Excellent $\frac{29}{30}$

الجامعة الأردنية	0301201 تفاضل وتكامل 3	الامتحان الأول: الأربعاء 2017/11/1
اسم الطالب: مرام محمد صبحي العتلة الرقم الجامعي:	$\frac{36}{37}$	مدرس المادة: سلام الناطيين وقت المحاضرة: 1.00 – 2.00

يتكون الامتحان من 6 أسئلة في 3 ورقات

$\boxed{14}$

[1] (4 marks) Find the distance between the sphere $x^2 + (y-2)^2 + (z+3)^2 = 25$ and the plane $3x + 2y - z + 20 = 0$.

4

center $(0, 2, -3)$

$\boxed{r = 5}$

Distance between plane and part (center)

$$D_1 = \frac{|3(0) + 2(2) + (3) + 20|}{\sqrt{(3)^2 + (2)^2 + (-1)^2}} = \frac{27}{\sqrt{14}}$$

Destance between plane and border of sphere

$$D_2 = D_1 - r \rightarrow \left|\frac{27}{\sqrt{14}} - 5\right|$$

D_2

[2] (6 marks) Consider the parallelogram (متوازي أضلاع) $ABCD$, with vertices: $A(2,-1,4), B(1,0,-1), C(1,2,3)$.

(a) Find D.

1.5

$|\vec{AB} \times \vec{BC}| = |\vec{AD} \times \vec{AB}|$

$\vec{AD} = \langle (x-2), (y+1), (z-4) \rangle$

$\vec{AB} = \langle -1, 1, -5 \rangle$

$\vec{BC} = \langle 0, 2, 4 \rangle$

$D_1 = D_2$

$\sqrt{(x-2)^2 + (y+1)^2 + (z-4)^2}$

$\boxed{(x-2)^2 + (y+1)^2 + (z-4)^2 = 13}$

$D = \sqrt{0 + 4 + 9} = \sqrt{13}$

حلك غير صحيح

(b) Find the area of ABCD

بالكلمة

4 Area $= |\vec{AB} \times \vec{BC}|$

صحح بالكلمة الضرب

$= |\langle 14i + 4j + 2k \rangle|$

$= \sqrt{(14)^2 + (4)^2 + (2)^2} = \sqrt{(14)^2 + 20}$

4

[3] (4 marks) Find parametric equations of the line passing through the point $(2,-1,3)$ that makes an angle $\frac{\pi}{6}$ with x-axis, and angle $\frac{\pi}{3}$ with the y-axis.

direction for the line. $d = \langle |v| \cos\alpha, |v| \cos\beta, |v| \cos\gamma \rangle$

zero

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\gamma = 1$

$\frac{3}{4} + \frac{1}{4} + \cos^2\gamma = 1$

$\cos^2\gamma = 0$

$\boxed{\gamma = \frac{\pi}{2}}$

assume $|v|$ any factor number $(|v| = 1)$

$d = (1)\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$, Point $(2,-1,3)$

$L: \; x = 2 + \frac{\sqrt{3}}{2}t$

$y = -1 + \frac{1}{2}t$

$z = 3$

Scanned by CamScanner

8

[4] (8 marks) Consider the lines:

L_1: $x = t;\ y = 3 - 3t;\ z = -2 - t,$

L_2: $x = 1 + s;\ y = 4 + s;\ z = -1 + s.$

(a) Find the point of intersection between the two lines.

$L_1 = L_2$

① $t = 1 + s$

② $3 - 3t = 4 + s$ $3 = 3$ $\boxed{s = -1}$ $\boxed{t = 0}$

③ $-2 - t = -1 + s$

$(0, 3, -2)$ Point of intersection.

① & ③

$t = 1 + s$

$-2 - t = -1 + s$

$-2 = 2s$ $\boxed{s = -1}$ $\boxed{t = 0}$

(b) Find the acute angle (زاوية حادة) between the two lines.

