1

12/3/2017

The University of Jordan
Department of Mathematics
Calculus I, First Exam

رقم التسلسلي : 43

Student's Name: ____ Student's Number ____

Instructor's Name: د. آيات عبابنة Lecture's time: 9 - 10

Q1) (10.5 marks) Fill the blank with the correct answer

1) $\text{Sin}^{-1}(\text{Sin}\frac{10\pi}{9})$ = $\boxed{\frac{10\pi}{9}}$ ≈ $(-)\frac{\pi}{9}$

2) If the range of f(x) is $[-3,5]$, and $h(x) = 4 - 2f(3x-1)$, then the range of h(x) is
Range h(x) = [-24, 24]

$-3 \le f(x) \le 5$
$-6 \le 2f(x) \le 10$
$6 \ge -2f(x) \ge 10$
$10 \ge 4 - 2f(x) \ge 14$ -6
$[-6, [10, 14]]$

3) The domain of $h(x) = \sqrt{9-x^2}\,\text{Sin}^{-1}(2x-5)$ is
$(-\infty, -3]$ OR $[3, \infty)$

4) If $\text{Log}_x(6x-5) = 2$, then x = $x = 5$ or $\cancel{x=1}$
$\boxed{6x - 5 = x^2}$

5) $\text{Sec}(\text{Sin}^{-1}(\frac{3}{5}) + \text{Cos}^{-1}(\frac{2}{5}))$ = $\frac{1}{90}$ ≈ $\frac{1}{\sin^{-1}(1)}$

6) Let $f(x) = x^2 + 2x + 4$, if h(x) is obtained from f(x) by shifting f(x) two units right and three units down, then h(x) = $x^2 + 2x + 5$

7) If $h(x) = 3^x$ and the domain of f(x) is $[2,8]$, then the domain of $(f \circ h)(x)$ is—

$[9, 3^8]$ $F(3^x)$ $2 \le f(x) \le 8$

6) $f(x-2) = 3$ $3^2 \le f(3^x) \le 3$

$h(x) = (x-2)^2 + 2(x-2) + 4 - 3$
$x^2 - 4x + \cancel{4} + 2x - \cancel{4} + \cancel{4} - 3$
$= x^2 - 2x + 1$

Q2) (3 marks) Let $f(x) = \frac{e^x}{3-e^x}$. Find $f^{-1}(x)$.

$$x = \frac{e^y}{3-e^y} = -x+3 = \frac{e^y}{e^y} \xrightarrow{\ln} \ln(-x+3) = y$$

$$\boxed{y = \ln -x+3}$$

Q3) (3 marks) Show that the function $f(x) = \frac{x^2 \operatorname{Cosx}}{x^3+x} + \operatorname{Ln}(\frac{1+x}{1-x})$ is an odd function.

$$-f(x) = -\left(\frac{x^2 \cos x}{x^3+x} + \ln \frac{1+x}{1-x}\right)$$

$$= \boxed{\frac{-(x^2 \cos x)}{-x^3 - x} - \ln \frac{1-x}{1+x}}$$

$f(x) \neq -f(x)$

so it's odd

Q4) (4 marks) If $\frac{4x^2-8x}{x^3-8} \leq 2f(x) - 3 \leq \frac{\sqrt{x+7}-3}{\sqrt{x+2}-2}$

on open interval near $x = 2$. Find $\operatorname{Lim}_{x \to 2} f(x)$.

$$\lim_{x \to 2^-} 2\left(\frac{4x^2-8x}{x^3-8}\right) - 3 = \frac{8x^2 - 16x - 3}{2x^3 - 13} = \frac{8(2)^2 - 16(2) - 3}{2(2)^3 - 13} = \frac{-3}{3}$$

$$= -1$$

$$\lim_{x \to 2^+} 2\left(\frac{\sqrt{x+7}-3}{\sqrt{x+2}-2}\right) - 3 = 2(\sqrt{9}$$

$$\lim_{x \to 2} f(x) = \boxed{-1}$$

Seat # 41

8.5

Jordan University

Mathematics Department

Calculus I, First Exam, 18/6/2017 III

Student's Name:

Student Number:

Lecture Time: 11:30 - 12:30 Seat # 40

Remark: **Show your supporting work.** **Final answers without supporting work receives no credit.**

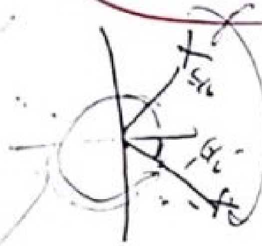
1) (1.5 points each) Fill in the blanks:

4.5

a) If $f(x) = \ln(5 - x) + \sqrt{4x+1}$, then domain$(f)$ = $(-\infty, 5)$.

b) $\tan^{-1}(\frac{-1}{\sqrt{3}}) = \frac{-\sqrt{1}}{1}$ $\frac{11\pi}{6}$ $\frac{11\pi}{6}$

c) $\sin(\cos^{-1}(\frac{2}{3}))$ = $\frac{\sqrt{5}}{3}$.



d) If $g(x) = 6 + x + e^x$, then $g^{-1}(7)$ = 0.

e) The solution of the equation $e^{x+3} = 7$ is x = -1.

f) The function $f(x) = \frac{x}{x^2+1}$ is symmetric about origion.

$-y = \frac{-x}{(x^2+1)} \rightarrow y = \frac{x}{x^2+1}$

$\sin(\frac{11}{1})\ \frac{1}{2}$

$\cos = \frac{\sqrt{3}}{2}$

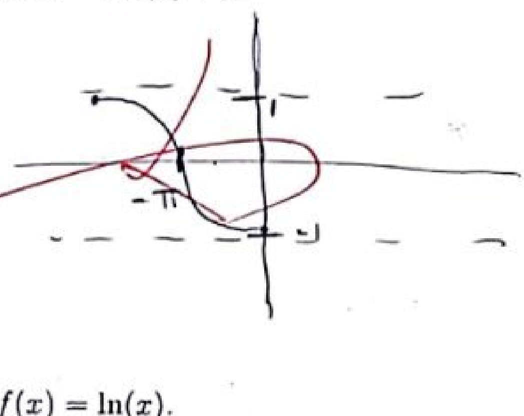
$\tan = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

2) Sketch the graph of

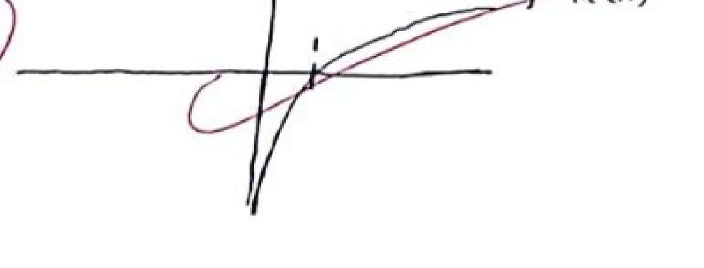
a) (2 points) $f(x) = \sec(x) + 2$.

