The values of a and b that will make the two planes
$2x + a\,z = 3 - y$ and $3x + b\,y = 7 + 6z$ parallel.

(A) $a = -4$, $b = \frac{3}{2}$

(B) $a = 4$, $b = \frac{3}{2}$

(C) $a = -4$, $b = -\frac{3}{2}$

(D) $a = -4$, $b = \frac{2}{3}$

(E) $a = -4$, $b = -\frac{2}{3}$

$2x + y + a z = 3$ $\qquad$ $3x + by - 6z = 7$

$\vec{n}_1 = \langle 2, 1, a \rangle$ $\qquad$ $\vec{n}_1 = \langle 3, b, -6 \rangle$

$\vec{n}_1 \times \vec{n}_2 = 0$

$$\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & a \\ 3 & b & -6 \end{matrix}$$

$(-6 - ab)\hat{i} - (-12 - 3a)\hat{j} + (2b - 3)\hat{k}$

$(-6 - ab)\hat{i} - (-12 - 3a)\hat{j} + (2b - 3)\hat{k} = 0$

$-12 = 3a$ $\qquad$ $2b = 3$

$a = -4$ $\qquad$ $b = 3/2$ $\qquad$ $\boxed{A}$

If the area of the parallelogram determined by $\vec{a}$ and $\vec{b}$ is equal to 3 , then $\|(2\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\|$

(A) -9

(B) 9

(C) 3

(D) $\vec{0}$

(E) -3

- (A)
- (B)
- (C)
- (D)
- (E)

$\vec{a}\times\vec{b} = 3$

$|(2\vec{a}+\vec{b})\times(\vec{a}-\vec{b})|$

$= (|2\cancel{\vec{a}\times\vec{a}}^{\,0}| - |2\vec{a}\times\vec{b}| + \vec{b}\times\vec{a} - \cancel{\vec{b}\times\vec{b}}^{\,0}|$

$= |2\vec{a}\times\vec{b} + \vec{b}\times\vec{a}|$

$= |2\times 3 + 3| = 9$ $\boxed{B}$

The angle between the two planes

$x - 2y + z = 1$ and $3x + 3y + z = 2$ is

(A) $\cos^{-1}\left(\frac{-2}{\sqrt{6}\sqrt{19}}\right)$

(B) $\cos^{-1}\left(\frac{10}{\sqrt{6}\sqrt{14}}\right)$

(C) $\cos^{-1}\left(\frac{-10}{\sqrt{6}\sqrt{14}}\right)$

(D) $\cos^{-1}\left(\frac{1}{\sqrt{6}\sqrt{14}}\right)$

(E) $\cos^{-1}\left(\frac{2}{\sqrt{6}\sqrt{19}}\right)$

$\vec{n}_1 = \langle 1, -2, 1 \rangle \quad \vec{n}_2 = \langle 3, 3, 1 \rangle$

$\langle 1, -2, 1 \rangle \cdot \langle 3, 3, 1 \rangle = \sqrt{1+4+1} \times \sqrt{9+9+1} \cos\theta$

$3 - 6 + 1 = \sqrt{6}\sqrt{19} \cos\theta$

$\cos\theta = \dfrac{-2}{\sqrt{6}\sqrt{19}} \longrightarrow \theta = \cos^{-1}\left[\dfrac{-2}{\sqrt{6}\sqrt{19}}\right]$

$\boxed{A}$

The curve defined by the vector equation
$\vec{r}(t) = \langle 2\cos t, -2\sin t, 1\rangle$ is

(A) A circle centered at (0,0,1) with radius 2 traversed in the clockwise direction.

(B) A circle centered at (0,0,1) with radius 2 traversed in the counterclockwise direction.

(C) A circle centered at (0,0,2) with radius 2 traversed in the clockwise direction.

(D) A helix with axis is the z- axis traversed in the upward direction.

(E) A helix with axis is the z- axis traversed in the downward direction.

(A)

(B)

$r(t) = \langle 2\cos t, -2\sin t, 1\rangle$

$x = 2\cos t \quad y = -2\sin t$

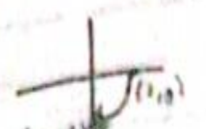
$x^2 = 4\cos^2 t \quad y^2 = 4\sin^2 t$

$x^2 + y^2 = 4\sin^2 t + 4\cos^2 t$

$x^2 + y^2 = 4 \quad \boxed{A}$ clockwise

let $t = 0 \Rightarrow (2,0)$

$t = \pi/2 \Rightarrow (0,-2)$



The equation $x^2 = y - z^2 + 2x - 2$ represents:

(A) A hyperbolic paraboloid centered at $(1,1,0)$ and axis is parallel to y-axis.

(B) A paraboloid with vertex $(1,1,0)$ and axis is parallel to y-axis

(C) A cone with vertex $(1,1,0)$ & axis is parallel to y-axis

(D) A hyperboloid of one sheet centered at $(1,1,0)$ & axis is parallel to y-axis.

