

The arc length function  $s(t)$ , for the curve  $\vec{r}(t) = \langle \cos e^t, \sin e^t, \sqrt{8} e^t \rangle$ , measured from the point  $Q$  where  $t = 0$ , in the direction of increasing  $t$  is:

(A)  $3e^{2t} - 3$

(B)  $3e^t - 3$

(C)  $\sqrt{8} e^t - 1$

(D)  $3e^t - 1$

(E)  $e^t - 1$

$$s(t) = \int_a^t \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \cdot dt$$

$$= \int_0^t \sqrt{e^{2t} (\sin^2(e^t) + \cos^2(e^t) + 8)} \cdot dt$$

$$= \int_0^t 3e^t \cdot dt$$

$$= 3e^t - 3$$

(B)

$$dx = -e^t \cdot \sin(e^t)$$

$$dy = e^t \cdot \cos(e^t)$$

$$dz = \sqrt{8} \cdot e^t$$

~

The domain of the function  $f(x, y) = \sqrt{x - 4} + \frac{1}{\sqrt{y - 2}}$  is

- A) All the points in the  $xy$ -plane that lie to the right of (or on) the line  $x = 4$  and below the line  $y = 2$ .
- B) All the points in the  $xy$ -plane that lie to the left of (or on) the line  $x = 4$  and below the line  $y = 2$ .
- C) All the points in the  $xy$ -plane except the point  $(4, 2)$ .
- D) All the points in the  $xy$ -plane that lie to the right of (or on) the line  $x = 4$  and above the line  $y = 2$ .
- E) All the points in the  $xy$ -plane that lie to the right of (or on) the line  $x = 4$  and above (or on) the line  $y = 2$ .

---

$$x - 4 \geq 0$$

$$\underline{x \geq 4}$$

①

$$y - 2 \geq 0$$

$y \geq 2$  . But the root is in the denominator

So it'll be  $\underline{y > 2}$

A vector function that represents the curve of intersection of the cylinder  $y^2 + z^2 = 16$  and the surface  $x = yz$  is

(A)  $\vec{r}(t) = \langle 4 \cos t, 4 \sin t, 8 \sin 2t \rangle$

(B)  $\vec{r}(t) = \langle 4 \cos t, \sin t, 8 \sin 2t \rangle$

(C)  $\vec{r}(t) = \langle 8 \sin 2t, 4 \cos 2t, 4 \sin 2t \rangle$

(D)  $\vec{r}(t) = \langle 8 \sin 2t, 4 \cos t, 4 \sin t \rangle$

(E)  $\vec{r}(t) = \langle 8 \cos 2t, 4 \cos 2t, 4 \sin t \rangle$

5

$$x = yz$$

$$y^2 + z^2 = 16$$

$$y = \sqrt{16 - z^2}$$

$$x = 2\sqrt{16 - z^2}$$

By trial & error

when  $z = 4 \sin(t)$

$$y = \sqrt{16 - 16 \sin^2 t}$$

$$y = 4 \cos(t)$$

$$x = 4 \cos(t) \cdot 4 \sin(t)$$

$$~~x = 16 \sin(2t)~~$$

$$x = 8 \sin(2t)$$

(D)

Suppose  $f$  is a differentiable function of  $x$  and  $y$ , and  $g(t, s) = f(x, y)$  where  $x = 4t - s^2$  and  $y = \frac{1}{2}se^{t-1}$ . Use the table of values to calculate

$g_s(1, 2)$ .

|          | $f$ | $g$ | $f_x$ | $f_y$ |
|----------|-----|-----|-------|-------|
| $(0, 1)$ | 3   | 10  | 5     | 4     |
| $(1, 2)$ | 10  | 2   | 1     | 3     |

- A) 4
- B) 8
- C) 14
- D) 24
- E) -18

From the table

$$\frac{dg}{ds} = \frac{\partial g}{\partial x} \cdot \frac{dx}{ds} + \frac{\partial g}{\partial y} \cdot \frac{dy}{ds}$$

$$= 5 \cdot (-4) + 4 \cdot \left(\frac{1}{2}\right)$$

$$= \underline{-18} \quad (\text{E})$$

$$g = f$$

$$t = 1$$

$$s = 2$$

$$x = 0$$

$$y = 1$$

$$\frac{dx}{ds} = -2s$$

$$\left. \frac{dx}{ds} \right|_{s=2} = -4$$

$$\frac{dy}{ds} = \frac{1}{2} e^{t-1}$$

$$\left. \frac{dy}{ds} \right|_{t=1} = \frac{1}{2}$$



**Parametric equations for the tangent line to the curve of the vector function:**

$\vec{r}(t) = \langle 2\sqrt{t}, t^2, -3t \rangle$  at the point  $(2, 1, -3)$  is:

