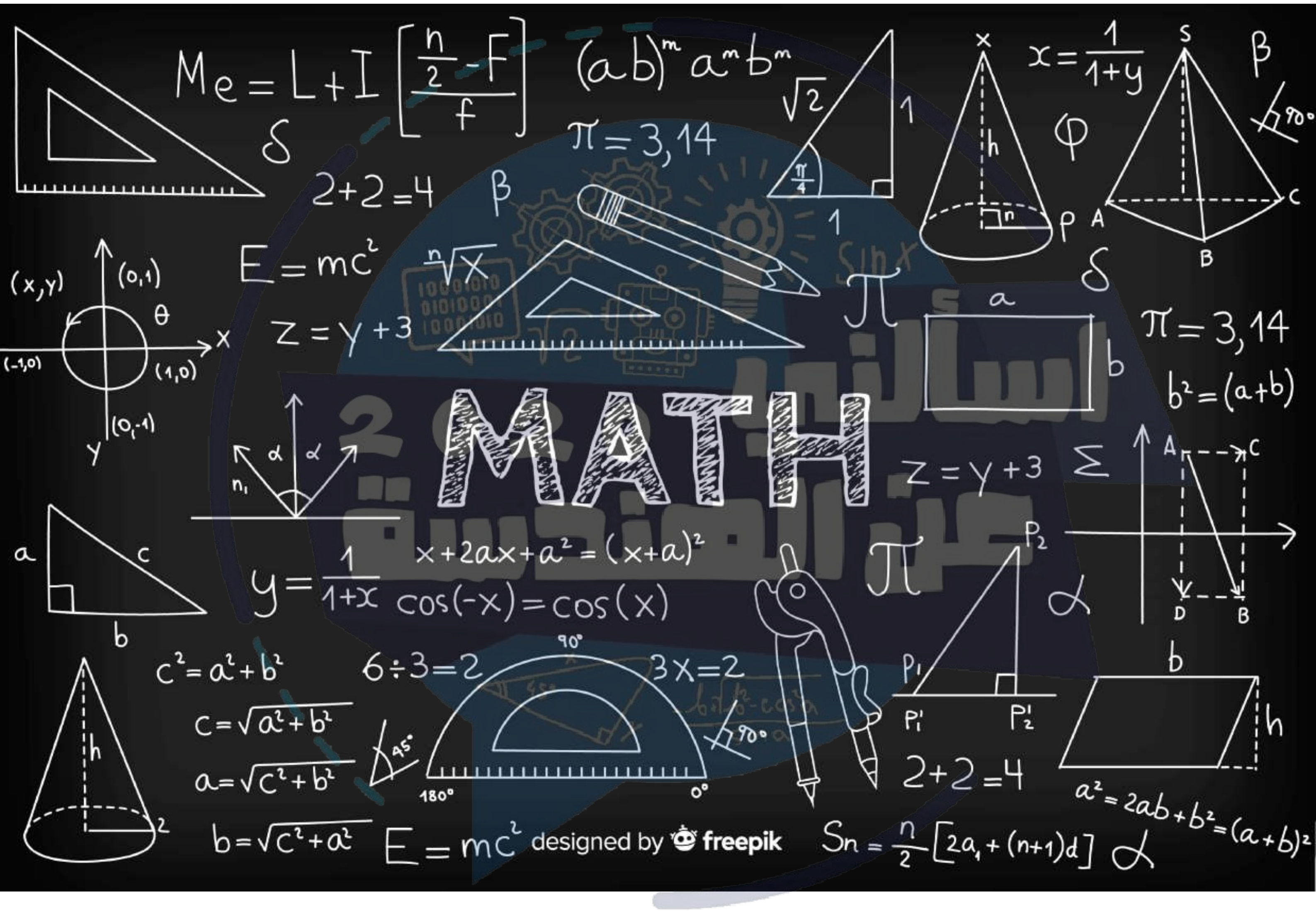
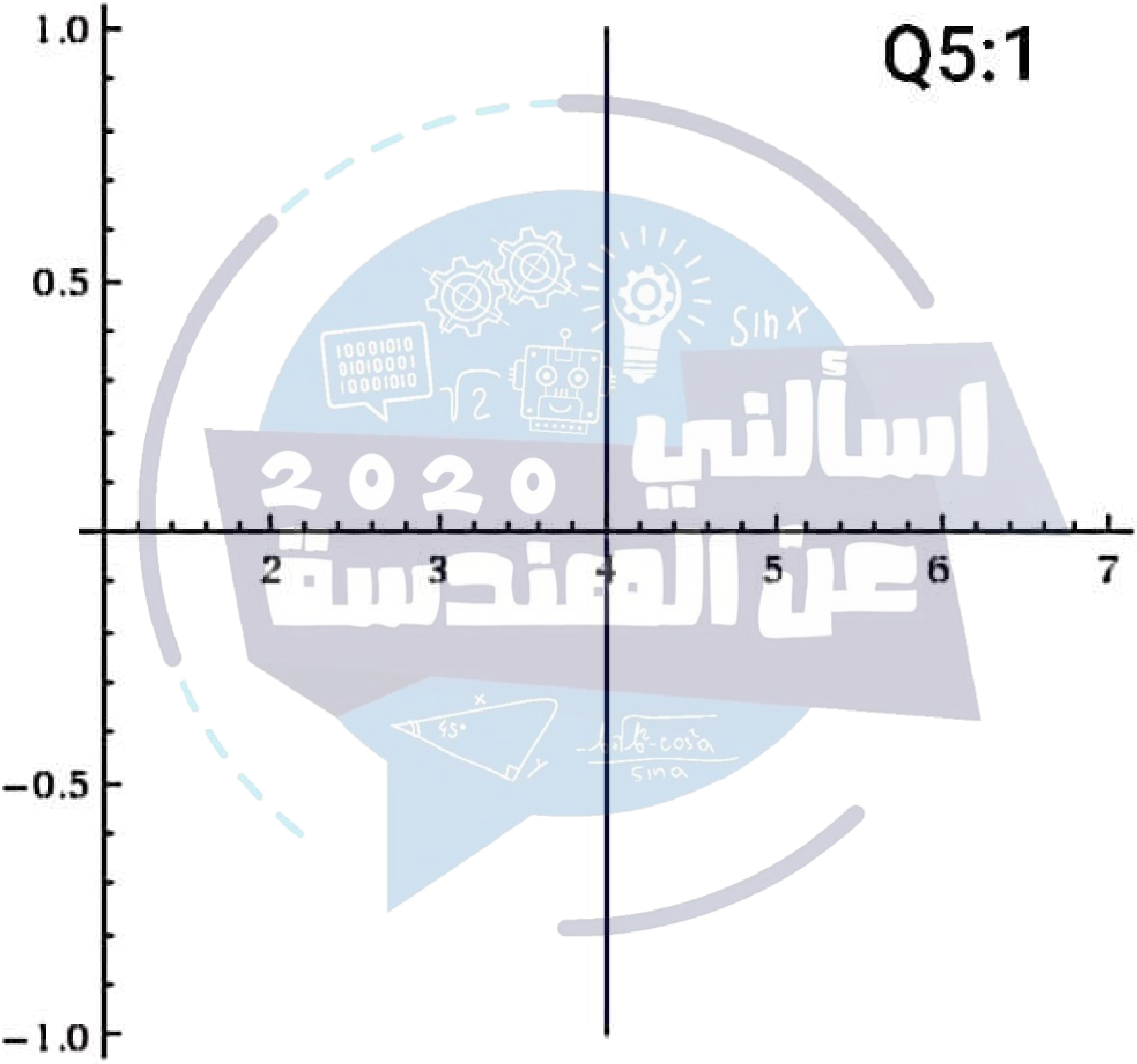

MATH
designed by freepik

Q3 :

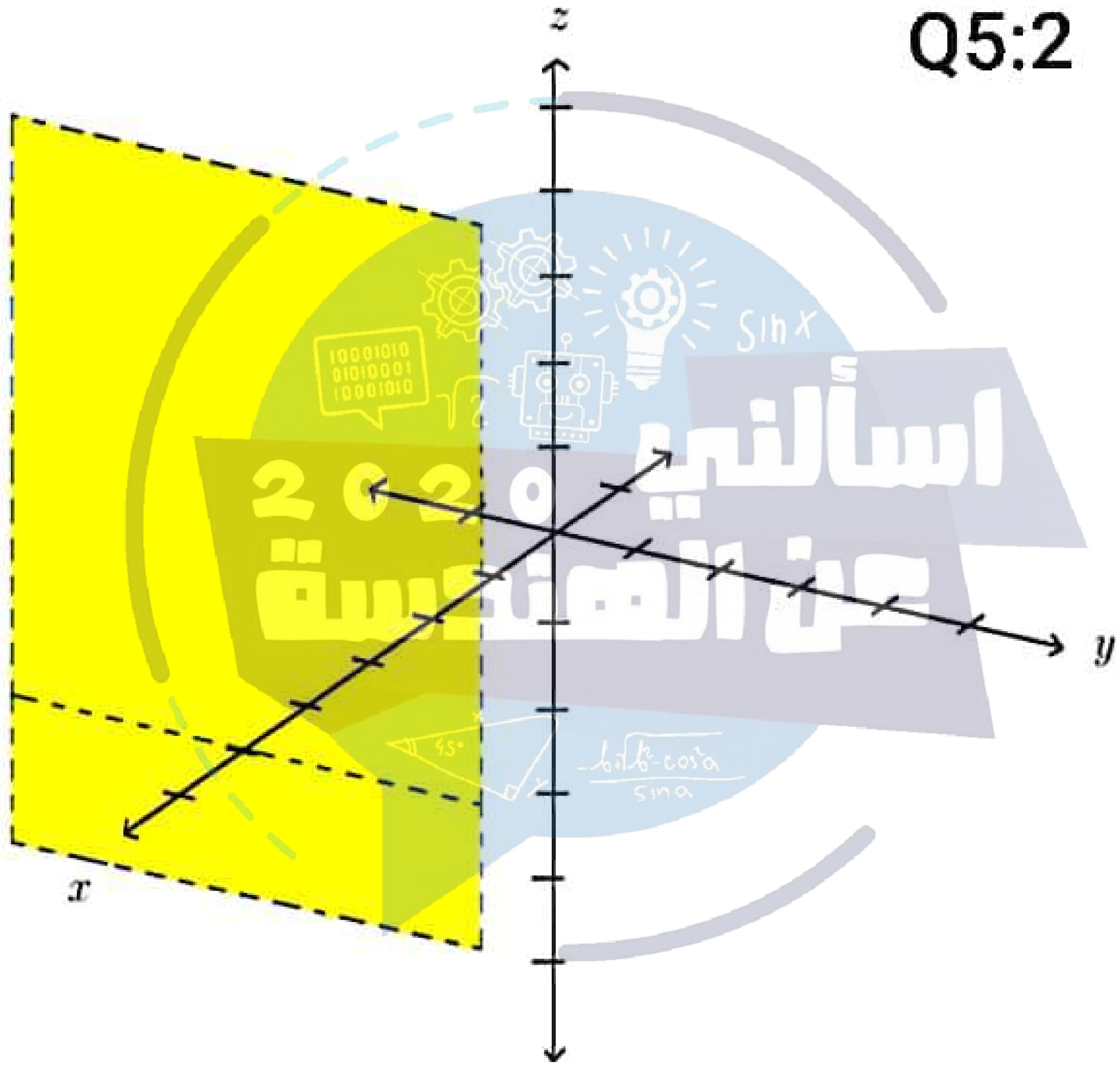
the point C is the closet to YZ - plane.

the point A on the XZ - plane.

