PAST PAPERS

CALCULUS 101

اسألني
الهندسة
2014

خدمة الطالب عبادة

The University of Jordan	**First Exam**	**SummerSemester**
Department of Mathematics	**Math 101**	**16/7/2011**

Name: .... **Section:** ....

Instructor name: ....

I. Write down only the answers of the questions (1 –8).

1. *The domain of the function* $y = 3\sin^{-1}(2x+4)+1$ *is* ....

2. The domain of $f(x) = \dfrac{\sqrt{x}}{x-3}$ is ....

3. $\lim\limits_{x \to 2^-} \dfrac{\ln x^2}{x-2} =$ ....

4. The discontinuities of $f(x) = \dfrac{4}{5+e^{(\frac{1}{x-7})}}$ is.............................

5. Is the following function $f(x) = \dfrac{x^3\,|x|}{x^2+1}$ even , odd or neither ?..........................

6. *The value of* $\cot(\cos^{-1}(\frac{3}{5}))$ *is* ....

7. Explain how the graph of $f(x) = x^2 - 4x + 1$ is obtained from the graph of

   $g(x) = x^2$: by........................and........................

8. The value of x that satisfies the equation $Log_3 x^3 - Log_3 x = 40$ is ....

Q2: If $\dfrac{(x^2-1)^2}{(x^2-2x+1)} \le f(x) \le 4\,\dfrac{\sin(x-1)}{(x-1)}$ Find $\lim\limits_{x\to 1} f(x)$?

إعداد : اسامة ابو مبارك

Q $\boxed{1}$

① $y = 3\sin^{-1}(2x+4) + 1$

domain $\sin^{-1}$ = range $\sin$

$R(\sin) = [-1, 1]$

$(-1 \le 2x + 4 \le 1) - 4$

$(-5 \le 2x \le -3)/2$

$$\boxed{\frac{-5}{2} \le x \le \frac{-3}{2}}$$

② $f(x) = \dfrac{\sqrt{x}}{x-3}$, $d = ??$

D(البسط) ∩ D(المقام)

$\sqrt{x} = 0$, $x = 0$

$x - 3 = 0$, $x = 3$

$$\boxed{D(f) = [0, \infty) - \{3\}}$$

③ $\lim\limits_{x \to 2^-} \dfrac{\ln x^2}{x-2} = \lim\limits_{x\to 2^-} \dfrac{\ln(2^-)^2}{2^- - 2} = \dfrac{\ln(4)}{0^-}$

$= \boxed{-\infty}$

④ $f(x) = \dfrac{4}{5 + e^{\left(\frac{1}{x-7}\right)}}$

$x - 7 = 0$, $x = 7$

$\lim\limits_{x \to 7^+} \dfrac{4}{5 + e^{\frac{1}{x-7}}} = \infty$

إعداد : اسامة الوصيلي

$$\lim_{x\to 7^-} \frac{4}{5+e^{\frac{1}{x-7}}} = \frac{4}{5+e^{-\infty}} = \frac{4}{5+0}$$

$$= \frac{4}{5}$$

$$\lim_{x\to 7^+} f(x) \neq \lim_{x\to 7^-} f(x)$$

$\boxed{\therefore\ 7 \text{ is discontinuities poin}}$

⑤ $f(x) = \dfrac{x^4|x|}{x^2+1}$, odd or even ???

$$f(-x) = \frac{(-x)^4|-x|}{(-x)^2+1} = \frac{x^4|x|}{x^2+1}$$

$$f(-x) = f(x)$$

$\boxed{\therefore\ f(x) \text{ even function}}$

⑥ $\cot\big(\underbrace{\cos^{-1}(\tfrac{3}{5})}_{\theta}\big) = ??$

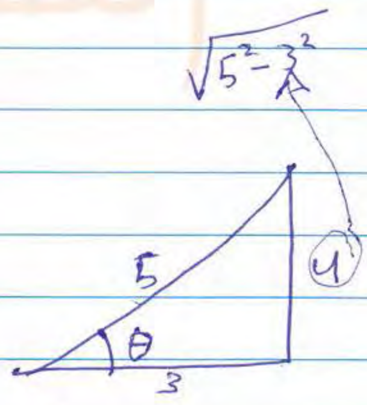
~~cos~~

$$\theta = \cos^{-1}\left(\frac{3}{5}\right)$$

$$\cos(\theta) = \cancel{\cos}\left(\cancel{\cos^{-1}}\left(\frac{3}{5}\right)\right)$$

$$\cos(\theta) = \frac{3}{5}$$

$\boxed{\cot\theta = \frac{3}{4}}$



اعداد .. اسامة ابو معليك

Q $\boxed{2}$

$$\frac{(x^2-1)^2}{(x^2-2x+1)} \leq f(x) \leq 4\,\frac{\sin(x-1)}{x-1}$$

$$\lim_{x\to 1} f(x) = ??$$

$\therefore$ squeez theorm

$$\lim_{x\to 1} \frac{(x^2-1)^2}{x^2-2x+1} \leq \lim_{x\to 1} f(x) \leq \lim_{x\to 1} 4\,\frac{\sin(x-1)}{x-1}$$

① $$\frac{(x^2-1)(x^2-1)}{x^2-2x+1} = \frac{\cancel{(x-1)}(x+1)\cancel{(x-1)}(x+1)}{\cancel{(x-1)}\cancel{(x-1)}}$$

$$\boxed{\lim_{x\to 1} (x+1)(x+1) = 4}$$

② $$\lim 4\,\frac{\sin(x-1)}{x-1}$$

$$z = x-1 \quad , \quad x \to 1$$
$$z \to 0$$

$$\lim_{z\to 0} 4\,\frac{\sin \cancel{z}}{\cancel{z}} = 4$$

$$4 \leq \lim_{x\to 1} f(x) \leq 4$$

$$\boxed{\therefore \lim_{x\to 4} f(x) = 4}$$

Jordan University	**First Exam**	**1st Semester**
Mathematics Department	**Math 101**	**2009/2010**

Name: .... **Section:** ....

Student Number: ....

1- Express the function $F(x) = \dfrac{1}{\sqrt{x + \sqrt{x}}}$ as a composition of three functions

$fogoh(x)$ where

$f(x) =$

$g(x) =$

$h(x) =$

2- $\sin^{-1}\left(\sin\left(\dfrac{5\pi}{2}\right)\right)$ is ....

3- $\cos(2\tan^{-1} x)$ is ....

4- The range of the function $f(x) = 3 + \cos 2t$ is ....

5- Explain how the graph of $f(x) = x^2 - 3x + 7$ is obtained from the graph of

$g(x) = x^2$

....

....

6- If $\lim\limits_{x \to 2} \dfrac{f(x) - 9}{x - 2} = 11$ then $\lim\limits_{x \to 2} f(x)$ is ....

7- Find the inverse of the function $f(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$

8- Solve the equation $2(9^x) - 14(3^x) = 36$

إعداد : اسامة ابو صعليك

① $F(x) = \dfrac{1}{\sqrt{x+\sqrt{x}}}$, $F(x) = (f \circ g \circ h)(x)$

$f(x) = \dfrac{1}{x}$ $\qquad\qquad$ $f(x) = \dfrac{1}{\sqrt{x+\sqrt{x}}}$

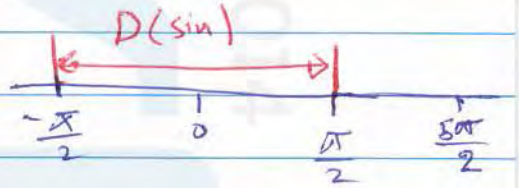
$g(x) = \sqrt{x}$ $\qquad$ or $\qquad$ $g(x) = x$

$h(x) = x + \sqrt{x}$ $\qquad\qquad$ $h(x) = x$

② $\sin^{-1}\left(\sin\left(\frac{5\pi}{2}\right)\right) = ??$

$f^{-1}(f(x)) = x \ , \ x \in D(f)$

$D(\sin) = \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$



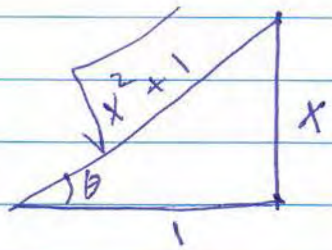
③ $\cos(2\underbrace{\tan^{-1}(x)}_{\theta}) = ??$

$\tan^{-1}(x) = \theta$

$\cancel{\tan}(\tan^{-1}(x)) = \tan\theta \longrightarrow \boxed{\tan\theta = x}$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$= \dfrac{1}{(\sqrt{x^2+1})^2} - \dfrac{x}{(\sqrt{x^2+1})^2}$



$= \boxed{\dfrac{1-x}{x^2+1}}$

إعداد : اسامة ابو جبلك

(4) $f(x) = 3 + \cos 2t \quad , \quad R = ??$

$$(-1 \leq \cos 2t \leq 1) + 3$$

$$\boxed{2 \leqslant 3 + \cos 2t \leqslant 4}$$

(5) $f(x) = \underset{a}{1}x^2 - \underset{b}{3}x + \underset{c}{7} \quad , \quad g(x) = x^2$

complete square (بإكمال مربع)

$$\pm\left(\frac{b}{2}\right)^2 = \pm\left(\frac{3}{2}\right)^2 = \pm \frac{9}{2}$$

$$x^2 - 3x + \frac{9}{2} - \frac{9}{2} + 7$$

$$\left(x - \frac{3}{2}\right)\left(x - \frac{3}{2}\right) - \frac{9}{2} + 7$$

$$\left(x - \frac{3}{2}\right)^2 + \frac{5}{2}$$

$\frac{3}{2}$ uint right , $\frac{5}{2}$ up ward

(6) $\lim_{x \to 2} \frac{f(x) - 9}{x - 2} = 11 \quad , \quad \lim_{x \to 2} f(x) = ??$

$$g(x) = \frac{f(x) - 9}{x - 2}$$

$$f(x) = (x-2)\, g(x) + 9$$

$$\lim_{x \to 2} f(x) = \underbrace{\lim_{x \to 2} g(x)(x-2)}_{0} + \underbrace{\lim_{x \to 2} 9}_{9}$$

اعداد : اسامة ابو مصيلك

$$\lim_{x \to 2} f(x) = 0 + 9$$

$$\boxed{\lim_{x \to 2} f(x) = 9}$$

(7) $f(x) = \ln(x + \sqrt{x^2+1})$

Inverse خطوات ال

$y = f(x)$

$x = y$ ~~...~~

$f^{-1}(y) = x$

$$y = \ln(x + \sqrt{x^2+1})$$

$$e^y = e^{\ln(x + \sqrt{x^2+1})}$$

$$e^y = x + \sqrt{x^2+1}$$

$$(e^y - x)^2 = (\sqrt{x^2+1})^2$$

$$(e^y)^2 - 2e^y x + \cancel{x^2} = \cancel{x^2} + 1$$

$$\frac{e^{2y} - 1}{2e^y} = \frac{\cancel{2e^y} x}{\cancel{2e^y}}$$

$$\boxed{x = \frac{e^{2y} - 1}{2e^y}}$$

$$f^{-1}(y) = x \quad , \quad f^{-1}(y) = \frac{e^{2y} - 1}{2e^y}$$

$$\boxed{f^{-1}(x) = \frac{e^{2x} - 1}{2e^x}}$$

اعداد : اسامة ابو صعيليك

8 $2(9^x) - 14(3^x) = 36$

$2((3^2)^x) - 14(3^x) = 36$

$(2(3^{2x}) - 14(3^x) = 36)/2$

$3^{2x} - 7(3^x) = 18$

$3^{2x} - 7(3^x) - 18 = 0$

$Z = 3^x$

$\rightarrow Z^2 - 7Z - 18 = 0$

$(Z-9)(Z+2) = 0$

$Z = 9$	$Z = -2$
$Z = 3^x$	$Z = 3^x$
$3^x = 9$	$\cancel{3^x = -2}$
$3^x = 3^2$	

$\boxed{X = 2}$ ✓

University of Jordan   Dept. of Math.   Math. 0301101   First Exam   27-10-2011

الاسم: الرقم الجامعي :

المدرس: الرقم المتسلسل :   وقت المحاضره:

Q_1) The following question contains seven multiple choice problems, each is 1.5 mark. Write (x) on the correct answer.

