Q1) (4 points)

a) **Let** $\vec{u} =< 2, 4, -1 >$, $\vec{v} =< 3, 2, 5 >$. **Find** $proj_{\vec{u}}\vec{v}$.

$|\bar{v}| = \sqrt{9+4+25} = \sqrt{38}$

$|\bar{u}| = \sqrt{4+16+1} = \sqrt{21}$

$$proj_{\bar{u}} \bar{v} = \frac{|\bar{v}.\bar{u}|}{|\bar{u}|} \cdot \frac{1}{|\bar{u}|}\bar{u}$$

$$= \frac{|\bar{u}.\bar{v}|}{|\bar{u}|^2} \cdot \bar{u} = \left| \frac{6+8-5}{21} \right| \cdot <2,4,-1>$$

2

$$= \frac{9 . <2,4,-1>}{21} = \frac{<18, 36, -9>}{21}$$

b) **Find the intersection of the line:** $x - 3 = 2y - 5 = 3z$ **with the** yz**-plane.**

$\{$line: $x-3=t$

$\boxed{x = t+3}$

$t+3 = 2y-5$

$2y = t+3+5$

$2y = t+8$

$\boxed{y = \frac{1}{2}t + 4}$

$t+3 = 3z$

$\boxed{z = \frac{1}{3}t + 1}$

yz-plane $\downarrow$ $\boxed{x=0}$

when $\boxed{x=0}$

$0 = t+3$

$\therefore \boxed{t = -3}$

So $\rightarrow$

$y = \frac{1}{2}(-3) + 4$

$\boxed{y = \frac{-3}{2} + 4}$

$z = \frac{1}{3}(-3) + 1$

$\boxed{z = 0}$

$*$ point of intersection

$(0, \frac{-3}{2}+4, 0)$

Q2) (4 points) Let $|\vec{a}| = 2$, $|\vec{b}| = 3$ **and** $|2\vec{a} - \vec{b}| = 5$, **find** $\vec{a} \cdot \vec{b}$.

$\bar{v}.\bar{v} = |\bar{v}|^2$

$$|2\bar{a} - \bar{b}|^2 = (2\bar{a} - \bar{b}).(2\bar{a} - \bar{b})$$

$$25 = 4\bar{a}.\bar{a} - 2\bar{a}.\bar{b} - 2\bar{a}\bar{b} + \bar{b}.\bar{b}$$

$$25 = 4|\bar{a}|^2 - 4\bar{a}.\bar{b} + |\bar{b}|^2$$

$$25 = 4(4) - 4\bar{a}.\bar{b} + 9$$

$$25 = 25 - 4\bar{a}.\bar{b} \rightarrow 0 = -4\bar{a}.\bar{b}$$

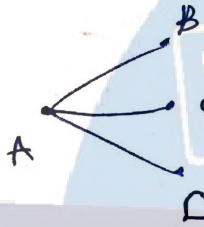
$$\underline{0 = \bar{a}.\bar{b}} \quad \therefore \bar{a} \perp \bar{b}$$

④

P.4 تكملة *

b) $3x - 3 + z - 4 = 0 \longrightarrow 3x + z - \underset{d}{\underline{\underline{7}}} = 0$ ✓

c)



$\overline{AB} = \langle 2, -1, -6 \rangle$

$\overline{AC} = \langle 1, -1, -3 \rangle$

$\overline{AD} = \langle 1, 2, 2 \rangle$

$\overline{AC} \times \overline{AB} = \langle 3, 0, 1 \rangle$

$\overline{AD} \cdot (\overline{AC} \times \overline{AB}) = \langle 1, 2, 2 \rangle \cdot \langle 3, 0, 1 \rangle = 3 + 0 + 2 = 5 \neq 0$ ✓

