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The University of Jordan Mathematics Department Calculus I Second Exam 19/8/2017

اسم مدرس المادة: د. منال علي ... اسم الطالب : محمد اليسع رمزي الرسلان

الرقم الجامعي: التخصص: صيدلة

Part (I): In questions 1 – 10, fill in the blank to get correct sentence.

(2 marks each)

(1) The function $f(x) = \frac{x+3}{x^2-9}$ has removable discontinuity at $x =$ ...-3...

(2) The function $f(x) = \frac{x+10}{(x-3)\cos x}$, $x \in [0,2\pi]$ has vertical asymptote(s) at $x =$ ...3... , $x = (2n+1)\frac{\pi}{2}$

(3) $\frac{d}{dx}(\cos(\tan^{-1}(1+10x))) =$ $-\sin(\tan^{-1}(1+10x))\left(\frac{1}{1+(1+10x)^2}\right)(10)$

(4) $\frac{d}{dx}(x^{4x}) =$ $x^{4x}\left(\frac{4x}{x} + 4\ln x\right)$

(5) If $g(x) = \frac{\sqrt{\tan x + 10}\,e^{...}}{(\sec^4 x)(4x+1)^{...}}$, then by using logarithmic differentiation

$g'(x) =$ ....

(6) The function $f(x) = \begin{cases} mx+b & if\ x < 4 \\ x^2 & if\ x \geq 4 \end{cases}$ is differentiable at 4

when $m =$ ..8.., and $b =$ ..-16

(7) If $f(x) = x^{11} + 6x + 4$, then $(f^{-1})'(4) =$ $\frac{1}{6}$

(8) By using linear approximation, $\sqrt[4]{80} \approx$ $3 - \frac{1}{4(27)}$ $= 3 - \frac{1}{108}$

(9) Equation of the tangent line to the curve of $y = x^4 + 4^x$ at the point (1,5) is

$(y-5) = 4 + 4\ln 4\,(x-1)$

(10) $\lim_{x\to\infty} \frac{e^{-x}-4e^x}{2e^{-x}+3e^x} =$ ....

(11) $\lim_{x\to -4} \frac{\sin(x^2+5x+4)}{x+4} =$ -3

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Part (II): In questions 12-14, give your answer in details

(12) (3- marks) Using the intermediate value theorem, show that the equation $x^3 - x - 1 = 0$ has a real root in the open interval (1,3).

$x^3 - x - 1 = 0$

$1 - 1 - 1$

$\lim_{x\to-\infty} \frac{x+\sqrt{x^2+3x}}{3x}$

$= \lim_{x\to-\infty} \frac{x + (-x)\sqrt{1+\frac{3x}{x^2}}}{3x}$

$= \lim_{x\to-\infty} \frac{x(1-1)}{x(3)}$

$= \frac{0}{3} = 0$

so $y = \frac{2}{3}$ is the only HA

(13) (4-marks) Find all horizontal asymptotes for $f(x) = \frac{1}{x-\sqrt{x^2+3x}}$

$\lim_{x\to\infty} \frac{1}{x-\sqrt{x^2+3x}} = \frac{1}{\infty-\infty}$!!

$= \lim_{x\to\infty} \frac{1}{x-|x|\sqrt{1+\frac{3x}{x^2}}}$ $|x|$, $x\to\infty$ = x

$\lim_{x\to\infty} \frac{1}{x-x\sqrt{1+\frac{3x}{x^2}}}$

$= \lim_{x\to\infty} \frac{1}{x\left(1-\sqrt{1+\frac{3x}{x^2}}\right)} = \frac{1}{\infty(0)}$!!

$= \frac{1}{x\left(1-\sqrt{1+\frac{3x}{x^2}}\right)}$

$\lim_{x\to\infty} \frac{1}{x-\sqrt{x^2+3x}} \cdot \frac{x+\sqrt{x^2+3x}}{x+\sqrt{x^2+3x}}$

$\lim_{x\to\infty} \frac{x+\sqrt{x^2+3x}}{x^2-x^2+3x}$

$= \lim_{x\to\infty} \frac{x+|x|\sqrt{1+\frac{3x}{x^2}}}{x(3)}$ $|x| = x$, $x\to\infty$

$= \frac{x(1+1)}{x(3)} = \frac{2}{3}$

so $y = \frac{2}{3}$ is H.A

3.5

(14) (3 marks) If $x^3 + 8y^3 = 2$, find y'' by **implicit differentiation**.

