\text{ kg}}{15.0\text{ kg} + 6.0\text{ kg}}\right)(9.80\text{ m/s}^2) = 7.00\text{ m/s}^2.$$

$T = m(g - a) = (15.0\text{ kg})(9.80\text{ m/s}^2 - 7.00\text{ m/s}^2) = 42.0\text{ N}.$

**(b)** $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(7.00\text{ m/s}^2)(10.0\text{ m})} = 11.8\text{ m/s}.$

**(c)** $a_y = 7.00\text{ m/s}^2$, $v_{0y} = 0$, $y - y_0 = 10.0\text{ m}$. $y - y_0 = v_{0y}t + \frac{1}{2}\alpha_yt^2$ gives

$$t = \sqrt{\frac{2(y-y_0)}{a_y}} = \sqrt{\frac{2(10.0\text{ m})}{7.00\text{ m/s}^2}} = 1.69\text{ s}$$

**(d)** $\sum F_y = ma_y$ applied to the cylinder gives $n - T - Mg = 0$ and

$n = T + mg = 42.0\text{ N} + (12.0\text{ kg})(9.80\text{ m/s}^2) = 160\text{ N}.$

**EVALUATE:** The tension in the rope is less than the weight of the bucket, because the bucket has a downward acceleration. If the rope were cut, so the bucket would be in free fall, the bucket would strike the water in $t = \sqrt{\frac{2(10.0\text{ m})}{9.80\text{ m/s}^2}} = 1.43\text{ s}$ and would have a final speed of 14.0 m/s. The presence of the cylinder slows the fall of the bucket.

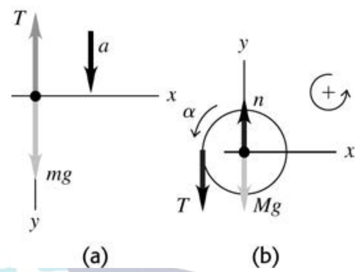



Figure 10.14

**10.15.** **IDENTIFY:** The constant force produces a torque which gives a constant angular acceleration to the wheel.

**SET UP:** $\omega_z = \omega_{0z} + \alpha_z t$ because the angular acceleration is constant, and $\sum \tau_z = I\alpha_z$ applies to the wheel.

**EXECUTE:** $\omega_{0z} = 0$ and $\omega_z = 12.0 \text{ rev/s} = 75.40 \text{ rad/s}$. $\omega_z = \omega_{0z} + \alpha_z t$, so

$\alpha_z = \dfrac{\omega_z - \omega_{0z}}{t} = \dfrac{75.40 \text{ rad/s}}{2.00 \text{ s}} = 37.70 \text{ rad/s}^2$. $\sum \tau_z = I\alpha_z$ gives

$I = \dfrac{Fr}{\alpha_z} = \dfrac{(80.0 \text{ N})(0.120 \text{ m})}{37.70 \text{ rad/s}^2} = 0.255 \text{ kg}\cdot\text{m}^2$.

**EVALUATE:** The units of the answer are the proper ones for moment of inertia.

**10.16.** **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each box and $\sum \tau_z = I\alpha_z$ to the pulley. The magnitude $a$ of the acceleration of each box is related to the magnitude of the angular acceleration $\alpha$ of the pulley by $a = R\alpha$.

**SET UP:** The free-body diagrams for each object are shown in Figure 10.16. For the pulley, $R = 0.250$ m and $I = \frac{1}{2}MR^2$. $T_1$ and $T_2$ are the tensions in the wire on either side of the pulley. $m_1 = 12.0$ kg, $m_2 = 5.00$ kg and $M = 2.00$ kg. $\vec{F}$ is the force that the axle exerts on the pulley. For the pulley, let clockwise rotation be positive.

**EXECUTE:** **(a)** $\sum F_x = ma_x$ for the 12.0 kg box gives $T_1 = m_1 a$. $\sum F_y = ma_y$ for the 5.00 kg weight gives $m_2 g - T_2 = m_2 a$. $\sum \tau_z = I\alpha_z$ for the pulley gives $(T_2 - T_1)R = (\frac{1}{2}MR^2)\alpha$. $a = R\alpha$ and $T_2 - T_1 = \frac{1}{2}Ma$.

Adding these three equations gives $m_2 g = (m_1 + m_2 + \frac{1}{2}M)a$ and

$a = \left(\dfrac{m_2}{m_1 + m_2 + \frac{1}{2}M}\right)g = \left(\dfrac{5.00 \text{ kg}}{12.0 \text{ kg} + 5.00 \text{ kg} + 1.00 \text{ kg}}\right)(9.80 \text{ m/s}^2) = 2.72 \text{ m/s}^2$. Then

$T_1 = m_1 a = (12.0 \text{ kg})(2.72 \text{ m/s}^2) = 32.6 \text{ N}$. $m_2 g - T_2 = m_2 a$ gives

$T_2 = m_2(g - a) = (5.00 \text{ kg})(9.80 \text{ m/s}^2 - 2.72 \text{ m/s}^2) = 35.4 \text{ N}$. The tension to the left of the pulley is 32.6 N and below the pulley it is 35.4 N.

**(b)** $a = 2.72 \text{ m/s}^2$

**(c)** For the pulley, $\sum F_x = ma_x$ gives $F_x = T_1 = 32.6$ N and $\sum F_y = ma_y$ gives

$F_y = Mg + T_2 = (2.00 \text{ kg})(9.80 \text{ m/s}^2) + 35.4 \text{ N} = 55.0 \text{ N}$.

**EVALUATE:** The equation $m_2 g = (m_1 + m_2 + \frac{1}{2}M)a$ says that the external force $m_2 g$ must accelerate all three objects.

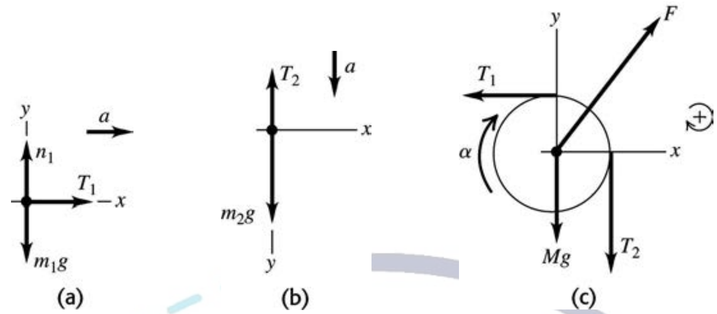



**Figure 10.16**

**10.17.** **IDENTIFY:** Since there is rolling without slipping, $v_{cm} = R\omega$. The kinetic energy is given by $K_{tot} = K_{cm} + K_{rot}$ where $K_{cm} = \frac{1}{2}Mv_{cm}^2$ and $K_{rot} = \frac{1}{2}I_{cm}\omega^2$. The velocity of any point on the rim of the hoop is the vector sum of the tangential velocity of the rim and the velocity of the center of mass of the hoop.

**SET UP:** $\omega = 2.60$ rad/s and $R = 0.600$ m. For a hoop rotating about an axis at its center, $I = MR^2$.

**EXECUTE:** **(a)** $v_{cm} = R\omega = (0.600\text{ m})(2.60\text{ rad/s}) = 1.56\text{ m/s}$.

**(b)** $K = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}(MR^2)(v_{cm}/R^2) = Mv_{cm}^2 = (2.20\text{ kg})(1.56\text{ m/s})^2 = 5.35\text{ J}$

**(c)** (i) $v = 2v_{cm} = 3.12$ m/s. $\vec{v}$ is to the right. (ii) $v = 0$

(iii) $v = \sqrt{v_{cm}^2 + v_{tan}^2} = \sqrt{v_{cm}^2 + (R\omega)^2} = \sqrt{2}v_{cm} = 2.21$ m/s. $\vec{v}$ at this point is at $45°$ below the horizontal.

**(d)** To someone moving to the right at $v = v_{cm}$, the hoop appears to rotate about a stationary axis at its center. (i) $v = R\omega = 1.56$ m/s, to the right. (ii) $v = 1.56$ m/s, to the left. (iii) $v = 1.56$ m/s, downward.

**EVALUATE:** For the special case of a hoop, the total kinetic energy is equally divided between the motion of the center of mass and the rotation about the axis through the center of mass. In the rest frame of the ground, different points on the hoop have different speed.

**10.18.** **IDENTIFY:** The tumbler has kinetic energy due to the linear motion of his center of mass plus kinetic energy due to his rotational motion about his center of mass.

**SET UP:** $v_{cm} = R\omega$. $\omega = 0.50\text{ rev/s} = 3.14\text{ rad/s}$. $I = \frac{1}{2}MR^2$ with $R = 0.50$ m. $K_{cm} = \frac{1}{2}Mv_{cm}^2$ and $K_{rot} = \frac{1}{2}I_{cm}\omega^2$.

**EXECUTE:** **(a)** $K_{tot} = K_{cm} + K_{rot}$ with $K_{cm} = \frac{1}{2}Mv_{cm}^2$ and $K_{rot} = \frac{1}{2}I_{cm}\omega^2$.

$v_{cm} = R\omega = (0.50\text{ m})(3.14\text{ rad/s}) = 1.57\text{ m/s}$. $K_{cm} = \frac{1}{2}(75\text{ kg})(1.57\text{ m/s})^2 = 92.4\text{ J}$.

$K_{rot} = \frac{1}{2}I_{cm}\omega^2 = \frac{1}{4}MR^2\omega^2 = \frac{1}{4}Mv_{cm}^2 = 46.2\text{ J}$. $K_{tot} = 92.4\text{ J} + 46.2\text{ J} = 140\text{ J}$.

**(b)** $\dfrac{K_{rot}}{K_{tot}} = \dfrac{46.2\text{ J}}{140\text{ J}} = 33\%$.

**EVALUATE:** The kinetic energy due to the gymnast's rolling motion makes a substantial contribution (33%) to his total kinetic energy.

**10.19.** **IDENTIFY:** Apply $K = K_{cm} + K_{rot}$.

**SET UP:** For an object that is rolling without slipping, $v_{cm} = R\omega$.

**EXECUTE:** The fraction of the total kinetic energy that is rotational is

$$\frac{(1/2)I_{cm}\omega^2}{(1/2)Mv_{cm}^2 + (1/2)I_{cm}\omega^2} = \frac{1}{1 + (M/I_{cm})v_{cm}^2/\omega^2} = \frac{1}{1 + (MR^2/I_{cm})}$$

**(a)** $I_{cm} = (1/2)MR^2$, so the above ratio is 1/3.

**(b)** $I_{\text{cm}} = (2/5)MR^2$ so the above ratio is 2/7.

**(c)** $I_{\text{cm}} = (2/3)MR^2$ so the ratio is 2/5.

**(d)** $I_{\text{cm}} = (5/8)MR^2$ so the ratio is 5/13.

**EVALUATE:** The moment of inertia of each object takes the form $I = \beta MR^2$. The ratio of rotational kinetic energy to total kinetic energy can be written as $\dfrac{1}{1+1/\beta} = \dfrac{\beta}{1+\beta}$. The ratio increases as $\beta$ increases.

**10.20. IDENTIFY:** Only gravity does work, so $W_{\text{other}} = 0$ and conservation of energy gives $K_1 + U_1 = K_2 + U_2$. $K_2 = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2$.

**SET UP:** Let $y_2 = 0$, so $U_2 = 0$ and $y_1 = 0.750$ m. The hoop is released from rest so $K_1 = 0$. $v_{\text{cm}} = R\omega$. For a hoop with an axis at its center, $I_{\text{cm}} = MR^2$.

**EXECUTE:** **(a)** Conservation of energy gives $U_1 = K_2$. $K_2 = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}(MR^2)\omega^2 = MR^2\omega^2$, so

$MR^2\omega^2 = Mgy_1$. $\omega = \dfrac{\sqrt{gy_1}}{R} = \dfrac{\sqrt{(9.80 \text{ m/s}^2)(0.750 \text{ m})}}{0.0800 \text{ m}} = 33.9$ rad/s.

**(b)** $v = R\omega = (0.0800 \text{ m})(33.9 \text{ rad/s}) = 2.71$ m/s

**EVALUATE:** An object released from rest and falling in free fall for 0.750 m attains a speed of $\sqrt{2g(0.750 \text{ m})} = 3.83$ m/s. The final speed of the hoop is less than this because some of its energy is in kinetic energy of rotation. Or, equivalently, the upward tension causes the magnitude of the net force of the hoop to be less than its weight.

**10.21. IDENTIFY:** Apply $\sum \vec{F}_{\text{ext}} = m\vec{a}_{\text{cm}}$ and $\sum \tau_z = I_{\text{cm}}\alpha_z$ to the motion of the ball.

**(a) SET UP:** The free-body diagram is given in Figure 10.21a.

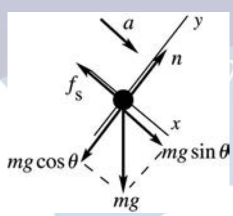



**Figure 10.21a**

**EXECUTE:** $\sum F_y = ma_y$

$n = mg\cos\theta$ and $f_s = \mu_s mg\cos\theta$

$\sum F_x = ma_x$

$mg\sin\theta - \mu_s mg\cos\theta = ma$

$g(\sin\theta - \mu_s\cos\theta) = a$ (Eq. 1)

**SET UP:** Consider Figure 10.21b.

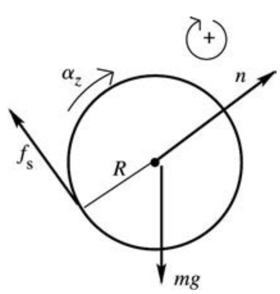



**Figure 10.21b**

*n* and *mg* act at the center of the ball and provide no torque.

**EXECUTE:** $\sum\tau = \tau_f = \mu_s mg\cos\theta R$; $I = \frac{2}{5}mR^2$

$\sum\tau_z = I_{\text{cm}}\alpha_z$ gives $\mu_s\, mg\cos\theta R = \frac{2}{5}mR^2\alpha$

No slipping means $\alpha = a/R$, so $\mu_s g\cos\theta = \frac{2}{5}a$ (Eq. 2)

We have two equations in the two unknowns $a$ and $\mu_s$. Solving gives $a = \frac{5}{7} g \sin\theta$ and

$\mu_s = \frac{2}{7}\tan\theta = \frac{2}{7}\tan 65.0° = 0.613.$

**(b)** Repeat the calculation of part (a), but now $I = \frac{2}{3}mR^2$. $a = \frac{3}{5} g \sin\theta$ and

$\mu_s = \frac{2}{5}\tan\theta = \frac{2}{5}\tan 65.0° = 0.858$

The value of $\mu_s$ calculated in part (a) is not large enough to prevent slipping for the hollow ball.

**(c) EVALUATE:** There is no slipping at the point of contact. More friction is required for a hollow ball since for a given $m$ and $R$ it has a larger $I$ and more torque is needed to provide the same $\alpha$. Note that the required $\mu_s$ is independent of the mass or radius of the ball and only depends on how that mass is distributed.

**10.22. IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the translational motion of the center of mass and $\sum \tau_z = I\alpha_z$ to the rotation about the center of mass.

**SET UP:** Let $+x$ be down the incline and let the shell be turning in the positive direction. The free-body diagram for the shell is given in Figure 10.22. From Table 9.2, $I_{cm} = \frac{2}{3}mR^2$.

**EXECUTE:** **(a)** $\sum F_x = ma_x$ gives $mg\sin\beta - f = ma_{cm}$. $\sum\tau_z = I\alpha_z$ gives $fR = (\frac{2}{3}mR^2)\alpha$. With $\alpha = a_{cm}/R$ this becomes $f = \frac{2}{3}ma_{cm}$. Combining the equations gives $mg\sin\beta - \frac{2}{3}ma_{cm} = ma_{cm}$ and

$a_{cm} = \dfrac{3g\sin\beta}{5} = \dfrac{3(9.80\ \text{m/s}^2)(\sin 38.0°)}{5} = 3.62\ \text{m/s}^2$. $f = \frac{2}{3}ma_{cm} = \frac{2}{3}(2.00\ \text{kg})(3.62\ \text{m/s}^2) = 4.83\ \text{N}$. The friction is static since there is no slipping at the point of contact. $n = mg\cos\beta = 15.45\ \text{N}$.

$\mu_s = \dfrac{f}{n} = \dfrac{4.83\ \text{N}}{15.45\ \text{N}} = 0.313.$

**(b)** The acceleration is independent of $m$ and doesn't change. The friction force is proportional to $m$ so will double; $f = 9.66\ \text{N}$. The normal force will also double, so the minimum $\mu_s$ required for no slipping wouldn't change.

**EVALUATE:** If there is no friction and the object slides without rolling, the acceleration is $g\sin\beta$. Friction and rolling without slipping reduce $a$ to 0.60 times this value.

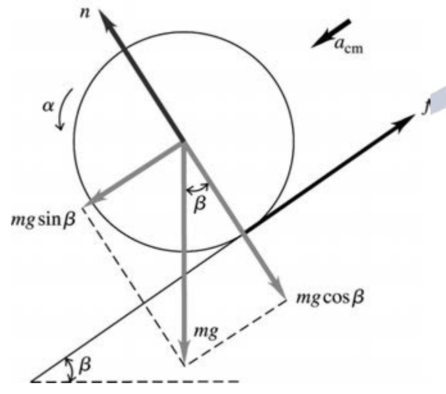
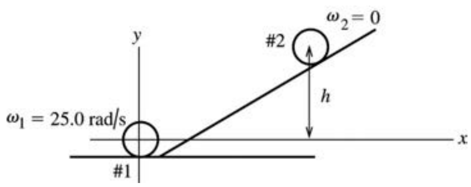



**Figure 10.22**

**10.23. IDENTIFY:** Apply conservation of energy to the motion of the wheel.

**SET UP:** The wheel at points 1 and 2 of its motion is shown in Figure 10.23.




Take $y = 0$ at the center of the wheel when it is at the bottom of the hill.

**Figure 10.23**

The wheel has both translational and rotational motion so its kinetic energy is $K = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}Mv_{cm}^2$.

**EXECUTE:** $K_1 + U_1 + W_{other} = K_2 + U_2$

$W_{other} = W_{fric} = -2600$ J (the friction work is negative)

$K_1 = \frac{1}{2}I\omega_1^2 + \frac{1}{2}Mv_1^2$; $v = R\omega$ and $I = 0.800MR^2$ so

$K_1 = \frac{1}{2}(0.800)MR^2\omega_1^2 + \frac{1}{2}MR^2\omega_1^2 = 0.900MR^2\omega_1^2$

$K_2 = 0$, $U_1 = 0$, $U_2 = Mgh$

Thus $0.900MR^2\omega_1^2 + W_{fric} = Mgh$

$M = w/g = 392\ \text{N}/(9.80\ \text{m/s}^2) = 40.0$ kg

$$h = \frac{0.900MR^2\omega_1^2 + W_{fric}}{Mg}$$

$$h = \frac{(0.900)(40.0\ \text{kg})(0.600\ \text{m})^2(25.0\ \text{rad/s})^2 - 2600\ \text{J}}{(40.0\ \text{kg})(9.80\ \text{m/s}^2)} = 14.0\ \text{m}.$$

**EVALUATE:** Friction does negative work and reduces $h$.

**10.24. IDENTIFY:** Apply conservation of energy to the motion of the marble.

**SET UP:** $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, with $I = \frac{2}{5}MR^2$. $v_{cm} = R\omega$ for no slipping.

Let $y = 0$ at the bottom of the bowl. The marble at its initial and final locations is sketched in Figure 10.24.

**EXECUTE:** **(a)** Motion from the release point to the bottom of the bowl: $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2 \text{ and } v = \sqrt{\frac{10}{7}gh}.$$

Motion along the smooth side: The rotational kinetic energy does not change, since there is no friction torque on the marble, $\frac{1}{2}mv^2 + K_{rot} = mgh' + K_{rot}$. $h' = \frac{v^2}{2g} = \frac{\frac{10}{7}gh}{2g} = \frac{5}{7}h$

**(b)** $mgh = mgh'$ so $h' = h$.

**EVALUATE:** **(c)** With friction on both halves, all the initial potential energy gets converted back to potential energy. Without friction on the right half some of the energy is still in rotational kinetic energy when the marble is at its maximum height.

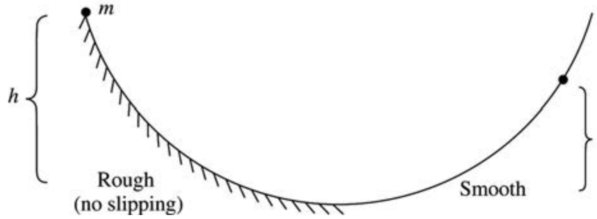



**Figure 10.24**

**10.25.** **IDENTIFY:** As the cylinder falls, its potential energy is transformed into both translational and rotational kinetic energy. Its mechanical energy is conserved.

**SET UP:** The hollow cylinder has $I = \frac{1}{2}m(R_a^2 + R_b^2)$, where $R_a = 0.200$ m and $R_b = 0.350$ m. Use coordinates where $+y$ is upward and $y = 0$ at the initial position of the cylinder. Then $y_1 = 0$ and $y_2 = -d$, where $d$ is the distance it has fallen. $v_{cm} = R\omega$. $K_{cm} = \frac{1}{2}Mv_{cm}^2$ and $K_{rot} = \frac{1}{2}I_{cm}\omega^2$.

**EXECUTE:** **(a)** Conservation of energy gives $K_1 + U_1 = K_2 + U_2$. $K_1 = 0$, $U_1 = 0$. $0 = U_2 + K_2$ and $0 = -mgd + \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$. $\frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}m[R_a^2 + R_b^2])(v_{cm}/R_b)^2 = \frac{1}{4}m[1 + (R_a/R_b)^2]v_{cm}^2$, so

$\frac{1}{2}(1 + \frac{1}{2}[1 + (R_a/R_b)^2])v_{cm}^2 = gd$ and $d = \dfrac{(1 + \frac{1}{2}[1 + (R_a/R_b)^2])v_{cm}^2}{2g} = \dfrac{(1 + 0.663)(6.66 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 3.76$ m.

**(b)** $K_2 = \frac{1}{2}mv_{cm}^2$ since there is no rotation. So $mgd = \frac{1}{2}mv_{cm}^2$ which gives

$v_{cm} = \sqrt{2gd} = \sqrt{2(9.80 \text{ m/s}^2)(3.76 \text{ m})} = 8.58$ m/s.

**(c)** In part (a) the cylinder has rotational as well as translational kinetic energy and therefore less translational speed at a given kinetic energy. The kinetic energy comes from a decrease in gravitational potential energy and that is the same, so in (a) the translational speed is less.

**EVALUATE:** If part (a) were repeated for a solid cylinder, $R_a = 0$ and $d = 3.39$ m. For a thin-walled hollow cylinder, $R_a = R_b$ and $d = 4.52$ cm. Note that all of these answers are independent of the mass $m$ of the cylinder.

**10.26.** **IDENTIFY:** Apply $\sum\tau_z = I\alpha_z$ and $\sum\vec{F} = m\vec{a}$ to the motion of the bowling ball.

**SET UP:** $a_{cm} = R\alpha$. $f_s = \mu_s n$. Let $+x$ be directed down the incline.

**EXECUTE:** **(a)** The free-body diagram is sketched in Figure 10.26.

The angular speed of the ball must decrease, and so the torque is provided by a friction force that acts up the hill.

**(b)** The friction force results in an angular acceleration, given by $I\alpha = fR$. $\sum\vec{F} = m\vec{a}$ applied to the motion of the center of mass gives $mg\sin\beta - f = ma_{cm}$, and the acceleration and angular acceleration are related by $a_{cm} = R\alpha$.

Combining, $mg\sin\beta = ma_{cm}\left(1 + \dfrac{I}{mR^2}\right) = ma_{cm}(7/5)$. $a_{cm} = (5/7)g\sin\beta$.

**(c)** From either of the above relations between $f$ and $a_{cm}$, $f = \dfrac{2}{5}ma_{cm} = \dfrac{2}{7}mg\sin\beta \le \mu_s n = \mu_s mg\cos\beta$.

$\mu_s \ge (2/7)\tan\beta$.

**EVALUATE:** If $\mu_s = 0$, $a_{cm} = mg\sin\beta$. $a_{cm}$ is less when friction is present. The ball rolls farther uphill when friction is present, because the friction removes the rotational kinetic energy and converts it to gravitational potential energy. In the absence of friction the ball retains the rotational kinetic energy that is has initially.

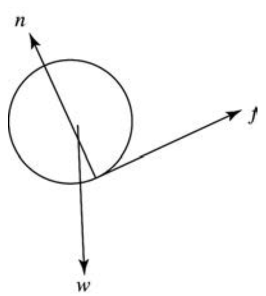



**Figure 10.26**

**10.27.** **IDENTIFY:** As the ball rolls up the hill, its kinetic energy (translational and rotational) is transformed into gravitational potential energy. Since there is no slipping, its mechanical energy is conserved.

**SET UP:** The ball has moment of inertia $I_{cm} = \frac{2}{3}mR^2$. Rolling without slipping means $v_{cm} = R\omega$. Use coordinates where $+y$ is upward and $y = 0$ at the bottom of the hill, so $y_1 = 0$ and $y_2 = h = 5.00$ m. The ball's kinetic energy is $K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$ and its potential energy is $U = mgh$.

**EXECUTE:** **(a)** Conservation of energy gives $K_1 + U_1 = K_2 + U_2$. $U_1 = 0$, $K_2 = 0$ (the ball stops).

Therefore $K_1 = U_2$ and $\frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 = mgh$. $\frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}(\frac{2}{3}mR^2)\left(\frac{v_{cm}}{R}\right)^2 = \frac{1}{3}mv_{cm}^2$, so

$\frac{5}{6}mv_{cm}^2 = mgh$. Therefore $v_{cm} = \sqrt{\frac{6gh}{5}} = \sqrt{\frac{6(9.80\text{ m/s}^2)(5.00\text{ m})}{5}} = 7.67$ m/s and

$$\omega = \frac{v_{cm}}{R} = \frac{7.67\text{ m/s}}{0.113\text{ m}} = 67.9\text{ rad/s}.$$

**(b)** $K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{3}mv_{cm}^2 = \frac{1}{3}(0.426\text{ kg})(7.67\text{ m/s})^2 = 8.35$ J.

**EVALUATE:** Its translational kinetic energy at the base of the hill is $\frac{1}{2}mv_{cm}^2 = \frac{3}{2}K_{rot} = 12.52$ J. Its total kinetic energy is 20.9 J, which equals its final potential energy:

$mgh = (0.426\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m}) = 20.9$ J.

**10.28.** **IDENTIFY:** At the top of the hill the wheel has translational and rotational kinetic energy plus gravitational potential energy. The potential energy is transformed into additional kinetic energy as the wheel rolls down the hill.

**SET UP:** The wheel has $I = MR^2$, with $M = 2.25$ kg and $R = 0.425$ m. Rolling without slipping means $v_{cm} = R\omega$ for the wheel. Initially the wheel has $v_{cm,1} = 11.0$ m/s. Use coordinates where $+y$ is upward and $y = 0$ at the bottom of the hill, so $y_1 = 75.0$ m and $y_2 = 0$. The total kinetic energy of the wheel is $K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$ and its potential energy is $U = mgh$.

**EXECUTE:** **(a)** Conservation of energy gives $K_1 + U_1 = K_2 + U_2$.

$K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}(mR^2)\left(\frac{v_{cm}}{R}\right)^2 = mv_{cm}^2$. Therefore $K_1 = mv_{cm,1}^2$ and $K_2 = mv_{cm,2}^2$.

$U_1 = mgy_1$, $U_2 = mgy_2 = 0$, so $mgy_1 + mv_{cm,1}^2 = mv_{cm,2}^2$. Solving for $v_{cm,2}$ gives

$v_{cm,2} = \sqrt{v_{cm,1}^2 + gy_1} = \sqrt{(11.0\text{ m/s})^2 + (9.80\text{ m/s}^2)(75.0\text{ m})} = 29.3$ m/s.

**(b)** From (b) we have $K_2 = mv_{cm,2}^2 = (2.25\text{ kg})(29.3\text{ m/s})^2 = 1.93 \times 10^3$ J.

**EVALUATE:** Because of the shape of the wheel (thin-walled cylinder), the kinetic energy is shared equally between the translational and rotational forms. This is *not* true for other shapes, such as solid disks or spheres.

**10.29.** **(a) IDENTIFY:** Use $\sum\tau_z = I\alpha_z$ to find $\alpha_z$ and then use a constant angular acceleration equation to find $\omega_z$.

**SET UP:** The free-body diagram is given in Figure 10.29.

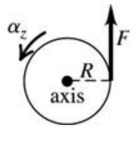



Figure 10.29

**EXECUTE:** Apply $\sum\tau_z = I\alpha_z$ to find the angular acceleration:

$FR = I\alpha_z$

$$\alpha_z = \frac{FR}{I} = \frac{(18.0\text{ N})(2.40\text{ m})}{2100\text{ kg}\cdot\text{m}^2} = 0.02057\text{ rad/s}^2$$

**SET UP:** Use the constant $\alpha_z$ kinematic equations to find $\omega_z$.

$\omega_z = ?$; $\omega_{0z}$ (initially at rest); $\alpha_z = 0.02057\ \text{rad/s}^2$; $t = 15.0\ \text{s}$

**EXECUTE:** $\omega_z = \omega_{0z} + \alpha_z t = 0 + (0.02057\ \text{rad/s}^2)(15.0\ \text{s}) = 0.309\ \text{rad/s}$

**(b) IDENTIFY and SET UP:** Calculate the work from $W = \tau_z \Delta\theta$, using a constant angular acceleration equation to calculate $\theta - \theta_0$, or use the work-energy theorem. We will do it both ways.

**EXECUTE:** (1) $W = \tau_z \Delta\theta$

$\Delta\theta = \theta - \theta_0 = \omega_{0z} t + \frac{1}{2}\alpha_z t^2 = 0 + \frac{1}{2}(0.02057\ \text{rad/s}^2)(15.0\ \text{s})^2 = 2.314\ \text{rad}$

$\tau_z = FR = (18.0\ \text{N})(2.40\ \text{m}) = 43.2\ \text{N}\cdot\text{m}$

Then $W = \tau_z \Delta\theta = (43.2\ \text{N}\cdot\text{m})(2.314\ \text{rad}) = 100\ \text{J}$.

*or*

(2) $W_{\text{tot}} = K_2 - K_1$

$W_{\text{tot}} = W$, the work done by the child

$K_1 = 0$; $K_2 = \frac{1}{2}I\omega^2 = \frac{1}{2}(2100\ \text{kg}\cdot\text{m}^2)(0.309\ \text{rad/s})^2 = 100\ \text{J}$

Thus $W = 100\ \text{J}$, the same as before.

**EVALUATE:** Either method yields the same result for $W$.

**(c) IDENTIFY and SET UP:** Use $P_{\text{av}} = \dfrac{\Delta W}{\Delta t}$ to calculate $P_{\text{av}}$.

**EXECUTE:** $P_{\text{av}} = \dfrac{\Delta W}{\Delta t} = \dfrac{100\ \text{J}}{15.0\ \text{s}} = 6.67\ \text{W}.$

**EVALUATE:** Work is in joules, power is in watts.

**10.30. IDENTIFY:** Apply $P = \tau\omega$ and $W = \tau\Delta\theta$.

**SET UP:** $P$ must be in watts, $\Delta\theta$ must be in radians, and $\omega$ must be in rad/s. $1\ \text{rev} = 2\pi\ \text{rad}$. $1\ \text{hp} = 746\ \text{W}$. $\pi\ \text{rad/s} = 30\ \text{rev/min}$.

**EXECUTE:** **(a)** $\tau = \dfrac{P}{\omega} = \dfrac{(175\ \text{hp})(746\ \text{W/hp})}{(2400\ \text{rev/min})\left(\dfrac{\pi\ \text{rad/s}}{30\ \text{rev/min}}\right)} = 519\ \text{N}\cdot\text{m}.$

**(b)** $W = \tau\Delta\theta = (519\ \text{N}\cdot\text{m})(2\pi\ \text{rad}) = 3260\ \text{J}$

**EVALUATE:** $\omega = 40\ \text{rev/s}$, so the time for one revolution is $0.025\ \text{s}$. $P = 1.306\times10^5\ \text{W}$, so in one revolution, $W = Pt = 3260\ \text{J}$, which agrees with our result.

**10.31. IDENTIFY:** Apply $\sum\tau_z = I\alpha_z$ and constant angular acceleration equations to the motion of the wheel.

**SET UP:** $1\ \text{rev} = 2\pi\ \text{rad}$. $\pi\ \text{rad/s} = 30\ \text{rev/min}$.

**EXECUTE:** **(a)** $\tau_z = I\alpha_z = I\dfrac{\omega_z - \omega_{0z}}{t}$.

$$\tau_z = \frac{\left((1/2)(2.80\ \text{kg})(0.100\ \text{m})^2\right)(1200\ \text{rev/min})\left(\dfrac{\pi\ \text{rad/s}}{30\ \text{rev/min}}\right)}{2.5\ \text{s}} = 0.704\ \text{N}\cdot\text{m}$$

**(b)** $\omega_{\text{av}}\Delta t = \dfrac{(600\ \text{rev/min})(2.5\ \text{s})}{60\ \text{s/min}} = 25.0\ \text{rev} = 157\ \text{rad}.$

**(c)** $W = \tau\Delta\theta = (0.704\ \text{N}\cdot\text{m})(157\ \text{rad}) = 111\ \text{J}.$

**(d)** $K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left((1/2)(2.80\ \text{kg})(0.100\ \text{m})^2\right)\left((1200\ \text{rev/min})\left(\dfrac{\pi\ \text{rad/s}}{30\ \text{rev/min}}\right)\right)^2 = 111\ \text{J}.$

the same as in part (c).

**EVALUATE:** The agreement between the results of parts (c) and (d) illustrates the work-energy theorem.

**10.32. IDENTIFY:** The power output of the motor is related to the torque it produces and to its angular velocity by $P = \tau_z \omega_z$, where $\omega_z$ must be in rad/s.

**SET UP:** The work output of the motor in 60.0 s is $\frac{2}{3}(9.00 \text{ kJ}) = 6.00 \text{ kJ}$, so $P = \frac{6.00 \text{ kJ}}{60.0 \text{ s}} = 100 \text{ W}$.

$\omega_z = 2500 \text{ rev/min} = 262 \text{ rad/s}$.

**EXECUTE:** $\tau_z = \frac{P}{\omega_z} = \frac{100 \text{ W}}{262 \text{ rad/s}} = 0.382 \text{ N} \cdot \text{m}$

**EVALUATE:** For a constant power output, the torque developed decreases when the rotation speed of the motor increases.

**10.33. (a) IDENTIFY and SET UP:** Use $P = \tau_z \omega_z$ and solve for $\tau_z$, where $\omega_z$ must be in rad/s.

**EXECUTE:** $\omega_z = (4000 \text{ rev/min})(2\pi \text{ rad/1 rev})(1 \text{ min/60 s}) = 418.9 \text{ rad/s}$

$$\tau_z = \frac{P}{\omega_z} = \frac{1.50\times10^5 \text{ W}}{418.9 \text{ rad/s}} = 358 \text{ N} \cdot \text{m}$$

**(b) IDENTIFY and SET UP:** Apply $\sum \vec{F} = m\vec{a}$ to the drum. Find the tension $T$ in the rope using $\tau_z$ from part (a). The system is sketched in Figure 10.33.

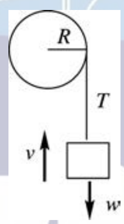



**Figure 10.33**

**EXECUTE:** $v$ constant implies $a = 0$ and $T = w$

$\tau_z = TR$ implies

$T = \tau_z / R = 358 \text{ N} \cdot \text{m}/0.200 \text{ m} = 1790 \text{ N}$

Thus a weight $w = 1790 \text{ N}$ can be lifted.

**(c) IDENTIFY and SET UP:** Use $v = R\omega$.

**EXECUTE:** The drum has $\omega = 418.9 \text{ rad/s}$, so $v = (0.200 \text{ m})(418.9 \text{ rad/s}) = 83.8 \text{ m/s}$.

**EVALUATE:** The rate at which $T$ is doing work on the drum is $P = Tv = (1790 \text{ N})(83.8 \text{ m/s}) = 150 \text{ kW}$. This agrees with the work output of the motor.

**10.34. IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ to the motion of the propeller and then use constant acceleration equations to analyze the motion. $W = \tau\Delta\theta$.

**SET UP:** $I = \frac{1}{12}mL^2 = \frac{1}{12}(117 \text{ kg})(2.08 \text{ m})^2 = 42.2 \text{ kg} \cdot \text{m}^2$.

**EXECUTE:** **(a)** $\alpha = \frac{\tau}{I} = \frac{1950 \text{ N} \cdot \text{m}}{42.2 \text{ kg} \cdot \text{m}^2} = 46.2 \text{ rad/s}^2$.

**(b)** $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ gives $\omega = \sqrt{2\alpha\theta} = \sqrt{2(46.2 \text{ rad/s}^2)(5.0 \text{ rev})(2\pi \text{ rad/rev})} = 53.9 \text{ rad/s}$.

**(c)** $W = \tau\theta = (1950 \text{ N} \cdot \text{m})(5.00 \text{ rev})(2\pi \text{ rad/rev}) = 6.13\times10^4 \text{ J}$.

**(d)** $t = \frac{\omega_z - \omega_{0z}}{\alpha_z} = \frac{53.9 \text{ rad/s}}{46.2 \text{ rad/s}^2} = 1.17 \text{ s}$. $P_{\text{av}} = \frac{W}{\Delta t} = \frac{6.13\times10^4 \text{ J}}{1.17 \text{ s}} = 52.5 \text{ kW}$.

**(e)** $P = \tau\omega = (1950 \text{ N} \cdot \text{m})(53.9 \text{ rad/s}) = 105 \text{ kW}$.

**EVALUATE:** $P = \tau\omega$. $\tau$ is constant and $\omega$ is linear in $t$, so $P_{\text{av}}$ is half the instantaneous power at the end of the 5.00 revolutions. We could also calculate $W$ from

$W = \Delta K = \frac{1}{2}I\omega^2 = \frac{1}{2}(42.2 \text{ kg} \cdot \text{m}^2)(53.9 \text{ rad/s})^2 = 6.13\times10^4 \text{ J}$.