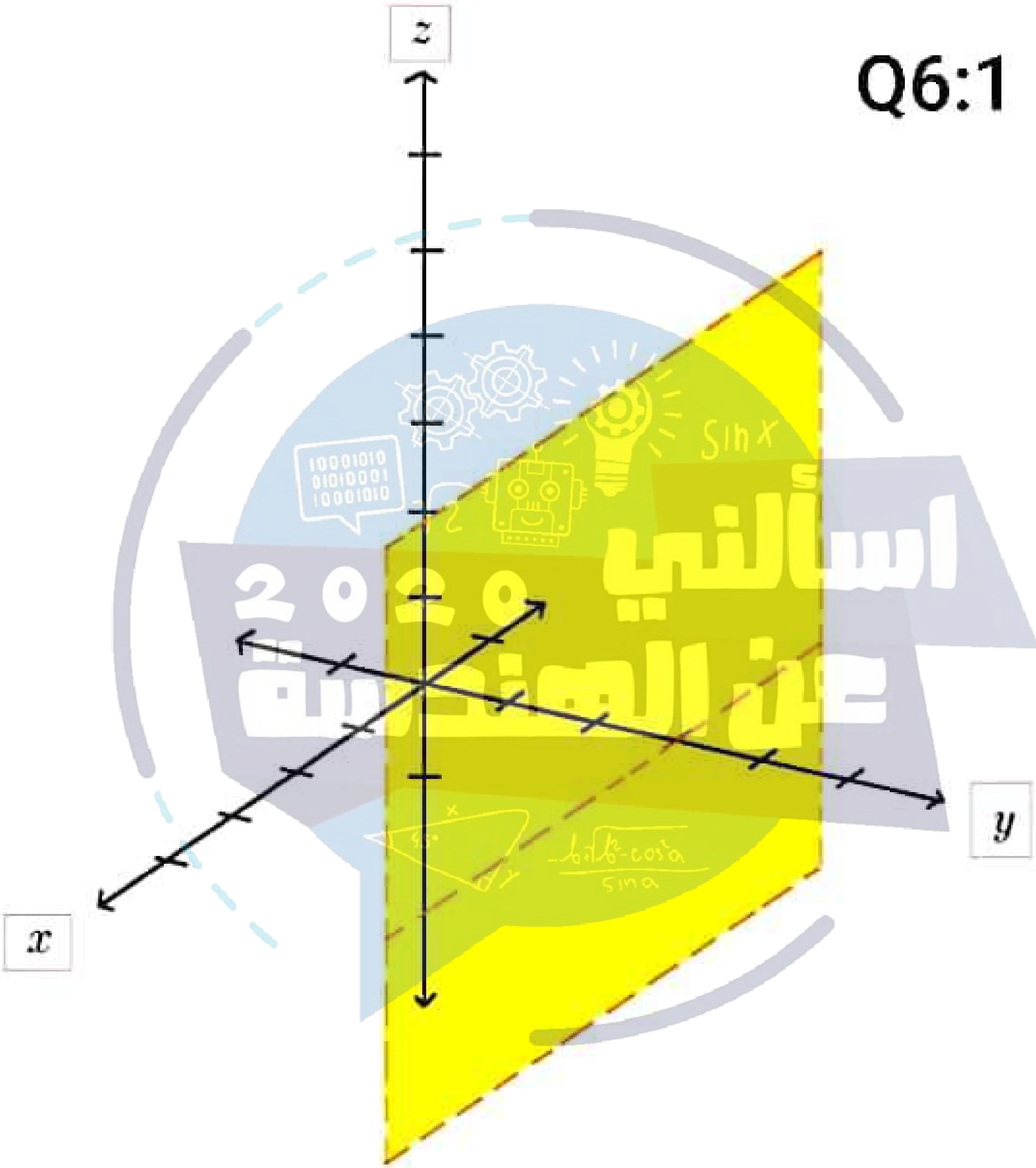
Q5:1



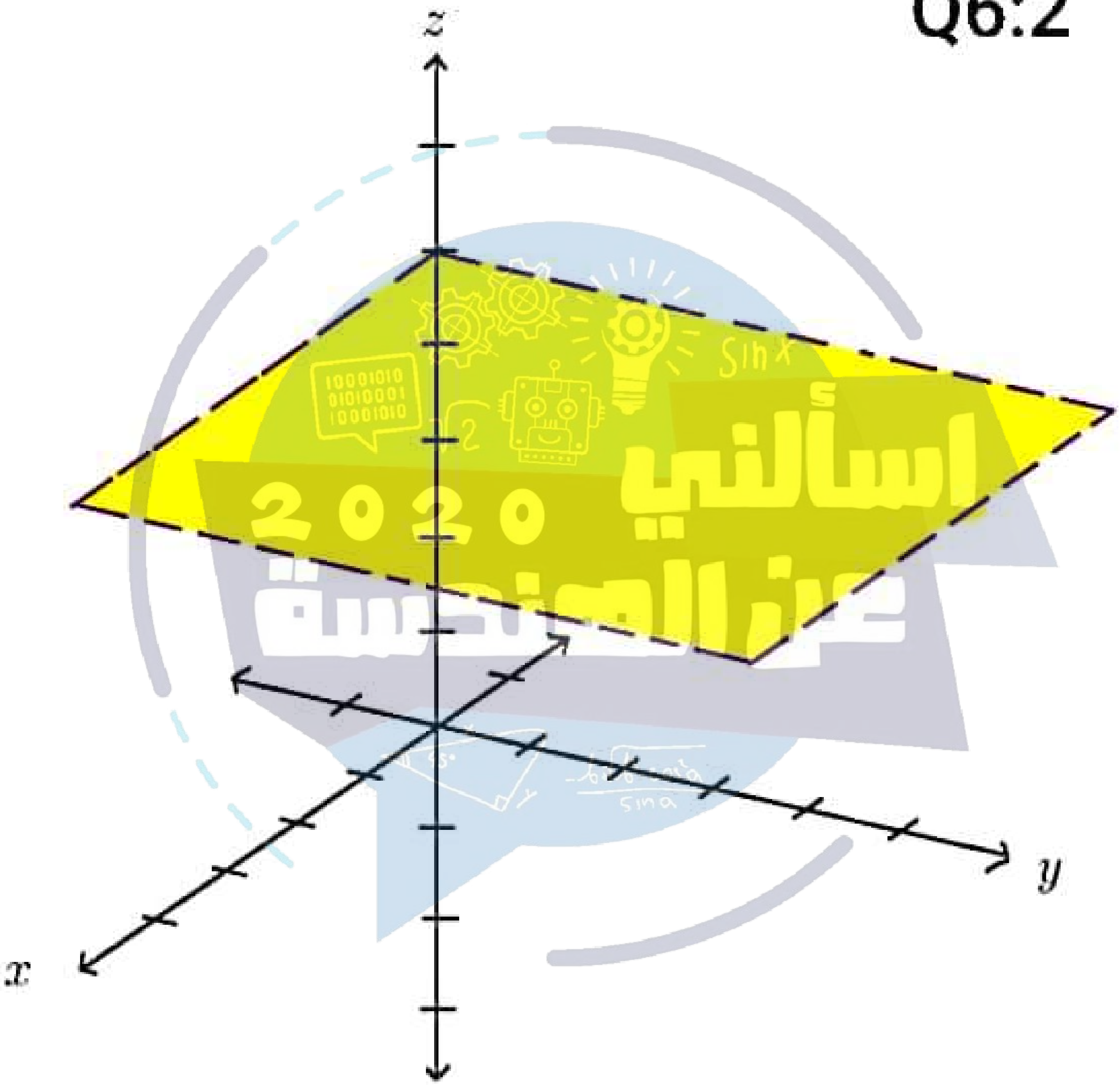
Q5:2



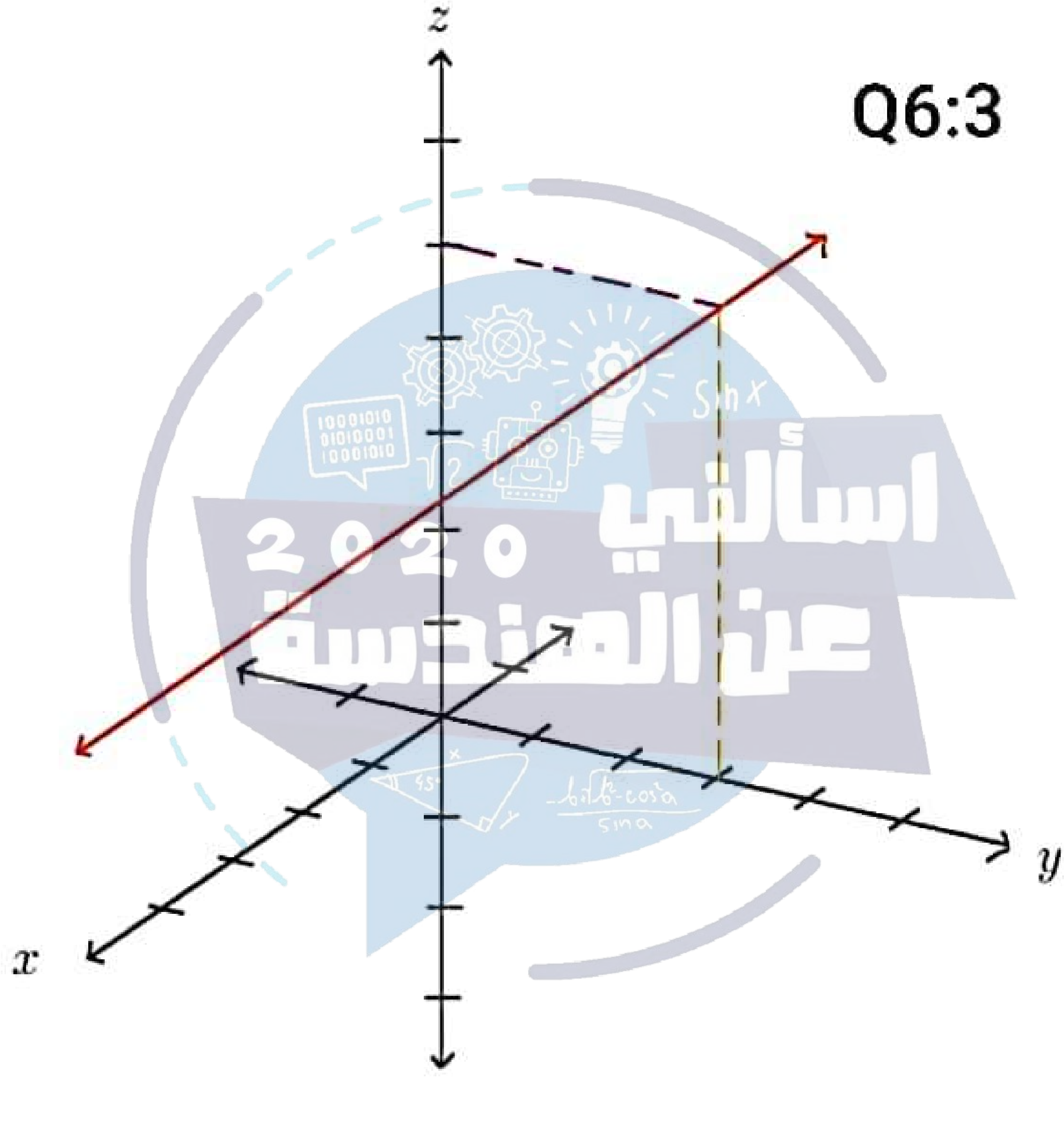
Q6:1



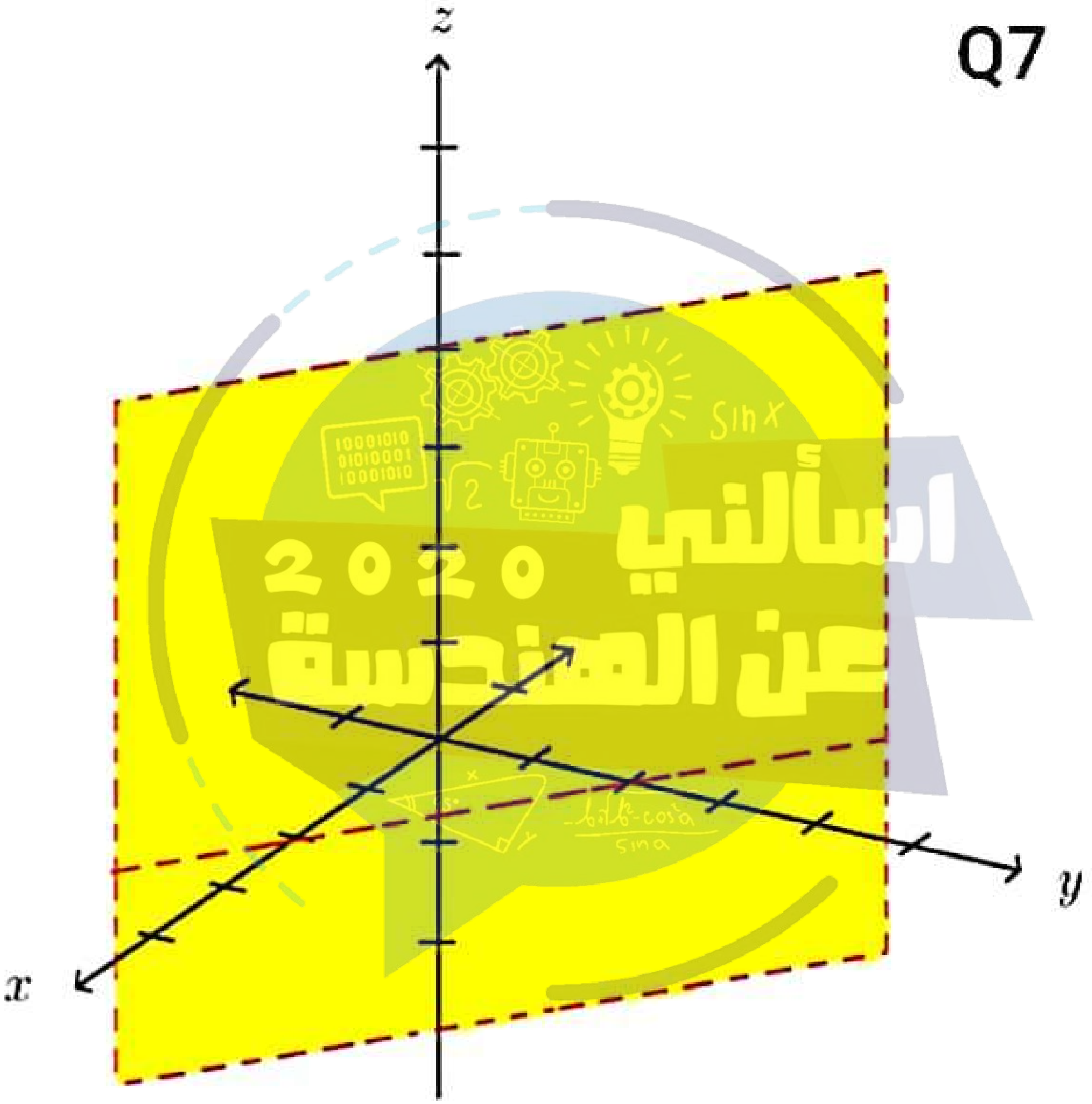
Q6:2



Q6:3



Q7



Q8

Q9:

P(3 , -2 , -3)

Q(7 , 0 , 1)

R(1 , 2 , 1)

$|PQ| = \sqrt{(7-3)^2 + (0+2)^2 + (1+3)^2} = \sqrt{36} = 6$

$|QR| = \sqrt{(1-7)^2 + (2-0)^2 + (1-1)^2} = \sqrt{40} = 2\sqrt{10}$

$|PR| = \sqrt{(1-3)^2 + (2+2)^2 + (1+3)^2} = \sqrt{36} = 6$

$\therefore |PQ| = |PR| = 6$

$\therefore$ PQR is isosceles ~~tran~~ triangle

Q_{11}:

a) $A(2,4,2)$
$B(3,7,-2)$
$C(1,3,3)$

$\rightarrow \frac{X_1 - X_2}{X_1 - X_3} = \frac{2-3}{2-1} = \boxed{-1}$

$\rightarrow \frac{y_1 - y_2}{y_1 - y_3} = \frac{4-7}{4-3} = \boxed{-3}$

$$\frac{X_1 - X_2}{X_1 - X_3} \neq \frac{y_1 - y_2}{y_1 - y_3}$$

$\therefore$ the Points are not collinear

b) $D(0,-5,5)$

$E(1,-2,4)$

$F(3,4,2)$

$$\frac{x_1-x_2}{x_1-x_3} = \frac{0-1}{0-3} = \frac{1}{3}$$

$$\frac{y_1-y_2}{y_1-y_3} = \frac{-5+2}{-5-4} = \frac{-3}{-9} = \frac{1}{3}$$

$$\frac{z_1-z_2}{z_1-z_3} = \frac{5-4}{5-2} = \frac{1}{3}$$

$$\frac{1}{3} = \frac{1}{3} = \frac{1}{3}$$

$\therefore$ the points are collinear

Q12: $P(4, -2, 6)$

a) 6

b) 4

c) 2

D) $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2} = \sqrt{40} = 2\sqrt{10}$

E) $= \sqrt{(4)^2 + 0 + (6)^2} = \sqrt{52} = 2\sqrt{13}$

F) $= \sqrt{(4)^2 + (-2)^2 + 0} = \sqrt{20} = 2\sqrt{5}$

(13) center $(-3, 2, 5)$, radus $= 4$

$\rightarrow (X+3)^2 + (Y-2)^2 + (Z-5)^2 = 16$

$\rightarrow$ intersection with YZ-plane $\rightarrow X=0$

$9 + (Y-2)^2 + (Z-5)^2 = 16$

$(Y-2)^2 + (Z-5)^2 = 7$

$\rightarrow$ 2D circle that radius $= \sqrt{7}$ and center $(0, 2, 5)$

(14) center $(2, -6, 4)$, radus $= 5$

$\rightarrow (X-2)^2 + (Y+6)^2 + (Z-4)^2 = 25$

$\ast$ intersection with XY-plane: $(Z=0)$

$(X-2)^2 + (Y+6)^2 + 16 = 25$

$(X-2)^2 + (Y+6)^2 = 9$

$\rightarrow$ 2D circle that radius $= 3$ and center $(2, -6, 0)$

$\ast$ intersection with XZ-plane: $(Y=0)$

$(X-2)^2 + 36 + (Z-4)^2 = 25$

$(X-2)^2 + (Z-4)^2 = -11$!!

