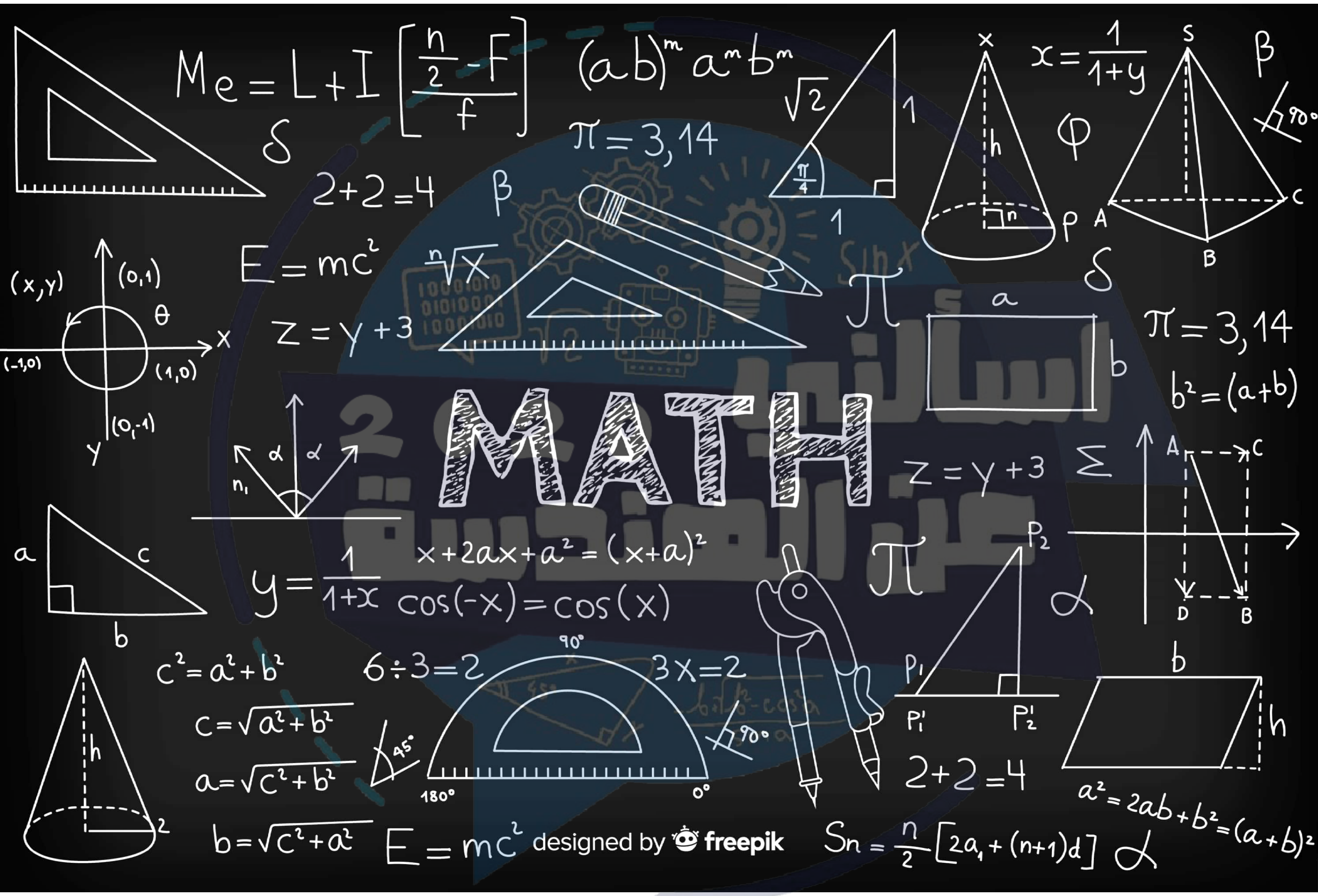

MATH
designed by freepik

Section

15.2

15.2

5) $\int_0^1 \int_0^{s^2} \cos(s^3)\, dt\, ds$.

$\rightarrow \int_0^1 \cos(s^3) t \Big]_0^{s^2} ds = \int_0^1 s^2 \cos(s^3)\, ds$.

$x = s^3$
$dx = 3s^2 ds$.
$ds = \frac{dx}{3s^2}$

$= \frac{1}{3}\int \cos(x)\, dx$

$= \frac{1}{3} \sin(s^3) \Big]_0^1$

$= \frac{1}{3}$

7) $\iint_D \frac{y}{x^2+1}\, dA$ $\quad D = \{ 0 \le x \le 4,\ 0 \le y \le \sqrt{x} \}$

$\rightarrow \int_0^4 \int_0^{\sqrt{x}} \left(\frac{y}{x^2+1}\right) dy\, dx = \int_0^4 \frac{1}{x^2+1} \cdot \frac{y^2}{2} \Big]_0^{\sqrt{x}} dx$

$= \frac{1}{2} \int_0^4 \frac{2x}{(x^2+1) \times 2}\, dx = \frac{1}{4} \ln|x^2+1| \Big]_0^4$

اكملوا حلها انتم

14) $\iint_D xy\, dA$ $\quad$ D is enclosed by $y = x^2$, $y = 3x$

$x = \mp\sqrt{y}$ $\quad x = \frac{y}{3}$

$x^2 = 3x \rightarrow x^2 - 3x = 0 \rightarrow x = 0,\ x = 3$

(1) $\int_0^9 \int_{y/3}^{\sqrt{y}} xy\, dx\, dy$.

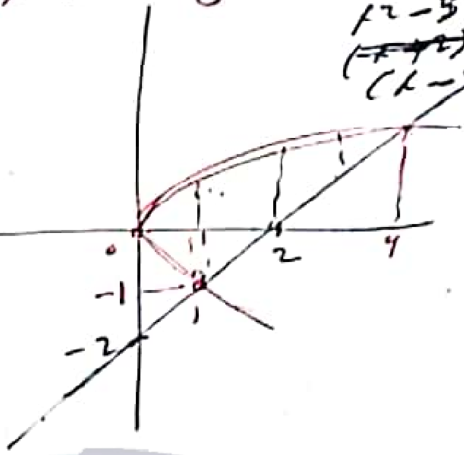
$= \int_0^9 \frac{x^2}{2} \Big]_{\frac{y}{3}}^{\sqrt{y}} y\, dy$

$= \int_0^9 \left(\frac{y}{2} - \frac{y^2}{2 \times 9}\right) y\, dy = \frac{y^3}{6} - \frac{y^4}{2 \times 36} \Big]_0^9$

(2) $\int_0^3 \int_{x^2}^{3x} xy\, dy\, dx$

5 $\iint_D y\,dA$ D is bounded by $y = x-2$, $x = y^2$.

$x = (x-2)^2$
$x = x^2 - 4x + 4$
$x^2 - 5x + 4 = 0$
$(x-4)(x-1) = 0$
$x = 4$
$x = 1$



① in dy dx.

$$\int_0^1\int_{-\sqrt{x}}^{\sqrt{x}} y\,dy\,dx + \int_1^4\int_{x-2}^{\sqrt{x}} y\,dy\,dx.$$

② in dx dy

$$\int_{-1}^{2}\int_{y^2}^{y+2} y\,dx\,dy. = \int_{-1}^{2} y(y+2-y^2)\,dy$$

$$= \int_{-1}^{2}(y^2+2y-y^3)\,dy = \frac{y^3}{3}+y^2-\frac{y^4}{4}\Big]_{-1}^{2}.$$

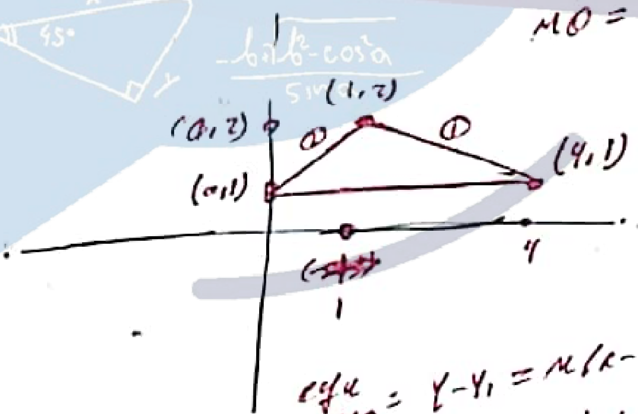
---

4 $\iint_D y^2\,dA$ D is $\triangle$ with vertices $(0,1)$ $(1,2)$ $(4,1)$

$$= \int_1^2\int_{y-1}^{-3y+7} y^2\,dx\,dy.$$

$M① = \frac{2-1}{1-4} = -\frac{1}{3}$

$M① = \frac{2-1}{1-0} = 1$



eqn $M①$: $y - y_1 = m(x-x_1)$
$y-2 = -\frac{1}{3}(x-1)$
$-3(y-2) = x-1$
$\boxed{x = -3y+7}$

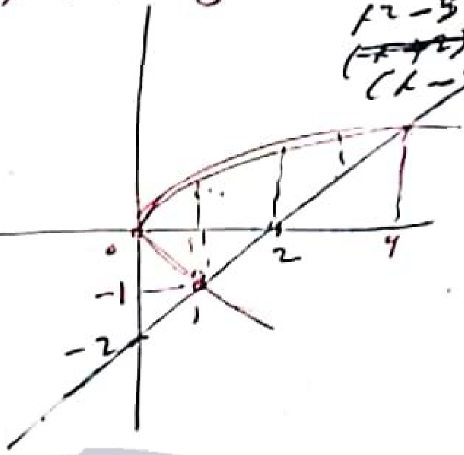
eqn $M①$: $y - y_1 = m(x - x_1)$
$\boxed{y-1 = x}$

15

[5] $\iint_D y\,dA$

D is bounded by $y = x - 2$, $x = y^2$.

