

Change the integral $\int_0^\pi \int_0^2 \int_r^2 r^3 dz dr d\theta$ from cylindrical to Cartesian coordinates:

(A) $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^2 \sqrt{x^2+y^2} dz dy dx$

(B) $\int_{-2}^2 \int_0^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^2 (x^2+y^2) dz dy dx$

(C) $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^2 (x^2+y^2)^{\frac{3}{2}} dz dy dx$

(D) $\int_{-2}^2 \int_0^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^2 (x^2+y^2)^{\frac{3}{2}} dz dy dx$

(E) $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-\sqrt{x^2+y^2}}^{\sqrt{x^2+y^2}} (x^2+y^2) dz dy dx$

$$z = r, 2$$

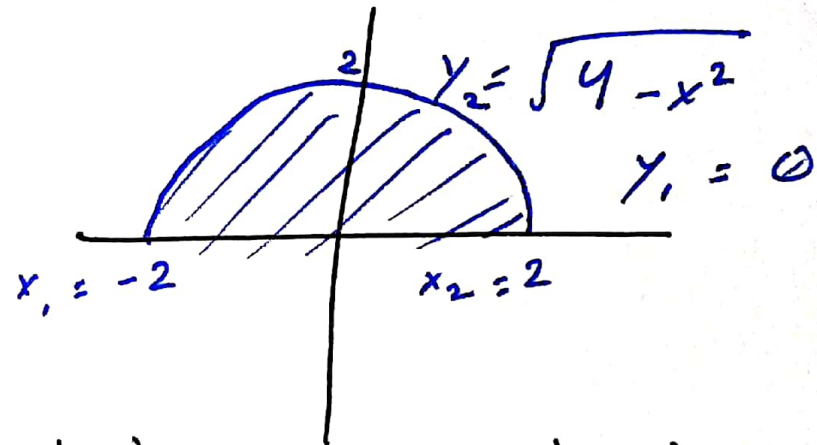
$$r \in [0, 2]$$

$$\theta \in [0, \pi]$$

$$z_2 = 2$$

$$z_1 = r, \quad r^2 = x^2 + y^2$$

$$z_1 = \sqrt{x^2 + y^2}$$



$$* (r) (dz dr d\theta) = dz dy dx$$

$r^3 \rightarrow (r^2)(r)$

$$\int_{-2}^2 \int_0^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^2 (x^2 + y^2) dz dy dx$$

(B)



To maximize and minimize $f(x, y) = e^{-xy}$, subject to the constraint $x^2 + 4y^2 = 1$ we need to solve these Lagrange equations:

(A) $-ye^{-xy} = 8\lambda y$

$$-xe^{-xy} = 2\lambda x$$

$$x^2 + 4y^2 = 1$$

(B) $-ye^{xy} = 2\lambda x$

$$-xe^{xy} = 8\lambda y$$

$$x^2 + 4y^2 = 1$$

(C) $-ye^{-xy} = 2\lambda x$

$$-xe^{-xy} = 8\lambda y$$

$$x^2 + 4y^2 = 1$$

(D) $ye^{-xy} = 2\lambda x$

$$xe^{-xy} = 8\lambda y$$

$$x^2 + 4y^2 = 1$$

(E) $e^{-xy} = \lambda x^2$

$$e^{-xy} = 4\lambda y^2$$

$$x^2 + 4y^2 = 1$$

$$f(x, y) = e^{-xy}$$

$$g(x, y) = x^2 + 4y^2 - 1$$

$$f_x = \lambda g_x$$

$$f_y = \lambda g_y$$

$$g(x, y) = k$$

$$-y e^{-xy} = \lambda (2x)$$

$$-x e^{-xy} = \lambda (8y)$$

$$x^2 + 4y^2 = 1$$

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Question 5

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The value of this integral $\int_{-2}^2 \int_0^{\sqrt{4-x^2}} \frac{x^2}{x^2+y^2} dy dx$ is:

(A) $\frac{\pi}{4}$

(B) π

(C) $\frac{\pi}{2}$

(D) 2π

(E) 0

$$y \leq 0, \sqrt{4-x^2}$$

$$x \in [-2, 2]$$

$$\frac{x^2}{x^2+y^2} \rightarrow \frac{r^2 \cos^2 \theta}{r^2}$$

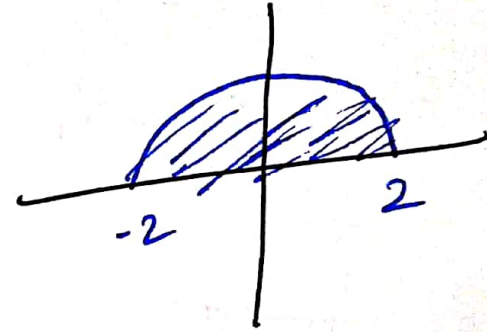
$$= \int_0^{\pi} \int_0^2 \cos^2 \theta \, r \, dr \, d\theta$$

$$= \int_0^{\pi} \frac{\cos^2 \theta}{2} [r^2]_0^2 \, d\theta$$

$$= \int_0^{\pi} 2 \cos^2 \theta \, d\theta$$

$$= \pi$$

(B)



Question 8

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The equation $\rho^2 = \sec 2\varphi$, is:

- (A) a hyperboloid of one sheets.
- (B) a hyperboloid of two sheet.
- (C) a paraboloid.
- (D) a hyperbolic paraboloid.
- (E) a cone.

$$\rho^2 = \sec(2\phi)$$

$$\rho^2 = \frac{1}{2\cos^2(\phi) - 1}$$

$$2 \frac{z^2 \rho^2}{\rho^2} - \rho^2 = 1$$

$$z^2 - x^2 - y^2 = 1$$

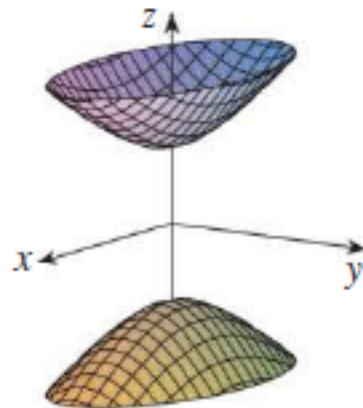
$$\sec(x) = \frac{1}{\cos(x)}$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\cos(\phi) = \frac{z}{\rho}$$

$$\rho^2 = x^2 + y^2 + z^2$$

Hyperboloid of Two Sheets



$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Horizontal traces in $z = k$ are ellipses if $k > c$ or $k < -c$.

Vertical traces are hyperbolas.

The two minus signs indicate two sheets.

Let $f(x, y) = \frac{4x^2 \tan^2 y}{x^2 + 2y^2}$. Then $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$

- A) does not exist since if (x, y) approaches $(0,0)$ along the line $x = 0$ the limit is 0 and if (x, y) approaches $(0,0)$ along the curve $x = y$ the limit is 2.
- B) does not exist since $\frac{0}{0}$ is an indeterminate form.
- C) exists and equals 0 by using Squeeze Theorem since $0 \leq \frac{4x^2 \tan^2 y}{x^2 + 2y^2} \leq 4 \tan^2 y$
and $\lim_{(x,y) \rightarrow (0,0)} 4 \tan^2 y = 0$.
- D) does not exist since if (x, y) approaches $(0,0)$ along the line $x = 2$ the limit is 0 and if (x, y) approaches $(0,0)$ along the line $y = 3$ the limit is ∞ .
- E) exists and equals 0 since $f(x, y)$ approaches $(0,0)$ along any line of the form $y = mx$ the limit is 0.