$\rightarrow$ is not possible

$\ast$ intersection with YZ-plane: $(X=0)$

$4 + (Y+6)^2 + (Z-4)^2 = 25$

$(Y+6)^2 + (Z-4)^2 = 21$

$\rightarrow$ 2D circle that radius $= 3$ and center $(0, -6, 4)$

(15) r is the ~~distance~~ lenghth between $(3,8,1)$ and $(4,3,-1)$

$r = \sqrt{(3-4)^2 + (8-3)^2 + (1+1)^2} = \sqrt{30}$

equ: $(x-3)^2 + (y-8)^2 + (z-1)^2 = 30$

(17)

$x^2 + y^2 + z^2 - 2x - 4y + 8z = 15$

$x^2 - 2x + \underline{1} + y^2 - 4y + \underline{4} + z^2 + 8z + \underline{16} = 15 + 16 + 4 + 1$

$(x-1)^2 + (y-2)^2 + (z+4)^2 = 36$

center: $(1, 2, -4)$, radius $= 6$

(20) $(3x^2 + 3y^2 + 3z^2 = 10 + 6y + 12z) \div 3$

$x^2 + y^2 + z^2 = \frac{10}{3} + 2y + 4z$

$x^2 + y^2 - 2y + \underline{1} + z^2 - 4z + \underline{4} = \frac{10}{3} + 1 + 4$

$x^2 + (y-1)^2 + (z-2)^2 = \frac{25}{3}$

center: $(0, 1, 2)$, radius $= \frac{5}{\sqrt{3}}$

21 $\boxed{a}$

$$P_1 \xleftarrow{\quad s \quad} M \xleftarrow{\quad s \quad} P_2$$

$$(x_1, y_1, z_1) \qquad (a, b, c) \qquad (x_2, y_2, z_2)$$

- $|\overline{P_1P_2}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
- $|s| = \sqrt{(x_2 - a)^2 + (y_2 - b)^2 + (z_2 - c)^2}$
- $|s| = \dfrac{|\overline{P_1P_2}|}{2}$

$$\therefore \sqrt{(x_2-a)^2 + (y_2-b)^2 + (z_2-c)^2} = \frac{1}{2}\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

$$\rightarrow (x_2-a)^2 + (y_2-b)^2 + (z_2-c)^2 = \frac{(x_2-x_1)^2}{4} + \frac{(y_2-y_1)^2}{4} + \frac{(z_2-z_1)^2}{4}$$

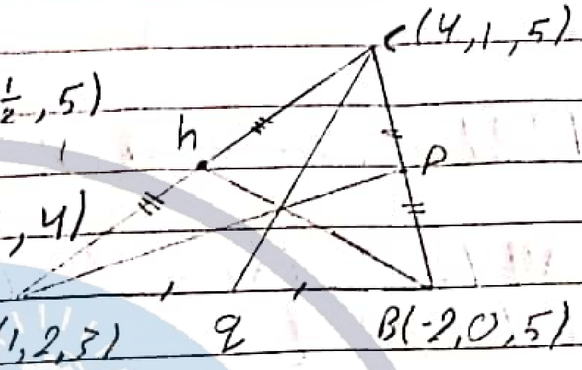
From compare each term of the equation

$(x_2-a)^2 = \dfrac{(x_2-x_1)^2}{4}$	$(y_2-b)^2 = \dfrac{(y_2-y_1)^2}{4}$	$(z_2-c)^2 = \dfrac{(z_2-z_1)^2}{4}$
$(x_2-a) = \dfrac{(x_2-x_1)}{2}$	$(y_2-b) = \dfrac{(y_2-y_1)}{2}$	$(z_2-c) = \dfrac{(z_2-z_1)}{2}$
$a = \dfrac{-x_2}{2} + x_2 + \dfrac{x_1}{2}$	$b = \dfrac{-y_2}{2} + y_2 + \dfrac{y_1}{2}$	$c = \dfrac{-z_2}{2} + z_2 + \dfrac{z_1}{2}$
$a = \dfrac{x_2 + x_1}{2}$	$b = \dfrac{y_2 + y_1}{2}$	$c = \dfrac{z_2 + z_1}{2}$

So

↳ The midpoint of the line segment from $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$ is $\left(\dfrac{x_2 + x_1}{2}, \dfrac{y_2 + y_1}{2}, \dfrac{z_2 + z_1}{2}\right)$

21 [b]



$\ast\ P: \left(\frac{-2+4}{2}, \frac{0+1}{2}, \frac{5+5}{2}\right) \rightarrow \left(1, \frac{1}{2}, 5\right)$

$q: \left(\frac{-2+1}{2}, \frac{0+2}{2}, \frac{5+3}{2}\right) \rightarrow \left(\frac{-1}{2}, 1, 4\right)$

$h: \left(\frac{4+1}{2}, \frac{1+2}{2}, \frac{5+3}{2}\right) \rightarrow \left(\frac{5}{2}, \frac{3}{2}, 4\right)$

$\ast\ |\overline{AP}| = \sqrt{(1-1)^2 + (2-\frac{1}{2})^2 + (3-5)^2} = \frac{5}{2}$

$\ast\ |\overline{Cq}| = \sqrt{(4+\frac{1}{2})^2 + (1-1)^2 + (5-4)^2} = \frac{\sqrt{85}}{2}$

$\ast\ |\overline{Bh}| = \sqrt{(-2-\frac{5}{2})^2 + (0-\frac{3}{2})^2 + (5-4)^2} = \frac{\sqrt{94}}{2}$

22

center : $\left(\frac{1+5}{2}, \frac{6+4}{2}, \frac{-9+3}{2}\right) \rightarrow (3, 5, -3)$

$|r| = \frac{1}{2}\sqrt{(5-1)^2 + (6-4)^2 + (3+9)^2} = \sqrt{41}$

eq $\rightarrow (x-3)^2 + (y-5)^2 + (z+3)^2 = 41$

25. A center $(2, -3, 6)$
radius $= 6$

equ $\rightarrow (X-2)^2 + (Y+3)^2 + (Z-6)^2 = 36$

B center $(2, -3, 6)$
radius $= 2$

equ $\rightarrow (X-2)^2 + (Y+3)^2 + (Z-6)^2 = 4$

C center $(2, -3, 6)$
radius $= 3$

equ $\rightarrow (X-2)^2 + (Y+3)^2 + (Z-6)^2 = 9$

21 Center $(5, 4, 9)$

$$(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = R^2$$

To largest (in the First octant)

$$(x - 5)^2 + (y - 4)^2 + (z - 9)^2 = 16$$

إذا كانت النقطة المختار = 9 أو 5

سوف يخرج عن الثمن الأول والسؤال طالبه أكبر دائرة

لكنه حصرها في الثمن الأول.

* Describe in Words

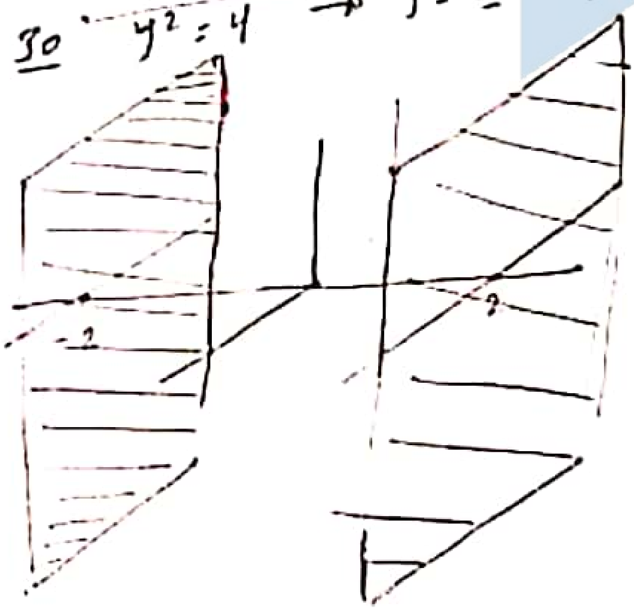
29 $0 \leq z \leq 6 \rightarrow x, y$ لا يوجد قيود على

إذن y و x = جميع النقاط

The inequality $0 \leq z \leq 6$ represents
→ The all points between (or on) $z = 0$ and $z = 6$

30 $y^2 = 4 \rightarrow y = \pm 2$, لا يوجد قيود على

x, z = جميع النقاط



The equation represents
All point on $y = 2, -2$

31.

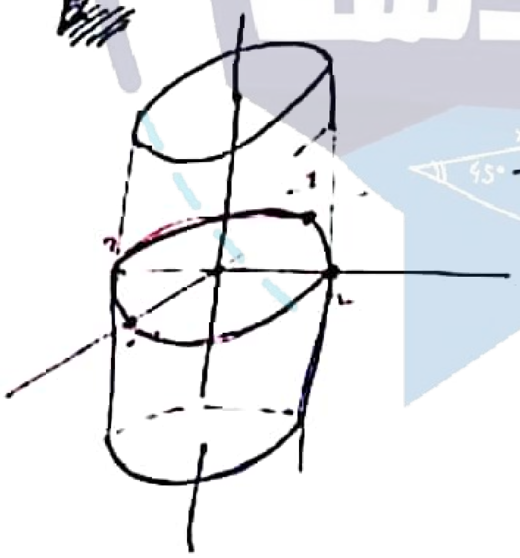
$x^2+y^2=4$, $z=-1$ lying on the plane $z=-1$

$R=2$, $C=(0,0,-1)$

$\Rightarrow$ a circle with center $(0,0,-1)$ and Radius $=2$

32 $x^2+y^2=4 \rightarrow R=2$, $C=(0,0,z)$

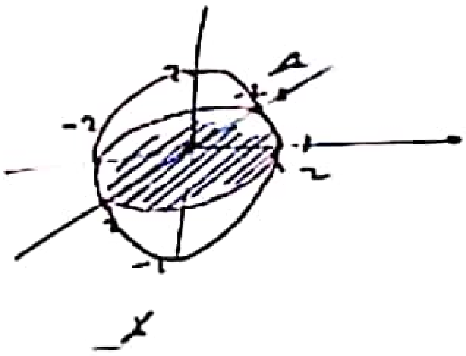
z بدون تقييد



$\Rightarrow$ a infinite cylinder with $R=2$ and z axis as its axis.

33 $x^2 + y^2 + z^2 = 4$

$\rightarrow R = 2, \quad C = (0,0,0)$

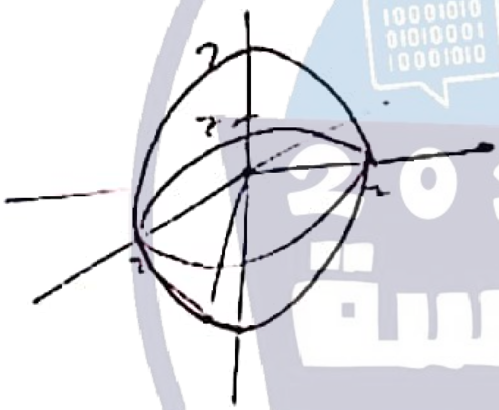


$\rightarrow$ A sphere with center $(0,0,0)$ and Radius $= 2$.

---

34 $x^2 + y^2 + z^2 \leq 4$

$C = (0,0,0) \quad R = 2$.



* A sphere with center $(0,0,0)$ and $R = 2$, and ~~A sphere~~ contains all points on and inside it

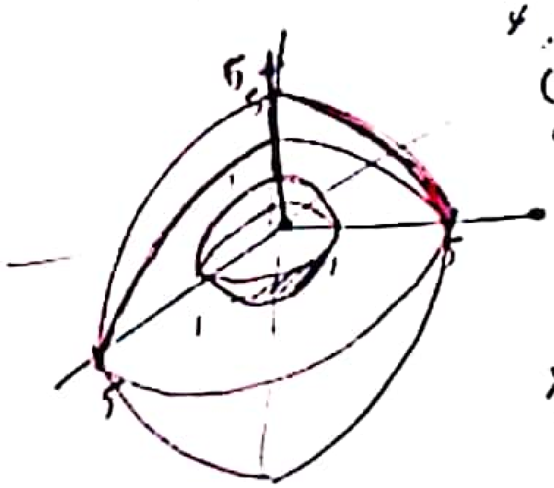
---

35 $1 \leq x^2 + y^2 + z^2 < 5$

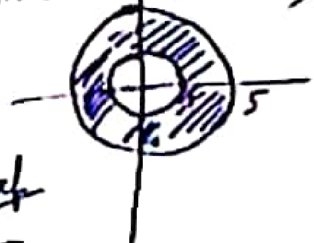
$\rightarrow x^2 + y^2 + z^2 = 5 \rightarrow C(0,0,0) . R = \sqrt{5}$

$x^2 + y^2 + z^2 = 1 \rightarrow C(0,0,0) . R = 1$

* every point between (or on) The two sphere of $R = 1$ and $\sqrt{5}$ and centred at $(0,0,0)$.

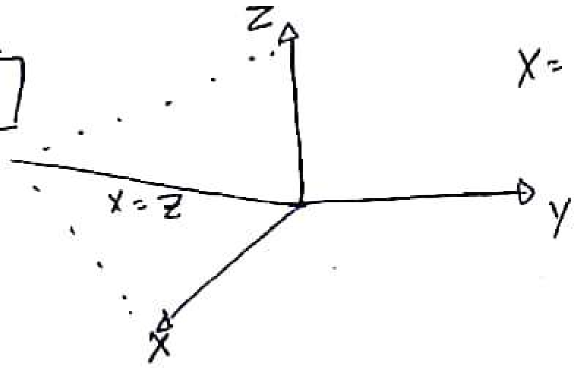


$\rightarrow$



* ~~every point between (or on) the~~

$\boxed{36}$



$x = z$

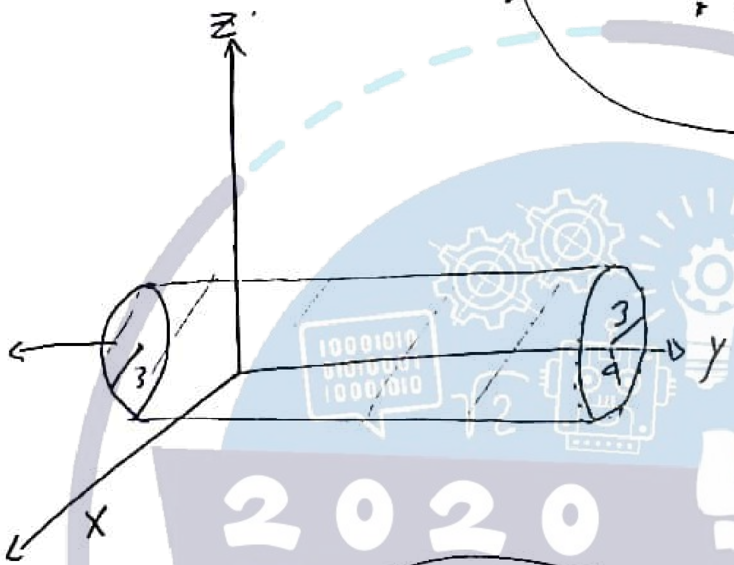
$\boxed{37}$

$x^2 + z^2 \le 4$

$x^2 + z^2 = 4$ (circle)

$r = 3$

cylinder



$\boxed{38}$

$x^2 + y^2 + z^2 > 2z$

$x^2 + y^2 + z^2 - 2z > 0$

$z^2 - 2z$

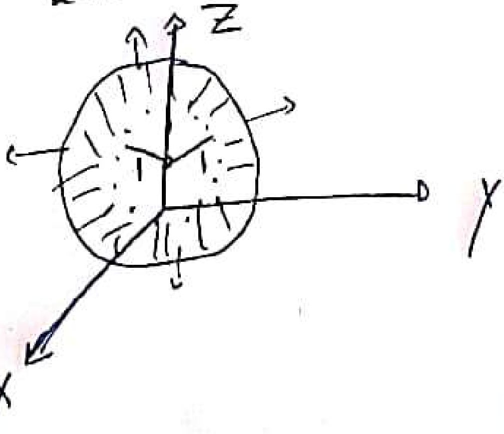
$(z^2 - 2z + 1) - 1$

$(z - 1)^2 - 1$

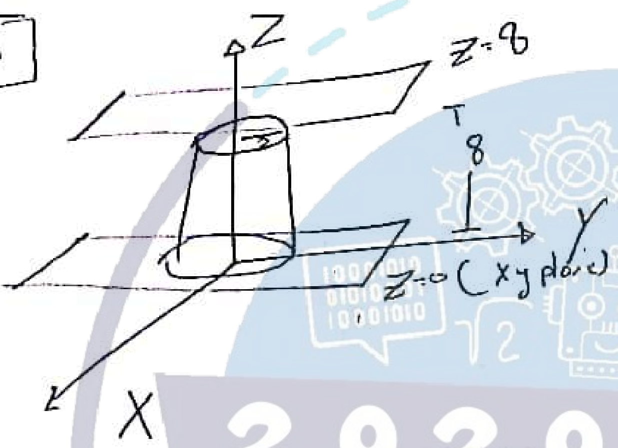
$x^2 + y^2 + (z-1)^2 - 1 > 0$

$x^2 + y^2 + (z-1)^2 > 1$

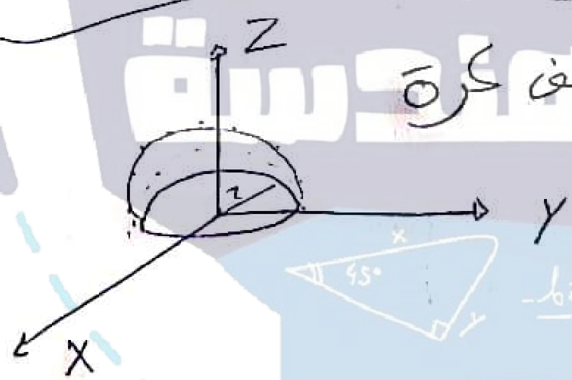
sphere center $(0,0,1)$



40



42



(24) $\boxed{44}$ $A(-1,5,3)$ $P(x,y,z)$

$B(6,2,-2)$

$|AP| = 2|BP|$

$$\sqrt{(x+1)^2+(y-5)^2+(z-3)^2} = 2\sqrt{(x-6)^2+(y-2)^2+(z+2)^2}$$

$$x^2+2x+1+y^2-10y+25+z^2-6z+9 = 4[x^2-12x+36+y^2-4y+4+z^2+4z+4]$$

$$x^2+2x+y^2-10y+z^2-6z+35 = 4x^2-48x+4y^2-16y+4z^2+16z+176$$

$$3x^2-50x+3y^2-6y+3z^2+22z = -141$$

$$x^2-\frac{50}{3}x+y^2-2y+z^2+\frac{22}{3}z = \frac{-141}{3}$$

اكمال مربع لكل متغير

$$\left(x-\frac{25}{3}\right)^2+(y-1)^2+\left(z+\frac{11}{3}\right)^2 = \frac{332}{9}$$