$x^3 + 8y^3 = 2$

diff

$3x^2 + 24y^2 y' = 0$

$24y^2y' = -3x^2$

$y' = \frac{-3x^2}{24y^2}$

$y' = \frac{-3x^2}{24y^2}$ diff

$y'' = \frac{24y^2(-3(2)x) - (-3x^2(48yy'))}{(24y^2)^2}$

$y' = \frac{-3x^2}{24y^2}$

$y'' = \frac{-144xy^2 + \frac{432yx^4}{24y^2}}{576y^4}$

3

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Jordan University
Mathematics Department
Calculus I, Second Exam, 5/7/2017

Student's Name: _ Student Number: _ ______.

Lecture Time: 12:30 - 2:00 #41

Remark: Show your supporting work. Final answers without supporting work receives no credit.

Q1) (3 points) Find the vertical asymptote of $f(x) = \frac{x^2+2x+1}{x^2-4x-5}$.

$x^2 - 4x - 5 \rightarrow (x-1)(x+5) \rightarrow x-1=0 = x=1$

$x+5=0 = x=-5$

VA at $x = -1$ and $x = 5$

Q2) (6 points) Find the horizontal asymptote of $f(x) = \sqrt{4x^2+7x} - 2x$

$\lim_{x \to \infty} \sqrt{4x^2+7x} - 2x$ $\begin{cases} x \geq 0 \\ -x < 0 \end{cases}$

$x \geq 0 \rightarrow 4x + 7 - 2x = 0 \rightarrow \frac{2x}{2} = \frac{-7}{2} = -\frac{7}{2}$

$-x < 0 \rightarrow -4x - 7 + 2x = 0 \rightarrow \frac{-2x}{-2} = \frac{7}{-2} = -\frac{7}{2}$

HA at $-\frac{7}{2}$

$$\lim_{x \to -5} \frac{x^2 - (-x) - 20}{x+5}$$ (1)

Q3) Find the following limits (Do not use L'Hospital's rule):

(1) a) (2 points) $\lim_{x \to -5} \frac{x^2 - |x| - 20}{x+5}$ $\to \frac{(x-4)(x+5)}{x+5}$

$\lim_{x \to -5} (x-4) = -5-4 = \boxed{-9}$ (1)

b) (3 points) $\lim_{x \to \infty} \frac{\sin^2(7x)}{x^2}$

Let $u = x^2$ $x = \sqrt{u}$

$\lim_{u \to 0} \frac{\sin^2(7\sqrt{u})}{u} = \boxed{\frac{7\cos^2(7x)}{2x}}$

$\frac{F(x) - F(a)}{x - a}$ (1) c) (2 points) $\lim_{x \to 0} \frac{5^x - 1}{x}$ $\to \frac{5^x - 1}{x - 0}$

$\boxed{5^x \cdot \ln 5}$ $\Big|_{x=0}$

(1.5) Q4) (4 points) If $e^y + 4xy = e$, then find equation of the tangent line at the point where $x = 0$.

$e^y + 4xy = e \to y'e^y + 4y + 4xy' = 0$
$\quad -4y \quad -4y$

$y'e^y + 4xy' = -4y \to y'(e^y + 4x) = -4y$

$\boxed{y' = \frac{-4y}{e^y + 4x}}$

4

Q5) Find the derivative of the following functions:

a) (2 points) $f(x) = \tan^{-1}(x^4)$

$$F'(x) = \frac{\sqrt{1-(x^4)^2}}{\sqrt{1+(x^4)^2}} = \frac{\sqrt{1-x^8}}{\sqrt{1+x^6}}$$

b) (2 points) $f(x) = 7^{(\frac{9}{x})}$

2

$$F'(x) = 7^{(\frac{9}{x})} \cdot -\frac{9}{x^2} \cdot \ln 7$$

$$\frac{9}{x} = \frac{u v' - v u'}{v^2} = \frac{-9}{x^2}$$

2

c) (2 points) $f(x) = \log_6(x^3 + 5x + 1)$

$$F'(x) = \frac{3x^2 + 5}{(x^3 + 5x + 1)(\ln 6)}$$

d) (4 points) $f(x) = (7x + 1)^{\cos^3(3x)}$

$$F'(x) = (7x+1)^{\cos^3(3x)} \cdot -3\sin^3(3x) \cdot \ln(7x+1)$$

$$\frac{d}{dx} \cos^3(3x) = -3\sin^3(3x)$$

رقم تسلسلي 15 **The University of Jordan** د. فراس يوسف

DEPARTMENT OF MATHEMATICS

4/30

Calculus I EXAM 2B / 1st SEMESTER 2016-2017 Date: 03/12/2016

وقت المحاضرة: (1:00 – 12:00) (...) الرقم الجامعي: (.........) .... الاسم:

Instructions: *The test one two-sided page; make sure you do both sides. You CANNOT use a calculator on any part of this exam. The point value of each problem is indicated in brackets. Finally, before you start to work a problem, be sure that you understand what is being asked.*

For questions 1 to 12, fill in the blank with the correct answer. **Only correct answers count. [1.5 pts each]**

?

1. If $x^2+2xy+4y^2=12$, then $\frac{dy}{dx}$ at the point $(2,1)$ is equal to ....14....

?

2. $\lim_{h\to 0}\frac{6(1+h)^6-6}{h}=$ ....$\pm\infty$.... 36

3. The equation of the tangent line to the graph of $y=\sin^{-1}\left(\frac{x}{3}\right)$ at the origin ....

4. $\frac{d}{dx}\left[\log_5\left(x^2+\pi^2\right)\right]=$ ....

5. $f(x)=\frac{\sin(x)}{\sqrt{9-x^2}}$ is continuous on ....$(-\infty,\infty)$.... $(-3,3)$

6. $\lim_{x\to\frac{\pi}{2}} e^{\sec(x)}=$ .... $\lim_{x\to\frac{\pi}{2}^+}$ $e^{\sec(x)}$ $e^{\frac{1}{\cos(x)}}=e^{\frac{1}{\cos(\frac{\pi}{2})}}=e^{\frac{1}{0}}=+\infty$ $=+\infty$

7. If $g(0)=3$, $g'(0)=2$ and $f(x)=\frac{5-2e^{3x}}{x+g(x)}$, then $f'(0)=$ ....

8. $\lim_{x\to 0}\frac{1-\cos^2(7x)}{x^2}=$ .... $\lim_{x\to 0}\frac{\sin^2(7x)}{x^2}=\sqrt{\frac{\sin^2(7x)}{x^2}}=\frac{\sin 7x}{x}=7$

9. If $f(x)=\sec^3\left(4^x\right)$, then $f'(x)=$ ....

?

10. If $f(x)=\ln(x+5)$, then $\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}=$ 1/7

11. The vertical asymptote of $f(x)=\frac{x-3}{x^2-5x+6}$ is $x=$....2....

12. $\lim_{x\to -1}\frac{2x+2}{|x|+1}=$ ....DNE.... $\frac{2x+2}{|-1|+1}=\frac{2(-1)+2}{2}=\frac{0}{2}=0$

For question 13, 14, and 15, sufficient work must be shown to receive credit.

?

13. [4 pts] Find $\lim_{x \to -\infty} \sqrt{x^2 + 5x} + x$.

$x^2 + 5x + 0$

$(x + 5)(x + 0)$

$\sqrt{(x+5)}$ $+ x$

14. [4 pts] Use the linear approximation to estimate $\sqrt[3]{1002}$.

$a = 1000$

$y = F(a) + (x - a) \cdot F'(a) = \frac{1}{30} + (1002 - 1000)^{+}(10)$

$= \frac{1}{30} + (2) + (10) = \frac{1}{30} + (20) = \frac{360}{30} = 12 \Rightarrow \sqrt[3]{1002} \approx 12$

2

15. [4 pts] Differentiate $f(x) = \dfrac{\sqrt[4]{x}\,\sin^6(x)}{(x-1)^8 e^{x^2}}$.

$F'(x) = \dfrac{\frac{1}{4\sqrt[4]{x}}\ 6\cos^5(x)}{8(x-1)^7\ e^{2x}} =$

Good Luck

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The University of Jordan Mathematics Department Calculus I Second Exam 19/8/2017

اسم مدرس المادة: د. ممال عامر

اسم الطالب: محمد اليسع رمزي الرسلان

الرقم الجامعي: 0161049

التخصص: هندسة

Part (I): In questions 1 – 10, fill in the blank to get correct sentence.

