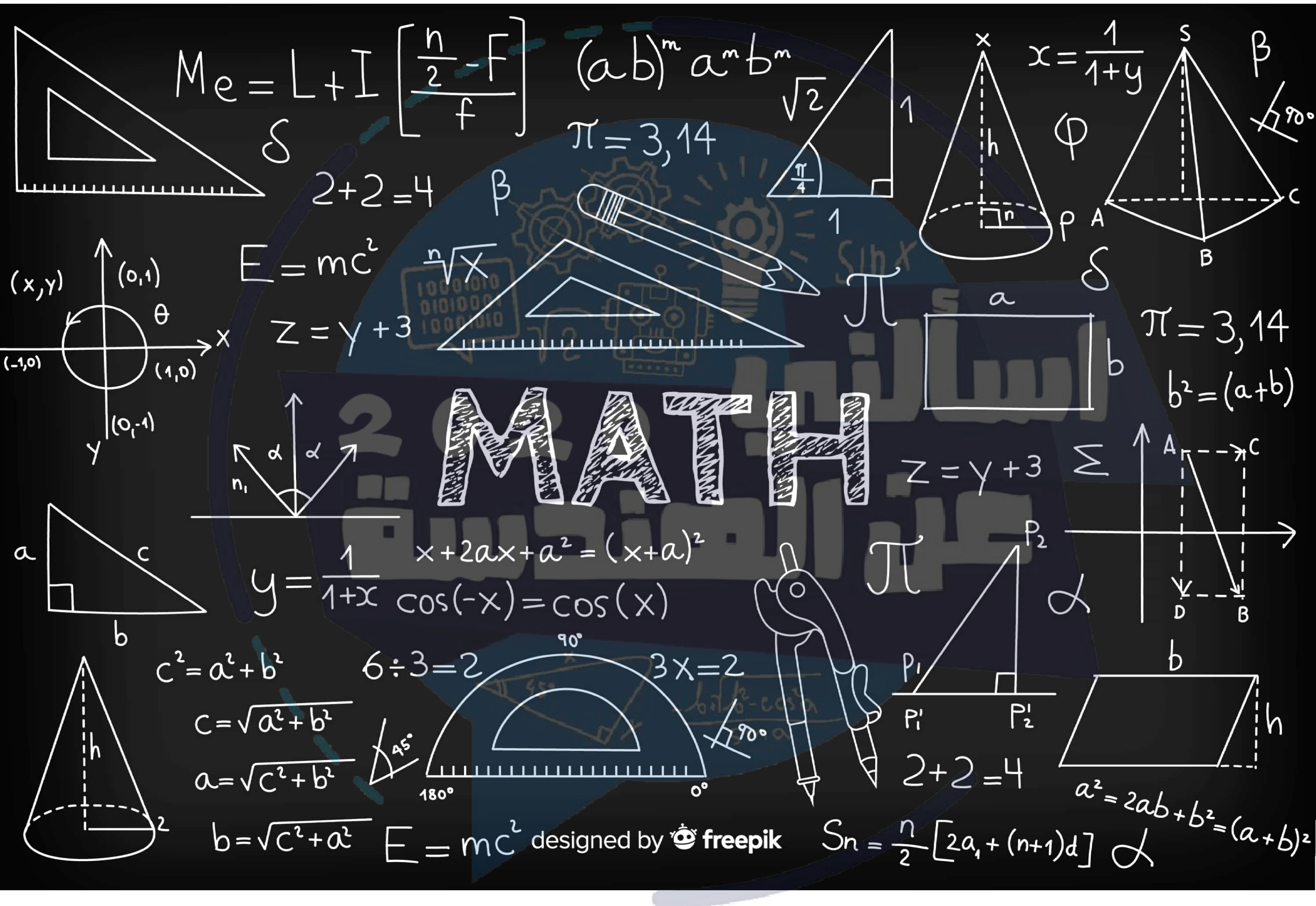


MATH
designed by freepik

Section 14.1

14.1

9 $g(x,y)=\cos(x+2y)$

(a) $g(2,-1) \rightarrow \cos(2-2)=1$

(b) R^2.

(c) $[-1,1]$

6 $f(x,y)=1+\sqrt{4-y^2}$.

(a) $f(3,1)=1+\sqrt{3}$.

(b) $4-y^2 \geq 0 \rightarrow -y^2 \geq -4 \rightarrow y^2 \leq 4$

$-2 \leq y \leq 2$

Range $=[3,1]$.

(c) If $y=0 \rightarrow f=3$

If $y=\pm 2 \rightarrow f=1$

12 $g(x,y,z)=x^3y^2z\sqrt{10-x-y-z}$

(a) $g(1,2,3)=1\times4\times3\sqrt{10-1-2-3}=12\times2=24$.

(b) $10-x-y-z \geq 0 \rightarrow 10 \geq \underline{x+y+z}$

all Points on and below The Plane.