Center $\left(\frac{25}{3}, 1, \frac{-11}{3}\right)$

radius $= \frac{\sqrt{332}}{3}$

$\boxed{45}$ $P(x, y, z)$

$A(-1, 5, 3)$ & $B(6, 2, -2)$

$|PA| = |PB|$

$$\sqrt{(x+1)^2 + (y-5)^2 + (z-3)^2} = \sqrt{(x-6)^2 + (y-2)^2 + (z+2)^2}$$

$$\cancel{x^2} + 2x + 1 + \cancel{y^2} - 10y + 25 + \cancel{z^2} - 6z + 9 = \cancel{x^2} - 12x + 36 + \cancel{y^2} - 4y + 4 + \cancel{z^2} + 4z + 4$$

$$14x + -6y - 10z = 9$$

The equation

Section 12.2

19 $a = \langle -3, 4 \rangle \quad b = \langle 9, -1 \rangle$

* $a + b = \langle -3+9, 4-1 \rangle$

$= \langle 6, 3 \rangle$

* $4a + 2b = \langle -3 \times 4 + 9 \times 2, 4 \times 4 - 1 \times 2 \rangle$

$= \langle -12 + 18, 16 - 2 \rangle$

$= \langle 6, 18 \rangle$

* $|a| = \sqrt{9+16} = 5$

* $a - b = \langle -3 - 9, 4 + 1 \rangle$

$= \langle -12, 5 \rangle$

$|a-b| = \sqrt{144 + 25} = 13$

---

21 $a = \langle 4, -3, 2 \rangle \quad b = \langle 2, 0, -4 \rangle$

* $a + b = \langle 4+2, -3+0, 2-4 \rangle = \langle 6, -3, -2 \rangle$

* $4a + 2b = \langle 4(4) + 2(2), -3(4), 2(4) - 4(2) \rangle$

$= \langle 16 + 4, -12, 8 - 8 \rangle = \langle 20, -12, 0 \rangle$

* $|a| = \sqrt{16 + 9 + 4} = \sqrt{29}$

* $a - b = \langle 4 - 2, -3, 2 + 4 \rangle = \langle 2, -3, 6 \rangle$

$|a-b| = \sqrt{4 + 9 + 36} = 7$

25)
$\vec{a} = 8\hat{i} - \hat{j} + 4\hat{k}$

$= \langle 8, -1, 4 \rangle$

$|\vec{a}| = \sqrt{64 + 1 + 16} = 9$

$\vec{U} = \dfrac{\vec{a}}{|a|} = \dfrac{1}{9} \langle 8, -1, 4 \rangle$

$= \langle \frac{8}{9}, \frac{-1}{9}, \frac{4}{9} \rangle$

---

26)
$\vec{a} = \langle 6, 2, -3 \rangle$

$|\vec{a}| = \sqrt{36 + 4 + 9} = 7$

$\vec{U} = \dfrac{\vec{a}}{|a|} = \dfrac{1}{7} \langle 6, 2, -3 \rangle$

$= \langle \frac{6}{7}, \frac{2}{7}, \frac{-3}{7} \rangle$

$\vec{b} = (4)\vec{U} = (4) \langle \frac{6}{7}, \frac{2}{7}, \frac{-3}{7} \rangle$

$= \langle \frac{24}{7}, \frac{8}{7}, \frac{-12}{7} \rangle$

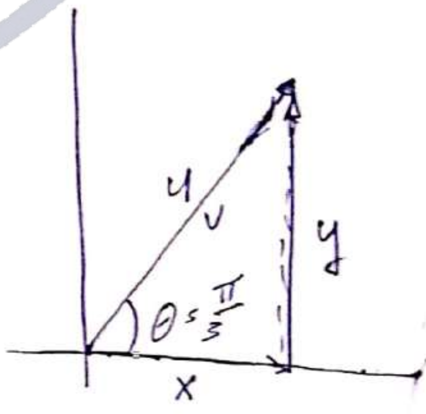
---

29)
$x = V \cos\theta$

$y = V \sin\theta$

$\rightarrow x = 4 \cos \frac{\pi}{3} = 4(\frac{1}{2}) = 2$

$y = 4 \sin \frac{\pi}{3} = 4 \frac{\sqrt{3}}{2} = 2\sqrt{3}$



$V = \langle 2, 2\sqrt{3} \rangle$

$\boxed{41}$

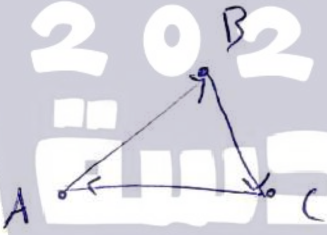
$$f(x) = x^2$$

$$f'(x) = 2x \longrightarrow f'(2) = 4$$

Direction of the tangent line is $\vec{a} = \pm(1,4)$

$$\hat{a} = \frac{\vec{a}}{|a|} = \pm\left(\frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}}\right)$$

$\boxed{43}$



Find $\vec{AB} + \vec{BC} + \vec{CA}$

$$\vec{AB} + \vec{BC} = \vec{AC}$$

So $\vec{AC} + \vec{CA} = 0$

Section
12.3

Section 12.3

$\boxed{1}$ a) number · c = X

b) ✓

c) ✓

d) ✓

e) X

F) X

$\boxed{2}$ $a = \langle 5, -2 \rangle$, $b = \langle 3, 4 \rangle$

find a.b?

$a \cdot b = \langle 5, -2 \rangle \cdot \langle 3, 4 \rangle = 15 - 8 = \boxed{7}$

$\boxed{8}$ $a = 3i + 2j - k$, $b = 4i + 5k$

find a.b? $a \cdot b = 12 + 0 - 5 = \boxed{7}$

$\boxed{10}$ $|a| = 80$, $|b| = 50$, $\theta = \frac{3\pi}{4}$

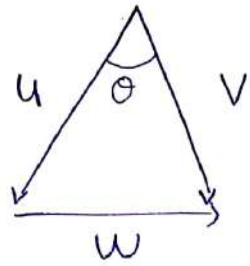
find a.b?

$a \cdot b = |a||b| \cos\theta = 80 * 50 * \cos \frac{3\pi}{4}$

$= 4000 * \frac{-1}{\sqrt{2}}$

$= \frac{2000 * \sqrt{2} * \sqrt{2}}{-\sqrt{2}} = -2000\sqrt{2}$

11



If u is a unit vector find $u \cdot v$ and $u \cdot w$

$|u| = |v| = |w|$ unit vectors

So $\theta = \frac{\pi}{3}$

$u \cdot v = |u| \cdot |v| \cdot \cos\theta$

$= 1 * 1 * \frac{1}{2} = \underline{\frac{1}{2}}$

$u \cdot w = |u| \cdot |w| \cdot \cos\theta = \underline{\frac{1}{2}}$

---

15 Find θ between vectors

$a = \langle 4, 3 \rangle, \quad b = \langle 2, -1 \rangle$

$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|a||b|} = \frac{8-3}{5 * \sqrt{5}} = \frac{5}{5\sqrt{5}} = \frac{1}{\sqrt{5}}$

$\theta = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right) \ldots$

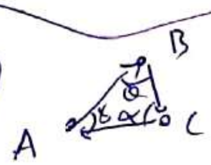
---

14 Find θ between vectors

$a = 4i - 3j + k, \quad b = 2i - k$

$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|a||b|}\right) = \cos^{-1}\left(\frac{8+0-1}{\sqrt{26} * \sqrt{5}}\right) = \cos^{-1}\left(\frac{7}{\sqrt{130}}\right)$

22



$\overline{AB} = \langle 2, -2, 1 \rangle$

$\overline{BC} = \langle -2, 5, 3 \rangle$

$\overline{CA} = \langle 0, 3, 4 \rangle$

* $\overline{AB} \cdot \overline{BC} = |AB| \cdot |BC| \cos\theta$

$\cos\theta = \frac{-4 - 10 + 3}{3 * \sqrt{38}} \Rightarrow \theta = \cos^{-1}\left(\frac{-11}{3\sqrt{38}}\right)$

* $\overline{BC} \cdot \overline{CA} = |BC||CA| \cos\alpha$

$\alpha = \cos^{-1}\left(\frac{0 + 15 + 12}{5 * \sqrt{38}}\right) \ldots$

$\gamma = 180 - (\theta + \alpha)$

24/a $\rightarrow u = \langle -5, 4, -2 \rangle$

$v = \langle 3, 4, -1 \rangle$

$\vec{u} \cdot \vec{v} = |\vec{u}|\,|\vec{v}| \cos\theta_{uv}$

$\rightarrow |\vec{v}| = \sqrt{9 + 16 + 1} = \sqrt{26}$

$\rightarrow |\vec{u}| = \sqrt{25 + 16 + 4} = \sqrt{45}$

$\rightarrow \vec{u} \cdot \vec{v} = -15 + 16 + 2 = 3$

$\Rightarrow 3 = \sqrt{45} \cdot \sqrt{26} \cdot \cos\theta_{uv}$

$\cos\theta_{uv} = 3 / \sqrt{45} \cdot \sqrt{26} \rightarrow \cos\theta = 0.08$

$\theta = 85.4^\circ \rightarrow$ The vectors are neither parallel or orthogonal.

24/b $\rightarrow u = 9i - 6j + 3k \rightarrow \langle 9, -6, 3 \rangle$

$V = -6i + 4j - 2k \rightarrow \langle -6, 4, -2 \rangle$

$\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cos\theta_{uv}$

$\rightarrow |\vec{v}| = \sqrt{36 + 16 + 4} = \sqrt{56}$

$\rightarrow |\vec{u}| = \sqrt{81 + 36 + 9} = \sqrt{126}$

$\rightarrow \vec{u} \cdot \vec{v} = -54 + -24 + -6 = -84$.

$-84 = \sqrt{56} \cdot \sqrt{126} \cos\theta_{uv}$

$\rightarrow \cos\theta = -1 \rightarrow \theta = 180^\circ \rightarrow$ The vectors are parallel.

1

24/c $\to u = \langle c, c, c\rangle$

$v = \langle c, 0, -c\rangle$

$\Rightarrow \vec{u}\cdot\vec{v} = |\vec{u}|\cdot|\vec{v}| \cos\theta_{uv}$

$\to |\vec{v}| = \sqrt{c^2+c^2} = \sqrt{2}\,c$

$\to |\vec{u}| = \sqrt{c^2+c^2+c^2} = \sqrt{3}\,c$

$\to \vec{u}\cdot\vec{v} = c^2 + 0 - c^2 = 0$

$\Rightarrow 0 = \sqrt{2}\,c \times \sqrt{3}\,c \cos\theta_{uv}$

$\cos\theta_{uv} = 0 \longrightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\to$ The vectors are orthogonal.

but $0 \leq \theta \leq \pi$

So, $\theta = \frac{\pi}{2}$

---

25 $P = (1, -3, -2)$, $Q = (2, 0, -4)$

$R = (6, -2, -5)$

$\to \vec{PQ} = \langle 2-1, 0--3, -4--2\rangle$

$= \langle 1, 3, -2\rangle$

$\to \vec{QR} = \langle 6-2, -2-0, -5+4\rangle$

$= \langle 4, -2, -1\rangle$

$\to \vec{RP} = \langle 1-6, -3+2, -2+5\rangle$

$= \langle -5, -1, 3\rangle$

$\vec{PQ}\cdot\vec{QR} = 4 - 6 + 2 = 0$

$\vec{RQ}\cdot\vec{PQ} = |\vec{RQ}||\vec{PQ}| \cos\theta_Q$

$0 = |\vec{RQ}|\,|\vec{PQ}| \cos\theta$

$\cos\theta = 0 \to \theta = \frac{\pi}{2}$.

So, The triangle is a Right triangle

26 - Values of x → $\theta = \frac{\pi}{4}$

$u = \langle 2, 1, -1 \rangle$, $v = \langle 1, x, 0 \rangle$

→ $\vec{u} \cdot \vec{v} = |\vec{u}| . |\vec{v}| \cos\theta_{uv}$

→ $|\vec{v}| = \sqrt{1 + x^2}$

→ $|\vec{u}| = \sqrt{4 + 1 + 1} = \sqrt{6}$

→ $\vec{u} \cdot \vec{v} = 2 + x$

⇒ $2 + x = \sqrt{6} \cdot \sqrt{1 + x^2} \cos\frac{\pi}{4}$

$(2 + x)^2 = \left(\sqrt{6} \cdot \sqrt{1 + x^2} \cdot \frac{1}{\sqrt{2}}\right)^2$

$(2 + x)^2 = \frac{\cancel{6}^{\,3} (1 + x^2)}{\cancel{2}_{\,1}}$

$\cancel{4}^{\,1} + 4x + \cancel{x^2} = \cancel{3} + 3x^2$ →

$2x^2 - 4x - 1 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

→ $x = 2.22$ or -0.22

27 - $u = i + j \rightarrow \langle 1,1,0 \rangle$

$v = i + k \rightarrow \langle 1,0,1 \rangle$

السؤال طالب إيجاد unit vector عمودي مع هذه المتجهات.

---

k is a unit vector:

$I \rightarrow i + j + k \rightarrow \langle x, y, z \rangle$

$\vec{u} \perp \vec{k}$

orthogonal $\rightarrow \cos\frac{\pi}{2} = 0$

$\vec{u} \cdot \vec{k} = 0 \rightarrow 1 \times x + 1 \times y + 0 \times z = 0$

$x + y = 0 \rightarrow x = -y$

$\vec{k} \cdot \vec{v} = 0 \rightarrow 1 \times x + 0 \times y + 1 \times z = 0$

$x + z = 0 \rightarrow x = -z$

$v = \langle x, y, z \rangle \rightarrow \langle -z, z, z \rangle$

$x = -z$

$x = -y$

$z = y$

$|\vec{v}| = 1 \rightarrow \sqrt{z^2 + z^2 + z^2} = 1$

unit vector

$\sqrt{3}\, z = 1 \rightarrow z = \frac{1}{\sqrt{3}}$.

so, $v = \langle \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$

or $\langle \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \rangle$

---

28 -

unit vector $\rightarrow |\vec{u}| = 1$, $\theta = 60°$, $\vec{v} = \langle 3, 4 \rangle$.

$\vec{v} \cdot \vec{u} = |\vec{u}|\,|\vec{v}| \cos\theta$ $\;\longrightarrow |\vec{v}| = \sqrt{9+16} = 5$

$= 5 \times 1 \cos 60 \rightarrow \frac{5}{2}$, $\vec{u} = \langle x, y \rangle$

$u = 1 = x^2 + y^2$

$\vec{v} \cdot \vec{u} = 3x + 4y$

$x^2 + y^2 = 1$

$y^2 = 1 - x^2$

$\rightarrow 3x + 4y = \frac{5}{2} \rightarrow 6x + 8y = 5$

$y = \pm\sqrt{1 - x^2}$

$3x + 4y = \frac{5}{2}$

$6x + 8y = 5$

$y = \frac{(5 - 6x)}{8}$

$6x + 8\left(\pm\sqrt{1 - x^2}\right) = 5$

$\left(8\left(\pm\sqrt{1 - x^2}\right)\right)^2 = (5 - 6x)^2$

$64(1 - x^2) = (5 - 6x)^2$

$\rightarrow 64 - 64x^2 = 25 - 60x + 36x^2$

$\rightarrow 100x^2 - 60x - 39 = 0$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = 0.99 , -0.39$.

$y = \frac{(5 - 6x)}{8}$ If $x = -0.39 \rightarrow y = 0.91$

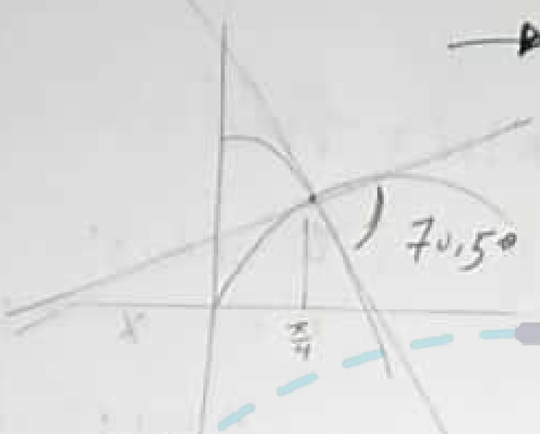
If $x = 0.99 \rightarrow y = -0.11$.

two unit vectors are 1- $\langle 0.99, -0.11 \rangle$

2- $\langle -0.39, 0.91 \rangle$.

