دفتر

Physics 1

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اسألني الهندسة 2014

خدمة الطالب عبادة

١٤ / / التاريخ .... عنوان الدرس

Chapter (1) Dimensional Analysis

→ $U = U_0 + a X$ هل هذه العلاقة صحيحة ؟؟

$m/s \quad m/s \quad m/s^2 \; * m$

اذن هذه العلاقة ليست صحيحة لعدم تساوي الوحدات

→ $y = (2m) \cos kx$

$m \quad m \quad$ dimensionless ليس لها وحدة

Ex :- show that the expression $U = at$ is dimensionally correct :-

$$U = a\,t \rightarrow so \ \frac{m}{s} = \frac{m}{s}$$

$\frac{m}{s} \quad \frac{m}{s\,s^2} \quad s$

so $U = at$ is correct

→ $\overline{U}_x = \frac{\Delta X}{\Delta t}$ = $\frac{\text{Particle's displacement}}{\text{time interval}}$ vector quantity

average velocity متوسط سرعة

coordinate of particular

increases in time: $(X_f > X_i) \rightarrow \Delta X$ positive, so $\overline{U}_x$ positive, so (motion in positive x direction)

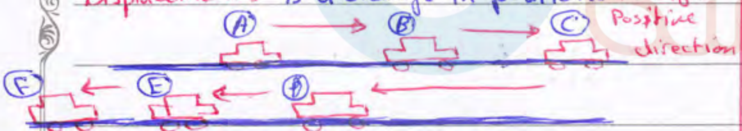
decreases in time: $(X_f < X_i) \rightarrow \Delta X$ negative, so → $\overline{U}_x$ negative, so (motion in negative X direction)

→ Average Speed $= \frac{\text{totall distance}}{\text{totall time}}$ → scalar quantity

Chapter (2) Motion in one dimension

(2.1) Position, Velocity & speed Page (24)

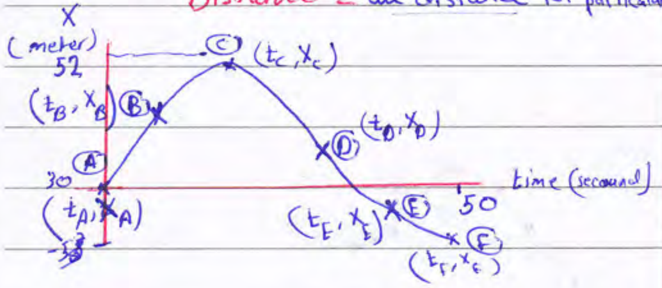
Displacement :- is a change in position.



so → Displacement $= \Delta X = X_f - X_i$

$= F - A$

Distance = all distance for particular



vector → physical quantity that requires the specification of bot < direction, magnitude

scalar → quantity that has only magnitude

→ Velocity, Displacement, Acceleration

→ Distance

→ In past example find (between A & F) displacement, $\overline{U}_x$, average speed

→ $\Delta X = X_f - X_i = -53 - 30 = -83$ m

→ $\overline{U}_x = \frac{\Delta X}{\Delta t} = \frac{-83}{(50.0)} = -1.66$ m/s

→ Average speed $= \frac{(52-30)+(52)+(53)}{(50)} = 2.54$ m/s

2.2 Instantenous velocity & speed.

→ Instantenous velocity

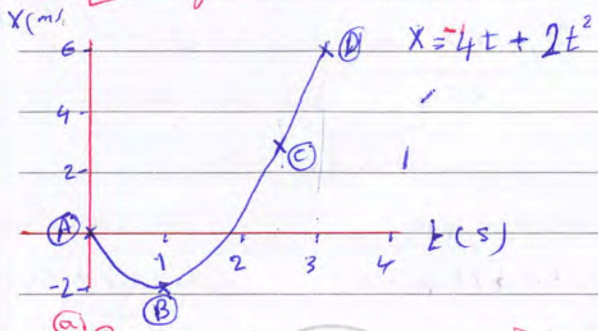
$$U_x = \lim_{\Delta t \to 0} \frac{\Delta X}{\Delta t} = \frac{dx}{dt}$$

→ Instanteneous speed

$|U_x|$ → maginitude of instantenous velocity.

→ Instantenous velocity has (no direction.)

Ex [2.2] page 29



$X = -4t + 2t^2$

ⓐ → Determine the displacement of the particular in the time interval

① $(t=0 \rightarrow t=1)$

$\Delta X = X_{(t=1)} - X_{(t=0)} = -2 - 0 = -2\ m$

② $(t=1 \rightarrow t=3)$

$\Delta X = X_{(t=3)} - X_{(t=1)} = 6 - (-2) = 8\ m$

→ ⓑ Find average velocity to same intervals

① $(0-1)s$

$\overline{U}_x = \frac{\Delta X}{\Delta t} = \frac{-2}{(1-0)} = -2\ m/s$

② $(1-3)s$

$\overline{U}_x = \frac{\Delta X}{\Delta t} = \frac{8}{(3-1)} = 4\ m/s$

→ ⓒ Find instanteneous velocity at:

$\boxed{t=1}$ $U_x = \frac{d}{dt}(-4t + 2t^2) = -4 + 4t \Big|_{t=1} = -4 + 4 = 0\ m/s$

$\boxed{t=2}$ $U_x \Big|_{t=2} = -4 + 4t = -4 + 8 = 4\ m/s$

$\boxed{t=2.5}$ $U_x \Big|_{t=2.5} = -4 + 4(2.5) = 6\ m/s$

$\boxed{t=1.5}$ $U_x \Big|_{t=1.5} = -4 + 4(1.5) = 2\ m/s$

2.3 Acceleration

$$\overline{a}_x = \frac{\Delta U_x}{\Delta t} = \frac{U_{xf} - U_{xi}}{t_f - t_i}$$ → average acceleration

$\boxed{t_i = 0s}$

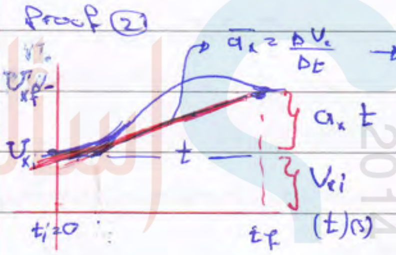
so $\overline{a}_x = \frac{U_f - U_i}{t}$ proof ①

$\boxed{U_{xf} = U_{xi} + \overline{a}_x t}$ → that is right at constant accelration

① معادلة

$\overline{a}_x = a_x$

Proof ②



→ slope $= a_x$

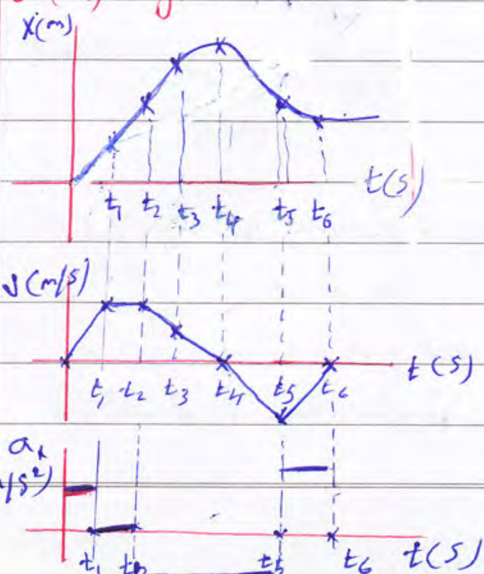
→ so $\boxed{U_{xf} = U_{xi} + a_x t}$ at constant acceleration

Instanteneous acceleration

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta U_x}{\Delta t} = \frac{dU_x}{dt}$$

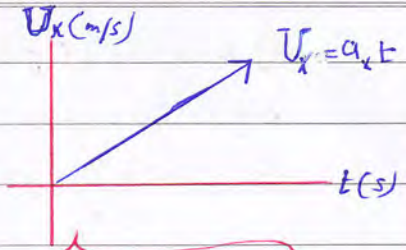
note → $a_x = \frac{dU_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$

Ex(2.3) Page 32



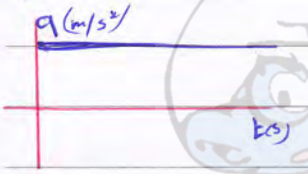
Ex (مثال)

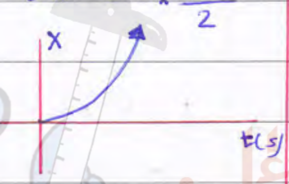
we have →



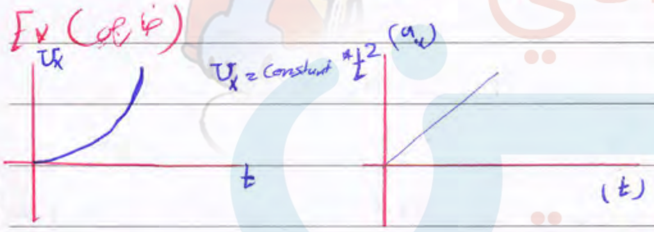
$$\int \frac{dx}{dt} = \int a_x t$$

$$\Delta X = a_x \frac{t^2}{2}$$





Ex (مثال)



$$\frac{d\,U_x}{dt} = \frac{d\,\text{Const.}\,t^2}{dt} \xrightarrow{\text{اشتقاق}} a_x = 2\,\text{Cons.}\,t$$

Kinematic expressions

$$U_{xf} = U_{xi} + a_x t \quad \text{— (1)} \quad \boxed{2.5} + \boxed{2.7}$$

$$\overline{U}_x = \frac{\Delta X}{t_f - t_i} = \frac{U_{xi} + U_{xf}}{2}$$

نعوض عنها بدلالة معادلة ①

t_i = تقريبا صفر

$$\frac{\Delta X}{t} = \frac{U_{xi}}{2} + \frac{U_{xi}}{2} + \frac{a_x t}{2}$$

$$\left(\frac{\Delta X}{t} = U_{xi} + \frac{a_x t}{2}\right) t$$

$$\Delta X = U_{xi}\, t + \frac{1}{2} a_x t^2 \quad \text{— (2)}$$

$$\boxed{t = \frac{U_{xf} - U_{xi}}{a_x}}$$ من معادلة ①

نعوض في معادلة ②

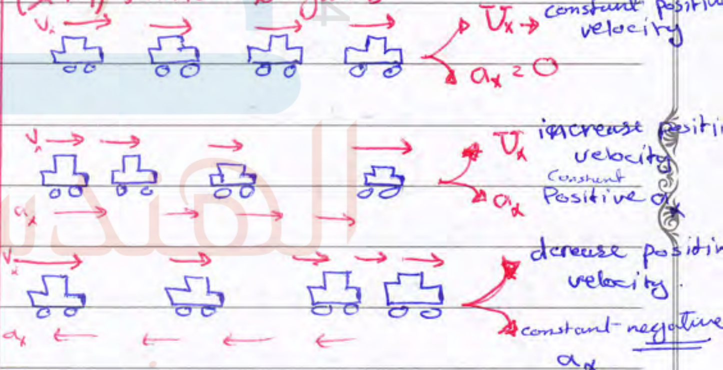
$$\left(\Delta X = U_{xi}\left(\frac{U_{xf} - U_{xi}}{a_x}\right) + \frac{1}{2} a_x \left(\frac{U_{xf} - U_{xi}}{a_x}\right)^2\right) 2$$

$$2\Delta X a_x = \cancel{2U_{xi}U_{xf}} - 2U_{xi}^2 + U_{xf}^2 - \cancel{2U_{xf}U_{xi}} + \cancel{U_{xi}^2}$$

$$= -U_{xi}^2$$

$$U_{xf}^2 = U_{xi}^2 + 2 a_x \Delta X \quad \text{— } \boxed{3}$$

(2.4) Motion Diagrams



(2.5) One-diemsional motion with constant a_x

شرح هذا الفصل سابق (معادلات الحركة) + امثلة الكتاب

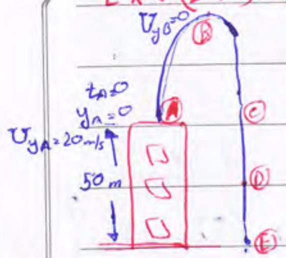
(2.6) Freely falling objects

→ it's any object moving freely under the influence of gravity alone.

$$\uparrow \quad g = 9.8 \simeq 10 \rightarrow \text{but } \boxed{a_y = -g}$$

unit vector:
- $\hat{i}$ = unit vector in the x-direction
- $\hat{j}$ = " " " " y- "
- $\hat{k}$ = " " " " z- "

Ex: (2.12) Page 42



→ [1] $t_B = ??$

$v_{yB} = v_{yA} - 10 * t_B$

$0 = 20 - 10 t_B$

$\boxed{t_B = 2\ s}$

→ [2] $y_B = ??$

$y_B - y_A = v_{yA} t - \frac{1}{2} 10 t^2$

$y_B - 0 = 20(2) - 5(4)$

$y_B = 20\ m$ from the roof of building

$y_B = 70\ m$ from the ground

→ [3] $t_C = ??$

$y_C - y_A = v_{yA} t_c - \frac{1}{2} 10 t_c^2$

$0 = 20 t_c - 5 t_c^2$

$5t\,(t_c - 4) = 0$

so $\boxed{t_c = 4}$

→ [4] $v_{Cy} = ??$

$v_{yc} = v_{yA} - 10 t$

$v_{yc} = 20 - 40 = -20\ m/s$

→ [5] v_y & y $|_{t=5s}$

* $v_{y(t=5)} = v_{yA} - 10(t)^5$

$v_{y(t=5)} = 20 - 50 = -30\ m/s$

* $(y_{t=5} - \overset{0}{y_A}) = v_{yA} t - \frac{1}{2} 10 t^2$

$y_{t=5} = 20(5) - \frac{1}{2} \overset{5}{10} (5)^2$

$y = 25(4 - 5) = -25\ m$

→ [6] $v_{yE} = ??$

$v_{yE}^2 = v_{yA}^2 - 2\,\Delta y(10)$

$v_{yE}^2 = (20)^2 - 2(-50)\,10$

$v_{yE}^2 = 20(20 + 50)$

$v_{yE}^2 = 20 * 70$

$v_{yE} = -37.1\ m/s$

→ [7] total time (flight time)

$\Delta y = v_{yA} t - \frac{1}{2}(10) t^2$

$\frac{-50 = 20t - 5t^2}{-5}$

$10 = -4t + t^2$

$t^2 - 4t - 10 = 0$

$t = \frac{4 + \sqrt{16+40}}{2}$ $\quad$ ~~$t = \frac{4 - \sqrt{16+40}}{2}$~~

$t = 5.75\ s$

Problems:

ex (1) (2.1) → Page 50

→ a) $\bar{v}_x = \frac{23 - 0}{(1-0)} = 2.3\ m/s$

→ b) $\bar{v}_x = \frac{57.5 - 9.2}{5.0 - 2.0} = \frac{48.3}{3} = 16.1\ m/s$

→ c) $\bar{v}_x = \frac{57.5 - 0}{(5 - 0)} = \frac{57.5}{5} = 11.5\ m/s$

ex (3) (2.1) → Page 50

→ a) $\bar{v}_x = \frac{10 - 0}{2 - 0} = 5\ m/s$ $\quad$ → b) $\bar{v}_x = \frac{5 - 0}{4 - 0} = 1.25\ m/s$

→ c) $\bar{v}_x = \frac{5 - 10}{4 - 2} = -2.5\ m/s$ $\quad$ → d) $\bar{v}_x = \frac{-5 - 5}{7 - 4} = -3.3$

→ e) $\bar{v}_x = \frac{0 - 0}{8 - 0} = 0\ m/s$

ex (5) (2.1) → Page 50

(a) → average velocity $= \frac{2x}{(\frac{x}{5}) + (\frac{x}{3})} = \frac{2x}{8x} \times 15 = 3.75\ m/s$

(b) → $\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{0}{\Delta t} = 0$

ex ⑪ → (2.2) Page 50

(a) → $\overline{U_x} = \frac{2-8}{4-1.5} = \frac{-6}{2.5} = -2.4$ m/s

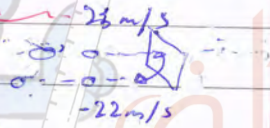
(b) → $U_x\big|_{t=2}$ = slope tangent line $= \frac{0-13}{3.5-0} = -3.7$ m/s

(c) when $\boxed{t=4s}$

ex ⑭ → (2.2) Page 50 U_x = slope of tangent line

a) 5 m/s b) -2.5 m/s c) 0 m/s d) 5

ex ⑬ (2.3) Page 51

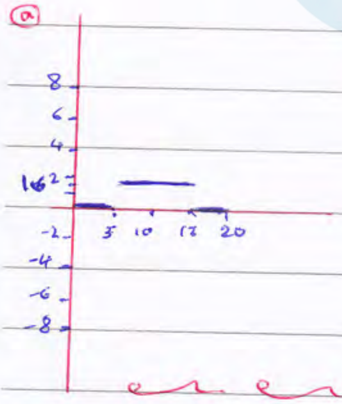


$U_i = 25$ m/s $U_f = -22$ m/s

$\bar{a} = \frac{\Delta U}{\Delta t} = \frac{-22-25}{3.5\times10^{-3}} = -1.34\times10^{4}$ m/s²

ex ⑮

ⓐ



ⓑ $\bar{a}\big|_{t=5 \text{ to } t=15} = 1.6$

$\bar{a}\big|_{t=0 \text{ to } t=20} = \frac{0+1.6}{2} = 0.8$ m/s²

ex ⑰ $x = 2 + 3t - t^2$

$U = 3 - 2t$

$a = -2$

ⓐ $x\big|_{t=3} = 2+9-9 = 2$ m

ⓑ $U\big|_{t=3} = 3 - 2*3 = -3$ m/s

ⓒ $a\big|_{t=3} = -2$

ex ㉓ $\Delta X = 40$ m $U_f = 2.8$ m/s $a_x = ??$

$t = 8.5$ $U_i = ??$

→ $\Delta X = U_i t + \frac{1}{2} a_x t^2$

$40 = 8.5\,U_i + 36.125\,a_x$ —— ①

→ $U_f = U_i + a_x t$

$-8.5\,(2.8 = U_i + 8.5\,a)$ —— ②

→ $40 = 8.5\,\cancel{U_i} + 36.125\,a_x$

$+\ -23.8 = -8.5\,U_i - 72.25\,a_x$

→ $16.2 = -36.125\,a_x$

$\boxed{a_x = -.448 \text{ m/s}^2}$

→ $2.8 = U_i + (8.5)(-0.448)$

$\boxed{U_i = 6.608 \text{ m/s}}$

ex ㊷ Page 54

→ $t = 1.5$ s

→ ⓐ $U_i = ??$ ⓑ $U_f = ??$

→ ⓐ $\Delta y = U_{yi} t - \frac{1}{2} g t^2$

$(4) = U_{yi}(1.5) - 5(1.5)^2$

$U_{yi} = 10.0$ m/s

→ ⓑ $U_{yf}^2 = U_{yi}^2 - 2(10)(\Delta y)$

$U_{yf}^2 = (10)^2 - 20(4)$

$U_{yf} = 4.47$ m/s

ex ㊼ Page 54

$t = 3$ s / $U_f = 0$ / $\boxed{U_i = ??}$ / ⓐ ⓑ

→ $U_f = U_i - 10(t)$

$0 = U_i - 30$ → $\boxed{U_i = 30 \text{ m/s}}$

→ $\Delta y = U_i t - \frac{1}{2}(10) t^2$

$y_f = (30)(3) - (5)(3)^2$

$y_f = 45$ m

ex [51] Page 54

$v_i = 15\ m/s \qquad v_f = 0$

(a) find t

$v_f = v_i - 10t$

$0 = 15 - 10t \qquad \boxed{t = 1.5\ s}$

(b) Find y_f

$y_f - \cancel{y_i}^{0} = v_i t - \frac{1}{2}(10)\,t^2$

$y_f = (15)(1.5) - 5\,(1.5)^2$

$\boxed{y_f = 11.25\ m}$

(c) at $\boxed{t = 2}$ find $\boxed{a}$ & (v)

$\rightarrow$

$v_f = v_i - (10)(t)$

$v_f = 15 - 20 \rightarrow \boxed{v_f = -5\ m/s}$

$\boxed{a_y = 9.8\ m/s}$

Chapter 3: Vectors.

(3.1) Coordinate Systems نظام الاحداثيات

Cartesian coordinate $\xrightarrow{\text{called}}$ rectangular coordinates

polar coordinate system (r, θ)

r is the distance from the origin to the point having cartesin coordinate (x, y)

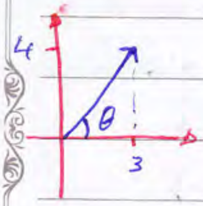
θ is the angle between r & a fixed axis

$\rightarrow x = r\cos(\theta)$ $\rightarrow r = \sqrt{x^2 + y^2}$

$y = r\sin(\theta)$

Ex:

$\vec{A} = (3, 4)$ Find the vector & its magnitude (Polar coordinate)



so $|\vec{A}| = \sqrt{(4)^2 + (3)^2} = 5$

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

(3.2) Vector & scalar quantity.

scalar quantity: ⇒ has just magnitude

ex: distance, density, speed.

vector quantity: ⇒ has both magnitude & direction

ex: displacement, velocity, accelaration

$\vec{A}$ → to expres about vector

$|A|$ → to " " magnitude.

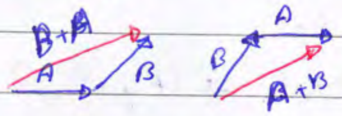
(3.3) Some properties of vectors

Equality of two vectors

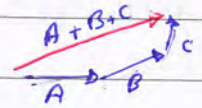
→ $\vec{A} = \vec{B}$ so they have same magnitude & direction

→ if A & B have same direction so their parallel lines.

→ Adding vectors :-

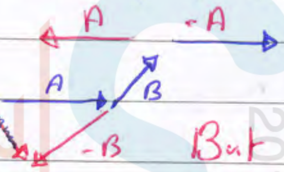


so $A+B = B+A$

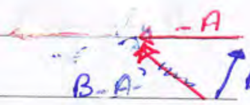


→ Subtracting vectors :-

Negative vector → it has same magnitude of (Positive vector) ~~but~~ opposite direction.