14 $f(x,y)=\sqrt[4]{x-3y}$

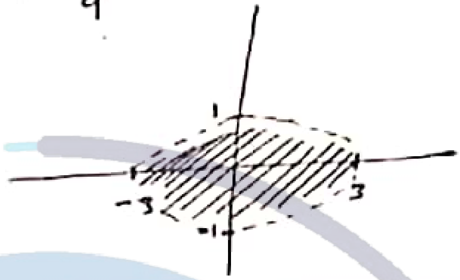
→ $x-3y\geq 0$ → $x\geq 3y$

15 $f(x,y)=\ln(9-x^2-9y^2)$

→ $9-x^2-9y^2>0$

$9>x^2+9y^2$

$1>\frac{x^2}{9}+y^2$ → ellipse



17 $g(x,y)=\frac{x-y}{x+y}$ → $\mathbb{R}$

$x+y>0$

→ $x>-y$

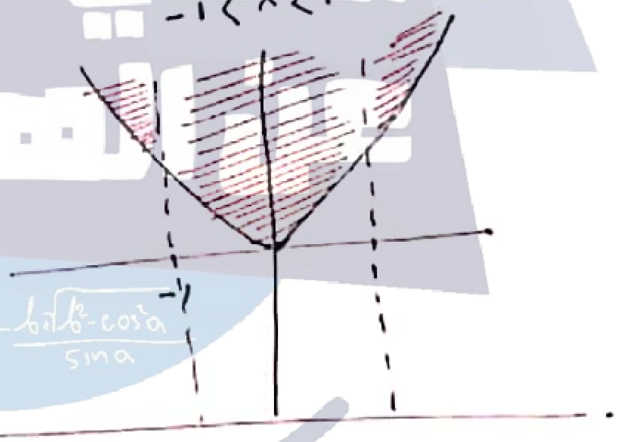
→ $x+y>0$

19 $f(x,y)=\frac{\sqrt{y-x^2}}{1-x^2}$

→ $y-x^2\geq 0$ → $y\geq x^2$

→ $1-x^2>0$ → $1>x^2$

$-1<x<1$



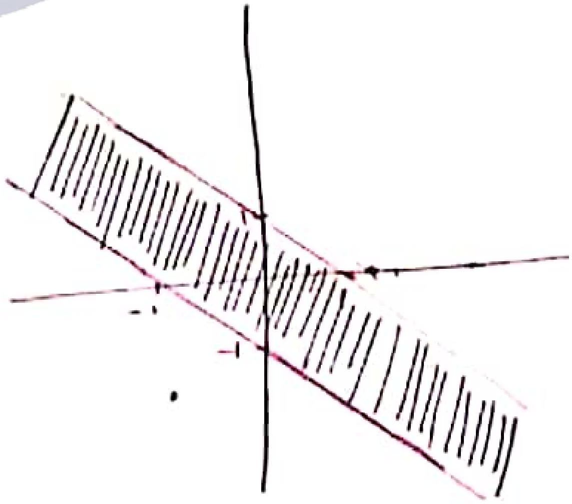
20 $f(x,y)=\sin^{-1}(x+y)$

Range $\sin(x+y)$ = Domain $\sin^{-1}(x+y)$

$-1\leq x+y\leq 1$

$-1\leq x+y$ → $x+y=-1$ → $y=-(1+x)$

$x+y\leq 1$ → $x+y=1$ → $y=1-x$



21) $f(x,y,z) = \sqrt{4-x^2} + \sqrt{9-y^2} + \sqrt{1-z^2}$

$4 - x^2 \geq 0 \rightarrow 4 \geq x^2 \rightarrow -2 \leq x \leq 2$

$9 - y^2 \geq 0 \rightarrow 9 \geq y^2 \rightarrow -3 \leq y \leq 3$

$1 - z^2 \geq 0 \rightarrow 1 \geq z^2 \rightarrow -1 \leq z \leq 1$

---

25) $f(x,y) = 10 - 4x - 5y$

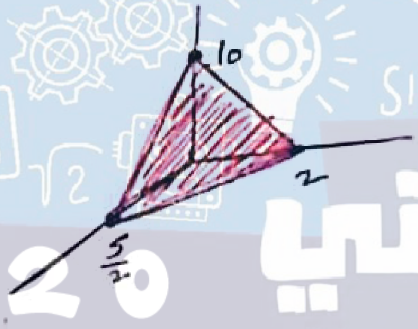
$\rightarrow z = 10 - 4x - 5y$

$z + 4x + 5y = 10$

$x=0, y=0 \rightarrow z = 10$

$z=0, y=0 \rightarrow x = \frac{10}{4}$

$x=0, z=0 \rightarrow y = 2$



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29) 3D سهل

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31) $f(x,y) = y^2+1 \rightarrow z = y^2+1$

48, 52, 54 → محذوف

Section

14.2

14.2

5 $\lim_{x,y\to(-3,2)} x^2y^3 - 4y^2 = 72 - 16 = 56$

---

7 $\lim_{x,y\to(\pi,\frac{\pi}{2})} y\sin(x-y) = \frac{\pi}{2}\sin(\pi-\frac{\pi}{2}) = \frac{\pi}{2}.$

---

9 $\lim_{x,y\to(0,0)} \frac{x^4-4y^4}{x^2+2y^2} = \frac{0}{0}!$

$\lim_{x,y\to(0,0)} \frac{(x^2-2y^2)\cancel{(x^2+2y^2)}}{\cancel{(x^2+2y^2)}} = 0-0 = 0.$

---

10 $\lim_{x,y\to(0,0)} \frac{x^2+\sin^2 y}{2x^2+y^2} = \frac{0}{0}!$

in $x=0 \to \lim_{y\to0} \frac{\sin^2 y}{y^2} = 1$

in $y=0 \to \lim_{x\to0} \frac{x^2}{2x^2} = \frac{1}{2}$

$\to$ D.N.E.

---

11 $\lim_{x,y\to(0,0)} \frac{xy\cos y}{3x^2+y^2} = \frac{0}{0}!$

in $x=0$

$\lim_{y\to0} \frac{zero}{y^2} = zero$

in $y=0$

$\lim_{y\to0} \frac{zero}{3x^2} = zero.$

in $y=x$.

or

$\lim_{y\to0} \frac{x^2\cos y}{4y^2} = \frac{1}{4}$ $\to$ D.N.E.

---

12) $$\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2} = \frac{0}{0}!$$

$$\to \lim_{(x,y)\to(0,0)} \frac{\cancel{(x^2+y^2)}(x^2-y^2)}{\cancel{(x^2+y^2)}} = \frac{0}{1}.$$

---

13) $$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = \frac{0}{0}!$$

$r^2 = x^2+y^2$

$x = r\cos\theta$

$y = r\sin\theta$.

in $x=0$ $\lim_{(x,y)\to(0,0)} \frac{0}{y} = 0$.

in $y=0$ $\lim_{(x,y)\to(0,0)} \frac{0}{x} = 0$.

$$\to \lim_{r\to 0} \frac{r^2\cos\theta\sin\theta}{\sqrt{2r^2}} = \lim_{r\to 0} \frac{r^{\cancel{2}}\cos\theta\sin\theta}{\sqrt{2}\cancel{r}} = \text{zero}. \quad \text{exist}$$

---

16) $$\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^4+y^4} = \frac{0}{0}!$$

$r^2 = x^2+y^2$

$x = r\cos\theta$

$y = r\sin\theta$.

in $x=0$ $\lim_{y\to 0} \frac{0}{y^4} = 0$, in $y=0$ $\lim_{x\to 0} \frac{0}{x^4} = 0$

$$\lim_{r\to 0} \frac{r^5\cos\theta\sin^4\theta}{r^4\cos^4\theta + r^4\sin^4\theta} = \lim_{r\to 0} \frac{\cancel{r^5}\cos\theta\sin^4\theta}{\cancel{r^4}(\cos^4\theta+\sin^4\theta)} = \text{zero}. \quad \text{exist}$$

---

18) $$\lim_{(x,y)\to(0,0)} \frac{x^2\sin^2 y}{x^2+2y^2} = \frac{0}{0}!$$

$r^2 = x^2+y^2$.

$x = r\cos\theta$

$y = r\sin\theta$.

in $x=0$ $\to \lim_{y\to 0} = 0$, in $y=0$ $\lim_{x\to 0} = 0$.

in $x=y$

$$\lim_{y\to 0} \frac{\cancel{y^2}\sin^2 y}{3\cancel{y^2}} = 0;$$

$$\lim_{r\to 0} \frac{\cancel{r^2}\cos^2\theta\,\sin^2(r\sin\theta)}{\cancel{r^2}(\cos^2\theta + 2\sin^2\theta)} = \frac{\cos^2\theta\,\sin^2(r\sin\theta)}{(\cos^2\theta+2\sin^2\theta)} = \text{D.N.E}$$

20) $\lim_{(x,y,z)\to(0,0,0)} \frac{xz}{x^2+4y^2+9z^2}$

$z = \rho\cos\phi$

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$\rho^2 = x^2+y^2+z^2$

$\rightarrow \lim_{\rho\to 0} \frac{\rho^2\cos\phi\sin\phi\cos\theta}{\rho^2(\sin^2\phi\cos^2\theta + 4\sin^2\phi\sin^2\theta + 9\cos^2\phi)}$

$\lim_{\rho\to 0} = \frac{\cos\phi\sin\phi\cos\theta}{\sin^2\phi\cos^2\theta + 4\sin^2\phi\sin^2\theta + 9\cos^2\phi}$