1) If $f(x) = \frac{x+1}{x^2+1}$, $g(x) = \sqrt{x^2+3x-1}$, then $(f \circ g)(2) =$

$(a)\ \frac{1}{2}$ $(b)\ \frac{5}{2}$ $(c)\ \frac{2}{5}$ $(d)\ \frac{3}{5}$ $(e)\ \frac{4}{5}$

2) If $f(x) = x^2$, $g(x) = \sqrt{x+1}$, then Domain $f \circ g =$

$(a)\ \mathbb{R}$ $(b)\ \mathbb{R}\setminus\{-1\}$ $(c)\ (-\infty,-1)$ $(d)\ (-\infty,1]$ $(e)\ [-1,\infty)$

3) If $\log_x(5x-4) = 2$, then $x =$

$(a)\ \{1,4\}$ $(b)\ \{1\}$ $(c)\ \{4\}$ $(d)\ \{1,3\}$ $(e)\ \{3\}$

4) If $f(x) = e^x + 3e^{-x} - 1$, then $f^{-1}(3) =$

$(a)\ \ln(1)$ $(b)\ \ln\left(\frac{3}{1}\right)$ $(c)\ \ln\left(\frac{3}{5}\right)$ $(d)\ \ln\left(\frac{4}{5}\right)$ $(e)\ \ln\left(\frac{3}{2}\right)$

5) If $x = \frac{1}{2}\ln 9 - \ln 2$, then $e^{2x} =$

$(a)\ e^3$ $(b)\ 3$ $(c)\ \frac{3}{2}$ $(d)\ \frac{9}{4}$ $(e)\ \frac{9}{2}$

6) If $f(x) = \frac{3}{1+2\cos x}$, then range $f =$

$(a)\ \left[\frac{1}{2},\frac{3}{4}\right]$ $(b)\ \left[\frac{1}{2},\frac{3}{2}\right]$ $(c)\ \left[\frac{1}{4},\frac{3}{2}\right]$ $(d)\ \left[\frac{1}{4},\frac{3}{4}\right]$ $(e)\ none$

7) Let $f(x) = x^2 + 2x$. If f is shifted 2units right, 3units up then reflected about the Y-axis, we obtained,

$(a)\ g(x) = x^2 + 2x + 3$ $(b)\ g(x) = x^2 - 2x + 3$ $(c)\ g(x) = x^2 + 2x - 3$

$(d)\ g(x) = x^2 - 2x - 3$ $(e)\ g(x) = x^2 - 6x + 3$

In the following questions **show** your work in details.

Q_2) $\left(3+2\ marks\right)$ Let $f(x) = 3 - 2\sin^{-1}(2x-1)$ Find,

a) The domain and range of f.

b) $f\left(\frac{1}{2} + \frac{1}{2}\sin\frac{5\pi}{4}\right)$

Q_3) $\left(3+2\ marks\right)$ Let $f(x) = \frac{e^{2x}-1}{e^{2x}+1}$. Find,

a) $f^{-1}(x)$

b) Classify $f^{-1}(x)$ as even, odd, or neither.

The University of Jordan	First Exam	First Semester
Department of Mathematics	Math 101	28.10.2010

Name: .... Section: ....

Student Number: ....

I. Write down only the answers of the following questions:

1- If $f(x)=\begin{cases} x-1 & x\le 3 \\ 3x-7 & x>3 \end{cases}$, then $\lim_{x\to 3} f(x)$ ....

2- If $f(x)=2x^3+3x+1$ and $f^{-1}(x)=1$, then $x=$ ....

3- The domain of $f(x)=\dfrac{1}{\sqrt{x^2-3x}}$ is ....

4- The range of $f(x)=2+\sqrt{9-x^2}$ is ....

5- The value of $(\log_2 3)(\log_3 4)(\log_4 5)\ldots(\log_{15} 16)$ is ....

6- $\tan\left(2\sin^{-1}\left(\dfrac{4}{5}\right)\right)=$ ....

7- The vertical asymptotes of $f(x)=\dfrac{(x^5-1)}{(x^2-1)(x+3)}$ are ....

II. Is the function $f(x)=\ln\left(x+\sqrt{x^2+1}\right)$ even or odd (Verify your answer).

III. Solve the inequality $\ln\left(x^2-2x-2\right)\le 0$

(1) the domain $f(x)=\sqrt{x-2}-\frac{1}{\sqrt{3-x}}$ is:

a) $[2.3)$ b) $(2,\infty\}$ c) $(-\infty.3)$ d) $(-\infty.\infty)$

2) the range of $f(x)=2-\sqrt{4..x^2}$ is:
a) $[0.2)$ b) $[1.2)$ c) $[-1.1)$ d) $[-2.0)$

3) $\lim_{x\to 0} x.\cot\frac{x}{3}$

a)1 b) $\frac{1}{3}$ c) 3 d) ∞

4) $\lim_{x\to 0}\frac{\sin^2 x}{1-\cos x}=$

a) 1 b) 0 c)2 d)-2

5- Given that $\lim_{x\to 3} f(x)=6, f(3)=5, g(5)=4, g(6)=7$

and g is continuous everywhere, then $\lim_{x\to 3} g(f(x))=$
a) 3 b) 4 c) 6 d)7

6) Let f(2x+3) = 6x+2, then f(x)=
a) 2x-3 b)3x-11 c) 3x-7 d) 2x+7

7- If f(x) = x.secx. then $f'(0)=$
a) -1 b)0 c)1 d)2

8- If xy^3-y=0, then slope of the tangent at the point (1.1) is
a) 2 b)-2 c) $\frac{1}{2}$ d) $-\frac{1}{2}$

9- the graph of f(x) = sin x was shifted horizontally $\frac{\Pi}{2}$ units to the right get g(x), then
a) g(x) =- cos x b) g(x)= cos x
c) g(x) = $\frac{\pi}{2}+\sin x$ d) g(x) = sin x- $\frac{\pi}{2}$

10) $f(x)=\begin{cases}\frac{x(x^2+1)}{\sin 2x}(x\neq 0)\\ k.(x=0)\end{cases}$

the value of k which makes f continuous at x-0 is
a) 0 b)0 c) $\frac{1}{2}$ d)2

11) Let $f(x)=\begin{cases}x^2+1x\leq 1\\ \\ 2x.x>4\end{cases}$ $,then$

a) f is cont. but not differentiable at x-1
b) f is cont. and differentiable at x =1
c) f has a removable discontinuity at x=1
d) f is neither cont. nor differentiable at x=1

12) Let $f(x)=\begin{cases} x^2\sin\frac{1}{4} \\ 0.(x=0) \end{cases}$,then

a) f is cont. at x=0 and $f'(0)=0$
b) f is cont. at x=0 and $f'(0)=1$
c) f is cont. at x=0 and $f'(0)$ does not exist
d) f is neither cont. nor differentiable at x=0

13) $\lim_{x\to 0}\frac{2\sin x}{x+\tan x}=$
a)0 b)1 c)2 d)-1

14) $\lim_{x\to\infty}\left(\sqrt{x^2+5}.\sin\frac{1}{x}\right)=$
a)1 b)-1 c)5 d) does not exist

15) $\lim_{x\to\infty}\frac{2-3x}{\pi x}\sin\left(\frac{\pi x}{2-3}\right)=$
a) 1 b) $\frac{-3}{\pi}$ c) $\frac{3\sqrt{3}}{2\pi}$ d) does not exist

16) the vertical asymptote (s) for $f(x)=\frac{\sin x}{x(x+1)}$ is (are)
a) x=0 only b) x=1 only
c) x=-1 only d)x=0,-1

17) the horizontal asymptote (s) for $f(x)=\frac{|2x|}{x+1}$ is
a) y=0 b) y=2 only c) y=-2 only d) y=2,-2

18) Given that f(1) =1 and $f'(x)=\sqrt{\cos\pi x+10}$, then a linear appoximation estimate for f(1.1) is.
a) 1.1 b) 1.2 c) 1.3 d) 1.4

19) If f(x) $2x^3$-x+1, $g'(x)=f'(x)$, and g(1)=4 then g(x)=
a) $2x^3$-x+1 b) $2x^3$-x +2
c) $2x^3$-x +3 d) $2x^3$ -x+4

20) If f(x) $\cos^3 2x$,then $f\left(\frac{\pi}{8}\right)=$

a) -9 b) $\frac{-3}{\sqrt{2}}$ c) $\frac{-3}{2\sqrt{2}}$

بسم الله الرحمن الرحيم

الامتحان الأول

قسم الرياضيات	تفاضل وتكامل -1--301101	الخميس: 1999/11/14
الجامعة الأردنية	الامتحان الأول	مدة الامتحان: 60 دقيقة
الاسم:	الرقم الجامعي	وقت المحاضرة:

(16 Pts) I- choose the correct answer

x	-3	-2	-1	0	1	2	3
F (x)	1	2	3	-1	-2	0	3
G(x)	2	-1	-4	0	4	1	-2

3- the domain of the function (fog) is:

a) $\{\pm 3, \pm 2, \pm 1, 0\}$ b) $\{\pm 3, \pm 2, \pm 2, 0\}$ c) $\{0, \pm 1, \pm 2\}$

d) $\{0, \pm 1, \pm 2, \pm 4\}$ e) None of the above

2- one of the following statement is true abut the above table:

a) f is ab odd function b) g is an even function

c) f is an even function d) g is an odd function.

e) None of the above statements is true.

3- Given the graph of f

a) [-2,3] b) [2,7] c) [1,9] d) [1,6] e) Nolle.

4- the family of curves that contains 3x6y = 1 as a member is:

a) $9x^2 + 36y^2 = c$ b) $y = mx + 1/6$ c) $2y = b - 4x$ d) $4y + 2xb$

f) None of the above

5- the curve that represent the paranetric equations

$x = \sin t$, $y = \cos^2 0 \le t \le \pi/2$ is:

6- fet $f(x) = \begin{cases} \dfrac{\sqrt{x^2+4-2}}{x} & ; x \neq 0 \\ 5 & ; x = 0 \end{cases}$ then

a) f is continuous at x=0 b) zero is not in the domain of f

b) f has a removable discontinuity at x=0

c) f has a nonremovable discontinuity at x=0

d) None of the above statements is true.

7- If $\lim\limits_{x \to 2} f(x) = 4$, $f(2) = 5$; $g(2) = 6$; $g(4) = 7$, $g(5) = 1$

and g is continuous on r, then $\lim\limits_{x \to 2} gof =$

a) 6 b)5 c) 1 d)7 None of the above

8- $\lim\limits_{x \to \infty} \dfrac{\sqrt{7x^2+5}}{x+3} =$

a) $\sqrt{5}$ b) $-\sqrt{5}$ c) $\sqrt{7}$ d) $-\sqrt{7}$ e) None

4) II: Let $f(x) = \begin{cases} x^2 + a & x \le 1 \\ cx & x > 1 \end{cases}$ find the value of a and C So thet f is differentiable at $x = 1$

3pts) III) Compute $\lim_{h \to 0} \frac{(x+2h)^4 - 16}{h}$ (if it ecn'sts)

3pts) IV: If the tangent line to curve of f(x) =w ax+x+c at (1,3) passes through the point (3,5); find a;c?

Jordan university first exam cal1/math101

Mathematics department

1-Find f o g o h .

F(x) = x+1; g(x) =2x ; h(x)= x-1

2-find the domain of each function .

a) F(x) = $\frac{1}{1+e^x}$ b) $\frac{1}{1-e^x}$ c) g(t) = sin(e^{-t})

c) f(t) = 3 + cos2t d) f(s)= $\frac{2}{(3x-1)}$ f) ln (x + 6)

3-find the formula for the inverse of the function

F(x) =$2x^3$ + 3

Y = ln(x + 3)

G(x) = $\sqrt{10-3x}$

F(x) = $\frac{4x-1}{2x+3}$

4- Evaluate the limit, if it exists .

a) $\lim_{x\to 2}\frac{x^2+x-6}{x-2}$ b) $\lim_{x\to -4}\frac{x^2-4x}{x^2-3x-4}$

c) $\lim_{x\to 0}\frac{x}{1-\sqrt{1+3x}}$ d) $\lim_{x\to -4}\frac{\frac{1}{4}+\frac{1}{x}}{4+x}$

5- find the limit.

a) $\lim_{x\to\infty} \frac{1}{2x+3}$

b) $\lim_{x\to\infty} \frac{x^3+5x}{2x^3-x^2+4}$

c) $\lim_{x\to\infty}(\sqrt{9x^2+x}-3x)$

d) $\lim_{x\to\infty} \cos x$

6- find an equation of the tangent line to the curve at the given point .

a) y= $\frac{x-1}{x-2}$,(3,2)

b) y= $2x^3 - 5x$,(-1,3)

c) y= $\sqrt{x}$, (1,1)

d) y= $\frac{2x}{(x+1)^2}$, (0,0)

7- if f(x) = x^3- x , find $f'(x)$,by the definition of derivative.

8- find f'(x) if f(x) = $\frac{1-x}{2+x}$, by the definition of derivative.

9-

First Exam ..