(A)  $x = 2 + 2t$  ,  $y = 1 + 2t$  ,  $z = -3 - 3t$

(B)  $x = 2 + t$  ,  $y = 1 - 2t$  ,  $z = -3 - 3t$

(C)  $x = 2 + t$  ,  $y = 1 + 2t$  ,  $z = -3 - 3t$

(D)  $x = 2 + t$  ,  $y = 1 + 2t$  ,  $z = -3 + 3t$

(E)  $x = -2 - t$  ,  $y = -1 - 2t$  ,  $z = 3 + 3t$

$$-3t = -3$$

$$\underline{t = 1}$$

$$r'(t) = \left\langle \frac{1}{\sqrt{t}}, 2t, -3 \right\rangle$$

$$\text{sub } t=1 \rightarrow r'(t) = \langle 1, 2, -3 \rangle \text{ @ } (2, 1, -3)$$

$$x = 2 + t$$

$$y = 1 + 2t$$

$$z = -3 - 3t$$

Ⓒ

Σ

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^3}{x^2 + y^6}$$

- (A) does not exist, since along the x-axis the limit is 0, and along  $x = y^3$  the limit is  $\frac{1}{2}$
- (B) does not exist, since along the y-axis the limit is 1, and along  $x = y^3$  the limit is  $\frac{1}{2}$
- (C) does not exist, since along the x-axis the limit is 0, and along  $y = x^3$  the limit is  $\frac{1}{2}$
- (D) does not exist, since along the  $x = my^3$ , the limit is  $\frac{m}{m+1}$
- (E) The limit exists and equals to  $\frac{1}{2}$ , since along  $x = y^3$  the limit is  $\frac{1}{2}$

$$\lim_{x, y \rightarrow 0, 0} \frac{xy^3}{x^2 + y^6} = \frac{0}{0}$$

$$\text{along } x=0 \rightarrow \lim \frac{0}{y^6} = 0$$

$$\text{along } y=0 \rightarrow \lim \frac{0}{x^2} = 0$$

$$\text{along } x=y^3 \rightarrow \lim \frac{\cancel{y^6}}{2y^6} = \boxed{\frac{1}{2}}$$

D.N.E (A)

At the point  $(0,0)$ , the linearization of the function

$$f(x, y) = \frac{2x+3}{1-4y} \text{ is}$$

(A)  $L(x, y) = -3 + 2x - 12y$

(B)  $L(x, y) = -3 - 2x - 12y$

(C)  $L(x, y) = -3 + 2x + 12y$

(D)  $L(x, y) = 3 - 2x + 12y$

(E)  $L(x, y) = 3 + 2x + 12y$

$$f(x, y) = \frac{2x+3}{1-4y} \quad (a, b) = (0, 0)$$

$$\begin{aligned} L(x, y) &= f(a, b) + f_x(a, b)(x-a) + f_y(a, b)(y-b) \\ &= 3 + 2x + 12y \end{aligned}$$

$$f(a, b) = 3$$

$$f_x(a, b) = 2$$

$$f_y(a, b) = 12$$

The arc length of the curve  $\vec{r}(t) = \langle 5 \sin t, 5 \cos t, 2 \rangle$  from  $t = -3$  to  $t = 3$  is

- (A) 15
- (B) 20
- (C) 30
- (D) 9
- (E) 5

Select one:

- A
- B
- C

$$s(t) = \int_a^t \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \cdot dt$$

$$= \int_{-3}^3 \sqrt{25(\sin^2 t + \cos^2 t)} \cdot dt$$

$$= 5(3 - (-3))$$

$$= 30 \quad (c)$$

$$dx = 5 \cos t$$

$$dy = -5 \sin t$$

$$dz = 0$$



The curvature of the curve of the function  
 $f(x) = 3 \ln x$ , is

(A)  $\frac{3}{x^2(1+\frac{9}{x^2})^{3/2}}$

(B)  $\frac{3}{x^2(1-\frac{9}{x^2})^{3/2}}$

(C)  $\frac{9}{x^2(1+\frac{3}{x^2})^{3/2}}$

(D)  $\frac{-3}{x^2(1-\frac{9}{x^2})^{3/2}}$

(E)  $\frac{-3}{x^2(1+\frac{9}{x^2})^{3/2}}$

(A)

(B)

(C)

(D)

$$K(x) = \frac{|f''(x)|}{(1 + (f'(x))^2)^{3/2}}$$

$$K(x) = \frac{+3}{x^2 \left(1 + \frac{9}{x^2}\right)^{3/2}}$$

(A)

$$f(x) = 3 \ln(x)$$

$$f'(x) = \frac{3}{x}$$

$$f''(x) = \frac{-3}{x^2}$$

↳ \_\_\_\_\_ .