**10.35. (a) IDENTIFY:** Use $L = mvr\sin\phi$.

**SET UP:** Consider Figure 10.35 (next page).

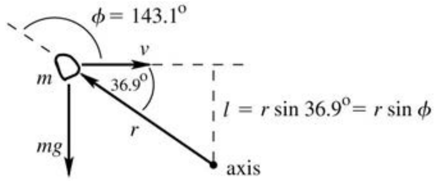



**Figure 10.35**

**EXECUTE:** $L = mvr\sin\phi =$
$(2.00\text{ kg})(12.0\text{ m/s})(8.00\text{ m})\sin 143.1°$
$L = 115\text{ kg}\cdot\text{m}^2/\text{s}$

To find the direction of $\vec{L}$ apply the right-hand rule by turning $\vec{r}$ into the direction of $\vec{v}$ by pushing on it with the fingers of your right hand. Your thumb points into the page, in the direction of $\vec{L}$.

**(b) IDENTIFY and SET UP:** By $\vec{\tau} = \dfrac{d\vec{L}}{dt}$ the rate of change of the angular momentum of the rock equals the torque of the net force acting on it.

**EXECUTE:** $\tau = mg(8.00\text{ m})\cos 36.9° = 125\text{ kg}\cdot\text{m}^2/\text{s}^2$

To find the direction of $\vec{\tau}$ and hence of $d\vec{L}/dt$, apply the right-hand rule by turning $\vec{r}$ into the direction of the gravity force by pushing on it with the fingers of your right hand. Your thumb points out of the page, in the direction of $d\vec{L}/dt$.

**EVALUATE:** $\vec{L}$ and $d\vec{L}/dt$ are in opposite directions, so $L$ is decreasing. The gravity force is accelerating the rock downward, toward the axis. Its horizontal velocity is constant but the distance $l$ is decreasing and hence $L$ is decreasing.

**10.36. IDENTIFY:** $L = I\omega$ and $I = I_{\text{disk}} + I_{\text{woman}}$.

**SET UP:** $\omega = 0.80\text{ rev/s} = 5.026\text{ rad/s}$. $I_{\text{disk}} = \frac{1}{2}m_{\text{disk}}R^2$ and $I_{\text{woman}} = m_{\text{woman}}R^2$.

**EXECUTE:** $I = (55\text{ kg} + 50.0\text{ kg})(4.0\text{ m})^2 = 1680\text{ kg}\cdot\text{m}^2$.

$L = (1680\text{ kg}\cdot\text{m}^2)(5.026\text{ rad/s}) = 8.4\times10^3\text{ kg}\cdot\text{m}^2/\text{s}$.

**EVALUATE:** The disk and the woman have similar values of $I$, even though the disk has twice the mass.

**10.37. IDENTIFY and SET UP:** Use $L = I\omega$.

**EXECUTE:** The second hand makes 1 revolution in 1 minute, so
$\omega = (1.00\text{ rev/min})(2\pi\text{ rad/1 rev})(1\text{ min/60 s}) = 0.1047\text{ rad/s}$.

For a slender rod, with the axis about one end,
$I = \frac{1}{3}ML^2 = \frac{1}{3}(6.00\times10^{-3}\text{ kg})(0.150\text{ m})^2 = 4.50\times10^{-5}\text{ kg}\cdot\text{m}^2$.

Then $L = I\omega = (4.50\times10^{-5}\text{ kg}\cdot\text{m}^2)(0.1047\text{ rad/s}) = 4.71\times10^{-6}\text{ kg}\cdot\text{m}^2/\text{s}$.

**EVALUATE:** $\vec{L}$ is clockwise.

**10.38. IDENTIFY:** $L_z = I\omega_z$

**SET UP:** For a particle of mass $m$ moving in a circular path at a distance $r$ from the axis, $I = mr^2$ and $v = r\omega$. For a uniform sphere of mass $M$ and radius $R$ and an axis through its center, $I = \frac{2}{5}MR^2$. The earth has mass $m_{\text{E}} = 5.97\times10^{24}\text{ kg}$, radius $R_{\text{E}} = 6.37\times10^6\text{ m}$ and orbit radius $r = 1.50\times10^{11}\text{ m}$. The earth completes one rotation on its axis in $24\text{ h} = 86{,}400\text{ s}$ and one orbit in $1\text{ y} = 3.156\times10^7\text{ s}$.

**EXECUTE:** **(a)** $L_z = I\omega_z = mr^2\omega_z = (5.97\times10^{24}\text{ kg})(1.50\times10^{11}\text{ m})^2\left(\dfrac{2\pi\text{ rad}}{3.156\times10^7\text{ s}}\right) = 2.67\times10^{40}\text{ kg}\cdot\text{m}^2/\text{s}$.

The radius of the earth is much less than its orbit radius, so it is very reasonable to model it as a particle for this calculation.

**(b)** $L_z = I\omega_z = (\frac{2}{5}MR^2)\omega = \frac{2}{5}(5.97\times10^{24}\text{ kg})(6.38\times10^6\text{ m})^2\left(\dfrac{2\pi\text{ rad}}{86{,}400\text{ s}}\right) = 7.07\times10^{33}\text{ kg}\cdot\text{m}^2/\text{s}$

**EVALUATE:** The angular momentum associated with each of these motions is very large.

**10.39. IDENTIFY:** $\omega_z = d\theta/dt$. $L_z = I\omega_z$ and $\tau_z = dL_z/dt$.

**SET UP:** For a hollow, thin-walled sphere rolling about an axis through its center, $I = \frac{2}{3}MR^2$. $R = 0.240$ m.

**EXECUTE:** **(a)** $A = 1.50\text{ rad/s}^2$ and $B = 1.10\text{ rad/s}^4$, so that $\theta(t)$ will have units of radians.

**(b)** (i) $\omega_z = \dfrac{d\theta}{dt} = 2At + 4Bt^3$. At $t = 3.00$ s,

$\omega_z = 2(1.50\text{ rad/s}^2)(3.00\text{ s}) + 4(1.10\text{ rad/s}^4)(3.00\text{ s})^3 = 128\text{ rad/s}$.

$L_z = (\frac{2}{3}MR^2)\omega_z = \frac{2}{3}(12.0\text{ kg})(0.240\text{ m})^2(128\text{ rad/s}) = 59.0\text{ kg}\cdot\text{m}^2/\text{s}$.

(ii) $\tau_z = \dfrac{dL_z}{dt} = I\dfrac{d\omega_z}{dt} = I(2A + 12Bt^2)$ and

$\tau_z = \frac{2}{3}(12.0\text{ kg})(0.240\text{ m})^2\left[2(1.50\text{ rad/s}^2) + 12(1.10\text{ rad/s}^4)(3.00\text{ s})^2\right] = 56.1\text{ N}\cdot\text{m}$.

**EVALUATE:** The angular speed of rotation is increasing. This increase is due to an acceleration $\alpha_z$ that is produced by the torque on the sphere. When *I* is constant, as it is here, $\tau_z = dL_z/dt = Id\omega_z/dt = I\alpha_z$.

**10.40. IDENTIFY and SET UP:** $\vec{L}$ is conserved if there is no net external torque.
Use conservation of angular momentum to find $\omega$ at the new radius and use $K = \frac{1}{2}I\omega^2$ to find the change in kinetic energy, which is equal to the work done on the block.

**EXECUTE:** **(a)** Yes, angular momentum is conserved. The moment arm for the tension in the cord is zero so this force exerts no torque and there is no net torque on the block.

**(b)** $L_1 = L_2$ so $I_1\omega_1 = I_2\omega_2$. Block treated as a point mass, so $I = mr^2$, where *r* is the distance of the block from the hole.

$mr_1^2\omega_1 = mr_2^2\omega_2$

$$\omega_2 = \left(\frac{r_1}{r_2}\right)^2\omega_1 = \left(\frac{0.300\text{ m}}{0.150\text{ m}}\right)^2(2.85\text{ rad/s}) = 11.4\text{ rad/s}$$

**(c)** $K_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}mr_1^2\omega_1^2 = \frac{1}{2}mv_1^2$

$v_1 = r_1\omega_1 = (0.300\text{ m})(2.85\text{ rad/s}) = 0.855\text{ m/s}$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.0250\text{ kg})(0.855\text{ m/s})^2 = 0.00914\text{ J}$

$K_2 = \frac{1}{2}mv_2^2$

$v_2 = r_2\omega_2 = (0.150\text{ m})(11.4\text{ rad/s}) = 1.71\text{ m/s}$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.0250\text{ kg})(1.71\text{ m/s}^2)^2 = 0.03655\text{ J}$

$\Delta K = K_2 - K_1 = 0.03655\text{ J} - 0.00914\text{ J} = 0.0274\text{ J} = 27.4\text{ mJ}$.

**(d)** $W_{\text{tot}} = \Delta K$

But $W_{\text{tot}} = W$, the work done by the tension in the cord, so $W = 0.0274$ J.

**EVALUATE:** Smaller *r* means smaller *I*. $L = I\omega$ is constant so $\omega$ increases and *K* increases. The work done by the tension is positive since it is directed inward and the block moves inward, toward the hole.

**10.41. IDENTIFY:** Apply conservation of angular momentum.

**SET UP:** For a uniform sphere and an axis through its center, $I = \frac{2}{5}MR^2$.

**EXECUTE:** The moment of inertia is proportional to the square of the radius, and so the angular velocity will be proportional to the inverse of the square of the radius, and the final angular velocity is

$$\omega_2 = \omega_1\left(\frac{R_1}{R_2}\right)^2 = \left(\frac{2\pi\text{ rad}}{(30\text{ d})(86{,}400\text{ s/d})}\right)\left(\frac{7.0\times10^5\text{ km}}{16\text{ km}}\right)^2 = 4.6\times10^3\text{ rad/s}.$$

**EVALUATE:** $K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega.$ $L$ is constant and $\omega$ increases by a large factor, so there is a large increase in the rotational kinetic energy of the star. This energy comes from potential energy associated with the gravity force within the star.

**10.42.** **IDENTIFY** and **SET UP:** Apply conservation of angular momentum to the diver.

**SET UP:** The number of revolutions she makes in a certain time is proportional to her angular velocity. The ratio of her untucked to tucked angular velocity is $(3.6\ \text{kg}\cdot\text{m}^2)/(18\ \text{kg}\cdot\text{m}^2)$.

**EXECUTE:** If she had not tucked, she would have made $(2\ \text{rev})(3.6\ \text{kg}\cdot\text{m}^2)/(18\ \text{kg}\cdot\text{m}^2) = 0.40\ \text{rev}$ in the last 1.0 s, so she would have made $(0.40\ \text{rev})(1.5/1.0) = 0.60\ \text{rev}$ in the total 1.5 s.

**EVALUATE:** Untucked she rotates slower and completes fewer revolutions.

**10.43.** **IDENTIFY:** Apply conservation of angular momentum to the motion of the skater.

**SET UP:** For a thin-walled hollow cylinder $I = mR^2$. For a slender rod rotating about an axis through its center, $I = \frac{1}{12}ml^2$.

**EXECUTE:** $L_\text{i} = L_\text{f}$ so $I_\text{i}\omega_\text{i} = I_\text{f}\omega_\text{f}$.

$I_\text{i} = 0.40\ \text{kg}\cdot\text{m}^2 + \frac{1}{12}(8.0\ \text{kg})(1.8\ \text{m})^2 = 2.56\ \text{kg}\cdot\text{m}^2.$ $I_\text{f} = 0.40\ \text{kg}\cdot\text{m}^2 + (8.0\ \text{kg})(0.25\ \text{m})^2 = 0.90\ \text{kg}\cdot\text{m}^2.$

$$\omega_\text{f} = \left(\frac{I_\text{i}}{I_\text{f}}\right)\omega_\text{i} = \left(\frac{2.56\ \text{kg}\cdot\text{m}^2}{0.90\ \text{kg}\cdot\text{m}^2}\right)(0.40\ \text{rev/s}) = 1.14\ \text{rev/s}.$$

**EVALUATE:** $K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega.$ $\omega$ increases and $L$ is constant, so $K$ increases. The increase in kinetic energy comes from the work done by the skater when he pulls in his hands.

**10.44.** **IDENTIFY:** Apply conservation of angular momentum to the collision.

**SET UP:** Let the width of the door be $l$. The initial angular momentum of the mud is $mv(l/2)$, since it strikes the door at its center. For the axis at the hinge, $I_\text{door} = \frac{1}{3}Ml^2$ and $I_\text{mud} = m(l/2)^2$.

**EXECUTE:** $\omega = \dfrac{L}{I} = \dfrac{mv(l/2)}{(1/3)Ml^2 + m(l/2)^2}.$

$$\omega = \frac{(0.500\ \text{kg})(12.0\ \text{m/s})(0.500\ \text{m})}{(1/3)(40.0\ \text{kg})(1.00\ \text{m})^2 + (0.500\ \text{kg})(0.500\ \text{m})^2} = 0.223\ \text{rad/s}.$$

Ignoring the mass of the mud in the denominator of the above expression gives $\omega = 0.225\ \text{rad/s}$, so the mass of the mud in the moment of inertia does affect the third significant figure.

**EVALUATE:** Angular momentum is conserved but there is a large decrease in the kinetic energy of the system.

**10.45.** **IDENTIFY** and **SET UP:** There is no net external torque about the rotation axis so the angular momentum $L = I\omega$ is conserved.

**EXECUTE:** **(a)** $L_1 = L_2$ gives $I_1\omega_1 = I_2\omega_2$, so $\omega_2 = (I_1/I_2)\omega_1$

$I_1 = I_\text{tt} = \frac{1}{2}MR^2 = \frac{1}{2}(120\ \text{kg})(2.00\ \text{m})^2 = 240\ \text{kg}\cdot\text{m}^2$

$I_2 = I_\text{tt} + I_\text{p} = 240\ \text{kg}\cdot\text{m}^2 + mR^2 = 240\ \text{kg}\cdot\text{m}^2 + (70\ \text{kg})(2.00\ \text{m})^2 = 520\ \text{kg}\cdot\text{m}^2$

$\omega_2 = (I_1/I_2)\omega_1 = (240\ \text{kg}\cdot\text{m}^2/520\ \text{kg}\cdot\text{m}^2)(3.00\ \text{rad/s}) = 1.38\ \text{rad/s}$

**(b)** $K_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}(240\ \text{kg}\cdot\text{m}^2)(3.00\ \text{rad/s})^2 = 1080\ \text{J}$

$K_2 = \frac{1}{2}I_2\omega_2^2 = \frac{1}{2}(520\ \text{kg}\cdot\text{m}^2)(1.38\ \text{rad/s})^2 = 495\ \text{J}$

**EVALUATE:** The kinetic energy decreases because of the negative work done on the turntable and the parachutist by the friction force between these two objects.

The angular speed decreases because $I$ increases when the parachutist is added to the system.

**10.46.** **IDENTIFY:** Apply conservation of angular momentum to the system of earth plus asteroid.

**SET UP:** Take the axis to be the earth's rotation axis. The asteroid may be treated as a point mass and it has zero angular momentum before the collision, since it is headed toward the center of the earth. For the earth, $L_z = I\omega_z$ and $I = \frac{2}{5}MR^2$, where $M$ is the mass of the earth and $R$ is its radius. The length of a day is $T = \dfrac{2\pi \text{ rad}}{\omega}$, where $\omega$ is the earth's angular rotation rate.

**EXECUTE:** Conservation of angular momentum applied to the collision between the earth and asteroid gives $\frac{2}{5}MR^2\omega_1 = (mR^2 + \frac{2}{5}MR^2)\omega_2$ and $m = \frac{2}{5}M\left(\dfrac{\omega_1 - \omega_2}{\omega_2}\right)$. $T_2 = 1.250T_1$ gives $\dfrac{1}{\omega_2} = \dfrac{1.250}{\omega_1}$ and

$\omega_1 = 1.250\omega_2$. $\dfrac{\omega_1 - \omega_2}{\omega_2} = 0.250$. $m = \frac{2}{5}(0.250)M = 0.100M$.

**EVALUATE:** If the asteroid hit the surface of the earth tangentially it could have some angular momentum with respect to the earth's rotation axis, and could either speed up or slow down the earth's rotation rate.

**10.47.** **(a) IDENTIFY** and **SET UP:** Apply conservation of angular momentum $\vec{L}$, with the axis at the nail. Let object $A$ be the bug and object $B$ be the bar. Initially, all objects are at rest and $L_1 = 0$. Just after the bug jumps, it has angular momentum in one direction of rotation and the bar is rotating with angular velocity $\omega_B$ in the opposite direction.

**EXECUTE:** $L_2 = m_A v_A r - I_B\omega_B$ where $r = 1.00$ m and $I_B = \frac{1}{3}m_B r^2$

$L_1 = L_2$ gives $m_A v_A r = \frac{1}{3}m_B r^2 \omega_B$

$\omega_B = \dfrac{3m_A v_A}{m_B r} = 0.120$ rad/s

**(b)** $K_1 = 0$;

$K_2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}I_B\omega_B^2 = \frac{1}{2}(0.0100 \text{ kg})(0.200 \text{ m/s})^2 + \frac{1}{2}(\frac{1}{3}(0.0500 \text{ kg})(1.00 \text{ m})^2)(0.120 \text{ rad/s})^2 = 3.2\times10^{-4}$ J.

**(c)** The increase in kinetic energy comes from work done by the bug when it pushes against the bar in order to jump.

**EVALUATE:** There is no external torque applied to the system and the total angular momentum of the system is constant. There are internal forces, forces the bug and bar exert on each other. The forces exert torques and change the angular momentum of the bug and the bar, but these changes are equal in magnitude and opposite in direction. These internal forces do positive work on the two objects and the kinetic energy of each object and of the system increases.

**10.48.** **IDENTIFY:** As the bug moves outward, it increases the moment of inertia of the rod-bug system. The angular momentum of this system is conserved because no unbalanced external torques act on it.

**SET UP:** The moment of inertia of the rod is $I = \dfrac{1}{3}ML^2$, and conservation of angular momentum gives $I_1\omega_1 = I_2\omega_2$.

**EXECUTE:** **(a)** $I = \dfrac{1}{3}ML^2$ gives $M = \dfrac{3I}{L^2} = \dfrac{3(3.00\times10^{-3} \text{ kg}\cdot\text{m}^2)}{(0.500 \text{ m})^2} = 0.0360$ kg.

**(b)** $L_1 = L_2$, so $I_1\omega_1 = I_2\omega_2$. $\omega_2 = \dfrac{v}{r} = \dfrac{0.160 \text{ m/s}}{0.500 \text{ m}} = 0.320$ rad/s, so

$(3.00\times10^{-3} \text{ kg}\cdot\text{m}^2)(0.400 \text{ rad/s}) = (3.00\times10^{-3} \text{ kg}\cdot\text{m}^2 + m_{\text{bug}}(0.500 \text{ m})^2)(0.320 \text{ rad/s})$.

$m_{\text{bug}} = \dfrac{(3.00\times10^{-3} \text{ kg}\cdot\text{m}^2)(0.400 \text{ rad/s} - 0.320 \text{ rad/s})}{(0.320 \text{ rad/s})(0.500 \text{ m})^2} = 3.00\times10^{-3}$ kg.

**EVALUATE:** This is a 3.00 mg bug, which is not unreasonable.

**10.49.** **IDENTIFY:** Apply conservation of angular momentum to the collision.

**SET UP:** The system before and after the collision is sketched in Figure 10.49. Let counterclockwise rotation be positive. The bar has $I = \frac{1}{3}m_2L^2$.

**EXECUTE:** **(a)** Conservation of angular momentum: $m_1v_0d = -m_1vd + \frac{1}{3}m_2L^2\omega$.

$$(3.00\text{ kg})(10.0\text{ m/s})(1.50\text{ m}) = -(3.00\text{ kg})(6.00\text{ m/s})(1.50\text{ m}) + \tfrac{1}{3}\left(\frac{90.0\text{ N}}{9.80\text{ m/s}^2}\right)(2.00\text{ m})^2\omega$$

$\omega = 5.88$ rad/s.

**(b)** There are no unbalanced torques about the pivot, so angular momentum is conserved. But the pivot exerts an unbalanced horizontal external force on the system, so the linear momentum is not conserved.

**EVALUATE:** Kinetic energy is not conserved in the collision.

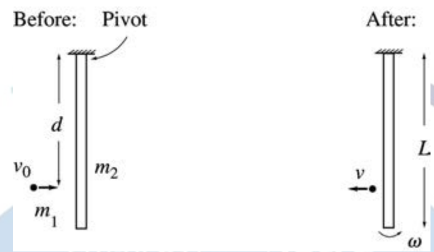



**Figure 10.49**

**10.50. IDENTIFY:** If we take the raven and the gate as a system, the torque about the pivot is zero, so the angular momentum of the system about the pivot is conserved.

**SET UP:** The system before and after the collision is sketched in Figure 10.50. The gate has $I = \frac{1}{3}ML^2$. Take counterclockwise torques to be positive.

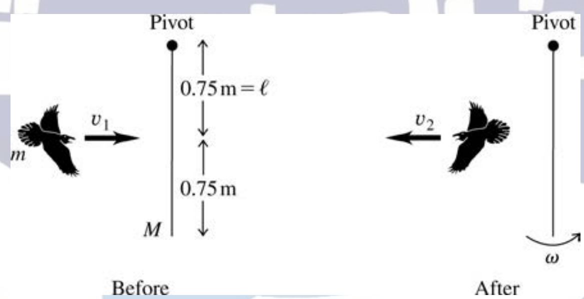



**Figure 10.50**

**EXECUTE:** **(a)** The gravity forces exert no torque at the moment of collision and angular momentum is conserved. $L_1 = L_2$. $mv_1 l = -mv_2 l + I_{\text{gate}}\omega$ with $l = L/2$.

$$\omega = \frac{m(v_1+v_2)l}{\frac{1}{3}ML^2} = \frac{3m(v_1+v_2)}{2ML} = \frac{3(1.1\text{ kg})(5.0\text{ m/s}+2.0\text{ m/s})}{2(4.5\text{ kg})(1.5\text{ m})} = 1.71\text{ rad/s}.$$

**(b)** Linear momentum is not conserved; there is an external force exerted by the pivot. But the force on the pivot has zero torque. There is no external torque and angular momentum is conserved.

**EVALUATE:** $K_1 = \frac{1}{2}(1.1\text{ kg})(5.0\text{ m/s})^2 = 13.8$ J.

$K_2 = \frac{1}{2}(1.1\text{ kg})(2.0\text{ m/s})^2 + \frac{1}{2}(\frac{1}{3}[4.5\text{ kg}][1.5\text{ m/s}]^2)(1.71\text{ rad/s})^2 = 7.1$ J. This is an inelastic collision and $K_2 < K_1$.

**10.51. IDENTIFY:** The precession angular velocity is $\Omega = \dfrac{wr}{I\omega}$, where $\omega$ is in rad/s. Also apply $\sum \vec{F} = m\vec{a}$ to the gyroscope.

**SET UP:** The total mass of the gyroscope is $m_r + m_f = 0.140\text{ kg} + 0.0250\text{ kg} = 0.165\text{ kg}$.

$$\Omega = \frac{2\pi\text{ rad}}{T} = \frac{2\pi\text{ rad}}{2.20\text{ s}} = 2.856\text{ rad/s}.$$

**EXECUTE:** **(a)** $F_p = w_{tot} = (0.165\text{ kg})(9.80\text{ m/s}^2) = 1.62\text{ N}$

**(b)** $\omega = \dfrac{wr}{I\Omega} = \dfrac{(0.165\text{ kg})(9.80\text{ m/s}^2)(0.0400\text{ m})}{(1.20\times10^{-4}\text{ kg}\cdot\text{m}^2)(2.856\text{ rad/s})} = 189\text{ rad/s} = 1.80\times10^3\text{ rev/min}$

**(c)** If the figure in the problem is viewed from above, $\vec{\tau}$ is in the direction of the precession and $\vec{L}$ is along the axis of the rotor, away from the pivot.

**EVALUATE:** There is no vertical component of acceleration associated with the motion, so the force from the pivot equals the weight of the gyroscope. The larger $\omega$ is, the slower the rate of precession.

**10.52. IDENTIFY:** The precession angular speed is related to the acceleration due to gravity by $\Omega = \dfrac{mgr}{I\omega}$, with $w = mg$.

**SET UP:** $\Omega_E = 0.50\text{ rad/s}$, $g_E = g$ and $g_M = 0.165g$. For the gyroscope, $m$, $r$, $I$, and $\omega$ are the same on the moon as on the earth.

**EXECUTE:** $\Omega = \dfrac{mgr}{I\omega}$. $\dfrac{\Omega}{g} = \dfrac{mr}{I\omega} = \text{constant}$, so $\dfrac{\Omega_E}{g_E} = \dfrac{\Omega_M}{g_M}$.

$$\Omega_M = \Omega_E\left(\frac{g_M}{g_E}\right) = 0.165\Omega_E = (0.165)(0.50\text{ rad/s}) = 0.0825\text{ rad/s}.$$

**EVALUATE:** In the limit that $g \to 0$ the precession rate $\to 0$.

**10.53. IDENTIFY:** An external torque will cause precession of the telescope.

**SET UP:** $I = MR^2$, with $R = 2.5\times10^{-2}\text{ m}$. $1.0\times10^{-6}\text{ degree} = 1.745\times10^{-8}\text{ rad}$.

$\omega = 19{,}200\text{ rpm} = 2.01\times10^3\text{ rad/s}$. $t = 5.0\text{ h} = 1.8\times10^4\text{ s}$.

**EXECUTE:** $\Omega = \dfrac{\Delta\phi}{\Delta t} = \dfrac{1.745\times10^{-8}\text{ rad}}{1.8\times10^4\text{ s}} = 9.694\times10^{-13}\text{ rad/s}$. $\Omega = \dfrac{\tau}{I\omega}$ so $\tau = \Omega I\omega = \Omega MR^2\omega$. Putting in the numbers gives $\tau = (9.694\times10^{-13}\text{ rad/s})(2.0\text{ kg})(2.5\times10^{-2}\text{ m})^2(2.01\times10^3\text{ rad/s}) = 2.4\times10^{-12}\text{ N}\cdot\text{m}$.

**EVALUATE:** The external torque must be very small for this degree of stability.

**10.54. IDENTIFY:** Apply $\sum\tau_z = I\alpha_z$ and constant acceleration equations to the motion of the grindstone.

**SET UP:** Let the direction of rotation of the grindstone be positive. The friction force is $f = \mu_k n$ and produces torque $fR$. $\omega = (120\text{rev/min})\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right) = 4\pi\text{ rad/s}$. $I = \frac{1}{2}MR^2 = 1.69\text{ kg}\cdot\text{m}^2$.

**EXECUTE:** **(a)** The net torque must be

$$\tau = I\alpha = I\frac{\omega_z - \omega_{0z}}{t} = (1.69\text{ kg}\cdot\text{m}^2)\frac{4\pi\text{ rad/s}}{9.00\text{ s}} = 2.36\text{ N}\cdot\text{m}.$$

This torque must be the sum of the applied force $FR$ and the opposing frictional torques $\tau_f$ at the axle and $fR = \mu_k nR$ due to the knife. $F = \dfrac{1}{R}(\tau + \tau_f + \mu_k nR)$.

$$F = \frac{1}{0.500\text{ m}}\left[(2.36\text{ N}\cdot\text{m}) + (6.50\text{ N}\cdot\text{m}) + (0.60)(160\text{ N})(0.260\text{ m})\right] = 67.6\text{ N}.$$

**(b)** To maintain a constant angular velocity, the net torque $\tau$ is zero, and the force $F'$ is

$$F' = \frac{1}{0.500\text{ m}}(6.50\text{ N}\cdot\text{m} + 24.96\text{ N}\cdot\text{m}) = 62.9\text{ N}.$$

**(c)** The time $t$ needed to come to a stop is found by taking the magnitudes in $\vec{\tau} = \dfrac{d\vec{L}}{dt}$, with $\tau = \tau_f$ constant; $t = \dfrac{L}{\tau_f} = \dfrac{\omega I}{\tau_f} = \dfrac{(4\pi \text{ rad/s})(1.69 \text{ kg}\cdot\text{m}^2)}{6.50 \text{ N}\cdot\text{m}} = 3.27 \text{ s}$.

**Evaluate:** The time for a given change in $\omega$ is proportional to $\alpha$, which is in turn proportional to the net torque, so the time in part (c) can also be found as $t = (9.00 \text{ s})\dfrac{2.36 \text{ N}\cdot\text{m}}{6.50 \text{ N}\cdot\text{m}}$.

**10.55. Identify:** Use the kinematic information to solve for the angular acceleration of the grindstone. Assume that the grindstone is rotating counterclockwise and let that be the positive sense of rotation. Then apply $\sum\tau_z = I\alpha_z$ to calculate the friction force and use $f_k = \mu_k n$ to calculate $\mu_k$.

**Set Up:** $\omega_{0z} = 850 \text{ rev/min}(2\pi \text{ rad/1 rev})(1 \text{ min/60 s}) = 89.0 \text{ rad/s}$

$t = 7.50$ s; $\omega_z = 0$ (comes to rest); $\alpha_z = ?$

**Execute:** $\omega_z = \omega_{0z} + \alpha_z t$

$$\alpha_z = \frac{0 - 89.0 \text{ rad/s}}{7.50 \text{ s}} = -11.9 \text{ rad/s}^2$$

**Set Up:** Apply $\sum\tau_z = I\alpha_z$ to the grindstone. The free-body diagram is given in Figure 10.55.

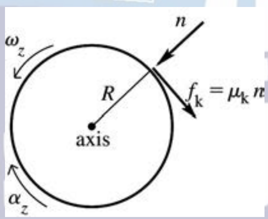



**Figure 10.55**

The normal force has zero moment arm for rotation about an axis at the center of the grindstone, and therefore zero torque. The only torque on the grindstone is that due to the friction force $f_k$ exerted by the ax; for this force the moment arm is $l = R$ and the torque is negative.

**Execute:** $\sum\tau_z = -f_k R = -\mu_k nR$

$I = \frac{1}{2}MR^2$ (solid disk, axis through center)

Thus $\sum\tau_z = I\alpha_z$ gives $-\mu_k nR = (\frac{1}{2}MR^2)\alpha_z$

$$\mu_k = -\frac{MR\alpha_z}{2n} = -\frac{(50.0 \text{ kg})(0.260 \text{ m})(-11.9 \text{ rad/s}^2)}{2(160 \text{ N})} = 0.483$$

**Evaluate:** The friction torque is clockwise and slows down the counterclockwise rotation of the grindstone.

**10.56. Identify:** Use a constant acceleration equation to calculate $\alpha_z$ and then apply $\sum\tau_z = I\alpha_z$.

**Set Up:** $I = \frac{2}{3}MR^2 + 2mR^2$, where $M = 8.40$ kg, $m = 2.00$ kg, so $I = 0.600 \text{ kg}\cdot\text{m}^2$.

$\omega_{0z} = 75.0 \text{ rpm} = 7.854 \text{ rad/s}$; $\omega_z = 50.0 \text{ rpm} = 5.236 \text{ rad/s}$; $t = 30.0$ s.

**Execute:** $\omega_z = \omega_{0_z} + \alpha_z t$ gives $\alpha_z = -0.08726 \text{ rad/s}^2$. $\tau_z = I\alpha_z = -0.0524 \text{ N}\cdot\text{m}$.

**Evaluate:** The torque is negative because its direction is opposite to the direction of rotation, which must be the case for the speed to decrease.

**10.57. Identify:** Use $\sum\tau_z = I\alpha_z$ to find the angular acceleration just after the ball falls off and use conservation of energy to find the angular velocity of the bar as it swings through the vertical position.

**SET UP:** The axis of rotation is at the axle. For this axis the bar has $I = \frac{1}{12}m_{\text{bar}}L^2$, where $m_{\text{bar}} = 3.80$ kg and $L = 0.800$ m. Energy conservation gives $K_1 + U_1 = K_2 + U_2$. The gravitational potential energy of the bar doesn't change. Let $y_1 = 0$, so $y_2 = -L/2$.

**EXECUTE:** **(a)** $\tau_z = m_{\text{ball}}g(L/2)$ and $I = I_{\text{ball}} + I_{\text{bar}} = \frac{1}{12}m_{\text{bar}}L^2 + m_{\text{ball}}(L/2)^2$. $\sum\tau_z = I\alpha_z$ gives

$$\alpha_z = \frac{m_{\text{ball}}g(L/2)}{\frac{1}{12}m_{\text{bar}}L^2 + m_{\text{ball}}(L/2)^2} = \frac{2g}{L}\left(\frac{m_{\text{ball}}}{m_{\text{ball}} + m_{\text{bar}}/3}\right) \text{ and}$$

$$\alpha_z = \frac{2(9.80\text{ m/s}^2)}{0.800\text{ m}}\left(\frac{2.50\text{ kg}}{2.50\text{ kg} + [3.80\text{ kg}]/3}\right) = 16.3\text{ rad/s}^2.$$

**(b)** As the bar rotates, the moment arm for the weight of the ball decreases and the angular acceleration of the bar decreases.

**(c)** $K_1 + U_1 = K_2 + U_2$. $0 = K_2 + U_2$. $\frac{1}{2}(I_{\text{bar}} + I_{\text{ball}})\omega^2 = -m_{\text{ball}}g(-L/2)$.

$$\omega = \sqrt{\frac{m_{\text{ball}}gL}{m_{\text{ball}}L^2/4 + m_{\text{bar}}L^2/12}} = \sqrt{\frac{g}{L}\left(\frac{4m_{\text{ball}}}{m_{\text{ball}} + m_{\text{bar}}/3}\right)} = \sqrt{\frac{9.80\text{ m/s}^2}{0.800\text{ m}}\left(\frac{4(2.50\text{ kg})}{2.50\text{ kg} + (3.80\text{ kg})/3}\right)}$$

$\omega = 5.70$ rad/s.

**EVALUATE:** As the bar swings through the vertical, the linear speed of the ball that is still attached to the bar is $v = (0.400\text{ m})(5.70\text{ rad/s}) = 2.28\text{ m/s}$. A point mass in free-fall acquires a speed of 2.80 m/s after falling 0.400 m; the ball on the bar acquires a speed less than this.

**10.58.** **IDENTIFY:** Newton's second law in its linear form applies to the elevator and counterweight, in its rotational form it applies to the pulley. We have constant acceleration, so we can use the standard linear kinematics formulas.

**SET UP:** For the pulley $I = \frac{1}{2}MR^2$. The elevator has mass $m_1 = \dfrac{22{,}500\text{ N}}{9.80\text{ m/s}^2} = 2300$ kg. The free-body diagrams for the elevator, the pulley, and the counterweight are shown in Figure 10.58. Apply $\Sigma\vec{F} = m\vec{a}$ to the elevator and to the counterweight. For the elevator take $+y$ upward and for the counterweight take $+y$ downward, in each case in the direction of the acceleration of the object. Apply $\sum\tau = I\alpha$ to the pulley, with clockwise as the positive sense of rotation. $n$ is the normal force applied to the pulley by the axle. The elevator and counterweight each have acceleration $a$, where $a = R\alpha$. $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ applies.

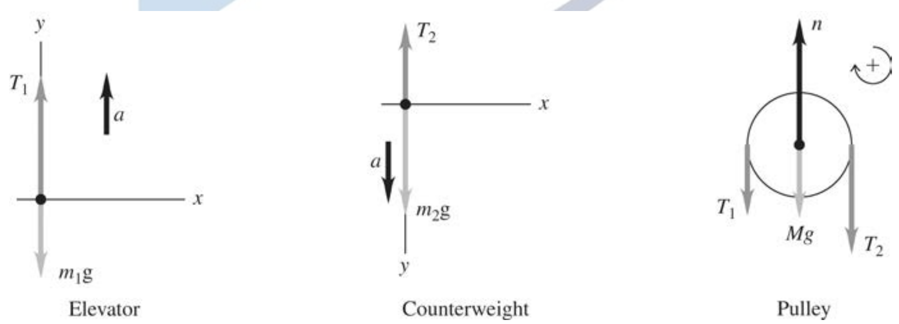



**Figure 10.58**

**EXECUTE:** Solve parts **(a)** and **(b)** together. Calculate the acceleration of the elevator:

$y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $a = \dfrac{2(y - y_0)}{t^2} = \dfrac{2(6.75\text{ m})}{(3.00\text{ s})^2} = 1.50\text{ m/s}^2$.

$\sum F_y = ma_y$ for the elevator gives $T_1 - m_1g = m_1a$ and

$$T_1 = m_1(a + g) = (2300\text{ kg})(1.50\text{ m/s}^2 + 9.80\text{ m/s}^2) = 2.60 \times 10^4\text{ N}$$

$\sum \tau = I\alpha$ for the pulley gives $(T_2 - T_1) = (\frac{1}{2}MR^2)\alpha$. With $\alpha = a/R$ this becomes $T_2 - T_1 = \frac{1}{2}Ma$.

$$T_2 = T_1 + \tfrac{1}{2}Ma = 2.60 \times 10^4 \text{ N} + \tfrac{1}{2}(875 \text{ kg})(1.50 \text{ m/s}^2) = 2.67 \times 10^4 \text{ N}$$

$\sum F_y = ma_y$ for the counterweight gives $m_2 g - T_2 = m_2 a$ and

$$m_2 = \frac{T_2}{g - a} = \frac{2.67 \times 10^4 \text{ N}}{9.80 \text{ m/s}^2 - 1.50 \text{ m/s}^2} = 3.22 \times 10^3 \text{ kg}$$

and $w = 3.16 \times 10^4$ N.

**EVALUATE:** The tension in the cable must be different on either side of the pulley in order to produce the net torque on the pulley required to give it an angular acceleration. The tension in the cable attached to the elevator is greater than the weight of the elevator and the elevator accelerates upward. The tension in the cable attached to the counterweight is less than the weight of the counterweight and the counterweight accelerates downward.

**10.59. IDENTIFY:** Blocks *A* and *B* have linear acceleration and therefore obey the linear form of Newton's second law $\sum F_y = ma_y$. The wheel *C* has angular acceleration, so it obeys the rotational form of Newton's second law $\sum \tau_z = I\alpha_z$.

**SET UP:** *A* accelerates downward, *B* accelerates upward and the wheel turns clockwise. Apply $\sum F_y = ma_y$ to blocks *A* and *B*. Let +*y* be downward for *A* and +*y* be upward for *B*. Apply $\sum \tau_z = I\alpha_z$ to the wheel, with the clockwise sense of rotation positive. Each block has the same magnitude of acceleration, *a*, and $a = R\alpha$. Call the $T_A$ the tension in the cord between *C* and *A* and $T_B$ the tension between *C* and *B*.