2



a) (1 point) $f(x) = \ln(x)$.

1



3) Let $f(x) = x^2 - 8x + 16, \quad x \geq 4$.

$(x-4)^2$; $x \geq 4$

$(x-4)^2$

4^{x^2}

a) (2 points) Sketch the graph of f

1

b) (2 points) Find $f^{-1}(x)$

$f^{-1}(x) = x = y^2 - 8y + 16 \rightarrow \frac{x-16}{-8} = \frac{y^2-8y}{-8} \rightarrow \sqrt{\frac{x+16}{8}} = \sqrt{y^2}$

$y = \frac{\sqrt{x+16}}{8} \rightarrow \boxed{f^{-1}(x) = \frac{\sqrt{x+16}}{8}}$

4) Let $f(x) = 7x^2$ and $g(x) = \sqrt{x+4}$.

a) (2 points) Find $f \circ g(x)$

2

$(f \circ g)(x) = 7(\sqrt{x+4})^2 \rightarrow 7(x+4) \rightarrow \boxed{7x+28}$

$\boxed{(f \circ g)(x) = 7x+28}$

b) (2 points) Find the domain of $f \circ g$

$7x + 28 = 0 \rightarrow 7x = -28 \rightarrow x = -4$.

$D_f = \boxed{\mathbb{R}, x \neq 4}$

(13 ~~10~~) # 77

The University of Jordan Mathematics Department Calculus I Second Exam 19/8/2017

اسم الطالب : اسم مدرس المادة: د. عبد الله الطلافحة

التخصص: الرقم الجامعي:

Part (I): In questions 1 – 10, fill in the blank to get correct sentence.

(2 marks each)

(1) The function $f(x) = \frac{x-2}{x^2-4}$ has removable discontinuity at $x =$ ....2.......

(2) The function $f(x) = \frac{x+5}{(x-2)\,sinx}$, $x \in [0,2\pi]$ has vertical asymptote(s) at $x =$ ....2.......

(3) $\frac{d}{dx}(\sin(tan^{-1}(5x+1)) =$ $\cos(\tan^{-1}(5x+1)) \cdot \frac{5}{2+25x}$

(4) $\frac{d}{dx}(x^{9x}) =$ $(9 + \ln x^9)(x^{9x})$

(5) If $g(x) = \frac{\sqrt{e^x+3tanx}}{(6x+1)^{\frac{5}{3}}(csc^{10}x)}$, then **by using logarithmic differentiation**

$g'(x) = \left(\frac{e^x + 3\sec^2 x}{2e^x + 6\tan x} - \frac{30}{18x+3} + \frac{-10\csc x \cot x}{\csc^{10} x}\right)\left(\frac{\sqrt{e^x+3\tan x}}{(6x+1)^{\frac{5}{3}}(\csc^{10} x)}\right)$

(6) The function $f(x) = \begin{cases} mx+b & if\ x<2 \\ x^2 & if\ x \geq 2 \end{cases}$ is differentiable at 2

when $m = \frac{4-b}{2}$, and $b =$ $4-2m$

(7) If $f(x) = x^{11} + 5x + 4$, then $(f^{-1})'(4) =$ $11(4)^{10} + 5$

(8) By using linear approximation, $\sqrt[4]{82} \approx$ 3.166

(9) Equation of the tangent line to the curve of $y = x^4 + 2^x$ at the point (1,3) is

$y = (4 + 2\ln 2)x + 2$

(10) $\lim_{x\to\infty} \frac{e^{-x}-4e^x}{2e^{-x}+5e^x} =$ $\frac{-4}{5}$

(11) $\lim_{x\to -3} \frac{\sin(x^2+4x+3)}{x+3} =$ $\boxed{-2}$

$(F^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$

(12 ~~9~~)

$x^{11} + 5x + 4 = 4$

$x^{11} + 5x = 0$ $\quad \frac{1}{f'(0)}$

$x = 0$ $\quad = 1$

Part (II): In questions 12-14, give your answer in details

(12) (3- marks) Using the intermediate value theorem, show that the equation $x^3 - x - 1 = 0$ has a real root in the open interval $(0, 3)$.

$\cancel{x^3 - x - 1 \to x(x-1)(x+1) \to x = 0,\ x = -1,\ x = 1}$

$\cancel{0 < x^3 - x - 1 < 3}$ $\Leftrightarrow$ $\cancel{x=0 \to -1}$

$+1 \quad +1 \quad +1$ $\qquad$ $\cancel{x=-1 \to -1}$

$\cancel{1 < x^3 - x < 4}$ $\qquad$ $\cancel{x=1 \to -1}$

$+x \quad +x \quad +x$

$\cancel{\sqrt[3]{1+x} < \sqrt[3]{x^3} < \sqrt[3]{4+x}} \to \cancel{\sqrt[3]{1+x} < x < \sqrt[3]{4+x}}$

By IUT, there ist al least 1 real root For $x^3 - x - 1$ in $(0, 3)$

(13) (4-marks) Find all horizontal asymptotes for $f(x) = \frac{1}{x - \sqrt{x^2+3x}}$

(14) (3 marks) If $9x^3 + y^3 = 16$, find y'' by **implicit differentiation**.

$y \Rightarrow 9x^3 + y^3 - 16 \to y' = 27x^2 + 3y^2y'$

$y' - 3y^2y' = 27x^2 \to \frac{y'(-3y^2)}{-3y^2} = \frac{27x^2}{-3y^2}$

$y' = \frac{27x^2}{-3y^2}$ $\qquad$ 1

$y'' = \frac{54x}{-6yy''} = y'' = \boxed{\frac{54x}{-6y}}$

Set No. 23

The University of Jordan

DEPARTMENT OF MATHEMATICS

$\frac{19}{20}$

Calculus I [redacted]TER 2016-2017 **Date:** 05/11/2016

الاسم : سجود عمر مصطفى اليوسف الرقم الجامعي: [redacted] وقت المحاضرة: (2 → 3:30)

Instructions: *The test one two-sided page; make sure you do both sides. You CANNOT use a calculator on any part of this exam. The point value of each problem is indicated in brackets. Finally, before you start to work a problem, be sure that you understand what is being asked.*

For questions 1 to 8, fill in the blank with the correct answer. Only correct answers count. [1.5 pts each]