But



so $A - B \neq B - A$

→ multiplying vector

* mutiply vector by scalar

ex: $\frac{1}{3}\vec{A}$ = one-third of magnitude of A

* → ~~cross~~ product (scalar) } ضرب
→ vector " } "

unit vector

3.4 Components of a vector & unit vector

Components of vector → the signs of the components depend on the angle ☺ تعتمد على الربع الموجود فيه اشارات المركبات

→ Unit vector → is a dimensionless vector having a magnitude of exactly 1.

$\hat{i}$ (i hat) → unit vector in the x - direction

$\hat{j}$ (J hat) → " " " " y - "

$\hat{k}$ (K hat) → " " " " z - "

ex → $\vec{A} = A_x\hat{i} + A_y\hat{j}$

$\vec{B} = B_x\hat{i} + B_y\hat{j}$

so $\vec{A} \pm \vec{B} = (A_x \pm B_x)\hat{i} + (A_y + B_y)\hat{j}$

$|\vec{A} \pm \vec{B}| = \sqrt{(A_x + B_x)^2\hat{i} + (A_y + B_y)^2\hat{j}}$

Scalar & Vector Product

(1) → Scalar Product (dot product)

$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}|\,\cos\theta$ ← الزاوية الصغرى بينهما

magnitude of $\vec{A}$ (A) ↳ magnitude of (B)

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

ex: Given $\vec{A} = \hat{i} + \hat{j}$ / $\vec{B} = 2\hat{j}$

Find the angle θ between them

→ Solution: $|\vec{A}| = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$

$|\vec{B}| = \sqrt{2^2 + (0)^2} = 2$

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

$\vec{A} \cdot \vec{B} = 0 + 2 + 0$

so $\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}|\,\cos\theta$

$\cos\theta = \dfrac{2}{\sqrt{2} \times 2}$ $\boxed{\theta = 45°}$

Find → $\hat{i} \cdot \hat{j} = 1\times0 + 0\times1 + 0\times0 = 0$

$\hat{i} \cdot \hat{k} = 1\times0 + 0\times0 + 0\times1 = 0$

← ناتج ضرب (أفقي) لأي متجه (unit vector) مختلفين = صفر

$\hat{i} \cdot \hat{i} = 1\times1 + 0\times0 + 0\times0 = 0$

← ناتج الضرب النقطي لأي متجه (unit vector) متشابهين = 1

(2) → Vector product (Cross product)

$\vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}|\,\sin\theta$

magnitude & direction of (A) — magnitude of B — magnitude

المعادلة غير مكتملة لكن نكتبها كالآتي:

$|\vec{A} \times \vec{B}| = |\vec{A}|\,|\vec{B}|\,\sin\theta$

← في هذا الضرب نهتم بالاتجاه إضافة إلى القيمة

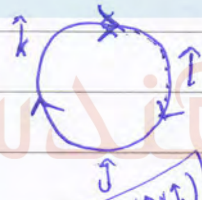
ويتم تحديد الاتجاه بناءً على (حركة اليد اليمنى)

$\vec{A} \times \vec{B}$ ← into page — unit vector — حيث يكون

out of the page

← ويشترط أن يكون اتجاه الناتج في (z-axis)

ويكون ذلك بناءً على ما يلي:



$\hat{i} \times \hat{j} \xrightarrow{\text{ينتج}} \hat{k}$

$\hat{j} \times \hat{i} \xrightarrow{\text{ينتج}} -\hat{k}$

$\hat{k} \times \hat{i} \xrightarrow{\text{ينتج}} \hat{j}$

$\hat{i} \times \hat{k} \xrightarrow{\text{ينتج}} -\hat{j}$

$\hat{k} \times \hat{j} \xrightarrow{\text{ينتج}} -\hat{i}$

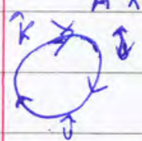
$\hat{j} \times \hat{k} \xrightarrow{\text{ينتج}} \hat{i}$

$\boxed{\text{so } \hat{i} \times \hat{j} = -(\hat{j} \times \hat{i})}$

Ex: Given $\vec{A} = \hat{i} + 2\hat{j}$ / $\vec{B} = \hat{i} + \hat{k}$

Find $\vec{A} \times \vec{B}$

$\vec{A} \times \vec{B} = (\hat{i} + 2\hat{j})(\hat{i} + \hat{k})$



$= \hat{i}\hat{i} + \hat{i}\hat{k} + 2\hat{j}\hat{i} + 2\hat{j}\hat{k}$

$= 0 + -\hat{j} + -2\hat{k} + 2\hat{i}$

$|\vec{A} \times \vec{B}| = \sqrt{(1)^2 + (2)^2 + (2)^2} = 3$

or $\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{vmatrix}$

$= i[2 - 0] - j[1 - 0] + k[0 - 2]$

$= 2i - j + -2\hat{k}$

General Examples :-

Given $\vec{A} = \hat{i} + \hat{j} + 0\hat{k}$

$\vec{B} = 2\hat{i} + 0\hat{j} + 3\hat{k}$

① Find the angle between $\vec{A}$ & $\vec{B}$

$\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$

$\vec{A}\cdot\vec{B} = (1\times2)\hat{i} + (0\times1)\hat{j} + (3\times0)\hat{k}$

$\vec{A}\cdot\vec{B} = 2$

$|\vec{A}| = \sqrt{2}$ $\quad |\vec{B}| = \sqrt{4+9} = \sqrt{13}$

So $\cos\theta = \dfrac{2}{\sqrt{2}\sqrt{13}} \longrightarrow \theta = 66.9°$

② Find by $(\vec{A}\times\vec{B})$ $\sin\theta$

$|\vec{A}\times\vec{B}| = |\vec{A}|\,|\vec{B}|.\sin\theta$

الأولى $|\vec{A}\times\vec{B}| = \begin{vmatrix} I & J & K \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$= I[A_yB_z - B_yA_z] - J[A_xB_z - A_zB_x] + k[A_xB_y - A_yB_x]$

$(\vec{A}\times\vec{B}) = \begin{vmatrix} I & J & k \\ 1 & 1 & 0 \\ 2 & 0 & 3 \end{vmatrix}$

$= I[3-0] - J[3-0] + k[0-2]$

$= 3I - 3J + -2k$

$|\vec{A}\times\vec{B}| = \sqrt{(3)^2 + (3)^2 + (2)^2} = \sqrt{22}$

$\sin\theta = \dfrac{|\vec{A}\times\vec{B}|}{|\vec{A}|\,|\vec{B}|} = \dfrac{\sqrt{22}}{\sqrt{2}*\sqrt{13}}$

$= \dfrac{\sqrt{11}}{\sqrt{13}}$

* * * * *

Note :-

① $(\vec{V_f} = \vec{V_i} + \vec{a}t)$

$V_{xf} = V_{xi} + a_x t$

$V_{yf} = V_{yi} + a_y t$

② $(\vec{\Delta r} = \vec{V_i}t + \frac{1}{2}\vec{a}t^2)$

$\Delta y = V_{yi}t + \frac{1}{2}a_y t^2$

$\Delta x = V_{xi}t + \frac{1}{2}a_x t^2$

③ $(\vec{V_f}^2 = \vec{V_i}^2 + 2\vec{\Delta r}\,\vec{a})$

$V_{yf}^2 = V_{yi}^2 + 2a_y\Delta y$

$V_{xf}^2 = V_{xi}^2 + 2a_x\Delta x$

* * * * *

Q :- Given $\vec{A} = 2\hat{i} + 3\hat{k}$

$\vec{B} = -\hat{i} - 2\hat{k}$

Then find $\vec{A}\times\vec{B}$

Solution :-

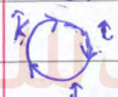
$\vec{A}\times\vec{B} = \begin{vmatrix} x & y & z \\ 2 & 0 & 3 \\ -1 & 0 & -2 \end{vmatrix}$

$= I[0-0] - \hat{J}[-4+3] + \hat{k}[0-0]$

$= +\hat{J}$

$\hat{i}\times\hat{k} = -\hat{J}$

or $\vec{A}\times\vec{B} = (2\hat{i} + 3\hat{k})(-\hat{i} - 2\hat{k})$

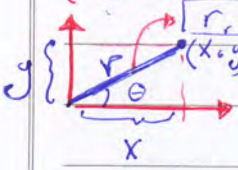


$= 0 + 4\hat{J} - 3\hat{J} + 0 = 1\hat{J}$

* * * * *

Chapter 4 :- Motion in two dimension

(4.1) The displacement, velocity & acceleration vector

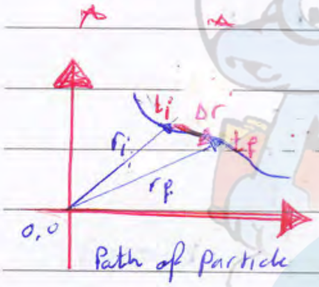


so $\vec{r} = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j}$

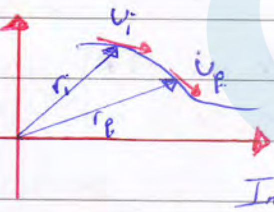
$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

→ to find $|\vec{r}| \rightarrow |\vec{r}|^2 = x^2 + y^2$

$|\vec{r}| = \sqrt{x^2 + y^2}$



$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$ → displacement vector of the particle

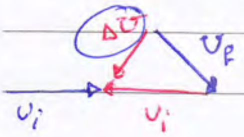


$\bar{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$

Inst. velocity $= \lim_{\Delta t \to 0} \bar{v}$

$= \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$

$= \frac{d\vec{r}}{dt}$



$\bar{a}_x = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$

Instantenous $a_x = \frac{d\vec{v}}{dt}$

(4.2) Two-diemensional motion with constant acceleration

(ع شرح هنا بدرس أكثر (vector) ول معادلات الحركة في بعدين .

Ex:(4.1)

$v_{xi} = 20\ m/s \qquad v_{yi} = -15\ m/s$

$a_x = 4.0\ m/s^2 \qquad a_y = 0$

→ Solution:-

→ $v_{xf} = v_{xi} + a_x t$

$\boxed{v_{xf} = 20 + 4t}$

→ $v_{yf} = v_{yi} + a_y t$

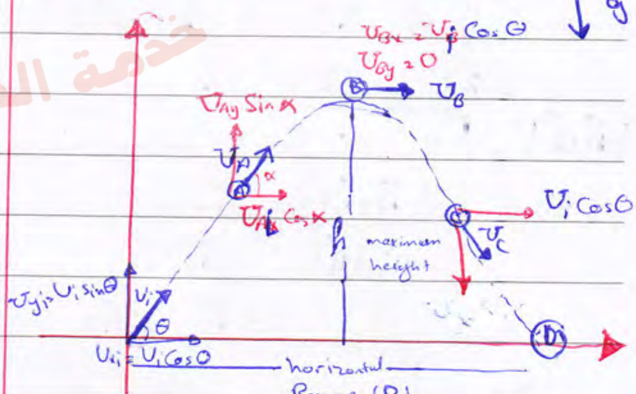
$\boxed{v_{yf} = -15 + 0 = -15}$

→ $v_f = v_{xf}\,\hat{i} + v_{yf}\,\hat{j}$

$v_f = [20 + 4t]\,\hat{i} + [-15]\,\hat{j}$

* * * * * * *

(4.3) Projectile motion



$v_{fx} = v_{ix} + \cancel{a_x}\, t$

$\boxed{v_{fx} = v_{ix}}$ لذلك

نستنتج v_{ix} ثابتة

max Range → $\theta = 45$

max high → $\theta = 90$

To find (h) maximum height

$\Delta y = v_{yi} t_B - \frac{1}{2} g t_B^2$

$y_{max} - 0 = v_{yi} t_B - \frac{1}{2} g t_B^2$

But $v_{yf} = v_{yi} - g t_B$

$0 = v_{yi} - g t_B$ so

$\boxed{t_B = \frac{v_{yi}}{g}}$

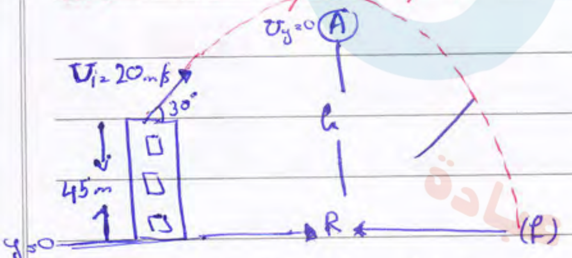
$y_{max} = v_{yi}\left(\frac{v_{yi}}{g}\right) - \frac{1}{2} g \frac{(v_{yi})^2}{g^2}$

$y_{max} = \frac{(v_{yi})^2}{g} - \frac{1}{2} \frac{(v_{yi})^2}{g}$

To find (R) horizontal Range

$R = (v_{xi}) \times t_0 \rightarrow R = (v_{xi} \cos\theta) \times t_0$

ex:- find t_f / R / h



[1] $\Delta y = v_{yi} t - \frac{1}{2} g t^2$

$\therefore -45 = 20 \sin 30\, t - 5 t^2$

$\frac{-45 = 10t - 5t^2}{5}$

$t^2 - 2t - 9 = 0 \rightarrow t = \frac{2 \pm \sqrt{4+36}}{2}$

$t = (2 + \sqrt{40})/2 = 4.16\,s$

[2] $R = v_i \cos(30)° \times t$

$R = 72\,m$

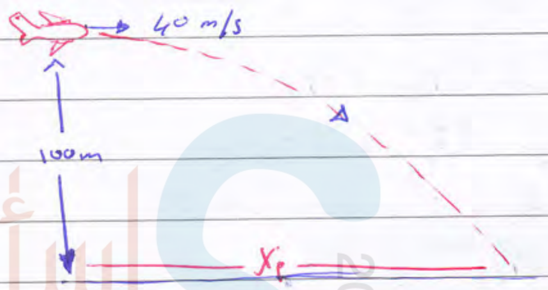
[3] $v_{y(A)}^2 = v_{yi}^2 - 2 g \Delta y$

$\therefore = (20 \sin\theta)^2 - 20 \Delta y$

$y_{(A)} - 45 = \frac{(20 \times \sin 30)^2}{20}$

$y_{(A)} = h = 5 + 45 = 50\,m$

Ex (4.6)



[1] Find (x_f)

$x_f = v_x \times t$

But $\Delta y = v_{yi} t - \frac{1}{2} g t^2$

$-100 = 0 - 5 t^2$

$t = \sqrt{20} = 4.47$

so $\rightarrow x_f = 40 \times 4.47 = 178.8$

[2] Find v_{xf} & v_{yf}

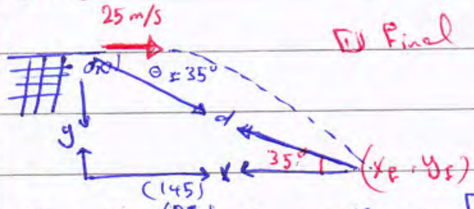
$\rightarrow v_{xf} = v_x = 40\,m/s$

$\rightarrow v_{yf} = v_{yi} - 10(4.47)$

$v_{yf} = -44.7\,m/s$

Ex (4.7)

[1] Find x_f, y_f



[1] $\rightarrow \tan(35) = \frac{-y_f}{x_f} = 0.7 \rightarrow \boxed{y_f = -0.7 x_f}$ --(1)

$x_f = 25 t - 0$

$y_f = 0 - \frac{1}{2} 10 t^2$

$y_f = -5t^2$ / $x_f = 25t$

$y_f = -5\left(\frac{x_f}{25}\right)^2$

$-0.7 x_f = -\frac{x_f^2}{125} \rightarrow (x_f^2 - 87.5 x_f) = 0$

$\boxed{x_f = 0}$ or

$\boxed{x_f = +87.5}$

$\boxed{y_f = -61.25}$

[2] Find : t_f / v_{yf}

$\rightarrow v_{yf}^2 = v_{yi}^2 - 2 \cdot 10 \cdot \Delta y$

$v_{yf}^2 = 0 - 20(-61.25)$

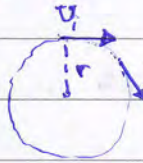
$\boxed{v_{yf} = 35\ m/s}$

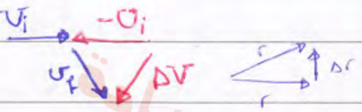
$\rightarrow x_f = 25t \rightarrow \frac{87.5}{25} = t = 3.5\ s$

* * * * * *

(4.4) Uniform circular motion

الحركة الدائرية المنتظمة (سرعة متساوية بالمقدار ومختلفة بالاتجاه)





acceleration radial $\rightarrow$ $a_r = \frac{v^2}{r}$ m/s^2 $\rightarrow$ centripetal acceleration

تسارع مركزي

$\bar{a} = \frac{\Delta v}{\Delta t}$ $\xrightarrow{\text{من تشابه المثلثات}}$ $\frac{\Delta v}{v} = \frac{\Delta r}{r}$

$\bar{a} = \frac{\Delta r}{\Delta t} \cdot \frac{v}{r} \rightarrow \boxed{a_r = \frac{v^2}{r}}$

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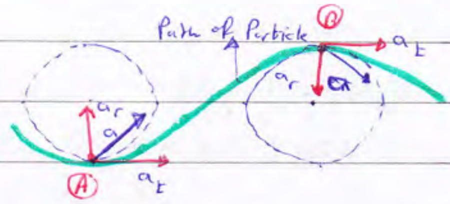
(4.5) Tangental & radial acceleration

→ tangential $(a)_t$: cause change in the speed of the particle

→ radial $(a)_r$: cause change in the direction of the velocity



$\vec{a} = (\vec{a_r}) + (\vec{a_t})$

$\left(\frac{v^2}{r}\right)$ $\frac{d|v|}{dt}$

$|a| = \sqrt{(a_r)^2 + (a_t)^2}$

→ a_r (large) → r (small)

a_r (small) → r (large)

→ direction of a_t

* as the same direction as v (if v is increasing)

* opposite (v) (if v is decreasing)

→ In uniform circular motion

$|v|$ → is constant $\xrightarrow{so}$ $\boxed{a_t = 0}$

$\boxed{a = a_r}$

لانه تغير سرعة بحركته تتغير في الاتجاه

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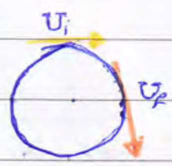
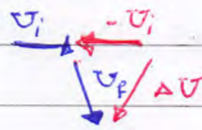
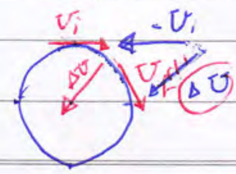
(6.2) Nonuniform circular motion

الحركة الدائرية غير المنتظمة ← بحيث تكون تتغير في مقدار الاتجاه





→ [1] we note that in (uniform circular motion) Δv indicate to center



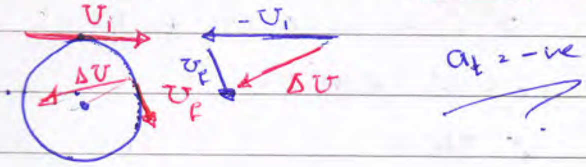
- constant speed

- changeing in direction of velocity

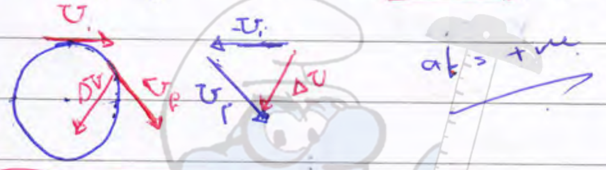
$a_t = 0$ $\boxed{a = a_r}$

[2] In nonuniform circular motion

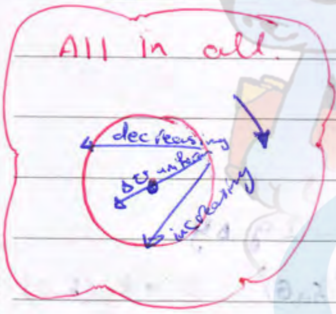
→ Changing in direction & (decreasing speed)



→ changing in direction & (increasing speed)

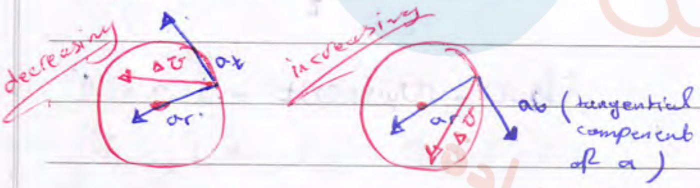


All in all.



$$a_t = \frac{d|v|}{dt}$$

→ In nonuniform C.M.