$y = (x-2)^2$
$x = x^2 - 4x + 4$
$x^2 - 5x + 4 = 0$
$(x-4)(x-1) = 0$
$x = 4$
$x = 1$



① in $dy\,dx$.

$$\int_0^1\int_{-\sqrt{x}}^{\sqrt{x}} y\,dy\,dx + \int_1^4\int_{x-2}^{\sqrt{x}} y\,dy\,dx.$$

② in $dx\,dy$

$$\int_{-1}^{2}\int_{y^2}^{y+2} y\,dx\,dy. = \int_{-1}^{2} y(y+2-y^2)\,dy$$

$$= \int_{-1}^{2}(y^2+2y-y^3)\,dy = \left.\frac{y^3}{3}+y^2-\frac{y^4}{4}\right]_{-1}^{2}.$$

19

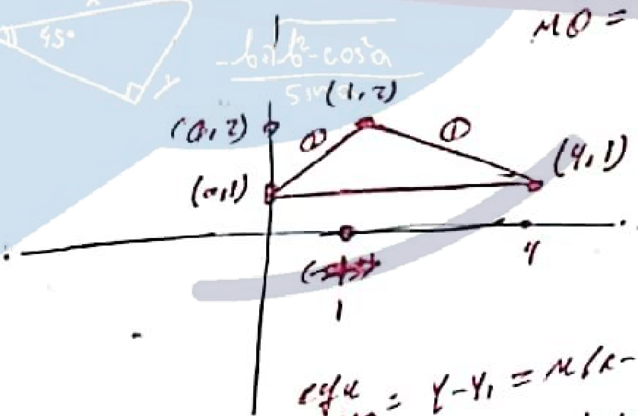
[9] $\iint_D y^2\,dA$

D is $\triangle$ with vertices $(0,1)$ $(1,2)$ $(4,1)$

$$= \int_1^2\int_{y-1}^{-3y+7} y^2\,dx\,dy.$$

$m_{①} = \frac{2-1}{1-4} = -\frac{1}{3}$

$m_{①} = \frac{2-1}{1-0} = 1$



eq. $m_{①}$: $y - y_1 = m(x - x_1)$
$y - 2 = -\frac{1}{3}(x-1)$
$-3(y-2) = x - 1$
$\boxed{x = -3y + 7}$

eq. $m_{①}$: $y - y_1 = m(x - x_1)$
$\boxed{y - 1 = x}$

2

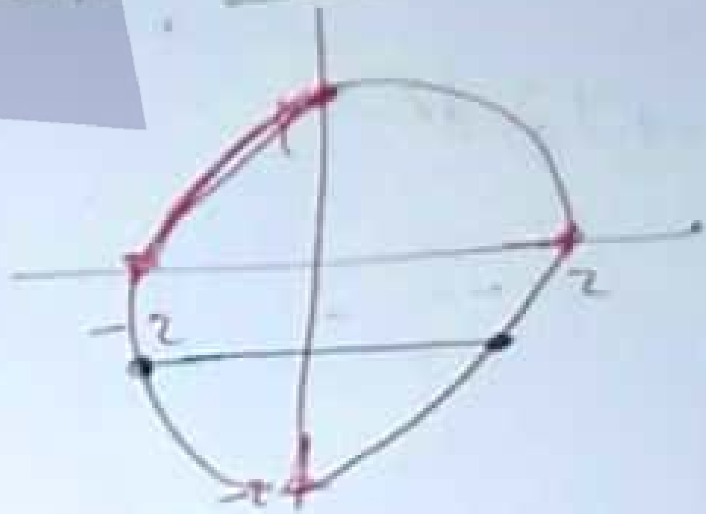
15.2

$\boxed{21}$ $\iint_D (2x-y)\,dA$

D is bounded by circle center $(0,0)$ $R=2$.

$\rightarrow \boxed{x^2+y^2=4}$

$$= \int_{-2}^{2}\int_{x_1}^{x_2} (2x-y)\,dx\,dy$$

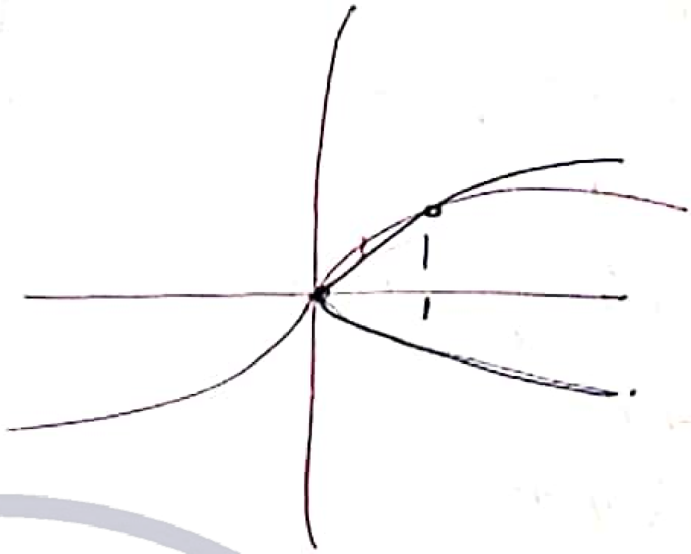


$x^2 = 4 - y^2$

$x = \pm\sqrt{4-y^2}$

$$\Rightarrow \int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{+\sqrt{4-y^2}} (2x-y)\,dx\,dy.$$

$\boxed{29}$ V under $z = 2x + y^2$.
and above the region.
bounded by $x = y^2,\ x = y^3$



$$V = \int_0^1 \int_{\sqrt{x}}^{\sqrt[3]{x}} (2x + y^2)\, dy\, dx.$$

$\boxed{26}$ V Enclosed by $z = x^2 + y^2 + 1$ and planes $x = 0,\ y = 0,\ z = 0,\ x + y = 2$.

$x = y - 2$. | $y = 2$.
$x = 0$

$$V = \iint \Delta z\, dA.$$

$$V = \int_0^2 \int_0^{2-y} (x^2 + y^2 + 1)\, dx\, dy$$

$\boxed{27}$ tetrahedron enclosed by Coordinate planes and $2x + y + z = 4$

$z = 4 - 2x - y$.

$4 - 2x - y = 0$ / $x = 0$.
$y = 4$

Δ
$x = 0$
$y = 0$
$z = 0$.

$4 - 2x - y = 0$
$\rightarrow 2x = 4 - y$
$x = \frac{4-y}{2}$.

$$V = \iint \Delta z\, dA \rightarrow \int_0^4 \int_0^{\frac{4-y}{2}} (4 - 2x - y)\, dx\, dy$$

28 Enclosed by $z = x^3 + 3y^2$.
and $x=0$, $y=1$, $x=y$, $z=0$

$$V = \iint \Delta z \, dA.$$

$$= \int_0^1 \int_0^y (x^3 + 3y^2) \, dx \, dy$$

---

31 Bounded by $x^2+y^2=1 \longrightarrow x = \pm\sqrt{1-y^2}$. in The first
and planes $y=z$, $x=0$, $z=0$
octant.

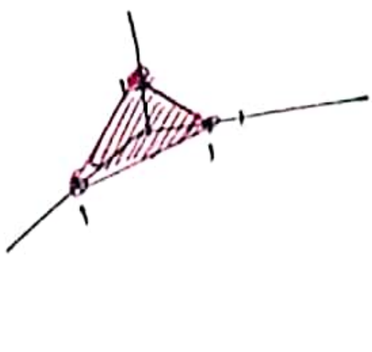
$$V = \iint \Delta z \, dA \rightarrow \int_0^1 \int_0^{\sqrt{1-y^2}} (y) \, dx \, dy$$

2D.

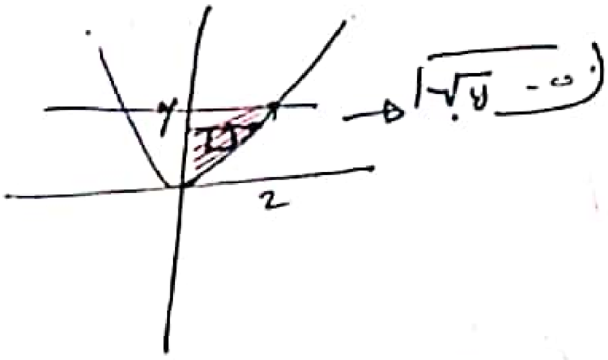
31
$$\int_0^1 \int_0^{1-x} (1-x-y) \, dy \, dx.$$
$\Delta z = 1-x-y$.

