اسألني عن الهندسة 2018

التفاضل والتكامل (1)

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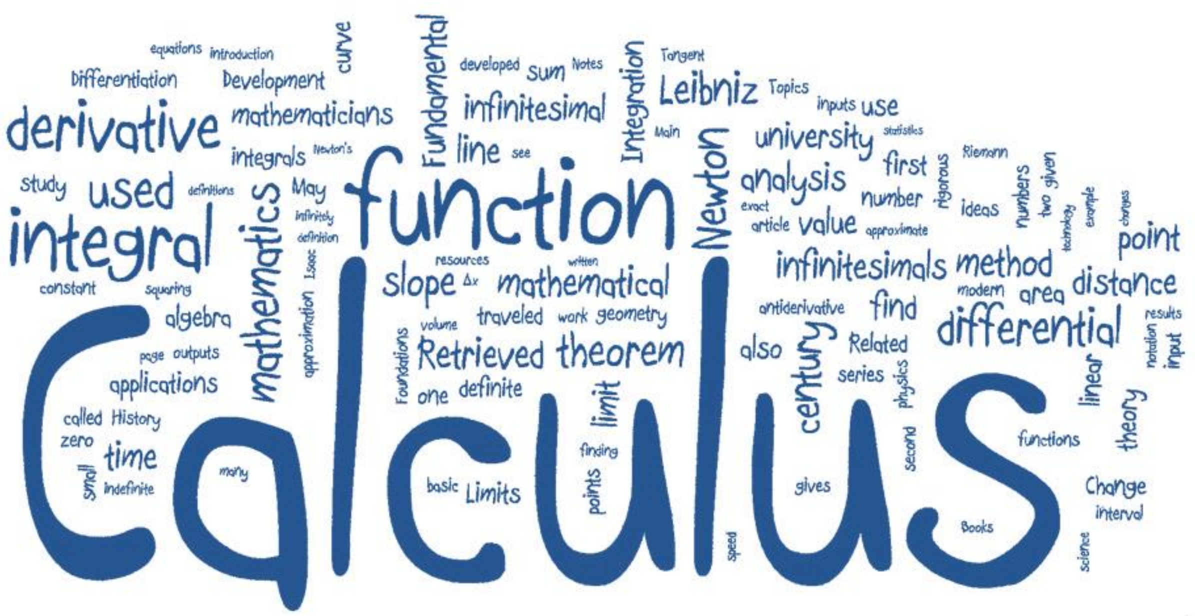
إعداد الطالب :أويس البوريني



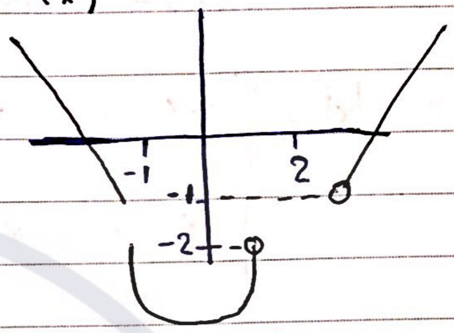
CAlCulus (1)

No.

Function : إذا قطع نقطة واحدة

لا يكونه Function إذا قطع أكثر من نقطة.

$f(x)$



Note

$\mathbb{R} = (-\infty, \infty)$

$D(f) = \mathbb{R} - \{2\} = (-\infty, 2) \cup (2, \infty)$

Domain (مجال)

مدى range $(f) = [-4, -2) \cup [-1, \infty)$

* **important rules**

$D(f.g) = D(f) \cap D(g)$

$D(f \pm g) = D(f) \cap D(g)$

$D\left(\frac{f}{g}\right) = D(f) \cap D(g) - \{\text{zeros of } g\}$

Ex (31): Find the domain of

$f(x) = \frac{x+4}{x^2-9}$

Sol: $D(x+4) = \mathbb{R}$

$D(x^2-9) = \mathbb{R} - \{-3, 3\}$

$x^2 - 9 = 0$ $\qquad D(f) = \mathbb{R} - \{-3, 3\}$

$\sqrt{x^2} = \sqrt{9}$

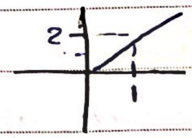
$x = \pm 3$

No.

ex(33): $f(x) = x + |x|$

Sol: $|x| = \begin{cases} -x, & x \leq 0 \\ x, & x > 0 \end{cases}$

$\begin{array}{ccc} -x & & +x \\ - & \big| & + \\ & x & \end{array}$



$x + |x| = \begin{cases} x - x, & x \leq 0 \\ x + x, & x > 0 \end{cases} \Rightarrow = \begin{cases} 0, & x \leq 0 \\ 2x, & x > 0 \end{cases}$

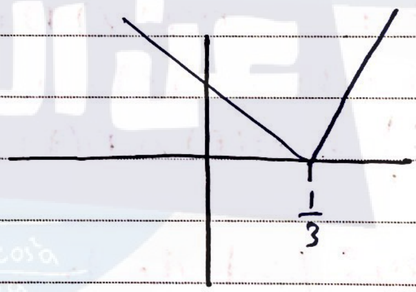
ex(47): $f(x) = |1 - 3x|$

$\begin{array}{ccc} + & & - \\ 1-3x & \big| & 3x-1 \\ & \frac{1}{3} & \end{array}$

Sol: $1 - 3x = 0$

$\frac{-3x}{-3} = \frac{-1}{-3} \Rightarrow \boxed{x = \frac{1}{3}}$

$f(x) = \begin{cases} 1 - 3x, & x \leq \frac{1}{3} \\ 3x - 1, & x > \frac{1}{3} \end{cases}$



Notes important

* $y = \pm\sqrt{\text{عدد} - x^2}$: equation of half circle

$\text{عدد} = r^2$

* عندما تكون y موضوع القانون فإن المعامل $x = \overset{\text{الميل}}{m}$ إذا كان الاقتران خطي "

* even (- تختفي السالبة)

odd (السالب يطلع برا فيصبح الاقتران كله مضروب بـ سالب)

No.

ex(36):

$f(u) = 1 + \frac{1}{u+1}$

$= \frac{u+2}{u+1}$

sol: $u+1=0$

$\boxed{u=-1}$ ×

$u+2=0$

$\boxed{u=-2}$

$D(f) = \mathbb{R} - \{-2,-1\}$

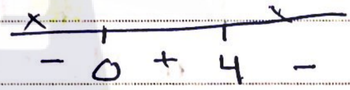
ex(37):

$f(p) = \sqrt{2-\sqrt{p}}$

Sol: $D(\sqrt{p}) = (\sqrt{p})^2 = (0)^2 \Rightarrow p = 0 = [0,\infty)$

$2-\sqrt{p} = 0 \Rightarrow (\sqrt{p})^2 = (2)^2 \Rightarrow \boxed{p=4} \Rightarrow =$ ~~[0,4]~~

$\boxed{D(f) = [0,4]}$

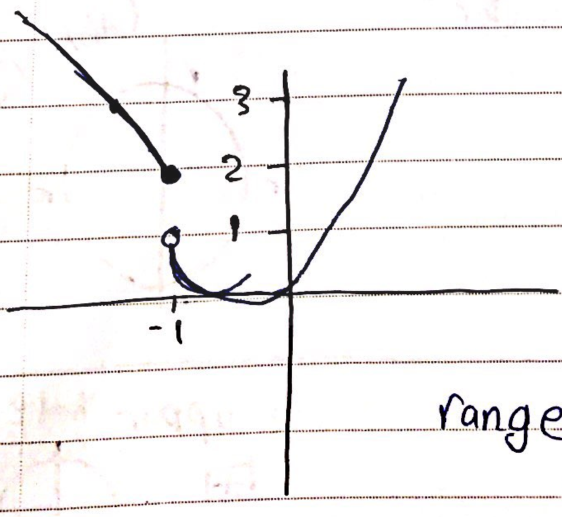


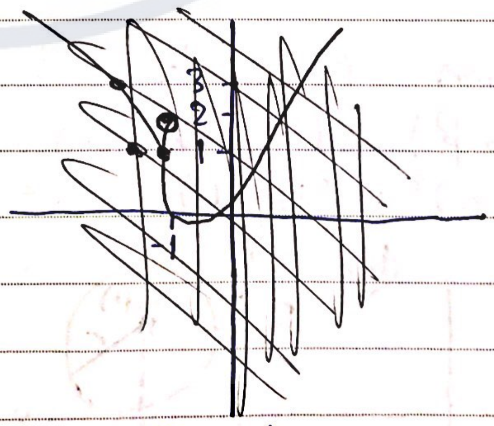
Piecewis function الاقتران المتشعب

ارسم

(Ex): Sketch $f(x) = \begin{cases} 1-x & , x \le -1 \\ x^2 & , x > -1 \end{cases}$

Sol: $D(f) = \mathbb{R}$





range(f) $= [0,\infty)$

smile for life

No.

Ex:

قيمة مطلقة

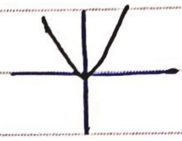
Absolute Value

$f(x) = |x|$

$-x$ (−) 0 (+) x

$\bullet\ x = 0$

$$f(x) = \begin{cases} -x, & x \leq 0 \\ x, & x > 0 \end{cases}$$



rules important

slope (m) الميل

center= مركز

radius (r) نصف قطر

* Equation of line passes through (x_0, y_0) & (x_1, y_1)

$$y - y_0 = m(x - x_0)$$

$$m = \frac{y_1 - y_0}{x_1 - x_0}$$

* Equation of line passes through (x_0, y_0) and with slope

$$y - y_0 = m(x - x_0)$$

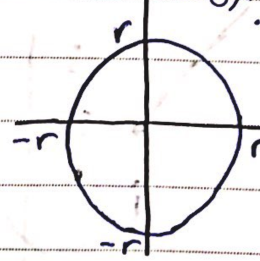
* Equation of circle with center $(0,0)$ and radius (r).

$$x^2 + y^2 = r^2$$

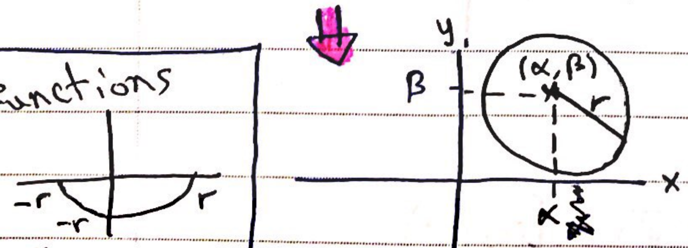
* Equation of circle with center (α, β) and radius (r).

$$(x - \alpha)^2 + (y - \beta)^2 = r^2$$

(Not. funs.)



Functions



$y = -\sqrt{r^2 - x^2}$

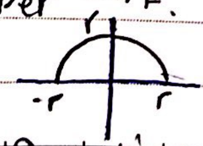
$x^2 + y^2 = r^2$

$\sqrt{y^2} = \pm\sqrt{r^2 - x^2}$

$\Rightarrow y = \pm\sqrt{r^2 - x^2}$

to upper half.

$\boxed{+}$



Functions

No.

Example (51):

Find an expression for the functions.

The line segment joining the points $(1,-3)$ & $(5,7)$

Sol: $m = \frac{7--3}{5-1} = \frac{10}{4} = \frac{5}{2} = 2.5$

$y - y_0 = m(x - x_0)$

$y + 3 = \frac{5}{2}(x-1) \Rightarrow y + 3 = \frac{5}{2}x - \frac{5}{2}$

$y = \frac{5}{2}x - \frac{11}{2} \Leftarrow y = \frac{5}{2}x - \frac{5}{2} - 3$

$1 \leq x \leq 5$

Example (53): قطع وكافئ

The bottom half of the parabola

$x + (y-1)^2 = 0$

Sol:

$\sqrt{(y-1)^2} = \sqrt{-x}$

$y - 1 = \pm\sqrt{-x} \Rightarrow \sqrt{-x}$ ⊗

نأخذ السالب
حسب السؤال

$\Rightarrow -\sqrt{-x}$ ✓

$y - 1 = -\sqrt{-x} \Rightarrow y = 1 - \sqrt{-x}$

Domain $= \mathbb{R} - \{-1\}$

Def : [1] $f(x)$ is even if $f(-x) = f(x)$

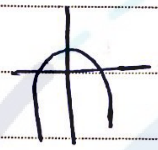
[2] $f(x)$ is odd if ~~f(x)~~ $f(-x) = -f(x)$

[about Notes]

ex :- 1) $f(x) = 1 - x^2$

$f(-x) = 1 - (-x)^2 = 1 - x^2 = f(x)$

$\boxed{\text{even}}$



example :

$f(x) = 1 - x^2$

$f(5) = 1 - 25 = -24$

$f(-5) = 1 - 25 = -24$ نفس القيمة

Note

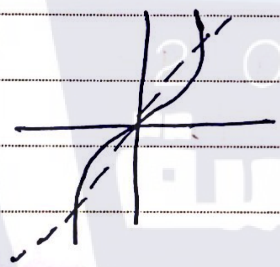
(1) If $f(x)$ is even than (f) is symmetric about y-axis.

(2) If $f(x)$ is odd than (f) is symmetric about origin.

2) : $f(x) = x^3$

$f(-x) = (-x)^3 = -x^3 = -f(x)$

odd



Example : (73) Determine f is even, odd or neither.

$f(x) = \dfrac{x}{x^2+1}$

sol: $f(-x) = \dfrac{-x}{x^2+1} = -f(x)$

odd

Example (75) : $f(x) = \dfrac{x}{x+1}$

sol : $f(-x) = \dfrac{-x}{-x+1} \neq -f(x)$ neither

$\boxed{\text{Notes}}$ odd × odd = even

even × even = even

odd × even = odd

$\dfrac{\text{odd}}{\text{even}}$ or $\dfrac{\text{even}}{\text{odd}}$ = odd

كذلك الجمع والطرح
حسب بالترتيب

Mathematical models كثيرات الحدود

No.

Polynsminds

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_{1x} + a_0$

$a_n, a_{n-1}, ---, a_1, a_0$. one consdants $n \in \mathbb{N}$.

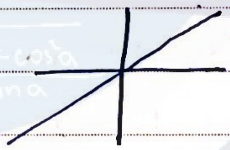
n = degree of $f(x)$.

1. $n=0 \Rightarrow f(x) = a$
   $D(f) = \mathbb{R}$ (constnet functions)
   range $= \{a\}$

2. $n=1 \Rightarrow f(x) = ax + h$ (linear functions)
   $D = \mathbb{R}$, range $= \mathbb{R}$

origin

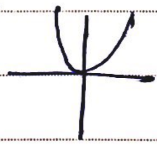
Example: $f(x) = x$



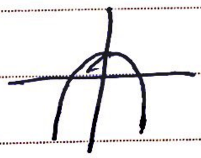
even أو odd أو neither

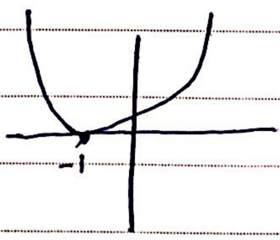
3. $n=2 \Rightarrow f(x) = ax^2 + bx + c$
   (qurdratin functions)

Example: * $f(x) = x^2$



* $f(x) = -x^2 + 1$



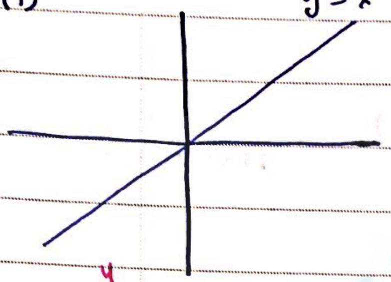


* $f(x) = x^2 + 2x + 1$

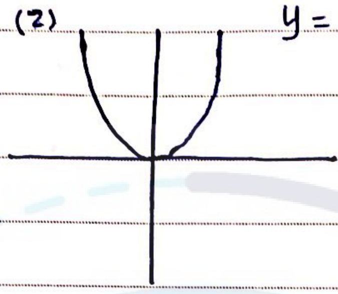
Mathmitecal models

No.

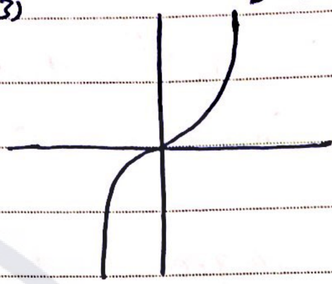
(1) $y = x$



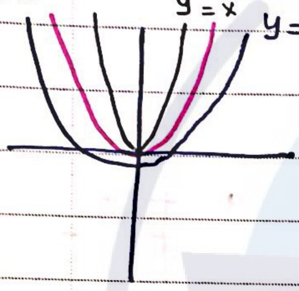
(2) $y = x^2$



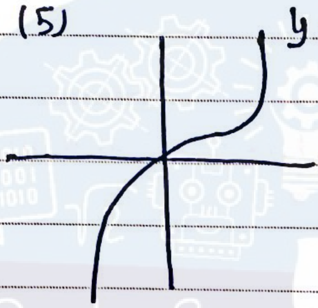
(3) $y = x^3$



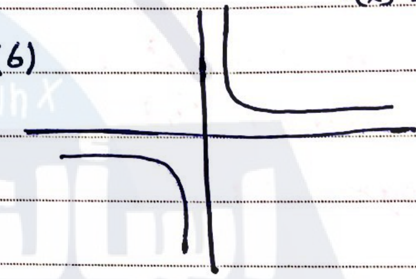
(4) $y = x^4$ $y = x^2$ $y = x^6$



(5) $y = x^5$



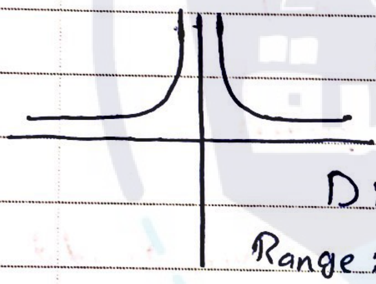
(6) $f(x) = \frac{1}{x}$



D: $\mathbb{R} - \{0\}$

Range: $\mathbb{R} - \{0\}$

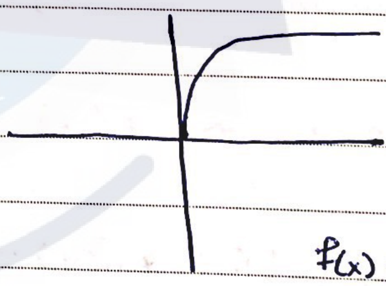
(7) $f(x) = \frac{1}{x^2}$



D: $\mathbb{R} - \{0\}$

Range: $(0, \infty)$

(8)

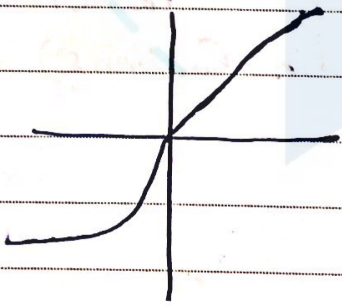


$f(x) = \sqrt{x}$

D: $(0, \infty)$

Range: $(0, \infty)$

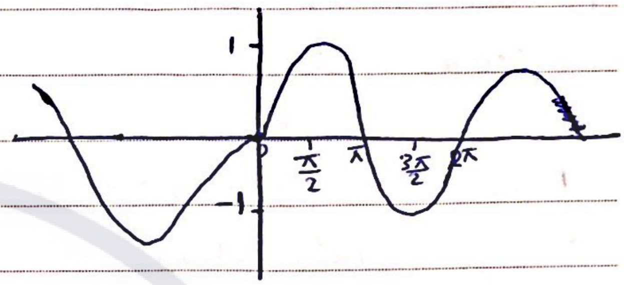
(9) $\sqrt[3]{x}$



Trigononetric functions

No.

1) $f(x) = \sin x$ (odd)



$D(\sin x) = \mathbb{R}$

$Range_{(\sin x)} = [-1, 1]$

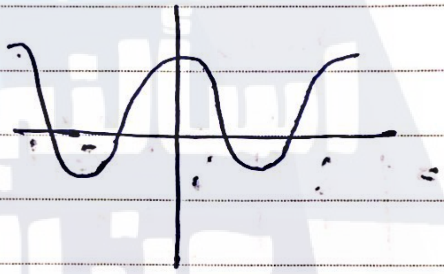
$\sin(-x) = -\sin(x) \Rightarrow$ odd

$-1 \leq \sin x \leq 1$

$|\sin x| \leq 1$

$\sin x = 0 \iff x = n\pi, \ n = 0, \pm 1, \pm 2, \ldots$

2) $f(x) = \cos x$ (even)



$D(\cos x) = \mathbb{R}$

$Range\ (\cos x) = [-1, 1]$

$\cos(-x) = \cos(x)$ (even)

$-1 \leq \cos(x) \leq 1$

$|\cos x| \leq 1$

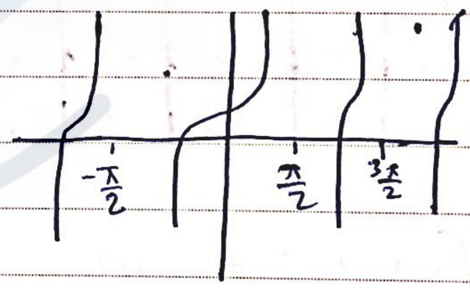
$\cos x = 0 \iff x = \frac{(2n+1)\pi}{2}$

$n = 0, \pm 1, \pm 2, \pm 3, \ldots$

3) $f(x) = \tan x$ (odd)

$D(\tan x) = \mathbb{R} - \{\pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\}$

$Range = \mathbb{R}$



$\tan x = \frac{\sin x}{\cos x}$

$\tan x = 0 \iff \sin x = 0$

$\Updownarrow$

$x = n\pi, \ n = 0, \pm 1, \ldots$

No.

$\boxed{4}$ $f(x) = \sec(x)$ (even)

$\sec(x) = \dfrac{1}{\cos x}$

$D(\sec x) = \mathbb{R} - \{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \dots\}$

Range $= \mathbb{R} - (-1, 1)$

$\boxed{5}$ $f(x) = \csc x$ (odd)

$\csc(x) = \dfrac{1}{\sin x}$

$D(\csc x) = \mathbb{R} - \{\pm\pi, 0, \pm 2\pi, \dots\}$

Range $= \mathbb{R} - (-1, 1)$

$\boxed{6}$ $\cot(x)$

$\cot(x) = \dfrac{\cos(x)}{\sin(x)}$

$D(\cot(x)) = \mathbb{R} - \{0, \pm\pi, \pm 2\pi, \dots\}$

Range $= \mathbb{R}$

(odd) [between $\frac{\pi}{2}$ and π]

The importa angles

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{3}{2}\pi$	2π	$\frac{3\pi}{4}$
$\sin x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	-1	0	$\frac{1}{\sqrt{2}}$
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	1	$-\frac{1}{\sqrt{2}}$
$\tan x$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	0	∞	0	-1
$\sec x$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-2	$-\frac{2}{\sqrt{3}}$	-1	∞	1	$-\sqrt{2}$
$\csc x$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	2	∞	-1	∞	$\sqrt{2}$
$\cot x$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	$\cancel{\infty}$ 0	$\frac{-1}{\sqrt{3}}$	$-\sqrt{3}$	∞	0	∞	-1

Ex (6) Page (33)

Find domain:

$f(x) = \dfrac{1}{1 - \tan x}$

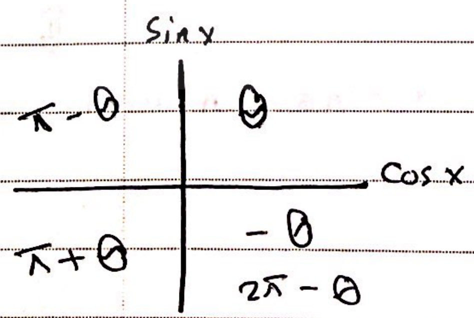
Solution:

$1 - \tan x = 0$

$\tan x = 1$

$x = \frac{\pi}{4} \pm n\pi$, $n = 0, \pm 1, \pm 2, \dots$

Domain $= \mathbb{R} - \{\frac{\pi}{4} \pm n\pi\}$



No.

Page (34)

Ex (11): Find an expression of a cubic function f if

$f(1)=6$, $f(-1)=f(0)=f(2)=0$

Solution:

$f(x) = ax^3+bx^2+cx+d$

$f(1) = a+b+c+d$

$6 = a+b+c+d \longrightarrow \boxed{1}$

$f(-1) = -a+b-c+d$

$0 = -a+b-c+d \longrightarrow \boxed{2}$

$f(0) = 0+0+0+d$

$0 = d \longrightarrow \boxed{3}$

$f(2) = 8a+4b+2c+d$

$0 = 8a+4b+2c+d \longrightarrow \boxed{4}$

3 with 1

$6 = a+b+c \longrightarrow \boxed{5}$

3 with 2

$0 = -a+b-c \longrightarrow \boxed{6}$

5 with 6

$6 = \cancel{a}+b+\cancel{c}$

$0 = \cancel{-a}+b\cancel{-c}$

$6 = 2b$

$\boxed{3=b}$

نعوض قيمة d,b في $\boxed{4}$ وفي $\boxed{12}$

$\cancel{2}/0 = 8a+12+2c$

$0 = 4a+6+c$

$-6 = 4a+c \longrightarrow \boxed{7}$

$\cancel{-}0 = -a+3-c$

$0 = a-3+c$

$3 = a+c \longrightarrow \boxed{8}$

7 with 8

$-6 = 4a+c$

$\boxed{-1}\times \quad 3 = a+c$

$\Rightarrow$

$-6 = 4a+\cancel{c}$

$-3 = -a\ \cancel{-c}$

$\frac{-9}{3} = \frac{\cancel{3}a}{\cancel{3}}$

$\boxed{-3=a}$

نعوض a و b و d في المعادلة $\boxed{1}$

$6 = \cancel{-3+3}+c$

$\boxed{6=c}$

$\therefore \boxed{f(x) = -3x^3+3x^2+6x}$

No.