$\frac{10.5}{12}$

1. $\tan\left(\frac{5\pi}{6}\right) =$ $\frac{-1}{\sqrt{3}}$

2. If $1+\log_2(x-5)=\log_2(x+4)$, then $x =$ 14

3. If $f(x)=\frac{\cos^{-1}(1-x)}{2-x}$, then Dom($f$) = $[0, 2)$

4. If $f(x)=\sqrt{16-x^2}+2$, then Range(f) = $[2, 6]$

5. $\sin^{-1}\sin\left(\frac{8\pi}{7}\right) =$ $\frac{-\pi}{7}$

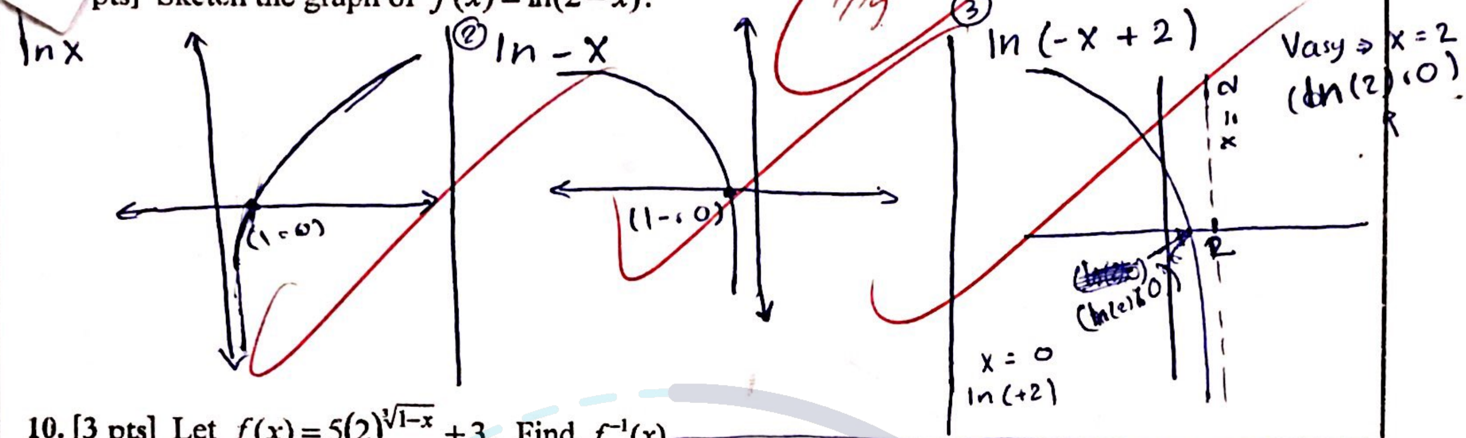
6. $\sin\left(2\tan^{-1}\left(\frac{2}{5}\right)\right) =$ $\frac{20}{29}$

7. If $f(x)=\ln x$, then Dom$(f\circ f)$ = ~~(0, ∞)~~

8. The function $f(x)=e^{(2+x)}-e^{(2-x)}$ is symmetric about the (0,0) oragin

...tion 9, 10, and 11, sufficient work must be shown to receive credit.

...pts] Sketch the graph of $f(x)=\ln(2-x)$.



10. [3 pts] Let $f(x)=5(2)^{\sqrt[3]{1-x}}+3$. Find $f^{-1}(x)$.

$y = 5(2)^{\sqrt[3]{1-x}} + 3$

$y - 3 = 5(2)^{\sqrt[3]{1-x}}$

$\ln(y-3) = \ln 5 + \sqrt[3]{1-x}\,\ln 2$

$\ln(y-3) - \ln 5 = \sqrt[3]{1-x}\,\ln 2$

$\left(\dfrac{\ln\frac{(y-3)}{5}}{\ln 2}\right)^3 = \left(\sqrt[3]{1-x}\right)^3$

$\left(\log_2 \dfrac{y-3}{5}\right)^3 = 1 - x \Rightarrow$

$x = 1 - \left(\log_2\left(\dfrac{y-3}{5}\right)\right)^3$

$\therefore f^{-1}(x) = 1 - \left(\log_2\left(\dfrac{x-3}{5}\right)\right)^3$

3/3

11. [2 pts] If $h(x)=\arctan x$ for $x\ge 0$, $g(x)=\cos x$, and $f(x)=(1-x^2)^{-1}$, then $(f\circ g\circ h)(x)=1+x^p$. Find the value of the number p.

$f\circ g\circ h = f(g(h(x)))$

$= f(g(\tan^{-1} x))$

$= f(\cos(\tan^{-1}x))$

$\theta = \tan^{-1} x$

$\tan\theta = x$

$\sqrt{x^2+1}$, x, 1

$\therefore \boxed{\cos\theta = \dfrac{1}{\sqrt{x^2+1}}}$

$\therefore f\left(\dfrac{1}{\sqrt{x^2+1}}\right) = \dfrac{1}{\left(1-\left(\frac{1}{\sqrt{x^2+1}}\right)^2\right)}$

$= \dfrac{1}{1-\frac{1}{x^2+1}}$

$= \dfrac{1}{\frac{x^2+\cancel{1}-\cancel{1}}{x^2+1}}$

$= \dfrac{x^2+1}{x^2}$

$= 1 + \dfrac{1}{x^2}$

$= 1 + x^{-2}$

$\therefore \boxed{p = -2}$ #

2/2

Good Luck

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The University of Jordan Mathematics Department Calculus I First Exam 6/8/2017

اسم الطالب : محمد اليسع رمزي أرسلان   اسم مدرس المادة: د. منال ع

التخصص: الرقم الجامعي:

Part (I): In questions 1 – 8, fill in the blank to get correct sentence.

(1.5 marks each)

8.5

(1) If $f(x) = \frac{9}{x+2}$, $g(x) = \sqrt{x-3}$, *and* $h(x) = x + 4$, *then*

$g \circ h \circ f(1) =$......2.................. ✓

(2) $\csc\cot^{-1}(\frac{-1}{2}) =$.....$\frac{\sqrt{4+1}}{-2}$ = $\frac{\sqrt{5}}{2}$ ✗

(3) The function $f(x) = x^6(2^{4-x} - 2^{4+x})$ is symmetric about..~~odd~~.. origin ✓

(4) The domain of the function $h(x) = \sqrt{2-x}\ \ln(x+5)$ is....(−5 , 2] ✓

(5) $\cos(2\tan^{-1}x) =$.. $\cancel{\frac{1-x}{\sqrt{1+x^2}}}$ = $\cancel{\frac{1-x}{1+|x|}}$ = $\cancel{\frac{1-x}{1+|x|}}$

(6) $\sin^{-1}\sin\frac{15\pi}{7} =$......$\frac{\pi}{7}$ ✓......................