The curvature of the curve  $\vec{r}(t) = \langle -t, t^2, t^3 \rangle$ , is

(A)  $\frac{\sqrt{36t^4+36t^2+4}}{(1+4t^2+9t^4)^3}$

(B)  $\frac{\sqrt{6t^2-6t+2}}{(1+4t^2+9t^4)^{3/2}}$

(C)  $\frac{\sqrt{36t^4+36t^2+4}}{(1+4t^2+9t^4)^{3/2}}$

(D)  $\frac{\sqrt{36t^4-36t^2+4}}{(1+4t^2+9t^4)^{3/2}}$

(E)  $\frac{\sqrt{36t^4+36t^2+4}}{(1+4t^2+9t^4)^{1/2}}$

$$K = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

$$K = \frac{\sqrt{36t^4 + 36t^2 + 4}}{(1 + 4t^2 + 9t^4)^{3/2}}$$

(c)

$$r'(t) = \langle -1, 2t, 3t^2 \rangle$$

$$r''(t) = \langle 0, 2, 6t \rangle$$

$$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ -1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix} = \langle 6t^2, 6t, -2 \rangle$$

Find the curvature of the curve

$C: \vec{r}(t) = \langle t^2, 2t, e^t \rangle$ : when  $t = 0$

(A) 0

(B)  $\frac{\sqrt{8}}{(\sqrt{5})^3}$

(C)  $\frac{\sqrt{24}}{5\sqrt{5}}$

(D)  $\frac{\sqrt{5}}{(\sqrt{24})^3}$

(E)  $\frac{\sqrt{24}}{125}$

Select one:

A

B

C

D

E

$$K = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

$$|r'(t)| = \sqrt{4t^2 + 4 + e^{2t}}$$

$$|r'(0)| = \sqrt{5}$$

~~$$|r''(t)| = \sqrt{4 + \dots}$$~~

$$|r'(t) \times r''(t)| = \sqrt{4e^{2t} + 4e^{2t}(t-1)^2 + 16}$$

$$\text{@ } t=0 \rightarrow \sqrt{24}$$

$$K = \frac{\sqrt{24}}{5\sqrt{5}} \quad \text{③}$$

$$r(t) = \langle t^2, 2t, e^t \rangle$$

$$r'(t) = \langle 2t, 2, e^t \rangle$$

$$r''(t) = \langle 2, 0, e^t \rangle$$

$$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 2t & 2 & e^t \\ 2 & 0 & e^t \end{vmatrix} = \langle 2e^t, \dots, -4 \rangle$$

$\downarrow$   
 $2e^t(t-1)$

The length of the curve

$\vec{r}(t) = \langle t, 2 \cos t, 2 \sin t \rangle, -2 \leq t \leq 2$ , is

(A)  $6\sqrt{5}$

(B)  $4\sqrt{5}$

(C)  $2\sqrt{5}$

(D)  $2\sqrt{3}$

(E) 4

(A)

(B)

(C)

(D)

(E)



$$L(t) = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \cdot dt$$

$$\int_{-2}^2 \sqrt{1 + 4(\sin^2 t + \cos^2 t)} \cdot dt$$

$$= 4\sqrt{5} \quad (B)$$

$$\frac{dx}{dt} = 1$$

$$\frac{dy}{dt} = -2 \sin t$$

$$\frac{dz}{dt} = 2 \cos t$$



The domain of the function  $f(x, y) = \frac{\sqrt{y-x^2}}{\sqrt{x^2-4}}$  is

- A)  $\{(x, y) | x \geq y^2, y \neq \mp 2\}$
- B)  $\{(x, y) | y \geq x^2, x \neq \mp 2\}$
- C)  $\{(x, y) | x > 2 \text{ or } x < -2\}$
- D)  $\{(x, y) | -2 < x < 2\}$
- E)  $\{(x, y) | y \geq x^2, |x| > 2\}$

Select one:

- A
- B
- C
- D
- E

2

$$y - x^2 \geq 0$$

$$\underline{y \geq x^2}$$

$$x^2 - 4 > 0$$

$$\underline{-2 < x < 2}$$

(E)

The domain of the function  $f(x, y) = \frac{\sqrt{25-x^2-y^2}}{9-x^2}$  is:

- (A) The set of points in  $\mathbb{R}^2$  but not on the lines  $x = 3$ ,  $x = -3$
- (B) The set of points  $(x, y)$  on the circle centered at  $(0, 0)$ , with radius 5, and not on the lines  $x = 3$ ,  $x = -3$
- (C) The set of points  $(x, y)$  inside the circle centered at  $(0, 0)$ , with radius 5, but not on the lines  $x = 3$ ,  $x = -3$
- (D) The set of points  $(x, y)$  on or inside the circle centered at  $(0, 0)$ , with radius 5
- (E) The set of points  $(x, y)$  on or inside the circle centered at  $(0, 0)$  but not on lines  $x = 3$ ,  $x = -3$