$=$ D.N.E

---

21) $\lim_{x,y,z} \frac{xy + yz^2 + xz^2}{x^2+y^2+z^4}$

$\rightarrow \rho^2 = x^2+y^2+z^2$

$x = \rho\cos\theta\sin\phi$

$y = \rho\sin\theta\sin\phi$

$z = \rho\cos\phi$

$\rightarrow \lim_{\rho\to 0} \frac{\rho^2(\cos\theta\sin\theta\sin^2\phi + \rho\sin\theta\sin\phi\cos^2\phi + \rho\sin\phi\cos^2\phi)}{\rho^2(\cos^2\theta\sin^2\phi + \sin^2\theta\sin^2\phi + \rho^2\cos^4\phi)}$

$\lim_{\rho\to 0} = \frac{\sin^2\phi\cos\theta\sin\theta}{\sin^2\phi(1)} = \cos\theta\sin\theta$

D.N.E

---

29) $f(x,y) = \frac{xy}{1+e^{x-y}} \rightarrow R^2$

>0

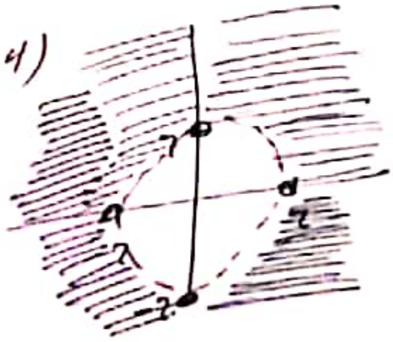
$1 + e^{x-y} = 0 \rightarrow e^{x-y} = -1$

no real sol.

$f(x,y)$ continuous for all x,y on R^2.

---

35) $g(x,y) = \ln(x^2+y^2-4)$



$x^2+y^2-4 > 0$.

$x^2+y^2 > 4$

37) $f(x,y) = \begin{cases} \dfrac{x^2y^3}{2x^2+y^2} \\ 1 \end{cases}$

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^3}{2x^2+y^2} \Rightarrow \lim_{r\to 0} \frac{r^5(\cos^2\theta \sin^3\theta)}{r^2(2\cos^2\theta + \sin^2\theta)} = \underline{0}$$

Continuous for all x and y except $x=0$, $y=0$

---

38) $f(x,y) = \begin{cases} \dfrac{xy}{x^2+xy+y^2} \\ 0 \end{cases}$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+yx+y^2} \rightarrow \begin{matrix} \text{in } x=0 \Rightarrow \lim = 0 \\ \text{in } y=0 \Rightarrow \lim = 0 \end{matrix}$$

in $x=y$

$$\lim_{y\to 0} \frac{\cancel{y^2}}{3\cancel{y^2}} \rightarrow \text{D.N.E}$$

Continuous for all x and y except $x=0, y=0$

---

40) $\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2)$

$r^2 = x^2+y^2$

$$\rightarrow \lim_{r\to 0} r^2 \ln r^2 \rightarrow 2\lim_{r\to 0} r^2 \ln r = \underline{\infty \times 0}$$

$$2\lim_{r\to 0} \frac{\ln r}{1/r^2} = \frac{\infty}{\infty}$$

$$\rightarrow 2\lim_{r\to 0} \frac{1/r}{-1/r^3}$$

$$= 2\lim_{r\to 0} \frac{r^{3\,2}}{-\cancel{r}} = 0$$

4/1 $\lim_{(x,y)\to(0,0)} \frac{e^{-x^2-y^2}-1}{x^2+y^2}$.

$r^2 = x^2+y^2$

$\to \lim_{r\to 0} \frac{e^{-r^2}-1}{r^2} = \frac{0}{0}$

$\to \lim_{r\to 0} \frac{-2re^{-r^2}}{2r} = -1$

Section

14.3

14.3

15

$f(x,y) = x^4 + 5xy^3$

* $f_x = 4x^3 + 5y^3$

* $f_y = 15xy^2$

21

$f(x,y) = \frac{x}{y}$

* $f_x = \frac{1}{y}$

* $f_y = \frac{-x}{y^2}$

26

$u(r,\theta) = \sin(r\cos\theta)$

* $u_r = \cos\theta . \cos(r\cos\theta)$

* $u_\theta = -r\sin\theta . \cos(r\cos\theta)$

28

$f(x,y) = x^y$

$f_x = y\,x^{(y-1)}$

$f_y = x^y . \ln x$

$\boxed{29}$

$$f(x,y) \int_y^x \cos(e^t)\,dt$$

$$g(x) = \int_a^x f(t)\,dt$$

$$g'(x) = f(x)$$

calcul

* $f_x = \cos(e^x)$

* $f_y = \left(-\int_x^y \cos(e^t)\,dt\right) \frac{df}{dy}$

$$= -\cos(e^y)$$

$\boxed{31}$

$$f(x,y,z) = x^3yz^2 + 2yz$$

* $f_x = 3x^2yz^2$

* $f_y = x^3z^2 + 2z$

* $f_z = 2x^3yz + 2y$

$\boxed{33}$ $w = \ln(x+2y+3z)$

* $f_x = \frac{1}{x+2y+3z}$

* $f_z = \frac{3}{x+2y+3z}$

* $f_y = \frac{2}{x+2y+3z}$

$\boxed{36}$

$$u = x^{\left(\frac{y}{z}\right)}$$

$$* \; u_x = \frac{y}{z} \cdot x^{\left(\frac{y}{z}\right)-1}$$

$$* \; u_y = \frac{1}{z} \cdot x^{\left(\frac{y}{z}\right)} \cdot \ln x$$

$$* \; u_z = \frac{-y}{z^2} \cdot x^{\left(\frac{y}{z}\right)} \cdot \ln x$$

$\boxed{39}$

$$u = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$

$$\Rightarrow u_{x_i} = \frac{2x_i}{2\sqrt{x_1^2 + x_2^2 + \dots + x_n^2}} \quad \text{for } i = 1, 2, \dots, n$$

$$= \frac{x_i}{\sqrt{x_1^2 + x_2^2 + \dots + x_n^2}}$$

$\boxed{41}$

$$R(s,t) = t e^{s/t} \; ; \; R_t(0,1)$$

$$R_t = e^{s/t} + \frac{-st}{t^2} e^{s/t}$$

$$R_t(0,1) = e^{0/1} + \frac{-0}{1} e^{0/1} = 1$$

$\boxed{43}$

$$f(x,y,z) = \ln\left[\frac{1-\sqrt{x^2+y^2+z^2}}{1+\sqrt{x^2+y^2+z^2}}\right]$$

$$= \ln\left[1-\sqrt{x^2+y^2+z^2}\right] - \ln\left[1+\sqrt{x^2+y^2+z^2}\right]$$

$$f_y = \frac{\cancel{2}y}{\cancel{2}\sqrt{x^2+y^2+z^2}} \cdot \frac{1}{1-\sqrt{x^2+y^2+z^2}} - \frac{\cancel{2}y}{\cancel{2}\sqrt{x^2+y^2+z^2}} \cdot \frac{1}{1+\sqrt{x^2+y^2+z^2}}$$

$$f_y(1,2,2) = \frac{2}{\sqrt{1+4+4}}\left[\frac{1}{1-\sqrt{9}} - \frac{1}{1+\sqrt{9}}\right]$$

