

→ Electricity & Magnetism  $\leadsto$  electromagnetism.  
the branch of physics that is concerned with electric & magnetic phenomena.

→ Historical view :-

1. electron  $\rightarrow$  the greek word for <sup>كهرمان</sup> amber becomes electrified when <sup>دلل</sup> rubbed
2. magnesia  $\rightarrow$  the name of a <sup>منطقة</sup> district where the magnetite  $Fe_3O_4$  <sup>شوه</sup> stone was first observed
3. colomb  $\rightarrow$  electrostatic force
4. compass <sup>الوصلة</sup> <sup>برة</sup> needle  $\rightarrow$  <sup>اخترع</sup> invented by Arab

$\leadsto$  electricity & magnetism are related phenomena.

1 - Orsted  $\rightarrow$  a compass needle when placed near an electric <sup>دائرة</sup> circuit will be <sup>انحرفت</sup> deflected.

2 - Faraday  $\rightarrow$  an electric current is established in a coil if it is <sup>ينشأ</sup> moved relative to magnet.  
<sup>تتحرف ابرة البوصلة عند تقريبها من تيار كهربائي (التيار الكهربائي يولد مجالاً مغناطيسياً)</sup>

3 - Maxwell  $\rightarrow$  4 equations for electromagnetism (electromagnetic theorem).  
<sup>المجال المغناطيسي يولد تياراً كهربائياً</sup>  
<sup>ظاهرة الحث المغناطيسي</sup>

23.1 properties of electric charges

→ Two types of electric charge :-

1) Positive charge

the charge on a glass rod rubbed with silk

هذه الأسلاك أمثلة لاصحية

تناقص مع + e + e

2) Negative charge

the charge on a rubber rod rubbed with fur.

تجاذب مع + e -

→ Properties of electric charges

1) like charges repel, unlike charges attract.

تنافر

2) The total electric charge in an isolated system is conserved.

- charge is not created by rubbing, but it is (transferred)

redistributed.

مثل الدلاء / لا يصبح الدلاء إلا المواد المتجانسة

3) electric charges can only occur on integral multiples

of a fundamental unit (charge) = electric charge is quantized.

مكملة

$$e = 1.6 \times 10^{-19} \text{ C (colomb)}$$

هذا يعني أن كولوم يكافئ عدد كبير من شحنات e

لذلك نستخدم وحدات صغيرة مثل Mc ...

→ q : charge (it comes from quantity of charge)

$$[q] = \text{colomb} = \text{C (capital)}$$

→ Examples :- (of charged particles)

1) electron  $\rightarrow q_e = -e$

2) proton  $\rightarrow q_p = +e$

3) neutron  $\rightarrow q_n = 0$

So for any  $q \Rightarrow q = \pm Ne$   
 integer

23.3 : Colomb's law

→ The force exerted between two point charges separated by a distance r.  $\vec{F}_{e \text{ electric } q_1, q_2}$

$|\vec{F}_e| \propto |q_1 q_2|$

$\propto \frac{1}{r^2} \text{ m}$

$\Rightarrow |\vec{F}_e| = \frac{\text{constant } |q_1 q_2|}{r^2} = \frac{K_e |q_1 q_2|}{r^2}$   
scalar فقط موجب

فقط يعتمد على الوسط

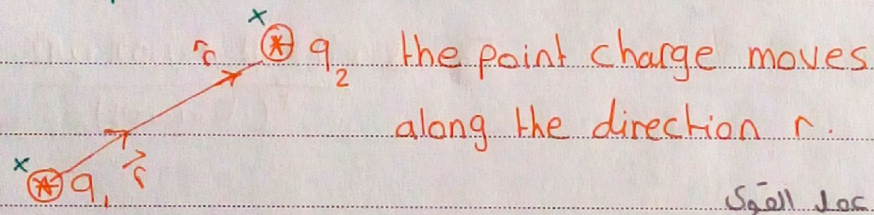
$K_e$  : Colomb constant =  $8.899 \times 10^9 \text{ N.m}^2/\text{C}^2 \approx 9 \times 10^9 \text{ N.m}^2/\text{C}^2$

$K_e = \frac{1}{4\pi\epsilon}$

$\epsilon$  : constant depends on the surrounding space  
الوسط المحيط

$\epsilon_0$  : فقدية permittivity of free (empty) space =  $8.85 \times 10^{-12} \text{ C}^2/\text{m}^2.\text{N}$

→ The direction for the colomb force is determine by the law of repulsion and attraction.



the point charge moves along the direction r.

الخط الواصل بين الشحنتين ← خط عمل القوى

$\vec{F}_e = \frac{K_e |q_1 q_2|}{r^2} \hat{r}$

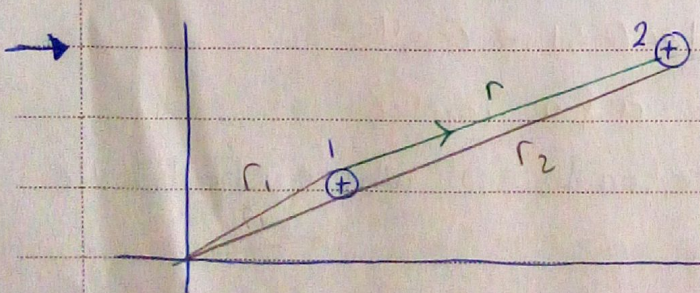
$\vec{F}_{e \text{ on } 2} = \frac{K_e |q_1 q_2|}{r^2} \hat{r}$

$\vec{F}_{e \text{ on } 1} = \frac{K_e |q_1 q_2|}{r^2} (-\hat{r})$

So ...

$|\vec{F}_{e \text{ on } 2}| = |\vec{F}_{e \text{ on } 1}|$  also...  $\vec{F}_{e \text{ on } 2} = -\vec{F}_{e \text{ on } 1}$

according to N. 3rd law. حتى لو كانت قوتها الشحنتان مختلفة



$r = \sqrt{\Delta x^2 + \Delta y^2}$

→ For a set of point charges  $q_1, q_2, \dots$

$$\vec{F}_{e \text{ on } q_1} = \sum \vec{F}_{ei} = \vec{F}_{e2 \text{ on } 1} + \vec{F}_{e3 \text{ on } 1} + \dots$$

vector equation (جمع متجهان)

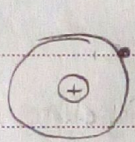
→ In some cases :-

إذا كانت الكتلة صغيرة فتعمل قوة الجاذبية لأنها ستكون صغيرة جداً بالقياس

net force = electric force + gravitational force  
 القوة الكهرواستاتيكية  
 $\approx \text{Zero}$

So... net force = electric force

→ See example 23.1 p. 695 Hydrogen atom



$m_e = 9.11 \times 10^{-31} \text{ Kg}$

$q_e = -e$

$m_p = 1.67 \times 10^{-27} \text{ Kg}$

$q_p = +e$

$m_n = 1.67 \times 10^{-27} \text{ Kg}$

$q_n = 0$

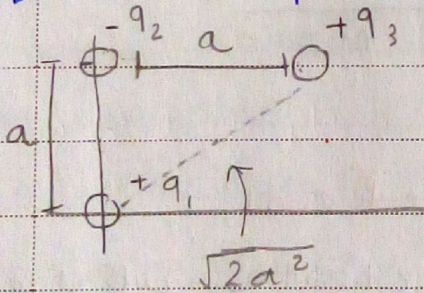
$F_e = \frac{K_e q_1 q_2}{r^2}$   
 $10^{10} \cdot 10^{10} \cdot 10^{38}$   
 $10^{-28}$

$F_g = \frac{G m_1 m_2}{r^2}$   
 $10^{-11} \cdot 10^{-30} \cdot 10^{-27}$   
 $10^{-68}$

$F_g \rightarrow$  ثابت الجذب \* ل1 \* ل2 \* (المسافة)<sup>-2</sup>

$\frac{F_e}{F_g} \approx \frac{10^{-28}}{10^{-68}} \approx 10^{40}$  So...  $|F_e| \gg |F_g|$  For elementary charges.

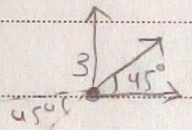
→ See example 23.2 p. 696 & 23.4 p. 698



$\vec{F}_{e \text{ on } q_3} = \vec{F}_{e1 \text{ on } 3} + \vec{F}_{e2 \text{ on } 3}$

$\vec{F}_{e2 \text{ on } 3} = \frac{K_e |q_2| |q_3|}{a^2} (-\hat{i})$

$\vec{F}_{e1 \text{ on } 3} = \frac{K_e |q_1| |q_3|}{(\sqrt{2}a)^2} [\cos\theta \hat{i} + \sin\theta \hat{j}]$



magnitude  $\rightarrow \sqrt{0^2 + 0^2}$

direction  $\rightarrow \tan^{-1}(\frac{y}{x})$

$\tan\theta = 1 \rightarrow \theta = 45^\circ$

a & a

→  $q_1^+$   $q_3^-$   $q_2^+$  the net force on  $q_3 = 0$  where should we

put it?  $F_{e \text{ on } q_3} = 0 = F_{e1 \text{ on } 3} + F_{e2 \text{ on } 3}$

→  $r = x$   $r = 2-x$

23.4 Analysis model: particle in a <sup>مجال</sup> Field (electric)

→ Field forces (Forces at a distance) :-

Forces acting in a space even if there is no "physical" contact between object.

- Example of a field forces :-

- 1. electric force.
- 2. magnetic force.

→ Electric Field  $\vec{E}$  (vector quantity).

\* the electric force  $\vec{F}_e$  acting on a positive test charge  $q_0$  divided by this test charge. شحنة اختبار موجبة

\*  $q_0$  is small (in order not to disturb the field of the source charge). اتجاه المجال هو اتجاه القوة الكهروستاتيكية (لأن  $q_0$  موجبة)

$$\vec{E} = \frac{\vec{F}_e}{q_0}$$

اتجاه المجال هو اتجاه القوة الكهروستاتيكية (لأن  $q_0$  موجبة)

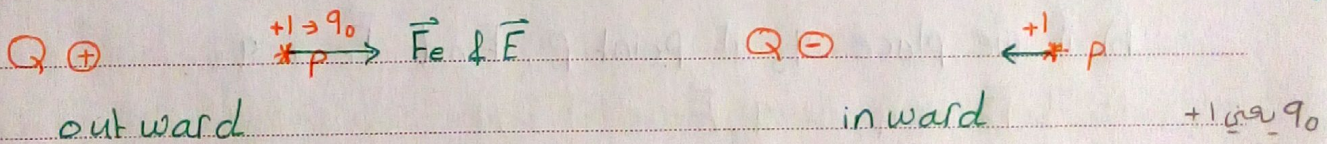
$q_0 =$  Positive test charge معرفة الشحنة السكونية للمجال

it serves as a detector of the field كاشف

\*  $[\vec{E}] = \frac{[F]}{[q]} = \frac{N}{C}$  in SI units &  $\frac{V}{m}$  in SI units.

$1 \frac{N}{C} = 1 \frac{V}{m}$  there is no conversion factor. تحويل

\*  $\vec{F}_e \text{ on } q_0 = q_0 \vec{E}$  → electric field at point P point along the same direction of electric force on +ve test charge



$$\vec{F}_e = \frac{K_e q_1 q_2 \hat{r}}{r^2} = \frac{K_e q_0 Q \hat{r}}{r^2}$$

\*  $\vec{E} = \frac{\vec{F}_e}{q_0} = \frac{K_e |Q| \hat{r}}{r^2} = \vec{E}$  → electric field at point P due to point charge Q.

يوجد فرق بين شحنة متأثرة & مؤثرة (مصدر)

\*For a set of point charges :-

$$\vec{E} \text{ at point } p = \sum \vec{E}_i = Ke \sum q_i \frac{\hat{r}_i}{r_i^2}$$

$\vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2} + \vec{E}_{Q_3} + \dots$  جمع متجهاته / الاتجاه حسب قانون التجاذب & التنافر  
 المجال الكهربائي عند نقطة لا يعتمد على الشحنة الموجودة في النقطة أما القوة الكهربائية فالعكس

Generally :-

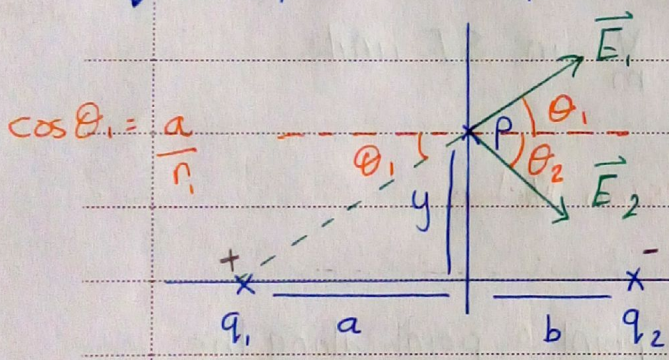
$$\vec{F}_e \text{ on } q = q \vec{E} \text{ at point } p \quad \text{vector equation}$$

affected charge ← due to source charge

$\sum \vec{F}_e \text{ on } q = q \sum \vec{E} \text{ at point } p$  ← لو فكينا القانون بطرح نفس قانون كولوم

$\rightarrow q$  if  $q$  is +ve  $\rightarrow \vec{F}_{\text{on } q} \parallel \vec{E}$   
 if  $q$  is -ve  $\rightarrow \vec{F}_{\text{on } q}$  opposite  $\vec{E}$

Example 23.6 p. 702



a)  $\vec{E}_1 = \frac{Keq_1}{r_1^2} [\cos \theta_1 (\hat{i}) + \sin \theta_1 (\hat{j})]$

$\vec{E}_2 = \frac{Keq_2}{r_2^2} [\cos \theta_2 (\hat{i}) + \sin \theta_2 (-\hat{j})]$

$\vec{E} \text{ at } p = \vec{E}_1 \text{ at } p + \vec{E}_2 \text{ at } p = \square \text{ N/C}$

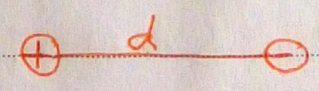
b) if we place  $q_3$  at point  $p$ , Find  $\vec{F}_e$  on  $q_3$

$\vec{F}_e \text{ on } q_3 = q_3 \vec{E} = \square \text{ N}$

نعوض بالإشارة  $\leftarrow +$  مع  $\leftarrow -$  عكس  
 يوجد مثال آخر بدل  $a < b$  سيكون  $\frac{1}{y^3} E \propto a$  إذا  $a >> y$  (وتكون المسافة في النصف)  $\leftarrow$  أكبر يوجد مجال  
 الشاطئ الكهربائي

Electrical dipole

a system of two charges (equal in magnitude but with opposite signs) separated by a distance.



23.5  $\vec{E}$  due to continuous charge distribution توزيع

Two types of continuous charge distribution.

1] Uniform con. ch. dis.