$$9 - x^2 = 0$$

$$x = 3, x = -3$$

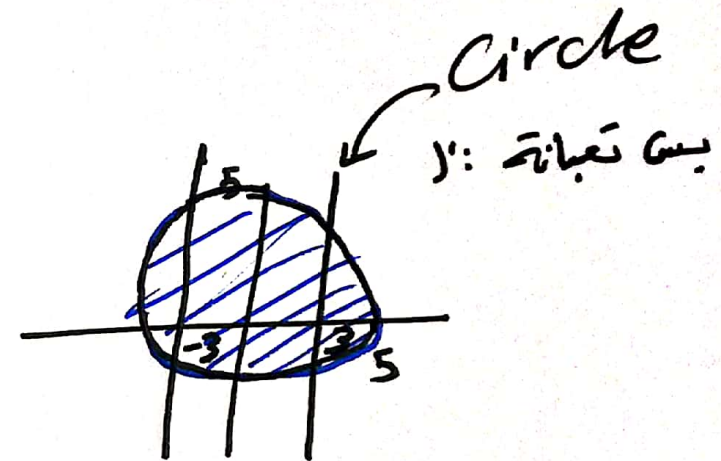
on or inside the circle

but not on  $x = 3, x = -3$

(E)

$$25 - x^2 - y^2 \geq 0$$

$$x^2 + y^2 \leq 25$$



The function  $f(x, y, z) = 2\sin\sqrt{3x^2 + 5y^2 + z^2}$  is continuous on

- A)  $\mathbb{R}$
- B)  $x \geq 0, y \geq 0, z \geq 0$
- C)  $\mathbb{R}^3$  (3-space)
- D)  $3x^2 + 5y^2 + z^2 > 0$
- E)  $\sqrt{3}x + \sqrt{5}y + z \geq 0$

Select one:

- A
- B
- C
- D
- E

$\mathbb{R}^3$  (3-space) ©

whatever the inputs are the root will be const.

← .

If  $\mathbf{r}'(t) = \langle 2 \sin 2t, 3 \cos 3t, \cos t \rangle$  and

$\mathbf{r}\left(\frac{\pi}{6}\right) = \langle 1, 1, 3 \rangle$  then  $\mathbf{r}(t) =$

A)  $\langle -\cos 2t + \frac{3}{2}, \sin 3t, \sin t + \frac{5}{2} \rangle$

B)  $\langle \sin t + \frac{1}{2}, -\cos 2t + \frac{3}{2}, \sin 3t + 2 \rangle$

C)  $\langle \sin t + \frac{1}{2}, \sin 3t, -\cos 2t + \frac{7}{2} \rangle$

D)  $\langle \sin 3t, \sin t + \frac{1}{2}, -\cos 2t + \frac{7}{2} \rangle$

E)  $\langle -\cos 2t + \frac{3}{2}, \sin 3t, -\sin t + \frac{7}{2} \rangle$

$$r'(t) = \langle 2 \sin 2t, 3 \cos 3t, \cos t \rangle$$

$$r(t) = \langle -\cos(2t) + C_1, \sin(3t) + C_2, \sin(t) + C_3 \rangle$$

$$r\left(\frac{\pi}{6}\right) = \langle 1, 1, 3 \rangle$$

$$-\frac{1}{2} + C_1 = 1$$

$$\underline{C_1 = \frac{3}{2}}$$

$$1 + C_2 = 1$$

$$\underline{C_2 = 0}$$

$$\frac{1}{2} + C_3 = 3$$

$$\underline{C_3 = \frac{5}{2}}$$

$$r(t) = \langle -\cos(2t) + \frac{3}{2}, \sin(3t), \sin(t) + \frac{5}{2} \rangle$$

(A)



The helix  $\vec{r}(t) = \langle t, \cos t, \sin t \rangle$  intersects the sphere  $x^2 + y^2 + z^2 = 17$  at  $t =$

- (A) 0, 4
- (B) 0, -4
- (C)  $\mp 4$
- (D) never intersect it
- (E)  $\mp 17$

(A)

(B)

(C)

(D)

(E)

$$x = t, y = \cos t, z = \sin t \rightarrow \text{sub in } x^2 + y^2 + z^2 = 17$$