(2 marks each)

(1) The function $f(x)=\frac{x+3}{x^2-9}$ has removable discontinuity at $x=$ -3

(2) The function $f(x)=\frac{x+10}{(x-3)\cos x}$, $x\in[0,2\pi]$ has vertical asymptote(s) at $x=$ 3 , $X=(2n+1)\frac{\pi}{2}$

(3) $\frac{d}{dx}(\cos(tan^{-1}(1+10x)))=$ $-\sin(\tan^{-1}(1+10x))\left(\frac{1}{1+(1+10x)^2}\right)(10)$

(4) $\frac{d}{dx}(x^{4x})=$ $x^{4x}\left(\frac{4x}{x}+4\ln x\right)$

(5) If $g(x)=\frac{\sqrt{\tan x+10e^{x}}}{(\sec^{8}x)(4x+1)^{\frac{5}{3}}}$, then by using logarithmic differentiation

$g'(x)=$ ....

(6) The function $f(x)=\begin{cases} mx+b & if\ x<4 \\ x^2 & if\ x\geq 4\end{cases}$ is differentiable at 4

when $m=$ 8, and $b=$ -16

(7) If $f(x)=x^{11}+6x+4$, then $(f^{-1})'(4)=$ $\frac{1}{6}$

(8) By using linear approximation, $\sqrt[4]{80}\approx$ $3-\frac{1}{4(27)}$ $= 3-\frac{1}{108}$

(9) Equation of the tangent line to the curve of $y=x^4+4^x$ at the point (1,5) is

$(y-5)=4+4\ln 4(x-1)$

(10) $\lim_{x\to\infty}\frac{e^{-x}-4e^{x}}{2e^{-x}+3e^{x}}=$ ....

(11) $\lim_{x\to-4}\frac{\sin(x^2+5x+4)}{x+4}=$ -3

17

Part (II): In questions 12-14, give your answer in details

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$x^3 - x - 1 = 0$

$1 - 1 - 1$

$$\lim_{x \to -\infty} \frac{x + \sqrt{x^2 + 3x}}{3x}$$

$$= \lim_{x \to -\infty} \frac{x + (-x)\sqrt{1 + \frac{3x}{x^2}}}{3x}$$

$$= \lim_{x \to -\infty} \frac{x(1 - 1)}{x(3)} = \frac{0}{3} = 0$$

so $y = \frac{2}{3}$ is the only HA

(13) (4-marks) Find all horizontal asymptotes for $f(x) = \frac{1}{x - \sqrt{x^2 + 3x}}$

$$\lim_{x \to \infty} \frac{1}{x - \sqrt{x^2 + 3x}} \cdot \frac{x + \sqrt{x^2 + 3x}}{x + \sqrt{x^2 + 3x}}$$

$$\lim_{x \to \infty} \frac{x + \sqrt{x^2 + 3x}}{x^2 - x^2 + 3x}$$

$$= \lim_{x \to \infty} \frac{x + |x|\sqrt{1 + \frac{3x}{x^2}}}{x(3)}$$

$$= \frac{x(1 + 1)}{x(3)} = \frac{2}{3}$$

$\lim_{x \to \infty} |x| = x$

so $y = \frac{2}{3}$ is H.A

3.5

(14) (3 marks) If $x^3 + 8y^3 = 2$, find y'' by **implicit differentiation**.

$x^3 + 8y^3 = 2$

diff

$3x^2 + 24y^2 y' = 0$

$24y^2 y' = -3x^2$

$y' = \frac{-3x^2}{24y^2}$

$y' = \frac{-3x^2}{24y^2}$ diff

$$y'' = \frac{24y^2(-3(2)x) - (-3x^2(48yy'))}{(24y^2)^2}$$

$$y' = \frac{-144xy^2 + \frac{432yx^4}{24y^2}}{576y^4}$$

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The University of Jordan

DEPARTMENT OF MATHEMATICS

Calculus I (34) EXAM 2B / 1st SEMESTER 2016-2017 Date: 03/12/2016

26/30

الاسم : ...نور هشام محمود الجابر...... الرقم الجامعي: (.......[redacted]..) وقت المحاضرة: (.........2 — 3:30....)

Instructions: *The test one two-sided page; make sure you do both sides. You CANNOT use a calculator on any part of this exam. The point value of each problem is indicated in brackets. Finally, before you start to work a problem, be sure that you understand what is being asked.*

For questions 1 to 12, fill in the blank with the correct answer. **Only correct answers count.** [1.5 pts each]

13.5/18

1. If $x^2+2xy+4y^2=12$, then $\dfrac{dy}{dx}$ at the point $(2,1)$ is equal to ......$-\frac{1}{2}$......

2. $\lim\limits_{h\to 0}\dfrac{6(1+h)^6-6}{h}=$ ......36......

