$\frac{33}{30}$ Excellent.

seat no. 38

The University of Jordan
Department of Mathematics

Calculus III, Second Exam

Student's Name: Rama Taki Darwesh   Student's Number:

Instructor's Name: Dr. Salam   Lecture's time: 1 → 2

9

Q1) (9 marks) Fill the blank with the correct answer.

a) If $f(x,y) = 4x\cos(x^2y) - 3xy^2 + y^3$, then the linear approximation for f at the point $(0,1)$ is: $\boxed{x+3y-2}$

$4\cos(xy^2) + \sin(xy^2)\cdot y^2\cdot 4x$

$\cancel{L(0,1) = f + f_x(x) + f_y(y-1)}$

$L(x,y) = f(0,1) + f_x\big|_{\substack{x=0\\y=1}}(x-0) + f_y\big|_{\substack{x=0\\y=1}}(y-1)$

$f_x\big|_{\substack{x=0\\y=1}} = 4\cos(x^2y) - 3y^2 - \sin(x^2y)\cdot 4x^2 = 4-3 = 1$

$f_y\big|_{\substack{x=0\\y=1}} = -4x^3\sin(x^2y) - 6yx + 3y^2 = 3$

$f(0,1) = 1$

$L(x,y) = 1 + 1(x-0) + 3(y-1)$

$= 1 + x + 3y - 3$

$= x + 3y - 2$

b) Let $z = f(x^3 - y^2, y^2 - x^3)$, if $a\dfrac{\partial z}{\partial x} + 3x^2\dfrac{\partial z}{\partial y} = 0$, then $a = \boxed{2y}$

$r = x^3 - y^2, \quad u = y^2 - x^3$

$z = f(r,u)$

$2y\left(3x^2\frac{dz}{dr} - 3x^2\frac{dz}{du}\right) + 3x^2\cdot 2y\left(-\frac{dz}{du} + \frac{dz}{dr}\right) = 0$

$\frac{dz}{dx} = \frac{\partial z}{\partial r}\cdot\frac{dr}{dx} + \frac{dz}{\partial u}\cdot\frac{du}{dx}$

$= 3x^2\frac{dz}{\partial r} - 3x^2\frac{dz}{\partial u}$

$\frac{dz}{dy} = \frac{dz}{dr}\cdot\frac{dr}{dy} + \frac{dz}{du}\cdot\frac{du}{dy}$

$= -2y\frac{dz}{dr} + 2y\frac{dz}{du}$

c) If $\vec{r}(t) = \langle e^t\cos t, \sin t, 2+\ln(t+1)\rangle$, then the unit tangent vector $T(0) = \boxed{\frac{1}{\sqrt{3}}\langle 1, 1, 1\rangle}$

$T = \dfrac{r'(t)}{|r'(t)|}$

$r'(t) = \langle e^t\cos t - \sin t\, e^t, \cos t, \frac{1}{t+1}\rangle$

$r'(0) = \langle 1, 1, 1\rangle$

$|r'(0)| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

3

Q2)(3 marks) Sketch the domain of $f(x,y) = \sqrt{y - |x|} + \ln(x^2 + y^2 - 9)$.

$D = \{(x,y),\ x^2+y^2 > 9,\ y \geq |x|\}$

$y - |x| \geq 0$

$y - |x| \geq 0$

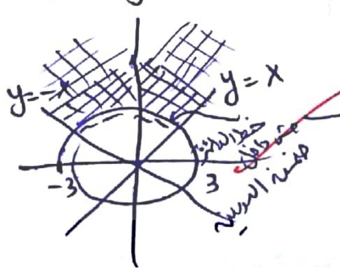
$y \geq |x|$

$y \geq x$

$y \geq -x$

$x^2 + y^2 - 9 > 0$

$x^2 + y^2 > 9$



$\boxed{7}$

u^{-1}

Q3) (7 marks) Let $\vec{r}(t) =< \cos(\ln t), \sin(\ln t), \sqrt{3}\ln t >$,

a) Find the arc length function $s(t)$ of $\vec{r}(t)$ measured from $t = 1$. $\boxed{2\ln t} = s(t)$

$$s(t) = \int_1^t |\vec{r}'(u)|\, du$$

$r'(u) \Rightarrow$

$x' = -\sin(\ln u) \cdot \frac{1}{u}$

$y' = \cos(\ln u) \cdot \frac{1}{u}$

$z' = \frac{\sqrt{3}}{t}$

$$\sqrt{\frac{\sin^2(\ln u)}{u^2} + \frac{\cos^2(\ln u)}{u^2} + \frac{3}{u^2}}$$

$$\sqrt{\frac{1}{u^2}\left(\underbrace{\sin^2(\ln u) + \cos^2(\ln u)}_{1} + 3\right)}$$

$$= \sqrt{\frac{4}{u^2}}$$

$$= \frac{2}{u}$$

$$\int_1^t \frac{2}{u}\, du = 2\ln u \Big]_1^t$$

$$= 2(\ln t - 0)$$

$$s(t) = 2\ln t$$

b) If the length of the curve from the point $P(1, 0, 0)$ to the point Q on the curve $\vec{r}(t)$ is two units. Find Q.