(7) If $2\,log_3 x - log_3(x+2) = log_3 7 + log_3(\frac{1}{7})$, then $x =$..~~−1 , 2~~......

(8) If $f(x) = 4 + \cos^{-1}(x+1)$, then $Range(f) =$ ....[4 , π+4] ✓

Part (II): In questions 9– 12, <u>give your answer in details.</u>

9- [3 marks] Find a function g that is obtained from $f(x) = = |x| + 3\sqrt{x}$ by the following steps. **Write the answer of each step.**

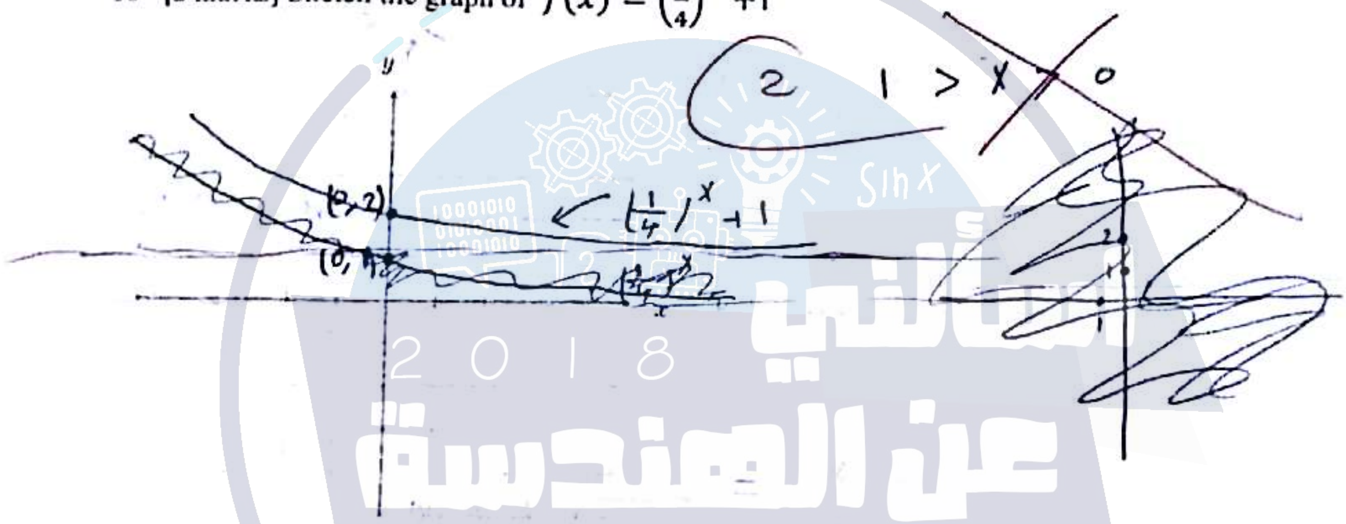
(i) **First step:** shift the graph $y = f(x)$ **7 units** to the left to get $y =$ ..$|x+7| + 3\sqrt{x+7}$ ....

(ii) **then** reflect it about y-axis to get $y =$ ......$|-x+7| + 3\sqrt{-x+7}$

3

(iii) **Finally,** shrink it vertically by a factor of 4 to get $g(x) = \frac{1}{4}\left(|-x+7| + 3\sqrt{-x+7}\right)$

10- [2 marks] Sketch the graph of $f(x) = \left(\frac{1}{4}\right)^x + 1$



11- [3 marks] If $f(x) = e^x + \frac{1}{e^x}$, find $f^{-1}(x)$

1

$y = e^x + \frac{1}{e^x}$

$x = e^y + \frac{1}{e^y}$

$x = \frac{e^{2y} + 1}{e^y}$

$\ln x = \ln \frac{e^{2y}+1}{e^y}$

$\ln x = \ln e^{2y} + 1 - \ln e^y$

$\ln x = \ln e^{2y} + \ln 1 - \ln e^y$

$\ln x = 2y - y$

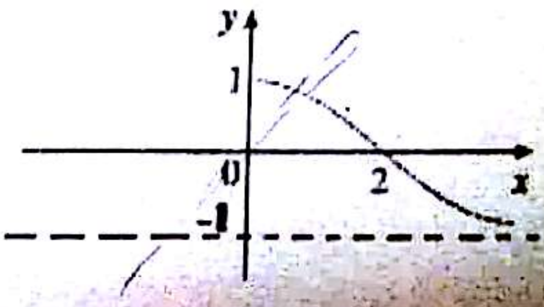
$\ln x = y$

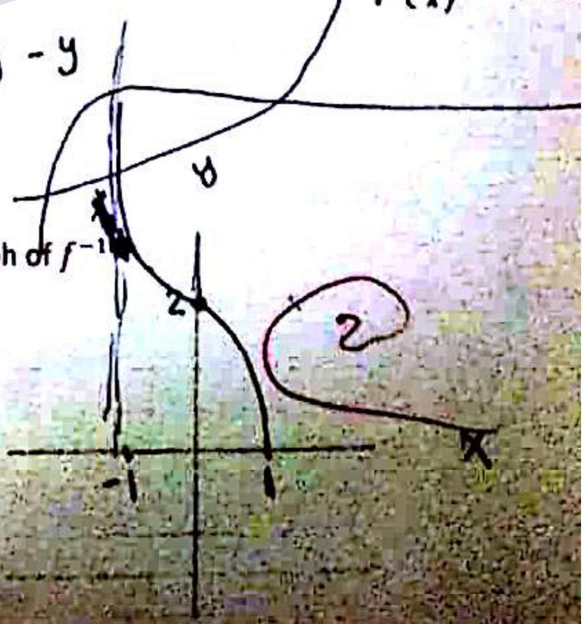
$y = \ln x$

$f^{-1}(x) = \ln x$

2

12- [2 marks] Use the given graph of f to sketch the graph of f^{-1}





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University of Jordan

DEPARTMENT OF MATHEMATICS

/20

Calculus I EXAM 1C / 1[st] SEMESTER 2016-2017 Date: 05/11/2016

الاسم .... الرقم الجامعي: (..0162892..) وقت المحاضرة: (............)