(E) A hyperboloid of 2-sheets centered at $(1,1,0)$ & axis is parallel to y-axis.

$$x^2 = y - z^2 + 2x - 2$$

$$x^2 - 2x - y + z^2 = -2$$

$$x^2 - 2x + 1 - 1 - y + z^2 = -2$$

$$(x-1)^2 - 1 - y + z^2 = -2$$

$$(x-1)^2 + z^2 - y = -1$$

$$(x-1)^2 + z^2 = y - 1$$ (B) Parapoloid

If $\vec{v} = \langle \sqrt{2}c, c, c \rangle$, where $c > 0$ then the direction cosine β (with positive y −axis) is

A) $\frac{\pi}{2}$

B) $\frac{\pi}{6}$

C) $\frac{\pi}{4}$

D) $\frac{\pi}{3}$

E) $\frac{2\pi}{3}$

$\vec{v} = \langle \sqrt{2}c, c, c \rangle$, $c > 0$

$\cos\beta = \frac{c}{\sqrt{2c^2 + c^2 + c^2}} \Rightarrow \cos\beta = \frac{c}{c\sqrt{4}}$

$\cos\beta = \frac{1}{2}$

$\beta = \pi/3$ $\boxed{D}$

If $\vec{b} = -3i - 2j$ and $\vec{a} = 2i + j$, then the vector projection of $\vec{b}$ onto $\vec{a}$, $proj_{\vec{a}}\vec{b}$ is

A) $\langle -\frac{16}{5}, -\frac{8}{5} \rangle$

B) $\langle -\frac{6}{5}, -\frac{8}{5} \rangle$

C) $-\frac{8}{\sqrt{5}}$

D) $-\frac{16}{\sqrt{5}}i - \frac{8}{\sqrt{5}}j$

E) $\frac{24}{13}i + \frac{16}{13}j$

$$\vec{b} = -3\hat{i} - 2\hat{j} \qquad \vec{a} = 2\hat{i} + \hat{j}$$

$$proj_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|a|^2}\,\vec{a}$$

$$= \frac{\langle -3, -2\rangle \cdot \langle 2, 1\rangle}{\left(\sqrt{4+1}\right)^2} \cdot \langle 2, 1\rangle$$

$$= \frac{-6-2}{\sqrt{5}} \langle 2, 1\rangle$$

$$= \left\langle \frac{-16}{\sqrt{5}^2}, \frac{-8}{\sqrt{5}^2} \right\rangle$$

$$= \left\langle \frac{-16}{5}, \frac{-8}{5} \right\rangle \qquad \boxed{A}$$

Let $x^2 + y^2 + z^2 - 2x - 4y - 6z + 10 = 0$ be the equation of a sphere then the center and radius are

A) center $(1,2,3)$, radius $= 2$
B) center $(-1,-2,-3)$, radius $= 2$
C) center $(-1,2,-3)$, radius $= 3$
D) center $(1,2,-3)$, radius $= 3$
E) center $(1,-2,3)$, radius $= 4$

$x^2 + y^2 + z^2 - 2x - 4y - 6z + 10 = 0$

$x^2 - 2x + y^2 - 4y + z^2 - 6z + 10 = 0$

$x^2 - 2x + 1 - 1 + y^2 - 4y - 4 + 4 + z^2 - 6z + 9 - 9 + 10 = 0$

$(x-1)^2 - 1 + (y-2)^2 - 4 + (z-3)^2 - 9 + 10 = 0$

$(x-1)^2 + (y-2)^2 + (z-3)^2 = 4$

center $(1,2,3)$, Radius of 2 $\boxed{A}$

If the direction cosines of a vector $\vec{v}$ satisfy

$\cos\alpha = \frac{\sqrt{5}}{4}$, $\cos\beta = \frac{\sqrt{2}}{2}$, $\cos\gamma < 0$, then the vector $\vec{w}$ that has the length 4 and the same direction of $\vec{v}$ is

(A) $\langle\sqrt{5}, \sqrt{2}, -\sqrt{3}\rangle$

(B) $\langle\frac{5}{4}, 2, -\frac{3}{4}\rangle$

(C) $\langle-\frac{1}{4}, \frac{13}{16}, 2\rangle$

(D) $\langle-\sqrt{5}, -2\sqrt{2}, 2\sqrt{3}\rangle$

(E) $\langle\sqrt{5}, 2\sqrt{2}, -\sqrt{3}\rangle$

1) $\vec{w} = |w| \cdot \hat{u}$

$= 4 \langle \frac{\sqrt{5}}{4}, \frac{\sqrt{2}}{2}$, less than Zero $\rangle$

$= \langle \sqrt{5}, 2\sqrt{2}$, less than Zero $\rangle$

$= \boxed{E}$

The distance between the line **L:** $\frac{x+1}{2} = y + 2 = z - 3$ and the plane $x - y - z = 4$