New functions from old function

Vertical and Horizandal Shifts

let $c>0$ then

إزاحة عامودية

$y = f(x) + c$ shift $f(x)$ c units upward] vertical shift

$y = f(x) - c$ shift $f(x)$ c units downward

إزاحة افقية

$y = f(x-c)$ shift $f(x)$ c units to the right] Horizontal shift

$y = f(x+c)$ shift $f(x)$ c units to the left

Ex(6): find domain $f(x) = \dfrac{1}{1-\tan x}$

Solution: $D(1) = \mathbb{R}$

$D(1-\tan x) = \mathbb{R} - \left\{\left(\frac{2n+1}{2}\right)\pi, \ n = 0, \pm1, \pm2, \dots\right\}$

zeroes of $(1-\tan x)$

$1-\tan x = 0 \Rightarrow \tan x = 1 \quad x = \frac{\pi}{4} + n\pi, \ n = 0, \pm1, \pm2, \dots \}$

Example: Find domain of $f(x) = \dfrac{1}{1-2\cos x}$

Solution: $D(1) = \mathbb{R}$

$D(1-2\cos x) = \mathbb{R}$

$D\left(\frac{1}{1-2\cos x}\right) = \mathbb{R} - \left\{\frac{\pi}{3} + 2n\pi, \ \frac{5\pi}{3} + 2n\pi, \ n = 0, \pm1, \pm2, \dots\right\}$

zeroes of $1-2\cos x = 0$

$\cos x = \frac{1}{2}$

$\Rightarrow x = \frac{\pi}{3} + 2n\pi \Rightarrow x = \frac{5\pi}{3} + 2n\pi$ or $-\frac{\pi}{3}$ }

$\boxed{\text{Notes}}$

$\sin(x+2n\pi) = \sin x$

$\cos(x+2n\pi) = \cos x$

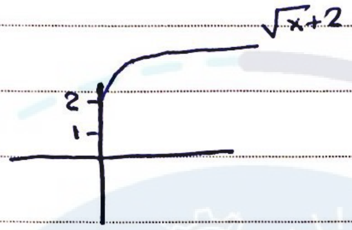
$\tan(x+n\pi) = \tan x$

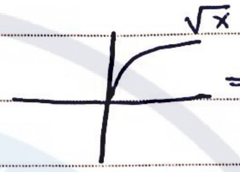
No.

Example : let $f(x) = \sqrt{x}$, sketch

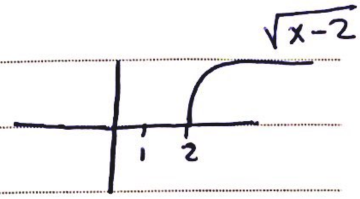
(1) $\sqrt{x} + 2$ (2) $\sqrt{x - 2}$

$\Rightarrow$





$\Rightarrow$



$\sqrt{x}$	$\sqrt{x}+2$	$\sqrt{x}$	$\sqrt{x-2}$
$D: [0, \infty)$	$D: [0, \infty)$	$D: [0, \infty)$	$D: [2, \infty)$
$R: [0, \infty)$	$R: [2, \infty)$	$R: [0, \infty)$	$R: [0, \infty)$

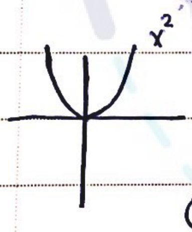
Example : Sketch $f(x) = x^2 + 6x + 10$.

Solution : $x^2 + 6x + 10$

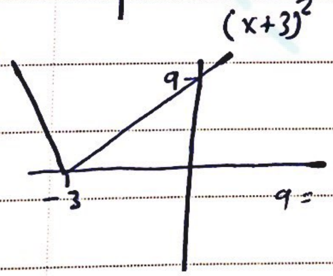
$(x^2 + 6 + 9)(-9 + 10)$

$f(x) = (x+3)^2 + 1$

(1)



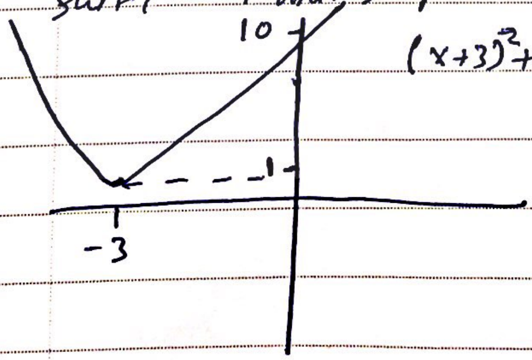
(2)



shift 3 units to left

$9 = (3)^2$ لأنه تربيع (تقطع عند 9)

(3) shift 1 units up



$(x+3)^2 + 1 = f(x)$

لعمل إكمال المربع

Complete square

$ax^2 + bx + c$

$a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$

1) نأخذ عامل مشترك لجعل x^2 منه غير معامل

٢) نضيف ونطرح $\left(\pm\left(\frac{b}{2a}\right)\right)^2$

$a\left(\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) - \left(\left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right)\right)$

$a\left[\left(x + \frac{b}{2a}\right)^2 + \left(\frac{c}{a} - \left(\frac{b}{2a}\right)^2\right)\right]$

Example $\Rightarrow$

No.

Example : Complete square

$f(x) = 2x^2 - 5x + 7$

$= 2\left(x^2 - \frac{5}{2}x + \frac{7}{2}\right) = 2\left(\left(x^2 - \frac{5}{2}x + \frac{25}{16}\right)\left(-\frac{25}{16} + \frac{7}{2}\right)\right)$

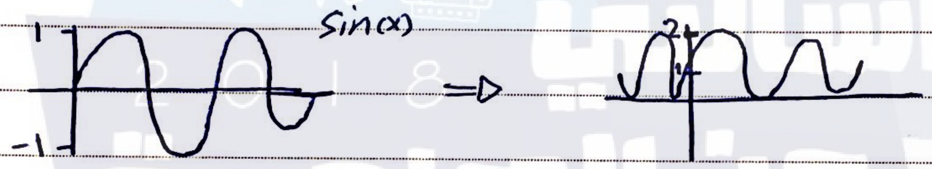
$= 2\left(\left(x - \frac{5}{4}\right)^2 + \frac{31}{16}\right) = 2\left(x - \frac{5}{4}\right)^2 + \frac{31}{8}$

Example : sketch

$f(x) = 1 + \sin(x)$

$1 + \sin x$

$\sin(x)$



[Composite function تركيب اقترانات]

$(f \circ g)(x) = f(g(x))$

$(g \circ f)(x) = g(f(x))$

$(g \circ f)(x) \neq (f \circ g)(x)$

Example (1) : $f(x) = x^2$, $g(x) = \sqrt{x-1}$

Find :

1) $(f \circ g)(x) = f(g(x)) = f(\sqrt{x-1}) = (\sqrt{x-1})^2$

$= x - 1$ #

2) $(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{x^2 - 1}$ #

Example (2) : $f(x) = \sqrt{x}$, $g(x) = \sqrt{2-x}$

Find 1) $(f \circ g)(x) = f(g(x)) = f(\sqrt{2-x}) = \sqrt{\sqrt{2-x}} = \sqrt[4]{2-x}$ #

2) $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{2-\sqrt{x}}$ #

3) $(f \circ g \circ g)(x) = f(g(g(x))) = f(g(\sqrt{2-x})) = f\left(\sqrt{2-\sqrt{2-x}}\right) = \left(\sqrt{2-\sqrt{2-x}}\right)^2$

$= 2 - \sqrt{2-x}$ #

No.

Example: let $F(x) = \cos^2(x-9)$, Find f, g, h such that:
$f \circ g \circ h = F$

Solution:

$f(x) = \cos^2 x$
$g(x) = x - 9$
$h(x) = x$

نجرب $\Longrightarrow$ هذه اعادة

$f(g(h(x))) =$
$f(g(x)) = f(x-9)$
$= \cos^2(x-9)$ #

فكرة السؤال
بيعطي الجوابه النهائي
ويتم من خلاله افراغ
الاقترانات (بالتجربة)

Ex (63), page (45):

(a) IF $g(x) = 2x+1$, $h(x) = 4x^2+4x+7$
find a function: $f(x)$ such that $f \circ g = h$

Solution:

$f(g(x)) = h(x)$
$f(2x+1) = 4x^2+4x+7 \Rightarrow f(x) = ax^2+bx+c$

$a(2x+1)^2 + b(2x+1) + c = 4x^2+4x+7$
$4ax^2 + 4ax + a + 2bx + b + c = 4x^2 + 4x + 7$

$x^2 \Rightarrow \dfrac{4ax^2}{4x^2} = \dfrac{4x^2}{4x^2} \Rightarrow \boxed{a=1}$

$x \Rightarrow 4ax + 2bx = 4x$
$2ax + bx = 2x$
$2x + bx = 2x$
$2 + b = 2$
$\boxed{b=0}$

zero
$x^0 \Rightarrow c+b+a = 7$
$c+0+1 = 7$
$\boxed{c=6}$

$\therefore \boxed{f(x) = x^2+6}$

(B) $f(x) = 3x+5$, $h(x) = 3x^2+3x+2$ find g such that $f \circ g = h$

Solution $\Rightarrow g(x) = x^2+x-1$ #

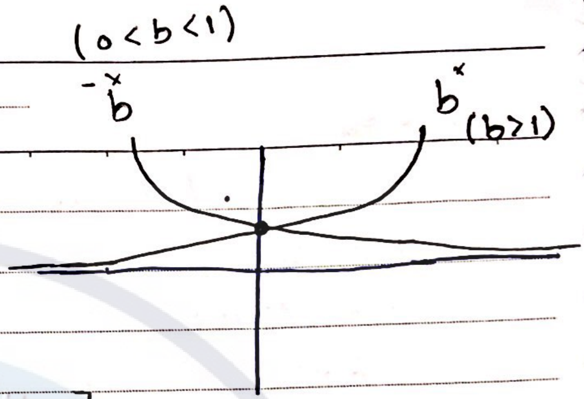
smile for life

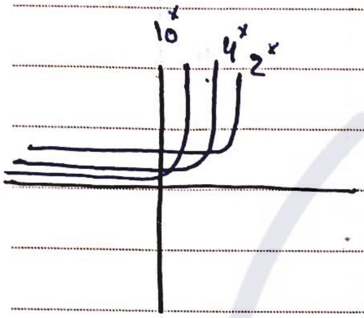
No.

(1.4) Exponential Functions

$f(x) = b^x$, $b > 0$ (b: constant)

$b \neq 1$





$D_{(b^x)} = \mathbb{R}$

Range $= (0, \infty)$

[Proplties]

1) $b^x \cdot b^y = b^{x+y}$

2) $\dfrac{b^x}{b^y} = b^{x-y}$

3) $(b^x)^y = b^{xy}$

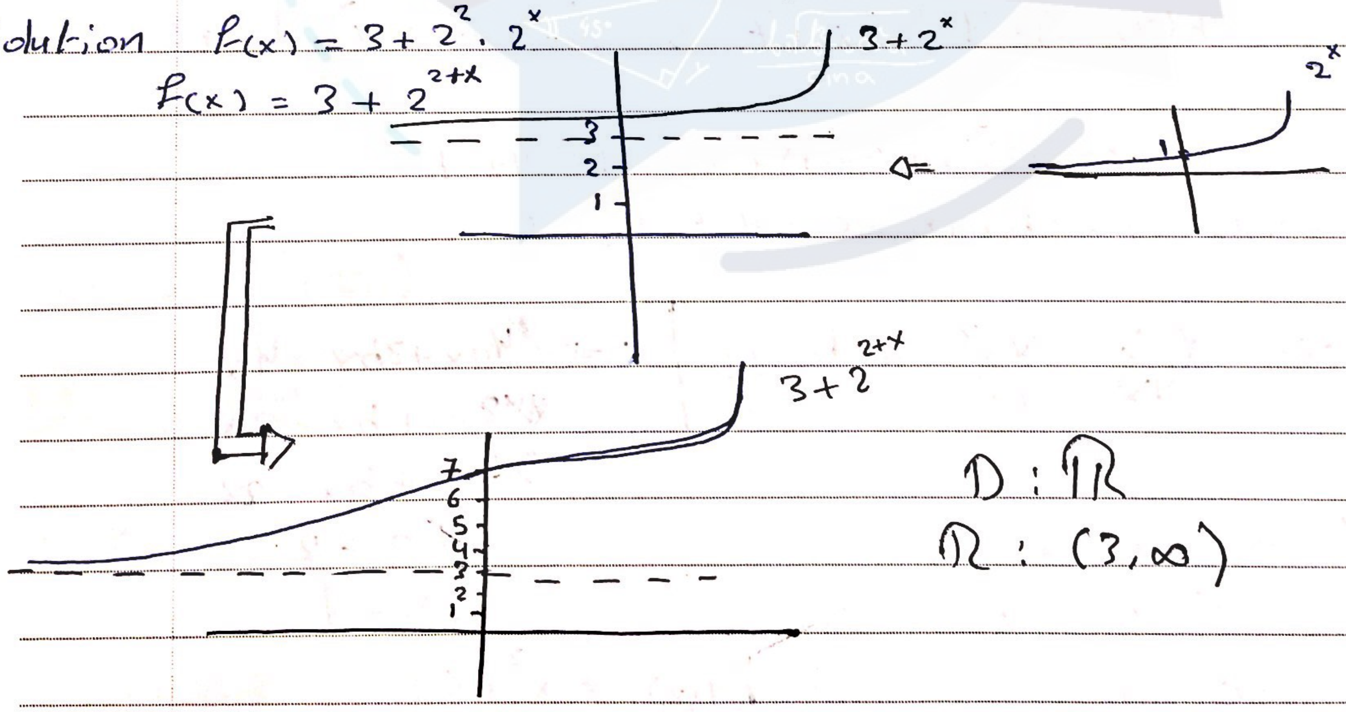
4) $(ab)^x = a^x \cdot b^x$

$2^{-x} = \dfrac{1}{2^x} = \left(\dfrac{1}{2}\right)^x$

Example: sketch $f(x) = 3 + 4 \times 2^x$ then find domain and range.

Solution $f(x) = 3 + 2^2 \cdot 2^x$

$f(x) = 3 + 2^{2+x}$



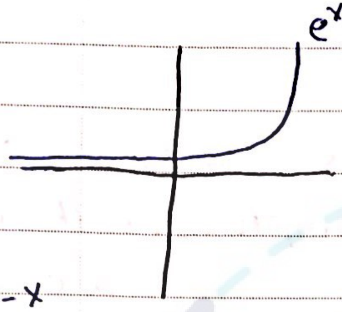
$D : \mathbb{R}$

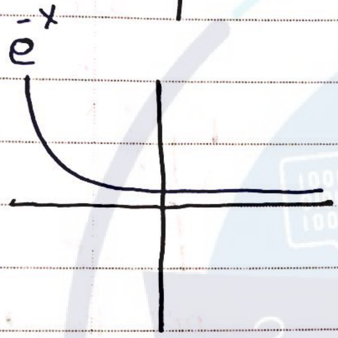
$R : (3, \infty)$

No.

$f(x) = e^{x}$ ← عدد نيبري

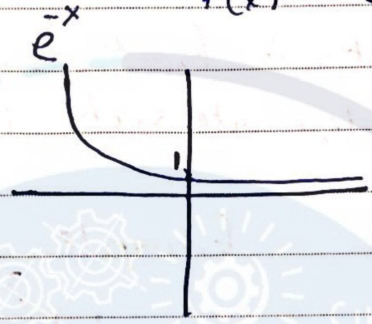
$e \approx 2.71828$



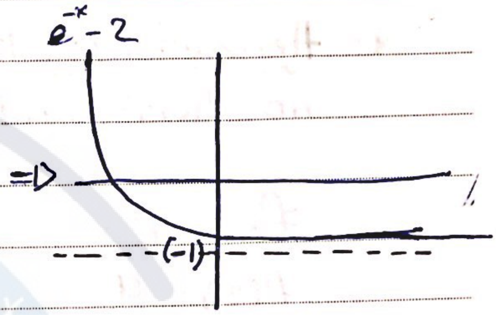


Example : sketch

$f(x) = e^{-x} - 2$



$\Rightarrow$



$x = 0$

$e^{0} - 2 = 1 - 2 = \boxed{-1}$

Ex(17) page(53)

Starting with $y = e^{x}$ write the equ. of the graph results.

(a) shift 2 units down $\Rightarrow$ $y = e^{x} - 2$

(b) shift 2 units to right $\Rightarrow$ $y = e^{x-2}$

Ex(19): Find domain of:

(a) $f(x) = \dfrac{1 - e^{x^2}}{1 - e^{1-x^2}}$

Sol: $D(1 - e^{x^2}) = \mathbb{R}$

$D(1 - e^{1-x^2}) = \mathbb{R}$

$1 - e^{1-x^2} = 0$

$e^{1-x^2} = 1$

$1 - x^2 = 0$

$x^2 = 1$

$x = \pm 1$

$D: \mathbb{R} - \{-1, 1\}$

or $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

(b) $f(x) = \dfrac{1 + x}{e^{\cos x}}$

Sol: $D_{(1+x)} = \mathbb{R}$

$D(e^{\cos x}) = \mathbb{R}$

$D(h(x)) = \mathbb{R}$

(c) $g(t) = \sqrt{10^{t} - 100}$

Sol: $\rightarrow 10^{t} - 100 = 0$

$10^{t} = 100$

$10^{t} = 10^{2}$

$\boxed{t = 2}$ $D(g) = [2, \infty)$

$\overset{-}{} \; 2 \; \overset{+}{}$

(d) $g(t) = \sin(e^{t} - 1)$

Sol:

Domain $= \mathbb{R}$

No.

(1.5) Inverse functions and logarithms

Def: A functions f is called one-to-one if $f(x_1) \neq f(x_2)$ wherever $x_1 \neq x_2$

* Horizontal line test: A functions f is one-to-one if no horizontal line intersects its graph more than once.

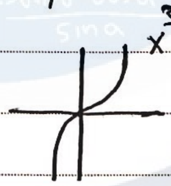
$f(x) = x^2$

$f(2) = 4$

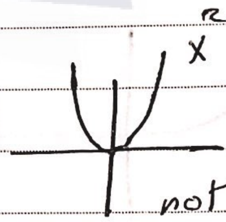
$f(-2) = 4$

$f^{-1}(4) \to \begin{cases} 2 \\ -2 \end{cases}$

Example



one-to-one



not one-to-one

Def: let $\boxed{f}$ be one-to-one with domain A and range B the it's inverse func. $\boxed{f^{-1}}$ has domain B and range A

$$f^{-1}(y) = x \iff f(x) = y$$

Functions

Notes

(1) $D(f^{-1}) = \text{range}(f)$

(3) $f^{-1}(x) \neq \frac{1}{f(x)}$

(2) $\text{range}(f^{-1}) = D(f)$

$(f(x))^{-1} = \frac{1}{f(x)}$

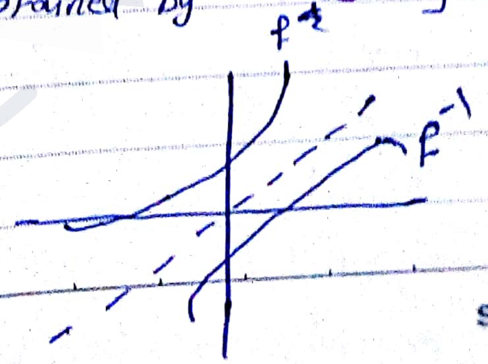
(4) $f^{-1}(f(x)) = x$

$x \in \boxed{D(f)} = \text{range}(f^{-1})$

(5) $f(f^{-1}(x)) = x$

$x \in D(f^{-1})$

Domain (6) The graph of f^{-1} is obtained by reflecting the graph of f about $y = x$.



No.

Example: $f(1)=5$, $f(3)=7$, $f(8)=-10$

Sol:

$f^{-1}(5)=1$, $f^{-1}(7)=3$, $f^{-1}(-10)=8$, $f^{-1}(8)=$ Does not exist.

$D(f)=\{1,3,8\}$, range $(f)=\{5,7,-10\}$

$\qquad\qquad\qquad\qquad\qquad\qquad \shortparallel$

$\qquad\qquad\qquad\qquad\qquad\quad D(f^{-1})$

* How to find f^{-1}

step (1): write $y=f(x)$

step (2): solve the equation for x in terms of y.

step (3): Interchange x & y, we have $y=f^{-1}(x)$.

Example: $f(x)=x^3+2$ find f^{-1}

Sol: $\boxed{1}$ $y=x^3+2$

لازم نخلي x موضوع قانون

$\boxed{2}$ $x^3=y-2$

$x=\sqrt[3]{y-2}$

$\boxed{3}$ $y=\sqrt[3]{x-2}$

$f^{-1}(x)=\sqrt[3]{x-2}$

Example: Find $f(x)=\sqrt{-1-x}$

f^{-1} and then sketch f & f^{-1}

Sol:

$\boxed{1}$ $y=\sqrt{-1-x}$

$\boxed{2}$ $y^2=-1-x$

$x=-1-y^2$

$\boxed{3}$ $y=\sqrt{-1-x^2}$

نلاحظ هنا

كل $\boxed{x}=y$

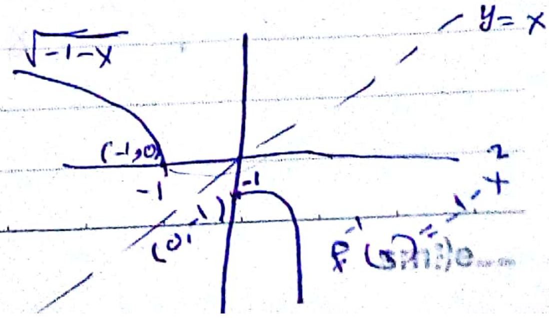
وكل $\boxed{y}=x$

$D(f_x)=(-\infty,-1]=\text{range } f^{-1}(x)$

$\text{range}(f)=[0,\infty)=\text{Domain } f^{-1}(x)$

عند الرسم لازم ننتبه للDomain

نتبع ال f^{-1}



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* Logarithmic Functions

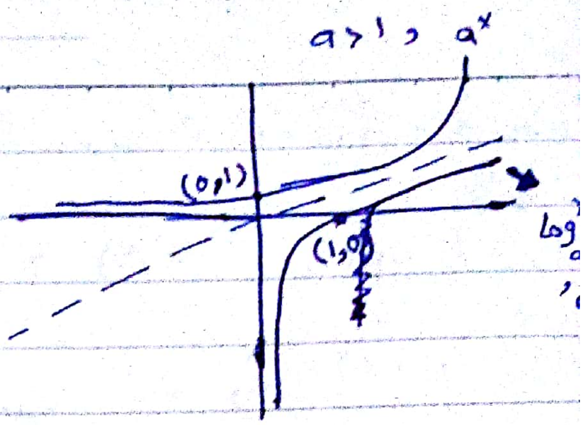
No.

$\log_b x = y \iff b^y = x$

يقرأ $\Downarrow$

logarithm

with base b.



laws

(1) $\log_b(1) = 0$ / $\log_b b = 1$

$D(\log_a x) = (0, \infty)$

range $(\log_a x) = \mathbb{R}$

$D(a^x) = \mathbb{R}$

range $(a^x) = (0, \infty)$

(2) $\log_b(b^x) = x$, $x = \mathbb{R}$

(3) $b^{\log_b x} = x$, $x > 0$.

(5) $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

(4) $\log_b(xy) = \log_b x + \log_b y$

(6) $\log_b x^r = r \log_b x$

(7) $\log_e x = \ln x$

(8) $\ln e = 1$

(9) $\ln e^x = X$

(10) $e^{\ln x} = x$

(11) $\ln x = y \iff e^y = x$

Example: simplify (بسط).

$\log_2 80 - \log_2 5 = \log_2\left(\frac{80}{5}\right) = \log_2 16 = \log_2 (4)^2 = 1$

$= \boxed{4}$

Example: Find x if $\ln x = 5$

$\Rightarrow \boxed{e^5 = x}$

Example: Find x (solue)

$$e^{5-3x} = 10$$

Sol: $e^{5-3x} = 10$ (take ln of both sides)

$$\ln e^{(5-3x)} = \ln 10$$

$$(5-3x)\ln e^1 = \ln 10$$

$$5-3x = \ln 10$$

$$3x = 5 - \ln 10$$

$$\boxed{x = \frac{5-\ln 10}{3}}$$

ex Express $\ln a + \frac{1}{2}\ln b$ as single lograthim.

→ $\ln a + \frac{1}{2}\ln b = \ln a + \ln b$

$\ln a + \ln\sqrt{b} = \ln a\sqrt{b}$,

$$\log_b^x = \frac{\ln x}{\ln b} \qquad b \neq 1$$

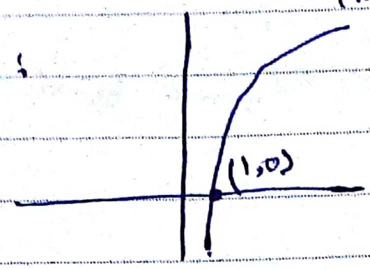
ex: find the valume of

$(\log_2^3)(\log_3^4)(\log_4^5)(\log_5^6)(\log_6^7) \ldots (\log_{15}^{16}) =$

$$= \frac{\cancel{\ln 3}}{\ln 2} \cdot \frac{\cancel{\ln 4}}{\cancel{\ln 3}} \cdot \frac{\ln 5}{\cancel{\ln 4}} \cdots \frac{\cancel{\ln 15}}{\ln 14} \cdot \frac{\ln 16}{\cancel{\ln 15}} = \frac{\ln 16}{\ln 2}$$

Ex: sketch $y = \ln(x-2) - 1$

Sol:



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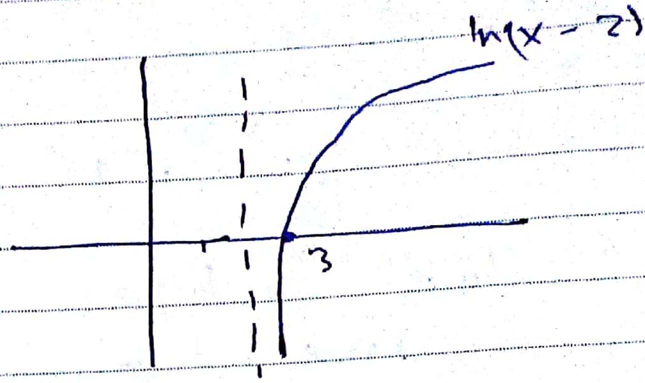
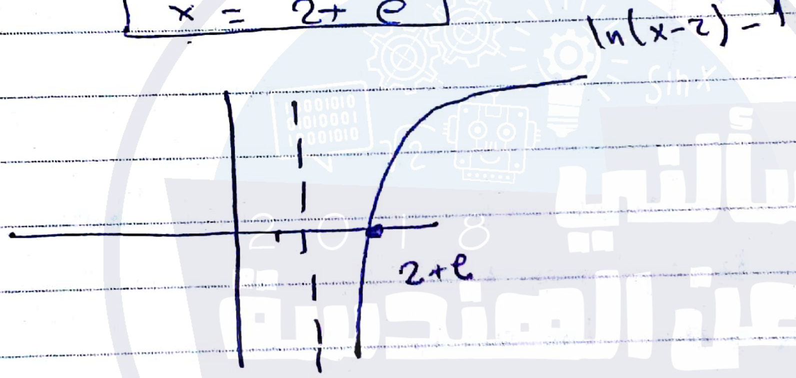


$\ln(x-2) - 1 = 0$

$\ln(x-2) = 1 \Rightarrow e^{\ln(x-2)} = e^{1}$

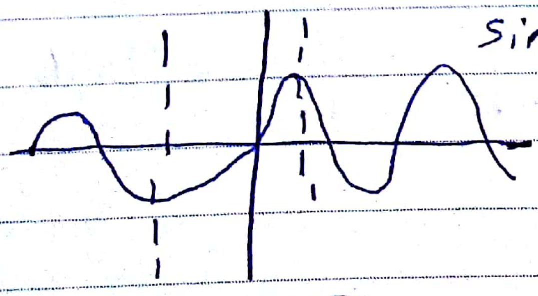
$x - 2 = e^{1}$

$\boxed{x = 2 + e}$



Inverse trigonometric funs.

