الفصل الأول

٢٠١٥ - ٢٠١٦

PHYSICS I
Notebook

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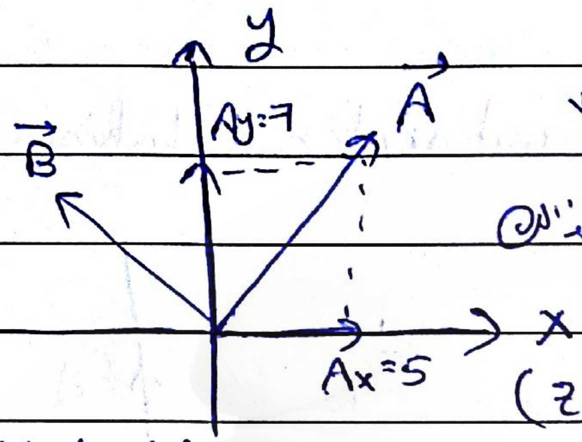
Chapter 3: Vectors Notation:-

No. ______________



$\underset{\text{vector}}{\vec{A}} = \underbrace{A_x + A_y}_{\text{Scaler quantity}}$

(ما بصير نكتب أنه Vector = Scaler لأنه)

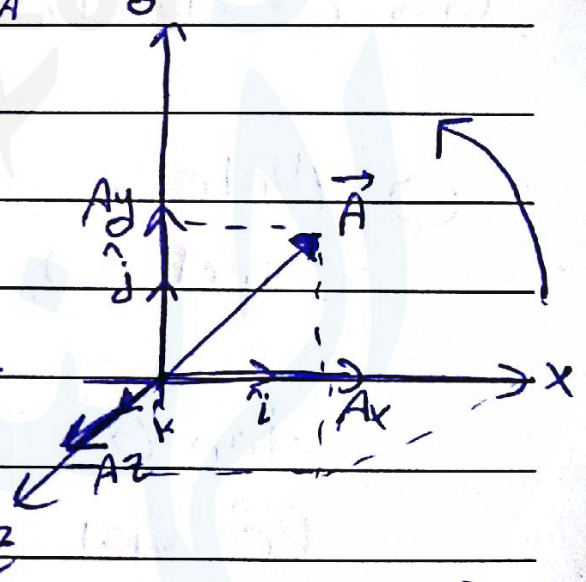
نحط ال ($\hat{i}$) لل x-axis و
ال ($\hat{j}$) لل y-axis, وال ($\hat{k}$) لل z-axis)

$(\hat{i}, \hat{j}, \hat{k}) \rightarrow$ unit vector notations.

$\circledast$ $\vec{A} = \underbrace{A_x \hat{i}}_{\text{x component of vector } \vec{A}} + \underbrace{A_y \hat{j}}_{\text{y component of vector } \vec{A}} + \underbrace{A_z \hat{k}}_{\text{z component of vector } \vec{A}}$

$\circledast$ What's the magnitude of $(\hat{i}, \hat{j}, \hat{k})$?

Answer: $(\underbrace{1 \text{ unit}}_{\text{Always}})$ to the positive (right) axis.



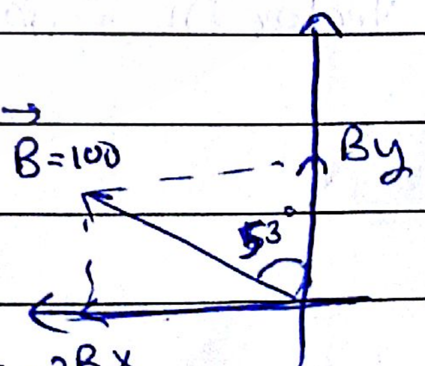
Example ①: find the x- and y- components of vector $\vec{B}$?

$B_x = B\sin 53°$

$= 80°$ → the magnitude of x-axis of vector B. (ما بنقدر نقول سالب)

$B_y = B\cos 53°$

$= 60°$



② Write vector $\vec{B}$ in unit vector notations?

$\vec{B} = -80\hat{i} + 60\hat{j}$

No. ______________

⊠ Example:-

Write vector $\vec{C}$ in unit vector notations:-

$C_y = C \sin 30$

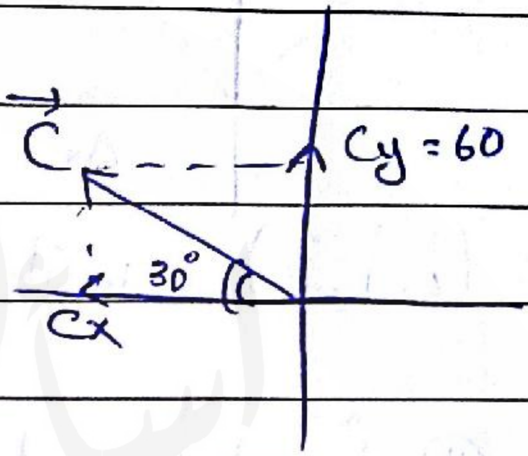
$60 = C \times 0.5$

$\vec{C} = 120$

$C_x = C \cos 30$

$C_x = 120 \times \frac{\sqrt{3}}{2}$

$C_x = 103.9$



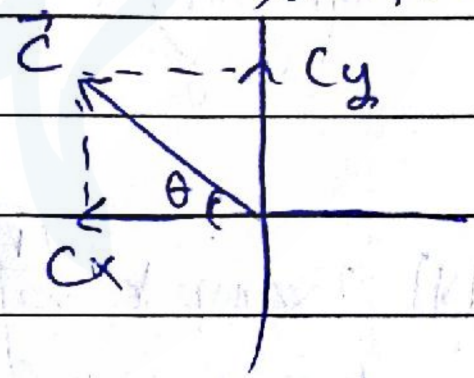
So → $\vec{C} = -103.9\,\hat{i} + 60\,\hat{j}$ #

Note :-

اذا طلب منك نرسم بمحصلة أو ال Vector :-
برسم المحاور

$\tan \theta = \frac{C_y}{C_x}$

($\vec{C}$ لايجاد اتجاه)



ولو طلب اتجاهه بس ما برسم
وبس بعوض من ال Vector
اللي عندي .

* when I'm given a vector, here are the questions:-

1 Magnitude (Length):-

$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$ → no ($\hat{i}, \hat{j}, \hat{k}$)
no (-ve signs)

$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$ unit

* $|\hat{i}| = |\hat{k}|$ ✓ (il absolute value (بيطل في Direction))

* $\hat{i} = \hat{k}$ ✗ Magnitude لازم يكونوا يساووا بعض بال
(& direction

2 Direction ($\vec{A} = A_x\hat{i} + A_y\hat{j}$)

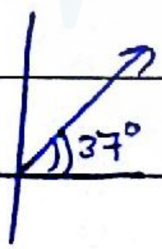
$\tan\theta = \frac{A_y}{A_x}$ → positive always

with nearest x-axis

اذا بسؤال كان (what's the direction) اذا هو نطلب Angle
و الزاوية اللي بكون بيه إياها تتكون مع ال positive x-axis
فقط !

* What's the direction of these vectors?

(A)



$\vec{A} = 3\hat{i} + 4\hat{j}$

$\tan\theta = \frac{4}{3} = 1.3$

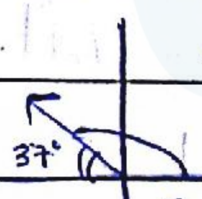
$\boxed{\theta = 37°}$

✓ وهيه نفس المطلوبة .

(B)

$180 - 37 = 143°$

هاد رح الجواب المطلوب لأنه لازم زاوية مع +x axis



$\vec{B} = -3\hat{i} + 4\hat{j}$

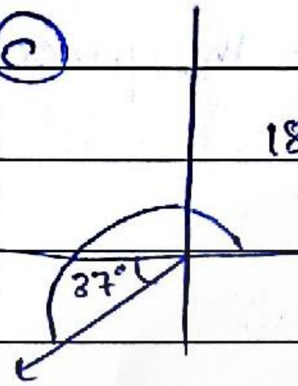
$\tan\theta = \frac{4}{3} = 1.3$

$\theta = 37°$

هادي الي مع ال
nearest x-axis.

No. ______

(C)



$\vec{C} = -3\hat{i} - 4\hat{j}$

$\tan\theta = \frac{4}{3} = 1.3$

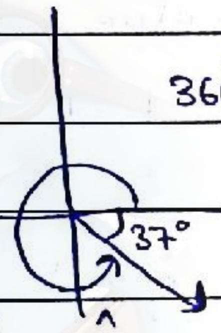
$\theta = 37°$

nearest x-axis ع

$180 + 37 = 217°$

هادي الزاوية الحلوة

(D)



$360° - 37° = 323°$

هاي الزاوية الحلوة

(لأنه احنا مشينا عكس عقارب الساعة

ما نوصل 323 ، لو مشينا مع

عقارب الساعة بس بنحط سالب جنب الزاوية

$-37° = 323°$

$\vec{D} = 3\hat{i} - 4\hat{j}$

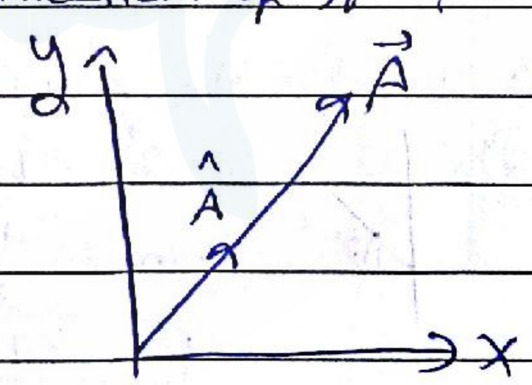
$\theta = 37$

nearest x-axis ع

$\boxed{3}$ Unit Vector in the direction of $\vec{A}$:-

$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$ ـ يكون بنفس اتجاه A

$|\hat{A}| = 1$



(*) دائماً أي Vector عليه Absolute Value بكون الجواب 1 unit وما بصير من غير علامات

المطلوب إني أجد إنه بتساوي 1 unit إلا إذا حكالك the magnitude بذكرك أو بس.

بس غير هيك لازم يكون في Absolute Value فالـ(^) تدل دائماً أنه طول

$\boxed{4}$. Absolute Value ولكن مع الـ 1 unit = الحجم :

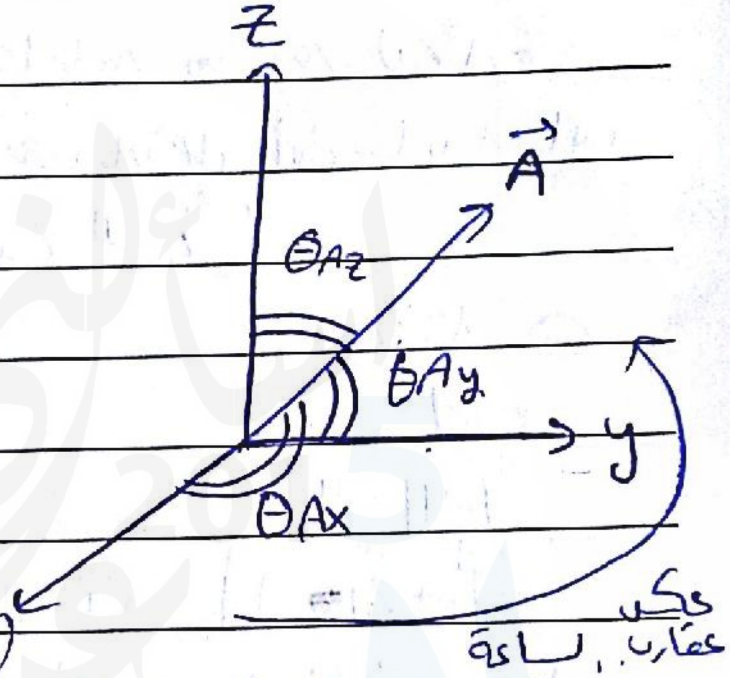
No. ____________

$\boxed{4}$ Angle of vector $\vec{A}$ and axes :-

محاور

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

* $\cos\theta_{Ax} \overset{\text{مركبة}}{=} \dfrac{A_x}{|\vec{A}|}$ لازم على ال | | عشان ما يصير vector / scaler



* $\cos\theta_{Ay} = \dfrac{A_y}{|\vec{A}|}$

* $\cos\theta_{Az} = \dfrac{A_z}{|\vec{A}|}$

(X) بنحط اتجاه ال x هون بس لازم ال y تكون بتجاه عكس عقارب الساعة أنه ما يصير اعتراض او مساقط لأ كلهم بتجبرها وراء بعض.

$\boxed{5}$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$
$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$
$$\vec{A} - \vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j} + (A_z - B_z)\hat{k}$$

(فكرة لما يعطيني متجهين لازم اطلع جمعهم أو طرحهم حسب المطلوب ونجمع ال $\hat{i}$ مع بعض وال $\hat{j}$ مع بعض وال $\hat{k}$ مع بعض)

[Note: Magnitude $\boxed{5}$ means absolute value وكأنه يصير لازم جذر]

$\boxed{6}$ $\vec{A} = \vec{B}$

$\boxed{A_x = B_x}$ & $\boxed{A_y = B_y}$ & $\boxed{A_z = B_z}$ 3 componants are equal

* $\vec{A} = -3\hat{i} + 5\hat{j}$

what's the A_x ?? -3, $A_y = 5$

(لما كانو مع بوجود مع الإشارة

ومنعرف بالأصل أنه هاي السالب

كانت لل $\hat{i}$)

Notes:-

$|3| = 3$

$|-4| = 4$

$|\hat{i} + 2\hat{j} - 3\hat{k}| = \sqrt{1^2 + 2^2 + (-3)^2}$

لازم أعمل جذر لأنه فيه Vectors.

Example:-

$\vec{A} = 4\hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{B} = 5\hat{i} + \hat{j} + 0\hat{k}$ مش عنه

Find:- ① ~~Direction of $\vec{B}$~~ Magnitude of $\vec{A}$ & $\vec{B}$

② Direction of $\vec{B}$

③ $\vec{A} + \vec{B}$

④ $|\vec{A} + \vec{B}|$

⑤ $|2\vec{B} - 3\vec{A}|$

⑥ $\hat{B}$

⑦ $|\hat{A}|$

⑧ angle between $\vec{A}$ and y-axis

⑨ " " $\vec{B}$ and z-axis

No. ______

Answers :-

① $|\vec{A}| = \sqrt{4^2 + 2^2 + 3^2} = \sqrt{29}$

$|\vec{B}| = \sqrt{5^2 + 1^2} = \sqrt{26}$

$|\hat{B}| = 1$ أشي ؟

$\hat{B} = \frac{\vec{B}}{|\vec{B}|} = \frac{5\hat{i} + \hat{j}}{\sqrt{26}}$

$\hat{B} = \frac{5}{\sqrt{26}}\hat{i} + \frac{1}{\sqrt{26}}\hat{j}$

$|\hat{B}| = \sqrt{\left(\frac{5}{\sqrt{26}}\right)^2 + \left(\frac{1}{\sqrt{26}}\right)^2}$

$|\hat{B}| = \sqrt{\frac{25}{26} + \frac{1}{26}} = \sqrt{\frac{26}{26}} = \sqrt{1} = 1$ #

② $\tan\theta = \frac{A_y}{A_x} = \frac{1}{5}$

$\theta = \tan^{-1}\left(\frac{1}{5}\right)$

وبطلع زاويه فش لأنه الاثنين موجبين

اذاً هيه بالربع الأول جهة شرق.

③ $\vec{A} + \vec{B} = 9\hat{i} - \hat{j} + 3\hat{k}$

④ $|\vec{A} + \vec{B}| = \sqrt{9^2 + (-1)^2 + 3^2} = \sqrt{91}$

بحسبه هاي أول

بعدين باخذ الجذر

ما بصير أوزع لانه الجذر

لا يوزع على الجمع

⑤ $2\vec{B} = 10\hat{i} + 2\hat{j} + 0\hat{k}$

$-3\vec{A} = 12\hat{i} - 6\hat{j} + 9\hat{k}$

$2\vec{B} - 3\vec{A} = -2\hat{i} + 8\hat{j} - 9\hat{k}$

$|2\vec{B} - 3\vec{A}| = \sqrt{(2)^2 + (8)^2 + (-9)^2} = \sqrt{\ldots}$

$$|\vec{A}+\vec{B}| = |\vec{A}| + |\vec{B}|$$

~~أيمتى لما يكونوا باتجاه واحد~~ !!

No. ______

(6) $\hat{B} = \dfrac{\vec{B}}{|\vec{B}|} = \dfrac{5\hat{i}+\hat{j}}{\sqrt{26}} = \dfrac{5}{\sqrt{26}}\hat{i} + \dfrac{1}{\sqrt{26}}\hat{j}$

(7) $|\hat{A}| = 1$ (دعوى. كلي ، والإثبات قبل الصفحة)

(8) θ_{Ay}? $\cos\theta_{Ay} = \dfrac{A_y}{|\vec{A}|} = \dfrac{-2}{\sqrt{29}}$

لهون رح نستخدم إشارة الـ ^ بـ

vector الي قبله إشارة $\theta_{Ay} = \cos^{-1}\left(\dfrac{-2}{\sqrt{29}}\right)$

(9) $\cos\theta_{Bz} = \dfrac{B_z}{|\vec{B}|} = \dfrac{0}{\sqrt{26}} = 0$

$$\theta_{Bz} = \tan^{-1}(0) = 90^\circ$$

(*) Example:-

if $\vec{A} = 3a\hat{i} - 2b\hat{j} + 9\hat{k}$

$\vec{B} = 12\hat{i} + 5\hat{j} - 3c^3\hat{k}$

find a, b, c if :-

① $\vec{A} = \vec{B}$

$3a = 12$	$-2b = 5$	$9 = -3c^3$
$a = 4$	$b = -2.5$	$c = \sqrt[3]{-3}$

② $2\vec{A} = -3\vec{B}$

$2\vec{A} = 6a\hat{i} - 4b\hat{j} + 18\hat{k}$

$-3\vec{B} = -36\hat{i} - 15\hat{j} + 9c^3\hat{k}$

$6a = -36$	$-4b = -15$	$18 = 9c^3$
$a = -6$	$b = 15/4$	$c = \sqrt[3]{2}$

(*) Example:-

if $\vec{A} = 2\hat{k} - 3\hat{i} + 4\hat{j}$

$\vec{B} = 2\hat{i} + \hat{j}$

find:- (1) $\hat{B}$

(2) $|2\vec{B} - \vec{A}|$

(3) angle between $\vec{A}$ and x-axis ?

Ansners:-

(1) $\hat{B} = \dfrac{\vec{B}}{|\vec{B}|} \rightarrow |\vec{B}| = \sqrt{4+1} = \sqrt{5}$

$\hat{B} = \dfrac{2\hat{i} + \hat{j}}{\sqrt{5}}$

(2) $2\vec{B} = 4\hat{i} + 2\hat{j} + 0\hat{k}$ أول شي نطلع $2\vec{B} - \vec{A}$

$- \quad \vec{A} = -3\hat{i} + 4\hat{j} + 2\hat{k}$ ثم كل مركبة $1 = \hat{i}, \hat{j}, \hat{k}$

$2\vec{B} - \vec{A} = 7\hat{i} + -2\hat{j} - 2\hat{k}$ ونربع كل منهم .

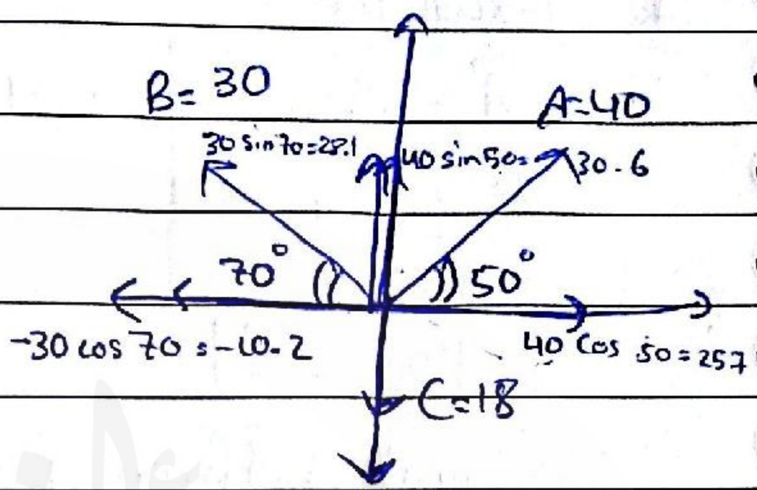
$|2\vec{B} - \vec{A}| = \sqrt{7^2 + (-2)^2 + (-2)^2} = \sqrt{57}$

(3) $\cos\theta_{Ax} = \dfrac{A_x}{|\vec{A}|} = \dfrac{-3}{\sqrt{29}}$

$\hookrightarrow |\vec{A}| = \sqrt{4+9+6} = \sqrt{29}$

Example :-

find the resultant of these vectors :-



$\vec{A} = 25.7\hat{i} + 30.6\hat{j}$

$\vec{B} = -10.2\hat{i} + 28.1\hat{j}$

$\vec{C} = 0\hat{i} - 18\hat{j}$

Resultant vector: $\vec{R} = 15.5\hat{i} + 40.7\hat{j}$

To find the Magnitude :- (Absolute value)

$$|\vec{R}| = \sqrt{(15.5)^2 + (40.7)^2}$$

$$\tan\theta = \frac{40.7 \cos\theta}{15.5 \sin\theta}$$

No. ______

* Vectors' product :-

1 Dot (scaler) product :-

بالترتيب غير مهم ← $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta_{AB}$ (مستخدم هاي إذا أعطاني ال Magnitude والزاوية)

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ (اذا أعطاني أرقام بس بضرب وطلعها)

(ما في بالضرب $\hat{i} / \hat{j} / \hat{k}$ لأنه رح يطلع الجواب Scaler وليس Vector quantity)

* $\hat{i} \cdot \hat{j} = 1 \cdot 1 \cos 90° = 0$

Magnitude

والزاوية 90

لأنه x/y عمل 90 له

* $\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = 0$ لأن $\theta = 90°$

$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$ لأن $\theta =$ صفر°

2 Cross (vector) Product :-

بالترتيب مهم ← $|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \theta_{AB}$

$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

(اللازم نحط ال | | عشان ما بصير Vector ≠ Scaler / رح يطلع الجواب Vector)

$\longrightarrow$

$$\begin{array}{c} \\ \vec{A}\times\vec{B} = \end{array}\begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \end{array}$$

→ (determinance محددة)
(وليست مصفوفة)

الـ B أول بتكتب الـ B والـ A فوق منها (تحت)

$$= \hat{i}\,(\underbrace{A_yB_z - A_zB_y}_{\text{داغا سالب}}) - \hat{j}\,(A_xB_z - A_zB_x) + \hat{k}\,(A_xB_y - A_yB_x)$$

لما بشال أعرف شو هو الـ $\hat{i}$
نشطب الـ $\hat{i}$ عمودي وافقي و
نضرب ضرب تبادلي من شمال لليمين

- ✱ $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$
- ✱ $\hat{i}\times\hat{j} = \hat{k}$

لازم نطلع بجواب Vector والي فتترا (1) بس
بكسير Vector مش ~~Vector~~ Scaler.

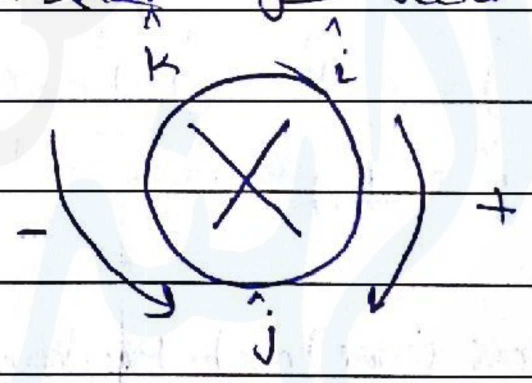
$\hat{i}\times\hat{k} = -\hat{j}$

$\hat{j}\times\hat{k} = \hat{i}$

$\hat{j}\times\hat{i} = -\hat{k}$

$\hat{k}\times\hat{i} = \hat{j}$

$\hat{k}\times\hat{j} = -\hat{i}$



- perpendecular → متعامد
- parallel → موازي

No. ____________

(*) Example :- if $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$

$\vec{B} = 2\hat{i} + 3\hat{j} - 5\hat{k}$

Find :-

1. $|\vec{A}|, |\vec{B}|$
2. $\vec{A} \cdot \vec{B}$
3. angle between $\vec{A}$ & $\vec{B}$
   dot product فوراً
4. $|\vec{A} \cdot \vec{B}|$
5. $2\vec{B} \cdot -3\vec{A}$
6. $\vec{A} \times \vec{B}$
7. $|\vec{A} \times \vec{B}|$
8. $2\vec{B} \times -3\vec{A}$
9. $|2\vec{B} * 3\vec{A}|$

- Answers :-

1. $|\vec{A}| = \sqrt{9+4+1} = \sqrt{14}$

   $|\vec{B}| = \sqrt{4+9+25} = \sqrt{38}$

2. $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$

لما نستخدم ال cos لأنه ما في زاوية

$= (3*2) + (-2*3) + (1*-5) = -5$ (الضرب للعاملات بمحض)

(ما حط $\hat{i}$ / $\hat{j}$ / $\hat{k}$ لأنه scaler)

No. ______________

$\boxed{3}$ $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos \theta_{AB}$

$-5 = \sqrt{14} * \sqrt{38} \cos \theta_{AB}$

$\cos \theta_{AB} = \dfrac{-5}{\sqrt{14}\sqrt{38}} \longrightarrow \theta = 102.5^{\circ}$

$\boxed{4}$ $|\vec{A} \cdot \vec{B}| = |-5| = 5$

(5) $2\vec{B} \cdot -3\vec{A} = -6(\vec{B} \cdot \vec{A})$

$= -6(-5) = 30$

لأنه في ضرب
نقدر أطلع معاملات
وأضربهم سوا
لعدم وضوح الأ.
الضرب لو كان بدل
3A كان.

(6) $\vec{A} \times \vec{B} = \begin{array}{ccc} + & - & + \end{array}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 2 & 3 & -5 \end{vmatrix}$$

$= \hat{i}(10-3) - \hat{j}(-15-2) + \hat{k}(9-(-4))$

$\vec{A} \times \vec{B} = 7\hat{i} + 17\hat{j} + 13\hat{k}$

$\boxed{7}$ $|\vec{A} \times \vec{B}| = \sqrt{7^2 + 17^2 + 13^2}$

No. ______________

8) $2\vec{B} \times -3\vec{A} = -6(\vec{B} \times \vec{A})$

$= -6(-\vec{A} \times \vec{B})$

$= 6(\vec{A} \times \vec{B})$

$= 6(7\hat{i} + 17\hat{j} + 13\hat{k})$

$= 42\hat{i} + 102\hat{j} + 78\hat{k}$

9) $|2\vec{B} \times 3\vec{A}| = \sqrt{(42)^2 + (102)^2 + (78)^2}$

* Homework :-

1) if $\vec{A} = 2a\hat{i} + 3\hat{j} - 2\hat{k}$

$\vec{B} = \hat{i} + 2\hat{j} + \hat{k}$

Find $\underline{\underline{a}}$ where $\vec{A}$ and $\vec{B}$ are perpendicular ?

$\theta_{AB} = 90°$

Answer :-

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos \theta_{AB}$

$\vec{A} \cdot \vec{B} = 0$

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

$= (2a * 1) + (3 * 2) + (-2 *)$

$= 2a + 6 - 2$

$= 2a + 4$

$0 = 2a + 4$

$\boxed{a = -2}$

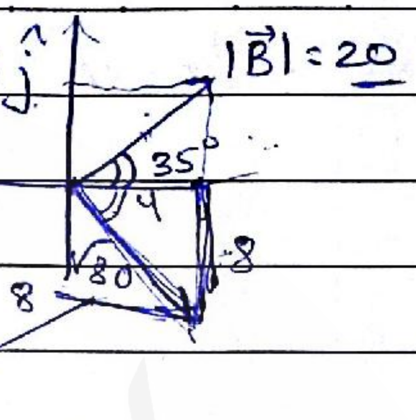
$\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta_{AB}$

$\vec{A} = 8.94\,\text{No}\,/\underline{-63.43}.$

$\boxed{2}$ if $\vec{A} = 4\hat{i} - 8\hat{j}$ $\quad \vec{B} = 16.4\hat{i} + 11.5\hat{j}$

find: ① $\vec{A}\cdot\vec{B}$

② $\vec{A}\times\vec{B}$



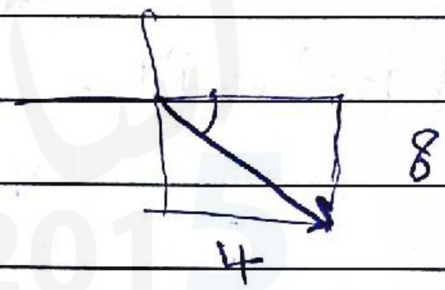
Answer:-

$\tan\theta = \frac{8}{4}$

$\tan\theta = 2$

$\theta = \tan^{-1}(2)$

$= 63.4$



ل (- or +) ؟ ولا أصغر صفر
وهي سالبة ولا موجبة ؟؟

$= \sqrt{16+64}$

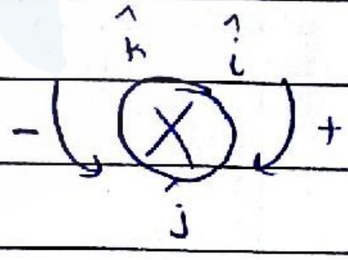
$= \sqrt{80} = \underline{8.94}$

$\theta = \cos^{-1}\left(\frac{8}{\sqrt{80}}\right)$

$\boxed{3}$ $\vec{A} = 8\hat{i}$

$\vec{B} = 3\hat{i} - 2\hat{j}$

Find: ① $\vec{B}\times\vec{A}$ ② $\vec{A}\times\vec{B}$



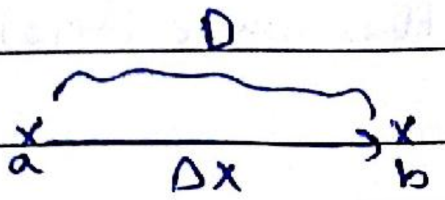
Answer:-

① $\vec{B}\times\vec{A} =$

$(3\hat{i} - 2\hat{j}) * 8\hat{i} = -16 * -\hat{k} = 16\hat{k}$

② $\vec{A}\times\vec{B} = 8\hat{i}\,(3\hat{i} - 2\hat{j}) = -16 * \hat{k} = -16\hat{k}$

Chapter two ; Motion in one dimension :-



مسافة | Distance (D) → scaler quantity (+ve)

إزاحة | Displacement (ΔX) → the difference between the initial

$(X_f - X_i)$ and final place (لا يعتمد على المسار)

(final) (initial)

* Velocity ($\vec{V}$) : is a vector quantity.