$y=0$, $y=1-x$
$x=1$, $x=0$

3D. $z = 1-x-y$
$z+x+y=1$



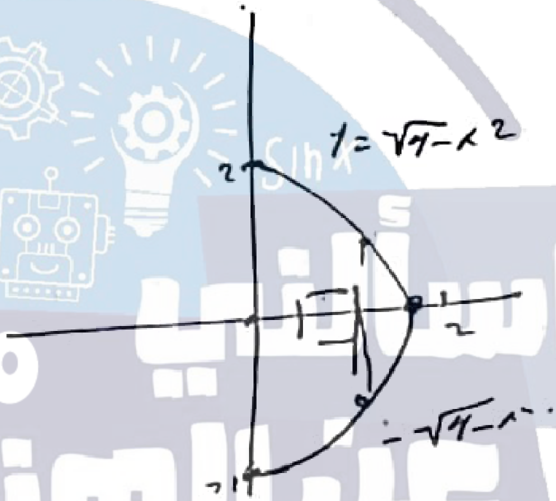
$\boxed{46}$ $\int_0^2\int_{x^2}^4 f(x,y)\,dy\,dx \to dx\,dy$



$x=2\ ,\ y\le 4$
$x=0\ ,\ y=x^2.$

$\to V=\int_0^4\int_0^{\sqrt{y}} f(x,y)\,dx\,dy.$

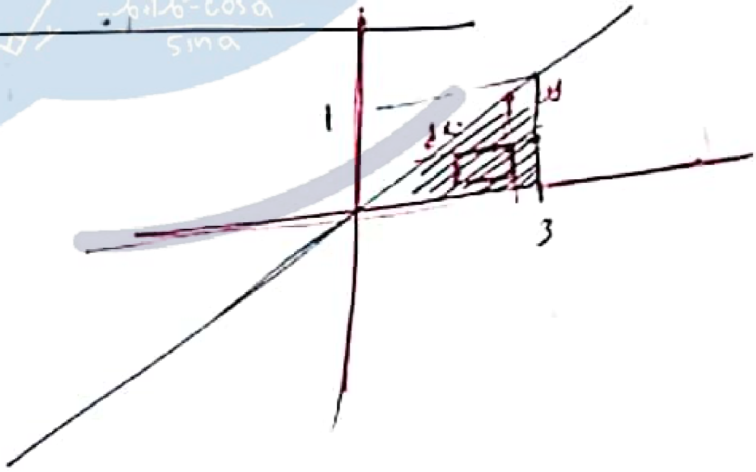
$\boxed{48}$ $\int_{-2}^{2}\int_0^{\sqrt{4-y^2}} f(x,y)\,dx\,dy.$



$y=\pm 2\ ,\ x=\sqrt{4-y^2}.$
$x=0.$

$V=\int_0^2\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x,y)\,dy\,dx.$

$\boxed{51}$ $\int_0^1\int_{3y}^3 e^{x^2}\,dx\,dy.$



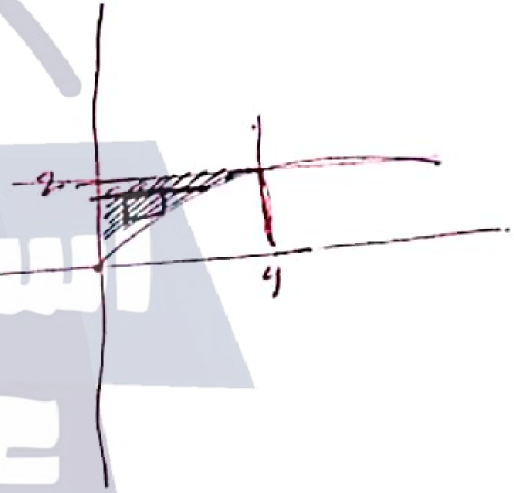
$y=0\ ,\ y=1$
$x=3,\ x=3y.$

$\to V=\int_0^3\int_0^{x/3} e^{x^2}\,dy\,dx.$

Scanned with CamScanner

55

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\,dy\,dx$$



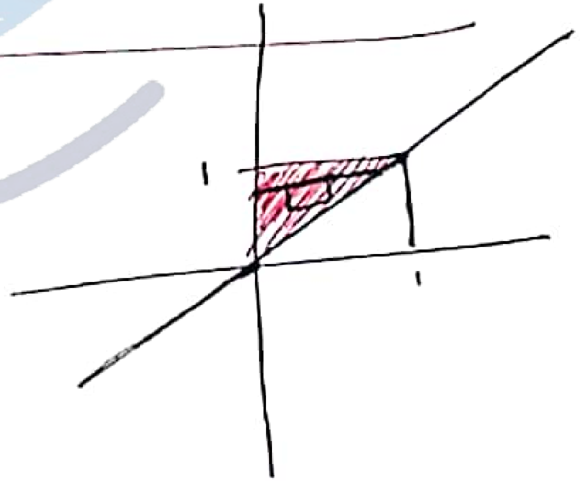
$x=0,\ x=4$

$y=2,\ y=\sqrt{x}$

$$V=\int_0^2 \int_0^{y^2} \frac{1}{(y^3+1)}\,dx\,dy.$$

56

$$\int_0^1 \int_x^1 e^{x/y}\,dy\,dx.$$



$x=0,\ y=1$

$x=1,\ y=x$

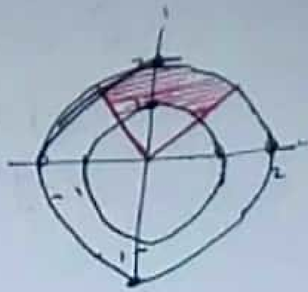
$$V=\int_0^1 \int_0^y e^{x/y}\,dx\,dy.$$

Section

15.3

15.3 $\rightarrow y=-x$

5) $\int_{\pi/4}^{3\pi/4}\int_{1}^{2} r\,dr\,d\theta$ $\rightarrow$



$\frac{\pi}{4}$ 1 → r صعد
صعد للزاوية /
Io
I ▷ x=y.

$\rightarrow \int_{\pi/4}^{3\pi/4}\int_{1}^{2} r\,dr\,d\theta \rightarrow \int_{\pi/4}^{3\pi/4} \left[\frac{r^2}{2}\right]_1^2 d\theta$

$5\left(2-\frac{1}{2}\right)\left(\frac{3\pi}{4}-\frac{\pi}{4}\right)$

7) $\iint_D x^2y\,dA$ D is The top half of disk with r=5, C(0,0).

$dA = r\,dr\,d\theta$
$x = r\cos\theta$
$y = r\sin\theta$

$\rightarrow \int_0^{\pi}\int_0^{5} r^3\cos^2\theta\sin\theta\, r\,dr\,d\theta \rightarrow$

$\rightarrow \int_0^{\pi}\int_0^{5} r^4\cos^2\theta\sin\theta\,dr\,d\theta$

hint:- ① $\int r^4\,dr = \frac{r^5}{5}+c.$

② $\int \cos^2\theta\sin\theta\,d\theta =$

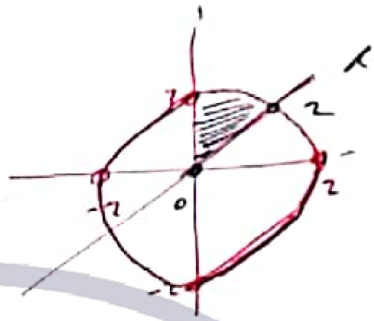
$u=\cos\theta$
$du=-\sin\theta\,d\theta$
$d\theta=\frac{du}{-\sin\theta} \rightarrow \int -u^2\,du = \frac{-u^3}{3}+c.$

Scanned with CamScanner

9) $\iint_R (2x-y)\,dA$

R is region in the first quad. enclosed by $x^2+y^2=4$ and $x=0$, $x=y$.

$x=y \to \theta=\frac{\pi}{4}$



$dA = r\,dr\,d\theta$

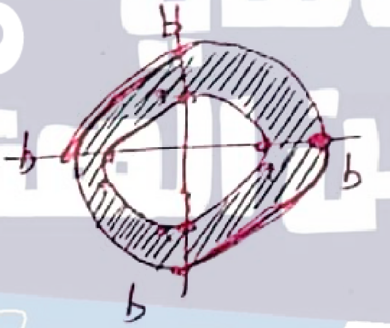
$x = r\cos\theta$

$y = r\sin\theta$

$\to \int_{\pi/4}^{\pi/2}\int_0^2 r^2(2\cos\theta-\sin\theta)\,dr\,d\theta$

10) $\iint_R \frac{y^2}{x^2+y^2}\,dA$

R is region that lies between $x^2+y^2=a^2$ & $x^2+y^2=b^2$. $0<a<b$.



$\to$ $dA = r\,dr\,d\theta$

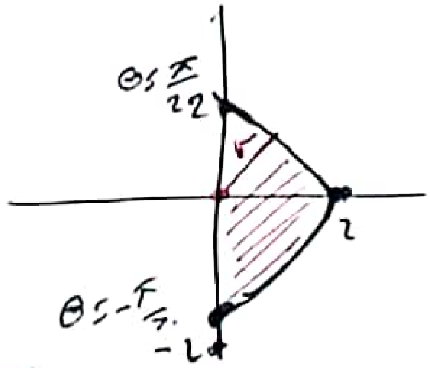
$y = r\sin\theta$

$x = r\cos\theta$

$= \int_0^{2\pi}\int_a^b \frac{\cancel{r^2}(\sin^2\theta)\,r\,dr\,d\theta}{\cancel{r^2}(\cos^2\theta+\sin^2\theta)}$

11) $\iint_D e^{-x^2-y^2}\,dA$

D is region bounded by $x=\sqrt{4-y^2}$ and y-axis.



