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The University of Jordan
Department of Mathematics
Calculus III, Second Exam

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Instructor's Name: Dr. Hassan Lecture's time: 9:30 - 11:00

Q1) (4 marks) Let $\vec{r}(t) = < \frac{t^2}{2}, \sqrt{2}\,t, \ln t >$, find the arc length of $\vec{r}(t)$, $2 \le t \le 5$.

$s = \int_{t_0}^{t_1} |r'(t)|\,dt \Rightarrow$ $r'(t) = < t, \sqrt{2}, \frac{1}{t} >$

$|r'(t)| = \sqrt{t^2 + (\sqrt{2})^2 + (\frac{1}{t})^2}$

$\Rightarrow s = \int_2^5 \frac{t^2}{\cancel{t}} + \frac{1}{t}\,dt$ $= \sqrt{t^2 + 2 + \frac{1}{t^2}}$

$= \frac{t^2}{2} + \ln t \Big|_2^5$ 4

$\sqrt{\frac{t^4 + 2t^2 + 1}{t^2}} = \frac{\sqrt{t^4 + 2t^2 + 1}}{t}$

$\boxed{= (\frac{25}{2} + \ln 5) - (\frac{4}{2} + \ln 2)}$

$= \frac{\sqrt{(t^2+1)^2}}{t} = \boxed{\frac{t^2+1}{t}}$

Q2)(4 marks) If $f(2,3) = 4$, $f_x(2,3) = 2$, $f_y(2,3) = -5$, use linear approximation to approximate $f(1.97, 3.05)$.

$L(x,y) = F(x,y) + f_x(x,y)\,(x-a) + f_y(x,y)\,(y-b)$

$L(2,3) = 4 + 2(x-2) + (-5(y-3))$

$L(2,3) = 4 + 2(-2.03) + (-5(-2.95))$

$= 4 + (-4.06) + (14.75) = 14.69$

$\Delta x = 1.97 - 2 = \frac{-3}{100}$

$\Delta y = 3.05 - 3 = \frac{5}{100}$

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Q3)(3+2 marks) Determine whether the following limits exist or not.

a) $\lim_{(x,y)\to(0,0)} \dfrac{x^2 y e^{xy}}{x^4+y^2}$

1) along x-axis $(y=0)$ $\quad \lim_{x\to 0} \dfrac{0}{x^4} = 0$

2) along $y=x^2$ $\quad \lim_{x\to 0} \dfrac{x^2 x^2 e^{x^3}}{x^4+x^4} = \lim_{x\to 0} \dfrac{x^4 e^{x^3}}{2x^4} = \dfrac{1}{2}$

not exist

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b) $\lim_{(x,y)\to(0,0)} \dfrac{x^4 - x + y - x^3 y}{x - y}$

Back

1) along y-axis $(x=0)$ $\quad \lim_{y\to 0} \dfrac{y}{-y} = -1$

2) along $x=1$ and $y=0$ $\quad \lim_{x\to 0} \dfrac{1-1}{1} = \dfrac{0}{1} = 0$

not exist

Q4)(4 marks) Let $f(x,y,z) = 4e^{xy}\cos z$, $p = (0, 1, \pi/4)$. Find the unit vector in the direction in which f increases most rapidly at p and find the rate of increase of f in that direction.

$0, 1, \frac{\pi}{4}$

$$\boxed{\begin{aligned} f_x &= 4y e^{xy} \cos z \\ f_y &= 4x e^{xy} \cos z \\ f_z &= -4e^{xy} \sin z \end{aligned}}$$

$\nabla f(x,y,z) = \langle 4y e^{xy}\cos z, 4x e^{xy}\cos z, -4e^{xy}\sin z\rangle$

$\nabla f(0,1,\frac{\pi}{4}) = \langle 2\sqrt{2}, 0, -2\sqrt{2}\rangle$

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$D_u f = |\nabla f(x,y,z)|$

max "rapidly"

$D_u f = \sqrt{(2\sqrt{2})^2 + 0 + (-2\sqrt{2})^2}$

$D_u f = \sqrt{8+8} = \sqrt{16} = 4$

Direction to the $\nabla f(x,y,z)$ vector

max rate

Q5)(4 marks) If $z = f(x, y)$ is differentiable function where $x = s^2 + t^3$, $y = 2st - 2t$ and $f_x(2,1) = 4$, $f_x(5,2) = 3$, $f_y(2,1) = -2$, $f_y(5,2) = 6$. Find $\frac{\partial z}{\partial t}$ at $s = 2$, $t = 1$.

$$\frac{dz}{dt} = \frac{dz}{dx} \cdot \frac{dx}{dt} + \frac{dz}{dy} \cdot \frac{dy}{dt}$$

$$\frac{dz}{dt} = 4 \cdot (3t^2) + (-2(2s-2))$$

$$\frac{dz}{dt} = 12t^2 - 4s + 4$$

$$\left.\frac{dz}{dt}\right|_{(2,1)} = 48 - 8 + 4 = 52$$

Q6) (6 marks) Find and classify the critical points for the function $f(x, y) = \frac{1}{4}x^4 - 3xy + \frac{1}{2}y^2$.

$$f_x = \frac{4x^3}{4} - 3y = 0 \Rightarrow x^3 = 3y \Rightarrow \frac{x^3}{3} = y$$

$$f_y = -3x + y = 0 \Rightarrow +3x = y$$

$$+3x = \frac{x^3}{3} \Rightarrow +9x = x^3$$

$$\Rightarrow x^3 - 9x = 0$$

$$x(x^2 - 9) = 0$$

$$x = (0, 3, -3)$$

$$f_{xx} = 3x^2$$

$$f_{yy} = 1$$

$$f_{xy} = -3$$

(3, 9)
(−3, −9) c.p
(0, 0) c.pt

	f_{xx}	f_{yy}	f_{xy}	D	result
(0,0)	0	1	−3	−9	saddle
(3,9)	27	1	−3	18	l. min
(−3,−9)	27	1	−3	18	l. min

$$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$$