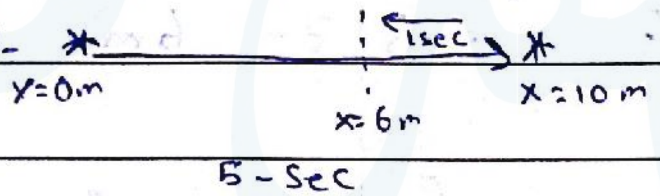
$$\vec{V} = \frac{\Delta X \text{ (Displacement)}}{\Delta t \text{ (time)}}$$

* Speed (S) : is a scaler quantity.

$$S = \frac{\text{Distance}}{\Delta t}$$

$\Delta t \to t_f - t_i$ (هنا فقط لا يعني أي اتجاه بحركة زمنية)

(*) Example :-



1) $X = 0 \to 10$

$D = 10$ meters.

$\Delta X = X_2 - X_1 = 10 - 0 = 10\,m$

(مش لازم أبدا أن تساوي الـ D)

بلا اتجاه $S = \frac{D}{t} = \frac{10}{5} = 2\,m/s$

$\vec{V} = \frac{\Delta X}{t} = \frac{10}{5} = 2\,m/s$

2) $X = 10 \to 6$

$D = 10 - 6 = 4$

$\Delta x = X_2 - X_1 = 6 - 10 = -4\,m$

معناها أنه -

$S = \frac{4}{1} = 4\,m/s$

$\vec{V} = \frac{-4}{1} = -4\,m/s$

No. ______

3) $x = 0 \rightarrow 6$ (the whole trip)

$D = 10 + 4 = 14$ m

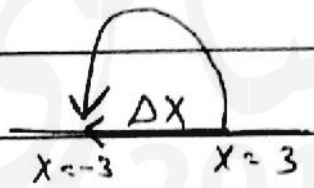
لا يهم المسار
الأول والأخير
هو الذي
(يعتمد)

$\Delta x = 6 - 0 = 6$ m

$S = \frac{14}{6}$ m/s

$\vec{V} = \frac{6}{6} = 1$ m/s

* Example:-



Find: ① distance ② displacement

Answer:-

1) Distance $= \frac{2\pi r}{2} = \pi r = 3\pi$ (مشي محيط ½ دائرة و r = 3)

2) $\Delta x = x_2 - x_1 = -3 - 3 = -6$ m

*) position as a function of time:-

x: position

$\Delta x = x_2 - x_1$

$\Delta t = t_2 - t_1$

$\Delta v = v_2 - v_1$

No. ____________

Origin : $x = 0$
$y = 0$

initial : $t = 0$ وليس $X = 0$ ($y = 0$) لأنه ممكن أنا بلشت حركة مش
من الـ 0 بس الـ time بكون 0).

rest : $V = 0$

Constant Velocity $\rightarrow a = 0$
acceleration (التسارع)

where: find $\underline{x}$ / When: find $\underline{t}$

(السرعة المتجهة)

* Velocity ($\vec{V}$) $\rightarrow$ Average Velocity $=$ $V_{av} = \dfrac{\Delta x}{\Delta t}$
($\vec{V}_{av}$)

$\rightarrow$ instantainuous velocity $= V_{ins} = \boxed{\dfrac{\partial x}{\partial t}}$ ($\vec{V}_{ins}$)

Partial d لأنه في أكثر من متغير زي
d لما يكون متغير واحد
لـ بشتق x بالنسبة لـ t مش بقسمهم عادي

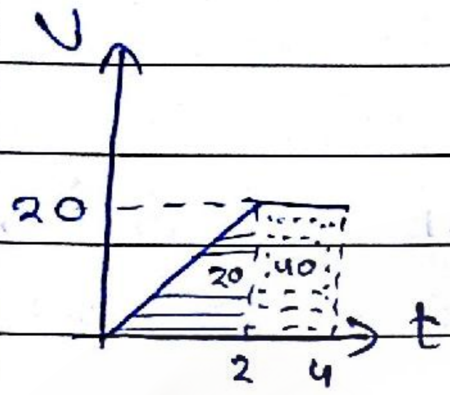
⊗ $S = |\vec{V}|$ ← هذا إذا كانت حركة واحدة لكن لو كانت في اتجاهين أجمع
مسافات ثم أقسم على الزمن (t) .

* Acceleration ($\vec{a}$) $\rightarrow$ Average acceleration ($\vec{a}_{av}$) $= \dfrac{\Delta V}{\Delta t}$

$\rightarrow$ instantainuous acceleration ($\vec{a}_{ins}$) $= \dfrac{\partial V}{\partial t}$
لحظي

$$\begin{tikzcd}
X \arrow[r, "\substack{\text{(slope)} \\ \text{(derivative)}}", "\text{مشتقة}"'] & V \arrow[l, bend left, "\substack{\text{(area) تكامل} \\ \text{integration}}"] \arrow[r, "\substack{\text{الميل} \\ \text{(slope) derivative}}"] & a \arrow[l, bend left, "\substack{\text{(area) تكامل} \\ \text{integration}}"]
\end{tikzcd}$$
ΔX $\qquad$ ΔV

No. ____________



لحظة لحظة بجد $V_{ins}/a_{ins}/x_{ins}$

ع سيرا بالكل .

(8) Example:-

if the position of an object is given by the expression :-

$$x = t^2 - t - 6 \quad , \text{ Find}$$

(1) displacement from $t=0$ to $t=2$ sec.

بيتوي الـ time إذا فترة Av وإذا نقطة ins (إذا لحظي)

(2) Average Velocity between $t=0$ and $t=2$sec.

(3) ins. Velocity at $t=3$ sec.

(4) av. acceleration from $t=0$ to $t=5$ sec.

(5) ins. acceleration at $t=2.25$ sec.

(6) initial position, velocity, speed, acceleration.

؟ (7) when is the object at rest?

(will the object reverse it's direction)? صفحة أخرى

(8) when is the object at origin?

(9) when is the velocity (+1 m/s)

(10) $\left(\frac{max}{min}\right)$ position?

No. ______

Answers :- $\boxed{1}$ $\Delta X = X_2 - X_1$

اول
شي
بطلعهم

$$\boxed{\begin{array}{l} X_{ins} = t^2 - t - 6 \\ V_{ins} = 2t - 1 \\ a_{ins} = 2 \end{array}}$$

$X_{1\,(t=0)} = (0)^2 - 0 - 6 = -6$

$X_{2\,(t=2)} = (2)^2 - 2 - 6 = -4$

$\Delta X = X_2 - X_1 = -4 - (-6) = 2$ meters. $(2\hat{i})$

$\boxed{2}$ $V_{av} = \dfrac{\Delta X}{\Delta t} = \dfrac{X_2 - X_1}{t_2 - t_1} = \dfrac{2}{2-0} = 1\ m/s\ (1\hat{i})$

$\boxed{3}$ $V_{ins} = 2(3) - 1 = 5\ m/s$

نعوض بالمشتقة لأنه كل وحدة معها تأثير -

$\boxed{4}$ $a_{av} = \dfrac{\Delta V}{\Delta t} = \dfrac{V_2 - V_1}{t_2 - t_1} = \dfrac{9 - (-1)}{5} = 2\ m/s^2$

$\boxed{5}$ $a_{ins} = 2\ m/s^2$

$\boxed{6}$ initial $\rightarrow t = 0$

$X_{ini} = -6m$ / $V_{ini} = -1 m/s$ / $S_{ini} = |-1| = 1\ m/s$

$a_{ini} = 2\ m/s^2$

$\boxed{7}$ $V = 2t - 1$ (rest $\rightarrow V = 0$)

$0 = 2t - 1 \rightarrow t = 1/2\ s$

$\boxed{8}$ $X = t^2 - t - 6$ (origin $\rightarrow X = 0$)

$0 = t^2 - t - 6$

$\boxed{t = 3}$ $\cancel{t = -2}$ تُهمل

$\boxed{9}$ $V = 2t - 1$

$1 = 2t - 1$

$t = 1$ sec $\rightarrow$

No. ____________

$\boxed{10}$ $x = t^2 - t - 6 \rightarrow x = (0.5)^2 - (0.5) - 6 = -6.25\,m$

نشتق $v = 2t - 1$ نرجع نعوضها — بطلع ال minimum

بساويها بالصفر $0 = 2t - 1$ — ما بطلع max و min

$t = 0.5\ sec$

حسب المعادلة مش الاتنين

مع بعض)

(11) Example:-

if $v = 8t^2 - 2t + 1$

Find:-

$\boxed{1}$ Average Velocity from $t=0$ to $t=1$?

$\boxed{2}$ Average Acceleration from $t=0$ to $t=1$?

$\boxed{3}$ <u>when</u> (find time) does the object have zero acceleration ?

$\boxed{4}$ instantainuous acceleration at $t=3$?

Answer:-

$$\boxed{\begin{aligned} x &= (8/3)t^3 - t^2 + t + C \\ v &= 8t^2 - 2t + 1 \\ a &= 16t - 2 \end{aligned}}$$

(1) $V_{av} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{\frac{8}{3} + \cancel{C} - \cancel{C}}{1 - 0} = \dfrac{8}{3}\ m/s$.

$x_{(t=0)} = C$

$x_{(t=1)} = \dfrac{8}{3} - \cancel{1} + \cancel{1} + C$

$= \dfrac{8}{3} + C$

(2) $a_{av} = \frac{\Delta V}{\Delta t} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{7-1}{1-0} = \frac{6}{1} = 6\ m/s^2$

$V_{(t=0)} = 1$

$V_{(t=1)} = 7$

(3) $0 = 16t - 2$

$16t = 2 \rightarrow t = \frac{1}{8}$ seconds.

(4) $a = 46\ m/s^2 \rightarrow$ (acceleration is a function of time)

*) $\vec{r} = x\hat{i} + y\hat{j}$ — قيمة, واتجاه ل $\vec{r}$

(*) Example:- if the position is given by:-

$$\vec{r} = (t^2 - 2t + 4)\hat{i} + (3t-1)\hat{j}$$

(لما يكون 2D (2 dimension) يصير $\vec{r}$ له قيمة, ل resultant ل (x, y).

Find :-

1. displacement from $t=0$ to $t=2$ sec
2. Magnitude of displacement from $t=0$ to $t=2$
3. Average velocity from $t=0$ to $t=2$ sec
4. Magnitude of av. velocity from $t=0$ to $t=2$ sec.
5. ins. velocity at $t=3$ sec.
6. ins. speed at $t=3$ sec
7. av. acc from $t=2$ to $t=4$ sec.
8. ins. acc at $t=6$ sec.

Answer:-

$\vec{r} = (t^2 - 2t + 4)\hat{i} + (3t - 1)\hat{j}$

$\vec{v}_{ins} = (2t - 2)\hat{i} + 3\hat{j}$

$\vec{a}_{ins} = 2\hat{i}$

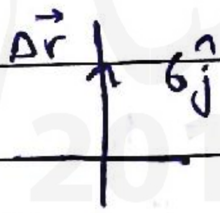
(1) $\Delta\vec{r} = \vec{r_2} - \vec{r_1}$

$\underset{(t=0)}{\vec{r}_1} = 4\hat{i} - \hat{j}$

$\underset{(t=2)}{\vec{r}_2} = (4 - 4 + 4)\hat{i} + (3 * 2 - 1)\hat{j}$

$= 4\hat{i} + 5\hat{j}$

$\Delta\vec{r} = r_2 - r_1 = 6\hat{j}$



(2) $|\Delta\vec{r}| = |6\hat{j}| = 6\,m$ $\sqrt{6^2}$

(3) $\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{6\hat{j}}{2} = 3\hat{j}$

(4) $|\vec{v}_{av}| = 3\,m/s$

(5) $\underset{(t=3)}{\vec{v}_{ins}} = (2 * 3 - 2)\hat{i} + 3\hat{j}$

$= 4\hat{i} + 3\hat{j}$ → this's a constant vector.

(6) $S_{ins} = |\vec{v}_{ins}| = \sqrt{4^2 + 3^2} = \sqrt{25} = 5\,m/s$

(7) $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{4\hat{i}}{4 - 2} = 2\hat{i}\ m/s^2$

$\underset{(t=2)}{\vec{v}_1} = 2\hat{i} + 3\hat{j}$ / $\underset{(t=4)}{v_2} = 6\hat{i} + 3\hat{j}$

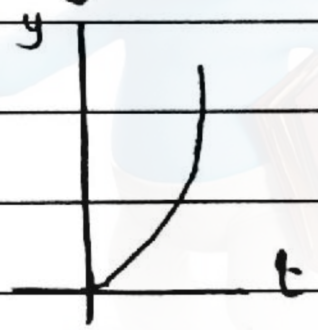
(8) $\vec{a}_{ins} = 2\hat{i}$ ((it's constant) إذاً ما بتعتمد على time أبداً)

(كل أنواع الـ acc constant إذاً ($a_{ins} = a_{av}$))

⊛ Graph - Based problems :-

$y \sim t^3$

$y \sim t^2$



معامل (t) إذا المشتقة

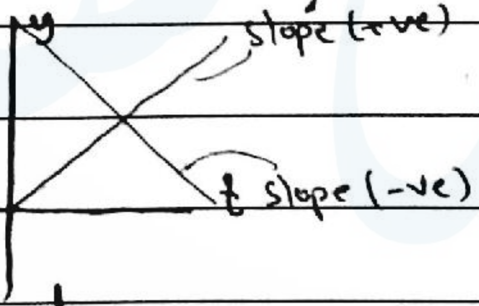
$y \sim t$

slope

$y = 2t - 1$

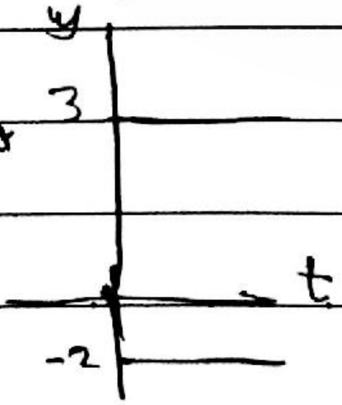
$y = -2t - 1$

slope

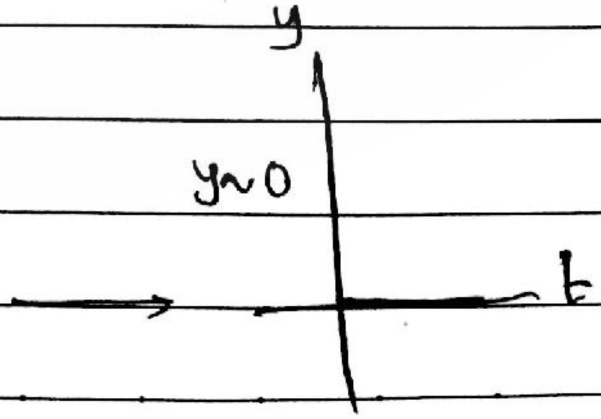


$y \sim C$

Constant



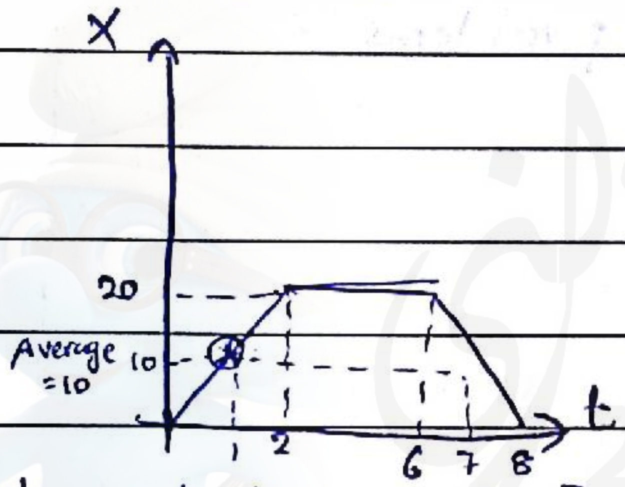
$y \sim 0$



No. ____________

(*) Example :-

if the relation between x and t is given in the following figure ; find :-



(1) position at t=0, 2, 6, 7, 8

(2) Displacement from (a) t=0 to t=2

(b) t=2 to t=6

(c) t=6 to t=8

(d) t=0 to t=8

(e) t=0 to t=6

(3) Average velocity from t=0, t=6

(4) Velocity at (a) t=1

(b) t=5.563

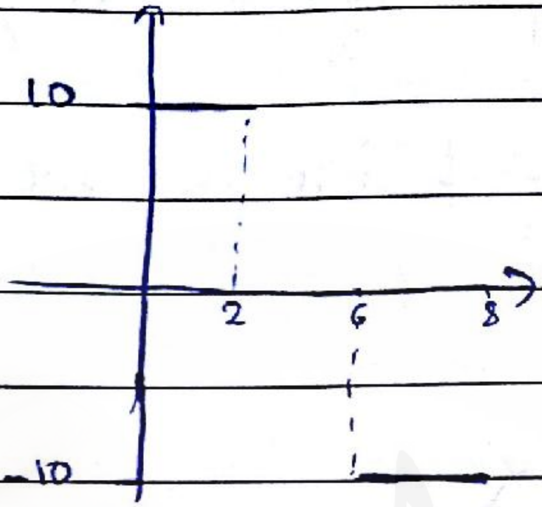
(c) t=7.5

(5) acceleration at t=3

??

Answer:-

$\frac{20+0}{2}$



① $X_{(t=0)} = 0m$ | $X_{(t=2)} = 20m$ | $X_{t=6} = 20m$ | $X_{t=7} = 10m$ | $X_{(t=8)} = 0m$

② $\Delta X = X_2 - X_1$

ⓐ $20 - 0 = 20m$

ⓑ $20 - 20 = 0m$

ⓒ $0 - 20 = (-20m$
(سالب) $-\hat{i}$

ⓓ $0 - 0 = 0m$

ⓔ $20 - 0 = 20m$

③ $\vec{V}_{av} = \frac{\Delta X}{\Delta t} = \frac{20}{6-0} = 20/6 \ m/s \ (+\hat{i})$

④ $V_{ins\,(t=1)} = V_{av}$ if X behaviour is linear.

ⓐ $V_{av\,(0\to2)} = V_{ins\,(t=1)} = \frac{20}{2} = 10 \ m/s.$

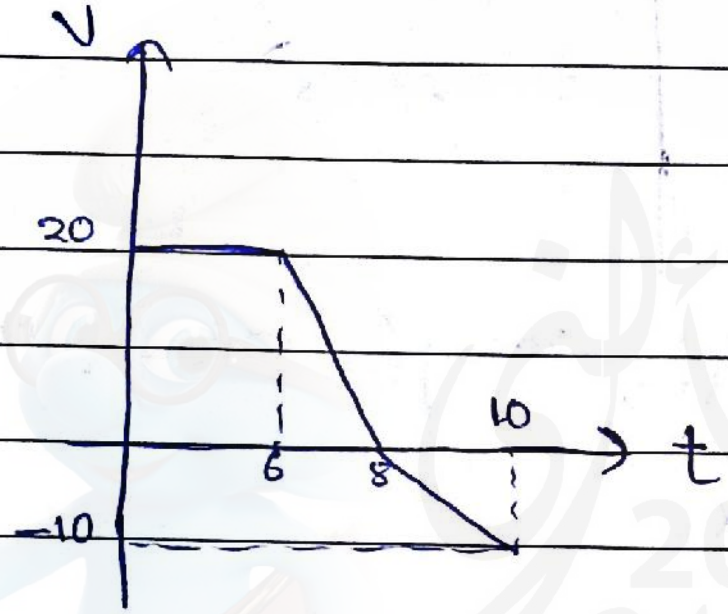
ⓑ $V_{av\,(2\to6)} = V_{ins\,(t=5.5(3)} = \frac{20-20}{4} = 0$

ⓔ $V_{ins\,(t=7.5)} = V_{av\,(6\to8)} = \frac{-20}{2} = -10 \ m/s.$ →

No. ______________

$\boxed{5}$ $\vec{a} = \frac{\Delta X}{\Delta t} = \frac{X_2 - X_1}{3} = \frac{20-20}{3} = 0$

(A) Exercise:- if $\vec{v}$ is a function of t;



find : ① av. acceleration from:-

a) $t=0 \to t=6$

b) $t=6 \to t=8$

c) $t=8 \to t=10$

d) $t=0 \to t=10$

② dis placement from:-

Area

?

a) $t=0 \to t=6$

b) $t=6 \to t=8$

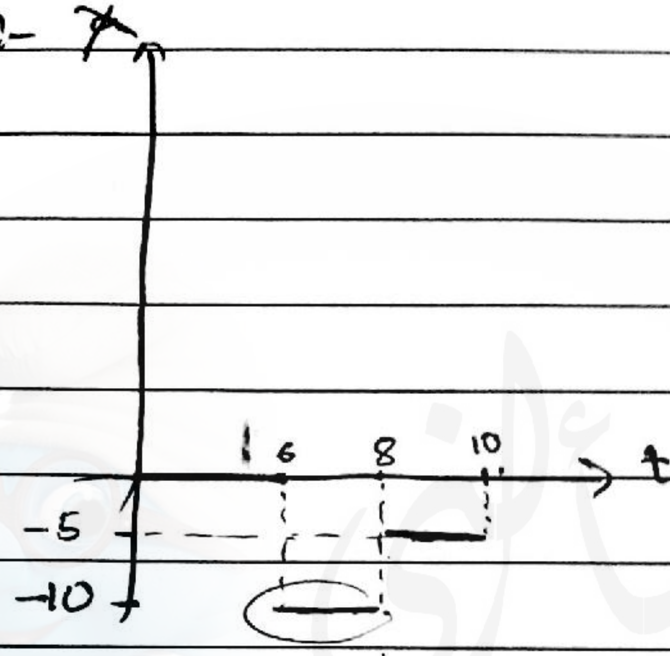
c) $t=8 \to t=10$

d) $t=6 \to t=10$

e) $t=0 \to t=10$

No. ______

Answer:-



Slope = acc محسوبة من خلال الميل

$= \frac{\Delta y}{\Delta x} = \frac{20}{2} = -10$ ← لأنه هابط

$\boxed{1}$ a) $\vec{a}_{av} = \frac{\Delta v}{\Delta t} = \frac{20-20}{6-0} = 0 \, m/s^2$

b) $\vec{a}_{av} = \frac{\Delta v}{\Delta t} = \frac{0-20}{2} = \frac{-20}{2} = -10 \, m/s^2$

c) $\vec{a}_{av} = \frac{-10-0}{2} = \frac{-10}{2} = -5 \, m/s^2$

d) $\vec{a}_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{\Delta t} = \frac{-10-20}{10} = \frac{-30}{10} = -3 m/s^2$

$\boxed{2}$ a) $\vec{v}_{av} = \frac{\Delta x}{\Delta t}$

$0 = \frac{\Delta x}{6} \rightarrow \Delta x = 0 \quad m$

No. ______________

(b) $\vec{V}_{av} = \dfrac{\Delta X}{\Delta t}$

$-10 = \dfrac{\Delta X}{2} \rightarrow \Delta X = -20 \text{ m}$

(c) $\vec{V}_{av} = \dfrac{\Delta X}{\Delta t}$

$-5 = \dfrac{\Delta X}{2} \rightarrow \Delta X = -10 \text{ m}$

(d) $\vec{V}_{av} = \dfrac{\Delta X}{\Delta t}$

$\dfrac{-30}{4} = \dfrac{\Delta X}{4} \rightarrow \Delta X = \dfrac{-120}{4} \text{ m} = -30 \text{ m}$

(e) $\vec{V}_{av} = \dfrac{\Delta X}{\Delta t}$

$-30 = \dfrac{\Delta X}{10} \rightarrow \Delta X = -300 \text{ m}$

No. ________

(A) Motion with constant acceleration:- (uniform)

*) if there's an object moving with constant velocity, there is no acceleration.

$$\boxed{a = 0}$$

المسافة المقطوعة (الازاحة)

$\Delta X = V * \Delta t$

displacement

*) if the velocity is changing in magnitude with a constant rate:

معدل ثابت (بانتظام)

$\boxed{\ } \ \Delta X \ \_\_ \ t \ \_ \ \boxed{\ }$

$V_1 \qquad a \qquad V_2$

* $V_2 = V_1 + at$ -- ΔX (لا يحتوي ΔX اذا سؤال ما فيها بتحتاج هذا القانون)
* $V_2^2 = V_1^2 + 2a\Delta X$ -- t (المعادلة الي ما فيها t بتأكد من كلها ترسيخ)
* $\Delta X = V_1 t + \frac{1}{2} a t^2$ --- V_2 (اذا كان المطلوب V_2 أو final speed بتحتاجي)

(كل معادلة فيها t هي نفسها Δt)

* $\Delta X = \underbrace{\left(\frac{V_1 + V_2}{2}\right)}_{\vec{V}_{av.}} t = \bar{V} t$

$\bar{V} = \frac{\Delta X}{\Delta t}$

هذا القانون فقط في حالة

$\boxed{\bar{V} = \frac{\Delta X}{\Delta t}}$ الحركة بسرعة ثابتة غير هيك

No. ____________

(*) Example:- Airplane is landing in a runway with speed of 100 m/s and coming to rest ($V_2 = 0$) (توقف) with $-5\ m/s^2$ (rate) (acceleration); Find:

① time needed to stop.

② Can this plane land on 0.8 km runway? (800 m)

Answer:-

① $V_1 = 100\ m/s$

$V_2 = 0\ m/s$

$a = -5\ m/s^2$

time ?? $\quad V_2 = V_1 + at$

$0 = 100 + (-5)t$

$t = 20$ seconds.

② $V_2^2 = V_1^2 + 2a\Delta X$

$0 = (100)^2 + 2(-5)\Delta X$

$10000 = 10\,\Delta X \longrightarrow \Delta X = 1000$ meters.

No, it can't

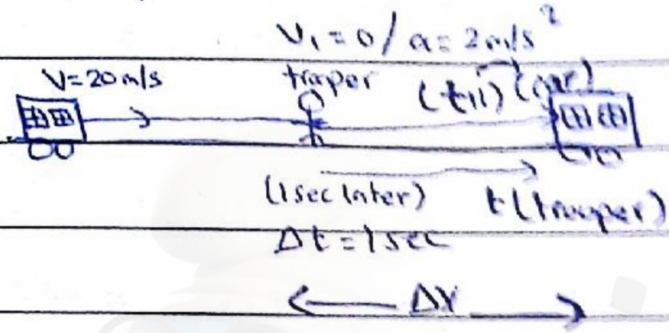
$\Delta X = V_1 t + \frac{1}{2}at^2$ (21)

$= 100 * 20 + \frac{1}{2} \times -5 * (20)^2$

$= 1000$ meters.

No. ________

Example :-



How long $\rightarrow t$

Find: (How far) does the trooper need to move to catch the car? (ΔX)

Car $\Delta X = Vt_{(car)}$

$\Delta X = 20(t+1)$

trooper $\Delta X = \cancel{V_1}t + \frac{1}{2}at^2$

$\Delta X = \frac{1}{2} * 2t^2$

$\Delta X = t^2$

$\Delta X_{car} = \Delta X_{trooper}$

$20t + 20 = t^2$

$t^2 - 20t - 20 = 0$

$\boxed{t = 21 \text{ sec}}$, $-1 \cancel{sec}$

$\Delta X_{car} = 20(t+1)$

$= 20 \times 22 = 440 m$

$\Delta X_{trooper} = t^2 = (21)^2 = 441 m$

(لازم يطلعوا متساويين ، ما بصير يفرقوا بس لأنه round off عادي مثل ما احنا قربنا) لأن الـ $t \neq 21$ بالضبط)

No. ______________

Example:- if an object started from $\vec{V}_1 = 2\hat{i} - 3\hat{j}$ with acceleration $\vec{a} = -2\hat{j}$, 2 secs later; Find:-

1. Final velocity.
2. Final Speed. → مشتبه ال Velocity، مجرد فقط ال Absolute value
3. displacement.
4. if the initial position is $\vec{r}_1 = (2\hat{i} + 2\hat{j})$, what's the final position?

Answer:-

$\vec{V}_1 = 2\hat{i} - 3\hat{j}$

$\vec{a} = -2\hat{j}$

$\Delta t = 2$

(1) $V_2 = V_1 + at$

$V_2 = (2\hat{i} - 3\hat{j}) + -2\hat{j}(2)$

$$\boxed{V_2 = 2\hat{i} - 7\hat{j} \ m/s}$$

(2) $S = |\vec{V}_2| = \sqrt{4+49} = \boxed{\sqrt{53} \ m/s}$

(3) $\vec{\Delta r} = \vec{V}_1 t + \frac{1}{2} a t^2$

$= (2\hat{i} - 3\hat{j}) + \frac{1}{2}(-2\hat{j})(4)$

$$\boxed{\vec{\Delta r} = 4\hat{i} - 10\hat{j} \ m}$$

(4) $\vec{\Delta r} = \vec{r}_2 - \vec{r}_1$

$\vec{r}_2 = \vec{\Delta r} + \vec{r}_1$

$= (4\hat{i} - 10\hat{j}) + (2\hat{i} + 2\hat{j})$

$$\boxed{= 6\hat{i} - 8\hat{j} \ m}$$

No. ____________

+ free falling motion:-

displacement $x \longrightarrow y$

acceleration $a \longrightarrow g = (9.8 \simeq 10\ m/s^2)$

(تسارع الجاذبية الأرضية)

$*\ V_2 = V_1 + gt$

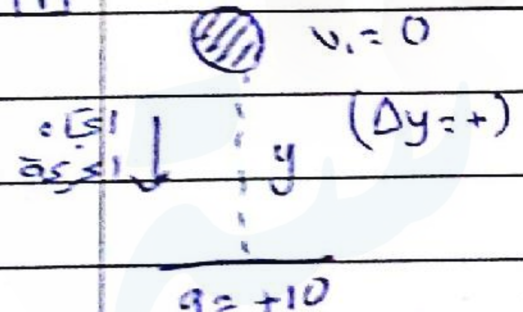
$*\ V_2^2 = V_1^2 + 2g\,\Delta y$

$*\ \Delta y = V_1 t + \frac{1}{2} g t^2$

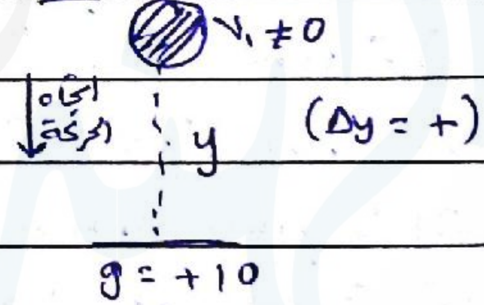
$\Delta y = \left(\frac{V_1 + V_2}{2}\right) t$

⊛ states of free falling:-

1 dropped/released (سقط)



2 fired أطلقت



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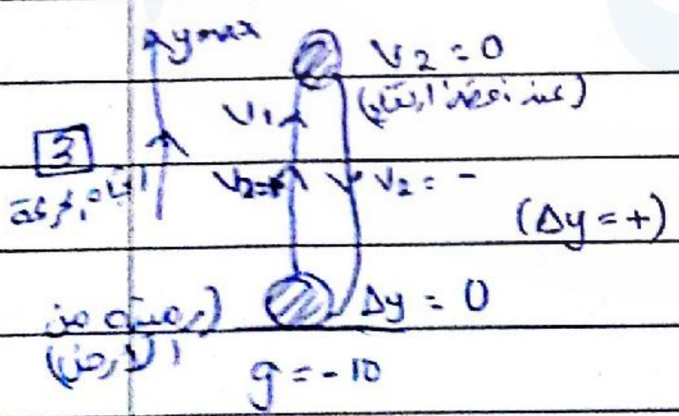
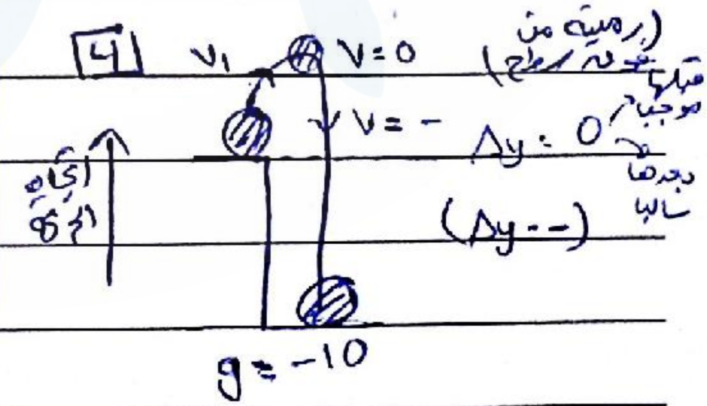


4



Δy : الخط المستقيم الواصل من البداية للنهاية.