$$t^2 + \frac{\cos^2 t + \sin^2 t}{1} = 17$$

$$t^2 = 16$$

$$t = 4, -4 \quad \textcircled{c}$$

~~~~~.

One of the following is a vector function that represents the curve of intersection of the cylinder  $x^2 + y^2 = a$ ,  $a > 0$  and the surface  $z = 6xy$  :-

- A)  $\langle acost, asint, 6acost\ sint \rangle$ ,  $0 \leq t \leq 2\pi$
- B)  $\langle acost, asint, 6a^2 cost\ sint \rangle$ ,  $0 \leq t \leq 2\pi$
- C)  $\langle \sqrt{a} cost, \sqrt{a} sint, 3a \sin(2t) \rangle$ ,  $0 \leq t \leq 2\pi$
- D)  $\langle cost, sint, 6cost\ sint \rangle$ ,  $0 \leq t \leq 2\pi$
- E)  $\langle \sqrt{a} cost, \sqrt{a} sint, acost\ sint \rangle$ ,  $0 \leq t \leq 2\pi$

$$x = \sqrt{a - y^2}$$

$$z = 6y\sqrt{a - y^2}$$

TRIAL ~~of~~ error

$$\langle \sqrt{a} \cos t, \sqrt{a} \sin t, 3a \sin(2t) \rangle \quad (c)$$



Question 11

Not yet  
answered

Marked out of

1.00  
Log  
question

$$\lim_{(x,y) \rightarrow (0,0)} \frac{e^{(x^2+y^2)} - 1}{x^2+y^2} =$$

- A. 1
- B. 4
- C. Does not exist
- D. 0
- E. -1

$$\lim_{x,y \rightarrow 0,0} \frac{e^{(x^2+y^2)} - 1}{x^2+y^2}$$

$$\rightarrow \lim_{r \rightarrow 0} \frac{e^{r^2} - 1}{r^2} = \frac{0}{0}$$

l'Hopital

$$\lim_{r \rightarrow 0} \frac{2r e^{r^2}}{2r} = \boxed{1} \quad \textcircled{A}$$

The value of  $\lim_{(x,y) \rightarrow (0,0)} \frac{2y^2 \sin^2 x}{x^4 + y^4}$  along  $y = x$  is

- A) 1
- B) 2
- C) does not exist
- D) 0
- E)  $\frac{1}{2}$

Select one:

- A
- B
- C
- D
- E

[Clear my choice](#)

$$\lim_{x, y \rightarrow 0, 0} \frac{2y^2 \sin^2 x}{x^4 + y^4} \quad \text{along } y = x$$

$$\lim_{x \rightarrow 0} \frac{2x^2 \sin^2 x}{2x^4} = \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)^2 = 1^2 = 1$$

(A)



Question 7

Not yet  
answered

Marked out of  
3.00

Flag  
question

$$\lim_{(x,y) \rightarrow (0,0)} \frac{5y}{\sqrt{x^2 + y^2}}$$

- (A) 0
- (B) 5
- (C)  $\frac{5}{\sqrt{2}}$
- (D) -5
- (E) Doesn't exist

$$\lim_{x,y \rightarrow 0,0} \frac{5y}{\sqrt{x^2+y^2}} = \frac{0}{0}$$

$$\text{along } x=0 \rightarrow \lim \frac{5x}{x} = \frac{5}{1}$$

$$\text{along } y=0 \rightarrow \lim \frac{0}{x} = 0$$

D.N.E (E)

The linear approximation at  $(0, 0)$  for the function  $f(x, y) = e^x \cos(xy)$

- A.  $1 + 3x$
- B.  $1 + 2x$
- C.  $1 - 2x$
- D.  $1 + x$
- E.  $1 - x$

The correct answer is:

$$1 + x$$

$$f(x, y) = e^x \cos(xy)$$

$$L(x, y) = f(a, b) + f_x(a, b)(x-a) + f_y(a, b)(y-b)$$

$$= 1 + x \quad \textcircled{D}$$

$$f(0, 0) = 1$$

$$f_x = e^x \cos(xy) - y e^x \sin(xy)$$

$$f_y = -x e^x \sin(xy)$$

$$f_x(0, 0) = 1$$

$$f_y(0, 0) = 0$$

The linear approximation of  $e^{xyz}$  at  $(2,1,0)$  is:

A)  $e^{xyz} \approx 1 + 2x$

B)  $e^{xyz} \approx 1 + 2z$

C)  $e^{xyz} \approx 1 + 2y$

D)  $e^{xyz} \approx 1 + 2x + 2y$

E)  $e^{xyz} \approx 1 + 2x + 2z$

Select one:

A

B

C

D

E

$$f(x, y, z) = e^{xyz} \text{ @ } (2, 1, 0)$$

$$L(x, y, z) = 1 + 2(z - 0)$$

$\rightarrow 1 + 2z \quad (B)$

$$f(2, 1, 0) = 1$$

$$f_x = yz e^{xyz} \text{ @ } 0$$

$$f_y = xz e^{xyz} \text{ @ } 0$$

$$f_z = xy e^{xyz} \text{ @ } 2$$

The equation of the normal line to the surface

$x^2 + 2y^2 - 3z^2 = 3$  at the point  $(-2, -1, 1)$  is

(A)  $\frac{x+2}{-4} = \frac{y+1}{4} = \frac{z-1}{-3}$

(B)  $\frac{x-2}{-4} = \frac{y+1}{-4} = \frac{z-1}{-3}$

(C)  $\frac{x+2}{4} = \frac{y+1}{-4} = \frac{z-1}{-6}$

(D)  $\frac{x+2}{-4} = \frac{y+1}{-4} = \frac{z-1}{-6}$

(E)  $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z+3}{-3}$

$$F_x = 2x$$

$$F_y = 4y$$

$$F_z = -6z$$

$$\rightarrow = -4$$

$$\textcircled{a} (-2, -1, 1) \rightarrow = -4$$

$$\rightarrow = -6$$

$$\frac{x+2}{-4} = \frac{y+1}{-4} = \frac{z-1}{-6}$$

①



If  $e^z = xyz$ , then  $\frac{\partial z}{\partial y} =$

(A)  $\frac{e^z - xy}{xz}$

(B)  $\frac{xz}{e^z - xy}$

(C)  $\frac{xy}{e^z - xy}$

(D)  $e^z - xy$

(E)  $\frac{yz}{e^z - xy}$

Select one:

A

B

C

D

E

$$e^z \cdot \frac{\partial z}{\partial y} = xy \cdot \frac{\partial z}{\partial y} + z \cdot x$$

$$\frac{\partial z}{\partial y} (e^z - xy) = xz$$