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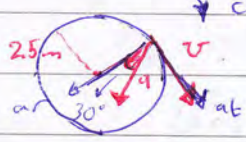
Ex (Problem 33 Page 104)

motion in clockwise



$a = 15\ m/s^2$

[1] find a_r

[2] find v

[3] find a_t

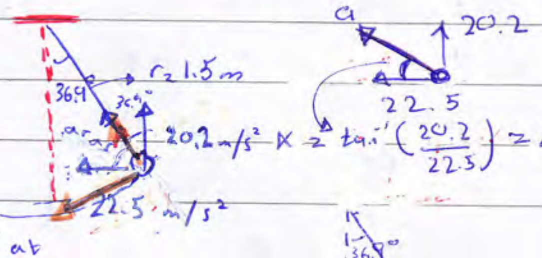
[1] $a_r = a * \cos(30) \rightarrow a_r = 13\ m/s^2$

[3] $a_t = a * \sin(30) \rightarrow a_t = 7.5\ m/s^2$

[2] $a_r = \frac{v^2}{r} \rightarrow v^2 = 13 * 2.5$

$v = 5.7\ m/s$

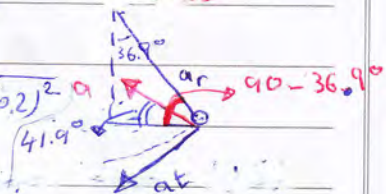
Ex (Problem 35 page 104)



$\alpha = \tan^{-1}\left(\frac{20.2}{22.5}\right) = 41.9°$

$|a| = \sqrt{(22.5)^2 + (20.2)^2}$

$|a| = 30.2$



حيث ان الزاوية بين a_t & a_r 90° .. بالتالي

فالزاوية بين a_r & a = $\boxed{90 - 36.9°} - 41.9°$

[1] find a_r → $a_r = a \cos(11.2)$

$a_r = 29.6\ m/s^2$

[2] find v → $v^2 = r a_r$ → $v = 6.7\ m/s$

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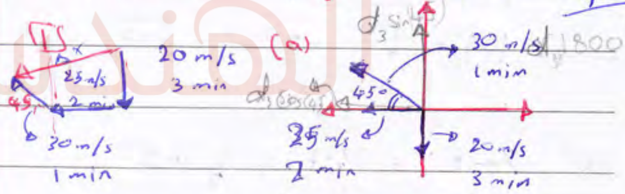
Problems → Page 101 (عند عدم ذكر الزاوية لكن شمال شرقي نعتبرها 45)

[1]



$d_2 = 25 \times 2 \times 60 = 3000\ m$

$d_1 = 20 \times 3 \times 60 = 3600\ m$

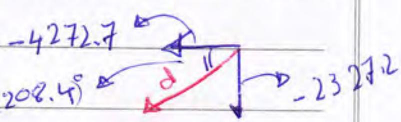
$d_3 = 1800$

→ $y = d_3 \sin(45) - d_1$

$= 1800 \times \frac{1}{\sqrt{2}} - 3600 = -2327.2\ m$

$x = -d_3 \cos(45) - d_2$

$= -1800 \times \frac{1}{\sqrt{2}} - 3000 = -4272.7$



$d = 4865.3$ (208.4°)

(b) average speed = $\frac{1800 + 3600 + 3000}{6 \times 60} = 23.3\ m/s$

(c) $\bar{v} = \frac{4865.3}{360} = 13.5\ m/s$ at 209°

3 (a) $\vec{r} = ((18\,m/s)t)\,i + ((4\,m/s)t - (4.9\,m/s^2)t^2)\,j$

(b) $\vec{v} = (18.0\,m/s)\,i + (4\,m/s - 9.8t)\,j$

(c) $\vec{a} = (0)\,i + (0 - 9.8)\,j$

$\rightarrow$ (d) $\rightarrow \vec{r}|_{t=3} = (54)\,i + (32.1)\,j$

(e) $\rightarrow \vec{v}|_{t=3} = (18)\,i - (25.4)\,j$

(f) $\rightarrow \vec{a} = -(9.8)\,j$

5 $v_i = (3\hat{i} - 2\hat{j})$

$v_{(t=3)} = (9\hat{i} + 7\hat{j})$

(a) $\rightarrow v_{y(t=3)} = v_{yi} + a_y t$

$7 = -2 + a_y(3)$

$\boxed{a_y = 3\,m/s^2}$

$\rightarrow v_{x(t=3)} = v_{xi} + a_x t$

$9 = 3 + 3a_x$

$\boxed{a_x = 2\,m/s^2}$

$\rightarrow |a| = \sqrt{(3)^2 + (2)^2} = \sqrt{13}\ m/s^2$

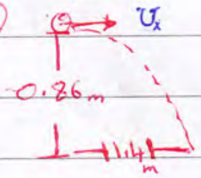
(b) $x_f - x_i = v_{xi} t + \frac{1}{2} a_x t^2$

$\boxed{X - 0 = 3t + t^2}$

$y_f - y_i = v_{yi} t + \frac{1}{2} a_y t^2$

$\boxed{y_f - 0 = -2t + 1.5t^2}$

4 $v_y = 0$, v_x



0.86 m

1.4 m

$\rightarrow \Delta y = v_y t - \frac{1}{2}(9.8)t^2$

$-0.86 = -4.9t^2$

$\boxed{t = 0.42\,s}$

$\rightarrow v_x = \frac{1.4}{t}$

$v_x = 3.34\ m/s$

18 $v = 300$, $55°$

(x, y)

$v_{yi} = 300\sin(55)$

$v_{xi} = 300\cos(55)$ $t = 42\,s$

$\rightarrow$ find (x, y)

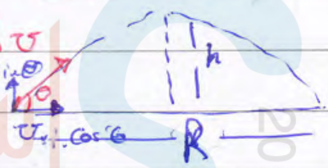
$x = v_{xi} t$

$= 300 \times \cos(55) \times 42 = 7227\ m$

$y = v_{yi} t - \frac{1}{2}(9.8)t^2$

$y = 1678\ m$

15 v



$v \cdot \sin\theta$, θ, h, $v_i \cdot \cos\theta$, R

$\boxed{R = 3h}$

$\rightarrow x = 3h$

$v_{yf}^2 = v_{yi}^2 - 2(h)(g)$

$h = \frac{v_{yf}^2 - v_{yi}^2}{-2g} \rightarrow \boxed{h = \frac{v^2 \sin^2\theta}{2g}}$

$\rightarrow v_{yf} = v_{yi} - g t_h$

$0 = v\sin\theta - g t_h \rightarrow \boxed{t_h = \frac{v\sin\theta}{g}}$

$\rightarrow R = v_{xi} \times (2t_h)$

$R = v\cos\theta\,(2t_h)$

$R = (v\cos\theta)\left(2\left(\frac{v\sin\theta}{g}\right)\right)$

$\Rightarrow R = 3h$

$\frac{2v^2\sin\theta\cos\theta}{g} = 3\left(\frac{v^2\sin^2\theta}{2g}\right)$

$\frac{\sin\theta}{\cos\theta} = \frac{4}{3}$

$\tan(\theta) = \frac{4}{3}$ $\theta = 53.1$

[1] Page 101

$x = -(5\ m)\ Sin(\omega t)$ $\omega \to s^{-1}$, $t \to s$

$y = (4\ m) - (5\ m)\ Cos(\omega t)$

(a) find $v_x|_{t=0}$, $v_y|_{t=0}$ / $a_x|_{t=0}$, $a_y|_{t=0}$

- $v_x = -5 \times Cos(\omega t)$

$v_x|_{t=0} = -5 \times Cos(0) = -5\ m/s$

- $v_y = +5\ Sin(\omega t)$

$v_y|_{t=0} = 0\ m/s$

- $a_x|_{t=0} = 5 \times Sin(\omega t) = 0\ m/s^2$

- $a_y|_{t=0} = 5\ Cos(\omega t) = 5\ m/s^2$

(b) find expression

$\vec{r} = (-5\ Sin(\omega t))\hat{i} + (4 - 5\ Cos(\omega t))\hat{j}$

$\vec{v} = (-5\ Cos(\omega t))\hat{i} + (5\ sin(\omega t))\hat{j}$

$\vec{a} = 5\ Sin(\omega t)\hat{i} + (5\ cos(\omega t))\hat{j}$

[7] $v_i = (4\hat{i} + 1\hat{j})\ m/s$

$r_i = (10\hat{i} - 4\hat{j})\ m$

→ fish swims with constant $(\vec{a})$ for 20 s

$v = (20\hat{i} - 5\hat{j})\ m/s$

(a) find (a_x) (a_y)

→ $v_{xf} = v_{xi} + a_x t$

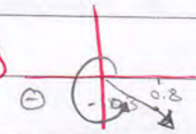
$(20) = (4) + a_x(20)$

$a_x = 0.8\ m/s^2$

→ $v_{yf} = v_{yi} + a_y t$

$-5 = 1 + a_y(20)$

$a_y = -0.3\ m/s^2$

(b) 

$= 339.5^\circ$

(c) $r|_{t=25}$ $a_x, a_y \to$ constant

→ $x_f - x_i = v_x t + \frac{1}{2} a_x t^2$

$x_f - 10 = 4(25) + \frac{1}{2}\, 0.8\, (25)^2$

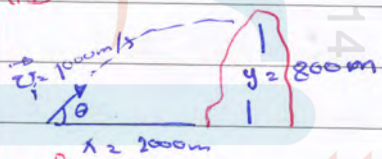
$x_f = 360\ m$

→ $y_f - y_i = v_y t + \frac{1}{2} a_y t^2$

$y_f + 4 = (1)25 + \frac{1}{2}(-0.3)(25)^2$

$y_f = -72.75\ m$

[17]



find θ →

$\Delta y = v_{yi} t - \frac{1}{2}(9.8) t^2$

$\boxed{800 = 1000 \times Sin\theta\ (t) - (4.9) t^2}$

$v_{xi} = \frac{x}{t}$

→ $t = \frac{2000}{v_i \cos\theta}$ → $t = \frac{\cancel{2000}\,2}{\cancel{1000}\cos(\theta)}$

$\boxed{t = \frac{2}{Cos\theta}}$

→ $\left(800 = 2000 \times \frac{Sin\theta}{Cos\theta} - 4.9 \times \frac{4}{Cos^2\theta}\right) \times \frac{1}{4}$

$200 = 500 \times tan\theta - \frac{4.9}{Cos^2\theta}$

Chapter 5 :- The laws of Motion

(5.1) The Concept of Force

→ Forces don't always cause motion

→ Force → which causes a body to accelerate

→ We have 2 type of forces :-

1. Contact force قوة تلامس

ex:- gas molecules, feet on the floor

2. Field force القوة عن بعد

⊕ → ← ⊖ { iron NIS } { m M }

$$\left(\sum_n \vec{F}_n = m\vec{a}\right) \rightarrow \text{Newton's 2nd laws}$$

→ if net force (total force) on an object $= 0$ then the $\boxed{\vec{a} = 0}$

← إذا كانت محصلة القوى على الجسم يساوي صفر، إذنه فإنه تسارع ذلك الجسم يساوي صفر (إذنه الجسم يسير بسرعة ثابتة)

وضع توازن (equilibrium) $\sum f = 0$

$$\Rightarrow \left\{ \sum_n F_n = m\left(\frac{d\vec{v}}{dt}\right) \right\}$$

Because forces are vector quantities, you must use the rules of addition.

(5.2) Newton's first law & inertial frames

Newton's first law of motion (In the absence of external forces, an object at rest remain at rest & an object in motion continues in motion with a constant v)

في غياب القوة الخارجية

$$\rightarrow \sum f = 0 \longrightarrow \boxed{\vec{a} = 0}$$

التصور الذاتي

Inertial frames :- (Newton's 1st laws)

(an inertial frame of reference is one that isn't accelerating).

(5.3) Mass

mass → scalar quantity

weight → vector

$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

(5.4) Newton's 2nd law

"(the acceleration of an object is directly proportional to the resultant (net) force acting on it & inversely proportional to its mass)

$$\sum F = m\,a$$

Newton → kg m/s^2

→ example Page 118

(5.5) The Force of gravity & weight

$F_g = mg =$ weight

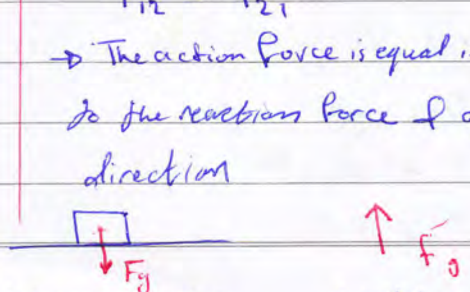
$g = 9.8 \simeq 10$

$\rightarrow \boxed{g = a}$, $\rightarrow \sum F = ma$, $a \rightarrow g$

(5.6) Newton's 3rd law

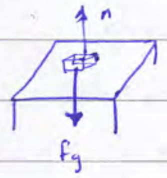
$$F_{12} = -F_{21}$$

→ The action force is equal in magnitude to the reaction force & opposite in direction



weight mg = action reaction قوة جذب الأرض للجسم

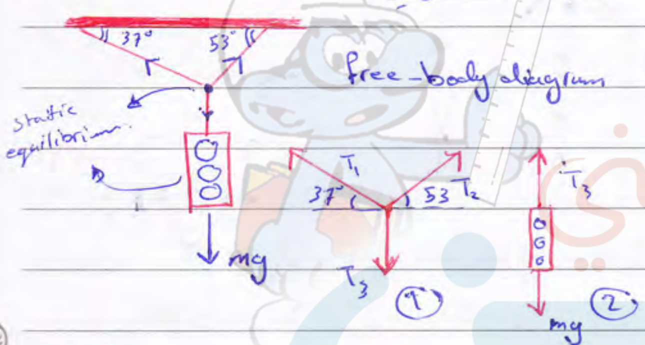
← في قانون نيوتن الثالث القوة ورد فعلها تؤثران في جسمين مختلفين وليس في جسم واحد.



n & F_g are not action & reaction

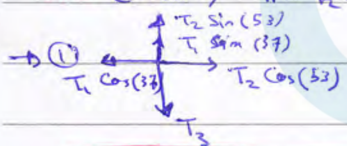
(5.7) Some Application of newton's laws

ex (5.4) Page 125 static equilibrium اتزان سكوني



Find (tension) $T_1 - T_2 - T_3$

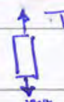
→ ①



$$\boxed{T_1 \cos(37) = T_2 \cos(53)} \quad ①$$

$$\boxed{T_2 \sin(53) + T_1 \sin(37) = T_3} \quad ②$$

→ ②



→ $T_3 = mg = \text{weight}$

$\boxed{T_3 = 125\ N}$ ✓

→② $(0.8\, T_2 + 0.6\, T_1 = 125) \times 4$

→① $(0.6\, T_2 - 0.8\, T_1 = 0) \times 3$

→ $3.2\, T_2 + \cancel{2.4\, T_1} = 500$

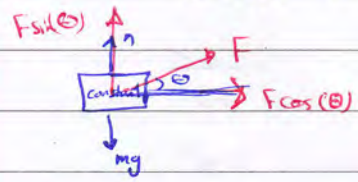
$1.8\, T_2 - \cancel{2.4\, T_1} = 0$

$5\, T_2 = 500$ → $\boxed{T_2 = 100\, N}$ ✓

→ $0.8(100) + 0.6\, T_1 = 125$

$\frac{0.6\, T_1}{0.6} = \frac{45}{0.6}$ $\boxed{T_1 = 75\, N}$ ✓

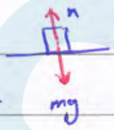
Note :- إذا كان الجسم ساكنة فإنه $\Sigma F = 0$ ولكن إذا الجسم غير ساكن فإنه $\Sigma F = ma$



So $\boxed{n + F \sin\Theta = mg}$

But $\boxed{F \cos(\Theta) = ma}$

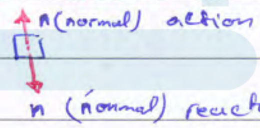
Note :-



↑ Position diagram

$n - mg = 0$ ← Free body diagram

Note: Newton's 3rd Law

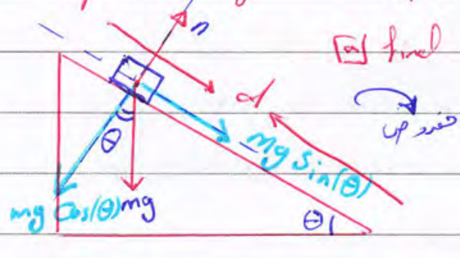


$|\vec{F_1}| = |\vec{F_2}|$

But $\vec{F_1} = -\vec{F_2}$

Example (5.6) Page 126 $V_i = 0$

Find $\vec{a_x}$



في هذه المسألة نفرض نحن اتجاه حركة 6 فإذا كانت النتائج (9) موجبة فالافتراض صحيح سالبة فالاتجاه عكس المفروض

→ $n - mg \cos(\Theta) = 0$

$\cancel{m} g \sin\Theta = \cancel{m} a_x$

$\boxed{a_x = g \sin\Theta}$

(b) Find t_f & v_f

$\rightarrow d = \cancel{v_i t} + \frac{1}{2} a_x t^2$

$d = \frac{1}{2} a_x t^2$

$t = \sqrt{\frac{2d}{a_x}} = \sqrt{\frac{2d}{g \sin\theta}}$

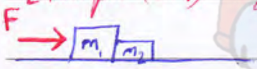
$\rightarrow v_f = \cancel{v_i} + a_x t$

$v_f = g \sin\theta \times \sqrt{\frac{2d}{g\sin\theta}}$

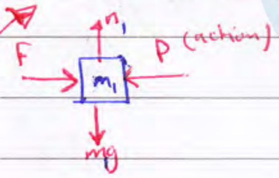
$= \sqrt{2d\, g \sin\theta}$

e.n.e.n.e.n.e.n

Example (5.7) Page 127

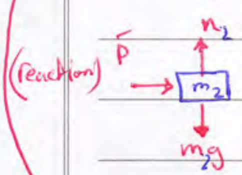


$\boxed{a_{total} = \frac{F}{(m_1+m_2)}}$ $\boxed{P = \bar{P}}$ N 3rd law



$n_1 - m_1 g = 0$ — ①

$F - P = m_1 a$ — ②



$n_2 - m_2 g = 0$ — ③

$\bar{P} = m_2 a$ — ④

$\rightarrow$ if we addition ④ + ②

$F - \cancel{P} + \cancel{\bar{P}} = (m_1+m_2) a$

$F > P$ سنة بالمعادلة يبين لذات

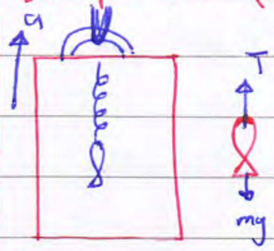
$\rightarrow$ if $m_1 = 4$ kg $m_2 = 3$ kg $F = 9$ N

find (a) & $\boxed{P}$

① $a = \frac{9}{7} = \boxed{1.3\ m/s^2}$

② $P = F - m_1 a \rightarrow \boxed{P = 3.8\ N}$

Example 5.8 (weighing a fish in Elevator)



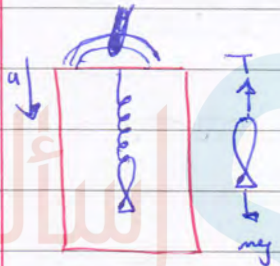
$T - mg = ma$

so $T = m(a+g)$

$T = mg\left(\frac{a}{g} + 1\right)$ أكبر من واحد

بالتالي $T >$ fish weight بالتالي يعطي الميزان

قراءة أكبر من قراءة الوزن الأصلي.



$T - mg = m(-a)$

$T - mg = -ma$

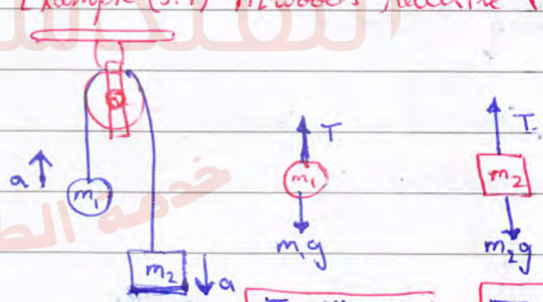
$T = mg\left(-\frac{a}{g} + 1\right)$ أقل من واحد

بالتالي $T <$ fish weight بالتالي يعطي الميزان قراءة

أقل من قراءة الوزن الأصلي.

e.n.e.n.e.n.e.n

Example (5.9) Atwood's Machine Page 129



$\boxed{T_1 - m_1 g = m_1 a}$ معادلة ①

$\boxed{T_2 - m_2 g = m_2 a}$ معادلة ②

If we add (1) to (2)

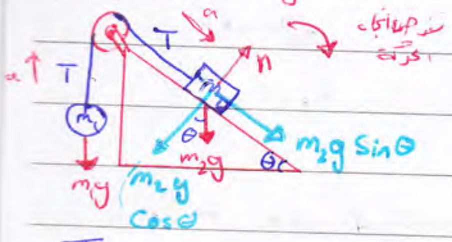
$\cancel{T} - m_1 g = m_1 a$

$+\ m_2 g - \cancel{T} = m_2 a$

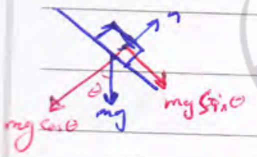
$a = \frac{g(m_2 - m_1)}{(m_1 + m_2)}$

so $a = \frac{\text{ق المسببة للحركة (الوزن)}}{\text{ك الكل المتحركة}}$

Example (5.10) Page 130



$T - m_1 g = m_1 a$ —— ①



$n - m_2 g \cos\Theta = 0$

$m_2 g \sin\Theta - T = m_2 a$ —— ②

if we add ① + ②

$\Rightarrow a = \dfrac{g(m_2 \sin\Theta - m_1)}{(m_1 + m_2)}$

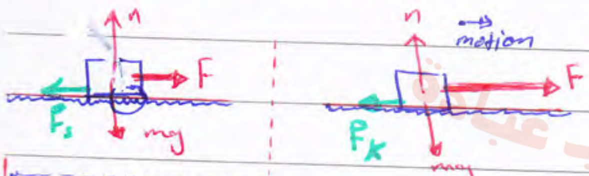
مجموع القوى المحركة

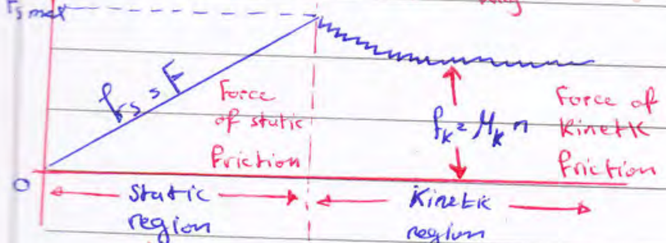
مجموع الكتل المتحركة

+ الافتراض صحيح

− عكس اتجاه الحركة

(5.8) Forces of Friction.





so $f_s \leq M_s n$

But $f_s = f_{s\,max} = M_s n$

coefficient of static friction

معامل الاحتكاك السكوني

$f_K = M_K n$

coefficient of Kinetic friction

معامل الاحتكاك الحركي

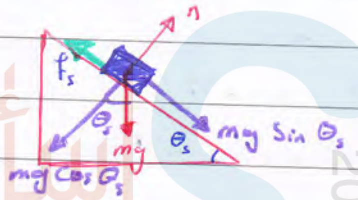
→ the values of M_s, M_K } depend on the nature of surfaces

→ In general $M_s > M_K$

→ Typical values range from around (0.03 — 1.0)

→ M_s & M_K → are dimensionless

Example (5.12) Page 134

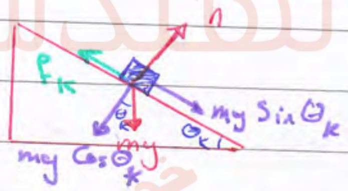


Equilibrium station

$\Rightarrow f_{s(max)} = n M_s \to \boxed{n M_s = mg \sin\Theta}$

$\cancel{mg} \cos\Theta \, M_s = \cancel{mg} \sin\Theta$

$\boxed{M_s = \tan\Theta_s}$



$\Rightarrow f_K = n M_K \to \boxed{n M_K = mg \sin\Theta_K}$

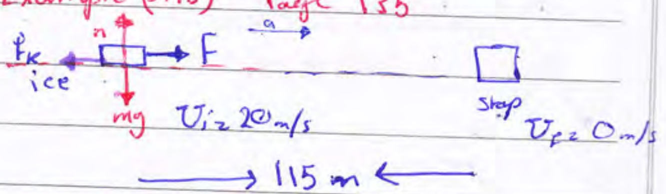
$\cancel{mg} \cos\Theta_K \, M_K = \cancel{mg} \sin\Theta_K$

$\boxed{M_K = \tan\Theta_K}$

→ so $\Theta_s > \Theta_K$

so $M_s > M_K$

Example (5.13) Page 135



find M_K ??