32

$F(x) = \sin x \quad , \quad g(x) = \cos x$



$\rightarrow \sin x = \cos x$ at $x = \frac{\pi}{4}$.

$\text{slope} = \frac{dF(x)}{dx} = \frac{\Delta y}{\Delta x}$

$\rightarrow F(x) = \sin x \rightarrow F'(x) = \cos x \Big|_{x=\frac{\pi}{4}} = \frac{1}{\sqrt{2}} = \frac{\Delta y}{\Delta x}$

$\rightarrow x = \sqrt{2} , y = 1$

as a vector $a = \langle \sqrt{2} , 1 \rangle$

$\rightarrow g(x) = \cos x \rightarrow g'(x) = -\sin x \Big|_{x=\frac{\pi}{4}}$

$g'(x) = \frac{-1}{\sqrt{2}} = \frac{\Delta y}{\Delta x} \rightarrow x = \sqrt{2} , y = -1$

as a vector $b = \langle \sqrt{2} , -1 \rangle$

$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta_{ab}$

$\rightarrow |\vec{b}| = \sqrt{2+1} = \sqrt{3}$

$\rightarrow |\vec{a}| = \sqrt{2+1} = \sqrt{3}$

$\rightarrow \vec{a} \cdot \vec{b} = \sqrt{2} \times \sqrt{2} + -1 \times 1 = 2 - 1 = 1$

$\rightarrow 1 = \sqrt{3} \times \sqrt{3} \cos\theta_{ab}$.

$\cos\theta_{ab} = \frac{1}{3} \rightarrow \theta = 70.5°$

34- $u = \langle 6, 3, -2 \rangle$

$$|\vec{u}| = \sqrt{36+9+4} = 7$$

$$\cos\alpha = \frac{i \cdot u}{|i|\,|\vec{u}|} = \frac{u_x}{|\vec{u}|} = \frac{6}{7}$$

$$\cos^{-1}\alpha = 31^\circ$$

$$\cos\beta = \frac{j \cdot u}{|j|\,|\vec{u}|} = \frac{u_y}{|\vec{u}|} = \frac{3}{7}$$

$$\cos^{-1}\beta = 64.6^\circ$$

$$\cos\gamma = \frac{k \cdot u}{|k|\,|\vec{u}|} = \frac{u_z}{|\vec{u}|} = \frac{-2}{7}$$

$$\cos^{-1}\gamma = 106.6^\circ$$

→ The direction cosines are:-

$\cos\alpha = \frac{6}{7}$, $\cos\beta = \frac{3}{7}$, $\cos\gamma = \frac{-2}{7}$

→ The direction angles are:-

$\alpha = 31^\circ$, $\beta = 64.6^\circ = 65^\circ$, $\gamma = 106.6^\circ = 107^\circ$

36- $u = \frac{1}{2} i + j + k \rightarrow \langle \frac{1}{2}, 1, 1 \rangle$.

$$|\vec{u}| = \sqrt{\frac{1}{4} + 1 + 1} = \frac{3}{2}$$

$$\cos\alpha = \frac{u_x}{|\vec{u}|} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$$

$$\cos^{-1}\alpha = 70.5^\circ$$

$$\cos\beta = \frac{v_y}{|\vec{v}|} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

$$\cos^{-1}\beta = 48.1^\circ$$

$$\cos\gamma = \frac{v_z}{|\vec{u}|} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

$$\cos^{-1}\gamma = 48.1^\circ$$

$\rightarrow$ The direction cosines are:-

$\cos\alpha = \frac{1}{3}$, $\cos\beta = \cos\gamma = \frac{2}{3}$

$\rightarrow$ The direction angles are:-

$\alpha = 70.5^\circ$ $\quad$ $\beta = \gamma = 48.1^\circ$

$= 71^\circ$ $\quad$ $= 48^\circ$

37- $\langle c, c, c \rangle$ where $c > 0$

$|\vec{u}| = \sqrt{c^2+c^2+c^2} \rightarrow |\vec{u}| = \sqrt{3}\,c$

$\cos\alpha = \frac{u_x}{|\vec{u}|} = \frac{\cancel{c}}{\sqrt{3}\,\cancel{c}} = \frac{1}{\sqrt{3}}$

$\cos\beta = \frac{u_y}{|\vec{u}|} = \frac{c}{\sqrt{3}\,c} = \frac{1}{\sqrt{3}}$

$\cos\gamma = \frac{u_z}{|\vec{u}|} = \frac{c}{\sqrt{3}\,c} = \frac{1}{\sqrt{3}}$

$\cancel{\cos\beta}$ $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.7^\circ$

$\rightarrow$ The direction cosines are:-

$\cos\alpha = \cos\beta = \cos\gamma = \frac{1}{\sqrt{3}}$

$\rightarrow$ The direction angles are:-

$\alpha = \beta = \gamma = 54.7^\circ \rightarrow 55^\circ$

38- $\alpha = \frac{\pi}{4}$, $\beta = \frac{\pi}{3}$ $\rightarrow$ find γ?

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\rightarrow \cos^2\frac{\pi}{4} + \cos^2\frac{\pi}{3} + \cos^2\gamma = 1$

$\frac{2 \times 1}{2 \times 2} + \frac{1}{4} + \cos^2\gamma = 1$

$\frac{3}{4} + \cos^2\gamma = 1$

$\cos^2\gamma = \frac{1}{4} \rightarrow \cos\gamma = \pm\frac{1}{2} \rightarrow \gamma = \frac{\pi}{3}$ or $\frac{2\pi}{3}$.

40 $a = \langle 1,4 \rangle \quad b = \langle 2,3 \rangle$

$\vec{a}\cdot\vec{b} = 2 + 12 = 14$

$|a| = \sqrt{1^2+4^2} = \sqrt{17}$

$$\text{Comp}_{\vec{a}}\vec{b} = \frac{a.b}{|a|} = \frac{14}{\sqrt{17}}$$

$$\text{proj}_{\vec{a}}\vec{b} = \frac{a.b}{|a|^2}\vec{a} = \frac{14}{17}\langle 1,4 \rangle$$

$$= \left\langle \frac{14}{17}, \frac{56}{17} \right\rangle$$

---

43 $a = \langle 3,-3,1 \rangle$

$b = \langle 2, 4, -1 \rangle$

$a.b = 6 + (-12) + (-1)$

$= -7$

$|a| = \sqrt{3^2+(-3)^2+1^2} = \sqrt{19}$

$$\text{Comp}_{a}\vec{b} = \frac{\vec{a}.\vec{b}}{|a|} = \frac{-7}{\sqrt{19}}$$

$$\text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a}.\vec{b}}{|a|^2}\vec{a} = \frac{-7}{19}\langle 3,-3,1 \rangle$$

$$= \left\langle \frac{-21}{19}, \frac{21}{19}, \frac{-7}{19} \right\rangle$$

---

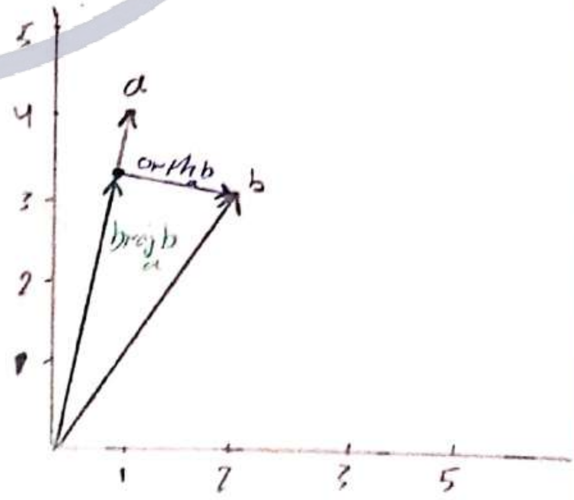
46 $a = \langle 1,4 \rangle \quad b = \langle 2,3 \rangle$

$\text{proj}_{\vec{a}}\vec{b} = \left\langle \frac{14}{17}, \frac{56}{17} \right\rangle$

$$\text{orth}_{\vec{a}}\vec{b} = \vec{b} - \text{proj}_{\vec{a}}\vec{b}$$

$$= \langle 2,3 \rangle - \left\langle \frac{14}{17}, \frac{56}{17} \right\rangle$$

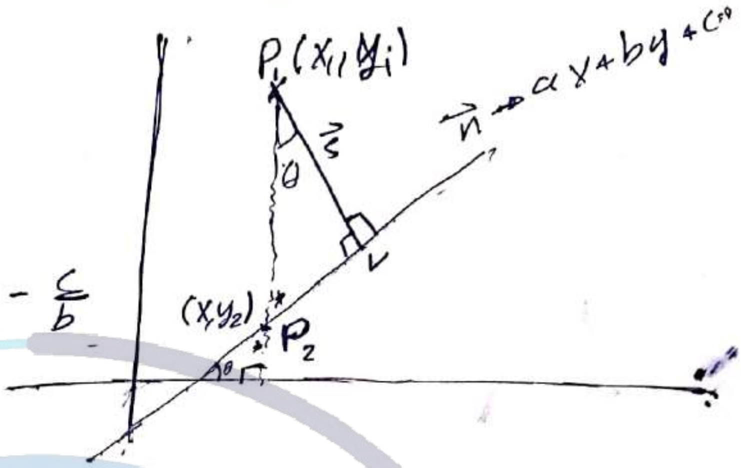
$$= \left\langle \frac{20}{17}, \frac{-5}{17} \right\rangle$$



53

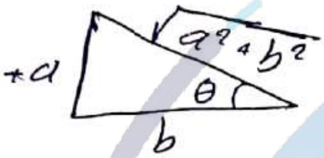
$$\cos\theta = \frac{\overline{P_1V}}{\overline{P_1P_2}}$$

$$\overline{P_1V} = \overline{P_1P_2}\cos\theta$$



$\rightarrow ax+by+c=0 \rightarrow y = \frac{-ax}{b} - \frac{c}{b}$

$\therefore$ slope $= \frac{-a}{b} = \tan\theta$



$$\rightarrow \boxed{\cos\theta = \frac{b}{\sqrt{a^2+b^2}}}$$

$\overline{P_1P_2} = y_1 - y_2$

$y_2 = \frac{-ax_1}{b} - \frac{c}{b}$

$$\boxed{\overline{P_1P_2} = y_1 + \frac{ax_1}{b} + \frac{c}{b}}$$

$$\overline{P_1V} = \frac{b}{\sqrt{a^2+b^2}}\left(y_1 + \frac{ax_1+c}{b}\right)$$

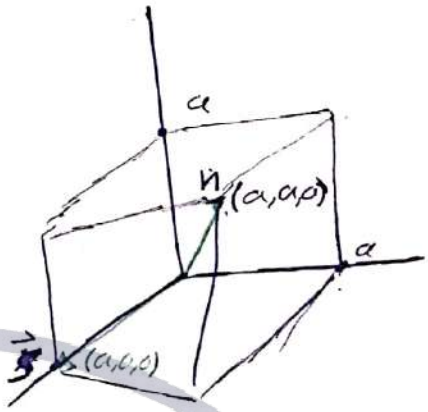
$$\overline{P_1V} = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} \quad \#$$

$P(-2,3) \quad 3x-4y+5=0$

$$\overline{PV} = \frac{|3(-2)-4(3)+5|}{\sqrt{3^2+4^2}} = \frac{|-13|}{5} = \frac{13}{5}$$

$\boxed{55}$ $\vec{n} = \langle a, a, a \rangle$

$\vec{S} = \langle a, 0, 0 \rangle$



$$\cos\theta = \frac{\vec{n} \cdot \vec{S}}{|\vec{n}||\vec{S}|}$$

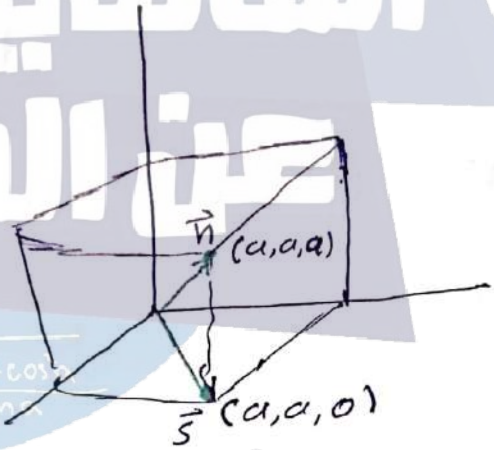
$$\cos\theta = \frac{\langle a, a, a \rangle \cdot \langle a, 0, 0 \rangle}{\sqrt{a^2+a^2+a^2} \times \sqrt{a^2}}$$

$$\cos\theta = \frac{\cancel{a^2}}{\cancel{a^2}\sqrt{3}} = \frac{1}{\sqrt{3}} \rightarrow \theta = \cos^{-1}\left[\frac{1}{\sqrt{3}}\right]$$

$$\theta = 54.74°$$

$\boxed{56}$ $\vec{n} \langle a, a, a \rangle$

$\vec{S} \langle a, a, 0 \rangle$



$$\cos\theta = \frac{\vec{n} \cdot \vec{S}}{|\vec{n}||\vec{S}|}$$

$$\cos\theta = \frac{\langle a, a, a \rangle \cdot \langle a, a, 0 \rangle}{\sqrt{a^2+a^2+a^2} \cdot \sqrt{a^2+a^2}}$$

$$\cos\theta = \frac{a^2 + a^2 + 0}{a\sqrt{3} \times a\sqrt{2}}$$

$$\cos\theta = \frac{2\cancel{a^2}}{\cancel{a^2}\sqrt{6}} = \frac{2}{\sqrt{6}} \rightarrow \theta = \cos^{-1}\left[\frac{2}{\sqrt{6}}\right]$$

$$\theta = 35.26°$$

$\boxed{64}$ $\because (u - v) \perp (u + v)$

$\therefore (u + v) \cdot (u - v) = 0$

$\rightarrow u \cdot u - \cancel{u \cdot v} + \cancel{u \cdot v} - v \cdot v = 0$

$|u|^2 = |v|^2$

So $|u| = |v|$

Section
12.4

12.4

1 $a = \langle 2, 3, 0 \rangle$

$b = \langle 1, 0, 5 \rangle$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 1 & 0 & 5 \end{vmatrix} = (3 \times 5 - 0)\hat{i} - (2 \times 5 - 0)\hat{j} + (0 - 3 \times 1)\hat{k}$$

$$= \boxed{15\hat{i} - 10\hat{j} - 3\hat{k}}$$

---

4 $a = 3\hat{i} + 3\hat{j} - 3\hat{k} = \langle 3, 3, -3 \rangle$

$b = 3\hat{i} - 3\hat{j} + 3\hat{k} = \langle 3, -3, 3 \rangle$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & -3 \\ 3 & -3 & 3 \end{vmatrix} = (9 - 9)\hat{i} - (9 + 9)\hat{j} + (-9 - 9)\hat{k}$$

$$= -81\hat{j} - 81\hat{k}$$

13 (a) $a\cdot(b\times c)$ ✓ → scalar ~~vector~~

(b) $a\times(b\cdot c)$ ✗ [Vector × scalar]

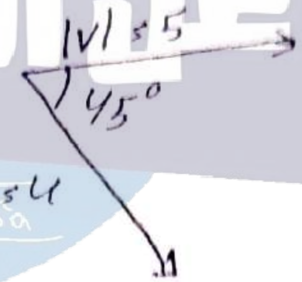
(c) $a\times(b\times c)$ ✓ → vector

(d) $a\cdot(b\cdot c)$ ✗ [Vector · scalar]

(e) $(a\cdot b)\times(c\cdot d)$ ✗ [scalar × scalar]