[1] $\sin^{-1} x$



not (1-1)

$D(\sin x) = \mathbb{R}$

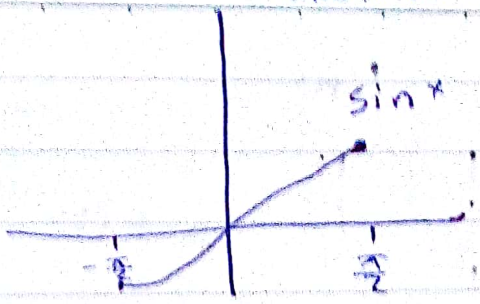
Range $= [-1, 1]$

Inverce trigonometric funs.

No.

$\Downarrow$ functions.

① $\sin^{-1} x \Rightarrow$



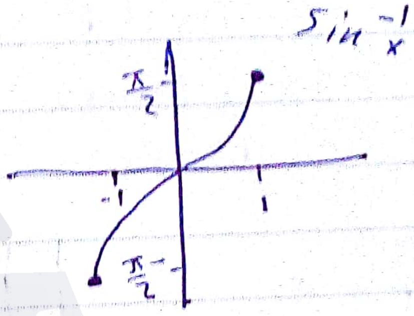
$(1,-1)$

$D = [-\frac{\pi}{2}, \frac{\pi}{2}]$

Range $= [-1, 1]$

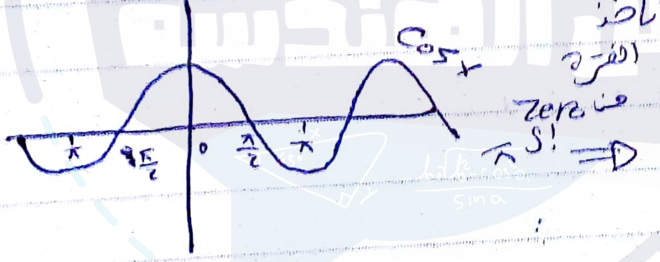
$D(\sin^{-1} x) = [-1, 1]$

Range $(\sin^{-1} x) = [-\frac{\pi}{2}, \frac{\pi}{2}] \Rightarrow$

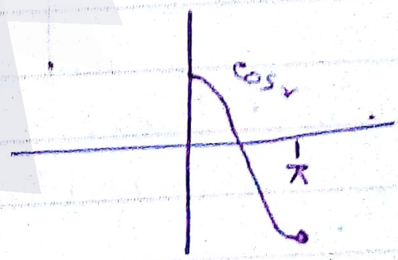


$\boxed{2}$ $\cos^{-1} x$

$\cos x \Rightarrow$



ناخذ الفترة من zero $\rightarrow \pi$ $\Rightarrow$



$D(\cos x) = \mathbb{R}$ not $(1,-1)$

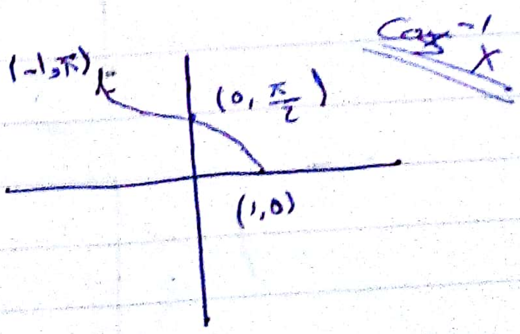
Range $= [-1, 1]$

$D(\cos x) = [0, \pi]$

Range $= [-1, 1]$

$\cos^{-1} x \Rightarrow$ $D(\cos^{-1} x) = [-1, 1]$

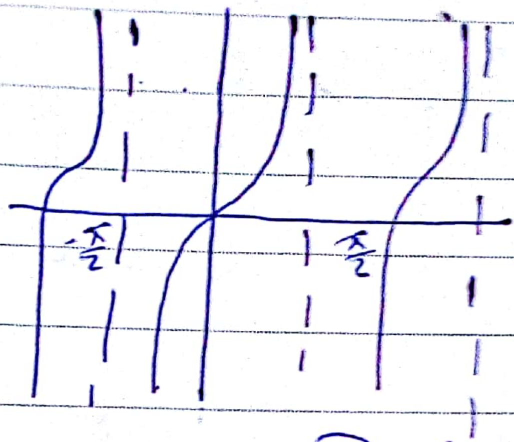
Range $= [0, \pi]$.



No.

~~[illegible]~~ $\tan^{-1} x$ $\tan x$

$\tan x \Rightarrow$



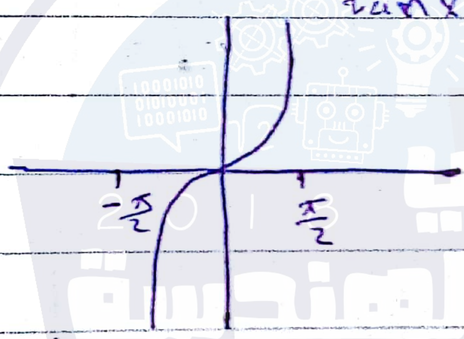
not (1-1)

$D(\tan x) = \mathbb{R} - \{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \dots\}$

Range $= \mathbb{R}$

هنضيق دورة

$\tan x \Rightarrow$



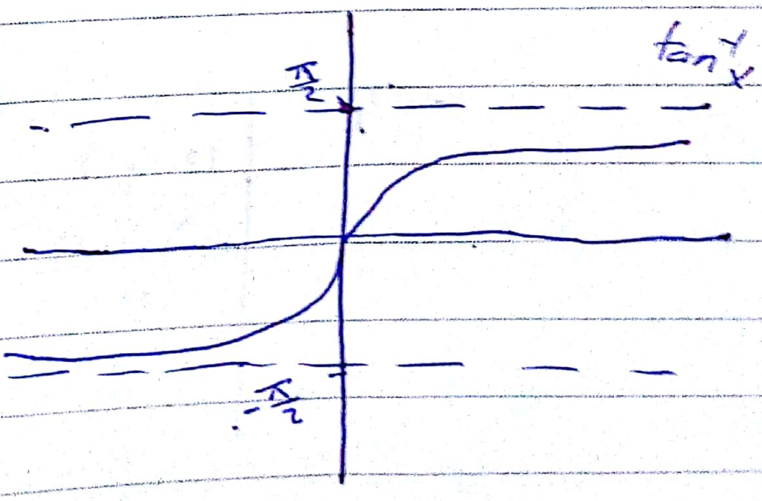
(1-1)

$D(\tan x) = (-\frac{\pi}{2}, \frac{\pi}{2})$

Range $(\tan x) = \mathbb{R}$

$\tan^{-1} x \Rightarrow D(\tan^{-1} x) = \mathbb{R}$

Range $(\tan^{-1} x) = (-\frac{\pi}{2}, \frac{\pi}{2})$



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No.

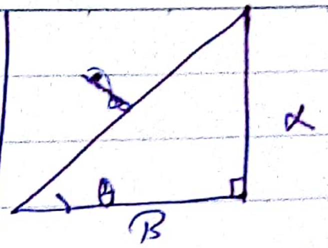
Example: Find $\sin^{-1}\frac{1}{2}$.

Sol: $y = \sin^{-1}\frac{1}{2}$

$\sin y = \sin(\sin^{-1}\frac{1}{2})$

$\sin y = \frac{1}{2}$

$$\boxed{y = \frac{\pi}{6}}$$



$\gamma^2 = \alpha^2 + \beta^2$

$\sin\theta = \frac{\alpha}{\gamma}$

$\cos\theta = \frac{\beta}{\gamma}$

$\tan\theta = \frac{\beta}{\alpha}$

$\csc\theta = \frac{\gamma}{\alpha}$

$\sec\theta = \frac{\gamma}{\beta}$ $\qquad$ $\cot\theta = \frac{\beta}{\alpha}$

Example: Evaluate $\tan(\arc\sin\frac{1}{3})$

$\arcsin = \sin^{-1}$

$\arccos = \cos^{-1}$

$\arctan = \tan^{-1}$

Sol:

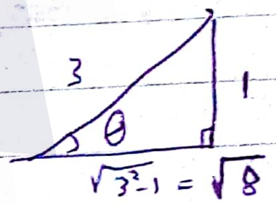
$\arcsin\frac{1}{3} = \boxed{\sin^{-1}\frac{1}{3} = \theta}$

$\sin\theta = \frac{1}{3}$ $\qquad$ $(3)^2 = (1)^2 + (\beta)^2$

$1 - 9 = (\beta)^2$

$\sqrt{8} = \sqrt{\beta^2}$

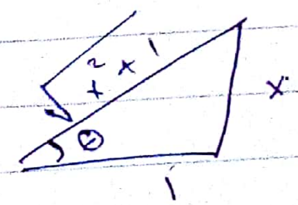
$\boxed{\beta = \sqrt{8}}$



$\tan(\sin^{-1}\frac{1}{3}) = \tan\theta$

$$= \boxed{\frac{1}{\sqrt{8}}}$$

Example: Simplify $\cos(\tan^{-1}x)$



Sol:

$\theta = \tan^{-1}x$

$\tan\theta = x$

$\cos(\tan^{-1}x) = \cos\theta$

$$= \boxed{\frac{1}{\sqrt{x^2+1}}}$$

Trigonometric identities

No.

1. $\sin^2 x + \cos^2 x = 1$
2. $\tan^2 x + 1 = \sec^2 x$
3. $1 + \cot^2 x = \csc^2 x$
4. $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
5. $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
6. $\cos(2x) = \cos^2 x - \sin^2 x$ نعكس

   $= 2\cos^2 x - 1$

   $= 1 - 2\sin^2 x$
7. $\sin(2x) = 2 \sin x \cos x$
8. $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
9. $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
10. $\sin x\ \cancel{\sin y}\ \cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$
11. $\cos x \times \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$
12. $\sin x \times \cancel{\cos}\ \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$

Ex:
(17)

No.

Page 66 $f(x) = 3 + x + e^{x}$, find $f^{-1}(4)$

* اعطانا الجواب (عدد صحيح) وهو
فبندور على عدد
نعوضه ونعطي الجواب

$x = 0$ نعوضها بالمعادلة

$\therefore f^{-1}(4) = 0$

Ex (22): Page:

find $f^{-1}(x)$

$f(x) = \frac{4x-1}{2x+3}$

$y = \frac{4x-1}{2x+3}$

$y(2x+3) = 4x-1$

$2yx + 3y = 4x - 1$

$1 + 3y = 4x - 2yx$

$1 + 3y = 2x(2-y)$

$\frac{2x}{2} = \frac{1+3y}{2(2-y)}$

$x = \frac{1+3y}{4-2y}$

$\Rightarrow y = \frac{1+3x}{4-2x}$

$$\boxed{f^{-1}(x) = \frac{1+3x}{4-2x}}$$

$f(x) = x + 1$ find $f^{-1}(2) \to y$

$f(f^{-1}(2)) = f(y)$

$2 = f(y)$

$f^{-1}(2) = 1$

$(1, 2)$

Ex (26) $f(x) = \frac{1-e^{-x}}{1+e^{-x}}$

Homework

Sol: $y = \frac{1-e^{-x}}{1+e^{-x}}$

$y + ye^{-x} = 1 - e^{-x}$

$-1 + y = -e^{-x}(y+1)$

$\frac{-e^{-x}}{-1} = \frac{-1+y}{-1(y+1)}$

$e^{-x} = \frac{1-y}{y+1}$

$\ln e^{-x} = \ln\left(\frac{1-y}{y+1}\right)$

$x = -\ln\left(\frac{1-y}{y+1}\right) \Rightarrow$

$\Leftarrow$ نعوضها $f^{-1}(x) = \ln\left(\frac{1+x}{1-x}\right)$

$y = -\ln\left(\frac{1-x}{x+1}\right)$

$f^{-1}(x) = -\ln\left(\frac{1-x}{x+1}\right)$

ln الـ

D R نوتز

$\boxed{50}$. $f(x) = \ln(x-1) - 1$, find $D(f)$, range(f) the sketch

Sol: $x - 1 > 0$

$x > 1 \Rightarrow D(f) = (1, \infty)$.

$R(f) = \mathbb{R}$

$\boxed{57}$ B find $D(f)$, $f(x) = \ln(e^x - 3)$

$e^x - 3 \neq 0$

$e^x = 3$

$\ln e^x = \ln 3$

$x = \ln 3$

--- +++ (at $\ln 3$)

$D(f) = (\ln 3, \infty)$

$\boxed{52}$ (b) $e^{2x} - 3e^x + 2 = 0$

$y^2 - 3y + 2 = 0$ $\quad y = e^x$

$(y-1)(y-2) = 0$

$\boxed{y = 1}$ or $\boxed{y = 2}$

$\ln e^x = \ln 1$ $\quad \ln e^x = \ln 2$

$\boxed{x = \text{zero}}$ or $\boxed{x = \ln 2}$

B

$\frac{1 \pm \sqrt{1+4e}}{2}$

$\boxed{53}$ (b) Solve

$\ln x + \ln(x-1) = 1$

$\ln x(x-1) = 1$

$\ln(x^2 - x) = e^1$

$x^2 - x = e$

$x^2 - x - e = 0$

ركز أكمل بالمسألة

No.

56 (a) solve the inequality

$$1 < e^{3x-1} < 2$$

Sol $\ln 1 < \cancel{\ln e}^{3x-1} < \ln 2$

$$\text{zero} < 3x-1 < \ln 2$$

$$1 < 3x < \ln 2 + 1$$

$$\frac{1}{3} < x < \frac{\ln 2+1}{3}$$

$$D(f) = \left(\frac{1}{3}, \frac{\ln 2+1}{3}\right)$$

63: Find (a) $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$, (b) $\cos^{-1}(-1) = \pi$

64: $\tan^{-1}\sqrt{3} = \frac{\pi}{3}$, $\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$

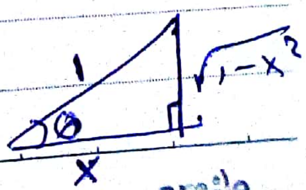
$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$, $\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \frac{-\pi}{4}$

$\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$, $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \pi - \frac{\pi}{4}$

لأن الزاوية بالربع الثاني تكون

72 Simplify $\sin(2 \arccos x)$

$$\theta = \cos^{-1} x \Rightarrow \cos\theta = \frac{x}{1}$$



$$= \boxed{\frac{2\sqrt{1-x^2}}{1}}$$

$$1 = x^2 + \alpha^2$$

$$\sqrt{1-x^2} = \sqrt{\alpha^2}$$

$$\alpha = \sqrt{1-x^2}$$

Homework

$$\cos\left(2\sin^{-1}\left(\frac{5}{13}\right)\right)$$

CHAPTER #2

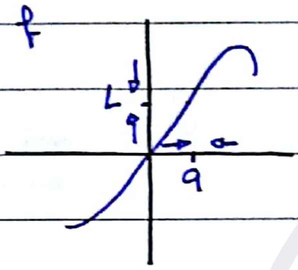
Limits and Derivates from § 2.1 $\xrightarrow{\text{to}}$ § 2.3

- $\lim\limits_{x \to a} f(x) = L$

النهاية تبحث فقط في جوار رقم
ولا تهتم لصورة رقم

x goes to a

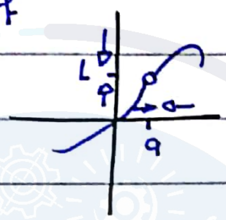
① f



$\lim\limits_{x \to a} f = L$

$a \in D_f$

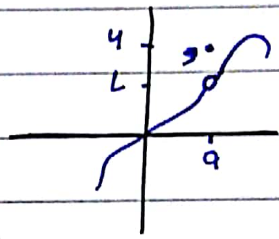
② f



$\lim\limits_{x \to a} f = L$

But $a \notin D_f$

③ f

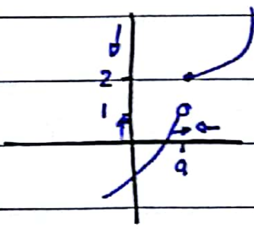


$\lim\limits_{x \to a} f = L$

$f(a) = 4$

1, 2, 3 Limits is exist

④ f

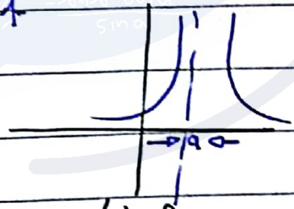


$\lim\limits_{x \to a^+} f = 2$

$\lim\limits_{x \to a^-} f = 1$

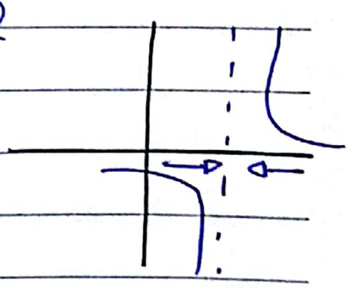
$\lim\limits_{x \to a} f$ = doesnot exist because right Lim ≠ Lift Lim

⑤ f



$\lim\limits_{x \to a} f = \infty$

⑥ f



a

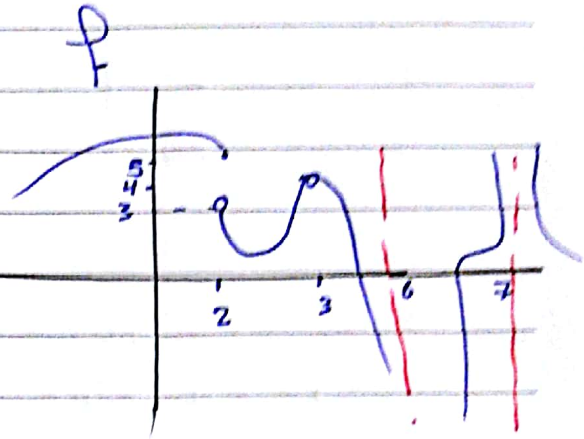
$\lim\limits_{x \to a^+} f = \infty$

$\lim\limits_{x \to a^-} f = -\infty$

$\lim\limits_{x \to a} f$ = doesnot exist

* Limits :-

Ex.



* From the graph f find the following

① $f(2) = 5$

② $\lim\limits_{x \to 2^-} f = 5$

③ $\lim\limits_{x \to 2^+} f = 3$

④ $\lim\limits_{x \to 2} f =$ does not exist

⑤ $f(3) =$ undifined

⑥ $\lim\limits_{x \to 3} f = 4$

⑦ $\lim\limits_{x \to 6^-} f = -\infty$

⑧ $\lim\limits_{x \to 6^+} f = -\infty$

⑨ $\lim\limits_{x \to 6} f = -\infty$

⑩ $\lim\limits_{x \to 7} f = \infty$

⑪ $f(7) = 2$

⑫ $f(6) =$ undifined

---

① $\lim\limits_{x \to 3} \frac{1}{x} = \frac{1}{3}$

② $\lim\limits_{x \to 0^+} \frac{1}{x} = \frac{1}{0^+} = +\infty$

ملاحظة لانه يمين الصفر ولا يساوي الصفر لكنه يقترب من الصفر

③ $\lim\limits_{x \to 0^-} \frac{1}{x} = \frac{1}{0^-} = -\infty$

" So in limits $= \frac{1}{0^\pm} = \pm\infty$ "

④ $\lim\limits_{x \to \infty} \frac{1}{x} = \frac{1}{\infty} = 0$

⑤ $\lim\limits_{x \to \infty^-} \frac{1}{x} = \frac{1}{\infty^-} = -0 = 0$ " So in limits $\frac{1}{\pm\infty} =$ Zero "

Ex. $f(x) = \dfrac{x^2-9}{x-3}$

1. $\lim\limits_{x\to1} \dfrac{x^2-9}{x-3} = \dfrac{1-9}{1-3} = \dfrac{-8}{-2} = 4$

2. $\lim\limits_{x\to3} \dfrac{x^2-9}{x-3} = \lim\limits_{x\to3} \dfrac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = 6$

$L_D = \dfrac{\text{zerooo}}{\text{zerooo}}$!! indeterminates

3. $f(3) =$ لا يوجد

ليست من اختصاصنا حاليا
وهي صفر مقام وتختلف عن النهاية

Ex. $f(x) = \begin{cases} x^2 , & x \geq 0 \\ x-2 , & x < 0 \end{cases}$

(a)

- $f(0) = 0$
- $f(2) = 2^2 = 4$
- $f(-1) = -1-2 = -3$

(b)

- $\lim\limits_{x\to3} f(x) = \lim\limits_{x\to3} x^2 = 9$
- $\lim\limits_{x\to-4} f(x) = \lim\limits_{x\to-4} x-2 = -6$

- $\lim\limits_{x\to0} f(x)$

$\lim\limits_{x\to0^+} x^2 = 0^2 = 0^+$

$\lim\limits_{x\to0^-} x-2 = 0-2 = -2$

$\neq$ so the limits does not exist

Ex. $f(x) = \begin{cases} \dfrac{x^3+8}{x+2} , & x \neq -2 \\ 1 , & x = -2 \end{cases}$

find

- $f(1) = -2$

- $\lim\limits_{x\to-2} f(x) \quad \lim\limits_{x\to-2} \dfrac{x^3+8}{x+2} = \dfrac{\text{zero}}{\text{zero}}$!!

$L_D \quad \lim\limits_{x\to-2} \dfrac{\cancel{(x+2)}(x^2-2x+4)}{\cancel{(x+2)}} = 4+4+4 = 12$

* Indeterminate forms :- الكميات الغير محددة
غير معينة

\# $\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$, $0 \cdot \infty$, 1^{∞} , 0^{0} , ∞^{0}

* $\frac{\infty}{0} = \infty$ * $\infty + \infty = \infty$

* $\frac{0}{\infty} = 0$ * $-\infty - \infty = -(\infty + \infty) = -\infty$

* $\frac{2}{\infty} = 0$ * $-\infty + \infty = \infty - \infty$!!

* $\frac{\frac{2}{\infty}}{0} = \infty$ * $\infty - \infty = ??$

$$\lim_{x \to \infty} 1^{x} = 1^{\infty} \text{ !!}$$ واحد اي رقم = $\boxed{1}$

$$= \lim_{x \to \infty} 1 = 1$$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x} = 1^{x} = 1^{\infty}$$

Ex. $f(x) = \begin{cases} 1 + \sin x , & x < 0 \\ \cos x , & 0 \leq x \leq \pi \\ \sin x , & x > \pi \end{cases}$

for what values of (a) $\lim_{x \to a} f(x)$ is exist ??

$\lim_{x \to \pi} f$ — $+$: $\lim \sin\pi = 0$; $-$: $\lim \cos\pi = -1$ → doesn't exist

نختبر نقاط التشعيب عند $a \leq 0$ ✓

عند $0 \leq a < \pi$ ✓

عند $a > \pi$ ✓

* عند تعويض العدد من اليمين واليسار يطلع الجواب (1) وهذا يعني ان النهاية موجودة عند (a) تقترب من الصفر ولكن عند الاختبار عند اقتراب ال (a) من π اليمين لا تساوي اليسار لذلك هي غير موجودة

so its exist for all

$a \in \mathbb{R}$ $a \neq \pi$

$D = \mathbb{R} - \{\pi\}$

Ex. $\lim_{x \to -3} \frac{x^2-9}{2x^2+7x+3} = \frac{zero}{zero}$!!

$\lim_{x \to -3} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}(2x+1)} = \lim_{x \to -3} \frac{x-3}{2x+1} = \frac{-6}{-5} = \frac{6}{5}$

OR

L.H $\lim_{x \to -3} \frac{2x}{4x+7} = \frac{-6}{-5} = \frac{6}{5}$

* قاعدة لوبيتال :- نشتق البسط و نشتق المقام و نعوض
~~نهاية~~ النهاية ولكن ((ليست لكل النهايات))

* Find the answer of following Limits:-

① $\lim_{t \to 2} \frac{\sqrt{4t+1} - 3}{t-2} = \frac{0}{0}$!!

نضرب بالمرافق

$\lim_{t \to 2} \frac{\sqrt{4t+1} - 3}{t-2} \times \frac{\sqrt{4t+1} + 3}{\sqrt{4t+1} + 3}$

$= \lim_{t \to 2} \frac{\overset{4}{\cancel{4t+1-9}}}{\cancel{t-2}} \times \frac{1}{\sqrt{4t+1}+3}$ $\quad \lim_{t \to 2} \frac{4}{\sqrt{4t+1}+3} = \frac{4}{6} = \frac{2}{3}$

② $\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x-3} = \frac{0}{0}$!!