Q1. If $x = \frac{1}{2}\ln 9 + \ln 2$, then $e^{2x} = 36$

Sol:

$$\frac{1}{2}\ln 9 + \ln 2 = \ln\sqrt{9} + \ln 2$$
$$= \ln 3*2 = \ln 6$$
$$x = \ln 6 \Rightarrow 2x = 2\ln 6$$
$$2x = \ln 6^2 \Rightarrow e^{2x} = e^{\ln 36}$$
$$e^{2x} = 36$$

Q2. If $f(x) = \frac{5}{3+2\cos x}$, then the range of f is: $[1, 5]$

Sol:

$$\underset{*2}{-1} \leq \underset{*2}{\cos x} \leq \underset{*2}{1}$$

$$\underset{+3}{-2} \leq \underset{+3}{2\cos x} \leq \underset{+3}{2}$$

$$1 \leq 3 + 2\cos x \leq 5$$
$$5 * 1 \geq \frac{1 * 5}{3+2\cos x} \geq \frac{1}{5} * 5$$

$$5 \geq \frac{5}{3+2\cos x} \geq 1$$

Q.3 the horizontal asymptote(s) of $f(x) = \frac{\sqrt{3x^2+1}}{7x-1}$ is (are):

Sol:

$$\lim_{x\to\infty} \frac{\sqrt{3x^2+1}}{7x-1} = \frac{\sqrt{3}}{7} \qquad \text{h.a at } x = \frac{\sqrt{3}}{7}$$

$$\lim_{x\to-\infty} \frac{\sqrt{3x^2+1}}{7x-1} = -\frac{\sqrt{3}}{7} \qquad \text{h.a at } x = -\frac{\sqrt{3}}{7}$$

Q4. The value of the constant K that makes $f(x) = \begin{cases} x^2 + 2Kx, & x \le 1 \\ \dfrac{K(1-\sqrt{x})}{1-x}, & x > 1 \end{cases}$

Continuous at $x = 1$ is : $\frac{-2}{3}$

Sol:

$$\lim_{x \to 1^+} = \lim_{x \to 1^-}$$

$$\frac{1+\sqrt{x}}{1+\sqrt{x}} * \frac{K(1-\sqrt{x})}{1-x} = x^2 + 2Kx$$

$$\lim_{x \to 1^+} \frac{K\cancel{(1-x)}}{\cancel{(1-x)}(1+\sqrt{x})} = \lim_{x \to 1^-} x^2 + 2Kx$$

$$\frac{K}{2} = 1 + 2K \Rightarrow K = 2 + 4K$$

$$3K = -2 \Rightarrow K = \frac{-2}{3}$$

Q5. If $f(x) = x^2 - 4x + 1$ and $x \le 2$, then $f^{-1}(x) = \cancel{\sqrt{x+3}}$ ~~2~~

Sol:

$$y = x^2 - 4x + 1$$

$$x = y^2 - 4y + 1$$

$$x - 1 = y^2 - 4y + 4 - 4 \quad \text{إكمال مربع}$$

$$\pm\sqrt{x+3} = \sqrt{(y-2)^2}$$

$$-\sqrt{x+3} = y - 2$$

$$y = 2 - \sqrt{x+3}$$

Q.6 Given that $f(x) = x^2$, and $g(x) = 5x + 1$, then the values of x wich $f \circ g(x) = g \circ f(x)$ are $\{-\frac{1}{2}, 0\}$

Sol:

$$f(g(x)) = g(f(x)) \Rightarrow (5x+1)^2 = 5x^2 + 1$$

$$25x^2 + 10x + \cancel{1} = 5x^2 + \cancel{1} \Rightarrow 20x^2 + 10x = 0$$

$$2x^2 + x = 0 \Rightarrow x(2x+1) = 0 \Rightarrow x = 0, \; x = -\frac{1}{2}$$

Q7) The value of $\sin(2\tan^{-1}(2))$ is : $\frac{4}{5}$

Sol :

$$\sin(2\tan^{-1}(2)) = 2\sin(\tan^{-1}2)\cos(\tan^{-1}2)$$

$\tan x = 2$

$\sin x = \frac{2}{\sqrt{5}}$, $\cos x = \frac{1}{\sqrt{5}}$

$$\sin(2\tan^{-1}(2)) = 2\left(\frac{2}{\sqrt{5}} * \frac{1}{\sqrt{5}}\right) = \frac{4}{5}$$

Q8. Let $g(x)$ be the function obtained from $f(x) = x^2 + x$ by shifting 3 units to the right, 2 units up, then reflected about y-axis. find $g(x)$.

Sol :

1. Shifting 3 units right $\rightarrow$ $f(x-3) = (x-3)^2 + (x-3)$
2. " 2 " up $\rightarrow$ $f(x-3)+2 = (x-3)^2 + (x-3) + 2$
3. Reflection $\rightarrow$ $f(-x-3)+2 = (-x-3)^2 + (-x-3) + 2$

$g(x) = x^2 + 6x + 9 - x - 3 + 2$

$g(x) = x^2 + 5x + 8$ ✱

Q.9 find the vertical asymptotes of $f(x) = \frac{x^2 - x - 6}{x^4 + 3x^3 + 2x^2}$

Sol :

$$f(x) = \frac{x^2 - x - 6}{x^4 + 3x^3 + 2x^2}$$

$$= \frac{(x-3)(x+2)}{x^2(x^2+3x+2)}$$

$$= \frac{(x-3)\cancel{(x+2)}}{x^2(x+1)\cancel{(x+2)}}$$

$x = 0$ / $x + 1 = 0 \Rightarrow x = -1$

$$\lim_{x \to 0^+} \frac{x^2 - x - 6}{x^4 + 3x^3 + 2x^2} = \frac{-6}{0} = \infty$$

$$\lim_{x \to -1} \frac{x^2 - x - 6}{x^4 + 3x^3 + 2x^2} = \frac{-4}{0} = \infty$$

V.A at $x = -1$, $x = 0$

Q10. Find the values of x at which $f(x) = \frac{x-1}{|x|-1}$ is not cont.

Then determine wether the discontinuity is removable.

$|x| - 1 = 0 \rightarrow x - 1 = 0 \Rightarrow x = 1$

$\qquad\qquad\quad \rightarrow -x - 1 = 0 \Rightarrow x = -1$

$\lim_{x \to 1^+} \frac{x-1}{x-1} = 1$

$\lim_{x \to 1^-} \frac{x-1}{-x-1} = 0$

$\lim_{x \to 1}$ D.N.E

$\left.\begin{array}{l} \lim_{x \to -1^+} \frac{x-1}{-x-1} = 0 \\ \\ \lim_{x \to -1^-} \frac{x-1}{-x-1} = 0 \end{array}\right\} \lim_{x \to -1} f(x) = 0$

Removable discon. at $x = -1$

$f(x)$ is not Cont. at $x = \pm 1$

The End

إعداد :- محمد عامر ياسين

سنة ثالثة رياضيات

(1) the domain $f(x)=\sqrt{x-2}-\frac{1}{\sqrt{3-x}}$ is:

a) $[2.3)$ b) $(\infty.\pounds]$ c) $(-\infty 3)$ d) $(-\infty\infty)$

2) the range of $f(x)=2-\sqrt{4..x^2}$ is:

a) $[0.2)$ b) $[1.2)$ c) $[-1.1)$ d) $[-2.0)$

3) $\lim_{x\to 0} x.\cot\frac{x}{3}$

a) 1 b) $\frac{1}{3}$ c) 3 d) ∞

4) $\lim_{x\to 0}\frac{\sin^2 x}{1-\cos x}=$

a) 1 b) 0 c) 2 d) -2

5- Given that $\lim_{x\to 3} f(x)=6, f(3)=5, g(5)=4, g(6)=7$

and g is continuous everywhere, then $\lim_{x\to 3} g(f(x))=$

a) 3 b) 4 c) 6 d) 7

6) Let f(2x+3) = 6x+2, then f(x)=

a) 2x-3 b) 3x-11 c) 3x-7 d) 2x+7

7- If f(x) = x.secx. then $f'(0)=$

a) -1 b) 0 c) 1 d) 2

8- If $xy^3-y=0$, then slope of the tangent at the point (1.1) is

a) 2 b) -2 c) $\frac{1}{2}$ d) $-\frac{1}{2}$

9- the graph of f(x) = sin x was shifted horizontally $\frac{\prod}{2}$ units to the right get g(x), then

a) g(x) =- cos x b) g(x)= cos x

c) $g(x)=\frac{\pi}{2}+\sin x$ d) g(x) = sin x- $\frac{\pi}{2}$

10) $f(x)=\begin{cases}\frac{x(x^2+1)}{\sin 2x} & (x\neq 0)\\ k.(x=0)\end{cases}$

the value of k which makes f continuous at x-0 is

a) 0 b) 0 c) $\frac{1}{2}$ d) 2

11) Let $f(x)=\begin{cases} x^2+1\, x\le 1 \\ 2x\ x>1\end{cases}$, then

a) f is cont. but not differentiable at x-1
b) f is cont. and differentiable at x =1
c) f has a removable discontinuity at x=1

إعداد :- محمد عامر ياسين

سنة ثالثة رياضيات

d) f is neither cont. nor differentiable at x=1

12) Let $f(x)=\begin{cases} x^2 \sin\frac{1}{4} \\ 0.(x=0) \end{cases}$ $, then$

a) f is cont. at x=0 and $f'(0)=0$
b) f is cont. at x=0 and $f'(0)=1$
c) f is cont. at x=0 and $f'(0)$ does not exist
d) f is neither cont. nor differentiable at x=0

13) $\lim\limits_{x\to 0}\frac{2\sin x}{x+\tan x}=$
a) 0 b) 1 c) 2 d) -1

14) $\lim\limits_{x\to\infty}\left(\sqrt{x^2+5}.\sin\frac{1}{x}\right)=$
a) 1 b) -1 c) 5 d) does not exist

15) $\lim\limits_{x\to\infty}\frac{2-3x}{\pi x}\sin\left(\frac{\pi x}{2-3}\right)=$
a) 1 b) $\frac{-3}{\pi}$ c) $\frac{3\sqrt{3}}{2\pi}$ d) does not exist

16) the vertical asymptote (s) for $f(x)=\frac{\sin x}{x(x+1)}$ is (are)
a) x=0 only b) x=1 only
c) x=-1 only d) x=0,-1

17) the horizontal asymptote (s) for $f(x)=\frac{|2x|}{x+1}$ is
a) y=0 b) y=2 only c) y=-2 only d) y=2,-2

18) Given that f(1) =1 and $f'(x)=\sqrt{\cos\pi x+10}$, then a linear appoximation estimate for f(1.1) is.
a) 1.1 b) 1.2 c) 1.3 d) 1.4

19) If f(x) $2x^3$-x+1, $g'(x)=f'(x)$, and g(1)=4 then g(x)=
a) $2x^3$-x+1 b) $2x^3$-x +2
c) $2x^3$-x +3 d) $2x^3$ -x+4

20) If f(x) $\cos^3 2x$,then $f'\left(\frac{\pi}{8}\right)=$

a) -9 b) $\frac{-3}{\sqrt{2}}$ c) $\frac{-3}{2\sqrt{2}}$

بسم الله الرحمن الرحيم

الامتحان الأول

قسم الرياضيات تفاضل وتكامل -١--١٠١١٠١٣٠ الخميس: ١٩٩٩/١١/١٤

إعداد : — محمد عامر ياسين

سنة ثالثة رياضيات

الجامعة الأردنية　　　الامتحان الأول　　　مدة الامتحان: ٦٠ دقيقة

الاسم:　　　الرقم الجامعي　　　وقت المحاضرة:

(16 Pts) I- choose the correct answer

x	-3	-2	-1	0	1	2	3
F (x)	1	2	3	-1	-2	0	3
G(x)	2	-1	-4	0	4	1	-2

3- the domain of the function (fog) is:

a) $\{\pm3,\pm2,\pm1,0\}$　　b) $\{\pm3,\pm2,\pm2,0\}$　　c) $\{0,\pm1,\pm2\}$

d) $\{0,\pm1,\pm2,\pm4\}$　　e) None of the above

2- one of the following statement is true abut the above table:

a) f is ab odd function　　b) g is an even function

c) f is an even function　　d) g is an odd function.

e) None of the above statements is true.

3- Given the graph of f

a) [-2,3]　b) [2,7]　c) [1,9]　d) [1,6]　e) Nolle.

4- the family of curves that contains 3x6y = 1 as a member is:

a) $9x^2 +36y^2=c$　b) y=mx+1/6　c) 2y=b-4x　d) 4y+2xb

f) None of the above

5- the curve that represent the paranetric equations

$x=\sin t,\ y=\cos^2 0 \le t \le \pi/2$ is:

6- fet $f(x)=\begin{cases}\dfrac{\sqrt{x^2+4-2}}{x}; x\neq 0\\ 5 \qquad ; x=0\end{cases}$ then

a) f is continuous at x=0 b) zero is not in the domain of f

b) f has a removable discontinuity at x=0

c) f has a nonremovable discontinuity at x=0

d) None of the above statements is true.