$\boxed{n = mg}$ — (1)

$\boxed{-f_k = ma}$ — (2)

$\boxed{-n\mu_k = ma}$

But: $v_{xf}^2 = v_{xi}^2 + 2\Delta x\, a$

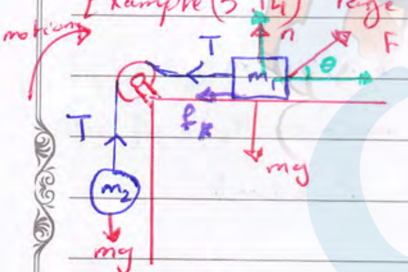
$0 = (20)^2 + 2(115)\, a$

$a = -1.74\ m/s^2$

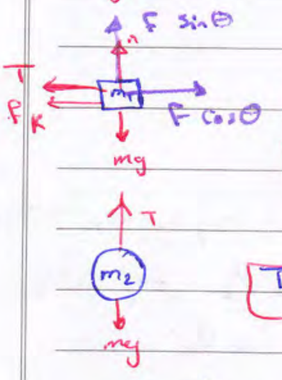
$-\cancel{m}g\,\mu_k = \cancel{m}a$

$\boxed{\mu_k = \frac{-1.7}{-10} = .174}$

Example (5.14) Page 136



find (a)



$\boxed{F\cos\theta - T - f_k = m_1 a}$ — (1)

$\boxed{F\sin\theta + n = mg}$ — (2)

$\boxed{T - m_2 g = m_2 a}$ — (3)

→ if we add (1) & (3)

$F\cos(\theta) - m_2 g - f_k = (m_1 + m_2)\, a$

$a = \frac{F\cos\theta - m_2 g - f_k}{(m_1 + m_2)}$

+ اتجاه الجسم

- عكس الاتجاه

Problems

$m_1 \rightarrow F$ $\quad$ $m_2 \rightarrow F$

$a_1 = 3\ m/s^2$ $\quad$ $a_2 = 1\ m/s^2$

(1) find the value of ratio m_1/m_2

$F = F$

$m_1 a_1 = m_2 a_2$

$\frac{m_1}{m_2} = \frac{a_2}{a_1} = \frac{1}{3}$

(2) if $[m_1|m_2] \rightarrow F$ find (a)

$F = (m_1 + m_2)\, a$

$a = \frac{F}{m_1 + m_2} = \frac{(m_1 a_1)/m_1}{(m_1 + m_2)/m_1}$

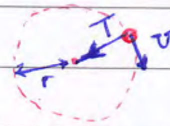
$= \frac{a_1}{1 + \frac{m_2}{m_1}} = \frac{3}{1+3} = \frac{3}{4}$

Chapter 6: Circular Motion

(6.1) Extending the particle: (U.C.M) Model

→ Circular Motion (in horizontal plane)

uniform circular Motion



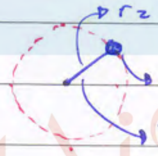
so $a = a_r = \frac{v^2}{r}$

so $T = m\frac{v^2}{r}$

↳ $v_{max} = \sqrt{\frac{Tr}{m}}$

→ Centripetal force → forces that cause centripetal Acceleration.

Example (6.2) Page 154



$T = ma \rightarrow 50 = 0.5 \times \frac{v^2}{1.5}$

$\rightarrow v = 12.2$ m/s

Example (6.4) Page 155

→ mass = 500 Kg r = 35 m

$v = ??$ $\mu_s = 0.5$

$\rightarrow f_s = m\frac{v^2}{r}$

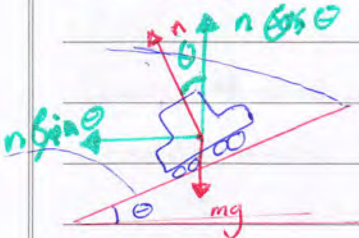
↳ $n\mu_s = \frac{500\,v^2}{35\,m}$

↳ $\frac{\cancel{m}g \times \mu_s \times 35\,m}{\cancel{m}} = v^2$

$v = \sqrt{10 \times 35 \times 0.5}$

$= 13.2$ m/s

Example (6.5) The Banked Exit Ramp



$v = 13.4\ m/s$

$r = 50\ m$

$\theta = ??$

$\rightarrow n\cos\theta = mg$ — ①

$n\sin\theta = ma$ — ②

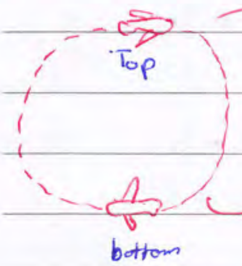
→ if we divided by ① & ②

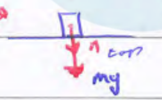
$\frac{1}{\tan\theta} = \frac{g}{a}$

$\frac{1}{\tan\theta} = \frac{g \times r}{v^2}$

$\frac{1}{\tan\theta} = \frac{10 \times 50}{(13.4)^2} \rightarrow \theta = 20°$

Example (6.7) Let's go loop-the-loop





$n_{top} + mg = m\left(\frac{v^2}{r}\right)$

$n_{bottom} - mg = m\left(\frac{v^2}{r}\right)$

$r = 2.7\ Km$ $v = 225\ m/s$

find n_{top} & n_{bottom}

$n_{top} + mg = m\left(\frac{(225)^2}{2.7\times10^3}\right)$
$-mg \qquad\qquad -mg$

$n_{top} = mg\left(\frac{(225)^2}{2.7\times10^3} \times \frac{1}{g} - 1\right)$

$\boxed{n_{top} = 0.9\ mg}$

$n_{bottom} = mg\left(\frac{v^2}{gr} + 1\right)$

$\boxed{n_{bottom} = 2.9\ mg}$

Nonuniform circular Motion

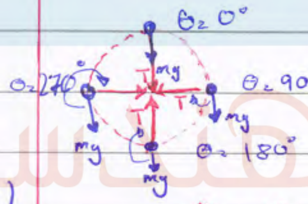
$\sum F_r = \left(T - mg\cos\theta = m\frac{v^2}{r}\right)$ (a_r)

$\sum F_t = (\cancel{m}g\sin\theta = \cancel{m}\,a_t)$

$\rightarrow T = m\left[\frac{v^2}{r} + g\cos\theta\right]$

$\boxed{a_t = g\sin\theta}$

* * * * *



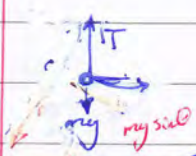
[15] Page 174

$m = 85\ Kg$ $r = 10\ m$

$v = 8\ m/s$

breaking strength T_{max} vines $= 1000\ N$

→ we must calculate T



$T - mg\cos\theta = m\,a_r$

$\boxed{T - mg\cos\theta = m\left(\frac{v^2}{r}\right)}$

$\theta = 0$

$T - 850 = 85\left(\frac{64}{10}\right)$

$T = 1394$

So $T_{apply} > T_{max}$

So he doesn't make it safely

Problems Page 173 / 174

[1]

→ d = 200 m t = 25 s

m = 1.5 $2\pi r = d$

$r = 31.8$

[1] Find average speed $= \frac{200}{25} = 8\ m/s$

[2] Find f_s $f_s = m a_r$

$= 1.5 \left(\frac{(8)^2}{31.8}\right) \approx 3\ N$

[3] $F = m a_r$ → $F = mg = 25(g)$ (10)

$m \frac{v^2}{r}$ mass = 3 kg

$r = 0.8\ m$

$v = \sqrt{\frac{g r m_1}{m_2}} = \sqrt{\frac{10(0.8)\,25}{3}}$

$v_{max} = 8.16\ m/s$ so any speed up to 8.16 m/s

[13]

$\theta = 5^\circ$ m = 80 kg

r = 10 m

mg = T = 800 N

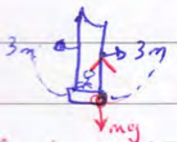
(a) $T_y = T\cos 5 = 800 \times \cos 5 = 796.9\ N$

$T_x = T\sin 5 = 800 \times \sin 5 = 69.7\ N$

(b) a_r $T\sin\theta = m a_r$

$a_r = \frac{69.7}{80} = 0.871\ m/s^2$

[17]



child sits → 40 kg

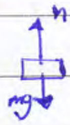
T = 350 N

→ find v → $2T - mg = m\left(\frac{v^2}{r}\right)$

$700 - 400 = 40 \frac{v^2}{3}$

$v = 4.7\ m/s$

b) Find (n)



$n - mg = m a_r$

$n - 392 = 308$ → $n = 700\ N$ → up

[19]

$F_r = m a_r$

$mg = m \frac{v^2}{r}$

$v^2 = g \times r$

$v = 3.13\ m/s$

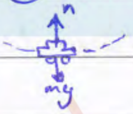
pail

[20]

mass = 500 kg

$v_A = 20\ m/s$

(a) Find (n)



F_r

→ $n - mg = m a_r$

$n - 500(9.8) = 500\left(\frac{(20)^2}{10}\right)$

$n = 20000 + 4900$

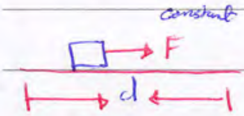
$n = 2.49 \times 10^4\ N$ → up

(b) Find v_B $mg = m\left(\frac{v^2}{r}\right)$

$v = 12.1\ m/s$

Chapter 7: Work & Kinetic Energy

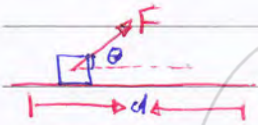
(7.2) Work done by a constant force



$W_F = (F) \cdot d$

$= F\, d \cos(\theta) \rightarrow \theta = 0$

$= F\, d \ (N.m) =$



$W_F = (F) \cdot (d)$

$= F\, d \cos\theta$

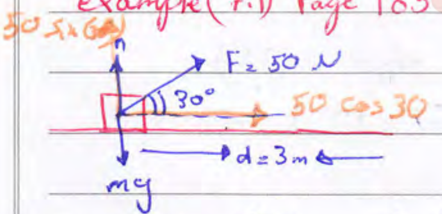
Work (is an energy transfer)

if energy is transfed to the system (object)
$W \rightarrow$ Positive

if energy is transfed from the system (object)
$W \rightarrow$ negative

$\rightarrow$ unit of work $= N.m =$ Joul. (J)

example (7.1) Page 185



① $W_F = (F\cos\theta)\, d$

$W = 50 \cos 30 \times 3$

$W = 130$ J

② if $d = 3$ $F = 32$ horizantal.

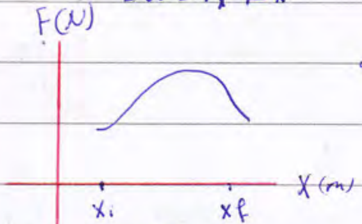
$W = F d$

$W = 32 \times 3 = 96$ J

(7.4) Work done by varying force.

$\Delta W = F_x \Delta x$

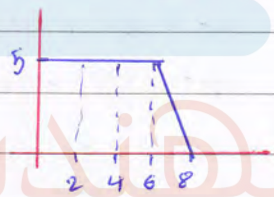
one dimension



$W = \int_{x_i}^{x_f} F_x\, dx =$ المساحة تحت المنحنى

$W = \int \vec{F} \cdot d\vec{r}$
$d\vec{r} = dx\hat{i} + dx\hat{j} + dx\hat{k}$
$\vec{F} = F\hat{i} + F\hat{j} + F\hat{k}$

Example (7.4) Page 189



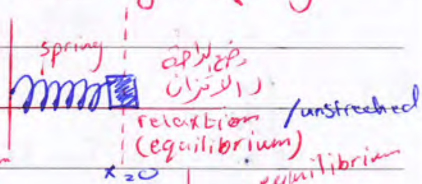
$W = \int_{x_i}^{x_f} F_x\, dx =$ Area

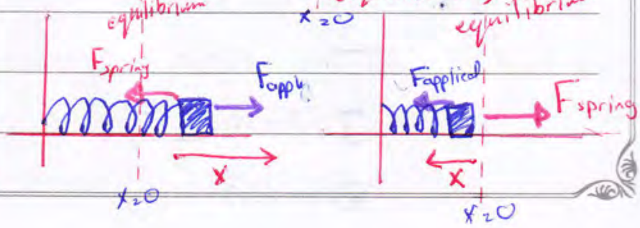
$= (6 \times 5) + \frac{1}{2} \times 2 \times 5$

$= 35$ N.m (J)

Example (7.5) Page 190

$\rightarrow$ Work done by a spring





→ $F_{spring} = -Kx$
$F_s = -kx$ } Hooke's law

x → is the displacement of the block

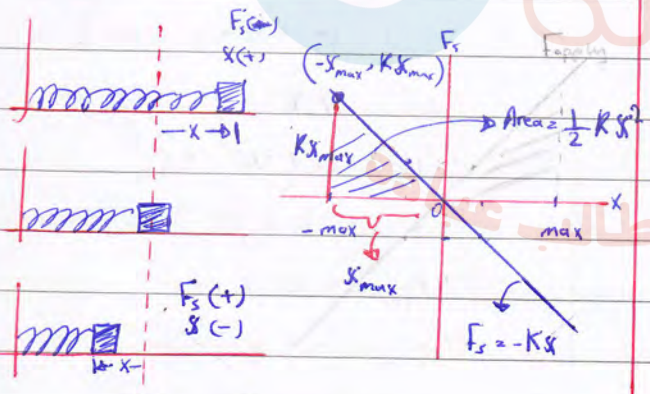
K → (positive constant) is called (force constant or spring constant)

- stiff springs → large K
- soft springs → small K

→ $W_{F_s} = \int F_s \, dx$

→ $F_s = -Kx$ → the negative sign indicate to that F_s is always directed opposite the displacement

→ $F_s = 0$ → the spring is unstretched



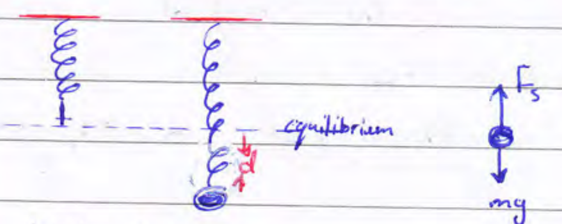
→ $W_{F_s} = \int_{x_i}^{x_f} (-Kx)\, dx$

$= \frac{1}{2}Kx_i^2 - \frac{1}{2}Kx_f^2$

→ $W_{F_{apply}} = \int_0^{x_{max}} F_{app}\, dx = \int_0^{x_{max}} Kx\, dx$

$= \frac{1}{2}Kx_{max}^2$

Example (7.6) Page 193 Measuring K for a spring



$|F_s| = K x$, $x \to d$

$Kd = mg$

$\boxed{K = \frac{mg}{d}}$ → how to measure (spring constant)

* Work done by constant force Example (7.3) Page (188)

ex:- Given $\vec{F} = (5\hat{i} + 2\hat{j})$ / $\vec{d} = (2\hat{i} + 3\hat{j})$
→ find W_F

Solution:- $W_F = \vec{F} \cdot \vec{d}$
→ $W_F = (10 + 6) = 16 \text{ J}$

* Work done by varying force

ex: Given $F = Kx^2$ $x_i = 1\text{m}$ $x_f = 3\text{m}$
find W

$W_F = \int_{x_i}^{x_f} Kx^2\, dx = \int_1^3 Kx^2$

$= \frac{Kx^3}{3}\Big|_1^3$

$= K\left(9 - \frac{1}{3}\right) = K\frac{26}{3}$ J

(7.5) Kinetic energy & the work - Kinetic energy theorem

if (ΣF) are constant F

$$\Sigma W = (\Sigma F)\, d$$

$$\Sigma W = (ma)\, d$$

but $d = \bar{v} t$

$$d = \frac{(v_f + v_i)}{2} t \qquad a = \frac{v_f - v_i}{t}$$

$$\rightarrow \Sigma W = m\left(\frac{v_f - v_i}{\cancel{t}}\right)\left(\frac{(v_f + v_i)\cancel{t}}{2}\right)$$

$$\Sigma W = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$\Sigma W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$\rightarrow K = \frac{1}{2} m v^2$$

→ Kinetic energy.

$$\text{so } \Sigma W = \Delta K = K_f - K_i$$

if (ΣF) are varying F_x

$$\Sigma W = \int_{x_i}^{x_f} (\Sigma F_x)\, dx$$

$$\Sigma W = \int_{x_i}^{x_f} (m a_x)\, dx$$

but $a_x = \frac{dv}{dt} \times \frac{dx}{dx}$

$$= \frac{dv}{dx} \frac{dx}{dt} \rightarrow v$$

$$= v \frac{dv}{dx}$$

$$\Sigma W = \int_{x_i}^{x_f} m \left(v \frac{dv}{\cancel{dx}}\right) \cancel{dx}$$

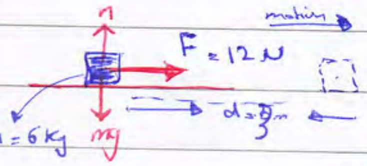
$$= \int_{v_i}^{v_f} m v \, dv$$

$$= \frac{m v^2}{2} \Bigg|_{v_i}^{v_f}$$

$$\Sigma W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

← التغير في الطاقة الحركية يعبر عن مقدار الطاقة المفقودة (قد تكون بسبب حدوث احتكاك)

Ex: (7.7) Page 196 (Block Pulled on a frictionless surface)



$v_i = 0$

$v_f = ??$

important

① → $F = m a_x$

$$a_x = \frac{12}{6} = 2 \text{ m/s}$$

$$\rightarrow v_f^2 = v_i^2 + 2 \Delta d\, a_x$$

$$v_f^2 = 2 \times 2 \times 3$$

$$v_f = 3.5 \text{ m/s}$$

② or $W = F d \cos(0)$

$$W = 12 * 3 * 1 = 36$$

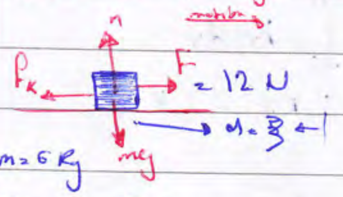
$$W = \Delta K$$

$$W = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$36 = \frac{1}{2} \times 6 (v_f^2)$$

$$v_f = 3.5 \text{ m/s}$$

Ex: (7.8) Page 197



$v_i = 0 \quad v_f = ??$

$\mu_k = 0.15$

$n = mg$

① → $\Sigma F = m a_x$

$$(F - f_k) = 6 a_x$$

$$(12 - (60)(0.15)) = 6 a_x$$

$$a_x = \frac{3}{6} = 0.5 \text{ m/s}$$

$v_f^2 = v_i^2 + 2a_x \Delta x$

$v_f^2 = \ddot{} + 2 \times \frac{1}{2} \times 3$

$v_f = 1.7\ m/s$

important

② or $\sum W = \Delta K$

$\sum W = W_{f_k} + W_F$

$\rightarrow W_{f_k} = f_k \cdot d$

$W_{f_k} = (n M_k)(3)\cos(180)$

$W_{f_k} = (60(0.15))(3)(-1)$

$W_{f_k} = -27$

$\rightarrow W_F = F \cdot d \cos 0$

$= 12 \times 3 \times 1 = 36$

$\rightarrow \sum W = 36 - 27$

$9 = \frac{1}{2} m v_f^2 - \cancel{\frac{1}{2} m v_i^2}$

$18 = 6 v_f^2$

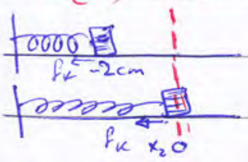
$v_f = 1.7\ m/s$

* * * * * *

→ note → work done by (n) or (mg) equal (zero) because $\boxed{\theta = 90^\circ}$

* * * * * *

Ex (7.11) A Block – spring system



$m = 1.6\ kg$

$k_s = 1 \times 10^3\ N/m$

$x_i = -2\ cm \quad x_f = 0$

ⓐ $\rightarrow W_s = \int (F_x) dx = \frac{1}{2} k x^2$

$= \frac{1}{2} \times 1000 \times (2 \times 10^{-2})^2$

$= 2 \times 10^{-1}\ J$

$W_s = \Delta K$

$0.2 = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$ (with $v_i = 0$)

$0.2 = \frac{1}{2}(1.6) v_f^2$ (0.8)

$0.25 = v_f^2 \rightarrow v_f = 0.5\ m/s$

ⓑ $f_k = 4.0\ N$

$\sum W = W_{f_k} + W_{F_s}$

$\rightarrow W_{f_k} = |f_k| \cdot |d| \cos(180)$

$W_{f_k} = 4 \times 0.02 \times (-1)$

$= -0.08\ J$

$\rightarrow W_{F_s} = 0.2\ J$

$\sum W = 0.2 - 0.08$

$= 0.12$

$\rightarrow \sum W = \Delta K$

$0.12 = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$ (with $v_i = 0$)

$0.12 = \frac{1}{2}(1.6)(v_f)^2$ (0.8)

$\frac{0.12}{0.8} = v_f^2 \rightarrow v_f = 0.39\ m/s$

* * * * * *

(8.4) (7.5) Power

Power (the time rate of doing work)

average power $= \bar{P} = \frac{\Delta W}{\Delta t} = J/s = watt = W$

instantaneous power $= P_{ins} = \frac{dW}{dt} = \vec{F} \cdot \frac{ds}{dt}$

$= \vec{F} \cdot \vec{v}$ velocity

F constant

ex (7.12) Power delivered by an Elevator Motor

$m_e = 1000$ kg
\+ $m_{passenger} = 800$ Kg
$f_k = 4000$ N

$P = ??$
$v = 3$ m/s
$a_y = 0$

$P = F \cdot v$

to left elevator so it's (T)

$\rightarrow T = f_k + mg$

$T = 4000 + (1800)(9.8)$

$T = 21640$ N

$\rightarrow P = 21640 \times 3 = 64920$ W

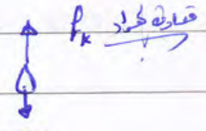
Problems:-

[1] $F = 5000$ N $d = 3$ km

$W_F = \vec{F} \cdot \vec{d} \rightarrow W_f = 5000 \times 3 \times 10^3$

$W_f = 15 \times 10^6$ $\boxed{W}$

[3]



$d = 100$ m
$m = 3.35 \times 10^{-5}$ g

$\rightarrow W_{f_k} = \vec{f_k} \cdot \vec{d}$.