$dA = r\,dr\,d\theta$.

$x = r\cos\theta$

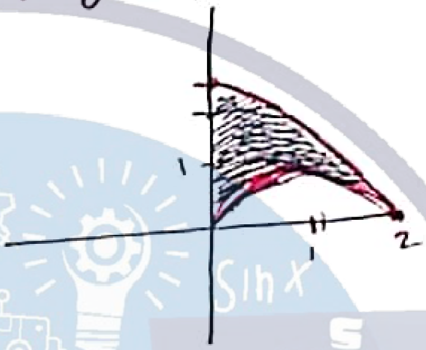
$y = r\sin\theta$

$\int_{-\pi/2}^{\pi/2}\int_0^2 e^{-r^2}\,r\,dr\,d\theta$

hint:- $\int e^{r^2}\,r\,dr \to \boxed{u=r^2}$

$\boxed{14}$ $\iint_D x\,dA$

D is region in the first quard. that lies between $x^2+y^2=4$, $x^2+y^2=2x$.



$\rightarrow x^2+y^2=2x$

$x^2-2x+y^2=0$

$(x-1)^2+y^2=1$

$R=1,\ c(1,0)$

$x^2+y^2=2x$

$r^2=2r\cos\theta$

$r=2\cos\theta$

$\rightarrow \int_0^{\frac{\pi}{2}}\int_{2\cos\theta}^{2} r\cos\theta \; r\,dr\,d\theta$

$=\int_0^{\frac{\pi}{2}}\int_{2\cos\theta}^{2} r^2\cos\theta\,dr\,d\theta$

$\rightarrow \int_0^{\frac{\pi}{2}} \left.\frac{r^3}{3}\right]_{2\cos\theta}^{2}\cos\theta\,d\theta = \int_0^{\frac{\pi}{2}}\left(\frac{8}{3}-\frac{8\cos^3\theta}{3}\right)\cos\theta\,d\theta$

$=\frac{8}{3}\left[\int_0^{\frac{\pi}{2}}\cos\theta\,d\theta - \int_0^{\frac{\pi}{2}}\cos^4\theta\,d\theta\right]$

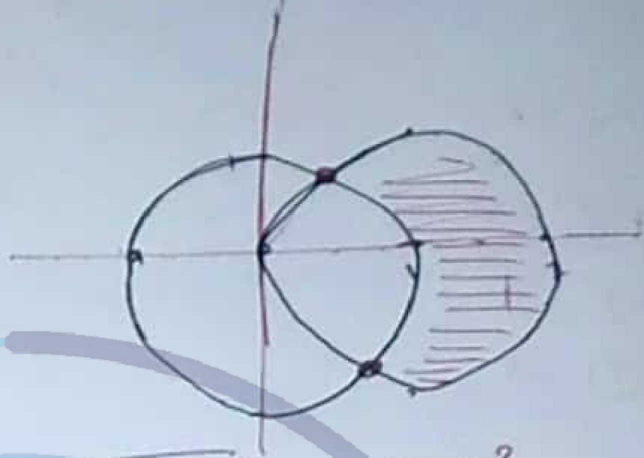
$\left.\sin\theta\right]_0^{\frac{\pi}{2}}$ $\quad$ A

$\rightarrow \int \cos^n\theta\,d\theta = \frac{1}{n}\cos^{n-1}\theta\sin\theta + \frac{n-1}{n}\int\cos^{n-2}\theta\,d\theta$

7 region inside $(x-1)^2+y^2=1$ and outside $x^2+y^2=1$

① $(x-1)^2+y^2=1 \rightarrow R=1,\ C(1,0)$

② $x^2+y^2=1$, $R=1,\ C(0,0)$



$\iint_D dA$

$= \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \int_{1}^{2\cos\theta} r\,dr\,d\theta$

$\theta? \rightarrow$ نقاط تقاطع

$\boxed{r_1=1}$ $r_2=?$

لما

$r_1=r_2$ لما

$x^2-2x+1+y^2=1$

$1=2\cos\theta$

$x^2+y^2=2x$

$\cos\theta=\frac{1}{2} \rightarrow$ الربع الأول والرابع

$r^2=r\,2\cos\theta$

$\theta=\frac{\pi}{3},\ -\frac{\pi}{3}$

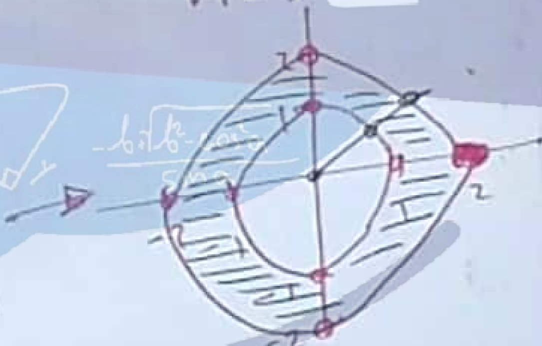
$r=2\cos\theta$

~~$\theta? \rightarrow$ نقاط تقاطع~~

~~$r_1=r_2$~~

20 Below $z=\sqrt{x^2+y^2}$

above $1\le x^2+y^2\le 4$



$V=\iint_D z\,dA \rightarrow r\,dr\,d\theta$, $x^2+y^2=r^2$

$= \int_0^{2\pi}\int_1^2 \left(\sqrt{r^2}\right) r\,dr\,d\theta$

$\rightarrow \int_0^{2\pi}\int_1^2 r^2\,dr\,d\theta$

21) Below $2x+y+z=4$ and above the disk $x^2+y^2 \le 1$

$$V = \iint_D \Delta z \, dA.$$

$z = -(2x+y)+4.$

$x = r\cos\theta, \; y = r\sin\theta$

$dA = r\,dr\,d\theta.$

$$\rightarrow \int_0^{2\pi}\int_0^1 \left(-(2r(\cos\theta + r\sin\theta) + 4\right) r \, dr \, d\theta$$

$$= \int_0^{2\pi}\int_0^1 \left(-r(2r\cos\theta + r\sin\theta) + 4r\right) dr \, d\theta.$$

22) Below the $z = 18 - 2x^2 - 2y^2$ and above the xy-plane. $z=0$

$z_1 = z_2.$

$18 - 2x^2 - 2y^2 = 0$

$\boxed{x^2+y^2 = 9}$

$z_1 = 18 - 2(x^2+y^2)$

$\iint_D \Delta z \, dA, \quad x = r\cos\theta, \; y = r\sin\theta$

$dA = r\,dr\,d\theta$

$$\rightarrow \int_0^{2\pi}\int_0^3 (18 - 2r^2) r \, dr \, d\theta.$$

25) Above $z = -\sqrt{x^2 - y^2}$ and below

The sphere $x^2 + y^2 + z^2 = 1$ $\rightarrow$ $z_1 = \pm\sqrt{1 - x^2 - y^2} \rightarrow \pm\sqrt{1 - r^2}$.

$z_2 = \sqrt{x^2 + y^2} \rightarrow \sqrt{r^2}$

$$V = \iint \Delta z \, dA \qquad dA \rightarrow r\,dr\,d\theta$$

$$\Delta z = \sqrt{1 - r^2} - r$$

$$V = \iint \left(\sqrt{1 - r^2} - r\right) r \, dr \, d\theta$$

$$\int_0^{2\pi} \int_0^{1/\sqrt{2}} \left(\sqrt{1 - r^2} - r\right) r \, dr \, d\theta$$

$z_1 = z_2$

$\rightarrow r = \sqrt{1 - r^2}$

$r^2 = 1 - r^2$

$2r^2 = 1$

$r = \pm 1/\sqrt{2}$

تقاطع ال cone وال sphere

26] Bounded by $z = 6 - x^2 - y^2$.
and $z = 2x^2 + 2y^2$.

$$\iint_D \Delta z \, dA \Rightarrow \iint 6 - x^2 - y^2 - 2x^2 - 2y^2 \, dA.$$

$$= \iint (6 - 3x^2 - 3y^2) \, dA$$

$z_1 = z_2$.
$6 - x^2 - y^2 = 2x^2 + 2y^2$.
$3x^2 + 3y^2 = 6$
$\boxed{x^2 + y^2 = 2}$

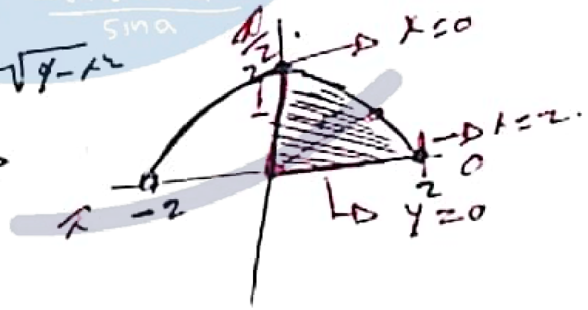
$x = r\cos\theta$, $dA = r\,dr\,d\theta$.
$y = r\sin\theta$

$$= \int_0^{2\pi} \int_0^{2} (6 - 3(r^2)) \, r \, dr \, d\theta$$

29] $$\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2} \, dy \, dx.$$

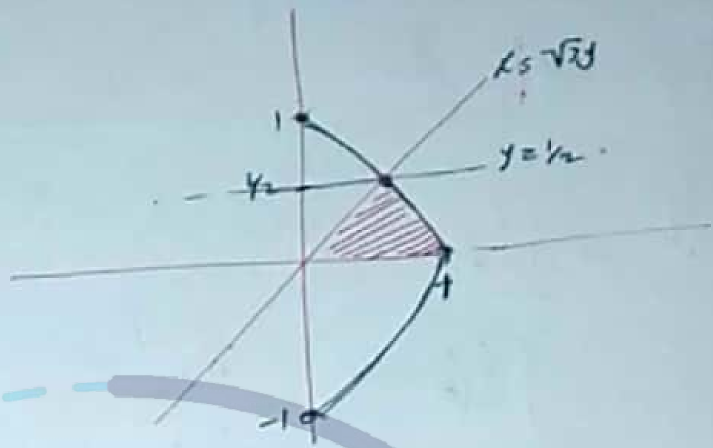
$dA = r\,dr\,d\theta$.
$x = r\cos\theta$
$y = r\sin\theta$.
$y = \sqrt{4-x^2}$

