الفيزياء (1)

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إعداد الطالب: قتيبة الروابدة

بسم الله الرحمن الرحيم

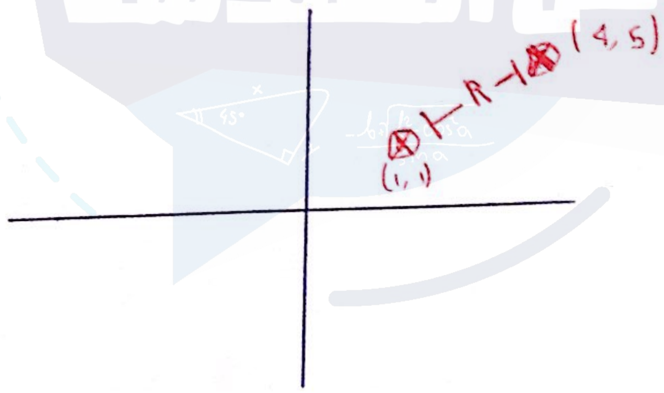
⇒ physics "1"

Chapter "1" vectors

⇒ physical quantities:-

1) Basic
- → time (s)
- → mass (Kg)
- → - displacement "Magnitude and direction" → (m)
- - distance "Magnitude" → (m)

- $\hat{i}$ = X-axis
- $\hat{j}$ = Y-axis
- $\hat{k}$ = Z-axis



$\vec{R} = \vec{\Delta x} + \vec{\Delta y}$

$\vec{R} = 3\hat{i} + 4\hat{j}$

$|\vec{R}| = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= 5$

⇒ Derived Quantities:

- velocity ⇒ displacement. ($\vec{v}$) (m/s)
- Speed ⇒ distance. (v) (m/s)
- acceleration ($\vec{a}$) (m/s^2)
- Momentum ($\vec{p}$)

$$\vec{p} = m . \vec{v}$$

mass velocity

- force

$$\vec{f} = m . \vec{a}$$

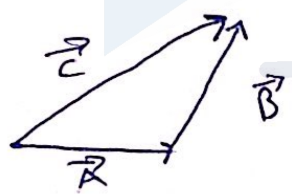
mass acceleration

* weight is a force

$$= m . \vec{g}$$

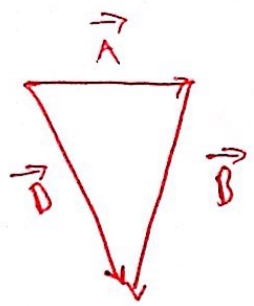
mass

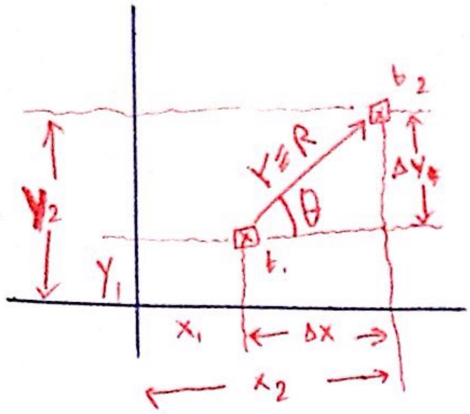
$$\vec{A} + \vec{B} = \vec{C}$$



$$\vec{A} - \vec{B} = \vec{D}$$

$$\vec{A} + (-1\vec{B}) = \vec{D}$$





$$R = r = |\vec{R}| = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$\sin\theta = \frac{\Delta y}{|\vec{R}|}$$

$$\cos\theta = \frac{\Delta x}{|\vec{R}|}$$

$$\tan\theta = \frac{\Delta y}{\Delta x}$$

$\Rightarrow$ Position Vector: $\vec{r} = x_1\hat{i} + y_1\hat{j} + \cancel{0\hat{k}}$

$$\vec{r} = x_2\hat{i} + y_2\hat{j} + \cancel{0}$$

$$\vec{\Delta r} = \vec{r_2} - \vec{r_1} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

$$= \Delta x\,\hat{i} + \Delta y\,\hat{j}$$

$\Rightarrow$ $\sin^2\theta + \cos^2\theta = 1$

$$\left(\frac{\Delta y}{|\vec{R}|}\right)^2 + \left(\frac{\Delta x}{|\vec{R}|}\right)^2 = 1$$

$$|\vec{R}|^2 = (\Delta y)^2 + (\Delta x)^2$$

$$|\vec{R}| = \sqrt{(\Delta y)^2 + (\Delta x)^2}$$

⇒ **Ex:** A (2, 3) $\Longrightarrow$ $\vec{A} = 2\hat{i} + 3\hat{j}$

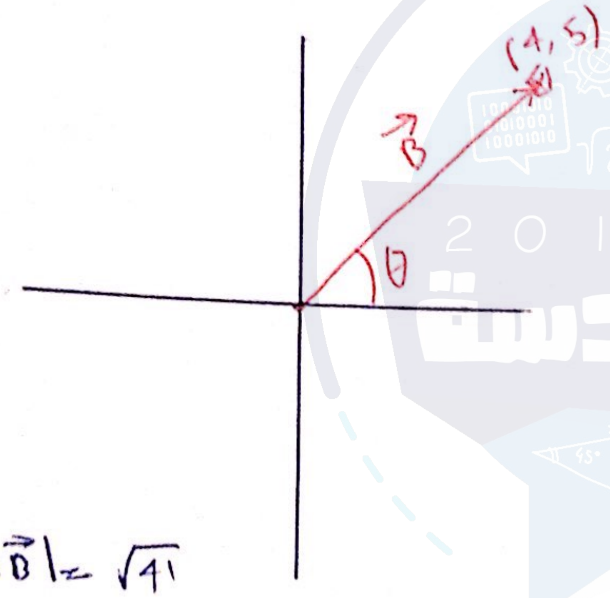
B (4, 5) $\qquad \vec{B} = 4\hat{i} + 5\hat{j}$

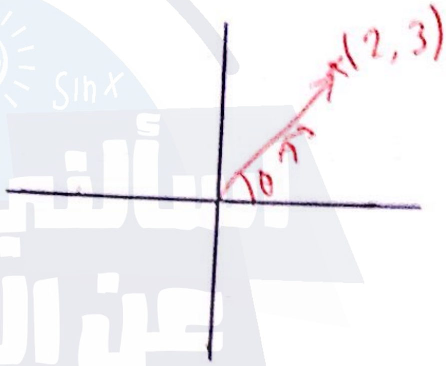
⇒ $\vec{C} = \vec{A} + \vec{B}$

$\vec{C} = 6\hat{i} + 8\hat{j} \qquad |\vec{C}| = \sqrt{(6)^2 + (8)^2} = \sqrt{100} = 10\ m$

$\vec{D} = \vec{B} - \vec{A}$

$\vec{D} = 2\hat{i} + 2\hat{j} \qquad |\vec{D}| = \sqrt{(2)^2 + (2)^2} = 2\sqrt{2}\ m$





$|\vec{A}| = \sqrt{13}$

$\sin\theta = \frac{3}{\sqrt{13}}$

$\cos\theta = \frac{2}{\sqrt{13}}$

$\tan\theta = \frac{3}{2}$

$|\vec{B}| = \sqrt{41}$

$\sin\theta = \frac{5}{\sqrt{41}}$

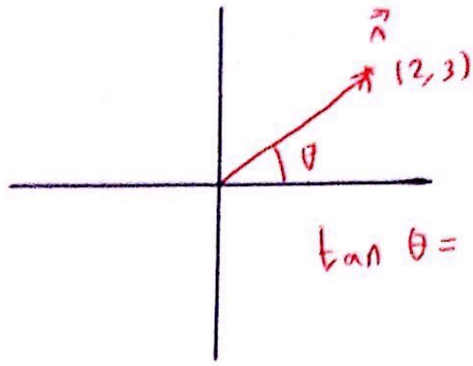
$\cos\theta = \frac{4}{\sqrt{41}}$

$\tan\theta = \frac{5}{4}$

⇒ **Scalar Product (Dot Product)**

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$

⇒ Ex:

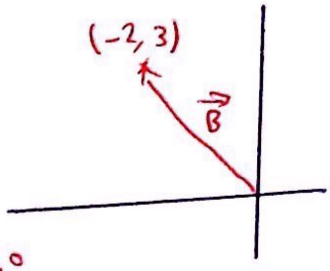


$\vec{A} = 2\hat{i} + 3\hat{j}$

$\tan\theta = \frac{3}{2} \Rightarrow \theta = 56.3^\circ$

⇒ $\vec{A}(2, 3)$ ⇒ Cartisian ~~Cartizan~~ Co-ordinates

⇒ $(r, \theta), (|\vec{A}|, \theta), (\sqrt{13}, 56.3^\circ)$ ⇒ polar co-ordinates



$\vec{B} = -2\hat{i} + 3\hat{j}$

$|\vec{B}| = \sqrt{(-2)^2 + (3)^2} = \sqrt{13}$ $\tan\theta = \frac{-2}{3}$

$\theta = 33.7 + 90 = 123.7^\circ$

⇒ $\vec{A} + \vec{B} = 6\hat{j} \equiv \vec{C}$ [(0, 6)

$\vec{A} - \vec{B} = 4\hat{i} \equiv \vec{D}$ [(4, 0)] ⇒ Cartizon

$\vec{C}$ (6, 90)

$\vec{D}$ (4, 0)] ⇒ Polar

⇒ Multiplication of vectors

1) Scalar product (dot) ⇒ $\vec{A} \cdot \vec{B}$

2) vectors product (cross) ⇒ $\vec{A} \times \vec{B}$

$\vec{A} \cdot \vec{B} = |\vec{A}| . |\vec{B}| . \cos\theta = \sqrt{13} . \sqrt{13} \cos 67.4$

⇒ Ex:

$\vec{A} = \hat{i}$

$\vec{B} = \hat{i}$

$\vec{A} \cdot \vec{B} = (1)(1)\cos 0 = 1$

$$\left\{ \begin{array}{l} \hat{i} \cdot \hat{i} = 1 \\ \hat{j} \cdot \hat{j} = 1 \\ \hat{k} \cdot \hat{k} = 1 \end{array} \right.$$

⇒ Ex: $\vec{A} = \hat{i}$, $B = \hat{j}$ $\longrightarrow$ $\vec{A} \cdot \vec{B} = (1)(1)\cos 90 = \text{Zero}$