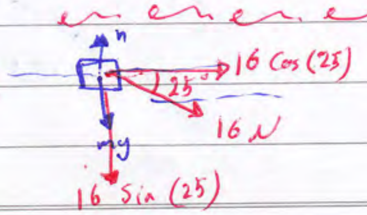
$W_{f_k} = W_g = mg(d)$

$= 3.35 \times 10^{-5} \times 10 \times 100$

$= 3.35 \times 10^{-2}$ J

so work done by all = zero

[5]



$m = 2.5$ kg

$d = 2.2$ m

[a] $F_{applied} \rightarrow W_{F_{appl}} = \vec{F} \cdot \vec{d}$

$= 16 \cos(25) \times 2.2$ m

$= 31.9$ J

[b] $W_n = \vec{n} \cdot \vec{d} = n \times d \times \cos(90) =$ zero

[c] $W_g =$ zero

[d] $W_{all} = W_{apply} = 31.9$ J

مسألة في app :

Q → Origin (0,0) $t = 0$

$\vec{v}_0 = 16\hat{i} - 12\hat{j}$

$\vec{a} = 3\hat{i} - 6\hat{j}$

→ Find v when $\boxed{t = 5}$

$v_{xf} = v_{xi} + a_x t$

$v_{xf} = 16 + 3(5) \rightarrow \boxed{v_{xf} = 31\ m/s}$

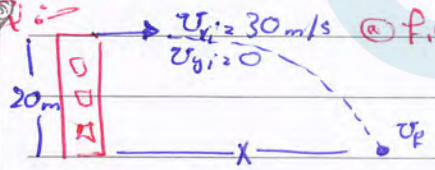
$v_{yf} = v_{yi} + a_y t$

$v_{yf} = -12 - 6(5) \rightarrow \boxed{v_{yf} = -42\ m/s}$

$\vec{v} = 31\hat{i} - 42\hat{j}$

$\boxed{v = 52.2\ m/s}$

Q :



(a) Find $\boxed{v_f}$

→ $v_{yf}^2 = v_{yi}^2 - 2(\Delta y)(g)$

$v_{yf}^2 = (0)^2 - 2(9.8)(-20)$

$\boxed{v_{yf} = -19.8\ m/s}$

$\boxed{v_{xf} = 30\ m/s}$

→ $v_f = 30\hat{i} + {}^-19.8\hat{j}$

$\boxed{v_f = 35.9\ m/s}$

(b) Find t_f

$v_{yf} = v_{yi} - gt$

$-19.8 = 0 - (9.8)t$

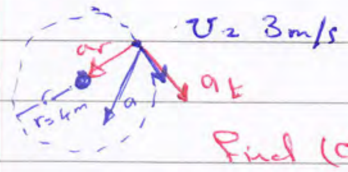
$\boxed{t = 2\ s}$

(c) Find (Range)

Range $= v_x \times (t)$

Range $= 30 \times (2) = 60\ m$

Q : →



Find (a)

rate $\frac{d}{dx}(\vec{v}_x) = a_t = 6\ m.s^{-2}$

$a = \sqrt{\left(\frac{v^2}{r}\right)^2 + (6)^2}$

$a = \sqrt{\left(\frac{9}{4}\right)^2 + 36}$

$a = 6.4\ m/s^2$

Theory of Dimension

L → length m

m → mass kg

T → time s

SI (m, kg, s)

→ m/s^2 → L/T^2

→ $\frac{1}{2}\rho v^2$ → $\frac{kg}{m^3} \times \frac{m^2}{s^2}$

dimensionless density

→ $\frac{m}{L\,T^2}$

Q : $x = 21 + 7t - t^2$

interval $(0, 3)$ s

$\bar{v}_x = \frac{33 - 21}{(3 - 0)} = 4\ m/s$

Potential Energy. E_p

Chapter 8 Conservation of Energy.

Q: $x = 2t - 10t^3$ (find a_x when particle change its direction, if it move in one dimension)

$x = 2t - 10t^3$ — when Particle change its direction, v_x = zero

$v_x = 2 - 30t^2$

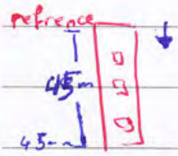
$a_x = -60t$

zero $= 2 - 30t^2$

$\boxed{t = 0.26\ s}$

$a_x = -60\,(0.26) = 15.6\ m/s^2$

Q2:



refrence

45m

45

$v_{yi} = -10\ m/s$

Find (t)

$\Delta y = v_{yi}\,t - \frac{1}{2}\,g\,t^2$

$-45 - 0 = -10(t) - \frac{1}{2}(\cancel{10})^{5}\,t^2$ (÷ -5)

$+9 = +2t + t^2$

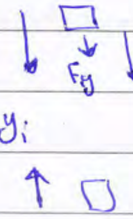
$t^2 + 2t - 9 = 0$

$t = \frac{-2 + \sqrt{4 + 36}}{2(1)}$

$\boxed{t = 2.16\ s}$

8.1 Potential Energy.

→ Gravitational Potential Energy.



F_g r ---initial level

y_i

---final level (refrence level)

gravitational potential energy = Zero

→ Gravitational Potential energy

$U_g = (mg)\,(y)$

الوزن الارتفاع

→ $W_g = \vec{F}\cdot\vec{r}$

$= |F|\cdot|r|$

الأكبر

$= mg\,(y_i - y_f)$

so → $W_g = (mg)\,y_i - (mg)\,y_f$

$W_g = -\Delta U_g$

Ex: $m = 12\ kg$ Find W_g

1m

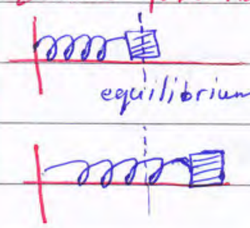
refrence level

→ $W_g = -\Delta U_g$

$W_g = mg\,(y_i - y_f)$

$= 12(10)\,(1 - 0) = 120\ J$

Elastic potential energy



equilibrium

$W_{spring} = \int_{x_i}^{x_f} F_{spring}\cdot dx$

$= \int_{x_i}^{x_f} -kx\ \cdot dx$

$= \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$

$= U_i - U_f$

$= -\Delta U_{spring}$

so Elastic potential Energy :⇒

$$U_{spring} = \frac{1}{2} K X^2$$

8.2 Conservative & nonconservative Forces

W: constant, we didn't care on path

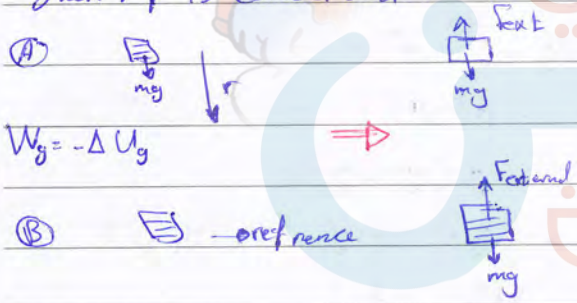
→ **Conservative Forces**

we can consider the force conservative if there isn't (loss energy) so there isn't change in mechanical energy (E)

so if → ① W doesn't depend on path

② W = zero if the particle move in closed path

then → F is conservative

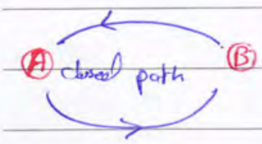


$W_g = -\Delta U_g$

so $W_{A\to B} + W_{B\to A} = \text{zero}$

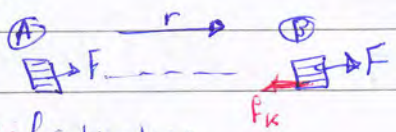
we don't care for path.

$$\Rightarrow W_{\text{conse. force}} = \int_A^B F\cdot dr + \int_B^A F\cdot dr = \text{zero}$$



→ **Nonconservative Forces**

we can consider the force nonconservative if there is (loss energy) so there is change in mechanical energy (E)



ex: frictionless

$W_{friction} = \vec{f}_k \cdot \vec{r} = -\mu_K\, n\, \Delta X$

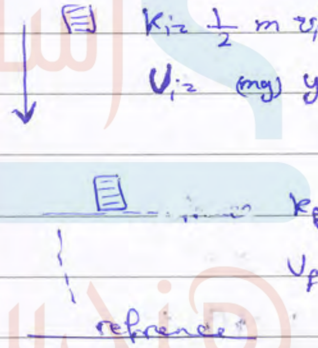
$= -\mu_K (mg)\, \Delta X_{A\to B}$

$W_{friction} = -\mu_K (mg)\, \Delta X_{B\to A}$

so $W_{friction} \neq 0$

so friction Force isn't conservative.

Conservative of mechanical energy



$K_i = \frac{1}{2} m v_i^2$

$U_i = (mg)\, y_i$

$K_f = \frac{1}{2} m v_f^2$

$U_f = (mg)\, y_f$

Mechanical energy → E

$E = U + K$

But $E_i = E_f$

$$\frac{1}{2} m v_i^2 + mg\, y_i = \frac{1}{2} m v_f^2 + mg\, y_f$$

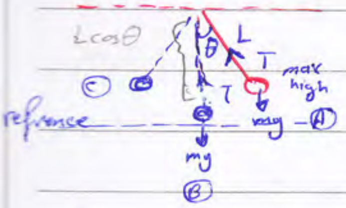
so $\Delta E = \Delta U + \Delta K$

$\Rightarrow v_f^2 - v_i^2 = 2g\,\Delta y$

$v_f^2 = v_i^2 + 2g\,\Delta y$

The pendulum.

ملاحظة: عندما تكون زاوية الميل صفر بالتالي البندول لا يتحرك بالتالي ($T = mg$)



$E_A = E_B = E_C$

$\Rightarrow \underline{E_A} = E_B$

$(mg)(L - L\cos\theta) = \frac{1}{2} m v_B^2$

[1] find v at lowest speed

$v_B^2 = 2gL(1-\cos\theta)$

$v_B = \sqrt{2gL(1-\cos\theta)}$

[2] find T at (B)

$T_B - mg = m\, a_r \rightarrow a_r \rightarrow$ لدينا حركة دائرية نصف قطرها (L)

$T_B - mg = m\frac{v_B^2}{L}$

$T_B = m\frac{v_B^2}{L} + mg$

$\Rightarrow$ if $L = 2m$ $M = 0.5\ kg$ $\theta = 30°$

find T_B & v_B

$v_B \Rightarrow$

$v_B = \sqrt{2gL(1-\cos\theta)}$

$= \sqrt{2(10)(2)(1-\cos 30)}$

$= 2.30\ m/s$

$T_B \Rightarrow T_B = m\frac{v_B^2}{L} + mg$

$= \frac{0.5(5.3)}{2} + 0.5(10)$

$= 6.32\ N$.

$\Rightarrow$ Conservative force

[1] gravitational $W_g = -\Delta U_g$

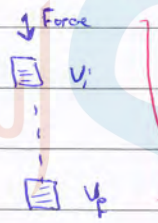
[2] elastic (spring, tension) $\Rightarrow W_{spring} = -\Delta U_{spring}$

[3] electrostatic (between charges)

$\Rightarrow$ Non Conservative force

frictionless

Work done by an applied force.



$W_{apply} + W_{g} = \Delta K$

$W_{apply} - \Delta U_g = \Delta K$

$W_{apply} = \Delta K + \Delta U_g$

$W_{apply} = \Delta E$

special case :-

If $W_{apply} = zero$ so $\Delta E = zero$

so $E_i = E_f$

so $K_i + U_i = K_f + U_f$

situation involving Kinetic Friction

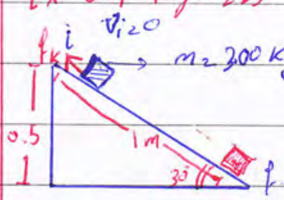
$W_{friction} = -f_k \cdot d$

$\Delta K_{friction} = -f_k \cdot d$

& $W_{friction} = \Delta E$

$-f_k \cdot d = \Delta K + \Delta U$

Ex 8.4 Page 225



$m = 300\ kg$ $f_k = 500N$

find v_f

isolated system $\Delta E = 0$

$W_{friction} = \vec{F_x} \cdot \vec{d} = |F||d| \cos(180)$

$= -f_k \cdot d \; (-1)$

$= 5 (1) = 500 \; N$

$\rightarrow W_{friction} = \Delta E$

$-5 = \left(\frac{1}{2} m v_f^2 + mg y_f\right) - \left(\frac{1}{2} m v_i^2 + mg y_i\right)$

$-5 = \frac{1}{2}(3)(v_f)^2 + (3)(10)(0)$
$\quad - \frac{1}{2}(3)(0) - 3(10)(0.5)$

$-5 = \frac{3}{2} v_f^2 - 15$

$v_f^2 = \frac{10 \times 2}{3}$

$v_f = 2.58 \; m/s$

$K_f = \frac{1}{2} m v_f^2$

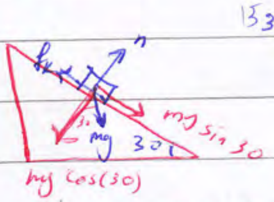
$= \frac{3}{2} \cdot \frac{10(2)}{3}$

$= 10 \; J$

Find a $\quad \rightarrow mg \sin 30 - f_k = ma$

$(3)(10)\left(\frac{1}{2}\right) - 5 = (3) a$

$\boxed{\frac{10}{3} = a} \; m/s^2$



If there isn't f_k

$E_f = E_i$

$\frac{1}{2} \cancel{m} v_f^2 = \cancel{m} g y_i$

$\frac{1}{2} v_f^2 = 10(0.5)$

$v_f = \sqrt{10} = 3.2 \; m/s$

8.6 Relationship between conservative forces & Potential energy

$\boxed{F_x = -\frac{dU}{dx}}$ العلاقة ما بين طاقة الوضع والقوة المحافظة

or $\boxed{F_y = -\frac{dU}{dy}}$

$\Rightarrow \int_{x_i}^{x_f} -dU = \int_{x_i}^{x_f} F_x \cdot dx$ → W for varying force

$-\Delta U = W_{force}$

⇒ for spring.

$F = -\frac{dU_{spring}}{dx}$ $\quad \frac{1}{2} k x^2$

$F_x = -\frac{d}{dx}\left(\frac{1}{2} k x^2\right)$

$F_x = -k x = F_{spring}$

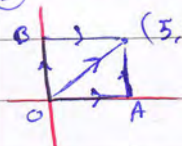
⇒ for gravity.

$F_y = -\frac{dU_{gravity}}{dy}$

$F_y = -\frac{d}{dy}(mgy) = -mg = F_{gravity}$

Problem:

(5)



$(5,5)\; m \qquad F_g = mg(-\hat{j})$ محور

$W_g = \int_{O \to A \to C} \vec{F} \cdot \vec{dr}$

$= \int_0^A mg(-\hat{j}) \cdot dx\,\hat{i} + \int_A^C mg(-\hat{j}) \cdot dy\,\hat{j}$

$= 0 - mg \int_A^C dy$

$= -(4)(10)(5) = -200$

تكرار (c) + (b)

$I = \Delta p = \bar{F}\,\Delta t = \int F \cdot dt$

Ch 9: Linear Momentum & Collisions.

9.1 linear Momentum & its conservation

الزخم الخطي

linear momentum :

$\vec{P} = m\vec{V}$ → velocity of particle

↳ mass of particle

→ linear momentum In 3D

$\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$

$\vec{P} = (mV_x)\hat{i} + (mV_y)\hat{j} + (mV_z)\hat{k}$

Relationship between F & P

$\frac{d\vec{P}}{dt} = \frac{d\,m\vec{V}}{dt}$

$\frac{d\vec{P}}{dt} = m\frac{d\vec{V}}{dt}$ (constant m) a

$\frac{dp}{dt} = \sum F$

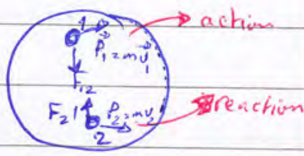
isolated system $\sum F = 0$

if $\sum F = \text{zero}$ then $\frac{dp}{dt} = \text{zero}$

so P → constant

Conservation of Momentum for a 2-particle system

$\sum F = \text{zero}$



$F_{21} = -F_{12}$

$\frac{dP_1}{dt} = -\frac{dP_2}{dt}$

isolated system

so $\frac{d}{dt}(P_1 + P_2) = 0$

so $P_1 + P_2 = \text{Constant}$] conservation linear momentum

so the total linear momentum is conservative

$P_{total} = P_1 + P \ldots = \text{constant}$

→ So the total p of an isolated system at all times equal its intial momentum

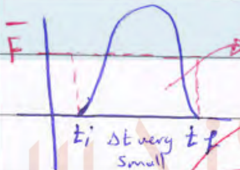
constant ثابت $\sum \vec{P}_i = \sum \vec{P}_f$

9.2 Impulse & Momentum

$\frac{dp}{dt} \approx F$

$\int dp = \int_{t_i}^{t_f} F \cdot dt$

$I = \Delta p = \int_{t_i}^{t_f} F\,dt$ Impuls force or (Impuls-momentum theorm)



→ Area = $\bar{F}\,\Delta t$

$\bar{F} = \frac{\bar{F}\Delta t}{\Delta t} = \frac{\int_{t_i}^{t_f} F\,dt}{\Delta t}$

$I = \Delta p = \bar{F}\Delta t = \int$

$\bar{F} = \frac{1}{\Delta t}\int_{t_i}^{t_f} F \cdot dt$

$\bar{F}\Delta t = I$

impuls approximation → one of the forces exerted on a particle acts for a short time but is much greater than any other force present

Ex (9.3)

$R = V_B\cos\theta\,(t)$

$0 = V_1\sin\theta - g t$

$R = \frac{V_B^2\sin(2\theta)}{g}$

$R = 200\,m$ / $\theta = 45°$

$m = 50g$

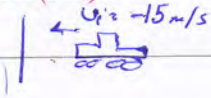
$V_A = V_A = \text{zero}$

$V_B = 44.27\,m/s$

$I = \Delta p = m(V_B - V_A)$

$= \frac{50}{1000}\,44.27 = 2.2\ kg\cdot m/s$

Ex(9.4)

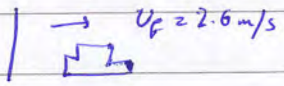


$\Delta t = 0.15$

$m = 1500$ Kg

Find (I)

$\bar{F}$



$\Rightarrow \Delta P = P_f - P_i$

$I = m(v_f - v_i)$

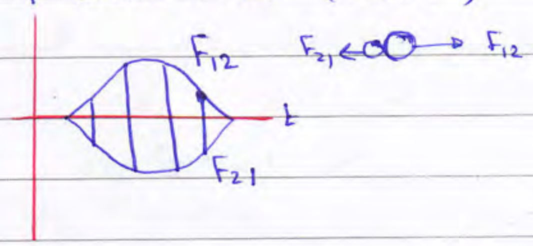
$I = 1500(2.6 - -15) = 2.6 \times 10^4$ Kg.m/s

$\Rightarrow \bar{F} = \frac{I}{\Delta t} = \frac{2.6 \times 10^4}{0.15} = 17.3 \times 10^4$ N

9.3 Collisons

→ Collisions: it's term to represent the event of two particle's coming together for a short time & thereby producing impulsive forces on each other (these force are assumed to be much greater than any external force represent)

F Newton's 3rd law (collision)



Law → The total momentum of an isolated system just before a collision equals the total momentum of the system just after collision

mathematic ⟹ $\sum \vec{P}_{\substack{before\\collision}} = \sum \vec{P}_{\substack{after\\collision}}$

Ex(9.5)

$m_2 = 1800$ Kg $m_1 = 900$ Kg

$v_i = 20$ m/s

$P_i = P_f$ to small car

$m_1 v_i = (m_1 + m_2) v_f$

$v_f = \frac{900(20)}{2700} = 6.67$ m/s

9.4 Elastic & inelastic collision in one dimension.

→ Elastic collision $\left\langle \begin{array}{l} \sum \vec{P}_{before} = \sum \vec{P}_{after} \\ \sum KE = \sum KE \end{array} \right.$

$\sum KE_{\substack{before\\collision}} = \sum KE_{\substack{after\\collision}}$ $K_f = K_i$

→ Inelastic collision $\left\langle \begin{array}{l} \sum \vec{P}_{before} = \sum \vec{P}_{after} \\ \sum KE \neq \sum KE \end{array} \right.$

$\sum KE_{\substack{before\\collision}} \neq \sum KE_{\substack{after\\collision}}$ $K_f < K_i$

ΔK

→ The momentum is constant in all collision, but KE is constant only in elastic collisions

→ Perfectly inelastic collisions

Before collision after collision

Ex ($\vec{P}$)

(2m) → (m)(m)

$\vec{P}_i = zero$ $\qquad$ $\vec{P}_f = mU_{1f} + mU_{2f}$

$\vec{V}_i = zero$ $\qquad$ $\vec{P}_f = m(U_{1f} + U_{2f})$

so $\vec{P}_p = zero$

$U_{1f} = -U_{2f}$

ملاحظة → $|P_1| = |P_2|$ if (2m) → (m₁)(m₂)

$P_i = zero$

so $P_f = zero$

so $P_1 = P_2$

$m_1 V_{1f} = m_2 V_{2f}$

$\Rightarrow \dfrac{m_1}{m_2} = \dfrac{V_{2f}}{U_{1f}}$

بينهم علاقة عكسية كلما زادت الكتلة تقل السرعة والعكس صحيح

* * * * *

Ex 9.5 Is the collision elastic or inelastic ??