$y = \sqrt{4-x^2}$, $x = 2$
$y = 0$, $x = 0$



$$\int_0^{\frac{\pi}{2}} \int_0^{2} e^{-r^2} \, r \, dr \, d\theta$$

31) $\int_0^{1/2}\int_{\sqrt{3}y}^{\sqrt{1-y^2}} xy^2\,\underline{dx\,dy} \to dA$



$x=\sqrt{3}y \quad , \quad y=1/2$

$x=\sqrt{1-y^2} \quad , \quad y=0$

$x=r\cos\theta$
$y=r\sin\theta \qquad dA = r\,dr\,d\theta$

$\sqrt{3}y = x$

$\to r\cos\theta = \sqrt{3}\,r\sin\theta$

$\tan\theta = \frac{1}{\sqrt{3}} \to \theta = \frac{\pi}{6}$

$\to \int_0^{\pi/6}\int_0^1 r\cos\theta\, r^2\sin^2\theta\, r\,dr\,d\theta$

32) $\int_0^a\int_{-\sqrt{a^2-y^2}}^0 x^2y\,\underline{dx\,dy} \to dA$

$x=0 \ , \ x=-\sqrt{a^2-y^2}$

$y=0 \ , \ y=a$

$dA = r\,dr\,d\theta$

$x=r\cos\theta \ , \ y=r\sin\theta$

$\to \int_{\pi/2}^{\pi}\int_0^a r^2\cos^2\theta\, r\sin\theta\, r\,dr\,d\theta$

Section

15.6

$\boxed{4}$ $\int_0^1\int_y^{2y}\int_0^{x+y} 6xy\,dz\,dx\,dy$

$\int_0^1\int_y^{2y} 6xyz\Big|_{z=0}^{z=x+y} dx\,dy$

$\int_0^1\int_y^{2y} 6x^2y+6xy^2\,dx\,dy$

$\int_0^1 (2x^3y+3x^2y^2)\Big|_{x=y}^{x=2y} dy \rightarrow \int_0^1 (16y^4+12y^4)-(2y^4+3y^4)\,dy$

$\cancel{\int_0^1 (4x^3y+12x^2y)}$

$\int_0^1 23y^4\,dy$

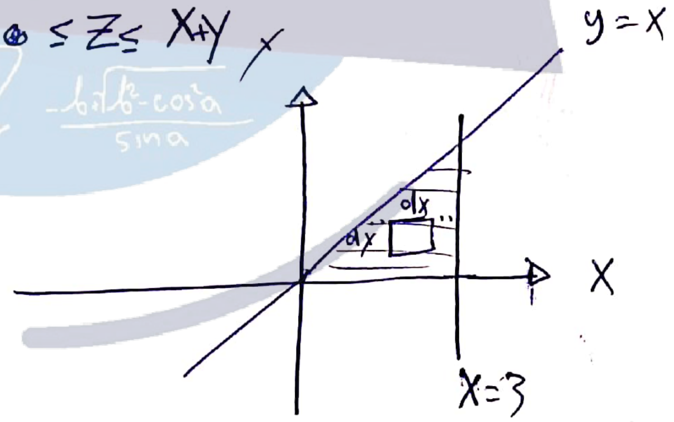
$\frac{23y^5}{5}\Big|_0^1 = \boxed{\frac{23}{5}}$