1) uniform linear charge density (1D)  $\lambda = \frac{Q}{l}$   $[\lambda] = \frac{C}{m}$

2) uniform surface charge density (2D)  $\sigma = \frac{Q}{A}$   $[\sigma] = \frac{C}{m^2}$   
area  $\rightarrow A$

3) uniform volume charge density (3D)  $\rho = \frac{Q}{V}$   $[\rho] = \frac{C}{m^3}$   
charge density here is constant

2] Non-uniform con. ch. dis.

1) non-uniform linear charge density (1D)  $\lambda = \frac{dQ}{dl}$

2) non-uniform surface charge density (2D)  $\sigma = \frac{dQ}{dA}$

3) non-uniform volume charge density (3D)  $\rho = \frac{dQ}{dV}$

if charge density =  $2 \text{ C/m}$  charge density per  $1 \text{ cm} =$

$$2 \frac{C}{m} \times \frac{m}{100 \text{ cm}} = .02 \text{ C/cm}$$

How to Find electric field due to con. ch. dist. :-

1) divide the charge distribution into small element of charge

$$\Delta q = dq$$

$$2) \Delta \vec{E} (d\vec{E}) = \frac{K_e \Delta q \hat{r}}{r^2} (dq)$$

3) total Field  $\vec{E}_{tot} = \sum \Delta \vec{E}$  if  $\Delta q$  is very small  $\Rightarrow \sum \rightarrow \int$

$$d\vec{E} = \frac{K_e dq \hat{r}}{r^2} \quad \vec{E}_{tot} = \int d\vec{E} = K_e \int \frac{dq \hat{r}}{r^2}$$

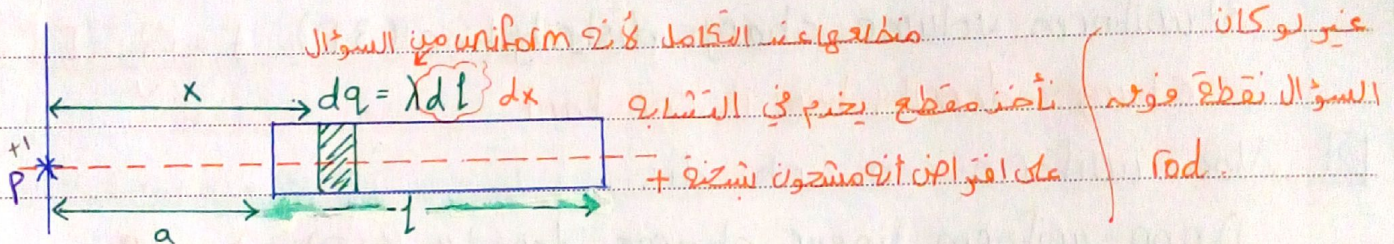
it must be one variable

4) to evaluate the integral use the symmetry of the problem to simplify the integral.

احسب المجال الكهربائي  $E$  في نقطة  $P$  على المحور الذي يمر به قضيب شحنة متساوية الطول  $l$  على مسافة  $a$  من أحد أقطابه.

### The Cases

□  $\vec{E}$  due to uniformly charge rod of finite length ( $l$ ) at a point  $P$  a distance  $a$  along the axis of the rod (example 23.7 p.705)



$$d\vec{E} = \frac{ke dq}{r^2} \hat{r} = \frac{ke \lambda dx}{x^2} (-\hat{i})$$

$$dE_x = \frac{ke \lambda dx}{x^2} \text{ magnitude}$$

$$E_{\text{total}} = \int dE_x = ke \lambda \int_{x=a}^{x=a+l} \frac{dx}{x^2}$$

$$= ke \lambda \left( -\frac{1}{x} \right) \Big|_a^{a+l} = ke \lambda \left[ -\frac{1}{a+l} + \frac{1}{a} \right] = \frac{ke \lambda l}{a(l+a)} Q$$

$\Rightarrow E = \frac{keQ}{a(l+a)}$  → electric field due to uniformly charged rod of length  $l$  at a point at distance  $a$  on the axis of rod.

→ Special cases :-

1) if  $a$  much greater than  $l$   $a \gg l$  (rod is  $a$ )

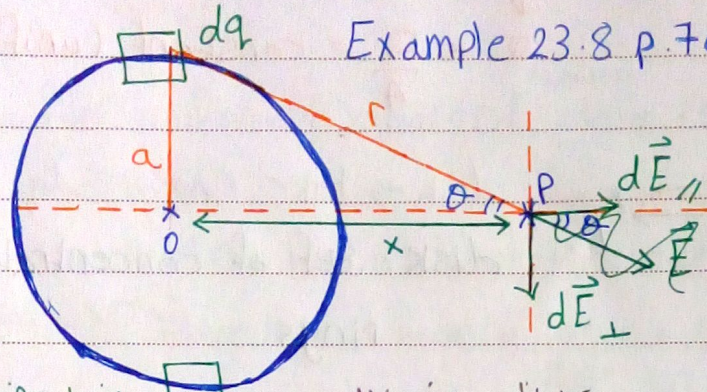
$$E \sim \frac{keQ}{a^2} \text{ (point charge)}$$

2) if  $a \rightarrow 0$  (rod is  $a$ )

$$E \sim \infty$$



2)  $\vec{E}$  due to uniformly charged ring of radius "a" at point P a distance x from the center of the ring. سلك مشحون دائري نصفه على مسك رأسيه



Example 23.8 p 706

فردن ان مشحون بشحنه +

x & a are the same for all dq's

كل تغير يلغي نظيره يلغوا بعض

$d\vec{E} = \frac{Ke dq}{r^2} \hat{r} = \frac{Ke dq}{x^2+a^2} \hat{r}$  by symmetry all  $dE_{\perp}$  add to zero. المركبات العمودية صفر

نحسب المركبات الافقيه فقط

$E_{tot} = \int dE_x = \int \frac{Ke dq}{x^2+a^2} \cos\theta \cdot \frac{x}{r} = \frac{x}{(x^2+a^2)^{3/2}}$

$= \int \frac{Ke dq x}{(x^2+a^2)^{3/2}} = \frac{Ke x}{(x^2+a^2)^{3/2}} \int_0^Q dq = \frac{Ke Q x}{(x^2+a^2)^{3/2}}$

$\Rightarrow E = \frac{Ke Q x}{(x^2+a^2)^{3/2}}$  N/c → E due to uniformly charged ring of radius "a" at point P a distance x from the center of the ring.

→ Special cases :-

1) at  $x=0$  (النقده على المركز)

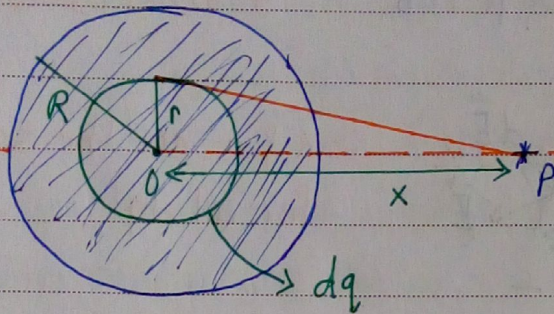
$E=0$

2)  $x \gg a$

$E \sim \frac{Ke Q}{x^2}$  (point charge).

[3]  $\vec{E}$  due to uniformly charged disk of radius  $R$  at point  $p$  a distance  $x$  from the center of the disk.

$$\sigma = \frac{Q}{A} = \text{constant (uniform)}$$



disk  $\rightarrow$  take ring as  $dq$   
 disk: set of concentric rings  
 متصان في المركز

$$dE = \frac{Kedq}{r^2} \text{ but we are going to use } E \text{ ring}$$

$$dq = \sigma * (\text{area of the ring}) = \sigma \cdot 2\pi r dr$$

$$\text{or } \sigma = \frac{Q}{A} = \frac{dq}{dA} \quad dq = \sigma dA \quad (A = \pi r^2) \quad dA = 2\pi r dr$$

$$= \sigma 2\pi r dr$$

$$\rightarrow E_{\text{tot}} = \int dE_{\text{ring}} = \int \frac{KeQx}{(x^2+r^2)^{3/2}} = Ke 2\pi \sigma \int \frac{x r dr}{(x^2+r^2)^{3/2}}$$

(Ering تسمى في الخ)

$x = \text{constant}$  for all  $dq$ 's (rings)

$$= Ke 2\pi \sigma x \int_{r=0}^{r=R} \frac{r dr}{(x^2+r^2)^{3/2}} \quad \text{بالقويض} \quad \text{let } x^2+r^2 = u$$

$$2r dr = du$$

$$\Rightarrow 2\pi Ke \sigma \left[ 1 - \frac{x}{(x^2+R^2)^{1/2}} \right]$$

$$\Rightarrow \boxed{2\pi Ke \sigma \left[ 1 - \frac{x}{(x^2+R^2)^{1/2}} \right]} = E \rightarrow E \text{ due to uniformly charged disk of radius } R \text{ at point } p \text{ a distance } x \text{ from its center.}$$

$\rightarrow$  Special cases :-

1) if  $x \gg R$

$$E \rightarrow 0$$

2) if  $R \gg x$  ( $x \rightarrow 0$  في) (infinitely large disk)

$$E \approx \frac{2\pi k_e \sigma}{4\pi \epsilon_0} = \frac{\sigma}{2\epsilon_0} = \text{constant}$$

لا يعتمد على dimension تبع السؤال

→ For uniformly charged <sup>صفيحة</sup> plate (surface) of infinite extent (R ≫ x):

مقصور نقطة قريبة

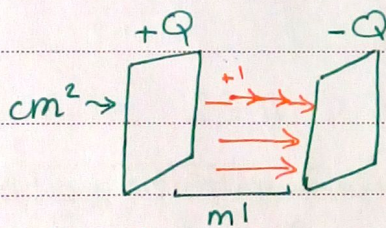
electric field above this surface

$$E \text{ above this surface} = \frac{\sigma}{2\epsilon_0} = \text{constant (uniform)}$$

- independant of distance from the surface

- independant of the shape of the plate

Example: <sup>electric</sup> capacity الواسع الكهربائي

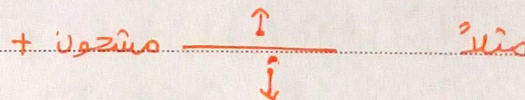



$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2$$

$$|\vec{E}| = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon} = \text{constant}$$

مزال منتظم بعيداً عن الأطراف

direction → مسبقاً الشحنة المشحون بها في موقع الشحنة



density = 

unit dimension (scalar).

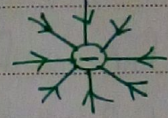
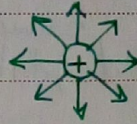
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NO. 23.6

### 23.6 $\vec{E}$ (electric field) lines

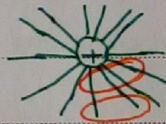
→ 2D - pictorial representation = the direction of  $F_e$  acting on the test charge

الخط الناتج عن وضع شحنة اختبار موجبة بالقرب من شحنة نقطية



→ Properties of  $\vec{E}$  lines :-

- 1) they originate from +ve charge & terminate into -ve charge.
- 2) number of lines  $\propto |Q|$  بغض النظر عن نوع الشحنة  $Q$
- 3) density of  $\vec{E}$  lines  $\propto |\vec{E}|$  عدد الخطوط لكل وحدة مساحة يدل على مقدار القوة (شدة المجال)  
أقرب  $\vec{E}$  أكبر  
number of lines per unit area.



4) No two lines can cross.

انا تقاطعوا يوجد اتجاهين في قوسنا هنا غير ممكن!

فوائد خطوط المجال الكهربائي :-

- ① معرفة اذنا المجال منتظم أم لا ، أي مجال يقبل على شكل خطوط متوازية و المسافات بينها متساوية هو مجال منتظم .

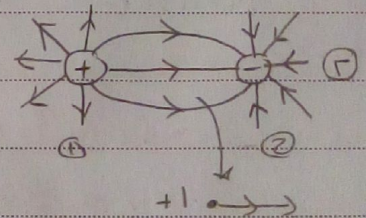
→ مجال غير منتظم (radial) مواضعه مجال منتظم بعيداً

عنا الأمثلة

عدد الخطوط الداخلة = عدد الخطوط الخارجة

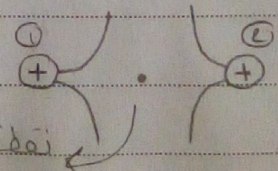
$$q_1 = q_2$$

لو عدد الداخلة = عدد الخارجة  $q_1 = 2q_2$



مستحيل بين disposc نقطة انضمام ممكن في الخارج

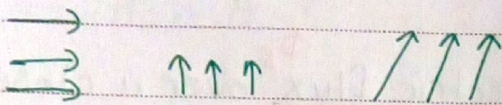
لو  $q_1 = q_2$  نقطة انضمام المجال في النصف



نقطة انضمام المجال = قوة = صفر

لو  $q_1 \neq q_2$

23.7 Motion of a charged particle in uniform electric field  $\vec{E}$   $\vec{E}$  is constant in magnitude & direction



$\vec{F}_e = q\vec{E} = \text{constant}$

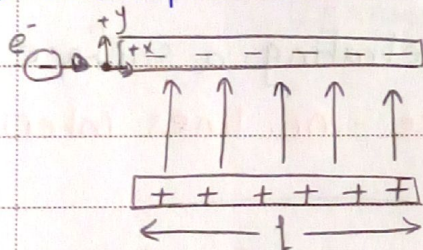
if  $\vec{E}$  is uniform &  $\Sigma \vec{F} = \vec{F}_e$  only  $F_g$  بكون  $\Rightarrow \vec{a} = \text{constant}$   
 motion of particle under cons.  $\vec{a}$   
 معادلات الحركة يتسارع ثابت

$\Sigma \vec{F} = m\vec{a}$      $\vec{F}_e = m\vec{a}$      $q\vec{E} = m\vec{a}$

$\vec{a} = \frac{q\vec{E}}{m}$   $\begin{cases} + \rightarrow a \text{ with the direction of } \vec{E} \\ - \rightarrow a \text{ with the opposite direction of } \vec{E} \end{cases}$

14/2/2016

Example 23.11 p.712



$q = -e$      $v_{ix} = 3 \times 10^6$      $E = 200$      $l = .1$

a?    t?     $y_p$ ?    معادلات الحركة  
 ①    ②    ③    معادلات  $\vec{F}_e$  و  $\vec{F}_g$

①  $\Sigma \vec{F} = m\vec{a}$      $\Sigma \vec{F} = \vec{F}_e + \vec{F}_g$  but  $|\vec{F}_g| \ll |\vec{F}_e| \rightarrow \Sigma \vec{F} = \vec{F}_e$

$\vec{F}_e = m\vec{a}$      $q\vec{E} = m\vec{a}$      $\vec{a} = \frac{q\vec{E}}{m} = \frac{1.6 \times 10^{-19} \times 200 (-\hat{i})}{9.11 \times 10^{-31}} =$   
 $\boxed{35.13 \times 10^{12} (-\hat{i}) \text{ m/s}^2}$

②  $\Delta x = .1$      $\Delta x = v_{ix}t + \frac{1}{2}at^2$  but  $a_x = 0$   
 $\Delta x = v_{ix}t$      $\boxed{t = .33 \times 10^{-7} \text{ s}}$