Instructions: *The test one two-sided page; make sure you do both sides. You* ***CANNOT*** *use a calculator on any part of this exam. The point value of each problem is indicated in brackets. Finally, before you start to work a problem, be sure that you understand what is being asked.*

For questions 1 to 8, fill in the blank with the correct answer. **Only correct answers count. [1.5 pts each]**

1. $\tan\left(\frac{5\pi}{6}\right) =$ $\frac{-\pi}{6}$ $= -\frac{1}{\sqrt{3}}$

$\frac{5 \times 180}{6} = 150$

$\tan 150 = -\tan 30 = -\frac{1}{\sqrt{3}}$

$180 - x = 150$

$x = 30$

2. If $1+\log_2(x-5)=\log_2(x+4)$, then $x =$ 1

3. If $f(x)=\frac{\cos^{-1}(1-x)}{2-x}$, then $\text{Dom}(f) =$ $[0, 2)$

4. If $f(x)=\sqrt{16-x^2}+2$, then $\text{Range}(f) =$ $[2, 6]$

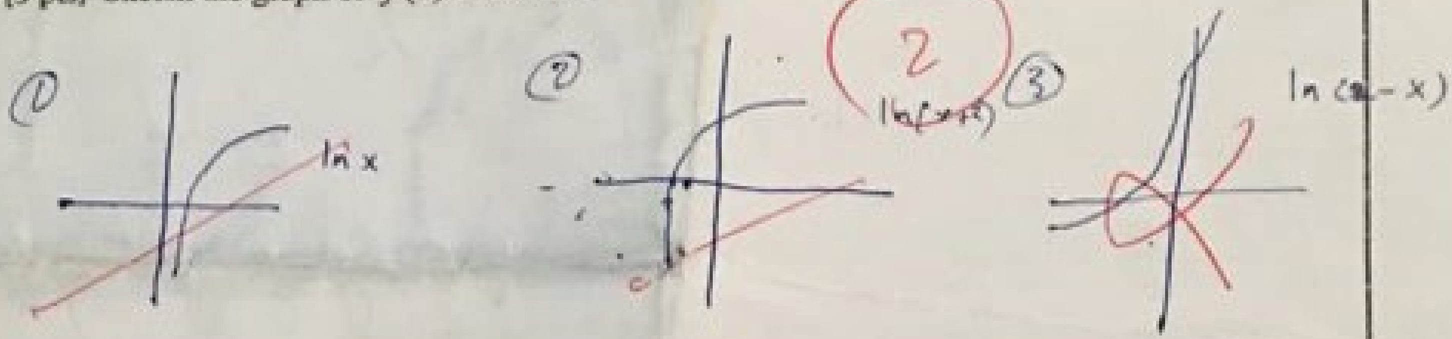
5. $\sin^{-1}\sin\left(\frac{8\pi}{7}\right) =$ $\frac{-\pi}{7}$

6. $\sin\left(2\tan^{-1}\left(\frac{2}{5}\right)\right) =$ $\frac{40}{2\sqrt{29}\sqrt{29}}$

7. If $f(x)=\ln x$, then $\text{Dom}(f \circ f) =$ $(0, \infty)$

8. The function $f(x)=e^{(2+x)}-e^{(2-x)}$ is symmetric about the origin ~~origan~~ $(0,0)$

9. [3 pts] Sketch the graph of $f(x) = \ln(2-x)$.



10. [3 pts] Let $f(x) = 5(2)^{\sqrt[3]{1-x}} + 3$. Find $f^{-1}(x)$.

$y = 5(2)^{\sqrt[3]{1-x}} + 3$

$y - 3 = 5(2)^{\sqrt[3]{1-x}}$

$\frac{y-3}{5} = 2^{(1-x)^{\frac{1}{3}}}$

$\log_2 \frac{y-3}{5} = (1-x)^{\frac{1}{3}}$

$\sqrt[3]{\log_2 \frac{y-3}{5}} \; \sqrt[3]{\frac{1}{(1-x)^3}}$

$\sqrt[3]{\log_2 \frac{y-3}{5}} = \frac{1}{1-x} \times 1-x$

2

$1 = \sqrt[3]{\log_2 \frac{y-3}{5}} - x\sqrt[3]{\log_2 \frac{y-3}{5}}$

$\frac{x\sqrt[3]{\log_2 \frac{y-3}{5}}}{\sqrt[3]{\log_2 \frac{y-3}{5}}} = \frac{\sqrt[3]{\log_2 \frac{y-3}{5}} - 1}{\sqrt[3]{\log_2 \frac{y-3}{5}}}$

11. [2 pts] If $h(x) = \arctan x$ for $x \geq 0$, $g(x) = \cos x$, and $f(x) = (1-x^2)^{-1}$, then $(f \circ g \circ h)(x) = 1 + x^p$. Find the value of the number p.

$h(x) = \tan^{-1}(x)$

$g(\tan^{-1}(x))$

$\cos \tan^{-1} x$

$F(x) = (1 - (\cos \tan^{-1} x)^2)^{-1}$

$= 1 - \cos \tan^{-1} x$

0.5

Good Luck

1- (2pts). Let $f(x) = x^3 + 3x^2 + 3x + 1$ and $g(x) = \frac{1}{x}$. Find

a) $(f \circ g)\left(\frac{1}{3}\right)$

$f(g(x)) \Rightarrow f\left(\frac{1}{x}\right) \Rightarrow \left(\frac{1}{x}\right)^3 + 3\left(\frac{1}{x}\right)^2 + 3\left(\frac{1}{x}\right) + 1$ $\boxed{64}$

$f\left(\frac{1}{\frac{1}{3}}\right) = 3^3 + 3\times(3)^2 + 3\times3 + 1$

$27 + 27 + 9 + 1 = \boxed{64}$

2

b) $f^{-1}(1)$ ~~$f(x) = y$~~

~~$y - 1 = x(x^2 + 3x + 3)$~~

$\boxed{= 0 \quad \text{zero}}$

2- (3pts). Let $f(x) = \sqrt{\frac{4-x}{x-1}} \cos^{-1} 2x$, find domain f.

$d\sqrt{\frac{4-x}{x-1}} = 1 < x \leq 4$

$(1, 4]$

$d \cos^{-1} 2x = [1, -1]$

1

$d \sqrt{\text{الجذر}} \cap d \cos^{-1}$

$\boxed{d f(x) = \phi}$

3- (2pts). Find the shifts by which $f(x) = x^2 + 2x + 2$ is obtained from $g(x) = x^2 + 4x + 2$.

$f(x) = x^2 + 2x + 2$ إكمال مربع $\left(\frac{\text{معامل } x}{2}\right)^2$. $x^2 + 2x + 1 - 1 + 2$

$(x+1)^2 + 1$

1.5

$g(x) = x^2 + 4x + 2$ إكمال مربع $\Rightarrow (x+2)^2 - 2$

± 4

shift 1 to the Lift then shift to up 3 unite

مقدار 1

4- (4pts). Find the range of the following functions.

a) $f(x) = x^2 + 4x + 3, -3 \leq x \leq 1$

9 -8 +3

$(x+3)(x+1)$

$x = -3$ $\quad$ $x = -1$

-3 -1 1

$R\ [0, 8]$

b) $g(x) = \frac{3x+1}{2x-1}$

R

5- (4pts). Let $f(x) = cosx, g(x) = sin^{-1}x$ and $h(x) = sinx$.