3. The equation of the tangent line to the graph of $y=\sin^{-1}\left(\dfrac{x}{3}\right)$ at the origin ......$y=\frac{1}{3}x$......

4. $\dfrac{d}{dx}\left[\log_5\left(x^2+\pi^2\right)\right]=$ ......$\dfrac{2x}{(x^2+\pi^2)\ln 5}$......

5. $f(x)=\dfrac{\sin(x)}{\sqrt{9-x^2}}$ is continuous on ......$(-3,3)$......

6. $\lim\limits_{x\to\frac{\pi}{2}^+} e^{\sec(x)}=$ ......∞......

7. If $g(0)=3$, $g'(0)=2$ and $f(x)=\dfrac{5-2e^{3x}}{x+g(x)}$, then $f'(0)=$ ......-3......

8. $\lim\limits_{x\to 0}\dfrac{1-\cos^2(7x)}{x^2}=$ ......49......

9. If $f(x)=\sec^3(4^x)$, then $f'(x)=$ ......$12\sec^3(4x)\tan(4x)$......

10. If $f(x)=\ln(x+5)$, then $\lim\limits_{x\to 2}\dfrac{f(x)-f(2)}{x-2}=$ ......$\frac{1}{7}$......

11. The vertical asymptote of $f(x)=\dfrac{x-3}{x^2-5x+6}$ is $x=$......2 (just)......

12. $\lim\limits_{x\to -1}\dfrac{2x+2}{|x|+1}=$ ......-2......

For question 13, 14, and 15, sufficient work must be shown to receive credit.

13. [4 pts] Find $\lim_{x\to-\infty} \sqrt{x^2+5x}+x$. $= \infty - \infty$!!

$\lim_{x\to-\infty} \sqrt{x^2+5x}+x \cdot \frac{\sqrt{x^2+5x}-x}{\sqrt{x^2+5x}-x} \Rightarrow \lim_{x\to-\infty} \frac{x^2+5x-x^2}{\sqrt{x^2+5x}-x}$

$\lim_{x\to-\infty} \frac{5x}{\sqrt{x^2(1+\frac{5}{x})}-x} \Rightarrow \lim_{x\to-\infty} \frac{5x}{|x|\sqrt{1+\frac{5}{x}}-x}$

$\lim_{x\to-\infty} \frac{5x}{x(-\sqrt{1+\frac{5}{x}}-1)} = \frac{5}{-\sqrt{1+0}-1} = \frac{5}{-1-1} = -\frac{5}{2}$

4/4

14. [4 pts] Use the linear approximation to estimate $\sqrt[3]{1002}$.

$b = 1002$

$a = 1000$

$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3(\sqrt[3]{x})^2}$

$f(1000) = \sqrt[3]{1000} = 10$

$f'(1000) = \frac{1}{3(\sqrt[3]{1000})^2} = \frac{1}{300}$

$f(b) \approx f(a) + f'(a)\cdot(b-a)$

$\sqrt[3]{1002} \approx 10 + \frac{1}{300}(1002-1000)$

$\sqrt[3]{1002} \approx 10 + \frac{1}{150}$

4/4

15. [4 pts] Differentiate $f(x) = \frac{\sqrt[4]{x}\,\sin^6(x)}{(x-1)^8 e^{x^2}}$.

$y = \frac{\sqrt[4]{x}\,\sin^6(x)}{(x-1)^8 e^{x^2}} \rightarrow \ln y = \ln\left(\frac{\sqrt[4]{x}\,\sin^6(x)}{(x-1)^8 e^{x^2}}\right)$

$\ln y = \ln x^{\frac{1}{4}} + \ln \sin^6(x) - \ln (x-1)^8 - \ln e^{x^2}$

$\frac{y'}{y} = \frac{\frac{1}{4}x^{-\frac{3}{4}}}{x^{\frac{1}{4}}} + \frac{6\sin^5(x)\cdot\cos(x)}{\sin^6(x)} - \frac{8(x-1)^7}{(x-1)^8} - \frac{2x\cdot e^{x^2}}{e^{x^2}}$