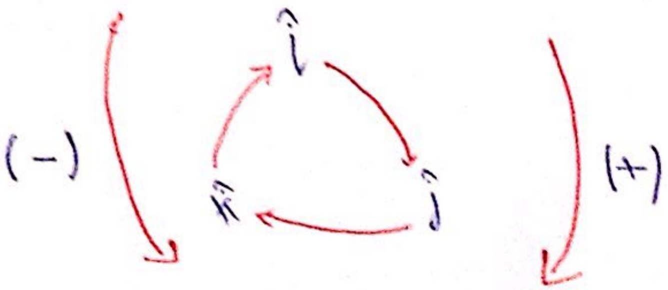
$$\left\{ \begin{array}{l} \hat{i} \cdot \hat{j} = \text{Zero} \\ \hat{j} \cdot \hat{k} = \text{Zero} \\ \hat{i} \cdot \hat{k} = \text{Zero} \end{array} \right.$$

⇒ Cross product

$\vec{A} \times \vec{B} = \overline{A} \times \overline{B} \times \sin\theta = \vec{C}$

عمودي على كل (A) و (B)

⇒ Ex: $\vec{A} = \hat{i}$, $\vec{B} = \hat{j}$ $\longrightarrow$ $\vec{C} = \vec{A} \times \vec{B} = \hat{i} \times \hat{j} \times \sin 90 = 1$



* $\hat{i} \times \hat{j} = \hat{k}$
  $\hat{j} \times \hat{k} = \hat{i}$
  $\hat{k} \times \hat{i} = \hat{j}$
  $\hat{k} \times \hat{j} = -\hat{i}$

$\hat{i} \times \hat{i} = 0$
$\hat{j} \times \hat{j} = 0$
$\hat{k} \times \hat{k} = 0$

$\Rightarrow$ <u>Ex</u>: $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$

$\vec{B} = \hat{i} + \hat{j} + \hat{k}$

$$\vec{C} = \vec{A} \times \vec{B} = 2\hat{i} \times (\hat{i} + \hat{j} + \hat{k}) + \hat{j} \times (\hat{i} + \hat{j} + \hat{k}) + \hat{k} \times (\hat{i} + \hat{j} + \hat{k})$$

$$= 0 + 2\hat{k} - 2\hat{j} - \hat{k} + 0 + \hat{i} + \hat{j} - \hat{i} + 0$$

$$\vec{C} = \hat{k} - \hat{j}$$

$$|\vec{C}| = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$$

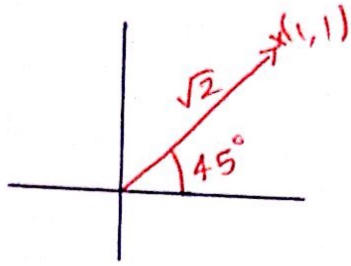
$\Rightarrow$ $\vec{A} \cdot \vec{C} = (2\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{k} - \hat{j})$

$$= \hat{k}(2\hat{i} + \hat{j} + \hat{k}) - \hat{j}(2\hat{i} + \hat{j} + \hat{k})$$

$$= 0 + 0 + 1 - 0 - 1 + 0 = 0$$

$\Rightarrow$ $(\vec{B} \times \vec{A}) = -(\vec{A} \times \vec{B})$

$\Rightarrow$ Let $\vec{A} = \hat{i} + \hat{j}$



$|\vec{A}| = \sqrt{1^2 + 1^2} = \sqrt{2}$

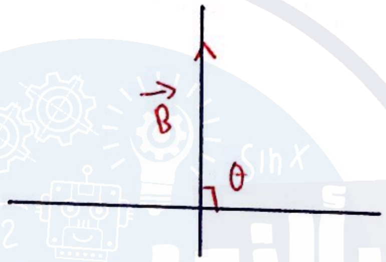
$\tan\theta = \frac{1}{1} = 1 \Rightarrow \theta = 45^\circ$

$\vec{A}$ $(1,1)$

$(\sqrt{2}, 45^\circ)$

$\vec{B} = 2\hat{j}$

$\theta = 90$



$|\vec{B}| = 2$

$\vec{B}$ $(0, 2)$

$(2, 90^\circ)$

$\Rightarrow$ $\vec{C} = \vec{A} + \vec{B}$

$= (\hat{i} + \hat{j}) + 2\hat{j}$

$\vec{C} = \hat{i} + 3\hat{j}$

$|\vec{C}| = \sqrt{1^2 + 3^2} = \sqrt{10}$

$\Rightarrow$ $\vec{D} = \vec{B} - \vec{A}$

$= 2\hat{j} - (\hat{j} + \hat{i})$

$= \hat{j} - \hat{i}$

$|\vec{D}| = \sqrt{2} \Rightarrow \vec{D}$ $(-1, 1)$ $\tan\theta = \frac{1}{-1} = -1 \Rightarrow \theta = 135^\circ$

$(\sqrt{2}, 135^\circ)$

$\vec{A}\cdot\vec{B} = (\hat{i}+\hat{j}).(2\hat{j})$

$= 2$

$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}| \cos 45 = 2 \cdot \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 2$

$\Rightarrow \vec{A}\times\vec{B} = (\hat{i}+\hat{j})\times(2\hat{j}) = \vec{E}$

$= 2\hat{k} + 0$

$\vec{E} = 2\hat{k}$

$|\vec{E}| = 2$

$\vec{A}\times\vec{B} = |\vec{A}|\times|\vec{B}|\times \sin\theta$

$= \sqrt{2}\times 2\times\frac{1}{\sqrt{2}} = 2 = |\vec{E}|$

$\Rightarrow \vec{A}\cdot\vec{E} = (\hat{j}+\hat{i}).(2\hat{k})$

$= 0 + 0$

$= 0$

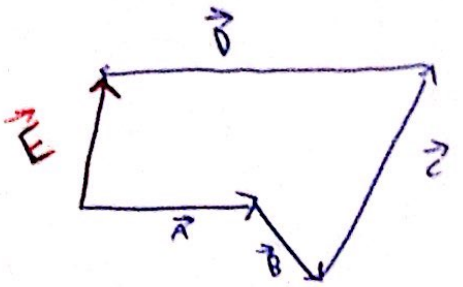
$\Rightarrow$ Ex: $\vec{A} = 2\hat{i} - y\hat{j}$

$\vec{B} = \hat{i}+\hat{j}$, if $\vec{A}\perp\vec{B}$ find y??

$\vec{A}\cdot\vec{B} = 0 \Rightarrow (2\hat{i} - y\hat{j}).(\hat{i}+\hat{j}) = 0$

$\Rightarrow (2 + 0 - 0 - y) = 0$

$\boxed{y = 2}$



$\vec{E}$: متجه إزاحة

$\vec{E} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$ [Resultant المحصلة]

⇒ Average Velocity " معدل السرعة "

$$(V_{av})_x = \frac{\Delta \vec{x}}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = (\bar{V})_x$$

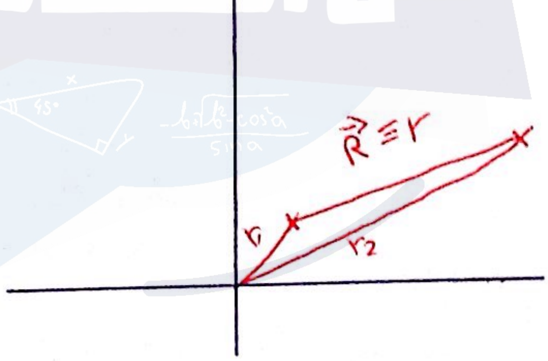
* Position Vector:

$\vec{r}_1 = x_1 \hat{i} + y_1 \hat{j}$

$\vec{r}_2 = x_2 \hat{i} + y_2 \hat{j}$

$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$

$= \Delta x \hat{i} + \Delta y \hat{j}$



$$\Rightarrow \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x \hat{i}}{\Delta t} + \frac{\Delta y \hat{j}}{\Delta t}$$

$$\vec{V}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

⇒ Instaneous Velocity " سرعة لحظية "

$$V = \lim_{\Delta t \to 0} \vec{V}_{av} = \lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j} \right) = \lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t} \right) \hat{i} + \lim_{\Delta t \to 0} \left(\frac{\Delta y}{\Delta t} \right) \hat{j}$$

$$= \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j}$$

$\Rightarrow x(t) = 2t^2 + 3 \quad , t: s$

$x: m$

$x\big|_{t=0} = 3m \quad , x\big|_{t=1} = 5m \quad , x\big|_{t=4} = 35m$

Find: 1) $(V_{av})_x$ bet. $t_1 = 0s$, $t_2 = 1s$

$x_1 = 3m$, $x_2 = 5m$

$$V_{av} = \frac{5-3}{1-0} = \frac{2}{1} = 2 \ m/s$$

2) $(V_{av})_x$ bet. $t_1 = 0_s$, $t_2 = 4_s$

$x_1 = 3m$, $x_2 = 35$

$$(V_{av}) = \frac{32}{4} = 8 \ m/s$$

3) Instaneous Velocity at : Ⓐ $t = 0_s$

Ⓑ $t = 1_s$

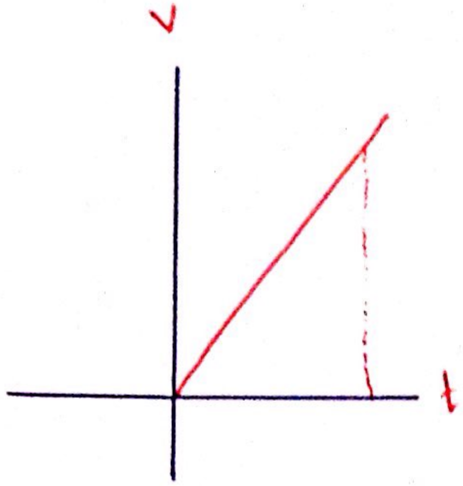
Ⓒ $t = 4_s$

$$\frac{dx}{dt} = 4t$$

Ⓐ $0 \times 4 = 0 \ m/s$

Ⓑ $4 \times 1 = 4 \ m/s$

Ⓒ $4 \times 4 = 16 \ m/s$



Displacement = Area

⇒ Average acceleration

$$(a_{av})_x = (\bar{a})_x = \frac{\Delta V}{\Delta t}$$

4) find $(\bar{a})$ bet. $t_1 = 0_s$ and $t_2 = 1_s$

$V_1 = 0$ m/s , $V_2 = 4$ m/s

$$(\bar{a}) = \frac{4}{1} = 4 \text{ m/s}^2$$

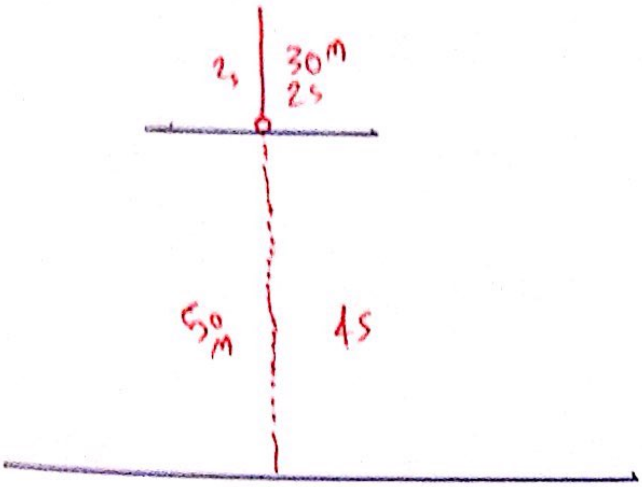
5) find $(\bar{a})$ bet. $t_1 = 1_s$, $t_2 = 4_s$