$V_i = 20\,m/s$ $\quad$ $V_i = zero$ $\qquad$ $U_f = 6.67\,m/s$

$\rightarrow KE_{before} = \frac{1}{2} m_1 V_{1i}^2 + \cancel{\frac{1}{2} m V_{2i}^2}$

$KE_{before} = \frac{1}{2}(900)(20)^2$

$= 180000\ J$

$= 18 \times 10^4\ J$

$KE_{after} = \frac{1}{2}(m_1 + m_2) U_f^2$

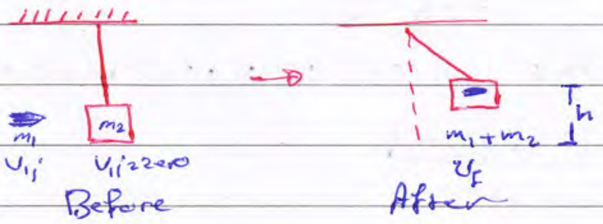
$= \frac{1}{2}(2700)(6.67)^2$

$= 6 \times 10^4\ J$

so $KE_{Befor} > KE_{after}$

so it isn't elastic collision.

Ex (9.6) The Blastic Pendulum



$P_i = P_f$

$m_1 V_{1i} + zero = (m_1 + m_2) U_f$

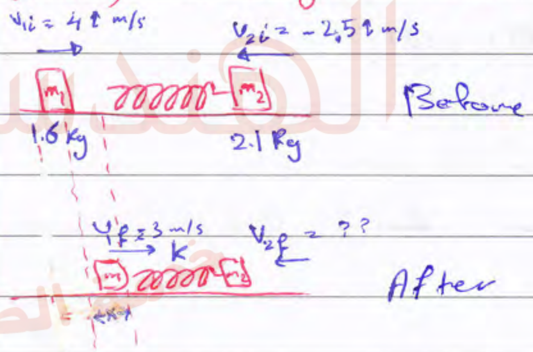
$\rightarrow U_f = \left(\dfrac{m_1}{m_1 + m_2}\right) V_{1i}$

$\rightarrow KE_i = \frac{1}{2} m V_{1i}^2$

$\rightarrow KE_f = \frac{1}{2}(m_1 + m_2) U_f^2$

* * * * *

Ex (9.7) Two-body collision with a spring



$K = 600\,N/m$

① Final V_{2f}

$\vec{P}_i = \vec{P}_f$

$m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1f} + m_2 V_{2f}$

$(1.6)(4) + (2.1)(-2.5) = (1.6)(3) + (2.1) V_{2f}$

$U_{2f} = -1.74\ m/s$

② Find X $\quad$ elastic collision

$KE_i = KE_f \begin{cases} m_1 \\ m_2 \\ spring \end{cases}$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 + \frac{1}{2} k x^2$$

$$(1.6)(4)^2 + (2.1)(-2.5)^2 = (1.6)(3)^2 + (2.1)(1.74)^2 + 600 x^2$$

$$\boxed{x = 0.173\ m}$$

9.5 Two-dimensional collisions

→ The isolated system (2 particle)

In One-dimension

1) Linear momentum is constant

if we have conservation of linear momentum

$$\sum \vec{P}_i = \sum \vec{P}_f$$

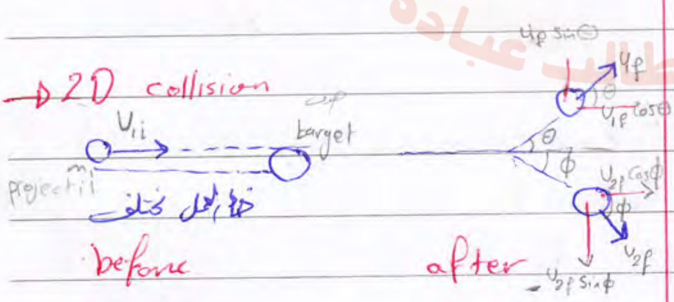
2) If $KE_i = KE_f$ so it's elastic collision

$$\sum KE_i = \sum KE_f$$

→ 1D collision

head on collision

→ 2D collision



1) Conservation of Linear Momentum

$$\sum P_i = \sum P_f$$

$$m_1 \vec{v}_{1i} + 0 = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

(x-component)

$$\boxed{m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi}$$

y-component

$$\boxed{0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi}$$

2) If collision elastic بدون vector

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

special case if $\boxed{m_1 = m_2}$

1) → so $(\vec{v}_{1i} = \vec{v}_{1f} + \vec{v}_{2f})^2$

$$\boxed{\vec{v}_{1i}^{\,2} = \vec{v}_{1f}^{\,2} + 2\vec{v}_{1f}\vec{v}_{2f} + \vec{v}_{2f}^{\,2}}$$

2) → $\boxed{v_{1i}^2 = v_{1f}^2 + v_{2f}^2}$

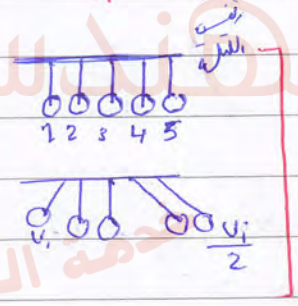
مع بعض يجب أن تكون الزاوية بينهم 90° أي أن

$$\boxed{90^\circ = \theta + \phi}$$

ضرب $(2\vec{v}_{1f} \cdot \vec{v}_{2f})$ يساوي صفر

$0 = |\vec{v}_{1f}||\vec{v}_{2f}| \cos(90)$ فيجب أن تكون

بالتالي $v_{1f} \perp v_{2f}$



1) $m v_i = 2m \frac{v_i}{2}$

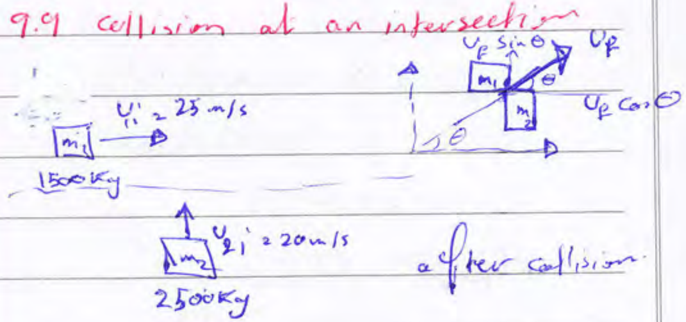
$m v_i = m v_i$

2) $\frac{1}{2} m v_i^2 \overset{??}{=} \frac{1}{2}(2m)\left(\frac{v_i}{2}\right)^2$

$KE_i \neq KE_f$

So it isn't elastic

9.9 collision at an intersection



before collision

after collision

Find $[v_f\ \&\ \theta]$

→ ① linear Momentum. اذا فيه v_f مختلفين

$$\sum \vec{P}_i = \sum \vec{P}_f$$

$\Delta KE = KE_f - KE_i$

→ x-component

$$(m_1 v_{1i} = (m_1+m_2) v_f \cos\theta)$$

$$1500(25) = (4000)\, v_f \cos\theta \quad \dots ①$$

→ y-component

$$m_2 v_{2i} = (m_1+m_2) v_f \sin\theta$$

$$2500(20) = (4000)\, v_f \sin\theta \quad \dots ②$$

→ we divide ② by ①

$$\frac{2500(20) = \cancel{4000}\, \cancel{v_f} \sin\theta}{1500(25) = (\cancel{4000})\, \cancel{v_f} \cos\theta}$$

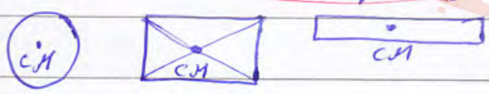
$\tan\theta = 1.3 \qquad \theta = 53.4°$

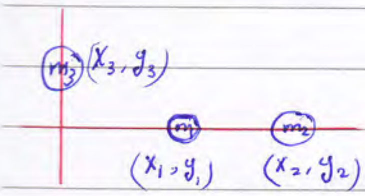
→ $1500(25) = 4000\, v_f\, (0.6)$

$v_f = 15.6\ m/s$

9.6 The Center of Mass

Center of Mass: C.M → special point & it's vector quantity



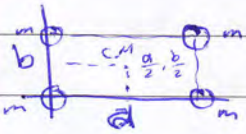


→ X-Component

$$X_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3}$$

$$y_{CM} = \frac{y_1 m_1 + y_2 m_2 + y_3 m_3}{m_1 + m_2 + m_3}$$

Ex:



$$x_{cm} = \frac{m(a) + m(a) + m(0) + m(0)}{4m} = \frac{2a\cancel{m}}{4\cancel{m}} = \frac{a}{2}$$

$$y_{CM} = \frac{m(b) + m(b) + m(0) + m(0)}{4m} = \frac{2mb}{4m} = \frac{b}{2}$$

In General

$$X_{CM} = \frac{\sum m_i x_i}{\sum m_i} \quad \Big|\quad y_{c.M} = \frac{\sum m_i y_i}{\sum m_i} \quad \Big|\quad Z_{cm} = \frac{\sum m_i z_i}{\sum m_i}$$

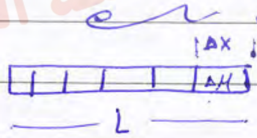
$$\Rightarrow r_{C.M} = X_{C.M}\, \hat{i} + y_{C.M}\, \hat{j} + Z_{C.M}\, \hat{k}$$

~~Q: $w_i = 0$~~
~~α → constant~~
~~$w_f = 12$ rad/s~~
~~$t = 3$ s~~
~~find α~~

~~$w_f = w_i + \alpha t$~~

~~$12 = 0 + \alpha(3)$~~

$$\boxed{\alpha = 4\ rad/s^2}$$



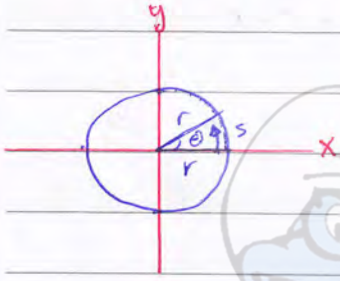
$$X_{C.M} = \frac{\sum (\Delta x)_i (\Delta m)_i}{\sum (\Delta m)} = \frac{1}{M} \times \int_0^L x\, \lambda\, dx$$

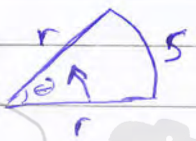
$$X_{C.M} = \frac{\lambda}{M}\left(\frac{L^2}{2}\right)$$

Chapter 10

Rotation of a Rigid object about a Fixed Axis.

10.1 Angular Displacement, velocity & Acceleration





$$\theta = \frac{s}{r} = \text{rad}$$

when $s = r$ $\theta = 1\,\text{rad}$

$1\,\text{rad} \rightarrow 57.3^\circ$

	Translation		Rotation	
①	t	$\rightarrow$	t	
②	x	$\rightarrow$	θ	
③	$\bar{v} = \frac{\Delta x}{\Delta t}$	$\rightarrow$	$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$	average angular speed
④	$v_{ins} = \frac{dx}{dt}$	$\rightarrow$	$\omega_{ins} = \frac{d\theta}{dt}$	instan. angular velocity
	$= \lim_{\Delta t \to 0} \bar{v}$		$\omega_{ins} = \lim_{\Delta t \to 0} \bar{\omega} = \frac{\Delta\theta}{\Delta t}$	
⑤	$\bar{a} = \frac{\Delta v}{\Delta t}$	$\rightarrow$	$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$	average angular acceleration
⑥	$a = \frac{dv}{dt}$		$\alpha = \frac{d\omega}{dt}$	instanten. angular acceleration
	$= \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \bar{a}$		$= \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \lim_{\Delta t \to 0} \bar{\alpha}$	

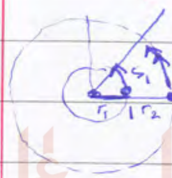
$\rightarrow$ $s = r\,\theta$ (ثابت, constant)

$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

$$\boxed{v = r\,\omega}$$ Tangential speed.

Relation ship between v & ω.

$\rightarrow$ very important note :-



every particle on a rigid object rotat (when rotating about fixed axis) $\theta = \frac{s_1}{r_1} = \frac{s_2}{r_2}$

$\rightarrow$ every particle has the same

θ, $\bar{\omega}$, $\bar{\alpha}$
angle, angular speed, angular acceleration

10.2 Rotational Kinematics (constant α)

The motion equations in Rotational.

$\omega_f = \omega_i + \alpha t$ — بدل x ← θ

$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$ — بدل v ← ω

$\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$ — بدل a ← α

Ex 10.1

$\alpha = 3.5\,\text{rad/s}^2$ constant

$t = 0 \rightarrow t = 2$

$\omega_i = 2\,\text{rad/s}$

① find $(\Delta\theta)$ at $t = 2$

$\rightarrow \Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$\Delta\theta = 2(2) + \frac{1}{2}(3.5)(4)$

$\Delta\theta = 11\,\text{rad} = 630.3$

No. of revulation $= \frac{630.3}{360^\circ} = 1.75\,\text{rev/s}$

② final ω_f at $t=2$

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f = 2 + (3.5)2 = 9 \text{ rad/s}$$

③ at interval (2-3) initial, final, final $\Delta\theta$

$$\Delta\theta = \underset{at\,(t=2)}{\omega_i}\, t + \frac{1}{2}\alpha t^2$$

$$\Delta\theta = (9)(3-2) + \frac{1}{2}(3.5)(3-2)^2$$

$$= 9 + 1.75 = 10.75 \text{ rad.}$$

10.3 Angular & Linear Quantities

$$\rightarrow \quad S = r\theta$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

$$\boxed{v = r\omega} \quad \text{— (1) Tangential speed}$$

$$\frac{dv}{dt} = r\frac{d\omega}{dt}$$

$$\boxed{a_t = r\alpha} \quad \text{— (2) Tangential acceleration}$$

$$a = \sqrt{a_t + a_r}$$

$$a = \sqrt{(r\alpha)^2 + \left(\frac{v^2}{r}\right)^2}$$

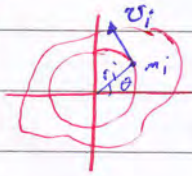
$$a = \sqrt{(r\alpha)^2 + \left(\frac{r^2\omega^2}{r}\right)^2}$$

$$a = \sqrt{r^2(\alpha^2 + \omega^4)}$$

$$a = r\sqrt{\alpha^2 + \omega^4}$$

10.4 Rotational energy.

$$K_i = \frac{1}{2} m_i v_i^2$$



$$K_R = \sum K_i$$

$$= \sum \frac{1}{2} m_i v_i^2$$

$$= \frac{1}{2}\sum m_i r_i^2 \omega_i^2$$

but $\omega \rightarrow$ constan for the same particle.

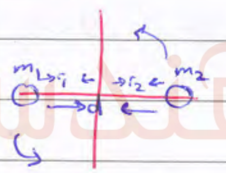
$$= \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

$$= \frac{1}{2} I \omega^2$$

So we called $(\sum m_i r_i^2)$ the mass of intia

Translation	Rotation
M (mass)	I (mass of intia)
$K = \frac{1}{2} m v^2$	$K = \frac{1}{2} I \omega^2$

Ex:



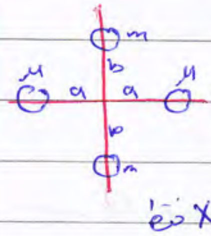
$$I = \sum m_i r_i^2$$

$$= m_1 r_1^2 + m_2 r_2^2$$

$$= m_1\left(\frac{d}{2}\right)^2 + m_2\left(\frac{d}{2}\right)^2$$

$m_1 = m_2$

$$= 2m\left(\frac{d}{2}\right)^2 = \frac{md^2}{2}$$



$$I_z = (mb^2) + (mb^2) + (Ma^2) + (Ma^2)$$

اي نأخذ كل النقاط (الموجودة حول z)

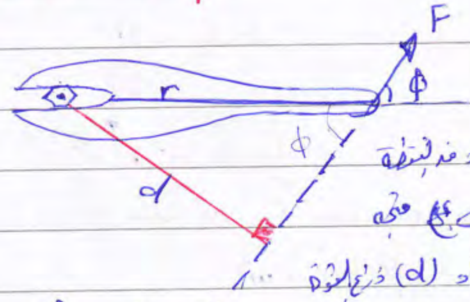
$$I_y = (Ma^2) + (Ma^2)$$

نأخذ النقاط الموجودة على X فقط
(الي موجودة حول y)

$$I_x = mb^2 + mb^2$$

نأخذ النقاط الموجودة على y فقط
(الي موجودة حول X)

10.6 Torque



نحتاج عمود من النقطة التي تتحرك فيه المتوقع لاتجاه (d) ذراع القوة

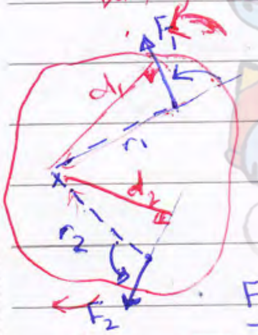
$\vec{\tau} = \vec{r} \times \vec{F}$

$|\vec{\tau}| = |\vec{r}|\,|\vec{F}| \cdot \sin\phi$ $\qquad \sin\theta = \frac{d}{r}$, $d = r\sin\phi$

$\boxed{|\tau| = d\,F}$ $\qquad \sum T = \sum F \cdot d$

ذراع القوة — ذراع القوة

* * * * *



if (F) move by clockwise so T = -ve

if (F) move by counter clockwise so T = +ve

وكذلك بناءً على سهم من (r) إلى F

$\sum T = T_1 + T_2$

$= F_1 d_1 - F_2 d_2$

→ By Ex:

if $F_1 = 5N \to d_1 = m$

$F_2 = 3N \to d_2 = 3m$

find $\sum T$

→ $\sum T = 5(1) - (3)(3)$

$= -4$ so the particle move by clockwise.

* * * * *

Ex:

$T_{F_1} \to$ -ve

$T_2 \to$ +ve

ⓐ $\sum T = T_1 + T_2$

$= \vec{r}_1 \vec{F}_1 \sin(90) + \vec{r}_2 \vec{F}_2 \sin(90)$

$= r_1 F_1 + r_2 F_2$

ⓑ if $F_1 = 5N \to R_1 = 1m$

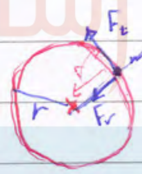
$F_2 = 15N \to R_2 = 0.5m$

$\sum T = -5(1) + (15)(0.5)$

$= 2.5\ N.m$

* * * *

Relationship between τ & α



$F_t = m a_t$

$F_r = m a_r$

$\sum T = F_t \times |\vec{r}| \sin(90) + F_r |\vec{r}| \cancel{\sin(180)}$

$T = F_t\, r$

$= m a_t r \qquad a_t = r\alpha$

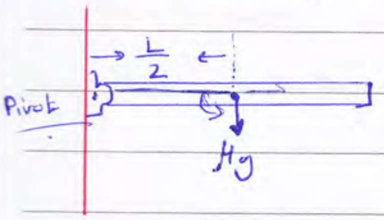
$= m(r\alpha) r$

$= (mr^2)\alpha \qquad I = mr^2$

$\tau = I\alpha$

Translation	Rotation
t →	t
x →	θ
v →	ω
a →	α
F →	T
$\sum F = ma$ →	$\sum T = I\alpha$

Ex (10.10) Rotating Rod



→ find α;

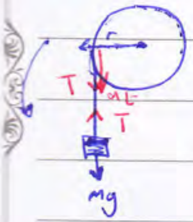
$\tau = F \cdot r = (Mg)\left(\frac{L}{2}\right)\sin(90)$

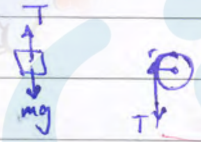
$\tau = I\alpha = Mg\left(\frac{L}{2}\right)$

$\alpha = \frac{MgL}{2I}$ معطاة

* * * * *

Ex (10.12)





$\boxed{mg - T = ma}$

$\boxed{a = \frac{mg - T}{m}}$

$\sum \tau = I\alpha$

$F \cdot r = I\alpha$

$TR = I\alpha$

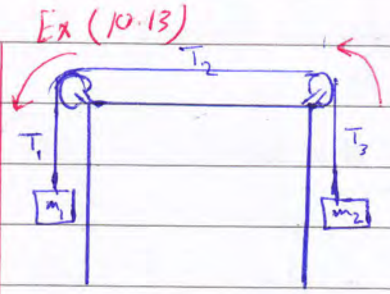
$\boxed{\alpha = \frac{TR}{I}}$

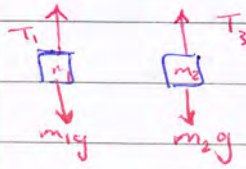
$a = R\alpha$

$a = R\left(\frac{TR}{I}\right) = \frac{TR^2}{I}$

$\frac{TR^2}{I} = \frac{mg - T}{m}$

Ex (10.13)





$\boxed{m_1 g - T_1 = m_1 a}$

$\boxed{T_3 - m_2 g = m_2 a}$

$\sum \tau = I\alpha$

$\boxed{R(T_2 - T_1) = I\alpha}$

$\sum \tau = I\alpha$

$R(T_3 - T_1) = I\alpha$

نربط بين كل المعادلات

$a = R\alpha$ قوام

* * * * *

10.8 Energy consideration in Rotational Motion

Translation	Rotation
F →	τ
$P = \frac{dW}{dt} = F \cdot v$ →	$P = \tau \cdot \omega$
	$dW = F \cdot ds$
	$\frac{dW}{dt} = \tau \cdot \frac{d\theta}{dt}$
	$P = \tau \cdot \omega$

$\sum \tau = I \cdot \alpha$

$\sum \tau = I \frac{d\omega}{dt}$

$\sum \tau = I \frac{d\omega}{d\theta} \frac{d\theta}{dt}$

$$\sum \tau = I \frac{d\omega}{d\theta}\,\omega$$

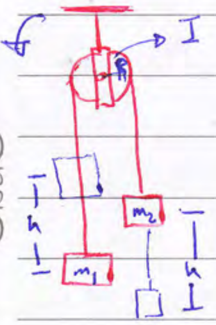
$$\sum \tau . d\theta = I\, d\omega . \omega$$

$$\sum W = \int_{\omega_i}^{\omega_f} I\,\omega . d\omega$$

$$\sum W = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2$$

$$\sum W = \Delta K$$

Ex (10.15) Connected Cylinders



$$\Delta K = K_f - K_i$$

$$\Delta K = \frac{1}{2} m_1 v_f^2 + \frac{1}{2} m_2 v_f^2 + \frac{1}{2} I \omega^2$$

$$\omega = \frac{v_f}{r}$$

$$\Delta K = \frac{1}{2}\left(m_1 + m_2 + \frac{I}{R^2}\right) v_f^2$$

Chapter 11: Rolling Motion & angular momentum.