(فقط إذا نزل بزيادة عن الخط (السطح) الذي أطلق منه بتصير $\Delta y(-)$)

⊛ [V_1 إذا (+) وΔy إذا موجبة ما بعد الحد]

No. ______

* اتجاه الحركة هو اتجاه الـ V_1 [illegible]

* Δy مسافتها الرأسية من نقطة البداية لنقطة الذروة.

$$\Delta y_{max} = \frac{V_1^2}{20 \rightarrow (2g)}$$

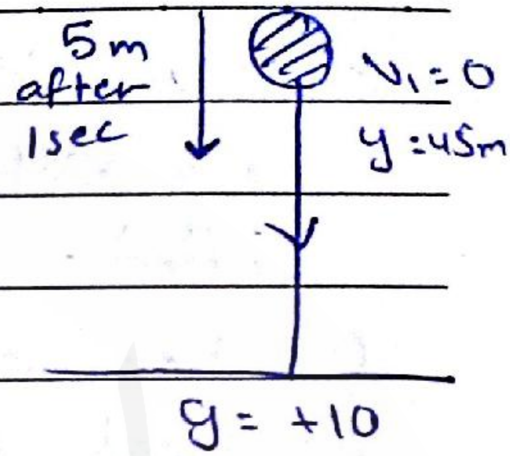
$$t_{max} = \frac{V_1}{10 \rightarrow (g)}$$

$$a_{max} = -10\ m/s^2$$

(*) Example:- an object is dropped from a top of a building of 45 m height, find:-

1) Flying time ?
2) Final speed ?
3) height after 1sec ?
4) Speed at 30 m height ?

Answer:-



① $\Delta y = v_1 t + \frac{1}{2} g t^2$

$45 = 0 + \frac{1}{2}(10)(t^2)$

$45 = 5t^2 \rightarrow \boxed{t = 3}$ seconds

② $v_2 = v_1 + g t$

$v_2 = 10 \times 3 = 30$ m/s.

③ $\Delta y = v_1 t + \frac{1}{2} g t^2$

$= 5(1)^2 = 5m$

$\therefore \underset{\text{(height)}}{h} = 45 - 5 = 40$ m

④ $\Delta y = 45 - 30 = 15$ m

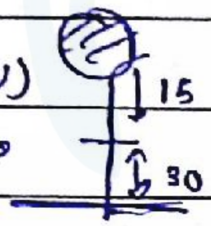
$v_2^2 = \cancel{v_1^2} + 2g\Delta y$

$v_2^2 = 2(10)(15)$

$v_2 = \sqrt{300} = 14$ m/s.

$\cancel{-14}$

(لانه اتجاه V_2 مع الحركة)



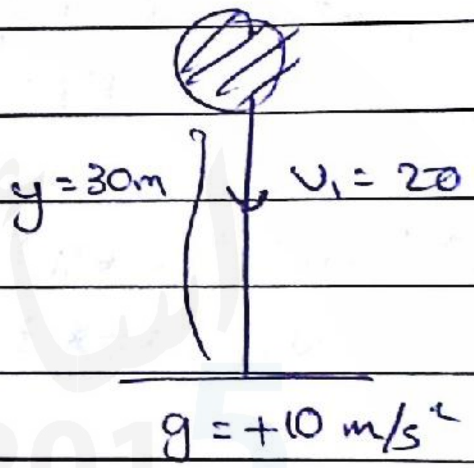
No.

و (بالحالة الثانية لأنه فوق لأسفل أما بالحالة الأولى لو كانت لفوق)

(*) Example:- A ball was thrown from a height = 30 meters with speed = 20 m/s (down-ward); find:-

① time of flight.

② Final speed.



Answer:-

1) $\Delta y = v_1 t + \frac{1}{2} g t^2$

$30 = 20t + \frac{1}{2}(10)t^2$

$0 = 5t^2 + 20t - 30$

$\boxed{t = 1.16}$, $\cancel{-5.16}$

2) $v_2 = v_1 + gt$

$v_2 = 20 + 10 * 1.16$

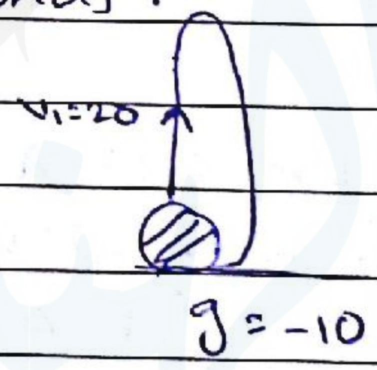
$= 31.6\ m/s$

$\vec{v}_2 = -31.6\,\hat{j} \longrightarrow$ as a cartisian vector.

No. ____________

Example:- A ball is thrown from ground (upword) with initial speed 20 m/s, find:-

1) maximum height.
2) time to reach maximum height.
3) total time.
4) displacement when reach ground.
5) Average velocity to ground.
6) Average speed to ground.
7) height after 1sec / 3 seconds.
8) Velocity after 1sec / 3 seconds.



Answer:-

① $y_{max} = \frac{v_1^2}{20} = \frac{400}{20} = 20\,m$

② $t_{max} = \frac{v_1}{10} = \frac{20}{10} = 2\,secs$

③ $t_{total} = 2 * 2 = 4\,secs$ → ما يطلع لأعلى نقطة بستغرق نفس الزمن

④ $\Delta y = zero$ (رجع لحد مكانه)

⑤ $\vec{v}_{av} = \frac{\Delta y}{t} = \frac{0-0}{4} = 0$

or $\frac{v_1 + v_2}{t} = \frac{20 + (-20)}{t} = 0$

No. ______________

(6) $S = \frac{D\,(\text{total})}{t\,(\text{total})} = \frac{20+20}{4} = 10\,m/s$

(7)

(1sec)

$\Delta y = v_1 t + \frac{1}{2} g t^2$

$= 20(1) - 5(1)^2$

$= 15\,m$

(3sec)

$\Delta y = v_1 t + \frac{1}{2} g t^2$

$= 20(3) - 5(3)^2$

$= 60 - 45 = 15\,m$

(8)

$v_2 = v_1 + gt$ (1sec)

$v_2 = 20 - 10(1)$

$= 10\,m/s$

$v_2 = v_1 + gt$ (3sec)

$= 20 - 10(3)$

$= -10\,m/s$

(*) Example:- A stone is thrown upword with speed 40 m/s from a top of a cliff of height 60m; Find!-

1. total time.
2. Final Speed.

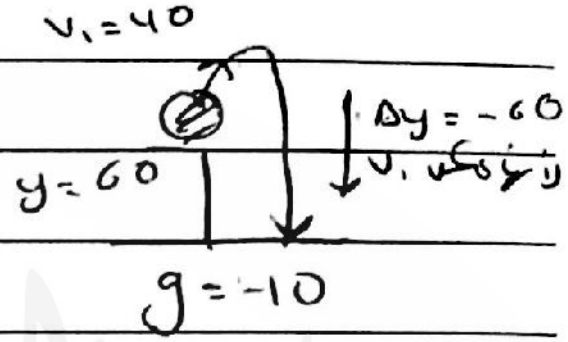
No. ____________

Answer:-

$\Delta y = v_1 t + \frac{1}{2} g t^2$

$-60 = 40t + \frac{1}{2}(-10)(t^2)$

$\boxed{t = 9.3}, -\cancel{1}.3$



طيب ليش وهو بإشارة موجبة دام الراجعة سالبة؟

$V_2 = V_1 + g t$

$= 40 - 10(9.3)$

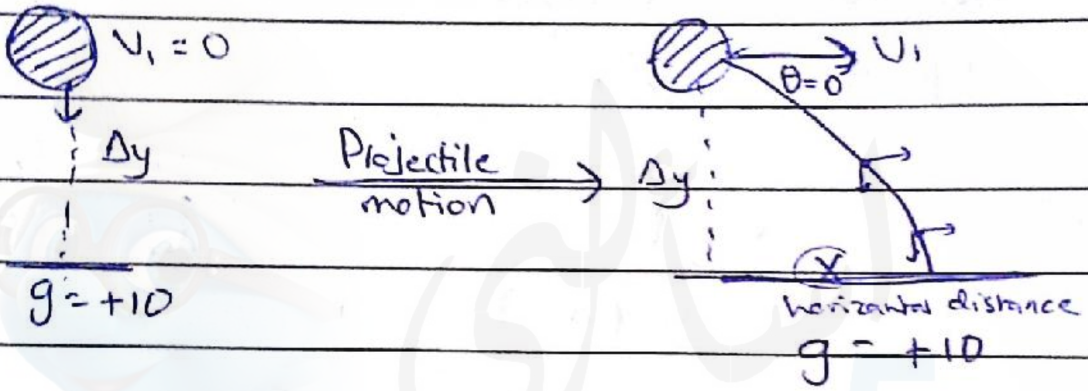
$= -53 \text{ m/s}$ هاي بـ velocity لكن هوه بده

ال speed اذاً 53+ لأنه $S = |V|$

Chapter 4 :- Motion in 2 dimensions :-

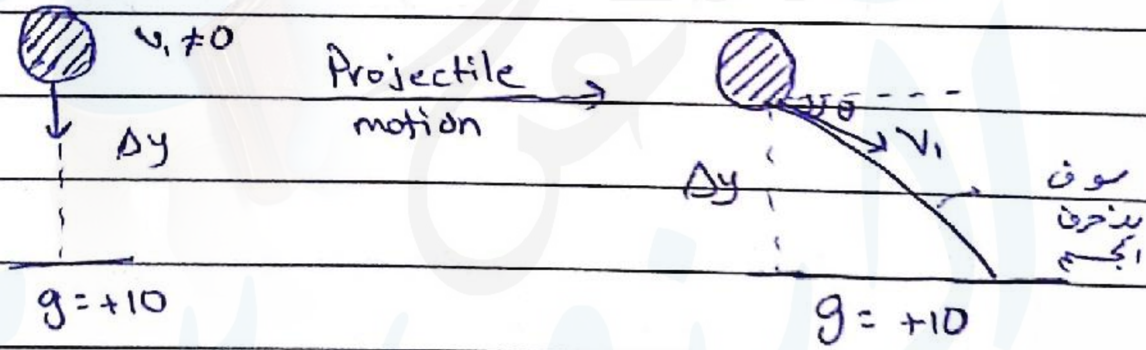
projectile motion :-

1



* θ هي زاوية ال v_1 مع ال x-axis

2



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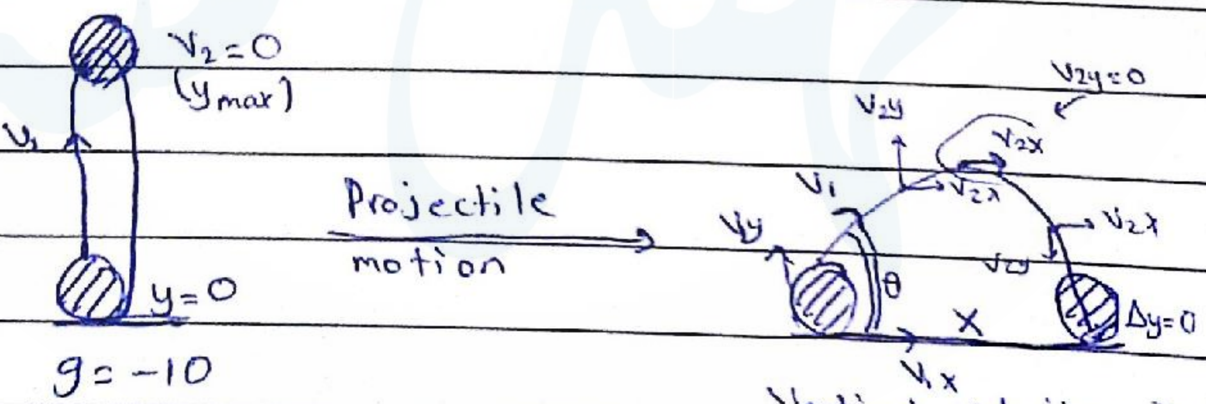
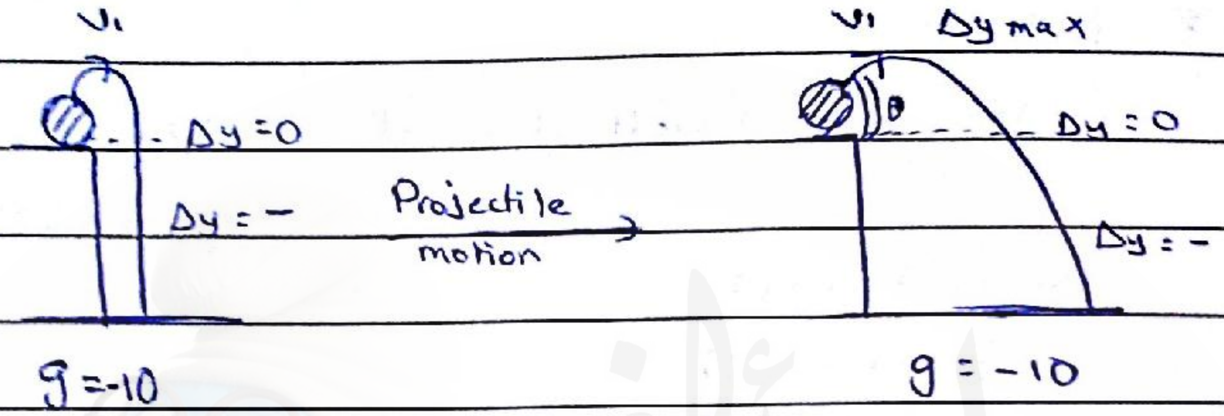


Vertical velocity = 0
horizontal " = constant

$g = -10$

4



X-axis	y-axis	
$V_{2x} = V_{1x}$ $\boxed{a=0}$	$V_{2y} = V_{1y} + gt$	①
$V_{2x}^2 = V_{1x}^2$ $V_{2x} = V_{1x}$	$V_{2y}^2 = V_{1y}^2 + 2g\Delta y$	②
$\Delta x = V_{1x}\, t$	$\Delta y = V_{1y}\, t + \frac{1}{2} g t^2$	③
بترجع نفسها تمامًا ←	$\Delta y = \left(\frac{V_{1y} + V_{2y}}{2}\right) t$	④
	$*y_{max} = \frac{V_{1y}^2}{20}$ / $t_{max} = \frac{V_{1y}}{10}$	

X-axis (V) constant ∴ a=0

بترجع نفسها

* نضيف x أو y جنب الـ V عشان أميزهم عن بعض.

(*) $*\vec{V} = \underbrace{30\hat{i}}_{V_{1x}} - \underbrace{20\hat{j}}_{V_{1y}}$ → هذه الحالة الثانية.

* Notes :-

أول وأهم خطوة في حل مسائل المقذوفات هي تحليل V_1 إلى مركبتيها.

$V_{1x} = V_1 \cos\theta$

$V_{1y} = V_1 \sin\theta$

* the horizontal (x) أو (R) distance is called : Range

* Final Velocity: $(\vec{V_2})$

$$\boxed{\vec{V_2} = V_{2x}\hat{i} \pm V_{2y}\hat{j}}$$

* Speed, $\sqrt{V_{2x}^2 + V_{2y}^2}$

أو $S = |\vec{V}|$

* Example:- An object is thrown horizontally with speed 10 m/s from 45 m height; Find :-

1. Flying time.
2. horizontal distance. (Range)
3. Final Velocity.

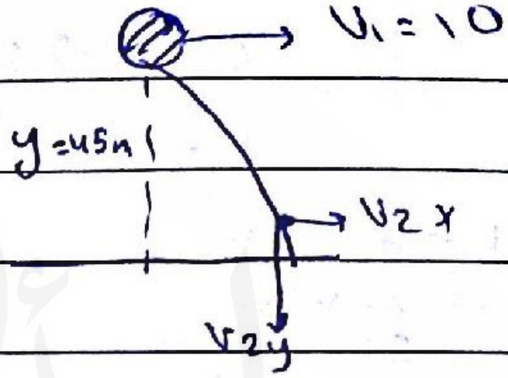
No. ____________

Answer:-

① $\Delta y = V_{iy} t + \frac{1}{2} g t^2$

$45 = 0 * t + \frac{1}{2} 10 t^2$

$\boxed{t = 3}$, $\cancel{-3}$



② $x = V_i * t$ $\qquad g = +10$

$= 10 * 3 = 30\ m$

③ $V_{2x} = V_{1x} = 10\ m/s$

$V_{2y} = V_{iy} + gt = 10*3 = 30\ m/s$

$\vec{V_2} = 10\hat{i} - 30\hat{j}$.

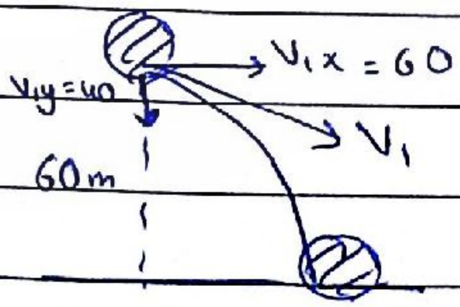
(*) Example: (رضه بكالم

A projectile is fired from a top of a building of 60 m height with velocity $60\hat{i} - 40\hat{j}$; Find:-

① Range.

② Final speed.

Answer:-



(this is parapolic & not circular)

قطع مكافئ

No. لإن ما بيجي سالب!

① $X = V_{1x}\, t$ → $\Delta y = V_{1y} t + \frac{1}{2} g t^2$

$= 60 * 1.29$ $\quad$ $60 = 40t + 5t^2$

$= 77.4\ m$ $\quad$ $5t^2 + 40t - 60 = 0$

Range (horizontal Δx) ✓ $\quad$ $\boxed{t = 1.29}$, $t = -\cancel{9}.29$

② $V_{2x} = V_{1x} = 60$

$V_{2y} = V_{1y} + gt$

$= 40 + 10 * 1.29$

$= 52.9\ m/s.$

$S = \sqrt{(60)^2 + (52.9)^2}$

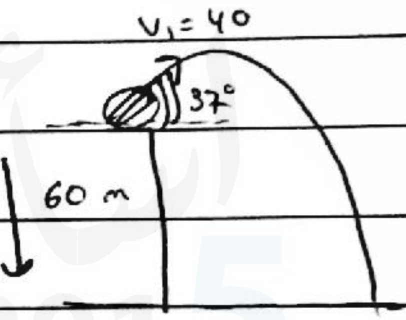
⊛ Example:-

No. ____________

Example:-

An object is thrown with speed 40 m/s from a top of a building of 60 m height with angle 37° above horizantal ; Find:-

اذا كانت الرمية لفوق أو below بتكون موجب



① Range. ② t_{max}

④ Final Speed. ③ y_{max}

Answer:-

$v_{1x} = v_1 \cos\theta$

$= 40 \cos 37$

$= 32$ m/s

$v_{1y} = v_1 \sin\theta$

$= 40 * \sin 37$

$= 24$ m/s

① $x = v_{1x}(t)$ $\qquad \Delta y = v_{1y}t + \frac{1}{2}gt^2$

$= 32 * 6.6$ $\qquad -60 = 24t + \frac{1}{2}(-10)t^2$

$= 211.2$ m $\qquad$ لأنها عكس ال v_1 $\qquad 0 = -5t^2 + 24t + 60$

$\boxed{t = 6.6}$, $\cancel{-1.8}$

total time نفسه ال

Flying time

time of flight

time to reach ground.
strike
hit

No. ____________

② $t_{max} = \frac{V_{1y}}{10} = \frac{24}{10} = 2.4$ seconds

③ $y_{max} = \frac{V_{1y}^2}{20} = \frac{(24)^2}{20} =$ -- meters.

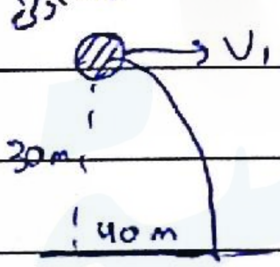
④ $V_{2x} = V_{1x} = 32$ m/s

$V_{2y} = V_{1y} + gt$

$= 24 - 10 * 6.6 = -42$ m/s

$\vec{V_2} = 32\hat{i} - 42\hat{j}$

$S = \sqrt{(32)^2 + (42)^2}$



Find V_1 ??

Answer: $V_{1x} = V_1 \cos 0 = V_1$

$V_{1y} = V_1 \sin 0 = 0$

$\Delta y = \cancel{V_{1y} t} + \frac{1}{2} g t^2$

$30 = 5t^2$

$t^2 = 6$

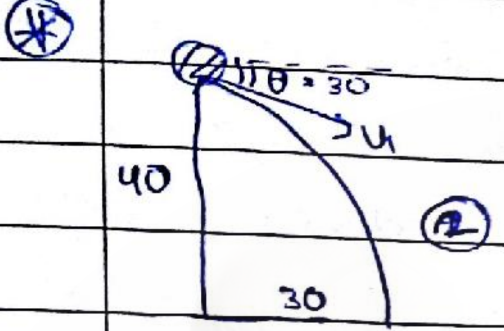
$t = \sqrt{6}$ s

$\Delta X = V_{1x} t$

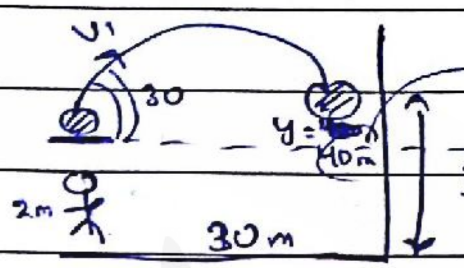
$40 = V_1 \sqrt{6}$

$V_1 = \frac{40}{\sqrt{6}}$ m/s

الحالة الثانية   No. ________   Find V_1 ??

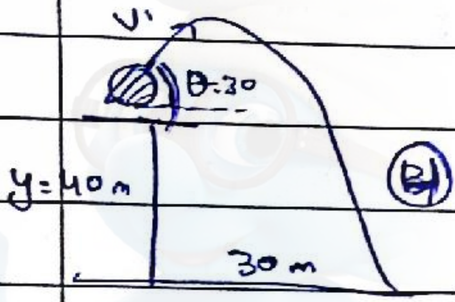


الحالة الثالثة



هون (1) بس
اهنا مشان
الارقام يكونوا زي بعض

(y هو الفرق بين رجل و ring)



الحالة الرابعة

Answer: كلهم بنحلوا زي بعض

$V_{1x} = V_1 \cos 30 = 0.86 V_1$

$V_{1y} = V_1 \sin 30 = 0.5 V_1$

$\Delta x = V_{1x} t$

$30 = 0.86 V_1 t$

$V_1 t = \frac{30}{0.86} = 34.6$

$\Delta y = V_{1y} t + \frac{1}{2} g t^2$

$40 = 0.5 V_1 t + \frac{1}{2}(10) t^2$ — ②

$40 = 0.5 V_1 t + \frac{1}{2}(-10) t^2$ — $\boxed{③}$

$-40 = 0.5 V_1 t + \frac{1}{2}(-10) t^2$ $\boxed{④}$

طالما كلهم زي بعض لكن باختيار ④

$-40 = 0.5(34.6) - 5t^2$

$-40 - 17.3 = -5t^2$

$-57.3 = -5t^2$

$t = 3.38$ seconds. $\longrightarrow V_1 t = 34.6$

$V_1 (3.38) = 34.6$

$V_1 = \frac{34.6}{3.38} = 10.2$ m/s.

No. ______

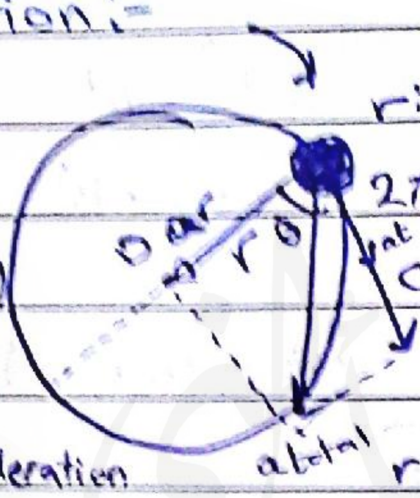
(*) Circular motion:-

r: radius

* revolution

$2\pi r$ = دورة =

$2\pi r$ = circumference (المحيط)



$C = 10$ / $2\pi r = 10$

$r = \frac{10}{2\pi}$

$r = \frac{D}{2}$

* ar: radial (centripetal) acceleration.

* at: tangential acceleration (rate)

$a_t = 0$ if the magnitude of Speed is constant.

* $a_{total} = \sqrt{(a_t)^2 + (a_r)^2}$

$\tan\theta = \frac{a_t}{a_r}$

$a_r = a_{total}\cos\theta$

$a_t = a_{total}\sin\theta$

* $a_r = \frac{V^2}{r}$ التسارع المركزي

$a_t = \frac{dV}{dt}$

(*) Example: if the speed of an object moving in a circular path is given by: $V = 2t^2 - 5$; Find:- total acceleration in $r = 2m$ at $t = 4$ seconds.

$\boxed{r = 5}$

(كيف نحول من rev/min ل m/s)

$V = 4 \text{ rev/m}$

$= \frac{4(2\pi r)}{60} = \frac{4(2\pi * 5)}{60} \text{ m/s}$

No. ______________

Answer:-

$a_r = \frac{V^2}{r} = \frac{(27)^2}{2} \text{ m/s}^2 = 364.5 \text{ m/s}^2$

$V = 2(4)^2 - 5 = 27 \text{ m/s}$

$a_t = \frac{dV}{dt} = 4t = 4(4) = 16 \text{ m/s}^2$

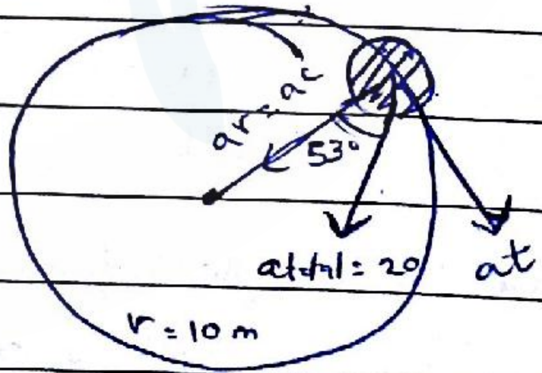
هاد مش مشتقة ولا تعوض
بمعناها انه بشتق بالنسبة ل t

$a_{total} = \sqrt{(364.5)^2 + (16)^2} \text{ m/s}^2$

(#) Example: In the figure; find:-

1. tangential acceleration.
2. centripital acceleration.
3. Speed in ($\overset{\text{revolution}}{\text{rev}}$/min)

Answer:-



(1) $a_t = a_{total} \sin 53$

$= 20 * 0.8$

$= 16 \text{ m/s}^2$

(2) $a_r = a_{total} \cos 53$

$= 20 * 0.6$

$= 12 \text{ m/s}^2$

No. ______

③ $a_r = \frac{v^2}{r}$

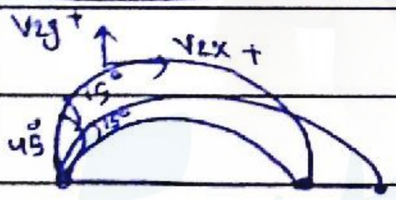
$v = \sqrt{r\,a_r}$

$= \sqrt{10 \times 12}$

$= \sqrt{120} \simeq 11 \text{ m/s}$

$v = \left(11 * \frac{60}{2\pi r}\right) \frac{\text{rev}}{\text{min}}$

$= \frac{330}{10\pi} = \frac{33}{\pi} \text{ m/s}$



لما أبعد عن ٤٥° بمقدار ثابت رح
تنزل نفس المكان.
(عند ~~x~~ ٤٥° رح يكون أبعد Range.
لأن أكبر Δx).

No. ____________

Chapter five :
Newton's Laws:

$$\sum \vec{F} = m\vec{a}$$
(force) (mass) (acc)

m: mass (kg)

Example : Two forces acted on an object of 2-kg in mass , $\vec{F_1} = 4\hat{i} - 3\hat{j}$ newoton
$\vec{F_2} = 6\hat{i} + 2\hat{j}$ newton.

Find ; 1) acceleration 2) Magnitude of acceleration

Answer :

① $\sum \vec{F} = m\vec{a}$

$\vec{F_1} + \vec{F_2} = m\vec{a}$

$10\hat{i} - \hat{j} = 2\vec{a}$

$\vec{a} = 5\hat{i} - 0.5\hat{j} \ m/s^2$

② $|\vec{a}| = \sqrt{5^2 + 0.5^2} = \sqrt{25.25} \ m/s^2$

* $\Sigma F_x = ma_x$
  $\Sigma F_y = ma_y$] لما تكون في حركة

∴ $\Sigma F_{\rightarrow} = \Sigma F_{\leftarrow}$ (on the x-axis if the object was equilibrized)
$\Sigma F_{\uparrow} = \Sigma F_{\downarrow}$ (on the y-axis ---)
[equilibrium] ← اتزان.