$V_1 = 4$ m/s , $V_2 = 16$ m/s

$$\bar{a} = \frac{16-4}{4-1} = \frac{12}{3} = 4 \text{ m/s}^2$$

⇒ Instaneous acceleration

$$a = \lim_{\Delta t \to 0} (\bar{a})_x = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \Rightarrow \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$



1) $\overrightarrow{\Delta y} = -50\,m$

2) $\Delta y = 110\,m$

3) average speed $= \frac{110}{8} = \frac{55}{4}\ m/s$

4) $(\bar{v}) = \frac{-50}{8}\ m/s$

$\Rightarrow$ acceleration = constant

average = Instancons

$$\bar{a} = a$$

$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$a = \frac{v_f - v_0}{t - 0} = \frac{v_f - v_0}{t}$$

$$v_f - v_0 = at$$

X-axis

$$v_{fx} = v_{0x} + at$$

$$v_{fx}^2 = v_{0x}^2 + 2a\,\Delta x$$

$$\Delta x = v_{0x} t + \tfrac{1}{2} a t^2$$

Y-axis

$$v_{fy} = v_{0y} + a_y t$$

$$v_{fy}^2 = v_{0y}^2 + 2a_y\,\Delta y$$

$$\Delta y = v_{0y} t + \tfrac{1}{2} a_y t^2$$

⇒ **free fall:** $a_y = -10\ m/s^2$

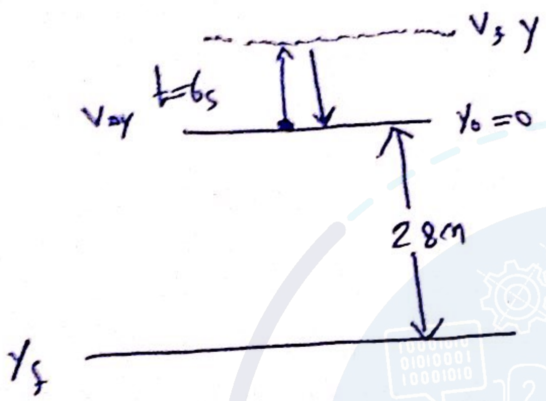
Kinematic equations
$a = const.$

$$\Rightarrow V_{fy} = V_{oy} - 10t$$

$$V_{fy}^2 = V_{oy}^2 - 20\,\Delta y$$

$$\Delta y = V_{oy}t - 5t^2$$

Ex:



$y_f = -28$

Find: 1) V_{oy}
2) total time

1) $(V_{fy}) = 0$

$V_{oy} - gt = 0$

$V_{oy} = 30\ m/s$

2) $\Delta y = y_f - y_o = -28$

$\Delta y = V_{oy}t - \frac{1}{2} \cdot 10 \cdot t^2$

$-28 = 30t - 5t^2$

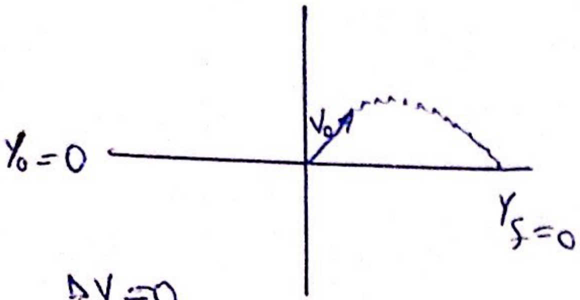
$t = \sim$

*** Projectile motion**

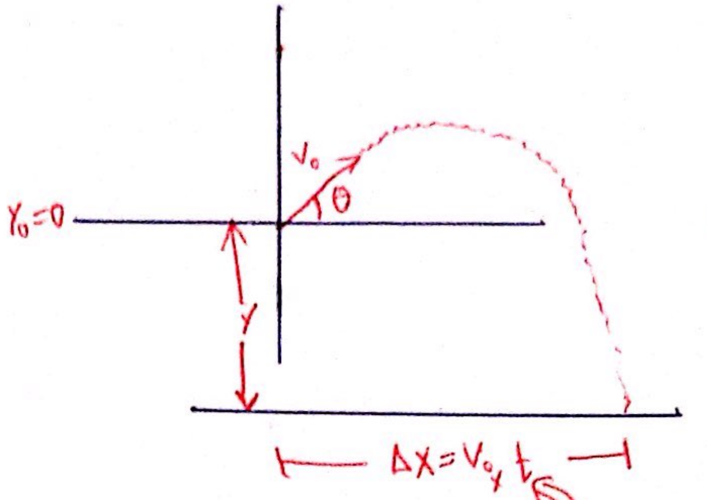
x-axis	y-axis (freefall)
$a_x = 0$ ⇐ الحركة ثابتة	$V_{fy} = V_{fo} - 10t$
$V_{fx} = V_{ox}$	$V_{fy}^2 = V_{oy}^2 - 20\Delta y$
$\Delta x = V_{ox}t$	$\Delta y = V_{oy}t - 5t^2$

⇒ Cases :-

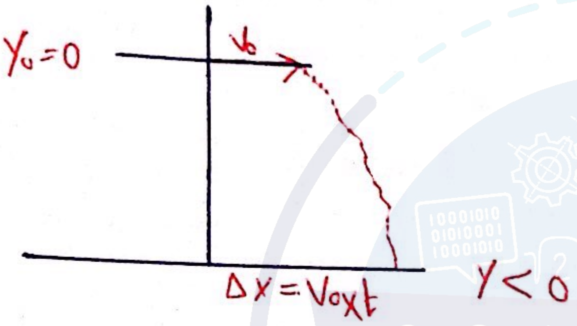
1)



$\Delta Y = 0$



3)



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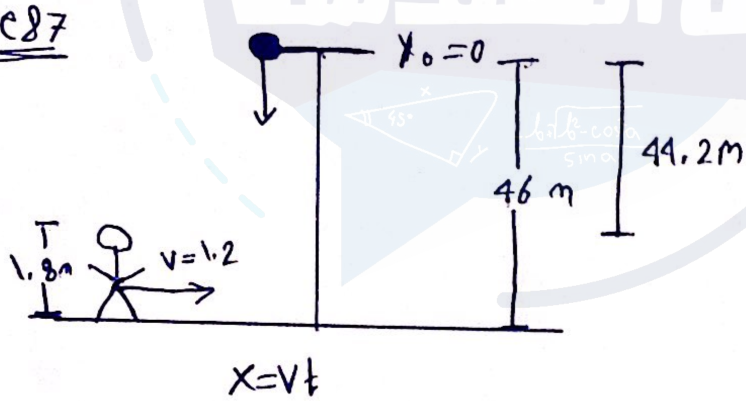


$$\Delta Y = V_{oy} t - 5t^2$$

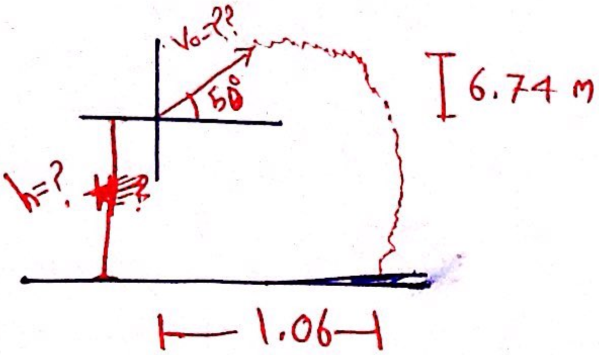
$$-44.2 = 0 - 5t^2$$

$$t = 3$$

$$X = 1.2\, t$$

$$\boxed{X = 3.6\, m}$$

Ex:



$$V_{ox} = V_0 \cos 50$$

$$V_{oy} = V_0 \sin 50$$

$$V_{fy} = V_{oy}^2 \sin^2 50 - 20 \times 6.74 = 0$$

$$V_{oy} = \frac{\sqrt{134.8}}{\sin 50}$$

$\Delta x = 1.06 = V_0 \cos 50\ t$

$1.06 = \frac{\sqrt{134.8}}{\sin 50} \cos 50\ t$

$t = \sim$

نعوّض بمعادلة ((٣))

$V = 1.5\ m/s$

$t = 1.25\ s$

$h = 4.95$

Ex: 1) cartisian $(-3, 3\sqrt{3})$

ربع ثاني

$90 < \theta < 180$

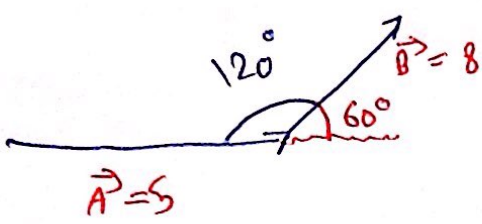
Find: polar co-ordinates.