2) let $f(x, y) = \frac{4x^2 \tan^2 y}{x^2 + 2y^2}$, find $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$

Solution:-

let $r = x \cos \theta, y = r \sin \theta$

$$\lim_{r \rightarrow 0} \left[\frac{4 (r \cos \theta)^2 \tan^2 (r \sin \theta)}{r^2 \cos^2 \theta + 2r^2 \sin^2 \theta} \right]$$

$$= \lim_{r \rightarrow 0} \left[\frac{r^2 \cdot 4 \cos^2 \theta \tan^2 (r \sin \theta)}{r^2 (\cos^2 \theta + 2 \sin^2 \theta)} \right]$$

$r=0 \Rightarrow \underline{0}$ C

The volume of the solid region enclosed by the plane $2x + 3y + z = 120$, the cylinder $y = x^2$, and the planes $y = 9$, $z = 0$, $x = 0$, in the first octant is:

(A) $\int_0^3 \int_{x^2}^9 \int_0^{120+2x+3y} dz dy dx$

(B) $\int_0^3 \int_{x^2}^9 \int_0^{120-2x-3y} dz dy dx$

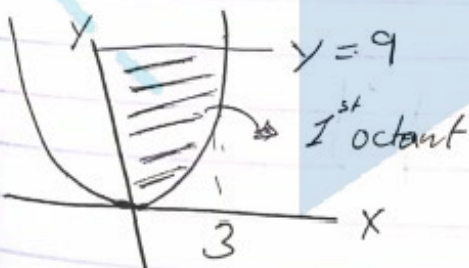
(C) $\int_{-3}^3 \int_{x^2}^9 \int_0^{120-2x-3y} dz dy dx$

(D) $\int_{-3}^3 \int_9^{x^2} \int_0^{120-2x-3y} dz dy dx$

(E) $\int_0^3 \int_{\sqrt{x}}^9 \int_0^{120-2x-3y} dz dy dx$

3) $2x + 3y + z = 120$, $y = x^2$, $x = 0$

$z = 120 - 2x - 3y$



$= \int_0^3 \int_{x^2}^9 \int_0^{120-2x-3y} dz dy dx$

$= \boxed{B}$

The Directional derivative of the function $f(x, y) = \ln(x^8 + y^8)$ at $(1, -1)$ in the direction of the unit vector that makes an angle $\frac{3\pi}{4}$ with the positive x -axis.

A) $2\sqrt{2}$.

B) $-2\sqrt{2}$.

C) $4\sqrt{2}$.

D) $-4\sqrt{2}$.

E) $3\sqrt{2}$.

6) $F(x, y) = \ln(x^8 + y^8)$

$$\nabla F = \left\langle \frac{8x^7}{x^8 + y^8}, \frac{8y^7}{x^8 + y^8} \right\rangle$$

$$\nabla F(1, -1) = \langle 4, -4 \rangle$$

$$\alpha = \frac{3\pi}{4}$$

$$\cos^2 \alpha + \cos^2 \beta = 1$$

$$\cos^2 \frac{3\pi}{4} + \cos^2 \beta = 1$$

$$\beta = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$u = \langle \cos \alpha, \cos \beta \rangle$$

$$D \cdot d = \langle 4, -4 \rangle \cdot \left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$= \frac{-8}{\sqrt{2}} = -4\sqrt{2} \quad \boxed{D}$$

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$$\frac{a^2 + b^2 - c^2}{2ab} = \cos \alpha$$

$$\frac{a^2 - b^2}{2ab} = \sin \alpha$$

Change the integral

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{3x^2+3y^2}}^{\sqrt{16-x^2-y^2}} \sin \sqrt{x^2 + y^2 + z^2} dz dy dx$$

from Cartesian to spherical coordinates:

(A) $\int_0^{\frac{\pi}{6}} \int_0^{2\pi} \int_0^4 \rho^2 \sin \rho \sin \phi d\rho d\phi d\theta$

(B) $\int_0^{2\pi} \int_0^{\frac{\pi}{6}} \int_0^4 \rho^2 \cos \rho \sin \phi d\rho d\phi d\theta$

(C) $\int_0^{\pi} \int_0^{\frac{\pi}{6}} \int_0^4 \rho^2 \sin \rho \sin \phi d\rho d\phi d\theta$

(D) $\int_0^{2\pi} \int_0^{\frac{\pi}{6}} \int_0^4 \rho^2 \sin \rho \sin \phi d\rho d\phi d\theta$