توحيد المقامات

$\lim_{x \to 3} \frac{\overset{-1}{\cancel{3-x}}}{3x(\cancel{x-3})}$

$\lim_{x \to 3} \frac{-1}{3x} = \frac{-1}{9}$

③ $\lim_{x \to -6} \frac{2x+12}{|x+6|} = \frac{0}{0}$!! $\quad \xleftarrow{-(x+6)} \; -6 \; \xrightarrow{x+6}$

$\lim_{x \to -6^+} \frac{2\cancel{(x+6)}}{\cancel{(x+6)}} = 2$

$\lim_{x \to -6^-} \frac{2\cancel{(x+6)}}{-\cancel{(x+6)}} = -2$

$\lim_{x \to -6} \frac{2x+12}{|x+6|}$ = does not exist

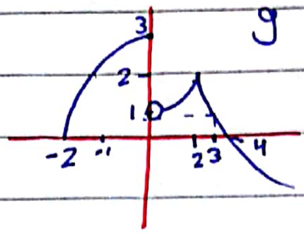
④ $\lim_{x \to -6} \frac{2x-12}{|x+6|} = \frac{-24}{0} = -\infty$

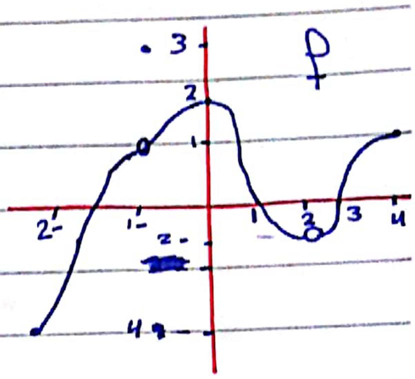
* rules of Limits :-

- $\lim_{x \to a} (f+g) = \lim_{x \to a} f + \lim_{x \to a} g$

- $\lim_{x \to a} \left(\frac{f}{g}\right) = \lim_{x \to a} f \div \lim_{x \to a} g$ $\quad$ وكذلك الضرب

Ex. Find the Limits from the following graph:-





① $\lim_{x \to 2} (f+g) = \lim_{x \to 2} f + \lim_{x \to 2} g$

$= -2 + 2 = \text{zero}$

② $\lim_{x \to 3} \frac{f}{g} = \frac{0}{\frac{1}{2}} = \text{zero}$

③ $\lim_{x \to 3} \frac{g}{f} = \frac{\frac{1}{2}}{0} = \infty$

④ $\lim_{x \to 2} x^2 . f(x) = \lim_{x \to 2} x^2 \times \lim_{x \to 2} f$

$= 4 \times -2 = -8$

⑤ $f(-1) + \lim_{x \to -1} g(x) = 3 + 2 = 5$

⑥ $\lim_{x \to 0^+} (f+g) = 2 + 1 = 3$

⑦ $\lim_{x \to 0^-} (f+g) = 2 + 3 = 5$

→ does not exist

⑧ $\lim_{x \to -1} (f.g) = 1 . 2 = 2$

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Ex. Find the following limits :-

① $\lim\limits_{x \to 0} \sin \frac{1}{x} = \sin \frac{1}{0} = \sin \infty$ ((it has many answer))

does not exist

because $-1 \leq \sin x \leq 1$

② $\lim\limits_{x \to 0} x^2 \sin \frac{1}{x} = 0 \cdot \sin \infty$ does not exist !!

** The squeeze theorem "sandwich theorem"

if $f(x) \leq g(x) \leq h(x)$ and $\lim\limits_{x \to a} f(x) = \lim\limits_{x \to a} h(x) = L$

Then $\lim\limits_{x \to a} g(x) = L$

$\lim\limits_{x \to 0} x^2 \cdot \sin \frac{1}{x}$

شرط الجيب $\quad -1 \leq \sin \frac{1}{x} \leq 1$

$\cdot x^2 \downarrow \quad \times x^2 \downarrow \quad \downarrow \cdot x^2$

نضرب كما هو مبين بالنهاية $\quad -x^2 \leq x^2 \sin \frac{1}{x} \leq x^2$

$x \to 0 \qquad\qquad x \to 0$

$= 0 \qquad\qquad = 0$ $\Rightarrow$ بما انهم متساويات
شي هذا محصورة لذا

$\boxed{0}$

by squeeze theorem

③ $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$\lim_{x \to 2} \frac{\sin|x-2|}{x-2} = 1$

* $\lim_{x \to \infty} \frac{\sin x}{x} = \frac{\sin \infty}{\infty} = \frac{\text{does not exist !!}}{\infty}$

$$-1 \leq \sin x \leq 1$$

$$\cdot \frac{1}{x} \qquad \cdot \frac{1}{x} \qquad \cdot \frac{1}{x}$$

$$\frac{-1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$$

$x \to \infty$ $\qquad$ $x \to \infty$

0 $\qquad$ so $\qquad$ 0

$= 0$

H.W find the following limits:-

① $\lim_{x \to 0} x^4 \cos \frac{2}{x}$

$$-1 \leq \cos \frac{2}{x} \leq 1$$

$$-x^4 \leq x^4 \cos \frac{2}{x} \leq x^4$$

$x \to 0$ $\qquad$ $x \to 0$

0 $\qquad$ 0

$= 0$

by squeeze Thm

The following

How find the limit of function $x \to 0$:- $\lim_{x \to 0} f$

$$1 - \frac{x^2}{2} \leq f \leq \cos x$$

The solution $\rightarrow$

$\lim_{x \to 0}$ $= \boxed{1}$ $\lim_{x \to 0}$ $= \boxed{1}$

so $= \boxed{1}$

* How to find the vertical and horisontal asymptotes:-

V. H. asy

① V. asy :

① اولاً "دجيب اصفار المقام ركي تساوي $(\mp\infty)$

② نختبر كل صفر مقام لوحده من طريق اخذ

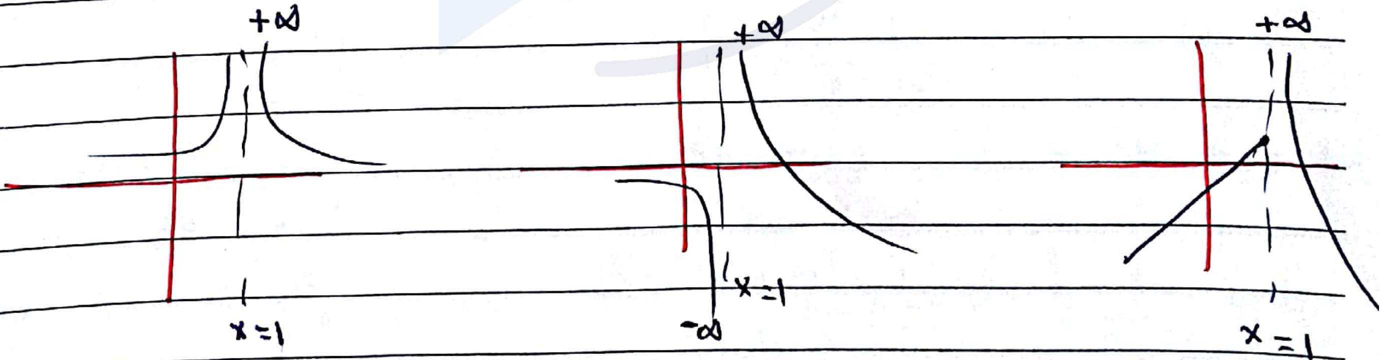
③ نهاية للرقم ((اي صفر المقام))

The Base $\lim_{x \to a^+} f = \mp\infty$

so $\rightarrow$ $x = a$ it is a V. asy

or

$\lim_{x \to a^-} f = \mp\infty \Rightarrow x = a$ it is a V. asy



it is a V. asy it is a V. asy it is a V. asy

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Ex. find the V. asy is it exist from the following :-

- $f(x) = \frac{x+1}{x^2-1}$

$x^2-1 = 1$

$\boxed{x = \pm 1}$ ← اصفار كمقام

① $\lim_{x \to 1} \frac{x+1}{x^2-1} = \frac{2}{0} = +\infty$

so $x=1$; it's a V. asy for f.

② $\lim_{x \to -1} \frac{x+1}{x^2-1} = \frac{0}{0}$!!

$\lim_{x \to -1} \frac{\cancel{(x+1)}}{\cancel{(x+1)}(x-1)}$

$\lim_{x \to -1} \frac{1}{x-1} = \frac{-1}{2} \neq \mp\infty$

so $x=-1$; it's not a V. asy

- $f(x) = \frac{x^2+1}{3x-2x^2}$

$3x - 2x^2 = 0$

$x(3-2x) = 0$

$x = 0$, $x = \frac{3}{2}$

① $\lim_{x \to 0} \frac{x^2+1}{3x-2x^2} = \frac{1}{0} = \mp\infty$

so $x=0$ (y-axis) it's a V. asy for f

② $\lim_{x \to \frac{3}{2}} \frac{x^2+1}{3x-2x^2} = \frac{\frac{9}{4}+1}{0} = \mp\infty$

so $x=\frac{3}{2}$ it's a V. asy for f

- $f(x) = \sin\left(\frac{1}{x}\right)$

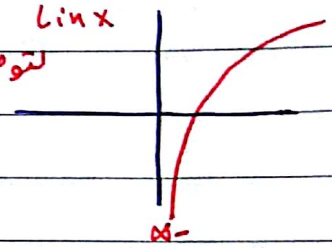
$\lim_{x \to 0} \sin\left(\frac{1}{x}\right) = \sin\left(\frac{1}{0}\right) = \sin\infty =$ does not exist

so $x = 0 \longrightarrow \neq \infty$ it's not a V. asy

* في اقتران ال ((Lin)) نأخذ الصفر ما داخل ال ((Lin)) ونساويها بالصفر

- $f(x) = \ln x$

Lin x
للتوضيح



since $\lim_{x \to 0^{+}} \ln x = -\infty$

So $x = 0$ (y-axis) is a V. asy

- $f(x) = \begin{cases} 2 & , x \geq 1 \\ \frac{1}{x-1} & , x < 1 \end{cases}$

* عند نقطة التشعيب ستكون احدى القاعدتين تساوي ((∞)) فقط نبحث من جهتها وبالنسبة لها ((نأخذ + او - بالنسبة للقاعدة التي تساوي ∞))
يمين يسار

$\lim_{x \to 1^{-}} \frac{1}{x-1} = \frac{1}{0}$ سالب $=$ سالب ∞

لأنه أخذنا من جهة اليسار
صح

So ; x = 1 is a V. asy

- $f(x) = \begin{cases} 3x & , x \geq 1 \\ \frac{1}{x-2} & , x < 1 \end{cases}$

* ٢ يقع صفر المقام ضمن الفترة لذلك لا يوجد ~~V. asy~~ V. asy
So it's not have a V. asy

يترا ظل
د ال
First * Horisontal asymptotes for the function:-

① $\lim\limits_{x \to \infty} f(x) = b < \infty$
↳ finite

$\Rightarrow$ $y = b$ is the H.asy

② $\lim\limits_{x \to -\infty} f(x) = C$ $\quad y = C$ is another H.asy

① أخذ النهاية للاقتران عندما x تقترب من +∞ ومرة عندما تقترب x من -∞

② إذا أعطى العنوان رقم Horisontal asymptotes فهو

Ex. find the Horisontal asymptotes for the following:-

- $f(x) = \dfrac{2}{x+1}$

ⓐ $\lim\limits_{x \to \infty} \dfrac{2}{x+1} = \dfrac{2}{\infty} = 0$

so $y = 0$ (x-axis) it's has a H.asy

ⓑ $\lim\limits_{x \to -\infty} \dfrac{2}{x+1} = \dfrac{2}{-\infty+1} = 0$

so $y = 0$ (x-axis) it's has a H.asy

- $f(x) = \tan^{-1} x$

ⓐ $\lim\limits_{x \to \infty} \tan^{-1} x = \tan^{-1} \infty = \dfrac{\pi}{2}$

there for $y = \dfrac{\pi}{2}$ is a H.asy

ⓑ $\lim\limits_{x \to -\infty} \tan^{-1} x = \tan^{-1} -\infty = -\dfrac{\pi}{2}$

There for $y = -\dfrac{\pi}{2}$ is another H.asy

Ex. find

- $\lim_{x \to 100} x^4 - 2x + 3$

$= (100)^4 - 2(100) + 3 =$ رقم

* قاعدة :- فقط لكثيرات الحدود عندما تقترب النهاية لـ $(-\infty)$ و $(+\infty)$ نحذف كل الحدود ونبقي الحد الذي له اكبر قوة وهو الذي سيحدد إشارة النهاية

- $\lim_{x \to \infty} x^4 - 2x + 3$

تم الحذف $\rightarrow \lim_{x \to \infty} x^4 = \infty$

- $\lim_{x \to \infty} -3x^4 + 2x^2 + 10x + 1$

تم الحذف $\rightarrow \lim_{x \to \infty} -3x^4 = -\infty$

مهم - $\lim_{x \to \infty} \dfrac{-10x^5 + 3x^2 + 1}{2x^5 - x^2 + 10x + 3}$

بالاقتران النسبي نعاملها عن القاعدة ولكن البسط عن حدا والمقام عن حدا

$\lim_{x \to \infty} \dfrac{-10\cancel{x^5}}{2\cancel{x^5}} = -5$ so it's a H. asy

- $\lim_{x \to \infty} \dfrac{3x^4 + 10x + 1}{-2x + 5}$

$\lim_{x \to \infty} \dfrac{3x^{\cancel{4}\,3}}{-2\cancel{x}} = -\infty$

* إذا كانت درجة البسط اقل من درجة المقام لذا الجواب سيكون صفراً في هذه الحالة

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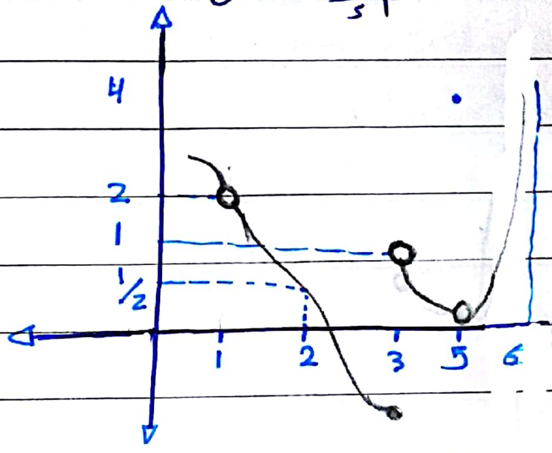
Calculus Limits

§2.5 Continuity

الإتصال



find :-

① $f(1)$ = undefined
- $\lim_{x \to 1} f = 2$

$\rightarrow$ f is not Continous

② - $f(2) = 1/2$
- $\lim_{x \to 2} f = 1/2$

$\rightarrow$ f is Continous

③ - $\lim_{x \to 3} f$ = does not exist
+ $\lim_{x \to 3^+} f = 1$
- $\lim_{x \to 3^-} f = -1$
- $f(3) = -1$

$\rightarrow$ f is not Continous

④ - $f(5) = 1$
$\lim_{x \to 5} f = 1/2$

$\rightarrow$ ~~f is Continous~~
f is not Continous

⑤ $f(6) = 3$
$\lim_{x \to 6^-} f = +\infty$

$\rightarrow$ f is not Continous

* Definition:-

if f is continous at $x = a$
① f is defined at a , $a \in D_f$
② $\lim_{x \to a} f$ $\rightarrow$ exist
③ $\lim_{x \to a} f = f(a)$

* The removable discontinuity :- at $x=a$

if

* $\lim_{x \to a} f$ exist but either f is not difined at a or $\lim_{x \to a} f \neq f(a)$

* النهاية موجودة ولكنها لا تساوي الصورة يسمى الانقطاع القابل للإزالة

* Un removable discontinuity :-

① jump ; if right Limit $\neq$ left limit

* النهاية من اليمين لا تساوي النهاية من اليسار تسمى قفزة , Jump

② infinite ; if $\lim_{x \to a} f = \mp \infty$ غير موجود

* النهاية تساوي $\pm\infty$ تسمى

Example ① $f(x) = \begin{cases} \cos x & , x < 0 \\ 0 & , x = 0 \\ 1 - x^2 & , x > 0 \end{cases}$

* Is f continous at $x=0$??

* نختبر النهاية من اليمين والنهاية من اليسار وصورة الرقم إذا تساوت فالنهاية موجودة

$\lim_{x \to 0^+} 1 - x^2 = 1$

$\lim_{x \to 0^-} \cos x = 1$

The Limit is exist ;

$f(0) = \text{Zero}$

but ; the Limit $\neq f(0)$

So f is not continous at $x=0$

and f is removable discontionity at $x=0$ since $\lim_{x \to 0} f = 1$

نوع عدم الاتصال

Example

$$f(x) = \begin{cases} x^2+1, & x \le 0 \\ x+1, & x > 0 \end{cases}$$

f is cont. at $x=0$??

$\Rightarrow$ $\lim_{x \to 0^+} x+1 = 1$

$\lim_{x \to 0^-} x^2+1 = 1$

$f(0) = 1$

So f is continuous at $x=0$ and f is continuous everywhere.

imp **Example**

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2}, & x < 2 \\ ax^2 - bx + 3, & 2 \le x < 3 \\ 2x - a + b, & x \ge 3 \end{cases}$$

* Find a and b so that f is cont. everywhere?

① f is continuous at $x=2$

so $\Rightarrow$ $\lim_{x \to 2^-} \dfrac{x^2-4}{x-2} = f(2)$

$\lim_{x \to 2^-} x+2 = a4 - 2b + 3$

$4 = a4 - 2b + 3$ —— ①

② f is continous at $x=3$

*So that:

$$\lim_{x \to 3^+} 2x - a + b = \lim_{x \to 3^-} ax^2 - bx + 3$$

$$\cancel{6} \cancel{-a} \cancel{+b} = 9a - 3b + 3$$
$$\cancel{-6} \cancel{+a} \cancel{-b} \quad +a \quad -b \quad -6$$

$$10a - 4b + -3 = 0 \quad \text{———} ②$$

بالحذف : from equ. ① $(4a - 2b + 3 = 4) \times -2$

$$-8a + \cancel{4b} - 6 = -8$$

$$2a \cancel{-9} = -8$$
$$\cancel{+9} \quad +9$$

$$\boxed{a = \tfrac{1}{2}}$$

بالتعويض : from equ. ① $\overset{2}{\cancel{4}} \times \frac{1}{\cancel{2}} - 2b + 3 = 4$

$$5 - 2b = 4$$
$$-5 \qquad -5$$

so $\boxed{b = \tfrac{1}{2}}$

Ex.

$$f(x) = \begin{cases} x - 1, & x \leq 1 \\ c(x-1)^2, & x > 1 \end{cases}$$

find $\underline{c}$ if f is continous every where??

$$\rightarrow \lim_{x \to 1^+} c(x-1)^2 = \lim_{x \to 1^-} (x-1)$$

$$c.(1-1) = 1-1$$

$$c.0 = 0$$

$$0 = 0$$

So $c \in \mathbb{R}$

بما انه اقتران متصل عند كل الأعداد

Ex. find the discontinuity for f and Classify them are removable, Jump or Vertical asymptotes for the following equasions??

① $f(x) = \dfrac{|x-2|}{x^2 + x - 6}$

* نقطة الإنفصال من نوع لـ infinite.
* نطلع اصفار المقام لانه غير معرف على اصفار المقام.

$f(x) = \dfrac{|x-2|}{(x-2)\ (x+3)}$

so $x = 2$, $x = -3$ are discontinuity Points

* نأخذ النهاية لنقاط عدم الإتصال.

a) $x = 2$; $\lim\limits_{x \to 2} \dfrac{x-2}{x^2 + x - 6} = \dfrac{0}{0}$!!

$\lim\limits_{x \to 2} \dfrac{\cancel{(x-2)}}{\cancel{(x-2)}\,(x+3)} = \dfrac{1}{5}$ so its exist

b) $x = -3$; $\lim\limits_{x \to -3} \dfrac{x-2}{x^2 + x - 6} = \dfrac{1}{0} = \infty$

So it's a Vertical asymptote

② $f(x) = \dfrac{2 + x}{2 - |x|}$

اصفار كمقام $2 - |x| = 0$

$|x| = 2$ so $\boxed{x = \pm 2}$ Points of discontinuity

a) $\lim\limits_{x \to 2} = \dfrac{2 + x}{2 - |x|} = \dfrac{4}{0} = \infty$

so $x = 2$ is a Vertical asymptotes Point

b) $\lim_{x \to -2} \frac{2+x}{2-|x|} = \frac{0}{0}$!!

$-x \quad x$

$\longleftarrow - \; - \; - \; + \; + \; + \longrightarrow$

$x \quad x = \frac{0}{0}$

$\lim_{x \to -2} \frac{2+x}{2--x} = \lim_{x \to -2} \frac{2+x}{2+x} = \boxed{1}$

So its exist and its Removable at $\boxed{x = -2}$

③ $f(x) = \begin{cases} x+1 , & x \geq 2 \\ 2x , & x < 2 \end{cases}$

$\lim_{x \to 2} f \;\; \begin{cases} \xrightarrow{+} \lim_{x \to 2^+} x+1 = 3 \\ \xrightarrow{-} \lim_{x \to 2^-} 2x = 4 \end{cases}$ $\neq$ does not exist

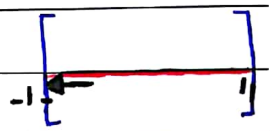
$\therefore$ So its not Continuous and it's has a form of Jump discontinuity at $x = 2$

Ex. Show f is continous on [-1, +1] :-

$f(x) = \sqrt{1 - x^2}$

1) f is continoues on $(-1, 1) \subseteq$ domain

لم نغطي جزء ((Subset))



* نبحث من جهة طرفي الفترة

2) f is continoues from Right at $x = -1$

$$\lim_{x \to -1^+} f = f(-1)$$

$$1 = 1$$

3) at $x = 1$; continous from the left

$$\lim_{x \to 1^-} f = f(1)$$

$$1 = 1$$

* Therom 1 :- شرح النظرية

$$\lim_{x \to a} g(x) = L$$

and f is Continous at L. شروط النظرية

then $\lim_{x \to a} f(g(x))$ exist

and $$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right)$$

but $= L$

$f(L)$

*Examples :-

① find the Limits as

$$\lim_{x \to 1} \underbrace{\arcsin}_{\sin^{-1}(\sim)}\left(\frac{1-\sqrt{x}}{1-x}\right)$$

$$\Rightarrow \; = \sin^{-1}\left(\frac{0}{0}\right)$$

So $$\lim_{x \to 1} \frac{1-\sqrt{x}}{1-x} = \lim_{x \to 1} \frac{\cancel{(1-\sqrt{x})}}{\cancel{(1-\sqrt{x})}(1+\sqrt{x})}$$

$$\lim_{x \to 1} \frac{1}{1+\sqrt{x}} = \boxed{1/2} \rightarrow \sin^{-1}\left(\tfrac{0}{0}\right)$$ بنعوضها في

$$\Rightarrow \sin^{-1}(1/2) = \frac{\pi}{6}$$

② $g(2)=3$, $\lim_{x \to 1} g(x) = 4$; if you know

* that f is continous everywhere, and $f(4)=5$

$f(3)=7$ then find:-

** $\lim_{x \to 1} f(g(x))$??

احد شروط النظرية $f\left(\lim_{x \to 1} g(x)\right) = f(4) = \boxed{5}$

* ليس شرطاً ان نستخدم جميع المعطيات في السؤال قد تكون فقط للتمويه.

" ——— " ——— " ——— " ——— " ——— " ——— " ——— "

* Therom 2 :- "Intermediate Value therom"

(I . V . T) الإختصار

→ Let f be continous on $[a,b]$

and let K between $f(a)$ and $f(b)$

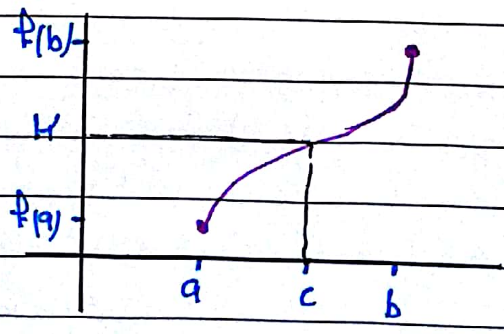
then there is ((at least)) the number $c \in [a,b]$ ثم النتيجه

So $f(c) = K$

* إذا الإقتران f متصل على $[a,b]$ وكان العدد K بين صورتي

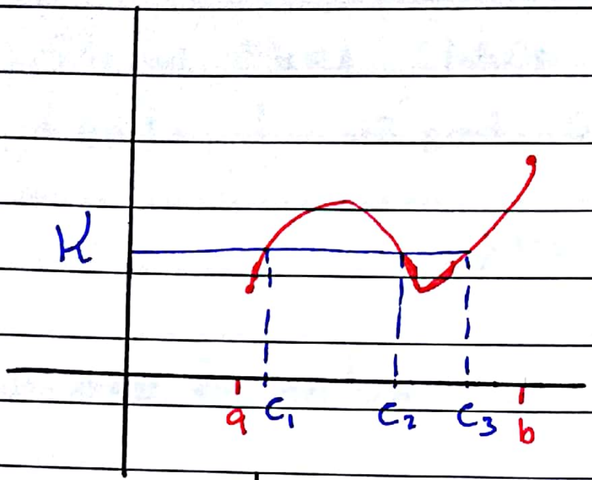
ال a و ال b إذن هنالك حل واحد على الأقل على فرض هو c

يكون محصور بين $[a,b]$ لذلك $f(c)=K$

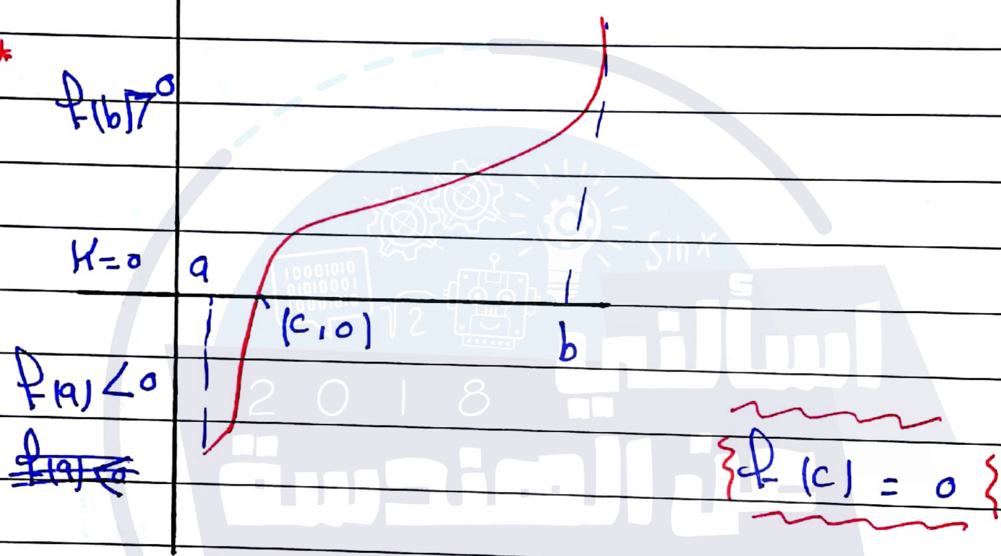


لتوضيح على الرسم البياني

لكل صورة تقع بين الصورتين $f(a)$ و $f(b)$ لها صورة على محور السينات وتكون بين صورتي ال a و b



*ملاحظة مهمة



$f(a) \cdot f(b) < 0$

*then $f(x) = 0$ has at least 1 real solution.