$r = \sqrt{9+27} = 6$

$\cos\theta = \frac{3\sqrt{3}}{6}$

$\boxed{\theta = 120^\circ}$

2)



$A \cdot B = |\vec{A}||\vec{B}| \cdot \cos 60$

$= 5 \cdot 8 \cdot \frac{1}{2}$

$= 20$

3) $\vec{A} = \hat{i} - 2\hat{j}$

$\vec{B} = -3\hat{i} + \hat{j} + 2\hat{k}$

$\vec{A}\cdot\vec{B} = (\hat{i}-2\hat{j})\cdot(-3\hat{i}+\hat{j}+2\hat{k})$

$= -3-2 = -5$

angle bet. the vector and;

1) x-axis $\Rightarrow \vec{A} = \hat{i}$

2) y-axis $\Rightarrow \vec{A} = \hat{j}$

3) z-axis $\Rightarrow \vec{A} = \hat{k}$

4) $\vec{B} = -3\hat{i} + \hat{j} + 2\hat{k}$

find the angle bet. vector $\vec{B}$ and x-axis.

let $\Rightarrow \vec{A} = \hat{i}$

$|\vec{B}| = \sqrt{9+1+4}$

$\boxed{|\vec{B}| = \sqrt{14}}$

$\boxed{|\vec{A}| = 1}$

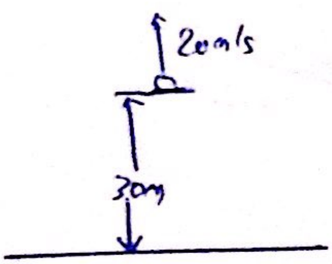
$\vec{A}\cdot\vec{B} = \hat{i}\cdot(-3\hat{i} + \hat{j} + 2\hat{k})$

$= -3$

$-3 = |\vec{A}|\,|\vec{B}|\cos\theta$

$-3 = 1 \cdot \sqrt{14} \cdot \cos\theta$

$\boxed{\cos\theta = \frac{-3}{\sqrt{14}}}$

5) 

$\Delta y = v_{oy} t - 5t^2$

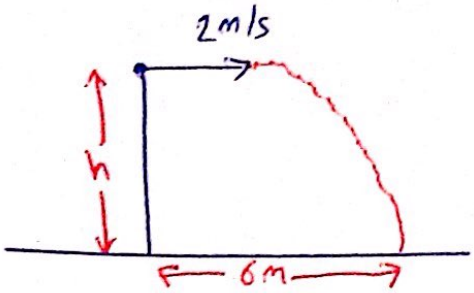
$-30 = 20t - 5t^2$

$5t^2 - 20t - 30 = 0$

$t = \frac{4 \pm \sqrt{16+24}}{2}$

$t = 2 \pm \sqrt{10} = 5.2\,s$

6)



find : h ??

$\Delta x = V_{ox}\, t$

$\boxed{t = 3}$

$\Delta y = V_{oy}\, t - 5t^2 = -h$

$-h = -45$

$\boxed{h = 45}$

⇒ Displacement :- Magnitude & direction

⇓

Change in velocity ⇒ Magnitude ⇒ acceleration
+ مع الحركة
− عكس الحركة

&

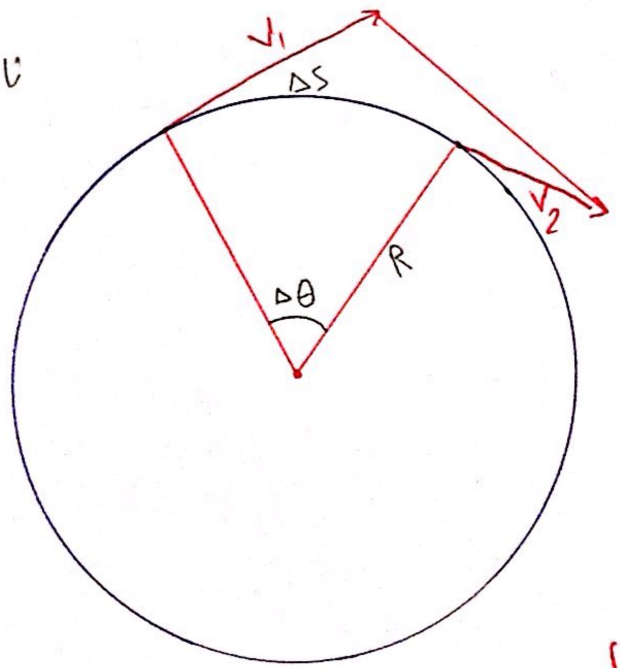
Direction → a_{rad}, $a_{\perp}$, a_{tan} عمودي على الحركة

→ $a_{\parallel}$ موازٍ للحركة

⇒ Uniform Circular motion

⇓

V ثابتة



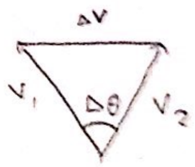
$a_{total} = \vec{a_{\parallel}} + \vec{a_{\perp}}$

$a_{total} = \sqrt{(a_{\parallel})^2 + (a_{\perp})^2}$

$\tan\theta_{a_{total}} = \frac{a_{\perp}}{a_{\parallel}}$

$\frac{\Delta V}{V} = \frac{\Delta S}{R}$

$\Delta S = R \cdot \Delta\theta$



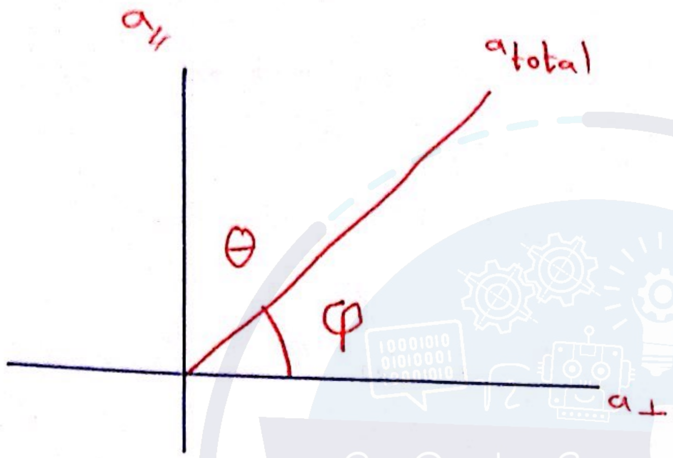
$$a_{rad} = \frac{\Delta V}{\Delta t} = \frac{V \cdot \frac{\Delta S}{R}}{\Delta t}$$

محيط $\rightarrow C = (2\pi)R$

$$\boxed{\Delta S = \Delta\theta \cdot R}$$

$$a_{rad} = \frac{V}{R}\left(\frac{\Delta S}{\Delta t}\right) \Rightarrow \left(\frac{\Delta S}{\Delta t}\right) = \bar{V}$$

$$\boxed{a_{rad} = \frac{V}{R} \cdot V = \frac{V^2}{R}}$$



$$\tan\theta = \frac{a_{\perp}}{a_{\parallel}}$$

$$\tan\varphi = \frac{a_{\parallel}}{a_{\perp}}$$

$$\vec{a}_{total} = \underbrace{\vec{a}_{\parallel}}_{\Downarrow} + \vec{a}_{\perp}$$

نحدها عن طريق معادلات الحركة

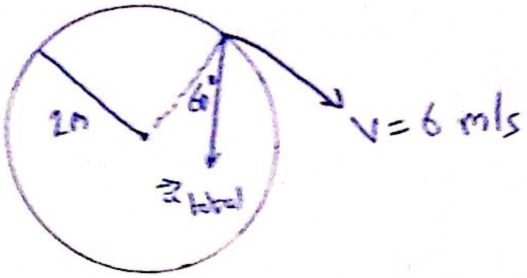
$\Rightarrow$ change in magnitude of V $\Rightarrow a_{\parallel}$

change in direction of V $\Rightarrow a_{\perp}$

$$\vec{a}_{total} = \vec{a}_{\parallel} + \vec{a}_{\perp}$$

$$a_{\perp} = \frac{V^2}{R}$$

Ex:



find : $a_{\parallel}$

$$a_{\perp} = \frac{V^2}{R} = \frac{36}{2} = 18 \, m/s^2 = a_{total} \cos 60^\circ$$

$$\boxed{a_{total} = 36}$$

$$a_{\parallel} = a_{total} \sin 60^\circ = 36 \times \frac{\sqrt{3}}{2} = 31.2$$

$$a_{total} = \sqrt{(a_{\parallel})^2 + (a_{\perp})^2}$$

$$= 36 \, m/s^2$$

Ex:

470 revolution per minute

$$V = \frac{470 (2\pi R)}{60} \; \frac{m}{s} \Big] \text{ velocity}$$

$$V = \frac{470 \; 2\pi \times 3}{60}$$

$$a_{\perp} = \frac{V^2}{R} = \left(\frac{470 \cdot 2\pi}{60}\right)^2$$

$\Rightarrow \vec{r}(t) = bt^2 \hat{i} + ct^3 \hat{j}$

$\theta = 45 \Rightarrow V$?

with x-axis or y-axis

$$\vec{r}(t) = bt^2\hat{i} + ct^3\hat{j} = x(t)\hat{i} + y(t)\hat{j}$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = 2bt\hat{i} + 3ct^2\hat{j}$$

$$\vec{v}(t)\cdot\hat{i} = |v|\cdot|\hat{i}|\cdot\cos 45^\circ$$

$$\sqrt{2}\cdot 2bt = |v|$$

$$\vec{v}(t)\cdot\hat{j} = |v|\cdot\hat{j}\cdot\cos 45^\circ$$

$$\sqrt{2}\cdot 3ct^2 = |v|$$

$$\cancel{\sqrt{2}}\cdot 2b\cancel{t} = \cancel{\sqrt{2}}\cdot 3ct^{\cancel{2}}$$

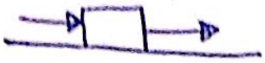
$$\boxed{t = \frac{2b}{3c}}$$

Ex:

⇒ Newton's Laws of motion

Dynamics

$\vec{Force}$ (magnitude + direction) "Newton"



Types of forces:

- Pull. (tension) سحب ، شد
- push. دفع
- friction. احتكاك

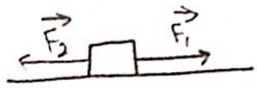
⇒ تلامس الجسمين

- gravity. (weight)

⇒ دون تلامس

Resultant force.

$$\vec{F}_R = \sum_{i=1} \vec{F}_i$$



$$\vec{F}_R = \vec{F}_1 + \vec{F}_2$$

Ex: $\vec{F}_1 = 3\hat{i} + 2\hat{j}$

$\vec{F}_2 = 2\hat{i} - 3\hat{j}$

$\vec{F}_R = 5\hat{i} - \hat{j}$ Newton

$|\vec{F}_R| = \sqrt{25+1} = \sqrt{26}$ Newton

→ Newton's 1st. Law (قانون الاتزان) (القصور الذاتي)

Static: $V = 0$

Dynamic "motion": $V \neq 0$, V constant $\Rightarrow \Delta V = 0$, $a = 0$

$\vec{F}_{ext} = 0$ (equilibrium) اتزان

⇒ Newton's 2nd. law

$\vec{F}_{ext} \neq 0$

$\Delta V \neq 0$ through Δt ⇒ $\frac{\Delta V}{\Delta t} = \bar{a}_{av} = \bar{a} = $ Const.