$$\frac{\partial z}{\partial y} = \frac{xz}{e^z - xy}$$

The parametric equations for the tangent line to the curve of  $\vec{r}(t) = \langle t, t^2, t^3 \rangle$  at the point  $(3, 9, 27)$

- A.  $x = -1 + t; y = 1 - 2t; z = -1 + 3t$
- B.  $x = 1 + t; y = 1 + 2t; z = 1 + 3t$
- C.  $x = 3 + t; y = 9 + 6t; z = 27 + 27t$
- D.  $x = 2 + t; y = 4 + 4t; z = 8 + 12t$
- E.  $x = -2 + t; y = 4 - 4t; z = -8 + 12t$

The correct answer is:

$$x = 3 + t; y = 9 + 6t; z = 27 + 27t$$

$$r(t) = \langle t, t^2, t^3 \rangle \quad @ (3, 9, 27)$$

$$r'(t) = \langle 1, 2t, 3t^2 \rangle$$

$$\underline{t=3}$$

$$r'(3) = \langle 1, 6, 27 \rangle$$

$$x = 3 + t, \quad y = 9 + 6t, \quad z = 27 + 27t \quad (c)$$

Parametric equations of the tangent line to the curve

$\vec{r}(t) = \langle 3t, e^t, e^{-t} \rangle$  at  $t = 0$  is

(A)  $x = 3t, y = 1 + t, z = 1 + t$

(B)  $x = 3 + 3t, y = 1 + t, z = 1 - t$

(C)  $x = 3 + 3t, y = 1 + t, z = 1 + t$

(D)  $x = 3 + 3t, y = 1 - t, z = 1 + t$

(E)  $x = 3t, y = 1 + t, z = 1 - t$

(A)

(B)

(C)

(D)

(E)



$$r(t) = \langle 3t, e^t, e^{-t} \rangle$$

$$r(0) = (0, 1, 1)$$

$$r'(t) = \langle 3, e^t, -e^{-t} \rangle$$

$$r'(0) = \langle 3, 1, -1 \rangle$$

$$x = 3t, \quad y = 1+t, \quad z = 1-t \quad \textcircled{E}$$

The equation of the tangent plane to the surface  $x^2 + 2y^2 - 3z^2 = 3$  at the point  $(2, -1, 1)$  is

(A)  $4(x - 2) - 4(y + 1) - 6(z - 1) = 0$

(B)  $4(x - 2) + 2(y + 1) - 3(z - 1) = 0$

(C)  $4(x + 2) - 4(y + 1) - 6(z + 1) = 0$

(D)  $2(x - 2) - 1(y + 1) + (z - 1) = 0$

(E)  $4(x - 2)^2 - 4(y + 1)^2 - 6(z - 1)^2 = 0$



$$x^2 + 2y^2 - 3z^2 = 3 \quad @ \quad (2, -1, 1)$$

$$F_x = 2x \quad \rightarrow = 4$$

$$F_y = 4y \quad @ \rightarrow = -4$$

$$F_z = -6z \quad \rightarrow = -6$$

$$4(x-2) - 4(y+1) - 6(z-1) = 0 \quad @$$

↪



Let  $f(x, y) = x^4 + 5xy^3$ . Find  $f_{xy}$

(A)  $5y^2$

(B)  $15y^2$

(C)  $5y$

(D)  $15y$

(E)  $12 + 15y^2$

Select one:

A

B

C

D

E

$$f(x, y) = x^4 + 5xy^3$$

$$f_x = 4x^3 + 5y^3$$

$$f_{xy} = 0 + 15y^2 \quad \textcircled{B}$$

The directional derivative of the function  $f(x, y) = x^2 e^{-y}$  at the point  $(-3, 0)$  in the direction of the vector  $v = 3i + 4j$  is

A)  $\frac{-18}{5}$

B)  $\frac{-54}{5}$

C)  $\frac{18}{5}$

D)  $\frac{-36}{5}$

E)  $\frac{4}{5}$

$$\vec{\nabla} f = \langle 2e^y x, -x^2 e^{-y} \rangle \rightarrow @(-3, 0) = \langle -6, -9 \rangle$$

$$|\vec{v}| = \sqrt{9+16} = 5$$

$$\hat{v} = \frac{3}{5} \hat{i} + \frac{4}{5} \hat{j}$$

$$\nabla f \cdot \hat{v} = \frac{-18}{5} - \frac{36}{5}$$

$$= \frac{-54}{5} \quad \textcircled{B}$$

If  $\vec{\nabla}f(3, 2) = \langle 10, -5 \rangle$  and  $\vec{a} = \langle 3, -4 \rangle$ , then  
the directional derivative of  $f$  at  $(3, 2)$  in the direction of  $\vec{a}$ ,  
 $D_{\vec{a}}f(3, 2) =$

- (A)  $\frac{10}{\sqrt{5}}$
- (B)  $\frac{2}{\sqrt{5}}$
- (C) 10
- (D) 50
- (E)  $-\frac{2}{\sqrt{5}}$

(A)

(B)

(C)

(D)

(E)

$$\hat{a} = \left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle$$

$$\nabla f \cdot \hat{a} = 6 - 4 = 10$$

(c)



If  $\vec{\nabla}f(-1, -2, 3) = \langle 1, -2, 2 \rangle$ , then

(A) The minimum directional derivative of the function  $f$  at the point  $(-1, -2, 3)$  is 3, and it occurs in the direction of  $(-1, -2, 2)$ .

(B) The minimum directional derivative of the function  $f$  at the point  $(-1, -2, 3)$  is 3, and it occurs in the direction of  $(-1, 2, -2)$ .

(C) The minimum directional derivative of the function  $f$  at the point  $(-1, -2, 3)$  is  $-3$ , and it occurs in the direction of  $\langle 1, -2, 2 \rangle$ .

(D) The minimum directional derivative of the function  $f$  at the point  $(-1, -2, 3)$  is  $-3$ , and it occurs in the direction of  $(-1, 2, -2)$ .

(E) The maximum directional derivative of the function  $f$  at the point  $(-1, -2, 3)$  is  $-3$ , and it occurs in the direction of  $(-1, 2, -2)$ .

(A)

(B)

(C)

(D)

(E)

max when  $\theta = 0^\circ$

$$\rightarrow |\vec{\nabla} f| = \sqrt{1+4+4} = (3)$$


min when  $\theta = 180^\circ$

$$\rightarrow (-3)$$

(D)