$$= \frac{2}{3}\left[\frac{1\times 2}{2\times 2} - \frac{1}{4}\right]$$

$$= \frac{\cancel{2}}{3} \cdot \frac{1}{\cancel{2}4} = \frac{1}{6}$$

$\boxed{46}$

$$f(x,y) = \frac{x}{x+y^2}$$

$$f_x = \lim_{h\to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$= \lim_{h\to 0}\left[\frac{x+h}{(x+h)+y^2} - \frac{x}{x+y^2}\right]\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)(x+y^2) - (x+h+y^2)(x)}{(x+h+y^2)(x+y^2)\,h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + \cancel{xy^2} + \cancel{hx} + hy^2 - \cancel{x^2} - \cancel{xh} - \cancel{xy^2}}{(x^2 + xy^2 + hx + hy^2 + xy^2 + y^4)\,\cancel{h}}$$

$$f_x = \frac{y^2}{x^2+2xy^2+y^4} = \frac{y^2}{(x+y^2)^2}$$

$$f_y = \lim_{h\to 0}\left[\frac{x}{x+(y+h)^2} - \frac{x}{x+y^2}\right]\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{x^2+xy^2 - [x^2 + x\overbrace{(y+h)^2}^{y^2+2yh+h^2}]}{[x+(y+h)^2](x+y^2)\,h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + \cancel{xy^2} - \cancel{x^2} - \cancel{xy^2} - 2xy\cancel{h} - xh^2}{(x^2+2xy^2+2xyh+y^2+2y^3h+h^2y^2)\,\cancel{h}}$$

...

$$\lim_{h \to 0} \frac{-2xy - xh}{x^2 + 2xy^2 + y^4 + 2xyh + xh^2 + 2y^3h + h^2y^2}$$

$$= \frac{-2xy}{(x+y^2)^2}$$

$\boxed{48}$ $x^2 - y^2 + z^2 - 2z = 4$

$\frac{dz}{dx} \Rightarrow \cancel{2}x + \cancel{2}z\frac{dz}{dx} - \cancel{2}\frac{dz}{dx} = 0$

$$\frac{(z-1)\frac{dz}{dx}}{z-1} = \frac{-x}{z-1}$$

$$\frac{dz}{dx} = \frac{-x}{z-1}$$

$\frac{dz}{dy} \Rightarrow -\cancel{2}y + \cancel{2}z\frac{dz}{dy} - \cancel{2}\frac{dz}{dy} = 0$

$$(z-1)\frac{dz}{dy} = y$$

$$\frac{dz}{dy} = \frac{y}{z-1}$$

$\boxed{49}$

$$e^z = xyz$$

$$\frac{dz}{dx} \Rightarrow \quad \frac{dz}{dx} e^z = y\left(z + x\frac{dz}{dx}\right)$$

$$\frac{dz}{dx} e^z = yz + yx\frac{dz}{dx}$$

$$\frac{dz}{dx} e^z - yx\frac{dz}{dx} = yz$$

$$(e^z - yx)\frac{dz}{dx} = yz$$

$$\frac{dz}{dx} = \frac{yz}{e^z - yx}$$

$$\frac{dz}{dy} \Rightarrow \quad \frac{dz}{dy} e^z = xz + xy\frac{dz}{dy}$$

$$(e^z - xy)\frac{dz}{dy} = xz$$

$$\frac{dz}{dy} = \frac{xz}{e^z - xy}$$

$\boxed{51}$

(a) $z = f(x) + g(y)$

* $\frac{dz}{dx} = f'$ * $\frac{dz}{dy} = g'(y)$

(b) $z = f(x+y)$

* $\frac{dz}{dx} = f'(x+y)$ * $\frac{dz}{dy} = f'(x+y)$

$\boxed{52}$

(a) $Z = f(x)g(y)$

* $\frac{dz}{dx} = f'(x)g(y)$ * $\frac{dz}{dy} = f(x)\,g'(y)$

(b) $z = f(xy)$

* $\frac{dz}{dx} = y f'(xy)$ * $\frac{dz}{dy} = x f'(xy)$

(c) $z = f(x/y)$

* $\frac{dz}{dx} = \frac{1}{y} f'(x/y)$ * $\frac{dz}{dy} = \frac{-x}{y^2} f'(x/y)$

$\boxed{57}$

$v = \sin(s^2 - t^2)$

$v_s = 2s.\cos(s^2 - t^2)$

$v_t = -2t\cos(s^2 - t^2)$

• $v_{ss} = 2\cos(s^2 - t^2) - 4s^2\sin(s^2 - t^2)$

• $v_{tt} = -2\cos(s^2 - t^2) + 4t^2\sin(s^2 - t^2)$

• $v_{st} = -4st.\sin(s^2 - t^2)$

• $v_{ts} = +4ts.\sin(s^2 - t^2)$

$\boxed{59}$

$u = x^4y^3 - y^4$

$u_x = 4x^3y^3$

• $u_y = 3y^2x^4 - 4y^3$

$u_{xy} = 4 \cdot 3x^3y^2$

$= 12x^3y^2$

• $u_{yx} = 3 \cdot 4y^2x^3$

$= 12y^2x^3$

So $u_{xy} = u_{yx} = 12y^2x^3$

$\boxed{63}$

$f(x,y) = x^4y^2 - x^3y$

- $f_x = 4x^3y^2 - 3x^2y$

$f_{xx} = 12x^2y^2 - 6xy$

$f_{xxx} = 24xy^2 - 6y$

- $f_x = 4x^3y^2 - 3x^2y$

$f_{xy} = 8x^3y - 3x^2$

$f_{xyx} = 24x^2y - 6x$

$\boxed{65}$

$f(x,y,z) = e^{xyz^2}$

- $f_x = yz^2 e^{xyz^2}$

$f_{xy} = z^2 e^{xyz^2} + xyz^4 e^{xyz^2}$

$f_{xyz} = 2z e^{(xyz^2)} + 2xyz^3 e^{(xyz^2)} + 4xyz^3 e^{(xyz^2)}$

$+ 2x^2y^2z^5 e^{(xyz^2)}$

$= e^{(xyz^2)}\left[2z + 6xyz^3 + 2x^2y^2z^5\right]$

$\boxed{72}$

$$g(x,y,z) = \sqrt{1+xz} + \sqrt{1-xy}$$

$$\frac{d^3 g}{dxyz} = \frac{d^3}{dxyz}\left(\sqrt{1+xz}\right) + \frac{d^3}{dxyz}\left(\sqrt{1-xy}\right)$$

$$\frac{d^3 g}{dxyz} = \frac{d^2}{dxz}\left(\underbrace{\frac{d}{dy}\sqrt{1+xz}}_{0}\right) + \frac{d^2}{dxy}\left(\underbrace{\frac{d}{dz}\sqrt{1-xy}}_{0}\right)$$

$$\frac{d^3 g}{dxyz} = 0$$

$\boxed{76}$

(a) $u = x^2 + y^2$

$\cdot\ u_x = 2x \qquad \cdot\ u_y = 2y$

$\cdot\ u_{xx} = 2 \qquad \cdot\ u_{yy} = 2$

So, $u_{xx} + u_{yy} \overset{?}{=} 0$

$2 + 2 \neq 0$

(b) $u = x^2 - y^2$

$\cdot\ u_{xx} = 2 \qquad \cdot\ u_{yy} = -2$

So, $u_{xx} + u_{yy} \overset{?}{=} 0 \rightarrow 2 - 2 = 0$ ✓

(c) $u = x^3 + 3xy^2$

$u_{xx} = 6x \qquad u_{yy} = 6x$

$6x + 6x \neq 0$

(d) $u = \ln\sqrt{x^2+y^2}$

$$u_x = \frac{x}{\sqrt{x^2+y^2}} \cdot \frac{1}{\sqrt{x^2+y^2}}$$

$$= \frac{x}{x^2+y^2}$$

$$u_{xx} = \frac{x^2+y^2-2x^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$u_y = \frac{y}{x^2+y^2}$$

$$u_{yy} = \frac{x^2+y^2-2y^2}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$$

$$\rightarrow u_{xx} + u_{yy} = \frac{y^2-x^2}{(x^2+y^2)^2} + \frac{x^2-y^2}{(x^2+y^2)^2} = 0 \checkmark$$