$\boxed{9}$ $\iiint y\,dV$ $\quad 0\le x\le 3$

$0\le y\le x$

$x-y\le z\le x+y$

$\int_0^3\int_y^3\int_{x-y}^{x+x} y\,dz\,dx\,dy$



خطوات
تكامل
عادية

$\frac{27}{2}$

or $\int_0^3\int_0^x\int_{x-y}^{x+y} y\,dz\,dy\,dx$

$\boxed{11}$ $\iiint \frac{z}{x^2+z^2}\, dV$

$1 \le y \le 4$

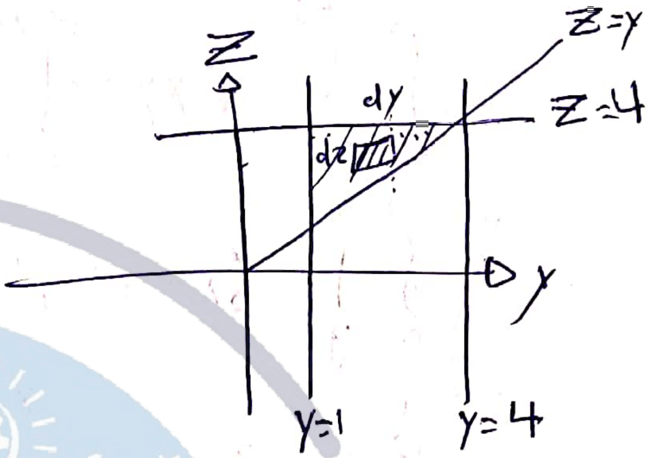
$0 \le x \le z$

$y \le z \le 4$

$$\int_1^4 \int_1^z \int_0^z \frac{z}{x^2+z^2}\, dx\, dy\, dz$$

⋮

$$\boxed{\frac{9\pi}{8}}$$



$\boxed{12}$ $\iiint \sin y\, dV$

$z = x$

$(0,0,0)$

$(\pi,0,0)$

$(0,\pi,0)$

$$\int_0^\pi \int_0^{\pi-x} \int_0^x \sin y\, dz\, dy\, dx$$

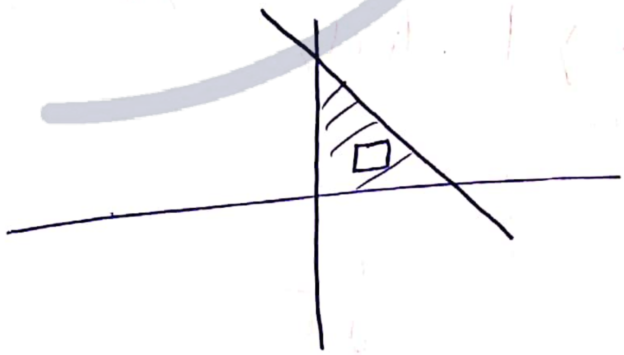
equation that joins $(0,\pi)$ $(\pi,0)$

xy plane

$x+y=\pi$ $\quad y=\pi-x$

⋮

$$\frac{\pi^2}{2}-2$$



~~14~~

$\boxed{15}$ $\iiint y^2\, dV$ $\quad x+y+z=2$

vertices $(0,0,0)$, $(2,0,0)$, $(0,2,0)$, $(0,0,2)$

$$\int_0^2\int_0^{2-x}\int_0^{(2-x-y)} y^2\, dz\, dy\, dx$$

بنثبت z

$(0,0)$, $(2,0)$, $(0,2)$

equation is $y+x=2$

$y=2-x$

⋮

~~$\frac{16}{3}$~~ $\frac{8}{15}$

~~15~~ $\boxed{14}$ $\iiint (x-y)\, dV$

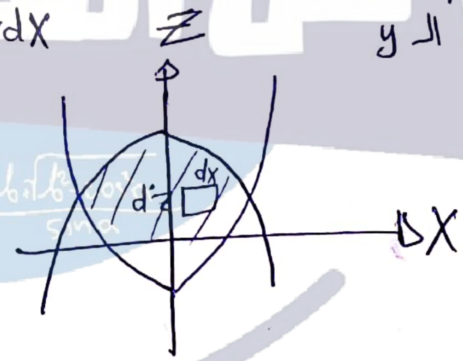
$z=x^2-1$

$z=1-x^2$

$y=0$, $y=2$

بنثبت الـ y

$$\int_{-1}^{+1}\int_{x^2-1}^{1-x^2}\int_0^2 (x-y)\, dy\, dz\, dx$$



⋮

$\frac{-16}{3}$

$z=z$

$x^2-1=1-x^2$

$2x^2=2$

$x^2=1$

$x=\pm 1$

$\boxed{17}$

$$\iiint x\,dV$$

$x_2 > x_1$

$x_1 = 4y^2 + 4z^2$

$x_2 = 4$

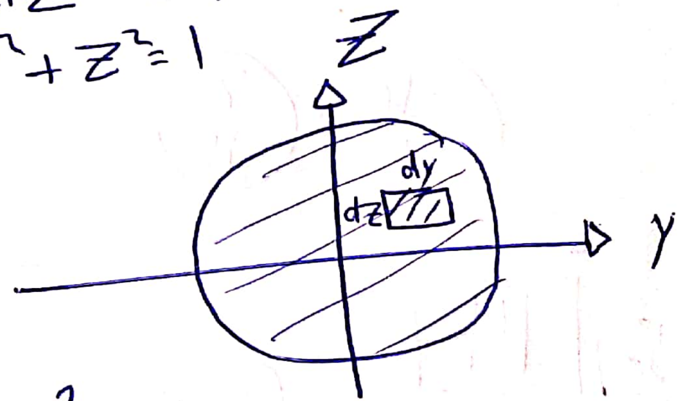
بنثبت ال X

$$\iiint_{-\sqrt{1-y^2}\;\;4y^2+4z^2}^{\sqrt{1-y^2}\;\;4} x\,dx\,dz\,dy$$

$x_1 = x_2$

$4y^2 + 4z^2 = 4$

$y^2 + z^2 = 1$



$\cancel{4z^2 = 4 -}$

$z^2 = 1 - y^2$

$z = \pm\sqrt{1 - y^2}$

$\boxed{17}$

$$\iiint x\, dV$$

$X_1 = 4y^2 + 4z^2$

$X_2 = 4$

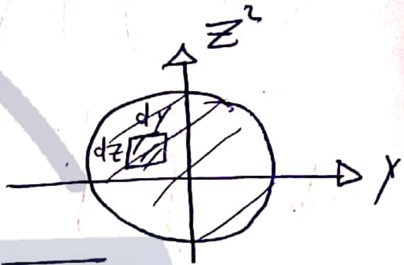
بنثبت ال X

$X_2 > X_1$

$$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{4y^2+4z^2}^{4} x\, dx\, dz\, dy$$

$X_1 = X_2$

$z^2 + y^2 = 1$



$z = \pm\sqrt{1-y^2}$

$\boxed{18}$

$$\iiint z\, dV$$

$y^2 + z^2 = 9$

In first octant

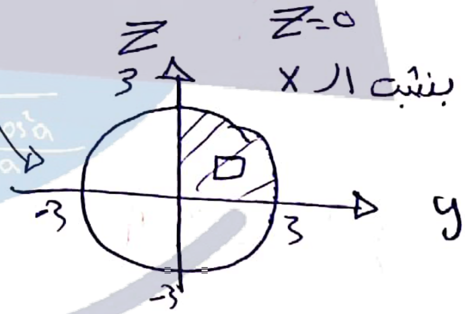
$x = 0$

$y = 3x \rightarrow x = \frac{y}{3}$

$z = 0$

بنثبت ال X

$$\int_{0}^{3} \int_{0}^{\sqrt{9-z^2}} \int_{0}^{y/3} z\, dx\, dy\, dz$$



$y = \pm\sqrt{9-z^2}$

يهمل $\ominus$

لأنه بالثمن الأول

$\boxed{19}$ coordinate planes

xy xz yz

$z=0$ $y=0$ $x=0$

plane $2x+y+z=4$ find Volume?

$$V=\iiint dV$$

$z_1=0$

$z_2=4-2x-y$

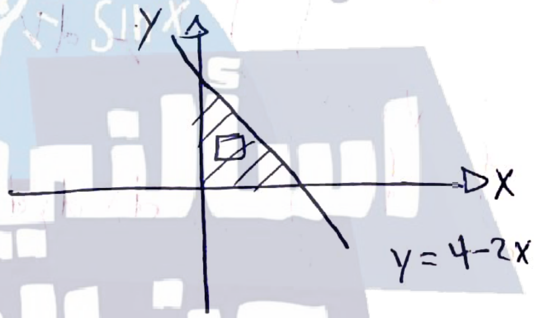
$z_2>z_1$

$$V=\int_0^2\int_0^{4-2x}\int_0^{\cancel{4-2x-y}}dz\,dy\,dx$$

$z_1=z_2$

$2x+y=4$

$y=4-2x$, $x=0$, $y=0$



$y=4-2x$

$\frac{16}{3}$

$\boxed{22}$ $x^2+z^2=4$

$y=-1$, $y+z=4$

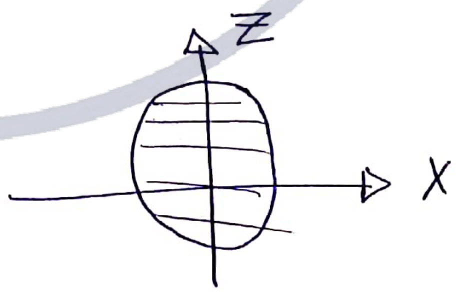
$y_2=4-z$

$y_1=-1$

$y_2>y_1$

$$V=\iiint dV$$

$$=\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{-1}^{4-z}dy\,dz\,dx$$

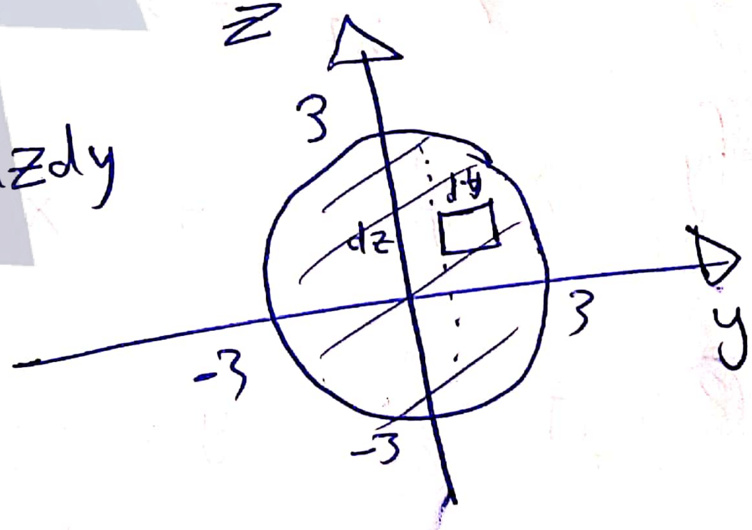


$z=\pm\cancel{4}\sqrt{4-x^2}$

$\boxed{30}$ $\iiint f(x,y,z)\,dV$

$y^2+z^2=9 \quad , \quad x=-2 \quad , \quad x=2$

$$\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}\int_{-2}^{2} f(x,y,z)\, d\cancel{z}x\, dz\, dy$$



$z = \pm\sqrt{9-y^2}$

Section 15.7

$\boxed{3}$ a) $(-1,1,1)$

$r^2 = x^2+y^2$ $\quad$ $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ $\quad$ $z = z$

$r^2 = 2$ $\quad$ $\theta = \tan^{-1}(-1)$

$r = \sqrt{2}$ $\quad$ $\theta = \frac{3\pi}{4}$

in polar $\left(\sqrt{2}, \frac{3\pi}{4}, 1\right)$

b) $(-2, 2\sqrt{3}, 3)$

$r^2 = x^2+y^2$ $\quad$ $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ $\quad$ $z = z$

$r^2 = 4+12$ $\quad$ $\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{-2}\right)$

$r^2 = 16$ $\quad$ $\theta = \tan^{-1}(-\sqrt{3})$

$r = 4$ $\quad$ $\theta = \frac{2\pi}{3}$

in polar $\left(4, \frac{2\pi}{3}, 1\right)$

$\boxed{5}$ $r = 2$

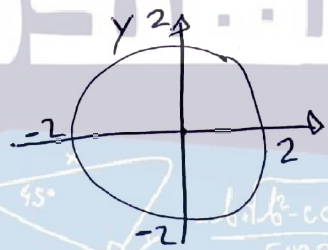
$\sqrt{x^2+y^2} = 2$

$x^2+y^2 = 4$

Circle

radius = 2

center $(0,0)$



$\boxed{8}$ $r = 2\sin\theta$ $\quad * r$

$r^2 = 2r\sin\theta$

$x^2 + y^2 = 2y$

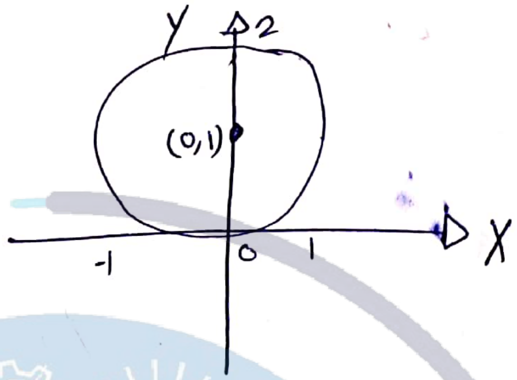
$x^2 + y^2 - 2y = 0$

$x^2 + y^2 - 2y + 1 - 1 = 0$

$x^2 + (y-1)^2 = 1$

circle $r = 1$

center $(0,1)$



$\boxed{9}$ a) $\underbrace{x^2 - x + y^2}_{r^2} + z^2 = 1$

$r^2 - r\cos\theta + z^2 = 1$

b) $z = x^2 - y^2$

$z = r^2\cos^2\theta - r^2\sin^2\theta$

$z = r^2(\cos^2\theta - \sin^2\theta)$

$z = r^2\cos 2\theta$

$\boxed{11}$ $r^2 \le Z \le 8-r^2$

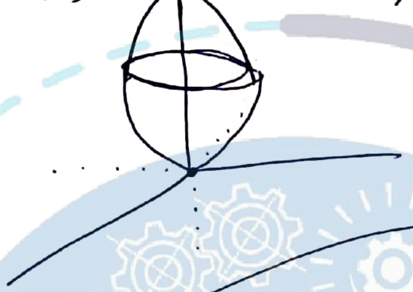
$Z = r^2$

$Z = x^2+y^2$ (paraboloid )