(f) $(a\times b)\cdot(c\times d)$ ✓ → scalar

14 $\vec{u}\times\vec{v} = |u||v|\sin\theta$



$= (4)(5)(\sin 45^\circ)$

$= 10\sqrt{2}$ out of the page

$\boxed{19}$ $a = \langle 3, 2, 1 \rangle$

$b = \langle -1, 1, 0 \rangle$

find two <u>unite</u> vectors orthogonal to (a) and (b)

$a \times b$

$a \times b = (a_2 b_3 - a_3 b_2) i + (a_3 b_1 - a_1 b_3) j + (a_1 b_2 - a_2 b_1) k$

$a \times b = (2*0 - 1*1) i + (1*-1 - 3*0) j + (3*1 - 2*-1) k$

$a \times b = -i - j + 5k$

$\vec{C} = -i - j + 5k$ as a vector, but question needs unite vector

$u = \frac{\vec{c}}{|\vec{c}|} = \frac{-i - j + 5k}{3\sqrt{3}}$

$|\vec{c}| = \sqrt{(-1)^2 + (-1)^2 + (5)^2} = \sqrt{27} = 3\sqrt{3}$

$u_1 = \langle \frac{-1}{3\sqrt{3}}, \frac{-1}{3\sqrt{3}}, \frac{5}{3\sqrt{3}} \rangle$

$\underline{\underline{\times -1}}$

$u_2 = \langle \frac{1}{3\sqrt{3}}, \frac{1}{3\sqrt{3}}, \frac{-5}{3\sqrt{3}} \rangle$

$\boxed{20}$ $\vec{a} = j - k$ find 2 unit vectors

$\vec{b} = i + j$ orthogonal to $\vec{a}$ and $\vec{b}$

$\vec{a} \times \vec{b} = \begin{vmatrix} \overset{+}{0} & \overset{-}{1} & \overset{+}{-1} \\ 1 & 1 & 0 \end{vmatrix}$

$\vec{c} = +i - j - k$

$|\vec{c}| = \sqrt{3}$

$u_1 = \langle \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \rangle$

$u_2 = \langle \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$

22 Show that $(a \times b) \cdot b = 0$

$$b \cdot (a \times b) = \begin{vmatrix} \overset{+}{b_1} & \overset{-}{b_2} & \overset{+}{b_3} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= b_1 \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - b_2 \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + b_3 \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$= b_1(a_2b_3 - a_3b_2) - b_2(a_1b_3 - a_3b_1) + b_3(a_1b_2 - a_2b_1)$$

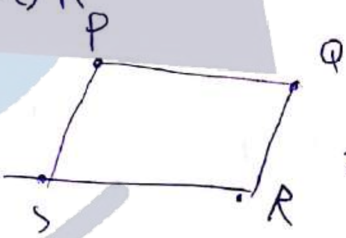
$$= \cancel{b_1a_2b_3} - \cancel{b_1a_3b_2} - \cancel{b_2a_1b_3} + \cancel{b_2a_3b_1} + \cancel{b_3a_1b_2} - \cancel{b_3a_2b_1}$$

$$= 0$$

28 Find the area of the parallelogram with vertices ~~A(-3,0)~~ $P(1,0,2)$, $Q(3,3,3)$, $R(7,5,8)$, $S(5,2,7)$

$$-\ \overrightarrow{PQ} = (3-1)i + (3-0)j + (3-2)k$$

$$\overrightarrow{PQ} = 2i + 3j + k$$



رسمة افتراضية

$$-\ \overrightarrow{QR} = 4i + 2j + 5k$$

$$\text{Area} = |\overrightarrow{PQ} \times \overrightarrow{QR}|$$

$$= 13i - 6j - 8k$$

$$\text{Area} = \sqrt{13^2 + 6^2 + 8^2} = \sqrt{269}$$

$\boxed{29}$ $P(1,0,1)$, $Q(-2,1,3)$, $R(4,2,5)$

a) find orthogonal vector

$$\overrightarrow{PQ} = (-3i + 1j + 2k)$$

$$\overrightarrow{QR} = (6i + 1j + 2k)$$

$$\underset{a}{\overrightarrow{PQ}} \times \underset{b}{\overrightarrow{QR}} = (a_2b_3 - a_3b_2)\,i + (a_3b_1 - a_1b_3)\,j + (a_1b_2 - a_2b_1)\,k$$

$$= (1*2 - 2*1)\,i + (2*6 - -3*2)\,j + (-3*1 - 1*6)\,k$$

$$\vec{C} = 18j - 9k$$

b) $\text{Area} = \frac{1}{2}\left|\vec{A} \times \vec{B}\right|$

$$= \frac{1}{2}\sqrt{0^2 + 18^2 + (-9)^2} = \frac{9\sqrt{5}}{2}$$

$\boxed{34}$ Find the volume of the parallelepiped

$a = i + j \quad , \quad b = j + k \quad , \quad c = i + j + k$

$$\text{Volume} = |a \cdot (b \times c)|$$

$$= \left| \begin{array}{ccc} + & - & + \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right|$$

$$= |1(0) - 1(-1) + 0(-1)|$$

$$= |1| = 1$$

$\boxed{35}$ PQ, PR, PS edges of parallelepiped
find volume.

$P(-2, 1, 0)$

$Q(2, 3, 2)$

$R(1, 4, -1)$

$S(3, 6, 1)$

* $\vec{PQ} = 4i + 2j + 2k$
* $\vec{PR} = 3i + 3j - k$
* $\vec{PS} = 2i + 2j + 2k$

$$\text{Volume} = |\vec{PQ} \cdot (\vec{PR} \times \vec{PS})|$$

$$= \left| \begin{array}{ccc} + & - & + \\ 4 & 2 & 2 \\ 3 & 3 & -1 \\ 2 & 2 & 2 \end{array} \right|$$

$$= 4(8) - 2(8) + 2(0)$$

$$= 32 - 16 = \boxed{16}$$

$\boxed{37}$ Coplanar vectors means that (على نفس السطح) $\vec{a}\cdot(\vec{B}\times\vec{C})=0$

Because $a(\vec{B}\times\vec{C}) \perp \vec{a}$

So $\theta = 90$

and $\vec{a}\cdot(\vec{B}\times\vec{C}) = |a|\cdot|B\times C|\cdot\cos\theta$

$(\cos 90 = 0)$

So $= 0$

$a = i + 5j - 2K$

$b = 3i - j$ are they coplanar??

$C = 5i + 4j - 4K$

$\vec{a}\cdot(\vec{B}\times\vec{C}) = ??$

$$= \begin{vmatrix} 1 & 5 & -2 \\ 3 & -1 & 0 \\ 5 & 9 & -4 \end{vmatrix}$$

$= 1(4-0) - 5(-12-0) - 2(27+5)$

$= 4 + 60 - 2 * 32$

$= 64 - 64 = \underline{\underline{0}}$ So they are coplanar

$\boxed{38}$ $A(1,3,2)$, $B(3,-1,6)$, $C(5,2,0)$

$D(3,6,-4)$

$\overrightarrow{AB} \cdot (\overrightarrow{BC} \times \overrightarrow{CD}) = ??$

$$= \begin{array}{ccc} + & - & + \end{array}$$
$$\begin{vmatrix} 2 & -4 & 4 \\ 2 & 3 & -6 \\ -2 & 4 & -4 \end{vmatrix}$$

$= 2(-12+24) + 4(-8-12) + 4(8+6)$

$= 24 - 80 + 56 = \underline{0}$

So they are coplanar

$\boxed{43}$ $a \cdot b = \sqrt{3}$ Find θ between a and b

$a \times b = \langle 1,2,2 \rangle$

$\rightarrow a \cdot b = \sqrt{3}$

$|a||b|\cos\theta = \sqrt{3}$

$\rightarrow a \times b = \langle 1,2,2 \rangle$

$|a||b|\sin\theta = \sqrt{9} = 3$

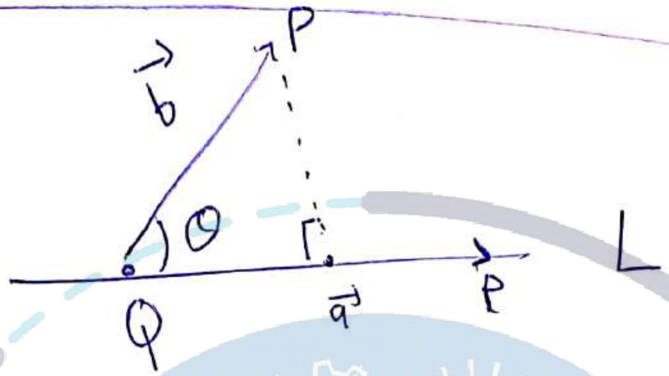
$$\frac{\cancel{|a|}\,\cancel{|b|}\cos\theta}{\cancel{|a|}\,\cancel{|b|}\sin\theta} = \frac{\sqrt{3}}{3}$$

$\cot\theta = \frac{\sqrt{3}}{3}$

$\tan\theta = \frac{3}{\sqrt{3}} = \frac{\cancel{\sqrt{3}} * \sqrt{3}}{\cancel{\sqrt{3}}} = \sqrt{3}$

$\theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

$\boxed{45}$



$$\sin\theta = \frac{d}{|b|}$$

$$d = |b| \sin\theta$$

$$|a \times b| = |a| \times |b| \times \sin\theta$$

$$|a \times b| = |a| \times d$$

$$d = \frac{|a \times b|}{|a|}$$

Section 12.5

3- $P_0(2, 2.4, 3.5) \rightarrow$ Parallel to $\vec{v} = 3i + 2j - k$

$\vec{v} = \vec{d} = \langle 3, 2, -1 \rangle$

L: Parametric

$x = 2 + 3t$

$y = 2.4 + 2t$

$z = 3.5 - t$

Vector equ $= (2+3t)i + (2.4+2t)j + (3.5-t)k$

$= \langle 2+3t, 2.4+2t, 3.5-t \rangle$

4- $P_0(10, 14, -10) \rightarrow$ Parallel to L: $x = -1 + 2t$

$y = 6 - 3t$

$z = 3 + 4t$

$\vec{d} = \langle 2, -3, 9 \rangle$

Parametric

L: $x = 0 + 2t$

$y = 14 - 3t$

$z = -10 + 9t$

vector equ:- $(2t)i + (14-3t)j + (-10+9t)k$

$= \langle 2t, 14-3t, -10+9t \rangle$

5- $P_0(1, 0, 6) \rightarrow$ Perpendicular to π: $x + 3y + z = 5$

$\vec{n} = \langle 1, 3, 1 \rangle$

Parametric

L: $x = 1 + t$

$y = 0 + 3t$

$z = 6 + t$

vector equ:- $(1+t)i + (3t)j + (6+t)k$

$= \langle 1+t, 3t, 6+t \rangle$

7. $P_1(0, \frac{1}{2}, 1)$ $\quad P_2(2, 1, -3)$

$\vec{d} = \overrightarrow{P_1P_2} = \langle 2, \frac{1}{2}, -4 \rangle$.

<u>Parametric</u>

$x = 0 + 2t$

$y = \frac{1}{2} + \frac{1}{2}t$

$z = 1 - 4t$

<u>symmetric</u>

$$\frac{x - 0}{2} = \frac{y - \frac{1}{2}}{1/2} = \frac{z - 1}{-4}$$

10. $P_0(2, 1, 0)$ $\rightarrow$ Perpendicular to $i + j$ and $j + k$.

$\hookrightarrow d_1 = \langle 1, 1, 0 \rangle$ $\quad \hookrightarrow d_2 = \langle 0, 1, 1 \rangle$

$\vec{d_1} \times \vec{d_2} = \vec{d_3}$

$$\vec{d_3} = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = (1-0)i - (1-0)j + (1-0)k = i - j + k = \langle 1, -1, 1 \rangle.$$

<u>Parametric</u>

$L: x = 2 + t, \quad y = 1 - t, \quad z = 0 + t$

<u>symmetric</u>

$$\frac{x - 2}{1} = \frac{y - 1}{-1} = \frac{z - 0}{1}$$

11- $P_0(-6,2,3) \to$ parallel to $L: \frac{x}{2} = \frac{y}{3} = z+1$

$\vec{d} = \langle 2,3,1 \rangle$

Parametric:

$L: x = -6 + 2t$

$y = 2 + 3t$

$z = 3 + t$

Symmetric:

$$\frac{x+6}{2} = \frac{y-2}{3} = \frac{z-3}{1}$$

12- intersection to $x+2y+3z=1 \to \vec{n}_1 \langle 1,2,3 \rangle$

and $x-y+z=1 \to \vec{n}_2 \langle 1,-1,1 \rangle$.

Point in intersection

$z=0 \to x+2y=1 \to \cancel{x}+2y=1$

$-(x-y=1) \quad -\cancel{x}-y=-1$

$3y=0 \to y=0$

$x-y=1 \to y=0 \to x=1$

Point is $(1,0,0)$

$\vec{n}_1 \times \vec{n}_2 = \vec{d}$

$$\vec{d} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{vmatrix}$$

$= \langle 5, 2, -3 \rangle$

Parametric:

$L: x = 1 + 5t$

$y = 0 + 2t$

$z = 0 - 3t$

Symmetric:

$$\frac{x-1}{5} = \frac{y-0}{2} = \frac{z-0}{-3}.$$

14- $P_1 (8,0,5)$ $P_2 (4,2,-3)$

↳ perpendicular to $P_3 (4,3,7)$ and $P_4 (7,1,3)$

$\overrightarrow{P_1P_2} = \langle -4, 2, -8 \rangle = \vec{a}$

$\overrightarrow{P_3P_4} = \langle 3, -2, -4 \rangle = \vec{b}$

perpendicular → $\vec{a} \cdot \vec{b} = 0$ → $\cos\theta = 0$

$\vec{a} \cdot \vec{b} = -12 - 4 + 32 = 16$, Not perpendicular

16- a) $P_0 (2,4,6)$ → perpendicular to

$x : x - y + 3z = 7$

$\vec{n} (1, -1, 3)$

parametric:

$L: x = 2 + t$

$y = 4 - t$

$z = 6 + 3t$

b) 1) in XZ plane

$x = 0$ → $t = -2$

$y = 4 - (-2) = 6$

$z = 6 + -2 \times 3 = 0$

2) in XY plane

$z = 0$ → $6 + 3t = 0$

$t = -2$

$x = 0$

$y = 6$

3) in XZ plane

$y = 0$ → $t = 4$

$x = 6$,

$z = 18$

in XZ plane

$P (6, 0, 18)$

in XY and ZY plane

$P (0, 6, 0)$

17- segment from (6, -1, 9) to (7, 6, 0)

$\vec{d} = \langle 1, 7, -9 \rangle$.

L: $x = 6 + t$ → vector equ: $(6+t)i + (-1+7t)j + (9-9t)k$

$y = -1 + 7t$

$z = 9 - 9t$ $= \langle 6+t, -1+7t, 9-9t \rangle$

19 - $L_1: x = 3+2t,\ y = 4-t,\ z = 1+3t \cdot \vec{v}_1 = \langle 2,-1,3\rangle$

$L_2: x = 1+4s,\ y = 3-2s,\ z = 4+5s\ \ \vec{v}_2 = \langle 4,-2,5\rangle$

Parallel ? → $\frac{x_{i1}}{x_{i2}} = \frac{y_{j1}}{y_{j2}} = \frac{z_{k1}}{z_{k2}}$.

→ $2/4 \overset{?}{=} \frac{-1}{-2} \overset{?}{=} 3/5 \Rightarrow \frac{1}{2} = \frac{1}{2} \neq \frac{3}{5}$ → not Parallel.

intersection ? → $x_1 = x_2,\ y_1 = y_2,\ z_1 = z_2$.

$y_1 = y_2$ $\qquad z_1 = z_2$

→ $4-t = 3-2s$ $\qquad 1+3t = 4+5s$

$= -2s + t = 1$ → $1+3(1+2s) = 4+5s$

$t = 1+2s$ $\qquad \cancel{1}+\cancel{3}+6s = \cancel{4}+5s$

$s = 0$

$t = 1$

$x_1 \overset{?}{=} x_2$.

→ $3+2\times1 \overset{?}{=} 1+4\times0$

$5 \neq 1$ → not intersecting. So, skew.