(e)

$u = \sin x \cosh y + \cos x \sinh y$

$u_x = \cos x \cosh y - \sin x \sinh y$

$u_{xx} = -\sin x \cosh y - \cos x \sinh y$

$u_y = \sin x \sinh y + \cos x \cosh y$

$u_{yy} = \sin x \cosh y + \cos x \cosh y$

$u_{xx} + u_{yy} = 0$ ✓

(f)

$u = e^{-x} \cos y - e^{-y} \cos x$

$u_x = -e^{-x} \cos y + e^{-y} \sin x$

$u_{xx} = e^{-x} \cos y + e^{-y} \cos x$

$u_y = -e^{-x} \sin y + e^{-y} \cos x$

$u_{yy} = -e^{-x} \cos y - e^{-y} \cos x$

$u_{xx} + u_{yy} = 0$

$\boxed{79}$

$$u(x,t) = f(x+at) + g(x-at)$$

$$u_x = f'(x+at) + g'(x-at)$$

$$\bullet\, u_{xx} = f''(x+at) + g''(x-at)$$

$$u_t = a f'(x+at) + (-a) g'(x-at)$$

$$\bullet\, u_{tt} = a^2 f''(x+at) + a^2 g''(x-at)$$

$$u_{tt} = a^2 \left(f''(x+at) + g''(x-at) \right)$$

$$u_{tt} = a^2 u_{xx} \quad \#$$

Section

14.4

$\boxed{1}$ $z = 2x^2 + y^2 - 5y$, $(1,2,-4)$

$f(x,y,z) = 2x^2 + y^2 - 5y - z$

$f_x = 4x = 4$

$f_y = 2y - 5 = -1$

$f_z = -1$

$f(1,2,-4) = 2 + 4 - 10 + 4$

$= 0$

T.P $\Rightarrow$ $f(x,y,z) + f_x(x-a) + f_y(y-b) + f_z(z-c)$

$= 4(x-1) \cancel{+} -(y-2) \cancel{+} -(z+4)$

$\boxed{4}$ $z = \frac{x}{y^2}$ $(-4,2,-1)$

$f(x,y,z) = \frac{x}{y^2} - z$

$f_x = \frac{1}{y^2} = \frac{1}{4}$ $f(-4,2,-1) = -1 + 1 = 0$

$f_y = \frac{-x\,2y}{y^4} = -1$

$f_z = -1$

Equation: $\frac{1}{4}(x+4) - (y-2) - (z+1)$

6

$Z = \ln(x-2y)$, $(3,1,0)$

$f(x,y,z) = \ln(x-2y) - z$

$f(3,1,0) = 0$

$f_x = \dfrac{1}{x-2y} = 1$

$f_y = \dfrac{-2}{x-2y} = -2$

$f_z = -1$

$T = (x-3) - 2(y-1) - z$

11

Domain of $f(x,y)$: $xy - 5 > 0$ at $(2,3)$

$xy > 5$

$f(x,y) = 1 + x\ln(xy-5)$

So at $x=2$, $y=3$ is Differentiable

$f_x = x \cdot \dfrac{y}{xy-5} \cancel{= 6} + \ln(xy-5) = 6$

$f_y = \dfrac{x \cdot x}{xy-5} = 4$

$f(2,3) = 1$

equation $\Rightarrow T = 1 + 6(x-2) + 4(y-3)$

12

$f(x,y) = \sqrt{xy}$, $(1,4)$

Domain $xy \geq 0$

so $(1,4)$:

$4 \geq 0$ ✓

Differentiable at $(1,4)$

$f_x = \dfrac{y}{2\sqrt{xy}} = 1$

$f_y = \dfrac{x}{2\sqrt{xy}} = \dfrac{1}{4}$

$f(1,4) = 2$

$T = 2 + (x-1) + \frac{1}{4}(y-4)$

14

$$f(x,y) = \frac{1+y}{1+x}, \quad (1,3)$$

$$f_x = \frac{-(1+y)}{(1+x)^2} = \frac{-4}{4} = -1$$

$$f_y = \frac{1}{1+x} = \frac{1}{2} \qquad f(1,3) = \frac{4}{2} = 2$$

$$T = 2 - (x-1) + \frac{1}{2}(y-3)$$

$\boxed{17}$

$f(x,y) = e^x \cos(xy)$ at $(0,0)$

$f_x = e^x(-\sin(xy)) \cdot y + \cos(xy)\, e^x$

$= 1$

$f_y = e^x x(-\sin xy)$

$= 0$

$f(0,0) = 1$

$T = 1 + (x-0) + 0$

$T = x+1$

Scanned with CamScanner

$\boxed{14}$ $f(2,5)=6$

$f_x(2,5)=1$

$f_y(2,5)=-1$

use linear aproximation to estimate $f(2.2,4.9)$

$a=2$
$b=5$

$L = f(x,y) + f_x(x-a) + f_y(y-b)$

$= 6 + (x-2) + -1(y-5)$

$= 6 + (x-2) - (y-5)$

$= 6 + (2.2-2) - (4.9-5)$

$= 6 + 0.2 + 0.1$

≈ 6.3

$\boxed{31}$ $z = 5x^2 + y^2$ (x,y) changes from $(1,2)$ $(1.05, 2.1)$ find Δz

$\Delta z = z_2 - z_1$

$=$

$z_2 =$ linear approx. of $(1.05, 2.1)$

$z_1 = 5(1)^2 + (2)^2$
$= 9$

$f(x,y) = 5x^2 + y^2$

$a = 1$
$b = 2$

$f_x = 10x = 10$

$f_y = 2y = 4$

$f(1,2) = 9$

$L = f(1,2) + 10(x-1) + 4(y-2)$

$= 9 + 10(1.01-1) + 4(2.1-2)$

$= 9 + 10(0.01) + 4(0.1)$

$= 9 + 0.1 + 0.4$

$= 9.5$

$\Delta z = 9.5 - 9 = \underline{\underline{0.5}}$

$\boxed{46}$ $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$

to check if is diff. we must check the continuity

So $\lim\limits_{x,y \to 0,0} f(x,y) = f(0,0)$

$$\lim_{x,y \to 0,0} \frac{xy}{x^2+y^2} \overset{?}{=} 0$$

at $y=0$

$\lim = 0$

at $x=0$

$\lim = 0$

at $x=y$

$$\lim_{y \to 0} \frac{y^2}{2y^2} = \frac{1}{2} \neq 0$$

D.N.E So

Not diff. at $(0,0)$

Section

14.6

14.6

4] $D_f = \nabla f \times U \rightarrow \theta = \frac{\pi}{3}$

$U = \langle \cos\theta, \sin\theta \rangle$

$U = \langle \cos\frac{\pi}{3}, \sin\frac{\pi}{3} \rangle$

$U = \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$

$\rightarrow f(x,y) = xy^3 - x^2 \quad (1,2)$

$f_x = y^3 - 2x \Big|_{1,2} = 8 - 2 = 6$

$f_y = 3y^2x - 0 \Big|_{1,2} = 3 \times 4 = 12$

$\nabla f = \langle 6, 12 \rangle$

$D_f = \nabla f \cdot U = \langle 6,12 \rangle \cdot \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle = 3 + 6\sqrt{3}.$