$$\Rightarrow \vec{F}_{ext} \propto \vec{a}$$

$$\vec{F}_{ext} = \underbrace{const}_{\Downarrow} \cdot a$$

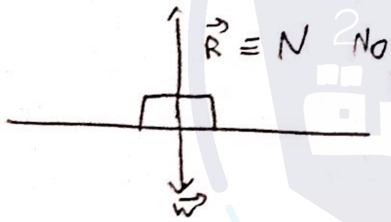
Mass $\Rightarrow$ mass (M) Kg $= \dfrac{\vec{F}_{ext}}{\vec{a}}$

$$\vec{F}_{ext} = m\vec{a}$$

gravity $\Rightarrow$ $\vec{F}_g = \vec{F}_a = \vec{W} = m_g$

$\Rightarrow$ Newton's 3rd. law

(Action – Reaction law)



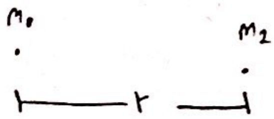
$\vec{R} \equiv N$ Normal force $\equiv \perp$

$$\vec{w} + \vec{R} = 0$$

$$\vec{w} = -\vec{R}$$

$$|\vec{w}| = |\vec{R}|$$

$\Rightarrow$ gravitational force



$$\vec{F} = G\frac{m_1 m_2}{r^2}$$

$$\vec{F}_g = \frac{G\,m\,M_E}{R_E^2} = m\vec{g} = \vec{W}$$

⇒ Density

1) Volume

$\rho_v = \frac{m}{vm}$

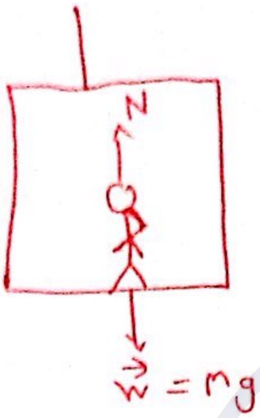
2) Areal (surface)

$\sigma = \frac{m}{area}$

3) linear

$\lambda = \frac{m}{l}$ ⇐ length

⇒ Ex: 1)



$v=0$ or $v=\text{constant}, \Delta v=0$

$N = W = mg$

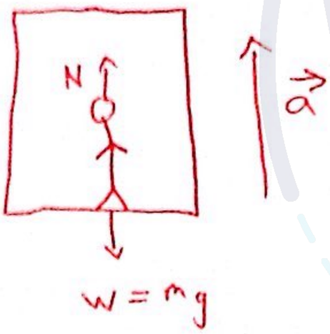
$\vec{N} + \vec{F_g} = 0$

$\vec{N} = -\vec{F_g}$

$N(\hat{j}) + mg(-\hat{j}) = 0$

$N = mg$

2)



$\vec{F}_{ext} = m\vec{a} = ma\hat{j}$

$N + \vec{F_g} = N(\hat{j}) + mg(-\hat{j})$

$N - mg = ma$

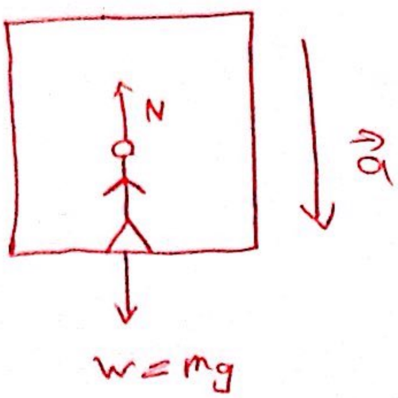
$N = m(g+a) = w_e$

زيادة في الوزن الظاهري

e: effective

ظاهري

3)

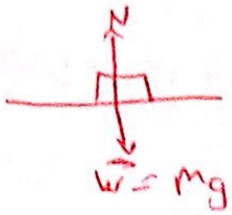


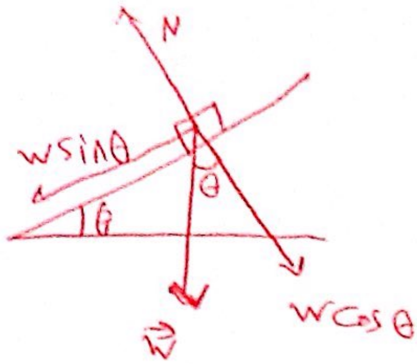
$mg - N = ma$

$w_e = N = m(g-a)$

نقصان في الوزن الظاهري

→ 3rd. law





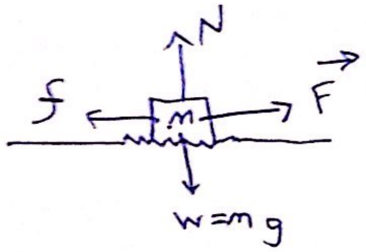
$N = W\cos\theta$

If $\theta = 0$

$\boxed{N = W}$

* First exam ; 8/11
5 → 6

⇒ Frictional force



Equilibrium

$\Sigma F = 0$

$\vec{F} - f = 0$

$\boxed{F = f}$

$f \propto$ Normal reaction force

$f \propto N$

$\boxed{f = \underbrace{\mu}_{\text{Ceffficient of friction}} . N}$

معامل الاحتكاك

⇒ $\vec{F} - f_s = 0$ (just moving) على وشك الحركة

$f_s = \underbrace{\mu_s}_{\text{static}} . N$

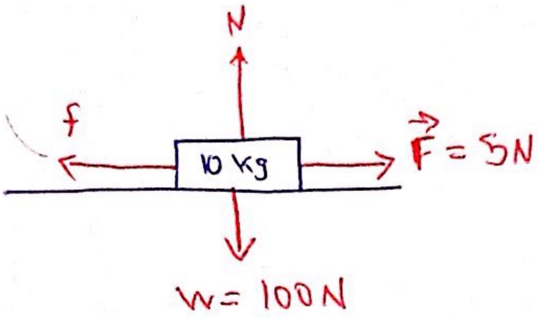
$f_K = \underbrace{\mu_K}_{\text{motion}} . N$

$\mu_K < \mu_s \Rightarrow f_K < f_s$

⇒ let $\mu_s = 0.5$

$\therefore \mu_K < \mu_s = 0.3$

Ex:



1) 5N is acting on the object; the object is at rest (stationary)

Find: f ??

$\boxed{f = 5N}$

2) if $\vec{F} = 8N$; object is still stationary. Find: f ??

$\boxed{f = 8N}$

3) $\vec{F} = 12$ N ; object is just moving.

$F - f_s = 0$

$12 - f_s = 0$

$\boxed{f_s = 12}$

$\Rightarrow$ find: μ_s

$12 = \mu_s \cdot 100$

$\mu_s = 0.12$

$\mu_K < \mu_s \Rightarrow$ let $\mu_K = 0.1$

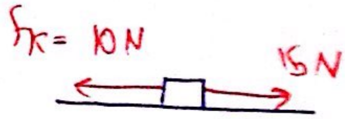
4) if F = 15 N. Find: f ??

$f_k = \mu_K \cdot N$

$= 0.1 \cdot 100$

$= 10$ N

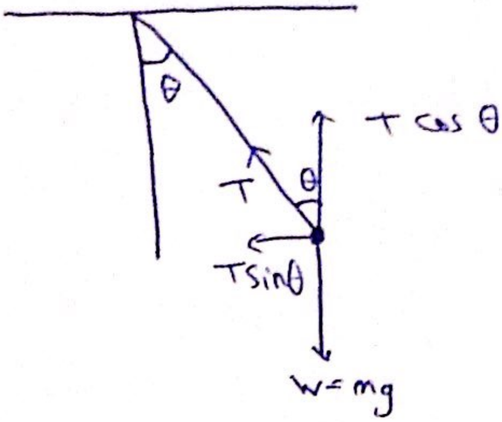
5) Find: a?



$15 - f_K = F_{net} = 15 - 10 = 5N = ma$

$\vec{F}_{ext}$

$a = \frac{5}{10} = \frac{1}{2}\ m/s^2$



$$a_{\perp} = \frac{v^2}{r}$$

$$\text{Force} = F_{\perp} = \frac{mv^2}{r}$$

$\boxed{T\cos\theta = mg}$ .... (1) $\Rightarrow$ $\boxed{T = \text{tension}}$

frictional force $= T\sin\theta$

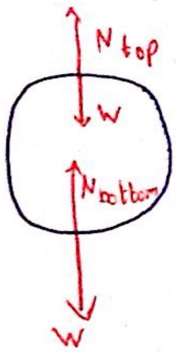
$$= \frac{mv^2}{r}$$

$\therefore$ $\boxed{\frac{mv^2}{r} = T\sin\theta}$ .... (2)

$$\frac{2}{1} \Rightarrow \tan\theta = \frac{v^2}{gr}$$

Uniform C.m

In a Vertical plane



$$N_{bottom} - mg = F_{net} = m a_{\perp} = \frac{mv^2}{r}$$

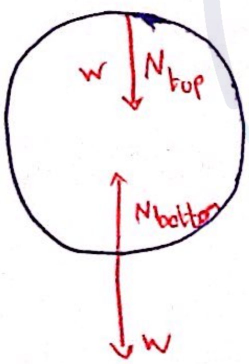
$$\boxed{N_{bottom} = \frac{mv^2}{r} + mg}$$

$$mg - N_{top} = F_{net} = \frac{mv^2}{r}$$

$$\boxed{N_{top} = mg - \frac{mv^2}{r}}$$

Non-uniform C.m

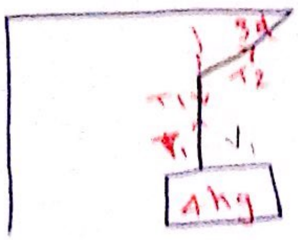
Vertical plane



$$N_{bottom} - mg = \frac{m v_b^2}{r} \Rightarrow N_{bottom} = mg + \frac{m v_b^2}{r}$$

$$N_{top} + mg = \frac{m v_t^2}{r} \Rightarrow N_{top} = \frac{m v_t^2}{r} - mg$$

Ex :



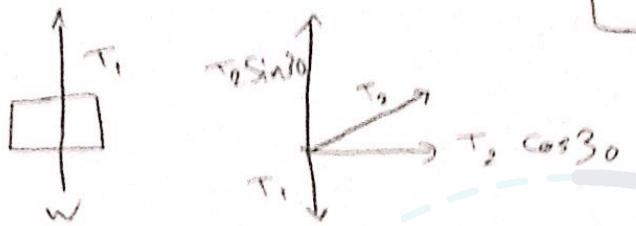
1) $T_1 = ??$

$T_1 = Mg = 4 \cdot 10 = 40N$

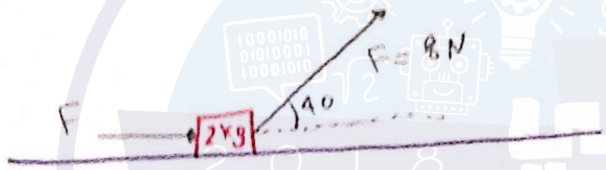
2) T_2 ??