$y' = \left(\frac{\frac{1}{4}x^{-\frac{3}{4}}}{x^{\frac{1}{4}}} + \frac{6\cos(x)}{\sin(x)} - \frac{8}{(x-1)} - 2x\right)\cdot\frac{\sqrt[4]{x}\cdot\sin^6(x)}{(x-1)^8 e^{x^2}}$

4/4

Good Luck

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الجامعه الاردنيه قسم الرياضيات رياضيات ١٠١ الامتحان الاول

الاسم:

ألرقم الجامعي:

ألمدرس: د. إيمان دعاوي وقت المحاضره: ١٢:٢٠ — ١٩،

1- (6pts). Find the following limits (Don`t use L`Hospital rule),

a) $\lim_{x\to 2^+}\left(\frac{1}{x-2}-\frac{4}{x^2-4}\right)$ b) $\lim_{x\to 1}\left(\frac{\sqrt[4]{x}-\sqrt{x}}{x-1}\right)$ $= \frac{0}{0}$

c) $\lim_{x\to\infty}\left(\sqrt{x^2-3x}-x\right)$

a) $\lim_{x\to 2^+}\left(\frac{1}{x-2}-\frac{4}{x^2-4}\right)$

$= \frac{1}{2^+-2} - \frac{4}{4^+-4}$

$= \frac{1}{+0} - \frac{4}{+0}$

$= +\infty - +\infty = +\infty$

$\lim_{x\to 1}\frac{\sqrt[4]{x}-\sqrt{x}}{x-1} = \frac{0}{0}$

$= \frac{x^{\frac{1}{4}} - x^{\frac{1}{2}}}{x-1}$ $= \frac{x^{\frac{1}{4}}}{x} = \frac{1}{4}$

→ Poly Nomual

$\frac{an^{a}}{bn^{a}}$ $an^{a} = bn^{a}$

$\frac{an^{a}}{bn^{a}} = \frac{a}{b}$

على قاعدة كيرات الحدود

c) $\lim_{x\to\infty}\sqrt{x^2-3x}-x = \infty-\infty$

$\sqrt{x^2-3x}-x \cdot \frac{\sqrt{x^2-3x}+x}{\sqrt{x^2+3x}+x}$

$\lim_{x\to\infty}\frac{x^2-3x-x^2}{\sqrt{x^2+3x}+x}$

$= \frac{-3x}{x\left(\sqrt{1+\frac{3}{x}}+1\right)}$

$= \boxed{\frac{-3}{2}}$

L.H:

$\lim_{x\to 1}\frac{\frac{1}{4\sqrt{x}} - \frac{1}{2\sqrt{x}}}{1} = \frac{1}{4} - \frac{2}{4} = \frac{1}{4}$

0.5

2

V. assy

(b)

~~$\lim_{x\to} f(x)$~~

$$\lim_{x\to 1} \frac{x^2 - x}{x(x) - x} = \lim \frac{x(x-1)}{x(x-1)} = 1$$

$$\lim_{x\to 1} \frac{x^2 - x}{x(-x) - x} = -1$$

$$\lim_{x\to -1} \frac{x^2 - x}{x(x) - x} = \frac{1+1}{-1-1} = \frac{2}{-2} = -1$$

$$\lim_{x\to -1} \frac{x^2 - x}{x(-x) - x} = \frac{1 - -1}{1-1} = \frac{2}{0} = \infty$$

V. ass at $x = -1$

$$f(x) = \frac{x^2 - x}{x(-x) - x}$$

H. assy:

$$\lim_{x\to\infty} \frac{x^2 - x}{x(x) - x} = 1$$

$$\lim_{x\to\infty} \frac{x^2 - x}{x(-x) - x} = -1$$

H. ass at $y = \mp 1$

$y = \mp 1$

$$\lim_{x\to -\infty} \frac{x^2 - x}{x(x) - x} = \frac{\infty + \infty}{\infty + \infty} = 1$$

$$\lim_{x\to -\infty} \frac{x^2 - x}{x(-x) - x} = \frac{\infty + \infty}{\infty + \infty} = 1$$

2-(3+4

2- (3+4pts). Let $f(x) = \begin{cases} \frac{x^2-x}{x|x|-x} & x \neq \pm1,0 \\ 2 & x = \pm1,0 \end{cases}$,

a) Find and classify the discontinuity of f.

b) Find the vertical and horizontal asymptotes of f.

a) classify:

Removable discontinuity

$\lim_{x\to +2} f(x) \neq \lim_{x \to 2} f(x)$

$\rightarrow \lim_{x\to 2} \frac{x^2-x}{x(-x)-x} = -1$

$\neq$

$\rightarrow \lim_{x\to 2} \frac{x^2-x}{x(x)-x} = 1$

unRemovable discontinuity

$\lim f(x) \neq f(x)$

$(1,-1) \neq 2$

$(1,-1) \neq 2$

v.asy

H. ass at $x = \mp 1$

b) H.ass at x ...