**EXECUTE:** For *A*, $\sum F_y = ma_y$ gives $m_A g - T_A = m_A a$. For *B*, $\sum F_y = ma_y$ gives $T_B - m_B g = m_B a$. For the wheel, $\sum \tau_z = I\alpha_z$ gives $T_A R - T_B R = I\alpha = I(a/R)w$ and $T_A - T_B = \left(\frac{I}{R^2}\right)a$. Adding these three equations gives $(m_A - m_B)g = \left(m_A + m_B + \frac{I}{R^2}\right)a$. Solving for *a*, we have

$$a = \left(\frac{m_A - m_B}{m_A + m_B + I/R^2}\right)g = \left(\frac{4.00 \text{ kg} - 2.00 \text{ kg}}{4.00 \text{ kg} + 2.00 \text{ kg} + (0.220 \text{ kg} \cdot \text{m}^2)/(0.120 \text{ m})^2}\right)(9.80 \text{ m/s}^2) = 0.921 \text{ m/s}^2.$$

$$\alpha = \frac{a}{R} = \frac{0.921 \text{ m/s}^2}{0.120 \text{ m}} = 7.68 \text{ rad/s}^2.$$

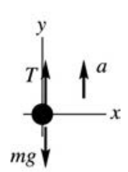
$$T_A = m_A(g - a) = (4.00 \text{ kg})(9.80 \text{ m/s}^2 - 0.921 \text{ m/s}^2) = 35.5 \text{ N}.$$

$$T_B = m_B(g + a) = (2.00 \text{ kg})(9.80 \text{ m/s}^2 + 0.921 \text{ m/s}^2) = 21.4 \text{ N}.$$

**EVALUATE:** The tensions must be different in order to produce a torque that accelerates the wheel when the blocks accelerate.

**10.60. IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the crate and $\sum \tau_z = I\alpha_z$ to the cylinder. The motions are connected by $a(\text{crate}) = R\alpha(\text{cylinder})$.

**SET UP:** The force diagram for the crate is given in Figure 10.60a.




**EXECUTE:** Applying $\sum F_y = ma_y$ gives $T - mg = ma$. Solving for *T* gives $T = m(g + a) = (50 \text{ kg})(9.80 \text{ m/s}^2 + 1.40 \text{ m/s}^2) = 560$ N.

**Figure 10.60a**

**SET UP:** The force diagram for the cylinder is given in Figure 10.60b.

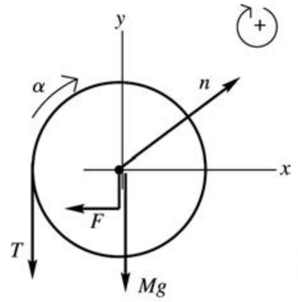


Figure 10.60b

**EXECUTE:** $\sum \tau_z = I\alpha_z$ gives $Fl - TR = I\alpha_z$, where $l = 0.12$ m and $R = 0.25$ m. $a = R\alpha$ so $\alpha_z = a/R$. Therefore $Fl = TR + Ia/R$.

$$F = T\left(\frac{R}{l}\right) + \frac{Ia}{Rl} = (560\text{ N})\left(\frac{0.25\text{ m}}{0.12\text{ m}}\right) + \frac{(2.9\text{ kg}\cdot\text{m}^2)(1.40\text{ m/s}^2)}{(0.25\text{ m})(0.12\text{ m})} = 1300\text{ N}.$$

**EVALUATE:** The tension in the rope is greater than the weight of the crate since the crate accelerates upward. If $F$ were applied to the rim of the cylinder ($l = 0.25$ m), it would have the value $F = 625$ N. This is greater than $T$ because it must accelerate the cylinder as well as the crate. And $F$ is larger than this because it is applied closer to the axis than $R$ so has a smaller moment arm and must be larger to give the same torque.

**10.61. IDENTIFY:** Apply $\sum \vec{F}_{\text{ext}} = m\vec{a}_{\text{cm}}$ and $\sum \tau_z = I_{\text{cm}}\alpha_z$ to the roll.

**SET UP:** At the point of contact, the wall exerts a friction force $f$ directed downward and a normal force $n$ directed to the right. This is a situation where the net force on the roll is zero, but the net torque is *not* zero.

**EXECUTE:** **(a)** Balancing vertical forces, $F_{\text{rod}}\cos\theta = f + w + F$, and balancing horizontal forces $F_{\text{rod}}\sin\theta = n$. With $f = \mu_k n$, these equations become $F_{\text{rod}}\cos\theta = \mu_k n + F + w$, $F_{\text{rod}}\sin\theta = n$. Eliminating $n$ and solving for $F_{\text{rod}}$ gives $F_{\text{rod}} = \dfrac{w+F}{\cos\theta - \mu_k\sin\theta} = \dfrac{(16.0\text{ kg})(9.80\text{ m/s}^2) + (60.0\text{ N})}{\cos 30° - (0.25)\sin 30°} = 293$ N.

**(b)** With respect to the center of the roll, the rod and the normal force exert zero torque. The magnitude of the net torque is $(F - f)R$, and $f = \mu_k n$ may be found by insertion of the value found for $F_{\text{rod}}$ into either of the above relations; i.e., $f = \mu_k F_{\text{rod}}\sin\theta = 36.57$ N. Then,

$$\alpha = \frac{\tau}{I} = \frac{(60.0\text{ N} - 36.57\text{ N})(18.0\times10^{-2}\text{ m})}{(0.260\text{ kg}\cdot\text{m}^2)} = 16.2\text{ rad/s}^2.$$

**EVALUATE:** If the applied force $F$ is increased, $F_{\text{rod}}$ increases and this causes $n$ and $f$ to increase. The angle $\theta$ changes as the amount of paper unrolls and this affects $\alpha$ for a given $F$.

**10.62. IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ to the flywheel and $\sum \vec{F} = m\vec{a}$ to the block. The target variables are the tension in the string and the acceleration of the block.

**(a) SET UP:** Apply $\sum \tau_z = I\alpha_z$ to the rotation of the flywheel about the axis. The free-body diagram for the flywheel is given in Figure 10.62a.

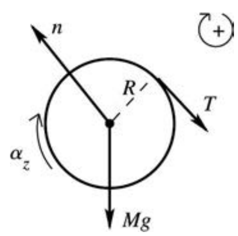


Figure 10.62a

**EXECUTE:** The forces $n$ and $Mg$ act at the axis so have zero torque.

$\sum \tau_z = TR$

$TR = I\alpha_z$

**SET UP:** Apply $\sum \vec{F} = m\vec{a}$ to the translational motion of the block. The free-body diagram for the block is given in Figure 10.62b.

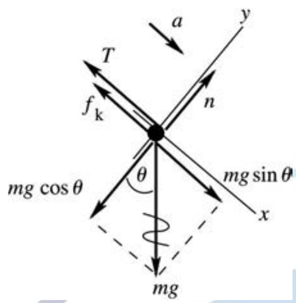



Figure 10.62b

**EXECUTE:** $\Sigma F_y = ma_y$

$n - mg\cos 36.9° = 0$

$n = mg\cos 36.9°$

$f_k = \mu_k n = \mu_k mg\cos 36.9°$

$\sum F_x = ma_x$

$mg\sin 36.9° - T - \mu_k mg\cos 36.9° = ma$

$mg(\sin 36.9° - \mu_k \cos 36.9°) - T = ma$

But we also know that $a_{\text{block}} = R\alpha_{\text{wheel}}$, so $\alpha = a/R$. Using this in the $\sum \tau_z = I\alpha_z$ equation gives $TR = Ia/R$ and $T = (I/R^2)a$. Use this to replace $T$ in the $\sum F_x = ma_x$ equation:

$mg(\sin 36.9° - \mu_k \cos 36.9°) - (I/R^2)a = ma$

$$a = \frac{mg(\sin 36.9° - \mu_k \cos 36.9°)}{m + I/R^2}$$

$$a = \frac{(5.00\text{ kg})(9.80\text{ m/s}^2)[\sin 36.9° - (0.25)\cos 36.9°]}{5.00\text{ kg} + 0.500\text{ kg}\cdot\text{m}^2/(0.200\text{ m})^2} = 1.12\text{ m/s}^2.$$

**(b)** $T = \dfrac{0.500\text{ kg}\cdot\text{m}^2}{(0.200\text{ m})^2}(1.12\text{ m/s}^2) = 14.0\text{ N}$

**EVALUATE:** If the string is cut the block will slide down the incline with $a = g\sin 36.9° - \mu_k g\cos 36.9° = 3.92\text{ m/s}^2$. The actual acceleration is less than this because $mg\sin 36.9°$ must also accelerate the flywheel. $mg\sin 36.9° - f_k = 19.6\text{ N}$. $T$ is less than this; there must be more force on the block directed down the incline than up the incline since the block accelerates down the incline.

**10.63. IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the block and $\sum \tau_z = I\alpha_z$ to the combined disks.

**SET UP:** For a disk, $I_{\text{disk}} = \frac{1}{2}MR^2$, so $I$ for the disk combination is $I = 2.25\times 10^{-3}\text{ kg}\cdot\text{m}^2$.

**EXECUTE:** For a tension $T$ in the string, $mg - T = ma$ and $TR = I\alpha = I\dfrac{a}{R}$.

Eliminating $T$ and solving for $a$ gives $a = g\dfrac{m}{m + I/R^2} = \dfrac{g}{1 + I/mR^2}$, where $m$ is the mass of the hanging block and $R$ is the radius of the disk to which the string is attached.

**(a)** With $m = 1.50$ kg and $R = 2.50\times 10^{-2}$ m, $a = 2.88\text{ m/s}^2$.

**(b)** With $m = 1.50$ kg and $R = 5.00\times 10^{-2}$ m, $a = 6.13\text{ m/s}^2$.

The acceleration is larger in case (b); with the string attached to the larger disk, the tension in the string is capable of applying a larger torque.

**EVALUATE:** $\omega = v/R$, where $v$ is the speed of the block and $\omega$ is the angular speed of the disks. When $R$ is larger, in part (b), a smaller fraction of the kinetic energy resides with the disks. The block gains more speed as it falls a certain distance and therefore has a larger acceleration.

**10.64.** **IDENTIFY:** Apply both $\sum \vec{F} = m\vec{a}$ and $\sum \tau_z = I\alpha_z$ to the motion of the roller. Rolling without slipping means $a_{cm} = R\alpha$. Target variables are $a_{cm}$ and $f$.

**SET UP:** The free-body diagram for the roller is given in Figure 10.64.

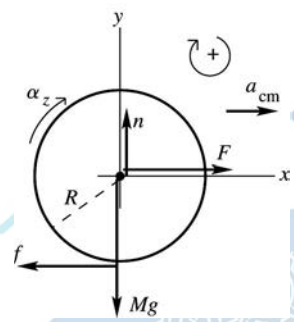



**Figure 10.64**

**EXECUTE:** Apply $\sum \vec{F} = m\vec{a}$ to the translational motion of the center of mass:

$\sum F_x = ma_x$

$F - f = Ma_{cm}$

Apply $\sum \tau_z = I\alpha_z$ to the rotation about the center of mass:

$\sum \tau_z = fR$

thin-walled hollow cylinder: $I = MR^2$

Then $\sum \tau_z = I\alpha_z$ implies $fR = MR^2\alpha$.

But $\alpha_{cm} = R\alpha$, so $f = Ma_{cm}$.

Using this in the $\sum F_x = ma_x$ equation gives $F - Ma_{cm} = Ma_{cm}$.

$a_{cm} = F/2M$, and then $f = Ma_{cm} = M(F/2M) = F/2$.

**EVALUATE:** If the surface were frictionless the object would slide without rolling and the acceleration would be $a_{cm} = F/M$. The acceleration is less when the object rolls.

**10.65.** **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each object and apply $\sum \tau_z = I\alpha_z$ to the pulley.

**SET UP:** Call the 75.0 N weight $A$ and the 125 N weight $B$. Let $T_A$ and $T_B$ be the tensions in the cord to the left and to the right of the pulley. For the pulley, $I = \frac{1}{2}MR^2$, where $Mg = 80.0\text{ N}$ and $R = 0.300\text{ m}$. The 125 N weight accelerates downward with acceleration $a$, the 75.0 N weight accelerates upward with acceleration $a$ and the pulley rotates clockwise with angular acceleration $\alpha$, where $a = R\alpha$.

**EXECUTE:** $\sum \vec{F} = m\vec{a}$ applied to the 75.0 N weight gives $T_A - w_A = m_A a$. $\sum \vec{F} = m\vec{a}$ applied to the 125.0 N weight gives $w_B - T_B = m_B a$. $\sum \tau_z = I\alpha_z$ applied to the pulley gives $(T_B - T_A)R = (\frac{1}{2}MR^2)\alpha_z$ and $T_B - T_A = \frac{1}{2}Ma$. Combining these three equations gives $w_B - w_A = (m_A + m_B + M/2)a$ and

$$a = \left(\frac{w_B - w_A}{w_A + w_B + w_{\text{pulley}}/2}\right)g = \left(\frac{125\text{ N} - 75.0\text{ N}}{75.0\text{ N} + 125\text{ N} + 40.0\text{ N}}\right)g = 0.2083g.$$

$T_A = w_A(1 + a/g) = 1.2083w_A = 90.62\text{ N}$. $T_B = w_B(1 - a/g) = 0.792w_B = 98.96\text{ N}$. $\sum \vec{F} = m\vec{a}$ applied to the pulley gives that the force $F$ applied by the hook to the pulley is $F = T_A + T_B + w_{\text{pulley}} = 270\text{ N}$. The force the ceiling applies to the hook is 270 N.

**EVALUATE:** The force the hook exerts on the pulley is less than the total weight of the system, since the net effect of the motion of the system is a downward acceleration of mass.

**10.66.** **IDENTIFY:** Newton's second law in its linear form applies to the person, and in its rotational form it applies to the wheel.

**SET UP:** $\Sigma\vec{F} = m\vec{a}$, $\Sigma\tau_z = I\alpha_z$, and $a_{\tan} = R\alpha$. For a uniform disk, $I = \frac{1}{2}MR^2$. Call $m$ the mass of the person and $M$ the mass of the wheel.

**EXECUTE:** **(a)** For the person, $\Sigma\vec{F} = m\vec{a}$ gives $mg - T = ma$, so $T = m(g - a)$.

For the wheel, $\Sigma\tau_z = I\alpha_z$ gives $TR = I\alpha_z = I(a/R)$, which gives $T = Ia/R^2$.

Combining the two expressions for $T$ and using $I = \frac{1}{2}MR^2$ gives

$(\frac{1}{2}MR^2)(a/R^2) = m(g-a)$. Solving for $M$ gives $M = 2m(g-a)/a$. Putting in $m$ = 90.0 kg and $a = g/4$ gives $M$ = 540 kg.

**(b)** As we saw in (a), $T = m(g-a) = 3mg/4 = 3(90.0\text{ kg})(9.80\text{ m/s}^2)/4 = 662\text{ N}$.

**EVALUATE:** The tension is $\frac{3}{4}$ the person's weight because it must reduce his acceleration by $\frac{3}{4}$.

**10.67. IDENTIFY:** Apply $\sum\vec{F}_{\text{ext}} = m\vec{a}_{\text{cm}}$ to the motion of the center of mass and apply $\sum\tau_z = I_{\text{cm}}\alpha_z$ to the rotation about the center of mass.

**SET UP:** $I = 2(\frac{1}{2}mR^2) = mR^2$. The moment arm for $T$ is $b$.

**EXECUTE:** The tension is related to the acceleration of the yo-yo by $(2m)g - T = (2m)a$, and to the angular acceleration by $Tb = I\alpha = I\dfrac{a}{b}$. Dividing the second equation by $b$ and adding to the first to eliminate $T$ yields $a = g\dfrac{2m}{(2m + I/b^2)} = g\dfrac{2}{2+(R/b)^2}$, $\alpha = g\dfrac{2}{2b + R^2/b}$. The tension is found by substitution into either of the two equations:

$$T = (2m)(g-a) = (2mg)\left(1 - \frac{2}{2+(R/b)^2}\right) = 2mg\frac{(R/b)^2}{2+(R/b)^2} = \frac{2mg}{(2(b/R)^2+1)}.$$

**EVALUATE:** $a \to 0$ when $b \to 0$. As $b \to R$, $a \to 2g/3$.

**10.68. IDENTIFY:** Apply conservation of energy to the motion of the shell, to find its linear speed $v$ at points $A$ and $B$. Apply $\sum\vec{F} = m\vec{a}$ to the circular motion of the shell in the circular part of the track to find the normal force exerted by the track at each point. Since $r \ll R$ the shell can be treated as a point mass moving in a circle of radius $R$ when applying $\sum\vec{F} = m\vec{a}$. But as the shell rolls along the track, it has both translational and rotational kinetic energy.

**SET UP:** $K_1 + U_1 = K_2 + U_2$. Let 1 be at the starting point and take $y = 0$ to be at the bottom of the track, so $y_1 = h_0$. $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. $I = \frac{2}{3}mr^2$ and $\omega = v/r$, so $K = \frac{5}{6}mv^2$. During the circular motion, $a_{\text{rad}} = v^2/R$.

**EXECUTE:** **(a)** $\sum\vec{F} = m\vec{a}$ at point $A$ gives $n + mg = m\dfrac{v^2}{R}$. The minimum speed for the shell not to fall off the track is when $n \to 0$ and $v^2 = gR$. Let point 2 be $A$, so $y_2 = 2R$ and $v_2^2 = gR$. Then $K_1 + U_1 = K_2 + U_2$ gives $mgh_0 = 2mgR + \frac{5}{6}m(gR)$. $h_0 = (2 + \frac{5}{6})R = \frac{17}{6}R$.

**(b)** Let point 2 be $B$, so $y_2 = R$. Then $K_1 + U_1 = K_2 + U_2$ gives $mgh_0 = mgR + \frac{5}{6}mv_2^2$. With $h = \frac{17}{6}R$ this gives $v^2 = \frac{11}{5}gR$. Then $\sum\vec{F} = m\vec{a}$ at $B$ gives $n = m\dfrac{v^2}{R} = \frac{11}{5}mg$.

**(c)** Now $K = \frac{1}{2}mv^2$ instead of $\frac{5}{6}mv^2$. The shell would be moving faster at $A$ than with friction and would still make the complete loop.

**(d)** In part (c): $mgh_0 = mg(2R) + \frac{1}{2}mv^2$. $h_0 = \frac{17}{6}R$ gives $v^2 = \frac{5}{3}gR$. $\sum\vec{F} = m\vec{a}$ at point $A$ gives $mg + n = m\dfrac{v^2}{R}$ and $n = m\left(\dfrac{v^2}{R} - g\right) = \frac{2}{3}mg$. In part (a), $n = 0$, since at this point gravity alone supplies the net downward force that is required for the circular motion.

**EVALUATE:** The normal force at $A$ is greater when friction is absent because the speed of the shell at $A$ is greater when friction is absent than when there is rolling without slipping.

**10.69. IDENTIFY:** As it rolls down the rough slope, the basketball gains rotational kinetic energy as well as translational kinetic energy. But as it moves up the smooth slope, its rotational kinetic energy does not change since there is no friction.

**SET UP:** $I_{cm} = \frac{2}{3}mR^2$. When it rolls without slipping, $v_{cm} = R\omega$. When there is no friction the angular speed of rotation is constant. Take $+y$ upward and let $y = 0$ in the valley.

**EXECUTE:** **(a)** Find the speed $v_{cm}$ in the level valley: $K_1 + U_1 = K_2 + U_2$. $y_1 = H_0$, $y_2 = 0$. $K_1 = 0$, $U_2 = 0$. Therefore, $U_1 = K_2$. $mgH_0 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$. $\frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}(\frac{2}{3}mR^2)\left(\frac{v_{cm}}{R}\right)^2 = \frac{1}{3}mv_{cm}^2$, so $mgH_0 = \frac{5}{6}mv_{cm}^2$ and $v_{cm}^2 = \dfrac{6gH_0}{5}$. Find the height $H$ it goes up the other side. Its rotational kinetic energy stays constant as it rolls on the frictionless surface. $\frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}I_{cm}\omega^2 + mgH$.

$H = \dfrac{v_{cm}^2}{2g} = \frac{3}{5}H_0$.

**(b)** Some of the initial potential energy has been converted into rotational kinetic energy so there is less potential energy at the second height $H$ than at the first height $H_0$.

**EVALUATE:** Mechanical energy is conserved throughout this motion. But the initial gravitational potential energy on the rough slope is not all transformed into potential energy on the smooth slope because some of that energy remains as rotational kinetic energy at the highest point on the smooth slope.

**10.70. IDENTIFY:** Apply conservation of energy to the motion of the ball as it rolls up the hill. After the ball leaves the edge of the cliff it moves in projectile motion and constant acceleration equations can be used.

**(a) SET UP:** Use conservation of energy to find the speed $v_2$ of the ball just before it leaves the top of the cliff. Let point 1 be at the bottom of the hill and point 2 be at the top of the hill. Take $y = 0$ at the bottom of the hill, so $y_1 = 0$ and $y_2 = 28.0$ m.

**EXECUTE:** $K_1 + U_1 = K_2 + U_2$

$\frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2 = mgy_2 + \frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2$

Rolling without slipping means $\omega = v/r$ and $\frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{2}{5}mr^2)(v/r)^2 = \frac{1}{5}mv^2$.

$\frac{7}{10}mv_1^2 = mgy_2 + \frac{7}{10}mv_2^2$

$v_2 = \sqrt{v_1^2 - \frac{10}{7}gy_2} = 15.26$ m/s

**SET UP:** Consider the projectile motion of the ball, from just after it leaves the top of the cliff until just before it lands. Take $+y$ to be downward. Use the vertical motion to find the time in the air:

$v_{0y} = 0$, $a_y = 9.80\text{ m/s}^2$, $y - y_0 = 28.0$ m, $t = ?$

**EXECUTE:** $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $t = 2.39$ s

During this time the ball travels horizontally
$x - x_0 = v_{0x}t = (15.26\text{ m/s})(2.39\text{ s}) = 36.5$ m.

Just before it lands, $v_y = v_{0y} + a_yt = 23.4$ m/s and $v_x = v_{0x} = 15.3$ m/s

$v = \sqrt{v_x^2 + v_y^2} = 28.0$ m/s

**(b) EVALUATE:** At the bottom of the hill, $\omega = v/r = (25.0\text{ m/s})/r$. The rotation rate doesn't change while the ball is in the air, after it leaves the top of the cliff, so just before it lands $\omega = (15.3\text{ m/s})/r$. The total kinetic energy is the same at the bottom of the hill and just before it lands, but just before it lands less of this energy is rotational kinetic energy, so the translational kinetic energy is greater.

**10.71. IDENTIFY:** Apply conservation of energy to the motion of the boulder.

**SET UP:** $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ and $v = R\omega$ when there is rolling without slipping. $I = \frac{2}{5}mR^2$.

**EXECUTE:** Break into two parts, the rough and smooth sections.

*Rough:* $mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. $mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$. $v^2 = \frac{10}{7}gh_1$.

*Smooth:* Rotational kinetic energy does not change. $mgh_2 + \frac{1}{2}mv^2 + K_{\text{rot}} = \frac{1}{2}mv_{\text{Bottom}}^2 + K_{\text{rot}}$.

$gh_2 + \frac{1}{2}\left(\frac{10}{7}gh_1\right) = \frac{1}{2}v_{\text{Bottom}}^2$. $v_{\text{Bottom}} = \sqrt{\frac{10}{7}gh_1 + 2gh_2} = \sqrt{\frac{10}{7}(9.80\text{ m/s}^2)(25\text{ m}) + 2(9.80\text{ m/s}^2)(25\text{ m})} = 29.0\text{ m/s}$.

**EVALUATE:** If all the hill was rough enough to cause rolling without slipping,

$v_{\text{Bottom}} = \sqrt{\frac{10}{7}g(50\text{ m})} = 26.5\text{ m/s}$. A smaller fraction of the initial gravitational potential energy goes into translational kinetic energy of the center of mass than if part of the hill is smooth. If the entire hill is smooth and the boulder slides without slipping, $v_{\text{Bottom}} = \sqrt{2g(50\text{ m})} = 31.3\text{ m/s}$. In this case all the initial gravitational potential energy goes into the kinetic energy of the translational motion.

**10.72. IDENTIFY:** Apply Newton's second law in its linear and rotational form to the cylinder. The cylinder does not slip on the surface of the ramp.

**SET UP:** $\sum\vec{F}_{\text{ext}} = M\vec{a}_{\text{cm}}$, $\Sigma\tau_z = I\alpha_z$, $I = \frac{1}{2}mR^2$, and $a_{\text{cm}} = R\alpha$ for no slipping. Take the $x$-axis parallel to the surface of the ramp; call up the ramp positive since that is the direction in which the cylinders must accelerate. Take the $y$-axis perpendicular to the surface. For uniform acceleration $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2$.

**EXECUTE: (a)** The forces balance in the $y$-direction, so the normal force $n$ is $n = mg\cos\theta$. In the $x$-direction, $\Sigma F_x = ma_x$ gives

$F - f_s - mg\sin\theta = ma$.

Now apply $\Sigma\tau_z = I\alpha_z$.

$f_sR = (\frac{1}{2}mR^2)(a/R)$, which gives $a = 2f_s/m$. Putting this result into the previous result gives

$F - f_s - mg\sin\theta = m(2f_s/m) = 2f_s$.

Solving for $F$ gives

$F = 3f_s + mg\sin\theta = 3\mu_sn + mg\sin\theta = 3\mu_smg\cos\theta + mg\sin\theta = mg(3\mu_s\cos\theta + \sin\theta)$

$F = (460\text{ kg})(9.80\text{ m/s}^2)[3(0.120)\cos 37° + \sin 37°] = 4010\text{ N}$.

**(b)** From part (a) we have

$a = 2f_s/m = (2\mu_smg\cos\theta)/m = 2\mu_sg\cos\theta$.

Linear kinematics using $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2$ gives

$$t = \sqrt{\frac{2(x-x_0)}{a}} = \sqrt{\frac{2(x-x_0)}{2\mu_sg\cos\theta}} = \sqrt{\frac{6.00\text{ m}}{(0.120)(9.80\text{ m/s}^2)\cos 37°}} = 2.53\text{ s}.$$

**EVALUATE:** Just lifting the 460-kg vertically would require a force of $mg = 4510$ N, so we don't do very much better by rolling them up the slope since friction opposes the linear motion.

**10.73. IDENTIFY:** Apply conservation of energy to the motion of the wheel.

**SET UP:** $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. No slipping means that $\omega = v/R$. Uniform density means $m_r = \lambda 2\pi R$ and $m_s = \lambda R$, where $m_r$ is the mass of the rim and $m_s$ is the mass of each spoke. For the wheel, $I = I_{\text{rim}} + I_{\text{spokes}}$. For each spoke, $I = \frac{1}{3}m_sR^2$.

**EXECUTE: (a)** $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. $I = I_{\text{rim}} + I_{\text{spokes}} = m_rR^2 + 6(\frac{1}{3}m_sR^2)$

Also, $m = m_r + m_s = 2\pi R\lambda + 6R\lambda = 2R\lambda(\pi + 3)$. Substituting into the conservation of energy equation gives $2R\lambda(\pi+3)gh = \frac{1}{2}(2R\lambda)(\pi+3)(R\omega)^2 + \frac{1}{2}\left[2\pi R\lambda R^2 + 6(\frac{1}{3}\lambda RR^2)\right]\omega^2$.

$$\omega = \sqrt{\frac{(\pi+3)gh}{R^2(\pi+2)}} = \sqrt{\frac{(\pi+3)(9.80\text{ m/s}^2)(58.0\text{ m})}{(0.210\text{ m})^2(\pi+2)}} = 124\text{ rad/s} \quad \text{and } v = R\omega = 26.0\text{ m/s}$$

**(b)** Doubling the density would have no effect because it does not appear in the answer. $\omega$ is inversely proportional to $R$ so doubling the diameter would double the radius which would reduce $\omega$ by half, but $v = R\omega$ would be unchanged.

**EVALUATE:** Changing the masses of the rim and spokes by different amounts would alter the speed $v$ at the bottom of the hill.

**10.74.** **IDENTIFY:** The rings and the rod exert forces on each other, but there is no net force or torque on the system, and so the angular momentum will be constant.

**SET UP:** For the rod, $I = \frac{1}{12}ML^2$. For each ring, $I = mr^2$, where $r$ is their distance from the axis.

**EXECUTE:** **(a)** As the rings slide toward the ends, the moment of inertia changes, and the final angular velocity is given by $\omega_2 = \omega_1 \dfrac{I_1}{I_2} = \omega_1 \left[\dfrac{\frac{1}{12}ML^2 + 2mr_1^2}{\frac{1}{12}ML^2 + 2mr_2^2}\right] = \omega_1 \left(\dfrac{5.00\times10^{-4}\ \text{kg}\cdot\text{m}^2}{2.00\times10^{-3}\ \text{kg}\cdot\text{m}^2}\right) = \dfrac{\omega_1}{4}$, so

$\omega_2 = 12.0$ rev/min.

**(b)** The forces and torques that the rings and the rod exert on each other will vanish, but the common angular velocity will be the same, 12.0 rev/min.

**EVALUATE:** Note that conversion from rev/min to rad/s was not necessary. The angular velocity of the rod decreases as the rings move away from the rotation axis.

**10.75.** **IDENTIFY:** Use conservation of energy to relate the speed of the block to the distance it has descended. Then use a constant acceleration equation to relate these quantities to the acceleration.

**SET UP:** For the cylinder, $I = \frac{1}{2}M(2R)^2$, and for the pulley, $I = \frac{1}{2}MR^2$.

**EXECUTE:** Doing this problem using kinematics involves four unknowns (six, counting the two angular accelerations), while using energy considerations simplifies the calculations greatly. If the block and the cylinder both have speed $v$, the pulley has angular velocity $v/R$ and the cylinder has angular velocity $v/2R$, the total kinetic energy is

$$K = \frac{1}{2}\left[Mv^2 + \frac{M(2R)^2}{2}(v/2R)^2 + \frac{MR^2}{2}(v/R)^2 + Mv^2\right] = \frac{3}{2}Mv^2.$$

This kinetic energy must be the work done by gravity; if the hanging mass descends a distance $y$, $K = Mgy$, or $v^2 = (2/3)gy$. For constant acceleration, $v^2 = 2ay$, and comparison of the two expressions gives $a = g/3$.

**EVALUATE:** If the pulley were massless and the cylinder slid without rolling, $Mg = 2Ma$ and $a = g/2$. The rotation of the objects reduces the acceleration of the block.

**10.76.** **IDENTIFY:** As Jane grabs the helpless Tarzan from the jaws of the hippo, the angular momentum of the Jane-Vine-Tarzan system is conserved about the point at which the vine swings. Before and after that, mechanical energy is conserved.

**SET UP:** Take $+y$ upward and $y = 0$ at the ground. The center of mass of the vine is 4.00 m from either end. Treat the motion in three parts: (i) Jane swinging to where the vine is vertical. Apply conservation of energy. (ii) The inelastic collision between Jane and Tarzan. Apply conservation of angular momentum. (iii) The motion of the combined object after the collision. Apply conservation of energy. The vine has $I = \frac{1}{3}m_{\text{vine}}l^2$ and Jane has $I = m_{\text{Jane}}l^2$, so the system of Jane plus vine has $I_{\text{tot}} = (\frac{1}{3}m_{\text{vine}} + m_{\text{Jane}})l^2$. Angular momentum is $L = I\omega$.

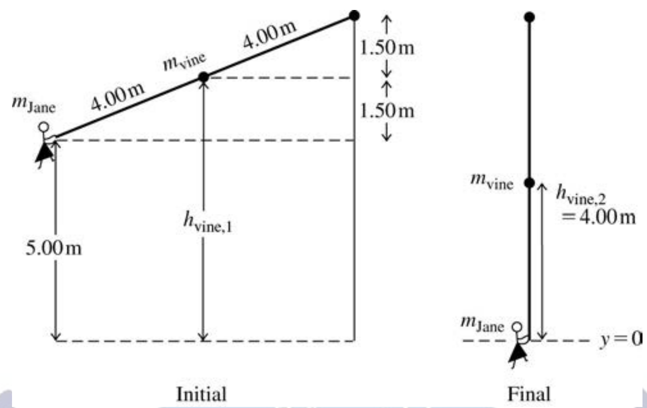



**Figure 10.76a**

**EXECUTE:** **(a)** The initial and final positions of Jane and the vine for the first stage of the motion are sketched in Figure 10.76a. The initial height of the center of the vine is $h_{\text{vine, 1}} = 6.50\text{ m}$ and its final height is $h_{\text{vine, 2}} = 4.00\text{ m}$. Conservation of energy gives $U_1 + K_1 = U_2 + K_2$. $K_1 = 0$ so $m_{\text{Jane}}g(5.00\text{ m}) + m_{\text{vine}}g(6.50\text{ m}) = m_{\text{vine}}g(4.00\text{ m}) + \frac{1}{2}I_{\text{tot}}\omega^2$. $\omega = \sqrt{\dfrac{2[m_{\text{Jane}}(5.00\text{ m}) + m_{\text{vine}}(2.50\text{ m})]g}{(\frac{1}{3}m_{\text{vine}} + m_{\text{Jane}})l^2}}$,

which gives $\omega = \sqrt{\dfrac{2[(60.0\text{ kg})(5.00\text{ m}) + (30.0\text{ kg})(2.50\text{ m})](9.80\text{ m/s}^2)}{\left[\frac{1}{3}(30.0\text{ kg}) + 60.0\text{ kg}\right](8.00\text{ m})^2}} = 1.28\text{ rad/s}$.

**(b)** Conservation of angular momentum applied to the collision gives $L_1 = L_2$, so $I_1\omega_1 = I_2\omega_2$.

$\omega_1 = 1.28\text{ rad/s}$.

$I_1 = \left[\frac{1}{3}(30.0\text{ kg}) + 60.0\text{ kg}\right](8.00\text{ m})^2 = 4.48\times10^3\text{ kg}\cdot\text{m}^2$.

$I_2 = I_1 + m_{\text{Tarzan}}l^2 = 4.48\times10^3\text{ kg}\cdot\text{m}^2 + (72.0\text{ kg})(8.00\text{ m})^2 = 9.09\times10^3\text{ kg}\cdot\text{m}^2$.

$\omega_2 = \left(\dfrac{I_1}{I_2}\right)\omega_1 = \left(\dfrac{4.48\times10^3\text{ kg}\cdot\text{m}^2}{9.09\times10^3\text{ kg}\cdot\text{m}^2}\right)(1.28\text{ rad/s}) = 0.631\text{ rad/s}$.

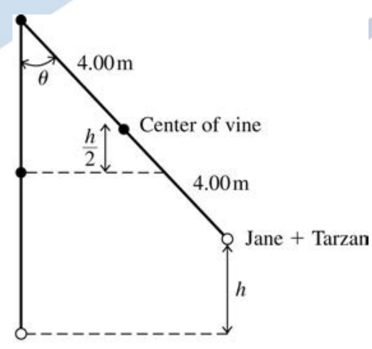



**Figure 10.76b**

**(c)** The final position of Tarzan and Jane, when they have swung to their maximum height, is shown in Figure 10.76b. If Tarzan and Jane rise to a height $h$, then the center of the vine rises to a height $h/2$. Conservation of energy gives $\frac{1}{2}I\omega^2 = (m_{\text{Jane}} + m_{\text{Tarzan}})gh + m_{\text{vine}}gh/2$, where $I = 9.09\times10^3\text{ kg}\cdot\text{m}^2$ and $\omega = 0.631\text{ rad/s}$, from part (b).

$$h = \frac{I\omega^2}{2(m_{\text{Jane}} + m_{\text{Tarzan}} + 0.5m_{\text{vine}})g} = \frac{(9.09\times 10^3\ \text{kg}\cdot\text{m}^2)(0.631\ \text{rad/s})^2}{2(60.0\ \text{kg}+72.0\ \text{kg}+15.0\ \text{kg})(9.80\ \text{m/s}^2)} = 1.26\ \text{m}.$$

**EVALUATE:** Mechanical energy is lost in the inelastic collision.

**10.77.** **IDENTIFY:** Apply conservation of energy to the motion of the first ball before the collision and to the motion of the second ball after the collision. Apply conservation of angular momentum to the collision between the first ball and the bar.

**SET UP:** The speed of the ball just before it hits the bar is $v = \sqrt{2gy} = 15.34$ m/s. Use conservation of angular momentum to find the angular velocity $\omega$ of the bar just after the collision. Take the axis at the center of the bar.

**EXECUTE:** $L_1 = mvr = (5.00\ \text{kg})(15.34\ \text{m/s})(2.00\ \text{m}) = 153.4\ \text{kg}\cdot\text{m}^2/\text{s}$

Immediately after the collision the bar and both balls are rotating together.

$L_2 = I_{\text{tot}}\omega$

$I_{\text{tot}} = \frac{1}{12}Ml^2 + 2mr^2 = \frac{1}{12}(8.00\ \text{kg})(4.00\ \text{m})^2 + 2(5.00\ \text{kg})(2.00\ \text{m})^2 = 50.67\ \text{kg}\cdot\text{m}^2$

$L_2 = L_1 = 153.4\ \text{kg}\cdot\text{m}^2/\text{s}$

$\omega = L_2/I_{\text{tot}} = 3.027$ rad/s

Just after the collision the second ball has linear speed $v = r\omega = (2.00\ \text{m})(3.027\ \text{rad/s}) = 6.055$ m/s and is moving upward. $\frac{1}{2}mv^2 = mgy$ gives $y = 1.87$ m for the height the second ball goes.

**EVALUATE:** Mechanical energy is lost in the inelastic collision and some of the final energy is in the rotation of the bar with the first ball stuck to it. As a result, the second ball does not reach the height from which the first ball was dropped.

**10.78.** **IDENTIFY:** Apply $\Sigma\vec{\tau} = \dfrac{d\vec{L}}{dt}$.

**SET UP:** The door has $I = \frac{1}{3}ml^2$. The torque applied by the force is $rF_{\text{av}}$, where $r = l/2$.

**EXECUTE:** $\sum\tau_{\text{av}} = rF_{\text{av},}$ and $\Delta L = rF_{\text{av}}\Delta t = rJ$. The angular velocity $\omega$ is then

$\omega = \dfrac{\Delta L}{I} = \dfrac{rF_{\text{av}}\Delta t}{I} = \dfrac{(l/2)F_{\text{av}}\Delta t}{\frac{1}{3}ml^2} = \dfrac{3}{2}\dfrac{F_{\text{av}}\Delta t}{ml}$, where $l$ is the width of the door. Substitution of the given numeral values gives $\omega = 0.514$ rad/s.

