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The University of Jordan
Department of Mathematics
Calculus III, Second Exam

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Instructor's Name: د. حسين النجار   Lecture's time: 9:30 – 11

Q1) (5 marks) Let $r(\vec{t}) =< 3cost, \sqrt{8}sint, sint >$

a) Find the curvature function $\kappa(t)$ for $r(\vec{t})$.

$\vec{r}'(t) = (-3sint, \sqrt{8}\cos t, \cos t)$

$\vec{r}''(t) = (-3cost, -\sqrt{8}\sin t, -\sin t)$

$k(t) = \frac{|\vec{r}' \times \vec{r}''|}{|r'|^3} = \frac{\sqrt{9+72}}{(\sqrt{9+8+1})^3}$

$= \boxed{\frac{9}{(18)^{3/2}}}$

$\vec{r}'' \times \vec{r}' = \begin{vmatrix} i & j & k \\ -3\sin t & \sqrt{8}\cos t & \cos t \\ -3\cos t & -\sqrt{8}\sin t & -\sin t \end{vmatrix}$

$= (-\sqrt{8}\sin t \cos t + \sqrt{8}\sin t \cos t) i - (3\sin^2 t + 3\cos^2 t)\hat{j} + (3\sqrt{8}\sin^2 t + 3\sqrt{8}\cos^2 t)\hat{k}$

$= \boxed{-3\hat{j} + 3\sqrt{8}\,\hat{k}}$

3.5

b) Describe the curve $r(\vec{t})$.

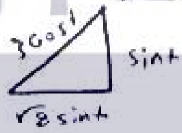
Real triangle

~~$x = 3\cos(t)$~~

~~$y = \sqrt{8}\sin(t) \Rightarrow \sin t = \frac{y}{\sqrt{8}}$~~

~~$z = \sin(t) \Rightarrow z = \frac{y}{\sqrt{8}}$~~

$y = \sqrt{8}z$



Q2)(3 marks) Determine whether the following limit exists or not.

$$\text{Lim}_{(x,y)\to(2,0)} \frac{xy^2 - 2y^2}{3y^4 + x^2 - 4x + 4} = \frac{0}{0} \,!$$

- along (x-axis) (y = 0)

$\lim_{x\to 0} \left(\frac{0}{x^2 - 4x + 4} \right) = 0$

- along (y = x − 2)

$\lim_{x\to 2} \left(\frac{x(x-2)^2 - 2(x-2)^2}{3(x-2)^4 + x^2 - 4x + 4} \right)$

$= \lim_{x\to 2} \left(\frac{(x-2)^2(x-2)}{3(x-2)^4 + (x-2)^2} \right)$

$= \lim_{x\to 2} \left(\frac{(x-2)^{\cancel{3}}}{(x-2)^2(3(x-2)^2 + 1)} \right) = \frac{0}{1} = 0$

(Not exist)

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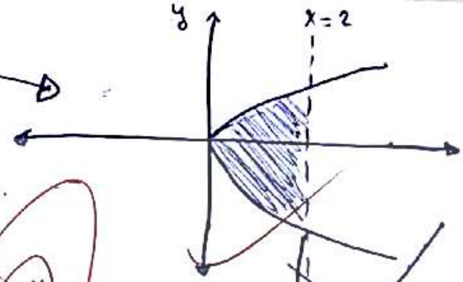
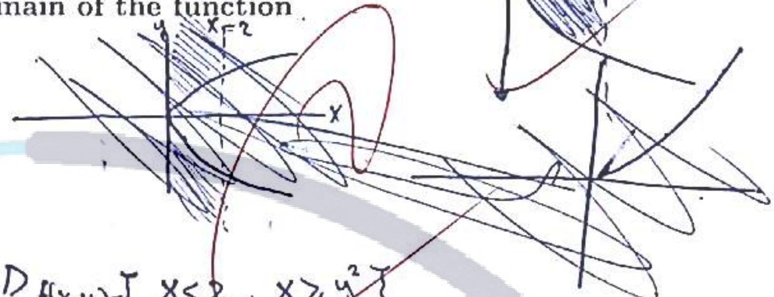


Q3) (2 marks) Sketch the domain of the function
$f(x, y) = \ln(2 - x) + \sqrt{x - y^2}$.



$2 - x > 0$ | $x - y^2 \geq 0$

$x < 2$ | $x \geq y^2$

$D_{f(x,y)} = \{ x < 2 , x \geq y^2 \}$

Q4)(3 marks) Use linear approximation to approximate $\frac{1}{\sqrt{(4.05)^2 + (2.93)^2}}$.