20- L_1: $x = 4 - 6t$, $y = 1 + 3t$, $z = -5 + 2t \rightarrow \vec{v}_1 \langle -6, 3, 2 \rangle$

L_2: $x = 5 + 18t$, $y = 6 - 9t$, $z = 3 - 6t \rightarrow \vec{v}_2 \langle 18, -9, -6 \rangle$

Parallel? $\rightarrow$ $\dfrac{x_{v_1}}{x_{v_2}} = \dfrac{y_{v_1}}{y_{v_2}} = \dfrac{z_{v_1}}{z_{v_2}}$

$\rightarrow -6/18 \overset{?}{=} \dfrac{3}{-9} \overset{?}{=} 2/-6$

$-1/3 = \dfrac{-1}{3} = \dfrac{-1}{3} \rightarrow$ Parallel.

21- L_1: $\dfrac{x-2}{1} = \dfrac{y-3}{-2} = \dfrac{z-1}{-3} \rightarrow \vec{v}_1 \langle 1, -2, -3 \rangle$

L_2: $\dfrac{x-3}{1} = \dfrac{y+4}{3} = \dfrac{z-2}{-7} \rightarrow \vec{v}_2 \langle 1, 3, -7 \rangle$

L_1: $x = 2 + t$, $y = 3 - 2t$, $z = 1 - 3t$

L_2: $x = 3 + s$, $y = -4 + 3s$, $z = 2 - 7s$

Parallel $\rightarrow$?

$\rightarrow 1/1 \overset{?}{=} \dfrac{-2}{3} \overset{?}{=} \dfrac{-3}{-7}$

$1 \neq -2/3 \neq +3/+7 \rightarrow$ not Parallel.

intersecting?

$\rightarrow$ $x_1 = x_2$ $\qquad$ $y_1 = y_2$

$2 + t = 3 + s$ $\qquad$ $3 - 2t = -4 + 3s$

$t = 1 + s$ $\qquad$ $-2t = -7 + 3s$

$-2(1+s) = -7 + 3s$

$-2 + -2s = -7 + 3s$

$5s = +5$

$s = 1$

$t = 2$

$z_1 = 1 - 3 \cdot 2$
$= -5$

$z_2 = 2 - 7 \cdot 1$
$= -5$

intersecting.

22- $L_1: \frac{x}{1} = \frac{y-1}{-1} = \frac{z-2}{3} \to \vec{v}_1 \langle 1, -1, 3 \rangle$

$L_2: \frac{x-2}{2} = \frac{y-3}{-2} = \frac{z}{7} \to \vec{v}_2 \langle 2, -2, 7 \rangle$

parallel? $\to \frac{1}{2} \stackrel{?}{=} \frac{-1}{-2} \stackrel{?}{=} \frac{3}{7}$

$\frac{1}{2} = \frac{1}{2} \neq \frac{3}{7} \Rightarrow$ not parallel.

intersecting?

$x_1 = x_2 , y_1 = y_2 , z_1 = z_2$

$L_1: x_1 = t$
$y_1 = 1 - t$
$z_1 = 2 + 3t$

$L_2: x_2 = 2 + \cancel{3}2s$
$y_2 = 3 - 2s$
$z_2 = 7s$

$x_1 = x_2$
$t = 2 + 2s$

$z_1 = z_2$
$2 + 3t = 7s$
$2 + 3(2 + 2s) = 7s$
$2 + 6 + 6s = 7s$
$s = 8$
$t = 18$

$y_1 = 1 - t$
$= 1 - 18 = -17$

$y_2 = 3 - 2s \Rightarrow 3 - 16 = -13$

not intersecting $\to$ skew.

$\boxed{24}$ $P(5,3,5)$ $\quad \vec{n} = 2\hat{i} + \hat{j} - \hat{k}$

$= \langle 2, 1, -1 \rangle$

$\rightarrow 2(x-5) + (y-3) - (z-5) = 0$

$\boxed{26}$ $P(2,0,1)$

$L: x = 3t,\ y = 2-t,\ z = 3+4t$

$\vec{n} = \langle 3, -1, 4 \rangle$

$\rightarrow 3(x-2) - y + 4(z-1) = 0$

$\boxed{27}$ $P(1,-1,-1)$

$\pi_1: 5(x) - y - z = 6$

$\vec{n}_1 = \langle 5, -1, -1 \rangle$

$\vec{n}_1 \parallel \vec{n}_2$

$\pi_2: 5(x-1) - (z+1) - (y+1) = 0$

30] $L_1 : x = 1 + t,\ y = 2 - t,\ z = 4 - 3t$

$\pi_2 : 5x + 2y + z = 1$

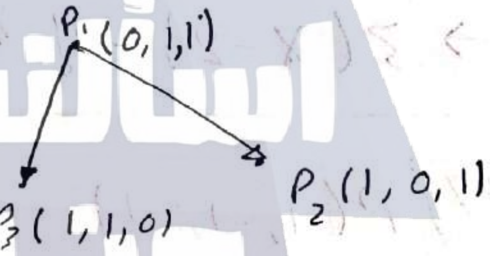
$P_1 : (1, 2, 4)$

$\vec{n}_2 = \langle 5, 2, 1 \rangle$

$\vec{n}_2 \parallel \vec{n}_1$

So $\to \pi_1 : 5(x - 1) + 2(y - 2) + 1(z - 4)$

31] $P_1 : (0, 1, 1),\ P_2 : (1, 0, 1),\ P_3 : (1, 1, 0)$



$\overrightarrow{P_1P_2} = \langle 1-0, 0-1, 1-1 \rangle$

$= \langle 1, -1, 0 \rangle$

$\overrightarrow{P_1P_3} = \langle 1-0, 1-1, 0-1 \rangle$

$= \langle 1, 0, -1 \rangle$

$$\vec{n} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = (1-0)\hat{i} - (-1-0)\hat{j} + (0+1)\hat{k}$$

$= 1\hat{i} + 1\hat{j} + 1\hat{k}$

$= \langle 1, 1, 1 \rangle$

$\pi : x + (y - 1) + (z - 1) = 0$

35] $P_1(3, 5, -1)$

$L: x = 4 - t,\ y = 2t - 1,\ z = -3t \rightarrow \vec{L} = \langle -1, 2, -3 \rangle,\ P_2(4, -1, 0)$

$\overrightarrow{P_1P_2} = \langle +1, -6, 1 \rangle$

$$\vec{n} = \overrightarrow{P_1P_2} \times \vec{L} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -6 & 1 \\ -1 & 2 & -3 \end{vmatrix} = \langle 16, 2, -4 \rangle$$

So $\rightarrow \pi: 16(x-3) + 2(y-5) - 4(z+1) = 0$

37] $P(3, 1, 4)$

$\pi_1: x + 2y + 3z - 1 = 0 \longrightarrow \vec{n}_1 = \langle 1, 2, 3 \rangle$

$\pi_2: 2x - y + z + 3 = 0 \longrightarrow \vec{n}_2 = \langle 2, -1, 1 \rangle$

$$\vec{n}_3 = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{vmatrix} = \langle 5, +5, -5 \rangle$$

but $x = 0 \rightarrow 2y + 3z = 1$

$\rightarrow (-y + z = -3) \times 2$

$5z = -5$

$\boxed{z = -1} \rightarrow \boxed{y = 2}$

$d : (0, 2, -1)$

$\vec{Pd} = \langle -3, 1, -5 \rangle$

$$\vec{n_3} = \vec{Pd} \times \vec{L} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & -5 \\ 5 & 5 & -5 \end{vmatrix} = \langle 20, 40, -20 \rangle$$

so $\to \pi_3 : 20(x-3) + 40(y-5) - 20(z+1) = 0$

$\boxed{38}$ $P_1(0, -2, 5)$, $P_2(-1, 3, 1)$

$\vec{P_1P_2} = \langle -1, 5, -4 \rangle$

$\pi_1 : 5x + 4y - 2z = 0 \to \vec{n_1} = \langle 5, 4, -2 \rangle$

$\pi_2 \perp \pi_1$ so $\to \pi_2 \parallel \vec{n_1}$

$$\vec{n_2} = \vec{P_1P_2} \times \vec{n_1} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 5 & -4 \\ 5 & 4 & -2 \end{vmatrix} = \langle 6, -22, -29 \rangle$$

$\pi_2 : 6(x+1) - 22(z-3) - 29(z-1) = 0$

$\boxed{39}$ $P(1, 5, 1)$

$\pi_1 : 2x + y - 2z = 2 \longrightarrow \vec{n}_1 = \langle 2, 1, -2 \rangle$

$\pi_2 : x + 3z = 4 \longrightarrow \vec{n}_2 = \langle 1, 0, 3 \rangle$

$$\vec{n}_3 = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 0 & 3 \end{vmatrix} = \langle 3, -8, -1 \rangle$$

$\pi_3 : 3(x-1) - 8(y-5) - (z-1) = 0$

40) $\pi_1 : x - z = 1 \longrightarrow \vec{n_1} = \langle 1, 0, -1 \rangle$

$\pi_2 : y + 2z = 1 \longrightarrow \vec{n_2} = \langle 0, 1, 2 \rangle$

$$\vec{L_1} = \vec{n_1} \times \vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{vmatrix} = \langle -1, -2, 1 \rangle$$

$\pi_3 = x + y - 2z = 1 \longrightarrow \vec{n_3} = \langle 1, 1, -2 \rangle$

$\boxed{\vec{n_3} \parallel \vec{L}}$

$$\rightarrow \vec{n_4} = \vec{n_3} \times \vec{L} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{vmatrix} = \langle -3, 1, -1 \rangle$$

in π_1 and π_3 but $x = 0$

$\rightarrow -z = 1$

$y + 2z = 1$

$\boxed{y = 3} \leftrightarrow \boxed{z = -1}$ the poin is $(0, 3, -1)$

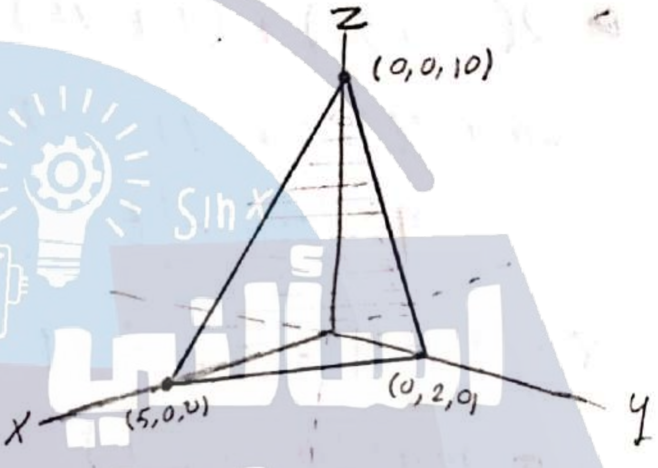
so $\pi_4 : -3(x) + (y - 3) - (z + 1) = 0$

41) $2x + 5y + z = 10$

x-intercept: $2x = 10 \rightarrow x = 5 \quad (5, 0, 0)$

y-intercept: $5y = 10 \rightarrow y = 2 \quad (0, 2, 0)$

z-intercept: $z = 10 \quad (0, 0, 10)$



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46) $L: x = t - 1,\ y = 1 + 2t,\ z = 3 - t$

$\pi: 3x - y + 2z = 5$

$\rightarrow 3(t-1) - (1+2t) + 2(3-t) = 5$

$3t - 3 - 1 - 2t + 6 - 2t = 5$

$-t = 3 \rightarrow \boxed{t = -3}$

$\rightarrow x = -4,\ \rightarrow y = -5,\ \rightarrow z = 6$

intersect point: $(-4, -5, 6)$

Scanned with CamScanner

48) $P_1(-3,1,0)$, $P_2(-1,5,6)$

$\overrightarrow{P_1P_2} = \langle 2,4,6 \rangle \rightarrow x = -3+2t$, $y = 1+4t$, $z = 6t$

$\pi = 2x + y - z = -2$

$\rightarrow 2(-3+2t) + (1+4t) - (6t) = -2$

$-6 + 4t + 1 + 4t - 6t = -2$

$2t = 3 \rightarrow \boxed{t = \frac{3}{2}}$

$x = 0$, $y = 7$, $z = 9$

$\rightarrow$ point $(0, 7, 9)$

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$\pi_1 : 9x - 3y + 6z = 2 \to \vec{n}_1 = \langle 9, -3, 6 \rangle$

$\pi_2 : 2y = 6x + 4z$

$\to 6x - 2y + 4z = 0 \to \vec{n}_2 = \langle 6, -2, 4 \rangle$

$\vec{n}_1 \cdot \vec{n}_2 = 9(6) + -3(-2) + 6(4)$

$= 84$

$|n_1| = \sqrt{126}$

$|n_2| = \sqrt{56}$

$\cos\theta = \dfrac{\vec{n}_1 \cdot \vec{n}_2}{|n_1||n_2|}$

$= \dfrac{84}{\sqrt{126 \cdot 56}} = 1 \to \cos^{-1} 1 = 0°$

Two planes are parallell

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$\pi_1 : 5x + 2y + 3z = 2$ | $\pi_2 : 4x - y - 6z = 0$

$\vec{n}_1 = \langle 5, 2, 3 \rangle$ | $\vec{n}_2 = \langle 4, -1, -6 \rangle$

$\cos\theta = \dfrac{\vec{n}_1 \cdot \vec{n}_2}{|n_1||n_2|} = \dfrac{20 - 2 - 18}{\sqrt{38} \cdot \sqrt{53}}$

$= 0 \to \cos^{-1} 0 = 90°$

Two planes are perpendicular

$\boxed{57}$ $\quad P_1: \; x + y + z = 1$

$P_2: \; x + 2y + 2z = 1$

let $z = 0$, $\quad x + y = 1 \longrightarrow y = 1 - x$

$$x + 2(1 - x) + 0 = 1$$

$$x + 2 - 2x = 1$$

$$-x = -1 \longrightarrow x = 1$$

$y = 0 \qquad P_0(1, 0, 0)$

$z = 0$

$$\vec{d} = \vec{n_1} \times \vec{n_2}$$

$$= \langle 1, 1, 1 \rangle \times \langle 1, 2, 2 \rangle$$

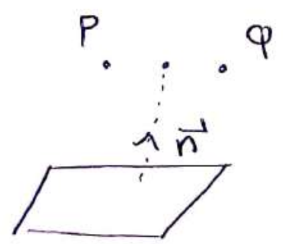
$$\vec{d} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix} = \langle 0, -1, 1 \rangle$$

a) equation of line : $\quad x = 1 \; , \; y = -t \; , \; z = t$

b) $$\theta = \cos^{-1}\left(\frac{|\vec{n_1} \cdot \vec{n_2}|}{|\vec{n_1}||\vec{n_2}|} \right) = \cos^{-1}\left(\frac{1 + 2 + 2}{\sqrt{3} \cdot \sqrt{9}} \right)$$

$$= \cos^{-1}\left(\frac{5}{3\sqrt{3}} \right)$$

$\boxed{61}$ we find midpoint



$P(1,0,-2)$

$Q(3,4,0)$

$m = (2,2,-1)$

$\vec{n} = PQ = \langle 2,4,2 \rangle$

$P: \quad ax + by + cz = 0$

$\cancel{2x\,2 + 4x + 2x}$

$2x + 4y + 2z = 0$

$x + 2y + z = 0$ ✓ اذا طلب المسافة للميديونية بعوض النقطة m

$\boxed{63}$

$P_1(a,0,0), \quad P_2(0,b,0), \quad P_3(0,0,c)$

$\vec{d_1} = \overline{P_1P_2} = \langle a, -b, 0 \rangle$

$\vec{d_2} = \overline{P_2P_3} = \langle 0, b, -c \rangle$

$\vec{n} = \vec{d_1} \times \vec{d_2} = \begin{vmatrix} a & -b & 0 \\ 0 & b & -c \end{vmatrix} = (bc)i - (-ac)j + (ab)k$

$\vec{n} = \langle bc, ac, ab \rangle$

$P_1(a,0,0)$

$P: \quad ax + by + cz = 0$

$bcx + acy + abz = 0$

$\boxed{64}$

$x = 1+t \ , \ y = 1-t \ , \ z = 2t$

a) $x = 2-s \ , \ y = s \ , \ z = 2$

$z = z$

$2t = 2 \rightarrow t = 1 \rightarrow s = 0$

intersect point $(2,0,2) \rightarrow P_3$

b) take $t=2$ $\quad P_1(3,-1,4)$

take $s=1$ $\quad P_2(1,1,2)$

$P_3(2,0,2)$

$\vec{d_1} = \overrightarrow{P_1P_3} = \langle -1, 1, -2 \rangle$

$\vec{d_2} = \overrightarrow{P_2P_3} = \langle 1, -1, 0 \rangle$

$$\vec{n} = \vec{d_1} \times \vec{d_2} = \begin{vmatrix} -1 & 1 & -2 \\ 1 & -1 & 0 \end{vmatrix} = (-2)i - (2)j + (0)k$$