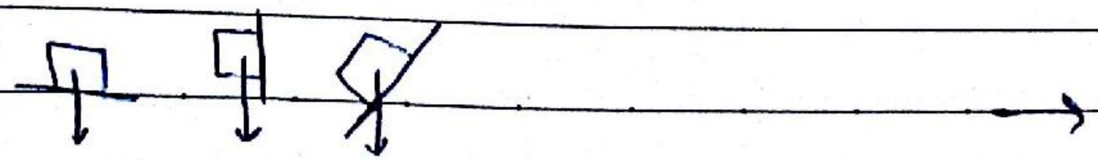
* Steps to answer Newton's problems :-

١) نعين اتجاه الحركة (إن وجد)
٢) نعين جميع القوى المؤثرة على الجسم.
٣) نحدد محاور متناسبة ونحلل أي قوة مائلة.
٤) أ) نطبق في الاتزان أولاً.
ب) نطبق في قانون نيوتن الثاني في حالة التسارع.
[أي قوة مع اتجاه الحركة (+) ، أي قوة عكس اتجاه الحركة (-)]

* Types of forces :-

1) weight (w)
$w = mg$

* اتجاه w دائماً للأسفل

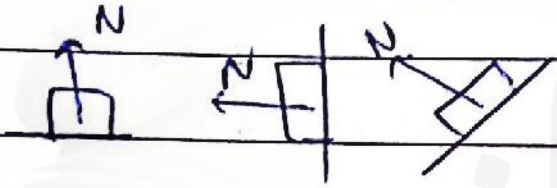


No. ________

← ما الها دخل بالوزن بس بالسطح

2) Normal force (N) (القوة العمودية)

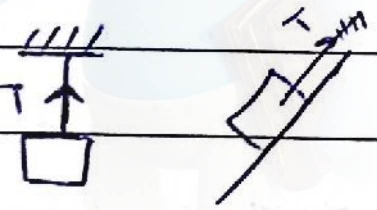
[قوة تنتج عن السطح فقط] * [اتجاه N دائماً عمودي على السطح]

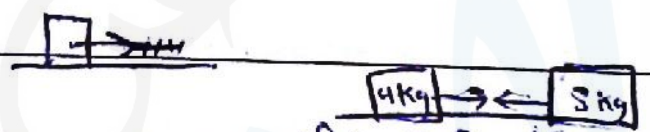


3) Tension (T) (قوة الشد)

Cord, wire, coil (ملف) [تنتج عن الحبال والخيوط فقط]

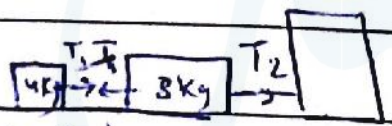
*[اتجاه T دائماً من الجسم إلى الخيط]





(نفس مقدار قوة الشد

لأنه لا يعتمد على الكتلة بل على الخيط فقط)



لا هدول مش متساويين لأن: $T_1 \neq T_2$

∴ قوة الشد غير متساوية

(دائماً عكس اتجاه الحركة) (وليس القوة)

4) Friction Force (قوة الاحتكاك)

a) Static friction (f_s) (احتكاك سكوني)

b) Kinetic friction (f_k) (الاحتكاك الحركي)

No. ______________

a) Static friction (f_s)

$f_{s(max)} = \mu_s * N \rightarrow$ if 1) $F \leq f_{s\,max}$ object will remain at rest

2) $F > f_{s\,max}$ object : start moving.

μ : cooefficient of f_s / f_k (معامل الاحتكاك السكوني أو الحركي)

(unit-less)

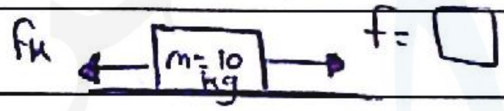
$0 \leq \mu_{s,k} \leq 1$ (ما يصير سالب)

$\mu_s > \mu_k$

b) Kinetic friction (f_k)

$f_k = \mu_k * N$

Example :- $N = 100$ new.



$\mu_s = 0.4$

$\mu_k = 0.3$

Find a, if ① $F = 35$ N. ② $F = 50$ N.

Answer :-

① $f_s = \mu_s N$ $\quad$ $f_s > F \quad \therefore a = 0$ (الجسم لن يتحرك)

$= 0.4 * 100$ $\quad$ $F = 35 < f_{x\,max}$

$= 40$ New. $\quad$ $f_s = F = 35$ new.

(زوض قوة احتكاك للجسم مع الأرض)

②

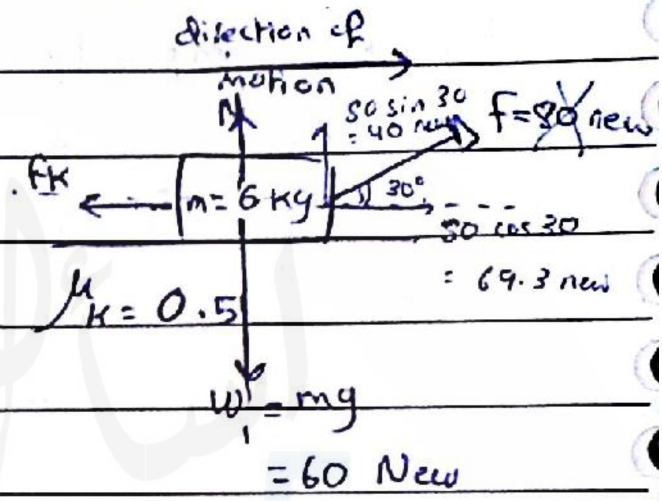
اتجاه ، حركة ، اتجاه "كا" السطح.

No. ____________

Example :- In the figure, find :-

1) Normal force.

2) acceleration.



Answer :-

$\Sigma F_{\uparrow} = \Sigma F_{\downarrow}$

$N + 40 = 60$

① $\boxed{N = 20 \text{ newton.}}$

$f_k = \mu_k * N$

$f_k = 0.5 * 20 = 10$ newton.

(h.w → f → [illegible]

(+) مع الحركة
(−) عكس [illegible]

$\Sigma f_x = m a_x$

$69.3 - 10 = 6a$

$a = \frac{59.3}{6}\ m/s^2$

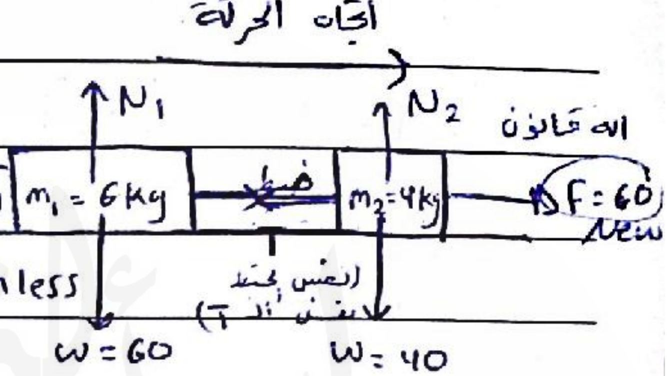
* Example :-

In the figure, Find :-

1) Tension. (T).

2) acceleration (a) كان F للعمش دليل T عكس عليه

لانه القوة المؤثرة على الجسم هي القوة الملامسة له فقط !!



Answer :-

m_2 :- y-axis

$\Sigma F_{\uparrow} = \Sigma F_{\downarrow}$

$N_2 = 40$ New

m_2 :- x-axis

$\Sigma F = m_2 a$

$\boxed{\underbrace{60 - T}_{\text{ضد الحركة}} = 4a}$

m_1 :- y-axis

$N_1 = 60$ New

m_2 :- x-axis

$\Sigma F = m_1 a$

$\boxed{T = 6a}$

②

$60 - \cancel{T} = 4a$

$+ \quad \cancel{T} = 6a$

$60 = 10a \longrightarrow a = 6 \, m/s^2$

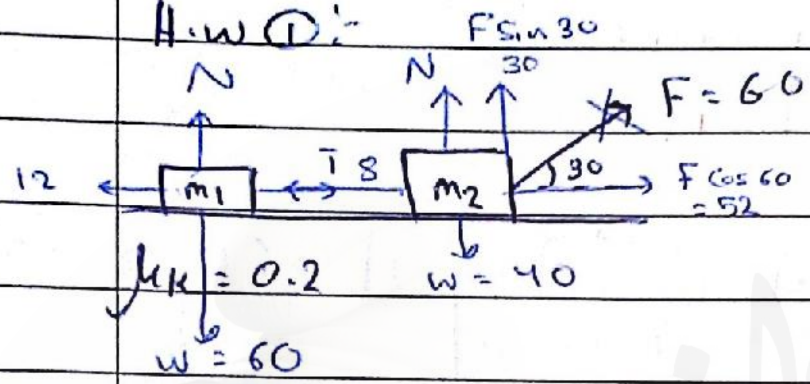
①

$T = 6a$

$T = 6 * 6 = 36$ New

No. ____________

H.w ①:-



Find:

1) Tension (T)

2) acceleration (a)

$f_{k_1} = \mu_k * N_1$

$= 0.2 * 40 = 8$ New

$f_{k_2} = \mu_k * N_2$

$= 0.2 * 60 = 12$ New

* $N_1 = 60$

$N_2 = 40$

* $\Sigma f_{x_1} = m a_{x_1}$ $\qquad$ $\Sigma f_{x_2} = m a_{x_2}$

$\boxed{T - 12 = 6a}$ $\qquad$ $52 - T - 8 = 4a$

$+ \quad \boxed{44 - T = 4a}$

② $\boxed{a = 3.2 \ m/s^2}$

① $T - 12 = 6a$

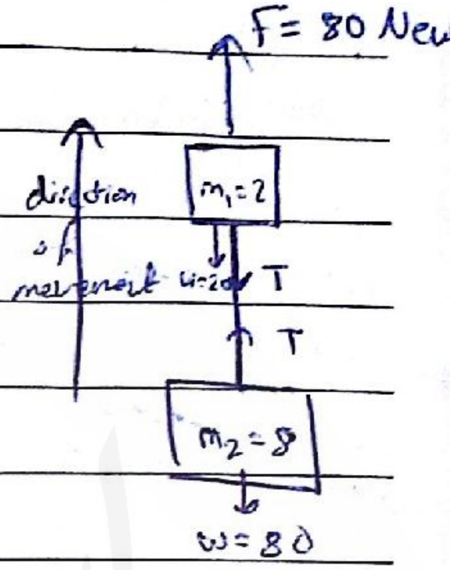
$T = 31.2$ New

→ physics. (ما في N لأنه شي فقط من السطوح
وهنا الجسم ما في سطوح علاقة الها)

No. ____________

* Example:- In the figure,

Find ① a ② T ??



Answer:-

m_1 : $\sum F_y = m_1 a$

$80 - T - 20 = 2a$

$$\boxed{60 - T = 2a}$$

m_2 : $\sum F_y = m_2 a$

$$\boxed{T - 80 = 8a}$$

$60 - \cancel{T} = 2a$

$-80 + \cancel{T} = 8a$

$-20 = 10a \rightarrow a = -2\ m/s^2$ لأنه بحكم لأعلى اذا a سالبة (تباطؤ)

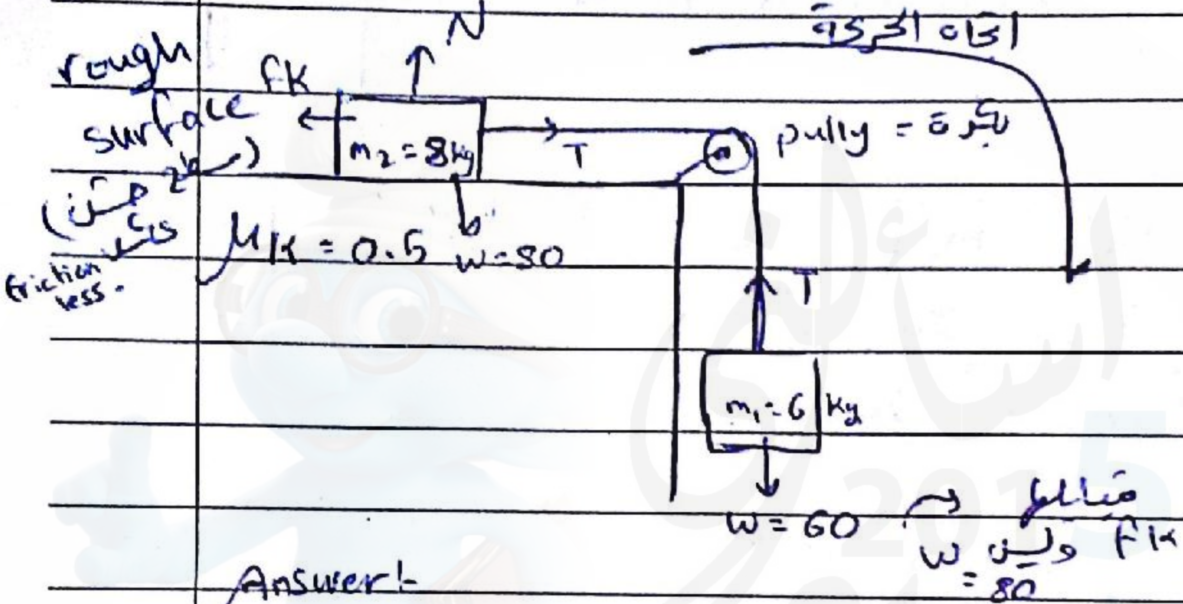
$T - 80 = 8a$

$T - 80 = 8 * -2$

$T = 64$ New (لازم T تطلع موجبة لأنه ما في قوة سالبة، السالب فقط للاتجاه)

No. ____________

** Example:- In the figure, Find:- ① a ② T



Answer:-

m_2:- y-axis

$\Sigma F\uparrow = \Sigma F\downarrow$ (ما عليه حركة) (بسبب الا N ما عليه حركة)

$\boxed{N = 80 \text{ New}}$

$f_k = \mu_k * N$

$f_k = 0.5 * 80$

$f_k = 40$ New

m_1 :-

$\Sigma F_y = m_1 a$

$\boxed{60 - T = 6a}$ +

m_2 :

$\Sigma F_x = m_2 a$

$\boxed{T - 40 = 8a}$

$= 20 = 14a$

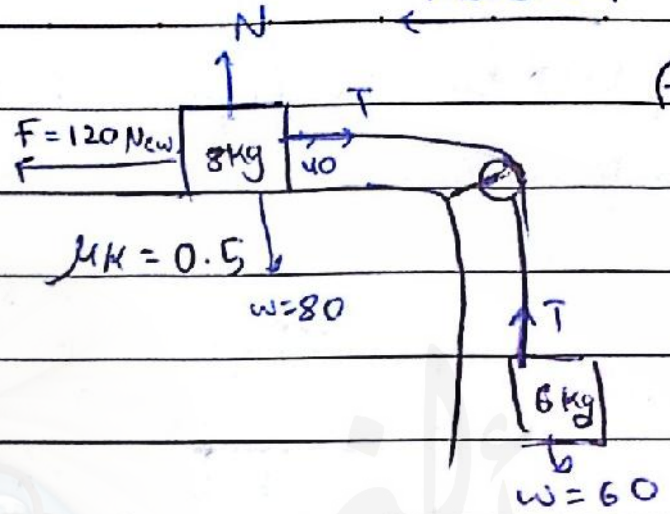
① $a = \frac{20}{14}$ m/s^2 $\simeq 1.5$ m/s^2

$T - 40 = 8 * \frac{20}{14}$

② $T = 52$ New

No. ______________ movement.

H.w (2):-



find: (1) a
(2) T

$\boxed{N = 80 \text{ newton}}$

$f_k = \mu_k * N$

$f_k = 0.5 * 80 = \boxed{40}$

$\Sigma F_x = m a_x$

$\Sigma F_y = \boxed{T - 60 = 6a}$

$120 - T - 40 = 8a$

$\boxed{80 - T = 8a}$

+

$a = 1.4 \ m/s^2$

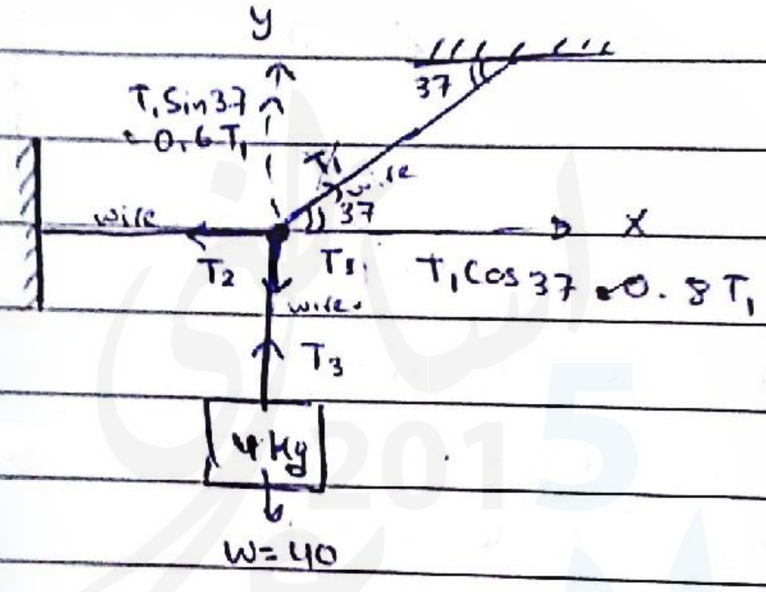
$80 - T = 8(1.4)$

T = ... newton.

No. ______________

* Example:- In the figure, Find Tension in each wire :-

(fully equilibrium)



$\boxed{4kg}$

$\Sigma F_{\uparrow} = \Sigma F_{\downarrow}$

$\boxed{T_3 = 40 \text{ New}}$

$\Sigma F_{\uparrow} = \Sigma F_{\downarrow}$

$T_1 \times 0.6 = 40$

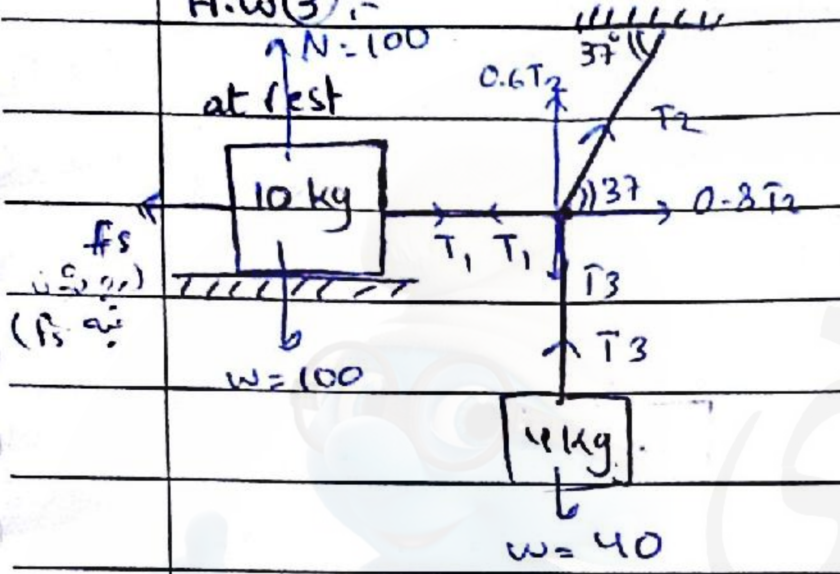
$\boxed{T_1 = \frac{40}{0.6} = 66.6 \text{ New.}}$

$\Sigma F_{\rightarrow} = \Sigma F_{\leftarrow}$

$0.8 T_1 = T2 \rightarrow T2 = 0.8 \times 66.6 = \boxed{T_2 = 53.3 \text{ New}}$

No. ____________

H.w(3):-



Find:

1) Tension in each wire.

2) μ_s.

① $\boxed{T_3 = 40 \text{ New}}$

$0.6\,T_2 = 40$

$\boxed{T_2 = 66.6 \text{ New}}$

$0.8\,T_2 = T_1$

$0.8\,(66.6) = T_1 \quad \boxed{53.3 = T_1}$

$f_s = T_1$

$T = 53.3 = f_s$

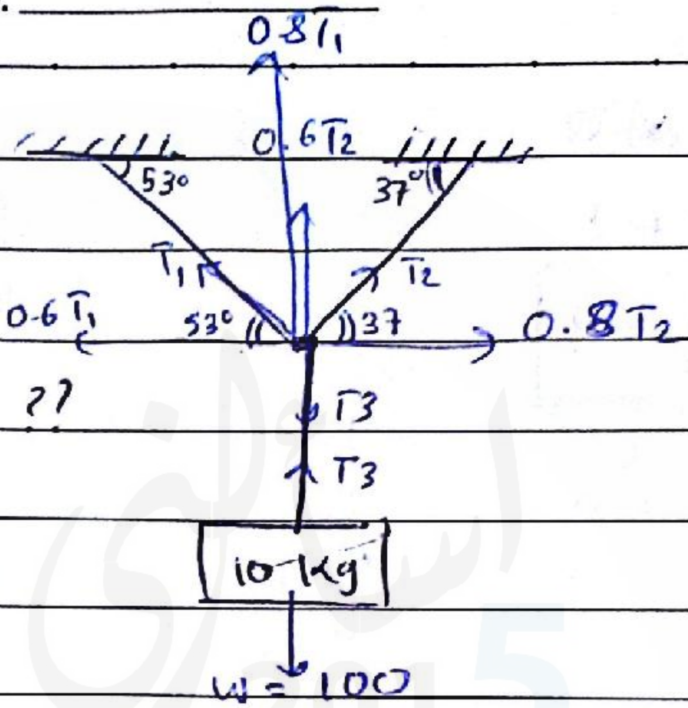
$f_s = \mu_s * 10$

② $\mu_s = 0.53$

No. ____

H.w (4):-

Find Tension in each wire ??



$\boxed{T_3 = 100 \text{ New}}$

$0.8T_1 + 0.6T_2 = T_3$

$0.8T_1 + 0.6T_2 = 100 \quad \ldots (1)$

$\frac{\cancel{0.6}T_1}{\cancel{0.6}} = \frac{0.8T_2}{0.6}$

$T = 1.3T_2$

$0.8(1.3T_2) + 0.6T_2 = 100$

$1.04T_2 + 0.6T_2 = 100$

$T_2 = 61$ New

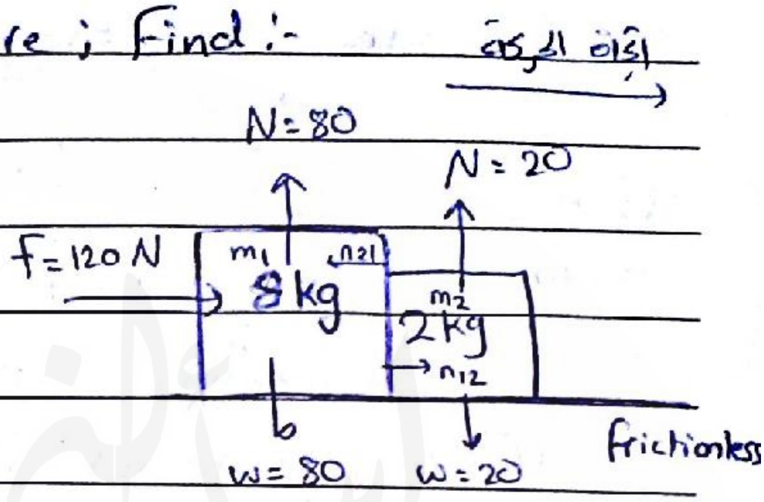
وعوضوا عنها في T_1

No. ____________

* Example: In the figure ; Find :-

1) acceleration.

2) internal forces



Answer:-

(m_1) $\sum F_x = m_1 a$

$\boxed{120 - n_{21} = 8a}$

(m_2) $\sum F_x = m_2 a$

$\boxed{n_{12} = 2a}$

$120 = 10a \;\xrightarrow{(1)}\; \boxed{a = 12\ m/s^2}$

(لوكان المطلوب فقط ال a
بسوي أنه جسم واحد 10kg
وهاي لأن القوتين n_{21}/n_{12} متعاكسة
كأنهم ما كانوا)

$|\vec{n}_{12}| = |\vec{n}_{21}|$ بالمقدار بساووا بعض

$\vec{n}_{12} = -n_{21}$ بالإتجاه ما بساووا بعض

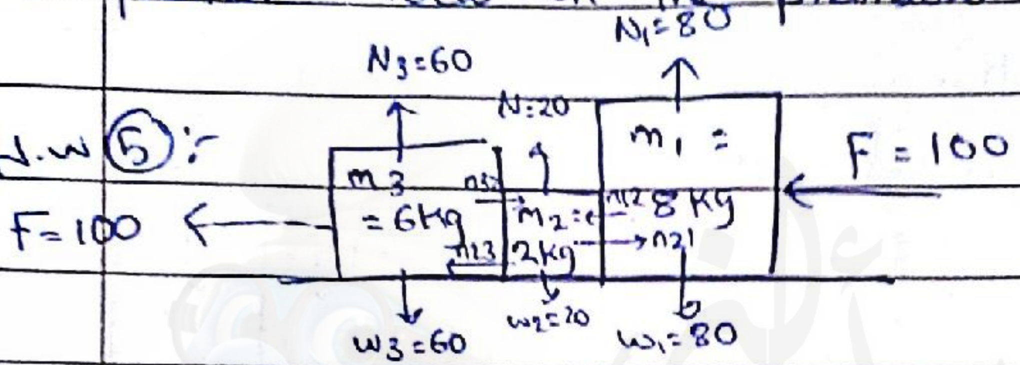
(2) $n_{12} = 2 \times 12 = 24$ new

$\boxed{n_{12} = n_{21} = 24 \text{ new}}$

No. ________________

* possible ideas on the previous question:-

H.w (5):-



Find: (1) internal forces (2) acceleration.

Answer:- (*) $\Sigma f_{x1} = m_1 a$

$n_{12} = 8a$ --(1)

(*) $\Sigma f_{x2} = m_2 a$

$n_{23} - n_{21} = 2a$ ---(2)

(*) $\Sigma f_{x3} = m_3 a$

$100 - n_{32} = 6a$ --(3)

$\rightarrow n_{23} - 8a = 2a$

$n_{23} = 10a$

$\rightarrow 100 - 10a = 6a$

(2) $\frac{100}{16} = a \rightarrow a = 6.25\ m/s^2$

(1) $n_{12} = n_{21} = 8a$

$= 8 * 6.25$

$= 50\ New.$

$n_{23} = n_{32} = 10a$

$10 * 6.25 = 62.5\ New$

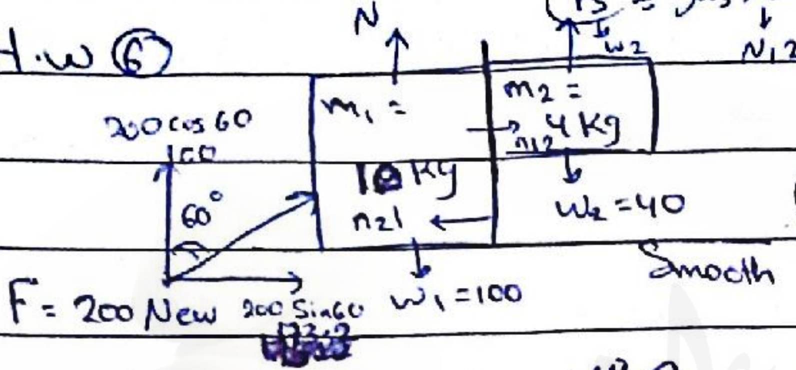
$n_{12} = n_{21}$ اتجاه الحركة →

No. اللي بلمتصه
بنزل

$\boxed{f_s} = \mu_s N$, $N \rightarrow N_{12}$

H.w ⑥

(الجدول الكسين مش
ملزقين ببعض بس لأنه
بيتحرك بسرعة فما بنزل لتحت)



Find μ_s? ~~[illegible]~~

Answer:-

m_1: $\sum F_\uparrow = \sum F_\downarrow$

$N + 100 = 100$

$\boxed{N = 0}$ ← لأنه رغم مش 2e بس صح ما بيطلع 0

m_2: $\sum F_\uparrow = \sum F_\downarrow$

$\boxed{f_s = 40}$

m_1: $\sum F_x = m_1 a$

$173.2 - n_{21} = 10a$

m_2: $\sum F_x = m_2 a$

$n_{12} = 4a$

$173.2 = 14a$

$\boxed{a = 12.4 \ m/s^2}$

$n_{12} = 4 * 12.4$

$n_{12} = 49.4$ New $= n_{21}$

$f_s = \mu_s * N^{\rightarrow n_{12}}$

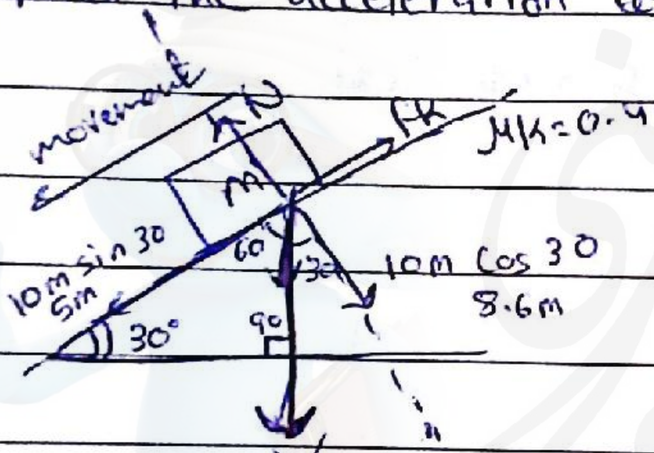
$40 = \mu_s * 49.4$

$\mu_s = \frac{40}{49.4} = 0.8$

No. ____________

* Example:- An object slides down an incline with angle 30° with the horizontal,
مستوى مائل

Find the acceleration of the object :-



$\Sigma F_y \uparrow = \Sigma F_y \downarrow$

$\boxed{N = 8.6\, m}$

$f_k = \mu_k * N$

$= 0.4 * 8.6$

$= 3.44 m$

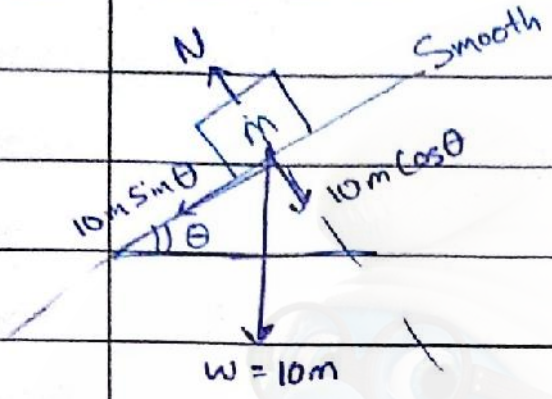
$\Sigma F_x = ma$

$5\cancel{m} - 3.44\cancel{m} = \cancel{m}a$

$\boxed{1.56\, m/s^2 = a}$

No. Home works

1. Find a :



$\sum f_y \uparrow = \sum f_y \downarrow$

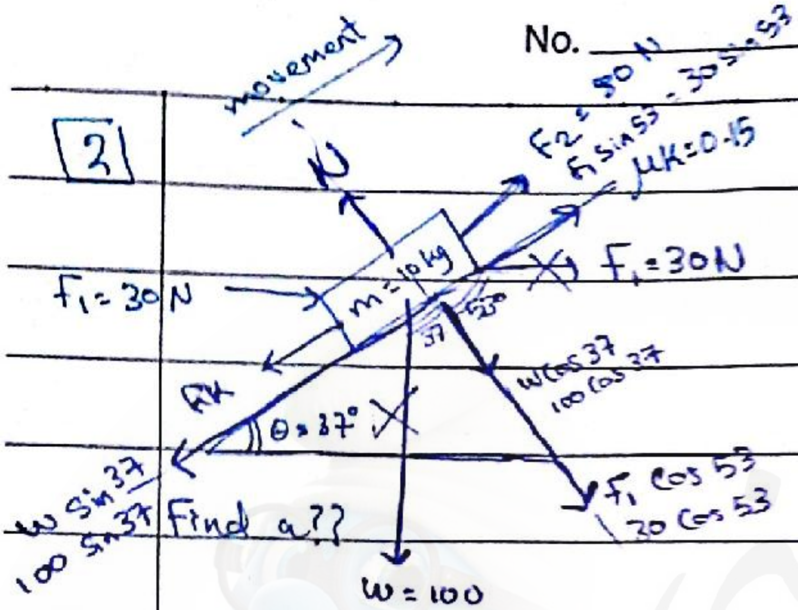
$\boxed{N = 10m \cos\theta}$

$\sum f_x = ma$

$10m \sin\theta = ma$.

No.

2



Find a??