7- If $\lim\limits_{x\to 2}$ f(x) =4, f(2) =5; g(2)=6; g(4)=7, g(5)=1

and g is continuous on r, then $\lim\limits_{x\to 2}$ gof =

a) 6 b)5 c) 1 d)7 None of the above

8- $\lim\limits_{x\to\infty} \dfrac{\sqrt{7x^2+5}}{x+3}=$

a) $\sqrt{5}$ b) - $\sqrt{5}$ c) $\sqrt{7}$ d)- $\sqrt{7}$ e) None

4) II: Let $f(x)=\begin{cases}x^2+a & x\leq 1\\ cx & x>1\end{cases}$ $\begin{matrix}\textit{find the value of a and C}\\ \textit{So thet f is differenti able at } x=1\end{matrix}$

3pts) III) Compute $\lim\limits_{h\to 0}\dfrac{(x+2h)^4-16}{h}$ (if it ecn'sts)

3pts) IV: If the tangent line to curve of f(x) =w ax+x+c at (1,3) passes through the point (3,5); find a;c?

15/20

THE UNIVERSITY OF **JORDAN**

0301101 Calculus I — First test — Nov. 4, 2012

Name: خلف معروف خلف   Number: 0127870

Instructor's name: د مأمون   Class days and time:

For instructor use only, please do not write in this table.

Q1 – Q7	Q8	Q9	Q10	Grade
6				

Questions 1 to 7, fill in the blanks with the answers only. Each question is worth 1.5 marks.

0 Q1) If $x = \frac{1}{2}\ln 9 + \ln 3$, then $e^{2x} =$ $9e^x$

0 Q2) If $f(x) = \frac{3}{4+2\cos x}$, then the range of f is $[\frac{1}{6}, \frac{1}{2}]$

1.5 Q3) The horizontal asymptote(s) of $f(x) = \frac{\sqrt{3x^2+1}}{2x-1}$ is (are) $y = \frac{\sqrt{3}}{2}$ $y = \frac{-\sqrt{3}}{2}$

1.5 Q4) The value of the constant k that makes $f(x) = \begin{cases} x^2 + 5kx & if\ x \le 1 \\ \frac{k(1-\sqrt{x})}{1-x} & if\ x > 1 \end{cases}$ continuous at $x = 1$ is $\frac{-2}{9}$

0 Q5) If $f(x) = x^2 - 4x + 5$ and $x \le 2$, then $f^{-1}(x)$ equals $= x^2 - 4x + 5$

1.5 Q6) Given that $f(x) = x^2$ and $g(x) = 2x + 1$, then the values of x at which $(f \circ g)(x) = (g \circ f)(x)$ are $x = 0$ $x = -2$

1.5 Q7) The value of $\sin(2\tan^{-1}(5))$ is $\frac{10}{26}$

$x = \ln 9$

$2\ln 81 = 2x$

$e^{2x} = 81$

$e^x = \ln 9$

$x = \frac{1}{2}\ln 9 + \ln 3$

$x = \ln 9^{\frac{1}{2}} + \ln 3$

$= \ln 3 + \ln 3$

$e^x = e^{\ln 9}$

$e^x \times e^x = 9 \times e^x$

$e^{2x} = 9e^x$

$1 \ge \cos x \ge -1$

$2 \ge 2\cos x \ge -2$

$6 \ge 4 + 2\cos x \ge 2$

$\frac{1}{6} \ge \frac{1}{4+2\cos x} \ge \frac{1}{2}$

$f(x) = \frac{\sqrt{3x^2+1}}{2x-1}$

$\lim_{x\to\infty} \frac{\sqrt{3x^2+1}}{2x-1} = \frac{\sqrt{3}\,|x|}{2x}$ → $+ \frac{\sqrt{3}}{2}$, $- \frac{\sqrt{3}}{2}$

$\lim_{x\to-\infty} \frac{\sqrt{3}\,|x|}{2}$

$x^2 + 5kx$

In questions 8, 9 and 10 write down every step of your work.

Q8) (3 marks) Let $g(x)$ be the function obtained from $f(x) = x^2 + x$ by shifting 2 units to the right, 3 units up then reflected about the y-axis. Find $g(x)$.

$F(x) = x^2 + x$

step1 $= (x-2)^2 + x - 2$

step 2 $= (x-2)^2 + (x-2) + 3$

$= x^2 - 4x + 4 + x - 2 + 3$

$= x^2 - 3x + 5$

step3 $(-x)^2 + 3x + 5$

$= x^2 + 3x + 5$

3

$f(x) = x^2 - 3x + 5$

$f(x) = (-x)^2 + 3x + 5$

$-f(x) = -x$

Q9) (3 marks) Find the vertical asymptotes of $f(x) = \frac{x^2-x-6}{x^4+5x^3+6x^2}$.

$x^4 + 5x^3 + 6x^2 = 0$

$\lim_{x \to 0} \frac{x^2 - x - 6}{x^4 + 5x^3 + 6x^2} = \frac{-6}{0} = -\infty$

$\lim_{x \to -2} \frac{x^2 - x - 6}{x^4 + 5x^3 + 6x^2} = \frac{0}{0}$

بالتحليل والاختصار ينتج عدد اذا ليس V.A

$(x-3)(x+2)$

V.A $\Rightarrow$ $x = 0$

2

Q10) (4 marks) Find the values of x at which $f(x) = \frac{x-4}{|x|-4}$ is not continuous then determine whether the discontinuity is removable.

$f(x) = \frac{x-4}{|x|-4}$

$\lim_{x \to 4^+} \frac{x-4}{x-4} = 1$

$\lim_{x \to -4^+} \frac{x-4}{-x-4}$

$f(x) =$ not defined

$\lim_{x \to 4^+} \neq \lim_{x \to 4^-} \neq f(x)$

~~$x = 4$ removable~~

$x = -4$

$\lim_{x \to -4^+} \frac{x-4}{-x-4} =$

not continous

$x = 4, -4$

على ظهر الورقة

4

$\lim_{x\to1} x^2 + 5kx = \lim_{x\to1} \frac{k(1-\sqrt{x})}{1-x} \times \frac{(1+\sqrt{x})}{(1+\sqrt{x})}$

$1+5k = \frac{k(1-x)}{2(1-x)}$

$2+10k = k$

$2+9k=0$

$9k=-2$

$k = \frac{-2}{9}$

$f(x) = x^2 - 4x + 5$ $\quad y^2 - 4y = x - 5$

$y = x^2 - 4x + 5$ $\quad y(y-4) =$

$x = y^2 - 4y + 5$

$\cancel{f^{-1}(x) = x^2 - 4x + 5}$

$f(x) = x^2 \quad g(x) = 2x+1$

$(f\circ g) = f(g(x)) = f(2x+1) = (2x+1)^2$

$g(f(x)) = g(x^2) = 2x^2+1$

$2x^2+1$

$4x + 1 = 2x^2 + 1$

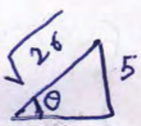
$2x^2 + 4x = 0$

$x(x+2) = 0$

$0, \; x = -2$

$\sin(2\tan^{-1}(5))$

$\tan\theta = \frac{5}{1}$



$\sin 2\theta = 2\cos\theta \sin\theta$

$= 2 \times \frac{1}{\sqrt{26}} \times \frac{5}{\sqrt{26}} = \frac{10}{26}$

$|x| - 4 = 0 \quad |x| = 4$

$f(x) = \frac{x-4}{|x|-4}$

$x = 4$

$\lim_{x\to4^+} \frac{x-4}{x-4} = 1$

$\lim_{x\to4^-} \frac{x-4}{-x-4} = -\infty$

$f(x)$ = non defined

$\lim_{x\to4^+} \neq \lim_{x\to4^-} \neq f(x)$

not con at $x=4$

$x=-4$

$x = -4$

$\lim_{x\to-4^+} \frac{x-4}{x-4} = 1$

$\lim_{x\to-4^-} \frac{x-4}{x-4} = 1$

$f(-4)$ = non defind

$\lim_{x\to4^+} \neq \lim_{x\to4^-} \neq f(-4)$

20

cole 1

The University of Jordan | **First Exam** | **Summer Semester**
Department of Mathematics | **Math 101** | **12/7/2010**

Name: .... [illegible] ....

0095311

Section: ~~[illegible]~~

Instructor name: ...... [illegible] ......

11:20 – 10:20 الساعة

Q1: Write down only the answers of the questions (1 –6). 12

1. Explain how the graph of $f(x)= x^2+6x+4$ is obtained from the graph of $g(x)=x^2$: by moving to the left 3 units and ~~[illegible]~~ downword 5 units

2. *The value of* $\cos^{-1}(\cos\frac{17\pi}{2})$ *is* .... $\frac{\pi}{2}$ ....

3. *The domain of the function* $y=2\cos^{-1}(2x-1)+5$ *is* ... $[0, 1]$ ...

4. The equation of the vertical asymptote for the function $f(x)=\ln(10x+5)$ is ... $x = \frac{-1}{2}$ ...

5. The value of the constant k such that $f(x) = \begin{cases} k & , x=2 \\ \dfrac{x^2-4}{\sin(x-2)} & , x\neq 2 \end{cases}$

will be continuous at $x = 2$ is .... 4 ....

6. $\lim_{x\to 0^-}(\frac{1}{x}+\frac{1}{x^2})$ = ... $-\infty \cdot (-\infty) = \boxed{\infty}$ ...

Q2: Solve $(16)^x + 4^x - 6 = 0$ $\Rightarrow$ $(4^x)^2 + 4^x - 6 = 0$ let $y = 4^x$

$\Rightarrow y^2 + y - 6 = 0$

3

$(y+3)(y-2) = 0$

$\Rightarrow y = -3$ | $y = 2$

$\boxed{4^x \neq -3}$ | $4^x = 2 = 4^{\frac{1}{2}}$

so $x = \frac{1}{2}$ #

Q3: Show that the equation $x^3 + x^2 - 2x = 1$ on the interval $[-1,1]$ has a solution

$f(x) = x^3 + x^2 - 2x - 1 = 0$ $\Rightarrow$ cont. on $[-1,1]$ since it's polynomial

$f(-1) = -1 + 1 + 2 - 1 = 1 > 0$

8

$f(1) = 1 + 1 - 2 - 1 = -1 < 0$

so by I.V.T there is a number $x \in [-1,1]$ such that $f(x) = 0$

Q4: Find $\lim\limits_{x\to 3} \dfrac{\sqrt{12-x} - 3}{\sqrt{4-x} - 1}$ $* \dfrac{\sqrt{12-x} + 3}{\sqrt{12-x} + 3} * \dfrac{\sqrt{4-x} + 1}{\sqrt{4-x} + 1}$

3

$\Rightarrow \lim\limits_{x\to 3} \left(\dfrac{12 - x - 9}{4 - x - 1} * \dfrac{\sqrt{4-x} + 1}{\sqrt{12-x} + 3} \right) = \lim\limits_{x\to 3} \left(\dfrac{\cancel{3-x}}{\cancel{3-x}} * \dfrac{\sqrt{4-x} + 1}{\sqrt{12-x} + 3} \right)$

$\Rightarrow \lim\limits_{x\to 3} \dfrac{\sqrt{4-x} + 1}{\sqrt{12-x} + 3} = \dfrac{2}{6} = \dfrac{1}{3}$

University of Jordan
Department of Mathematics

Math 101 **First Exam**

19 1/2

Name: [redacted] **Number:** [redacted] **Section:** 3 **25/10/2008**

Seat no. 21

Q1. Let $f(x) = x^2 + 2x$. If $g(x)$ is obtained by shifting $f(x)$ 2 units to the right and then 3 units upward, then $g(x) = (x-2)^2 + 2(x-2) + 3 = x^2 - 2x + 3$

Q2. Let $\mathrm{Dom}(f) = [1,5]$. If $g(x) = 2f(x-3)$, then $\mathrm{Dom}(g) = [4,8]$

Q3. If $f(x) = \dfrac{x+1}{x+2}$, then $\mathrm{Dom}(f \circ f) = R - \{-2, -\frac{5}{3}\}$

Q4. $\cos^{-1}(\cos \frac{4\pi}{3}) = \frac{2\pi}{3}$

Q5. If $f(x) = \cos^2 x - \sin^2 x + 3$, then $\mathrm{Range}(f) = [2,4]$

Q6. If $f(x) = \ln x + \sqrt{3-x}$, then $\mathrm{Dom}(f) = (0,3]$

Q7. If $f(x) = x^3 + x + a$ is an odd function, then $a = 0$ If cons. ~~OR x^5~~

Q_2: $D_f = [1,5]$

$g(x) = 2f(x-3)$

$1 \le x-3 \le 5$

$4 \le x \le 8$

Q_3: $\frac{x+1}{x+2}$ $D \to$

$\boxed{60}$ $\frac{\pi}{240}$

$\cos^2 x - \sin^2 x + 3$

$\cos(2x) + 3$

$2 \le \cos(2x) + 3 \le 4$

Q8. Solve $\log_2 x + \log_2(x-3) = \log_3 9$

$$\log_2(x*(x-3)) = \log_3 9$$

$$\log_2(x^2-3x) = 2$$

$$(2)^2 = x^2 - 3x$$

$$4 = x^2 - 3x$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$x - 4 = 0 \quad \underline{\text{OR}} \quad x + 1 = 0$$

$$x = 4 \qquad\qquad x = -1$$

$$x \in \{4, \cancel{-1}\}$$

Q9. Let $f(x) = \dfrac{e^x - 1}{e^x + 2}$. Find $f^{-1}(x)$.