$\therefore$ Not on the Same plane

✓

Q3) (4 points) Identify and sketch the following surfaces

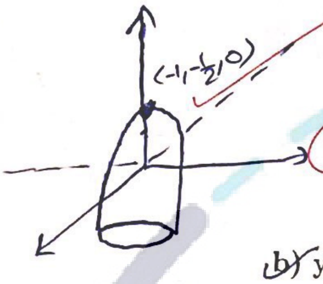
[illegible] a) $x^2 + 2y^2 + 2x + 2y + z = 0.$

$x^2+2x+1+2y^2+2y+1+z=0+1+1$

$(x+1)^2+(y+\frac{1}{2})^2+z=2$

$(x+1)^2+(y+\frac{1}{2})^2=2-z$

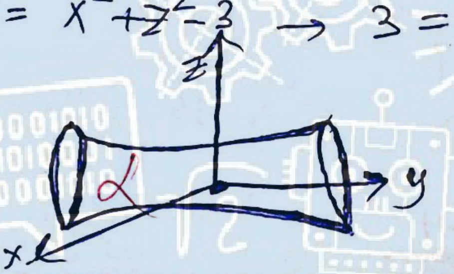
2



elliptic
parabolaid
~~separated~~
on Z-axis
at center $(-1,-\frac{1}{2},0)$

b) $y = \sqrt{x^2 + z^2 - 3}.$

$y^2= x^2+z^2-3 \rightarrow 3=x^2+z^2-y^2$



2

Hyperboloid
of one sheet
about y-axis
at center (0,0,0)

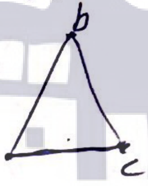
Q4) (7 points) Consider the points A(1, 3, 4), B(3, 2, −2), C(2, 2, 1) and D(2, 5, 6).

a) Find the area of the triangle ABC.

b) Find the equation of the plane that contains the points A, B and C.

c) Determine whether the points A, B, C and D lie on the same plane.

a)



Area = $\frac{1}{2}\,|\overline{AC}\times\overline{AB}|$

$\overline{AC}=\langle 1,-1,-3\rangle$

$\overline{AB}=\langle 2,-1,-6\rangle$

$\rightarrow \overline{AC}\times\overline{AB}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -3 \\ 2 & -1 & -6 \end{vmatrix}$

$=(6-3)\hat{i}-(-6+6)\hat{j}+(-1+2)\hat{k}$

$=3\hat{i}+\hat{k}=\langle 3,0,1\rangle$

$|\overline{AC}\times\overline{AB}|=\sqrt{9+0+1}=\sqrt{10}$

$\therefore$ Area $=\frac{\sqrt{10}}{2}$

4

b) $\overline{AC}\times\overline{AB}=\langle \overset{a}{3},\overset{b}{0},\overset{c}{1}\rangle$

equation of the plane : ~~[illegible]~~

$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

$3(x-1)+0(y-3)+1(z-4)=0$

2

← التكملة في الصفحة المجاورة

Q5) (6 points) Consider the line L: $x = 2 + 3t$, $y = 1 + t$, $z = 5 - 2t$ and the plane P: $3x - y + 4z = 2$. $\rightarrow 3X - y + 4Z - 2 = 0$

(a) Show that the line L is parallel to the plane P.

(b) Find the distance between the line L and the plane P.

a)

from the line $\rightarrow U_1 = < 3, 1, -2 >$

from the plane $\rightarrow V_2 = < 3, -1, 4 >$

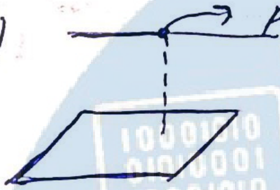
$\rightarrow \bar{V}_1 \cdot \bar{V}_2 = 9 - 1 - 8 = 9 - 9 = 0$

$\therefore \bar{V}_1 \perp \bar{V}_2$

3

$\therefore$ line is parallel to the plane

b)



$t = 0 \rightarrow (\underset{x}{2}, \underset{y}{1}, \underset{z}{5})$

distance between point and plane $= \dfrac{|a(x) + b(y) + c(z) + d|}{\sqrt{a^2 + b^2 + c^2}}$