③  $y_p = y_i + v_{iy}t + \frac{1}{2}at^2$  but  $v_{iy} = 0$   
 $y_p = 0 + 0 + \frac{1}{2} \times 35.13 \times 10^{12} \times .11 \times 10^{-14} = \boxed{-1.95 \times 10^{-2} \text{ m}}$

See example 23.10 p.711

طريقتين : 1/ معادلات الحركة  
 2/ حفظ الطاقة  $w = \Delta K$

NO. Chapter 24

Chapter 24 : Gauss's law

- convenient to calculate  $\vec{E}$  for highly symmetric charge distribution.
- it connects (relates) the Electric flux  $\Phi_E$  over a closed surface to the net charge enclosed the surface to find  $\vec{E}$ .  
 $q_{net, enclosed}$

24.1 Electric Flux  $\Phi_E$  scalar quantity

- the product of the  $|\vec{E}|$  and the surface area  $\perp$  to it.
- Line density (density of  $\vec{E}$  lines)  $\propto |\vec{E}|$   
 $\downarrow$   
 $\frac{\text{number of lines}}{\text{area}} \propto |\vec{E}|$

$|\vec{E}| \times \text{area} \propto \text{no. of lines}$

$\Phi_E \propto \text{net number of lines penetrating a surface}$  (leaving)

number of lines leaving the surface - no. lines entering

→ For uniform  $\vec{E}$

$\Phi_E = \vec{E} \cdot \vec{A} = |\vec{E}| |\vec{A}| \cos\theta = EA \cos\theta$  scalar = vector  $\cdot$  vector

$\theta$ : angle between  $\vec{E}$  &  $\vec{A}$

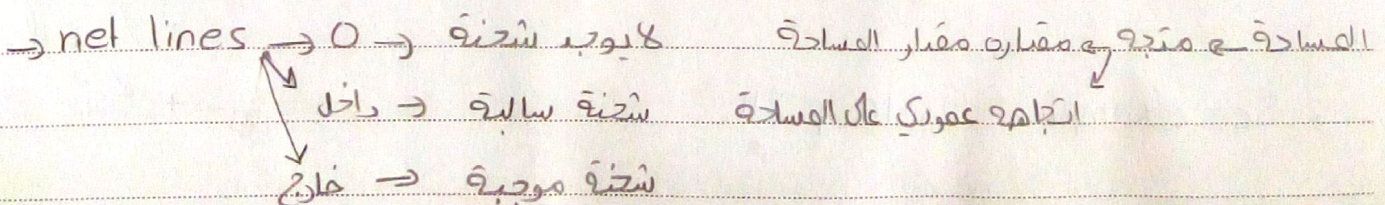
$\vec{A}$ : a vector of magnitude equal to area

$|\vec{A}|$ : area & it's direction is normal ( $\perp$  = perpendicular) to surface.

مفهوم الاتجاه المتجه المساعدة دائماً للأعلى (دائماً في كل من مساعدة و دالة) ليدل.

$[\Phi_E] = \frac{N}{c} \cdot m^2$  in SI units &  $\frac{V}{m} \cdot m^2 = V \cdot m$

دقة السطح ولكن أسهل



- $\Phi_E = 0$  if  $\theta = 90^\circ \rightarrow (\vec{E} \perp \vec{A})$  ( $\vec{E} \parallel \text{surface}$ ) المجال لا يخترق السطح  
 $\Phi_E = EA$  if  $\theta = 0^\circ \rightarrow (\vec{E} \parallel \vec{A})$  ( $\vec{E} \perp \text{surface}$ ) أكبر ما يمكن  
 $\Phi_E = -EA$  if  $\theta = \pi \rightarrow (\vec{E} \text{ opposite } \vec{A})$  ( $\vec{E} \perp \text{surface}$ )  
antiparallel

$\Rightarrow$  For non uniform  $\vec{E}$

$$\Phi_E = \int_{\text{surface}} \vec{E} \cdot d\vec{A} \quad \text{2D not 1D}$$

مجال غير منتظم  $\rightarrow$  تقسم إلى مساحات صغيرة  
(كل مجال منتظم)  $\rightarrow$   $\infty$  تكامل

For a closed surface  $\Phi_{E, \text{net}} = \oint \vec{E} \cdot d\vec{A}$

→ Gauss law :-

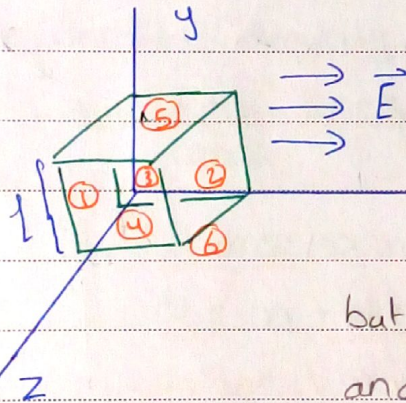
The net electric flux ( $\Phi_E$ ) over a closed surface (Gaussian surface) is equal to the net charge enclosed by the surface ( $q_{net\ enclosed} = q_{net, in}$ ) (inside the surface) divided by  $\epsilon_0$ .

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{net\ enclosed}}{\epsilon_0}$$

→ useful to find  $\vec{E}$  for highly symmetric charge distributions (volume, surface (planar), linear).

$\Phi_E$  يعتمد فقط على  $q_{net, in}$  وليس على أي شيء آخر.

→ Example 24.1



$\vec{E} = E\hat{i}$  ← uniform electric field

Find  $\Phi_E = \oint \vec{E} \cdot d\vec{A}$

$$\vec{E}_{tot} = \int_1 \vec{E} \cdot d\vec{A} + \int_2 \vec{E} \cdot d\vec{A} + \int_3 \vec{E} \cdot d\vec{A} + \int_4 \vec{E} \cdot d\vec{A} + \int_5 \vec{E} \cdot d\vec{A} + \int_6 \vec{E} \cdot d\vec{A}$$

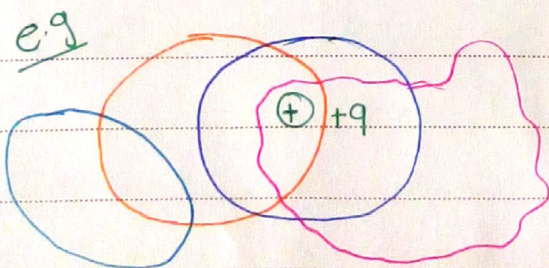
but  $\vec{E}$  on surface 3+4+5+6 = zero

and  $\vec{E}$  is uniform

$$\vec{E}_{tot} = E \int_1 d\vec{A} + E \int_2 d\vec{A} = E l^2 \cos 180 + E l^2 \cos 0 = zero$$

- لو كان المجال متساويًا داخل الجسم  $\Phi_E = q / \epsilon_0$  & لا يهم موقع السطح أو الحجم أو الشكل.

→  $\Phi_{net\ E}$  over a closed surface depends on  $q_{net\ in}$  (net of lines leaving the surface) & it is independent of the shape of the surface or the location of the charge or  $r$  (distance).



$$\Phi_E = \Phi_E = \Phi_E$$

$$\Phi_E = zero$$



## → Gaussian surface (G.S)

a mathematical construction (closed surface) & does not necessary coincide with a real surface.

→ to simplify the integral  $\oint \vec{E} \cdot d\vec{A}$  gaussian surface must have one or more of the following properties :-

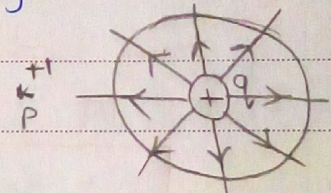
- 1) if  $|\vec{E}| = \text{constant}$  over the surface (gaussian surface).
- 2)  $|\vec{E}| = 0$  over the surface (gaussian surface).
- 3)  $\vec{E} \cdot d\vec{A} = E dA \rightarrow \vec{E} \parallel \vec{A}$  over the surface (gaussian surface).
- 4)  $\vec{E} \cdot d\vec{A} = 0 \rightarrow \vec{E} \perp \vec{A}$  over the surface (gaussian surface).

مراجعة قوانين المساحة & الأحجام (كرة، اسطوانة، ...)

→ Example : Find  $\vec{E}$  due to a point charge using G. law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{net, in}}}{\epsilon_0} \quad \text{but } \theta = 0$$

$$E \int d\vec{A} = E (4\pi r^2) = \frac{q}{\epsilon_0}$$



G.S : a sphere

$$E = \frac{q}{4\pi r^2 \epsilon_0} = \frac{K_e q}{r^2} \quad \# \text{ the same as colomb's law.}$$

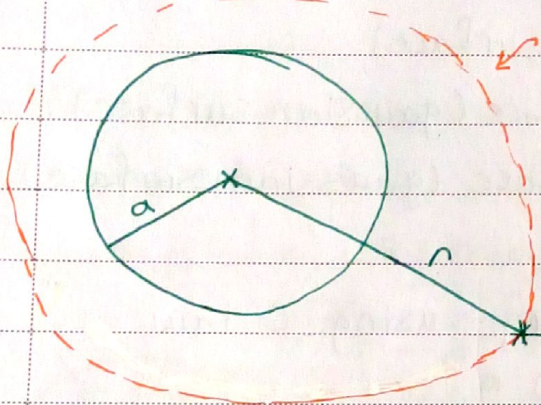
of radius r  
concentric with q

→ See example 24.2 conceptual question.

Applications of G. law to various charge distribution

- 1]  $\vec{E}$  due to uniformly charged insulating solid sphere of radius (a)  
 (Example 24.3 p. 731) (non-conducting) (dielectric)  
 $\rho = \frac{Q}{V} = \text{constant}$  (non-hollow) (non-shell)

[a]  $\vec{E}$  at  $r > a$  (outside the insulating solid sphere)



G.S: a sphere of  $r$  concentric with the charged sphere.

من أجل حساب المجال خارج الكرة، يلزمنا بـ G.S أكبر من الكرة الأصلية & متحدة معها في المركز

$\oint \vec{E} \cdot d\vec{A} = \frac{q_{net, in}}{\epsilon_0}$  but  $\theta = 0$   
 $\cos \theta = 1$

$\oint E dA$  but  $E$  is constant  $\rightarrow E \int dA = E(4\pi r^2) = \frac{Q}{\epsilon_0}$

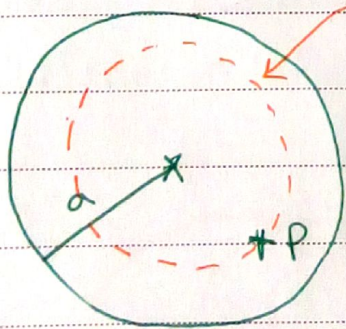
$E = \frac{KeQ}{r^2}$   $r > a$  looks like a point charge

$Q = \rho V = \rho \frac{4}{3} \pi a^3$

$E = \frac{Ke \rho \frac{4}{3} \pi a^3}{r^2} \propto \frac{1}{r^2}$

الكرة الأصلية (المتحدة معها)

[b]  $\vec{E}$  at  $r < a$  (inside the insulating solid sphere)



G.S: a sphere of  $r < a$  concentric with the solid sphere.

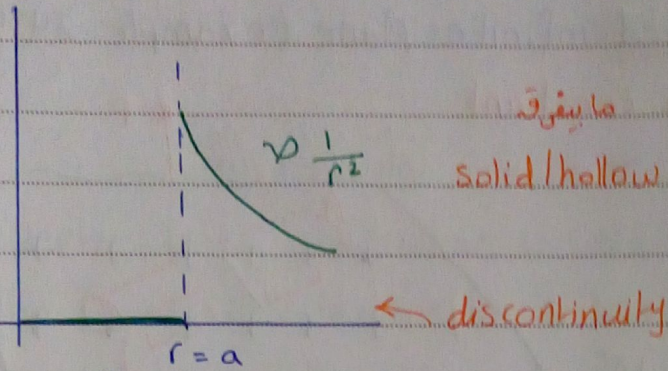
$\oint \vec{E} \cdot d\vec{A} = \frac{q_{net, in}}{\epsilon_0} = E \int dA$   $E$  is constant  
 $\theta = 0$

$E \int dA = E(4\pi r^2) = \frac{q_{net, in}}{\epsilon_0}$   $q_{net, in} = \rho V_{G.S.} = \rho \frac{4}{3} \pi r^3$

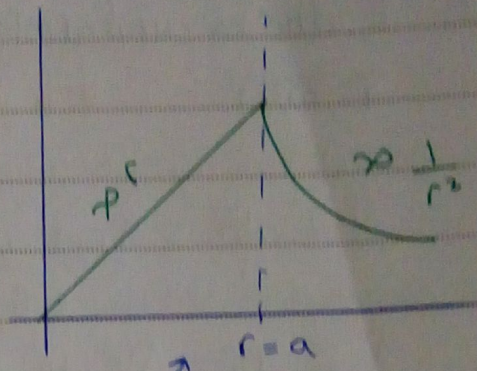
$E(4\pi r^2) = \frac{\rho \frac{4}{3} \pi r^3}{\epsilon_0}$

$E = \rho r = \frac{Q r}{4\pi \epsilon_0 a^3}$   $r < a$   $\propto r$

$q_{net, in} = \rho V_{G.S.} = \rho \frac{4}{3} \pi r^3$



$\vec{E}$  conducting sphere uniformly charged.



$\vec{E}$  solid insulating uniformly charged.

21/2/2015

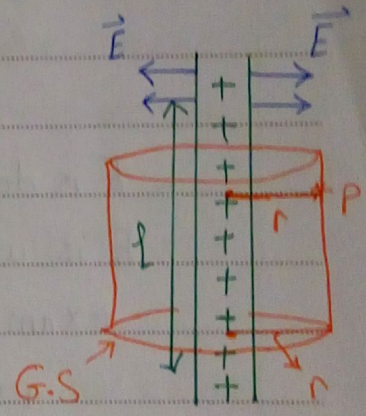
2  $\vec{E}$  at a distance  $r$  from a uniformly charged rod of infinite length (example 24.4 p.733 cylindrical charge distribution).

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{net, in}}{\epsilon_0}$$

$$\lambda = \frac{Q}{L}$$

$$\int_{\text{curved part}} \vec{E} \cdot d\vec{A} + \int_{\text{zero}} \vec{E} \cdot d\vec{A} = E \int dA = E(2\pi r L)$$

$$\Rightarrow E(2\pi r L) = \frac{\lambda L}{\epsilon_0}$$



$$E \text{ at } r = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k_e \lambda}{r}$$

أو  $\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \hat{r}$

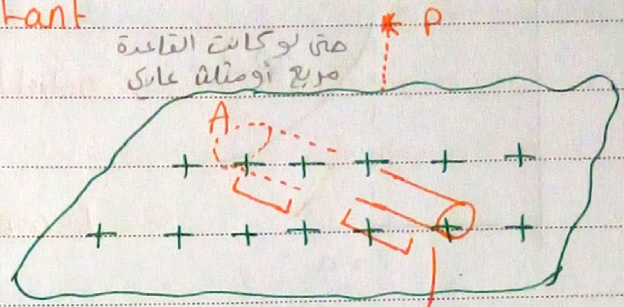
أو  $\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \hat{r}$  أو  $\vec{E} = \frac{2k_e \lambda}{r} \hat{r}$

3]  $\vec{E}$  due to uniformly charged infinite plane (example 24.5 p. 734)

$\sigma = \frac{Q}{A_{total}} = \text{constant}$

مربع لو كانت القاعدة مربع أو مثلث على

$\oint \vec{E} \cdot d\vec{A} = \frac{q_{net.in}}{\epsilon_0}$



Pill-box shaped cylinder with 2 ends of area A equidistant from the plane.