(A) $\frac{6}{\sqrt{3}}$

(B) $\frac{2}{\sqrt{3}}$

(C) $\frac{4}{\sqrt{3}}$

(D) $\frac{8}{\sqrt{3}}$

(E) $\frac{3}{\sqrt{3}}$

2) Point on the line (let $t=0$) $\Rightarrow (-1, -2, 3)$

$$D = \frac{|1(-1) + -1(-2) + -1(3) \cancel{+} - 4|}{\sqrt{1+1+1}}$$

$$= \frac{6}{\sqrt{3}} = \boxed{A}$$

$2x^2 + y^2 + 3z^2 - 2y = 4$, represents

(A) cone

(B) hyperboloid of one sheet

(C) hyperboloid of two sheets

(D) ellipsoid

(E) paraboloid

3) $2x^2 + y^2 + 3z^2 - 2y = 4$

$2x^2 + y^2 - 2y + 1 - 1 + 3z^2 = 4$

$2x^2 + (y-1)^2 + 3z^2 = 5$

$\frac{2x^2}{5} + \frac{(y-1)^2}{5} + \frac{3z^2}{5} = 1 \Rightarrow$ ellipsoid $\boxed{D}$

The set of all points that lie between the xz −plane and the vertical plane $y = 4$ and inside the sphere with center $(0,0,-1)$ and radius 6 can be represented by the inequalities

(A) $xz < y < 4$ and and $x^2 + y^2 + z^2 + 2z \leq 36$.

(B) $0 < y < 4$ and $x^2 + y^2 + z^2 - 2z \leq 36$.

(C) $0 < y < 4$ and $x^2 + y^2 + z^2 + 2z = 35$.

(D) $0 < y < 4$ and $x^2 + y^2 + z^2 + 2z < 35$.

(E) $0 \leq y \leq 4$ and $x^2 + y^2 + z^2 - 2z \leq 35$.

4) inside the sphare $(0,0,-1)$:

$$x^2 + y^2 + (z+1)^2 < 36$$

$$x^2 + y^2 + z^2 + 2z + 1 < 36$$

$$x^2 + y^2 + z^2 + 2z < 35 \qquad \boxed{D}$$

Find the projection of $\overrightarrow{BC}$ onto $\overrightarrow{AB}$, $proj_{\overrightarrow{AB}}\overrightarrow{BC}$

where $A(1,2)$, $B(4,6)$, $C(5,5)$

(A) $\langle \frac{21}{25}, \frac{28}{25} \rangle$

(B) $\langle -\frac{1}{2}, \frac{1}{2} \rangle$

(C) $-\frac{3}{25}i - \frac{4}{25}j$

(D) $-\frac{1}{5}$

(E) $\langle -\frac{3}{5}, -\frac{4}{5} \rangle$

6) $\overrightarrow{BC} = \langle 1, -1 \rangle$, $\overrightarrow{AB} = \langle 3, 4 \rangle$

$$proj = \frac{\langle 1,-1 \rangle \cdot \langle 3, 4 \rangle}{\sqrt{3^2+4^2}} \cdot \langle 3, 4 \rangle$$

$$= \langle \frac{-3}{5}, \frac{-4}{5} \rangle \quad \boxed{E}$$

An equation of the plane through the point$(-2, 2, 1)$ and parallel to the plane $5x + z = 4 + 2y$, is

(A) $5(x-2) - 2(y+2) + (z+1) = 0$

(B) $5(x+2) + (y-2) - 2(z-1) = 0$

(C) $5(x-2) + 2(y+2) + (z+1) = 0$

(D) $5(x+2) - 2(y-2) + (z-1) = 0$

(E) $5(x+2) - 2(y-2) - (z-1) = 0$

7) $5(x+2) - 2(y-2) + z - 1 = 0$ $\boxed{D}$

Parametric equations of the line passing through the point $(2,-1,-3)$, and perpendicular to the two lines
L1: $x = 1+t$, $y = -2$, $z = -t$
L2: $x = 3$, $y = 2-2s$, $z = 2+s$ are

(A) $x = 2+2t$, $y = -1-t$, $z = -3-2t$

(B) $x = 2-2t$, $y = -1+t$, $z = -3+2t$

(C) $x = 2-2t$, $y = -1+t$, $z = -3-2t$

(D) $x = -2-2t$, $y = -1-t$, $z = -3+2t$

(E) $x = 2-2t$, $y = -1-t$, $z = -3-2t$

8) $\vec{v}_1 = \langle 1,0,-1 \rangle$ $\vec{v}_2 = \langle 0,-2,1 \rangle$

$\vec{v}_1 \times \vec{v}_2 \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 0 & -2 & 1 \end{vmatrix}$

$= -2\hat{i} - \hat{j} - 2\hat{k}$, through $(2,-1,-3)$

$x = 2-2t$
$y = -1-t$
$z = -3-2t$

$\boxed{E}$