(E) $\int_0^{2\pi} \int_0^{\frac{\pi}{3}} \int_0^4 \rho^2 \sin \rho \sin \phi d\rho d\phi d\theta$

$$x = \pm 2$$

$$y = \pm \sqrt{4 - x^2}$$

full circle

$$z = \sqrt{3x^2 + 3y^2}, \sqrt{16 - x^2 - y^2}$$

upper cone

$$z^2 = 16 - x^2 - y^2$$

$$x^2 + y^2 + z^2 = 16$$

$$\rho^2 = 16$$

$$\rho = \pm 4$$

$$0 \leq \theta \leq 2\pi$$

$$\sin \sqrt{x^2 + y^2 + z^2} = \sin \rho$$

$$2\pi \quad \pi/6 \quad 4$$

$$\int_0^{2\pi} \int_0^{\pi/6} \int_0^4 \sin \rho (\rho^2 \sin \theta) d\rho d\theta d\phi$$

$$z = \sqrt{3} r$$

$$\theta = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

$$\theta = \pi/6$$

(D)

Question 11

Not yet answered

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The point of intersection between the plane $P: x + 2y + 6z = 6$ and the Line $L: x = 8 + 6t, y = -2t, z = 1 + t$ is

- A) $(2, 2, 0)$.
- B) $(-4, -4, 3)$.
- C) $(10, -2, 0)$.
- D) $(40, -8, -3)$.
- E) $(8, 8, -3)$.

① plane : $x + 2y + 6z = 6$

② Line : $x = 8 + 6t$, $y = -2t$, $z = 1 + t$

Sub ② in ①

$$8 + 6t + 2(-2t) + 6(1+t) = 6$$

$$t = -1$$

$$x = 8 + 6(-1) = \underline{2}$$

$$y = -2(-1) = \underline{2}$$

$$z = 1 - 1 = \underline{0}$$

①

Question 14

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The curvature of the function $y = \cos 3x$ is

(A) $\frac{-9 \cos 3x}{(1+9 \sin^2 3x)^{\frac{3}{2}}}$

(B) $\frac{3|\sin 3x|}{(1+9 \cos^2 3x)^{\frac{3}{2}}}$

(C) $\frac{9 \cos 3x}{(1+9 \sin^2 3x)^{\frac{3}{2}}}$

(D) $\frac{9|\cos 3x|}{(1+9 \sin^2 3x)^{\frac{3}{2}}}$

(E) $\frac{9|\cos 3x|}{(1-9 \sin^2 3x)^{\frac{3}{2}}}$

$$y = \cos(3x)$$

$$y' = -3 \sin(3x)$$

$$y'' = -9 \cos(3x)$$

$$K(x) = \frac{|f''(x)|}{(1 + (f'(x))^2)^{3/2}}$$

$$K(x) = \frac{9 |\cos(3x)|}{(1 + 9 \sin^2(3x))^{3/2}} \quad \text{①}$$

$\int_0^{\ln 3} \int_{e^y}^3 f(x, y) dx dy$, when we reverse the order of integration, we will get:

(A) $\int_{e^y}^3 \int_0^{\ln 3} f(x, y) dy dx$

(B) $\int_1^3 \int_0^{\ln x} f(x, y) dy dx$

(C) $\int_0^{\ln 3} \int_{e^x}^3 f(x, y) dy dx$

(D) $\int_1^3 \int_0^{\ln 3} f(x, y) dy dx$

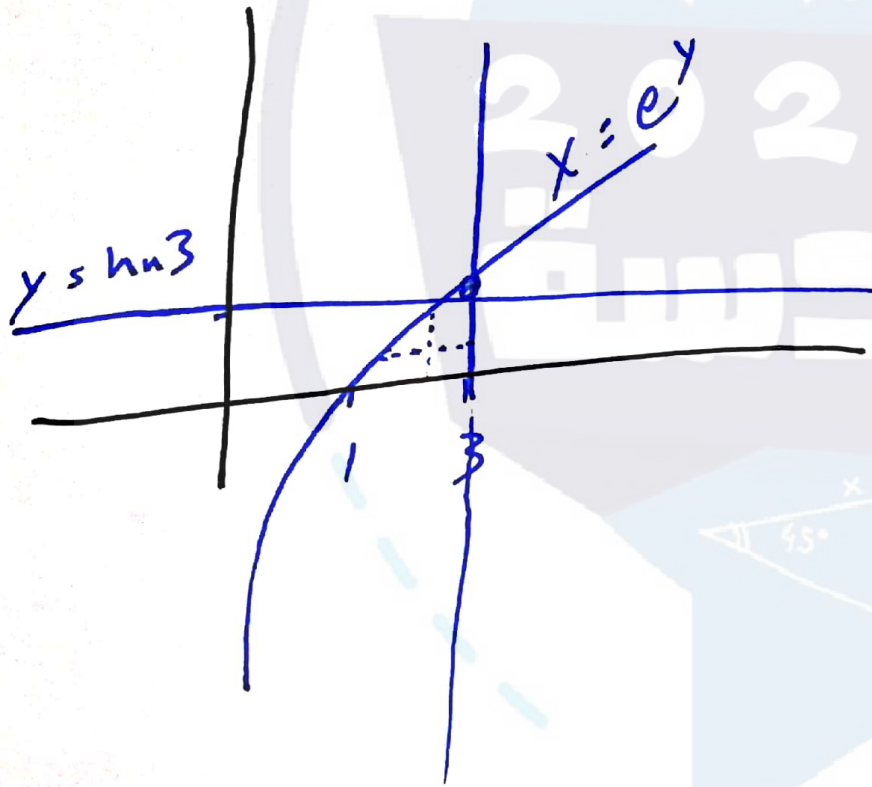
(E) $\int_0^3 \int_0^{\ln x} f(x, y) dy dx$

$$x = e^y$$

$$x = 3$$

$$y = 0$$

$$y = \ln 3$$



$$\int_0^{\ln 3} \int_1^3 f(x,y) dy dx$$

$$x \ln x$$

$$\ln x \ln y$$

(B)

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If $x = \frac{y^2 - x \ln z}{z}$, then $\frac{\partial x}{\partial y}$ is:

(A) $\frac{2y}{z - \ln z}$

(B) $-\frac{z + \ln z}{2y}$

(C) $\frac{z + \ln z}{2y}$

(D) $-\frac{2y}{z + \ln z}$

(E) $\frac{2y}{z + \ln z}$

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$$x = \frac{y^2 - x \ln z}{z}$$

$$\frac{\partial x}{\partial y} = \frac{2y - \frac{\partial x}{\partial y} (\ln z)}{z}$$

$$\frac{\partial x}{\partial y} (z + \ln z) = 2y$$

$$\frac{\partial x}{\partial y} = \frac{2y}{z + \ln z} \quad \text{(E)}$$

To find the shortest distance between the cone $z^2 = x^2 + y^2$ and the point $(-4, 1, -2)$ we can use Lagrange multipliers method to solve the following problem:

- A) Minimize $f(x, y, z) = (x - 4)^2 + (y + 1)^2 + (z - 2)^2$ subject to the constraint $x^2 + y^2 - z^2 = 0$.
- B) Minimize $f(x, y, z) = (x + 4)^2 + (y - 1)^2 + (z + 2)^2$ subject to the constraint $x^2 + y^2 - z^2 = 0$.
- C) Minimize $f(x, y, z) = \sqrt{x^2 + y^2 - z^2}$ subject to the constraint $(x - 4)^2 + (y + 1)^2 + (z - 2)^2 = 0$.
- D) Maximize $f(x, y, z) = (x + 4)^2 + (y - 1)^2 + (z + 2)^2$ subject to the constraint $z^2 = x^2 + y^2$.
- E) Minimize $f(x, y, z) = \sqrt{x^2 + y^2 - z^2}$ subject to the constraint $(x + 4)^2 + (y - 1)^2 + (z + 2)^2 = 0$.