$$\frac{y}{1} = \frac{e^x - 1}{e^x + 2}$$

$$e^x - 1 = ye^x + 2y$$

$$e^x - ye^x = 1 + 2y$$

$$e^x(1-y) = 1 + 2y$$

$$e^x = \frac{1+2y}{1-y}$$

$$\ln e^x = \ln\left(\frac{1+2y}{1-y}\right)$$

$$x = \ln\left(\frac{1+2y}{1-y}\right)$$

$$y = \ln\left(\frac{1+2x}{1-x}\right)$$

$$f^{-1}(x) = \ln\left(\frac{1+2x}{1-x}\right)$$

CALCULUS 1

FIRST EXAM 2008

1) let $f(x) = x^2 + 2x$. If $g(x)$ is obtained by shifting $f(x)$ 2 units to the Right and then 3 units upwords, then $g(x) =$

Sol:

$$\begin{aligned} g(x) &= f(x-2) + 3 \\ &= (x-2)^2 + 2(x-2) + 3 \\ &= (x^2 + 4x + 4) + (2x - 4) + 3 \\ &= x^2 - 2x + 3 \end{aligned}$$

2) Let Domain$(f) = [1,5]$. If $g(x) = 2f(x-3)$, then Dom$(g) =$

Sol :-

$$1 \le x - 3 \le 5$$

$$4 \le x \le 8$$

3) if $f(x) = \frac{x+1}{x+2}$, then the Domain $(f \circ f) =$

Sol :-

$$f \circ f(x) = f\left(\frac{x+1}{x+2}\right)$$

$$= \frac{\left(\frac{x+1}{x+2}\right) + 1}{\left(\frac{x+1}{x+2}\right) + 2} = \frac{(x+1) + (x+2)}{(x+1) + (2x+4)}$$

$$= \frac{2x+3}{3x+5}$$

$$D = \mathbb{R} - \{-\tfrac{5}{3}, -2\}$$

CALCULUS 1

FIRST EXAM 2008

4) $\cos^{-1}(\cos \frac{4\pi}{3}) = \frac{\pi}{3}$

5) If $f(x) = \cos^2 x - \sin^2 x + 3$, then Range(f) =

Sol :-

$f(x) = \cos 2x + 3$ (متطابقة)

$-1 \leq \cos 2x \leq 1$

$2 \leq \cos 2x + 3 \leq 4$ (بجمع 3)

The Range $[2,4]$

6) If $f(x) = \ln x + \sqrt{3-x}$, then Dom(f) =

Sol :-

- $\ln x$

$x > 0$

- - - - - o + + + + +
0

$\sqrt{3-x}$

$3 - x \geq 0$

+ + + + + + • - - -
3

- - o + + + • - - -
0 3

$\therefore$ Dom(f) = $(0,3]$

7) If $f(x) = x^3 + x + a$, is an odd function, then a =

$f(-x) = -f(x)$

$-x^3 - x + a = -(x^3 + x + a)$

$\cancel{-x^3} \cancel{-x} + a = \cancel{-x^3} \cancel{-x} - a$

$a = -a$

$a = \text{zero}$

CALCULUS 1

FIRST EXAM 2008

8) Solve $\log_2 x + \log_2 (x-3) = \log_3 9$:

Sol :

$$\log_2 x(x-3) = \log_3 9$$

$$\log_2 (x^2-3x) = 2$$

$$x^2 - 3x = 4$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$x = 4$, $x = -1 \rightarrow$ مرفوض

9) Let $f(x) = \dfrac{e^x - 1}{e^x + 2}$. Find $f^{-1}(x)$

Sol. $y = \dfrac{e^x - 1}{e^x + 2}$

* Interchange x with y

$$x = \frac{e^y - 1}{e^y + 2}$$

$$xe^y + 2x = e^y - 1 \qquad \text{(ضرب تبادلي)}$$

$$xe^y - e^y = -1 - 2x$$

$$e^y(x-1) = -1 - 2x$$

$$e^y = \frac{-1-2x}{x-1}$$

$$\ln e^y = \ln\left(\frac{-1-2x}{x-1}\right)$$

$$y = \ln\left(\frac{-1-2x}{x-1}\right)$$

$$f^{-1}(x) = \ln\left(\frac{-1-2x}{x-1}\right)$$

CALCULUS 1
FIRST EXAM 2009

1) Express The function $f(x) = \dfrac{1}{\sqrt{x + \sqrt{x}}}$ as a composition of three functions $f \circ g \circ h\,(x)$,

where :

$f(x) = \dfrac{1}{\sqrt{x}}$

$g(x) = x^2 + x$

$h(x) = \sqrt{x}$

2) $\sin^{-1}\left(\sin \dfrac{5\pi}{2}\right)$ is $\dfrac{\pi}{2}$

3) $\cos(2\tan^{-1}x)$ is

Sol.

Let $u = \tan^{-1}x$

$\tan u = x = \dfrac{x}{1}$

$\cos(2u) = \cos^2 u - \sin^2 u$ ⇐ (متطابقة)

$= \left(\dfrac{1}{\sqrt{1+x^2}}\right)^2 - \left(\dfrac{x}{\sqrt{1+x^2}}\right)^2 = \dfrac{1 - x^2}{1 + x^2}$

4) The range of the function $f(x) = 3 + \cos 2t$ is

Sol.

$-1 \le \cos x \le 1$

$2 \le 3 + \cos x \le 4$ add 3

∴ Range $= [2, 4]$

CALCULUS 1
FIRST EXAM 2009

5) Explain how the graph of $f(x) = x^2 - 3x + 7$ is obtained from the graph of $g(x) = x^2$.

Sol.

$$f(x) = x^2 - 3x + 7 = \left(x^2 - 3x + \frac{9}{4}\right) + 7 - \frac{9}{4}$$

$$= \left(x - \frac{3}{2}\right)^2 + \frac{19}{4}$$

Unit Right Unit up

6) If $\lim\limits_{x \to 2} \frac{f(x) - 9}{x - 2} = 11$, then $\lim\limits_{x \to 2} f(x)$ is

Sol.

$$\lim f(x) - 9 = 0$$

$$\therefore \lim_{x \to 2} f(x) = 9$$

7) $f(x) = \ln(x + \sqrt{x^2 + 1})$. Find the inverse of the $\underline{fn}$:

Sol.

$$y = \ln(x + \sqrt{x^2 + 1})$$

$$x = \ln(y + \sqrt{y^2 + 1})$$

$$e^x = y + \sqrt{y^2 + 1}$$

$$e^x - y = \sqrt{y^2 + 1}$$

$$(e^x - y)^2 = (\sqrt{y^2 + 1})^2$$

$$e^{2x} - 2e^x y + y^2 = y^2 + 1$$

$$e^{2x} - 1 = 2e^x y$$

$$y = \frac{e^{2x} - 1}{e^{2x}}$$

CALCULUS 1

FIRST EXAM 2009

(1) Solve $2(9^x) - 14(3^x) = 36$.

Sol.

Let $u = 3x \rightarrow 9^x = u^2$

$2u^2 - 14u = 36$

$2u^2 - 14u - 36 = 0$

$u^2 - 7u - 18 = 0$

$(u-9)(u+2) = 0$

$u = 9$, $u = -2 \rightarrow$ مرفوض

$3^x = 9 \Rightarrow 3^x = 3^2 \Rightarrow x = 2$

CALCULUS 1
FIRST EXAM 2010

1) If $f(x) = \begin{cases} x-1, & x \le 3 \\ 3x-7, & x > 3 \end{cases}$ then $\lim_{x \to 3} f(x) =$

Sol :

$\lim_{x \to 3^+} 3x-7 = 2$

$\lim_{x \to 3^-} x-1 = 2$ $\quad \therefore \lim_{x \to 3} f(x) = 2$

2) If $f(x) = 2x^3 + 3x + 1$ and $f(x)^{-1} = 1$, then $x =$

Sol: $\quad f^{-1}(f(1)) = 1$ $\quad$ ([illegible])

$\therefore f(1) = x$

$f(1) = 2 + 3 + 1 = 6$

$\therefore x = 6$

3) The Domain of $f(x) = \dfrac{1}{\sqrt{x^2 - 3x}}$ is

Sol :

$x^2 - 3x > 0$

$x(x-3) > 0$

x: $- - - - - \; 0 \; + + + +$

$x-3$: $- - - - - - - - - \; 3 \; + +$

$+ + + \; 0 \; - - - \; 3 \; + + +$

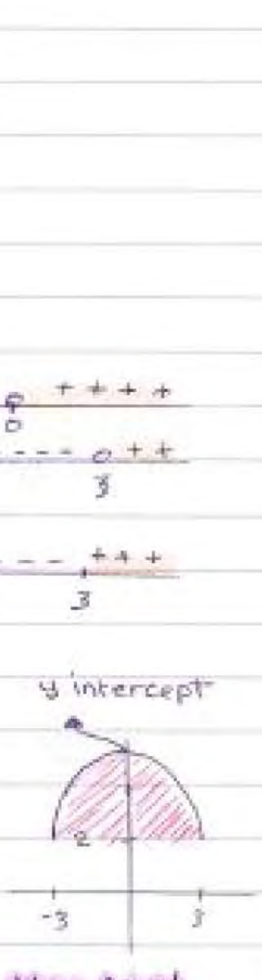
$\therefore$ The Domain $= (-\infty, 0) \cup (3, \infty)$

4) The Range of $f(x) = 2 + \sqrt{9 - x^2}$ is

The Domain of (f) is $[-3, 3]$

$f(0) = 5$, $f(-3) = 2$

$\therefore$ the Range of $(f) = [2, 5]$



the graph

CALCULUS 1

FIRST EXAM 2010

5) The value of $(\log_2 3)(\log_3 4)(\log_4 5)\dots(\log_{15} 16)$ is

Sol:

$$\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdot (\dots) \cdot \log_{15} 16 = \log_2 16 = \log_2 2^4 = 4$$

6) $\tan\left[2\sin^{-1}\left(\frac{4}{5}\right)\right] =$

Sol: $\theta = \sin^{-1}\frac{4}{5}$

$\sin\theta = \frac{4}{5}$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ (متطابقة)

$$= \frac{2\cdot\frac{4}{3}}{1-\frac{16}{9}} = \frac{\frac{8}{3}}{-\frac{7}{9}} = \frac{24}{-7}$$

7) The vertical asymptotes of $f(x) = \dfrac{(x^5-1)}{(x^2-1)(x+3)}$ are

vertical asymptotes $\Rightarrow$ أصفار المقام (التي النهاية)
عندها تساوي ∞

$\lim_{x\to 1} \dfrac{(x^5-1)}{(x^2-1)(x+3)} = \dfrac{0}{0}$

$\lim_{x\to -3} \dfrac{x^5-1}{(x^2-1)(x+3)} = \dfrac{\text{رقم}}{0} = \infty$

$\lim_{x\to -1} \dfrac{(x^5-1)}{(x^2-1)(x+3)} = \dfrac{\text{عدد}}{0} = \infty$

$\therefore x=-3$ and $x=-1$ are vertical asy

CALCULUS 1

FIRST EXAM 2010

II) is The Function $f(x) = \ln(x + \sqrt{x^2+1})$ even or odd.

$$f(-x) = \ln(-x + \sqrt{(-x)^2+1}) = \ln(-x + \sqrt{x^2+1})$$

$$-f(x) = -\ln(x + \sqrt{x^2+1}) = \ln(x + \sqrt{x^2+1})^{-1}$$

$$= -\ln(x + \sqrt{x^2+1})$$

$$= \ln\left(\frac{1}{x + \sqrt{x^2+1}}\right)$$

$$= \ln\left(\frac{1}{x + \sqrt{x^2+1}} * \frac{x - \sqrt{x^2+1}}{x - \sqrt{x^2+1}}\right)$$

$$= \ln\left(\frac{x - \sqrt{x^2+1}}{x^2 - (x^2+1)}\right) = \ln\left(\frac{x - \sqrt{x^2+1}}{-1}\right)$$

$$= \ln(-x + \sqrt{x^2+1}) \Rightarrow f(-x) = -f(x)$$

$\Rightarrow$ odd

III) Solve The inequality $\ln(x^2-2x-2) \le 0$.