**EVALUATE:** The final angular velocity of the door is proportional to both the magnitude of the average force and also to the time it acts.

**10.79.** **IDENTIFY:** Apply conservation of angular momentum to the collision. Linear momentum is not conserved because of the force applied to the rod at the axis. But since this external force acts at the axis, it produces no torque and angular momentum is conserved.

**SET UP:** The system before and after the collision is sketched in Figure 10.79.

**EXECUTE:** **(a)** $m_{\text{b}} = \frac{1}{4}m_{\text{rod}}$

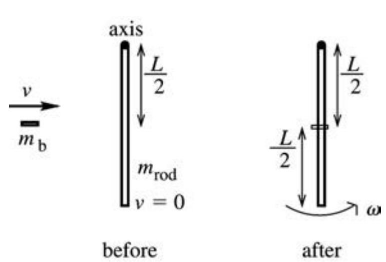



Figure 10.79

**EXECUTE:** $L_1 = m_{\text{b}}vr = \frac{1}{4}m_{\text{rod}}v(L/2)$

$L_1 = \frac{1}{8}m_{\text{rod}}vL$

$L_2 = (I_{\text{rod}} + I_{\text{b}})\omega$

$I_{\text{rod}} = \frac{1}{3}m_{\text{rod}}L^2$

$I_{\text{b}} = m_{\text{b}}r^2 = \frac{1}{4}m_{\text{rod}}(L/2)^2$

$I_{\text{b}} = \frac{1}{16}m_{\text{rod}}L^2$

Thus $L_1 = L_2$ gives $\frac{1}{8}m_{\text{rod}}vL = (\frac{1}{3}m_{\text{rod}}L^2 + \frac{1}{16}m_{\text{rod}}L^2)\omega$

$\frac{1}{8}v = \frac{19}{48}L\omega$

$\omega = \frac{6}{19}v/L$

**(b)** $K_1 = \frac{1}{2}mv^2 = \frac{1}{8}m_{\text{rod}}v^2$

$K_2 = \frac{1}{2}I\omega^2 = \frac{1}{2}(I_{\text{rod}} + I_{\text{b}})\omega^2 = \frac{1}{2}(\frac{1}{3}m_{\text{rod}}L^2 + \frac{1}{16}m_{\text{rod}}L^2)(6v/19L)^2$

$K_2 = \frac{1}{2}(\frac{19}{48})(\frac{6}{19})^2 m_{\text{rod}}v^2 = \frac{3}{152}m_{\text{rod}}v^2$

Then $\dfrac{K_2}{K_1} = \dfrac{\frac{3}{152}m_{\text{rod}}v^2}{\frac{1}{8}m_{\text{rod}}v^2} = 3/19.$

**EVALUATE:** The collision is inelastic and $K_2 < K_1$.

**10.80.** **IDENTIFY:** As you walk toward the center of the turntable, the angular momentum of the system (you plus turntable) is conserved. By getting closer to the center, you are decreasing the moment of inertia of the system. Newton's second law applies to you, and static friction provides the centripetal force on you.

**SET UP:** $I_0\omega_0 = I_2\omega_2$, $I = mr^2$ for a point mass, $a_{\text{rad}} = r\omega^2$, $f_{\text{s}}^{\text{max}} = \mu_{\text{s}}n$, and $\Sigma\vec{F} = m\vec{a}$.

**EXECUTE:** At the closest distance, the friction force is

$f_{\text{s}} = \mu_{\text{s}}n = \mu_{\text{s}}mg$

Newton's second law gives

$f_{\text{s}} = ma = mr\omega^2$

Combining these two equations gives

$\mu_{\text{s}}mg = mr\omega^2$

Conservation of angular momentum gives $\omega = \dfrac{I_0}{I}\omega_0 = \left(\dfrac{I_{\text{t}} + mr_0^2}{I_{\text{t}} + mr^2}\right)\omega_0$. Solving the earlier equation for $\mu_{\text{s}}$ and using the previous result gives $\mu_{\text{s}} = \dfrac{\omega^2 r}{g} = \left(\dfrac{I_{\text{t}} + mr_0^2}{I_{\text{t}} + mr^2}\right)^2 \dfrac{\omega_0^2 r}{g}$. Putting in $m$ = 70.0 kg, $r$ = 3.00 m, and $I_{\text{t}}$ = 1200 kg · m$^2$, and using $\omega_0$ = 2π/(8.0 s), we get $\mu_{\text{s}}$ = 0.780.

**EVALUATE:** This coefficient of static friction is a physically reasonable.

**10.81.** **IDENTIFY:** As the disks are connected, their angular momentum is conserved, but some of their initial kinetic energy is converted to thermal energy. The 2400 J of thermal energy is equal to the loss of rotational kinetic energy.

**SET UP:** $I_1\omega_1 = I_2\omega_2$, $K = \frac{1}{2}I\omega^2$.

**EXECUTE:** Angular momentum conservation gives $I_A\omega_A = (I_A + I_B)\omega \rightarrow \omega = \dfrac{I_A\omega_A}{I_A + I_B}$. The loss of kinetic energy is $\Delta K = K_1 - K_2 = \frac{1}{2}I_A\omega_0^2 - \frac{1}{2}(I_A + I_B)\omega^2$. Combining these two equations gives

$\Delta K = \dfrac{I_A\omega_0^2}{2}\left(1 - \dfrac{I_A}{I_A + I_B}\right)$. The loss of kinetic energy should be no more than 2400 J, so

$\dfrac{I_A\omega_0^2}{2}\left(1 - \dfrac{I_A}{I_A + I_B}\right) \le 2400$ J. The quantity $\dfrac{I_A\omega_0^2}{2}$ is the kinetic energy of $A$, $K_A$. Therefore we can solve the inequality for $K_A$, giving $K_A \le (2400 \text{ J})\left(\dfrac{I_A + I_B}{I_B}\right)$. Since $I_A = I_B/3$, the maximum kinetic energy of $A$ is 3200 J.

**EVALUATE:** This situation is the rotational analog to a collision in which one object is initially at rest and they stick together. As in that situation, the momentum (angular in this case) is conserved but the kinetic energy is not.

**10.82. IDENTIFY:** This is a collision in which one object is initially stationary and they stick together. The rod is pivoted at one end, so it can only rotate after it is struck. The puck has angular momentum, some of which is transferred to the rod, but the angular momentum of the puck-rod system is conserved.

**SET UP:** The initial angular momentum of the puck is $mvr$, the final angular momentum of the rod is $I\omega$, and $I_{rod} = \frac{1}{3}ML^2$.

**EXECUTE:** After the collision, $\omega = 2\pi/T$, where $T = 0.736$ s, $r = L$, and $I = I_{rod} + I_{puck}$. Conservation of angular momentum gives $mvr = (\frac{1}{3}ML^2 + mL^2)\omega$. Solving for $v$ gives $v = \dfrac{(\frac{1}{3}ML^2 + mL^2)\left(\frac{2\pi}{T}\right)}{mL}$. Putting in $m = 0.163$ kg, $M = 0.800$ kg, $L = 2.00$ m, $T = 0.736$ s gives $v = 45.0$ m/s.

**EVALUATE:** This situation is the rotational analog to a collision in which one object is initially at rest and they stick together. As in that situation, the momentum (angular in this case) is conserved but the kinetic energy is not.

**10.83. IDENTIFY:** We must break this problem up into three parts: the motion on the waterslide, the collision with the pole, and the swing of the pole after the collision. On the slide, mechanical energy is conserved. During the collision with the pole, angular momentum is conserved. During the swing of the pole after the collision, mechanical energy is conserved.

**SET UP:** On the slide and after the collision, $K_1 + U_1 = K_2 + U_2$ is valid. $I_{pole} = \frac{1}{3}ML^2$, the initial angular momentum of the person is $mvr = mvL$, the final angular momentum of the pole-person system is $(I_{person} + I_{pole})\omega = \left(mL^2 + \frac{ML^2}{3}\right)\omega$. The kinetic energy after the collision is $K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(mL^2 + \frac{ML^2}{3}\right)\omega^2$.

**EXECUTE:** We work backward, starting with the swinging motion after the collision. We take the zero of potential energy to be the bottom of the pole just after the person grabs it. $K_1 + U_1 = K_2 + U_2$ gives

$$K_1 + U_{person,1} + U_{pole,1} = 0 + U_{pole,2} + U_{person,2}$$

$$\frac{1}{2}I\omega^2 + 0 + Mg\frac{L}{2} = mgL(1-\cos\theta) + Mg\left[\frac{L}{2} + \frac{L}{2}(1-\cos\theta)\right]$$

Solving for $\omega$ and using the moment of inertia of the person-plus-pole we get

$$\omega = \sqrt{\frac{gL(1-\cos\theta)(2m+M)}{\frac{1}{3}ML^2 + mL^2}} = \sqrt{\frac{(9.80\text{ m/s}^2)(6.00\text{ m})(1-\cos 72.0°)[2(70.0\text{ kg} + 24.0\text{ kg})]}{\frac{1}{3}(24.0\text{ kg})(6.00\text{ m})^2 + (70.0\text{ kg})(6.00\text{ m})^2}}$$

$\omega = 1.54$ rad/s.

Now we use conservation of angular momentum during the collision to find the speed of the person just before the collision.

$$mvL = (I_{person} + I_{pole})\omega = \left(mL^2 + \frac{ML^2}{3}\right)\omega$$

Solving for $v$ and putting in the numbers gives

$$v = \frac{\omega}{mL}(\tfrac{1}{3}ML^2 + mL^2) = \frac{\omega L}{3m}(3m + M) = (1.54\text{ rad/s})(6.00\text{ m})[3(70.0\text{ kg}) + 24.0\text{ kg}]/[3(70.0\text{ kg})]$$

$v = 10.30$ m/s.

Now use energy conservation to find the initial height $h$.

$0 + U_1 = K_2 + 0$

$mgh = \frac{1}{2}mv^2$

$h = v^2/2g = (10.30\text{ m/s})^2/[2(9.80\text{ m/s}^2)] = 5.41$ m.

**EVALUATE:** The final height reached by the person at the end of the swing is $h_f = (6.00 \text{ m})(1 - \cos 72°) =$ 4.15 m, which is less than the original heighgt of 5.41 m. Part of the reason for the decreased height is the fact that the pole also swings up, and part is due to the loss of kinetic energy during the inelastic collision. We cannot do this problem in a single step because different conservation laws are involved.

**10.84. IDENTIFY:** Angular momentum is conserved, so $I_0\omega_0 = I_2\omega_2$.

**SET UP:** For constant mass the moment of inertia is proportional to the square of the radius.

**EXECUTE:** $R_0^2\omega_0 = R_2^2\omega_2$, or $R_0^2\omega_0 = (R_0+\Delta R)^2(\omega_0+\Delta\omega) = R_0^2\omega_0 + 2R_0\Delta R\omega_0 + R_0^2\Delta\omega$, where the terms in $\Delta R\Delta\omega$ and $(\Delta\omega)^2$ have been omitted. Canceling the $R_0^2\omega_0$ term gives $\Delta R = -\frac{R_0}{2}\frac{\Delta\omega}{\omega_0} = -1.1 \text{ cm}$.

**EVALUATE:** $\Delta R/R_0$ and $\Delta\omega/\omega_0$ are each very small so the neglect of terms containing $\Delta R\Delta\omega$ or $(\Delta\omega)^2$ is an accurate simplifying approximation.

**10.85. IDENTIFY:** Apply conservation of angular momentum to the collision between the bird and the bar and apply conservation of energy to the motion of the bar after the collision.

**SET UP:** For conservation of angular momentum take the axis at the hinge. For this axis the initial angular momentum of the bird is $m_{\text{bird}}(0.500 \text{ m})v$, where $m_{\text{bird}} = 0.500 \text{ kg}$ and $v = 2.25 \text{ m/s}$. For this axis the moment of inertia is $I = \frac{1}{3}m_{\text{bar}}L^2 = \frac{1}{3}(1.50 \text{ kg})(0.750 \text{ m})^2 = 0.281 \text{ kg}\cdot\text{m}^2$. For conservation of energy, the gravitational potential energy of the bar is $U = m_{\text{bar}}gy_{\text{cm}}$, where $y_{\text{cm}}$ is the height of the center of the bar. Take $y_{\text{cm},1} = 0$, so $y_{\text{cm},2} = -0.375 \text{ m}$.

**EXECUTE: (a)** $L_1 = L_2$ gives $m_{\text{bird}}(0.500 \text{ m})v = (\frac{1}{3}m_{\text{bar}}L^2)\omega$.

$$\omega = \frac{3m_{\text{bird}}(0.500 \text{ m})v}{m_{\text{bar}}L^2} = \frac{3(0.500 \text{ kg})(0.500 \text{ m})(2.25 \text{ m/s})}{(1.50 \text{ kg})(0.750 \text{ m})^2} = 2.00 \text{ rad/s}.$$

**(b)** $U_1 + K_1 = U_2 + K_2$ applied to the motion of the bar after the collision gives

$$\tfrac{1}{2}I\omega_1^2 = m_{\text{bar}}g(-0.375 \text{ m}) + \tfrac{1}{2}I\omega_2^2.\quad \omega_2 = \sqrt{\omega_1^2 + \frac{2}{I}m_{\text{bar}}g(0.375 \text{ m})}.$$

$$\omega_2 = \sqrt{(2.00 \text{ rad/s})^2 + \frac{2}{0.281 \text{ kg}\cdot\text{m}^2}(1.50 \text{ kg})(9.80 \text{ m/s}^2)(0.375 \text{ m})} = 6.58 \text{ rad/s}.$$

**EVALUATE:** Mechanical energy is not conserved in the collision. The kinetic energy of the bar just after the collision is less than the kinetic energy of the bird just before the collision.

**10.86. IDENTIFY:** Angular momentum is conserved, since the tension in the string is in the radial direction and therefore produces no torque. Apply $\sum\vec{F} = m\vec{a}$ to the block, with $a = a_{\text{rad}} = v^2/r$.

**SET UP:** The block's angular momentum with respect to the hole is $L = mvr$.

**EXECUTE:** The tension is related to the block's mass and speed, and the radius of the circle, by $T = m\frac{v^2}{r}$.

$$T = mv^2\frac{1}{r} = \frac{m^2v^2}{m}\frac{r^2}{r^3} = \frac{(mvr)^2}{mr^3} = \frac{L^2}{mr^3}.$$ The radius at which the string breaks is

$$r^3 = \frac{L^2}{mT_{\max}} = \frac{(mv_1r_1)^2}{mT_{\max}} = \frac{\left[(0.130 \text{ kg})(4.00 \text{ m/s})(0.800 \text{ m})\right]^2}{(0.130 \text{ kg})(30.0 \text{ N})},$$ from which $r = 0.354 \text{ m}$.

**EVALUATE:** Just before the string breaks, the speed of the rock is $(4.00 \text{ m/s})\left(\frac{0.800 \text{ m}}{0.354 \text{ m}}\right) = 9.04 \text{ m/s}$. We can verify that using $T = mv^2/R$ that $v = 9.04 \text{ m/s}$ and $r = 0.354 \text{ m}$ do give $T = 30.0 \text{ N}$.

**10.87. IDENTIFY:** Apply conservation of momentum to the system of the runner and turntable.

**SET UP:** Let the positive sense of rotation be the direction the turntable is rotating initially.

**EXECUTE:** The initial angular momentum is $I\omega_1 - mRv_1$, with the minus sign indicating that runner's motion is opposite the motion of the part of the turntable under his feet. The final angular momentum is $\omega_2(I + mR^2)$, so $\omega_2 = \dfrac{I\omega_1 - mRv_1}{I + mR^2}$.

$$\omega_2 = \frac{(80\ \text{kg}\cdot\text{m}^2)(0.200\ \text{rad/s}) - (55.0\ \text{kg})(3.00\ \text{m})(2.8\ \text{m/s})}{(80\ \text{kg}\cdot\text{m}^2) + (55.0\ \text{kg})(3.00\ \text{m})^2} = -0.776\ \text{rad/s}.$$

**EVALUATE:** The minus sign indicates that the turntable has reversed its direction of motion. This happened because the man had the larger magnitude of angular momentum initially.

**10.88. IDENTIFY:** We use the power and angular velocity to calculate the torque.

**SET UP:** $P = \tau\omega$, 1 hp = 746 W.

**EXECUTE: (a)** First make the necessary conversions: 1 ft · lb = (0.3048 m)(4.448 N) = 1.356 N · m

1 rpm = 1 rev/min = (2π rad)/(60 s) = 0.1047 rad/s.

Solve for torque and use the above conversions:

$\tau = P/\omega$ = [(285 hp)/(5300 rpm)]{(746 W/hp)/[(0.1047 rad/s)/rpm]} = 383 N · m = 283 ft · lb.

As we can see, 283 ft · lb is less than the maximum 305 ft · lb.

**(b)** $P = \tau\omega$ = (305 ft · lb) (3900 rpm)(1.356 N · m / ft · lb) [(0.1047 rad/s)/rpm] = 169 kW = 226 hp.

The power of 226 hp is smaller than the maximum of 285 hp.

**(c)** Make the following conversions:

$$\text{hp} = \tau(\text{ft}\cdot\text{lb})\,\omega(\text{rpm})\left(\frac{1.356\ \text{N}\cdot\text{m}}{1\ \text{ft}\cdot\text{lb}}\right)\left(\frac{0.1047\ \text{rad/s}}{1\ \text{rpm}}\right)\left(\frac{1\ \text{hp}}{746\ \text{W}}\right) = 1.9031 \times 10^{-4}\ \tau(\text{ft}\cdot\text{lb})\,\omega(\text{rpm}),$$ so $1/c$ = $1.9031 \times 10^{-4}$, which gives $c$ = 5254.

**(d)** From (c), $P = \tau\omega$ gives 580 hp = $\tau$ (6000 rpm)/5254, so $\tau$ = 508 ft · lb.

**EVALUATE:** Torque, power, and angular velocity are often expressed in diverse units, so conversions are frequently necessary.

**10.89. IDENTIFY:** All the objects have the same mass and start from rest at the same height *h*. They roll without slipping, so their mechanical energy is conserved. Newton's second law, in its linear and rotational forms, applies to each object. Since the objects have different mass distributions, they will take different times to reach the bottom of the ramp.

**SET UP:** $K_1 + U_1 = K_2 + U_2$, $\sum \vec{F}_{\text{ext}} = M\vec{a}_{\text{cm}}$, $\Sigma\tau = I\alpha$, $K_{\text{tot}} = K_{\text{cm}} + K_{\text{rot}}$, $K_{\text{cm}} = \frac{1}{2}Mv_{\text{cm}}^2$, $K_{\text{rot}} = \frac{1}{2}I_{\text{cm}}\omega^2$.

**EXECUTE: (a)** We can express the moment of inertia of a round object as $I = cmR^2$, where *c* depends on the shape and mass distribution. Energy conservation gives $K_1 + U_1 = K_2 + U_2$, so

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}cmR^2\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}cmR^2\left(\frac{v}{R}\right)^2 = \tfrac{1}{2}v^2(1+c).$$ Solving for $v^2$ gives

$v^2 = \dfrac{2gh}{1+c}$. This *v* is the speed at the bottom of the ramp. The object with the greatest speed *v* will also have the greatest average speed down the ramp and will therefore take the shortest time to reach the bottom. Thus the object with the smallest *c* will have the greatest *v* and therefore the shortest time in the bar graph shown with the problem. For a solid cylinder, $I = \frac{1}{2}mR^2$ so *c* = ½, for a hollow cylinder, $I = mR^2$, so *c* = 1, and likewise we get *c* = 2/5 for a solid sphere and *c* = 2/3 for a hollow sphere. The smallest value of *c* is 2/5 for a solid sphere, so that object must take the shortest time, which makes it object *A*. The largest value of *c* is 1 for a hollow cylinder, so that object takes the longest time, which makes it object *D*. The hollow sphere has a larger *c* than the solid cylinder, so it takes longer than the solid cylinder, so *C* must be the hollow sphere and *B* the solid cylinder. Summarizing these results, we have

*A:* solid sphere, *c* = 2/5
*B:* solid cylinder, *c* = 1/2
*C:* hollow sphere, *c* = 2/3
*D:* hollow cylinder, *c* = 1

**(b)** All the objects start from rest at the same initial height and roll without slipping, so they all have the same kinetic energy at the bottom of the ramp.

**(c)** Using $K_{\text{rot}} = \frac{1}{2}I_{\text{cm}}\omega^2$, we have $K_{\text{rot}} = ½\,(cmR^2)(v/R)^2 = ½\,mcv^2$. Using our result for $v^2$ from (a) gives

$K_{\text{rot}} = \frac{1}{2}mc\left(\frac{2gh}{1+c}\right) = mgh\left(\dfrac{1}{1+\dfrac{1}{c}}\right)$. From this result, we see that the object with the largest $c$ has the largest rotational kinetic energy because the denominator in the parentheses is the smallest. Therefore the hollow cylinder, with $c = 1$, has the largest rotational kinetic energy.

**(d)** Apply Newton's second law. Perpendicular to the ramp surface, we get $n = mg\cos\theta$ for the normal force. Parallel to the surface, with down the ramp as positive, we get $mg\sin\theta - f_s = ma$. Taking torques about the center of the rolling object gives $fR = I\alpha = (mcR^2)(a/R)$, which gives $f_s = mca$, so $ma = f_s/c$. Putting this into the previous equation gives $mg\sin\theta - f_s = f_s/c$, which can be written as $mg\sin\theta = f_s(1 + 1/c)$. We want the minimum coefficient of friction to prevent slipping, so $f_s = \mu_s n = \mu_s mg\cos\theta$. Putting this into the previous equation gives $mg\sin\theta = (\mu_s mg\cos\theta)(1 + 1/c)$.

Solving for $\mu_s$ gives $\mu_s = \dfrac{\tan\theta}{1+\dfrac{1}{c}}$. We want $\mu_s$ such that none of the objects will slip, so we must find the maximum $\mu_s$. That will occur when $c$ has its largest value since that will make the denominator smallest, and that is for the hollow cylinder for which $c = 1$. This gives $\mu_s = (\tan 35.0°)/2 = 0.350$.

**EVALUATE:** As a check, part (a) could be solved using Newton's second law, as we did in part (d). As a check in part (d), find $\mu_s$ for the solid sphere which has the smallest value of $c$. This gives

$\mu_s = \dfrac{\tan 35.0°}{1+\dfrac{1}{2/5}} = \dfrac{\tan 35.0°}{3.5} = 0.200.$ This is less than the 0.350 we found in (d), so a coefficient of friction of 0.350 is more than enough to prevent slipping of the solid sphere.

**10.90. IDENTIFY:** The work done by the force $F$ is equal to the kinetic energy gained by the flywheel. This work is the area under the curve in a $F$-versus-$d$ graph.

**SET UP:** $W = Fd$, $K = \frac{1}{2}I\omega^2$, $v = r\omega$.

**EXECUTE:** **(a)** The pull is constant, so the linear and angular accelerations are constant. Therefore $v = 2v_{\text{av}} = 2(d/t)$, so $\omega = v/R = 2d/tR$. The work done is equal to the kinetic energy of the flywheel, so

$Fd = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{2d}{tR}\right)^2$. Solving for $I$ gives

$I = Ft^2R^2/2d = (25.0\text{ N})(2.00\text{ s})^2(0.166\text{ m})^2/[2(8.35\text{ m})] = 0.165\ \text{kg}\cdot\text{m}^2$.

**(b)** The kinetic energy gained is equal to the work done which is equal to the area under the curve on the $F$-$d$ graph. This gives
$K = (60.0\text{ N})(3.00\text{ m}) + ½\,(60.0\text{ N})(3.00\text{ m}) = 270\text{ J}$.

**(c)** $K = \frac{1}{2}I\omega^2$ so $\omega = \sqrt{\dfrac{2K}{I}} = \sqrt{\dfrac{2(270\text{ J})}{0.165\text{ kg}\cdot\text{m}^2}}$ = 57.2 rad/s. Converting to rpm gives
(57.2 rad/s)[(60 s)/(1 min)][(1 rev)/(2π rad)] = 546 rpm.

**EVALUATE:** In this case, we could have deduced the equation for $F$ as a function of $d$ from the graph and integrated to find the work. But for a more complicated $F$-$d$ dependence, that would have been impossible, but we could still estimate the area quite accurately from the graph.

**10.91. IDENTIFY:** The answer to part (a) can be taken from the solution to Problem 10.86. The work-energy theorem says $W = \Delta K$.

**SET UP:** Problem 10.86 uses conservation of angular momentum to show that $r_1v_1 = r_2v_2$.

**EXECUTE:** **(a)** $T = mv_1^2r_1^2/r^3$.

**(b)** $\vec{T}$ and $d\vec{r}$ are always antiparallel. $\vec{T}\cdot d\vec{r} = -T\,dr$.

$$W = -\int_{r_1}^{r_2} T\,dr = mv_1^2 r_1^2 \int_{r_2}^{r_1} \frac{dr}{r^3} = \frac{mv_1^2}{2} r_1^2 \left[\frac{1}{r_2^2} - \frac{1}{r_1^2}\right].$$

**(c)** $v_2 = v_1(r_1/r_2)$, so $\Delta K = \frac{1}{2}m(v_2^2 - v_1^2) = \frac{mv_1^2}{2}\left[(r_1/r_2)^2 - 1\right]$, which is equal to the work found in part (b).

**EVALUATE:** The work done by $T$ is positive, since $\vec{T}$ is toward the hole in the surface and the block moves toward the hole. Positive work means the kinetic energy of the object increases.

**10.92. IDENTIFY:** Apply $\sum \vec{F}_{\text{ext}} = m\vec{a}_{\text{cm}}$ and $\sum \tau_z = I_{\text{cm}}\alpha_z$ to the motion of the cylinder. Use constant acceleration equations to relate $a_x$ to the distance the object travels. Use the work-energy theorem to find the work done by friction.

**SET UP:** The cylinder has $I_{\text{cm}} = \frac{1}{2}MR^2$.

**EXECUTE:** **(a)** The free-body diagram is sketched in Figure 10.92. The friction force is $f = \mu_k n = \mu_k Mg$, so $a = \mu_k g$. The magnitude of the angular acceleration is $\frac{fR}{I} = \frac{\mu_k MgR}{(1/2)MR^2} = \frac{2\mu_k g}{R}$.

**(b)** Setting $v = at = \omega R = (\omega_0 - \alpha t)R$ and solving for $t$ gives $t = \frac{R\omega_0}{a + R\alpha} = \frac{R\omega_0}{\mu_k g + 2\mu_k g} = \frac{R\omega_0}{3\mu_k g}$,

and $d = \frac{1}{2}at^2 = \frac{1}{2}(\mu_k g)\left(\frac{R\omega_0}{3\mu_k g}\right)^2 = \frac{R^2\omega_0^2}{18\mu_k g}$.

**(c)** The final kinetic energy is $(3/4)Mv^2 = (3/4)M(at)^2$, so the change in kinetic energy is

$$\Delta K = \frac{3}{4}M\left(\mu_k g \frac{R\omega_0}{3\mu_k g}\right)^2 - \frac{1}{4}MR^2\omega_0^2 = -\frac{1}{6}MR^2\omega_0^2.$$

**EVALUATE:** The fraction of the initial kinetic energy that is removed by friction work is $\frac{\frac{1}{6}MR\omega_0^2}{\frac{1}{4}MR\omega_0^2} = \frac{2}{3}$.

This fraction is independent of the initial angular speed $\omega_0$.

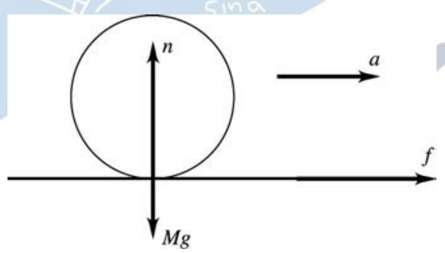



**Figure 10.92**

**10.93. IDENTIFY:** The vertical forces must sum to zero. Apply $\Omega = \frac{\tau_z}{L_z} = \frac{wr}{I\omega}$.

**SET UP:** Denote the upward forces that the hands exert as $F_L$ and $F_R$. $\tau = (F_L - F_R)r$, where $r = 0.200$ m.

**EXECUTE:** The conditions that $F_L$ and $F_R$ must satisfy are $F_L + F_R = w$ and $F_L - F_R = \Omega\frac{I\omega}{r}$, where the second equation is $\tau = \Omega L$, divided by $r$. These two equations can be solved for the forces by first adding and then subtracting, yielding $F_L = \frac{1}{2}\left(w + \Omega\frac{I\omega}{r}\right)$ and $F_R = \frac{1}{2}\left(w - \Omega\frac{I\omega}{r}\right)$. Using the values

$w = mg = (8.00 \text{ kg})(9.80 \text{ m/s}^2) = 78.4 \text{ N}$ and

$$\frac{I\omega}{r} = \frac{(8.00 \text{ kg})(0.325 \text{ m})^2(5.00 \text{ rev/s} \times 2\pi \text{ rad/rev})}{(0.200 \text{ m})} = 132.7 \text{ kg} \cdot \text{m/s} \text{ gives}$$

$F_L = 39.2 \text{ N} + \Omega(66.4 \text{ N} \cdot \text{s})$, $F_R = 39.2 \text{ N} - \Omega(66.4 \text{ N} \cdot \text{s})$.

**(a)** $\Omega = 0$, $F_L = F_R = 39.2$ N.

**(b)** $\Omega = 0.05 \text{ rev/s} = 0.314 \text{ rad/s}$, $F_L = 60.0$ N, $F_R = 18.4$ N.

**(c)** $\Omega = 0.3 \text{ rev/s} = 1.89 \text{ rad/s}$, $F_L = 165$ N, $F_R = -86.2$ N, with the minus sign indicating a downward force.

**(d)** $F_R = 0$ gives $\Omega = \dfrac{39.2 \text{ N}}{66.4 \text{ N} \cdot \text{s}} = 0.590$ rad/s, which is 0.0940 rev/s.

**EVALUATE:** The larger the precession rate $\Omega$, the greater the torque on the wheel and the greater the difference between the forces exerted by the two hands.

**10.94. IDENTIFY:** The rotational form of Newton's second law applies.

**SET UP:** $\Sigma\tau = I\alpha$ and $\omega_z = \omega_{0z} + \alpha_z t$.

**EXECUTE:** $\Sigma\tau = I\alpha = \Delta\omega/\Delta t$, where $I = I_{\text{person}} + I_0$. Solving for $I_{\text{person}}$ gives $I_{\text{person}} = \tau/\alpha - I_0$.

$$I_{\text{person}} = \frac{2.5 \text{ N} \times \text{m}}{\left(\dfrac{1.0 \text{ rad/s}}{3.0 \text{ s}}\right)} - 1.5 \text{ kg} \times \text{m}^2 = 6.0 \text{ kg} \times \text{m}^2,$$ which is choice (b).

**EVALUATE:** The moment of inertia of the turntable is considerably less than that of the person, which is a good thing. If the moment of inertia of the table were much greater than that of the person, the person's body would have a small effect on the angular acceleration of the table, making it hard to get an accurate measurement.

**10.95. IDENTIFY and SET UP:** Moment of inertia depends on the distribution of mass.

**EXECUTE:** Extending her legs increases the person's moment of inertia to increase. With a constant torque on the turntable, this would decrease her angular acceleration, which is choice (c).

**EVALUATE:** The person being studied should be told to lie still during the procedure.

**10.96. IDENTIFY and SET UP:** The torque is the product of the force times the lever arm, and $\Sigma\tau = I\alpha$.

**EXECUTE:** Doubling the lever arm with a constant force doubles the torque, which then doubles the angular acceleration, so choice (b) is correct.

**EVALUATE:** Doubling the diameter of the pulley would also allow the tension to be decreased by a factor of 2 and still keep the same original angular acceleration.

**10.97. IDENTIFY and SET UP:** The parallel-axis theorem, $I = I_{\text{cm}} + md^2$, applies to the person.

**EXECUTE:** The measured moment of inertia would be $I$, but this would be greater than $I_{\text{cm}}$, so the measured value would be too large, choice (a).

**EVALUATE:** Care is essential to position the person properly on the turntable.

# 11

## EQUILIBRIUM AND ELASTICITY

**11.1.** **IDENTIFY:** Use $x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \dots}{m_1 + m_2 + m_3 + \dots}$ to calculate $x_{cm}$. The center of gravity of the bar is at its center and it can be treated as a point mass at that point.

**SET UP:** Use coordinates with the origin at the left end of the bar and the $+x$-axis along the bar. $m_1 = 0.120$ kg, $m_2 = 0.055$ kg, $m_3 = 0.110$ kg.

**EXECUTE:** $x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(0.120\text{ kg})(0.250\text{ m}) + 0 + (0.110\text{ kg})(0.500\text{ m})}{0.120\text{ kg} + 0.055\text{ kg} + 0.110\text{ kg}} = 0.298$ m. The fulcrum should be placed 29.8 cm to the right of the left-hand end.

**EVALUATE:** The mass at the right-hand end is greater than the mass at the left-hand end. So the center of gravity is to the right of the center of the bar.

**11.2.** **IDENTIFY:** Use $x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \dots}{m_1 + m_2 + m_3 + \dots}$ to calculate $x_{cm}$ of the composite object.

**SET UP:** Use coordinates where the origin is at the original center of gravity of the object and $+x$ is to the right. With the 1.50 kg mass added, $x_{cm} = -2.20$ cm, $m_1 = 5.00$ kg and $m_2 = 1.50$ kg. $x_1 = 0$.

**EXECUTE:** $x_{cm} = \frac{m_2x_2}{m_1 + m_2}$. $x_2 = \left(\frac{m_1 + m_2}{m_2}\right)x_{cm} = \left(\frac{5.00\text{ kg} + 1.50\text{ kg}}{1.50\text{ kg}}\right)(-2.20\text{ cm}) = -9.53$ cm.

The additional mass should be attached 9.53 cm to the left of the original center of gravity.

**EVALUATE:** The new center of gravity is somewhere between the added mass and the original center of gravity.

**11.3.** **IDENTIFY:** Treat the rod and clamp as point masses. The center of gravity of the rod is at its midpoint, and we know the location of the center of gravity of the rod-clamp system.

**SET UP:** $x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$.

**EXECUTE:** $1.20\text{ m} = \frac{(1.80\text{ kg})(1.00\text{ m}) + (2.40\text{ kg})x_2}{1.80\text{ kg} + 2.40\text{ kg}}$.

$$x_2 = \frac{(1.20\text{ m})(1.80\text{ kg} + 2.40\text{ kg}) - (1.80\text{ kg})(1.00\text{ m})}{2.40\text{ kg}} = 1.35\text{ m}$$

**EVALUATE:** The clamp is to the right of the center of gravity of the system, so the center of gravity of the system lies between that of the rod and the clamp, which is reasonable.

**11.4.** **IDENTIFY:** Apply the first and second conditions for equilibrium to the trap door.

**SET UP:** For $\sum \tau_z = 0$ take the axis at the hinge. Then the torque due to the applied force must balance the torque due to the weight of the door.

**EXECUTE:** **(a)** The force is applied at the center of gravity, so the applied force must have the same magnitude as the weight of the door, or 300 N. In this case the hinge exerts no force.

**(b)** With respect to the hinges, the moment arm of the applied force is twice the distance to the center of mass, so the force has half the magnitude of the weight, or 150 N.

The hinges supply an upward force of $300\text{ N} - 150\text{ N} = 150\text{ N}$.
**EVALUATE:** Less force must be applied when it is applied farther from the hinges.

**11.5.** **IDENTIFY:** Apply $\sum \tau_z = 0$ to the ladder.

**SET UP:** Take the axis to be at point $A$. The free-body diagram for the ladder is given in Figure 11.5. The torque due to $F$ must balance the torque due to the weight of the ladder.
**EXECUTE:** $F(8.0\text{ m})\sin 40° = (3400\text{ N})(10.0\text{ m})$, so $F = 6.6\text{ kN}$.

**EVALUATE:** The force required is greater than the weight of the ladder, because the moment arm for $F$ is less than the moment arm for $w$.

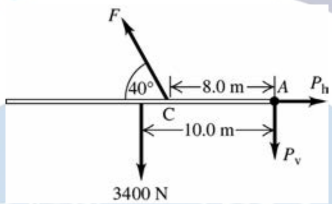



**Figure 11.5**

**11.6.** **IDENTIFY:** Apply the first and second conditions of equilibrium to the board.
**SET UP:** The free-body diagram for the board is given in Figure 11.6. Since the board is uniform its center of gravity is 1.50 m from each end. Apply $\sum F_y = 0,$ with $+y$ upward. Apply $\sum \tau_z = 0$ with the axis at the end where the first person applies a force and with counterclockwise torques positive.
**EXECUTE:** $\sum F_y = 0$ gives $F_1 + F_2 - w = 0$ and $F_2 = w - F_1 = 160\text{ N} - 60\text{ N} = 100\text{ N}.$ $\sum \tau_z = 0$ gives

$F_2 x - w(1.50\text{ m}) = 0$ and $x = \left(\dfrac{w}{F_2}\right)(1.50\text{ m}) = \left(\dfrac{160\text{ N}}{100\text{ N}}\right)(1.50\text{ m}) = 2.40\text{ m}.$ The other person lifts with a force of 100 N at a point 2.40 m from the end where the other person lifts.
**EVALUATE:** By considering the axis at the center of gravity we can see that a larger force is applied by the person who pushes closer to the center of gravity.

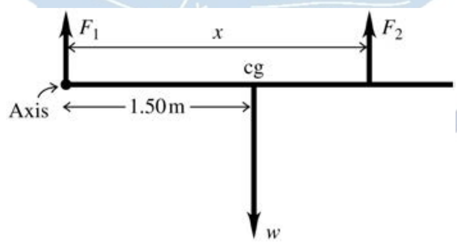



**Figure 11.6**

**11.7.** **IDENTIFY:** Apply $\sum F_y = 0$ and $\sum \tau_z = 0$ to the board.

**SET UP:** Let $+y$ be upward. Let $x$ be the distance of the center of gravity of the motor from the end of the board where the 400 N force is applied.
**EXECUTE:** **(a)** If the board is taken to be massless, the weight of the motor is the sum of the applied forces, $1000\text{ N}.$ The motor is a distance $\dfrac{(2.00\text{ m})(600\text{ N})}{(1000\text{ N})} = 1.20\text{ m}$ from the end where the 400 N force is applied, and so is 0.800 m from the end where the 600 N force is applied.
**(b)** The weight of the motor is $400\text{ N} + 600\text{ N} - 200\text{ N} = 800\text{ N}.$ Applying $\sum \tau_z = 0$ with the axis at the end of the board where the 400 N acts gives $(600\text{ N})(2.00\text{ m}) = (200\text{ N})(1.00\text{ m}) + (800\text{ N})x$ and $x = 1.25\text{ m}.$ The center of gravity of the motor is 0.75 m from the end of the board where the 600 N force is applied.
**EVALUATE:** The motor is closest to the end of the board where the larger force is applied.

