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The University of Jordan
Department of Mathematics
Calculus III, Second Exam

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Q1) (4 marks) Let $\vec{r}(t) = < \frac{t^2}{2}, \sqrt{2}\, t, \ln t >$, find the arc length of $\vec{r}(t)$, $2 \le t \le 5$.

$r'(t) = < t, \sqrt{2}, \frac{1}{t} >$

$|r'(t)| = \sqrt{t^2 + 2 + \frac{1}{t^2}} = \sqrt{\frac{t^4 + 2t^2 + 1}{t^2}}$

$= \frac{\sqrt{(t^2+1)^2}}{\sqrt{t^2}} = \frac{t^2+1}{t} = t + \frac{1}{t}$

$\int_2^5 t + \frac{1}{t} \, dt = \frac{t^2}{2} + \ln t \Big]_2^5 = \frac{25}{2} + \ln 5 - 2 - \ln 2$

$= \frac{21}{2} + \ln\left(\frac{5}{2}\right)$

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Q2)(4 marks) If $f(2,3) = 4$, $f_x(2,3) = 2$, $f_y(2,3) = -5$, use linear approximation to approximate $f(1.97, 3.05)$.

$L = f_x(x - x_0) + f_y(y - y_0) + Z_0$

$L = 2(x-2) + -5(y-3) + 4$

$L = 2(1.97 - 2) - 5(3.05 - 3) + 4$

$(x,y) = (1.97, 3.05)$

$L = 2(-0.03) - 5(0.05) + 4$

$L = 3.69$

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Q3)(3+2 marks) Determine whether the following limits exist or not.

a) $\lim_{(x,y)\to(0,0)} \frac{x^2ye^{xy}}{x^4+y^2}$ $\Rightarrow$ along $y = x^2$

$\frac{0}{0}$

$\lim_{x\to0} \frac{x^4 e^{x^3}}{x^4+x^4} = \lim_{x\to0} \frac{\cancel{x^4} e^{x^3}}{2\cancel{x^4}} = \frac{1}{2}$

$\Rightarrow$ along $y = 2x^2$

$\lim_{x\to0} \frac{x^2(2x^2) e^{2x^3}}{x^4+4x^4} = \lim_{x\to0} \frac{2\cancel{x^4} e^{2x^3}}{5\cancel{x^4}} = \frac{2}{5}$

So the limit, Not exist

~~b) $\lim_{(x,y)\to(0,0)} \frac{x^4-x+y-x^3y}{x-y}$~~ $= \frac{0}{0}$

$= \lim_{(x,y)\to(0,0)} \frac{x(x^3-1) - y(x^3-1)}{x-y}$

$= \lim_{(x,y)\to(0,0)} \frac{(x^3-1)(\cancel{x-y})}{\cancel{x-y}} = -1$ exist

Q4)(4 marks) Let $f(x, y, z) = 4e^{xy}\cos z$, $p = (0, 1, \pi/4)$. Find the unit vector in the direction in which f increases most rapidly at p and find the rate of increase of f in that direction.

$\nabla f = \langle 4y e^{xy} \cos z, 4x e^{xy} \cos z, -4 e^{xy} \sin z \rangle$

$\nabla f \big|_{(0,1,\frac{\pi}{4})} = \langle \frac{4}{\sqrt{2}}, 0, \frac{-4}{\sqrt{2}} \rangle$

$|\nabla f| = \sqrt{\frac{16}{2}+\frac{16}{2}} = \sqrt{\frac{32}{2}} = \sqrt{16} = 4$

Unit vector $= \frac{\nabla f}{|\nabla f|} = \frac{\langle \frac{4}{\sqrt{2}}, 0, \frac{-4}{\sqrt{2}} \rangle}{4} = \langle \frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}} \rangle$

$\Rightarrow$ the rate of increase of f in that direction is $= \underline{4}$

∇f // unit vector $\quad D_{\text{unit vector}} f = \nabla f \cdot$ unit vector

$\hookrightarrow$ the direction is $\langle \frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}} \rangle$ that make maximum rate. $\Rightarrow D_{u.v} f = |\nabla f|$

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Q5)(4 marks) If $z = f(x,y)$ is differentiable function where $x = s^2 + t^3$, $y = 2st - 2t$ and $f_x(2,1) = 4$, $f_x(5,2) = 3$, $f_y(2,1) = -2$, $f_y(5,2) = 6$. Find $\frac{\partial z}{\partial t}$ at $s = 2$, $t = 1$.

$s = 2 \quad t = 1$

$x = 5 \quad y = 2$

$$\frac{\partial z}{\partial t} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$\frac{\partial z}{\partial t} = \frac{\partial f}{\partial x}\cdot 3t^2 + \frac{\partial f}{\partial y}\cdot(2s-2)$$

$$\left.\frac{\partial z}{\partial t}\right|_{(x,y)=(5,2)} = f_x(5,2)\cdot 3 + f_y(5,2)\cdot 2$$

$$= (3)(3) + (6)(2)$$

$$= 9 + 12 = 21$$

$$\left.\frac{\partial z}{\partial t}\right|_{(x,y)=(5,2)}$$

Q6) (6 marks) Find and classify the critical points for the function $f(x,y) = \frac{1}{4}x^4 - 3xy + \frac{1}{2}y^2$.

$F_x = x^3 - 3y \qquad F_y = -3x + y$

$x^3 - 3y = 0 \qquad 3 \times (-3x + y = 0)$

$-9x + 3y = 0$

$x^3 - 9x = 0$

$x(x^2 - 9) = 0$

$x = 0 \quad x = \pm 3$

If $x = 0 \rightarrow y = 0$

If $x = 3 \rightarrow y = 9$

If $x = -3 \rightarrow y = -9$

Critical Points $\rightarrow (0,0)$, $(3,9)$, $(-3,-9)$

$f_{xx} = 3x^2$

$f_{yy} = 1$

$f_{xy} = -3$

$x^3 - 3y = 0 \qquad -3x + y = 0$

	f_{xx}	f_{yy}	f_{xy}	D	result
(0,0)	0	1	-3	−	saddle
(3,9)	27	1	-3	+	L. min
(-3,-9)	27	1	-3	+	L. min