Find a) $(f \circ g)\left(\frac{1}{2}\right)$ b) $(g \circ h)\left(\frac{11\pi}{3}\right)$

$f(g(\frac{1}{2})) = f(\sin^{-1}(\frac{1}{2}))$

$= \cos(\sin^{-1}(\frac{1}{2})) = \frac{2}{\sqrt{3}}$

$g(h(\frac{11\pi}{3})) = g(\sin(\frac{11\pi}{3}))$

$\sin^{-1}(\sin(\frac{11\pi}{3})) = \boxed{-\frac{\pi}{3}}$

6- (2pts). Find the value of x which satisfies $log_x(4x - 3) = 2$

$x^{\log_x(4x-3)} = x^2$

$4x - 3 = x^2 - 4x + 3$

$(x-1)(x-3)$

$\boxed{x=1}$ $\boxed{x=3}$

7- (2pts). Let $f(x) = \frac{e^x-2}{e^x+1}$. Find $f^{-1}(x)$.

$f(x) = y$ نفرض

$y = \frac{e^x - 2}{e^x + 1}$

$e^x - 2 = ye^x + y$

$e^x - ye^x = y + 2$

$\frac{e^x(1-y)}{1-y} = \frac{y+2}{1-y}$

$e^x = \frac{y+2}{1-y}$

$\ln e^x = \ln\left(\frac{y+2}{1-y}\right)$

$x = \ln\left(\frac{y+2}{1-y}\right)$

$f^{-1}(x) = \ln\left(\frac{x+2}{1-x}\right)$

8- (2pts). Show that $f(x) = ln\frac{x+1}{x-1}$ is an odd function.

simtrec of the origen

$f(-x) = -\ln\frac{x+1}{x-1}$

$= \ln\left(\frac{x+1}{x-1}\right)^{-1}$

THE UNIVERSITY OF JORDAN

0301101 Calculus I — First test — Oct. 31, 2015

Student's name:

Student's number:

Instructor's name:

Class time:

For instructor use only, please do not write in this table.

Q1	Q2	Q3	Q4	Grade
9	1	6	[illegible]	19

Q1) Fill in the blanks with the answers only. Each part is worth 1.5 marks.

1) The solution of the equation $\ln(x) - \ln(3x-2) = 0$ is ....$x = 1$....

2) $\sin\left(\frac{4\pi}{3}\right) = $ ....$\frac{-\sqrt{3}}{2}$....

3) The domain of $f(x) = \ln\left(\frac{x-1}{x-3}\right)$ is .... $D_f = (3, \infty)$ .... $+ \mathbb{R} - [1, 3]$

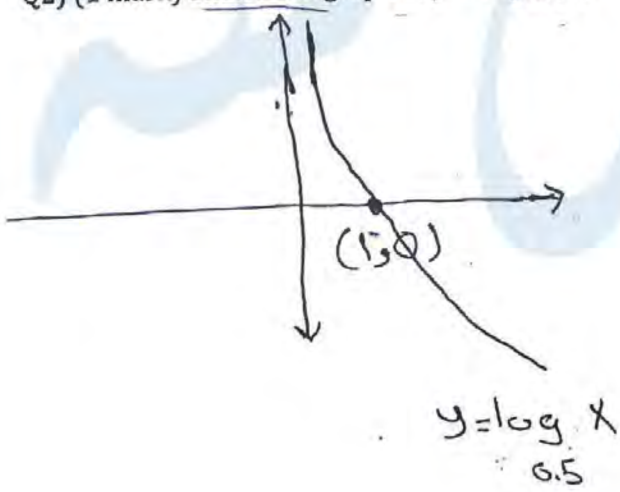
4) The range of $g(x) = 1 - \sqrt{x}$ is .... $R_f = (-\infty, 1]$ ....

5) $\tan\left(\cos^{-1}\left(\frac{-1}{3}\right)\right) = $ .... $\boxed{-\sqrt{8}}$ ....

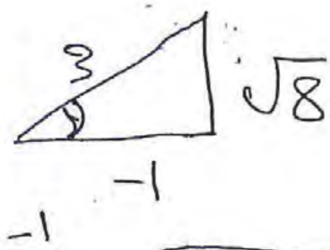
6) If $f(x) = \frac{2}{x}$ and $g(x) = \frac{x+1}{x+3}$, then the domain of $g \circ f$ is .... $D_{(g \circ f)} = \mathbb{R} - \{0, \frac{-2}{3}\}$

7) $\cos^{-1}\left(\cos\left(\frac{27\pi}{7}\right)\right) = $ ....$\cos^{-1}\left(\cos \frac{\pi}{7}\right) = \boxed{\frac{\pi}{7}}$....

Q2) (1 mark) Sketch the graph of $y = \log_{0.5}(x)$.



$y = \log_{0.5} x$



$\cos^{-1} \frac{-1}{3}$

$\sqrt{9-1} = \sqrt{8}$

In questions 3 and 4, write every step of your work.

Q3) Let $f(x) = \left(\frac{1+e^x}{1-e^x}\right)$.

1) (3 marks) Find $f^{-1}(x)$.

$$y = \frac{1+e^x}{1-e^x} \Rightarrow y - ye^x = 1 + e^x$$

$$\Downarrow$$

$$y - 1 = e^x + ye^x$$

$$\Downarrow$$

$$y - 1 = (1+y)e^x$$

$$e^x = \frac{y-1}{1+y}$$

$$\Downarrow$$

$$\ln e^x = \ln\left(\frac{y-1}{1+y}\right)$$

$$\Downarrow$$

$$x = \ln\left(\frac{y-1}{1+y}\right) \Longrightarrow \text{Finally } \boxed{f^{-1}(x) = \ln\left(\frac{x-1}{1+x}\right)}$$

3

2) (3 marks) Show that f is symmetric with respect to the origin.

$$f(-x) = \frac{1+e^{-x}}{1-e^{-x}} = \left(\frac{1+e^{-x}}{1-e^{-x}}\right)\frac{e^x}{e^x}$$