$d_1 = \langle 1, -3, -1 \rangle$

$d_2 = \langle 1, 1, 1 \rangle$

$$\cos\theta = \frac{d_1 \cdot d_2}{|d_1||d_2|} = \frac{1 + (-3) + (-1)}{\sqrt{1+9+1} \cdot \sqrt{3}} = \frac{-3}{\sqrt{11}\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{+3}{\sqrt{11}\sqrt{3}}\right)$$

(c) Find the plane containing the two lines. — intersected so $d_1 \times d_2 = \vec{n}$

$$d_{L_1} \times d_{L_2} = \vec{n} = \begin{vmatrix} i & j & k \\ 1 & -3 & -1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(-3 + 1) - \hat{j}(1 + 1) + \hat{k}(1 + 3)$$

$$\vec{n} = -2\hat{i} - 2\hat{j} + 4\hat{k} \rightarrow \langle -2, -2, 4 \rangle$$

Point on the plan = Point on the L_1 $(0, 3, -2)$

eq. $-2(x) + (-2)(y - 3) + 4(z + 2) = 0$

$$\boxed{-2x - 2(y - 3) + 4(z + 2) = 0}$$

$y = \sqrt{r-x}$

4

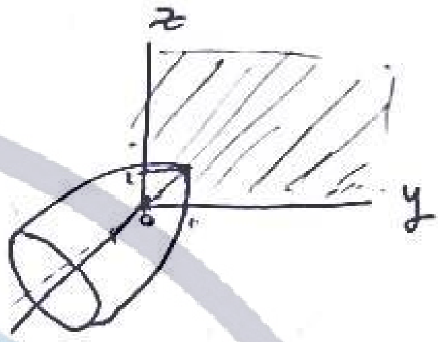
[5] (4 marks) Name and sketch the surfaces:

(a) $x = y^2 + z^2 - 2y - 4z + 5$

$x = (y^2 - 2y + 1) - 1 + (z^2 - 4z + 4) - 4 + 5$

$x = (y-1)^2 + (z-2)^2$

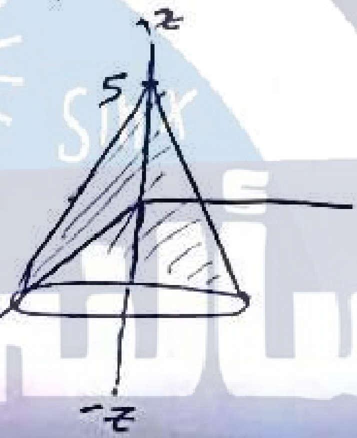
Paraboloid in x-axis. at center $(0,1,2)$



(b) $z = 5 - \sqrt{x^2 + y^2}$

$z = -\sqrt{x^2+y^2} + 5$

the lower cone in -z axis and shift up to $\boxed{z=5}$



3

[6] (4 marks) Let $\vec{r}(t)$ be the vector valued function determined by the curve of intersection of the two surfaces: the semi sphere $x^2 + y^2 + z^2 = 4$, $y \geq 0$, and the cylinder $x^2 + z^2 = 1$.

(a) Find $\vec{r}(t)$

$\boxed{x^2 + z^2 = 1}$

$x^2 + y^2 + z^2 = 4$

$1 + y^2 = 4$

$y^2 = 3$

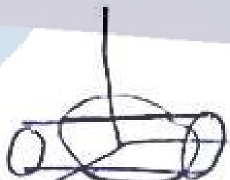
$\boxed{y = \sqrt{3}}$ at $y \geq 0$ $\boxed{y = +\sqrt{3}}$

assume $\boxed{x = t}$

$z = \sqrt{1 - x^2}$

$\boxed{z = \sqrt{1-t^2}}$

$\vec{r}(t) = \langle t, \sqrt{3}, \sqrt{1-t^2} \rangle$



(b) Sketch the curve of $\vec{r}(t)$.

$\vec{r}(t) = \langle t, \sqrt{3}, \pm\sqrt{1-t^2} \rangle$

$x = t$

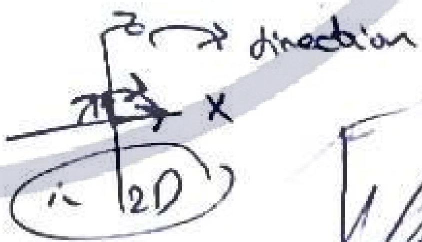
$y = \sqrt{3}$

$z = \pm\sqrt{1-t^2}$

$\boxed{z = \sqrt{1-x^2}}$.

half circle with $\boxed{r=1}$ cut $(0,\sqrt{3},0)$

at $y = \sqrt{3}$



at $t=0$ $(0, \sqrt{3}, 1)$

$t=1$ $(1, \sqrt{3}, 0)$.