Step 1 $x^2-2x-2 \le e^0$

$x^2-2x-2 \le 1$

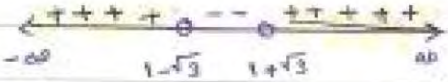
$x^2-2x-3 \le 0$

$(x-3)(x+1) \le 0$

$x = 3$, $x = -1$

Step 2 $x^2-2x-2 > 0$

$$x = \frac{-b \mp \sqrt{b^2 \mp 4ac}}{2a}$$



$$= \frac{2 \mp 2\sqrt{3}}{2a}$$

$= 1 \mp \sqrt{3}$

$[-1, 1-\sqrt{3}) \cup (1+\sqrt{3}, 3]$

CALCULUS 1

FIRST EXAM 2011

1) Let $f(x) = \frac{x^3}{x^2+1}$, If $F^{-1}(x) = 2$, then $x =$

a _ 5/8   c . 1

b _ 4   d _ 8/5

Sol.

$F^{-1}(x) = 2$

$x = f(2)$

$x = \frac{8}{4+1} = \frac{8}{5}$

2) Let $F(x)$ be a function with Domain $[-3, 5]$ and Range $[2, 8)$.
Then the Domain of $g(x) = 2 - 4f(1-2x)$ is .

a _ $[-2, 2)$ c _ $(-2, 2]$

b _ $[-30, -6)$ d _ $(-30, -6]$

Sol.

$-3 < 1 - 2x \le X$ (طرح 1)

$-4 < -2x \le 4$ (نقسم على -2)

$2 > x \ge -2$

3) To get the graph of $y = 3 + \sqrt[3]{2x+1}$ from the graph of $y = \sqrt[3]{2x}$,
you have to make the following shifts .

a _ 1/2 right and 3 down   c _ 1 left and 3 up

b _ 1/2 left and 3 up   d _ 1 right and 3 down

Sol.

$y = 3 + \sqrt[3]{2(x + \frac{1}{2})}$

$\downarrow$ up 3 $\downarrow$ Left $\frac{1}{2}$

CALCULUS 1

FIRST EXAM 2011

4) The Domain of $g(x) = \dfrac{\sqrt{x-2}}{\sqrt{x-3}}$ is.

a. $(3, \infty)$ c. $[3, \infty)$

b. $(-\infty, 2] \cup (3, \infty)$ d. $(-\infty, 2] \cup [3, \infty)$

Sol. $\sqrt{x-2}$

$x - 2 \geq 0$

$x \geq 2$

$\sqrt{x-3}$

$x > 3$

المنطقة المشتركة

5) The Range of the function $f(x) = 3 - 2\sin\left[2x + \frac{3\pi}{2}\right]$ is

a. $[-5, 5]$ c. $[-1, 1]$

b. $[1, 5]$ d. $[-2, 2]$

Sol.

$-1 \leq \sin x \leq 1$

$2 \geq -2\sin x \geq -2$ ضرب -2

$5 \geq 3 - 2\sin x \geq 1$ جمع 3

$[1, 5]$

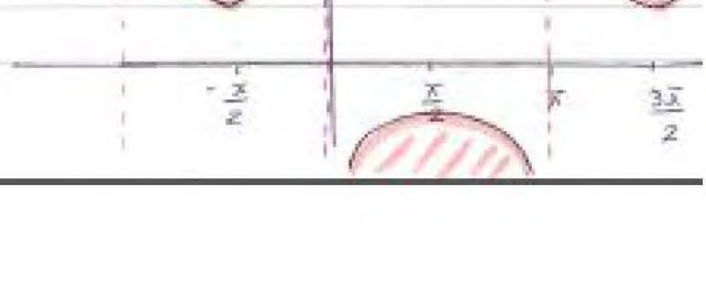
6) Domain $\sec\left[x + \frac{\pi}{2}\right]$ is

a. $\mathbb{R} - \{\ldots, \frac{-5\pi}{2}, \frac{-3\pi}{2}, \ldots\}$ c. $\mathbb{R}$

b. $\mathbb{R} - \{\ldots, -3\pi, -2\pi, -\pi, \ldots\}$ d. $\mathbb{R} - \{-1, 1\}$

Sol.

الجواب من الرسمة



CALCULUS 1

FIRST EXAM 2011

7) Let $f(x) = \sqrt[5]{4x+1}$, Then $f^{-1}(x) =$

a - $\frac{1}{4}x^5 + 1$

b - $\sqrt[5]{4x+1}$

c - $\frac{x^5 - 1}{4}$

d - $\frac{1}{\sqrt[5]{4x+1}}$

Sol.

$y = \sqrt[5]{4x+1}$

$x = \sqrt[5]{4y+1}$

$x^5 = 4y + 1$

$y = \frac{x^5 - 1}{4}$

8) let $f(x)$ be a function with Domain $[-3,5]$ and Range $[2,8]$. Then the Range of $g(x) = 2 - 4f(1-2x)$ is

a - $[-30, -6)$

b - $[-2, 2)$

c - $(-2, 2]$

d - $(-30, -6]$

Sol.

$2 \leq f \leq 8$

$-8 \geq -4f \geq -32$ (-4 يقلب)

$-6 \geq 2 - 4f \geq -30$ (2 يُجمع)

$[-30, -6]$

9) $\sin\left[x - \frac{\pi}{2}\right] =$

a - $-\cos x$

b - $\sin x$

c - $\cos x$

d - $-\sin x$

Sol.

$\sin x \cdot \cos\frac{\pi}{2} - \cos x \sin\frac{\pi}{2}$

a - $\cos x$

CALCULUS 1
FIRST EXAM 2011

10) $\cos^{-1}\left(\cos \frac{5\pi}{4}\right) =$

a. $\pi/4$ c. $3\pi/4$
b. $5\pi/4$ d. $-\pi/4$

Sol.

$$\cos(\quad) = \cos \frac{5\pi}{4}$$

$$x = \frac{5\pi}{4} = \frac{\pi}{4}$$

11) $\tan(\sin^{-1} x) =$

a $\frac{x}{\sqrt{1-x^2}}$ c $\sqrt{1-x^2}$

b $\frac{\sqrt{1-x^2}}{x}$ d. $\frac{1}{\sqrt{1-x^2}}$

12) one of the following function is one-to-one function

a. $x^2 - 2x + 2$ c. $|x|$
b. $\sin x$ d. $9x - 2$

13) let $f(x) = 5x - x^2$, $x \geq 6$. Then the Domain of $f^{-1}(x)$ is

a. $(-\infty, \frac{25}{4}]$ c. $(-\infty, -6]$
b. $[6, \infty)$ d. $[\frac{25}{4}, \infty)$

14) one of the following function is not an odd function.

a. $\csc x$ c. $x|x|$
b. $x^3 + x + 1$ d. $x \cos x$

45

الأربعاء 2013/11/6	الامتحان الأول: تفاضل وتكامل -1	الجامعة الأردنية
مدرس المادة: مجد مجيلان	اسم الطالب: آلاء سليمان محمد الجعفري	
الشعبة: 44	الرقم الجامعي: 0134923	

$-1 \le 3x+3 \le 1$

$-4 \le 3x \le -2$

$\frac{-4}{3} \le x \le \frac{-2}{3}$

20

excellent, thank you

In questions 1 to 7 fill in the blanks (2 marks each):

[1] The domain for $f(x) = \frac{1}{\sin^{-1}(3x+3)}$ is ..$\left[\frac{-4}{3}, \frac{-2}{3}\right]$..

[2]Write using one logarithm to the base 3:

$\log_3 10 + \log_9 16 =$ ..$\log_3 40$.......

$\frac{\ln 10}{\ln 3} + \frac{\ln 4}{\ln 3} = \frac{\ln 40}{\ln 3}$

[3] $\cos(\tan^{-1} \frac{-2}{5}) =$ ....$\frac{+5}{\sqrt{29}}$..........................

[4] The graph of $y = x\cos x$ is symmetric around ...the origin

$f(-x) = -x\cos x = -f(x)$ ∴ odd

14

[5] $\lim_{x\to 4^-} \frac{x^2-2x-8}{|x-4|} =$ ....-6..........................

$\lim_{x\to 4} \frac{(x-4)(x+2)}{(4-x)} = -6$

[6] The vertical asymptotes for $y = \frac{x^2+2x}{|x|(x-2)(x+2)}$ is (are) .$x=2$.

[7] $\cos^{-1}(\cos\frac{7\pi}{6}) =$ ..$\frac{5\pi}{6}$.......................

$\frac{5\pi}{6}$

In questions 8 and 9 solve and show your work

[8] (3 marks) Let $f(x) = \frac{e^x}{1-4e^x}$

e^x $D = \mathbb{R}$
$R = (0, \infty)$

(a) Find domain (f)

$1 - 4e^x = 0 \rightarrow 4e^x = 1$

$e^x = \frac{1}{4} \rightarrow \ln e^x = \ln\frac{1}{4} \rightarrow x = \ln\frac{1}{4}$

$\therefore D_f = \mathbb{R} - \{\ln\frac{1}{4}\}$

(b) Find f^{-1}.

$f(x) = \frac{e^x}{1-4e^x}$

$y = \frac{e^x}{1-4e^x}$

$x = \frac{e^y}{1-4e^y}$

$x - 4x(e^y) = e^y$

$x = 4x(e^y) + e^y$

$x = e^y(4x+1)$

$e^y = \frac{x}{4x+1}$

$\ln e^y = \ln\frac{x}{4x+1}$

$y = \ln\frac{x}{4y+1}$

$f^{-1} = \ln\frac{x}{4x+1}$ #

3

(c) Find Range (f)

$R_f = D_{f^{-1}}$

$D_{f^{-1}} \rightarrow$

$\frac{x}{4x+1} > 0$

~~$f(x) = \frac{e^x}{1-4e^x}$~~ ~~$1-4e^x < 1$~~ ~~$R_f : (-\infty, 1)$~~ ~~$-\infty < -4e^x < 0$~~

[9] (3 marks) Sketch the graph of $y = -3x^2 + 6x + 3$

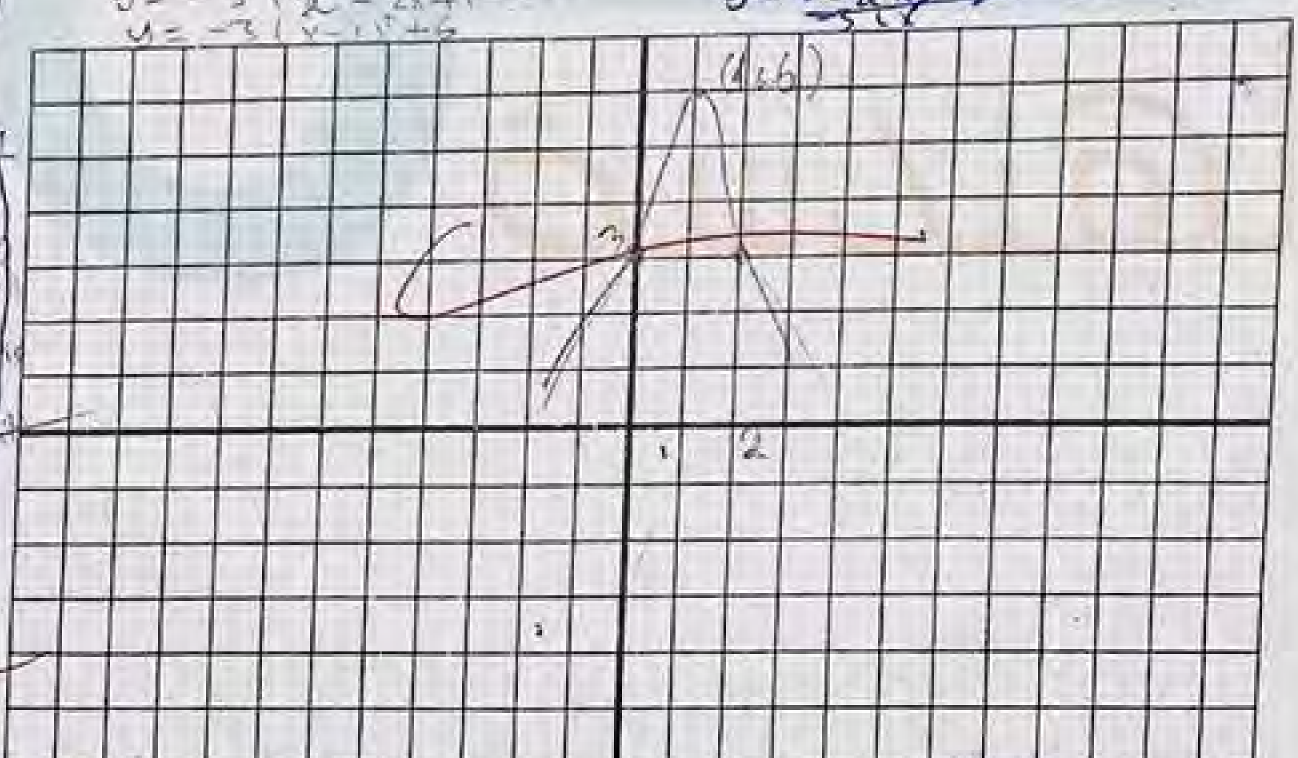
$y = -3(x^2 - 2x - 1)$

$y = -3(x^2 - 2x + 1 - 1 - 1)$

$y = -3(x-1)^2 + 6$

~~$y = -3x^2 + 6x + 9 - 9 + 3$~~

① x^2
② shift to the right 1
③ compress horizontally
④ reflect about x-axis
⑤ up 6 units



3

[8] c)

$4x+1=0$
$x = \frac{-1}{4}$

$- \quad +$
0

$- \quad +$
$-\frac{1}{4}$

~~$D_{f^{-1}} = (0, \infty) = R_f$~~

$+ \quad - \quad +$
$-\frac{1}{4} \quad 0$

$D_{f^{-1}} = (-\infty, 4) \cup (0, \infty)$

$D_{f^{-1}} = R_f = (-\infty, 4) \cup (0, \infty)$

$\frac{20.5}{20}$

Jordan University
Mathematics Department
Calculus I, First Exam, 5/4/2014

Student's Name: Lama Muclan. Student Number: 2130520.