11.2 Angular Momentum: The Non-isolated system

$$\tau = r \times F$$

$$\left(\sum \vec{\tau} = r \times \sum \vec{F}\right)$$

$$\sum \vec{\tau} = r \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt}\,\vec{p}$$

$$\frac{d\vec{r}}{dt}\,\vec{p} = 0 \qquad \vec{v} \times \vec{p} \qquad \rightarrow \vec{p} \quad \rightarrow \vec{v}$$

$$v \times p \sin(0) = \text{zero}$$

$$\sum \tau = \frac{d(\vec{r} \times \vec{p})}{dt}$$

Translation	Rotation
$\sum F = \frac{d\vec{p}}{dT}$	$\sum \tau = \frac{d(\vec{r} \times P)}{dt}$
So Linear Momentum $\vec{P}$	Angular Momentum $\vec{L} = \vec{r} \times \vec{P}$

So the Relationship between L & P

$$\boxed{L = rp}$$

if $\sum \tau = 0$ so L_{total} = constant

→ to conservation angular Momentum for isolated system

so $\sum L_i = \sum L_f$

$$I_i \omega_i = I_f \omega_f$$

11.3 Angular Momentum of Rotating Rigid object

$\vec{L} = r \times \vec{p}$

$L = r \times mv$

$L = r\, m\, r\omega$

$\sum L_i = (\sum m_i r_i^2)\, \omega$

$\sum L_i = I\, \omega$

Translation	Rotation
$\vec{P} = m\vec{v}$	$\vec{L} = I\omega$

$\frac{dL_i}{dt} = \frac{d(I\omega)}{dt}$

$\frac{dL}{dt} = I \frac{d\omega}{dt}$

$\frac{dL}{dt} = I\alpha$

$\frac{dL}{dt} = \sum \tau$

T	R
$\frac{dP}{dt}$	$\frac{dL}{dt}$

Different Questions about Ch10 & ch 11

Q: $\theta = 5 + 10t + 2t^2$ $\quad \omega = 10 + 4t$

find (θ, ω, α) $\quad \alpha = 4$

① at $\boxed{t=0}$

$\theta_{(t=0)} = 5$ rad

$\omega_{(t=0)} = 0 + 10 + 4(0)$

$= 10$ rad/s

$\alpha = 4$ rad/s^2

② at $\boxed{t = 3s}$

$\theta_{(t=3)} = 5 + 10(3) + 2(9) = 53$ rad

$\omega_{(t=3)} = 10 + 4(3) = 22$ rad/s

$\alpha = 4$ rad/s^2

Q: $\alpha = 10 + 6t$ $\quad \omega_i = 0$ $\quad \theta_i = 0$

find θ when $\boxed{t = 4s}$

$\int \alpha = \int 10 + 6t$

$\rightarrow \omega = 10t + 3t^2 + C$

when $t=0 \rightarrow \omega = 0 \rightarrow C = 0$

$\rightarrow \int \omega = \int 10t + 3t^2$

$\theta = \frac{10t^2}{2} + t^3 + C$

when $t=0 \rightarrow \theta_i = 0 \rightarrow C = 0$

$\theta_{(t=4)} = 5(4)^2 + (4)^3$

$= 16(9) = 144$ rad.

Ch 12 :- Static equilibrium & Elasticity.

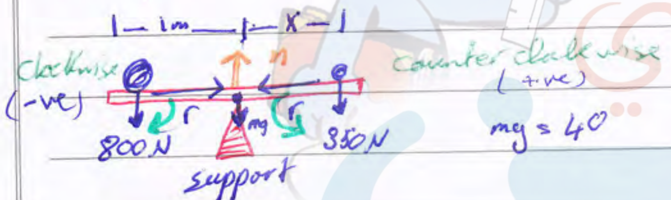
(12.1) Analysis : Rigid object in Equilibrium.

→ we have two condition for static equilibrium :-

[1] $\sum \vec{F} = 0 \; < \; \begin{matrix} \sum F_x = 0 \\ \sum F_y = 0 \end{matrix}$

[2] $\sum \vec{\tau} = 0$ about any axis

Example :-



→ find n & x ?

→ solution :-

[1] $\sum F = 0$

$n - 800 - mg - 350 = 0$

$n - 800 - 40 - 350 = 0$

$\boxed{n = 1190\ N}$

(2) $\sum \tau = 0$

لابد من تحديد نقطة الارتكاز.
وهنا نأخذها من المنتصف فنهمل
اي قوى تقع عند هذه النقطة

$350\,(x) - 800\,(1) = 0$

$x = \dfrac{800}{350} = 2.29\ m$

* * * *

(12.2) More on the Center of Mass

$$\Rightarrow x_{c.g} = \frac{m_1 x_1 g + m_2 x_2 g \ldots}{(m_1 + m_2 + m \ldots)\, g}$$

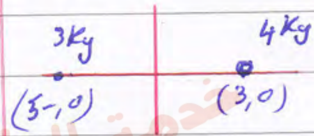
$$x_{c.g} = \frac{\cancel{g}\,(m_1 x_1 + m_2 x_2 \ldots)}{\cancel{g}\,(m_1 + m_2 + m \ldots)}$$

So that

→ Center of mass = Center of gravity

$$\begin{bmatrix} x_{c.g} = x_{c.M} \\ y_{c.g} = y_{c.M} \\ z_{c.g} = z_{c.M} \end{bmatrix}$$

Example :-



find the c.g ?

→ C.M = C.G

we have only x - component

$$x_{c.g} = \frac{4(3) + 3(-5)}{(4+3)}$$

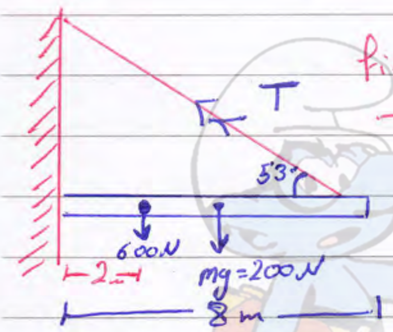
$= \dfrac{-3}{7} = -0.43$

12.3 Examples of Rigid objects in Static equilibrium.

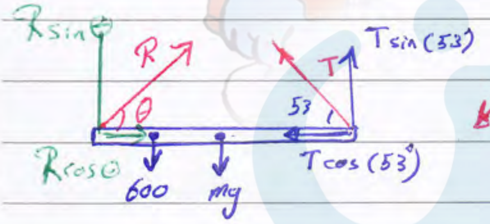
→ Keep in mind that:

$\sum \tau = 0 \;,\; \sum \vec{F} = 0$

Example :- standing on a horizontal beam



find T & R & θ



R → force result from the wall

$\sum \vec{F} = 0 \rightarrow \sum F_x = 0$

$T\cos(53°) - R\cos\theta = 0$

$\boxed{T(0.6) = R\cos\theta}$ ---(1)

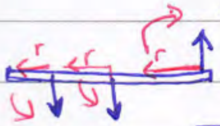
$\rightarrow \sum F_y = 0$

$T\sin(53°) + R\sin\theta - 600 - 200 = 0$

$\boxed{R\sin\theta = -0.8T + 800}$ (2)

$\sum \tau = 0$ R تحدد نقطة الارتكاز عند القوى

& نجعل اتجاه (r) لكل قوى باتجاه نقطة الارتكاز هاي.



$\boxed{\tau = rF}$

$\sum \tau = 0$

$600(2) + 200(4) - T\sin 53\,(8) = 0$

$2000 = T(0.8)(8)$

$\boxed{T = 312.5\,N}$

$\Rightarrow$ (1) $\rightarrow 312.5(0.6) = R\cos\theta$

(2) $\rightarrow -0.8(312.5) + 800 = R\sin\theta$

$\Rightarrow$ then we divided by (2)/(1)

$$\frac{R\sin\theta = 550}{R\cos\theta = 187.5}$$

$\rightarrow \tan\theta = 2.93$

$\boxed{\theta = 71.16°}$

$\Rightarrow R\sin(71.6°) = 550$

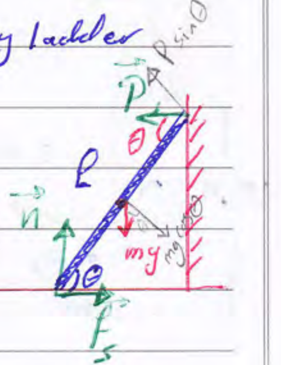
$\boxed{R = 579.6\,N}$

Example :- The leaning ladder

we have (m) &

$\mu_s = 0.4$

find θ_{min} which the ladder doesn't slip?



$\Rightarrow \sum \vec{F} = 0$

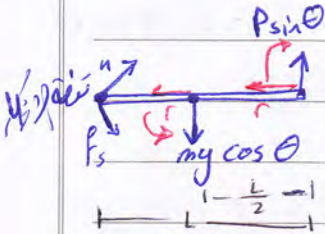
$\sum \vec{F}_x = 0 \rightarrow P - f_s = 0 \rightarrow P = f_s$

$\boxed{P = n\mu_s}$

$\sum F_y = 0 \rightarrow n - mg = 0 \rightarrow \boxed{n = mg}$

→ $\sum T = 0$

خذ نقطة الارتكاز ولتكن النقطة الملاصقة للارض والتي تقع عليها اكثر قوى $(n \; f \; f_s)$



$\sum T = 0$

$\left(\frac{L}{2} mg\cos\theta\right) - (L P \sin\theta) = 0$

→ $mg\cos\theta = 2P\sin\theta$

→ $\frac{\cos\theta}{\sin\theta} = \frac{2P}{mg} \overset{f_s}{=} \frac{2n\mu_s}{mg}$

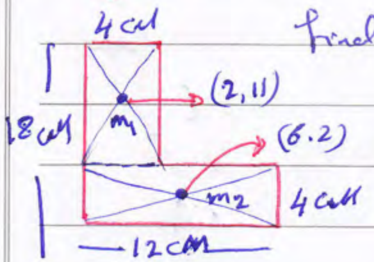
$= \frac{2mg\,\mu_s}{mg}$

→ $\tan\theta = 1/0.8$

→ $\boxed{\theta = 51.3^\circ}$

Different Question about Chapter 11 :

Q3:-



find the (C.g)

$m = \rho V$

$= \rho\,(\text{Area})(\text{thickness})$

→ $m_1 = \rho A_1 (\text{thickness})$

$m_2 = \rho A_2 (\text{thickness})$

ρ constant for same matter & thickness constant.

$A_1 = (14)(4) = 56$

$A_2 = (12)(4) = 48$

→ $X_{c.g} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

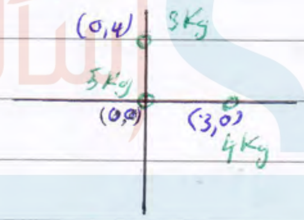
$X_{c.g} = \frac{2(56\,\cancel{\rho\,\text{thickness}}) + 6(48\,\cancel{\rho\,\text{thickness}})}{(56+48)\,\cancel{\rho\,\text{thickness}}}$

$= 3.8$ cm.

$Y_{c.g} = \frac{11(56\,\cancel{\rho\,\text{thickness}}) + 2(48\,\cancel{\rho\,\text{thickness}})}{(56+48)\,\cancel{\rho\,\text{thickness}}}$

$= 6.8$ cM.

Q4:



Where should $m_4 = 8$Kg be placed so that the C.g of four object arrangement will be at (0,0)?

⇒ $X_{c.g} = 0$

$\frac{8x + 5(0) + 4(3) + 3(0)}{20} = 0$

→ $\boxed{x = -1.5}$

⇒ $Y_{c.g} = 0$

$\frac{8y + 5(0) + 4(0) + 4(3)}{20}$

$\boxed{y = -1.5}$

Ch13 Universal Gravitation (Problems)

(13.1) Newton's law of (U G)

m_1 —— r —— m_2

$\left(F_{g_{12}} = -F_{g_{21}}\right)$ consistent with Newton's 3rd law

$\rightarrow F_g = G \frac{m_1 m_2}{r^2}$

→ we must know that :-

F_g between any two particle have approximately the same mass it's very very small.

لا نهتم بقوة جذب الأجسام لبعضها لأنها صغيرة جداً مهملة ، ولكنه لا نهمل هذه القوى للأجسام الكبيرة مثل قوة جذب الأرض لنا.

(13.2) Free-Fall Acceleration & Gravitational force.

$\rightarrow F_g = G \frac{M_E m}{R_E^2}$

M_E → mass of Earth, R_E → radius of Earth

$\cancel{m} g = G \frac{M_E \cancel{m}}{R_E^2}$

$\Rightarrow g = \frac{G M_E}{R_E^2}$

$\simeq 9.8 \ m/s^2$

ملاحظة : تأتي صيغة السؤال على قوة الجذب الكلي مشابهة لأسئلة القوة الكهربائية.

$\rightarrow \left[M_E = \frac{g R_E^2}{G}\right]$ نعوضه في الثانية

Different Question for this chapter :-

$\boxed{3}$ find ΣF

(a) ① 300 kg → F_{13} 4 m F_{23} ← ② 200 kg

|— 2m —③— 2m —| 50 kg

$\Sigma \vec{F} = \vec{F}_{13} - \vec{F}_{23}$

$= \frac{G m_1 m_3}{r^2} - \frac{G m_2 m_3}{r^2}$

$= \frac{G}{r^2}\left(m_3 (m_1 - m_2)\right)$

$= \frac{G}{2^2}\left(50(500-200)\right)$

$= \frac{6.674 \times 10^{-11} (50(300))}{4}$

$= 2.5 \times 10^{-7} \ N$

(b) Find d if $\Sigma F = 0$

50 kg must be between ① & ②

500 kg ① ③ — d — ② 200 kg

|—— 4m ——| 50 kg

$\Sigma \vec{F} = zero$

$F_{13} - F_{23} = zero$

$\cancel{G} \frac{m_3 m_1}{(4-d)^2} - \cancel{G} \frac{m_3 m_2}{d^2} = zero$

$\frac{500}{(4-d)^2} = \frac{200}{d^2}$

$\rightarrow 5d^2 = 2(4-d)^2$

$5d^2 = 2(16 - 8d + d^2)$

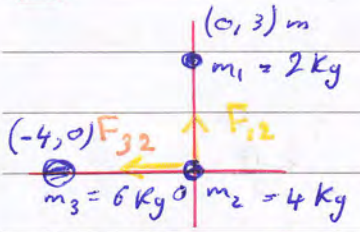
$5d^2 = 32 - 16d + 2d^2$

$3d^2 + 16d - 32 = 0$

$d = \frac{-16 \pm \sqrt{(16)^2 - (4)(3)(-32)}}{6}$

$\boxed{d = 1.5 \ m}$

[6]



find ΣF net force (magnitude & direction)

$$F_{12} = \frac{G\, m_1 m_2}{r^2}$$

$$F_{12} = \frac{6.674 \times 10^{-11} (2)(4)}{9}$$

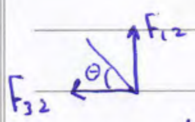
$$= 5.93 \times 10^{-11}$$

$$F_{32} = \frac{6.674 \times 10^{-11} (4)(6)}{16}$$

$$\approx 10 \times 10^{-11}$$

$$\Sigma F = \left(\sqrt{(10)^2 + (5.93)^2}\right) \times 10^{-11}$$

$$= 11.6 \times 10^{-11}\ N$$

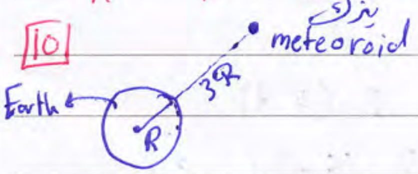


$$\tan\theta = \frac{5.93 \times 10^{-11}}{10 \times 10^{-11}}$$

$$\boxed{\theta = 30.6^\circ}$$

★ ★ ★ ★ ★

[10]



find (a) for meteoroid ??

$$F_g = \frac{G\, M_E\, m_{نيزك}}{r^2}$$

$$\cancel{m_{نيزك}}\, a = \frac{G\, M_E\, \cancel{m_{نيزك}}}{r^2}$$

$$g = \frac{(6.674 \times 10^{-11})\, M_E}{(4 R_{Earth})^2}$$

$$g = \frac{6.674 \times 10^{-11} (5.97 \times 10^{24})}{16 (6.37 \times 10^6)^2}$$

$$g = 0.06 \times 10\ m/s$$

$$g = 0.6$$

★ ★ ★ ★ ★

[12] $$g_{moon} = \frac{g_{Earth}}{6}$$

$$R_{moon} = 0.25\, R_E$$

$$R_{Earth} = 6.37 \times 10^6\ m$$

find $\dfrac{\rho_{moon}}{\rho_{Earth}}$

$$\rightarrow \frac{\rho_{moon}}{\rho_{Earth}} = \frac{M_{moon} \quad V_{moon}}{M_{Earth} \quad V_{earth}}$$

$$= \frac{\left(\dfrac{g_{moon} R^2_{moon}}{\cancel{G}}\right) \Big/ \left(\cancel{\tfrac{4}{3}\pi} (R_{moon})^3\right)}{\left(\dfrac{g_e R^2_{earth}}{\cancel{G}}\right) \Big/ \left(\cancel{\tfrac{4}{3}\pi} (R_{earth})^3\right)}$$

$$= \overset{\frac{1}{6}}{\frac{g_{moon}}{g_{earth}}} \cdot \frac{R_{earth}}{R_{moon}}$$

$$= \frac{1}{6} \cdot \frac{\cancel{R_{earth}}}{0.25\, \cancel{R_{earth}}}$$

$$= 0.667$$

Ch 14: Fluid Mechanics

14.1 Pressure

Pressure: is the force which exerted on an area unit

$$P = \frac{|F|}{A}$$

→ P is a scalar quantity because it's proportional to the magnitude of the force on the piston.

⇒ If P varies over an area

$$P = \frac{dF}{dA} \Rightarrow dF = P\,dA$$

if P → constant

$$\int dF = P \int dA$$

→ special case

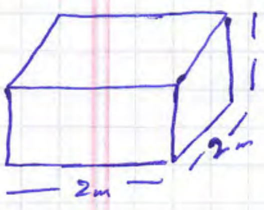
$$\left[\; P = \frac{F}{A} \;\right]$$

⇒ The unit of pressure

N/m^2 → 1 Pa (Pascal)

⇒ Atmospheric $P \equiv P_o$

Example → Water bed



find the weight of the water & the Pressure?

the water density (ρ) = 1000 kg / m³

→ Volume = (2)(2)(0.3)

$= 1.2\ m^3$

→ Mass $= \rho\, V = 1000 \frac{kg}{m^3} (1.2)\, m^3$

$= 1200$ kg

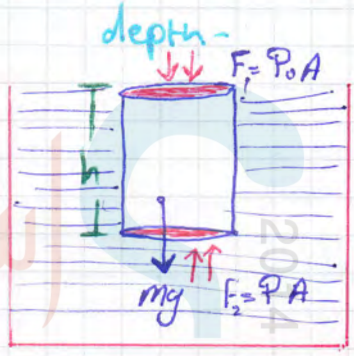
→ weigh = Mg

$= 1200\,kg\ (9.8\ \frac{m}{s^2}) = 1.18 \times 10^4\ N$

→ $P = \frac{F}{A}$ $A = (2)(2) = 4\ m^2$

$= \frac{1.18 \times 10^4}{4} = 2.95 \times 10^3$ Pa.

14.2 Variation of pressure with depth



$V = Ah$

$$\sum \vec{F} = 0$$

$PA - P_oA - mg = 0$ $m = V\rho$
(mass of liquid)

$PA - P_oA - V\rho g = 0$ $V = Ah$
(volume of liquid)

$\cancel{P}\cancel{A} - P_o\cancel{A} - h\cancel{A}\rho\, g = 0$

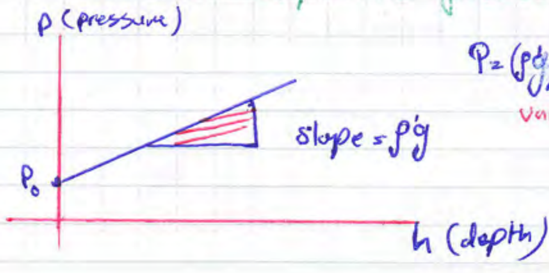
$P - P_o - h\rho g = 0$

$$\boxed{P = P_o + \rho h g}$$

$P - P_o = \rho h g \equiv$ gauge pressure

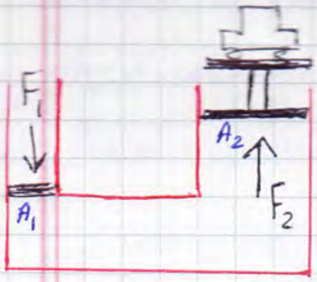
$P \equiv$ absolute pressure

$P_o \equiv$ atmospheric pressure



$P = (\rho g)\, h + P_o$
(h: variable, P_o: constant)

→ Hydraulic press
(important application of Pascal's law)

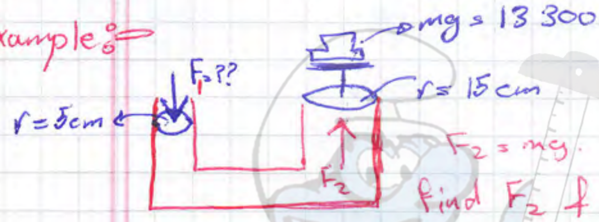


at point ① & ② there are the same pressure.

$$\boxed{P = \frac{F_1}{A_1} = \frac{F_2}{A_2}}$$

* * * * *

Example:



find F_2 & P ?