$\int_{end1} \vec{E} \cdot d\vec{A} + \int_{end2} \vec{E} \cdot d\vec{A} + \int_{curved\ part} \vec{E} \cdot d\vec{A}$

$E \parallel A (\theta = 0)$

$E \perp A (\theta = 90^\circ)$

$\int_{end1} E dA + \int_{end2} E dA$

$EA + EA = 2EA$

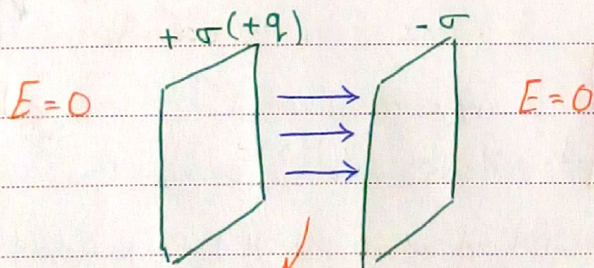
$2EA = \frac{\sigma A}{\epsilon_0}$

$E = \frac{\sigma}{2\epsilon_0}$

constant & independent of location of p; (distance of p from the plane), dimension and the shape of plane.

- E is downward (p below the plane) if it have +ve charge
- E is upward (p above the plane)

example:  $\vec{E}$  between 2 uniformly charged plates



$E = \frac{\sigma}{\epsilon_0}$

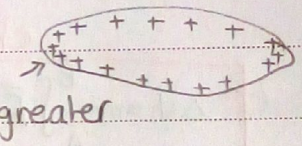
### Conductors in electrostatic equilibrium

there is no net motion of charges inside the conductor

لو كان هناك net motion لكانت تيار كهربائي ← يوجد  $\vec{E}$

#### → Properties of a conductor in electrostatic equilibrium

- 1) The  $\vec{E}$  inside a conductor is always zero whether the conductor is solid or hollow.
- 2) IF the conductor is charged, the charge will instantly reside on it's surface.
- 3) The electric field just outside the conductor is  $\perp$  to surface and has a magnitude  $\frac{\sigma}{\epsilon_0}$  (constant)
- 4) For an irregularly shaped conductor, the surface charge density ( $\sigma$ ) is greatest when the radius of curvature is smallest (later we will prove it).



#### Proof of the properties

1)  $\vec{E}$  (external) because there are free charge carrier  
 $\vec{E}$  induced  
 when  $\vec{E}_{induced} = \vec{E}_{external}$   $E_{inside} = zero$  (equilibrium)

2)  $\oint \vec{E} \cdot d\vec{A}$  (inside) =  $\frac{q_{net}}{\epsilon_0}$   
 zero  $\uparrow$  zero  $\uparrow$   $\epsilon_0$

3)  $\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$   $EA = \frac{\sigma A}{\epsilon_0}$  There is no  $\vec{E}$  in curved part  
 $E$  above conductor =  $\frac{\sigma}{\epsilon_0}$   
 (لو يوجد لتحركت الشحنات)

25.1 Electric potential & Potential difference

→ E. Force  $\vec{F}_e = q \vec{E}$

it is conservative

الطاقة الكهروستاتيكية النظام محفوظ  
النقل المسار مغلق = مسير (لا يعتمد النقل على المسار)  
it is associated with potential energy.

$\Delta U_{eF} = -W_{eF}$  - لو قوة خارجية نرفع + بدل -

$\Delta U_E = -W_{F_e} = -\int F_e \cdot d\vec{s} = -q \int \vec{E} \cdot d\vec{s}$

$\frac{\Delta U_E}{q} = -\int_A^B \vec{E} \cdot d\vec{s}$  x, y & z

$\Delta V =$  electric potential difference / potential difference (P.D) / voltage.

= electrostatic potential energy (P.E) divided by test charge. Scalar quantity

P.D  $\left[ \Delta V = -\int_A^B \vec{E} \cdot d\vec{s} \right]$  → line integral (path) (not as  $\int \vec{E} \cdot d\vec{A}$ )

$[\Delta V] = \frac{J}{C} = \text{volt} = V$      $1V = \frac{J}{C}$  & another SI unit

For electric field is  $\frac{V}{m}$      $\frac{N}{C} = \frac{V}{m}$  \* مقدار الطاقة التي اكتسبها شحنة اختبار صغيرة موجبة اذا تحركت تحت فعله = افولت

→ another unit for energy: e.V = electronvolt (a non-SI unit of energy).

energy gained / lost if a charge of magnitude "e" moves in P.D of 1 V.

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ C} \cdot 1 \text{ V} = 1.6 \times 10^{-19} \text{ J}$      $J > \text{eV}$

$\Delta U_E = q \Delta V$

$$\Delta V = - \int \vec{E} \cdot d\vec{s} = \frac{\Delta u}{q}$$

→ Electric potential : electrostatic potential energy divided by test charge  $q = e \cdot P.E$  per unit charge.

$$V = \frac{u}{q}$$

لا نستطيع معرفة مقدار  $V_A/V_B$  بشكل دقيق، إذا أردنا معرفة ذلك يجب أن نعرف الظروف الابتدائية.

⇒ Potential difference in uniform  $\vec{E}$  :-

$$\Delta V = - \int_A^B \vec{E} \cdot d\vec{s} = V_B - V_A \quad \text{if } \vec{E} \text{ is uniform (path independent)}$$

$$\Delta V = -E \cdot \int d\vec{s} = -E \cdot s$$

$$\Delta V = -\vec{E} \cdot \vec{s} \quad \text{for uniform } E$$

$\vec{E}$  lines always point

كلما سرتنا مع خطوط المجال يقل الجهد. with decreasing potential.

$$\Delta V = -\vec{E} \cdot \vec{s} = 0 \quad \text{if } \vec{E} \perp \vec{s} \quad E=0$$

$$V_B = V_A = \text{constant} \quad \theta = 90^\circ$$

"equipotential surface" مقيمين للتنبؤ بخطوط المجال

$\vec{E}$  lines always  $\perp$  to equipotential surfaces.

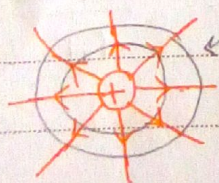
→ Equipotential surface :- مقيمين أيضاً

- surfaces with point of equal electric potential ( $\Delta V = 0$ )
- $\vec{E}$  lines  $\perp$  on them
- no work is done to move any  $q$  on equipotential surface.

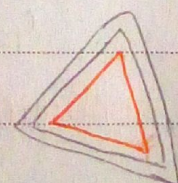
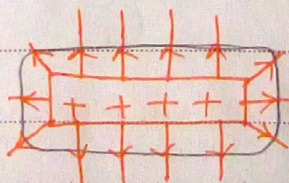
e.g. conductor in electrostatic equilibrium

$\vec{E}$  inside conductor = 0  $\Delta V = 0$  → the conductor is equipotential surface.

See example 25.1 p.750 & 25.2 p.751



equipotential line (surface)



خطوط المجال تتجه نحو تناقص الجهد  
خطوط تساوي الجهد تأخذ شكل الجسم المتشوه.

NO. 25. 3

⇒ Electric Potential & Potential energy due to point charges

$$\Delta V = - \int_A^B \vec{E} \cdot d\vec{s} \quad \text{For a point charge } \vec{E} = \frac{Kq}{r^2} \hat{r}$$

$$\Delta V = - \int_A^B \frac{Kq}{r^2} \hat{r} \cdot d\vec{s} \quad dr$$

$$\Delta V = -Kq \int_{r_A}^{r_B} \frac{dr}{r^2} = -Kq \left( \frac{-1}{r} \right) \Big|_{r_A}^{r_B} = +Kq \left( \frac{1}{r} \right) \Big|_{r_A}^{r_B}$$

$$V_B - V_A = \frac{Kq}{r_B} - \frac{Kq}{r_A} \quad \text{choose } V_A = 0 \text{ at } r_A = \infty$$

(reference configuration)

$$V_B = \frac{Kq}{r_B} \quad \text{Point charge} \rightarrow V = \frac{Kq}{r}$$

$V \rightarrow$  +ve for +veq  
 $V \rightarrow$  -ve for -veq  
 (with  $V(\infty) = 0$ )  
 V is scalar

$$\Delta u = q \Delta V \quad \text{P.E for a point charge } u = qV$$

$$(u \text{ for charge } Q \text{ at Point } P = qV_P) \quad u = QV$$

$$V_P = \frac{Kq}{r}$$

→ P.E between 2 Point charges

$$u = \frac{Kq_1 q_2}{r}$$

if  $q_1 q_2 > 0$  +ve repulsion

if  $q_1 q_2 < 0$  -ve attraction

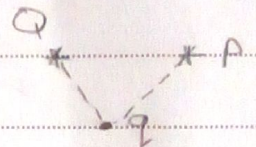
→ For a set of point charges

$U_{\text{system}} = W$  done to bring these charges from infinity (external force)

$$U_{\text{system}} = \frac{Kq_1 q_2}{r_{12}} + \frac{Kq_1 q_3}{r_{13}} + \frac{Kq_2 q_3}{r_{23}} = K \sum_{i,j} \frac{q_i q_j}{r_{ij}}$$

$i=j \equiv$  configuration energy. (Keep track of signs)

28/2





## 25.3 Electric potential &amp; Potential energy due to point charge

→ For a set of point charges

$$V_p = k_e \sum \frac{q_i}{r_i} \quad (\text{Keep track of signs}) \quad \text{دالة الجهد}$$

$$\boxed{W \text{ transfer a charge} = q \Delta V = q (V_B - V_A) \text{ From A to B}}$$

Example: if  $\vec{E} = -E \hat{j}$

$$\Delta V = - \int_A^B \vec{E} \cdot d\vec{s} = - \int_A^B E \hat{j} \cdot dy \hat{j} = - \int_A^B E dy = -E \int_A^B dy = -E d$$

$$-Ed$$

$$\boxed{\text{For uniform } \vec{E} \quad \Delta V = -Ed}$$

$E$  is pointing in direction of decreasing  $V$ .

$$\boxed{\Delta U = -qEd} \quad \leftarrow \Delta U = q \Delta V = -qEd \text{ for uniform } \vec{E}$$

↳ if  $q$  is +ve  $\rightarrow \Delta U$  -ve because when +ve charge moves in  $\vec{E}$  P.E  $\downarrow$  & K.E  $\uparrow$

↳ if  $q$  is -ve  $\rightarrow \Delta U$  +ve because when -ve charge moves opposite to  $\vec{E}$  P.E  $\uparrow$  K.E  $\downarrow$

⇒ 25.4 Obtaining the value of  $\vec{E}$  (magnitude) From e. Potential.

→  $\Delta V = - \int_A^B \vec{E} \cdot d\vec{s}$      $\vec{E} = -$  derivative of E.P (V) with respect to coordinates

→ if  $V = V(x, y, z)$  (function)

$$E_x = \frac{-\partial V(x, y, z)}{\partial x} \quad E_y = \frac{-\partial V(x, y, z)}{\partial y} \quad E_z = \frac{-\partial V(x, y, z)}{\partial z}$$

$$\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k} \quad \text{magnitude} = \sqrt{E_x^2 + E_y^2 + E_z^2}$$

if  $V = V(r)$      $E_r = \frac{-\partial V}{\partial r} \Rightarrow \vec{E} = E_r \hat{r}$

Example: For the Point charge

$$V = \frac{K_e q}{r} \quad E_r = \frac{-\partial V}{\partial r} = -K_e q \frac{d}{dr} \left( \frac{1}{r} \right) = \frac{K_e q}{r^2}$$

$$\vec{E} = \frac{K_e q}{r^2} \hat{r} \quad \#$$

⇒ 25.5 electric potential due to continuous charge distribution

We have two methods :-

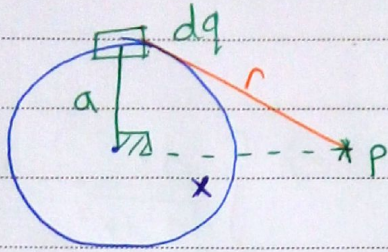
1) For known charge distribution     $dV = \frac{K_e dq}{r}$

$$V = K_e \int \frac{dq}{r} \quad \text{one variable} \\ \text{(integral of scalar)}$$

2) if  $\vec{E}$  is known for the distribution (for highly symmetric charge distribution).