Lecture Time: ______________

$\boxed{12}$ 1) (1.5 points each) Fill in the blanks:

1.5 a) If $f(x) = \ln(\frac{x-2}{x^2+x+1})$, then domain$(f)$ = $(2,\infty)$.

1.5 b) If $g(x) = 2\cos^2(x) + 3$, then range(f) = $[3,5]$.

1.5 c) $\csc(\frac{7\pi}{6})$ = -2.

1.5 d) The solution of the equation $\ln(2x-5) - \ln(x) = 0$ is x = 5.

1.5 e) $\lim_{x\to 0} x\sin\left(\frac{3}{x}\right)$ = 0.

1.5 f) $\cos(\sin^{-1}(\frac{-1}{3}))$ = $\frac{\sqrt{8}}{3}$.

1.5 g) $\tan^{-1}(\tan(\frac{16\pi}{5}))$ = $\frac{\pi}{5}$.

1.5 h) The vertical asymptote of $f(x) = \frac{x-1}{x^2-5x+4}$ is x = 4.

2) Let $f(x) = \frac{e^x}{e^x+4}$.

4

a) (4 points) Find $f^{-1}(x)$

$f(x) = \frac{e^x}{e^x+4}$

$y = \frac{e^x}{e^x+4}$

$y(e^x+4) = e^x$

$ye^x + 4y = e^x$

$ye^x - e^x = -4y$

$e^x(y-1) = -4y$

$e^x = \frac{-4y}{y-1}$

$\ln e^x = \ln \frac{-4y}{y-1}$

$x = \ln \frac{-4y}{y-1}$

$\boxed{y = \ln \frac{-4x}{x-1} = f^{-1}(x)}$

b) (1 points) Find Range($f^{-1}(x)$)

1

$R(f^{-1}(x)) = D(f(x))$

$\Rightarrow D(f(x)) = \mathbb{R} - \{x : e^x + 4 = 0\}$

2-b

$D(f(x)) = \mathbb{R} - \{x : e^x = -4\}$

$D(f(x)) = \mathbb{R} - \{x : \ln e^x = \ln -4\}$

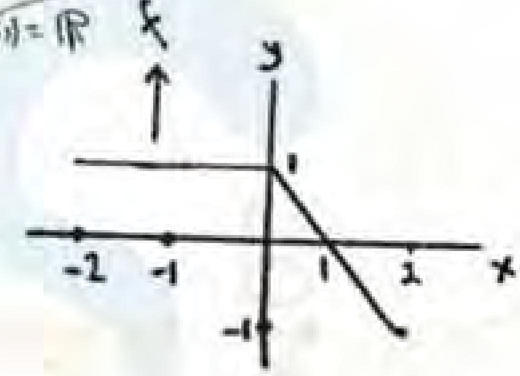
$D(f(x)) = \mathbb{R} - \{x : x = \ln -4\}$

$D(f(x)) = \mathbb{R} - \{x : x = \emptyset\}$

$D(f(x)) = \mathbb{R}$

$\Rightarrow R(f^{-1}(x)) = \mathbb{R}$

3) The graph of f is given. Sketch the graph of

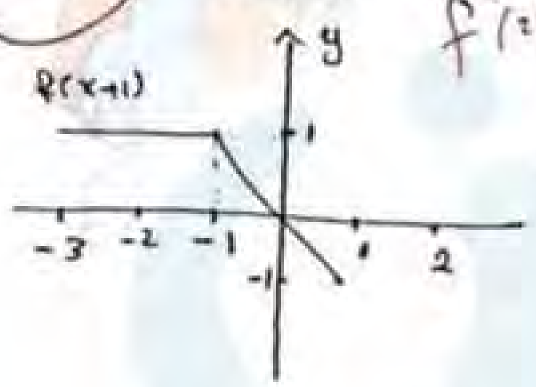


3

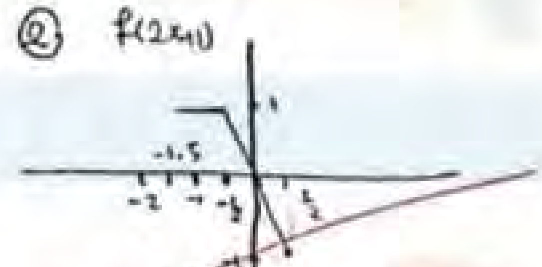
a) (3 points) $f(2x+1)$ (Show your work)

$f(2(x+\frac{1}{2}))$

① $f(x+1)$



② $f(2x+1)$

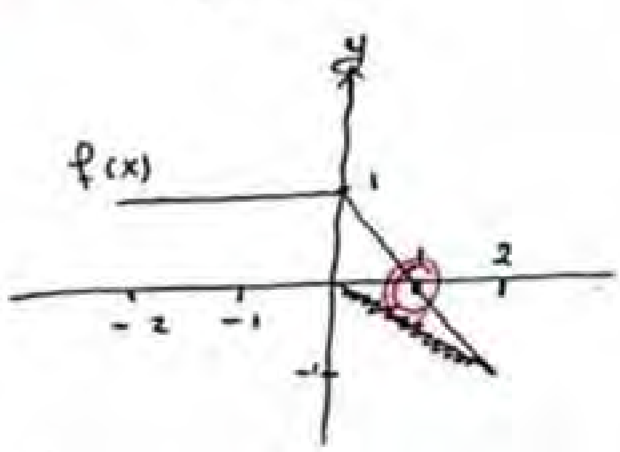


b) (1 point) $\frac{(x-1)f(x)}{x-1}$

05

$\frac{\cancel{(x-1)}f(x)}{\cancel{(x-1)}} = f(x)$

$f(x)$



18/20

The University of Jordan
Department of Mathematics

First Exam
Math 101

First Semester
28.10.2010

Name: Sondos Al-Rashaideh

Section: الثين ، اربعاء 3:30 – 5:00

Student Number: 0100542

I. Write down only the answers of the following questions:

1- If $f(x)=\begin{cases} x-1 & x\le 3 \\ 3x-7 & x>3 \end{cases}$, then $\lim\limits_{x\to 3} f(x)$ ... 2

2- If $f(x)=2x^3+3x+1$ and $f^{-1}(x)=1$, then $x=$ ... 6

3- The domain of $f(x)=\dfrac{1}{\sqrt{x^2-3x}}$ is ... $(-\infty,0)\cup(3,\infty)$

4- The range of $f(x)=2+\sqrt{9-x^2}$ is ... $[2,\infty)$

5- The value of $(\log_2 3)(\log_3 4)(\log_4 5)\ldots(\log_{15} 16)$ is ... 4

6- $\tan\left(2\sin^{-1}\left(\frac{4}{5}\right)\right)=$ ... $\frac{24}{-7}$

7- The vertical asymptotes of $f(x)=\dfrac{(x^5-1)}{(x^2-1)(x+3)}$ are ... $-3, -1$

II. Is the function $f(x)=\ln\left(x+\sqrt{x^2+1}\right)$ even or odd (Verify your answer).

$f(-x) = \ln(-x+\sqrt{(-x)^2+1})$

$f(-x) = \ln(-x+\sqrt{x^2+1})$ $\quad f(-x) \neq f(x)$ not even

$-f(x) = -\ln(x+\sqrt{x^2+1})$ ~~not odd~~

$-f(x) = \ln(x+\sqrt{x^2+1})^{-1}$

$-f(x) = \ln \dfrac{1}{x+\sqrt{x^2+1}}$

4

$-f(x) = \ln 1 - \ln(x+\sqrt{x^2+1})$

$-f(x) = 0 - \ln(x+\sqrt{x^2+1})$

$-f(x) = -\ln(x+\sqrt{x^2+1})$

~~f(x) is odd~~ $f(x)$ is odd

$-f(x) = -\ln(x+\sqrt{x^2+1})$

$f(-x) = \ln(\sqrt{x^2+1}-x)\cdot\dfrac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}$

(الضرب بالمرافق)

$= \ln\dfrac{(x^2+1-x^2)}{\sqrt{x^2+1}+x}$

$= \ln\left(\dfrac{1}{\sqrt{x^2+1}+x}\right)$

$= \ln 1 - \ln\sqrt{x^2+1}+x$

$= -\ln\sqrt{x^2+1}+x$

$f(-x) = -f(x)$

$f(x)$ is odd

III. Solve the inequality $\ln(x^2-2x-2)\le 0$

$\ln_e(x^2-2x-2) \le 0_e$

$(x^2-2x-2) \le 1$

$x^2-2x-3 \le 0$

$(x-3)(x+1) \le 0$

$x=3 \qquad x=-1$

$+ \quad - \quad +$

$-1 \qquad 3$

$x \in [-1, 3]$??

2

?

$\frac{17.5 + 1.5}{30}$

The University of Jordan
Department of Mathematics

Second Exam
Math 101

First Semester
09.12.2010

Name: ......سندس الرساليه.............

Section: ...3:30 – 5:00

Student Number: ...0100542...

I. Write down only the answers of the following questions:

1- The function $f(x) = \frac{x-3}{|x|-3}$ has removable discontinuity at $x =$ ....~~3~~ 0....

2- $\lim_{x\to0} \frac{x \sin 2x}{1-\cos x} =$ .....$\frac{1}{2}$..... 4

3- The derivative of $\sqrt{x^2+1}$ with respect to x^3 is ......

$$\frac{\frac{x(x^2+1)}{\sqrt{x^2+1}} - 2x\sqrt{x^2+1}}{\left((x^2+1)(\sqrt{x^2+1})\right)^2} = \frac{3x^3 - 3x}{\sqrt{x^2+1}\left((x^2+1)(\sqrt{x^2+1})\right)}$$

4- If $y = \sin 2x$ then $\frac{d^{21}y}{dx^{21}}$ is ...$-(2)^{21} \sin 2x$... $\cos 2x$

1.5 5- $\lim_{x\to0} \frac{\sin(3+x)^2 - \sin 9}{x} =$ ...$2x\cos(x^2)$...

$\tan^{-1}3x$

2 6- If $y = (\ln x)^{\tan^{-1}3x}$ then $\frac{dy}{dx} = \left[\frac{\tan^{-1}3x}{x\ln x} + \frac{3\ln(\ln x)}{1+9x^2}\right] \cdot (\ln x)^{\tan^{-1}3x}$

7- If $\frac{d}{dx}(f(3x)) = 6x^2$ then $f'(x) =$ ...$4x$... $6x^2$

2 8- The linear approximation of $\sqrt[3]{1+x}$ near $x = 0$ is ...$L(x) = 1 + \frac{1}{3}x$...

9- The equation of the tangent line to the curve $y = \sqrt{1+4\sin x}$ at the point $(0, 1)$

2 is ...$y = 2x + 1$...

10- The values of c such that the function $f(x) = \begin{cases} x+1 & x \le c \\ x^2 & x > c \end{cases}$ is continuous

2 on $(-\infty, \infty)$ are ...$\frac{1+\sqrt{5}}{2}$, $\frac{1-\sqrt{5}}{2}$...