**11.8.** **IDENTIFY:** Apply the first and second conditions of equilibrium to the shelf.
**SET UP:** The free-body diagram for the shelf is given in Figure 11.8. Take the axis at the left-hand end of the shelf and let counterclockwise torque be positive. The center of gravity of the uniform shelf is at its center.
**EXECUTE:** **(a)** $\sum \tau_z = 0$ gives $-w_t(0.200\text{ m}) - w_s(0.300\text{ m}) + T_2(0.400\text{ m}) = 0$.

$$T_2 = \frac{(25.0\text{ N})(0.200\text{ m}) + (50.0\text{ N})(0.300\text{ m})}{0.400\text{ m}} = 50.0\text{ N}$$

$\sum F_y = 0$ gives $T_1 + T_2 - w_t - w_s = 0$ and $T_1 = 25.0$ N. The tension in the left-hand wire is 25.0 N and the tension in the right-hand wire is 50.0 N.
**EVALUATE:** We can verify that $\sum \tau_z = 0$ is zero for any axis, for example for an axis at the right-hand end of the shelf.

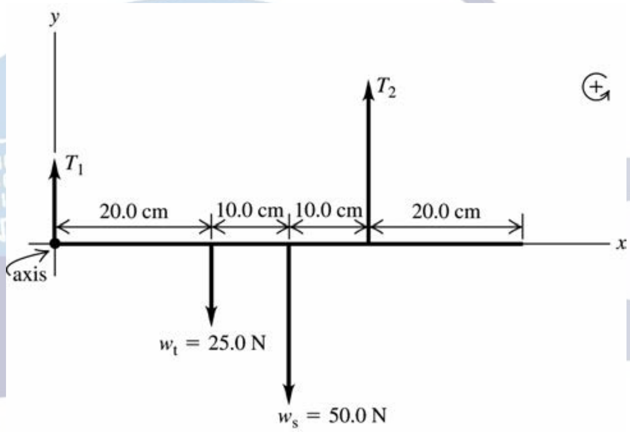



**Figure 11.8**

**11.9.** **IDENTIFY:** Apply the conditions for equilibrium to the bar. Set each tension equal to its maximum value.
**SET UP:** Let cable $A$ be at the left-hand end. Take the axis to be at the left-hand end of the bar and $x$ be the distance of the weight $w$ from this end. The free-body diagram for the bar is given in Figure 11.9.
**EXECUTE:** **(a)** $\sum F_y = 0$ gives $T_A + T_B - w - w_{\text{bar}} = 0$ and

$w = T_A + T_B - w_{\text{bar}} = 500.0\text{ N} + 400.0\text{ N} - 350.0\text{ N} = 550\text{ N}$.

**(b)** $\sum \tau_z = 0$ gives $T_B(1.50\text{ m}) - wx - w_{\text{bar}}(0.750\text{ m}) = 0$.

$x = \dfrac{T_B(1.50\text{ m}) - w_{\text{bar}}(0.750\text{ m})}{w} = \dfrac{(400.0\text{ N})(1.50\text{ m}) - (350\text{ N})(0.750\text{ m})}{550\text{ N}} = 0.614\text{ m}$. The weight should be placed 0.614 m from the left-hand end of the bar (cable $A$).
**EVALUATE:** If the weight is moved to the left, $T_A$ exceeds 500.0 N and if it is moved to the right $T_B$ exceeds 400.0 N.

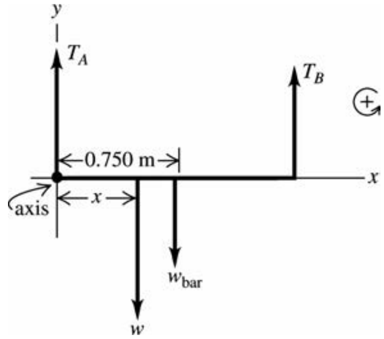



**Figure 11.9**

**11.10.** **IDENTIFY:** Apply the first and second conditions for equilibrium to the ladder.

**SET UP:** Let $n_2$ be the upward normal force exerted by the ground and let $n_1$ be the horizontal normal force exerted by the wall. The maximum possible static friction force that can be exerted by the ground is $\mu_s n_2$.

**EXECUTE:** **(a)** Since the wall is frictionless, the only vertical forces are the weights of the man and the ladder, and the normal force $n_2$. For the vertical forces to balance, $n_2 = w_1 + w_m = 160\text{ N} + 740\text{ N} = 900\text{ N}$, and the maximum frictional force is $\mu_s n_2 = (0.40)(900\text{ N}) = 360\text{ N}$.

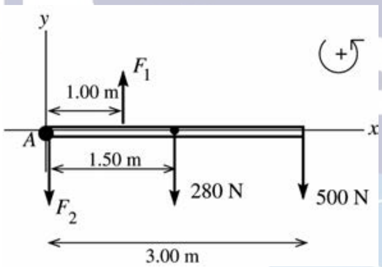
**(b)** Note that the ladder makes contact with the wall at a height of 4.0 m above the ground. Balancing torques about the point of contact with the ground,
$(4.0\text{ m})n_1 = (1.5\text{ m})(160\text{ N}) + (1.0\text{ m})(3/5)(740\text{ N}) = 684\text{ N}\cdot\text{m}$, so $n_1 = 171.0\text{ N}$. This horizontal force must be balanced by the friction force, which must then be 170 N to two figures.

**(c)** Setting the friction force, and hence $n_1$, equal to the maximum of 360 N and solving for the distance $x$ along the ladder, $(4.0\text{ m})(360\text{ N}) = (1.50\text{ m})(160\text{ N}) + x(3/5)(740\text{ N})$, so $x = 2.7\text{ m}$.

**EVALUATE:** The normal force exerted by the ground doesn't change as the man climbs up the ladder. But the normal force exerted by the wall and the friction force exerted by the ground both increase as he moves up the ladder.

**11.11.** **IDENTIFY:** The system of the person and diving board is at rest so the two conditions of equilibrium apply.

**(a) SET UP:** The free-body diagram for the diving board is given in Figure 11.11. Take the origin of coordinates at the left-hand end of the board (point $A$).




$\vec{F}_1$ is the force applied at the support point and $\vec{F}_2$ is the force at the end that is held down.

**Figure 11.11**

**EXECUTE:** $\sum\tau_A = 0$ gives $+F_1(1.0\text{ m}) - (500\text{ N})(3.00\text{ m}) - (280\text{ N})(1.50\text{ m}) = 0$

$$F_1 = \frac{(500\text{ N})(3.00\text{ m}) + (280\text{ N})(1.50\text{ m})}{1.00\text{ m}} = 1920\text{ N}$$

**(b)** $\sum F_y = ma_y$

$F_1 - F_2 - 280\text{ N} - 500\text{ N} = 0$

$F_2 = F_1 - 280\text{ N} - 500\text{ N} = 1920\text{ N} - 280\text{ N} - 500\text{ N} = 1140\text{ N}$

**EVALUATE:** We can check our answers by calculating the net torque about some point and checking that $\sum\tau_z = 0$ for that point also. Net torque about the right-hand end of the board:
$(1140\text{ N})(3.00\text{ m}) + (280\text{ N})(1.50\text{ m}) - (1920\text{ N})(2.00\text{ m}) = 3420\text{ N}\cdot\text{m} + 420\text{ N}\cdot\text{m} - 3840\text{ N}\cdot\text{m} = 0$, which checks.

**11.12.** **IDENTIFY:** Apply the first and second conditions of equilibrium to the beam.

**SET UP:** The boy exerts a downward force on the beam that is equal to his weight.

**EXECUTE:** **(a)** The graphs are given in Figure 11.12.

**(b)** $x = 6.25\text{ m}$ when $F_A = 0$, which is 1.25 m beyond point $B$.

**(c)** Take torques about the right end. When the beam is just balanced, $F_A = 0$, so $F_B = 900\text{ N}$.

The distance that point $B$ must be from the right end is then $\dfrac{(300\text{ N})(4.50\text{ m})}{(900\text{ N})} = 1.50\text{ m}$.

**EVALUATE:** When the beam is on the verge of tipping it starts to lift off the support $A$ and the normal force $F_A$ exerted by the support goes to zero.

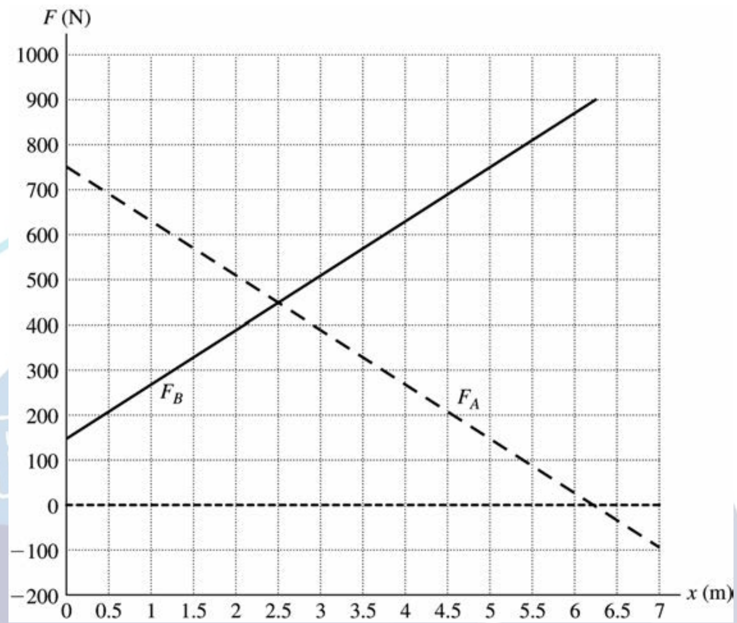



**Figure 11.12**

**11.13. IDENTIFY:** Apply the first and second conditions of equilibrium to the strut.

**(a) SET UP:** The free-body diagram for the strut is given in Figure 11.13a. Take the origin of coordinates at the hinge (point A) and $+y$ upward. Let $F_h$ and $F_v$ be the horizontal and vertical components of the force $\vec{F}$ exerted on the strut by the pivot. The tension in the vertical cable is the weight $w$ of the suspended object. The weight $w$ of the strut can be taken to act at the center of the strut. Let $L$ be the length of the strut.

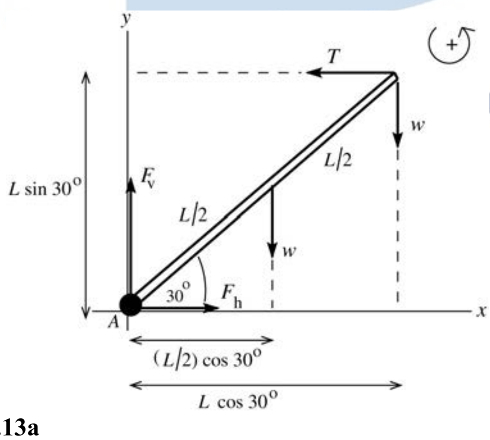



**Figure 11.13a**

**EXECUTE:**

$\sum F_y = ma_y$

$F_v - w - w = 0$

$F_v = 2w$

Sum torques about point $A$. The pivot force has zero moment arm for this axis and so doesn't enter into the torque equation.

$\tau_A = 0$

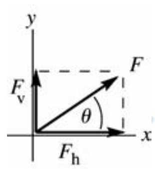
$TL\sin 30.0° - w((L/2)\cos 30.0°) - w(L\cos 30.0°) = 0$

$T\sin 30.0° - (3w/2)\cos 30.0° = 0$

$$T = \frac{3w\cos 30.0°}{2\sin 30.0°} = 2.60w$$

Then $\sum F_x = ma_x$ implies $T - F_{\text{h}} = 0$ and $F_{\text{h}} = 2.60w.$

We now have the components of $\vec{F}$ so can find its magnitude and direction (Figure 11.13b).




$$F = \sqrt{F_{\text{h}}^2 + F_{\text{v}}^2}$$

$$F = \sqrt{(2.60w)^2 + (2.00w)^2}$$

$$F = 3.28w$$

$$\tan\theta = \frac{F_{\text{v}}}{F_{\text{h}}} = \frac{2.00w}{2.60w}$$

$$\theta = 37.6°$$

**Figure 11.13b**

**(b) SET UP:** The free-body diagram for the strut is given in Figure 11.13c.

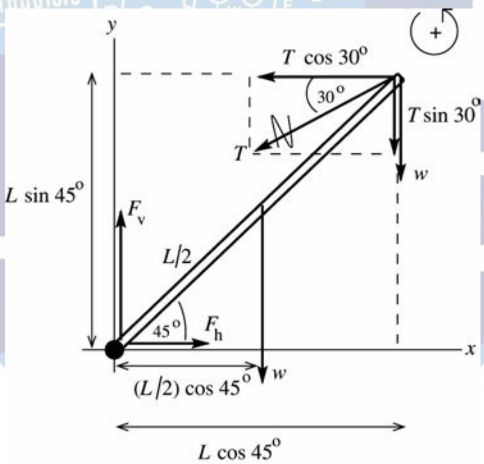



**Figure 11.13c**

The tension $T$ has been replaced by its $x$ and $y$ components. The torque due to $T$ equals the sum of the torques of its components, and the latter are easier to calculate.

**EXECUTE:** $\sum \tau_A = 0 + (T\cos 30.0°)(L\sin 45.0°) - (T\sin 30.0°)(L\cos 45.0°) -$

$$w\left[(L/2)\cos 45.0°\right] - w(L\cos 45.0°) = 0$$

The length $L$ divides out of the equation. The equation can also be simplified by noting that $\sin 45.0° = \cos 45.0°$.

Then $T(\cos 30.0° - \sin 30.0°) = 3w/2.$

$$T = \frac{3w}{2(\cos 30.0° - \sin 30.0°)} = 4.10w$$

$$\sum F_x = ma_x$$

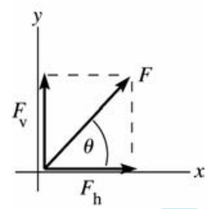
$$F_{\text{h}} - T\cos 30.0° = 0$$

$$F_{\text{h}} = T\cos 30.0° = (4.10w)(\cos 30.0°) = 3.55w$$

$$\sum F_y = ma_y$$

$$F_{\text{v}} - w - w - T\sin 30.0° = 0$$

$F_v = 2w + (4.10w)\sin 30.0° = 4.05w$




From Figure 11.13d,

$F = \sqrt{F_h^2 + F_v^2}$

$F = \sqrt{(3.55w)^2 + (4.05w)^2} = 5.39w$

$\tan\theta = \frac{F_v}{F_h} = \frac{4.05w}{3.55w}$

$\theta = 48.8°$

**Figure 11.13d**

**EVALUATE:** In each case the force exerted by the pivot does not act along the strut. Consider the net torque about the upper end of the strut. If the pivot force acted along the strut, it would have zero torque about this point. The two forces acting at this point also have zero torque and there would be one nonzero torque, due to the weight of the strut. The net torque about this point would then not be zero, violating the second condition of equilibrium.

**11.14. IDENTIFY:** Apply the first and second conditions of equilibrium to the beam.

**SET UP:** The free-body diagram for the beam is given in Figure 11.14. $H_v$ and $H_h$ are the vertical and horizontal components of the force exerted on the beam at the wall (by the hinge). Since the beam is uniform, its center of gravity is 2.00 m from each end. The angle $\theta$ has $\cos\theta = 0.800$ and $\sin\theta = 0.600$. The tension *T* has been replaced by its *x*- and *y*-components.

**EXECUTE: (a)** $H_v$, $H_h$ and $T_x = T\cos\theta$ all produce zero torque. $\sum\tau_z = 0$ gives $-w(2.00\text{ m}) - w_{load}(4.00\text{ m}) + T\sin\theta(4.00\text{ m}) = 0$ and

$$T = \frac{(190\text{ N})(2.00\text{ m}) + (300\text{ N})(4.00\text{ m})}{(4.00\text{ m})(0.600)} = 658.3\text{ N},$$ which rounds to 658 N.

**(b)** $\sum F_x = 0$ gives $H_h - T\cos\theta = 0$ and $H_h = (658.3\text{ N})(0.800) = 527\text{ N}$. $\sum F_y = 0$ gives $H_v - w - w_{load} + T\sin\theta = 0$ and $H_v = w + w_{load} - T\sin\theta = 190\text{ N} + 300\text{ N} - (658\text{ N})(0.600) = 95\text{ N}$.

**EVALUATE:** For an axis at the right-hand end of the beam, only *w* and $H_v$ produce torque. The torque due to *w* is counterclockwise so the torque due to $H_v$ must be clockwise. To produce a clockwise torque, $H_v$ must be upward, in agreement with our result from $\sum F_y = 0$.

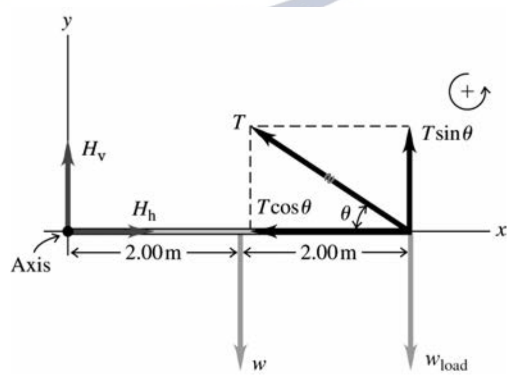



**Figure 11.14**

**11.15. IDENTIFY:** The boom is at rest, so the forces and torques on it must each balance.

**SET UP:** $\sum\tau = 0$, $\sum F_x = 0$, $\sum F_y = 0$. The free-body is shown in Figure 11.15 (next page). Call *L* the length of the boom.

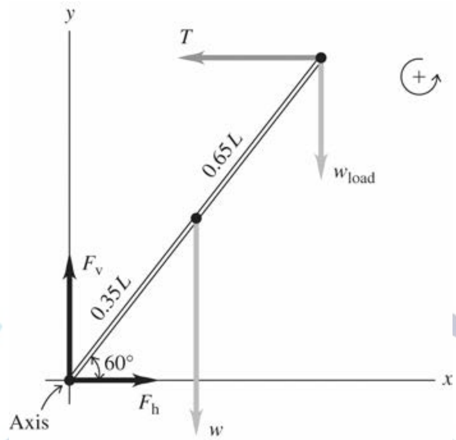



**Figure 11.15**

**EXECUTE: (a)** $\sum\tau = 0$ gives $T(L\sin 60.0°) - w_{load}(L\cos 60.0°) - w(0.35L\cos 60.0°) = 0$ and

$$T = \frac{w_{load}\cos 60.0° + w(0.35\cos 60.0°)}{\sin 60.0°} = \frac{(5000\text{ N})\cos 60.0° + (2600\text{ N})(0.35\cos 60.0°)}{\sin 60.0°} = 3.41\times 10^3\text{ N}.$$

**(b)** $\sum F_x = 0$ gives $F_h - T = 0$ and $F_h = 3410$ N.

$\sum F_y = 0$ gives $F_v - w - w_{load} = 0$ and $F_v = 5000\text{ N} + 2600\text{ N} = 7600\text{ N}$

**EVALUATE:** The bottom of the boom is the best point about which to take torques because only one unknown (the tension) appears in our equation. Using the top (or the center of mass) would give a torque equation with two (or three) unknowns.

**11.16. IDENTIFY:** Apply the conditions of equilibrium to the wheelbarrow plus its contents. The upward force applied by the person is 650 N.

**SET UP:** The free-body diagram for the wheelbarrow is given in Figure 11.16. $F = 650$ N, $w_{wb} = 80.0$ N and $w$ is the weight of the load placed in the wheelbarrow.

**EXECUTE: (a)** $\sum\tau_z = 0$ with the axis at the center of gravity gives $n(0.50\text{ m}) - F(0.90\text{ m}) = 0$ and

$n = F\left(\frac{0.90\text{ m}}{0.50\text{ m}}\right) = 1170$ N. $\sum F_y = 0$ gives $F + n - w_{wb} - w = 0$ and

$w = F + n - w_{wb} = 650\text{ N} + 1170\text{ N} - 80.0\text{ N} = 1740\text{ N}.$

**(b)** The extra force is applied by the ground pushing up on the wheel.

**EVALUATE:** You can verify that $\sum\tau_z = 0$ for any axis, for example for an axis where the wheel contacts the ground.

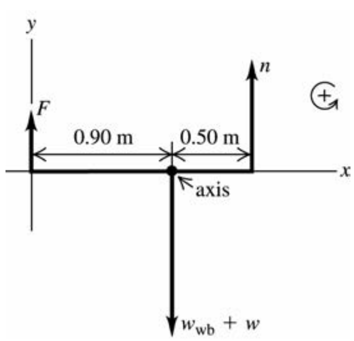



**Figure 11.16**

**11.17. IDENTIFY:** The beam is at rest so the forces and torques on it must each balance.

**SET UP:** $\sum\tau = 0,\ \sum F_x = 0,\ \sum F_y = 0.$ The distance along the beam from the hinge to where the cable is attached is 3.0 m. The angle $\phi$ that the cable makes with the beam is given by $\sin\phi = \frac{4.0\text{ m}}{5.0\text{ m}}$, so $\phi = 53.1°$. The center of gravity of the beam is 4.5 m from the hinge. Use coordinates with $+y$ upward and $+x$ to the right. Take the pivot at the hinge and let counterclockwise torque be positive. Express the hinge force as components $H_v$ and $H_h$. Assume $H_v$ is downward and that $H_h$ is to the right. If one of these components is actually in the opposite direction we will get a negative value for it. Set the tension in the cable equal to its maximum possible value, $T = 1.00$ kN.

**EXECUTE:** **(a)** The free-body diagram is shown in Figure 11.17, with $\vec{T}$ resolved into its *x*- and *y*-components.

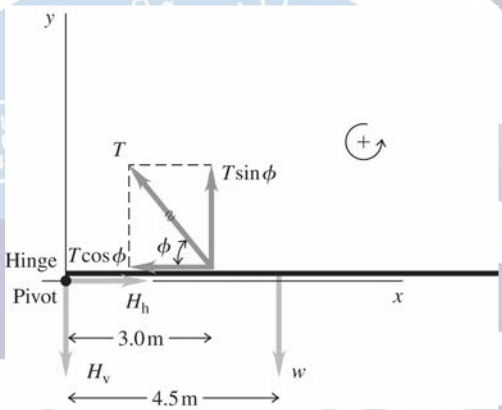



**Figure 11.17**

**(b)** $\sum\tau = 0$ gives $(T\sin\phi)(3.0\text{ m}) - w(4.5\text{ m}) = 0$

$$w = \frac{(T\sin\phi)(3.00\text{ m})}{4.50\text{ m}} = \frac{(1000\text{ N})(\sin 53.1°)(3.00\text{ m})}{4.50\text{ m}} = 533\text{ N}$$

**(c)** $\sum F_x = 0$ gives $H_h - T\cos\phi = 0$ and $H_h = (1.00\text{ kN})(\cos 53.1°) = 600\text{ N}$

$\sum F_y = 0$ gives $T\sin\phi - H_v - w = 0$ and $H_v = (1.00\text{ kN})(\sin 53.1°) - 533\text{ N} = 267\text{ N}.$

**EVALUATE:** $T\cos\phi$, $H_v$ and $H_h$ all have zero moment arms for a pivot at the hinge and therefore produce zero torque. If we consider a pivot at the point where the cable is attached we can see that $H_v$ must be downward to produce a torque that opposes the torque due to $w$.

**11.18. IDENTIFY:** Apply the conditions for equilibrium to the crane.

**SET UP:** The free-body diagram for the crane is sketched in Figure 11.18 (next page). $F_h$ and $F_v$ are the components of the force exerted by the axle. $\vec{T}$ pulls to the left so $F_h$ is to the right. $\vec{T}$ also pulls downward and the two weights are downward, so $F_v$ is upward.

**EXECUTE:** **(a)** $\sum\tau_z = 0$ gives $T([13\text{ m}]\sin 25°) - w_c([7.0\text{ m}]\cos 55°) - w_b([16.0\text{ m}]\cos 55°) = 0.$

$$T = \frac{(11{,}000\text{ N})([16.0\text{ m}]\cos 55°) + (15{,}000\text{ N})([7.0\text{ m}]\cos 55°)}{(13.0\text{ m})\sin 25°} = 2.93\times10^4\text{ N}.$$

**(b)** $\sum F_x = 0$ gives $F_h - T\cos 30° = 0$ and $F_h = 2.54\times10^4$ N.

$\sum F_y = 0$ gives $F_v - T\sin 30° - w_c - w_b = 0$ and $F_v = 4.06\times10^4$ N.

**EVALUATE:** $\tan\theta = \frac{F_v}{F_h} = \frac{4.06\times10^4\text{ N}}{2.54\times10^4\text{ N}}$ and $\theta = 58°$. The force exerted by the axle is not directed along the crane.

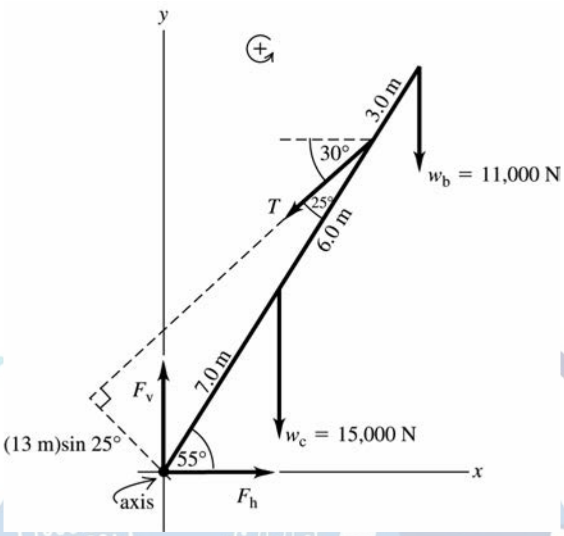



**Figure 11.18**

**11.19. IDENTIFY:** Apply the first and second conditions of equilibrium to the rod.

**SET UP:** The force diagram for the rod is given in Figure 11.19.

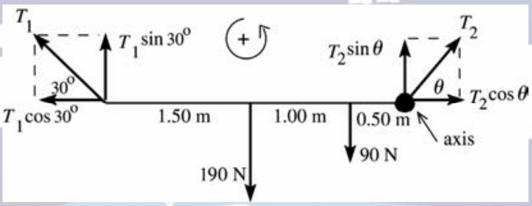



**Figure 11.19**

**EXECUTE:** $\sum \tau_z = 0,$ axis at right end of rod, counterclockwise torque is positive

$(190\text{ N})(1.50\text{ m}) + (90\text{ N})(0.50\text{ m}) - (T_1 \sin 30.0^\circ)(3.00\text{ m}) = 0$

$T_1 = \dfrac{285\text{ N}\cdot\text{m} + 45\text{ N}\cdot\text{m}}{1.50\text{ m}} = 220\text{ N}$

$\sum F_x = ma_x$

$T_2 \cos\theta - T_1 \cos 30^\circ = 0$ and $T_2 \cos\theta = (220\text{ N})(\cos 30^\circ) = 190.5\text{ N}$

$\sum F_y = ma_y$

$T_1 \sin 30^\circ + T_2 \sin\theta - 190\text{ N} - 90\text{ N} = 0$

$T_2 \sin\theta = 280\text{ N} - (220\text{ N})\sin 30^\circ = 170\text{ N}$

Then $\dfrac{T_2 \sin\theta}{T_2 \cos\theta} = \dfrac{170\text{ N}}{190.5\text{ N}}$ gives $\tan\theta = 0.89239$ and $\theta = 41.7^\circ$

And $T_2 = \dfrac{170\text{ N}}{\sin 41.7^\circ} = 255\text{ N}.$

**EVALUATE:** The monkey is closer to the right rope than to the left one, so the tension is larger in the right rope. The horizontal components of the tensions must be equal in magnitude and opposite in direction. Since $T_2 > T_1,$ the rope on the right must be at a greater angle above the horizontal to have the same horizontal component as the tension in the other rope.

**11.20.** **IDENTIFY:** Apply the first and second conditions for equilibrium to the beam.
**SET UP:** The free-body diagram for the beam is given in Figure 11.20.
**EXECUTE:** The cable is given as perpendicular to the beam, so the tension is found by taking torques about the pivot point; $T(3.00\text{ m}) = (1.40\text{ kN})(2.00\text{ m})\cos 25.0° + (5.00\text{ kN})(4.50\text{ m})\cos 25.0°$, and $T = 7.64\text{ kN}$. The vertical component of the force exerted on the beam by the pivot is the net weight minus the upward component of $T$, $6.00\text{ kN} - T\cos 25.0° = -0.53\text{ kN}$. The vertical component is downward. The horizontal force is $T\sin 25.0° = 3.23\text{ kN}$.
**EVALUATE:** The vertical component of the tension is nearly the same magnitude as the total weight of the object and the vertical component of the force exerted by the pivot is much less than its horizontal component.

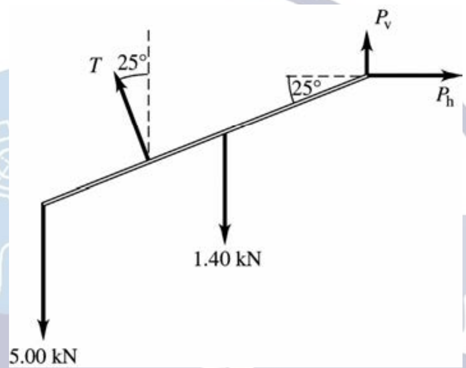



**Figure 11.20**

**11.21.** **(a) IDENTIFY** and **SET UP:** Use $\tau = Fl$ to calculate the torque (magnitude and direction) for each force and add the torques as vectors. See Figure 11.21a.

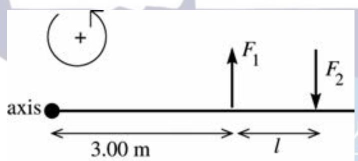



**Figure 11.21a**

**EXECUTE:**

$\tau_1 = F_1 l_1 = +(8.00\text{ N})(3.00\text{ m})$

$\tau_1 = +24.0\text{ N}\cdot\text{m}$

$\tau_2 = -F_2 l_2 = -(8.00\text{ N})(l + 3.00\text{ m})$

$\tau_2 = -24.0\text{ N}\cdot\text{m} - (8.00\text{ N})l$

$\sum\tau_z = \tau_1 + \tau_2 = +24.0\text{ N}\cdot\text{m} - 24.0\text{ N}\cdot\text{m} - (8.00\text{ N})l = -(8.00\text{ N})l$

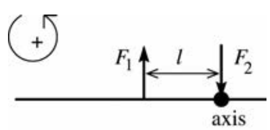
Want $l$ that makes $\sum\tau_z = -6.40\text{ N}\cdot\text{m}$ (net torque must be clockwise)

$-(8.00\text{ N})l = -6.40\text{ N}\cdot\text{m}$

$l = (6.40\text{ N}\cdot\text{m})/8.00\text{ N} = 0.800\text{ m}$

**(b)** $|\tau_2| > |\tau_1|$ since $F_2$ has a larger moment arm; the net torque is clockwise.

**(c)** See Figure 11.21b.




$\tau_1 = -F_1 l_1 = -(8.00\text{ N})l$

$\tau_2 = 0$ since $\vec{F}_2$ is at the axis

**Figure 11.21b**

$\sum\tau_z = -6.40\text{ N}\cdot\text{m}$ gives $-(8.00\text{ N})l = -6.40\text{ N}\cdot\text{m}$

$l = 0.800\text{ m}$, same as in part (a).

**EVALUATE:** The force couple gives the same magnitude of torque for the pivot at any point.

**11.22.** **IDENTIFY:** The person is in equilibrium, so the torques on him must balance. The target variable is the force exerted by the deltoid muscle.

**SET UP:** The free-body diagram for the arm is given in Figure 11.22. Take the pivot at the shoulder joint and let counterclockwise torques be positive. Use coordinates as shown. Let $F$ be the force exerted by the deltoid muscle. There are also the weight of the arm and forces at the shoulder joint, but none of these forces produce any torque when the arm is in this position. The forces $F$ and $T$ have been replaced by their $x$- and $y$-components. $\sum \tau_z = 0$.

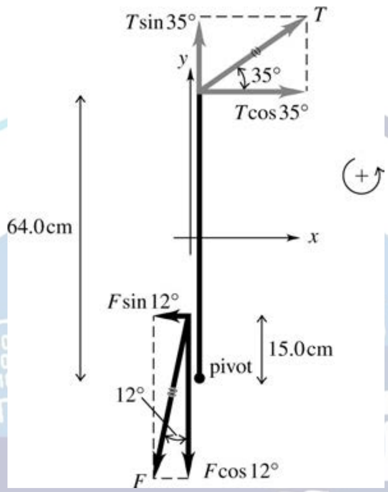



**Figure 11.22**

**EXECUTE:** $\sum \tau_z = 0$ gives $(F\sin 12.0°)(15.0 \text{ cm}) - (T\cos 35°)(64.0 \text{ cm}) = 0$.

$$F = \frac{(36.0 \text{ N})(\cos 35°)(64.0 \text{ cm})}{(\sin 12.0°)(15.0 \text{ cm})} = 605 \text{ N}.$$

**EVALUATE:** The force exerted by the deltoid muscle is much larger than the tension in the cable because the deltoid muscle makes a small angle (only 12.0°) with the humerus.

**11.23. IDENTIFY:** The student's head is at rest, so the torques on it must balance. The target variable is the tension in her neck muscles.

**SET UP:** Let the pivot be at point $P$ and let counterclockwise torques be positive. $\sum \tau_z = 0$.

**EXECUTE:** **(a)** The free-body diagram is given in Figure 11.23.

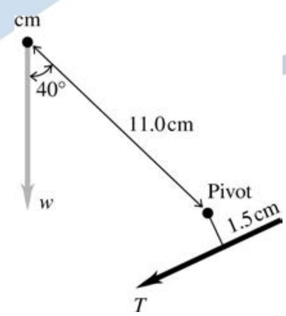



**Figure 11.23**

**(b)** $\sum \tau_z = 0$ gives $w(11.0 \text{ cm})(\sin 40.0°) - T(1.50 \text{ cm}) = 0$.

$$T = \frac{(4.50 \text{ kg})(9.80 \text{ m/s}^2)(11.0 \text{ cm})\sin 40.0°}{1.50 \text{ cm}} = 208 \text{ N}.$$

**EVALUATE:** Her head weighs about 45 N but the tension in her neck muscles must be much larger because the tension has a small moment arm.

**11.24. IDENTIFY:** Use $Y = \dfrac{l_0 F_\perp}{A\Delta l}$.

**SET UP:** $A = 50.0\ \text{cm}^2 = 50.0\times10^{-4}\ \text{m}^2$.

**EXECUTE:** relaxed: $Y = \dfrac{(0.200\ \text{m})(25.0\ \text{N})}{(50.0\times10^{-4}\ \text{m}^2)(3.0\times10^{-2}\ \text{m})} = 3.33\times10^4\ \text{Pa}$

maximum tension: $Y = \dfrac{(0.200\ \text{m})(500\ \text{N})}{(50.0\times10^{-4}\ \text{m}^2)(3.0\times10^{-2}\ \text{m})} = 6.67\times10^5\ \text{Pa}$

**EVALUATE:** The muscle tissue is much more difficult to stretch when it is under maximum tension.

**11.25.** **IDENTIFY** and **SET UP:** Apply $Y = \dfrac{l_0F_\perp}{A\Delta l}$ and solve for $A$ and then use $A = \pi r^2$ to get the radius and $d = 2r$ to calculate the diameter.

**EXECUTE:** $Y = \dfrac{l_0F_\perp}{A\Delta l}$ so $A = \dfrac{l_0F_\perp}{Y\Delta l}$ ($A$ is the cross-section area of the wire)

For steel, $Y = 2.0\times10^{11}$ Pa (Table 11.1)

Thus $A = \dfrac{(2.00\ \text{m})(700\ \text{N})}{(2.0\times10^{11}\ \text{Pa})(0.25\times10^{-2}\ \text{m})} = 2.8\times10^{-6}\ \text{m}^2$.

$A = \pi r^2$, so $r = \sqrt{A/\pi} = \sqrt{2.8\times10^{-6}\ \text{m}^2/\pi} = 9.44\times10^{-4}\ \text{m}$

$d = 2r = 1.9\times10^{-3}\ \text{m} = 1.9\ \text{mm}$.

**EVALUATE:** Steel wire of this diameter doesn't stretch much; $\Delta l/l_0 = 0.12\%$.

**11.26.** **IDENTIFY:** Apply $Y = \dfrac{l_0F_\perp}{A\Delta l}$.

**SET UP:** From Table 11.1, for steel, $Y = 2.0\times10^{11}$ Pa and for copper, $Y = 1.1\times10^{11}$ Pa. $A = \pi(d^2/4) = 1.77\times10^{-4}\ \text{m}^2$. $F_\perp = 4000$ N for each rod.

**EXECUTE:** **(a)** The strain is $\dfrac{\Delta l}{l_0} = \dfrac{F}{YA}$. For steel $\dfrac{\Delta l}{l_0} = \dfrac{(4000\ \text{N})}{(2.0\times10^{11}\ \text{Pa})(1.77\times10^{-4}\ \text{m}^2)} = 1.1\times10^{-4}$.

Similarly, the strain for copper is $2.1\times10^{-4}$.

**(b)** Steel: $(1.1\times10^{-4})(0.750\ \text{m}) = 8.3\times10^{-5}\ \text{m}$. Copper: $(2.1\times10^{-4})(0.750\ \text{m}) = 1.6\times10^{-4}\ \text{m}$.

**EVALUATE:** Copper has a smaller $Y$ and therefore a greater elongation.

**11.27.** **IDENTIFY:** Apply $Y = \dfrac{l_0F_\perp}{A\Delta l}$.

**SET UP:** $A = 0.50\ \text{cm}^2 = 0.50\times10^{-4}\ \text{m}^2$

**EXECUTE:** $Y = \dfrac{(4.00\ \text{m})(5000\ \text{N})}{(0.50\times10^{-4}\ \text{m}^2)(0.20\times10^{-2}\ \text{m})} = 2.0\times10^{11}\ \text{Pa}$

**EVALUATE:** Our result is the same as that given for steel in Table 11.1.

**11.28.** **IDENTIFY:** Apply $Y = \dfrac{l_0F_\perp}{A\Delta l}$.

**SET UP:** $A = \pi r^2 = \pi(3.5\times10^{-3}\ \text{m})^2 = 3.85\times10^{-5}\ \text{m}^2$. The force applied to the end of the rope is the weight of the climber: $F_\perp = (65.0\ \text{kg})(9.80\ \text{m/s}^2) = 637\ \text{N}$.