Ex. Show that this equasion has at least 1 real solution :- $x^4 + x = 3$

① اجعل طرف المعادلة بالصفر (ساويها بالصفر)
② نسمي العلاقة كاقتران as a function
③ نأخذ أرقام وأولها ~~الصفر~~ الصفر ثم 1 ونأخذ حتى ينتج معنا احد الأجوبة سالب والآخر موجب.

الحل ⇒ ① $x^4 + x - 3 = 0$

②

Let $f(x) = x^4 + x - 3$

at $x = 0$ $f(0) = -3 < 0$

at $x = 1$ $f(1) = 1 + 1 - 3 = -1 < 0$

at $x = 2$ $f(2) = 16 + 1 - 3 = 14 > 0$

and f is continuous

So $f(1) \cdot f(2) < 0$

by the (I.V.T) there is at least $c \in (1,2)$

so $f(c) = 0$ has at least one real root or solution

Ex. show that ~~the~~ this equasion $\cos x = x^3$ at least $\underline{\underline{1}}$ real root in the $(0, \pi)$??

* على حسب "real solution" نأخذ حلين اثنان سالب واثنان موجب

① $\cos x = x^3 \longrightarrow \cos(x) - x^3 = 0$

$D_f = \mathbb{R}$ $D_f = \mathbb{R}$

② $f(x) = \cos(x) - x^3$ on $[0, \pi]$

So f is continous لأنها من مجال الاقتران

تابع $\Rightarrow$ $f(0) = 1 - 0 = \boxed{1} > 0$

$f(\pi) = -1 - \pi^3 = \boxed{-1-\pi^3} < 0$

So by the (I.V.T) there is (c) that $f(c) = 0$

that is $\cos x = x^3$ has at least 1 Real solution.

Ex. if $f(a) \cdot f(b) < 0$ and f is continuous on $[a, b]$, How many rules are there for the function f in (a, b) ??

~~a)~~ at least 1 — أكيد في حل
b) at most 1 — على الأكثر حل
c) exactly 1 — حل واحد

ذاكر $\Rightarrow$ § 2.5

V. asy مطلوب — الامتحان
H. asy غير مطلوب

Five Apple

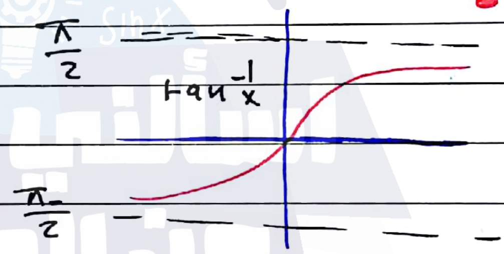
§ 2.6 Horizontal asymptotes and infinite Limits

* Examples:-

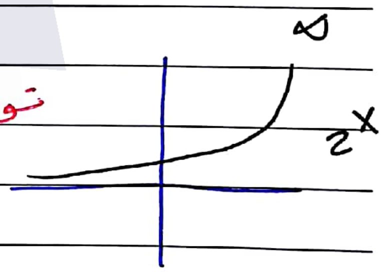
a: find the Limit of the following exercises:-

1- $\lim\limits_{x \to 2^+} \arctan\left(\frac{1}{x-2}\right)$

$\Rightarrow \tan^{-1} \frac{1}{0^+} = \tan^{-1} \infty = \frac{\pi}{2}$ لتذكير



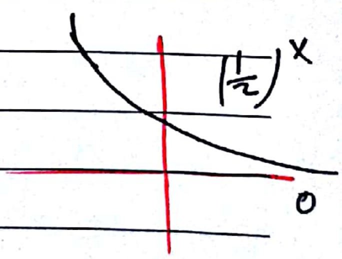
2. $\lim\limits_{x \to \infty} e^x = e^\infty = \infty$ توضيح



3. $\lim\limits_{x \to \infty} a^x \begin{cases} \infty , & a > 1 \\ 0 , & 1 > a > 0 \end{cases}$

$\lim\limits_{x \to \infty} (1/2)^x = (1/2)^\infty =$ zeroo توضيح

$= \frac{1}{2^\infty} = \frac{1}{\infty} = 0$



4- $\lim\limits_{x \to \infty} e^{-x} = e^{-\infty} = \frac{1}{e^{\infty}} =$ zero

So $e^{-\infty} = 0$

5- $\lim\limits_{x \to \infty} \sin x = \sin \infty$ Has many Answers

So the limit does not exist

6- $\lim\limits_{x \to \infty} -3x^5 + 2x^2 - 1 = -\infty$

اكبر قوة هي اللي تحدد الإشارة

ما في (H. asy)

7- $\lim\limits_{x \to -\infty} -3x^5 + 2x^2 - 1 = -(-\infty) = +\infty$

8- $\lim\limits_{x \to -\infty} \sqrt{x^2+1} - x = \infty - (-\infty) = \infty + \infty$

$= \infty$

9- imP

خلصنا كلّي بعد هالصفحة

C=

Be happy

9- $\lim\limits_{x\to\infty} \sqrt{x^2+1} - x = \infty - \infty$!!

الضرب بالمرافق

$\rightarrow \lim\limits_{x\to\infty} \left(\sqrt{x^2+1} - x\right) \times \left(\frac{\sqrt{x^2+1} + x}{\sqrt{x^2+1} + x}\right)$

$\lim\limits_{x\to\infty} \frac{x^2+1-x^2}{\sqrt{x^2+1}+x} \Rightarrow \lim\limits_{x\to\infty} \frac{1}{\sqrt{x^2+1}+x}$

$= \frac{1}{\infty} = 0$

" —— " —— " —— " —— " —— " —— " —— " —— "

Ex. Find the Horizontal asymptotes (1) and the vertical asymptotes (2) from the function:-

$f = \frac{\sqrt{4x^2+1}}{2x-5}$

H.asy (1) a) $\lim\limits_{x\to\infty} \frac{\sqrt{4x^2+1}}{2x-5}$ ناخد x^2 عامل مشترك $= \frac{|x|\sqrt{4+\frac{1}{x^2}}}{2x-5}$

$\rightarrow \lim\limits_{x\to\infty} \frac{{}^{*}\cancel{x}\sqrt{4+\frac{1}{x^2}}}{\cancel{x}\left(2-\frac{5}{x}\right)}$

* وطلعنا (x) لأنه تؤول إلى عدد موجب

$= \frac{\sqrt{4+\frac{1}{\infty}}}{2-\frac{5}{\infty}} = \frac{\sqrt{4}}{2} = \boxed{1}$

So $y=1$ is a H. asy

Five Apple

b) $$\lim_{x \to -\infty} \frac{\sqrt{4x^2+1}}{2x-5} =$$

$$\lim_{x \to -\infty} \frac{|x| \sqrt{4 + 1/x^2}}{x\left(2 - \frac{5}{x}\right)}$$

$$\lim_{x \to -\infty} \frac{-^{*}\cancel{x}}{\cancel{x}} \times \frac{\sqrt{4 + \frac{1}{x^2}}}{2 - \frac{5}{x}}$$

* وضعنا سالب لأن x تؤول لأي عدد سالب

$$= -1 \cdot \frac{\sqrt{4}}{2} = -1$$

So $y = -1$ its another Horizontal asymptotes

V. asy 2

عن طريق صفر المقام

$$2x - 5 = 0$$

$$x = \frac{5}{2}$$

$$\lim_{x \to \frac{5}{2}} \frac{\sqrt{4x^2+1}}{2x-5} = \frac{\sqrt{4\left(\frac{25}{4}\right)+1}}{5-5} = \frac{\sqrt{26}}{0} = \infty$$

So $x = \frac{5}{2}$ is a Vertical asymptotes.

§ 2.7, 2.8 Derivatives

* Defination: ① f is diffrentiable at $x = a$

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$ So its exist ((محدود، متغير))

$$= f'(a)$$

Ex. find the $f'(4)$ by using the defination:- $f(x) = x^2$

$$\lim_{x \to 4} \frac{f(x) - f(4)}{x - 4} = \lim_{x \to 4} \frac{\overbrace{\cancel{x^2 - 16}}^{(x+4)}}{\cancel{x - 4}}$$

$$\lim_{x \to 4} x + 4 = \boxed{8}$$

② $$f'(a) \overset{def}{=} \lim_{h \to 0} \frac{f(a+h) - f(a)}{a + h - a}$$

$$= \lim \frac{f(a+h) - f a}{h}$$

$$\to f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

* by ① $$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x}$$

Ex. find the $f'(x)$ by using definition for $f(x) = \sqrt{2x-1}$

$\Rightarrow f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x}$

الموافق

$= \lim_{z \to x} \frac{\sqrt{2z-1} - \sqrt{2x-1}}{z - x} \times \frac{\sqrt{2z-1} + \sqrt{2x-1}}{\sqrt{2z-1} + \sqrt{2x-1}}$

$= \lim_{z \to x} \frac{2z-1-(2x-1)}{z-x \cdot (\sqrt{2z-1} + \sqrt{2x-1})}$

$= \lim_{z \to x} \frac{\overset{2}{\cancel{2z - 2x}}}{\cancel{z - x} \cdot (\sqrt{2z-1} + \sqrt{2x-1})}$

$\lim_{z \to x} \frac{2}{\sqrt{2z-1} + \sqrt{2x-1}} = \frac{\cancel{2}}{\cancel{2}\sqrt{2x-1}}$

$\Rightarrow \frac{1}{\sqrt{2x-1}}$

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CHAPTER #3

§ 3.1 , 3.2 , 3.3

→ The Rules of Differential:- $f'(x) = \dfrac{df}{dx}$

	f	f'
1	$(f \mp g)'$	$f' \mp g'$
2	$(f \cdot g)'$	$f \cdot g' + g \cdot f'$
3	$\left(\dfrac{f}{g}\right)'$	$\dfrac{g f' - f g'}{g^2}$
4	$\left(\dfrac{1}{g}\right)'$	$\dfrac{g'}{g^2}$
5	$(c f)'$	$c f'$

as $c \equiv$ Constant

$f_{(x)}$	$f'_{(x)}$
* C	0
* $x^n \quad n \in \mathbb{R}$	$n\,x^{n-1}$
* $\sin x$	$\cos x$
* $\cos x$	$-\sin x$
* $\tan x$	$\sec^2 x$
* $\cot x$	$-\csc^2 x$
* $\sec x$	$\sec x \cdot \tan x$
* $\cos x$	$-\cos x \cdot \cot x$

"imp table"

$f_{(x)}$	$f'_{(x)}$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\cot^{-1} x$	$\frac{-1}{1+x^2}$

مسح لا يتحمل

$f_{(x)}$	$f'_{(x)}$
* e^x	e^x
* a^x	$a^x \ln a$
* $\ln x$	$\dfrac{1}{x}$
* $\ln(f_{(x)})$	$\dfrac{f'_{(x)}}{f_{(x)}}$
* $\log_a x$	$\dfrac{1}{x \ln a}$

find $\left(\log_3 x\right)' = \dfrac{1}{x \ln 3}$

Be the changer, make it difference and wish the good things to all People and help the needy..

This is the ...

أسألني عن هندسة

* find: $f(x) = \begin{cases} x \sin \frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

① is f differentiable at x=0 ?
② is f continuous at x=0 ?

* so we will use the def of $f'(0)$

① → $f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$

$f'(0) = \lim_{x \to 0} \frac{x \sin \frac{1}{x} - 0}{x - 0}$

$= \lim_{x \to 0} \sin \frac{1}{x}$

$= \sin \infty =$ has many answer
so its does not exisit
$f'(0) =$ undefind

* لأن النهاية غير موجودة إذن هو غير قابل للإشتقاق عند x = zero
So the function isn't differentiable at x=0

② → $\lim_{x \to 0} f(x) = \lim_{x \to 0} x \sin \frac{1}{x}$ تم النهاية بجوار الرقم ي تؤول اليه

Part 2. Squeez Theram $= 0.$ does not exist
$= 0$ since $\lim_{x \to 0}$
and $\sin \frac{1}{x}$ is bounded

* الإقترانات المحصورة بين Horizontal asymptotes تطابق نظرية Squeez Theram

ويجب أن يكون الإقترانات محدود لذا صفر x اقتران محدود = صفر

$= f(0)$

there for f is continous at $x=0$

Ex. $f(x) = \begin{cases} x^2 \sin \frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

a) it is differinable at $x=0$?

b) it is Countinuous at $x=0$?

$$f'(0) = \lim_{x \to 0} \frac{x^2 \sin \frac{1}{x} - 0}{x - 0}$$

$$= \lim_{x \to 0} \frac{x^{\cancel{2}} \sin \frac{1}{x}}{\cancel{x}}$$

$$= \lim_{x \to 0} \left(x \sin \frac{1}{x}\right)$$

$$= \underbrace{\lim_{x \to 0} x}_{0 = 0} \cdot \lim_{x \to 0} \sin \frac{1}{x}$$ ← bounded

$=$ Zero

So $Df = Df' = \mathbb{R}$

* Therom :- f is diffeniition at $x=0$
so f is continous at $x=0$

since f is diffenition at $x=0$ ⟹ اثبات
f is continous at $x=0$

الإثبات the metod :-
f is continous at $x=0$

$$\lim_{x\to 0} f = f(a)$$

$$\lim x^2 \sin\frac{1}{x} \overset{??}{=} 0$$

$0 =$ لأ ⟶ bounded

so $0 = 0$

المعطى
⟹ If f is diffenition at $x=a$
then f is continous at $x=a$

المطلوب
the Prove
f is continous at $x=a$

نريد أن نثبت أن $\lim_{x\to a} f = f(a)$? ⟹ $\lim_{x\to a} (f(x) - f(a)) = 0$

التوضيح ⟶ $f(x) - f(a) = f(x) - f(a)$

$$\lim_{x\to 2} f(x) = 3$$

$$\lim_{x\to 2} f(x) - 3 = 0$$

$$\lim_{x\to 2} (f(x) - 3) = 0$$

نأخذ النهاية

$$f(x) - f(a) = \frac{f(x) - f(a)}{x - a} \times \frac{x-a}{x-a}$$

$$\lim_{x\to a} f(x) - f(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a} \times \lim_{x\to a} x - a$$

$$\lim_{x\to a} (f(x) - f(a)) = f'(a) \cdot \text{Zerrrooo}$$

اهم خطوة في البرهان

$$\lim_{x\to a} f(x) - f(a) = 0$$

$$\text{So } \lim_{x\to a} f(x) = f(a) \quad \#$$

The Result differinition $\Longrightarrow$ Continous

الشرط A $\Longrightarrow$ B النتيجة

يكافئها

not B $\Longrightarrow$ not A

* نفي النتيجة تعطي نفي الشرط *

Not Continous $\Longrightarrow$ Not ~~differintiable~~ differinitable

Ex.

$$f(x) = \begin{cases} x^2 + 1 & , \; x \le 1 \\ 2x + 5 & , \; x > 1 \end{cases}$$

is f diffirentable at $x = 1$? and find f' ?

نبحث الإتصال → f Continuous at $x = 1$?

$\Rightarrow \lim_{x \to 1^+} 2x + 5 = \lim_{x \to 1^-} x^2 + 1$

$\Rightarrow 2 + 5 = 1 + 1$

$7 \neq 2$

so f ~~is~~ does not exist at $x = 1$ and f isn't differentiable at $x = 1$

$$f' = \begin{cases} 2x & , \; x < 1 \\ 2 & , \; x > 1 \end{cases}$$

* لا تأخذ المشتقة قبل البحث في الإتصال

Ex. $f(x) = \begin{cases} x^2 & , x \leq 0 \\ 2x & , x > 0 \end{cases}$

is f differinan at x=0 ?

$\rightarrow \lim_{x \to 0^-} x^2 = \lim_{x \to 0^+} 2x$

$0 = 0$ So it's Continous at x=0

$f'(x) = \begin{cases} 2x & , x < 0 \\ 2 & , x > 0 \end{cases}$

بدون مساواة عند الإشتقاق

$f'_+(0) \stackrel{?}{=} f'_-(0)$

$2 \neq 0$ So it's not differntiable at x=0

Ex. $f(x) = \begin{cases} x^2+1 & , x \leq 1 \\ 2x & , x > 1 \end{cases}$ is f differntiable at x=0?

$\lim_{x \to 1^-} x^2+1 = \lim_{x \to 1^+} 2x$

$2 = 2$ So its Continous at x=1

$f'(x) = \begin{cases} 2x & , x < 1 \\ 2 & , x > 1 \end{cases}$

Thu. 15,11,018

Ex find $f'(x)$:-

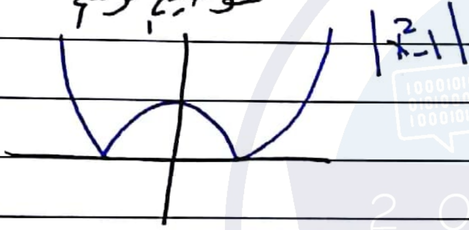
$$f(x) = |x^2 - 1|$$

$$x^2 - 1 = 0$$

$$\boxed{x = \pm 1}$$

$$f(x) = \begin{cases} x^2 - 1 , & x \geq 1 \text{ or } x \leq -1 \\ 1 - x^2 , & 1 > x > -1 \end{cases}$$

لتوضيح الرسم



$$f'(x) = \begin{cases} 2x , & x > 1 , x < 1 \\ -2x , & 1 > x > -1 \end{cases}$$

$$\left.\begin{array}{l} f'_+(1) = 2 \\ \qquad\qquad \neq \\ f'_-(1) = -2 \end{array}\right\} \to \text{so } f'(1) \text{ does not exist}$$

$$\left.\begin{array}{l} f'_+(-1) = 2 \\ \qquad\qquad \neq \\ f'_-(-1) = -2 \end{array}\right\} \to \text{so } f'(-1) \text{ does not exist}$$

* نقاط عدم الإشتقاق هي -1 و 1 ، لا ننسى مساواة كل الأرقام عند كتابة مشتقة الإقتران المتشعب

Five Apple

* The Higer derivatives

- $y = f(x)$
- $y' = f'(x) = \dfrac{df}{dx}$
- $y'' = f''(x) = \dfrac{d^2f}{dx^2}$ = the 2nd derviatives
- $y''' = f'''(x) = \dfrac{d^3f}{dx^3}$ = the 3rd derviatives
* - $y'''' = f''''(x) = \dfrac{d^4f}{dx^4} = f^{(4)}$ = the 4th derviatives

يجب وضع الأقواس لتمييزها عن
الرمز مرفوع للقوة

* $\dfrac{d^n y}{dx^n} = y^{(n)} =$ nth derivatives

*Ex. find $f^{(6)}(x)$ $\quad f(x) = x^5 + 3x^2 + 1$

$f'(x) = 5x^4 + 6x$

$f''(x) = 5.4x^3 + 6$

$f'''(x) = 5.4.3x^2$

$f^{(4)}(x) = 5.4.3.2.x$

$f^{(5)}(x) = 5.4.3.2.1 = 5! = 120$

تم المضروب

$f^{(6)}(x) = 0$

* $f(x) = x^n$

n:- is positive integer

$f^{(n)}(x) = n!$

$f^{(n+1)}(x) = $ zero

Ex. find the following limits

① $\lim_{x \to 0} \frac{3^x - 1}{x} = \frac{0}{0}$!!

عبر قاعدة لوبيتال $\Rightarrow \lim_{x \to 0} \frac{3^x \ln 3}{1} = 1 . \ln 3 = \ln 3$

OR

عن طريق تعريف المشتقة

$\lim_{x \to 0} f(x) = f'(0)$

Such that $f(x) = 3^x$

$f'(x) = 3^x \ln 3$

$f'(0) = 1 . \ln 3 = \ln 3$ #

للتوضيح

$\lim_{x \to 0} \frac{3^x - 1}{x} = f'(0) = \lim_{x \to 0} \frac{3^x - 1}{x - 0}$

تعريف المشتقة

$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

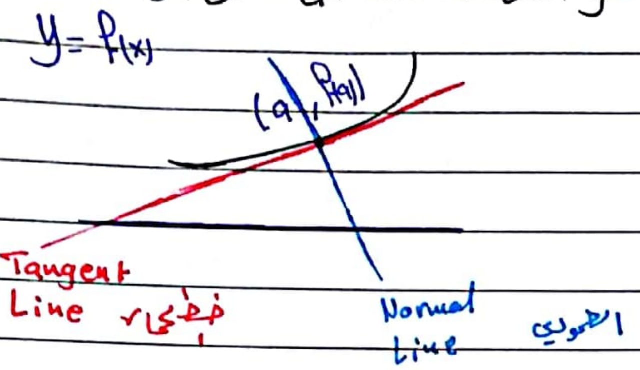
$f(x) = 3^x$

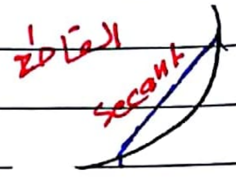
$f(a) = 1$

$a = 0 \overset{?}{=} f(0)$

$f(0) = 3^0 = 1$

* Geometrical meaning of $f'(a)$:- المعنى الهندسي للمشتقة عند (a)





* So $f'(a)$ in the slope of the Tangent Line at point $(a, f(a))$

T Line :- $(y - f(a)) = f'(x -)$

∴ there for the normal Line of the tangent line

N Line :- $(y - f(a)) = \frac{-1}{f'(a)}(x - a)$

* لكتابة اي معادلة خط مستقيم يجب ان نطلع الحيل والرقم والصورة للرقم والاستخدام لحساب الحيل المشتقة نقطة التماس .

Ex. Find the equation of Tangent Line and Normal Line of the function $f(x) = x^2$ at $x = 3$??

$y = f(3) = 9 \quad (3, 9) \quad x = 3$

$f'(3) = 2.3 = 6$ ميل مماس

معادلة المماس الخط المستقيم T :- $(y - 9) = 6(x - 3)$

$y - 6x + 9 = 0$ or $y + 9 = 6x$

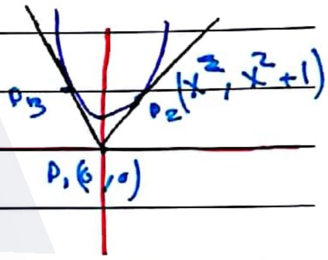
N.: $y - 9 = \frac{-1}{6}(x - 3)$

* To equal the slope from tangent line equasion :-

slope $= \frac{-\text{agent } x}{\text{agent } y}$ اكيد ، $\frac{-\text{مشتقة } x}{\text{مشتقة } y}$

V.imp

Ex. write the equasion of the tangent of $f(x) = x^2 + 1$ that passes through the point $(0,0)$?



$m = \frac{\Delta y}{\Delta x}$, $m = f'(x)$

$f'(x) = \frac{x^2 + 1 - 0}{x - 0}$

$2x = \frac{x^2 + 1}{x} \Longrightarrow 2x^2 = x^2 + 1$

$x^2 = 1$

$\boxed{x = \pm 1}$ نعوضها في الإقتران لإيجاد نقاط التماس

$P_2 = (1, 2)$

$P_3 = (-1, 2)$

T_1:- $y - 0 = f'(1)(x - 0)$
$y = 2x$

T_2:- $y - 0 = f'(-1)(x - 0)$
$y = -2x$

Five Apple

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Ex. $y = x^{3/2}$ Write the equasion to the tangent and Normal line if the tangent of curve is parrelled to the line $y - 3x = 1$??

* T Line :- $m = f'(x) = \frac{3}{2} x^{1/2}$

The slope :- $\frac{--3}{1} = \boxed{3}$

$T \parallel L \Rightarrow m_t = m_L$

$\cancel{\frac{2}{3}} \times \cancel{\frac{3}{2}} x^{1/2} = \cancel{3} \times \frac{2}{\cancel{3}}$

So $x^{1/2} = 2 \longrightarrow \boxed{x = 4}$

$y = 4^{3/2} = 8$ The point $(4, 8)$

T_L :- $y - 8 = 3(x - 4)$

N_L : $y - 8 = \frac{-1}{3}(x - 4)$

imp Ex. $f(x) = (x-2)^{2/3} + 1$ find the equasion of the tangent at $x = 2$?