উচ্চ সিমট্রিক

1] V due to uniformly charged ring of radius  $a$  at point  $p$  a distance  $x$  from the center of the ring on the axis passing through its center.   
 $\lambda = \frac{Q}{L} = \text{constant}$   $dq = \lambda dl$



$$dU = \frac{K_e dq}{r} \quad V = K_e \int \frac{\lambda dl}{\sqrt{x^2 + a^2}} =$$

$$\frac{K_e}{\sqrt{x^2 + a^2}} \int dq = \frac{K_e Q}{\sqrt{x^2 + a^2}}$$

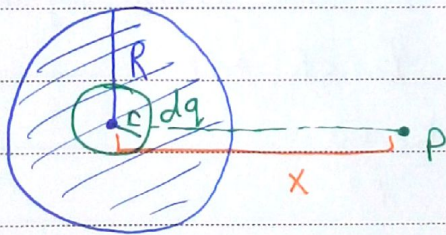
$$V_p = \frac{K_e Q}{\sqrt{x^2 + a^2}}$$

$$E_x = \frac{K_e Q x}{(x^2 + a^2)^{3/2}}$$

$$E_x = -\frac{\partial V}{\partial x} = -K_e Q (x^2 + a^2)^{-3/2} (2x) = -2K_e Q x (x^2 + a^2)^{-3/2}$$

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2] V due to uniformly charged disk of radius " $R$ " at point " $p$ " a distance  $x$  passing through its center. (Example 25.6)



$$\sigma = \frac{Q}{A} = \frac{dq}{dA}$$

$$dU = \frac{K_e dq}{r}$$

$dq = \text{ring}$  (نوعه تين نفس كورس)  
ring

$$dU = d_{\text{ring}} = \frac{K_e dq}{\sqrt{x^2 + r^2}} \quad dq_{\text{ring}} = \sigma 2\pi r dr$$

$$dU = \frac{K_e \sigma 2\pi r dr}{\sqrt{x^2 + r^2}} \quad \text{but } x \text{ is constant} \quad V = K_e \sigma 2\pi \int_0^R \frac{r dr}{\sqrt{x^2 + r^2}}$$

$$\text{let } x^2 + r^2 = u \quad 2r dr = du \quad \int_{x^2}^{x^2 + R^2} \frac{du}{2u^{1/2}} = u^{1/2} \Big|_{x^2}^{x^2 + R^2} = \sqrt{x^2 + R^2} - x$$

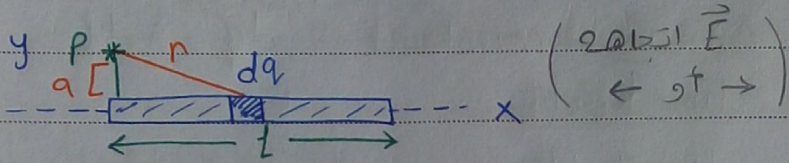
$$V = K_e \sigma 2\pi (\sqrt{x^2 + R^2} - x) \quad \text{Function of } x \text{ only / } \vec{E} \text{ along } x \text{ only}$$

$$E_x = -\frac{\partial V}{\partial x}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

for infinitely charged disk  
طرفه تين

3] V due to uniformly charged rod of finite length "l" at Point P a distance "a"  $\perp$  rod.  $dq = \lambda dl$   $dq = \lambda dx$



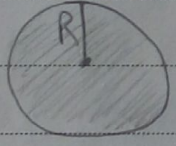
$$dV = \frac{K_e dq}{r} \quad dV = \frac{K_e \lambda dx}{\sqrt{a^2 + x^2}} \quad V = K_e \lambda \int \frac{dx}{\sqrt{a^2 + x^2}} \ln(x + \sqrt{x^2 + a^2})$$

$$V = K_e \lambda \left[ \ln(x + \sqrt{x^2 + a^2}) \right]_0^l = K_e \lambda (\ln(l + \sqrt{l^2 + a^2}) - \ln a)$$

$$V = K_e \lambda \ln \left[ \frac{l + \sqrt{l^2 + a^2}}{a} \right] = \frac{K_e Q}{l} \ln \left[ \frac{l + \sqrt{l^2 + a^2}}{a} \right]$$

$E = -\frac{dV}{da}$  (نسبة التغير في الجهد مع المسافة) لو كان المجال الكهربائي غير متجانس

Example: V due to uniformly charged solid conducting sphere of radius R



$\Delta V = -\int \vec{E} \cdot d\vec{s}$  but  $E_{\text{inside conducting sphere}} = 0$   
 $E_{r < R} = 0$

$E_{r > R} \text{ solid sphere} = \frac{K_e Q}{r^2}$

→ Outside the sphere  $\vec{E}_{r > R} = \frac{K_e Q}{r^2} \hat{r}$

$$\Delta V = -\int \vec{E} \cdot d\vec{s} \quad V_{\infty} - V_r = -\int_r^{\infty} \frac{K_e Q}{r^2} dr$$

$$\Delta V = -K_e Q \int_r^{\infty} \frac{dr}{r^2} = \frac{K_e Q}{r} \Big|_r^{\infty} = \frac{K_e Q}{\infty} - \frac{K_e Q}{r}$$

$$V_{\infty} - V_r = \frac{K_e Q}{\infty} - \frac{K_e Q}{r}$$

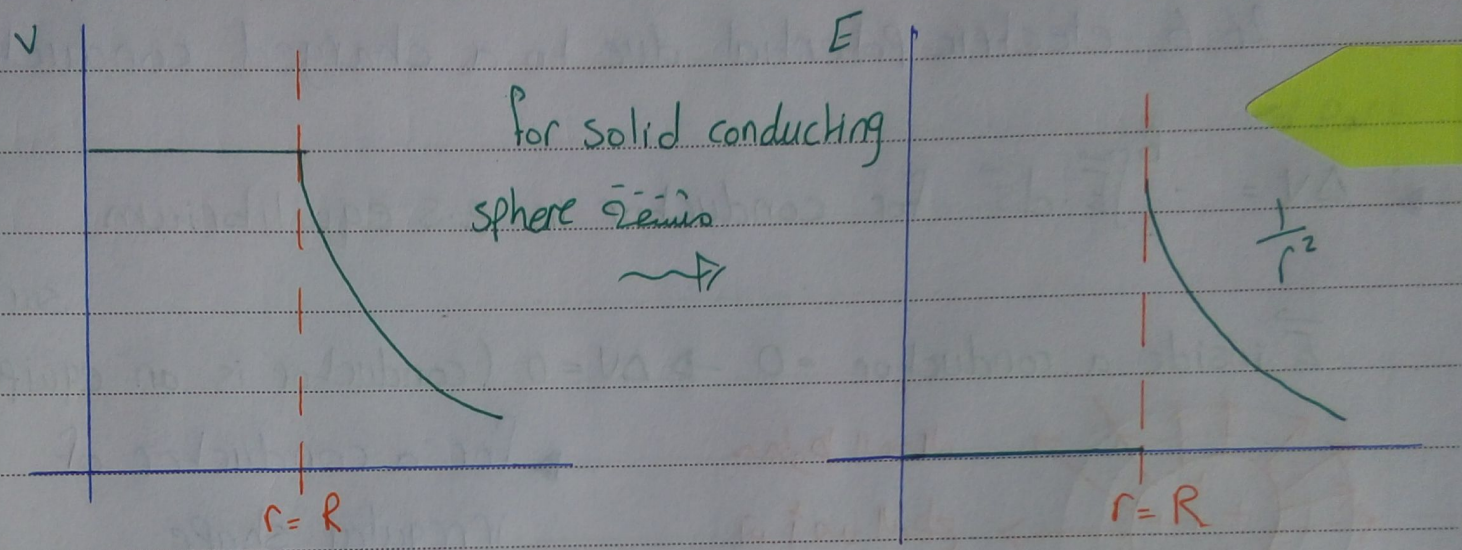
$$V_r = \frac{K_e Q}{r} \text{ look like a point charge}$$

solid shell conductor سواد كمان موصل

$V_r = \frac{K_e Q}{R} \leftarrow r = R \rightarrow$

11/3/2016

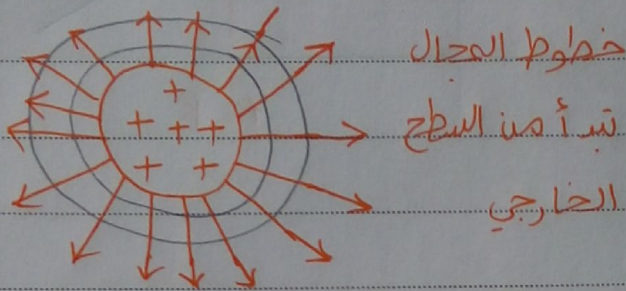
NO. 25.5



26.6 electric potential due to a charged conductor

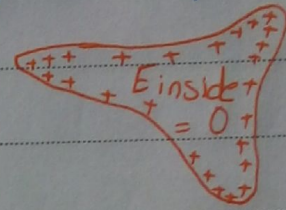
$\Delta V = - \int_A^B \vec{E} \cdot d\vec{s}$  for conductor in e.s equilibrium (surface)

$\vec{E}$  inside a conductor = 0  $\rightarrow \Delta V = 0$  (conductor is an equipotential)



خطوط المجال تبدأ من السطح الخارجي

For a conductor of irregular shape



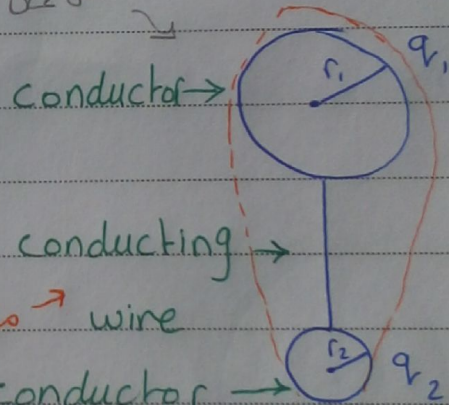
إنبان رقم E من حيث كثافة المجال

$|\vec{E}| \propto \sigma = \frac{Q}{A}$   $\sigma$  is greatest for smallest radius of curvature

Example 25.8 p.762

العديد من الكرات الكبيرة من أنصاف أقطارهم so they look like point charge

$\sigma$  على 2 أكبر



$\frac{E_1}{E_2} ??$

من أجل أن يتساوى wire

$V_1 = V_2$

العهد وليس الشحنة

$\frac{Kq_1}{r_1} = \frac{Kq_2}{r_2}$

$\frac{q_1}{r_1} = \frac{q_2}{r_2}$

$\frac{E_1}{E_2} = \frac{Kq_1/r_1^2}{Kq_2/r_2^2}$

$= \frac{q_1}{q_2} \frac{r_2^2}{r_1^2} = \frac{r_1}{r_2} \frac{r_2^2}{r_1^2} \Rightarrow \frac{E_1}{E_2} = \frac{r_2}{r_1}$

$E_{outside} = \frac{\sigma}{\epsilon_0}$   $E \propto \sigma$  &  $\sigma \propto \frac{1}{\text{radius of curvature}}$

ظاهرة أن العواقر على الأسلاك ذات الجهد الكهربائي لا تسحب بينا يسحب الإنسان؟  
 العواقر  $\Delta V = 0$  لا يسحب تياره لا تسحب الإنسان  $\Delta V \neq 0$  يسحب تياره كهربائي لا تسحب لأنه واقف على الأرض

## NO. Chapter 26

## Chapter 26 : Capacitance &amp; Dielectrics

## 26.1 Definition of capacitance

⇒ Capacitor :-

a device used to store electric charge consist of 2 conductors seperated by a dielectric (insulating = non conducting material) carrying charges of equal in magnitude but opposite sign when attached to a potential difference.

⇒ What is <sup>قدرة</sup> capacitance ?

it is a major of how much charge is stored on a capacitor at certain potential difference.

$$C = \frac{Q}{\Delta V}$$

C: capacitance (it is scalar & always tve).

Q: charge on a capacitor (the quantity of charge on any plate of the capacitor)

$$\Delta V : |\Delta V| = |P.D|$$

$$[C] = \frac{[Q]}{[V]} = \frac{C}{V} = \text{Farad} = F \leftarrow \text{كمية كبيرة جدا}$$

→ micro Farad =  $\mu F = mF$  مقياس الميزال micro

→ micromicro Farad = picofarad =  $pF = 10^{-12} F = mmF$

→ nanoFarad =  $nF = 10^{-9} F$

$$C = \frac{Q}{\Delta V} \leftarrow \text{النسبة ثابتة} \quad \begin{matrix} \uparrow \\ \downarrow \end{matrix} = \text{same (constant)}$$

⇒ Capacitance depends on :-

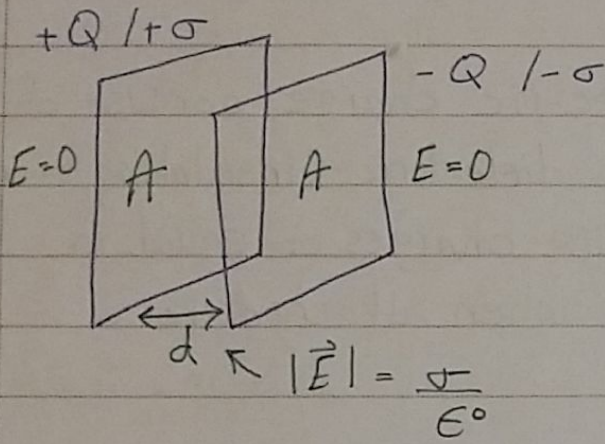
1) geometry of capacitor (size & shape) . . . . .

2) dielectric material. . . . .

C is independent of Q &  $\Delta V$  لا تعتمد على الشحنة أو فرق الجهد

26.2 Calculating the capacitance

1 Parallel plate capacitor



2 parallel conducting plates each of area  $A$ , separated by  $d$

( $d \ll$  length & width of the plate)

$$C = \frac{Q}{\Delta V} \quad \Delta V = - \int_A^B \vec{E} \cdot d\vec{s}$$

For uniform  $\vec{E}$   $\Delta V = -Ed$   $|\Delta V| = Ed$

$$C = \frac{Q}{Ed} \quad \text{but } E = \frac{\sigma}{\epsilon_0} \rightarrow C = \frac{Q}{\frac{\sigma}{\epsilon_0} d} = \frac{Q\epsilon_0}{\frac{Q}{A} d} = \frac{\epsilon_0 A}{d}$$

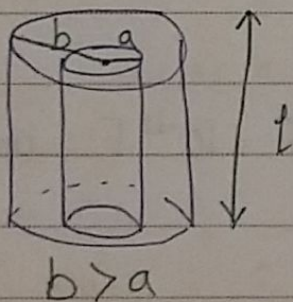
$$\frac{Q\epsilon_0}{\frac{Q}{A} d} = \frac{\epsilon_0 A}{d}$$

$$C = \frac{\epsilon_0 A}{d}$$

$\propto A$   
 $\propto \frac{1}{d}$  ← practical to increase  $C$

↑  $C \propto \frac{Q\epsilon_0}{\frac{Q}{A} d} = \frac{\epsilon_0 A}{d}$

2 C for a cylindrical capacitor



2 cylindrical conductors of a solid cylinder of charge  $Q$  coaxial with a cylindrical shell of charge  $-Q$  the length =  $l$  ( $l \gg a, b$ ).

$$C = \frac{Q}{\Delta V} \rightarrow V_B - V_A$$

$$\Delta V = V_b - V_a = - \int_a^b \vec{E} \cdot d\vec{s} = - \int_a^b \frac{2K\epsilon_0 \lambda}{r} dr =$$

$$- \int_a^b \frac{\lambda}{2\pi\epsilon_0 r} dr = \frac{-\lambda}{2\pi\epsilon_0} \int_a^b \frac{dr}{r} = \frac{-\lambda}{2\pi\epsilon_0} \ln\left(\frac{b}{a}\right) =$$



$$-\frac{2KeQ}{l} \ln\left(\frac{b}{a}\right)$$

$b > a \rightarrow +ve$

$\frac{a}{b}$  ...

$$|\Delta V| = \frac{2KeQ}{l} \ln\left(\frac{b}{a}\right) \quad b > a$$

$$C = \frac{Q}{\frac{2KeQ}{l} \ln\left(\frac{b}{a}\right)} = \frac{l}{2Ke \ln\left(\frac{b}{a}\right)}$$

$$Ke = \frac{1}{4\pi\epsilon^0}$$

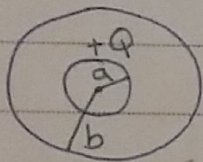
$$C = \frac{l}{2Ke \ln\left(\frac{b}{a}\right)}$$

C for cylindrical capacitor

→ capacitance for unit length  
more practical

$$\frac{C}{l} = \frac{1}{2Ke \ln\left(\frac{b}{a}\right)}$$

3 C for a spherical capacitor



$(b > a)$

2 spherical conductors separated by dielectric / a spherical conducting shell ( $b, -Q$ ) concentric with a solid conducting sphere ( $a, +Q$ )

$$C = \frac{Q}{\Delta V}$$

$$\Delta V = -\int_a^b \vec{E} \cdot d\vec{s}$$

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s}$$

$$= -\int_a^b \frac{KeQ}{r^2} dr = -KeQ \int_a^b \frac{dr}{r^2} = \frac{KeQ}{r} \Big|_a^b =$$

$$KeQ \left( \frac{1}{b} - \frac{1}{a} \right) = \frac{KeQ(b-a)}{ab} = |\Delta V| \quad b > a$$

$$C = \frac{Qab}{KeQ(b-a)}$$

$$C = \frac{ab}{Ke(b-a)} = \frac{4\pi\epsilon^0 ab}{b-a}$$

For spherical capacitor

... b ... C ...