8] $f(x,y) = x^2 \ln y \quad P(3,1) \quad u = \frac{-5}{13} i + \frac{12}{13} j \rightarrow$ unit vector

ⓐ $\nabla f = \langle f_x, f_y \rangle$

$f_x = 2x \ln y, \; f_y = \frac{x^2}{y}$

$\nabla f = \langle 2x \ln y, \frac{x^2}{y} \rangle$

ⓑ $\nabla f = \langle 2x \ln y, \frac{x^2}{y} \rangle \Big|_{3,1}$

$\nabla f = \langle 0, 9 \rangle$

ⓒ $D_f = \nabla f \cdot U$

$= \langle 0,9 \rangle \cdot \langle \frac{-5}{13}, \frac{12}{13} \rangle = 0 + \frac{9 \times 12}{13} = \frac{108}{13}.$

11] $f(x,y) = e^x \sin y \quad (0, \frac{\pi}{3}) \quad V = \langle -6, 8 \rangle.$

$\rightarrow$ not a unit vector.

$\nabla f = \langle f_x, f_y \rangle$

$f_x = e^x \sin y \Big|_{0,\frac{\pi}{3}} = \frac{\sqrt{3}}{2}.$

$f_y = e^x \times \cos y \Big|_{0,\frac{\pi}{3}} = \frac{1}{2}.$

$U_v = \frac{\langle -6, 8 \rangle}{\sqrt{36+64}} = \langle \frac{-6}{10}, \frac{8}{10} \rangle$

$D_f = \nabla f \cdot U_v$

$= \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle \cdot \langle \frac{-6}{10}, \frac{8}{10} \rangle$

$= \frac{-6\sqrt{3}}{20} + \frac{8}{20} = \frac{8 - 6\sqrt{3}}{20}.$

17) $f(x,y,z) = xe^y + ye^z + ze^x \quad (0,0,0) \qquad v = \langle 5,1,-2\rangle$ → to make a unit vector

$D_f = \nabla f \cdot U_v$

$\nabla f = \langle f_x, f_y, f_z \rangle$

$f_x = e^y + ze^x \Big|_{0,0,0} = 1$

$f_y = xe^y + e^z \Big|_{0,0,0} = 1$

$f_z = ye^z + e^x \Big|_{0,0,0} = 1$

$\nabla f = \langle 1,1,1 \rangle$

$U_v = \dfrac{\langle 5,1,-2\rangle}{\sqrt{30}} = \left\langle \dfrac{5}{\sqrt{30}}, \dfrac{1}{\sqrt{30}}, \dfrac{-2}{\sqrt{30}} \right\rangle$

$D_f = \nabla f \cdot U_v$

$= \langle 1,1,1\rangle \cdot \left\langle \dfrac{5}{\sqrt{30}}, \dfrac{1}{\sqrt{30}}, \dfrac{-2}{\sqrt{30}} \right\rangle$

$= \dfrac{5+1-2}{\sqrt{30}} = \dfrac{4}{\sqrt{30}}$

---

20) D_f? → $f(x,y,z) = xy^2z^3$ at $P(2,1,1)$ → Direction of $Q(0,-3,5)$.

$D_f = \nabla f \cdot U_{PQ}$, $PQ = \langle -2,-4,4 \rangle$

$\nabla f = \langle f_x, f_y, f_z \rangle$

$f_x = y^2z^3 \Big|_{2,1,1} = 1$

$f_y = 2xyz^3 \Big|_{2,1,1} = 4$

$f_z = 3xy^2z^2 \Big|_{2,1,1} = 6$

$\nabla f = \langle 1,4,6 \rangle$

$U_{PQ} = \dfrac{\langle -2,-4,4 \rangle}{\sqrt{36}}$

$D_f = \nabla f \cdot U_{PQ}$

$= \langle 1,4,6 \rangle \cdot \left\langle \dfrac{-2}{\sqrt{36}}, \dfrac{-4}{\sqrt{36}}, \dfrac{4}{\sqrt{36}} \right\rangle$

$= \dfrac{-2-16+24}{\sqrt{36}}$

$= \dfrac{6}{6} = 1$

21) Maximum Rate of change:

$f(x,y) = 4y\sqrt{x}$, $(4,1)$

$f_x = 4y \cdot \frac{1}{2\sqrt{x}} \Big|_{4,1} = \frac{4}{4} = 1$

$f_y = 4\sqrt{x} \Big|_{4,1} = 8$

$\nabla f = \langle 1, 8 \rangle$

Max Rate $\rightarrow D_f = \nabla f \cdot U$

$\nabla f \cdot U = |\nabla f|\,|U| \cos\theta$

$\underbrace{\nabla f \cdot U}_{\to D_f}$, $|U| = 1$, $\cos\theta \to$ Max $\Rightarrow \theta = \text{zero}$

$D_f = |\nabla f| = \sqrt{1+64} = \sqrt{65}$.

$\rightarrow$ Max. Rate $= \sqrt{65}$ in The direction of $\nabla f = \langle 1, 8 \rangle$.

---

25) $f(x,y,z) = \frac{x}{y+z}$ $(8,1,3)$

$\nabla f = \langle f_x, f_y, f_z \rangle$

$f_x = \frac{1}{y+z} \Big|_{8,1,3} = \frac{1}{4}$

$f_y = \frac{-x}{(y+z)^2} \Big|_{8,1,3} = \frac{-8}{16} = \frac{-1}{2}$.

$f_z = \frac{-x}{(y+z)^2} = \frac{-8}{16} = \frac{-1}{2}$.

$|\nabla f| = \sqrt{\frac{1}{16} + \frac{1}{4} + \frac{1}{4}}$

Max Rate $\rightarrow D_f = \nabla f \cdot U$

$\nabla f \cdot U = |\nabla f|\,|U| \cos\theta$

$|U| = 1$, $\theta = \text{zero}$

$D_f = |\nabla f|$

$D_{f\,\text{Max}} = \frac{3}{4}$, In The Direction $\nabla f = \langle \frac{1}{4}, \frac{-1}{2}, \frac{-1}{2} \rangle$.