$\lim_{x\to\infty} \frac{x^2-x}{x(x)-x} = \frac{x(x-1)}{x(x-1)} = 1$

$\lim_{x\to -\infty} \frac{x^2-x}{x(-x)-x} = \frac{x(x-1)}{-x^2-x} = -1$

قلب الورقة

2

2

H.asy

3- (4pts). Find the derivative of the following functions.

a) $f(x) = \frac{ln(sin2x)}{x-1}$ b) $g(x) = sin^{-1}(e^{3x})$

$\frac{1}{\sqrt{1-x^2}}$

$f'(x) = \frac{\frac{2\cos 2x}{\sin 2x} \cdot (x-1) - \ln(\sin 2x)}{(x-1)^2}$

$= \frac{2\cot x(x-1) - \ln(\sin 2x)}{(x-1)^2}$

2

$g'(x) = \frac{3e^{3x}}{\sqrt{1-(e^{3x})^2}}$

2

5

$f(x) = \sqrt{x+3} \quad x = 1$

$f(x) = \sqrt{x+3}$

$\Delta x = 3.98 - 3 = 0.98$

$f(1) = \sqrt{1+3} = 2$

$f'(x) = \frac{1}{2\sqrt{x+3}} =$

$f'(1) = \frac{1}{2 \times 2} = \frac{1}{4}$

$f(x) \cong 2 + \frac{1}{4}(0.98)$

$\sqrt{3.98} \cong 2 + \frac{1}{4}(0.98)$

$\cong$

---

31 → odd   2n + 1 = 31
2n = 30
n = 15 → $(-1)^{15+1}$ sin α

4- (4pts). Let $f(x) = cos2x + e^{-x}$, find $f^{(31)}(x)$

2n+1 = 31
2n = 30
n = 15

$(-1)^{15+1}$ sin α = 2 sin x + − e^{-x}
= $(-1)^{16}$ 2 sin α − e^{-x}
= 2 sin α − e^{-x}
= 2 sin α + e^{-x}

3

5- (4pts). Find the local linear approximation of $f(x) =$ $\sqrt{x+3}$ at $x = 1$, then approximate $\sqrt{3.98}$.

3.5

f(a) = $\sqrt{x+3}$
Δx = 3.98 − 3 = 0.98
f(3) = $\sqrt{6}$ = [illegible]
f(a) = f(a) + f′(a)(x − a)
f′(a) = $\frac{1}{2\sqrt{x+3}}$
f′(a) = $\frac{1}{2\sqrt{6}}$ = $\frac{1}{2\sqrt{x}}$ f(a) = $\frac{1}{2\sqrt{6}}$ + $\sqrt{6}$ (0.98)
$\sqrt{3.98}$ = $\frac{1}{2\sqrt{6}}$ + $\sqrt{6}$ (0.98)

6- (3+2pts). Let $f(x) = x^5 + 8x + 3$.

a) Show that f has exactly one real root.

b) Find $(f^{-1})'(3)$

a → f(a) = $x^5 + 8x + 3$
f(−1) = $(-1)^5$ + (−8) + 3 = −6 < 0
f(1) = 1 + 8 + 3 = 12 > 0
by M.V.thm:
f is cont at [−1, 1]
f is diff of (−1, 1)
f(−1) · f(1) < 0
by M.V.thm. then there ∃ c ∈ (−1, 1) s.t. f(c) = 0

2.5

b (f⁻¹)′(3)
$x^5 + 8x + 3 = 3$
$x^5 + 8x = 0$
$x(x^4 + 8) = 0$
x = 0

b f′(x) = $5x^4 + 8$
f′(0) = 0 + 8 = 8
$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{8}$

2

THE UNIVERSITY OF **JORDAN**

0301101 Calculus I — Second test — Dec. 5, 2015

الاسم :

الرقم الجامعي :

وقت المحاضرة :

اسم المدرس:

For instructor use only, please do not write in this table.

Q1	Q2	Q3	Grade
16.5	5	7	29

Q1) Fill in the blanks with the answers only. Each part is worth 1.5 marks.