$Z = 8-r^2$

$Z = 8-x^2-y^2$

$Z-8 = -(x^2+y^2)$ (paraboloid )



$\boxed{17}$ $\iiint \sqrt{x^2+y^2}\, dV$

$x^2+y^2=16$

$Z=-5$

$Z=4$

$$\int_0^{2\pi}\int_0^4\int_{-5}^{4} r \cdot r\, dz\, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^4 Zr^2\Big|_{-5}^{4}\, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^4 9r^2\, dr\, d\theta$$

$$\int_0^{2\pi} 3r^3\Big|_0^4\, d\theta$$

$$\int_0^{3\pi} 192\, d\theta$$

$\boxed{384\pi}$

19

$$\iiint (x+y+z)\,dV$$

$$\int_0^{\frac{\pi}{2}}\int_0^{2}\int_0^{4-r^2} (r\cos\theta + r\sin\theta + z)\, r\,dz\,dr\,d\theta$$

first octant

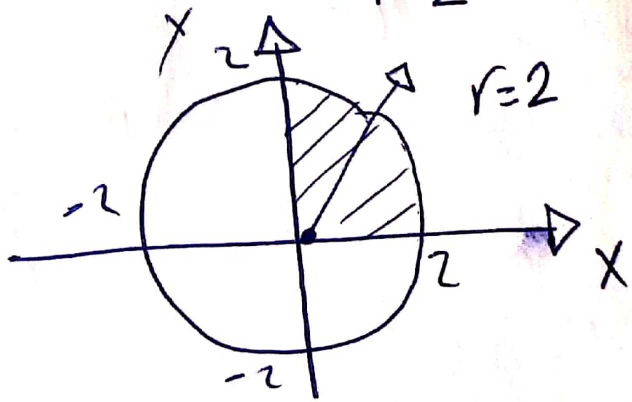
under $z = 4 - x^2 - y^2$

$z = 4 - r^2$

$0 = 4 - r^2$

$r^2 = 4$

$r = 2$



$\boxed{21}$ $\iiint x^2 \, dV$

$x^2+y^2=1$

above $z=0$, below $z^2=4x^2+4y^2$

$z=0$, $z^2=4r^2$

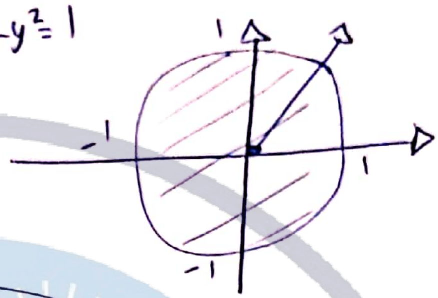
$z=2r$

$$\int_0^{2\pi}\int_0^{1}\int_0^{2r} x^2 \, r \, dz \, dr \, d\theta$$

$\vdots$

$= \frac{2}{5}\pi$

$x^2+y^2=1$



$\boxed{22}$

$V = \iiint r \, dz \, dr \, d\theta$

$x^2+y^2=1$

$x^2+y^2+z^2=4$

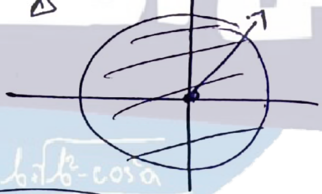
$z^2 = 4-(x^2+y^2)$

$z = \pm\sqrt{4-r^2}$

$$V = \int_0^{2\pi}\int_0^{1}\int_{-\sqrt{4-r^2}}^{+\sqrt{4-r^2}} r \, dz \, dr \, d\theta$$

$\vdots$

$= \frac{32}{3}\pi - 4\sqrt{3}\,\pi$



$\boxed{23}$

$V = \iiint r \, dz \, dr \, d\theta$

enclosed by cone

$z=r \leftarrow z=\sqrt{x^2+y^2}$

$z=+\sqrt{2-r^2} \leftarrow x^2+y^2+z^2=2$

$z_1 = z_2$

$x^2+y^2+\sqrt{x^2+y^2}^{\,2}=2$

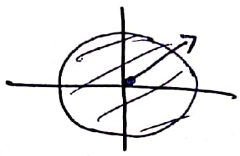
$2x^2+2y^2=2$

$x^2+y^2=1$

$$V = \int_0^{2\pi}\int_0^{1}\int_{r}^{\sqrt{2-r^2}} r \, dz \, dr \, d\theta$$

$\vdots$

$V = \frac{4\pi}{3}(\sqrt{2}-1)$



$\boxed{29}$

$$\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{\sqrt{x^2+y^2}}^{2} xz\,dz\,dx\,dy$$

$x = r\cos\theta$

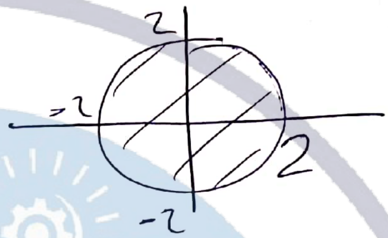
$z_1 = 2 \quad , z_2 = r$

$x_1 = -\sqrt{4-y^2}$

$x_2 = \sqrt{4-y^2}$

$x^2 = 4-y^2$

$x^2+y^2 = 4$



$$\int_0^{2\pi}\int_0^{2}\int_r^{2} r^2 z\cos\theta\,dz\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^{2} \frac{r^2z^2}{2}\cos\theta\Big|_{z=r}^{z=2} dr\,d\theta$$

$$\int_0^{2\pi}\int_0^{2} 2\cancel{4}r^2\cos\theta - \frac{r^4}{2}\cos\theta\,dr\,d\theta$$

$$\int_0^{2\pi}\left(\frac{2r^3}{3}\cos\theta - \frac{r^5}{10}\cos\theta\right)\Big|_0^{2} d\theta$$

$$\int_0^{2\pi}\left(\frac{16}{3}\cos\theta - \frac{32}{10}\cos\theta\right)-(0)\,d\theta$$

$$\frac{16}{3}\sin\theta - \frac{32}{10}\sin\theta\Big|_0^{2\pi} = 0$$

$\boxed{30}$

$$\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}}\int_{0}^{9-x^2-y^2}\sqrt{x^2+y^2}\,dz\,dy\,dx$$

$z=0 \rightarrow z=9-x^2-y^2$

$\underline{z=9-r}$

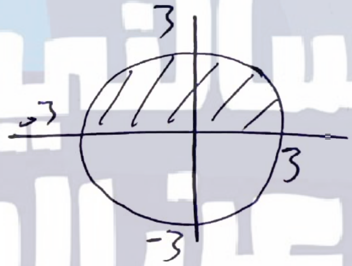
$y=0 \rightarrow y=\sqrt{9-x^2}$

$x=-3 \rightarrow x=3$

$$\int_{0}^{\pi}\int_{0}^{3}\int_{0}^{9-r} r^2\,dz\,dr\,d\theta$$

$y^2=9-x^2$

$x^2+y^2=9$



لأنها ليست من $-\sqrt{9-x^2}$ إلى $\sqrt{9-x^2}$ بل من 0 إلى $\sqrt{9-x^2}$

$$\int_{0}^{\pi}\int_{0}^{3} zr^2\Big|_{z=0}^{z=9-r} dr\,d\theta$$

$$\int_{0}^{\pi}\int_{0}^{3} 9r^2-r^3\,dr\,d\theta$$

$$\int_{0}^{\pi}\left(3r^3-\frac{r^4}{4}\right)\Big|_{r=0}^{r=3} d\theta$$

$$\int_{0}^{\pi}\left(81-\frac{81}{4}\right)d\theta$$

$$\left(81\theta-\frac{81}{4}\theta\right)\Big|_{0}^{\pi}$$

$$\boxed{81\pi-\frac{81}{4}\pi}$$

Section 15.8

5] $\phi = \frac{\pi}{3}$ $\rightarrow$ $x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$z = \rho \cos\phi$

$\cos\frac{\pi}{3} = \frac{1}{2}$.

$z = \frac{1}{2}\rho \rightarrow \rho^2 = x^2 + y^2 + z^2$.

$\rho = 2z \qquad 4z^2 = x^2 + y^2 + z^2$.

$3z^2 = x^2 + y^2$.

Cone $\rightarrow$ ~~half~~ upper half

7] $\rho\cos\phi = 1$

$z = \rho\cos\phi = 1 \rightarrow$ plane Parallel to xy-plane at $z = 1$

8] $\rho = \cos\phi$

$\cancel{z =}\ z = \rho\cos\phi \rightarrow z = \rho^2$.

$\rho^2 = x^2 + y^2 + z^2$.

$z = x^2 + y^2 + z^2$.

$z^2 - z + x^2 + y^2 = 0 \rightarrow (z - \frac{1}{2})^2 + x^2 + y^2 = \frac{1}{4}$

sphere $C(0, 0, \frac{1}{2})$ with $R = \frac{1}{2}$.