Point $(2, 1)$

$f'(x) = \frac{2}{3}(x-2)^{-1/3}$

$f'(2) = \frac{2}{3} \times \frac{1}{\sqrt[3]{0}} = \frac{2}{0} = \infty$

$f' \xrightarrow[x \to 2]{} \infty$

So its Vertical tangent

so $x = 2$

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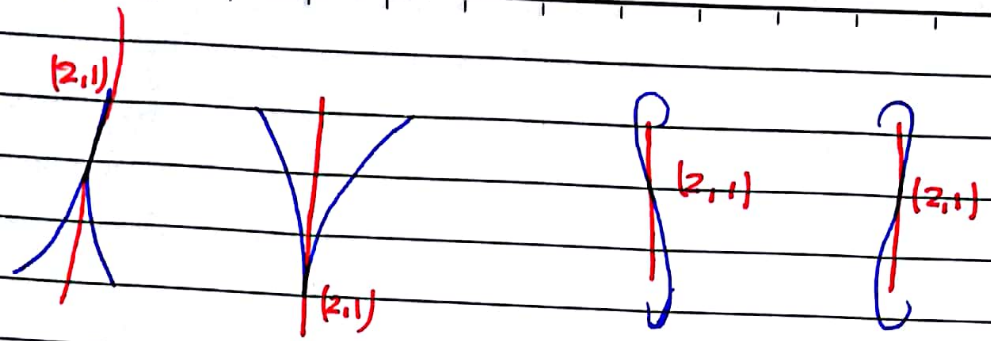
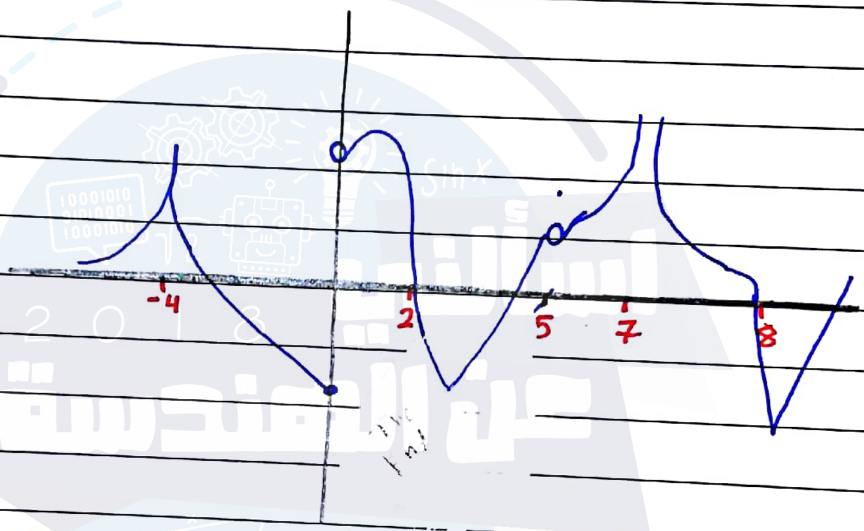


*المشتقة غير موجودة
عند هذه النقاط

*** Ex.** Find the number ~~for~~ from the graph for which f isn't differntiable :-



* X = -4 ; V. Tangent
* X = 0 ; not continous
* X = 2 ; V. Tangent
* X = 5 ; not continous
* X = 7 ; not continous ((undifined))
* X = 8 ; ~~V. Tangent~~ Corner

قاعدة السلسلة والاشتقاق الضمني

§ 3.4 , 3.5 Chain Rule and the implicit differentiation

1 Chain Rule

قاعدة السلسلة

$F(x) = g(h(x))$

① $\frac{dF(x)}{dx} = F'(x) = g'(h(x)) \cdot h'(x)$

② $y = f(u)$ and $u = g(x)$

$y \longrightarrow u \longrightarrow x$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Ex. ① $y = \tan^3(x^2 + 3x)$ find $\frac{dy}{dx}$?

$\rightarrow \frac{dy}{dx} = 3\tan^2(x^2+3x) \cdot \sec^2(x^2+3x) \cdot (2x+3)$

② $y = \cos^2 x^2$, $x = t^3$ find $\frac{dy}{dt}$?

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = -\sin x^2 \cdot 2x \cdot 3t^2$$

18. Nov. 2018 Sun

③ $y = 2^x$ find $\dfrac{dy}{dx^2}$?

نفرض $u = x^2 \longrightarrow \dfrac{dy}{du}$

$\Longrightarrow \dfrac{dy}{du} = \dfrac{dy}{dx} \cdot \dfrac{dx}{du} \Rightarrow 2^x \ln 2 \cdot \dfrac{1}{2x}$

بس $\Rightarrow \dfrac{du}{dx}$ مشتقتها بالنسبة ل

Ex. $h(x) = \sqrt{4 + 3f(x)}$ $f(1) = 7$ $f'(1) = 4$ find $h'(1)$??

$h'(x) = \dfrac{3f'(x)}{2\sqrt{4+3f(x)}} \Longrightarrow h'(1) = \dfrac{3f'(1)}{2\sqrt{4+3f(1)}}$

$h'(1) = \dfrac{12}{2\sqrt{4+21}} = \boxed{\dfrac{6}{5}}$

This is Moaris

3.5 Implicit differentaiable الاشتقاق الضمني

∴ when the relation between the Variables is implicit (equasion) then the differentiation is implicit.

Ex.1

$$x^2 + 3xy^2 + y = 5 \qquad \text{Find } \frac{dy}{dx} \text{ ??}$$

→ differentiat side w.r to x

$$\Rightarrow \cancel{2x + 3\cdot 2yy' + y' = 0} \qquad 2x + 3x\cdot 2yy' + y^2\cdot 3 + y' = 0$$

$$\cancel{2x + 6yy' + y' = 0}$$

بالترتيب

$$2x + 6xyy' + 3y^2 + y' = 0$$

$$2x + 3y^2 = -6xyy' - y'$$

$$\frac{2x + 3y^2}{-6xy-1} = \frac{+y'(-6xy-1)}{\cancel{-6xy-1}}$$

$$y' = \frac{dy}{dx} = \frac{2x + 3y^2}{-6xy-1} \quad \#$$

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22. Nov. 2018

Ex. 2

$$e^{\frac{x}{y}} = x - y \qquad \text{find } \frac{dy}{dx} \;??$$

$$\rightarrow e^{\frac{x}{y}} \cdot \left(\frac{y \cdot 1 - x \cdot y'}{y^2}\right) = 1 - y'$$

$$\frac{e^{\frac{x}{y}} \cdot y \;\; - e^{\frac{x}{y}} x \cdot y'}{y^2} = \frac{1 - y'}{1}$$

بالضرب بالتبادل

$$y^2 - y^2 y' = e^{\frac{x}{y}} \cdot y - e^{\frac{x}{y}} x y'$$

$$\rightarrow e^{\frac{x}{y}} x y' - y^2 y' = e^{\frac{x}{y}} \cdot y - y^2$$

$$\frac{y' \cancel{\left(e^{\frac{x}{y}} x - y^2\right)}}{\cancel{e^{\frac{x}{y}} x - y^2}} = \frac{e^{\frac{x}{y}} \cdot y - y^2}{e^{\frac{x}{y}} x - y^2}$$

$$y' = \frac{e^{\frac{x}{y}} y - y^2}{e^{\frac{x}{y}} x - y^2} \quad \#$$

Ex.3 If you have $f(x) + x^2 (f(x))^3 = 10$ and you know $f(1) = 2$ so find $f(1)$

$$\rightarrow f'(x) + x^2 \cdot 3(f(x))^2 \cdot f'(x) + \cancel{(f(x))^3 \cdot 2x} = 0$$

$$\cancel{-2x (f(x))^3} \qquad\qquad -2x (f(x))^3$$

$$f'(x) + 3x^2 (f(x))^2 \cdot f'(x) = -2x (f(x))^3$$

$$\frac{f'(x)\left(1 + 3x^2 (f(x))^2 \cdot 1\right)}{\cancel{1 + 3x^2 (f(x))^2}} = \frac{-2x(f(x))^3}{1 + 3x^2 (f(x))^2}$$

$$f'(x) = \frac{-2x(f(x))^3}{1 + 3x^2 (f(x))^2}$$

$$f'(1) = \frac{-2 \cdot 1 (f(1))^3}{1 + 3 \cdot 1 (f(1))^2}$$

$$f'(1) = \frac{-2 \cdot 8}{1 + 3 \cdot 4} = \frac{-16}{13}$$

Ex.4 $g(x) + x \sin(g(x)) = x^2$ Find $g'(0) = ??$

* طبعاً هنا لم يعطِ صورة $g(0)$ لذا منطلعها احنا

$g(0) + 0 \cancel{\sin(g(0))} = 0^2$

$g(0) + 0 = 0$

so $\boxed{g(0) = 0}$

$g'(x) + x \cos(g(x)) \cdot g'(x) + \sin(g(x)) \cdot 1 = 2x$

$g'(0) + \cancel{0 \cdot \cos(g(0)) \cdot g'(0)} + \sin(g(0)) = 0$

$g'(0) + \sin 0 = 0$

so $\boxed{g'(0) = 0}$

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Ex. 5 $\quad \sin y + \cos x = 1 \qquad$ Find y'' ??

$$\cos y \cdot y' + - \sin x = 0$$

$$\cancel{\cos y \cdot y'' +}\ y' = \frac{\sin x}{\cos y}$$

$$y'' = \frac{\cos y \cdot \cos x - \sin x \cdot - \sin y \cdot y'}{(\cos y)^2}$$

but $y' = \frac{\sin x}{\cos y}$ $\quad$ so $\quad y'' = \dfrac{\cos y \cos x - \sin x \sin y \cdot \frac{\sin x}{\cos y}}{(\cos y)^2}$

ويمكننا تبسيط الجواب

* $\cos^2 y = (\cos y)^2$

Ex. 6 $\quad x^2 + y^2 = (2x^2 + 2y^2 - x)^2 \qquad$ Find the equasion of the tangent at P.t $(0, \frac{1}{2})$??

$$2x + 2yy' = 2(2x^2 + 2y^2 - x)^1 \cdot (4x + 4yy' - 1)$$ نعوض $(0, \frac{1}{2})$

$$0 + \cancel{2}y' = 2\left(0 + \cancel{2} \cdot \frac{1}{\cancel{2}4} - 0\right) \cdot \left(0 + 4 \cdot \tfrac{1}{2} y' - 1\right)$$

$$\frac{y'}{-2y'} = \cancel{2} \cdot \frac{1}{\cancel{2}} \cdot \frac{(2y' - 1)}{\cancel{-2y'}}$$

$$-y' = -1$$

so $y' = 1 \Rightarrow$ Slope of the tangent

$$T.E:- \ (y - \tfrac{1}{2}) = 1\,(x - 0)$$

$$y - \tfrac{1}{2} = x$$

$$* \ (f^{-1})'(y) = \frac{1}{f'(x)} \qquad , \qquad y = f(x)$$

لو صورة

أصل الصورة

$f^{-1}(x) \rightarrow$ صورة

$f(x) \rightarrow$ الأصل

$$* \ (f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} \qquad (f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))}$$

لو صورة

أصل الصورة

Ex. given that ① $f(4) = 5$, $f'(4) = \frac{2}{3}$, $f'(5) = 2$ Find $(f^{-1})'(5) = ??$

$$\Rightarrow (f^{-1})'(5) = \frac{1}{f'(4)} = \frac{1}{\frac{2}{3}} = 3/2$$

② $f(x) = x^3 + x + 1$ Find $\left.\dfrac{d f^{-1}(x)}{dx}\right|_{x=3} = ??$

صورة الاقتران

$$f'(x) = 3x^2 + 1$$

$$f'(1) = 3 + 1 = \boxed{4}$$

لذا ←

$$x^3 + x + 1 = 3$$

$$1 + 1 + 1 = 3$$

$$3 = 3 \quad \text{so } \boxed{x = 1}$$

$$(f^{-1})'(3) = \frac{1}{f'(x)} = \frac{1}{f'(1)} = \frac{1}{4}$$

* Find the Limits :-

① $\lim_{x \to \infty} e^{x - x^2} = \lim_{x \to \infty} e^{-x^2} = e^{-\infty} = \frac{1}{e^{\infty}} = 0$

there is a Horizontal asymptotes

② $\lim_{x \to \infty} e^{x + x^2} = \lim_{x \to \infty} e^{x^2} = \cancel{\epsilon} e^{\infty} = \infty$

Ex. find y' for the following:-

① $y = \sin^{-1}(x^3 + 2x)$

$$y' = \frac{1}{\sqrt{1 - (x^3 + 2x)}} \cdot (3x^2 + 2) = \frac{3x^2 + 2}{\sqrt{1 - (x^3 + 2x)}}$$

② $y = \sqrt{\tan^{-1}(x^3)}$

$$y' = \frac{1}{2\sqrt{\tan^{-1}(x^3)}} \times \frac{3x^2}{(1 + (x^3)^2)}$$

③ $y = \ln(e^{-x} + x e^{-x})$

$$y' = \frac{\cancel{-e^{-x}} + x \cdot -e^{-x} \cancel{+ e^{-x}}}{e^{-x} + x e^{-x}} = \frac{-x e^{-x}}{\cancel{e^{-x}}(1 + x)} = \frac{-x}{1 + x}$$

④ $y = \ln(1 + \ln x)$

$$y' = \frac{\cancel{0} + 1}{(1 + \ln x) \cdot x}$$

Five Apple

25 . Nov. 2018

Ex. Write the Equasion of the tangent Line at $(3, 0)$ for $y = \ln(x^2 - 3x + 1)$??

$$y' = \frac{2x - 3}{x^2 - 3x + 1} \Bigg|_{x=3}$$

$$y' = \frac{2.3 - 3}{9 - 9 + 1} = 3 \to \text{The slope of the tangent}$$

T.E $(y - 0) = 3(x - 3)$

Ex. Find y' ?? $\quad y = \ln\left(\dfrac{\sqrt{x^2+1} \cdot (1+x)^2}{x^3 \, (e^{4x})}\right)$

$\to y = \ln\sqrt{x^2+1} + \ln(1+x)^2 - \ln x^3 - \ln e^{4x}$ $\qquad$ Ln +

$y = \frac{1}{2}\ln x^2+1 + 2\ln 1+x - 3\ln x - 4x$

$$y' = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}x}{x^2+1} + \frac{2.1}{1+x} - \frac{3.1}{x} - 4$$

$$y' = \frac{x}{x^2+1} + \frac{2}{1+x} - \frac{3}{x} - 4$$

§3.6 Logarithmic differentiation

Ex. find y' for this equasion :- $y = \dfrac{\sqrt{x} \cdot e^{x^2-x} \cdot (x+1)^{2/3}}{(x^2+3)^2}$

* لنستطيع الحل نطبق اللوغارتم ع الطرفين :-

نوزع $\ln y = \ln\sqrt{x} + \ln e^{x^2-x} + \ln(x+1)^{2/3} - \ln(x^2+3)^2$

$\ln y = \frac{1}{2}\ln x + x^2 - x + \frac{2}{3}\ln(x+1) - 2\ln x^2+3$

نشتق

$\dfrac{y'}{y} = \dfrac{1}{2x} + 2x - 1 + \dfrac{2 \cdot 1}{3x+3} - \dfrac{2 \cdot (2x)}{x^2+3}$

$y' = \dfrac{\sqrt{x} \cdot e^{x^2-x} \cdot (x+1)^{2/3}}{(x^2+3)^2} \left(\dfrac{1}{2x} + 2x - 1 + \dfrac{2}{3x+3} - \dfrac{4x}{x^2+3} \right)$

Ex.

① $y = x^2$

$y' = 2x$

② $y = 2^x$

$y = 2^x \ln 2$

③ $y = x^x$

* You have to Use Logarithmic differentation *

$\ln y = \ln x^x \longrightarrow \ln y = x \ln x$

$\dfrac{y'}{y} = x \cdot \dfrac{1}{x} + \ln x \cdot 1$

$y' = x^x(1 + \ln x)$

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④ $y = x^{\sin x}$

$\ln y = \ln x^{\sin x} \longrightarrow \ln y = \sin x \cdot \ln x$

$\frac{y'}{y} = \sin x \cdot \frac{1}{x} + \ln x \cdot \cos x$

$y' = x^{\sin x} \left(\frac{\sin x}{x} + \ln x \cdot \cos x \right)$

imp

Ex. $x^y = y^x$ find $y' = ??$

$\ln x^y = \ln y^x$

$y \ln x = x \ln y$

$y \cdot \frac{1}{x} + \ln x \cdot y' = x \cdot \frac{y'}{y} + \ln y \cdot 1$

$\frac{-x y'}{y} + y' \ln x = \ln y - \frac{y}{x}$

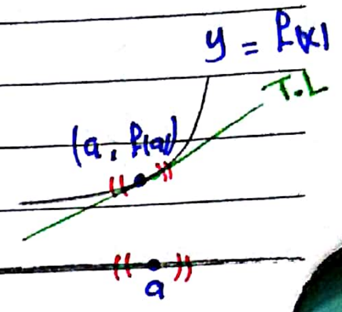
$$\frac{y' \left(\cancel{-\frac{x}{y} + \ln x} \right)}{\cancel{-\frac{x}{y} + \ln x}} = \frac{\ln y - \frac{y}{x}}{\ln x - \frac{x}{y}}$$

$$y' = \frac{\ln y - \frac{y}{x}}{\ln x - \frac{x}{y}}$$

ويمكنا تبسيط وتوحيد
المقامات

25 Nov 2018

§ 3.10 Linear approximation



* المعادلة :- تكون مخربطة بين x و y
* الإقتران: يكون ال y في جهة وال x في جهة app.
* يمكن تحويل المعادلة الخطية إلى إقتران

T.L :- $(y - f(a)) = f'(a)(x - a)$ "معادلة خط"

$$\underbrace{y = f(a) + f'(a)\cdot(x - a)}$$ "اقتران"

$$L(x) = f(a) + f'(a)\cdot(x - a)$$

$L(x)$ = Linearaisation of f

التقريب الخطي

$$y_p \cong y_T$$

so $f(x) \cong f(a) + f'(a)(x - a)$

Ex. ① Find the Linearaisation of $f(x) = \sqrt{x}$ at $a = 4$?
$L(x)$

$$L(x) = f(a) + f'(a)(x - a)$$

$$L(x) = f(4) + f'(4)(x - 4)$$

$f(4) = \sqrt{4} = 2$

$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$$L(x) = 2 + \frac{1}{4}(x - 4)$$

② find the equation of the tangent

$$y = 2 + 1/4\,(x - 4)$$

③ find approximation f(x) linearty at $x = 4$

$$f(x) \cong 2 + 1/4\,(x - 4)$$

$$\sqrt{x} \cong 2 + 1/4\,(x - 4)$$

4) Find $\sqrt{4.1}$?

$\cong 2 + \frac{1}{4}(4.1 - 4)$

$\cong 2 + \frac{1}{4} \cdot \frac{1}{10}$

$= \frac{2 \times 40}{40 \times 1} + \frac{1}{40} = \frac{81}{40}$

~~This is the ...~~

Ex. Find ~~the~~ approximate Linerty for the following :-

1) $\sqrt[3]{1001}$?

نفرض الجذر a) $f(x) = \sqrt[3]{x}$, $a = 1000$ نقرب لأقرب رقم تسهل عنده الحسابات

صورة التقريب b) $f(1000) = \sqrt[3]{1000} = 10$

نشتق الجذر c) $f'(x) = \frac{1}{3} \cdot x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$

نجد صورة التقريب في المشتقة d) $f'(1000) = \frac{1}{3(\sqrt[3]{1000})^2} = \frac{1}{300}$

$f(x) \cong f(a) + f'(a)(x - a)$

$f(x) \cong 10 + \frac{1}{300}(1001 - 1000)$

$\sqrt[3]{1001} \cong 10 + \frac{1}{300} = \frac{3001}{300}$

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2) $\cos 28°$

Let

a) $f(x) = \cos x \quad , \quad a = 30$

للتحويل

$180° \longrightarrow \pi$

$1° \longrightarrow ?$

$1° = \frac{\pi}{180}$

$f(30) = \cos 30 = \frac{\sqrt{3}}{2}$

$f'(x) = -\sin x$

$f'(30) = -\sin 30 = \frac{-1}{2}$

$\Delta x = |28 - 30| \frac{\pi}{180} = \frac{-2\pi}{180}$

للتحويل إلى راديان

$f(x) \simeq f(a) + f'(a)\underbrace{(x - a)}^{\Delta x}$

$\cos 28 \simeq \frac{\sqrt{3}}{2} + \frac{-1}{2}\left(\frac{-2\pi}{180}\right)$

$\cos 28 \simeq \frac{\sqrt{3}}{2} + \frac{\pi}{180}$

Ex. $e^x \cos x \simeq 1 + x \quad , \quad x = 0 \quad$ Verify it?

Let $f(x) = e^x \cos x \quad , a + x = 0$

* $f(0) = \underline{\underline{1}}$
* $f'(x) = -e^x \sin x + \cos x \, e^x$
* $f'(0) = -1 \cdot 0 + 1 \cdot 1 = \underline{\underline{1}}$

$f(x) \simeq f(a) + f'(a)(x - a)$

$f(x) \simeq 1 + 1(x - 0) = 1 + x$

$f(x) \simeq 1 + x \quad \#$

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§ 3.11 Hyperbolic Function:-

* hyperbolic $\sin x \equiv \sinh x$

① $\sinh x = \dfrac{e^x - e^{-x}}{2}$ $D_f = \mathbb{R}$, $R_f = \mathbb{R}$

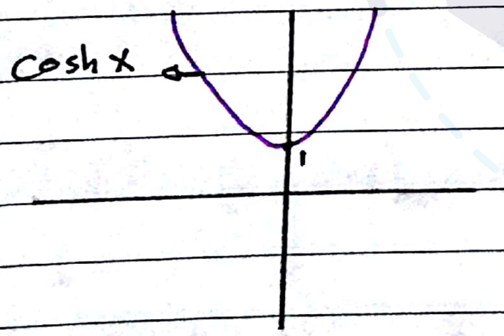
② $\cosh x = \dfrac{e^x + e^{-x}}{2}$ $D_f = \mathbb{R}$, $R_f = [1, \infty)$

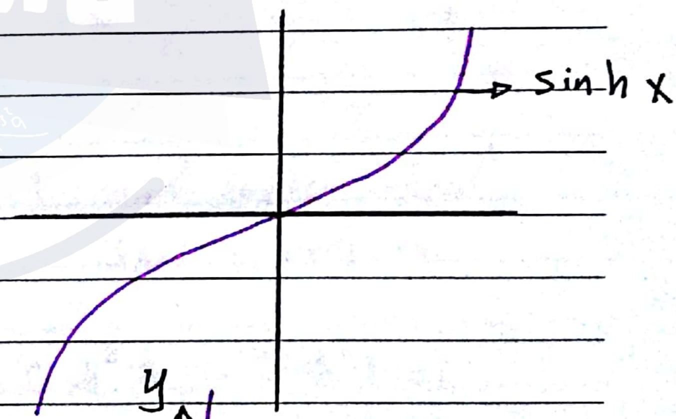
③ $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ they Have H.asy $= \boxed{1}$

④ $\coth x = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

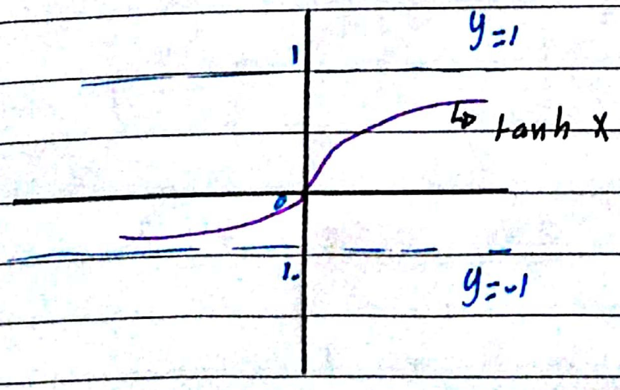
⑤ $\operatorname{sech} x = \dfrac{1}{\cosh x} = \dfrac{2}{e^x + e^{-x}}$

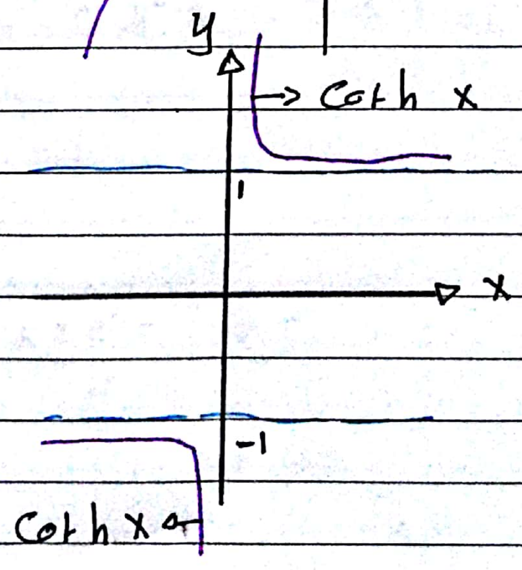
⑥ $\operatorname{csch} x = \dfrac{1}{\sinh x} = \dfrac{2}{e^x - e^{-x}}$



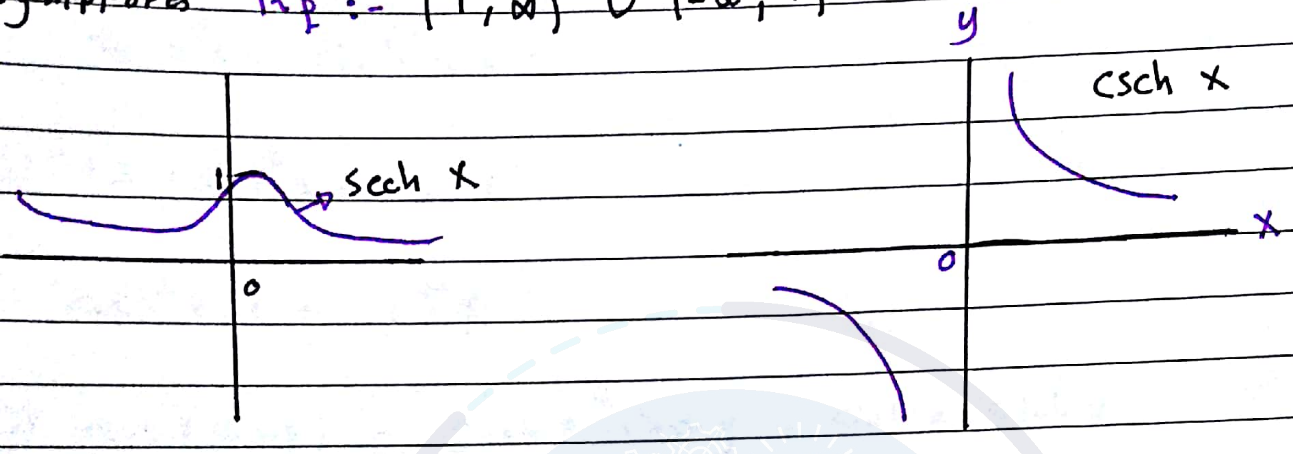


$\cosh(0) = 1$



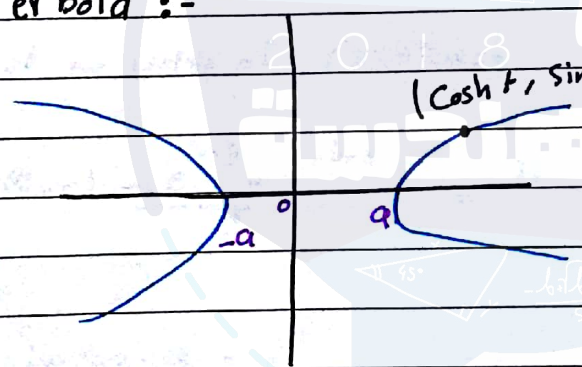


* The coth x have a vertical asymptotes and Horizontal asymptotes R_f :- $(1, \infty) \cup (-\infty, -1)$



* hyperbola :-

القطع الزائد



* $\frac{x^2}{a} - \frac{y^2}{b} = 1$

If $(a=b=1)$ so $x^2 - y^2 = 1$ and thats called unit hyperbola

The Proff $x^2 - y^2 = 1 \longrightarrow \cosh^2 t - \sinh^2 t = 1$

$$\left(\frac{e^t + e^{-t}}{2}\right)^2 - \left(\frac{e^t - e^{-t}}{2}\right)^2 = 1$$

فك التربيع

$$\frac{\cancel{e^{2t}} + 2\cancel{e^t e^{-t}}^{=1} + \cancel{e^{-2t}}}{4} - \frac{\cancel{e^{2t}} + 2\cancel{e^t e^{-t}}^{=1} + \cancel{e^{-2t}}}{4} = 1$$

$$\frac{2+2}{4} = 1 \qquad 1 = 1 \;\#$$

* Identities

① $\cosh^2 x - \sinh^2 x = 1$

② $1 - \tanh^2 x = \operatorname{sech}^2 x$

③ $\coth^2 x - 1 = \operatorname{csch}^2 x$

*④ $\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

* متطابقات الـ cos في جمع متطابقاتها في هذه الحالة نضع بدل (-) بـ (+).