1) The vertical asymptote(s) of $f(x) = \frac{x+1}{x^2-3x-4}$ is (are) $x = 4$ only V. ASY

2) $\lim_{x\to-2} \frac{x^2+2|x|-8}{x+2} = \lim_{x\to-2} \frac{x^2-2x-8}{x+2} = \lim_{x\to-2} \frac{(x-4)(x+2)}{(x+2)} = -6$

3) $\lim_{x\to0}(4 + x^2 \sin(\frac{2}{x^2})) = 4$

4) If $f(x) = 5x^{\sqrt{3}}$, then $\lim_{x\to1} \frac{f(x)-5}{x-1} = \frac{5}{3} * \frac{1}{(\sqrt[3]{5})^2} = \frac{5}{3 * \sqrt[3]{25}}$

5) If the tangent line to $f(x)$ at $x = 4$ is $y - 5x = 1$, then $f'(4) = 5$

6) $\lim_{x\to\infty} e^{x-x^2} = \lim_{x\to\infty} e^{x^2(\frac{1}{x}-1)} = e^{\lim_{x\to\infty} x^2(\frac{1}{x}-1)} = e^{-\infty} = 0$

7) If $f(x) = \tan^{-1}(e^{5x})$, then $f'(x) = \frac{5(e^{5x})}{1+e^{10x}}$

8) If $f(x) = 3^{\sin(x)}$, then $f'(x) = \cos x (3^{\sin x}) \cdot \ln 3 \Rightarrow \cos(x) (3^{\sin(x)}) \cdot \ln 3$

9) If $f(x) = (5x+1)^{x^2}$, then $f'(x) = (5x+1)^{x^2} \cdot \left[\frac{5x^2}{5x+1} + 2x \cdot \ln(5x+1)\right]$

10) $\lim_{a\to0} \frac{\ln(x^5+a)-\ln(x^5)}{a} = \frac{5x^4}{x^5}$

11) Let $f(x) = \frac{1}{\sin(x)-3} - \frac{x}{(x-5)}$. Then f is discontinuous at $x =$ $x = 5$ AND $x = \sin^{-1} 3$

12) If $f(x) = g(x^4+4)$ and $g'(5) = -5$ then $f'(1) = -20$

13) $\lim_{x\to2} \frac{\sin(x^2-4)}{x-2} = (\sin(x^2-4))'\big|_{x=2} = (\cos(x^2-4) \cdot 2x)\big|_{x=2} = 1 \cdot 4 = 4$

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Q2) (5 points) Find $\lim_{x\to-\infty} f(x) = \sqrt{x^2+2x} - \sqrt{x^2+1}$. indeterminate

$\lim_{x\to-\infty} \sqrt{x^2+2x} - \sqrt{x^2+1} = \sqrt{\infty - \infty} \ldots !!$

$= \lim_{x\to-\infty} \frac{x^2+2x-x^2-1}{\sqrt{x^2+2x}+\sqrt{x^2+1}}$

$= \lim_{x\to-\infty} \frac{2x-1}{|x|\left(\sqrt{1+\frac{2}{x}}+\sqrt{1+\frac{1}{x^2}}\right)}$

$\Rightarrow \lim_{x\to-\infty} \frac{x(2-\frac{1}{x})}{-x\left(\sqrt{1+\frac{2}{x}}+\sqrt{1+\frac{1}{x^2}}\right)}$

$= \frac{-1(2-\frac{1}{-\infty})}{\sqrt{1+\frac{2}{\infty}}+\sqrt{1+\frac{1}{\infty}}}$

$= \frac{-2}{1+1} = \frac{-2}{2} = \boxed{-1}$

5

Q3) (7 points) Find the points on the curve $2x^2+xy+y^2=14$ where the normal line is parallel to the line $y=x$.

7

Slope N $= y' = 1$

Slope of T $= -1 \Rightarrow y' = -1 \ldots (1)$

نعوض

$4x + x \cdot y' + y + 2y\, y' = 0$

$4x - x + y - 2y = 0$

$4x - x - y = 0$

$3x - y = 0$

نعوض في الاصل $\boxed{3x = y}$

$2x^2 + x(3x) + (3x)^2 = 14$

$2x^2 + 3x^2 + 9x^2 = 14$

$14x^2 = 14$

$x^2 = 1$

$x = \pm 1$

Points: $(1, 3)$ / $(-1, -3)$