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NO. 26.2

→ Special case

isolated spherical conductor (when  $b \rightarrow \infty$ ) →

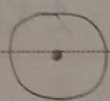
C isolated spherical conductor =  $\lim_{b \rightarrow \infty} C$  spherical conductor

$$= \lim_{b \rightarrow \infty} \frac{4\pi\epsilon_0 ab}{b-a} \approx 4\pi\epsilon_0 \frac{a}{b}$$

C isolated spherical conductor of radius  $a$  =

$$4\pi\epsilon_0 \frac{a}{K_e} = \frac{a}{K_e}$$

radius of conductor



$$V = \frac{K_e q}{r}$$

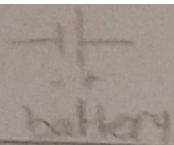
$$= \frac{K_e q}{a}$$

$$C = \frac{q}{K_e q}$$

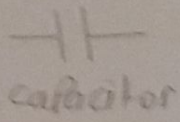
$$= \frac{a}{K_e} \quad \#$$

$$= \frac{a}{K_e} \quad \#$$

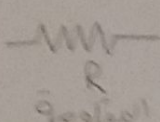
#



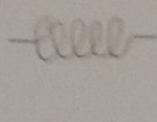
battery



capacitor



R  
resistor



inductor

(No. 26.3)

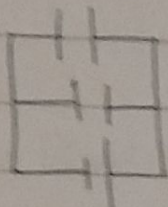
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## 26.3 Combination of capacitors

المواضع بين المتكافئ

⇒ Parallel combination (المطابقة المتوازية)



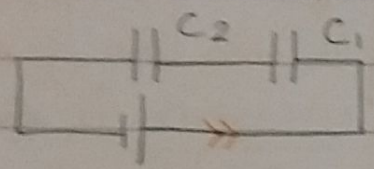
1 ⇒  $\Delta V_1 = \Delta V_2 = \Delta V = \dots$   
 2 ⇒  $C_{\text{equ. p}} = C_1 + C_2 + \dots$   
 →  $C_{\text{equ. p}} > \text{largest } C$

نفس الجهد في التار يقابل

proof of 2

$Q_{\text{tot}} = Q_1 + Q_2 + \dots$  but  $C = \frac{Q}{\Delta V}$   
 $C \Delta V = C_1 \Delta V_1 + C_2 \Delta V_2 + \dots$   
 but  $\Delta V = \Delta V_1 = \Delta V_2 \rightarrow C_{\text{equ. p}} = C_1 + C_2 + \dots$

⇒ Series combination (opposite polarity with each other).



1 ⇒  $\Delta V_{\text{tot}} = \Delta V_1 + \Delta V_2 + \dots$   
 2 ⇒  $\frac{1}{C_{\text{equ. s}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$

→  $C_{\text{equ. s}} < \text{least } C$

proof of 2

$\Delta V_{\text{tot}} = \Delta V_1 + \Delta V_2 + \dots$   
 $\frac{Q}{C_{\text{equ. s}}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \dots$  but  $Q = Q_1 = Q_2$

The inverse

Remember to take

→  $\frac{1}{C_{\text{equ. s}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$

→ For 2 capacitors in series

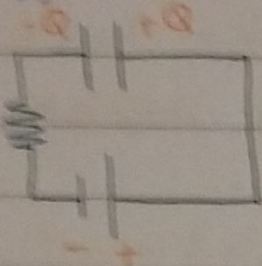
$$C_{\text{equ. s}} = \frac{C_1 C_2}{C_1 + C_2}$$

Just for 2 capacitors in series.

See example 26.3 p 785

Series → same current → same charge.  
 Parallel → same voltage

⇒ energy stored in a charged capacitor



$$dW = \Delta V dq \quad (dq \text{ variable with time})$$

$$W = \int \Delta V dq \quad \Delta V = \frac{q}{C}$$

$$= \int \frac{q}{C} dq \quad \text{but } C \text{ is constant}$$

$$= \frac{1}{C} \int_0^Q q dq = \frac{1}{C} \left[ \frac{q^2}{2} \right]_0^Q = \frac{1}{2} \frac{Q^2}{C} = \frac{Q^2}{2C}$$

capital  $U = \frac{Q^2}{2C}$  the energy stored in a charged capacitor

but  $C = \frac{Q}{\Delta V}$

$$U = \frac{1}{2} C (\Delta V)^2$$

← it looks like  $\frac{1}{2} m v^2$

$$U = \frac{1}{2} Q \Delta V$$

$$[U] = [\text{energy}] = \text{J}$$

⇒ Energy density  $\equiv$  energy per unit volume

$$u = \frac{U}{\text{Volume}} \quad [\text{energy density}] = \text{J/m}^3$$

e.g. parallel plate capacitor

$$C = \frac{\epsilon_0 A}{d} \quad \Delta V = -Ed$$

$$U = \frac{1}{2} \frac{\epsilon_0 A}{d} E^2 d^2 = \frac{1}{2} \epsilon_0 E^2 (Ad) \xrightarrow{\text{volume}}$$

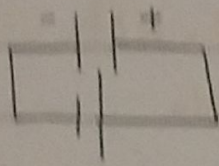
↑  
magnitude

$$u = \frac{1}{2} \epsilon_0 E^2$$

valid for any capacitor &  $u = \frac{1}{2} \epsilon_0 E^2$

کروائیے بجوں و سولہ کرمانیہ

26.5 Capacitors with dielectrics



$V_{\text{capacitor}} = V_{\text{battery}} \rightarrow$  open circuit

\* Dielectric / non conducting material

e.g. air / wax / rubber / wood / ...

→ To improve the capacitance of a capacitor (parallel plate capacitor)

$C = \frac{\epsilon A}{d}$  by decreasing  $d$  (separation between 2 conductors)

by inserting dielectric (دielektrik)

[A] → if dielectric is inserted

$C = KC_0$   $C_0$ : without dielectric (dielectric of air = 1)

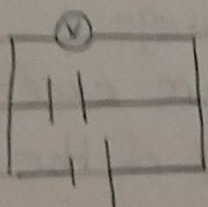
$C > C_0$   $K: K > 1$  dielectric constant

$[K] =$  unitless (بى بىمىرلىق)

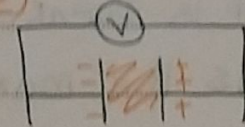
$C = \frac{K\epsilon_0 A}{d} = \frac{\epsilon A}{d}$   $\epsilon$ : permittivity of substance

$C = Q/\Delta V$   $C_0 = Q_0/\Delta V_0$

① → inserting dielectric in a capacitor while the battery is disconnected. Potential difference will change /  $Q$  same (decrease)

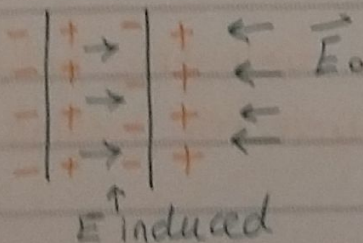


$C_0 = \frac{Q_0}{\Delta V_0}$  (decrease)



$C = KC_0 \rightarrow Q = Q_0$

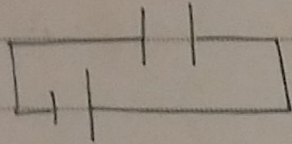
$\Delta V \downarrow \Rightarrow \Delta V = \frac{\Delta V_0}{K}$



$\vec{E} = \vec{E}_0 - \vec{E}_{\text{induced}} / \times d$   $\text{die } E = 0 \text{ in conductor}$   
 $\vec{V} = \vec{V}_0 - \vec{V}_{\text{induced}}$   $\text{conductor jin kis carriers}$

② → inserting dielectric in a capacitor while the battery is still connected  $Q$  increases  $Q = KQ_0$

$\Delta V$  remains =  $\Delta V_0$



$$C = \frac{KQ_0}{\Delta V_0} = KC_0$$

لازم في السؤال نعرف أي العاليتين يقصد

عند الصدمات متساوي  
open circuit ←  
التيار الكهربائي

→ examples of K :-

أكثرها الهواء ٣-٤ مرات

Free space

K (dry air) = 1.00059

K (nylon) = 3.4

K (vacuum) = 1.000000

See table Page 791

[B] → C can be improved (increased) by inserting dielectric or by decreasing d

this way is limited (districted)

$$\Delta V = -Ed \quad E = \frac{\Delta V}{d}$$

constant ↑  
d ↓

إذا قلنا المسافة في conductor يصبح  $\Delta V$  كبيرة جداً وتنفذ المادة عازليتها

→  $\Delta V > \Delta V_{max}$  → material will loose its properties by the breaks down voltage of the capacitor.

\* What is breakdown voltage ≡ operating voltage ?

it is the max voltage of a capacitor before exceeding the dielectric strength of the insulating of the dielectric material.

the maximum electric field on a dielectric before breaking down occurs (before dielectric begins to conduct).

10/3/2016

NO. 26.5

$$E = \frac{\Delta V}{d} < E_{\max} \quad \checkmark$$

$$E = \frac{\Delta V}{d} > E_{\max} \quad \times$$

ينهار نظام المواسع  
الكهربي

when we decrease d

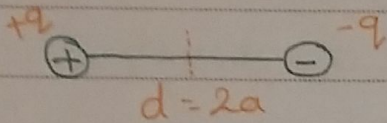
see example 26.5 p. 793

## 26.6 Electric dipole in an uniform electric field

"101" = 101 & "1,100" = 1,100

⇒ electric dipole :-

a system of 2 charges equal in magnitude with opposite sign (+q, -q) separated by a distance (d=2a)

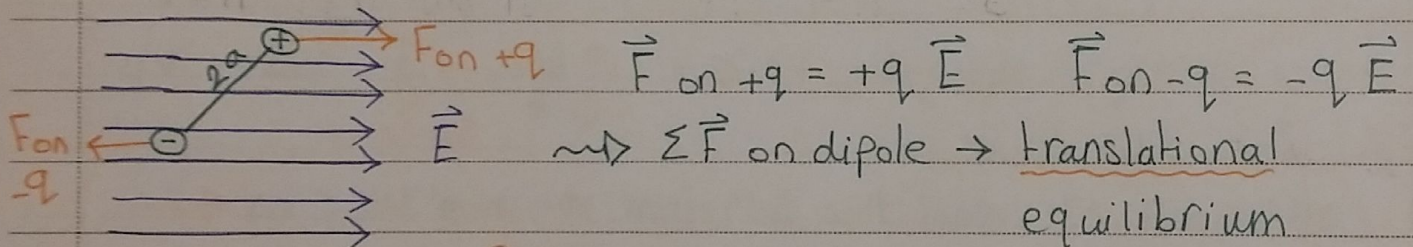


⇒ electric dipole moment (P) vector

$$\vec{P} = q \vec{d} = q 2\vec{a} \quad |\vec{P}| = qd = q2a$$

$[P] = C \cdot m$  & it points from -q to +q

⇒ insert an electric dipole in uniform  $\vec{E}$  makes an angle  $\theta$  with  $\vec{E}$



But the 2 forces have a net torque.

So ... rotationally .. the system is not in equilibrium.

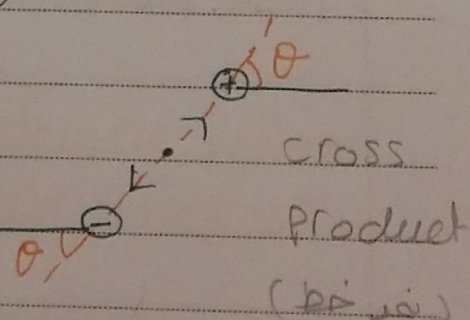
$$\vec{\tau} = \vec{r} \times \vec{F} \quad |\vec{\tau}| = r F \sin \theta \quad \tau_1 = a q E \sin \theta$$

$$\tau_2 = a q E \sin \theta \quad \Sigma \tau = 2 a q E \sin \theta$$

$$\Sigma \tau = P E \sin \theta$$

$$\tau = \vec{P} \times \vec{E} \quad \vec{P} = q \vec{d}$$

$$N \cdot m \quad \frac{N}{C} = \frac{V}{m}$$





13/3/2016

NO. 26.6

⇒ energy stored in e-dipole -  $\vec{E}$  system

$$dU = dW = \tau d\theta \quad (\text{if it is external force} \rightarrow +)$$

( $\vec{E}$  is external)

$$W = \int \tau d\theta = \int PE \sin\theta d\theta = PE \int \sin\theta d\theta =$$

$$U = -PE \cos\theta$$

$$U = -\vec{P} \cdot \vec{E}$$

NO. Chapter 27

Chapter 27 : Current & resistance

27.1 Electric current  $\equiv$  current  $I$

$\Rightarrow$  Current :- <sup>charge carriers</sup>  
the rate of flow of electric charges on a region.

$\hookrightarrow$  Average current  $\bar{I} \equiv I_{avg}$

$$\frac{\Delta Q}{\Delta t}$$

$\hookrightarrow$  inst. current  $I$

$$\lim_{\Delta t \rightarrow 0} I_{avg} = \frac{dq}{dt}$$

$$[I] = \frac{[Q]}{[t]} = \frac{C}{s} \rightarrow \text{Ampere} = A = \text{Amp}$$

$I$  is scalar & it has a direction. (التيار كمية قياسية ولها اتجاه)

$\rightarrow$  the direction of current (conventional) <sup>الاتجاه</sup>

it is the same as the direction of the flow of the charges.

$\rightarrow$  current is produced by moving charges (mobile charge carriers) due to potential difference.

if  $\rightarrow$  P.D =  $\Delta V = 0 \rightarrow I = 0$

$\rightarrow$  P.D =  $\Delta V \neq 0 \rightarrow I \neq 0$

$\rightarrow$  mobile charge carriers are 3 types :-

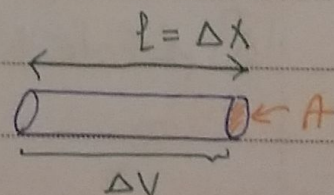
in particle accelerators

- 1) +ve protons /  $H^+$  / any atom when we remove  $e^-$
- 2) -ve in most of conductors (like metals) <sup>المعادن</sup>
- 3) both in gases and electrolytes <sup>الغازات</sup>

$\Rightarrow$  Microscopic model of electric current

a <sup>قطعة</sup> segment of uniform conductor

$$I_{avg} = \frac{\Delta Q}{\Delta t}$$



→  $\Delta Q$ : no. of free charges carriers x charge of each charge carrier.

$$\Delta Q = Ne = ne \text{ Volume}$$

$n$ : number density of charge carriers

→  $n = \frac{\text{total no. of charge carriers}}{\text{Volume}} = \frac{N}{\text{Volume}} \quad [n] = m^{-3}$

$$\text{Volume} = A \Delta x \quad \Delta Q = neA \Delta x$$

$$I_{\text{avg}} = \frac{neA \Delta x}{\Delta t} \quad \text{سرعة الانسياب} \quad \boxed{I_{\text{avg}} = ne v_d A} \quad (*)$$

▷ Drift velocity =  $v_d$  = average velocity of the mobile charge carriers on conductor.