$T_2 \sin 30 = T_1$

$\boxed{T_2 = 80\ N}$



Ex



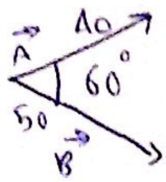
find: a??

$$\frac{F + F\cos 40}{m} = \frac{ma}{m}$$

$$a = \frac{F(1 + \cos 40)}{m}$$

$$a = \frac{8(1+\cos 40)}{2} \quad m/s^2$$

⇒ Ex:



$|\vec{C}| = A^2 + B^2 + 2AB\cos\theta$

$= 1600 + 2500 + 2000$

$= 6100$

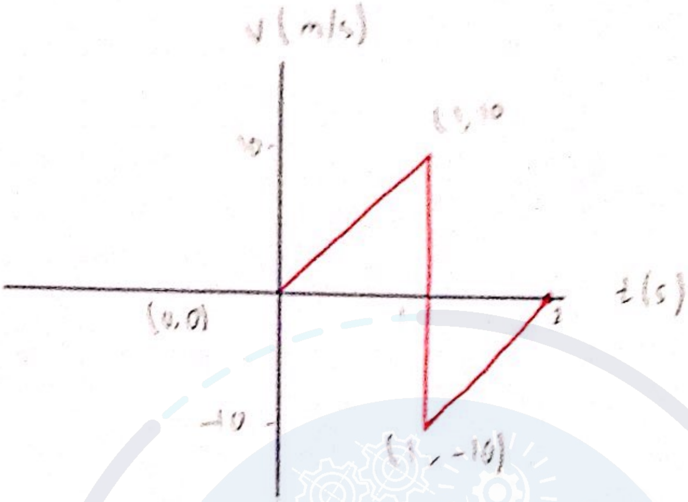
$\vec{C} = \vec{A} - \vec{B}$

$|\vec{C}| = ??$

$\Rightarrow \quad F \propto \frac{m_1 m_2}{r^2}$ ME R_E

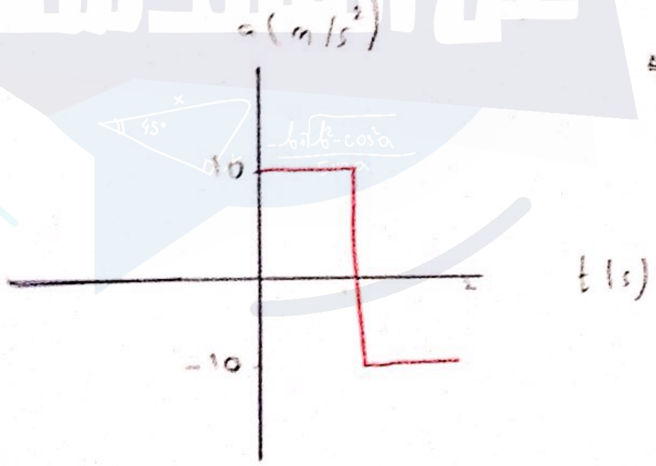
$$F = \frac{G m_1 M_E}{R_E^2} = m_1 g \quad (g \simeq 10\, m/s^2)$$

Ex.



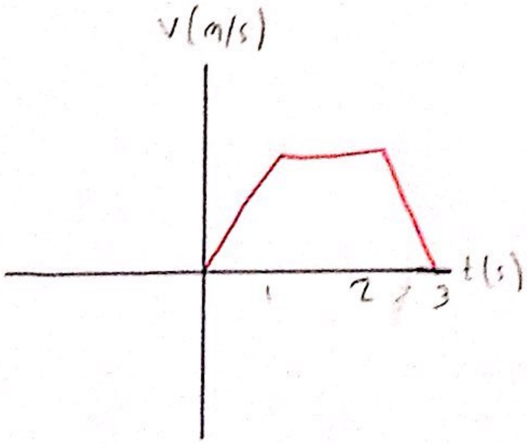
$x, (t=0 \;\&\; t=1s)$

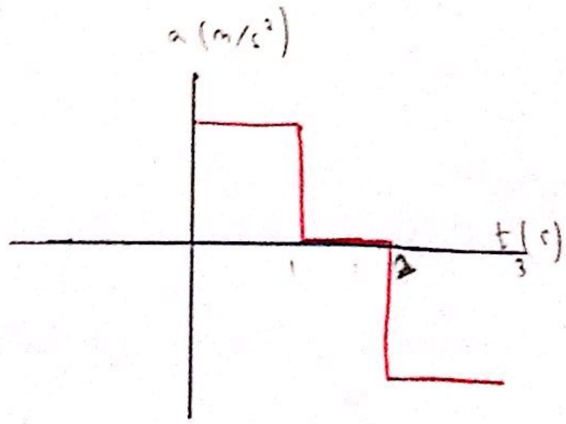
$= \frac{1}{2} \cdot 1 \cdot 10 = 5m$



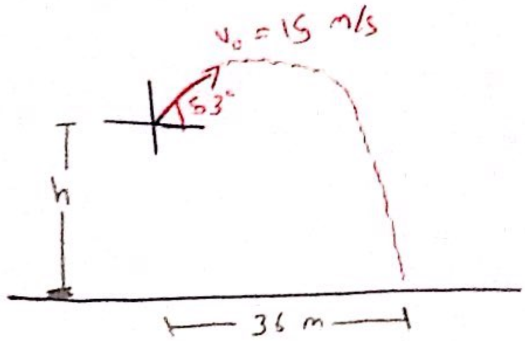
$\frac{\Delta v}{\Delta t} = \frac{10}{1} = 10\, m/s^2$

Ex:





Ex:



find: h??

$x = V_0 \cos 53 \; t.$

$t \simeq 4 s$

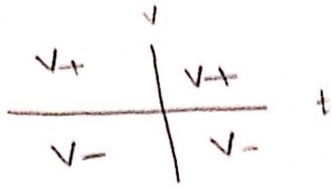
$\Delta y = V_{0y} \, t - \frac{1}{2} g \, t^2$

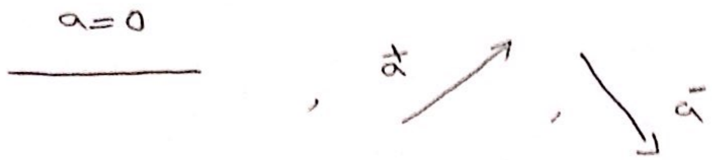
$-h = 15 \sin 53° \, t - 5 t^2$

← ملاحظة خارجية:

← لحتى نفهم الرسمات أكثر:

* في منحنى (v) و(t)





* في منحنى (a) و(t)

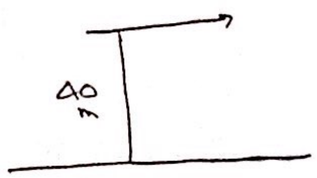
Ex: $x(t) = 21 + 20t - 4t^2$

$\vec{V}_{av}$ = ?? between $t=1$, $t=3$

$$\frac{(21+60-36)-(21+20-4)}{3-1} = \frac{8}{2} = 4 \text{ m/s}$$

Ex: Projectile is thrown from the building.

$V_0 = 40$ m/s positive x.



How fast will the ball be moving.

$\Delta x = V_{0x} t$

$\Delta y = V_{0y} t - 5t^2$

$-40 = -5t^2$

$t = \sqrt{8}$

$V_{fy} = V_{0y} - 10t$

$= -10\sqrt{8}$

$V_{fx} = 40$ m/s

$\vec{V_f} = 40\hat{i} - 10\sqrt{8}\hat{j}$

Ex: a ball is released from rest falls a distance h_1 m during the first second of time. The distance of fall (in meters) ~~during~~ at the next second of time

$$\Delta y = V_{oy} t - 5t^2$$

$$-h_1 = -5$$

$$\boxed{h_1 = 5\,m}$$

$$-h_2 = -20$$

$$\boxed{h_2 = 20\,m}$$

$$\boxed{\Delta h = 15\,m}$$

⇒ Chapter "6"

work: الشغل

$\vec{Force}$ & $\vec{Displacement}$

Dot product of

$$W \equiv \vec{F} \cdot \Delta\vec{S}$$

$$W_F = \vec{F} \cdot \Delta\vec{S} = |\vec{F}|\,|\Delta\vec{S}| \cdot \cos\theta$$

$$\Downarrow$$

$$(F_x\hat{i} + F_y\hat{j} + F_z\hat{k}) \cdot (\Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k})$$

$$= (F_x \Delta x + F_y \Delta y + F_z \Delta z)$$

Maximum / (parallel (+), anti-parallel (−))

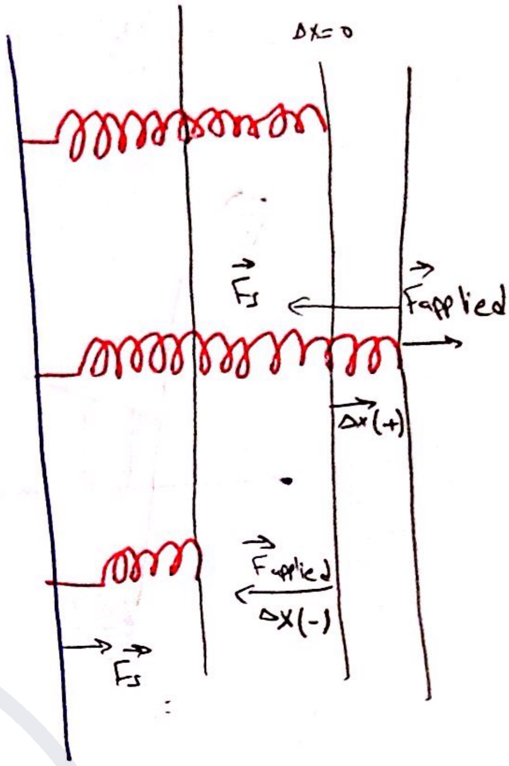
ΔX , $F_{applied}$ « Same direction »