**EXECUTE:** $Y = \dfrac{(45.0\ \text{m})(637\ \text{N})}{(3.85\times10^{-5}\ \text{m}^2)(1.10\ \text{m})} = 6.77\times10^8\ \text{Pa}$

**EVALUATE:** Our result is a lot smaller than the values given in Table 11.1. An object made of rope material is much easier to stretch than if the object were made of metal.

**11.29.** **IDENTIFY:** Use the first condition of equilibrium to calculate the tensions $T_1$ and $T_2$ in the wires (Figure 11.29a, next page). Then use Eq. (11.10) to calculate the strain and elongation of each wire.

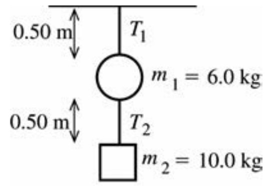
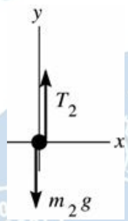



**Figure 11.29a**

**SET UP:** The free-body diagram for $m_2$ is given in Figure 11.27b.




**EXECUTE:**

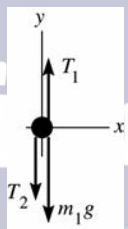
$\sum F_y = ma_y$

$T_2 - m_2 g = 0$

$T_2 = 98.0 \text{ N}$

**Figure 11.29b**

**SET UP:** The free-body-diagram for $m_1$ is given in Figure 11.29c.




**EXECUTE:**

$\sum F_y = ma_y$

$T_1 - T_2 - m_1 g = 0$

$T_1 = T_2 + m_1 g$

$T_1 = 98.0 \text{ N} + 58.8 \text{ N} = 157 \text{ N}$

**Figure 11.29c**

**(a)** $Y = \dfrac{\text{stress}}{\text{strain}}$ so $\text{strain} = \dfrac{\text{stress}}{Y} = \dfrac{F_\perp}{AY}$

upper wire: $\text{strain} = \dfrac{T_1}{AY} = \dfrac{157 \text{ N}}{(2.5\times10^{-7} \text{ m}^2)(2.0\times10^{11} \text{ Pa})} = 3.1\times10^{-3}$

lower wire: $\text{strain} = \dfrac{T_2}{AY} = \dfrac{98 \text{ N}}{(2.5\times10^{-7} \text{ m}^2)(2.0\times10^{11} \text{ Pa})} = 2.0\times10^{-3}$

**(b)** $\text{strain} = \Delta l / l_0$ so $\Delta l = l_0(\text{strain})$

upper wire: $\Delta l = (0.50 \text{ m})(3.1\times10^{-3}) = 1.6\times10^{-3} \text{ m} = 1.6 \text{ mm}$

lower wire: $\Delta l = (0.50 \text{ m})(2.0\times10^{-3}) = 1.0\times10^{-3} \text{ m} = 1.0 \text{ mm}$

**EVALUATE:** The tension is greater in the upper wire because it must support both objects. The wires have the same length and diameter, so the one with the greater tension has the greater strain and elongation.

**11.30. IDENTIFY:** Apply $\text{stress} = \dfrac{F_\perp}{A}$, $\text{strain} = \dfrac{\text{stress}}{Y}$, $Y = \dfrac{l_0 F_\perp}{A\Delta l}$.

**SET UP:** The cross-sectional area of the post is $A = \pi r^2 = \pi(0.125 \text{ m})^2 = 0.0491 \text{ m}^2$. The force applied to the end of the post is $F_\perp = (8000 \text{ kg})(9.80 \text{ m/s}^2) = 7.84\times10^4 \text{ N}$. The Young's modulus of steel is $Y = 2.0\times10^{11}$ Pa.

**EXECUTE:** **(a)** $\text{stress} = \dfrac{F_\perp}{A} = -\dfrac{7.84\times10^4 \text{ N}}{0.0491 \text{ m}^2} = -1.60\times10^6 \text{ Pa}$. The minus sign indicates that the stress is compressive.

**(b)** strain $= \dfrac{\text{stress}}{Y} = -\dfrac{1.60\times10^{6}\ \text{Pa}}{2.0\times10^{11}\ \text{Pa}} = -8.0\times10^{-6}$. The minus sign indicates that the length decreases.

**(c)** $\Delta l = l_0(\text{strain}) = (2.50\ \text{m})(-8.0\times10^{-6}) = -2.0\times10^{-5}\ \text{m}$

**EVALUATE:** The fractional change in length of the post is very small.

**11.31. IDENTIFY:** The amount of compression depends on the bulk modulus of the bone.

**SET UP:** $\dfrac{\Delta V}{V_0} = -\dfrac{\Delta p}{B}$ and $1\ \text{atm} = 1.01\times10^{5}\ \text{Pa}$.

**EXECUTE:** **(a)** $\Delta p = -B\dfrac{\Delta V}{V_0} = -(15\times10^{9}\ \text{Pa})(-0.0010) = 1.5\times10^{7}\ \text{Pa} = 150\ \text{atm}$.

**(b)** The depth for a pressure increase of $1.5\times10^{7}\ \text{Pa}$ is 1.5 km.

**EVALUATE:** An extremely large pressure increase is needed for just a 0.10% bone compression, so pressure changes do not appreciably affect the bones. Unprotected dives do not approach a depth of 1.5 km, so bone compression is not a concern for divers.

**11.32. IDENTIFY:** Apply $\dfrac{\Delta V}{V_0} = -\dfrac{\Delta p}{B}$.

**SET UP:** $\Delta V = -\dfrac{V_0 \Delta p}{B}$. $\Delta p$ is positive when the pressure increases.

**EXECUTE:** **(a)** The volume would increase slightly.
**(b)** The volume change would be twice as great.
**(c)** The volume change is inversely proportional to the bulk modulus for a given pressure change, so the volume change of the lead ingot would be four times that of the gold.

**EVALUATE:** For lead, $B = 4.1\times10^{10}\ \text{Pa}$, so $\Delta p/B$ is very small and the fractional change in volume is very small.

**11.33. IDENTIFY and SET UP:** Use $\dfrac{\Delta V}{V_0} = -\dfrac{\Delta p}{B}$ and $k = 1/B$ to calculate $B$ and $k$.

**EXECUTE:** $B = -\dfrac{\Delta p}{\Delta V/V_0} = -\dfrac{(3.6\times10^{6}\ \text{Pa})(600\ \text{cm}^3)}{(-0.45\ \text{cm}^3)} = +4.8\times10^{9}\ \text{Pa}$

$k = 1/B = 1/4.8\times10^{9}\ \text{Pa} = 2.1\times10^{-10}\ \text{Pa}^{-1}$

**EVALUATE:** $k$ is the same as for glycerine (Table 11.2).

**11.34. IDENTIFY:** Apply $\dfrac{\Delta V}{V_0} = -\dfrac{\Delta p}{B}$. Density $= m/V$.

**SET UP:** At the surface the pressure is $1.0\times10^{5}\ \text{Pa}$, so $\Delta p = 1.16\times10^{8}\ \text{Pa}$. $V_0 = 1.00\ \text{m}^3$. At the surface $1.00\ \text{m}^3$ of water has mass $1.03\times10^{3}\ \text{kg}$.

**EXECUTE:** **(a)** $B = -\dfrac{(\Delta p)V_0}{\Delta V}$ gives $\Delta V = -\dfrac{(\Delta p)V_0}{B} = -\dfrac{(1.16\times10^{8}\ \text{Pa})(1.00\ \text{m}^3)}{2.2\times10^{9}\ \text{Pa}} = -0.0527\ \text{m}^3$

**(b)** At this depth $1.03\times10^{3}\ \text{kg}$ of seawater has volume $V_0 + \Delta V = 0.9473\ \text{m}^3$. The density is

$\dfrac{1.03\times10^{3}\ \text{kg}}{0.9473\ \text{m}^3} = 1.09\times10^{3}\ \text{kg/m}^3$.

**EVALUATE:** The density is increased because the volume is compressed due to the increased pressure.

**11.35. IDENTIFY:** The forces on the cube must balance. The deformation $x$ is related to the force by $S = \dfrac{F_{\parallel}}{A}\dfrac{h}{x}$.

$F_{\parallel} = F$ since $F$ is applied parallel to the upper face.

**SET UP:** $A = (0.0600\ \text{m})^2$ and $h = 0.0600\ \text{m}$. Table 11.1 gives $S = 4.4\times10^{10}\ \text{Pa}$ for copper and $0.6\times10^{10}\ \text{Pa}$ for lead.

**EXECUTE:** **(a)** Since the horizontal forces balance, the glue exerts a force $F$ in the opposite direction.

**(b)** $F = \dfrac{AxS}{h} = \dfrac{(0.0600\text{ m})^2(0.250\times10^{-3}\text{ m})(4.4\times10^{10}\text{ Pa})}{0.0600\text{ m}} = 6.6\times10^5\text{ N}$

**(c)** $x = \dfrac{Fh}{AS} = \dfrac{(6.6\times10^5\text{ N})(0.0600\text{ m})}{(0.0600\text{ m})^2(0.6\times10^{10}\text{ Pa})} = 1.8\text{ mm}$

**EVALUATE:** Lead has a smaller $S$ than copper, so the lead cube has a greater deformation than the copper cube.

**11.36. IDENTIFY:** Apply $S = \dfrac{F_\parallel}{A}\dfrac{h}{x}$.

**SET UP:** $F_\parallel = 9.0\times10^5$ N. $A = (0.100\text{ m})(0.500\times10^{-2}\text{ m})$. $h = 0.100$ m. From Table 11.1, $S = 7.5\times10^{10}$ Pa for steel.

**EXECUTE:** **(a)** Shear strain $= \dfrac{F_\parallel}{AS} = \dfrac{(9\times10^5\text{ N})}{[(0.100\text{ m})(0.500\times10^{-2}\text{ m})][7.5\times10^{10}\text{ Pa}]} = 2.4\times10^{-2}$.

**(b)** Since shear strain $= x/h$, $x = (\text{Shear strain})\cdot h = (0.024)(0.100\text{ m}) = 2.4\times10^{-3}$ m.

**EVALUATE:** This very large force produces a small displacement; $x/h = 2.4\%$.

**11.37. IDENTIFY:** The force components parallel to the face of the cube produce a shear which can deform the cube.

**SET UP:** $S = \dfrac{F_\text{P}}{A\phi}$, where $\phi = x/h$. $F_\parallel$ is the component of the force tangent to the surface, so $F_\parallel = (1375\text{ N})\cos 8.50° = 1360$ N. $\phi$ must be in radians, $\phi = 1.24° = 0.0216$ rad.

**EXECUTE:** $S = \dfrac{1360\text{ N}}{(0.0925\text{ m})^2(0.0216\text{ rad})} = 7.36\times10^6$ Pa.

**EVALUATE:** The shear modulus of this material is much less than the values for metals given in Table 11.1 in the text.

**11.38. IDENTIFY:** The breaking stress of the wire is the value of $F_\perp/A$ at which the wire breaks.

**SET UP:** From Table 11.3, the breaking stress of brass is $4.7\times10^8$ Pa. The area $A$ of the wire is related to its diameter by $A = \pi d^2/4$.

**EXECUTE:** $A = \dfrac{350\text{ N}}{4.7\times10^8\text{ Pa}} = 7.45\times10^{-7}\text{ m}^2$, so $d = \sqrt{4A/\pi} = 0.97$ mm.

**EVALUATE:** The maximum force a wire can withstand without breaking is proportional to the square of its diameter.

**11.39. IDENTIFY and SET UP:** Use stress $= \dfrac{F_\perp}{A}$.

**EXECUTE:** Tensile stress $= \dfrac{F_\perp}{A} = \dfrac{F_\perp}{\pi r^2} = \dfrac{90.8\text{ N}}{\pi(0.92\times10^{-3}\text{ m})^2} = 3.41\times10^7$ Pa

**EVALUATE:** A modest force produces a very large stress because the cross-sectional area is small.

**11.40. IDENTIFY:** The proportional limit and breaking stress are values of the stress, $F_\perp/A$. Use $Y = \dfrac{l_0 F_\perp}{A\Delta l}$ to calculate $\Delta l$.

**SET UP:** For steel, $Y = 20\times10^{10}$ Pa. $F_\perp = w$.

**EXECUTE:** **(a)** $w = (1.6\times10^{-3})(20\times10^{10}\text{ Pa})(5\times10^{-6}\text{ m}^2) = 1.60\times10^3$ N.

**(b)** $\Delta l = \left(\dfrac{F_\perp}{A}\right)\dfrac{l_0}{Y} = (1.6\times10^{-3})(4.0\text{ m}) = 6.4$ mm

**(c)** $(6.5\times10^{-3})(20\times10^{10}\text{ Pa})(5\times10^{-6}\text{ m}^2) = 6.5\times10^3$ N.

**EVALUATE:** At the proportional limit, the fractional change in the length of the wire is 0.16%.

**11.41. IDENTIFY:** The elastic limit is a value of the stress, $F_\perp/A$. Apply $\sum\vec{F} = m\vec{a}$ to the elevator in order to find the tension in the cable.

**SET UP:** $\dfrac{F_\perp}{A} = \frac{1}{3}(2.40\times10^8\ \text{Pa}) = 0.80\times10^8\ \text{Pa}$. The free-body diagram for the elevator is given in Figure 11.41. $F_\perp$ is the tension in the cable.

**EXECUTE:** $F_\perp = A(0.80\times10^8\ \text{Pa}) = (3.00\times10^{-4}\ \text{m}^2)(0.80\times10^8\ \text{Pa}) = 2.40\times10^4\ \text{N}$. $\sum F_y = ma_y$ applied to the elevator gives $F_\perp - mg = ma$ and $a = \dfrac{F_\perp}{m} - g = \dfrac{2.40\times10^4\ \text{N}}{1200\ \text{kg}} - 9.80\ \text{m/s}^2 = 10.2\ \text{m/s}^2$

**EVALUATE:** The tension in the cable is about twice the weight of the elevator.

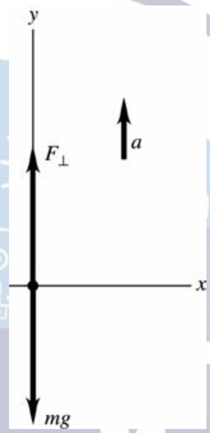



**Figure 11.41**

**11.42. IDENTIFY:** Apply the first and second conditions of equilibrium to the door.

**SET UP:** The free-body diagram for the door is given in Figure 11.42. Let $\vec{H}_1$ and $\vec{H}_2$ be the forces exerted by the upper and lower hinges. Take the origin of coordinates at the bottom hinge (point $A$) and $+y$ upward.

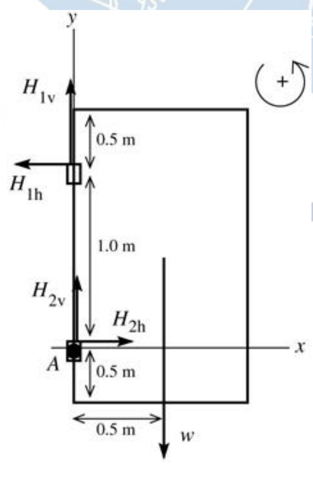



**Figure 11.42**

**EXECUTE:**
We are given that
$H_{1v} = H_{2v} = w/2 = 165\ \text{N}$.
$\sum F_x = ma_x$
$H_{2h} - H_{1h} = 0$
$H_{1h} = H_{2h}$
The horizontal components of the hinge forces are equal in magnitude and opposite in direction.

Sum torques about point $A$. $H_{1v}$, $H_{2v}$, and $H_{2h}$ all have zero moment arm and hence zero torque about an axis at this point. Thus $\sum \tau_A = 0$ gives $H_{1h}(1.00\ \text{m}) - w(0.50\ \text{m}) = 0$

$$H_{1h} = w\left(\frac{0.50\ \text{m}}{1.00\ \text{m}}\right) = \frac{1}{2}(330\ \text{N}) = 165\ \text{N}.$$

The horizontal component of each hinge force is 165 N.

**EVALUATE:** The horizontal components of the force exerted by each hinge are the only horizontal forces so must be equal in magnitude and opposite in direction. With an axis at *A*, the torque due to the horizontal force exerted by the upper hinge must be counterclockwise to oppose the clockwise torque exerted by the weight of the door. So, the horizontal force exerted by the upper hinge must be to the left. You can also verify that the net torque is also zero if the axis is at the upper hinge.

**11.43. IDENTIFY:** The center of gravity of the combined object must be at the fulcrum. Use $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$ to calculate $x_{cm}$.

**SET UP:** The center of gravity of the sand is at the middle of the box. Use coordinates with the origin at the fulcrum and $+x$ to the right. Let $m_1 = 25.0$ kg, so $x_1 = 0.500$ m. Let $m_2 = m_{sand}$, so $x_2 = -0.625$ m. $x_{cm} = 0$.

**EXECUTE:** $x_{cm} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2} = 0$ and $m_2 = -m_1\dfrac{x_1}{x_2} = -(25.0\text{ kg})\left(\dfrac{0.500\text{ m}}{-0.625\text{ m}}\right) = 20.0$ kg.

**EVALUATE:** The mass of sand required is less than the mass of the plank since the center of the box is farther from the fulcrum than the center of gravity of the plank is.

**11.44. IDENTIFY:** Apply $\sum\tau_z = 0$ to the bridge.

**SET UP:** Let the axis of rotation be at the left end of the bridge and let counterclockwise torques be positive.

**EXECUTE:** If Lancelot were at the end of the bridge, the tension in the cable would be (from taking torques about the hinge of the bridge) obtained from

$T(12.0\text{ m}) = (600\text{ kg})(9.80\text{ m/s}^2)(12.0\text{ m}) + (200\text{ kg})(9.80\text{ m/s}^2)(6.0\text{ m})$, so $T = 6860$ N.

This exceeds the maximum tension that the cable can have, so Lancelot is going into the drink. To find the distance *x* Lancelot can ride, replace the 12.0 m multiplying Lancelot's weight by *x* and the tension $T$ by $T_{max} = 5.80\times10^3$ N and solve for *x*;

$$x = \frac{(5.80\times10^3\text{ N})(12.0\text{ m}) - (200\text{ kg})(9.80\text{ m/s}^2)(6.0\text{ m})}{(600\text{ kg})(9.80\text{ m/s}^2)} = 9.84\text{ m.}$$

**EVALUATE:** Before Lancelot goes onto the bridge, the tension in the supporting cable is

$T = \dfrac{(6.0\text{ m})(200\text{ kg})(9.80\text{ m/s}^2)}{12.0\text{ m}} = 980$ N, well below the breaking strength of the cable. As he moves along the bridge, the increase in tension is proportional to *x*, the distance he has moved along the bridge.

**11.45. IDENTIFY:** Apply the conditions of equilibrium to the climber. For the minimum coefficient of friction the static friction force has the value $f_s = \mu_s n$.

**SET UP:** The free-body diagram for the climber is given in Figure 11.45. $f_s$ and *n* are the vertical and horizontal components of the force exerted by the cliff face on the climber. The moment arm for the force *T* is $(1.4\text{ m})\cos10°$.

**EXECUTE:** **(a)** $\sum\tau_z = 0$ gives $T(1.4\text{ m})\cos10° - w(1.1\text{ m})\cos35.0° = 0$.

$$T = \frac{(1.1\text{ m})\cos35.0°}{(1.4\text{ m})\cos10°}(82.0\text{ kg})(9.80\text{ m/s}^2) = 525\text{ N}$$

**(b)** $\sum F_x = 0$ gives $n = T\sin25.0° = 222$ N. $\sum F_y = 0$ gives $f_s + T\cos25° - w = 0$ and

$f_s = (82.0\text{ kg})(9.80\text{ m/s}^2) - (525\text{ N})\cos25° = 328$ N.

**(c)** $\mu_s = \dfrac{f_s}{n} = \dfrac{328\text{ N}}{222\text{ N}} = 1.48$

**EVALUATE:** To achieve this large value of $\mu_s$ the climber must wear special rough-soled shoes.

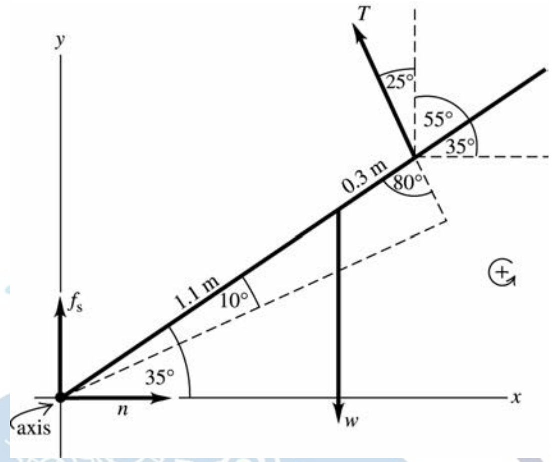



Figure 11.45

**11.46. IDENTIFY:** The beam is at rest, so the forces and torques on it must balance.

**SET UP:** The weight of the beam acts 4.0 m from each end. Take the pivot at the hinge and let counterclockwise torques be positive. Represent the force exerted by the hinge by its horizontal and vertical components, $H_h$ and $H_v$. $\sum F_x = 0$, $\sum F_y = 0$ and $\sum \tau_z = 0$.

**EXECUTE:** **(a)** The free-body diagram for the beam is given in Figure 11.46a.

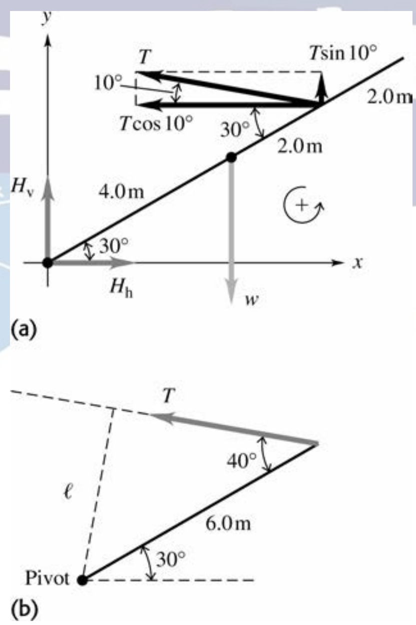



Figure 11.46

**(b)** The moment arm for $T$ is sketched in Figure 11.46b and is equal to $(6.0\text{ m})\sin 40.0°$. $\sum \tau_z = 0$ gives

$T(6.0\text{ m})(\sin 40.0°) - w(4.0\text{ m})(\cos 30.0°) = 0$. $T = \dfrac{(1150\text{ kg})(9.80\text{ m/s}^2)(4.0\text{ m})(\cos 30.0°)}{(6.0\text{ m})(\sin 40.0°)} = 1.01\times 10^4\text{ N}$.

**(c)** $\sum F_x = 0$ gives $H_h - T\cos 10.0° = 0$ and $H_h = T\cos 10.0° = 9.97\times 10^3\text{ N}$.

**EVALUATE:** The tension is less than the weight of the beam because it has a larger moment arm than the weight force has.

**11.47. IDENTIFY:** In each case, to achieve balance the center of gravity of the system must be at the fulcrum. Use $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$ to locate $x_{cm}$, with $m_i$ replaced by $w_i$.

**SET UP:** Let the origin be at the left-hand end of the rod and take the $+x$-axis to lie along the rod. Let $w_1 = 255$ N (the rod) so $x_1 = 1.00$ m, let $w_2 = 225$ N so $x_2 = 2.00$ m and let $w_3 = W$. In part (a) $x_3 = 0.500$ m and in part (b) $x_3 = 0.750$ m.

**EXECUTE:** **(a)** $x_{cm} = 1.25$ m. $x_{cm} = \dfrac{w_1x_1 + w_2x_2 + w_3x_3}{w_1 + w_2 + w_3}$ gives $w_3 = \dfrac{(w_1 + w_2)x_{cm} - w_1x_1 - w_2x_2}{x_3 - x_{cm}}$ and

$$W = \frac{(480\text{ N})(1.25\text{ m}) - (255\text{ N})(1.00\text{ m}) - (225\text{ N})(2.00\text{ m})}{0.500\text{ m} - 1.25\text{ m}} = 140\text{ N}.$$

**(b)** Now $w_3 = W = 140$ N and $x_3 = 0.750$ m.

$x_{cm} = \dfrac{(255\text{ N})(1.00\text{ m}) + (225\text{ N})(2.00\text{ m}) + (140\text{ N})(0.750\text{ m})}{255\text{ N} + 225\text{ N} + 140\text{ N}} = 1.31$ m. $W$ must be moved $1.31\text{ m} - 1.25\text{ m} = 6$ cm to the right.

**EVALUATE:** Moving $W$ to the right means $x_{cm}$ for the system moves to the right.

**11.48. IDENTIFY:** Apply $\sum \tau_z = 0$ to the hammer.

**SET UP:** Take the axis of rotation to be at point $A$.

**EXECUTE:** The force $\vec{F}_1$ is directed along the length of the nail, and so has a moment arm of $(0.080\text{ m})\sin 60°$. The moment arm of $\vec{F}_2$ is 0.300 m, so

$$F_2 = F_1 \frac{(0.0800\text{ m})\sin 60°}{(0.300\text{ m})} = (400\text{ N})(0.231) = 92.4\text{ N}.$$

**EVALUATE:** The force $F_2$ that must be applied to the hammer handle is much less than the force that the hammer applies to the nail, because of the large difference in the lengths of the moment arms.

**11.49. IDENTIFY:** Apply the conditions of equilibrium to the horizontal beam. Since the two wires are symmetrically placed on either side of the middle of the sign, their tensions are equal and are each equal to $T_w = mg/2 = 137$ N.

**SET UP:** The free-body diagram for the beam is given in Figure 11.49. $F_v$ and $F_h$ are the vertical and horizontal forces exerted by the hinge on the beam. Since the cable is 2.00 m long and the beam is 1.50 m long, $\cos\theta = \dfrac{1.50\text{ m}}{2.00\text{ m}}$ and $\theta = 41.4°$. The tension $T_c$ in the cable has been replaced by its horizontal and vertical components.

**EXECUTE:** **(a)** $\sum \tau_z = 0$ gives $T_c(\sin 41.4°)(1.50\text{ m}) - w_{beam}(0.750\text{ m}) - T_w(1.50\text{ m}) - T_w(0.60\text{ m}) = 0.$

$$T_c = \frac{(16.0\text{ kg})(9.80\text{ m/s}^2)(0.750\text{ m}) + (137\text{ N})(1.50\text{ m} + 0.60\text{ m})}{(1.50\text{ m})(\sin 41.4°)} = 408.6\text{ N},$$ which rounds to 409 N.

**(b)** $\sum F_y = 0$ gives $F_v + T_c\sin 41.4° - w_{beam} - 2T_w = 0$ and

$F_v = 2T_w + w_{beam} - T_c\sin 41.4° = 2(137\text{ N}) + (16.0\text{ kg})(9.80\text{ m/s}^2) - (408.6\text{ N})(\sin 41.4°) = 161$ N. The hinge must be able to supply a vertical force of 161 N.

**EVALUATE:** The force from the two wires could be replaced by the weight of the sign acting at a point 0.60 m to the left of the right-hand edge of the sign.

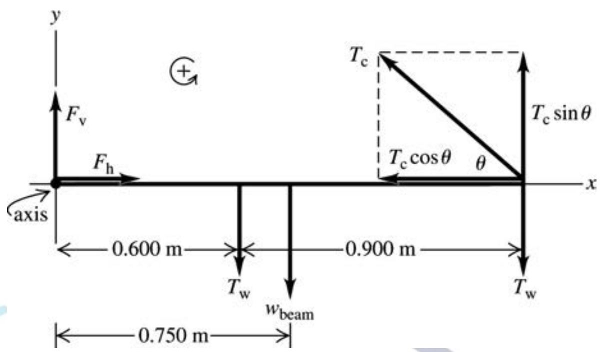



**Figure 11.49**

**11.50.** **IDENTIFY:** Apply the first and second conditions of equilibrium to the bar.

**SET UP:** The free-body diagram for the bar is given in Figure 11.50. $n$ is the normal force exerted on the bar by the surface. There is no friction force at this surface. $H_{\text{h}}$ and $H_{\text{v}}$ are the components of the force exerted on the bar by the hinge. The components of the force of the bar on the hinge will be equal in magnitude and opposite in direction.

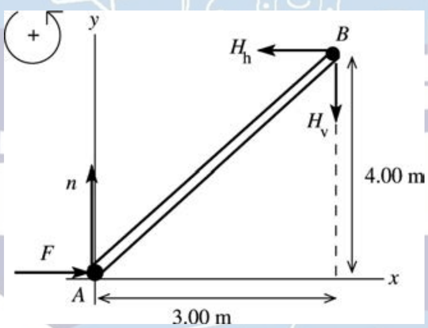



**Figure 11.50**

**EXECUTE:**

$\sum F_x = ma_x$

$F = H_{\text{h}} = 220\text{ N}$

$\sum F_y = ma_y$

$n - H_{\text{v}} = 0$

$H_{\text{v}} = n$, but we don't know either of these forces.

$\sum \tau_B = 0$ gives $F(4.00\text{ m}) - n(3.00\text{ m}) = 0.$

$n = (4.00\text{ m}/3.00\text{ m})F = \frac{4}{3}(220\text{ N}) = 293\text{ N}$ and then $H_{\text{v}} = 293\text{ N}.$

Force of bar on hinge:
horizontal component 220 N, to right
vertical component 293 N, upward

**EVALUATE:** $H_{\text{h}}/H_{\text{v}} = 220/293 = 0.75 = 3.00/4.00$, so the force the hinge exerts on the bar is directed along the bar. $\vec{n}$ and $\vec{F}$ have zero torque about point $A$, so the line of action of the hinge force $\vec{H}$ must pass through this point also if the net torque is to be zero.

**11.51.** **IDENTIFY:** We want to locate the center of mass of the leg-cast system. We can treat each segment of the leg and cast as a point-mass located at its center of mass.

**SET UP:** The force diagram for the leg is given in Figure 11.51 (next page). The weight of each piece acts at the center of mass of that piece. The mass of the upper leg is $m_{\text{ul}} = (0.215)(37\text{ kg}) = 7.955\text{ kg}$. The mass of the lower leg is $m_{\text{ll}} = (0.140)(37\text{ kg}) = 5.18\text{ kg}$. Use the coordinates shown, with the origin at the hip and the $x$-axis along the leg, and use $x_{\text{cm}} = \dfrac{x_{\text{ul}}m_{\text{ul}} + x_{\text{ll}}m_{\text{ll}} + x_{\text{cast}}m_{\text{cast}}}{m_{\text{ul}} + m_{\text{ll}} + m_{\text{cast}}}$.

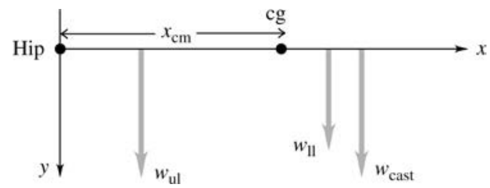



**Figure 11.51**

**EXECUTE:** Using $x_{cm} = \dfrac{x_{ul}m_{ul} + x_{ll}m_{ll} + x_{cast}m_{cast}}{m_{ul} + m_{ll} + m_{cast}}$, we have

$$x_{cm} = \frac{(18.0\text{ cm})(7.955\text{ kg}) + (69.0\text{ cm})(5.18\text{ kg}) + (78.0\text{ cm})(5.50\text{ kg})}{7.955\text{ kg} + 5.18\text{ kg} + 5.50\text{ kg}} = 49.9\text{ cm}$$

**EVALUATE:** The strap is attached to the left of the center of mass of the cast, but it is still supported by the rigid cast since the cast extends beyond its center of mass.

**11.52. IDENTIFY:** Apply the first and second conditions for equilibrium to the bridge.

**SET UP:** Find torques about the hinge. Use $L$ as the length of the bridge and $w_T$ and $w_B$ for the weights of the truck and the raised section of the bridge. Take $+y$ to be upward and $+x$ to be to the right.

**EXECUTE:** **(a)** $TL\sin 70° = w_T(\tfrac{3}{4}L)\cos 30° + w_B(\tfrac{1}{2}L)\cos 30°$, so

$$T = \frac{(\tfrac{3}{4}m_T + \tfrac{1}{2}m_B)(9.80\text{ m/s}^2)\cos 30°}{\sin 70°} = 2.84\times 10^5\text{ N}.$$

**(b)** Horizontal: $T\cos(70° - 30°) = 2.18\times 10^5\text{ N}$ (to the right).

Vertical: $w_T + w_B - T\sin 40° = 2.88\times 10^5\text{ N}$ (upward).

**EVALUATE:** If $\phi$ is the angle of the hinge force above the horizontal,

$\tan\phi = \dfrac{2.88\times 10^5\text{ N}}{2.18\times 10^5\text{ N}}$ and $\phi = 52.9°$. The hinge force is not directed along the bridge.

**11.53. IDENTIFY:** The leg is not rotating, so the external torques on it must balance.

**SET UP:** The free-body diagram for the leg is given in Figure 11.53. Take the pivot at the hip joint and let counterclockwise torque be positive. There are also forces on the leg exerted by the hip joint but these forces produce no torque and aren't shown. $\sum\tau_z = 0$ for no rotation.

**EXECUTE:** **(a)** $\sum\tau_z = 0$ gives $T(10\text{ cm})(\sin\theta) - w(44\text{ cm})(\cos\theta) = 0$.

$T = \dfrac{4.4w\cos\theta}{\sin\theta} = \dfrac{4.4w}{\tan\theta}$ and for $\theta = 60°$, $T = \dfrac{4.4(15\text{ kg})(9.80\text{ m/s}^2)}{\tan 60°} = 370\text{ N}.$

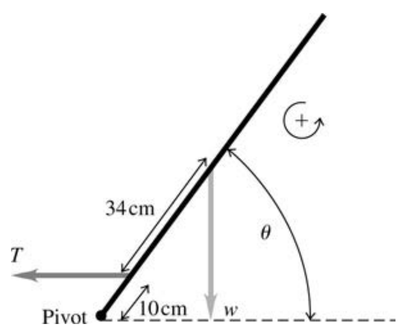



**Figure 11.53**

**(b)** For $\theta = 5°$, $T = 7400$ N. The tension is much greater when he just starts to raise his leg off the ground.
**(c)** $T \to \infty$ as $\theta \to 0$. The person could not raise his leg. If the leg is horizontal so $\theta$ is zero, the moment arm for $T$ is zero and $T$ produces no torque to rotate the leg against the torque due to its weight.

**EVALUATE:** Most of the exercise benefit of leg-raises occurs when the person just starts to raise his legs off the ground.

**11.54. IDENTIFY:** The arm is stationary, so the forces and torques must each balance.

**SET UP:** $\sum\tau = 0$, $\sum F_x = 0$, $\sum F_y = 0$. Let the forearm be at an angle $\phi$ below the horizontal. Take the pivot at the elbow joint and let counterclockwise torques be positive. Let $+y$ be upward and let $+x$ be to the right. Each forearm has mass $m_{\text{arm}} = \frac{1}{2}(0.0600)(72\text{ kg}) = 2.16\text{ kg}$. The weight held in each hand is $w = mg$, with $m = 7.50\text{ kg}$. $\vec{T}$ is the force the biceps muscle exerts on the forearm. $\vec{E}$ is the force exerted by the elbow and has components $E_{\text{v}}$ and $E_{\text{h}}$.

**EXECUTE:** **(a)** The free-body diagram is shown in Figure 11.54.

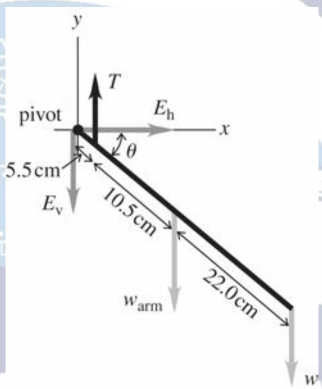



**Figure 11.54**

**(b)** $\sum\tau = 0$ gives $T(5.5\text{ cm})(\cos\theta) - w_{\text{arm}}(16.0\text{ cm})(\cos\theta) - w(38.0\text{ cm})(\cos\theta) = 0$

$$T = \frac{16.0w_{\text{arm}} + 38.0w}{5.5} = \frac{16.0(2.16\text{ kg})(9.80\text{ m/s}^2) + 38.0(7.50\text{ kg})(9.80\text{ m/s}^2)}{5.5} = 569\text{ N}$$

**(c)** $\sum F_x = 0$ gives $E_{\text{h}} = 0$. $\sum F_y = 0$ gives $T - E_{\text{v}} - w_{\text{arm}} - w = 0$, so

$$E_{\text{v}} = T - w_{\text{arm}} - w = 569\text{ N} - (2.16\text{ kg})(9.80\text{ m/s}^2) - (7.50\text{ kg})(9.80\text{ m/s}^2) = 474\text{ N}$$

Since we calculate $E_{\text{v}}$ to be positive, we correctly assumed that it was downward when we drew the free-body diagram.

**(d)** The weight and the pull of the biceps are both always vertical in this situation, so the factor $\cos\theta$ divides out of the $\sum\tau = 0$ equation in part (b). Therefore the force $T$ stays the same as she raises her arm.

**EVALUATE:** The biceps force must be much greater than the weight of the forearm and the weight in her hand because it has such a small lever arm compared to those two forces.

**11.55. IDENTIFY:** The presence of the fetus causes the woman's center of mass to shift forward. Figure 11.55 (next page) shows the cylinder and sphere model suggested in the problem.

**SET UP:** $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$. The mass of each object can be considered as located at its center of mass, at its geometrical center. Use coordinates that have the origin at the center of the cylinder and the $x$-axis horizontal.

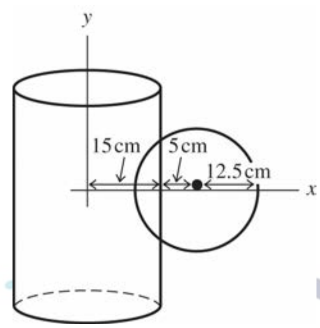



**Figure 11.55**

**EXECUTE:** **(a)** Before pregnancy, $x_{\text{cm,i}} = 0$. The center of mass of the pregnant woman is at

$x_{\text{cm,f}} = \dfrac{0 + (20 \text{ cm})(10 \text{ kg})}{60 \text{ kg} + 10 \text{ kg}} = 2.9$ cm. Her center of mass moves a horizontal distance of 2.9 cm forward.