↪ if P.D  $\neq 0 \rightarrow v_d \neq 0 \rightarrow I \neq 0$

↪ if P.D = 0  $\rightarrow v_d = 0 \rightarrow I = 0$

← كلما كانت الدارة أكثر موصولة  $\uparrow I \quad \uparrow v_d$

explanation of (\*)

$n$ : number density of charge carrier  $[n] = m^{-3}$

$e$ :  $1.6 \times 10^{-19} \text{ C}$  قيمة الشحنة

$A$ : cross sectional area: مساحة المقطع العرضي

See example 27.1 P.811 important.

⇒ Current density  $\equiv \vec{J}$

current per unit area  $|\vec{J}| = J = \frac{I}{A} \quad [J] = \text{Amp}/m^2$

## 27.2 Resistance = R

a major of how much <sup>اقباله</sup> opposition a material exhibits to electric current.

R = P.D between the ends of a conductor  
current passing through it

قوة  
التيار  
التي  
تتدفق  
في  
القطب

$$R = \frac{\Delta V}{I}$$

$$[R] = \frac{\text{Volt}}{\text{Amp}} = \text{Ohm} = \Omega$$

⇒ Ohm's law :-

For many conductors (e.g materials); the ratio of the current density & the electric field through this conductor is constant =  $\sigma$  & it is independent of  $|\vec{E}|$

$$\vec{J} \propto \vec{E}$$

$$\vec{J} = \text{const.} \cdot \vec{E}$$

$$\vec{J} \propto \vec{E} \text{ في جميع الحالات}$$

$$\vec{J} = \sigma \vec{E}$$

$$[\sigma] = (\Omega \cdot m)^{-1}$$

$$[J] = \text{Amp} / m^2$$

$$[E] = \frac{N}{C} = \frac{V}{m}$$

if  $J \propto E \rightarrow$  ohmic material  
if not  $\rightarrow$  not Ohmic material

$$I_{avg} = neV_d A$$

$$J = \frac{I}{A} = neV_d$$

⇒ for a segment of uniform conductor (metal)

$$J = \sigma E$$

$$\frac{I}{A} = \frac{\sigma \Delta V}{l}$$

$$R = \frac{\Delta V}{I} = \frac{l}{\sigma A}$$

if it is ohmic

$$R = \frac{l}{\sigma A}$$

R depends on 1) the material of a conductor  
2) geometry of a conductor.

$$R \propto l$$

$$R \propto \frac{1}{A}$$

$$[R] = (\Omega \cdot m)^{-1}$$

$$R = \frac{l}{\sigma A} = \rho \frac{l}{A}$$

$\sigma$ : conductivity

$$[\sigma] = (\Omega \cdot m)^{-1}$$

$\rho$ : resistivity

$$[\rho] = (\Omega \cdot m)$$

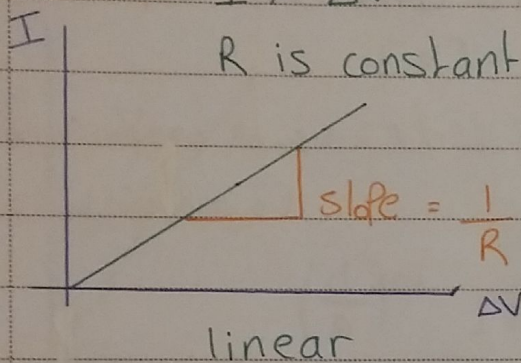
$$\rho = \frac{1}{\sigma}$$

$$\frac{I}{\Delta V} = \frac{A \sigma}{l} = \frac{A}{\rho l} = \frac{1}{R}$$

if the material is ohmic  $\rightarrow \frac{I}{\Delta V} = \text{constant} = \frac{1}{R}$

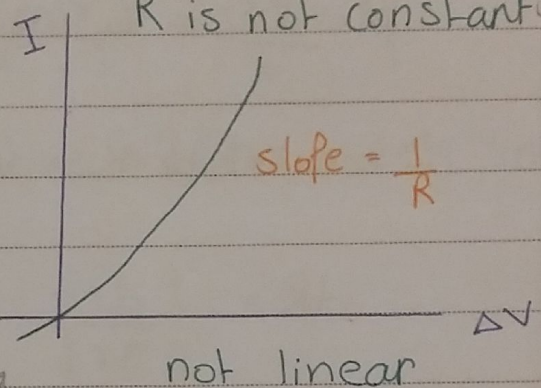
Ohmic material

$I \propto \Delta V$   
R is constant



Non-Ohmic material

R is not constant  
نقطة عمل الجهاز التيار



high R at +ve  $\Delta V$   
low R at -ve  $\Delta V$   
e.g. diode  
من الجهتين  
من الجهتين

$\Rightarrow$  Resistor

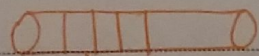
a device that controls current in a circuit.

$\rightarrow$  There are two types of resistors :-

① composition resistor مكوّن من مادّة

e.g. carbon resistor  $\rightarrow$  color coding of carbon resistors

a: brown  
b: red  
c: orange



a b c  $\rightarrow$  uncertainty

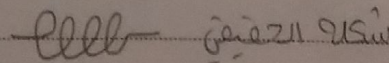
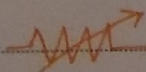
gold  $\pm 5\%$   
silver  $\pm 10\%$   
colorless  $\rightarrow \pm 20\%$

$R = ab \times 10^c \pm \text{uncertainty}$

See page 813

$12 \times 10^3 \pm 5\%$  (if it is gold for example) = 12 K $\Omega$

② wire-wound resistor من الينابيع

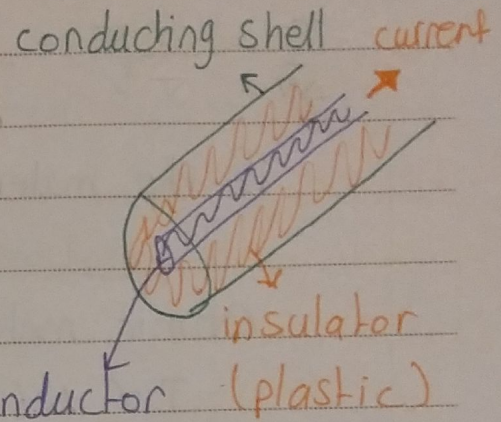


من الينابيع

→ Example 27.3 p. 815

a → smaller radius b → larger radius

→ current must be conducted along the length of coaxial cable (not through the insulator)  $\rho_{ins} = \infty$  R?



$$R = \frac{\rho l}{a}$$

solid conductor

element of R

$$dR = \frac{\rho dr}{A}$$

dr:  $\rho_{ins} = \infty$  (أجزاء صغيرة)

$$dR = \frac{\rho dr}{2\pi r l}$$

$$R = \frac{\rho}{2\pi l} \ln\left(\frac{b}{a}\right)$$

current is more shielded ← R زائد لاس

27.3 is not included

R يتغير بتغير مساحة المقطع

ρ يتغير مع الحرارة / نوع المادة

↑ حرارة ↓ حرارة ↑ حرارة

### 27.4 Resistance & Temperature

For conductors

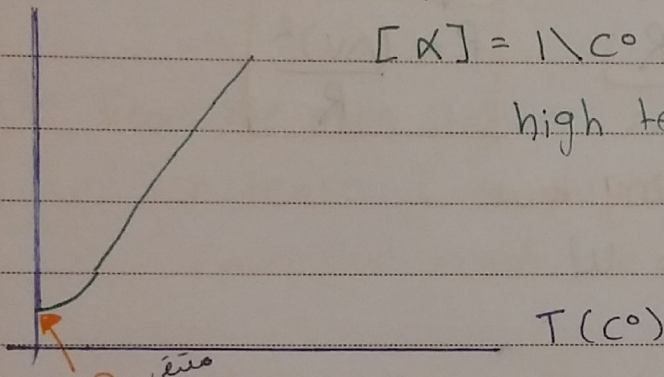
$$\rho = \rho_0 [1 + \alpha (T - T_0)] \rightarrow \text{dimensionless} \quad \text{الخاصة لدرجة الحرارة}$$

$$\rho_0 = \rho \text{ (at } T = T_0 = 20^\circ\text{C)}$$

$\rho$  (Ω.m)  $\alpha$ : temperature coefficient of resistivity

$$[\alpha] = 1/^\circ\text{C} = (^\circ\text{C})^{-1}$$

high temperature  $\rightarrow$  approximately linear



$\rho$  residual due to collision with impurities in the material.

بقية  $\rho$  تعتمد على الشوائب / الشوائب عيوب بالنسبة للموصلات والعكس صحيح

$$\rho = \rho_0 [1 + \alpha (T - T_0)] \quad \alpha \frac{1}{A}$$

$$R_0 = R, \text{ if } T_0 = T$$

$$R = R_0 [1 + \alpha (T - T_0)]$$

$$\alpha = \frac{1}{\rho_0} \frac{\Delta \rho}{\Delta T}$$

كيفية تباين  $\rho$  مع  $T$  - اذا زادت  $T$  تزيد  $\rho$   $\leftarrow \alpha$   
see page 814

$\rightarrow \alpha$  always positive for conductors

$$\Delta T \uparrow \quad \Delta \rho \uparrow$$

$\rightarrow$  For semi conductors  $\alpha$  always negative

$$T \uparrow \quad \rho \downarrow$$

17/3/2016

NO. 27.5

27.5 electrical Power <sup>قدرة</sup>  
electric energy produced (transferred / consumed) <sup>نقل</sup> / <sup>مستهلك</sup>  
per unit time.

$$\Delta U = q \Delta V \quad dU = dq \Delta V$$

$$\text{power} = \frac{dU}{dt} = \frac{dq}{dt} \Delta V \quad \boxed{P = I \Delta V}$$

$$R = \frac{\Delta V}{I}$$

$$\boxed{P = I^2 R}$$

$$\boxed{P = \frac{(\Delta V)^2}{R}}$$

Joule heating law  
الطاقة الحرارية

$$[P] = \frac{J}{s} = \text{Watt} = W$$



⇒ Transporting  $E$  energy through power lines

\* We have high power & -

$$P = I \Delta V$$

→ at low current & high voltage

أوفر اقتصادياً

→ at high current & low voltage

أكثر أماناً

chosen for economic reason.

→ Power lost =  $I^2 R$  through the cable (wires) minimized.

e.g From the main station  $\Delta V \approx 700 \text{ kV}$

increase  $I \rightarrow \Delta V$  reduced to  $4 \text{ kV}$  by transformer

Power that reach the consumer =  $240 \text{ V}$   $\downarrow V \uparrow I$

Chapter 28 : Direct - current circuits

⇒ Two types of current :-

1] direct current (DC)

current that is constant in direction

↳ steady state current (constant in magnitude & direction)

2] alternating current (AC)

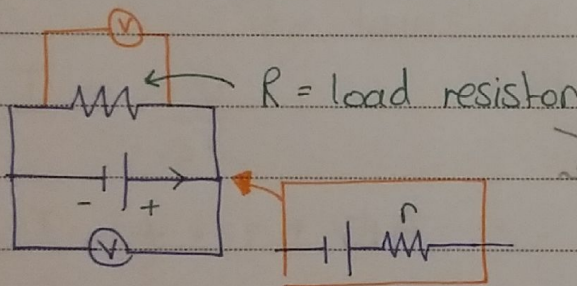
current that changes its direction periodically.

28.1 electromotive force (emf =  $\mathcal{E}$  =  $E$ )

القوة الدافعة الكهربائية

it is not a force

[emf] = volt it is a potential difference (voltage).



$V = \mathcal{E}$

تسمى كذلك لأنها تعبر البطارية على اختلاف فرق الجهد وتحتوي الخلايا المتواليات في البنية الكهربائية

$\mathcal{E} = I_r + IR = I(r+R)$

$\mathcal{E} = I(r+R)$

→ electromotive force : أكبر شغل علينا تكون البنية مفتوحة أو  $r=0$  (المستحيل)

work done by the battery per unit charge = max P.D

that a battery can provide to circuit = terminal voltage

when the circuit is open. قراءة  $V$  أقصى  $\mathcal{E}$  (بفتح الهمزة)

→ terminal voltage = P.D through the external resistance =

P.D measured by a voltmeter =  $\mathcal{E}$  if  $r=0$

↓  
open circuit.

$$\mathcal{E} = Ir + IR \quad / * I$$

$$I \cdot \mathcal{E} = I^2 r + I^2 R$$

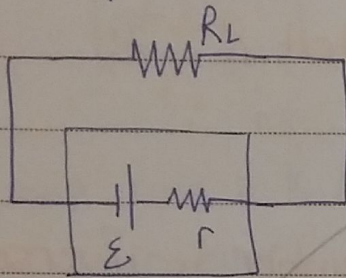
$$P_{\mathcal{E}} = P_r + P_R$$

total  $\leftarrow$  lost by the internal resistance  
 produced by  $\mathcal{E}$  consumed by the device

ر : مقاومة أي شيء في  
 العبارة ما عدا الجهاز المقصود  
 (الأسلاك، أجهزة أخرى، ...)

→ See example 28.1

→ example 28.2 p. 835 important [matching load] Power transfer



Find  $R_L$  for which  $P_L$  is maximum?

$$P_L = I^2 R_L \quad \text{maximum} \rightarrow \text{نصف موزون}$$

$$P_L = I^2 R_L \quad (\text{one variable})$$

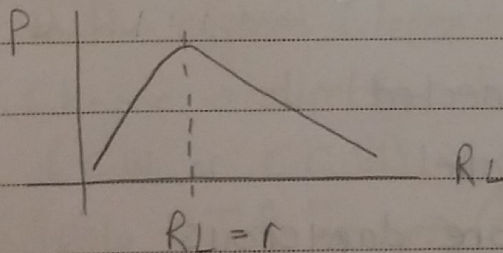
$$\mathcal{E} = Ir + IR_L = I(r + R_L)$$

$$I = \frac{\mathcal{E}}{r + R_L} \quad I \text{ depends on } R_L$$

$$P_L = \frac{\mathcal{E}^2 R_L}{(r + R_L)^2}$$

$$\frac{dP_L}{dR_L} = 0 = \frac{[2\mathcal{E}(r + R_L)^2 + \mathcal{E}^2 2(r + R_L)]}{(r + R_L)^4}$$

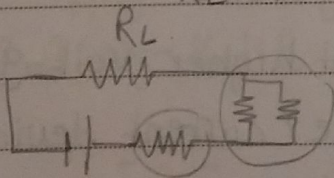
$$[ ] = 0 \rightarrow \boxed{R_L = r}$$



لو يوجد نقطة أخرى أيضاً نشوف مرة أخرى  
 يجب أن تكون مقاومة الجهاز مساوية للمقاومة الأخرى

r : internal resistance  $\equiv$  equivalent resistance of all resistors in the circuit except the device.