Answer :-

$\Sigma f_y \uparrow = \Sigma f_y \downarrow$

$N = 100 \cos 37 + 30 \cos 53$

$N = 100 * 0.8 + 30 * 0.6$

$N = 80 + 18 = 98$ Newton.

$f_K = \mu_K * N$

$= 0.15 * 98 = 14.7$ Newton.

$\Sigma f_x = ma$

$80 + 30 \sin 53 - (f_K + 100 \sin 37) = 10a$

$80 + 24 - 14.7 - 60 = 10a$

$\frac{29.3}{10} = \frac{\cancel{10} a}{\cancel{10}}$

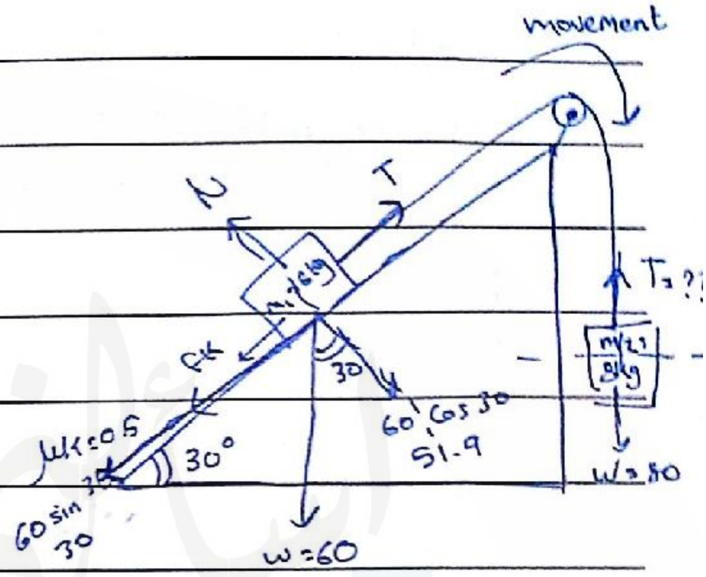
$a = 2.93 \ m/s^2$.

No. ____________

(الحركة مع جسم المعلق بالبولي)

* Example:

Find a?? T??



$\Sigma f_{\uparrow} = \Sigma f_{\downarrow}$.

N = 51.9 Newton

fk = μk * N

= 0.5 * 51.9

≃ 26 Newton.

(m_1) $\Sigma f_x = m_1 a$

$T - 30 - 26 = 6a$

$\boxed{T - 56 = 6a}$

(m_2) $\Sigma f_y = m_2 a$

$\boxed{80 - T = 8a}$

+

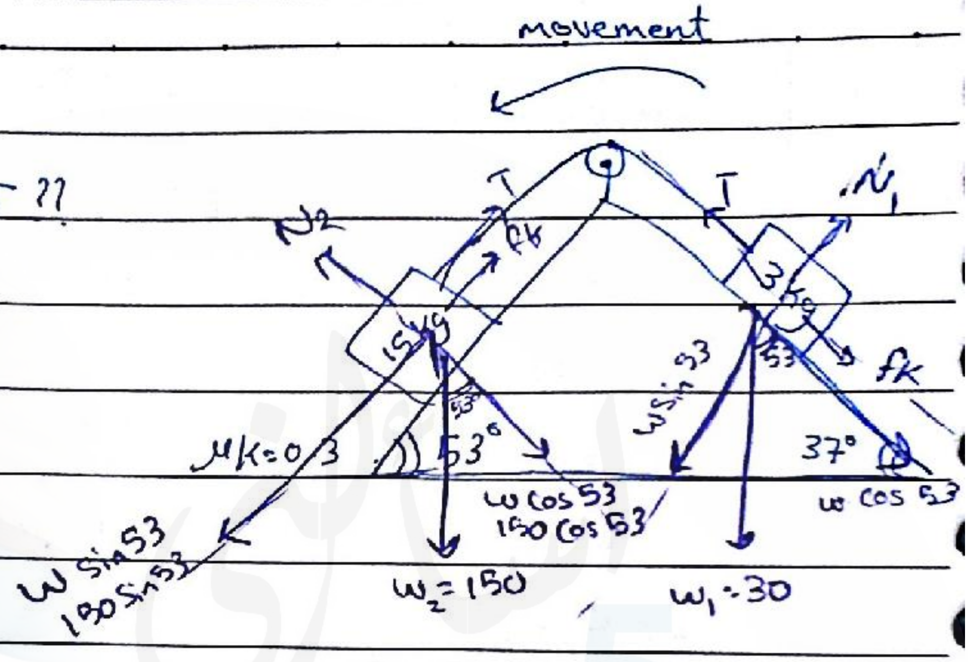
$24 = 14a \rightarrow a = \frac{24}{14}$ m/s²

$T - 56 = 6\left(\frac{24}{14}\right) \rightarrow T =$ —— Newton.

No. ____________

H.w :-

Find a?? T??



Answer :-

$\Sigma fy\uparrow = \Sigma fy\downarrow$

$N_1 = 30 \sin 53$

$= 30 * 0.8$

$N_1 = 24$ Newton

$fk_1 = \mu k * N_1$

$= 0.3 * 24 = 7.2$

$\Sigma fy\uparrow = \Sigma fy\downarrow$

$N_2 = 150 \cos 53$

$= 150 * 0.6$

$N_2 = 90$ Newton

$fk_2 = \mu k * N_2$

$= 0.3 * 90$

$= 27.$

$\Sigma fx_1 = m_1 a$

$T - fk_1 - 30 \cos 53 = 3a$

$T - 7.2 - 27 = 3a$

$T + 19.8 = 3a$ --- (1)

$\Sigma fx_2 = m_2 a$

$150 \sin 53 - T - fk_2 = 15a$

$90 - T - 27 = 15a$

$63 - T = 15a$ --- (2)

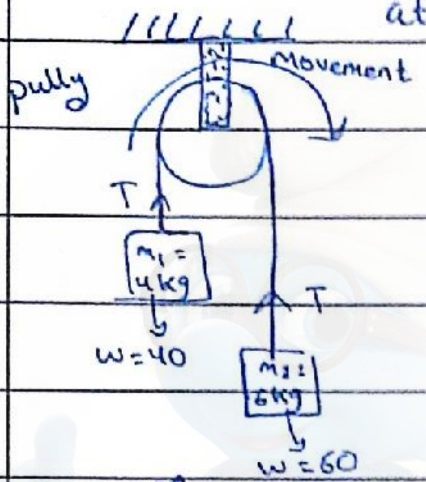
[illegible] T و a [illegible].

No. ______

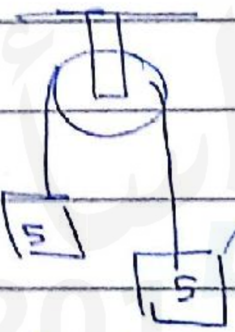
* Example :-

Find a, T ??

atwood machine



ما بصير $\Sigma F \uparrow = \Sigma F \downarrow$ لأنه الجسم غير متزن !!



uLS ↓

this is equilibrium. then the objects aren't moving. then a = 0.

a = 0

T = 50

Answer:-

$\Sigma f_y = m_1 a$

$\boxed{T - 40 = 4a}$

$\Sigma f_y = m_2 a$

$\boxed{60 - T = 6a}$

+

$20 = 10a$

$a = 2\ m/s^2$

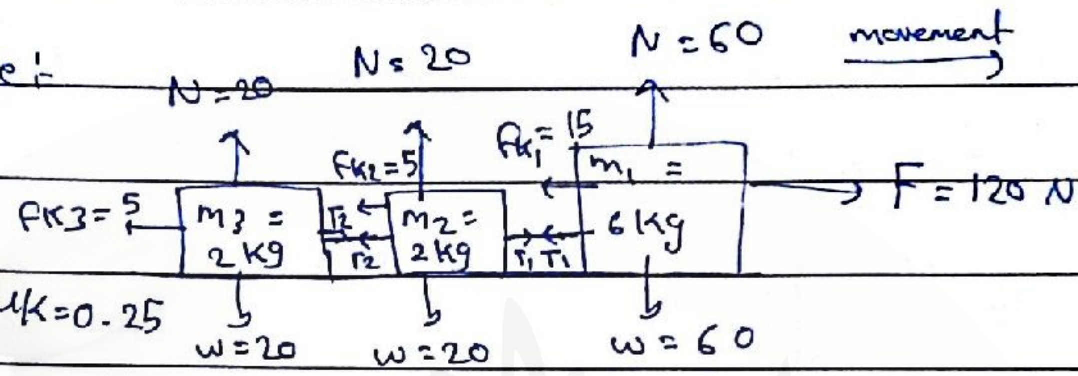
$T - 40 = 4(2)$

$T - 40 = 8$

$\boxed{T = 48\ \text{newton}}$

No. ____________

* Example :-



(m_1) $120 - 15 - \cancel{T_1} = 6a$

(m_2) $-\cancel{T_2} - 5\ \cancel{T_1} = 2a$

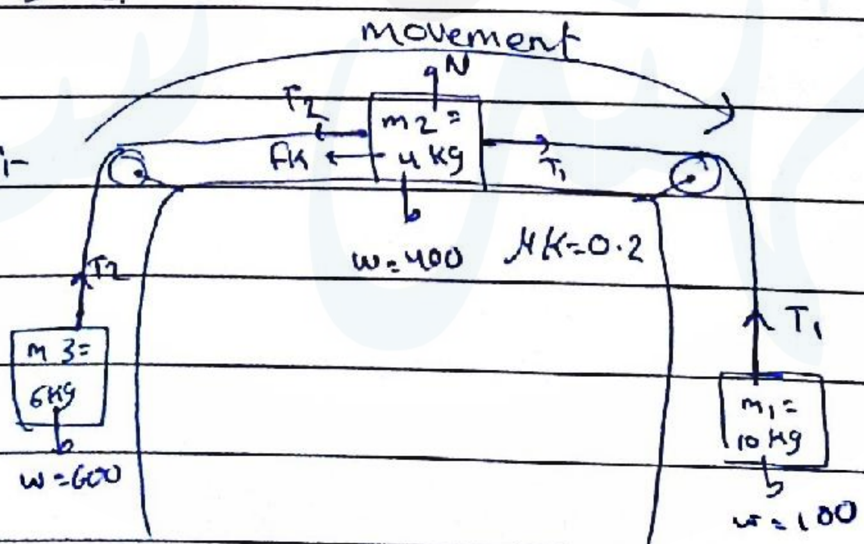
(m_3) $\cancel{T_2} - 5 = 2a$

$95 = 10a$

$a = 9.5\ m/s^2$

T_2 , T_1 بعوض بحوض منطلع

H.w :-



Find T_1 , T_2 , a ??

No. ____________

$\Sigma F_y = N = 400$ Newton

$\hookrightarrow F_K = \mu_K * N$

$= 0.2 * 400 = 80$ Newton.

$\Sigma F_{x2} = m_2 a$

$T_1 - f_K - T_2 = 4a$

$T_1 - 80 - T_2 = 4a$ -- (1)

$\Sigma F_{y1} = m_1 a$

$100 - T_1 = 10a$ -- (2)

$\Sigma F_{y3} = m_3 a$

$T_2 - 600 = 6a$ -- (3) وحل المعادلات نطلع T_1 / T_2 , a

$\hookrightarrow T_2 = 6a + 600$

$T_1 - 80 + 600 = 4a + 6a$

$-T_1 + 100 = 10a$

$620 = 20a \longrightarrow a = 31\ m/s^2$

$T_2 = 6(31) + 600 = ...$ Newton.

$100 - T_1 = 10(31) = ...$ Newton.

No. ________

$\Sigma F_y = N = 400$ Newton

$\Rightarrow f_K = \mu_K * N$

$= 0.2 * 400 = 80$ Newton.

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$T_1 - f_K - T_2 = 4a$

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$\Sigma F_{y3} = m_3 a$

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$T_1 - 80 + 600 = 4a + 6a$

$-T_1 + 100 = 10a$

$620 = 20a \longrightarrow a = 31\ m/s^2$

$T_2 = 6(31) + 600 = \ldots$ Newton.

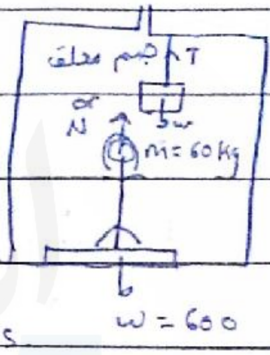
$100 - T_1 = 10(31) = \ldots$ Newton.

* تباطؤ للأعلى نفسه التسارع للأسفل
* تسارع للأعلى نفسه التباطؤ للأسفل

No. ______

* Example: A man of 60 kg mass inside elevator, find the appearant weight in the following cases:-

نفسه الـ Normal force تحت الجسم ولو كان جسم معلق بكون نفسه الـ Tension



[1] at rest → $\Sigma F_y = \Sigma F_\uparrow$: $N = 600$ → نفسه الـ ~~apperant~~ appearant weight

[2] moving upward with constant speed 5 m/s.

[3] moving downard with constant speed 5 m/s

[4] moving upward with constant acceleration $5 m/s^2$

[5] moving downward with constant acceleration $5 m/s^2$

[6] moving upward with constant deceleration $5 m/s^2$ (تباطؤ)

[7] moving downword with constant deceleration $5 m/s^2$

* [1+2+3] → $\Sigma F_\uparrow = \Sigma F_\downarrow$

$N = 60$ → نفسه الـ appearant weight.

* [4] $\Sigma F = ma$

$N - 600 = 60(5)$

$N = 900$ Newton → appearant weight. " "

* [5] $600 - N = 60(5)$

$N = 300$ Newton → " " " "

* [6] $N - 600 = 60(-5)$

$N = 300$ Newton " " " "

* [7] $600 - N = 60(-5)$

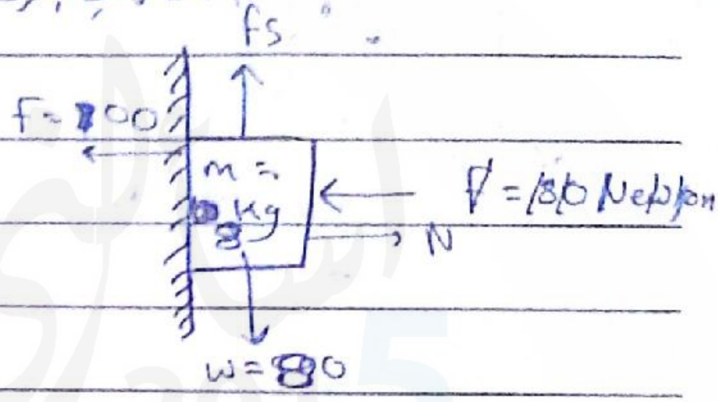
$N = 900$ Newton " " " "

No. ______

* Example:- Find the value of the Coefficient of Static friction that prevents the block from sliding down.

ساكن
حركة:
هذه
مسألة
اتزان

ما قبل ابداء ,الحركة ,الموشكة .



$N = 100$ newton

$f_s = 80$ newton

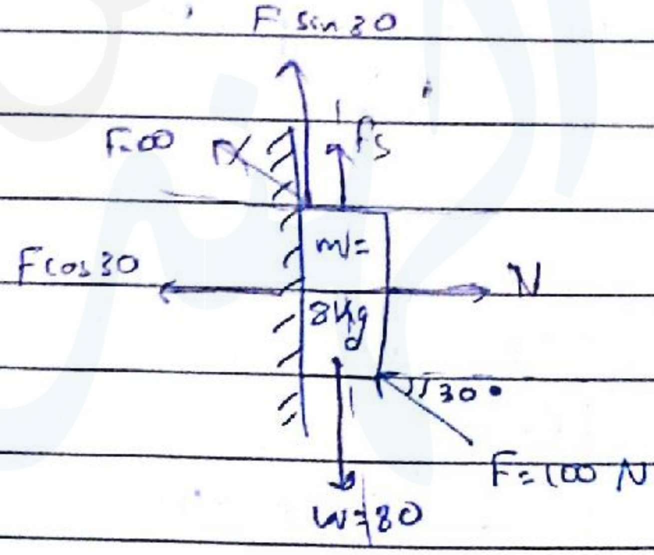
$f_s = \mu_s * N$

$80 = \mu_s 100$

$\boxed{\mu_s = 0.8}$

H.w :-

Find μ_s ??



$\sum F_{x\leftarrow} = \sum F_{x\rightarrow}$

$N = F \cos 30$

$N = 100 * 0.8$

$N = 80$ Newton.

* $f_s + F \sin 30 = 80$

$f_s + 50 = 80$

$f_s = 30$ Newton

$f_s = \mu_s * N$

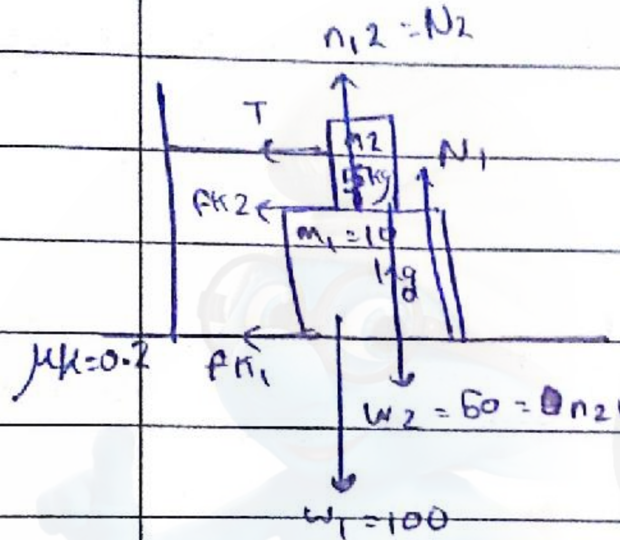
$30 = \mu_s * 80$

$\mu_s = 0.375$

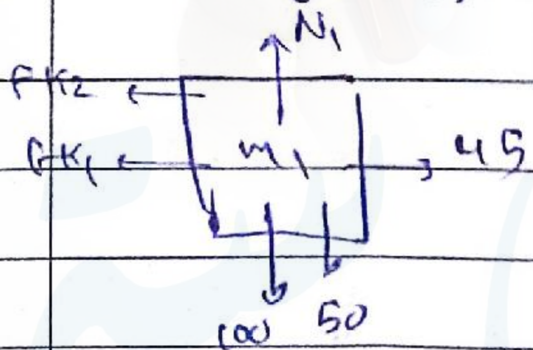
No. ______

* Example:

* أكيد حل لسؤال لو
ما كان في خيط



m1 Body diagram (1)



(y)

$\Rightarrow N_1 = 150$ New

$f_{k_1} = \mu_k * N_1$

$= 0.2 * 150$

$= 30$ Newton.

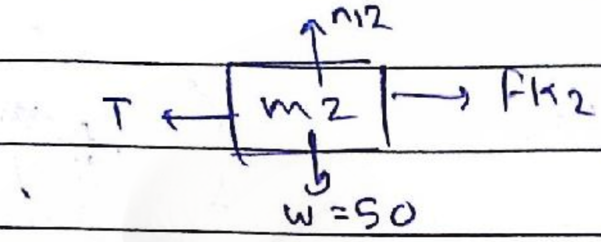
(x) $\sum F_x = m_1 a$

$45 - 30 - f_{k_2} = 10a$

No. ____________

m2: Body diagram (2)

هذا الجسم لا يتحرك لأنه متزن .



m2 (y) $n_{12} = 50$ New

$fk_2 = n_{12} * \mu k$

$= 50 * 0.2$

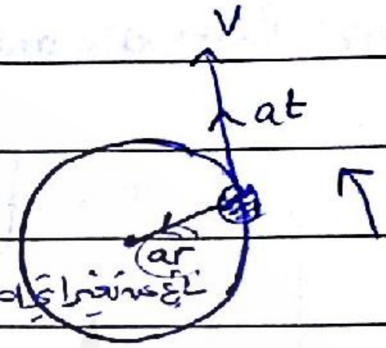
$fk_2 = 10$ Newton

(x) $T = fk_2 = 10$ Newton.

Chapter Six :-

No. ________________

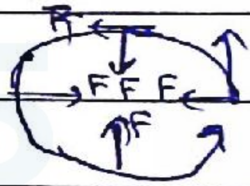
chapter 6 :- Circular Motion :-



$a_{total} = \sqrt{(a_r)^2 + (a_t)^2}$

uniform circular motion , $a_t = 0$

because the $|V|$ is constant .



$$\boxed{\Sigma F_r = m a_r}$$

$$\Sigma F_r = m \frac{V^2}{r}$$

F toward center (+) F outward center (-)

Periodic time. T_p

$$\boxed{V = \frac{2\pi r}{T_p}}$$

دائماً يعتبر القوة الي باتجاه المركز تكون موجبة والـ F مش بالمركز و الـ $F\cos\theta$ الي هي القوة باتجاه المركز هي الـ F_r.

No. ______________

خطوات الحل :

١) نعين جميع القوى

٢) نحدد محاور باتجاه المركز وعمودية عليه.

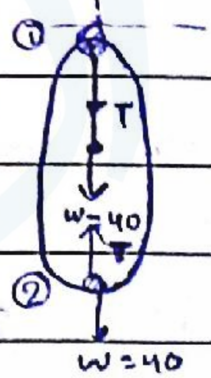
٣) نحلل أي قوة غير منطبقة على المحاور

٤) نطبق بـ لا تزان إن وجد

[القوى العمودية على محور الدائرة ⊥ $\Sigma F_{\uparrow} = \Sigma F_{\downarrow}$]

٥) نطبق في قانون الحركة المركزية $\quad \Sigma F_r = \frac{mv^2}{r}$

(*) Example: An object of mass 4 kg is (attached) subjected to a wire of length 2 m, and rotates in a vertical circle with constant speed $\xrightarrow{}$ 10 m/s, find: T ?



1) at top.

$$\Sigma F_r = \frac{mv^2}{r}$$

$$T + 40 = \frac{4 * 100}{2}$$

central force
كل القوى باتجاه المركز نعطيها + و - اذا عكس الـ center

$$\boxed{T = 160\,N}$$

2) at bottom.

$$\Sigma F_r = \frac{mv^2}{r}$$

$$T - 40 = \frac{4 * 100}{2}$$

$$\boxed{T = 240\,N}$$

→

No. ______

3) $\Sigma F_r = m\frac{v^2}{r}$

$\boxed{T = 200\,N}$

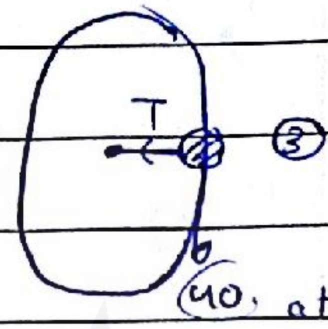
$\Sigma F_t = m a_t$

$\underline{40} = ---$

(at المطلوب)

هذا لو السرعة بتتغير

لكن هو constant speed.



(40, at هي سبب

ΣFr لسب

centerلل خطا

4) $\Sigma F_r = m\frac{v^2}{r}$

$T + 40\cos 60 = 200$

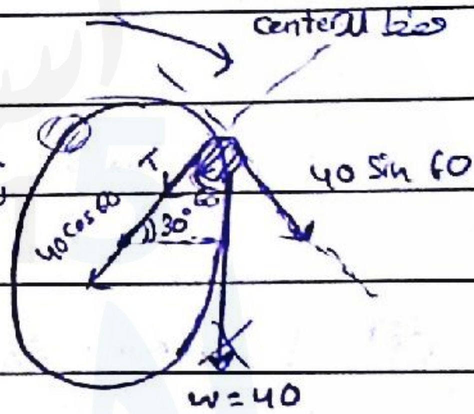
$T + 20 = 200$

$\boxed{T = 180 \text{ Newton}}$

$\Sigma F_t = m a_t$

$40 \sin 60 = ---$

نفس الشي

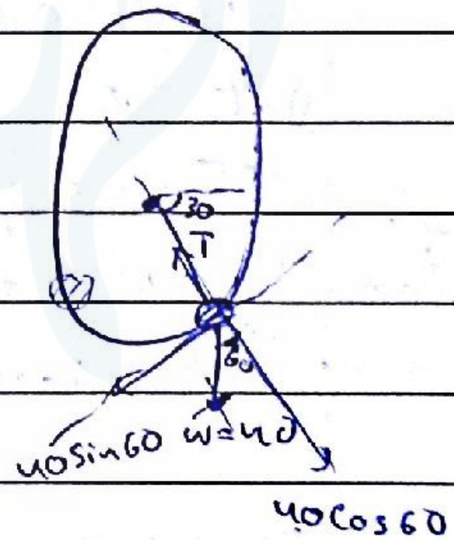


5) $\Sigma F_r = m\frac{v^2}{r}$

$T - 40\cos 60 = 200$

$T - 20 = 200$

$\boxed{T = 220 \text{ Newton}}$

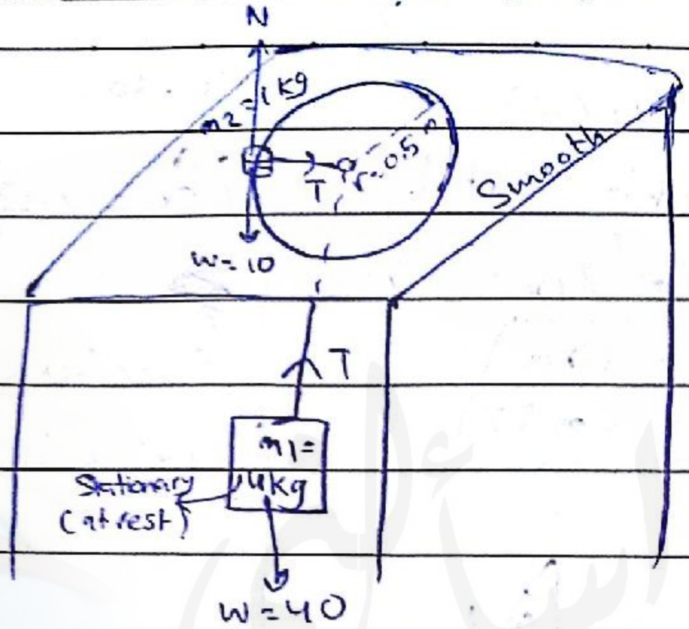


لو كان فيه friction
كيف تكون الحبل والقوى

No. ______ وحد اتجاه الحركة ؟ هل في N ؟

* Example:

Find V :-



Answer:

$\Sigma F_r = \frac{mv^2}{r}$

$T = \frac{1 v^2}{0.5} \rightarrow \boxed{T = 2v^2}$

$\Sigma F_\uparrow = \Sigma F_\downarrow$

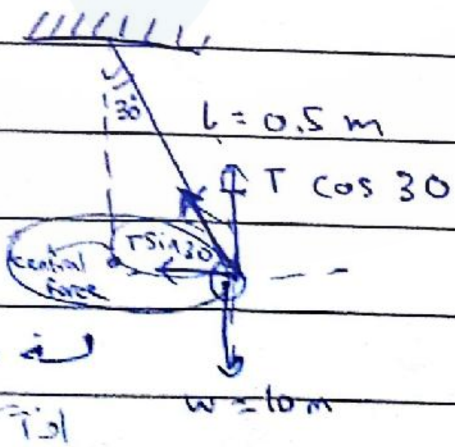
$\boxed{T = 40}$

$40 = 2v^2$

$v = \sqrt{20}$ m/s.

* Example: Conical pendulum :- (المخروط البندولي)

Find Speed of the mass ?



center لـ ماش force باتجاه الـ
اذا الحركة كلها object X

No. ______

Answer:

$\sum F_{\uparrow} = \sum F_{\downarrow}$

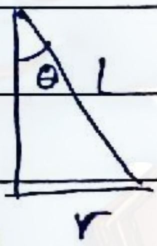
$T\cos 30 = 10m$

$\boxed{T = \dfrac{10m}{\cos 30}}$

$\sin\theta = \dfrac{r}{1}$

$r = 0.5 * 0.5$

$\boxed{r = 0.25\ m}$



$T\sin 30$

$\sum F_r = m\dfrac{v^2}{r}$

$T\sin 30 = \dfrac{mv^2}{r}$

$\dfrac{10\cancel{m}}{\cos 30} * \sin 30 = \dfrac{\cancel{m}v^2}{0.25}$

$\dfrac{5}{0.86} = \dfrac{v^2}{0.25}$

$V = \text{--}\ m/s.$

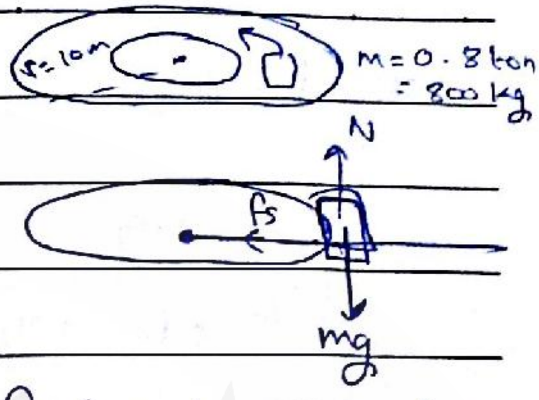
* Note specifically on the previous question only.

$\boxed{T = \dfrac{mg}{\cos\theta}}$

$V = \sqrt{rg\tan\theta}$

$\boxed{v^2 = rg\tan\theta}$

No. ____________



* Example :- Find :

1) μ_s if $v = 5$ m/s

2) V if $\mu_s = 0.2$

fs towards the center of the circle فاد الدائرة ما تنزلق هو الي بتجذبها لجوة.

Answer :

$N = mg$

$\Sigma F_r = \frac{mv^2}{r}$ أو $\mu_s * N$ بساوي ما بتكون شرط μ_s max

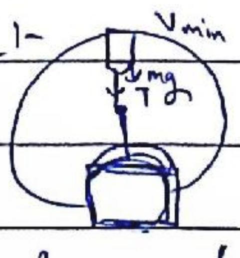
$f_s = \frac{mv^2}{r}$

$\mu_s * N = \frac{mv^2}{r}$

$\mu_s * \cancel{m}g = \frac{\cancel{m}v^2}{r} \longrightarrow \boxed{v^2 = rg\mu_s}$ مهم بجوز الطالب يطلبها

قانون للسيارة اللي بطلع منحنى.