28| Direction? → $D_f = 2$ at $P(2,1)$

$f(x,y) = x^2 + xy^3$ $\qquad U = \langle x, y \rangle$

$D_f = \nabla f \cdot U$ $\qquad$ To $U = 1$ → $\sqrt{x^2+y^2} = 1$ — ①

$2 = D_f$ $\qquad$ ~~① $x^2+y^2=1$~~

$2 = \nabla f \cdot U$

$2 = \langle 5, 6 \rangle \cdot \langle x, y \rangle$

$2 = 5x + 6y$ — ②

$\nabla f = \langle f_x, f_y \rangle$

$f_x = 2x + y^3 \Big|_{2,1} = 4 + 1 = 5$

$f_y = 3y^2 x \Big|_{2,1} = 6$

→ ① $x^2 + y^2 = 1$

② $5x + 6y = 2$ → $x = \left(\frac{2-6y}{5}\right)$

$\left(\frac{2-6y}{5}\right)^2 + y^2 = 1$, $\left(\frac{4 - 24y + 36y^2}{25} + y^2 = 1\right) 25$

$4 - 24y + 36y^2 + 25y^2 = 25$

$61y^2 - 24y - 21 = 0$

$y = 0{,}837$ → $x = -0{,}6$

or $y = -0{,}411$ → $x = 0{,}8932$

37) $T(x,y,z) = 200e^{-x^2-3y^2-9z^2} \rightarrow 200e^{-x^2} \times e^{-3y^2} \times e^{-9z^2}$

ⓐ D_f of $P_1(2,-1,2) \rightarrow P_2(3,-3,3)$

$D_f = \nabla f \cdot U_a$, $a = \langle 1,-2,1 \rangle$, $U_a = \frac{\langle 1,-2,1 \rangle}{\sqrt{6}}$

$P_1P_2 = a$

$\rightarrow \nabla f = \langle f_x, f_y, f_z \rangle$

$f_x = -400x\, e^{-x^2} \cdot e^{-3y^2} \times e^{-9z^2} \Big|_{2,-1,2} = -800e^{-43}$

e^{-43} ثابت

$f_y = 200 \times -6y\, e^{-43} \Big|_{2,-1,2} = 1200e^{-43}$

ثابت ومستقل

$f_z = 200 \times -18z \times e^{-43} \Big|_{2,-1,2} = -7200e^{-43}$

$\rightarrow D_f = \nabla f \cdot U_a = \langle -800e^{-43}, 1200e^{-43}, -7200e^{-43} \rangle \cdot \frac{\langle 1,-2,1 \rangle}{\sqrt{6}}$

$= \frac{10400e^{-43}}{\sqrt{6}}$

ⓑ in the direction of $\nabla f = \langle \frac{-800e^{-43}}{1}, \frac{1200e^{-43}}{1}, \frac{-7200e^{-43}}{1} \rangle$

ⓒ Max Rate $\rightarrow D_f = |\nabla f| \rightarrow = 400e^{-43}\sqrt{337}$

$\boxed{42}$ a $x = y^2 + z^2 + 1$ $(3, 1, -1)$

$f(x,y,z) = -x + y^2 + z^2 + 1$

a) ~~tangent plane~~

$f_x = -1$

$f_y = 2y|_{y=1} = 2$

$f_z = 2z|_{z=-1} = -2$

→ a) tangent plane

$-1(x-3) + 2(y-1) + -2(z+1) = 0$

→ b) normal line

$\frac{x-3}{-1} = \frac{y-1}{2} = \frac{z+1}{-2}$

$\boxed{43}$ $xy^2z^3 = 8$ $(2, 2, 1)$

$f(x,y,z) = xy^2z^3 - 8$

$f_x = y^2z^3|_{2,2,1} = 4 \times 1 = 4$

$f_y = 2xyz^3|_{2,2,1} = 8$

$f_z = 3xy^2z^2|_{2,2,1} = 24$

a) tangent plane

$4(x-2) + 8(y-2) + 24(z-1) = 0$

b) normal line

$\frac{x-2}{4} = \frac{y-2}{8} = \frac{z-1}{24}$

54| What point?

① $x^2+y^2+2z^2=1 \rightarrow f_1 = x^2+y^2+2z^2-1$

② $x+2y+z=1 \rightarrow f_2 = x+2y+z-1$

Parallel $\rightarrow \nabla f_1 \parallel \nabla f_2$

$\Rightarrow \nabla f_1 = k\nabla f_2$
i.e. k is a constant.

$\nabla f_1 = \langle 2x, 2y, 4z \rangle$ $\rightarrow \nabla f_1 = k\nabla f_2$

$\nabla f_2 = \langle 1, 2, 1 \rangle$ $\rightarrow \langle 2x, 2y, 4z \rangle = \langle k, 2k, k \rangle$

① $2x = k \rightarrow x = \frac{k}{2}$

② $2y = 2k \rightarrow y = k$

③ $4z = k \rightarrow z = \frac{k}{4}$

$k^2 \rightarrow$

$\frac{k^2}{4} + k^2 + \frac{2k^2}{16} = 1$

$\rightarrow 2k^2 + 8k^2 + k^2 = 8$

$11k^2 = 8$

$k = \pm\sqrt{\frac{8}{11}}$

$\rightarrow x = \pm\frac{\sqrt{8/11}}{2}$

$y = \pm\sqrt{8/11}$

$z = \pm\frac{\sqrt{8/11}}{4}$

55| ① $x^2-y^2-z^2=1 \rightarrow f_1 = x^2-y^2-z^2-1$, $\nabla f_1 = \langle 2x, -2y, -2z \rangle$

② $z = x+y \rightarrow f_2 = x+y-z$, $\nabla f_2 = \langle 1, 1, -1 \rangle$

$\nabla f_1 \parallel \nabla f_2$

$\rightarrow \nabla f_1 = k\nabla f_2$

$\langle 2x, -2y, -2z \rangle = \langle k, k, -k \rangle$

$x = \frac{k}{2}$, $y = \frac{-k}{2}$, $z = \frac{k}{2}$

$\rightarrow \frac{k^2}{4} - \frac{k^2}{4} - \frac{k^2}{4} = 1$

$k^2 = -4$
i.e. no real sol.

64) $y + z$ [illegible] intersects to $x^2 + y^2 = 5$

$P(1, 2, 1)$

tangent line [illegible]

$f = y + z$

$\nabla T = \langle 2x, 2y, 0 \rangle \Big|_{(1,2,1)} = \langle 2, 4, 0 \rangle$

$\nabla f = \langle 0, 1, 1 \rangle$

$$\nabla f \times \nabla T = \begin{vmatrix} i & j & k \\ 0 & 1 & 1 \\ 2 & 4 & 0 \end{vmatrix} = \langle -4, 2, -2 \rangle$$

→ Parametric:

L: $x = 1 - 4t$

$y = 2 + 2t$

$z = 1 - 2t$

Section

14.8

14.8

3) $f(x,y) = x^2 - y^2 \qquad x^2+y^2=1 \to g = x^2+y^2-1$

$f_x = \lambda g_x \quad , \quad f_y = \lambda g_y$

$2x = \lambda 2x \quad , \quad -2y = \lambda 2y$

$\frac{\cancel{2}x}{-\cancel{2}y} = \frac{\cancel{\lambda}\cancel{2}x}{\cancel{\lambda}\cancel{2}y} \to xy = -xy$

$xy = 0 \to y=0 \,,\, x = \pm 1$

$x = 0 \,,\, y = \pm 1$

C.P. $(1,0), (-1,0)$

$(0,1), (0,-1)$

$f(x,y) = x^2 - y^2$

$f|_{1,0} = 1 \,,\, f|_{-1,0} = 1 \to$ Max

$f|_{0,1} = -1 \,,\, f|_{0,-1} = -1 \to$ min

4) $f(x,y) = e^{xy} \,,\, x^3+y^3=16 \to g = x^3+y^3-16$

$f_x = \lambda g_x \quad , \quad f_y = \lambda g_y$

$ye^{xy} = \lambda 3x^2 \,,\, xe^{xy} = \lambda 3y^2$

$\frac{y\cancel{e^{xy}}}{x\cancel{e^{xy}}} = \frac{\cancel{\lambda}\cancel{3}x^2}{\cancel{\lambda}\cancel{3}y^2} \to y^3 = x^3$