~~~~~



Question \*   
(3 Points)

The critical points of  $f(x, y) = x^3 - 3x + y^2 - 2y$  are

- (A) (1,1) and (-1,-1)
- (B) (1,1) and (-1,1)
- (C) (-1,1) and (-1,-1)
- (D) (1,-1) and (-1,-1)
- (E) (-1,-1) and (-1,1)

(A)

(B)

(C)

(D)

(E)

$$f_x = 3x^2 - 3 = 0$$

$$x = \pm 1$$

$$f_y = 2y - 2 = 0$$

$$y = 1$$

$$(1, 1)$$

$$(-1, 1)$$

(B)

$f(x, y)$  has a continuous second partial derivatives at the point  $(1, 2)$  :

$f_{xx}(1, 2) = -3$  ,  $f_{yy}(1, 2) = 4$  and  $f_{xy}(1, 2) = 2$   
then at  $(1, 2)$   $f$  has

- (A) Local maximum value
- (B) Local minimum value
- (C) Saddle point
- (D) Inflection point
- (E) 8 is the minimum value of the directional derivative of  $f$

$$D = f_{xx} \cdot f_{yy} - f_{xy}^2$$

$$= -12 - 4$$

$\Rightarrow -16$ , then saddle point (c)

Let  $f(x, y) = y^2 - 2y \cos x$ . Then

- (A)  $f$  has a local minimum value at  $\left(\frac{3\pi}{2}, 0\right)$
- (B)  $f$  has a local maximum value at  $\left(\frac{3\pi}{2}, 0\right)$
- (C)  $\left(\frac{3\pi}{2}, 0\right)$  is not a critical point of  $f$ .
- (D)  $\left(\frac{3\pi}{2}, 0\right)$  is a saddle point of  $f$ .
- (E)  $\left(\frac{3\pi}{2}, 1\right)$  is a critical point of  $f$ .

Select one:

- A
- B
- C
- D
- E

$$f_x = 2y \sin x$$

$$f_y = 2y - 2 \cos x \rightarrow y = \cos x$$

$$f_{xx} = 2y \cos x$$

$$f_{yy} = 2$$

$$f_{xy} = 2 \sin x$$

$$2 \sin x \cos x = 0$$

$$x = 0, \pi/2, \pi, 3\pi/2, 2\pi$$

$$y = 1, 0, -1, 0, 1$$

$$\text{sub} \left( \frac{3\pi}{2}, 0 \right)$$

$\left( \frac{3\pi}{2}, 0 \right)$  is Critical

$$D = f_{xx} f_{yy} - f_{xy}^2$$

$$= 0 \cdot 2 - (-2)^2$$

$$D = -4 \rightarrow \text{saddle point} \quad (D)$$

~

Suppose  $f$  is a differentiable function in  $x$  and  $y$ , and

$g(u, v) = f(u + v, u - v)$ , where  $x = u + v$  and  $y = u - v$ .