* Example :-

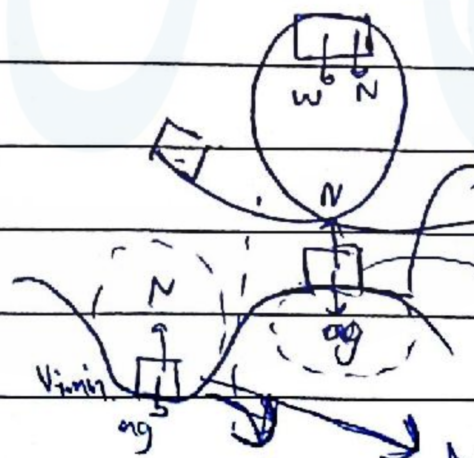


$\Sigma F_r = \frac{mv^2}{r}$

$\cancel{T}$ أو $\cancel{N} + \cancel{m}g = \frac{\cancel{m}v^2}{r}$

عند الـ V_{min} بنعدم

$\boxed{v_{min}^2 = rg}$



$\Sigma F_r = \frac{mv^2}{r}$

$mg - N = \frac{\cancel{m}v^2}{r}$

V_{max} عشان السيارة ما تطير

$\boxed{v_{max}^2 = rg}$

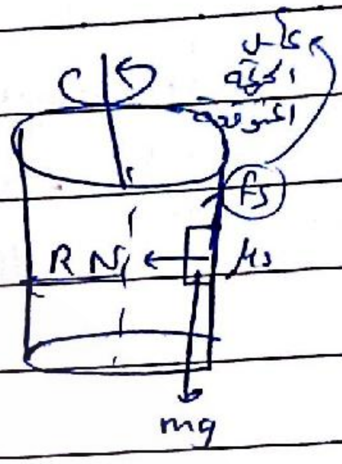
$N - mg = \frac{mv^2}{r}$

$N \neq mg$

لأنه في circular motion

No. ______

* Example :-



$f_s = mg$

$$\boxed{\mu_s * N = mg}$$

$$\Sigma F_r = \frac{mv^2}{r}$$

$$N = \frac{mv^2}{R}$$

$$\frac{mg}{\mu_s} = \frac{mv^2}{R}$$

$$\frac{Rg}{\mu_s} = \frac{4\pi^2 R^2}{T_P^{\,2}}$$

$$T_P^{\,2} = \frac{4\pi^2 R \cdot \mu_s}{g} \longrightarrow T_P = \sqrt{\frac{4\pi^2 R \mu_s}{g}} \ \#$$

اشتق T_P =

$$T_P = \sqrt{\frac{4\pi^2 R \mu_s}{g}}$$

$$V = \frac{2\pi R}{T_P} \rightarrow V^2 = \frac{4\pi^2 R^2}{T^2}$$

$$T_P = \frac{2\pi R}{V}$$

Chapter 7/8 :-

No. ______________

(Work الشغل بيكون عددي بس)
magnitude without direction (scaler)

Chapter 7)8 Work and energy :-

* Kinetic energy (K.E) → K

$$\boxed{K = \frac{1}{2} m v^2}$$ (ع ½)

change in K.E → ΔK

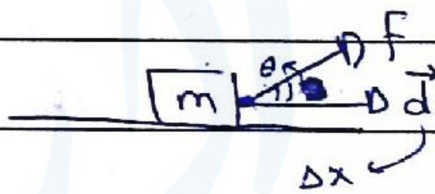
$$\Delta K = \underset{(f)}{K_2} - \underset{(i)}{K_1}$$
$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

* Work :-

[1] Constant force in direction and magnitude.

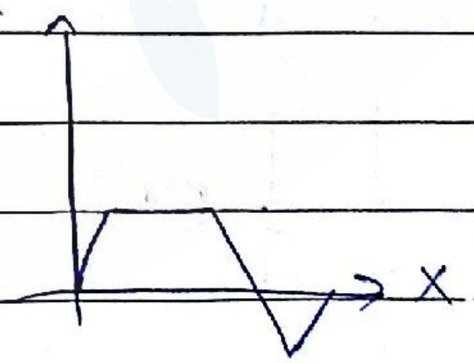
* $W_F = \vec{F} \cdot \vec{d}$ (dot) scaler product.

$$= Fd \cos \underset{Fd}{\theta}$$



[2] if we have changable force in magnitude.

* Work = Area under the curve.



No. ____________

معناها x,y,z مع بعض.

[3] If we have a variable force as a function :-

$F = 2r^2 + 3r + 1$

* $W = \int F_{\hat{i}}\,dx + \int F_{\hat{j}}\,dy + \int F_{\hat{k}}\,dz + \int F\,dr$

variable (x,y,z or t) ← $\vec{F} = 5x^2y\,\hat{i} - 3zxy^3\,\hat{j}$

constant ← $\vec{F} = 2\hat{i} + 3\hat{j}$

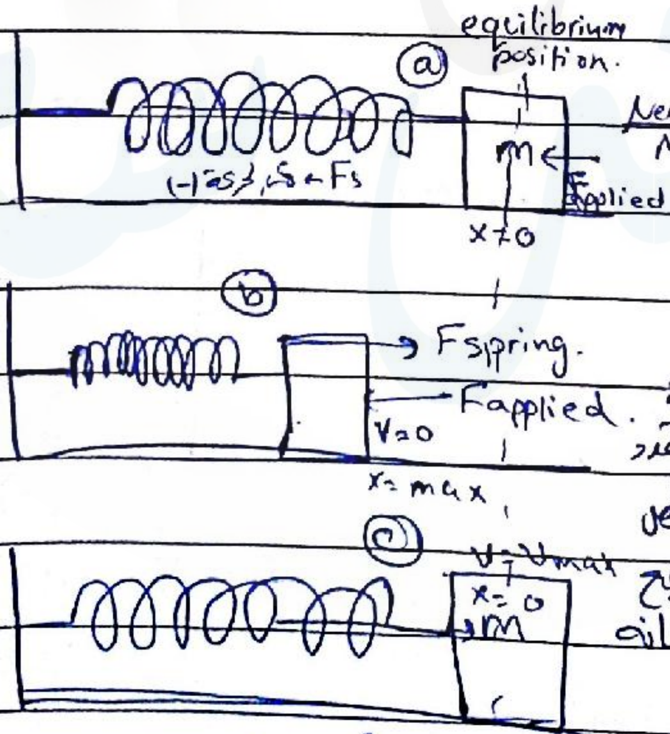
ونعوض بس الي موجود بالسؤال.

$\vec{F} = 3xy\,\hat{i} - 2z\cancel{x}\,\hat{j}$

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$\vec{dr} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

[4] Work of spring force :-



(k) : Spring constant / force constant

Newton / M.

$F_{spring} = \pm kx$ ← the compression distance of the spring.

(Hook's law)

spring force with move (+)

" " عكس (-)

لو تحرك → v = 0, x = max

نقطة البداية ←

* $W_{F\,applied} = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$

* $W_{F\,spring} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$

اذا تحركت الـ Spring باتجاه نقطة الاتزان → (Ks +)

No. ____________

* Work and Energy :- (Sum-up).

$K.E = \frac{1}{2} m v^2$

$\Delta K = K_2 - K_1$

$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

$$\boxed{\Delta K = W_{total}}$$

$v_2^2 = v_1^2 + 2ax$

$\frac{1}{2} v_2^2 - \frac{1}{2} v_1^2 = ax$

$W_{tot} = \Sigma F \, x$ ($ma\,x$) #

* Work :- → displacement.

1) $W_F = Fd\cos\theta_{Fd} = \vec{F} \cdot \vec{d}$

2) W = Area (if you have a diagram between F-x)

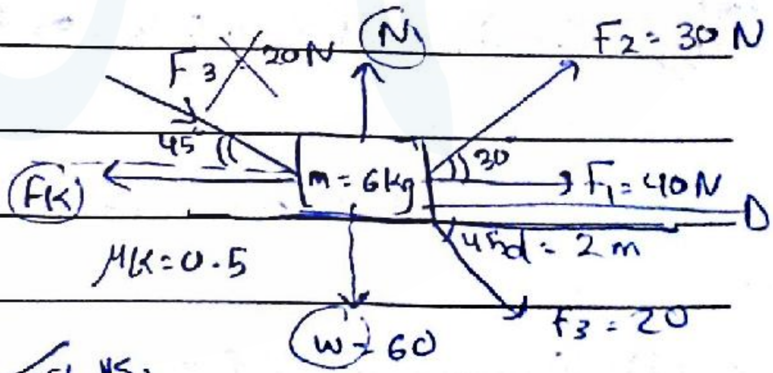
3) $W = \int F dx \,\hat{i} + \int F dy \,\hat{j} + \int F dz \,\hat{k} + \int F dr$

(if you have f as a function of position)

(+) Example :- In the figure, Find :-

① Total Work.

② if $V_1 = 0$, find final Speed.



$30 \sin 30 = 15$

(N) لحتى حطينا بحسابنا لأنه
الأعلى و لفوق
زي تحت

$N + 15 = 60 + 14$

$N = 59$ New →

No. ______

$F_K = \mu_K * N$ (أنا بقدر أطلع ال work

$F_K = 29.5$ Newton. بدون تحليل لأنه أصلاً

مقانون ال work فيه θ

لكن أنا بدي ال F_K علاشان

أطلع).

external forces. ($W_F = F d \cos\theta_{Fd}$).

① $W_{F_1} = 40 * 2 * \cos 0$ أصغر زاوية بين القوة وال d.

$= 80\ J$ → جول.

هو بس بدفع م وضح أو وضح أنا بس طلع فيش ال FK.

$W_{F_2} = 30 * 2 * \cos 30$

$= 52\ J.$

$W_{F_3} = 20 * 2 * \cos 45$

$= 28\ J.$

دائماً

$W_N = N * 2 \cos 90 = 0$

$W_w = 60 * 2 * \cos 90 = 0$ (العكس دائماً)

lost energy. $W_{fK} = 29.5 * 2 * \cos 180 = -59\ J.$

$W_{total} = 80 + 52 + 28 + 0 + 0 + -59 = 101\ J.$

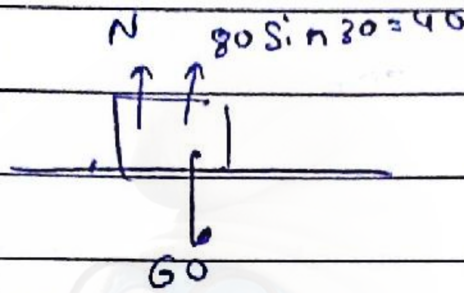
② $W_{total} = \Delta K$

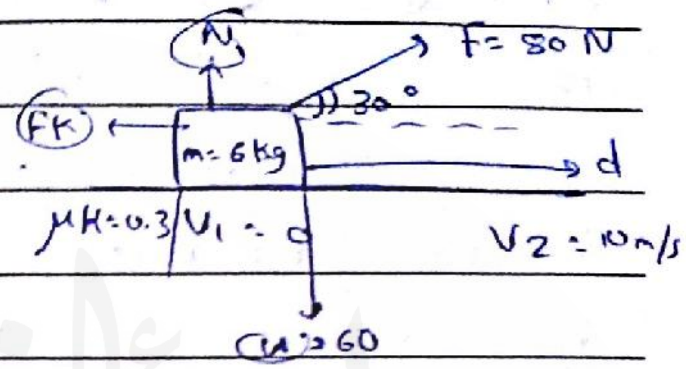
$101 = \frac{1}{2} m V_2^2 - \cancel{\frac{1}{2} m V_1^2}$ ($V_1 = 0$)

$101 = \frac{1}{2}(6)(V_2^2) \longrightarrow V_2 = \sqrt{\frac{101}{3}}$ m/s.

No. ______

Example:- Find d:-





$N + 40 = 60$

$N = 20$

$\boxed{FK = 6 \text{ Newton}}$

$W_{total} = \Delta K$

$\underset{\theta=90}{\cancel{W_N}} + \underset{\theta=90}{\cancel{W_w}} + W_F + W_{fk} = \frac{1}{2} m v_2^2 - \cancel{\frac{1}{2} m v_1^2}$

$(80 * d \cos 30) + (6 * d \cos 180) = \frac{1}{2}(6)(10)^2$

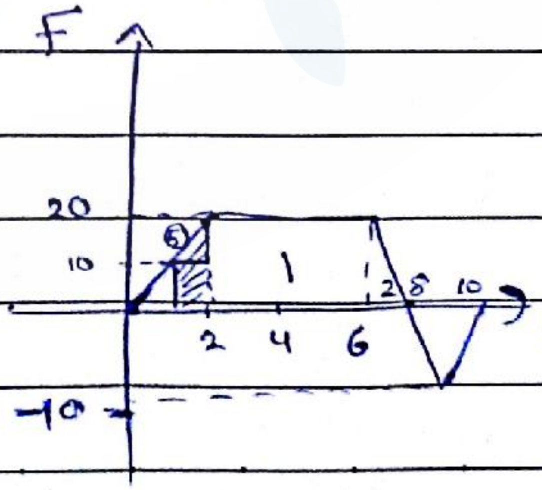
$69.3d - 6d = 300$

$d = ✓ \text{ m.}$

Example:- In the figure shown, find Work from:-

1) $x = 0 \to 2$
2) $x = 2 \to 8$
3) $x = 8 \to 10$
4) total Work.
5) $x = 1 \to 2$
6) if $v_1 = 10$ at $x = 2$

Find v at $x = 8$
$(m = 2 \text{kg})$



No. ____________

① $W = \frac{1}{2} * 2 * 20 = 20\ J.$

② $W = \overset{(1)}{\text{Rectangle}} + \overset{(2)}{\text{triangle}}.$

$= (4 * 20) + (\frac{1}{2} \times 2 \times 20) = 100\ J.$

③ $W = \frac{1}{2} * 2 * -10 = -10\ J.$

④ $20 + 100 - 10 = 110\ J.$

⑤ $W = (1 * 10) + (\frac{1}{2} * 1 * 10) = 15\ J.$

أو الشكل الكبير صحيح أو قانون شبه المنحرف (½ * مجموع القاعدتين * ع)
$(1 * (5+10) * \frac{1}{2}) =$

⑥ $W_{total\,(2\to 8)} = \Delta K$

$110 \neq$

لازم بس من $2 \to 8$

من 2×10

$100 = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

$100 = \frac{1}{2}(2)(v_2^2) - \frac{1}{2}(2)(10)^2$

$v_2 = \sqrt{200} \to v_2 = 14.14\ m/s$

No. ____________

(*) Example :- Find Work from $r = 1$ to $r = 2\,m$ if $F = \frac{2}{r^2}$

Functions & integration.

So :- $W = \int F\,dr = \int_1^2 \frac{2}{r^2}\,dr = \int_1^2 2r^{-2}\,dr = -2r^{-1}\Big]_1^2$

$= \frac{-2}{r}\Big]_1^2$

$= \left(\frac{-2}{2}\right) - \left(\frac{-2}{1}\right)$

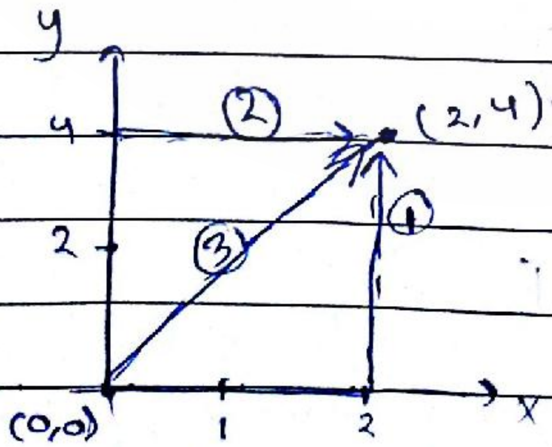
$= 1\,J$

(*) Example :- if $\vec{F} = 2x^2y\,\hat{i} - 3xz\,\hat{j} + 2z^3y\,\hat{k}$ find Work?

$$W = \int 2x^2y\,dx + \int -3xz\,dy + \int 2z^3y\,dz$$

$$= \frac{2x^3y}{3} - 3xzy + \frac{2yz^4}{4} + C$$

(*) Example :- if $\vec{F} = 6xy\,\hat{i} + 2xy\,\hat{j}$ Find work along the path ① 1 ② 2 ③ 3.



$$\frac{y-0}{x-0} = \frac{4-0}{2-0}$$

$$\frac{y}{x} = 2$$

$\therefore \boxed{y = 2x}$

لازم أطلع معادلة الخط المستقيم للفرع ③

لأنه y.N مش constant فلازم أكتبها كلها بدلالة الـ x

* Friction force → non-conservative force.

→ this isn't a conservative force because it's not the same work along the paths.

No.

① $W = \int_0^2 6xy\,dx + \int_0^4 2xy\,dy$ $(y=0)$, $(x=2)$

$= 0 + \frac{2xy^2}{2}\Big|_0^4 = 2[16-0] = 32\ J.$ $(x=2)$

② $W = \int_0^2 6xy\,dx + \int_0^4 2xy\,dy$ $(y=4)$, $(x=0)$

$= \frac{6x^2y}{2}\Big|_0^2 = 12[4-0] = 48\ J.$ $(y=4)$

③ $W = \int_0^2 6xy\,dx + \int_0^4 2xy\,dy$

$= \int_0^2 6x(2x)\,dx + \int_0^4 2\left(\frac{y}{2}\right)y\,dy$

$= \int_0^2 12x^2\,dx + \int_0^4 y^2\,dy$

$= \frac{12x^3}{3}\Big]_0^2 + \frac{y^3}{3}\Big]_0^4$

$= \left(32 + \frac{64}{3}\right) J.$

No. ______

Continuing chapter 8:-

* Potential and mechanical energy.

$$K = \frac{1}{2} m v^2$$

potential (طاقة) تخزين

$$U = mgh + \frac{1}{2} k x^2$$

mechanical energy

$$E = K + U$$

is constant.

$\mu_k N$

$- f_k d$ (rough surface بس)

work of friction W_{f_k}

$E_i = E_f$ (conservative system)

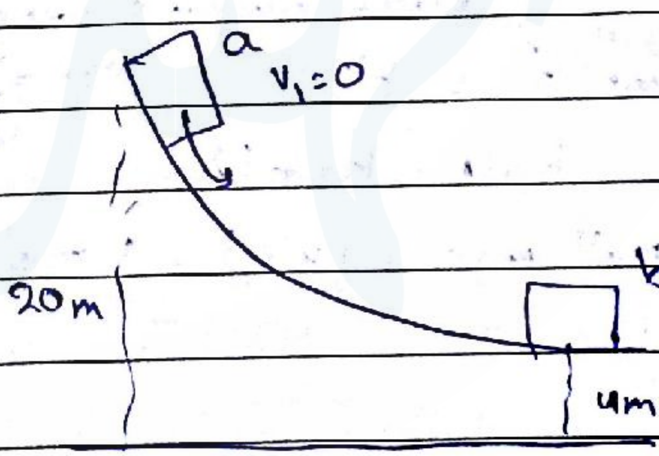
$K_i + U_i = K_f + U_f$ (بالحركة lost energy يعني ما في U_i)

$$\frac{1}{2} m v_i^2 + \frac{1}{2} k x_i^2 + mgh_i = \frac{1}{2} m v_f^2 + \frac{1}{2} k x_f^2 + mgh_f$$

* Example:-

Find Speed at b??

لانه سطح منحني لازم نطبق قانون الطاقة الميكانيكية ما بنعرف أعمل forces لحساب work وهيك



No. ____________

Answer:-

[هلا انطا؟ حافظنا زي ما هو fk
وما في μ_k ولا أعطانا إياها)

$$E_a = E_b$$

$$\cancel{m}gh_a + \tfrac{1}{2}\cancel{m}v_a^2 = \cancel{m}gh_b + \tfrac{1}{2}\cancel{m}v_b^2$$

الوكان spring كان رح
نضيف المسافة (m) خابط $\frac{1}{2}Kx^2$

$$10(20) + \tfrac{1}{2}(0) = 10(4) + \tfrac{1}{2}(v_b^2)$$

$$200 = 40 + \tfrac{1}{2}v_b^2$$

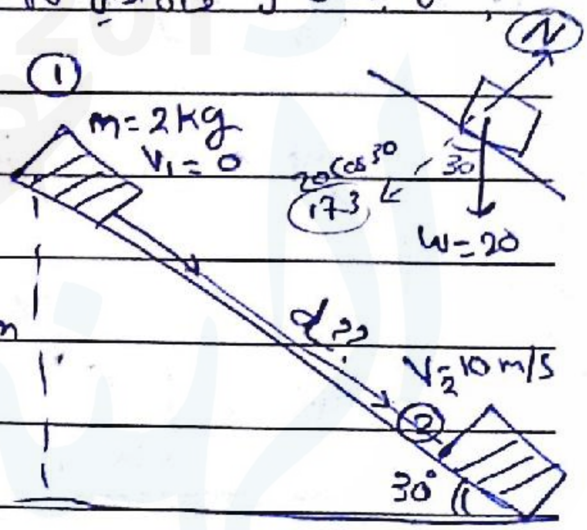
$$\boxed{V_b = \sqrt{320}\ m/s.}$$

لما بصير أصغر الـ g
لأنه لازم تكون
بكل الحدود)

يكال بساوي 17.3 فإن لاقي N

Example:-

Find μ_k??



نستخدم الـ work
حتى لو فيه F
تحسبها
علي كل
الحكاية
أي

$$\sin 30 = \frac{10}{d}$$

$$\boxed{d = 20}$$

Answer:-

$$\boxed{W_{fk}} + E_1 = E_2$$

$$mgh_1 + \cancel{\tfrac{1}{2}mv_1^2} = \cancel{mgh_2} + \tfrac{1}{2}mv^2$$

$$2(10)(10) + \tfrac{1}{2}(2)(0) = 2(0) + \tfrac{1}{2}(2)(10)^2$$

$$-f_k d + 200 = 100\ !!$$

بسبب W_{fk} اللي هو lost energy

$$-\mu_k N * 20$$

$$-\mu_k * 20 * 17.3 + 200 = 100$$

$$0 < \boxed{\mu_k = \frac{100}{346}} < 1$$

إذا في حل فواش   والعوض بالقانون $\frac{1}{2}kx^2 \leftarrow$ spring في نحطها

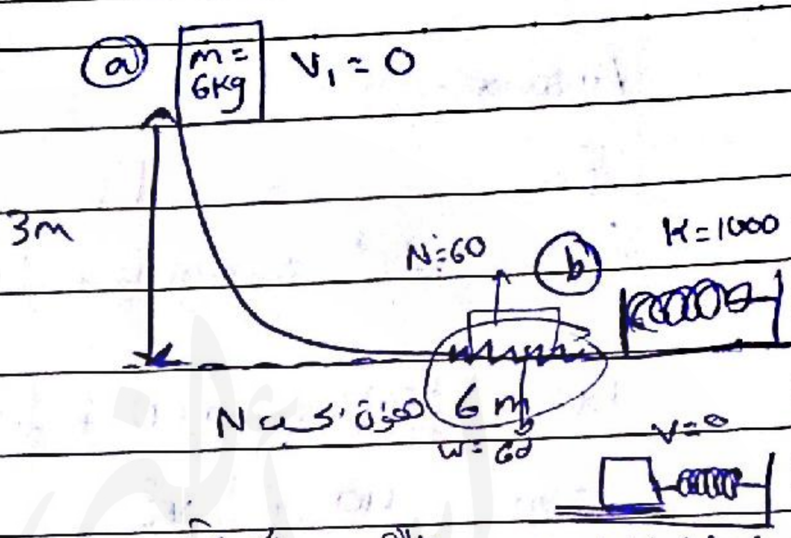
لما الـ work نطلع الـ work $fk \leftarrow$ نحط W_{fk} في

(F في حكاية

No. ______

Example :-

Find coofficient of Kinetic friction (μ_K) ??



Answer :

$$W_{fk} + E_a = E_b$$

الجسم مش موجود عشان   ما له ارتفاع   الـ spring انضغط

$$-(\mu_K * N) * d_{rough} + mgh_a + \cancel{\tfrac{1}{2}mv_a^2} + \cancel{\tfrac{1}{2}kx_a^2} = \cancel{mgh_b} + \cancel{\tfrac{1}{2}mv_b^2} + \tfrac{1}{2}kx_b^2 \quad 0.1\ m$$

(الطاقة التي انضغطها) (لا تقاس ... على ... سرعة) b

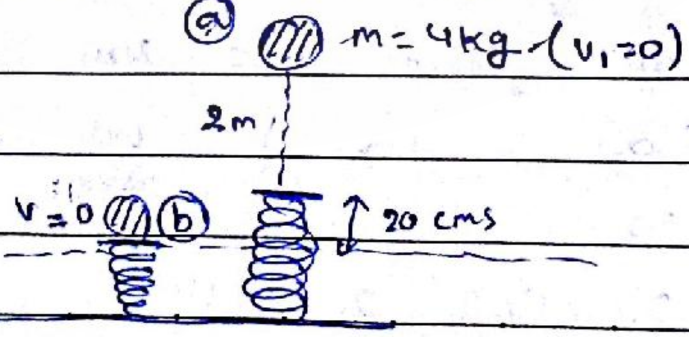
$$-\mu_K * 60 * 6 + 6(10)(3) = \frac{1}{2}(1000)(0.1)^2$$

$$-360\,\mu_K = 5 - 180$$

$$\boxed{\mu_K = \frac{175}{360}}$$

Example :-

In the figure, Find Spring constant (K) if the max compression is 20 cms ?



صفر (h)
مكان يصل له الجسم
(الطاقة الحركة المرجع تكون سالبة)

No. ________________

* Answer:-

$$E_a \overset{\text{الطاقة عند الارتفاع}}{=} E_b \quad \text{عند اقصى انضغاط}$$

$$v=0$$

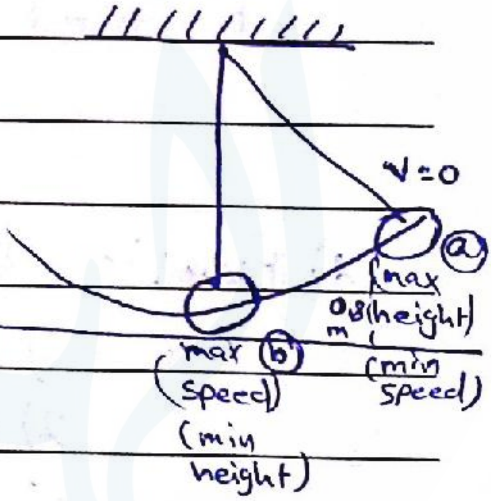
$$mgh_a + \cancel{\tfrac{1}{2}mv_a^2} + \cancel{\tfrac{1}{2}kx_a^2} = \cancel{mgh_b} + \cancel{\tfrac{1}{2}mv_b^2} + \tfrac{1}{2}kx_b^2$$

$$4(10)(2.2) = \tfrac{1}{2}k(0.2)^2$$

$$88 = 0.02\,k \rightarrow \boxed{k = 4400\ N/m}$$

* Example:

if the max height is 0.8m,
Find the max speed ??



$$E_a = E_b$$

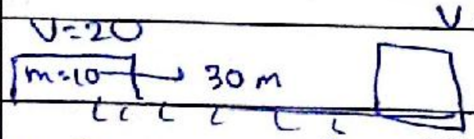
$$\cancel{m}gh_a + \cancel{\tfrac{1}{2}mv_a^2} = \cancel{mgh_b} + \tfrac{1}{2}\cancel{m}v_b^2$$

$$10(0.8) = \tfrac{1}{2}v_b^2$$

$$\boxed{v_b = 4\ m/s}$$

* Quick Ex:-

Find μ_k ??



$$W_{fk} + \tfrac{1}{2}mx^2 = 0$$

ما في سرعة
ولا h
ولا اشي
[illegible]

$$-(\mu_k * \underset{(mg)}{N} * d) = -\tfrac{1}{2}m\,v_1^2$$

No. ______________

* if the Force is given as a function of Position, then:-

$$W = \int F dx \text{ ---}$$

$$U = -W$$

* If U is given by a function of position, then:-

partial derivative

$$\vec{F} = -\hat{i}\frac{\partial U}{\partial x} - \hat{j}\frac{\partial U}{\partial y} - \hat{k}\frac{\partial U}{\partial z} \quad \text{or} \quad \boxed{\vec{F} = \hat{r}\frac{dU}{dr}}$$

(*) Example: If $U = 3x^2yz^3 - 5xz$

Find :- 1) Force as a function of position.

2) magnitude of force at (1,1,2)

3) if $m = 2\,kg$, Find magnitude of a.

Answer:-

1) $\frac{\partial U}{\partial x} = 6xyz^3 - 5z$

[illegible]

$\frac{\partial U}{\partial y} = 3x^2z^3 - 0$ [illegible]

$\frac{\partial U}{\partial z} = 9x^2yz^2 - 5x$

$\longrightarrow$

No. ______________

$$\vec{F} = -\hat{i}(6xyz^3 - 5z) - \hat{j}(3x^2z^3) - \hat{k}(9z^2x^2y - 5x)$$

2) $$\vec{F}_{(1,1,2)} = -\hat{i}(48-10) - \hat{j}(24) - \hat{k}(36-5)$$

$$\vec{F} = -38\hat{i} - 24\hat{j} - 31\hat{k}$$

$$|\vec{F}| = \sqrt{(-38)^2 + (-24)^2 + (-31)^2} = --- N.$$

3) $$|\vec{F}| = m|\vec{a}|$$

$$\sqrt{---} = 2a \rightarrow a = \frac{\sqrt{---}}{2} \; m/s^2$$

(مبذول exerted)

No. ____________

Chapter 9 :-

Linear Momentum and collisions :-

الزخم

momentum (vector) $\vec{p} = m\vec{v}$ (N.s) وحدة

change in momentum $\rightarrow \Delta\vec{p} = \vec{p}_f - \vec{p}_i$

$= m\vec{v}_f - m\vec{v}_i$

Impulse $(\vec{I})$. قبل التصادم مباشرة / بعد التصادم مباشرة

imp

$\vec{I} = \Delta\vec{p} = p_f - p_i = m\vec{v}_f - m\vec{v}_i = \underbrace{\vec{F}}_{\text{external}} \Delta t$ ← time of impact (contact) التلامس

سرعتين قبل الاصطدام وبين الرجة، لازم انا أطبق فيها

أرقام بالسؤال بتجيبه

$$= \int_{t_1}^{t_2} F\,dt =$$ 

$$\vec{F} = \frac{d\vec{P}}{dt} \text{ (functions)}, \quad \vec{F}_{ext} = \frac{\Delta\vec{P}}{\Delta t} \text{ (numbers)}$$

Example :- if $F = 6t - 3$ find impulse from $t = 0$ to $t = 2$

Answer :-

$$I = \int_0^2 (6t-3)\,dt = 3t^2 - 3t \Big|_0^2 = 6 \text{ Newton.sec}$$

No. ____________

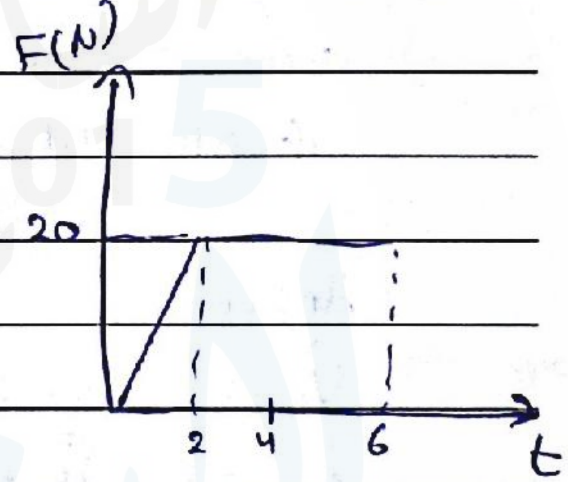
(*) Example:- if $P = 12t^2 - 5t + 1$, Find F at $t=4$.

$F = \frac{dP}{dt} = 24t - 5 = 91$ Newton.

(*) Example: In the figure, Find:-

1) Impulse from $t=0$ to $t=6$

2) if $m = 2$ kg find V at $t=6$ if $v=0$ at $t=0$



1) $I =$ Area

$= (\frac{1}{2} * 2 * 20) + (4 * 20) = 100$ N.S

2) $I = mv_2 - mv_1$

$100 = 2v_2 - 2(0)$

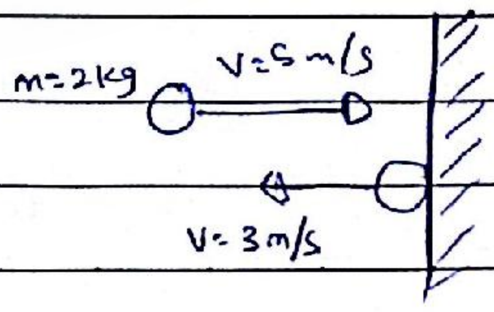
$v_2 = 50$ m/s.