$\to 2x^3 = 16 \to x^3 = 8$

$x = 2$

$y = 2$

$f(2,2) = e^4 \to$

7] $f(x,y,z) = x^2+y^2+z^2$, $x+y+z=12$

$g = x+y+z-12$

$f_x = \lambda g_x$, $f_y = \lambda g_y$, $f_z = \lambda g_z$

$2x = \lambda$ $\quad 2y = \lambda$ $\quad 2z = \lambda$

$\frac{2x}{2y} = \frac{\lambda}{\lambda} \rightarrow x = y$, $\frac{2x}{2z} = \frac{\lambda}{\lambda} \rightarrow x = z$

$x+x+x = 12$

$\rightarrow x+x+x = 12 \rightarrow x = 4$

$3x = 12$

$y = 4$

$z = 4$

$f(x,y,z) = x^2+y^2+z^2$

$f\Big|_{4,4,4} = 48 \rightarrow$

z

$\ldots y^2 + z^2 \quad 4 = x^4 + y^4 + z^4 = 1$

21 $f(x,y) = x^2 + y^2 + 4x - 4y \;,\; x^2+y^2 \le 4.$

$f_x = \lambda g_x \;,\; f_y = \lambda g_y$

$2x+4 = \lambda 2x \qquad 2y-4 = \lambda 2y$

$$\frac{2x+4}{2y-4} = \frac{\lambda 2x}{\lambda 2y} \rightarrow \cancel{xy} + 2y = \cancel{xy} - 2x$$

$x = -y$

$\rightarrow 2x^2 = 4 \rightarrow x = \frac{\pm 2}{\sqrt{2}} \;,\; y = \pm\frac{2}{\sqrt{2}}.$

$f_x = 2x+4 \rightarrow = 0$

$2x = -4 \rightarrow x = -2$

$f_y = 2y-4 \rightarrow y = 2$

$C.P. \rightarrow (-2,2) \;,\; \left(\frac{\pm 2}{\sqrt{2}} \;,\; \frac{\pm 2}{\sqrt{2}}\right)$

$f = x^2+y^2+4x-4y.$

نعوض فيها من الاقتران

23 $\rightarrow$ Ex. 14.7

$\rightarrow g(x,y,z) = x^2 + y^2 - z^2.$

$f(x,y,z) = (x-4)^2 + (y-2)^2 + z^2$

$f_x = \lambda g_x \;,\; f_y = \lambda g_y \;,\; f_z = \lambda g_z$

$2(x-4) = \lambda 2x \;,\; 2(y-2) = \lambda 2y \;,\; 2z = \lambda(-2)z$

$$1/2 \rightarrow \frac{2(x-4)}{2(y-2)} = \frac{\lambda 2x}{\lambda 2y}$$

$yx - 4y = xy - 2x$

$y = \frac{1}{2}x.$

when $x = 2 \;,\; y = 1$

$$1/3 \rightarrow \frac{2(x-4)}{2z} = \frac{\lambda 2x}{-2\lambda z}$$

$-zx + 4z = xz.$

$2xz = 4z \rightarrow xz = 2z \rightarrow x = 2.$

~~$xz - 2z = 0$~~

~~$z(x-2) = 0$~~

~~$z = 0, x = 2$~~

$g(x,y,z) = 0$

$\rightarrow 4 + 1 - z^2 = 0 \rightarrow z = \pm\sqrt{5}$, point is $(2,1,\pm\sqrt{5})$.

47) $f(x,y,z) = xyz$, $g = x^2+y^2+z^2-r^2$. $\hookrightarrow$ على الكرة ... القطر $= 2r$

$f_x = \lambda g_x$, $f_y = \lambda g_y$, $f_z = \lambda g_z$

$yz = \lambda 2x$, $xz = \lambda 2y$, $xy = \lambda 2z$.

$\frac{yz}{xy} = \frac{\lambda 2x}{\lambda 2z} \rightarrow x^2 = z^2 \rightarrow x = \pm z$, but both are positive.

$\rightarrow x = z$.

$\frac{xz}{xy} = \frac{\lambda 2y}{\lambda 2z} \rightarrow z^2 = y^2 \rightarrow z = y$

$\hookrightarrow y = x$.

$3x^2 = r^2 \rightarrow x = \frac{2r}{\sqrt{3}} = y = z$

Max volume $= \left(\frac{2r}{\sqrt{3}}\right)^3$

---

38) $\rightarrow$ 13, 14.7

$g = xyz = 1000\,cm^3$.

$f = 2(xy + yz + zx)$ $\hookrightarrow$ surface area of box

$f_x = \lambda g_x$, $f_y = \lambda g_y$, $f_z = \lambda g_z$

$2(y+z) = \lambda yz$, $2(x+z) = \lambda xz$, $2(x+y) = \lambda xy$.

$1/2 \rightarrow \frac{\cancel{2}(y+z)}{\cancel{2}(x+z)} = \frac{\cancel{\lambda} yz}{\cancel{\lambda} xz}$

$\cancel{xy} + xz = \cancel{yx} + yz$

$1.\ \boxed{x = y}$

$1/3 \rightarrow \frac{\cancel{2}(y+z)}{\cancel{2}(x+y)} = \frac{\cancel{\lambda} yz}{\cancel{\lambda} xy}$

$xy + \cancel{yz} = \cancel{zx} + zy$

$x\cancel{y} = z\cancel{y}$

$\boxed{x = z}$

$\rightarrow x^3 = 1000\,cm^3$.

$x = \underline{10\,cm} = y = z$

14.7

$f(x,y,z) = xyz \quad , g(x,y,z) = 2(xy + yz + xz) - 64$

$f_x = \lambda g_x , \quad f_y = \lambda g_y , \quad f_z = \lambda g_z$

$yz = \lambda 2(y+z) , \quad xz = \lambda 2(x+z) , \quad xy = \lambda 2(x+y)$

$1/2 \rightarrow \frac{yz}{xz} = \frac{\lambda 2(y+z)}{\lambda 2(x+z)}$

$\cancel{xy} + xz = \cancel{yx} + yz$

$\boxed{x = y}$

$1/3 \rightarrow \frac{yz}{xy} = \frac{\lambda 2(y+z)}{\lambda 2(x+y)}$

$\cancel{zx} + zy = xy + \cancel{xz}$

$\boxed{z = x}$

$2(3x^2) = 64 \rightarrow 6x^2 = 64$

$x^2 = \frac{32}{3} , \quad x = \sqrt{\frac{32}{3}} = y = z$

$41 \rightarrow 51$
14.7

$f(x,y,z) = xyz$

$g(x,y,z) = 4x + 4y + 4z - c$

$f_x = \lambda g_x , \quad f_y = g_y \lambda , \quad f_z = \lambda g_z$

$yz = \lambda 4 , \quad xz = \lambda 4 , \quad yx = \lambda 4$

$1/2 \rightarrow \frac{yz}{xz} = \frac{\lambda 4}{\lambda 4}$

$\boxed{x = y}$

$1/3 \rightarrow \frac{yz}{yx} = \frac{\lambda 4}{\lambda 4}$

$\boxed{x = z}$

$\rightarrow 4 \times 3x = c \rightarrow x = \frac{c}{12} = y = z$