Ex. find the value of $\cosh(\ln 5)$

$\rightarrow \cosh(\ln 5) = \dfrac{e^{\ln 5} + e^{-\ln 5}}{2} = \dfrac{5 + e^{\ln \frac{1}{5}}}{2} = \dfrac{5 + \frac{1}{5}}{2} = \dfrac{13}{5}$

① $\cosh x + \sinh x = e^{x}$

② $\cosh x - \sinh x = e^{-x}$

Ex. - $(\cosh x + \sinh x)^{10} = (e^{x})^{10} = e^{6x}$

- $(\cosh \ln 2 + \sinh \ln 2)^4 = (e^{\ln 2})^4 = 16$

Ex. given that $\cosh x = \frac{5}{4}$ and , $x > 0$ find $\sinh x = ?$

$\rightarrow \cosh^2 x - \sinh^2 x = 1$

$\left(\frac{5}{4}\right)^2 - \sinh^2 x = 1 \longrightarrow \frac{25}{16} - \frac{1 \times 16}{16} = \sinh^2 x$

$\sinh^2 x = \frac{9}{16}$

so $\sinh x = \pm \frac{3}{4}$

since $x > 0 \rightarrow$ then $\sinh x = + \frac{3}{4}$

Ex given that $\tan x = \frac{-3}{5}$ find ① $\sinh x = ?$

② $\cosh x = ?$

①

$$\coth^2 x - 1 = \operatorname{csch}^2 x$$

$$\frac{25}{9} - \frac{1 \times 9}{9 \times 1} = \frac{1}{\sinh^2 x} \longrightarrow \frac{1}{\sinh^2 x} = \frac{16}{9}$$

$$\sinh^2 x = \frac{9}{16} \longrightarrow \sinh x = \pm\frac{3}{4} \longrightarrow \sinh x = \frac{-3}{4}$$

الذي يحدد إشارة tanhx هو ال sinhx وأيضاً "coshx دائماً" موجب.

②

$$\cosh^2 x - \sinh^2 x = 1$$

$$\cosh^2 x - \frac{9}{16} = 1$$

$$\cosh^2 x = \frac{25}{16} \longrightarrow \cosh x = +\frac{5}{4}$$

Coshx دائماً موجب

* Derivatives

f	f'
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\coth x$	$-\operatorname{csch}^2 x$
$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
$\operatorname{csch} x$	$-\operatorname{csch} x \coth x$

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Ex. find y' for the following :-

① $y = \ln(\cosh x) + e^{\cosh 3x}$

$y' = \frac{\sinh x}{\cosh x} + e^{\cosh 3x} \sinh \cancel{x}(3x) \cdot 3$

$y' = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} + \cancel{\cos}\, e^{\cosh 3x} \cdot \sinh 3x \cdot 3$

② $y = \sin(\cosh x^2)$

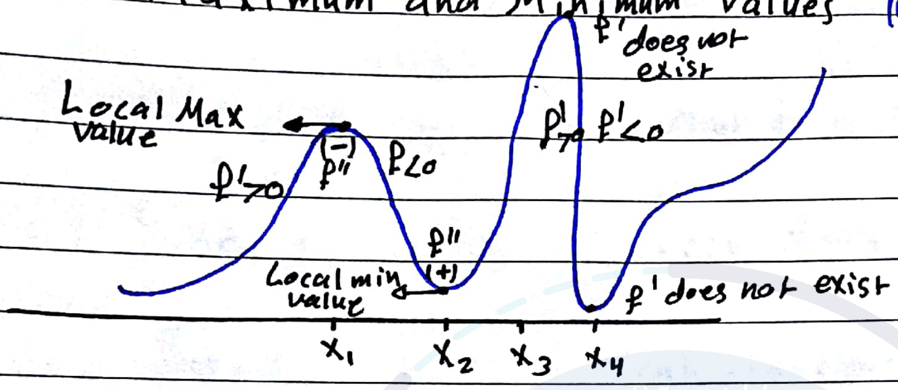
$y' = \cos(\cosh x^2) \cdot \sinh x^2 \cdot 2x$

The End of chapter #3

& This is Me

Chapter #4 Applications of Defferintation

§ 4.1 Maximum and Minimum Values ((extrem Values))



* Def₁ $x_0 \in D_f$ at x_0 ; f has a critical number (p.t) if $f'(x_0) = 0$ or $f'(x_0)$ does not exist

* Def₂ a) f is increasing on the ~~internat~~ interval I if f' is positive $f' > 0$ on I
b) f is decreasing on the interval I if $f' < 0$ on I

* To classify the criticals for Local Max and Local Min use the first dervitives test

The first derivatives test :-

1. if f' change from +Ve to −Ve around x_1 + | − (x_1)
then $f(x_1)$ is a Local (relative) Max. value f ((inc → dec))

2. if f' changes from −Ve to +Ve around x_1 − | + (x_1)
then $f(x_1)$ is Local Min. Value f ((dec → inc))

The second derivatives test :-

$f'(x_1) = 0$

a) if $f''(x_1) > 0 \Longrightarrow f(x_1)$ is a local Min

b) if $f''(x_1) < 0 \Longrightarrow f(x_1)$ is a Local Max

c) if $f''(x_1) = 0 \Longrightarrow$ test fails

Ex. $f(x) = x^2$ find critical, increasing and decreasing poin

$\rightarrow f'(x) = 2x$

$2x = 0$

$\boxed{x = 0}$ critical $\qquad \underset{0}{- \quad +} \qquad f' = 2x$

* f is increasing on $[0, \infty)$

* f is decreasing on $(-\infty, 0]$

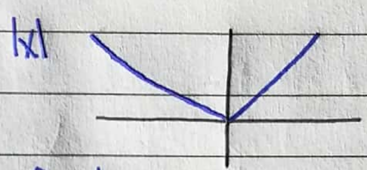
* $f(0) = 0$ is local min value

* $(0,0)$ critical

* Def ①:- $f(c)$ is an absolute Max. value for the function $f(x)$ if $f(c) \geq f(x)$ for all $x \in D_f$

* Def ②:- if $f(c) \leq f(x)$ for all $x \in D_f$ so $f(c)$ is an absolute min. value for $f(x)$

Ex. ① $f(x) = |x|$; find the absolute Max. and Min. if it exist?

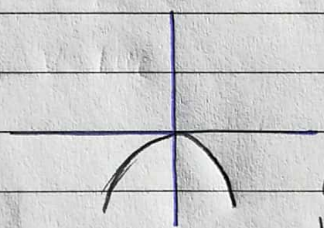


$|x| = 0$

so $x = 0$; at $x=0$ f has a critical since $f'(0)$ does not exist

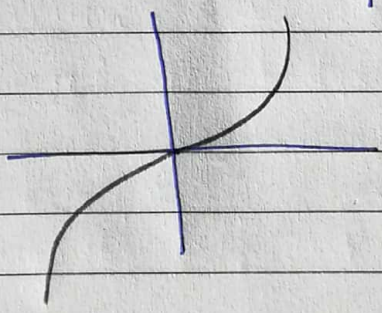
$\therefore f(0) = 0$ is an absolute min Value or at $x=0$ is an absolute min. Value ((and it's local)).

② $f(x) = -x^2$



at $x=0$ has a Local and absolute Max. Value

③ $f(x) = x^3$



at $x=0$, critical number

$f'(x) = 3x^2$

$f'(0) = 0 \longrightarrow f'(0) \geq 0 \; , \; x \in \mathbb{R}$

$\therefore x^3$ is alawys increas.

$\therefore x=0$ is neither Local (abso) min nor Local (absoulte) max

Five Apple

* **Therom** :- the function f is continous on a closed interval $[a,b]$ then f has absolute min and absoulte max. inside the interval $[a,b]$.

* The Steps to equal :-

① find criticals in (a,b)

② make a list $f(a)$, $f(b)$, $f(c)$

Ex. find the absolute max and min Values for this equasion :-

① $f(x) = x^3 - 3x^2 + 1$ on $[-\frac{1}{2}, 4]$

→ $f'(x) = 3x^2 - 6x$

$f'(x) = 0 \longrightarrow 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0 \quad x = 2$ } → محقق الحل

	x	$f(x) = x^3 - 3x^2 + 1$
critical point	0	1
	2	-3
End point	$-\frac{1}{2}$	$\frac{-1}{8} - \frac{3}{4} + 1 = \frac{1}{8}$
	4	$(4)^3 - 3(4)^2 + 1 = 17$

* the absolute min Value is -3
* f has min Value at $x = 2$
* the absolute max. Value is 17
* f has max Value at $x = 4$

2 Dec 2018

② $f(x) = \ln(x^2 + x + 1)$ on $[-1, 1]$

$f'(x) = \dfrac{2x+1}{x^2+x+1}$

البسط 1. $2x + 1 = 0$ — المقام $x^2 + x + 1 = 0$ ليس له أصفار

$2x = -1$

$\boxed{x = \frac{-1}{2}}$

نعوض بالأطراف

$\ln\frac{3}{4}$: the absolute min Value

$\ln 3$: the absolute max Value

x	$f(x) = \ln x^2 + x + 1$
$\frac{-1}{2}$	$\ln \frac{1}{4} - \frac{1}{2} + 1 = \ln\frac{3}{4}$
-1	$\ln 1 - 1 + 1 = \ln 1 = 0$
1	$\ln 1 + 1 + 1 = \ln 3$

a) $f(x) = \dfrac{1}{x}$ critical numbers ?

$f'(x) = \dfrac{-1}{x^2}$ → اصفار البسط α

x^2 → اصفار المقام $x^2 = 0$ → $x = 0$ it's not a critical because the zero don't Belong to domain

b) $f(x) = x^{\frac{3}{5}}(4 - x)$

$f(x) = 4x^{\frac{3}{5}} - x^{\frac{8}{5}}$

$f'(x) = 4 \cdot \frac{3}{5} x^{\frac{-2}{5}} - \frac{8}{5} x^{\frac{3}{5}}$

$f'(x) = \dfrac{12}{5x^{\frac{+2}{5}}} - \dfrac{8x^{\frac{3}{5}} \times x^{\frac{+2}{5}}}{5 \times x^{\frac{+2}{5}}}$

توحيد مقامات

$f'(x) = \dfrac{12 - 8x}{5x^{\frac{+2}{5}}}$

Five Apple

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①

$\longrightarrow 12 - 8x = 0$

اصفار البسط $x = \frac{3}{2}$

② does not exist :- $x^{\frac{2}{5}} = 0 \longrightarrow x = 0$

$f'(0)$ does not exist

$\therefore$ critical number $x = \frac{3}{2}$, $x = 0$

$\therefore$ Critical Points $\left(\frac{3}{2}, f\left(\frac{3}{2}\right)\right)$, $(0, f(0))$

§ 4.2 Mean Value therom

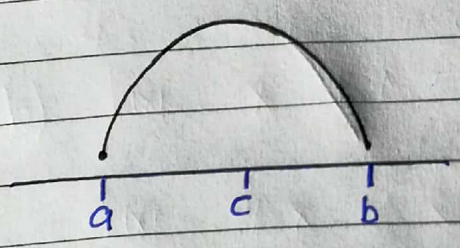
$\boxed{1}$ Rolles theorm

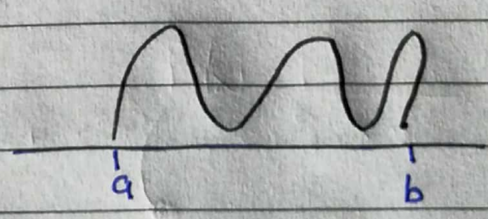
I) f Continous on $[a,b]$

II) f is differentiable on (a,b)

III) and also $f(a) = f(b)$

$\Longrightarrow$ then there is at Least a number $c \in (a,b)$ $f'(c) = 0$ and it's the same as a H. tangent $f' = 0$ has at Least one root.





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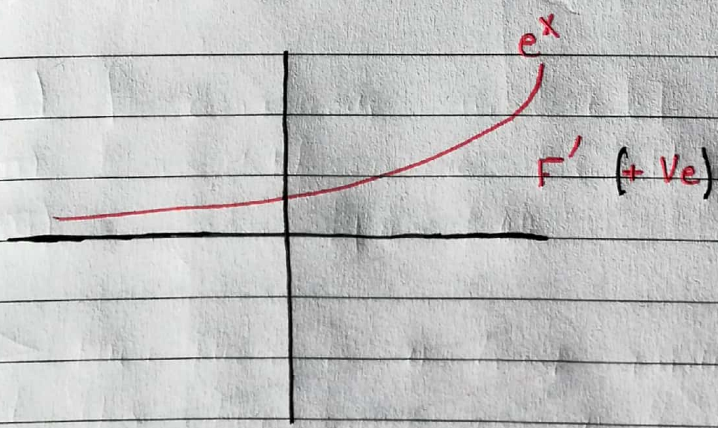
⇒ Ex. $F(x) = 3x^2 - 12x + 5$ Find the number ((C)) $\in (1,3)$ that satisfies Rolles Therom?

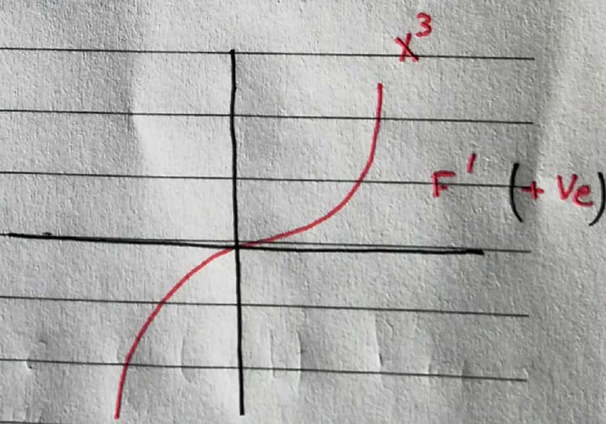
→ apply rolles therom on $[1,3]$

$F(1) = 3 - 12 + 5 = -4$
$F(3) = 27 - 36 + 5 = -4$ → $f(1) = f(3) = -4$

∴ there is ((C)) $\in (1,3) \xrightarrow{So} f'(C) = zero$

$F'(x) = 6x - 12$
$6x - 12 = 0$
$\boxed{x = 2}$ so $\boxed{C = 2}$





* if $F' > 0$ (if increasing on it's domain) → $F(x) = 0$ has at most on real root

* if $F' < 0$ (f if decreasing on it's domain) $F(x) = 0$ has one real root

⇒ show that the equasion $x^3 + x - 1 = 0$ has exactly one real root?

i) I.V.T show at least one

ii) Rolles (2) show that it has at most one] exactly one

Let $f(x) = x^3 + x - 1$

$F(0) = -1 < 0$

$F(1) = 1 > 0$

i) there is at least $c \in (0,1)$:- $F(c) = 0$

ii) $F'(x) = 3x^2 + 1$ allways Positive $\xrightarrow{\text{Rolles}}$ $x^3 + x - 1 > 0$

$F(x) = 0$ has at most one real ~~solution~~ root

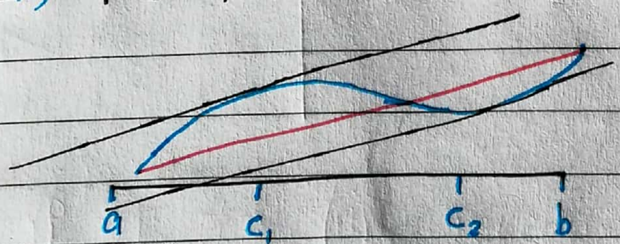
$F(x) = 0$ has exactly one $x = c$ $0 < c < 1$

* Mean Value Therom :-

i) $F(x)$ is continous on $[a,b]$

ii) $F(x)$ is differentiable on (a,b)

iii) then there is $c \in (a,b)$:- $f'(c) = \dfrac{f(b) - f(a)}{b - a}$



ميل المماس = ميل القاطع

∴ المماس // القاطع

$$F'(c) = \frac{f(b) - f(a)}{b - a}$$

the slope of the tangent

→ the slope of the secant

Five Apple

Ex: $F(x) = \sqrt{25 - x^2}$ on $[-5, 3]$ Find ((C)) that is obtained From the M. V. T ?

F is Continuous on $[-5, 3]$ and differentiable at $(-5, 3)$

→ There is $c \in (-5, 3)$ $\quad f'(c) = \dfrac{f(3) - f(-5)}{3 - -5}$

$$f'(c) = \frac{4 - 0}{8} = \frac{1}{2}$$

$$-2x = \sqrt{25 - x^2}$$

$$4x^2 = 25 - x^2$$

$$5x^2 = 25$$

$$\text{so } x = \pm\sqrt{5}$$

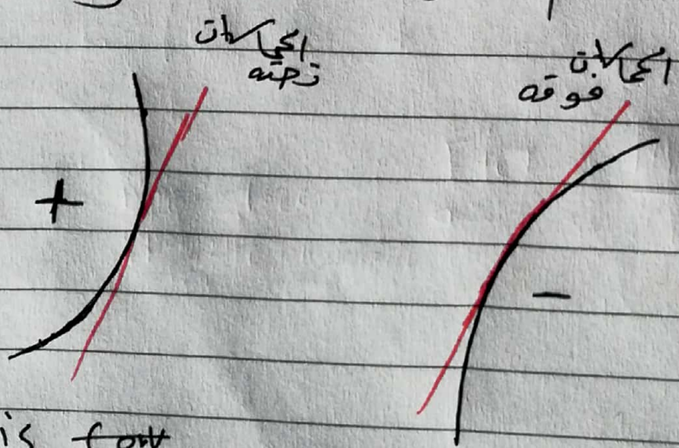
* * لما رسمنا غيرنا بالمعادلة رجعنا بنختبر اللي طلع معنا
تهمل

$$-2 \cdot \sqrt{5} \overset{?}{=} \sqrt{25 - (\sqrt{5})^2}$$

$$-2 \cdot -\sqrt{5} \overset{?}{=} \sqrt{25 - (-\sqrt{5})^2}$$

$\therefore$ $C = -\sqrt{5}$

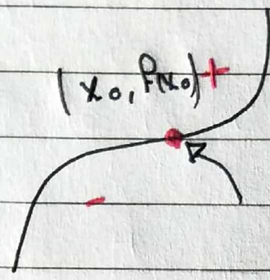
* Concavity التقعّر



f is ~~Con~~ Concavity UP

f is Concavity down

⇒ Thm:- ① if $f'' > 0$ on I ⇒ f is concave up
② if $f'' < 0$ on I ⇒ f is concave down



inflection point نقطة انعطاف
وهي يتغير عندها التقعر

$F'' = 0$, F'' does not exist

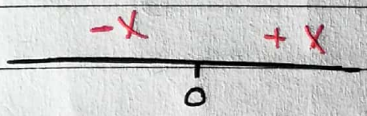
→ at x_0 F has an inflection point if F'' change its concavity around it, $f''(x_0) = 0$, $f''(x_0)$ does not exist

⇒ Ex:- $f(x) = 3x^4 - 4x^3 - 12x^2 + 4$

$f'(x) = 12x^3 - 12x^2 - 24x$

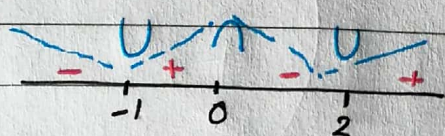
$12x(x^2 - x - 2) = 0$

Criticals ⇒ $(0, f(0))$, $(2, f(2))$, $(-1, f(-1))$



Local minimum at $x = -1, 2$

Local maximum at $x = 0$



F is increasing on $(-1, 0)$, $(2, 0)$
F is decreasing on $(-\infty, -1)$, $(0, 2)$

~~Ex: F(x) = ...~~

$f''(x) = 36x^2 - 24x - 24$

$= 12(3x^2 - 2x - 2)$

$$\begin{array}{ccccc} + & & - & & + \\ \hline & r_1 & & r_2 & \end{array}$$

Concave up — Concave down — Concave up

inflection $\rightarrow \{r_1, r_2\}$

* M.V.T

Ex ⇒ supose that $f(0) = -3$, $f'(x) \leq 5 \quad \forall x$ (for all x)

How Large can $f(2)$ Possible be?

⇒ so we apply M.V.T on $[0, 2]$

→ there is "C" :- $f'(c) = \dfrac{f(2) - f(0)}{2 - 0}$

$$\frac{f(2) + 3}{2} \leq 5$$

$$f(2) \leq 7$$

{Large value $f(2) = 7$ }

Ex. is there a differintiable function if such that $f(0) = -1$, $f(2) = 4$, $f'(x) \leq 2, \forall x$

→ there is a "C" such that $f'(c) = \dfrac{f(2) - f(0)}{2 - 0}$

→ $\dfrac{4 - -1}{2} = \dfrac{5}{2}$ ⇒ $f'(c) = \dfrac{5}{2} > 2$ لسه أقل من "2" اشترط أنه أقل من "2"

So there is no

- Thearm ① if $f'(x)=0 \quad \forall x$ then $f(x) = c \equiv$ Constant Function

Ex. supose that $f(x) = \tan^{-1}x + \cot^{-1}x$

i) show that the $f(x)$ is a constant function?

ii) find that constant?

→

$$f'(x) = \frac{1}{1+x^2} + \frac{-1}{1+x^2} = \frac{0}{1+x^2} = 0$$

there for $\therefore f(x) = \text{constant}$

$\therefore \tan^{-1}x + \cot^{-1}x = c$

→ Let $x=1$

$\tan^{-1}1 + \cot^{-1}1 = c$

$c = \frac{\pi}{4} + \frac{\pi}{4}$

$c = \frac{\pi}{2}$

$\therefore f(x) = \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

- Thearm ② if $f'(x) = g'(x)$, $\forall x$ then $f(x) - g(x) = c$ but they differ by a constant

i.e $f(x) = g(x) + c$

Ex. $f(x) = x^2 + x + 3$ find $g(x)$ such that ~~$g(1)=4$~~ $g'(x) = f'(x)$ and $g(1) = 4$?

because $g'(x) = f'(x)$

then $g(x) = f(x) + c \longrightarrow g(x) = x^2 + x + 3 + c$

$g(1) = 1 + 1 + 3 + c$

$4 = 5 + c$

$\boxed{c = -1}$

So $g(x) = x^2 + x + 2$

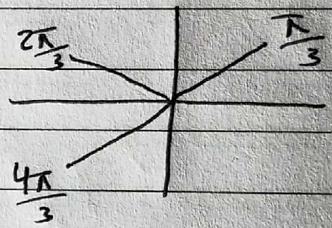
Ex $f(x) = x + 2\sin x \quad , \quad 0 \leq x \leq 2\pi$

Find and classify criticals for local and absolute extrems ??