**(b)** The woman must shift her upper body backward to keep her center of mass from extending past her feet.
**(c)** The unnatural posture and curved back strains the back muscles.
**EVALUATE:** Observation of a pregnant woman walking should confirm the results found here.

**11.56.** **IDENTIFY:** Apply the first and second conditions of equilibrium to each rod.
**SET UP:** Apply $\sum F_y = 0$ with $+y$ upward and apply $\sum \tau_z = 0$ with the pivot at the point of suspension for each rod.
**EXECUTE:** **(a)** The free-body diagram for each rod is given in Figure 11.56.
**(b)** $\sum \tau_z = 0$ for the lower rod: $(6.0 \text{ N})(4.0 \text{ cm}) = w_A(8.0 \text{ cm})$ and $w_A = 3.0$ N.
$\sum F_y = 0$ for the lower rod: $S_3 = 6.0 \text{ N} + w_A = 9.0$ N.

$\sum \tau_z = 0$ for the middle rod: $w_B(3.0 \text{ cm}) = (5.0 \text{ cm})S_3$ and $w_B = \left(\dfrac{5.0}{3.0}\right)(9.0 \text{ N}) = 15.0$ N.

$\sum F_y = 0$ for the middle rod: $S_2 = 9.0 \text{ N} + S_3 = 24.0$ N.

$\sum \tau_z = 0$ for the upper rod: $S_2(2.0 \text{ cm}) = w_C(6.0 \text{ cm})$ and $w_C = \left(\dfrac{2.0}{6.0}\right)(24.0 \text{ N}) = 8.0$ N.

$\sum F_y = 0$ for the upper rod: $S_1 = S_2 + w_C = 32.0$ N.

In summary, $w_A = 3.0$ N, $w_B = 15.0$ N, $w_C = 8.0$ N. $S_1 = 32.0$ N, $S_2 = 24.0$ N, $S_3 = 9.0$ N.
**(c)** The center of gravity of the entire mobile must lie along a vertical line that passes through the point where $S_1$ is located.
**EVALUATE:** For the mobile as a whole the vertical forces must balance, so $S_1 = w_A + w_B + w_C + 6.0$ N.

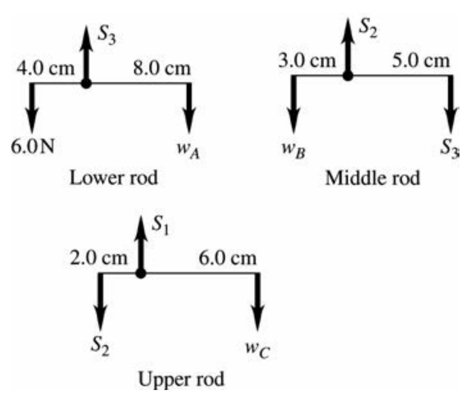



**Figure 11.56**

**11.57. IDENTIFY:** Apply $\sum \tau_z = 0$ to the beam.

**SET UP:** The free-body diagram for the beam is given in Figure 11.57.

**EXECUTE:** $\sum \tau_z = 0$, axis at hinge, gives $T(6.0\text{ m})(\sin 40°) - (6490\text{ N})(3.75\text{ m})(\cos 30°) = 0$ and $T = 5500\text{ N}$.

**EVALUATE:** The tension in the cable is less than the weight of the beam. $T\sin 40°$ is the component of $T$ that is perpendicular to the beam.

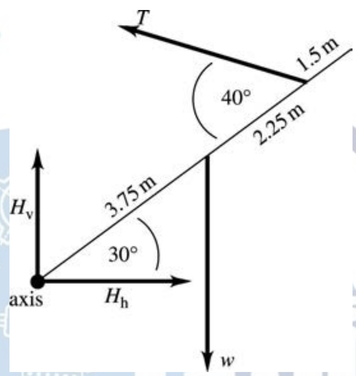



**Figure 11.57**

**11.58. IDENTIFY:** Apply the first and second conditions of equilibrium to the drawbridge.

**SET UP:** The free-body diagram for the drawbridge is given in Figure 11.58 (next page). $H_v$ and $H_h$ are the components of the force the hinge exerts on the bridge. In part (c), apply $\sum \tau_z = I\alpha$ to the rotating bridge and in part (d) apply energy conservation to the bridge.

**EXECUTE: (a)** $\sum \tau_z = 0$ with the axis at the hinge gives $-w(7.0\text{ m})(\cos 37°) + T(3.5\text{ m})(\sin 37°) = 0$ and

$$T = 2w\frac{\cos 37°}{\sin 37°} = 2\frac{(45{,}000\text{ N})}{\tan 37°} = 1.19\times 10^5\text{ N}.$$

**(b)** $\sum F_x = 0$ gives $H_h = T = 1.19\times 10^5$ N. $\sum F_y = 0$ gives $H_v = w = 4.50\times 10^4$ N.

$H = \sqrt{H_h^2 + H_v^2} = 1.27\times 10^5$ N. $\tan\theta = \dfrac{H_v}{H_h}$ and $\theta = 20.7°$. The hinge force has magnitude $1.27\times 10^5$ N and is directed at 20.7° above the horizontal.

**(c)** We can treat the bridge as a uniform bar rotating around one end, so $I = 1/3\, mL^2$. $\sum \tau_z = I\alpha_z$ gives $mg(L/2)\cos 37° = 1/3\, mL^2\alpha$. Solving for $\alpha$ gives $\alpha = \dfrac{3g\cos 37°}{2L} = \dfrac{3(9.80\text{ m/s}^2)\cos 37°}{2(14.0\text{ m})} = 0.839\text{ rad/s}^2$.

**(d)** Energy conservation gives $U_1 = K_2$, giving $mgh = 1/2\, I\omega^2 = (1/2)(1/3\, mL^2)\omega^2$. Trigonometry gives $h = L/2\,\sin 37°$. Canceling $m$, the energy conservation equation gives $g(L/2)\sin 37° = (1/6)L^2\omega^2$. Solving for $\omega$ gives $\omega = \sqrt{\dfrac{3g\sin 37°}{L}} = \sqrt{\dfrac{3(9.80\text{ m/s}^2)\sin 37°}{14.0\text{ m}}} = 1.12\text{ rad/s}$.

**EVALUATE:** The hinge force is not directed along the bridge. If it were, it would have zero torque for an axis at the center of gravity of the bridge and for that axis the tension in the cable would produce a single, unbalanced torque.

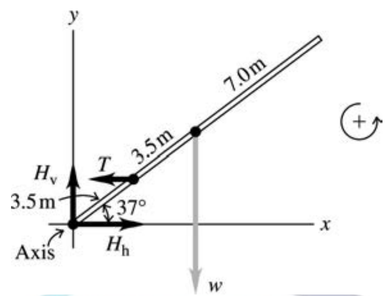



**Figure 11.58**

**11.59.** **IDENTIFY:** The amount the tendon stretches depends on Young's modulus for the tendon material. The foot is in rotational equilibrium, so the torques on it balance.

**SET UP:** $Y = \dfrac{F_T/A}{\Delta l/l_0}$. The foot is in rotational equilibrium, so $\sum \tau_z = 0$.

**EXECUTE:** **(a)** The free-body diagram for the foot is given in Figure 11.59. $T$ is the tension in the tendon and $A$ is the force exerted on the foot by the ankle. $n = (75\text{ kg})g$, the weight of the person.

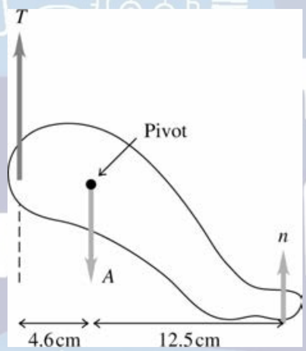



**Figure 11.59**

**(b)** Apply $\sum \tau_z = 0$, letting counterclockwise torques be positive and with the pivot at the ankle:

$T(4.6\text{ cm}) - n(12.5\text{ cm}) = 0$. $T = \left(\dfrac{12.5\text{ cm}}{4.6\text{ cm}}\right)(75\text{ kg})(9.80\text{ m/s}^2) = 2000\text{ N}$, which is 2.72 times his weight.

**(c)** The foot pulls downward on the tendon with a force of 2000 N.

$$\Delta l = \left(\frac{F_T}{YA}\right)l_0 = \frac{2000\text{ N}}{(1470\times 10^6\text{ Pa})(78\times 10^{-6}\text{ m}^2)}(25\text{ cm}) = 4.4\text{ mm}.$$

**EVALUATE:** The tension is quite large, but the Achilles tendon stretches about 4.4 mm, which is only about 1/6 of an inch, so it must be a strong tendon.

**11.60** **IDENTIFY:** Apply $\sum \tau_z = 0$ to the beam.

**SET UP:** The center of mass of the beam is 1.0 m from the suspension point.

**EXECUTE:** **(a)** Taking torques about the suspension point,

$w(4.00\text{ m})\sin 30° + (140.0\text{ N})(1.00\text{ m})\sin 30° = (100\text{ N})(2.00\text{ m})\sin 30°.$

The common factor of $\sin 30°$ divides out, from which $w = 15.0$ N.

**(b)** In this case, a common factor of $\sin 45°$ would be factored out, and the result would be the same.

**EVALUATE:** All the forces are vertical, so the moments are all horizontal and all contain the factor $\sin\theta$, where $\theta$ is the angle the beam makes with the horizontal.

**11.61.** **IDENTIFY:** Apply $\sum \tau_z = 0$ to the flagpole.

**SET UP:** The free-body diagram for the flagpole is given in Figure 11.61. Let clockwise torques be positive. $\theta$ is the angle the cable makes with the horizontal pole.

**EXECUTE:** **(a)** Taking torques about the hinged end of the pole

$(200\text{ N})(2.50\text{ m}) + (600\text{ N})(5.00\text{ m}) - T_y(5.00\text{ m}) = 0$. $T_y = 700$ N. The $x$-component of the tension is then

$T_x = \sqrt{(1000 \text{ N})^2 - (700 \text{ N})^2} = 714 \text{ N}$. $\tan\theta = \dfrac{h}{5.00 \text{ m}} = \dfrac{T_y}{T_x}$. The height above the pole that the wire must be attached is $(5.00 \text{ m})\dfrac{700}{714} = 4.90 \text{ m}$.

**(b)** The $y$-component of the tension remains 700 N. Now $\tan\theta = \dfrac{4.40 \text{ m}}{5.00 \text{ m}}$ and $\theta = 41.35°$, so $T = \dfrac{T_y}{\sin\theta} = \dfrac{700 \text{ N}}{\sin 41.35°} = 1060 \text{ N}$, an increase of 60 N.

**EVALUATE:** As the wire is fastened closer to the hinged end of the pole, the moment arm for $T$ decreases and $T$ must increase to produce the same torque about that end.

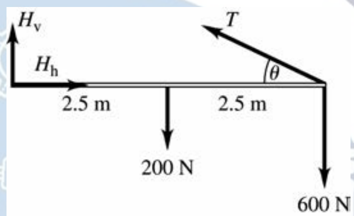



**Figure 11.61**

**11.62.** **IDENTIFY:** Apply $\sum \vec{F} = 0$ to each object, including the point where $D$, $C$, and $B$ are joined. Apply $\sum \tau_z = 0$ to the rod.

**SET UP:** To find $T_C$ and $T_D$, use a coordinate system with axes parallel to the cords.

**EXECUTE:** $A$ and $B$ are straightforward, the tensions being the weights suspended: $T_A = (0.0360 \text{ kg})(9.80 \text{ m/s}^2) = 0.353 \text{ N}$ and $T_B = (0.0240 \text{ kg} + 0.0360 \text{ kg})(9.80 \text{ m/s}^2) = 0.588 \text{ N}$. Applying $\sum F_x = 0$ and $\sum F_y = 0$ to the point where the cords are joined, $T_C = T_B \cos 36.9° = 0.470 \text{ N}$ and $T_D = T_B \cos 53.1° = 0.353 \text{ N}$. To find $T_E$, take torques about the point where string $F$ is attached.

$T_E(1.00 \text{ m}) = T_D \sin 36.9°(0.800 \text{ m}) + T_C \sin 53.1°(0.200 \text{ m}) + (0.120 \text{ kg})(9.80 \text{ m/s}^2)(0.500 \text{ m})$ and $T_E = 0.833 \text{ N}$.

$T_F$ may be found similarly, or from the fact that $T_E + T_F$ must be the total weight of the ornament. $(0.180\text{kg})(9.80\text{m/s}^2) = 1.76 \text{ N}$, from which $T_F = 0.931 \text{ N}$.

**EVALUATE:** The vertical line through the spheres is closer to $F$ than to $E$, so we expect $T_F > T_E$, and this is indeed the case.

**11.63.** **IDENTIFY:** The torques must balance since the person is not rotating.

**SET UP:** Figure 11.63a (next page) shows the distances and angles. $\theta + \phi = 90°$. $\theta = 56.3°$ and $\phi = 33.7°$. The distances $x_1$ and $x_2$ are $x_1 = (90 \text{ cm})\cos\theta = 50.0 \text{ cm}$ and $x_2 = (135 \text{ cm})\cos\phi = 112 \text{ cm}$. The free-body diagram for the person is given in Figure 11.63b. $w_l = 277 \text{ N}$ is the weight of his feet and legs, and $w_t = 473 \text{ N}$ is the weight of his trunk. $n_f$ and $f_f$ are the total normal and friction forces exerted on his feet and $n_h$ and $f_h$ are those forces on his hands. The free-body diagram for his legs is given in Figure 11.63c. $F$ is the force exerted on his legs by his hip joints. For balance, $\sum \tau_z = 0$.

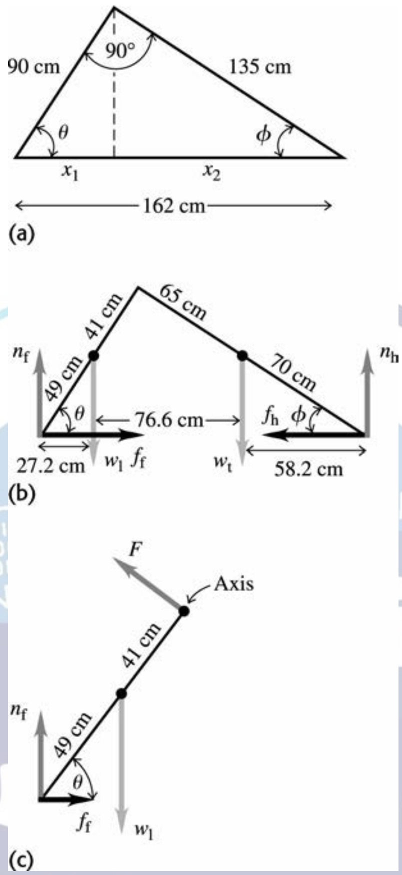



**Figure 11.63**

**EXECUTE:** **(a)** Consider the force diagram of Figure 11.63b. $\sum \tau_z = 0$ with the pivot at his feet and counterclockwise torques positive gives $n_h(162\text{ cm}) - (277\text{ N})(27.2\text{ cm}) - (473\text{ N})(103.8\text{ cm}) = 0.$ $n_h = 350\text{ N},$ so there is a normal force of 175 N at each hand. $n_f + n_h - w_l - w_t = 0$ so $n_f = w_l + w_t - n_h = 750\text{ N} - 350\text{ N} = 400\text{ N},$ so there is a normal force of 200 N at each foot.

**(b)** Consider the force diagram of Figure 11.63c. $\sum \tau_z = 0$ with the pivot at his hips and counterclockwise torques positive gives $f_f(74.9\text{ cm}) + w_l(22.8\text{ cm}) - n_f(50.0\text{ cm}) = 0.$

$f_f = \dfrac{(400\text{ N})(50.0\text{ cm}) - (277\text{ N})(22.8\text{ cm})}{74.9\text{ cm}} = 182.7\text{ N}.$ There is a friction force of 91 N at each foot.

$\sum F_x = 0$ in Figure 11.63b gives $f_h = f_f,$ so there is a friction force of 91 N at each hand.

**EVALUATE:** In this position the normal forces at his feet and at his hands don't differ very much.

**11.64.** **IDENTIFY:** The bar is in equilibrium until the cable breaks, so the forces and torques on it must all balance.

**SET UP:** Look at the bar when the cable is just ready to break. At that time, the tension in it is 455 N. $\Sigma F_x = 0,$ $\Sigma F_y = 0,$ $\sum \tau_z = 0.$

**EXECUTE:** **(a)** Take torques about the hinge, calling $L$ your distance from the hinge. $\sum \tau_z = 0$ gives $(455\text{ N})(\sin 37.0°)(8.00\text{ m}) - (65.0\text{ kg})(9.80\text{ m/s}^2)L(\cos 64.0°) - (30.0\text{ kg})(9.80\text{ m/s}^2)(4.00\text{ m})(\cos 64.0°) = 0.$ Solving for $L$ gives $L = 6.00$ m from the hinge, which is 2.00 m from the upper end of the bar.

**(b)** Calling $H$ the magnitude of the hinge force, $\Sigma F_x = 0$ gives $H_x = (455\text{ N})(\cos 27.0°) = 405.4$ N.

$\Sigma F_y = 0$ gives $H_y = (65.0\text{ kg})(9.80\text{ m/s}^2) + (30.0\text{ kg})(9.80\text{ m/s}^2) + (455\text{ N})(\sin 27.0°) = 1138$ N.

$H = \sqrt{H_x^2 + H_y^2} = \sqrt{(405.4 \text{ N})^2 + (1138 \text{ N})^2} = 1210 \text{ N}.$

The angle that $H$ makes above the horizontal is $\phi = \arctan\dfrac{H_y}{H_x} = \arctan\dfrac{1138 \text{ N}}{405.4 \text{ N}} = 70.4°.$

**EVALUATE:** The bar rises at 64.0° above the horizontal but the hinge force points at 70.4° above the horizontal, so the hinge force does not point along the bar.

**11.65. IDENTIFY:** Apply the equilibrium conditions to the crate. When the crate is on the verge of tipping it touches the floor only at its lower left-hand corner and the normal force acts at this point. The minimum coefficient of static friction is given by the equation $f_s = \mu_s n.$

**SET UP:** The free-body diagram for the crate when it is ready to tip is given in Figure 11.65.

**EXECUTE:** **(a)** $\sum \tau_z = 0$ gives $P(1.50 \text{ m})\sin 53.0° - w(1.10 \text{ m}) = 0.$

$$P = w\left(\frac{1.10 \text{ m}}{[1.50 \text{ m}][\sin 53.0°]}\right) = 1.15 \times 10^3 \text{ N}$$

**(b)** $\sum F_y = 0$ gives $n - w - P\cos 53.0° = 0.$

$$n = w + P\cos 53.0° = 1250 \text{ N} + (1.15 \times 10^3 \text{ N})\cos 53° = 1.94 \times 10^3 \text{ N}$$

**(c)** $\sum F_y = 0$ gives $f_s = P\sin 53.0° = (1.15 \times 10^3 \text{ N})\sin 53.0° = 918 \text{ N}.$

**(d)** $\mu_s = \dfrac{f_s}{n} = \dfrac{918 \text{ N}}{1.94 \times 10^3 \text{ N}} = 0.473$

**EVALUATE:** The normal force is greater than the weight because $P$ has a downward component.

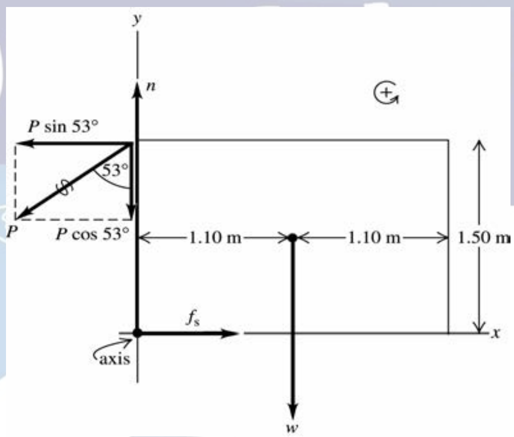



**Figure 11.65**

**11.66. IDENTIFY:** Apply $\sum \tau_z = 0$ to the meterstick.

**SET UP:** The wall exerts an upward static friction force $f$ and a horizontal normal force $n$ on the stick. Denote the length of the stick by $l$. $f = \mu_s n.$

**EXECUTE:** **(a)** Taking torques about the right end of the stick, the friction force is half the weight of the stick, $f = w/2.$ Taking torques about the point where the cord is attached to the wall (the tension in the cord and the friction force exert no torque about this point), and noting that the moment arm of the normal force is $l\tan\theta$, $n\tan\theta = w/2.$ Then, $(f/n) = \tan\theta < 0.40,$ so $\theta < \arctan(0.40) = 22°.$

**(b)** Taking torques as in part (a), $fl = w\dfrac{l}{2} + w(l - x)$ and $nl\tan\theta = w\dfrac{l}{2} + wx.$ In terms of the coefficient of friction $\mu_s$, $\mu_s > \dfrac{f}{n} = \dfrac{l/2 + (l - x)}{l/2 + x}\tan\theta = \dfrac{3l - 2x}{l + 2x}\tan\theta.$ Solving for $x$, $x > \dfrac{l}{2}\dfrac{3\tan\theta - \mu_s}{\mu_s + \tan\theta} = 30.2$ cm.

**(c)** In the above expression, setting $x = 10$ cm and $l = 100$ cm and solving for $\mu_s$ gives

$$\mu_s > \frac{(3 - 20/l)\tan\theta}{1 + 20/l} = 0.625.$$

**EVALUATE:** For $\theta = 15°$ and without the block suspended from the stick, a value of $\mu_s \geq 0.268$ is required to prevent slipping. Hanging the block from the stick increases the value of $\mu_s$ that is required.

**11.67. IDENTIFY:** Apply the first and second conditions of equilibrium to the crate.

**SET UP:** The free-body diagram for the crate is given in Figure 11.67.

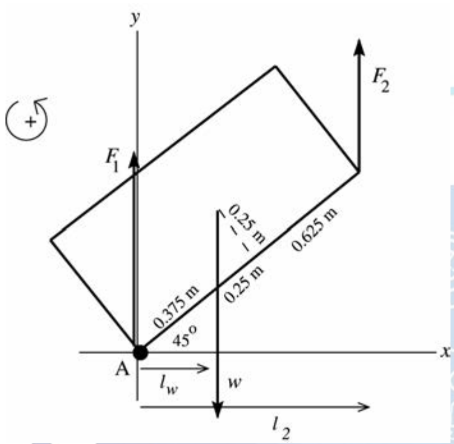



**Figure 11.67**

$l_w = (0.375\text{ m})\cos 45°$

$l_2 = (1.25\text{ m})\cos 45°$

Let $\vec{F}_1$ and $\vec{F}_2$ be the vertical forces exerted by you and your friend. Take the origin at the lower left-hand corner of the crate (point A).

**EXECUTE:** $\sum F_y = ma_y$ gives $F_1 + F_2 - w = 0$

$F_1 + F_2 = w = (200\text{ kg})(9.80\text{ m/s}^2) = 1960\text{ N}$

$\sum \tau_A = 0$ gives $F_2 l_2 - w l_w = 0$

$$F_2 = w\left(\frac{l_w}{l_2}\right) = 1960\text{ N}\left(\frac{0.375\text{ m}\cos 45°}{1.25\text{ m}\cos 45°}\right) = 590\text{ N}$$

Then $F_1 = w - F_2 = 1960\text{ N} - 590\text{ N} = 1370\text{ N}.$

**EVALUATE:** The person below (you) applies a force of 1370 N. The person above (your friend) applies a force of 590 N. It is better to be the person above. As the sketch shows, the moment arm for $\vec{F}_1$ is less than for $\vec{F}_2$, so must have $F_1 > F_2$ to compensate.

**11.68. IDENTIFY:** Apply the first and second conditions for equilibrium to the forearm.

**SET UP:** The free-body diagram is given in Figure 11.68a, and when holding the weight in Figure 11.68b. Let $+y$ be upward.

**EXECUTE:** **(a)** $\sum \tau_{\text{Elbow}} = 0$ gives $F_B(3.80\text{ cm}) = (15.0\text{ N})(15.0\text{ cm})$ and $F_B = 59.2\text{ N}$.

**(b)** $\sum \tau_{\text{Elbow}} = 0$ gives $F_B(3.80\text{ cm}) = (15.0\text{ N})(15.0\text{ cm}) + (80.0\text{ N})(33.0\text{ cm})$ and $F_B = 754\text{ N}$. The biceps force has a short lever arm, so it must be large to balance the torques.

**(c)** $\sum F_y = 0$ gives $-F_E + F_B - 15.0\text{ N} - 80.0\text{ N} = 0$ and $F_E = 754\text{ N} - 15.0\text{ N} - 80.0\text{ N} = 659\text{ N}$.

**EVALUATE:** **(d)** The biceps muscle acts perpendicular to the forearm, so its lever arm stays the same, but those of the other two forces decrease as the arm is raised. Therefore the tension in the biceps muscle *decreases*.

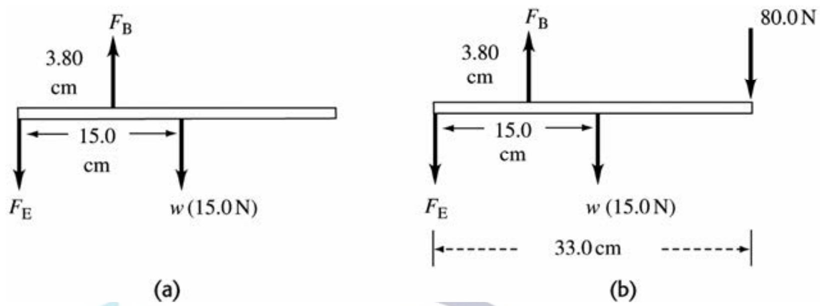



**Figure 11.68**

**11.69. IDENTIFY:** Apply $\sum \tau_z = 0$ to the forearm.

**SET UP:** The free-body diagram for the forearm is given in Figure 11.10 in the textbook.

**EXECUTE: (a)** $\sum \tau_z = 0$, axis at elbow gives

$wL - (T \sin\theta)D = 0.\ \sin\theta = \dfrac{h}{\sqrt{h^2+D^2}}$ so $w = T\dfrac{hD}{L\sqrt{h^2+D^2}}$.

$$w_{\text{max}} = T_{\text{max}}\frac{hD}{L\sqrt{h^2+D^2}}.$$

**(b)** $\dfrac{dw_{\text{max}}}{dD} = \dfrac{T_{\text{max}}h}{L\sqrt{h^2+D^2}}\left(1-\dfrac{D^2}{h^2+D^2}\right)$; the derivative is positive.

**EVALUATE: (c)** The result of part (b) shows that $w_{\text{max}}$ increases when $D$ increases, since the derivative is positive. $w_{\text{max}}$ is larger for a chimp since $D$ is larger.

**11.70. IDENTIFY:** The beam is at rest, so the forces and torques on it must all balance.

**SET UP:** The cables could point inward toward each other or outward away from each other. We shall assume they point away from each other. Call $d$ the distance of the center of gravity from the left end, call $w$ the weight of the beam, and call $T$ the tension in the right-hand cable. $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\sum \tau_z = 0$.

**EXECUTE:** $\Sigma F_x = 0$ gives (620 N)(sin30.0°) – $T$(sin50.0°) = 0, so $T$ = 404.68 N.

$\Sigma F_y = 0$ gives (620 N)(cos30.0°) + (404.68 N)(cos50.0°) – $w$ = 0, so $w$ = 797 N.

Taking torques about the left end, $\sum \tau_z = 0$ gives (404.68 N)(cos50.0°)(4.00 m) – (797 N)$d$ = 0, so $d$ = 1.31 m from the left end of the beam, or 2.69 m from the right end.

**EVALUATE:** The center of gravity is closer to the cable having the greater tension. The answer would be no different if we assumed that the cables pointed inward toward each other.

**11.71. IDENTIFY:** The beam is at rest, so the forces and torques on it must all balance.

**SET UP:** $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\sum \tau_z = 0$. Look at the situation where the cable is just about to break, in which case the tension in it is 650 N.

**EXECUTE: (a)** Taking torques about the hinge, with $L$ the length of the beam, $\sum \tau_z = 0$ gives (650 N)(sin30.0°)$L$ – $mg$($L$/2)(cos22.0°) = 0, which gives $m$ = 71.535 kg, which rounds to 71.5 kg.

**(b)** Now $m$ = 61.5 kg. Taking torques about the hinge and calling $T$ the tension, we have $LT$(sin30.0°) = (61.5 kg)(9.80 m/s$^2$)($L$/2)(cos22.0°), so $T$ = 559 N.

Call $H$ the magnitude of the hinge force. $\Sigma F_x = 0$ gives $H_x$ = (559 N)(sin38.0°) = 344.24 N

$\Sigma F_y = 0$ gives $H_y$ + (559 N)(cos38.0°) – (61.5 kg)(9.80 m/s$^2$), so $H_y$ = 162.1 N.

$H = \sqrt{H_x^2 + H_y^2} = \sqrt{(344.24\text{ N})^2 + (162.1\text{ N})^2}$ = 380 N. The angle that the hinge force makes above the horizontal is $\phi = \arctan\dfrac{H_y}{H_x} = \arctan\dfrac{162.1\text{ N}}{344.24\text{ N}} = 25.2°$.

**EVALUATE:** The hinge force points at 25.2° above the horizontal but the beam makes an angle of 22.0° below the horizontal, so the hinge force does not point along the beam. We never needed to know the length of the beam since it always canceled out in the equations.

**11.72. IDENTIFY:** Apply $\sum \tau_z = 0$ to the wheel.

**SET UP:** Take torques about the upper corner of the curb.

**EXECUTE:** The force $\vec{F}$ acts at a perpendicular distance $R-h$ and the weight acts at a perpendicular distance $\sqrt{R^2-(R-h)^2} = \sqrt{2Rh-h^2}$. Setting the torques equal for the minimum necessary force,

$$F = mg\frac{\sqrt{2Rh-h^2}}{R-h}.$$

**(b)** The torque due to gravity is the same, but the force $\vec{F}$ acts at a perpendicular distance $2R-h$, so the minimum force is $(mg)\sqrt{2Rh-h^2}/(2R-h)$.

**EVALUATE:** **(c)** Less force is required when the force is applied at the top of the wheel, since in this case $\vec{F}$ has a larger moment arm.

**11.73. IDENTIFY:** Apply the first and second conditions of equilibrium to the gate.

**SET UP:** The free-body diagram for the gate is given in Figure 11.73.

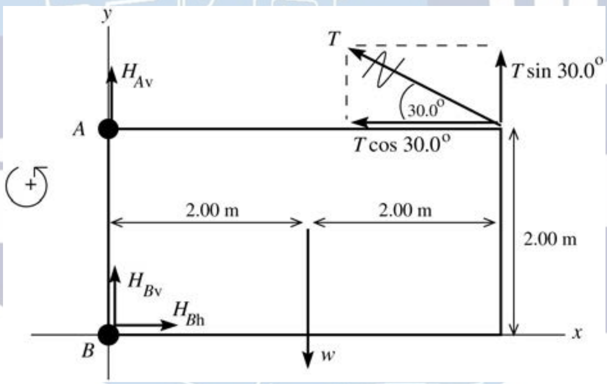



**Figure 11.73**

Use coordinates with the origin at $B$. Let $\vec{H}_A$ and $\vec{H}_B$ be the forces exerted by the hinges at $A$ and $B$. The problem states that $\vec{H}_A$ has no horizontal component. Replace the tension $\vec{T}$ by its horizontal and vertical components.

**EXECUTE:** **(a)** $\sum \tau_B = 0$ gives $+(T\sin 30.0°)(4.00\text{ m}) + (T\cos 30.0°)(2.00\text{ m}) - w(2.00\text{ m}) = 0$

$T(2\sin 30.0° + \cos 30.0°) = w$

$$T = \frac{w}{2\sin 30.0° + \cos 30.0°} = \frac{700\text{ N}}{2\sin 30.0° + \cos 30.0°} = 375\text{ N}.$$

**(b)** $\sum F_x = ma_x$ says $H_{Bh} - T\cos 30.0° = 0$

$H_{Bh} = T\cos 30.0° = (375\text{ N})\cos 30.0° = 325\text{ N}.$

**(c)** $\sum F_y = ma_y$ says $H_{Av} + H_{Bv} + T\sin 30.0° - w = 0$

$H_{Av} + H_{Bv} = w - T\sin 30.0° = 700\text{ N} - (375\text{ N})\sin 30.0° = 512\text{ N}.$

**EVALUATE:** $T$ has a horizontal component to the left so $H_{Bh}$ must be to the right, as these are the only two horizontal forces. Note that we cannot determine $H_{Av}$ and $H_{Bv}$ separately, only their sum.

**11.74. IDENTIFY:** Use $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$ to locate the $x$-coordinate of the center of gravity of the block combinations.

**SET UP:** The center of mass and the center of gravity are the same point. For two identical blocks, the center of gravity is midway between the center of the two blocks.

**EXECUTE:** **(a)** The center of gravity of the top block can be as far out as the edge of the lower block. The center of gravity of this combination is then $3L/4$ to the left of the right edge of the upper block, so the overhang is $3L/4$.

**(b)** Take the two-block combination from part (a), and place it on top of the third block such that the overhang of $3L/4$ is from the right edge of the third block; that is, the center of gravity of the first two blocks is above the right edge of the third block. The center of mass of the three-block combination, measured from the right end of the bottom block, is $-L/6$ and so the largest possible overhang is $(3L/4)+(L/6)=11L/12.$ Similarly, placing this three-block combination with its center of gravity over the right edge of the fourth block allows an extra overhang of $L/8$, for a total of $25L/24$.

**(c)** As the result of part (b) shows, with only four blocks, the overhang can be larger than the length of a single block.

**EVALUATE:** The sequence of maximum overhangs is $\dfrac{18L}{24}, \dfrac{22L}{24}, \dfrac{25L}{24}, \ldots$ The increase of overhang when one more block is added is decreasing.

**11.75.** **IDENTIFY:** Apply the first and second conditions of equilibrium, first to both marbles considered as a composite object and then to the bottom marble.

**(a) SET UP:** The forces on each marble are shown in Figure 11.75.

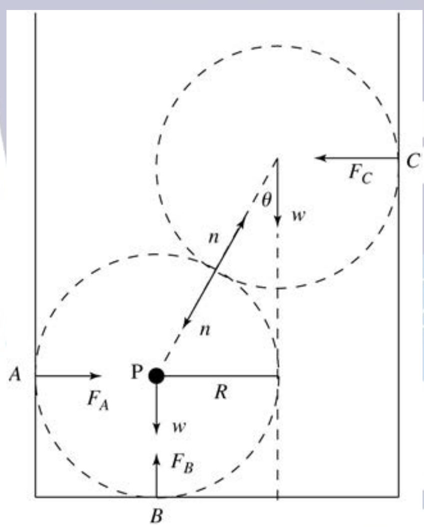



**Figure 11.75**

**EXECUTE:**

$F_B = 2w = 1.47$ N

$\sin\theta = R/2R$ so $\theta = 30°$

$\sum \tau_z = 0$, axis at $P$

$F_C(2R\cos\theta) - wR = 0$

$$F_C = \frac{mg}{2\cos 30°} = 0.424 \text{ N}$$

$F_A = F_C = 0.424$ N

**(b)** Consider the forces on the bottom marble. The horizontal forces must sum to zero, so $F_A = n\sin\theta$.

$$n = \frac{F_A}{\sin 30°} = 0.848 \text{ N}$$

Could use instead that the vertical forces sum to zero

$F_B - mg - n\cos\theta = 0$

$$n = \frac{F_B - mg}{\cos 30°} = 0.848 \text{ N, which checks.}$$

**EVALUATE:** If we consider each marble separately, the line of action of every force passes through the center of the marble so there is clearly no torque about that point for each marble. We can use the results we obtained to show that $\sum F_x = 0$ and $\sum F_y = 0$ for the top marble.

**11.76. IDENTIFY:** Apply $\sum \tau_z = 0$ to the right-hand beam.

**SET UP:** Use the hinge as the axis of rotation and take counterclockwise rotation as positive. If $F_{\text{wire}}$ is the tension in each wire and $w = 260\text{ N}$ is the weight of each beam, $2F_{\text{wire}} - 2w = 0$ and $F_{\text{wire}} = w$. Let $L$ be the length of each beam.

**EXECUTE:** **(a)** $\sum \tau_z = 0$ gives $F_{\text{wire}} L \sin\frac{\theta}{2} - F_{\text{c}}\frac{L}{2}\cos\frac{\theta}{2} - w\frac{L}{2}\sin\frac{\theta}{2} = 0$, where $\theta$ is the angle between the beams and $F_{\text{c}}$ is the force exerted by the cross bar. The length drops out, and all other quantities except $F_{\text{c}}$ are known, so $F_{\text{c}} = \dfrac{F_{\text{wire}}\sin(\theta/2) - \frac{1}{2}w\sin(\theta/2)}{\frac{1}{2}\cos(\theta/2)} = (2F_{\text{wire}} - w)\tan\dfrac{\theta}{2}$. Therefore

$$F_{\text{c}} = (260\text{ N})\tan\frac{53^\circ}{2} = 130\text{ N}.$$

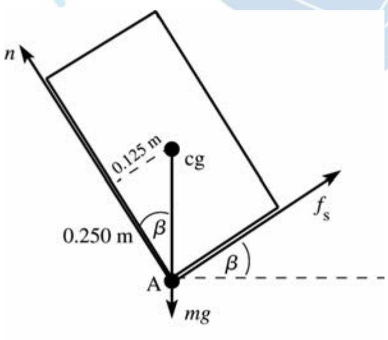
**(b)** The crossbar is under compression, as can be seen by imagining the behavior of the two beams if the crossbar were removed. It is the crossbar that holds them apart.

**(c)** The upward pull of the wire on each beam is balanced by the downward pull of gravity, due to the symmetry of the arrangement. The hinge therefore exerts no vertical force. It must, however, balance the outward push of the crossbar. The hinge exerts a force 130 N horizontally to the left for the right-hand beam and 130 N to the right for the left-hand beam. Again, it's instructive to visualize what the beams would do if the hinge were removed.

**EVALUATE:** The force exerted on each beam increases as $\theta$ increases and exceeds the weight of the beam for $\theta \geq 90^\circ$.