(*) In the figure, Find:-

1) Impulse

2) if time of impact is 0.2 sec, Find the force exerted from Wall on the ball?

Answer:-



1) $\vec{I} = m\vec{v_2} - m\vec{v_1}$

$= 2(-3) - 10$

$= -16$ N.S

$\hookrightarrow -\hat{i}$

(to the left)

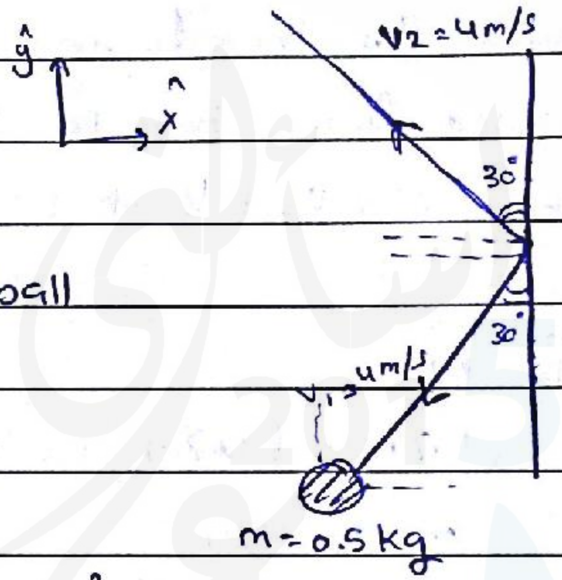
$V_2 \rightarrow$ الجهة الي انطلق فيه الكرة
$V_1 \rightarrow$ صفر

No. ____________

② $I = F\,\Delta t$

$-16 = F(0.2)$

$F = -80$ Newton
$\hookrightarrow -\hat{i}$



(*) Find:-

1) Impulse.

2) Force on the ball if $\Delta t = 0.1$

Answer:-

Supposedly

$\vec{V}_1 = (\quad)\hat{i} + (\quad)\hat{j}$

$\vec{V}_2 = -(\quad)\hat{i} + (\quad)\hat{j}$

$\vec{V}_2 - \vec{V}_1 = -(\quad)\hat{i}$

وال $\hat{j}$ يروح كلهم.

ولازم كلل V_1 V_2 لـ $\hat{i}$ و $\hat{j}$

ونجيب $V_2 - V_1$

(وفلهم بنحل السؤال
$\vec{I} = m\vec{V}_2 - m\vec{V}_1$
منطقي فيه بس)

{ لأنه مقدار السرعة نفسه

اذا $\hat{i}$, $\hat{j}$ رح يكون اتجاه الـ I للـ $(-\hat{i})$

بس لانهم نفس الـ سرعة)

No. ______

* Collisions between two objects :-

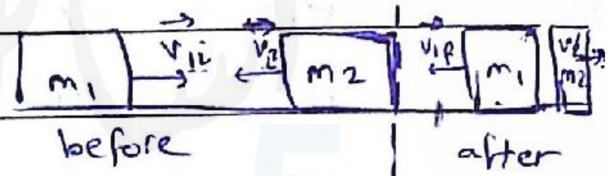
Emphlse $(I = F_{ext}\,\Delta t = 0$

$\Delta\vec{P} = \vec{P}_f - \vec{P}_i = 0$

momentum $\vec{P}_i = \vec{P}_f$ → [conservation of momentum].

$(m\vec{v})$

1) Elastic collision :- (تصادم مرن)



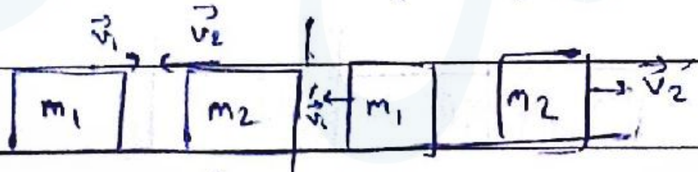
$\vec{P}_i = \vec{P}_f$

$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$ (لحظة التصادم لا يوجد lost energy)

$K_i = K_f$

$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$

2) In-elastic collision :- (تصادم غير مرن)



(يوجد lost energy لحظة التصادم)

$\vec{P}_i = \vec{P}_f$

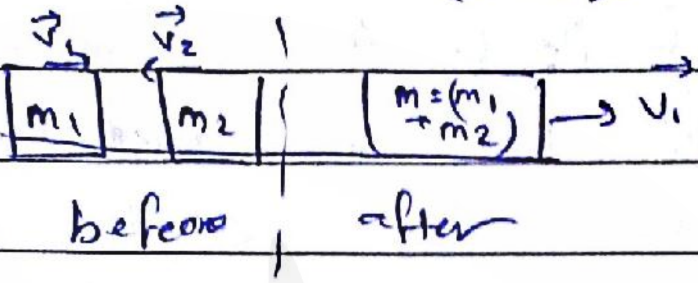
$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$

lost energy due to collision.

$\Delta K = K_f - K_i = (-)$ الطاقة بعد التصادم لازم تكون أقل لأنه في طاقة ضائعة.

No. ________________

3) Completely in-elastic collision (يلتحم الجسمان ويصيروا واحد)

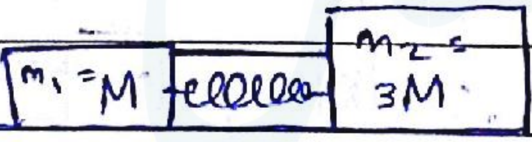


$$\vec{P}_i = \vec{P}_f$$

$$m_1\vec{v}_1 + m_1\vec{v}_2 = (m_1+m_2)\vec{v}'$$

$$\Delta K = K_f - K_i = \left(\frac{1}{2}(m_1+m_2)V'^2\right) - \left(\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2\right)$$

(*) Example :- In the figure, Find the speed $\underline{V_2}$ of m_2 if v_1 of m_1 is $\underline{V}$ to the left [after burning the string]



Answer :-

$$P_i = P_f$$

$$M_1(0) + 3M_2(0) = M(-V) + 3M(\vec{V}_2)$$

$$0 = -MV + 3M\vec{V}_2$$

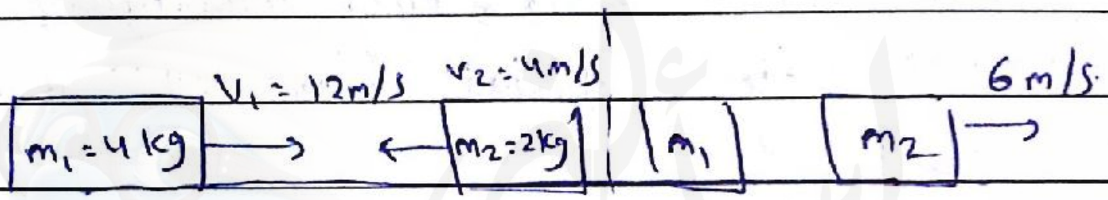
$$\cancel{M}V = 3\cancel{M}\vec{V}_2$$

$\vec{V}_2 = \frac{V}{3}$ إذاً اتجاهها لليمين لأنهم عكس بعض رح يروحوا

No. ____________

Example: In the figure, Find:

1) Velocity of m_1 after collision.
2) is the collision elastic or in-elastic?



Answer:

1) $P_i = P_f$

$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$ (لما جداء A بـ لتي بدك (x))

$4(12) + 2(\bar{4}) = 4(V_1') + (2)(6)$

لأنه يسار — لأنه يمين

$\boxed{V_1' = 7 \ m/s.}$ it will move to the right direction.

بدنا نحكي عشان نعرف اذا تم lost energy أو لا.

2) $K_i = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2$

$= \frac{1}{2}(4)(12)^2 + \frac{1}{2}(2)(4^2)$

$= 304 \ J.$

$K_f = \frac{1}{2} m_1 V_1'^2 + \frac{1}{2} m_2 V_2'^2$

$= \frac{1}{2}(4)(7^2) + \frac{1}{2}(2)(6)^2$

$= 98 + 36 = 134 \ J$

in-elastic collision because there is lost energy and $K_i \neq K_f$.

$\Delta K = K_f - K_i$

$= -(\quad) \ J.$

(lost energy الطاقة الضائعة)

$(3.5)^2 = \begin{array}{r} 3.5 \\ 4.5 \\ \hline 12.25 \end{array}$

لحساب ضرب الأرقام الي بآخرها 0.5
نحط الرقم والي بعده ونضربهم
بس كل واحد بالي تحته

No. ____________

(*) Example:- A block of mass $m_1 = 2kg$ is moving to the right with speed $4 m/s$, another block $m_2 = 3kg$ is moving to left with speed $6 m/s$, find Velocity of each one after collision if the collision is elastic?

Answer:-

$$P_i = P_f$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$2(4) + 3(-6) = 2v_1' + 3v_2'$$

$$2v_1' + 3v_2' = -10 \quad --- (1) \quad \rightarrow \boxed{v_1' = -5 - 1.5v_2'}$$

ونعوض v_1' بالمعادلة الثانية

$$K_i = K_f$$

$$\cancel{\tfrac{1}{2}}(m_1 v_1^2) + \cancel{\tfrac{1}{2}}(m_2 v_2^2) = \cancel{\tfrac{1}{2}}(m_1 v_1'^2) + \cancel{\tfrac{1}{2}}(m_2 v_2'^2)$$

$$2(4)^2 + 3(-6)^2 = 2(v_1')^2 + 3(v_2')^2$$

$$2v_1'^2 + 3v_2'^2 = 140 \quad -- (2)$$

نطلع قيمة v_2' ثم نرجع نطلع v_1' ونعوضه.

No. ____________

* Elastic collision :-

memo

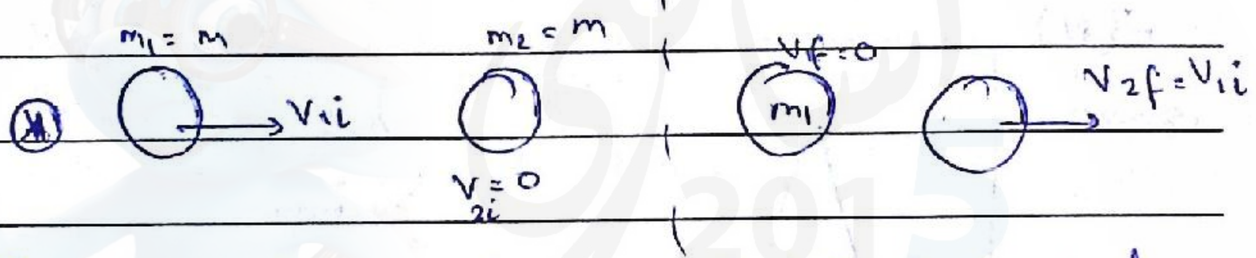
$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

الكل عليم

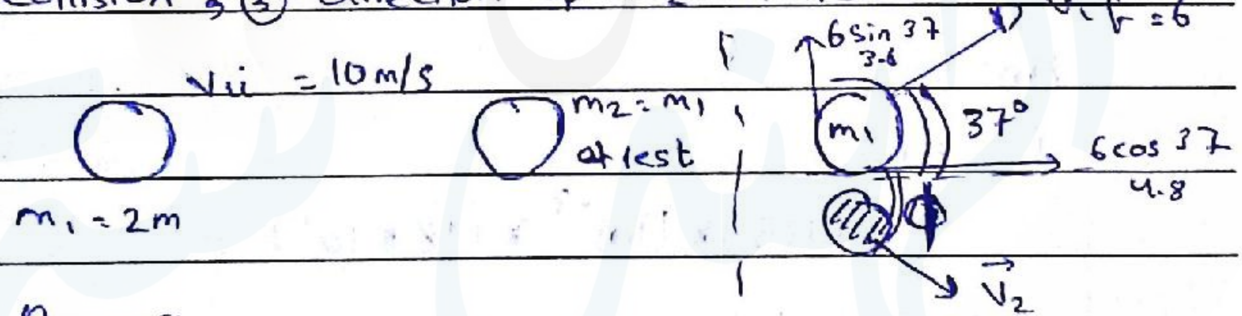
$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

التوالي، علامن تاسم

عامل (رياضيات)



* Example :- In the figure, Find ① velocity of m_2 after collision, ② speed of "m_2" after collision, ③ direction of m_2 motion.



Answer:

$$\vec{P_i} = \vec{P_f}$$

$$m_1\vec{v}_{1i} + \cancel{m_2\vec{v}_{2i}} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$2\cancel{m}(10\hat{i}) = 2\cancel{m}(4.8\hat{i} + 3.6\hat{j}) + \cancel{m}\vec{v}_{2f}$$

$$20\hat{i} = 7.2\hat{j} + 9.6\hat{i} + \vec{v}_{2f}$$

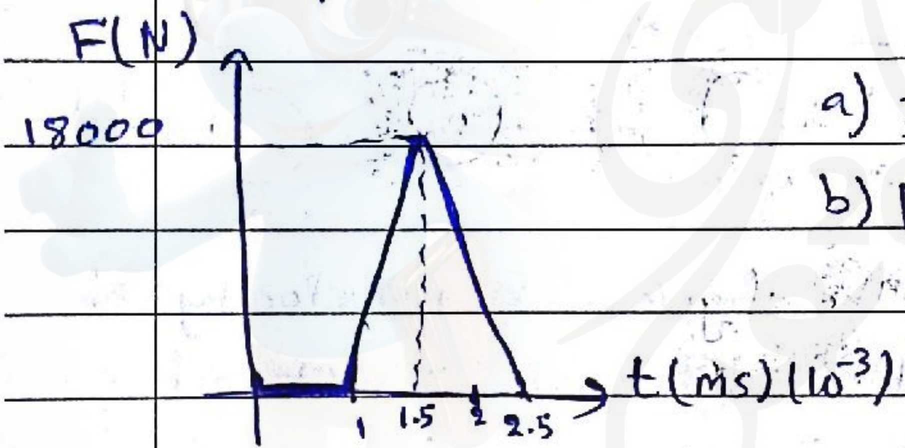
① $\boxed{\vec{v}_{2f} = 10.4\hat{i} - 7.2\hat{j}}$

No. ______

② Speed: $|\vec{V_2}| = \sqrt{(10.4)^2 + (7.2)^2} = ----$

③ $\tan\phi = \frac{7.2}{10.4} = --$

④ Example:- Chapter 9. problem 13.



a) I
b) Fav.

Answer:-

ⓐ I = Area

$= \frac{1}{2} * (1.5) * (10)^{-3} * (18 * 10^3)$

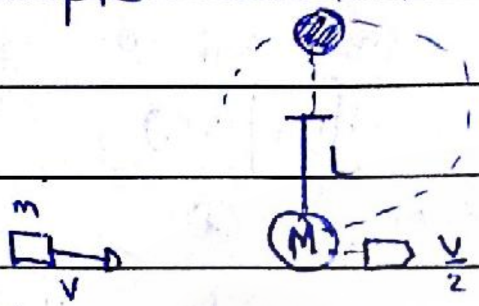
$= 13.5$ N/s.

ⓑ $F_{av} = I/\Delta t$

$= 13.5 / 2.5 * 10^{-3}$ N.

No. ____________

Example :- Ch. 9 P. 30



Answer :-

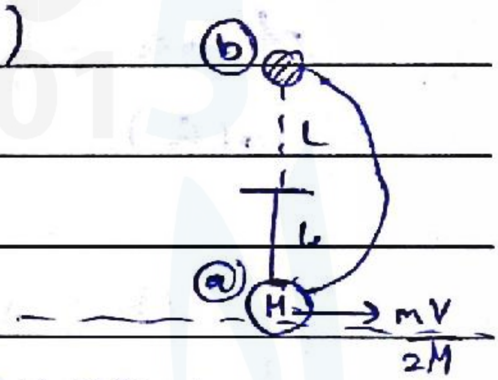
$$P_i = P_f$$

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

$$mv + \cancel{M(0)} = m\left(\frac{v}{2}\right) + M(v_2')$$

$$\frac{mv}{2} = Mv_2'$$

$$\boxed{v_2' = \frac{mv}{2M}}$$



(مكمل السؤال ع الطاقة الميكانيكية)

(لأولها تكون Collision مفيش mech. energy)

$$E_a = E_b$$

$$\frac{1}{2}\cancel{M}v_a^2 + \cancel{Mgh_a} = \cancel{M}gh_b + \frac{1}{2}\cancel{M}v_b^2$$

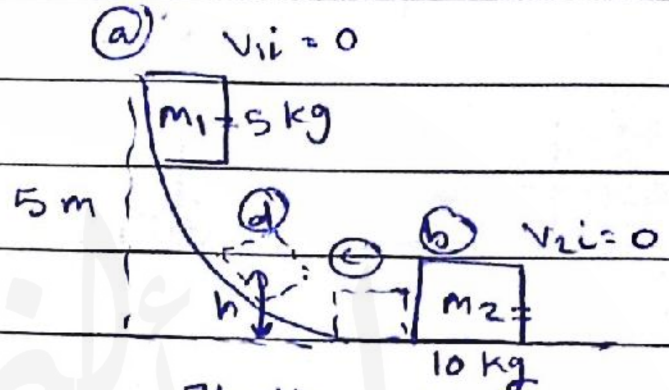
$$\frac{1}{2}\left(\frac{m^2v^2}{4M^2}\right) + g(0) = g(2L) + \frac{1}{2}(0)$$

$$v^2 = \frac{16\,gLM^2}{m^2}$$

$$v_{min} = \sqrt{\frac{16\,gLM^2}{m^2}} \text{ m/s}$$

No. ____________

(*) Example :- Ch.9 prob 33.



Elastic collision.

Answer:-

$$E_a = E_b$$

$$\cancel{m}gh_a + \cancel{\frac{1}{2}mv_a^2} = \cancel{mgh_b} + \frac{1}{2}\cancel{m}v_b^2$$

$$10(5) = \frac{1}{2}v_b^2$$

$$v_b = 10 \text{ m/s}$$

$$\to v_{1i} = 10$$

$$v_{2i} = 0$$

$$P_i = P_f$$

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

(بس هاي لسى 2 بالفوق) (مطبق ال formula

$$v_{1f} = \left(\frac{5-10}{5+10}\right)10 + \cancel{\left(\frac{2m_2}{m_1+m_2}\right)(0)}$$

$$v_{1f} = \frac{-10}{3} \text{ m/s.}$$

No.

$$E_c = E_d$$

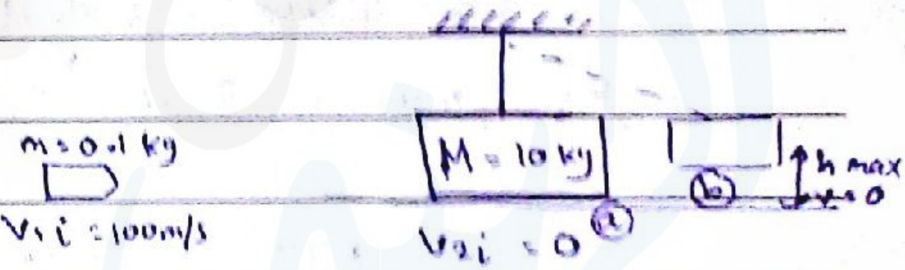
$$\cancel{mgh_c} + \frac{1}{2}\cancel{m}v_c^2 = \cancel{m}gh_d + \cancel{\frac{1}{2}mv_d^2}$$

$$\frac{1}{2}\left(\frac{-10}{3}\right)^2 = 10h$$

$$\boxed{h = \frac{10}{6}\ m}$$

Example: In the figure, Find:

1) Speed of system just after collision.
2) max. height the block-bullet will reach.



completely in-elastic collision.

Answer:-

$$P_i = P_f$$

$$m_1 v_{1i} + \cancel{m_2 v_{2i}} = (m_1 + m_2) V_f$$

$$0.1 * 100 = 10.1 * V_f$$

$$V_f = 0.99\ m/s.$$

No. ________________

$$E_a = E_b$$

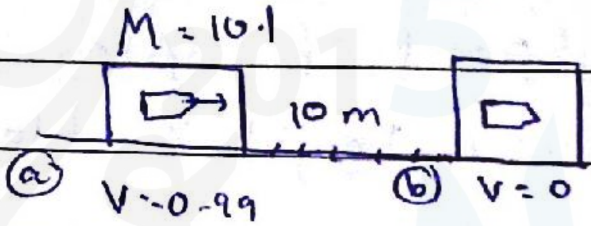
$$\cancel{mgh_a} + \frac{1}{2}\cancel{m}v_a^2 = \cancel{mgh_b} + \cancel{\frac{1}{2}mv_b^2}$$

$$\frac{1}{2}(0.99)^2 = 10h$$

$$h = 0.05\,m = 5\,cm.$$

Example :-

find μ_k??



$$E_a + W_{fk} = E_b$$

$$\cancel{mgh_a} + \frac{1}{2}mv_a^2 + -f_k d = \cancel{mgh_b} + \cancel{\frac{1}{2}mv_b^2}$$

$$\frac{1}{2}(10.1)(0.99)^2 = \mu_k * N(10)$$

$$\frac{1}{2}(\cancel{10.1})(1) = \mu_k(10)\underset{(\cancel{10.1})(g)}{(mg)}$$

$$\mu_k = 0.005.$$

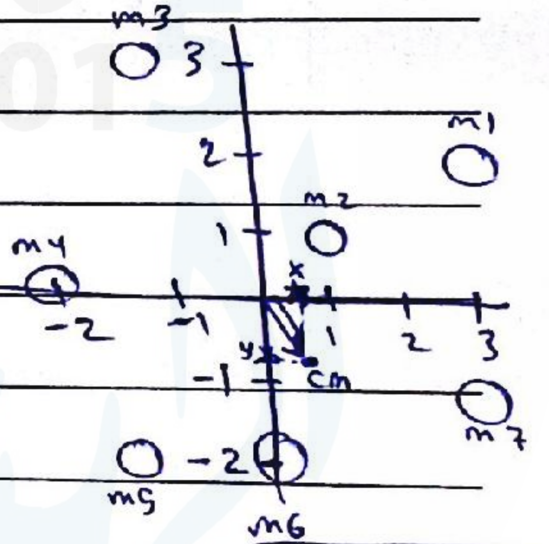
Center of Mass:-

$$X_{(cm)} = \frac{m_1x_1 + m_2x_2 + \cdots}{M}$$

center of mass

$$X = \frac{1*(3) + 2*(1) + (3)(-1) + (4*-2) + (5)(-1) + (6*0) + (7*(3))}{1+2+3+4+5+6+7}$$

$$= \frac{10}{28}\ m$$



$$y_{(cm)} = \frac{1*2 + (2(1)) + (3(3)) + (4(0)) + (5*-2) + (6*-2) + (7(-1))}{28}$$

$$= \frac{-16}{28}\ m$$

$$\boxed{\vec{r}_{cm} = X_{cm}\hat{i} + y_{cm}\hat{j}}$$

No. ____________

* Chapter 10 ; Rational motion.

x: position

Δx: displacement

v: speed

a: acceleration

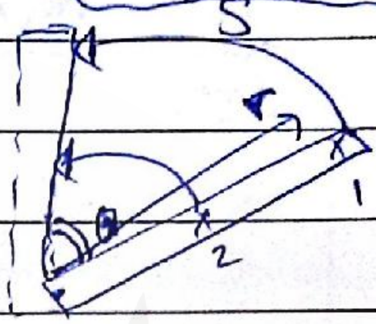
t: time

m: mass

F: Force

$\tan\theta = \frac{S}{r}$

$\theta = \frac{S}{r} \rightarrow \boxed{\theta r = S}$



θ: angular position

$\Delta\theta$: angular displacement

ω: angular velocity.

α: angular acceleration

t: time

(m ﺩﻭ) I: moment of Inertia.

(F ﺩﻭ) τ: Torque.

$\omega_{av} = \frac{\Delta\theta}{\Delta t}$

$\omega_{ins} = \frac{d\theta}{dt}$

$\alpha_{av} = \frac{\Delta\omega}{\Delta t}$

$\alpha_{ins} = \frac{d\omega}{dt}$

$\omega_2 = \omega_1 + \alpha t$

$\omega_2^2 = \omega_1^2 + 2\alpha\Delta\theta$

$\Delta\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

$\Delta\theta = \left(\frac{\omega_1+\omega_2}{2}\right) t$

No. ________________

$S = \boxed{x = r\theta}$

$\boxed{v = r\omega}$

$\boxed{a_t = r\,\alpha_t} \rightarrow$ لكن لا
ينطبق على r

~~where~~

$\alpha_t = \frac{d\omega}{dt}$

$a_r = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = \frac{r^2\omega^2}{r} \rightarrow \boxed{a_r = r\omega^2}$

$a_{total} = \sqrt{a_r^2 + a_t^2}$

$= \sqrt{(r\omega^2)^2 + (r\alpha)^2}$

$= r\sqrt{\omega^4 + \alpha^2}$

* θ : rad

$1\theta \text{ (rad)} = \frac{180}{\pi}$ degree

* ω = rad / s
* α : rad / s^2

(4) Example:- A disk started rotation from $\theta = 30°$ to $90°$ from rest - if the rotation takes 10 sec, find:-

1) final angular speed.
2) if the radins is 0.5 m, Find the Linear speed.
3) angular and Linear acceleration.

No.

Answer:-

$\Delta\theta = 60^\circ$ $\quad w_1 = 0$ $\quad t = 10\text{ sec}$ $\quad r = 0.5$

$= 1.05 \text{ rad}$ $\quad v_1 = 0$

1) $2\Delta\theta = (w_1 + w_2)t$ $\qquad \Delta\theta = 60 * \frac{\pi}{180} = \frac{\pi}{3}$

$2.1 = w_2 * 10$ $\qquad = 1.05$

$w_2 = 0.21 \text{ rad/sec}$

2) $v_2 = r w_2$

$= 0.5 * 0.21$

$= 0.105 \text{ m/s}$

3) $w_2 = w_1 + \alpha t$

$0.21 = \alpha(10)$

$\alpha = 0.021 \text{ rad/s}^2$

$a = r\alpha = 0.5 * 0.021 = 0.0105 \text{ m/s}^2$

Example: if $\theta = 2t^2 + 3t + 1$, Find:-

1) angular displacement from $t=0 \rightarrow t=1$
2) average angular velocity from $t=0 \rightarrow t=1$
3) instantainous angular velocity at $t = 3$ sec.
4) avergge angular acceleration from $t=0 \rightarrow t=2$
5) instantainous angular acceleration at $t = 4$.

No. ____________

Answer :- $\theta = 2t^2 + 3t + 1$

$\omega = 4t + 3$

$\alpha = 4$

1) $\theta_1 = 1$

$\theta_2 = 2(1)^2 + 3(1) + 1 = 6$

$\Delta\theta = 6 - 1 = 5$ rad.

2) $\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{5}{1} = 5$ rad/s

3) $W_{ins} = 4(3) + 3 = 15$ rad/sec

4) $\alpha_{av} = \frac{\Delta\omega}{\Delta t} = \frac{11-3}{2-0} = 4$ rad/s^2

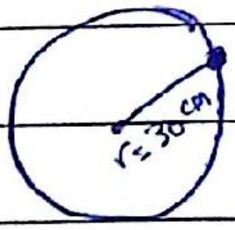
$\omega_1 (t=0) = 0 + 3 = 3$

$\omega_2 (t=2) = 4(2) + 3 = 11$

5) $\alpha_{ins} = 4$ rad/s^2

(*) Example :-

Find total acceleration?



$\omega = 2$ rad/sec (constant)

No. ______________

Answer:-

$a_r = r\omega^2$

$= (0.3)(2)^2$

$= 1.2\ m/s^2$

$a_t = r\alpha = \text{zero}$

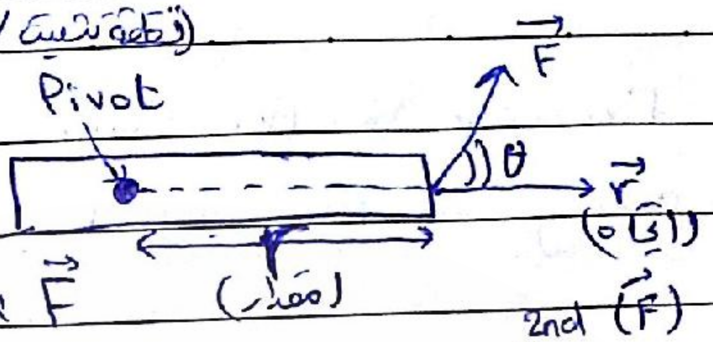
$\left(\alpha = \frac{d\omega}{dt} = 0\right)$

$a_{total} = a_r = 1.2\ m/s^2$

No. ______

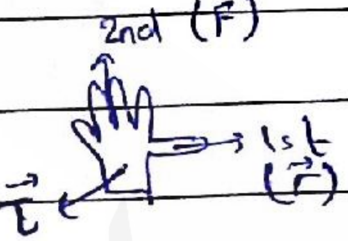
(عزم القوة) (نقطة تثبيت / محور دوران)

* Torque : $(\vec{\tau})$



$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= r F \sin\theta_{rF}$$



(C.C.W) → counter clock wise (عكس عقارب الساعة)

$\boxed{\tau = +\hat{k}}$ → لأنه خارج

لو τ بتكون عكس z .

$\boxed{\tau = -\hat{k}}$ → مع عقارب الساعة

* Example :- if $\vec{r} = 3\hat{i} + 2\hat{j}$

$\vec{F} = -2\hat{i} + 9\hat{j}$

Find τ ??

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 0 \\ -2 & 9 & 0 \end{vmatrix}$$

$$= \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(27+4)$$

$\vec{\tau} = 31\hat{k}$.

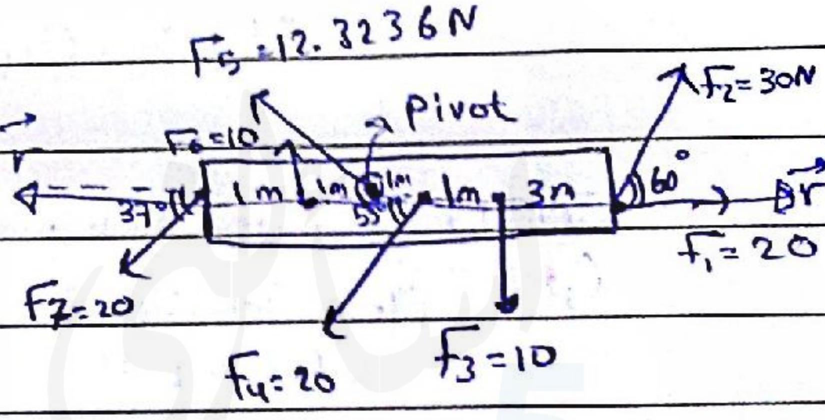
$\tau = |\vec{\tau}| = 31$ N.m (القيمة تعتمد وجهة)

(ال work لكن لا نستخدم جول)

No. ______

Example: In the figure, Find the net Torque acting on the object? (بنحكيلهم لوين بده يدور؟)

(نحسب T ومنشوف شو اشارتها)



Answer:-

$T_1 = 5 * 20 * \sin 0 = 0$

$T_2 = 5 * 30 \sin 60 = +150\frac{\sqrt{3}}{2} = +130$ (عكس +)

$T_3 = 10 * 2 * \sin 90 = -20$ N.m (مع --) → متعلق بالحتي (-) للأسفل اذاً مع.

