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The University of Jordan
Department of Mathematics
Calculus III, First Exam

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Instructor's Name: د. حسن النجار Lecture's time: 9:30 - 11:00

Q1) (5 marks) Consider the points $A(3,5,-1)$, $B(4,3,7)$ and $C(2,1,6)$. Find

a) $Comp_{\vec{BC}}\ \vec{BA}$.

$\vec{BA} = (-1, 2, -8)$

$\vec{BC} = (-2, -2, -1)$

$comp\ \vec{BA}_{\vec{BC}} = \frac{\vec{BA}\cdot\vec{BC}}{|\vec{BC}|} = \frac{2 - 4 + 8}{\sqrt{9}} = \frac{6}{3} = \boxed{2}$

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b) The area of the triangle ABC.

$\vec{AB} = (1, -2, 8)$

$\vec{AC} = (-1, -4, 7)$

$A = \frac{1}{2}|\vec{AB}\times\vec{AC}| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 8 \\ -1 & -4 & 7 \end{vmatrix} = \frac{1}{2}|18\hat{i} - 15\hat{j} + 6\hat{k}|$

$= \frac{1}{2}\sqrt{(18)^2 + (15)^2 + (6)^2}$

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Q2)(5 marks) Find the equation of the plane (P_2) passes through the point (1,2,1) that is perpendicular to the plane $z = 3x - 2y + 5$ (P_1) and parallel to the line (L_1) $x = 3t,\ y = 1 + 2t,\ z = 2 + 4t$

$\Rightarrow n_{P_1} = (3, -2, -1) \Rightarrow\ //\ P_2$

$\Rightarrow V_{L_1} = (3, 2, 4) \Rightarrow\ //\ P_2$

$\Rightarrow n_{P_1}\times V_{L_1} = n_{P_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & -1 \\ 3 & 2 & 4 \end{vmatrix} = (-6, -15, 12)$

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$\Rightarrow$ equ. of the plane: $-6(x-1) - 15(y-2) + 12(z-1) = 0$

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Q3) (3 marks) Find the intersection of the line $x = 4-2t$, $y = 5+5t$, $z = 3-t$ with xz−plane.

$y=0 \Rightarrow 0 = 5+5t \Rightarrow \boxed{t=-1}$

$x = 4+2 = 6$

$z = 3+1 = 4$

the point of intersection $(6,0,4)$

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Q4)(5 marks) Consider the Plane P : $x + 2y - 2z = 0$ and the sphere $x^2 + y^2 + z^2 - 2x - 6y + 2z + 2 = 0$.

a) Show that the plane P touches the sphere.

$\cancel{(x-1)^2 + (y+1)^2 + (z+1)^2 = 1}$

$\cancel{x + 2y - 2z = 0}$

$x=0$

$\Rightarrow 1 + (y+1)^2 + (z+1)^2 = 4$

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b) Find the point where the plane P touches the sphere. (Hint: the radius of the sphere is perpendicular to the plane P at the touching point)

$n_P = (1, 2, -2)$

Center $(1, 3, -1)$

$\boxed{x=1}$

$2y - 2z = -1 \rightarrow ①$

$2y = 2z - 1$

$\boxed{y = z - \frac{1}{2}} \rightarrow ①$

$(1-1)^2 + (y-3)^2 + (z+1)^2 = a$

$(z - \frac{1}{2} - 3)^2 + (z+1)^2 = a$

$z^2 - 7z + \frac{49}{4} + z^2 + 2z + 1 - a =$

$2z^2 - 5z + \frac{17}{4} = 0$

بنجيب قيمة z وبنعوضها

وبنجيب y

$(1, y, z) \rightarrow$ 11

$\vec{a} \perp \vec{b}$

Q5)(5 marks) Let $\vec{a}$ and $\vec{b}$ be orthogonal vectors such that $|\vec{a}| = 3$ and $|\vec{b}| = 2$, let $\vec{c} = 2\vec{a} + 4\vec{b}$. Find the angle between $\vec{b}$ and $\vec{c}$.

$\vec{a} \cdot \vec{b} = |a||b| \cos 90 = 0$

$\Rightarrow \vec{a} = \frac{1}{2}\vec{c} - 2\vec{b}$

$(\frac{1}{2}\vec{c} - 2\vec{b}) \cdot \vec{b} = 0$

$\frac{1}{2}\vec{c} \cdot \vec{b} = 2|b|^2$

$\vec{c} \cdot \vec{b} = 4(4)$

$\vec{c} \cdot \vec{b} = 16$

$|\vec{c}||\vec{b}| \cos\theta = 16$

$\Rightarrow |\vec{c}| = 2|\vec{a}| + 4|\vec{b}| = 2(3) + 4(2) = \boxed{14}$

$\Rightarrow 14(2)\cos\theta = 16$

$\cos\theta = \frac{\overset{4}{\cancel{16}}}{\underset{7}{\cancel{28}}}$

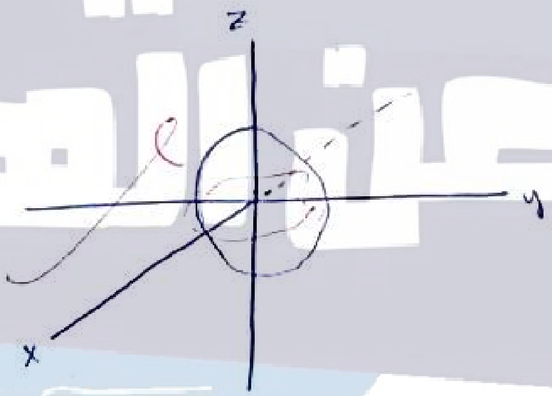
$\theta = \cos^{-1}\left(\frac{4}{7}\right)$

Q6) (4 marks) Classify and sketch the following surfaces

a) $x^2 + y^2 = 3 - 4z^2$.

$x^2 + y^2 + 4z^2 = 3$

Ellipsoid along z-axis



b) $y^2 = 2 + x^2 - 5z^2$.

$5z^2 + y^2 - x^2 = 2$

elliptical hyperboloid of one sheet along x-axis