3

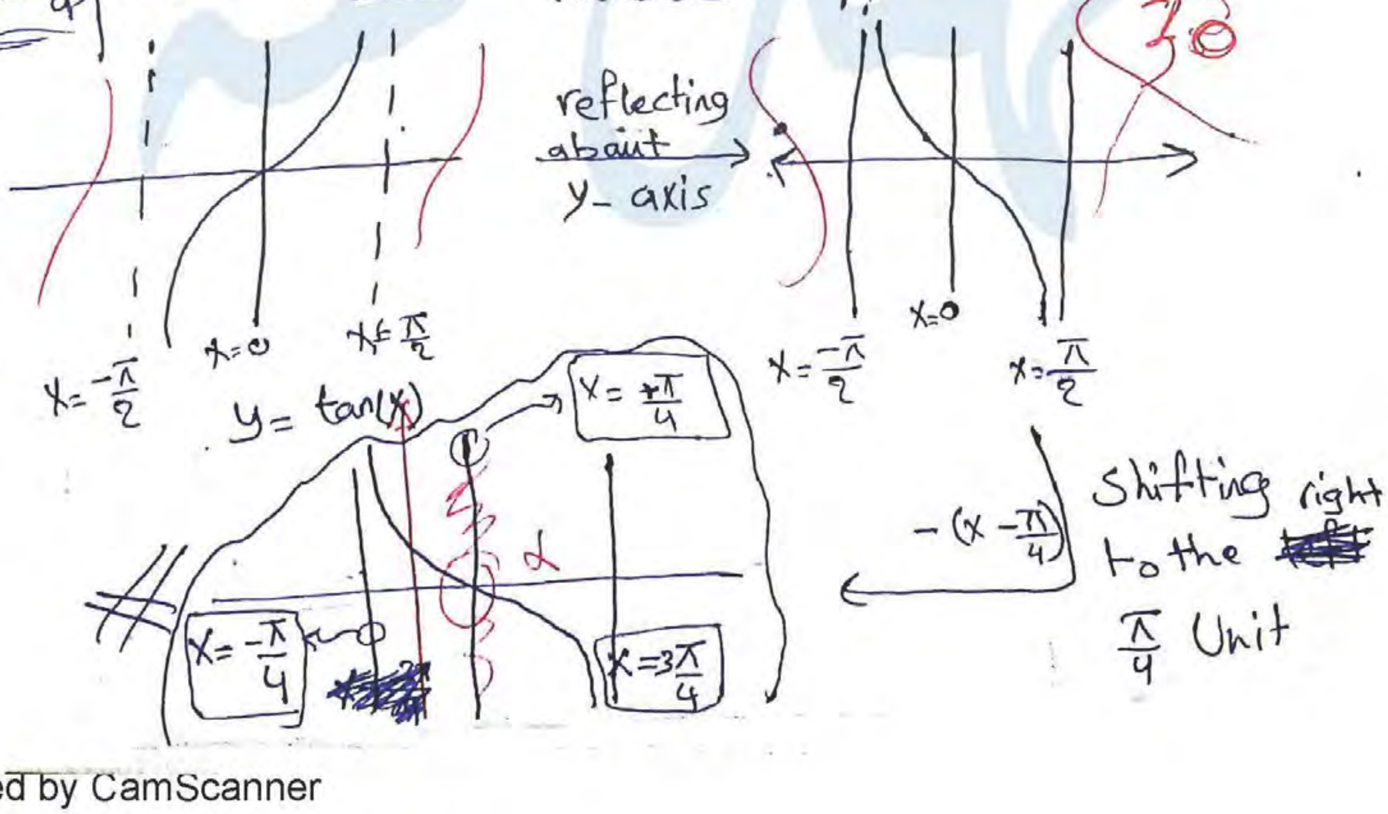
$$= \frac{e^x+1}{e^x-1} = \frac{e^x+1}{-(1-e^x)} = \frac{-(1+e^x)}{1-e^x} = -f(x)$$

So $f(-x) = -f(x)$

So it's odd function So that it is symmetric with respect to the origin.

Q4)(3 marks) Sketch the graph of $y = \tan\left(\frac{\pi}{4} - x\right)$ using the graph of $y = \tan(x)$.

By the way $-\left(x - \frac{\pi}{4}\right)$



3

$\frac{13.5}{20}$

In questions (1) to (10), fill in the blanks to get a true statement:

10

1) The solution(s) of equation $e^{x^2}e^{4x+4} = 1$ is (are) $x =$ ......-2..........

2) $\cos\left(2\sin^{-1}\left(\frac{2}{3}\right)\right) =$ ....1...........

3) Is the function $f(x) = \frac{\sin(x^2)}{x^3-2x}$ even, odd or neither .even

4) If $f(x) = \frac{1}{\sqrt{x-1}}$ and $g(x) = x^2 - 8$, then domain of $(f \circ g)$ is $(-\infty, -3) \cup (3, \infty)$

5) The solution(s) of $|\tan x| = 1$ in the interval $\pi \le x \le 2\pi$ is (are) $x =$

......$\frac{3\pi}{2}$..........

6) If $f(x) = -3\cos^{-1}(2x+7) + 5$, then range$(f)$ is ..$[-8\pi, 5]$

7) Let $f(x) = \sqrt{1-x}\ln x$, then domain $f(x)$ is $(0, 1] =$ domain

8) The value of $5^{\log_5 3 - 2\log_5 4}$ ...$\frac{3}{16}$......

9) If $g(x)$ is obtained from translating the graph of $f(x) = \frac{1}{1-x}$ 1- unit to the right and 2- units upward then $g(x) = \left(\frac{1}{1-(x-1)}\right) + 2$

10) Let $f(x) = \ln(3-x) + x^2$, $f^{-1}(4) =$ ....2...

10) Let $y = (\cos x)^{\ln x}$, find $\frac{dy}{dx}$.

$\ln y = \ln x(\ln \cos x)$

$\frac{y'}{y} = \frac{1}{x}.(\ln \cos x) + \ln x\left(\frac{-\sin x}{\cos x}\right)$

$y' = (\cos x)^{\ln x}\left(\frac{1}{x}.\ln \cos x + \ln x.\frac{-\sin x}{\cos x}\right)$

4

11) Prove that the equation $4^x = 2 - 3x$ has at least one real root.

$\cancel{f(x) = 4^x + 3x - 2}$

$\cancel{f'(x) = 4^x.\ln 4 + 3}$

$\cancel{f''(x) = 4^x.(\ln 4)^2}$

$\cancel{4^x = 2 - 3x}$

$\cancel{x = \log_4 2 - 3x}$

$4^x.\ln 4 = -3$

$4^x = \frac{-3}{\ln 4}$

$x = \log_4\left(\frac{-3}{\ln 4}\right)$

Zero

12) Let $f(x) = tan^{-1}(e^{3x})$ find equation of the tangent line at $x = 0$.

$f'(x) = \frac{3.e^{3x}}{1+(e^{3x})^2}$

$f'(0) = \frac{3.1}{1+1}$

$f'(x) = \frac{3}{2} = \text{slope}$

$f(0) = \tan^{-1}(e^{3x})$

$= \tan^{-1}(1)$

$y = 0$

$(y-0) = \frac{3}{2}(x-0)$

$y = \frac{3}{2}x$

2.5

Good luck..