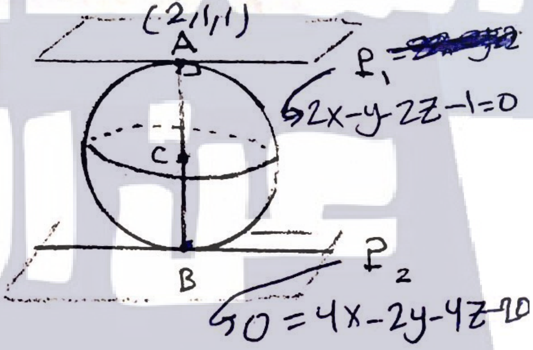
التكملة في المقابل

Q6) (7 points) Consider the two planes P_1: $2x - y - 2z = 1$ and P_2: $4x - 2y - 4z = 20$. Let S be a sphere between the two planes that touches the plane P_1 at the point $A(2, 1, 1)$ and touches the plane P_2 at the point B.

(a) Show that the two planes are parallel.

(b) Find the point B.

(c) Find the equation of the sphere S.



a) plane 1 $\rightarrow \bar{U}_1 = < 2, -1, -2 >$

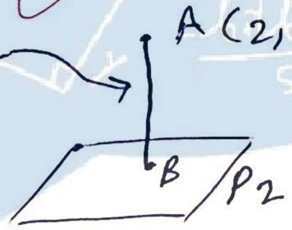
plane 2 $\rightarrow \bar{U}_2 = < 4, -2, -4 >$

$\frac{2}{4} \overset{?}{=} \frac{-1}{-2} \overset{?}{=} \frac{-2}{-4} \rightarrow \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$ ✓

$\therefore \bar{U}_1 \parallel \bar{U}_2$ $\therefore$ they are parallel (the planes) ✓

✓

b) if I want to put a line then the equation of the line is:



$X = x_0 + at = 2 + 4t$

$y = y_0 + bt = 1 - 2t$

$Z = z_0 + ct = 1 - 4t$

$\longrightarrow$ then we use them in the equation of the plane:

$0 = 4(2 + 4t) - 2(1 - 2t) - 4(1 - 4t) - 2$

$0 = 8 + 16t - 2 + 4t - 4 + 16t - 20$

التكملة في المقابل $\longleftarrow$ $20 = 2 + 36t \rightarrow 18 = 36t$

$\boxed{\frac{1}{2} = t}$

Q.5 [illegible] *

b)

distance $= \dfrac{|3(2)+4(5)-1(1)-2|}{\sqrt{9+1+16}} = \dfrac{|23|}{\sqrt{26}} = \dfrac{23}{\sqrt{26}}$ ✓

③

Q6 [illegible] *

b) $X = 2+4(\frac{1}{2}) = 4$

$y = 1-2(\frac{1}{2}) = 0$ $\therefore B = (4,0,-1)$ ✓

$Z = 1-4(\frac{1}{2}) = -1$

c) Midpoint between A and B is: $\boxed{A(2,1,1)}$

$\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}, \dfrac{z_1+z_2}{2}\right) = \left(\dfrac{6}{2}, \dfrac{1}{2}, 0\right) = (\underset{a}{3}, \underset{b}{\tfrac{1}{2}}, \underset{c}{0})$

and this is the center of the sphere

→ distance between A and the center is:

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$$= \sqrt{(-1)^2+(\tfrac{1}{2})^2+(1)^2} = \sqrt{1+\tfrac{1}{4}+1} = \sqrt{2+\tfrac{1}{4}}$$

⑥

$$= \sqrt{\tfrac{8}{4}+\tfrac{1}{4}} = \sqrt{\tfrac{9}{4}} = \dfrac{\sqrt{9}}{2}$$

and this is the radius

So the equation of the sphere is:

$r^2 = (x-a)^2+(y-b)^2+(z-c)^2$

$\frac{9}{4} = (x-3)^2+(y-\frac{1}{2})^2+(z)^2$ ✓