$T_4 = 1 * 20 * \sin 127 \; (53) = -16$ N.m (مع --)

$T_5 = 0$

$T_6 = 1 * 10 * \sin 90 = -10$ N.m (مع عقارب)

$T_7 = 2 * 20 * \sin 37 = +24$ N.m (عكس +)

Net Torque $= 130 - 20 - 16 - 10 + 24 = \oplus$

(عكس عقارب الساعة) c.c.w

الشكل صفحة ٣.٤

No. ______

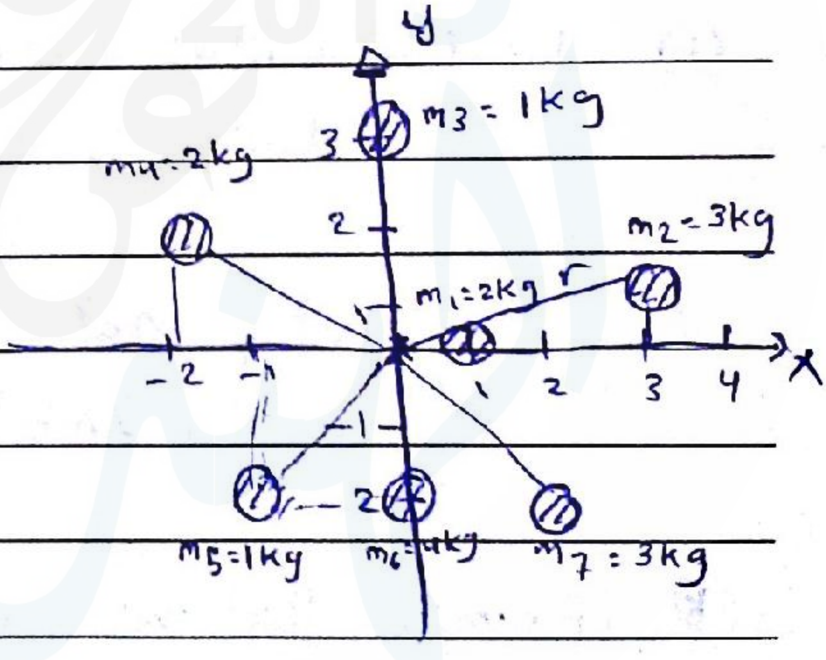
* moment of inertia (I) مانعة الجسم للدوران .

$$I = \begin{cases} \sum_{i=1}^{n} m_i r_i^2 \longrightarrow \text{discreat objects} \\ \int r^2 dm. \longrightarrow \text{continuous objects} \end{cases}$$

أجسام منفصلة

أجسام متصلة

* Example:- In the figure, Find the moment of inertia around the origin?



Answer:-

$I = \overset{m_1}{(2)}\overset{r}{(1)}^2 + \overset{m_2}{(3)}(\overset{2}{3}+1^2)$

origin لبعدها عن ال

$+ \overset{m_3}{(1)}(3^2) + \overset{m_4}{(2)}(2^2+2^2)$

$+ \overset{m_5}{(1)}(1^2+2^2) + \overset{m_6}{(4)}(2)^2$

$+ \overset{m_7}{(3)}(2^2+2^2)$

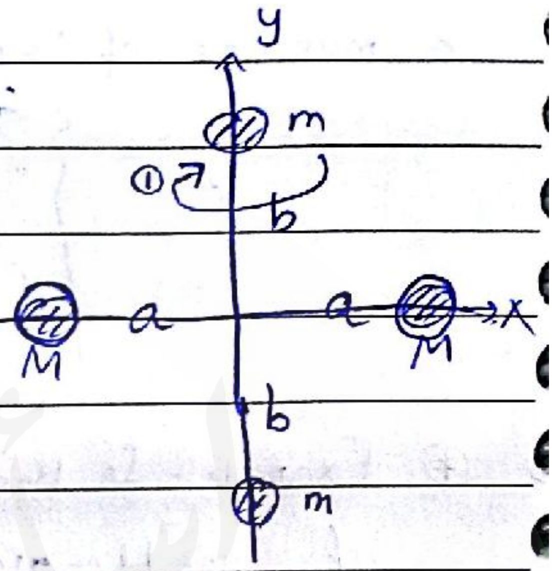
$= 2 + 30 + 9 + 16 + 5 + 16 + 24$

$I = 102 \ kg.m^2$

★ Example!

Find I if the rotation is about:-

1) y - axis.
2) z - axis.



Answer:-

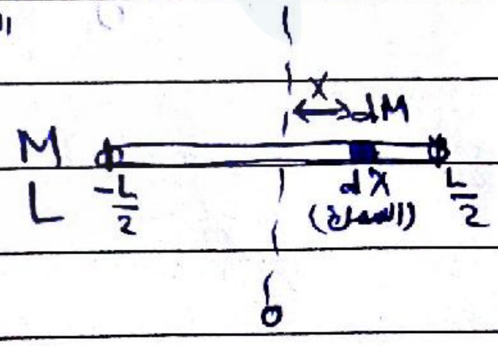
① $I = Ma^2 + Ma^2 + 0 + 0$
$= 2Ma^2$.

② $I = Ma^2 + Ma^2 + mb^2 + mb^2$
$= 2Ma^2 + 2mb^2$

★ Example:- find the moment of inertia for a rod of length L rotates around it's center of mass. المنتصف تماماً

Answer:- $I = \int r^2\, dm$

$\int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \lambda\, dx$

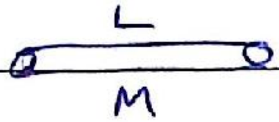


$\lambda = \frac{M}{L}$

$= \lambda \left[\frac{1}{3} x^3 \right]_{-\frac{L}{2}}^{\frac{L}{2}}$

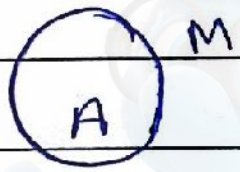
$= \frac{\lambda}{3} \times \left[\frac{L^3}{8} - \left(-\frac{L^3}{8} \right) \right) = \frac{\lambda L^3}{12} = \frac{M}{L} \frac{L^3}{12}$ $M = \lambda L$ $\boxed{dM = \lambda dx}$ $= \boxed{\frac{1}{12} * ML^2}$

No. ____



$M = \lambda L$

$\lambda = \frac{M}{L} \longrightarrow \lambda$: linear mass density..



$M = \sigma A$

$\sigma = \frac{M}{A} \longrightarrow \sigma$: Surface mass density.

$M = \rho V$

$\rho = \frac{M}{V} \longrightarrow \rho$: Volumetric mass density.
(Ro)

CH: 11 Angular Momentum

د. أحمد عوض

فيزيا 101

* أشياء لا بد من معرفتها أولاً :

1) cross product :

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$|\vec{C}| = |\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta_{AB}$$

$$\vec{C} = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}(A_yB_z - A_zB_y) - \hat{j}(A_xB_z - A_zB_x) + \hat{k}(A_xB_y - A_yB_x)$$

$$\Rightarrow C_x = A_yB_z - A_zB_y \qquad C_y = -(A_xB_z - A_zB_x) \qquad C_z = (A_xB_y - A_yB_x)$$

$$\boxed{\vec{C} = C_x\hat{i} + C_y\hat{j} + C_z\hat{k}}$$

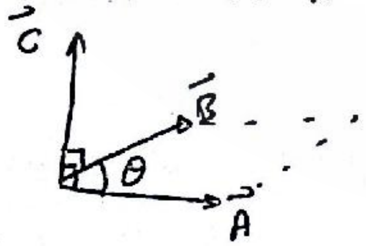
* $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

$\hat{i}\times\hat{j} = \hat{k}$

$\hat{i}\times\hat{k} = -\hat{j}$

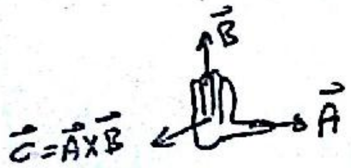
$\hat{j}\times\hat{k} = \hat{i}$

* يكون اتجاه المتجه $\vec{C}$ عمودي على المستوى الذي يحتوي $\vec{A}$, $\vec{B}$.



* إذا كان $\vec{A}$ و $\vec{B}$ يقعان في المستوى xy ، فإن $\vec{C}$ سيكون باتجاه (+Z) أو (-Z)

ويتم تحديد ذلك باستخدام قاعدة اليد اليمنى ، حيث نضع الابهام باتجاه المتجه الأول ($\vec{A}$) ، والاصابع الاربعة باتجاه ($\vec{B}$) فيكون ($\vec{A}\times\vec{B} = \vec{C}$) باتجاه باطن الكف .

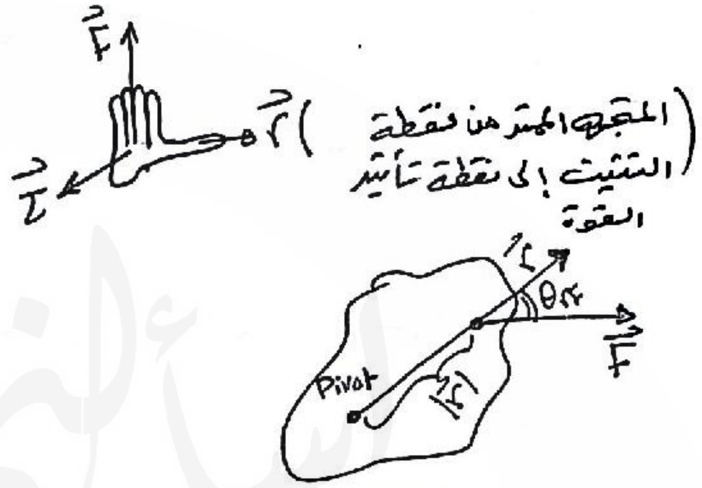


) Torque($\vec{\tau}$) :-

(*) إذا أثرت قوة على جسم (ممتد) له أبعاد مثبت من نقطة معينة (Pivot)، فإنه سيدور حول هذا المحور، وهذا يعني أن لهذه القوة عزم دوران (Torque) حيث

$$\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta_{rF}$$

$\vec{r}$ ← البعد عن نقطة التثبيت



(*) If many forces act on the object, then the net force will be:

$$\boxed{\vec{\tau}_{net} = \sum_i \vec{\tau}_i = \sum_i \vec{r}_i \times \vec{F}_i}$$

$$\boxed{\sum \vec{\tau} = I\vec{\alpha}} \qquad \left[I = \sum_i m_i r_i^2 \right]$$

⇓

ملاحظة: τ تؤثر على جسم ينتج عنه دوران الجسم بتسارع زاوي (α)

(*) remember (from ch:9) : $\vec{F} = \frac{d\vec{P}}{dt}$ (P: momentum)

$$\Rightarrow \sum \vec{\tau} = \frac{d}{dt}(\vec{r}\times\vec{P}) = \vec{r}\times \underbrace{\frac{dP}{dt}}_{\vec{F}}$$

$\boxed{\vec{L} = \vec{r}\times\vec{P}}$ * يسمى المقدار : $\underline{\underline{\vec{r}\times\vec{P}}}$ بـ angular momentum ($\vec{L}$) : الزخم الزاوي

Linear Momentum :- ($\vec{P}$)

$$\vec{P} = m\vec{v} \Rightarrow \Delta\vec{P} = \vec{P}_2 - \vec{P}_1 = \int \vec{F}\,dt = \vec{F}\Delta t$$

مهم : (اتجاه $\vec{P}$ هو نفس اتجاه $\vec{v}$) .

$$\vec{F} = \frac{d\vec{P}}{dt}$$

$$\boxed{\vec{L}} \Leftarrow \begin{cases} \vec{C} = \vec{A}\times\vec{B} & (1 \\ \text{Torque} & (2 \\ \text{linear momentum} \end{cases}$$

الآن سندمج هذه الأشياء معاً

$$\vec{L} = \vec{r}\times\vec{P} \qquad \vec{L} : \text{angular momentum}$$

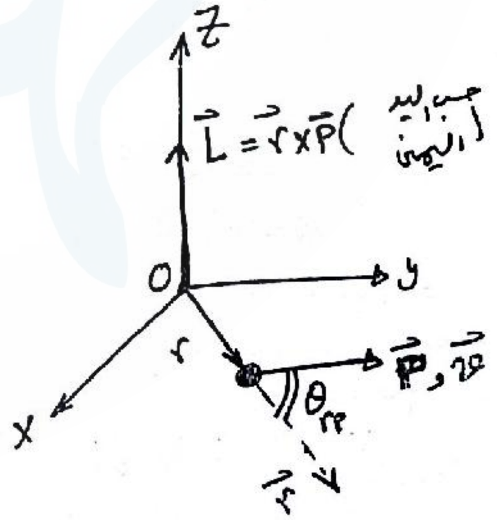
$$L = rP\sin\theta_{rp} \quad \text{if } \theta_{rp} = 90^\circ \Rightarrow \boxed{L = rP = rmv = mrv}$$

* تذكر : r : بعد الكتلة m عن محور الدوران (نقطة التثبيت)

$$\boxed{\sum_{ext} \vec{\tau} = \frac{d\vec{L}_{tot}}{dt}} \Rightarrow \vec{L} = \int_{t_1}^{t_2} \vec{\tau}\,dt$$

خارجي

عزم الدوران الناتج عن قوى خارجية

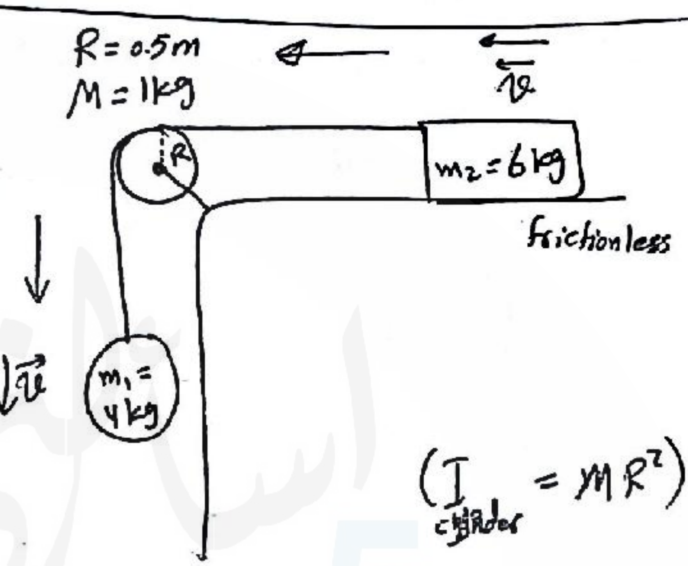


* أي جسم يتحرك في مسار دائري تكون الزاوية بين $\vec{r}$ و ($\vec{P}$ أو $\vec{v}$) هي $90^\circ \Leftarrow \boxed{\sin\theta_{rp} = 1}$

$$\Rightarrow L = rp = mrv .$$

For system of Particles ((أي أكثر من جسم)

$$\vec{L}_{tot} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \dots = \sum_{i=1}^{N} \vec{L}_i$$



$\left(I_{cylinder} = MR^2\right)$

Ex: يتحرك هذا النظام لليسار، ولاحظ 6
عند لحظة ما كانت بسرعة هي $\vec{v}$ 6
احسب التسارع الخطي (linear acceleration)
$\boxed{a}$ في هذه اللحظة بطريقتين:

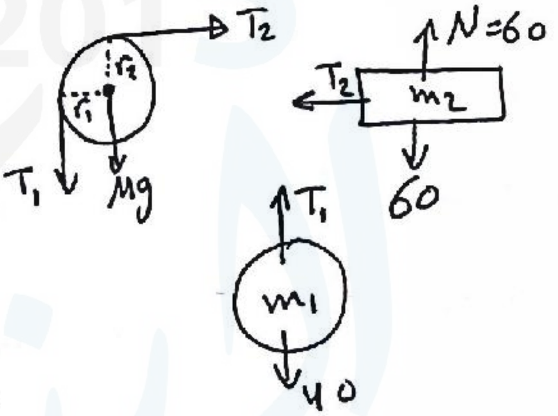
) Newton's law : (ch:10)

المعادلات

$$\sum \tau_{Pully} = I\alpha \quad —① $$

$$\sum F_y = m_1 a \quad —②$$

$$\sum F_x = m_2 a \quad —③$$



الآن نعوض

③ $\Rightarrow$ $\boxed{T_2 = 6a}$ —③

② $\Rightarrow$ $\boxed{40 - T_1 = 4a}$ —②

① pully $\Rightarrow$ $+RT_1 - RT_2 = (MR^2)\alpha \quad \left(\alpha = \frac{a}{R}\right)$

$R(T_1 - T_2) = MR^2\frac{a}{R}$

$\Rightarrow \boxed{T_1 - T_2 = Ma = 1a}$ —①

$\tau = rF\sin\theta$

$\tau_1 = T_1 \times R\sin 90 \quad (r_1 = R = r_2)$

$= RT_1$ (يعكس العقارب)

$\tau_2 = T_2 R\sin 90 = RT_2$ (مع العقارب)

$\tau_3 = (Mg)(0) = 0$

$40 = 11a$!جح

$\boxed{a = 3.64 \text{ m/s}^2}$

لكن لم يطلب ⇐ (ومن معرفة a يمكن إيجاد T_1, T_2, α)

2) Angular Momentum : (CH: 11)

$$\sum \tau_{ext} = \frac{dL_{tot}}{dt}$$

① بالنسبة لـ $\underline{\tau_{tot}}$

انتبه : هنا τ_{ext} أي الـ عزم المؤثر على البكرة من القوى الخارجية ، والقوة الخارجية

الوحيدة : هنا هي $(m_1 g)$ وبعدها عن المحور هو $\underline{R}$:

(لو كان يوجد احتكاك مثلاً لأصبح له τ_{ext})

(لا يعتبر الـ τ الناتج عن الحبال خارجياً لأن الحبال هنا تعامل معاملة القوى الداخلية التي تلغي بعضها بعضاً).

$$\Rightarrow \tau_{tot} = \tau_{m_1 g} = (m_1 g) R \sin 90 = m_1 g R = 4 \times 10 \times 0.5 = 20 \text{ N.m}$$

© بالنسبة لـ $\underline{L}$:

يوجد لدينا ٣ أجسام متحركة حول المحور وهي m_1 و m_2 و (البكرة $\underline{M}$) .

$$L_1 = r_1 m_1 v \sin\theta_1 = R m_1 v \sin 90 = R(4)\, v = 4 * 0.5 * v = 2v$$

$$L_2 = r_2 m_2 v \sin\theta_2 = R m_2 v \sin 90 = R(6)\, v = 6 * 0.5 * v = 3v$$

$$L_3 = r_3 M v \sin\theta_3 = R M v \sin 90 = R(1)\, v = 1 * 0.5\, v = 0.5 v$$

(لأن كل جزء على البكرة يبعد عن المركز مسافة $\underline{R}$ فتعامل معاملة جسم واحد)

$$\Rightarrow \sum \tau_{ext} = \frac{dL_{tot}}{dt} \Rightarrow (20) = \frac{d}{dt}(2v + 3v + 0.5v) \Rightarrow 20 = \frac{d}{dt}(5.5v)$$

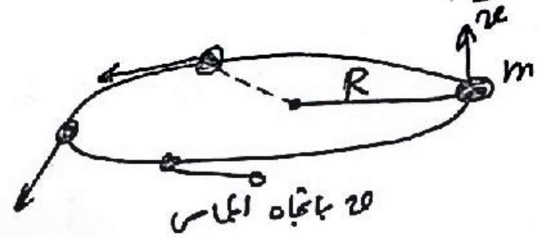
$$\Rightarrow 20 = 5.5 \frac{dv}{dt} \quad \boxed{\left(\frac{dv}{dt}\right) = a} \Rightarrow \boxed{a = \frac{20}{5.5} = \frac{40}{11} = 3.64 \text{ m/s}^2}$$

3: Angular Momentum of a Rotating Rigid object :-

* إذا كان لدينا كرة مثلاً متصلة بخيط وتدور في مسار دائري ، فإن لها :

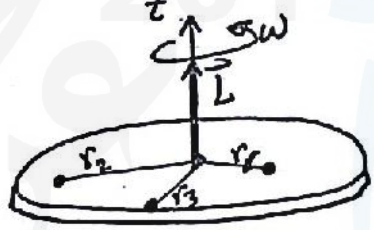
$\vec{P} = m\vec{v}$

$\vec{L} = r m \vec{v}$



وهنا لا مشكلة في حساب $\underline{L}$ لأن $(R = r)$ ، فالجسم (كل الجسم) يبعد مسافة $\underline{R}$ عن محور الدوران ، لكن :

لو كان لدينا جسم ممتد (مثل القرص) ، فإن كل جزء من اجزائه يبعد $\underline{r}$ مختلفة عن محور الدوران



$(R = r_3) > r_2 > r_1$

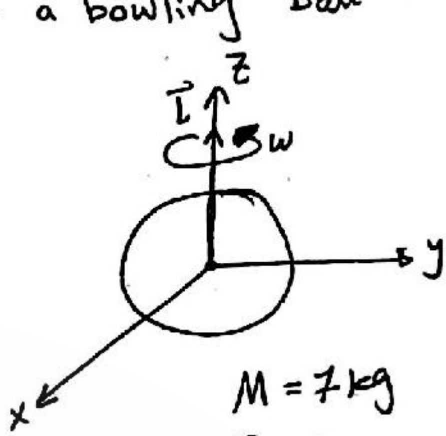
لكن هناك شيئاً مشتركاً بين هذه الاجسام الصغيرة المكونة للقرص وهو : gular speed (ω)

حيث : $\boxed{r_i \omega = v_i}$ ، لذا في هذه الحالة نستعمل العلاقة التالية لحساب $\boxed{\underline{L}}$

$$\Longrightarrow \boxed{L_z = I\omega} \qquad \left(I = \sum_i m_i r_i^2\right)$$

$$\Longrightarrow \boxed{\sum \tau_{ext} = I_z \alpha = \frac{dL_z}{dt}}$$ → Rotational form of Newton's 2nd law

Ex: Calculate the angular momentum L of a bowling ball spinning (يدور) at 10 rev/sec.



$M = 7\,kg$
$R = 12\,cm$

* rev: دورة $\Rightarrow$ $1\ rev = (2\pi R)$ meter

$\Rightarrow \quad v = 10 * 2\pi R \quad m/s$

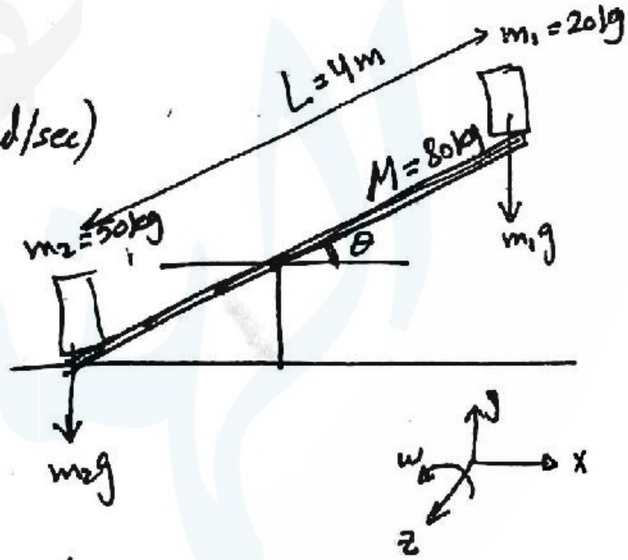
$\quad = 20\pi R \Longrightarrow \omega = \frac{v}{R} = 20\pi$

Sol $\quad L = I\omega \qquad \left(I_{\substack{solid\\sphere}} = \frac{2}{5} MR^2 \right)$

$= \left[\frac{2}{5} * 7 * (0.12)^2\right] * (20 * 3.14)$

$= 2.53\ kg \cdot m^2/sec.$

Ex (Seesaw ) $(\omega = 0.5\ rad/sec)$



يمكن اعتبار النظام المكون من : m_1, m_2, M
عبارة عن ← Rigid body (جسم متماسك)

$\Rightarrow L = I\omega$

now: $I = I_{m_1} + I_{m_2} + I_M$ (ch:10) من $\qquad$ (تذكر $I = mR^2$)

$= m_1 \left(\frac{L}{2}\right)^2 + m_2 \left(\frac{L}{2}\right)^2 + \frac{1}{12} ML^2$

$= 20(2)^2 + 50(2)^2 + \frac{1}{12} 80(4)^2$

$= 386.67\ kg \cdot m^2.$

$\Rightarrow L = I\omega = 386.67 * 0.5 = \frac{386.67}{2} = 193.33\ kg \cdot m^2/s$

Isolated system (نظام دوراني معزول)

* ويقصد بمعزول هنا : لا يوجد مؤثرات خارجية عليه .

Torques

* مثلاً : (متزلج على الجليد يدور حول نفسه في الهواء بدون قوى خارجية).

٢) (قرص يطير في الهواء بدون قوى خارجية) .

$$\Rightarrow \sum \tau_{ext} = \frac{d\vec{L}}{dt}$$

$$0 = \frac{d\vec{L}}{dt} \Rightarrow L = \text{Constant}$$

$$\Rightarrow \boxed{\vec{L}_{initial} = \vec{L}_{final}}$$

$$\Downarrow$$

$$\boxed{I_i \cdot \omega_i = I_f \, \omega_f}$$

إذا كنا نتكلم عن جسم واحد .

* $E_i = E_f \Rightarrow$ If there are no energy transfers across the system boundary (CH:8).

جدران النظام

$\vec{P}_i = \vec{P}_f \Rightarrow$ If the net external force on the system is zero. (CH:9)

$\vec{L}_i = \vec{L}_f \Rightarrow$ If the net external torque on the system is zero

Ex: A [solid sphere] rotates about an axis through its center with a period of 5-sec, it had a radius of 50cm before its radius became 10cm, Find the new Period of rotation.

period (T_p) : زمن إكمال دورة واحدة كاملة.

Sol:

بما أنه لا توجد قوى خارجية ⇐ isolated system. (جسم واحد) ⇐

$$\omega = \frac{2\pi}{T_p}$$

$$v = \frac{2\pi R}{T_p}$$

$$L_i = L_f$$

$$I_i \omega_i = I_f \omega_f$$

$$\left(v = \frac{2\pi R}{T_p}\right)$$

$$\Rightarrow \omega = \frac{v}{R} = \frac{2\pi \cancel{R}}{\cancel{R} T_p} = \frac{2\pi}{T_p}$$

$$\cancel{\tfrac{2}{5}} \cancel{M} R_1^2 \omega_1 = \cancel{\tfrac{2}{5}} \cancel{M} R_2^2 \omega_2$$

$$I_{\text{solid sphere}} = \frac{2}{5} M R^2$$

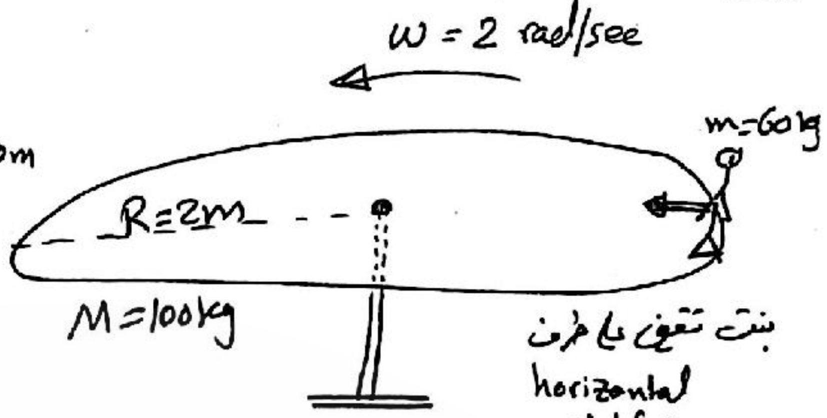
$$\Rightarrow R_1^2 \frac{\cancel{2\pi}}{T_1} = R_2^2 \frac{\cancel{2\pi}}{T_2}$$

$$\left.\begin{aligned} R_1 &= 50\,cm = 0.5\,m \\ R_2 &= 10\,cm = 0.1\,m \end{aligned}\right\} \Rightarrow T_2 = T_1 \left(\frac{R_2^2}{R_1^2}\right) = 5 \left[\frac{0.1}{0.5}\right]^2$$

$$\boxed{T_2 = 0.2\ sec}$$

لاحظ ، عندما اصبحت نفس الكرة بنفس الكتلة لها نصف قطر اصغر (حجم اصغر) اصبح دورانها أسرع ، فهي كانت تكمل دورة واحدة كاملة كل (5 sec) أما الآن فهي تكمل الدورة في (0.2 sec) فقط.

Ex: Find $\underline{\omega}$ when the lady reaches a point $\underline{r = 0.5\,m}$ from the center.



بنت تقف على طرف horizontal platform... وتريد الحركه باتجاه المركز

$$\vec{I}_{i\,(r=2m)} = \vec{I}_{f\,(r=0.5m)}$$

$$\begin{bmatrix} \omega_m = \omega_M = 2\ \text{rad/s} \\ \omega'_m = \omega'_M = ?? \end{bmatrix}$$

$$I_m \omega_m + \underset{\text{disk}}{I_M} \omega_M = I_m \omega'_m + I_M \omega'_M$$

$$mR^2\omega_i + \underset{\text{من CH:10}}{\left(\tfrac{1}{2}MR^2\right)}\omega_i = m r^2 \omega'_m + \left(\tfrac{1}{2}MR^2\right)\omega'_M$$

$$60(2)^2(2) + \left(\tfrac{1}{2}\times 100 \times (2)^2\right)(2) = 60(0.5)^2\,\omega'_m + \tfrac{1}{2}(100)\times(2)^2\,\omega'_M$$

$$\Longrightarrow \boxed{\omega'_m = \omega'_M = 4.1\ \text{rad/sec}}$$

سؤال اضافي: هل البنت تحتاج لبذل شغل لتتحرك إلى $r = 0.5m$ أما لا؟

الجواب: نجد K_i و K_f ونقارن، فإذا كانت $K_i < K_f$ ⟸ تحتاج لبذل شغل وإلا لا

$$K_i = \tfrac{1}{2} I_i \omega_i^2 = \tfrac{1}{2}\left(\underset{60}{m}\underset{2}{R^2} + \tfrac{1}{2}\underset{100}{M}\underset{2}{R^2}\right)(2)^2 = 880\ \text{J}$$

$$K_f = \tfrac{1}{2} I_f \omega_f^2 = \tfrac{1}{2}\left(\underset{60}{m}\underset{(0.5)}{r^2} + \tfrac{1}{2}\underset{100}{M}\underset{(2)}{R^2}\right)(4.1)^2 = 1807\ \text{J}$$

$$K_f > K_i$$

البنت بحاجة لبذل جهد لزيادة الطاقة الحركية للنظام، وهذا سوف تستهلكه من الطاقة الكيميائية المختزنة في عضلاتها.