$\vec{F_s} \propto \vec{X}$

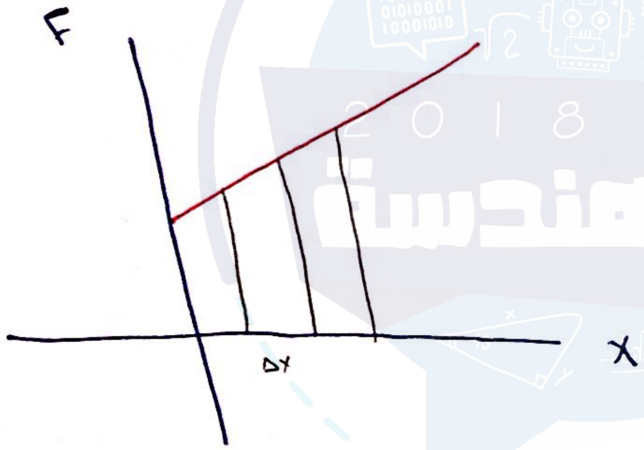
$\vec{F_s} = - K \cdot \vec{X}$

لأنهم عكس إشارة

$\vec{F}_{applied} = K \vec{X}$



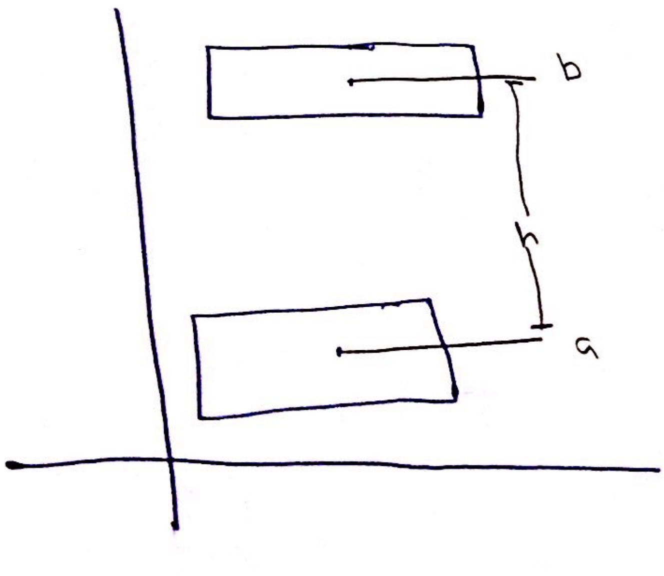
K: Spring constant $\left(\frac{Newton}{meter}\right) \left(\frac{N}{m}\right)$



$W = \int_{x_i}^{x_f} \vec{F} \cdot dx$

$W = \vec{F} \cdot \int_{x_i}^{x_f} dx$

$W = \vec{F} \cdot (x_f - x_i) = \vec{F} \cdot \Delta X$



$\vec{F} = \vec{F_g} = mg(-\hat{j})$

1) object at b:-

وبنزل إلى a

$$W_{b\to a} = \vec{F} \cdot \Delta x$$

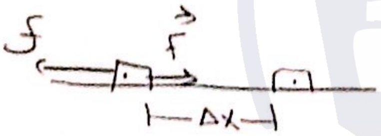
$$= mg(-\hat{j}) \cdot h(-\hat{j}) = mgh$$

2) object at a:-

وبطلع إلى b

$$W_{a\to b} = mg(-\hat{j}) \cdot h(\hat{j}) = -mgh$$

Frictional force



$$W_f = ??$$

$$= (\mu \cdot mg) \cdot (\Delta x)(\cos 80^\circ)$$

$$= -\mu \cdot mg \cdot \Delta x.$$

Work

$$\vec{F} \cdot \vec{\Delta s} \equiv |\vec{F}| \cdot |\vec{\Delta s}| \cdot \cos\theta$$

- $\oplus$ $0 < \theta < 90$
- $\boxed{90 = \theta}$
- $\ominus$ $90 < \theta < 180$

$$\vec{\Delta s} = (\Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k})$$

$$\vec{F} = (F_x \hat{i} + F_y \hat{j} + F_z \hat{k}) \cdot$$

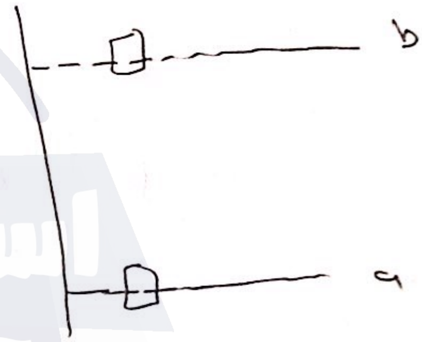
$$= F_x \Delta x + F_y \Delta y + F_z \Delta z$$

$\Rightarrow$ Force :- Constant $\Rightarrow$ $\vec{F_g} = \vec{W} = m\vec{g}$

$$W_{a \to b} = - mgh \qquad (mg(-\hat{j})) \cdot h(\hat{j})$$

$$- \quad (y_a - y_b)$$

$$W_{b \to a} = mgh$$



$\Rightarrow$ $m\vec{a} = \vec{F}_{net} = \vec{F_g} + \vec{f} + \vec{F_s} + F_{others}$

$$W = m\vec{a} \cdot \vec{\Delta x} \qquad V_f^2 = V_i^2 + 2a\Delta X$$

$$\vec{F}_{net} = m\left(\frac{V_f^2 - V_0^2}{2\Delta X}\right) \cdot \Delta X$$

$$= \frac{1}{2} m (V_f^2 - V_0^2) = \frac{1}{2} m V_f^2 - \frac{1}{2} m V_0^2 = K_f - K_i = \Delta K$$

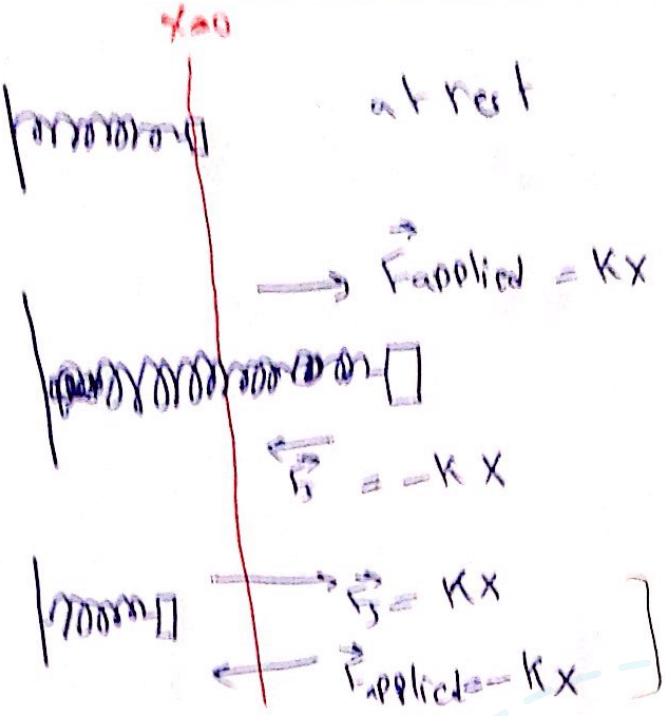
$\Rightarrow$ Varying force

$$\vec{F} \neq \text{Const.}$$

$$\vec{F}_{net} = \vec{F_g} + \vec{F_s} + \vec{F}_{other} + \vec{F_f}$$

$$W_{F_{net}} = \Delta K = W_{\vec{F_g}} + W_{\vec{f}} + W_{\vec{F_s}} + W_{F_{others}}$$

$$\Delta K = \text{Energy} = (\text{Work})_{total}$$



K ≡ Spring const. (N/m)

$$W_{F_a} = \int_{x_i}^{x_f} \vec F_a \cdot d\vec x$$

$$W_{F_a} = \int_{x_i}^{x_f} kx \, dx = \frac{k}{2}(x_f^2 - x_i^2) = \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2$$

$$W_{F_s} = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2$$

⇒ $W_{F_{net}} = \vec F_{net} \cdot \Delta \vec s = \Delta K = \frac{1}{2} m (v_f^2 - v_i^2)$

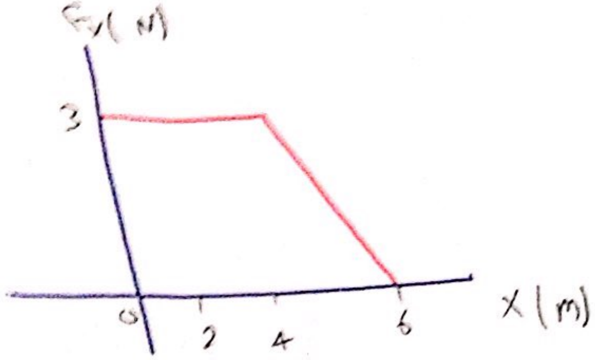
Ex: Consider $\vec F = (4\hat i - 2\hat j - 4\hat k)$ Newton

$\Delta \vec x$ = 5 Newton along x-positive direction.

what is the work??

$W_F = \vec F \cdot \Delta \vec s$ = 20 (Newton.m) (Joule (J))

Ex:



w ?? | x=0 to x=6

$$W_F = Area = \frac{1}{2}(6+4) \cdot 3 = 15 J$$

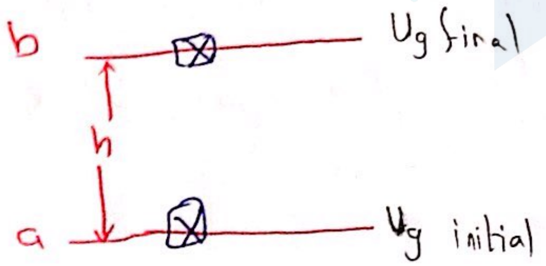
Ex: $m = 2 Kg$

at $t=0 \Rightarrow \vec{v_i} = (4\hat{i} - 3\hat{j})$

at $t=3 \Rightarrow \vec{v} = (2\hat{i} + 3\hat{j})$

During this time interval find the net work done on the particle?

$$\vec{W}_{Fnet} = \Delta K = \frac{1}{2}(2)(13-25)$$
$$= -12 J$$



$$\vec{F_g} = mg(-\hat{j})$$

1) a: initially, b: finally

$$W_{Fg} = mg(y_i - y_f) = mg(-h)$$
$$= -\Delta U_g$$

$$\vec{F}_{net} = \vec{F_g}$$

$$W_{Fnet} = \Delta K = W_{Fg} = -\Delta U_g$$

$$\Delta K + \Delta U_g = 0$$

$$\underbrace{(K_f - K_i) + (U_{gf} - U_{gi})}_{} = 0$$

$$E_f - E_i = 0 = \Delta E$$

Conservation of mechanical energy.