10 (a) $z = x^2 + y^2$

$\rightarrow \rho\cos\phi = \rho^2\cos^2\theta\sin^2\phi + \rho^2\sin^2\theta\sin^2\phi$

$\cancel{\rho}\cos\phi = \rho^{\cancel{2}}\sin^2\phi(\sin^2\theta + \cos^2\theta)$

$\rho = \frac{\cos\phi}{\sin\phi} \cdot \frac{1}{\sin\phi} \rightarrow \rho = \csc\phi\cot\phi$

(b) $z = x^2 - y^2$

$\rightarrow \cancel{\rho}\cos\phi = \rho^{\cancel{2}}\sin^2\phi(\underbrace{\cos^2\theta - \sin^2\theta}_{\cos 2\theta})$

$\rho = \csc\phi\cot\phi\sec 2\theta$

13 Drawing.

18 $\int_0^{\pi/4}\int_0^{2\pi}\int_0^{\sec\phi} \rho^2\sin\phi \, d\rho \, d\theta \, d\phi.$

$0 \le \rho \le \sec\phi$

$0 \le \cos\phi\rho \le 1$ $\rightarrow$ z

$\rightarrow 0 \le z \le 1$

$\rightarrow$ cone.

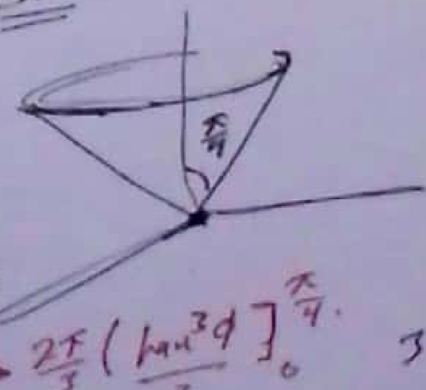
$\rightarrow \int_0^{\pi/4}\int_0^{2\pi} \frac{\rho^3}{3}\sin\phi \Big]_0^{\sec\phi} d\theta \, d\phi$ | $\sec^2\phi\sin\phi = \tan\phi\sec\phi$

$= \frac{1}{3}\int_0^{\pi/4}\int_0^{2\pi} \sec^2\phi\sin\phi \, d\theta \, d\phi$

$= \frac{1}{3}\int_0^{\pi/4} \sec^2\phi\tan\phi \Big]_0^{2\pi} d\phi$

$= \frac{2\pi}{3}\int_0^{\pi/4} \sec^2\phi\tan^2\phi \, d\phi$

$u = \tan\phi$
$d\phi = \frac{du}{\sec^2\phi}$ $\rightarrow \int u^2 \, du = \frac{u^3}{3}$



$\rightarrow \frac{2\pi}{3}\left(\frac{\tan^3\phi}{3}\right]_0^{\pi/4}$

3

21) $\iiint_B (x^2+y^2+z^2)^2 \, dV$

$\rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

$B \to$ Ball $C(0,0,0)$ with $R=5$.

$x^2+y^2+z^2=25$

$\rho^2=25$

$\rho=5$.

$$\int_0^{\pi}\int_0^{2\pi}\int_0^{5} \rho^4 \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi.$$

full $\to$ sphere

$0 \le \phi \le \pi$

$$\to \int_0^{\pi}\int_0^{2\pi} \sin\phi \, \frac{\rho^7}{7}\Big]_0^5 \, d\theta \, d\phi.$$

$$\int_0^{\pi} \frac{2\pi(5)^7}{7} \sin\phi \, d\phi \to -\frac{2\pi(5)^7}{7}\cos\phi\Big]_0^{\pi}$$

---

22) $\iiint_E y^2 z^2 \, dV$

E lies above The cone $\phi=\frac{\pi}{3}$, below ~~the~~ sphere $\rho=1$.

$$\to \int_0^{\pi/3}\int_0^{2\pi}\int_0^{1} (\rho\sin\theta\sin\phi)^2(\rho\cos\phi)^2 \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

$y=\rho\sin\theta\sin\phi$

$z=\rho\cos\phi$.

26 $\iiint_E \sqrt{x^2+y^2+z^2}\, dv$

E lies above.

$z = \sqrt{x^2+y^2}$ and ~~between~~

between $x^2+y^2+z^2 = 1$

$x^2+y^2+z^2 = 4$.

$1 \leq \rho \leq 2$

$\rightarrow \int_0^{\frac{\pi}{4}} \int_0^{2\pi} \int_1^{2} \rho \times \rho^2 \sin\phi \, d\rho\, d\theta\, d\phi$

$\rightarrow z = \sqrt{x^2+y^2}$

$z^2 = x^2+y^2$.

$z^2 = r^2$.

$\rho^2 \cos^2\phi = \rho^2 \sin^2\phi$

$\tan\phi = 1$

$\phi = \frac{\pi}{4}$.

30 lies within $x^2+y^2+z^2 = 4$ above xy-plane

and below the cone.

$z = \sqrt{x^2+y^2}$

$V = \iiint_E dv \rightarrow x^2+y^2+z^2 = 4$.

$\rho^2 = 4$

$\rho = 2 \rightarrow 0 \leq \rho \leq 2$

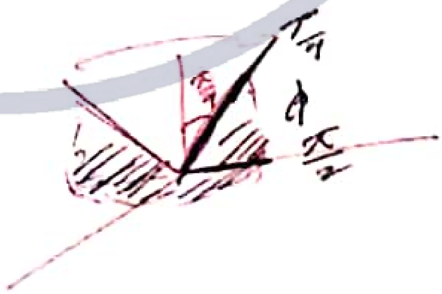
$\rightarrow z = \sqrt{x^2+y^2}$.

$z^2 = r^2$.

$\rho^2 \cos^2\phi = \rho^2 \sin^2\phi$

$\phi = \frac{\pi}{4}$

$V = \int_0^{2\pi} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{2} \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$



$$\int_0^a \int_0^{\sqrt{a^2-y^2}} \int_0^{\sqrt{a^2-x^2-y^2}}$$

46

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy \, dz \, dy \, dx.$$

$\rightarrow x=0 , \; y=0 , \; z=\sqrt{2-x^2-y^2}$

$x=1 , \; y=\sqrt{1-x^2} , \; z=\sqrt{x^2+y^2}$

$z=\sqrt{2-x^2-y^2}.$

$z^2=2-r^2 \rightarrow \rho^2\cos^2\phi+\rho^2\sin^2\phi=2.$

$\rho^2=2 \rightarrow \rho=\sqrt{2}.$

$x^2+y^2=1$

$z=\sqrt{x^2+y^2}$ $\quad 0 \le \rho \le \sqrt{2}$

$\rightarrow \rho=0$

$0 \le \theta \le \frac{\pi}{2}$

$x=0, \; y=0$

$\rightarrow z^2=r^2.$

$\rho^2\cos^2\phi=\rho^2\sin^2\phi$

$\tan\phi=1$

$\phi=\frac{\pi}{4}.$

$$\rightarrow \int_0^{\frac{\pi}{4}} \int_0^{\frac{\pi}{2}} \int_0^{\sqrt{2}} \rho^2 \sin\theta\cos\theta \sin^2\phi \; d\rho \, d\theta \, d\phi.$$

$[\rho^2 \sin\phi]$

42) $\int_{-a}^{a}\int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}}\int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} (x^2z+y^2z+z^3)\,dz\,dx\,dy$

$\hookrightarrow dv = \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$

$\hookrightarrow z(x^2+y^2+z^2)$

$z = \pm\sqrt{a^2-x^2-y^2}$

$\hookrightarrow z^2 = a^2-x^2-y^2$

$z^2+x^2+y^2 = a^2 \rightarrow 0 \le \rho \le a$

$\hookrightarrow \rho^2 = a^2 \rightarrow \rho = a$

$x = \pm\sqrt{a^2-y^2}$

$x^2+y^2=a^2 \rightarrow$ $0 \le \theta \le 2\pi$

full sphere $\rightarrow \phi$

$\hookrightarrow 0 \le \phi \le \pi$

$\rightarrow \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \rho\cos\phi\, \rho^2\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

43) $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{2-\sqrt{4-x^2-y^2}}^{2+\sqrt{4-x^2-y^2}} (x^2+y^2+z^2)^{\frac{3}{2}}\,dz\,dy\,dx$

$\hookrightarrow dv = \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

$z = \pm 2 \pm \sqrt{4-x^2-y^2} \rightarrow (z-2)^2 = 4-x^2-y^2$

$(z-2)^2+x^2+y^2=4$

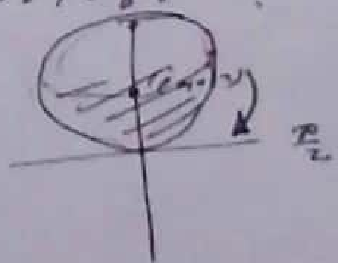
$\cancel{(z-2)^2 = 4-x^2-y^2}$

$z^2 - \cancel{4z+4} + x^2+y^2 = \cancel{4}$

$\rho^2 = 4z$

$\rho^2 = 4\rho\cos\phi \rightarrow \rho = 4\cos\phi$

$\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{4\cos\phi} (\rho^2)^{\frac{3}{2}}\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$



$\phi ? \rightarrow$

$0 \le \phi \le \frac{\pi}{2}$

$y = \pm\sqrt{4-x^2} \rightarrow y^2+x^2=4$, $0 \le \theta \le 2\pi$

Scanned with CamScanner