If  $f_x(0, -2) = 1$ ,  $f_y(0, -2) = 3$ , then  $g_u(-1, 1) - g_v(-1, 1) =$

(A) 4

(B) 6

(C) 2

(D) 1

(E) 3

Select one:

A

B

C

D

E

$$u = -1$$

$$v = 1$$

$$x = 0$$

$$y = -2$$

$$\frac{dx}{du} = 1$$

$$\frac{dx}{dv} = 1$$

$$\frac{dy}{du} = 1$$

$$\frac{dy}{dv} = -1$$

$$\frac{dg}{du} = \frac{dg}{dx} \cdot \frac{dx}{du} + \frac{dg}{dy} \cdot \frac{dy}{du}$$

$$1 \cdot 1 + 3 \cdot 1 = 4$$

$$\frac{dg}{dv} = \frac{dg}{dx} \cdot \frac{dx}{dv} + \frac{dg}{dy} \cdot \frac{dy}{dv}$$

$$1 \cdot 1 + 3 \cdot (-1) = -2$$

$$\frac{dg}{du} - \frac{dg}{dv} = 4 - (-2) = 6$$



To find the points on the cone  $z^2 = x^2 + y^2$ , that are closest to the point  $(2, 4, 0)$ , we need to solve these Lagrange equations

(A)  $x - 2 = \lambda x, y - 4 = \lambda y, z = \lambda z, x^2 + y^2 - z^2 = 0.$

~~(B)  $x + 2 = \lambda x, y + 4 = \lambda y, z = \lambda z, x^2 + y^2 - z^2 = 0.$~~

~~(C)  $x + 4 = \lambda x, y + 2 = \lambda y, z = -\lambda z, x^2 + y^2 - z^2 = 0.$~~

~~(D)  $x - 2 = \lambda x, y - 4 = \lambda y, z = -\lambda z, x^2 + y^2 - z^2 = 0.$~~

(E)  $x - 2 = \lambda x, y - 4 = \lambda y, z = -\lambda z, (x - 2)^2 + (y - 4)^2 - z^2 = 0.$

$$f(x, y, z) = (x-2)^2 + (y-4)^2 + z^2$$

$$g(x, y, z) = x^2 + y^2 - z^2 \quad \text{--- (4)}$$

$$2(x-2) = \lambda 2x \quad \text{--- (1)}$$

$$2(y-4) = \lambda 2y \quad \text{--- (2)}$$

$$2z = -\lambda 2z \quad \text{--- (3)}$$

Divide (1), (2), (3) by 2

(E)

Using language multipliers method. The maximum value of  $f(x, y, z) = xyz$ , subject to the constraint  $x^2 + y^2 + z^2 = 3$  is

- (A) 1
- (B) 2
- (C) 6
- (D) 4
- (E) 8

---

$$f(x, y, z) = xyz$$

$$g(x, y, z) = x^2 + y^2 + z^2 = 3$$

$$yz = \lambda 2x$$

$$xz = \lambda 2y$$

$$xy = \lambda 2z$$

$$x^2 = y^2$$

$$y^2 = z^2$$

$\rightarrow x^2 = y^2 = z^2 \rightarrow 9$  possible points

$$f(1, 1, 1) = \textcircled{1} \quad \textcircled{A}$$

The minimum value of the function  $f(x, y) = 3x + y$  on the circle  $x^2 + y^2 = 10$  is (Hint: use the Lagrange method)

- A) 10 and  $-10$
- B)  $-(6 + \sqrt{6})$  at  $(-2, -\sqrt{6})$
- C) 10 at  $(3, 1)$
- D)  $-10$  at  $(-3, -1)$
- E)  $-4\sqrt{5}$  at  $(-\sqrt{5}, -\sqrt{5})$

$$f(x, y) = 3x + y$$

$$g(x, y) = x^2 + y^2 - 10$$

$$\left. \begin{array}{l} 3 = 2 \cdot 2x \\ 1 = 2 \cdot 2y \end{array} \right\} \rightarrow x = 3y \xrightarrow{\text{sub in } g} 10y^2 = 10$$

$$y = \pm 1$$

$$x = \pm 3$$

sub in  $f$

$$(3, 1) \rightarrow 10$$

$$(3, -1) \rightarrow 8$$

$$(-3, 1) \rightarrow -8$$

$$\boxed{(-3, -1) \rightarrow -10}$$

(D)