**11.77. IDENTIFY:** Apply the first and second conditions of equilibrium to the bale.

**(a) SET UP:** Find the angle where the bale starts to tip. When it starts to tip only the lower left-hand corner of the bale makes contact with the conveyor belt. Therefore the line of action of the normal force $n$ passes through the left-hand edge of the bale. Consider $\Sigma\tau_z = 0$ with point A at the lower left-hand corner. Then $\tau_n = 0$ and $\tau_f = 0$, so it must be that $\tau_{mg} = 0$ also. This means that the line of action of the gravity must pass through point A. Thus the free-body diagram must be as shown in Figure 11.77a.




**EXECUTE:** $\tan\beta = \dfrac{0.125\text{ m}}{0.250\text{ m}}$

$\beta = 27^\circ$, angle where tips

**Figure 11.77a**

**SET UP:** At the angle where the bale is ready to slip down the incline $f_{\text{s}}$ has its maximum possible value, $f_{\text{s}} = \mu_{\text{s}} n$. The free-body diagram for the bale, with the origin of coordinates at the cg is given in Figure 11.77b.

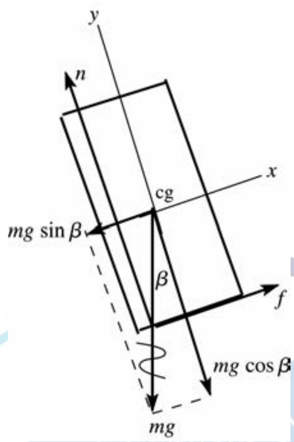



Figure 11.77b

**EXECUTE:**
$\sum F_y = ma_y$
$n - mg\cos\beta = 0$
$n = mg\cos\beta$
$f_s = \mu_s mg\cos\beta$
($f_s$ has maximum value when bale ready to slip)
$\sum F_x = ma_x$
$f_s - mg\sin\beta = 0$
$\mu_s mg\cos\beta - mg\sin\beta = 0$
$\tan\beta = \mu_s$
$\mu_s = 0.60$ gives that $\beta = 31°$

$\beta = 27°$ to tip; $\beta = 31°$ to slip, so tips first
**(b)** The magnitude of the friction force didn't enter into the calculation of the tipping angle; still tips at $\beta = 27°$. For $\mu_s = 0.40$ slips at $\beta = \arctan(0.40) = 22°$.
Now the bale will start to slide down the incline before it tips.
**EVALUATE:** With a smaller $\mu_s$ the slope angle $\beta$ where the bale slips is smaller.

**11.78.** **IDENTIFY:** Apply the equilibrium conditions to the pole. The horizontal component of the tension in the wire is 22.0 N.
**SET UP:** The free-body diagram for the pole is given in Figure 11.78. The tension in the cord equals the weight $W$. $F_v$ and $F_h$ are the components of the force exerted by the hinge. If either of these forces is actually in the opposite direction to what we have assumed, we will get a negative value when we solve for it.
**EXECUTE:** **(a)** $T\sin 37.0° = 22.0\text{ N}$ so $T = 36.6\text{ N}$. $\sum\tau_z = 0$ gives $(T\sin 37.0°)(1.75\text{ m}) - W(1.35\text{ m}) = 0$.

$$W = \frac{(22.0\text{ N})(1.75\text{ m})}{1.35\text{ m}} = 28.5\text{ N}.$$

**(b)** $\sum F_y = 0$ gives $F_v - T\cos 37.0° - w = 0$ and $F_v = (36.6\text{ N})\cos 37.0° + 55.0\text{ N} = 84.2\text{ N}$. $\sum F_x = 0$ gives $W - T\sin 37.0° - F_h = 0$ and $F_h = 28.5\text{ N} - 22.0\text{ N} = 6.5\text{ N}$. The magnitude of the hinge force is $F = \sqrt{F_h^2 + F_v^2} = 84.5\text{ N}$.
**EVALUATE:** If we consider torques about an axis at the top of the pole, we see that $F_h$ must be to the left in order for its torque to oppose the torque produced by the force $W$.

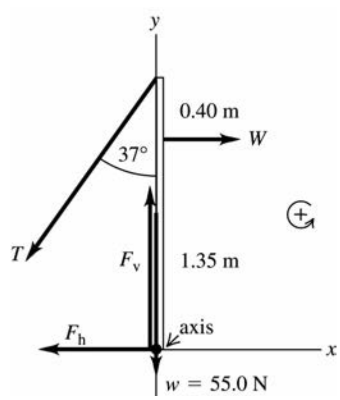



Figure 11.78

**11.79.** **IDENTIFY:** Apply the first and second conditions of equilibrium to the door.

**(a) SET UP:** The free-body diagram for the door is given in Figure 11.79.

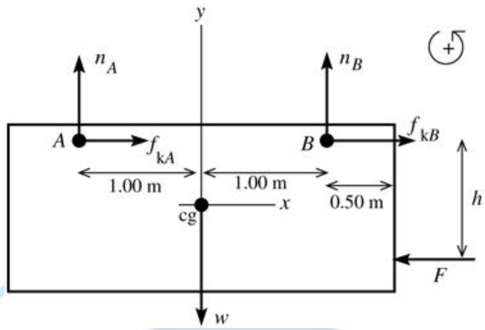



**Figure 11.79**

Take the origin of coordinates at the center of the door (at the cg). Let $n_A$, $f_{kA}$, $n_B$, and $f_{kB}$ be the normal and friction forces exerted on the door at each wheel.

**EXECUTE:** $\sum F_y = ma_y$

$n_A + n_B - w = 0$

$n_A + n_B = w = 950\text{ N}$

$\sum F_x = ma_x$

$f_{kA} + f_{kB} - F = 0$

$F = f_{kA} + f_{kB}$

$f_{kA} = \mu_k n_A$, $f_{kB} = \mu_k n_B$, so $F = \mu_k(n_A + n_B) = \mu_k w = (0.52)(950\text{ N}) = 494\text{ N}$

$\sum \tau_B = 0$

$n_B$, $f_{kA}$, and $f_{kB}$ all have zero moment arms and hence zero torque about this point.

Thus $+w(1.00\text{ m}) - n_A(2.00\text{ m}) - F(h) = 0$

$$n_A = \frac{w(1.00\text{ m}) - F(h)}{2.00\text{ m}} = \frac{(950\text{ N})(1.00\text{ m}) - (494\text{ N})(1.60\text{ m})}{2.00\text{ m}} = 80\text{ N}$$

And then $n_B = 950\text{ N} - n_A = 950\text{ N} - 80\text{ N} = 870\text{ N}$.

**(b) SET UP:** If $h$ is too large the torque of $F$ will cause wheel $A$ to leave the track. When wheel $A$ just starts to lift off the track $n_A$ and $f_{kA}$ both go to zero.

**EXECUTE:** The equations in part (a) still apply.

$n_A + n_B - w = 0$ gives $n_B = w = 950\text{ N}$

Then $f_{kB} = \mu_k n_B = 0.52(950\text{ N}) = 494\text{ N}$

$F = f_{kA} + f_{kB} = 494\text{ N}$

$+w(1.00\text{ m}) - n_A(2.00\text{ m}) - F(h) = 0$

$$h = \frac{w(1.00\text{ m})}{F} = \frac{(950\text{ N})(1.00\text{ m})}{494\text{ N}} = 1.92\text{ m}$$

**EVALUATE:** The result in part (b) is larger than the value of $h$ in part (a). Increasing $h$ increases the clockwise torque about $B$ due to $F$ and therefore decreases the clockwise torque that $n_A$ must apply.

**11.80. IDENTIFY:** Apply $\sum \tau_z = 0$ to the slab.

**SET UP:** The free-body diagram is given in Figure 11.80a. $\tan\beta = \dfrac{3.75\text{ m}}{1.75\text{ m}}$ so $\beta = 65.0°$.

$20.0° + \beta + \alpha = 90°$ so $\alpha = 5.0°$. The distance from the axis to the center of the block is

$$\sqrt{\left(\frac{3.75\text{ m}}{2}\right)^2 + \left(\frac{1.75\text{ m}}{2}\right)^2} = 2.07\text{ m}.$$

**EXECUTE:** **(a)** $w(2.07\text{ m})\sin 5.0° - T(3.75\text{ m})\sin 52.0° = 0.$ $T = 0.061w.$ Each worker must exert a force of $0.012w,$ where $w$ is the weight of the slab.

**(b)** As $\theta$ increases, the moment arm for $w$ decreases and the moment arm for $T$ increases, so the worker needs to exert less force.

**(c)** $T \to 0$ when $w$ passes through the support point. This situation is sketched in Figure 11.80b.

$\tan\theta = \dfrac{(1.75\text{ m})/2}{(3.75\text{ m})/2}$ and $\theta = 25.0°.$ If $\theta$ exceeds this value the gravity torque causes the slab to tip over.

**EVALUATE:** The moment arm for $T$ is much greater than the moment arm for $w$, so the force the workers apply is much less than the weight of the slab.

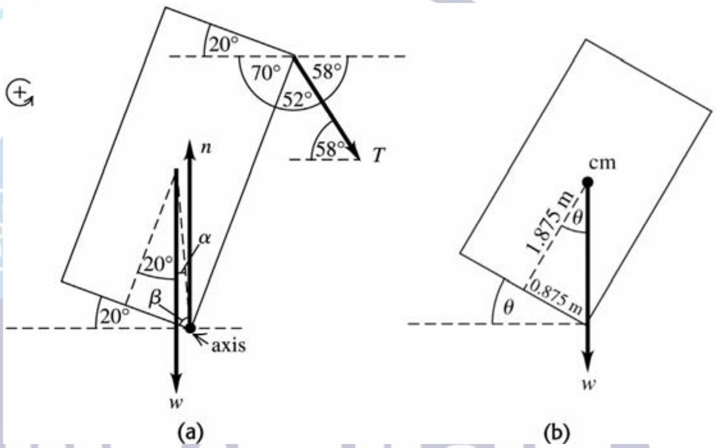
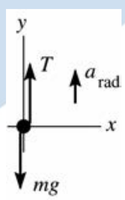



**Figure 11.80**

**11.81.** **IDENTIFY:** Apply Newton's second law to the mass to find the tension in the wire. Then apply $Y = \dfrac{l_0 F_\perp}{A\Delta l}$ to the wire to find the elongation this tensile force produces.

**(a) SET UP:** Calculate the tension in the wire as the mass passes through the lowest point. The free-body diagram for the mass is given in Figure 11.81a.




The mass moves in an arc of a circle with radius $R = 0.70$ m. It has acceleration $\vec{a}_{\text{rad}}$ directed in toward the center of the circle, so at this point $\vec{a}_{\text{rad}}$ is upward.

**Figure 11.81a**

**EXECUTE:** $\sum F_y = ma_y$

$T - mg = mR\omega^2$ so that $T = m(g + R\omega^2).$

But $\omega$ must be in rad/s:

$\omega = (120\text{ rev/min})(2\pi\text{ rad/1 rev})(1\text{ min/60 s}) = 12.57\text{ rad/s}.$

Then $T = (12.0\text{ kg})\left[9.80\text{ m/s}^2 + (0.70\text{ m})(12.57\text{ rad/s})^2\right] = 1445\text{ N}.$

Now calculate the elongation $\Delta l$ of the wire that this tensile force produces:

$$Y = \frac{F_\perp l_0}{A\Delta l} \text{ so } \Delta l = \frac{F_\perp l_0}{YA} = \frac{(1445\text{ N})(0.70\text{ m})}{(7.0\times 10^{10}\text{ Pa})(0.014\times 10^{-4}\text{ m}^2)} = 0.0103\text{ m} = 1.0\text{ cm}.$$

**(b) SET UP:** The acceleration $\vec{a}_{rad}$ is directed in toward the center of the circular path, and at this point in the motion this direction is downward. The free-body diagram is given in Figure 11.81b.

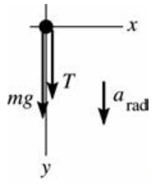



Figure 11.81b

**EXECUTE:**
$\sum F_y = ma_y$
$mg + T = mR\omega^2$
$T = m(R\omega^2 - g)$

$T = (12.0\text{ kg})\left[(0.70\text{ m})(12.57\text{ rad/s})^2 - 9.80\text{ m/s}^2\right] = 1210\text{ N}.$

$\Delta l = \dfrac{F_\perp l_0}{YA} = \dfrac{(1210\text{ N})(0.70\text{ m})}{(7.0\times 10^{10}\text{ Pa})(0.014\times 10^{-4}\text{ m}^2)} = 8.6\times 10^{-3}\text{ m} = 0.86\text{ cm}.$

**EVALUATE:** At the lowest point $T$ and $w$ are in opposite directions and at the highest point they are in the same direction, so $T$ is greater at the lowest point and the elongation is greatest there. The elongation is at most 1.4% of the length.

**11.82. IDENTIFY:** For a spring, $F = kx$. $Y = \dfrac{F_\perp l_0}{A\Delta l}$.

**SET UP:** $F_\perp = F = W$ and $\Delta l = x$. For copper, $Y = 11\times 10^{10}$ Pa.

**EXECUTE:** **(a)** $F = \left(\dfrac{YA}{l_0}\right)\Delta l = \left(\dfrac{YA}{l_0}\right)x$. This in the form of $F = kx$, with $k = \dfrac{YA}{l_0}$.

**(b)** $k = \dfrac{YA}{l_0} = \dfrac{(11\times 10^{10}\text{ Pa})\pi(6.455\times 10^{-4}\text{ m})^2}{0.750\text{ m}} = 1.9\times 10^5\text{ N/m}$

**(c)** $W = kx = (1.9\times 10^5\text{ N/m})(1.25\times 10^{-3}\text{ m}) = 240\text{ N}$

**EVALUATE:** For the wire the force constant is very large, much larger than for a typical spring.

**11.83. IDENTIFY:** Use the second condition of equilibrium to relate the tension in the two wires to the distance $w$ is from the left end. Use stress $= \dfrac{F_\perp}{A}$ and $Y = \dfrac{l_0 F_\perp}{A\Delta l}$ to relate the tension in each wire to its stress and strain.

**(a) SET UP:** stress $= F_\perp/A$, so equal stress implies $T/A$ same for each wire.

$T_A/2.00\text{ mm}^2 = T_B/4.00\text{ mm}^2$ so $T_B = 2.00T_A$

The question is where along the rod to hang the weight in order to produce this relation between the tensions in the two wires. Let the weight be suspended at point $C$, a distance $x$ to the right of wire $A$. The free-body diagram for the rod is given in Figure 11.83.

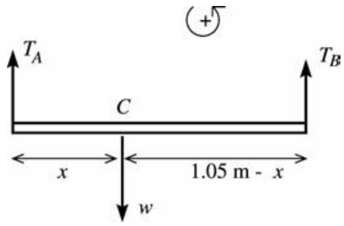



Figure 11.83

**EXECUTE:**
$\sum \tau_C = 0$
$+T_B(1.05\text{ m} - x) - T_A x = 0$

But $T_B = 2.00T_A$ so $2.00T_A(1.05\text{ m} - x) - T_A x = 0$

$2.10\text{ m} - 2.00x = x$ and $x = 2.10\text{ m}/3.00 = 0.70\text{ m}$ (measured from $A$).

**(b) SET UP:** $Y = \text{stress/strain}$ gives that strain $= \text{stress}/Y = F_\perp/AY$.

**EXECUTE:** Equal strain thus implies

$$\frac{T_A}{(2.00\text{ mm}^2)(1.80\times10^{11}\text{ Pa})}=\frac{T_B}{(4.00\text{ mm}^2)(1.20\times10^{11}\text{ Pa})}$$

$$T_B=\left(\frac{4.00}{2.00}\right)\left(\frac{1.20}{1.80}\right)T_A=1.333T_A.$$

The $\sum\tau_C=0$ equation still gives $T_B(1.05\text{ m}-x)-T_Ax=0.$

But now $T_B=1.333T_A$ so $(1.333T_A)(1.05\text{ m}-x)-T_Ax=0.$

$1.40\text{ m}=2.33x$ and $x=1.40\text{ m}/2.33=0.60\text{ m}$ (measured from $A$).

**EVALUATE:** Wire $B$ has twice the diameter so it takes twice the tension to produce the same stress. For equal stress the moment arm for $T_B$ (0.35 m) is half that for $T_A$ (0.70 m), since the torques must be equal. The smaller $Y$ for $B$ partially compensates for the larger area in determining the strain and for equal strain the moment arms are closer to being equal.

**11.84. IDENTIFY:** Apply $Y=\dfrac{l_0F_\perp}{A\Delta l}$ and calculate $\Delta l$.

**SET UP:** When the ride is at rest the tension $F_\perp$ in the rod is the weight 1900 N of the car and occupants. When the ride is operating, the tension $F_\perp$ in the rod is obtained by applying $\sum\vec{F}=m\vec{a}$ to a car and its occupants. The free-body diagram is shown in Figure 11.84. The car travels in a circle of radius $r=l\sin\theta$, where $l$ is the length of the rod and $\theta$ is the angle the rod makes with the vertical. For steel, $Y=2.0\times10^{11}$ Pa. $\omega=12.0\text{ rev/min}=1.2566\text{ rad/s}.$

**EXECUTE:** **(a)** $\Delta l=\dfrac{l_0F_\perp}{YA}=\dfrac{(15.0\text{ m})(1900\text{ N})}{(2.0\times10^{11}\text{ Pa})(8.00\times10^{-4}\text{ m}^2)}=1.78\times10^{-4}\text{ m}=0.18\text{ mm}$

**(b)** $\sum F_x=ma_x$ gives $F_\perp\sin\theta=mr\omega^2=ml\sin\theta\omega^2$ and

$$F_\perp=ml\omega^2=\left(\frac{1900\text{ N}}{9.80\text{ m/s}^2}\right)(15.0\text{ m})(1.2566\text{ rad/s})^2=4.592\times10^3\text{ N}.$$

$$\Delta l=\left(\frac{4.592\times10^3\text{ N}}{1900\text{ N}}\right)(0.18\text{ mm})=0.44\text{ mm}.$$

**EVALUATE:** $\sum F_y=ma_y$ gives $F_\perp\cos\theta=mg$ and $\cos\theta=mg/F_\perp$. As $\omega$ increases $F_\perp$ increases and $\cos\theta$ becomes small. Smaller $\cos\theta$ means $\theta$ increases, so the rods move toward the horizontal as $\omega$ increases.

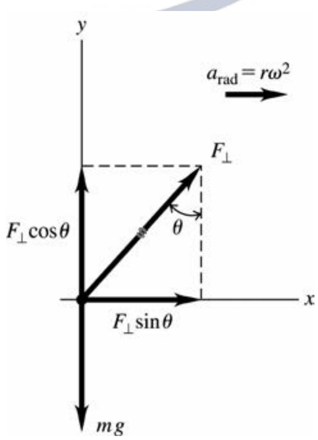



**Figure 11.84**

**11.85.** **IDENTIFY:** Apply $\dfrac{F_\perp}{A}=Y\left(\dfrac{\Delta l}{l_0}\right)$. The height from which he jumps determines his speed at the ground. The acceleration as he stops depends on the force exerted on his legs by the ground.

**SET UP:** In considering his motion take $+y$ downward. Assume constant acceleration as he is stopped by the floor.

**EXECUTE:** **(a)** $F_\perp = YA\left(\dfrac{\Delta l}{l_0}\right)=(3.0\times10^{-4}\ \text{m}^2)(14\times10^9\ \text{Pa})(0.010)=4.2\times10^4\ \text{N}$

**(b)** As he is stopped by the ground, the net force on him is $F_{\text{net}}=F_\perp - mg$, where $F_\perp$ is the force exerted on him by the ground. From part (a), $F_\perp = 2(4.2\times10^4\ \text{N})=8.4\times10^4\ \text{N}$ and

$F=8.4\times10^4\ \text{N}-(70\ \text{kg})(9.80\ \text{m/s}^2)=8.33\times10^4\ \text{N}$. $F_{\text{net}}=ma$ gives $a=1.19\times10^3\ \text{m/s}^2$.

$a_y=-1.19\times10^3\ \text{m/s}^2$ since the acceleration is upward. $v_y=v_{0y}+a_yt$ gives

$v_{0y}=-a_yt=(-1.19\times10^3\ \text{m/s}^2)(0.030\ \text{s})=35.7\ \text{m/s}$. His speed at the ground therefore is $v=35.7\ \text{m/s}$.

This speed is related to his initial height $h$ above the floor by $\frac{1}{2}mv^2=mgh$ and

$$h=\frac{v^2}{2g}=\frac{(35.7\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)}=65\ \text{m}.$$

**EVALUATE:** Our estimate is based solely on compressive stress; other injuries are likely at a much lower height.

**11.86.** **IDENTIFY:** The graph gives the change in length of the wire as a function of the weight hanging from it, which is equal to the tension in the wire. Young's modulus $Y$ applies to the stretching of the wire. Energy conservation and Newton's second law apply to the swinging sphere.

**SET UP:** $Y=\dfrac{l_0F_\perp}{A\Delta l}$, $K_1+U_1=K_2+U_2$, $\Sigma\vec{F}=m\vec{a}$, $a_{\text{rad}}=v^2/R$.

**EXECUTE:** **(a)** Solve $Y=\dfrac{l_0F_\perp}{A\Delta l}$ for $\Delta l$ and realize that $F_\perp=mg$: $\Delta l=\dfrac{gl_0}{AY}m$. Therefore, in the graph of $\Delta l$ versus $m$, the slope is equal to $gl_0/AY$. The equation of the graph is given in the problem as $\Delta l=(0.422\ \text{mm/kg})m$, so the slope is 0.422 mm/kg, so $gl_0/AY = 0.422$ mm/kg $= 4.22\times10^{-4}$ m/kg. Solving for $Y$ gives $Y=\dfrac{gl_0}{A(4.22\times10^{-4}\ \text{m/kg})}$. Using $A=\pi r^2$ and putting in the given numbers gives

$$Y=\frac{(9.80\ \text{m/s})(22.0\ \text{m})}{\pi(4.30\times10^{-4}\ \text{m})^2(4.22\times10^{-4}\ \text{m/kg})}=8.80\times10^{11}\ \text{Pa}.$$

**(b)** Use energy conservation to find the speed of the sphere. $K_1+U_1=K_2+U_2$ gives

$mgL(1-\cos\theta)=\dfrac{1}{2}mv^2$. Solving for $v$ using $\theta=36.0°$ and $L=22.0$ m gives $v=9.075$ m/s.

Now apply Newton's second law to the sphere at the bottom of the swing. $\Sigma\vec{F}=m\vec{a}$ and $a_{\text{rad}}=v^2/R$ give $T-mg=mv^2/L$, so $T=mv^2/L+mg=(9.50\ \text{kg})(9.075\ \text{m/s})^2/(22.0\ \text{m})+(9.50\ \text{kg})(9.80\ \text{m/s}^2)=129$ N. Using the value of $Y$ found in part (a), we have

$$\Delta l=\frac{F_\perp l_0}{AY}=(129\ \text{N})(22.0\ \text{m})/[\pi(4.30\times10^{-4}\ \text{m})^2(8.80\times10^{11}\ \text{Pa})]=0.00554\ \text{m}=5.54\ \text{mm}.$$

**EVALUATE:** For a wire 22 m long, 5.5 mm is a very small stretch, 0.0055/22 = 0.025%.

**11.87.** **IDENTIFY:** The bar is at rest, so the forces and torques on it must all balance.

**SET UP:** $\Sigma F_y=0$, $\sum\tau_z=0$.

**EXECUTE:** **(a)** The free-body diagram is shown in Figure 11.87a, where $F_p$ is the force due to the knife-edge pivot.

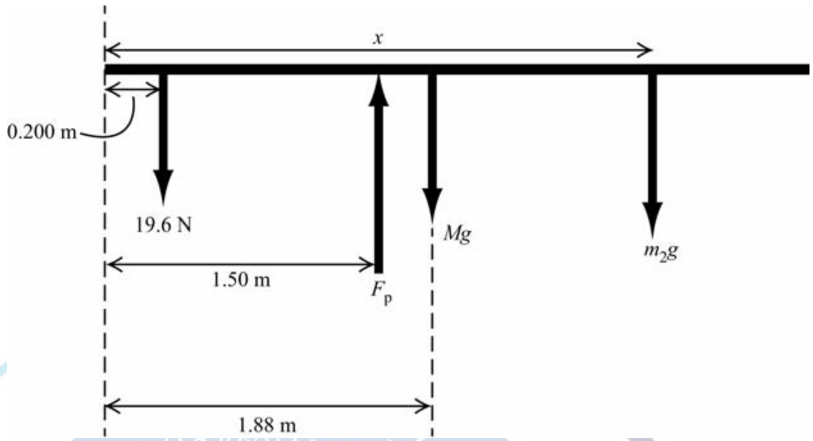



**Figure 11.87a**

**(b)** $\sum\tau_z = 0$, with torques taken about the location of the knife-edge pivot, gives
$(2.00\text{ kg})g(1.30\text{ m}) - Mg(0.38\text{ m}) - m_2g(x - 1.50\text{ m}) = 0$
Solving for $x$ gives
$x = [(2.00\text{ kg})(1.30\text{ m}) - M(0.38\text{ m})](1/m_2) + 1.50\text{ m}$
The graph of this equation ($x$ versus $1/m_2$) is a straight line of slope $[(2.00\text{ kg})(1.30\text{ m}) - M(0.38\text{ m})]$.
**(c)** The plot of $x$ versus $1/m_2$ is shown in Figure 11.87b. The equation of the best-fit line is $x = (1.9955\text{ m}\cdot\text{kg})/m_2 + 1.504\text{ m}$. The slope of the best-fit line is $1.9955\text{ m}\cdot\text{kg}$, so $[(2.00\text{ kg})(1.30\text{ m}) - M(0.38\text{ m})] = 1.9955\text{ m}\cdot\text{kg}$, which gives $M = 1.59$ kg.

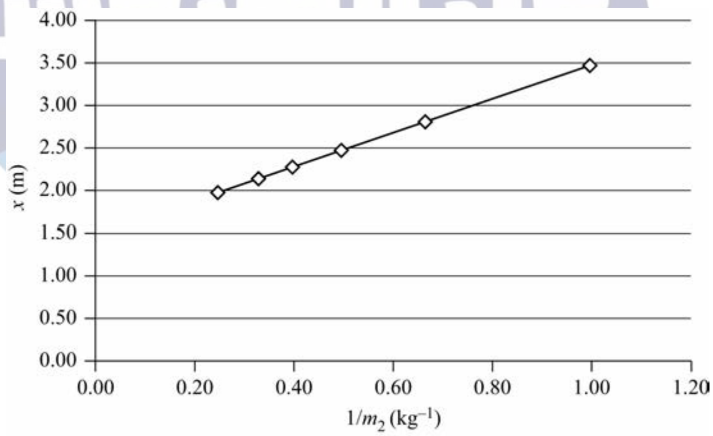



**Figure 11.87b**

**(d)** The $y$-intercept of the best-fit line is 1.50 m. This is plausible. As the graph approaches the $y$-axis, $1/m_2$ approaches zero, which means that $m_2$ is getting extremely large. In that case, it would be much larger than any other masses involved, so to balance the system, $m_2$ would have to be at the knife-point pivot, which is at $x = 1.50$ m.

**EVALUATE:** The fact that the graph gave a physically plausible result in part (d) suggests that this graphical analysis is reasonable.

**11.88. IDENTIFY:** The bar is at rest, so the forces and torques on it must all balance.

**SET UP:** $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\sum\tau_z = 0$.

**EXECUTE:** **(a)** Take torques about the hinge, calling $m$ the mass of the bar and $L$ its length. $\sum\tau_z = 0$ gives $xT\sin\theta = mg\dfrac{L}{2}$. Solving for $T$ gives $T = \dfrac{mgL/2}{x\sin\theta}$. Therefore the alternative having the largest value of $x\sin\theta$ will have the smallest tension, and the one with the smallest value of $x\sin\theta$ will have the greatest tension. Calculating $x\sin\theta$ for each alternative gives the following values. A: 1.00 m, B: 1.30 m, C: 0.451 m, D: 0.483 m. Therefore alternative B gives the smallest tension and C produces the largest tension

**(b)** Calling $H$ the magnitude of the hinge force, $\Sigma F_x = 0$ gives $H_x = T\cos\theta$. Using the value of $T$ from part (a), we get $H_x = \dfrac{mg\,L/2}{x\sin\theta}.\cos\theta = \dfrac{mg\,L/2}{x\tan\theta}$. From this result, we can see that $H_x$ is greatest when $x\tan\theta$ is the smallest, and $H_x$ is least when $x\tan\theta$ is greatest. Calculating $x\tan\theta$ for each alternative gives A: 1.15 m, B: 2.60 m, C: 0.565 m, D: 1.87 m. Therefore alternative C gives the greatest $H_x$ and B gives the smallest $H_x$.

**(c)** Taking torques about the point where the cable is connected to the bar, $\sum\tau_z = 0$ gives $H_y x = mg(x - L/2)$. Solving for $H_y$ gives $H_y = mg(1 - L/2x)$. Since $H_y$ could be positive or negative, we should calculate all four possibilities. For alternative A, we have $H_y = mg\left(1 - \dfrac{2.00\text{ m}}{4.00\text{ m}}\right) = 0.500mg$. For B we have $H_y = mg\left(1 - \dfrac{2.00\text{ m}}{3.00\text{ m}}\right) = 0.333mg$, and likewise we get $H_y = -0.333mg$ for C and $H_y = -1.00mg$ for D. Therefore alternative D gives the largest $H_y$ and B and C both give the smallest value.

**(d)** Alternative B is clearly optimal because it results in the smallest values for $T$, $H_x$, and $H_y$. It might be a good idea to avoid alternative C because it has the greatest $T$ and $H_x$.

**Evaluate:** As a check, part (c) could be solved by using $\Sigma F_y = 0$.

**11.89.** **Identify:** Apply the equilibrium conditions to the ladder combination and also to each ladder.

**Set Up:** The geometry of the 3-4-5 right triangle simplifies some of the intermediate algebra. Denote the forces on the ends of the ladders by $F_L$ and $F_R$ (left and right). The contact forces at the ground will be vertical, since the floor is assumed to be frictionless.

**Execute:** **(a)** Taking torques about the right end, $F_L(5.00\text{ m}) = (480\text{ N})(3.40\text{ m}) + (360\text{ N})(0.90\text{ m})$, so $F_L = 391\text{ N}$. $F_R$ may be found in a similar manner, or from $F_R = 840\text{ N} - F_L = 449\text{ N}$.

**(b)** The tension in the rope may be found by finding the torque on each ladder, using the point $A$ as the origin. The lever arm of the rope is 1.50 m. For the left ladder, $T(1.50\text{ m}) = F_L(3.20\text{ m}) - (480\text{ N})(1.60\text{ m})$, so $T = 322.1\text{ N}$ (322 N to three figures). As a check, using the torques on the right ladder, $T(1.50\text{ m}) = F_R(1.80\text{ m}) - (360\text{ N})(0.90\text{ m})$ gives the same result.

**(c)** The horizontal component of the force at $A$ must be equal to the tension found in part (b). The vertical force must be equal in magnitude to the difference between the weight of each ladder and the force on the bottom of each ladder, $480\text{ N} - 391\text{ N} = 449\text{ N} - 360\text{ N} = 89\text{ N}$. The magnitude of the force at $A$ is then $\sqrt{(322.1\text{ N})^2 + (89\text{ N})^2} = 334\text{ N}$.

**(d)** The easiest way to do this is to see that the added load will be distributed at the floor in such a way that $F_L' = F_L + (0.36)(800\text{ N}) = 679\text{ N}$, and $F_R' = F_R + (0.64)(800\text{ N}) = 961\text{ N}$. Using these forces in the form for the tension found in part (b) gives

$$T = \frac{F'_L(3.20\text{ m}) - (480\text{ N})(1.60\text{ m})}{(1.50\text{ m})} = \frac{F'_R(1.80\text{ m}) - (360\text{ N})(0.90\text{ m})}{(1.50\text{ m})} = 937\text{ N}.$$

**Evaluate:** The presence of the painter increases the tension in the rope, even though his weight is vertical and the tension force is horizontal.

**11.90.** **Identify:** Apply $\sum\tau_z = 0$ to the post, for various choices of the location of the rotation axis.

**Set Up:** When the post is on the verge of slipping, $f_s$ has its largest possible value, $f_s = \mu_s n$.

**Execute:** **(a)** Taking torques about the point where the rope is fastened to the ground, the lever arm of the applied force is $h/2$ and the lever arm of both the weight and the normal force is $h\tan\theta$, and so

$$F\frac{h}{2} = (n - w)h\tan\theta.$$

Taking torques about the upper point (where the rope is attached to the post), $fh = F\frac{h}{2}$. Using $f \le \mu_s n$ and solving for $F$, $F \le 2w\left(\frac{1}{\mu_s} - \frac{1}{\tan\theta}\right)^{-1} = 2(400\text{ N})\left(\frac{1}{0.30} - \frac{1}{\tan 36.9°}\right)^{-1} = 400\text{ N}.$

**(b)** The above relations between $F$, $n$ and $f$ become $F\frac{3}{5}h = (n-w)h\tan\theta, f = \frac{2}{5}F,$ and eliminating $f$ and $n$ and solving for $F$ gives $F \le w\left(\frac{2/5}{\mu_s} - \frac{3/5}{\tan\theta}\right)^{-1}$, and substitution of numerical values gives 750 N to two figures.

**(c)** If the force is applied a distance $y$ above the ground, the above relations become $Fy = (n-w)h\tan\theta,\ F(h-y) = fh,$ which become, on eliminating $n$ and $f$, $w \ge F\left[\frac{(1-y/h)}{\mu_s} - \frac{(y/h)}{\tan\theta}\right].$

As the term in square brackets approaches zero, the necessary force becomes unboundedly large. The limiting value of $y$ is found by setting the term in square brackets equal to zero. Solving for $y$ gives
$$\frac{y}{h} = \frac{\tan\theta}{\mu_s + \tan\theta} = \frac{\tan 36.9°}{0.30 + \tan 36.9°} = 0.71.$$

**EVALUATE:** For the post to slip, for an axis at the top of the post the torque due to $F$ must balance the torque due to the friction force. As the point of application of $F$ approaches the top of the post, its moment arm for this axis approaches zero.

**11.91.** **IDENTIFY:** Apply $Y = \frac{l_0 F_\perp}{A\Delta l}$ to calculate $\Delta l$.

**SET UP:** For steel, $Y = 2.0\times10^{11}$ Pa.

**EXECUTE:** **(a)** From $Y = \frac{l_0 F_\perp}{A\Delta l}$, $\Delta l = \frac{(4.50\text{ kg})(9.80\text{ m/s}^2)(1.50\text{ m})}{(20\times10^{10}\text{ Pa})(5.00\times10^{-7}\text{ m}^2)} = 6.62\times10^{-4}$ m, or 0.66 mm to two figures.

**(b)** $(4.50\text{ kg})(9.80\text{ m/s}^2)(0.0500\times10^{-2}\text{ m}) = 0.022$ J.

**(c)** The magnitude $F$ will vary with distance; the average force is $YA(0.0250\text{ cm}/l_0) = 16.7$ N, and so the work done by the applied force is $(16.7\text{ N})(0.0500\times10^{-2}\text{ m}) = 8.35\times10^{-3}$ J.

**(d)** The average force the wire exerts is $(4.50\text{ kg})g + 16.7\text{ N} = 60.8$ N. The work done is negative, and equal to $-(60.8\text{ N})(0.0500\times10^{-2}\text{ m}) = -3.04\times10^{-2}$ J.

**(e)** The equation $Y = \frac{l_0 F_\perp}{A\Delta l}$ can be put into the form of Hooke's law, with $k = \frac{YA}{l_0}$. $U_{el} = \frac{1}{2}kx^2$, so $\Delta U_{el} = \frac{1}{2}k(x_2^2 - x_1^2)$. $x_1 = 6.62\times10^{-4}$ m and $x_2 = 0.500\times10^{-3}\text{ m} + x_1 = 11.62\times10^{-4}$ m. The change in elastic potential energy is
$$\frac{(20\times10^{10}\text{ Pa})(5.00\times10^{-7}\text{ m}^2)}{2(1.50\text{ m})}\left[(11.62\times10^{-4}\text{ m})^2 - (6.62\times10^{-4}\text{ m})^2\right] = 3.04\times10^{-2}\text{ J},$$ the negative of the result of part (d).

**EVALUATE:** The tensile force in the wire is conservative and obeys the relation $W = -\Delta U$.

**11.92.** **IDENTIFY and SET UP:** The forces and torques on the competitor must balance, so $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\sum \tau_z = 0$.

**EXECUTE:** Take torques about his feet, giving $(T_1 - T_2)(1.5\text{ m})(\cos 30°) = mg(1.0\text{ m})(\sin 30°)$. Solving for $T_2$ gives $T_2 = 1160\text{ N} - [(80.0\text{ kg})(9.80\text{ m/s}^2)/(1.5\text{ m})]\tan 30° = 858\text{ N} \approx 860$ N, which is choice (c).

**EVALUATE:** We find $T_2 < T_1$ as expected.

**11.93.** **IDENTIFY and SET UP:** The forces and torques on the competitor must balance, so $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\sum \tau_z = 0$.

**EXECUTE:** As in the previous problem, $T_1 - T_2$ is proportional to $\tan\theta$, so as $\theta$ increases, so does $\tan\theta$ and so does $T_1 - T_2$, which makes choice (a) correct.

**EVALUATE:** The result is physically reasonable. As he leans back, the ropes get lower, which reduces their moment arm, and his weight also gets lower, which increases its moment arm. Therefore to keep balance, the diffrerence in the tensions must be greater than before.

**11.94.** **IDENTIFY and SET UP:** Apply $\tau = Fl$.

**EXECUTE:** The moment arm for $T_1$ has increased, so $T_1$ can be smaller and still produce the same torque needed to balance the torque due to gravity, so choice (c) is correct.

**EVALUATE:** If the rope is held too high, it will be hard for the competitor to hold it, so there is a limit on how much the holding height can be effectively increased.

**11.95.** **IDENTIFY and SET UP:** The competitor will slip if the static friction force would need to be greater than its maximum possible value. $f_s^{max} = \mu_s n$.

**EXECUTE:** From earlier work, we know that $T_1 - T_2 = 1160\text{ N} - 858\text{ N} = 302\text{ N}$. The maximum static friction force is $f_s^{max} = \mu_s n = (0.50)(80.0\text{ kg})(9.80\text{ m/s}^2) = 392\text{ N}$. He needs only 302 N to balance the tension difference, yet the static friction force could be as great as 392 N, so he is not even ready to slip. Therefore he will not move, choice (d).

**EVALUATE:** The friction force is 302 N, not 392 N, because he is not just ready to slip.