$\vec{F}_{net} = \vec{F}_g$ (conservative force)

$F_a = kx$

$F_s = -kx$

$$W_{F_s} = \int_{x_i}^{x_f} -kx(\hat{i}) \cdot dx(\hat{i})$$

$$= -k \cdot \frac{x^2}{2} \Big]_{x_i}^{x_f}$$

$$= \frac{-k}{2}(x_f^2 - x_i^2)$$

$$= \frac{k}{2}(x_i^2 - x_f^2) = -\Delta U_s \rbrace \rightarrow \text{spring}$$

$$\Rightarrow \Delta U_s = \frac{k}{2}(x_f^2 - x_i^2)$$

$$\boxed{\therefore W_{F_s} = -\Delta U_s}$$

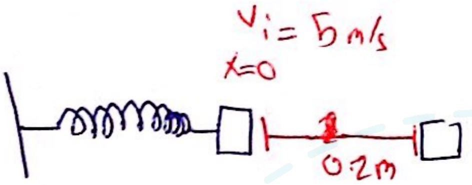
$$\vec{F}_{net} = \vec{F_g} + \vec{F_s} + \vec{F_f}$$

$$W_{F_{net}} = \left(W_{F_g} + W_{F_s}\right) + W_f$$

$$\Downarrow \qquad\qquad \downarrow$$

$$W_{F_{net}} = W_{const.} + W_{non.const.} = \Delta K$$ التغير في الطاقة الحركية

Ex:



$K = 1000\ N/m$

$m = 2\ Kg$

$\Rightarrow$ frictionless $\therefore \vec{F}_f = 0$

Horizontal $\therefore \vec{F}_g = 0$

what is the Kinematic energy of the block??

$$W_{F_{net}} = W_{\vec{F_s}} = \Delta K$$

$$-\Delta U_s = \Delta K \Rightarrow \Delta K + \Delta U_s = 0$$

$$\frac{1}{2}(m)(V_f^2 - V_i^2) + \frac{1}{2}(m)(X_f^2 - X_i^2) = 0$$

$$\frac{1}{2}\cdot(2)(V_f^2 - 25) + \frac{1}{2}(1000)(0.2^2 - 0) = 0$$

$$V_f^2 - 25 + 20 = 0$$

$$V_f^2 = 5$$

$$\boxed{V_f = \sqrt{5}}$$

$$K_f = \frac{1}{2} m V_f^2 = \frac{1}{2}\cdot 2 \cdot \sqrt{5}^2 = 5\ J$$

$W_F = -\Delta U$

$$\left.\begin{array}{l}\vec{F}: \ \vec{F}_g = W_{F_g} = \vec{F}_g \cdot \Delta \vec{y} \\ \qquad \vec{F}_s \end{array}\right] W_{Con}$$

$$\left.\begin{array}{l}\qquad \vec{F}_f \end{array}\right] W_{non\text{-}Con}$$

$$\qquad \vec{F}_{others}$$

$W = \vec{F} \cdot \Delta \vec{s}$

$\vec{F}_{net} = \vec{F}_g + \vec{F}_s + \vec{F}_f + \vec{F}_{others}$

$W_{F_{net}} = \Delta K = \frac{1}{2} m (v_f^2 - v_0^2)$

Ex: $U(x, y, z) = 2x^2 y^2 + 3y^2 z^3$ $\quad \vec{F}$?? $\Big|_{(1,2,1)}$

$F_x = \frac{-\partial U}{\partial x} = -(4xy^2 - 0) = -4(1)(2)^2 = -16$

$F_y = \frac{-\partial U}{\partial x} = -(4x^2y + 6yz^3) = -(4.1.2 + 6.2.1) = -20$

$F_z = \frac{-\partial U}{\partial z} = -9y^2 z^2 = -9.4.1 = -36$

$\vec{F} = -16\hat{i} - 20\hat{j} - 36\hat{k}$

$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$

$\Delta \vec{s} = \Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k}$

$W_F = \vec{F} \cdot \Delta \vec{s} = F_x \Delta x + F_y \Delta y + F_z \Delta z$

$$F_x \Delta x \Big|_{\substack{\Delta y \\ \Delta z = 0}} = -\Delta U = F_x = -\frac{\Delta U}{\Delta x}\Big|_{\substack{\Delta y \\ \Delta z = 0}} \Rightarrow F_x = -\lim_{\Delta x \to 0} \frac{\Delta U}{\Delta x} = \frac{-\partial U}{\partial x}$$

$$F_y \Delta y \Big|_{\substack{\Delta x \\ \Delta z = 0}} = -\Delta U \Rightarrow F_y = \frac{-\Delta U}{\Delta y}\Big|_{\substack{\Delta x \\ \Delta z = 0}} \Rightarrow F_y = \frac{-\partial U}{\partial y}$$

$$F_z \Delta z \Big|_{\substack{\Delta x \\ \Delta y = 0}} = -\Delta U \Rightarrow F_z = \frac{-\Delta U}{\Delta z}\Big|_{\substack{\Delta x \\ \Delta y = 0}} \Rightarrow F_z = \frac{-\partial U}{\partial z}$$

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right] = -\vec{\nabla} U(x,y,z)$$

$$-\underbrace{\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)}_{\nabla :\ \text{gradian}} U(x,y,z)$$

$$\underbrace{W_c}_{} + W_{nc} = \Delta K$$

⇓

Conservative

⇓

Force. Conservative

⇒ Question 33 ; Page 254 a small block is moving in x-y plane

The F is described by $U(x,y) = \underset{6\ J/m^2}{5.85}\, x^2 - \underset{4\ J/m^3}{3.65}\, y^3$

Find the magnitude and direction of $\vec{a}\Big|_{(0.3,\,0.6)}$??

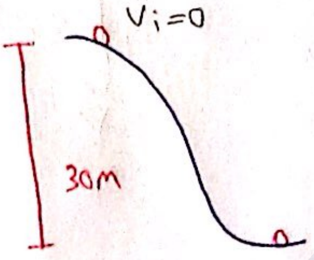
$$F_x = \frac{-\partial u}{\partial x} = -12x$$

$$F_y = \frac{-\partial u}{\partial y} = 12y^2$$

$$\vec{F} = F_x\hat{i} + F_y\hat{j} = -12x\hat{i} + 12y^2\hat{j}$$

$$a = \frac{\vec{F}}{m}$$

Ex: 1)



$m = 2kg$

* الوضع تحول لحركة

$$mgh = \frac{1}{2} m V_f^2$$

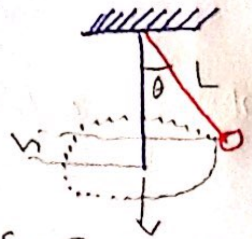
$$600 = V_f^2$$

$$V_f = \sqrt{600} \; m/s$$

Ex: 2)

$$\frac{1}{2} m V_f^2 = \frac{1}{2} K x_f^2 - \frac{1}{2} k x_i^2$$

Ex:



$V = 0$

$u = mg\underbrace{h}_{L - L\cos\theta}$

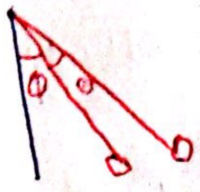
$= mg(L(1-\cos\theta))$

الطاقة الحركية أكبر ما يمكن
طاقة الوضع = صفر

$\frac{1}{2} m v^2$

$$\frac{1}{2} m v^2 = mg\,(L(1-\cos\theta))$$

$$V = \sqrt{2gL(1-\cos\theta)}$$



$$\frac{1}{2} m (v_f^2 - v_i^2) = mgL(1-\cos\theta)$$

الصغيرة

⇒ Power القدرة

$$\bar{P} = \vec{F} \cdot \vec{V}_{av}$$

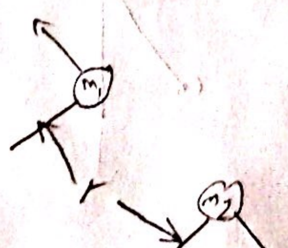
$$P = \vec{F} \cdot \bar{v} = m a \cdot \vec{V} = m \cdot \frac{d\vec{v}}{dt} \cdot \vec{V} = \frac{mv}{dt} \cdot dv$$

$$P\,dt = mv\,dv$$

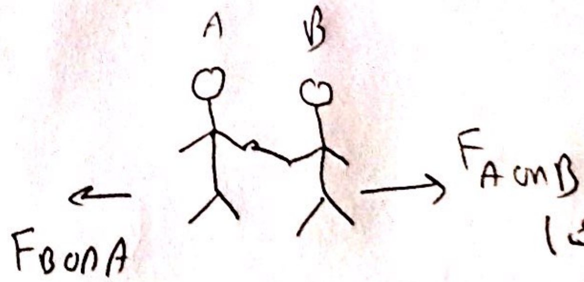
$$\int_{u_i}^{u_f} -du = W_F = m \int (v \cdot dv) = \frac{mv^2}{2}\Big]_{v_i}^{v_f} = \underbrace{\frac{1}{2} m (v_f^2 - v_i^2)}_{\Delta K}$$

⇒ Chapter "8"

Interaction



$$F = G \frac{m_1 m_2}{r^2}$$



$$\vec{F} = m\vec{a}$$

$$= m \cdot \frac{d\vec{v}}{dt}$$

$$\vec{F} = \frac{d}{dt} m\vec{v} = \frac{d\vec{p}}{dt}$$

$m\vec{v}$ = momentum

الزخم (كمية التحرك)

(kg.m/s)

$$\vec{F}_{A\,on\,B} + \vec{F}_{B\,on\,A} = 0$$

$$\frac{dP_B}{dt} + \frac{dP_A}{dt} = 0 = \frac{d}{dt}(\vec{P}_B + \vec{P}_A) = 0$$

$$\vec{P}_A + \vec{P}_B = \text{Const.}$$

momentum is Conserved

$m_A \neq m_B$

$\vec{V}_A \neq \vec{V}_B$

$m_A \vec{V}_A + m_B \vec{V}_B = \text{Const}$

$\Downarrow$

In 2-D

$$(m_A V_{Ax} + m_B V_{Bx})_i = (m_A V_{Ax} + m_B V_{Bx})_f$$

$\Delta P = 0$

$$\int_{P_i}^{P_f} d\vec{P} = \int \vec{F} \cdot dt = \vec{F} \int_{t_i}^{t_f} dt = \vec{F} \cdot \Delta t$$

$P_f - P_i = \Delta \vec{P}$

$$\vec{F} \cdot \Delta t = \Delta \vec{P}$$

الدفع Impulse

$$J = \Delta \vec{P} \quad (N \cdot S \equiv Kg \cdot m/s$$