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\rightarrow F_1 = \frac{A_1}{A_2} F_2$$

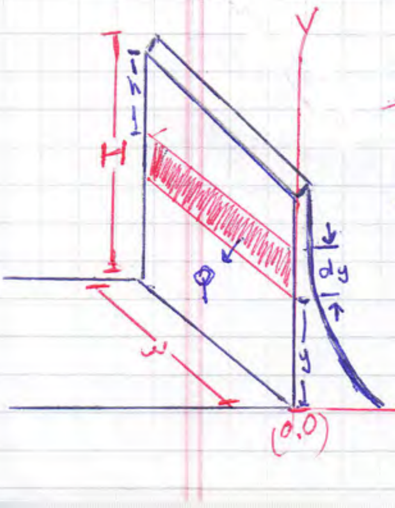
$$F_1 = \frac{\cancel{\pi}(5)^2(\cancel{10^{-2}})^2}{\cancel{\pi}(15)^2(\cancel{10^{-2}})^2}(13300)$$

$$F_1 = 1.48 \times 10^3 \ N$$

$$\rightarrow P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3}{\pi (5)^2 \, 10^{-4}}$$

$$= 1.88 \times 10^5 \ Pa.$$

→ The force on a dam



$$P = P_0 + \rho g (H-y)$$

$$\Rightarrow \int dF = P \cdot dA$$

$$\int dF = \int P_0 + \rho g (H-y) \cdot dA$$

$$F = \int (P_0 + \rho g (H-y)) \cdot w\,dy$$

$$F = \int w P_0 + \rho w g (H-y) \cdot dy \quad \Rightarrow$$

$$F = \int_0^H w P_0 + \rho w g H - \rho w g y \cdot dy$$

$$= \int_0^H w P_0 + \int_0^H \rho w g \underbrace{H}_{\text{constant}} - \int_0^H \rho w g y \, .$$

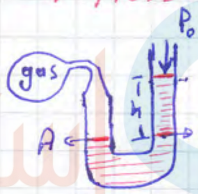
$$= w P_0 y \Big]_0^H + \rho w g H y \Big]_0^H - \frac{\rho w g y^2}{2} \Big]_0^H$$

$$= w P_0 H + \rho w g H^2 - \frac{1}{2} \rho w g H^2$$

$$F = w P_0 H + \frac{1}{2} \rho w g H^2$$

14.3 Pressure Measurement

⇒ Pressure Measurement for any gas.

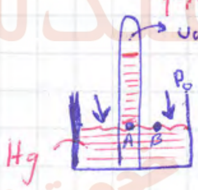


Pressure of gases $= P_A = P_B$

$P_B = P_0 + \rho g h$

→ Pressure of gas.

⇒ Pressure Measurement for atmospheric Pressure:



$P_A = P_B = P_0$

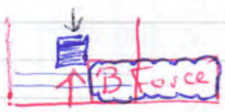
$P_0 = \rho_{Hg} g h$

$P_0 = 13.6 \times 10^3 \frac{Kg}{m^3} (9.8 \frac{m}{s^2})(0.76 \ m)$

$P_0 = 1.013 \times 10^5 \ Pa = 1 \ atm$

14.4 Buoyant force & Archimede's Principle.

⇒ Buoyant force: The upward force exerted by a fluid on any immersed object

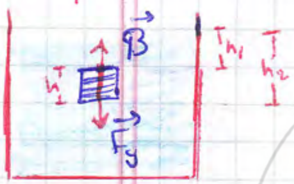


Archimede's principle :-

the magnitude of the buoyant force on an object always equals the weight of the fluid displaced by the object

$\Longrightarrow B = (Mg) \rightarrow$ for fluid displaced

Proof :-



$$B = \Delta P \, A_{object}$$
$$= (P_{bot} - P_{top}) A_{object}$$
$$= (\rho_{fluid} g h_2 - \rho_{fluid} g h_1) A_{object}$$
$$= g \rho_{fluid} (h_2 - h_1) A_{object}$$
$$= g \rho_{fluid} \boxed{(h) A}_{object}$$
$$= g \rho_{fluid} V_{object}$$

$V_{object} = V_{displacement}$

$$= g \boxed{\rho_{fluid} V}_{dis. fluid}$$

$$B = M_{fluid} \, g$$

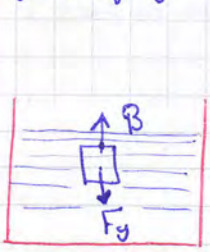
* We have two common situation :-

[1] Totally Submerged غطس كلي

$\rightarrow V_{obj} = V_{disp}$

$\rightarrow B = \rho_{fluid} \, g \, V_{obj}$

$\rightarrow F_g = \rho_{obj} \, g \, V_{obj}$



object in equilibrium

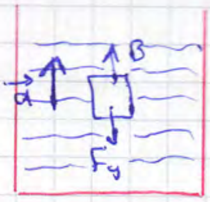
$B = F_g$

$\rho_{fluid} = \rho_{obj}$

object accelerate ...

$F_g > B$

$\rho_{obj} > \rho_{fluid}$



object accelerate ...

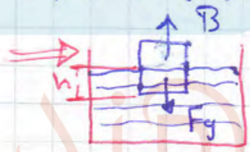
$B > F_g$

$\rho_{fluid} > \rho_{obj}$

[2] Floating object (partially submerged) (طفو) عائم غطس جزئي

في هذا الجزء نهتم فقط بحجم الجزء المغمور بالماء

$\rightarrow$ In this case, the object in static equilibrium floating on the surface of a fluid. $(\rho_{obj} < \rho_{fluid})$



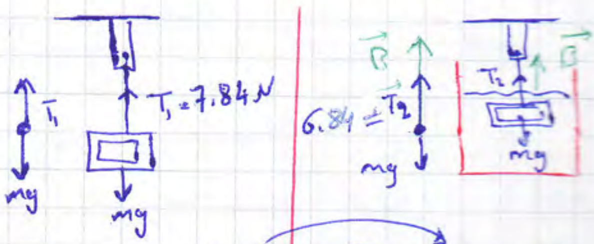
$B = F_g$

$\rho_{fluid} \cancel{g} V_{i} = \rho_{obj} \cancel{g} V_{obj}$

$\rho_{fluid} V_{disp.} = \rho_{obj} V_{obj}$

$$\rightarrow \frac{V_{disp}}{V_{obj}} = \frac{\rho_{obj}}{\rho_{fluid}}$$

Example :- Eureka !



$T_1 = mg = 7.84 \, N$

$B = mg - T_2$

$B = 7.84 - 6.84 = 1 \, N$

Continous $\triangleright$

$$V_{disp.} = V_{obj}$$

$\rightarrow B = \rho_{water}\, g\, V_{disp}$

$$V_{disp} = \frac{B}{\rho_{water}\, g}$$

$$\frac{B}{\rho_{water}\, g} = V_{obj}$$

$$\frac{B}{\rho_{water}\, g} = M_{obj}\,(\rho_{obj})$$

$$\frac{1\,N}{1000\,\frac{Kg}{m^3}} = (M_{obj}\, g)\,(\rho_{obj})$$

weight of object = 7.84

$$\frac{1}{1000} = (7.84)\,\rho_{obj}$$

$\Longrightarrow \rho_{obj} = 7.84 \times 10^3\ Kg/m^3$

But $\rho_{gold} = 19.3 \times 10^3\ Kg/m^3$

→ so that the previous it isn't gold

☹

14.5 Fluid Dynamic

→ The properties of ideal fluid :-

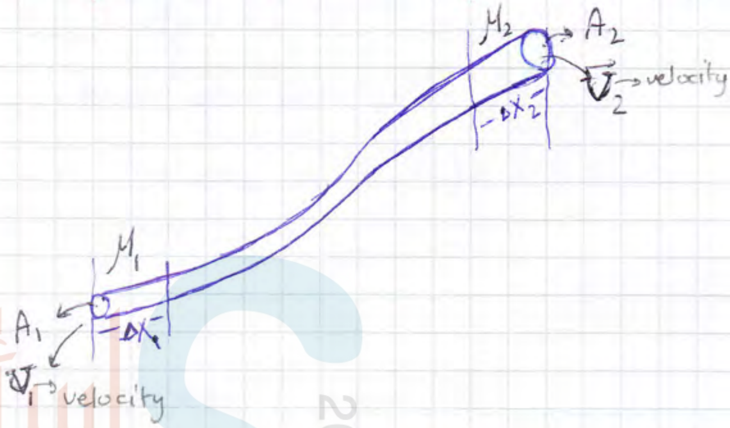
1. The fluid is nonviscous غير لزج
2. The flow is steady التدفق ثابت
3. The fluid is incompressible
   غير قابل للإنضغاط
4. The flow is irrotational
   (have no angular momentum)
   غير دوراني

→ **Streamline :-**

The path taken by a fluid particle under steady flow خطوط الإنسياب

خطوط الإنسياب = خطوط المجال الكهربائي

Equation of continuity for fluids :-



$$M_1 = M_2$$

$\cancel{\rho_1} V_1 = \cancel{\rho_2} V_2 \rightarrow \rho_1 = \rho_2$ to same fluid

$$V_1 = V_2$$

$A_1 \Delta X_1 = A_2 \Delta X_2 \rightarrow \Delta X_1 = v_1 t$

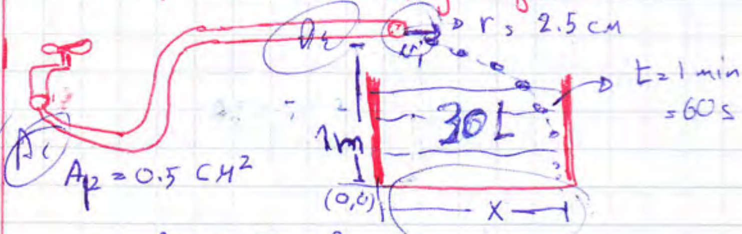
$A_1 (v_1 \cancel{t}) = A_2 (v_2 \cancel{t})$ $\Delta X_2 = v_2 t$

$$\boxed{A_1 v_1 = A_2 v_2}$$

Continuity equation

→ we note that if (A) increase, v will decrease & vice versa is true.

Example :- watering a garden



$\rightarrow A_1 = \pi r^2$

$A_1 = \pi \left(\frac{2.5}{2}\right)^2 cm = 4.91\ cm$

continuous

$A_1 v_1 = 30\ L/1\,min$

$A_1 v_1 = \frac{30 \times 10^3\ cm^3}{60\ s} = 0.5 \times 10^3\ cm^3/s$

$v_1 = \frac{0.5 \times 10^3\ cm^3/s}{4.91\ cm^2}$

$v_1 = 102\ cm/s$

$= 1.02\ m/s$

$\rightarrow A_1 v_1 = A_2 v_2$

$(4.91\ cm^2)(1.02\ \frac{m}{s}) = (0.5\ cm^2)\, v_2$

$\rightarrow v_2 = 10.02\ m/s$

$\rightarrow \Delta y = \cancel{v_{iy}}^{\,0}\, t - \frac{1}{2} g t^2$

$-1 = -\frac{1}{2}(9.8)\, t^2$

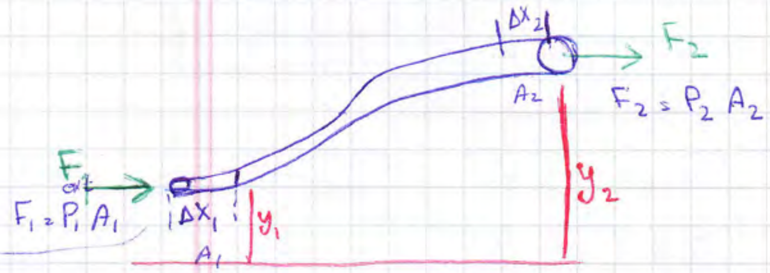
$\boxed{t = 0.45\ s}$

$\rightarrow \Delta x = v_{x2}\, t$

$x_f - \cancel{x_i}^{\,0} = 10.02\ \frac{m}{s}\ (0.45)\,s$

$x_f = 4.5\ m$

14.6 Bernoulli's Equation



$\rightarrow W_{ext} = \Delta P\ V$ من الكتاب

$\Delta E = (P_1 - P_2)\, V$

$\Delta K + \Delta U = (P_1 - P_2)\, V$

$(\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2) + (mgy_2 - mgy_1) = (P_1 - P_2)\, V$

$\rightarrow (P_1 - P_2)\, V = \frac{1}{2} m (v_2^2 - v_1^2) + mg(y_2 - y_1)$

$\div V$

$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) + \rho g (y_2 - y_1)$

باعادة ترتيب المعادلة نحصل على :

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2$$

→ so we can note that

$$P + \frac{1}{2} \rho v^2 + mgy = constant$$

The previous equations is called (Bernoulli's equation)

There is two examples in book @ page 417 & 418

Problems:-

① $M = 95\ Kg$ $r = 0.5\ cm$

Find P ??

$\Rightarrow P = \frac{weight}{Area} = \frac{95\,(9.8)}{\pi (0.5 \times 10^{-2})^2}$

$= \frac{931}{0.79 \times 10^{-4}}$

$= 1178.5 \times 10^4\ Pa$

② $V = (4.5\ cm)(11\ cm)(26\ cm)$

$\rho_{gold} = 19.3 \times 10^3\ kg/m^3$

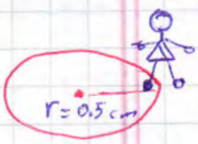
Find M ?

$M = V \rho_{gold} = 1287 \times 10^{-6}\,(19.3 \times 10^3)$

$= 24.8\ Kg.$

(5)

$M_{girl} = 50\ Kg$



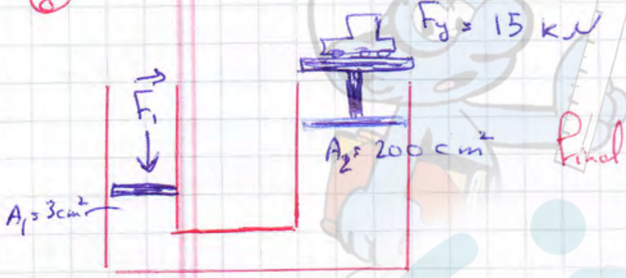
a) find P ?

$$P = \frac{\text{weigh of women}}{\text{Area}}$$

$$= \frac{50\ (9.8)}{\pi\ (0.5 \times 10^{-2})^2}$$

$$= 623.9 \times 10^4\ Pa.$$

(8)



find F_1 ?

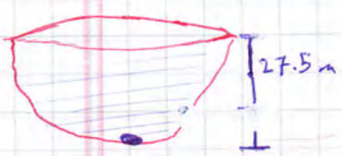
$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\rightarrow F_1 = \frac{A_1}{A_2} F_2$$

$$= \frac{3\ \cancel{cm^2}}{200\ \cancel{cm^2}}\ 15\ kN$$

$$\boxed{F_1 = 225\ N}$$

(9)



$\rho = 1000\ Kg/m^3$

$P_o = 101.3\ KPa$

(a) $P = P_o + \rho g h$

$P = 101.3 \times 10^3 + 10^3\ (9.8)(27.5)$

$= 370.8 \times 10^3\ Pa$

(b) find F_{exited} → Area $= \pi r^2$

$= \pi (17.5)^2 cm^2$

$= 0.096$

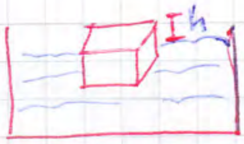
$F = P \cdot A$

$F = 370.8 \times 10^3\ (0.096)$

$= 35.6 \times 10^3\ N$

(25)

$\rho = 650\ \frac{Kg}{m^3}$



(a) find h (distance between top of cube & water level)

$F_g = B$

$\rho_{obj}\ \cancel{g}\ V_{obj} = \rho_{water}\ \cancel{g}\ V_{disp}$

$(650)(0.2)^3 = 1000\ V_{disp}$

حجم الجزء

$V_{disp} = 5.2 \times 10^{-3}\ m^3$

Area $(0.2 - h) = 5.2 \times 10^{-3}$

$(0.2 - h) = \frac{5.2 \times 10^{-3}}{0.04}$

$0.2 - h = 0.130$

$\boxed{h = 0.07\ m}$

(b) $B = m g$ — m of lead

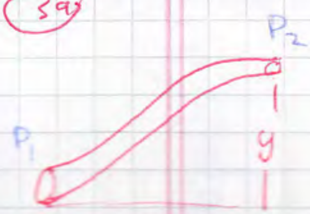
$\rho_{water}\ \cancel{g}\ V_{disp} = m\ \cancel{g}$

$10^3\ (\text{Area } h) = m_{lead}$

$m_{lead} = (0.04)(0.07)(10^3)$

$\boxed{m_{lead} = 2.8\ Kg}$

(39)



$P_1 = 1.75 \times 10^4$ Pa

$r_1 = 3$ cm

$y = 0.25$ m

$P_2 = 1.2 \times 10^4$ Pa

$r_2 = 1.5$ cm

$\rightarrow A_1 V_1 = A_2 V_2$

$\cancel{\pi} r_1^2 v_1 = \cancel{\pi} r_2^2 v_2$

$(0.03)^2 v_1 = (0.015)^2 v_2$ — ①

$\rightarrow P_1 + \frac{1}{2}\rho v_1^2 + \cancel{\rho g y_1} = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y$

$1.75 \times 10^4 + 500 v_1^2 = 1.2 \times 10^4 + 500 v_2^2 + \rho g y$

$\rightarrow 1.75 \times 10^4 + 500 v_1^2 = 1.2 \times 10^4 + 500 v_2^2 + (1000)(9.8)(0.25)$

$\rightarrow 1.75 \times 10^4 + 500 v_1^2 = 1.2 \times 10^4 + 0.245 \times 10^4 + 500 v_2^2$

$1.75 \times 10^4 + 500 v_1^2 = 1.445 \times 10^4 + 500 v_2^2$

$0.305 \times 10^4 = -500 v_1^2 + 500 v_2^2$

$\frac{0.305 \times 10^4}{500} = \frac{500}{500}(v_2^2 - v_1^2)$

$6.1 = \left(\frac{(0.03)^2}{(0.015)^2} v_1\right)^2 - v_1^2$

$6.1 = 16 v_1^2 - v_1^2$

$6.1 = 15 v_1^2$

$v_1^2 = 0.407$

$\rightarrow v_1 = 0.64$ m/s

$= 64$ cm/s

$\rightarrow v_2 = \left(\frac{0.03}{0.015}\right)^2 (0.64)$

$= 2.56$ m/s

مسألة خارجية / 1

① $2r = 3 \times 10^{-2}$ m

$r = 1.5 \times 10^{-2}$ m

$\rho = 2.7 \times 10^3 \ \frac{Kg}{m^3}$ for ball

Find M ?

$M = \rho V$

$M = \rho \frac{4}{3}\pi r^3$

$= 2.7 \times 10^3 \ \frac{4}{3} \pi (1.5)^3 \times 10^{-6}$

$= 38.17 \times 10^{-3}$ Kg

Find M of atmospheric

$F = mg$

$F = P_{atm} A_{Earth}$

$R_E = 6.37 \times 10^6$ m

$M_{\cancel{Earth}}\, g = P_{atm} A_{Earth}$

$M_{\cancel{Earth}} = \frac{(1.013 \times 10^5)\, 4\pi r^2}{g}$

$M_{\cancel{Earth}} = 52.7 \times 10^{17}$ Kg

~~$K = 10^3$ N/m~~

~~$x = 0.5 \times 10^{-2}$ m~~

~~$r = 10^{-7}$ m~~

~~$\rho_{water} = 10^3 \ \frac{Kg}{m^3}$~~

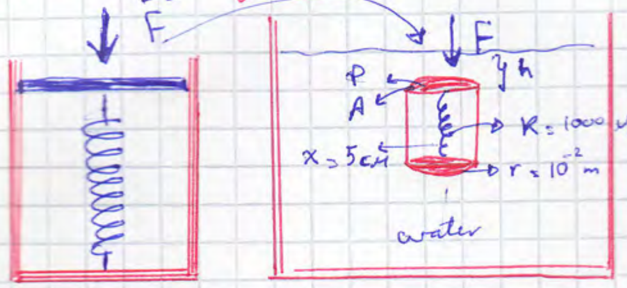
Find h

~~$\rightarrow F = P A$~~

gauge pressure

~~$Kx = (\rho h g)(\pi r^2)$~~

Pressure gauge



$F_{spring} = PA$ — find (h)

$Kx = (\rho g h) \cdot A$

$Kx = (\rho_{water}\, g\, h)\ Area$

$1000\ \frac{N}{m}\ (5\times10^{-2}) = 1000\ \frac{kg}{m^3}\ (9.8)(h)(\pi r^2)$

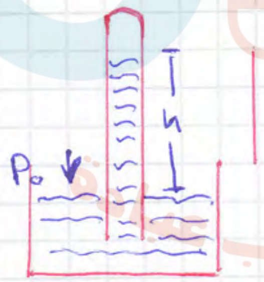
$50 = 1000\ (9.8)\ h\ (\pi(5\times10^{-2})^2)$

$$\boxed{h = 0.65\ m}$$

Barometer to another matter

$P_0 = \rho g h$

تختلف حسب نوع السائل



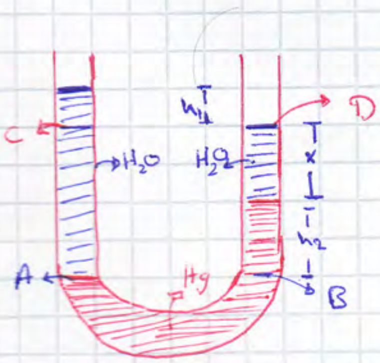
$\Rightarrow\ P_0 = P_0$

$\rho_1 \cancel{g} h_1 = \rho_2 \cancel{g} h_2$

$\rightarrow \frac{\rho_1}{\rho_2} = \frac{h_2}{h_1}$

→ it's mean that the relationship between ρ & h inversable proportional

U-tube



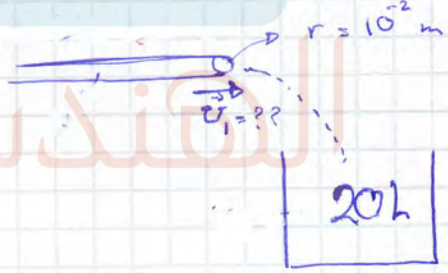
$P_A = P_B$

$\rho_{water}\, g\,(h_1 + x + h_2) = \rho_{Hg}\, g\, h_2 + \rho_{H_2O}\, g\, X$

$P_D = P_0$

$P_0 = P_C$

$P_0 = g\, \rho_{H_2O}\, h_1$



$t = 60\ s$

$\vec{v} = \frac{\text{length of tube}}{t}$

طول الأنبوب

حجم / مساحة مقطع الأنبوب

$= \frac{V/A}{t}$

$= \frac{\frac{20\times10^{-3}\ m^3}{\pi(10^{-2})^2\ m^2}}{60\ s} = \frac{m}{s}$

$= 1.06\ m/s$