$\rightarrow f'(x) = 1 + 2\cos x$

$f'(x) = 0 \rightarrow 1 + 2\cos x = 0 \qquad \cos x = -\tfrac{1}{2}$

Critical number So $x = \frac{2\pi}{3}, \frac{4\pi}{3}$



critical pt $\left(\frac{2\pi}{3}, \frac{2\pi}{3} + \sqrt{3}\right)$

$\left(\frac{4\pi}{3}, \frac{4\pi}{3} - \sqrt{3}\right)$

$\rightarrow$ +   −   + $f'(x)$

$0 \quad \frac{2\pi}{3} \quad \frac{4\pi}{3} \quad 2\pi \rightarrow f(2\pi) = 2\pi$

$f(0) = 0$

Local max $= \frac{2\pi}{3} + \sqrt{3}$ Local min $= \frac{4\pi}{3} - \sqrt{3}$

	f
0	0 $\rightarrow$ absolute min
$\frac{2\pi}{3}$	$\frac{2\pi}{3} + \sqrt{3}$
$\frac{4\pi}{3}$	$\frac{4\pi}{3} - \sqrt{3}$
2π	2π $\rightarrow$ absolute max

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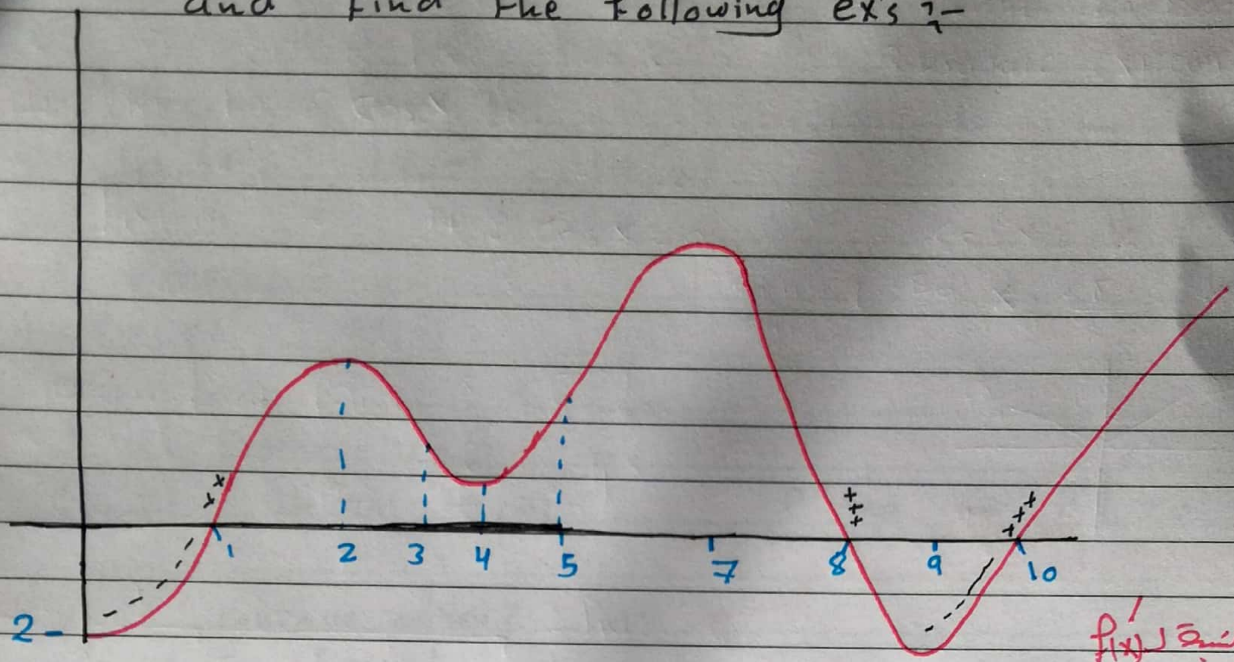
Ex. given this graph of $f'(x)$ and find the following ex's :-



بالنسبة لـ $f'(x)$

① $f'(x)$ has a critical at $x = ?$

$x = 2 , 7 , 9 , 4$

عند المماسات الأفقية

② f' has local max at $x = ?$

$x = 2 , 7$

واضحة ليش

③ f has a critical at $x = ??$

$x = 1 , 8 , 10$

عند المشتقة = صفر

اي يقطع محور السينات

④ f increas on ?

on $(1, 8)$, $(10, \infty)$

- الاقتران يكون متزايد فوق محور السينات
- الاقتران يكون متناقصاً تحت محور السينات

⑤ f decresaing on ?

on $(a, 1)$, $(8, 10)$

⑥ f has a local min at $x = ?$ عند التغير من الإشارة السالبة إلى الإشارة

الموجبة

$x = 1$, $x = 10$

⑦ f has local max at $x = ??$

$x = 8$

عند التغير من موجب إلى سالب

⑧ f' increasing on ?

on $(0,2)$, $(4,7)$, $(9,\infty)$

⑨ f' decreasing on ?

on $(2,4)$, $(7,9)$

⑩ f is Concave up ?

$(0,2)$, $(4,7)$, $(9,\infty)$ $\qquad f'' > 0 \rightarrow (f')' > 0$

يجب أن تكون متزايدة لأعلى

لا متناقصة ولا سفل

⑪ f is concave down ?

$(2,4)$, $(7,9)$

there for f has inflection Points at $x = 2, 4, 7, 9$

This is ff. $\qquad$ لأن المشتقة الثانية عندهم تساوي صفر

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9. Dec. 2018

* Grap of the Function :-

imp

Ex.- Find intervals of increasing ~~and~~, decreasing , Concavity and the inflection point for this Function :- $f(x) = x^4 - 4x^3$

$\rightarrow f'(x) = 4x^3 - 12x^2$

هون يصير الأس ٢ مش ٣

$4x^{2}(x-3) = 0$

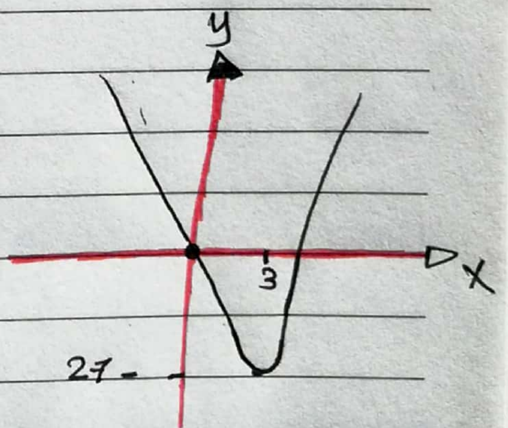
- $x = 0$, $x = 3$ $\longrightarrow$ critical ~~Point~~ number

- $(0,0)$, $(3,-27)$ $\longrightarrow$ critical Point

$$\begin{array}{ccc} - - - & - - - - - & + + + \\ & 0 & 3 \end{array}$$

- f is decreasing on $(-\infty, 3)$
- f is increasing on $(3, \infty)$
- $f(3) = -27$ is a Local min Value

the graph of f' $\longrightarrow$



* Concavity

$f''(x) = 12x^2 - 24x$

$f''(x) = 0$

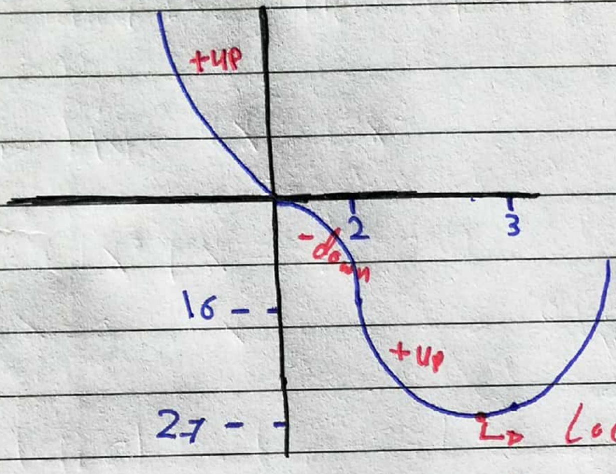
$12x^2 - 24x = 0 \longrightarrow 12x(x-2) = 0$

$x = 0$, $x = 2$

V , U , V critical Point

$$\begin{array}{cccc} + + + & - - - & + + + & f'' \\ & 0 & 2 & \end{array}$$

→ - f is concave up on $(-\infty, 0)$, $(2, \infty)$
- f is concave down on $(0, 2)$
- the inflection points are $(0, 0)$, $(2, -16)$ } عند الرسم نراعي التقعر



→ Local and absolute min point

D_f $[-27, \infty)$

§ 4.4 Indeterminate forms and L'Hopitals Rule (L.H)

* the indeterminate value :- $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, $\infty - \infty$, 1^{∞} , 0^0 , ∞^0

1. $\frac{0}{0}$ and $\frac{\infty}{\infty}$ } for this problem we use the L'Hopitals Rule

* $\lim\limits_{x \to a} \frac{f}{g} = \frac{0}{0}$ or $\frac{\infty}{\infty}$

then $\xrightarrow{L.H}$ $\lim\limits_{x \to a} \frac{f'}{g'}$

2. $0 \cdot \infty = \frac{\infty}{\frac{1}{0}} = \underset{\text{صورة لوبيتال}}{\frac{\infty}{\infty}}$ or $\frac{0}{\frac{1}{\infty}} = \underset{\text{صورة لوبيتال}}{\frac{0}{0}}$

هذي شكل معلوم بنحولها لقاعدة لوبيتال

3. $\infty - \infty$ we use i) Conjugate
ii) take common denominator

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Ex. Find the limits

① $\lim\limits_{x \to 1} \dfrac{\ln x}{x-1} = \dfrac{0}{0}$!!

$\overset{L,H}{\Longrightarrow} \lim\limits_{x \to 1} \dfrac{\frac{1}{x}}{1} = \boxed{1}$

② $\lim\limits_{x \to \infty} \dfrac{\ln x}{x-1} = \dfrac{\infty}{\infty}$!!

$\overset{L,H}{\Longrightarrow} \lim\limits_{x \to \infty} \dfrac{\frac{1}{x}}{1} = \dfrac{1}{\infty} = \text{Zero}\ \boxed{0}$

③ $\lim\limits_{x \to \infty} \dfrac{e^x}{x^2} = \dfrac{\infty}{\infty}$!!

$\overset{L,H}{\Longrightarrow} \lim\limits_{x \to \infty} \dfrac{e^x}{2x} = \dfrac{\infty}{\infty}$!! another time $\overset{L,H}{\Longrightarrow} \lim\limits_{x \to \infty} \dfrac{e^x}{2} = \dfrac{e^\infty}{2} = \infty$

* الاقترانات الأسية (e^x) fast من كثيرات الحدود فهو يصل إلى ∞ أول

④ $\lim\limits_{x \to \infty} \dfrac{\ln x}{\sqrt{x}} = \dfrac{\infty}{\infty}$!!

$\overset{L,H}{\Longrightarrow} \lim\limits_{x \to \infty} \dfrac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim\limits_{x \to \infty} \dfrac{2}{\sqrt{x}} = \dfrac{2}{\infty} = \boxed{0}$

⑤ $\lim\limits_{x \to 0} \dfrac{\tan x - x}{x^3} = \dfrac{0}{0}$!!

$\overset{L,H}{\Longrightarrow} \lim\limits_{x \to 0} \dfrac{\sec^2 x - 1}{3x^2} = \dfrac{1-1}{0} = \dfrac{0}{0}$!!

$\overset{L,H_2}{\Longrightarrow} \lim\limits_{x \to 0} \dfrac{\cancel{2} \sec x \sec x \tan x}{3 \cancel{6} x} = \lim\limits_{x \to 0} \sec^2 x * \lim\limits_{x \to 0} \dfrac{\tan x}{3x}$

$= 1 * \dfrac{1}{3} = \boxed{1/3}$

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⑥ $\lim\limits_{x \to \pi^-} \frac{\sin x}{1-\cos x} = \frac{0}{1--1} = 0$ ■

⑦ $\lim\limits_{x \to 0^+} x \ln x = 0 \cdot \ln 0^+ = 0 \cdot \infty$!!

$\lim\limits_{x \to 0^+} \frac{\ln x}{\frac{1}{x}} \overset{LH}{\Longrightarrow} \lim\limits_{x \to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim\limits_{x \to 0^+} \frac{-1}{\frac{\cancel{x}}{x^{\cancel{2}}}}$

$\lim\limits_{x \to 0^+} -x = 0$ ■

Ex. $\lim\limits_{x \to 1^+} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$

$= \frac{1}{0^+} - \frac{1}{0} = \infty - \infty$!!

توحيد $\Longrightarrow \lim\limits_{x \to 1^+} \left(\frac{x-1-\ln x}{(x-1)(\ln x)} \right) = \frac{0}{0}$!!

$\overset{L,H}{\Longrightarrow} \lim\limits_{x \to 1^+} \left(\frac{1-\frac{1}{x}}{\ln x + (x-1)\frac{1}{x}} \right) = \frac{0}{0}$!!

$\overset{L,H}{\Longrightarrow} \lim\limits_{x \to 1^+} \frac{\frac{x-1}{\cancel{x}}}{\frac{x \ln x}{\cancel{x}} + \frac{x-1}{\cancel{x}}} \longrightarrow \lim\limits_{x \to 1^+} \frac{x-1}{x \ln x + x - 1}$

$\overset{L,H}{\Longrightarrow} \lim\limits_{x \to 1^+} \frac{1}{\frac{x}{x} + \ln x + 1} = \lim\limits_{x \to 1^+} \frac{1}{1 + \ln x + 1} = \frac{1}{1+0+1} = \boxed{1/2}$

Five Apple

11 Dec 2018

→ Ex. ① $\lim_{x \to \infty} e^{x} - x = \infty - \infty$!!

→ $\lim_{x \to \infty} x\left(\frac{e^{x}}{x} - 1\right)$

نجزء الحل

→ $\lim_{x \to \infty} x = \infty$, $\lim_{x \to \infty} \frac{e^{x}}{x} = \frac{\infty}{\infty} \xrightarrow{LH} \lim_{x \to \infty} \frac{e^{x}}{1} = \infty$, $\lim_{x \to \infty}\left(\frac{e^{x}}{x} - 1\right) = \infty$

then $\lim_{x \to \infty} x\left(\frac{e^{x}}{x} - 1\right) = \infty \cdot \infty = \infty$

4- 1^{∞} , 0^{∞} , ∞^{0}

نحول كي

* $2^{x} = e^{x \ln 2}$
* $a^{x} = e^{x \ln a}$
* $F(x)^{G(x)} = e^{G(x) \ln F(x)}$

Ex. find the limits :-

① $\lim_{x \to 0^+} x^{x} = 0^{0}$!!

⇒ $\lim_{x \to 0^+} e^{x \ln x} = e^{0 \cdot -\infty}$ L.H $0 < a < 1 = -\infty$ for $\ln f(x)$

there for $\xrightarrow{L.H} \lim_{x \to 0^+} e^{\frac{\ln x}{\frac{1}{x}}}$ ⇒ $\lim_{x \to 0^+} e^{\frac{\frac{1}{x}}{\frac{-1}{x^2}}}$

⇒ $\lim_{x \to 0^+} e^{-x} = e^{-0} = \frac{1}{e^0} = \boxed{1}$

Five Apple

② $\lim_{x \to 0^+} (1 + \sin 4x)^{\cot x} = 1^{\infty}$!!

$* \cot 0 = \frac{1}{\tan 0} = \frac{1}{0} = \infty$

$\Rightarrow \lim_{x \to 0^+} e^{\cot x \ln(1 + \sin 4x)}$

$\lim_{x \to 0^+} e^{\frac{\ln(1+\sin 4x)}{\frac{1}{\cot x}}} \Rightarrow \lim_{x \to 0^+} e^{\frac{\ln(1+\sin 4x)}{\tan x}} = * \frac{0}{0}$!! LH

$\Rightarrow \lim_{x \to 0^+} e^{\frac{\frac{4\cos 4x}{1+\sin 4x}}{\sec^2 x}} = e^{\frac{1}{1} \frac{4*1}{1}} = \boxed{e^4}$

③ $\lim_{x \to \infty} x^{\frac{1}{x}} = \infty^0$!!

$\lim_{x \to \infty} e^{\frac{1}{x} \ln x} = \lim_{x \to \infty} e^{\frac{\ln x}{x}}$ LH

$\lim_{x \to \infty} e^{\frac{1}{x}} = e^0 = 1$

~~Zi first~~ $* \lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^x = (1 + 0)^{\infty} = 1^{\infty}$!!

$\lim_{x \to \infty} e^{x \ln\left(1 + \frac{a}{x}\right)} \longrightarrow \lim_{x \to \infty} e^{\frac{\ln\left(1 + \frac{a}{x}\right)}{\frac{1}{x}}}$

by L.H $= \lim_{x \to \infty} e^{\frac{\frac{-a}{x^2}}{1+\frac{a}{x}} \Big/ \frac{-1}{x^2}} \Rightarrow \lim_{x \to \infty} e^{\frac{a}{1+\frac{a}{x}}} = \boxed{e^a}$

$- \lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^x = e^2$

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Ex. $\lim_{x \to \infty} \left(1 - \frac{3}{2x}\right)^{5x+1}$

نحاول أن نجعلها كالصورة الاستنتاج.

$$\lim_{x \to \infty} \left(1 + \frac{\frac{-3}{2}}{x}\right)^{5x+1}$$

$$= \lim_{x \to \infty} \left(1 + \frac{\frac{-3}{2}}{x}\right)^{5x} * \lim_{x \to \infty} \left(1 + \frac{\frac{-3}{2}}{x}\right)^{1}$$

$$= \lim_{x \to \infty} \left(\underbrace{\left(1 + \frac{\frac{-3}{2}}{x}\right)^{x}}_{\text{نفس صورة الاستنتاج لذا}}\right)^{5} * 1$$

$$\Rightarrow = \left(e^{\frac{-3}{2}}\right)^{5} = e^{\frac{-15}{2}}$$

$$\Rightarrow \lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^{bx} = e^{ab}$$

** $\lim_{x \to \infty} \left(\frac{2x - 3}{2x + 5}\right)^{2x+1} = 1^{\infty} \;!!$

الصورة الاستنتاج.

$$\Rightarrow \lim_{x \to \infty} \frac{\left(\frac{2x-3}{2x}\right)^{2x+1}}{\left(\frac{2x+5}{2x}\right)^{2x+1}} = \lim_{x \to \infty} \frac{\left(1 + \frac{\frac{-3}{2}}{x}\right)^{2x+1}}{\left(1 + \frac{\frac{5}{2}}{x}\right)^{2x+1}}$$

لـ الصورة الاستنتاج.

This is the key!

$$= \frac{e^{\left(\frac{-3}{2}\right)2}}{e^{\left(\frac{5}{2}\right)2}} = \frac{e^{-3}}{e^{5}} = \boxed{e^{-8}}$$

Ex.

a- $\lim_{x \to \infty} \left(2 + \frac{3}{x}\right)^{x} = 2^{\infty} = \infty$

b- $\lim_{x \to \infty} \left(\frac{1}{2} + \frac{3}{x}\right)^{x} = \frac{1}{2}^{\infty} = \text{zero}$

11 . Dec . 20

CHAPTER #5 Integration

Sum $\equiv \int$

* Indefinint Integrals :- $\int f_{(x)}\, d_{(x)} = F(x) + c$

*such that $F'(x) = f(x)$

* untiderivatives :- $F(x)$

تكامل

$F(x) \equiv$ untiderivatives of $f_{(x)}$

$F(x) + c \equiv$ General derivaatives

Ex. ① $\int x\, d_x = \frac{x^2}{2} + c$

find the general unit derivatives of x

② $\int x\, d_x = \frac{x^2}{2} + c$

③ find an untiderivatives of x

$\int x\, d_x = \frac{x^2}{2} + 0$

يمكن وضع اي رقم ما عندنا

* Rules *

$\boxed{1}$ $\int f(x) \pm g(x)\, dx = \int f(x)\, dx \pm \int g(x)\, dx$

$\boxed{2}$ $\int a\, f(x)\, dx = a \int f(x)\, dx$

$\boxed{3}$ $\int df = \int f'(x)\, dx = f(x) + c$

① $\int x^n\, dx = \frac{x^{n+1}}{n+1} + c$

② $\int x^{-1}\, dx = \int \frac{1}{x}\, dx = \ln x + c$

③ $\int \frac{f'}{f}\, dx = \ln f(x) + c$

④ $\int e^x\, dx = e^x + c$

⑤ $\int a^x\, dx = \frac{a^x}{\ln a} + c$

⑥ $\int \sin x\, dx = -\cos x + c$

⑦ $\int \cos x\, dx = \sin x + c$

⑧ $\int \sec^2 x\, dx = \tan x + c$

⑨ $\int \csc^2 x\, dx = -\cot x + c$

⑩ $\int \sec x \tan x\, dx = \cancel{\tan}\ \sec x + c$

⑪ $\int \csc x \cot x\, dx = -\csc x + c$

⑫ $\int \tan x\, dx = -\ln(\cos x) + c \rightarrow \ln(\cos x)^{-1} + c$

$\rightarrow \ln \sec + c$

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(13) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$

or $-\cos^{-1} x + c$

* $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$

Ex. $\int \frac{dx}{\sqrt{4 - x^2}} = \sin^{-1}\left(\frac{x}{2}\right) + c$

(14) $\int \frac{dx}{1+x^2} = \tan^{-1} x + c$

* $\int \frac{dx}{a^2 + x^2} = \tan^{-1}\left(\frac{x}{a}\right) + c$

Examples :-

1- $\int \frac{x^3 + 2x + \sqrt{x}}{\sqrt{x}} \, dx =$

$\to \int \frac{x^3}{\sqrt{x}} + \frac{2\cancel{x}^{\sqrt{x}}}{\cancel{\sqrt{x}}} + 1 \, dx \Rightarrow \int x^{\frac{5}{2}} + 2\sqrt{x} + 1 \, dx$

$\to \frac{2}{7}\sqrt{x^7} + \frac{4}{3}\sqrt{x^3} + x + c$

2- $\int x\sqrt{1+x^2} \, dx$

Let $u = 1 + x^2$

$\frac{du}{\cancel{dx}} = 2x \, dx$

$\int \sqrt{u} \, \frac{du}{2}$

$= \frac{1}{3}\sqrt{u^3} + c \longrightarrow \frac{1}{3}\sqrt{(1+x^2)^3} + c$

$= \frac{1}{3}\left(\sqrt{1+x^2}\right)^3 + c$

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* Integration by subslisturaint

$$\int f_{(g(x))}\, g'_{(x)}\, dx$$

أي انه إذا كان في الإقتران وبجانبه مشتقه فإنه يستخدم التكامل بالتعويض

then $u = g_{(x)}$

$du = g'_{(x)}\, dx$

$\rightarrow \int f_{(u)}\, du$

Ex. ① $\int \frac{(\ln x)^2}{x}\, dx$ $\quad u = \ln x$, $\quad du = \frac{1}{x}\, dx$

$\rightarrow \int u^2\, du$

$\frac{u^3}{3} + c \longrightarrow \frac{(\ln x)^3}{3} + c$

② $\int x^2 \sqrt{x^3+1}\, dx$ $\quad u = x^3 + 1$, $\quad du = 3x^2\, dx$

$\rightarrow \int \sqrt{u}\, \frac{du}{3} \rightarrow \frac{1}{3} * \frac{2}{3} \sqrt{u^3} + c$

$\frac{2}{9} \sqrt{(x^3+1)^3} + c$

imp ③ $\int x \sqrt{x-1}\, dx$ $\quad u = x-1$, $\quad u+1 = x$, $\quad du = dx$

$\rightarrow \int (u+1)\sqrt{u}\, du$

$\int u\sqrt{u} + \sqrt{u}\, du \rightarrow \frac{2}{5}\sqrt{u^5} + \frac{2}{3}\sqrt{u^3} + c$

$\rightarrow \frac{2}{5}\left(\sqrt{x-1}\right)^5 + \frac{2}{3}\sqrt{(x-1)^3} + c$

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(4) $\int e^x \sqrt{1+e^x}\, dx$ $u = 1+e^x$, $du = e^x\, dx$

$\int \sqrt{u}\, du$

$\frac{2}{3}\sqrt{u^3} + c \longrightarrow \frac{2}{3}\left(\sqrt{1+e^x}\right)^3 + c$

imp

Ex. $\int \frac{1+x}{\sqrt{3-x^2}}$

$\rightarrow \int \frac{1}{\sqrt{3-x^2}} + \frac{x}{\sqrt{3-x^2}}\, dx$ → by subslisttutaint

$\rightarrow \int (3-x^2)^{-1/2} dx + \int \frac{x}{\sqrt{3-x^2}}\, dx$ $u = 3-x^2$, $du = -2x\, dx$

$\rightarrow \int \frac{1}{\sqrt{3-x^2}} dx + \int \frac{du}{-2\sqrt{u}}$

$\sin^{-1}\left(\frac{x}{\sqrt{3}}\right) + \cancel{-\frac{1}{2}} * \cancel{2}\sqrt{u} + c$

$\rightarrow \sin^{-1}\left(\frac{x}{\sqrt{3}}\right) - \sqrt{3-x^2} + c$

Ex. $\int \frac{2+x}{4+x^2}\, dx$

$\rightarrow \int \frac{2}{4+x^2} + \frac{x}{4+x^2}\, dx$

$\rightarrow 2\cdot\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{2}\ln(4+x^2) + c$

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Ex. $\int \frac{e^{\frac{1}{x}}}{x^2}\,dx$ $\qquad u = \frac{1}{x}$

$du = \frac{-1}{x^2}\,dx$

$\rightarrow \int -e^{u}\,du$

$= -e^{u} + c \Rightarrow -e^{\frac{1}{x}} + c$

Ex. $\int \frac{e^{4x}}{1+e^{4x}}\,dx$

$= \frac{1}{4}\ln\left(1+e^{4x}\right) + c$

Ex. $\int \frac{e^{2x}}{1+e^{4x}}\,dx$ $\qquad u = e^{2x}$

للتوضيح $\qquad du = 2e^{2x}\,dx$

$= \int \frac{e^{2x}}{1+\left(e^{2x}\right)^2}\,dx \rightarrow \int \frac{1}{2} \frac{du}{(1+u^2)}$

$= \frac{1}{2}\int \frac{du}{(1+u^2)} = \frac{1}{2}\tan^{-1}\left(\frac{u}{1}\right) + c = \frac{1}{2}\tan^{-1}\left(\frac{e^{2x}}{1}\right) + c$

* إذا كانت درجة البسط اكبر من او تساوي درجة المقام نستخدم طريقة القسمة.

Ex. $\int \frac{u^2+3}{u^2+2}\,du$

$$\begin{array}{r|l} & 1 \\ \hline u^2+2 & u^2+3 \\ & -(u^2+2) \\ \hline & 1 \end{array}$$

So $\int 1 + \frac{1}{u^2+2}\,du$

$\rightarrow u + \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + c$

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* definit integrals :-

* f is continous on interval $[a, b]$

then $\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b$

$= F(b) - F(a) \in \mathbb{R}$ and its a real number

Ex. $\int_e^{e^2} \frac{dx}{x\sqrt{\ln x}}$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int_1^2 \frac{du}{\sqrt{u}}$

$x = e^2 \longrightarrow u = 2$

$x = e \longrightarrow u = 1$

$= 2\sqrt{u}\Big|_1^2 = (2 * \sqrt{2}) - (2 * \sqrt{1})$

$= 2\sqrt{2} - 2$

$- \int_{e^2}^{e} \frac{dx}{x\sqrt{\ln x}} = -2\sqrt{2} + 2$ ((العكس))

* properities of difinit integral :-

① $\int_a^a f(x) = \text{zero}$

② $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

③ $\int_c^b f(x)\,dx + \int_a^c f(x)\,dx = \int_a^b f(x)\,dx$

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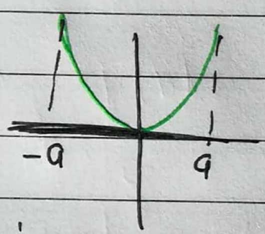
(4) $$\int_{-a}^{a} f(x)\, dx$$

if it's even

$= 2 \int f(x)\, dx$



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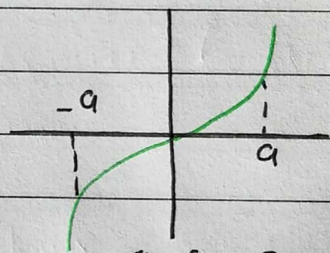
الموجب لذا تساوي ضعف المساحة

if it's odd

= zero , f is odd



المساحات متساوية ولكن
واحد مساحة موجبة
وواحد مساحة سالبة
لذا تساوي الصفر