$\vec{n} = \langle -2, -2, 0 \rangle$

$P: \quad -2(x-2) - 2(y-0) + 0 = 0$

$x + y = 2$

68

$\vec{d_1} \langle 6, -3, 12 \rangle$

$\vec{d_2} \langle 2, 1, 4 \rangle$

$\vec{d_3} \langle \frac{1}{2}, \frac{-1}{4}, 1 \rangle$

$\vec{d_4} \langle 4, 2, 8 \rangle$

L_1, L_3 parallel ~~identical~~

L_2, L_4 parallel

70

$P_0(0,1,3)$ $\quad x = 2t,\ y = 6-2t,\ z = 3+t$

$P_1(0,6,3)$

$\vec{d} \langle 2, -2, 1 \rangle$

$\vec{a} = \overline{P_0P_1} = \langle 0, 5, 0 \rangle$

$$d = \frac{|\vec{a}\cdot\vec{d}|}{|\vec{d}|} = \frac{|0-10+1|}{\sqrt{9}} = \frac{9}{3} = 3$$

72

$P_0(-6,3,5)$, $x-2y-4z-8=0$

$\vec{n} \langle 1, -2, -4 \rangle$

$$d = \frac{|ax_0 + by_0 + cz_0 - d|}{|\vec{n}|} = \frac{|-6-6-20-8|}{\sqrt{1+4+16}}$$

$$= \frac{40}{\sqrt{21}}$$

$\boxed{73}$ $P_1: 2x - 3y + z = 4$

$P_2: 4x - 6y + 2z = 3$

Distance between point and plane

from P_1 let $x, y = 0$ so $P_0(0, 0, 4)$

$\vec{n} < 4, -6, 2 >$

$$d = \frac{|ax_0 + by_0 + cz_0 + d|}{|\vec{n}|} = \frac{|0 + 0 + 8 - 3|}{\sqrt{16+36+4}}$$

$$= \frac{5}{\sqrt{56}}$$

$\boxed{75}$ Show that distance between parallel planes $ax + by + cz + d_1 = 0$, $ax + by + cz + d_2 = 0$

$$D = \frac{|d_1 - d_2|}{\sqrt{a^2+b^2+c^2}}$$

Sol $P_0\left(0, 0, \frac{-d_1}{c}\right)$

$\vec{n_2} < a, b, c >$

$$d = \frac{|ax_0 + by_0 + cz_0 + d|}{|\vec{n_2}|} = \frac{\left|0 + 0 + \frac{-d_1}{c} * c + d_2\right|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{|d_2 - d_1|}{\sqrt{a^2+b^2+c^2}} = \frac{|d_1 - d_2|}{\sqrt{a^2+b^2+c^2}}$$

//

$\boxed{76}$

$P_1: \; x + 2y - 2z = 1$

$P_2: \; ax + by + cz + d = 0$

$P_1 \parallel P_2 \Rightarrow \vec{n_1} = \vec{n_2} = \langle 1, 2, -2 \rangle$ $\cancel{P_0(0, 0, -\tfrac{1}{2})}$

$P_0(x, y, z)$

$$d = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2+b^2+c^2}}$$

$$2 = \frac{|x + 2y - 2c + -1|}{\sqrt{1+4+4}}$$

$$2 = \frac{|x + 2y - 2c + -1|}{3}$$

$$|x + 2y - 2c - 1| = 6$$

or

$x + 2y - 2c - 1 = 6$ $\qquad$ $x + 2y - 2c - 1 = -6$

$x + 2y - 2c - 7 = 0$ $\qquad$ $x + 2y - 2c + 5 = 0$

$\boxed{78}$ $\vec{d_1} = <1, 6, 2>$

$\vec{d_2} = <2, 13, 6>$

$\vec{n} = \vec{d_1} \times \vec{d_2} = <6, -2, 3>$

let $s = 0$ so $P_0(1, 5, -2)$

equation of plane is : $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$6(x-1) + -2(y-5) + 3(z+2) = 0$

$6x - 2y + 3z + 10 = 0$

let $t = 0$ $P(1, 1, 0)$

$$d = \frac{|ax + by + cz + d|}{|\vec{n}|} = \frac{|6 - 2 + 0 + 10|}{\sqrt{36 + 4 + 9}} = \frac{14}{7} = 2$$

74)

$\vec{d_1} = \langle 2, 0, -1 \rangle$ not parallel

$\vec{d_2} = \langle 3, 2, 2 \rangle$

$L_1: \quad x = 2t, \quad y = 0, \quad z = -t$

$L_2: \quad x = 4 + 3t, \quad y = 1 + 2t, \quad z = 3 + 2t$

Search for intersect $\quad y = y \Rightarrow 0 = 1 + 2t$

$t = \frac{-1}{2}$

$x \overset{?}{=} x$

$2 * \frac{-1}{2} \overset{?}{=} 4 + 3 * \frac{-1}{2}$

$-1 \neq 2.5$

not intersect

So they are skew.

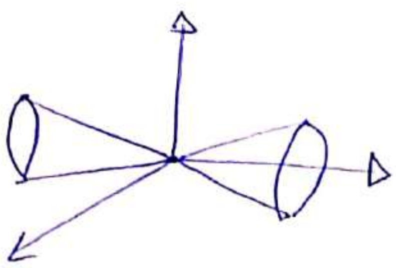
نفس خطوات السؤال الي قبله

بعض حلول سكشن

12.6

$\boxed{31}$ $y^2 = x^2 + \frac{z^2}{9}$

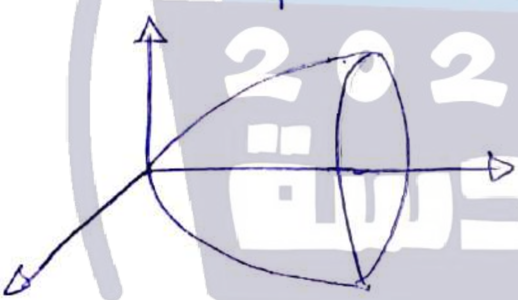
elliptic cone on $\underline{y}$ axis



$\boxed{32}$ $4x^2 - y + 2z^2 = 0$

$y = 4x^2 + 2z^2$

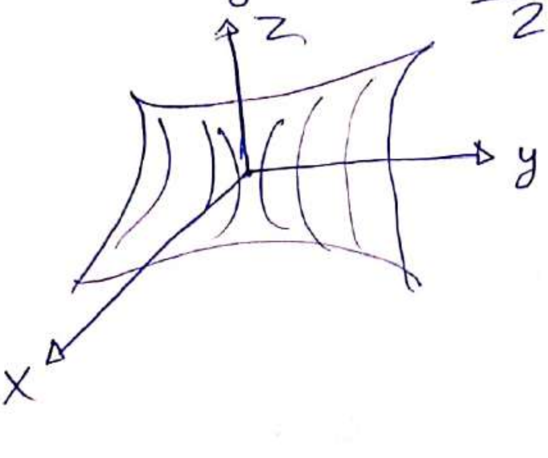
$\frac{y}{4} = x^2 + \frac{z^2}{2} \longrightarrow$ Elliptic paraboloid on $\underline{y}$ axes



$\boxed{33}$ $x^2 + 2y - 2z^2 = 0$

$2y = 2z^2 - x^2$

$y = z^2 - \frac{x^2}{2} \longrightarrow$ hyperbolic paraboloid

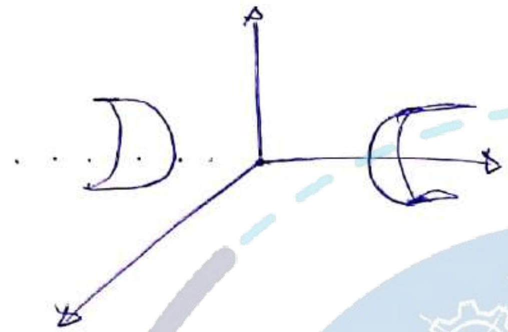


* صعب رسمها

$\boxed{34}$ $y^2 = x^2 + 4z^2 + 4$

$y^2 - x^2 - 4z^2 = 4$

$\frac{y^2}{4} - \frac{x^2}{4} - z^2 = 1$ → hyperboloid of 2 Sheet

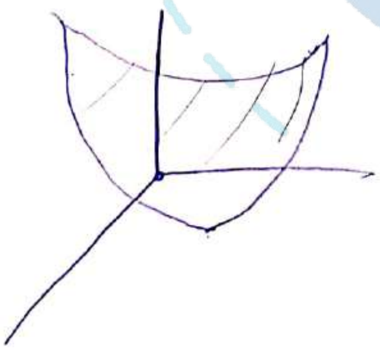


$\boxed{35}$ $x^2 + y^2 - 2x - 6y - z + 10 = 0$

$x^2 - 2x + y^2 - 6y - z + 10 = 0$ اكمال مربع

$z = (x-1)^2 - 1 + (y-3)^2 - 9 + 10$

$z = (x-1)^2 + (y-3)^2$ → elliptic paraboloid



$\boxed{36}$

$$x^2 - y^2 - z^2 - 4x - 2z + 3 = 0$$

$$x^2 - 4x + 4 - 4 - y^2 - z^2 - 2z + 3 = 0$$

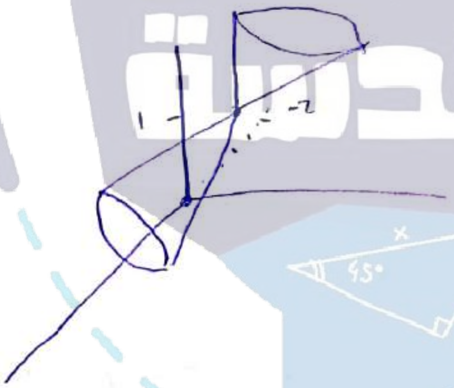
$$(x-2)^2 - y^2 - (z^2 + 2z + 1 - 1) - 1 = 0$$

$$(x-2)^2 - y^2 - (z+1)^2 = 1 - 1$$

$$(x-2)^2 - y^2 - (z+1)^2 = 0$$

$$(x-2)^2 = y^2 + (z+1)^2$$

elliptic cone on $\underline{x}$ axis



37

$$X^2 - y^2 + Z^2 - 4X - 2Z = 0$$

$$X^2 - 4X - y^2 + Z^2 - 2Z = 0$$

$$X^2 - 4X + 4 - 4 - y^2 + Z^2 - 2Z + 1 - 1 = 0$$

$$(X-2)^2 - y^2 + (Z-1)^2 = 5$$

hyperboloid of one sheet on y-axis

center (2,0,1)

38

$$4X^2 + y^2 + Z^2 - 24X - 8y + 4Z + 55 = 0$$

$$4X^2 - 24X + y^2 - 8y + Z^2 + 4Z + 55 = 0$$

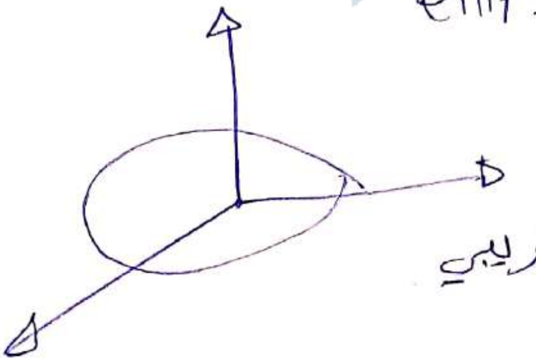
$$4(X^2 - 6X + 9 - 9) + (y^2 - 8y + 16) - 16 + (Z^2 + 4Z + 4) - 4 + 55 = 0$$

$$4(X-3)^2 - 36 + (y-4)^2 - 16 + (Z+2)^2 - 4 + 55 = 0$$

$$4(X-3)^2 + (y-4)^2 + (Z+2)^2 = 1$$

$$(X-3)^2 + \frac{(y-4)^2}{4} + \frac{(Z+2)^3}{4} = 1$$

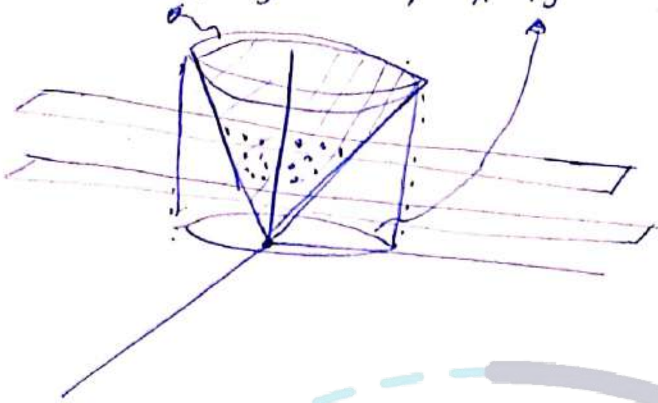
ellipsoid center (3,4,-2)



تقريبي

$\boxed{43}$ Sketch region bounded between

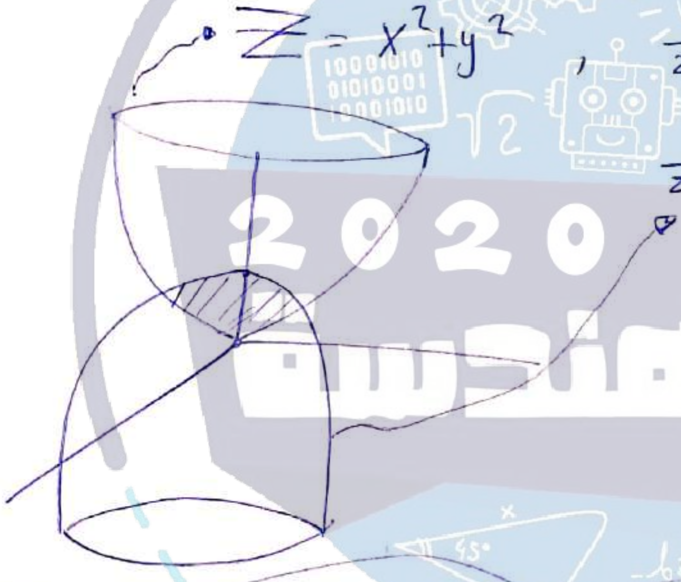
$Z=\sqrt{x^2+y^2}$, $x^2+y^2=1$ $\quad 1 \le Z \le 2$



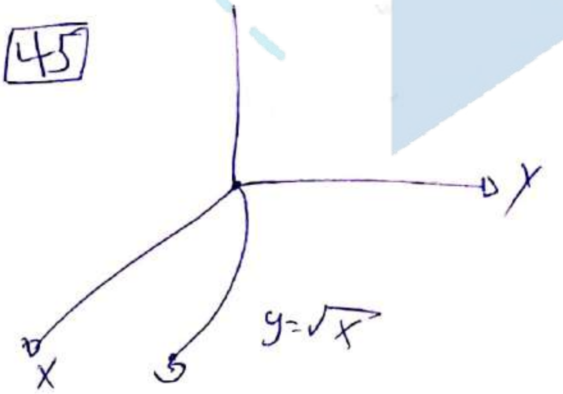
$\boxed{44}$ Sketch region bounded between

$Z=x^2+y^2$, $Z=2-x^2-y^2$

$Z-2=-(x^2+y^2)$



$\boxed{45}$



if we rotate $y=\sqrt{x}$
we get a circle parallel
to yz plane

note:
$x=r$

So :

$y^2+z^2=r^2$

$y^2+z^2=(\sqrt{x})^2$

$x=y^2+z^2$

elliptic paraboloid

$\boxed{46}$

If we rotate that point about z-axis we get circle parallel to xy plane So:

$x^2 + y^2 = r^2 \quad (r = \frac{z}{2})$

$x^2 + y^2 = (\frac{z}{2})^2$

$x^2 + y^2 = \frac{z^2}{4}$ ⟶ elliptic cone