25

The University of Jordan
Department of Mathematics
Calculus III, First Exam

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Q1) (4 marks) Determine whether the points $A(1,2,-1)$, $B(2,3,1)$, $C(3,-1,2)$ and $D(2,8,2)$ lie on the same plane.

$\vec{AB} = \langle 1, 1, 2 \rangle \quad \vec{AC} = \langle 2, -3, 3 \rangle$

$\vec{AB} \times \vec{AC} = \begin{vmatrix} 1 & 1 & 2 \\ 2 & -3 & 3 \end{vmatrix} = \langle 9, 1, -5 \rangle$ Point $(1, 2, -1)$

eq of the plane $\Rightarrow 9(x-1) + (y-2) - 5(z+1) = 0$

is D on the plane?? $D(2,8,2)$

$9 + 6 - 15 = 0$

$0 = 0$

these Points are in the same plane

4

Q2)(4 marks) Consider the line $L: x = 2y = 4z$ and the plane $P: 2x - 2y - 4z = 5$. Show that the line L is parallel to the plane P and find the distance between them.

$x = t$
$y = \frac{1}{2}t$
$z = \frac{1}{4}t$

$L \parallel P \rightsquigarrow V_L \perp n_P \Rightarrow V_L \cdot n_P = 0$

$V_L = \langle 1, \frac{1}{2}, \frac{1}{4} \rangle \quad n_P = \langle 2, -2, -4 \rangle$

$V_L \cdot n_P = (2 - 1 - 1) = 0$ so ~~they are~~ $L \parallel P$

2

Point on the line : $P(0,0,0)$

$\Rightarrow d = \dfrac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}} = \dfrac{|-5|}{\sqrt{24}} = \dfrac{5}{\sqrt{24}}$

$\Rightarrow$ the distance between

Q3) (4 marks) Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$ and the angle between them is $\frac{\pi}{3}$. Find the angle between $\vec{u}$ and $\vec{v}$ where $\vec{u} = 5\vec{a} + \vec{b}$ and $\vec{v} = \vec{a} - 2\vec{b}$.

$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|u||v|} = \frac{(5a+b)\cdot(a-2b)}{|5a+b||a-2b|} \Rightarrow \frac{5|a|^2 - 9(a.b) - 2|b|^2}{\sqrt{301} * 7}$

$|5a+b|^2 = (5a+b)\cdot(5a+b)$
$= 25|a|^2 + 10a.b + |b|^2$
$= (25)9 + 10(6) + 16$
$= 301$
$|5a+b| = \sqrt{301}$

$|a-2b|^2 = (a-2b)(a-2b)$
$= |a|^2 - 4a.b + 4|b|^2$
$= 9 - 24 + 64$
$= 49$
$|a-2b| = 7$

$\Rightarrow \frac{45 - 54 - 32}{\sqrt{301} * 7} = \frac{-41}{\sqrt{301} * 7}$

4

$\cos\theta = -0.337$

$\theta = 109.7$

تحويلة ($\to 180 - \theta = 70.26$

$\to$ angle between them

Q4)(4 marks) Find parametric equations of the line tangent to the curve $\vec{r}(t) = < t^2, \frac{3}{1-t}, 4 - t^2 >$ at the point $(4, 1, 0)$.

$r'(t) = < 2t, \frac{+3}{(1-t)^2}, -2t >$

$4 = t^2$
$\pm 2 = t$
$1 = \frac{3}{1-t}$ $\quad 1-t = 3$ $\quad 1 - 3 = t$
$\boxed{t = -2}$

$r'(-2) = < -4, \frac{1}{3}, 4 >$

$\Rightarrow$ eq of the line: $x = 4 - 4t$

4 $\quad y = 1 + \frac{1}{3}t$

$z = 4t$

Q5)(2 marks) Let $\vec{r}(t) = t^3\,\vec{u}(2t-1)$, $\vec{u}(3) = < 2, -1, 5 >$ and $\vec{u}'(3) = < 2, 2, -3 >$. Find $\vec{r}'(2)$.

$r'(t) = (3t^2)(\vec{u}(2t-1)) + 2u'(2t-1)(t^3)$

$r'(2) = (12)(\vec{u}(3)) + 2u'(3)(8)$

2

$r'(2) = (12)< 2, -1, 5 > + 16 < 2, 2, -3 >$

$r'(2) = < 24, -12, 60 > + < 32, 32, -48 >$

$r'(2) = < 56, 20, 12 >$

Q6) (4 marks) Find the equation of the plane contains the two lines $x = 1 - t,\ y = 2t,\ z = 4 + t$ and $x = 2s,\ y = 1 - 4s,\ z = 3 - 2s.$

$\vec{VL_1} = \langle -1, 2, 1 \rangle$ $\quad \vec{VL_2} = \langle 2, -4, -2 \rangle$

Point on L_1 $\Rightarrow P(1, 0, 4)$ $\quad \Rightarrow$ Point on L_2 $Q(0, 1, 3)$

$L_1 \parallel L_2$

$\vec{PQ} = \langle -1, 1, -1 \rangle$

$\vec{PQ} \times \vec{VL_1} = \begin{vmatrix} -1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix} = \langle 3, 2, -1 \rangle$ $\vec{n}_p$

$P(1, 0, 4)$

eq of the plane $\Rightarrow 3(x-1) + 2(y) - (z-4) = 0$

4

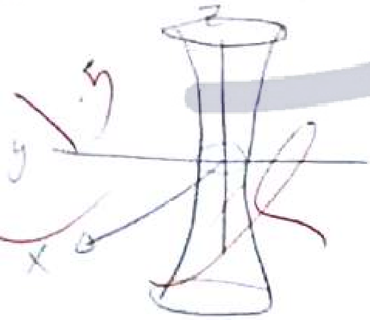
Q7) (4 marks) Classify and sketch the following surfaces

a) $y + 5x^2 = 3.$ $\rightarrow$ $y = 3 - 5x^2$ $\rightarrow$ parabola along the z axis

1.5

b) $z = \sqrt{x^2 + y^2 - 1}.$ $\quad z^2 = x^2 + y^2 - 1 \rightarrow x^2 + y^2 - z^2 = 1$

hyperboloid of one sheet along z axis



1.5