$f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$, $P(4, 3)$

$f_x = \frac{-\frac{2x}{2\sqrt{x^2+y^2}}}{(\sqrt{x^2+y^2})^2} = \frac{-x}{(x^2+y^2)^{3/2}} \Rightarrow f_x(4,3) = \boxed{\frac{-4}{125}}$

$f_y = \frac{-\frac{2y}{2\sqrt{x^2+y^2}}}{(\sqrt{x^2+y^2})^2} = \frac{-y}{(x^2+y^2)^{3/2}} \Rightarrow f_y(4,3) = \boxed{\frac{-3}{125}}$

$f(4,3) = \boxed{\frac{1}{5}}$

Q5)(3 marks) If $\nabla f(a, b) = < 8, -6 >$ and $D_v f(a, b) = 10$. Find v.

$\hat{V} = (V_1, V_2)$

$D_v f(a,b) = \nabla f(a,b) \cdot (V_1, V_2)$

$10 = (8, -6) \cdot (V_1, V_2)$

$\boxed{10 = 8V_1 - 6V_2}$

$\Rightarrow \boxed{V_2 = \frac{4}{3}V_1 - \frac{5}{3}} \rightarrow ①$

$\boxed{V_1^2 + V_2^2 = 0} \rightarrow ②$

$V_1^2 + \left(\frac{4}{3}V_1 - \frac{5}{3}\right)^2 = 0$

$V_1^2 + \frac{16}{9}V_1^2 - \frac{40}{9}V_1 + \frac{25}{9} = 0$

$9 \times \left(\frac{25}{9}V_1^2 - \frac{40}{9}V_1 + \frac{25}{9}\right) = 0 \times 9$

$25V_1^2 - 40V_1 + 25 = 0$

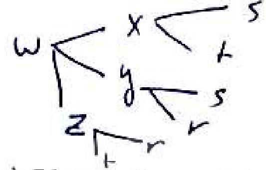
$5V_1^2 - 8V_1 + 5 = 0$

$(5V_1 - 3)(V_1 - 1) = 0$

$V_1 = \frac{3}{5}$, $V_2 = \frac{-13}{15} \Rightarrow \hat{V}_1 = \left(\frac{3}{5}, \frac{-1}{1}\right.$

$V_1 = 1$, $V_2 = \frac{-1}{3} \Rightarrow \hat{V}_2 = \left(1, \frac{-1}{3}\right.$

$V = \hat{V} \cdot |V|$ ...



Q6)(4 marks) If $w = f(x, y, z)$ is differentiable function where $x = ts$, $y = sr$ and $z = rt$. Show that

$$t\frac{\partial w}{\partial t} + s\frac{\partial w}{\partial s} - r\frac{\partial w}{\partial r} = 2st\frac{\partial f}{\partial x}.$$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial t} = \boxed{s\frac{\partial w}{\partial x} + r\frac{\partial w}{\partial z}}$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial s} = \boxed{t\frac{\partial w}{\partial x} + r\frac{\partial w}{\partial y}}$$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial r} = \boxed{s\frac{\partial w}{\partial y} + t\frac{\partial w}{\partial z}}$$

$$\Rightarrow st\frac{\partial w}{\partial x} + \cancel{rt\frac{\partial w}{\partial z}} + ts\frac{\partial w}{\partial x} + \cancel{rs\frac{\partial w}{\partial y}} - \cancel{rs\frac{\partial w}{\partial y}} - \cancel{rt\frac{\partial w}{\partial z}}$$

$$= 2st\frac{\partial w}{\partial x} = 2st\frac{\partial f}{\partial x}$$

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Q7) (5 marks) Find and classify the critical points for the function $f(x, y) = yx^2 - 2y^2 - 4x^2$.

$f_x = \boxed{2yx - 8x = 0} \to ①$

$f_y = \boxed{x^2 - 4y = 0} \to ②$

$\boxed{y = \frac{x^2}{4}}$

عوض في ①

$\Rightarrow 2\left(\frac{x^2}{4}\right)x - 8x = 0$

$\frac{1}{2}x^3 - 8x = 0$

$x\left(\frac{1}{2}x^2 - 8\right) = 0$

$\boxed{x = 0}$

$2 \times \frac{1}{2}x^2 = 8 \times 2$

$x^2 = 16$

$\boxed{x = 4}$, $\boxed{x = -4}$

critical Points

(0, 0)

(4, 4)

(-4, 4)

4

~~$f_{xx} = 2y - 8$~~

$f_{xx} = 2y - 8$

$f_{yy} = -4$

$f_{xy} = 2y$

C.P	f_{xx}	f_{yy}	f_{xy}	D	result
(0,0)	-8	-4	0	32	L.Max
(4,4)	0	-4	8	-64	saddle
(-4,4)	-16	-4	8	0	test fai