$\times\ y_2,\ \sqrt{3}\ln t$

$z = 0 \longrightarrow t = 1$

الابتدائية

$r(t)$

$$L = \int_0^Q \frac{2}{u}\, du$$

$$2 = 2\ln t \Big]_1^t$$

$$2 = 2(\ln Q - \ln 1)$$

$$1 = \ln Q \qquad \boxed{Q = e}$$

$t = e$

$\hookrightarrow <\cos \ln e,\ \sin \ln e,\ \sqrt{3}\ln e>$

$$\boxed{Q = (\cos 1,\ \sin 1,\ \sqrt{3})}$$

$\boxed{4}$

Q4) (4 marks) Find the curvature for $\vec{r}(t) =< e^t, e^{-t}, t >$ at the point $(1, 1, 0)$.

$\boxed{t=0}$

$r'(t) = <e^t, -e^{-t}, 1>$ $\qquad r'(0) = <1, -1, 1>$ $\longrightarrow |r'(0)| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$r''(t) = <e^t, e^{-t}, 0>$ $\qquad r''(0) = <1, 1, 0>$

$$r'(0) \times r''(0) \rightarrow \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$0 - 1$, $-(0 - 1)$, $(1 - -1)$

$-i + j + 2k$

$<-1, 1, 2>$

$= \sqrt{-1^2 + 1^2 + 2^2} = \sqrt{6}$

$$k = \frac{|r'(0) \times r''(0)|}{|r'(0)|^3} = \frac{\sqrt{6}}{(\sqrt{3})^3}$$

5

Q5)(3+2 marks) Determine whether the following limits exist or not.

a) $\lim_{(x,y)\to(0,0)} \dfrac{x^2y}{\sqrt{x^2+y^2}}$

$x = r\sin\theta$
$y = r\cos\theta$
$r^2 = x^2 + y^2$
$r \to 0$

$$\lim_{r\to 0} \frac{r^3 \sin^2\theta \cdot \cos\theta}{\sqrt{r^2}}$$

$$\lim_{r\to 0} \frac{r^{\cancel{3}} \sin^2\theta \cdot \cos\theta)}{\cancel{r}} = \lim_{r\to 0} 0 \times (\sin^2\theta \cdot \cos\theta) = 0$$

bounded

$\boxed{\text{Limit exist}}$

b) $\lim_{(x,y)\to(0,0)} \dfrac{3xy}{5x^2-2y^2}$

along x-axis $y=0$

$$\lim_{(x,y)\to(0,0)} \frac{0}{5x^2} = 0$$

along $y=x$

$$\lim_{x\to 0} \frac{3x^2}{5x^2-2x^2} = \lim_{x\to 0} \frac{3x^2}{3x^2} = 1$$

D.N.E → not the same from different Path.

5

Q6)(5 marks) If $w = z^3e^{\cos(xy)} + xyz$, $x = r\cos\theta$, $y = r\sin\theta$, and $z = r\theta$. Use chain rule to find $\boxed{\dfrac{\partial w}{\partial r}}$ at $(r,\theta) = (2, \frac{\pi}{2})$.

w → x, y, z; x → r, θ; y → r, θ; z → r, θ

$$\left.\frac{\partial w}{\partial r}\right|_{\substack{r=2\\ \theta=\frac{\pi}{2}\\ x=0\\ y=2\\ z=\pi}} = \frac{\partial w}{\partial x}\cdot\frac{dx}{dr} + \frac{\partial w}{\partial y}\cdot\frac{dy}{dr} + \frac{\partial w}{\partial z}\cdot\frac{dz}{dr}$$

$$= 2\pi \cdot 0 + 0 \cdot 1 + 3\pi^2 e \cdot \frac{\pi}{2} = \boxed{\frac{3\pi^3 e}{2}}$$

$\left.\dfrac{dx}{dr}\right|_{r=2,\ \theta=\frac{\pi}{2}} = \cos\theta \to 0$

$\left.\dfrac{dy}{dr}\right|_{r=2,\ \theta=\frac{\pi}{2}} = \sin\theta \to 1$

$\left.\dfrac{dz}{dr}\right|_{r=2,\ \theta=\frac{\pi}{2}} = \theta \to \dfrac{\pi}{2}$

$r=2,\ s=\frac{\pi}{2}$ $\xrightarrow{\text{تعويض}}$ $x=0,\ y=2,\ z=\pi$

$$\frac{\partial w}{\partial x} = -z^3 e^{\cos(xy)} \cdot \sin(xy)\cdot y + yz = 2\pi$$

$$\frac{\partial w}{\partial y} = -z^3 e^{\cos(xy)} \cdot \sin(xy)\cdot x + xz = 0$$

$$\frac{\partial w}{\partial z} = 3z^2 e^{\cos(xy)} + xy = 3\pi^2 e$$