3) $f(x) = \sqrt{x^2+1}$

$f'(x) = \frac{x}{\sqrt{x^2+1}}$

$f''(x) = \frac{1}{(x^2+1)(\sqrt{x^2+1})}$

$f^{(3)}(x) = \frac{(x^2+1)\sqrt{x^2+1} - \left((2x)\cdot\sqrt{x^2+1} + (x^2+1)\cdot\frac{x}{\sqrt{x^2+1}}\right)}{\left((x^2+1)\sqrt{x^2+1}\right)^2}$

II. Find the horizontal asymptote of the function $f(x) = x + \sqrt{x^2 + 2x}$.

4.5

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x + \sqrt{x^2+2x} \cdot \frac{x - \sqrt{x^2+2x}}{x - \sqrt{x^2+2x}}$$

$$= \lim_{x\to\infty} \frac{x^2 - (x^2+2x)}{x - \sqrt{x^2+2x}} = \lim_{x\to\infty} \frac{\cancel{x^2} - \cancel{x^2} - 2x}{x - \sqrt{x^2+2x}}$$

$$= \lim_{x\to\infty} \frac{-2x}{x - \sqrt{x^2-2x}} = \lim_{x\to\infty} \frac{-2x}{x - |x|\sqrt{1-\frac{2}{x}}}$$

$$= \lim_{x\to\infty} \frac{-2\cancel{x}}{\cancel{x}\left(1-\sqrt{1-\frac{2}{x}}\right)} = \lim_{x\to\infty} \frac{-2}{1-\sqrt{1-\frac{2}{x}}^{\,0}} = \frac{2}{0} = \infty$$

$$\lim_{x\to-\infty} \frac{2}{1+\sqrt{1-\frac{2}{x}}} = \frac{2}{2} = 1$$

نفس الاختلاف $x \to +\infty$ لأن نعوض مكان $|x| = -x$

مافي وساع لأكمل الحل!!

$$\lim_{x\to-\infty} = \frac{+2}{1+1} = +1 \quad \therefore \boxed{y = +1} \text{ is the H.A}$$

III. Find the points at which the curve $x^2 - xy + y^2 = 3$ crosses the $x-axis$ and show that the tangent lines at these points are parallel.

crosses x-axis $\longrightarrow$ $\boxed{y=0}$

5

$x^2 - 0 + 0 = 3$

$x^2 = 3$

$x = \pm\sqrt{3}$

$m_1 = y'|_{(\sqrt{3},0)} = \frac{2\sqrt{3}}{-\sqrt{3}} = -2$

$m_2 = y'|_{(-\sqrt{3},0)} = \frac{2\sqrt{3}}{\sqrt{3}} = 2$

$x^2 - xy + y^2 = 3$

$2x - (xy' + y) + 2yy' = 0$

$2x - xy' - y + 2yy' = 0$

$$y' = \frac{-2x + y}{-x + 2y}$$

$m_1 = m_2$

$\therefore m_1 \parallel m_2$

The tangent points $(\sqrt{3}, 0)$, $(-\sqrt{3}, 0)$

University of Jordan
Department of Mathematics
Student Name:..........................
Student Number:..................

Calculus I
First Exam

Class Number:.......
Instructor's Name:...........
Date: 21/3/2011

Q1) Fill in the blanks in each of the following 7 problems:

1) The domain of $f(x) = \ln(x-1) + \sqrt{4-x}$ is ....

2) The vertical asymptote(s) of the graph of $f(x) = \dfrac{x^2 - x - 2}{x^2 + 4x + 3}$ is (are) ....

3) $\cos(2\sin^{-1} x) =$ ....

4) $\ln(\frac{3}{2}) + \ln(\frac{4}{3}) + \ln(\frac{5}{4}) + \ldots + \ln(\frac{64}{63}) =$ ....

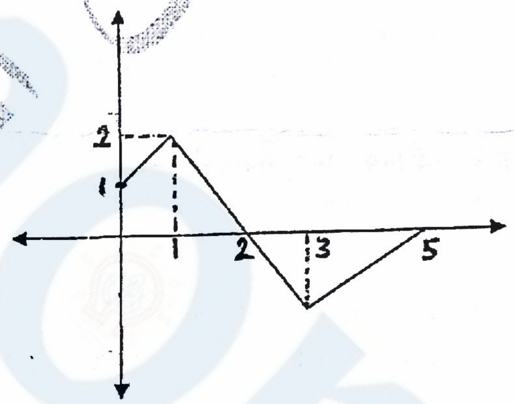
5) If $4 + x - \dfrac{x^3}{6} \le f(x) \le 4 + \sin x$, then $\lim\limits_{x \to 0} f(x) =$ ....

6) If g is an odd function with $g(2) = -2$, then $(g \circ g)(-2) =$ ....

7) If $f(x) = 2x + \sin x$, then $f^{-1}(2\pi) =$ ....

Q2) a) Let $f(x) = \dfrac{e^{x} - 3e^{-x}}{2}$. Find $f^{-1}(x)$.

b) The graph of a function f is given. Use it to sketch the graph of $1 - f(x+4)$



University of Jordan Dept. of Math. Math. 0301101 First Exam 27-10-2011

الاسم:: عمر حسن عبد ...

0110358 : الرقم الجامعي

المدرس: محمد شوقي

2 - 3:30 (اثن - أر) :وقت المحاضره

7 : الرقم المتسلسل

Q₁) The following question contains seven multiple choice problems, each is 1.5 mark. Write (x) on the correct answer.

1) If $f(x) = \frac{x+1}{x^2+1}$, $g(x) = \sqrt{x^2+3x-1}$, then $(f \circ g)(2) =$

(a) $\frac{1}{2}$ (b) $\frac{5}{2}$ $\boxed{(c)}$ $\frac{2}{5}$ (d) $\frac{3}{5}$ (e) $\frac{4}{5}$

2) If $f(x) = x^2$, $g(x) = \sqrt{x+1}$, then Domain $f \circ g =$

(a) $\mathbb{R}$ (b) $\mathbb{R}\backslash\{-1\}$ (c) $(-\infty,-1)$ (d) $(-\infty,1]$ $\boxed{(e)}$ $[-1,\infty)$

3) If $\log_x(5x-4) = 2$, then $x =$

(a) $\{1,4\}$ (b) $\{1\}$ $\boxed{(c)}$ $\{4\}$ (d) $\{1,3\}$ (e) $\{3\}$

x

11

10.5

4) If $f(x) = e^x + 3e^{-x} - 1$, then $f^{-1}(3) =$

$\boxed{(a)}$ $\ln(1)$ (b) $\ln(\frac{3}{4})$ (c) $\ln(\frac{3}{5})$ (d) $\ln(\frac{4}{5})$ (e) $\ln(\frac{3}{2})$

5) If $x = \frac{1}{2}\ln 9 - \ln 2$, then $e^{2x} =$

(a) e^3 (b) 3 (c) $\frac{3}{2}$ $\boxed{(d)}$ $\frac{9}{4}$ (e) $\frac{9}{2}$

6) If $f(x) = \frac{3}{4+2\cos x}$, then range $f=$

(a) $[\frac{1}{2},\frac{3}{4}]$ $\boxed{(b)}$ $[\frac{1}{2},\frac{3}{2}]$ (c) $[\frac{1}{4},\frac{3}{2}]$ (d) $[\frac{1}{4},\frac{3}{4}]$ (e) none

7) Let $f(x) = x^2 + 2x$, If f is shifted 2units right, 3units up then reflected about the Y-axis, we obtain,

$\boxed{(a)}$ $g(x) = x^2+2x+3$ (b) $g(x) = x^2-2x+3$ (c) $g(x) = x^2+2x-3$

(d) $g(x) = x^2-2x-3$ (e) $g(x) = x^2-6x+3$

In the following questions **show** your work in details.

$Q_2)$ $(3+2\ marks)$ Let $f(x) = 3 - 2\sin^{-1}(2x-1)$ Find,

a) The domain and range of f.

b) $f\left(\frac{1}{2} + \frac{1}{2}\sin\frac{5\pi}{4}\right)$

a) – The domain :

$1 \geq 2x - 1 \geq -1$

$2 \geq 2x \geq 0$

$1 \geq x \geq 0$

$\Rightarrow D_f = [0, 1]$

– The Range :

$\frac{\pi}{2} \geq \sin^{-1}(2x-1) \geq \frac{-\pi}{2}$

$\pi \geq 2\sin^{-1}(2x-1) \geq -\pi$

$-\pi \leq -2\sin^{-1}(2x-1) \leq \pi$

$3+\pi \geq 3 - 2\sin^{-1}(2x-1) \geq 3 - \pi$

$\Rightarrow R_f = [3-\pi, \pi+3]$

b)

$f(x) = 3 - 2\sin^{-1}\left(2\left(\frac{1}{2} + \frac{1}{2}\sin\frac{5\pi}{4}\right) - 1\right)$

$f(x) = 3 - 2\sin^{-1}\left(1 + \sin\frac{5\pi}{4} - 1\right).$

$f(x) = 3 - 2\sin^{-1}\left(\sin\frac{5\pi}{4}\right)$

$f(x) = 3 - 2\sin^{-1}\left(\sin\left(\frac{5\pi}{4} - \pi\right)\right)$

$f(x) = 3 + 2\sin^{-1}\left(\sin\frac{\pi}{4}\right)$

$f(x) = 3 + 2 \times \frac{\pi}{4}$

$f(x) = \frac{6+\pi}{2}$

$Q_3)$ $(3+2\ marks)$ Let $f(x) = \frac{e^{2x}-1}{e^{2x}+1}$. Find,

a) $f^{-1}(x)$

b) Classify $f^{-1}(x)$ as even, odd, or neither.

a) $y = \frac{e^{2x}-1}{e^{2x}+1}$

$e^{2x}y + y = e^{2x} - 1$

$e^{2x}y - e^{2x} = -y - 1$

$e^{2x}(y-1) = -y - 1$

$e^{2x} = \frac{-y-1}{y-1}$

$\ln e^{2x} = \ln\frac{-y-1}{y-1}$

$2x = \ln\frac{-y-1}{y-1}$

$x = \ln\sqrt{\frac{-y-1}{y-1}}$

$f^{-1}(x) = \ln\sqrt{\frac{-x-1}{x-1}}$

b) $f^{-1}(x) = \frac{1}{2}\ln\frac{-x-1}{x-1}$

$f^{-1}(-x) = \frac{1}{2}\ln\frac{x-1}{-x-1}$

$f^{-1}(-x) = \frac{1}{2}\ln\left(\frac{-x-1}{x-1}\right)^{-1}$

$f^{-1}(-x) = \frac{-1}{2}\ln\frac{-x-1}{x-1}$

$\Rightarrow f^{-1}(-x) = -f^{-1}(x)$

So it's an odd function

اسئلة سنوات
Past Papers

إعداد :

zenon

مجموعة من طلبة الهندسة - الجامعة الأردنية

بالتعاون.. نمضي..

bank.engzenon.com

Calculus
First

University of Jordan
Department of Mathematics

Math 101 First Exam

$19\frac{1}{2}$

Name: [redacted] Number: [redacted] Section: 3 25/10/2008

Seat no. 21

Q1. Let $f(x) = x^2 + 2x$. If $g(x)$ is obtained by shifting $f(x)$ 2 units to the right and then 3 units upward, then $g(x) =$ $(x-2)^2 + 2(x-2) + 3 = x^2 - 2x + 3$

Q2. Let $\mathrm{Dom}(f) = [1, 5]$. If $g(x) = 2f(x-3)$, then $\mathrm{Dom}(g) =$ $[4, 8]$

Q3. If $f(x) = \dfrac{x+1}{x+2}$, then $\mathrm{Dom}(f \circ f) =$ $R - \{-2, -\frac{5}{3}\}$

Q4. $\cos^{-1}(\cos \frac{4\pi}{3}) =$ $\frac{2\pi}{3}$

Q5. If $f(x) = \cos^2 x - \sin^2 x + 3$, then $\mathrm{Range}(f) =$ $[2, 4]$

Q6. If $f(x) = \ln x + \sqrt{3-x}$, then $\mathrm{Dom}(f) =$ $(0, 3]$

Q7. If $f(x) = x^3 + x + a$ is an odd function, then $a =$ 0 (if cons.) ~~OR~~ x^5

Q_2: $D_f = [1, 5]$

$g(x) = 2f(x-3)$

$1 \leq x - 3 \leq 5$

$4 \leq x \leq 8$

Q_3: $\dfrac{x+1}{x+2}$ $D \to$

$\boxed{60}$ $\frac{\pi}{240}$

$\cos^2 x - \sin^2 x + 3$

$\cos(2x) + 3$

$2 \leq \cos(2x) + 3 \leq 4$

Q8. Solve $\log_2 x + \log_2(x-3) = \log_3 9$

$$\log_2 (x * (x-3)) = \log_3 9$$

$$\log_2 (x^2 - 3x) = 2$$

$$(2)^2 = x^2 - 3x$$

$$4 = x^2 - 3x$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$x - 4 = 0$ $\quad$ OR $\quad$ $x + 1 = 0$

$x = 4$ $\quad\quad\quad\quad$ $x = -1$

$$x \in \{4, \cancel{-1}\}$$

Q9. Let $f(x) = \dfrac{e^x - 1}{e^x + 2}$. Find $f^{-1}(x)$.

$$\frac{y}{1} = \frac{e^x - 1}{e^x + 2}$$

$$e^x - 1 = ye^x + 2y$$

$$e^x - ye^x = 1 + 2y$$

$$e^x(1-y) = 1 + 2y$$

$$e^x = \frac{1+2y}{1-y}$$

$$\ln e^x = \ln\left(\frac{1+2y}{1-y}\right)$$

$$x = \ln\left(\frac{1+2y}{1-y}\right)$$

$$y = \ln\left(\frac{1+2x}{1-x}\right)$$

$$f^{-1}(x) = \ln\left(\frac{1+2x}{1-x}\right)$$