17

The University of Jordan
Department of Mathematics
Calculus III, First Exam

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Instructor's Name: Dr. Hassan Lecture's time: 9:30 - 11:00

Q1) (4 marks) Determine whether the points $A(1,3,-1)$, $B(2,3,2)$, $C(3,1,2)$ and $D(2,6,2)$ lie on the same plane.

$|\vec{AB}| = |\langle 1,0,3\rangle| = \sqrt{10}$

$|\vec{AC}| = |\langle 2,-2,3\rangle| = \sqrt{17}$

$|\vec{AD}| = |\langle 1,3,3\rangle| = \sqrt{19}$

Not on the same plane

Q2)(4 marks) Consider the line L : 2x = y = 4z and the plane P : 2x − 2y + 4z = 3. Show that the line L is parallel to the plane P and find the distance between them.

4

$L;\ x = \frac{1}{2}t$

$y = t$

$z = \frac{1}{4}t$

$V_L = \langle \frac{1}{2}, 1, \frac{1}{4} \rangle$

$n_P = \langle 2, -2, 4 \rangle$

$\frac{x}{\frac{1}{2}} = \frac{y}{1} = \frac{z}{\frac{1}{4}}$

$L \parallel P$ if $V_L \perp n_P$

$V_L \cdot n_P = 0$

$(\frac{1}{2} \cdot 2) + (1 \cdot -2) + \frac{1}{4} \cdot 4$

$1 + -2 + 1 = 0$

so its $\perp$ then $L \parallel P$

تكملة السؤال خلف الورقة لحل المسافة بينهما

Q3) (4 marks) Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and the angle between them is $\frac{\pi}{6}$. Find the angle between $\vec{u}$ and $\vec{v}$ where $\vec{u} = 3\vec{a} + \vec{b}$ and $\vec{v} = \vec{a} - 4\vec{b}$.

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} \Rightarrow \cos\theta = \frac{(3\vec{a}+\vec{b})\cdot(\vec{a}-4\vec{b})}{|3a+b|\,|\vec{a}-4b|}$$

$a \cdot b = |a||b|\cos\theta$

$a \cdot b = \sqrt{3} \cdot 2 \cos\frac{\pi}{6}$

$a \cdot b = 3$

$$\Rightarrow \frac{(3|a|^2 - 12\,a\cdot b) + (\vec{b}\cdot\vec{a} - 4|b|^2)}{(3\sqrt{3}+2)\cdot(\sqrt{3}-8)}$$

$$\Rightarrow \cos\theta = \frac{(9-6)+(3-16)}{(3\sqrt{3}+2)\cdot(\sqrt{3}-8)} = \frac{-10}{(3\sqrt{3}+2)(\sqrt{3}-8)}$$

Q4) (4 marks) Find parametric equations of the line tangent to the curve $\vec{r}(t) = < t^2, \frac{4}{1-t}, 9-t^2 >$ at the point $(9, 1, 0)$.

$r'(t) = < 2t, \frac{4}{(1-t)^2}, -2t >$

$4(1-t)^{-1}$

$-4(1-t)^{-2} \cdot -1$

$= (1-t)^{-2}$

we need solution give point $(9,1,0)$ from $\vec{r}(t)$

$x = t = \pm 3$

$y = -3$

$z = \pm 3$

$\Rightarrow$ المطلوب تحل

بنوخذ $\boxed{-3}$ بنعوضها ب $r'(t)$

$\Rightarrow r'(t) = < -6, \frac{4}{16}, 6 >$

Q5) (2 marks) Let $\vec{r}(t) = t^2\,\vec{u}(2t-1)$, $\vec{u}(5) = < 2, -1, 4 >$ and $\vec{u}'(5) = < 2, 3, -3 >$. Find $\vec{r}'(3)$.

$r'(t) = 2t \cdot \vec{u}(2t-1) + t^2 \cdot u'(2t-1)\,2$

$r'(3) = 6 \cdot \vec{u}(5) + 9 \cdot u'(5) \cdot 2$

$r'(3) = <12, -6, 24> + 18\,(2, 3, -3)$

$r'(3) = <12, -6, 24> + <36, 54, -54> = <48, 48, -30>$

Q4) parametric equation

$x = 9 - 6s$

$y = 1 + \frac{1}{4}s$

تكملة الحل

Q6) (4 marks) Find the equation of the plane contains the two lines $x = 1-2t,\ y = 3t,\ z = 4+t$ and $x = 4s,\ y = 1-6s,\ z = 3-2s$.

Plane equation

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

$VL \times VL = n$ [illegible]

This two planes is parallel

$$\boxed{\begin{aligned} V_{L_1} &= \langle -2, 3, 1\rangle \\ V_{L_2} &= \langle 4, -6, -2\rangle \end{aligned}} \times = \begin{array}{c} \begin{matrix} i & j & k \end{matrix} \\ \begin{vmatrix} -2 & 3 & 1 \\ 4 & -6 & -2 \end{vmatrix} \end{array} = \langle 0, 0, 0\rangle$$

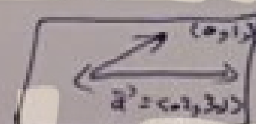
$VL_1 \parallel VL_2$

$i(-6+6) - j(4-4) + k(12-12)$

$0 - 0 + 0$

[illegible]

$\Longrightarrow$ كملة الصفحة حل السؤال



3

Q7) (4 marks) Classify and sketch the following surfaces

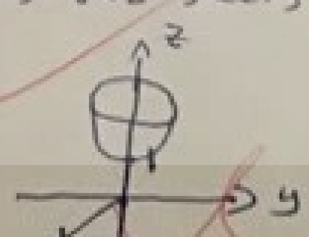
a) $y^2 + 5x^2 = 3$.

$$\frac{y^2}{3} + \frac{5x^2}{3} = 1 \qquad \text{Ellipse}$$

b) $z = \sqrt{x^2+y^2+1}$.

$z^2 = x^2+y^2+1$

$z^2 - x^2 - y^2 = 1$ hyperbolid of two sheets along z-axis



5