↪

$$(\text{Distance})^2 = (x+4)^2 + (y-1)^2 + (z+2)^2$$

Constraint is the point (x, y, z) is on the cone

So (B)

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If $f(x, y)$ has a continuous second partial derivatives and

$$f_x = 2y - 2x^4, \quad f_y = 2x - 2y^4, \text{ then}$$

- A) f has a saddle point at $(0,0)$ and a local minimum value at $(1, 1)$, $(-1,1)$ and $(1, -1)$.
- B) f has a saddle point at $(0,0)$ and a local maximum value at $(1, 1)$ and $(-1, -1)$.
- C) f has a saddle point at $(0,0)$ and a local maximum value at $(1,1)$.
- D) f has a local maximum at $(0,0)$ and a local minimum value at $(1, 1)$.
- E) f has a saddle point at $(1, 1)$, a local minimum value at $(0,0)$.

$$F_x = 2y - 2x^4$$

$$f_y = 2x - 2y^4$$

$$F_{xx} = -8x^3$$

$$F_{yy} = -8y^3$$

$$F_{xy} = 2$$

$$F_x = 0$$

$$y = x^4$$

$$x = x^{16}$$

$$F_y = 0$$

$$x = y^4$$

$$x(1-x^{15}) = 0$$

$$x = 0, x = 1$$

$$(0, 0), (1, 1)$$

		f_{xx}	f_{yy}	f_{xy}	$D = (f_{xx})(f_{yy}) - (f_{xy})^2$
0,0	(0,0)	0	0	2	-4
1,1	(1,1)	-8	-8	2	60

(0,0) → Saddle

(1,1) → local max

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The equation of the tangent plane to the surface $x^2y^3 + 2xz^3 = 4 + y^4z$ at the point $(-1, 1, -1)$ is

A) $-4(x - 1) + 5(y + 1) + 7(z - 1) = 0.$

B) $-4(x - 1) + 7(y + 1) - 7(z - 1) = 0.$

C) $-4(x + 1) + 7(y - 1) - 3(z + 1) = 0.$

D) $-4(x + 1) + 7(y - 1) - 7(z + 1) = 0.$

E) $-4(x - 1) + 5(y + 1) - 9(z - 1) = 0.$

$$x^2 y^3 + 2xz^3 = 4 + y^4 z$$

$$x^2 y^3 + 2xz^3 - y^4 z - 4 = 0$$

$$F_x = 2xy^3 + 2z^3$$

$$F_x = -4$$

$$F_y = 3y^2 x^2 - 4y^3 z \quad @(-1, 1, -1) \quad F_y = 7$$

$$F_z = 6xz^2 - y^4 \quad F_z = -7$$

$$-4(x+1) + 7(y-1) - 7(z+1) = 0$$

ⓐ

Question 13

Not yet answered

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Let $C_1: \vec{r}_1(t) = \langle 3 - t, t - 2, t^2 \rangle$ and $C_2: \vec{r}_2(k) = \langle k, 1 - k, k^2 + 3 \rangle$ be two curves intersect at the point $P(1,0,4)$. Then the following vector \vec{n} is **perpendicular** to both of the tangent lines of C_1 and C_2 at $P(1,0,4)$.

- A) $\vec{n} = \langle 2, 6, -2 \rangle$.
- B) $\vec{n} = \langle 6, 6, 0 \rangle$.
- C) $\vec{n} = \langle 10, 6, -2 \rangle$.
- D) $\vec{n} = \langle -6, 6, 0 \rangle$.
- E) $\vec{n} = \langle -8, 8, 0 \rangle$.

$$r_1(t) = \langle 3-t, t-2, t^2 \rangle \quad @ (1,0,4)$$

$$r_1'(t) = \langle -1, 1, 2t \rangle \quad t=2$$

$$\underline{r_1'(2) = \langle -1, 1, 4 \rangle}$$

$$r_2(k) = \langle k, 1-k, k^2+3 \rangle \quad k=1$$

$$r_2'(k) = \langle 1, -1, 2k \rangle$$

$$r_2'(1) = \langle 1, -1, 2 \rangle$$

$$r_1'(2) \times r_2'(1) = \langle 6, 6, 0 \rangle$$

(13)