17

THE UNIVERSITY OF JORDAN

0301101 Calculus I | First test | Oct. 25, 2014

Name: | Number:

Instructor's name: | Class days and time:

Questions 1 to 8: fill in the blanks with the answers only. Each question is worth 1.5 marks.

___ Q1) If the domain and range of $f(x)$ are $[-1,3]$ and $[0,4]$, respectively, then the range of $g(x) = 1 - 4f^{-1}(3x)$ is $[-11, 5]$

___ Q2) If the domain of $f(x)$ is $[-4,3)$ and $g(x) = 2x + 1$, then the domain of $(f \circ g)(x)$ is ~~$[-7, 7)$~~

___ Q3) If $Log_2(7x - 1) - Log_2 x = 1$, then $x =$ $\frac{1}{5}$

___ Q4) $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right) =$ $\frac{5\pi}{6}$

-3

___ Q5) $\cos\left(2\tan^{-1}\left(\frac{2}{3}\right)\right) =$ $\frac{5}{13}$

___ Q6) If $f(x) = \frac{3x^2+1}{\ln(3-x)}$, then the domain of $f(x)$ is $\mathbb{R} -$ ~~$(3, \infty)$~~

0 ___ Q7) The graph of $y = x^2 \tan x$ is symmetric about origine (نقطة الأصل)

___ Q8) If $16^x - 4^x - 6 = 0$, then $x =$ $\log_4 3$

~~* D f(x) = [-4, 3)~~

~~D g(x) = $\mathbb{R}$~~

Questions 9 and 10: Each question is worth 4 marks.

____ Q9) Find the function g that is obtained from $f(x) = |4x + 1|$ by the following steps. Write the answer of each step.

a) Reflecting the graph of $y = f(x)$ about the y-axis $\cancel{f(x) = -|4x+1|}$ then $|-4x+1|$

b) shifting 5 units down $\cancel{-|4x+1|-5}$ then $|-4x+1|-5$

c) stretching vertically by a factor of 2 units $\cancel{-|8x+1|-5}$ then $\cancel{|-8x+1|-5}$

d) reflecting about the x-axis $\cancel{-1}$ $\cancel{-|-8x+1|+5}$

c) $2|-4x+1|-10$

d) $-2|-4x+1|+10$

____ Q10) ***Solve and show your work.*** Let $f(x) = \ln(4x + 1) - \ln(1 - 4x)$

a) Determine whether the function f is even, odd or neither.

$f(-x) = \ln(-4x+1) - \ln(1+4x)$

$f(-x) \neq f(x)$ not even

$-f(x) = -\ln(4x+1) + \ln(1-4x)$

$f(-x) = -f(x)$ $\therefore$ $f(x)$ odd

b) Find $f^{-1}(x)$.

$f(x) = \ln(4x+1) - \ln(1-4x)$

$y = \ln(4x+1) - \ln(1-4x)$

$x = \ln(4y+1) - \ln(1-4y)$

$e^{x} = e^{\ln \frac{4y+1}{1-4y}}$

$e^{x} = \dfrac{4y+1}{1-4y}$

$e^{x} - 4e^{x}y = 4y + 1$

$e^{x} - 1 = 4e^{x}y + 4y$

$e^{x} - 1 = y(4e^{x} + 4)$

$y = \dfrac{e^{x}-1}{4e^{x}+4} = f^{-1}(x)$

قـ ا ـم ـي

ق أحل أ

The University of Jordan **First Exam** **Second Semester**

Department of Mathematics **Math 101** **21/3/2015**

الرقم الجامعي: **اسم الطالب:**

وقت المحاضرة: **اسم المدرس:**

Q1: Write down only the answers of the questions (1-8)

1. If the function $f(x)=\sqrt{5x-1}-\sqrt{3-x}$, then the domain of $f(x)$ is

2. $\log_4 32+\log_4 50-\log_4 25=$

3. $\cos^{-1}(\cos(4\pi/3))=$

4. If f(x) is even function, g(x) is odd function such that f(3)= -4 a g(1) =3, then (fog)(-1) =

5. The graph of f(x) = ln(3x-5) is obtained from the graph of g(x) = ln(x) by shifting . then

6. The range of the function $y=2\cos^{-1}(3x+1)-\pi$ is

7. The solution of $\ln(3x-1)-\ln(x-5)=1$ is

8. If $g(x)=5-2\cos(\pi x)+e^{x-1}$, the $g^{-1}(8)=$

Q2. If $f(x)=\dfrac{x^2}{x^2+5}, x \geq 0$,

a) find $f^{-1}(x)$

b) find the range of $f(x)$.

Q3. Solve $e^{2x}+8e^{x+1}=e^2$

In questions (1) to (10), fill in the blanks to get a true statement:

1) The domain of the function $f(x) = \sqrt{\frac{x^2-9}{x-3}}$ is

2) The range of the function $f(x) = |x| + 3|x-2|$ is

3) Let $f(x) = \frac{x^3}{x^3+1}$. If $f^{-1}(x) = 3$, then $x =$

4) If $f(x) = 3\ln\sqrt[3]{x-2}$, then $f^{-1}(x) =$.

5) $\sin\left(\cos^{-1}\left(\frac{-2}{5}\right)\right) =$

6) If $f(x) = \begin{cases} 1/(x+2) & x < -2 \\ x^2 - 5 & -2 < x < 4 \\ \sqrt{x+14} & x > 4 \end{cases}$, then

$\lim_{x\to 4^-} f(x) =$

7) Let $f(x) = \begin{cases} x^2 + 5 & x > 2 \\ a(x+1) + b & -1 < x \le 2 \\ 2x^3 + x + 7 & x \le -1 \end{cases}$

Then the values of a and b that will make the function $f(x)$ continuous are $a =$ and $b =$

8) $\lim_{x\to 0}\left(\frac{1}{x} - \frac{2}{x^2+2x}\right) =$.

9) If $\ln\left(\frac{1}{x}\right) + \ln(2x^3) = \ln 5$, then $x =$

10) If $\sin(x) = \frac{2}{5}$, then $\sin\left(x + \frac{\pi}{6}\right) =$

Solve questions (11) and (12). Show your work:

11) Let $f(x) = \frac{x^3-8}{x-2}$

(a) Find $\lim_{x \to 2} f(x)$.

(b) Sketch the graph of $f(x)$.

12) Assume that $f(x)$ is continuous, and $\lim_{x \to 1} \frac{2f(x)-1}{x-1} = 2$.

(a) Find $\lim_{x \to 1} f(x)$.

(b) Find $f(1)$.