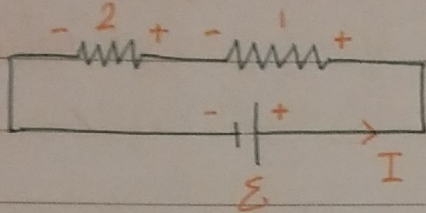
$R_L$  = other resistors to have high power



28.2 Resistors in series & parallel

⇒ Series combination of resistors سلسلة المقاومات

إذا فرغ  
وصرف  
التيار  
تلفى



$\mathcal{E} = \Delta V_1 + \Delta V_2$

$\mathcal{E} = I_1 R_1 + I_2 R_2$  but  $I$  is equal

$R_{equ.s} = R_1 + R_2$

current same

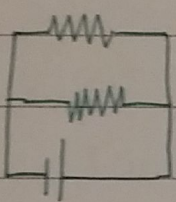
$R_{equ.s} = R_1 + R_2 + \dots$

voltage seperated

$R_{series} > \text{greatest } R$

⇒ Parallel combination of resistors متوازية

إذا فرغ  
وأمر  
يغرب  
جزء



$\frac{1}{R_{equ.p}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$

remember to inverse

$R_{equ.p} < \text{least } R$

only for 2 resistors in parallel  $R_{equ.p} = \frac{R_1 R_2}{R_1 + R_2}$

→ in household combination

parallel combination of resistors → operate independantly

→ light strings combined (connected)

series ← parallel

safer

more dangerous

same current / voltage divided

operated at higher voltage

operated at lower voltage

same voltage / current divided

less brightness / lower temp.

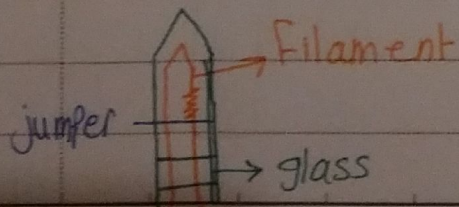
brighter / hotter

dependently operated

independently operated

→ we use miniature light strings wired in series

see example 28.4 & 28.5



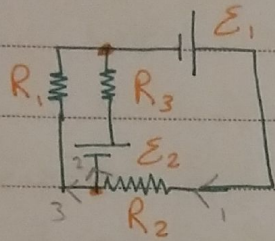
التيار يفتقر، المسار ذو الأقل مقاومة

عندما يتفرغ يصبح كالأسلاك وسلك التيار المسار الآخر & تزداد شدة الاضاءة وتزداد درجة الحرارة / يجذب صليحها

## 28.3 Kirchoff's Rules (to analyze complicated circuits)

1 Junction rule  $\equiv$  K.J.R  $\equiv$  conservation of electric charge

$\sum I = 0$      $\sum I_{in} = \sum I_{out}$      $(I = \frac{dq}{dt})$



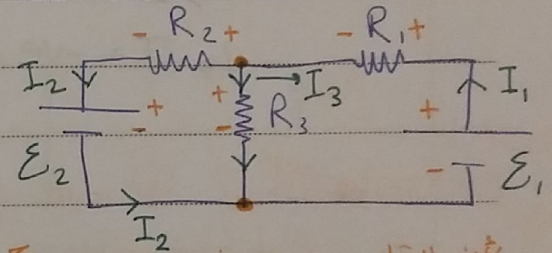
3 loops     $I_1 = I_2 + I_3$   
 2 nodes  $\rightarrow$  one independent node

2 Loop rule  $\equiv$  K.L.R  $\equiv$  conservation of energy

For every closed loop  $\rightarrow \sum \Delta V = 0 \rightarrow \Delta U/q$

$\Rightarrow$  Steps to apply K-Rules in analyzing electric circuits :-

1. Assign a direction for every current in every node.
2. Label the polarity for every element in the current (depends on step 1)
3. Choose a direction for your path (C.W or C.C.W) independent
4. Apply K.J.R for every node in the circuit
5. Apply K.L.R for every loop.
6. solve the equations of unknowns



لأن التيار يتجه من اليمين لليسار  
 الجهد الأول، أو الجهد الآخر  
 $I_1 = I_2 + I_3$

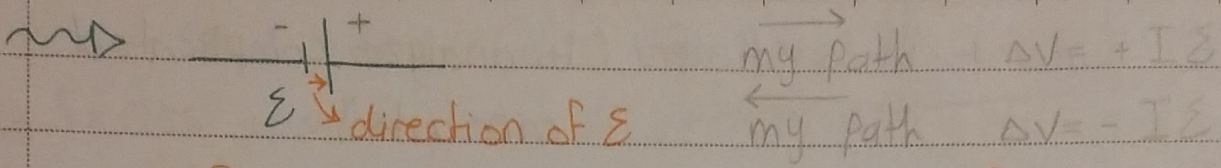
$\rightarrow$   $\xrightarrow{I} \text{---} R \text{---} \leftarrow$  my path  $\Delta V = -IR$      $\xleftarrow{\text{my path}} \Delta V = +IR$

$\rightarrow$  if resistor is transerved along the direction of I  $\rightarrow$  Potential drop. (-)

$\rightarrow$  if resistor is transerved at the opposite direction of I  $\rightarrow$  Potential gain. (+)

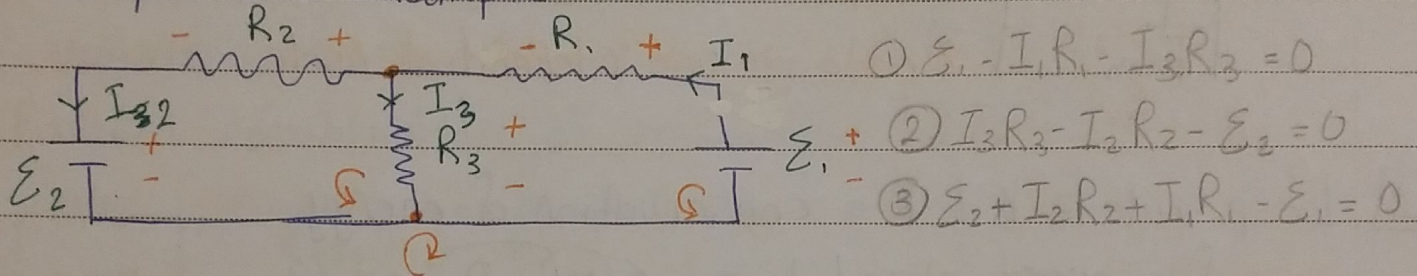
\* no. equations = no. unknowns لا عدد معادلات = لا عدد متغيرات

\*  $\rightarrow$  independent nodes بقية العقد من العقد المتبقية في الدارة



- if an e.m.f is traversed along emf → potential gain
- if an e.m.f is traversed opposite to emf → potential drop

- the previous example

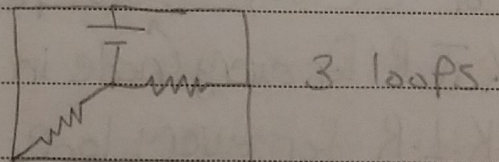


\* Result :

- ↳  $I = +ve$  → you guess for the correct direction.
- ↳  $I = -ve$  → magnitude is correct but in the opposite direction.

\* If  $I$  is  $-ve$  you should continue to use  $-ve$  value in all subsequent calculations.

→ see example 28.6 & 28.7  
closed path → loop



### 28.4 R-C circuits

circuits with resistor & capacitors connected in series.

→ R-C circuits are two types :-

#### 1. Charging R.C circuits

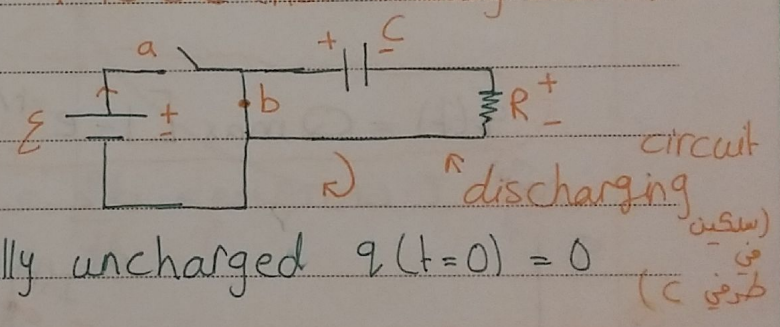
- capacitor is being charged.
- with power supply

#### 2. Discharging R.C circuits

- capacitor is being discharged
- without battery

→ I in R.C circuit depends on time & changing in magnitude and constant in direction). it is DC (not steady state).

$$I = I(t) \neq \text{constant}$$



→ R-C charging circuit

at  $t=0$  capacitor is initially uncharged  $q(t=0) = 0$

at any  $t \rightarrow$  apply K.L.R

$$\sum \Delta V = 0 \quad \varepsilon - \Delta V_C - \Delta V_R = 0$$

$\frac{q}{C}$   $IR$   $q \& I$  not constant

$$\varepsilon - \frac{q}{C} - IR = 0 \quad \text{but } I = \frac{dq}{dt}$$

جولس فیق  $\frac{q}{C}$   $IR$   $q \& I$  not constant

$$\varepsilon - \frac{q(t)}{C} - \frac{dq(t)}{dt} R = 0 \quad \varepsilon - \frac{q}{C} = \frac{dq}{dt} R$$

$$\frac{\varepsilon C - q}{C} = \frac{dq}{dt} R \quad \frac{\varepsilon C - q}{CR} = \frac{dq}{dt}$$

$$\frac{1}{RC} dt = \frac{dq}{\varepsilon C - q} \quad \frac{-1}{RC} dt = \frac{dq}{q - \varepsilon C}$$

→ initial conditions

①  $t=0 \rightarrow \mathcal{E} = I(t=0)R = I_0 R$

$$I_0 = \frac{\mathcal{E}}{R} \quad \underline{I_{max}}$$

$$\Delta V_R = \mathcal{E}$$

②  $t \rightarrow \infty \rightarrow$  capacitor becomes fully charged  $q = Q_{max} = \mathcal{E}C$

$$\Delta V_C = \mathcal{E} \quad I(t \rightarrow \infty) = 0$$

$$\text{but } C = \frac{q}{\Delta V}$$

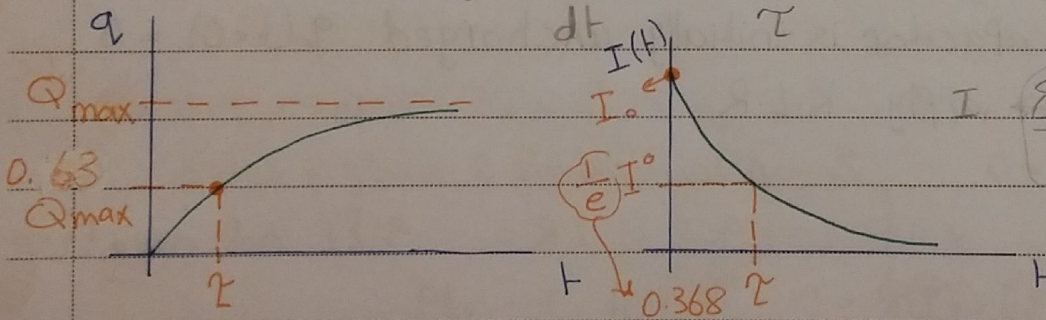
$$\int_0^t \frac{-1}{RC} dt = \int_0^q \frac{dq}{q - \mathcal{E}C} \quad \frac{-t}{RC} = \ln |q - \mathcal{E}C|_0^q$$

$$\frac{-t}{RC} = \ln \left[ \frac{q - \mathcal{E}C}{-\mathcal{E}C} \right] \quad e^{\frac{t}{RC}} = \frac{q - \mathcal{E}C}{-\mathcal{E}C}$$

$$\mathcal{E}C - \mathcal{E}C e^{\frac{t}{RC}} = q(t) \quad q(t) = \mathcal{E}C (1 - e^{-\frac{t}{\tau}})$$

$$q(t) = Q_{max} [1 - e^{-t/\tau}] \quad I = I_0 e^{-t/\tau}$$

$$I_{\text{charging}} = \frac{dq}{dt} = \frac{Q_{max}}{\tau} e^{-t/\tau} = \frac{\mathcal{E}C}{\tau} e^{-t/\tau}$$



$$I = \frac{\mathcal{E}C}{RC} e^{-t/\tau} = I_0 e^{-t/\tau}$$

→ what is  $\tau$ ?

R-C time constant, time needed for  $I$  in R-C circuit to reach  $\frac{1}{e}$  of its max value ( $I_0$ ) or time needed for  $q$  to reach 0.63 of its max value ( $q_0$ ) ( $Q_{max}$ )

\* time  $\rightarrow e^{-t/\tau}$   $e^{\omega}$  → unitless unitless =  $-t \rightarrow$  time

$$\text{or } \tau = [R][C] = \Omega \cdot F = \frac{V}{A} \frac{C}{V} = \tau \text{ time}$$

$$\frac{S}{C} C = S \neq$$

\* constant  $\rightarrow \tau = RC$  R & C are independent of time. FIVE APPLE



→ if we want  $\Delta V_R(t) = IR$

$$\Delta V_R = \frac{q_{ch}(t)}{c} R = \mathcal{E} e^{-t/\tau}$$

$$\Delta V_R = \mathcal{E} e^{-t/\tau}$$

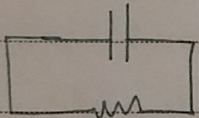
→ if we want  $\Delta V_C(t) = q/c$

$$\Delta V_C = \frac{q_{ch}(t)}{c} = \mathcal{E} (1 - e^{-t/\tau})$$

$$\Delta V_C = \mathcal{E} (1 - e^{-t/\tau})$$

→ we can also find the energy =  $\frac{1}{2} \frac{q^2}{c}$

⇒ R-C discharging circuit

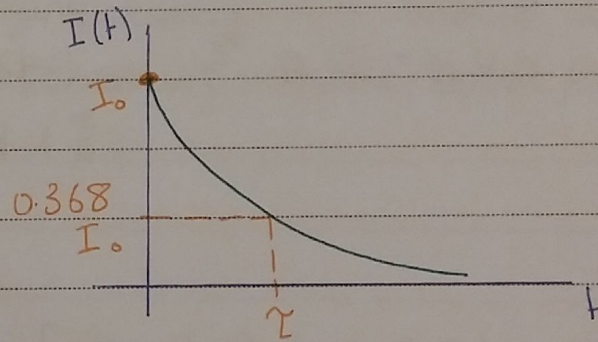
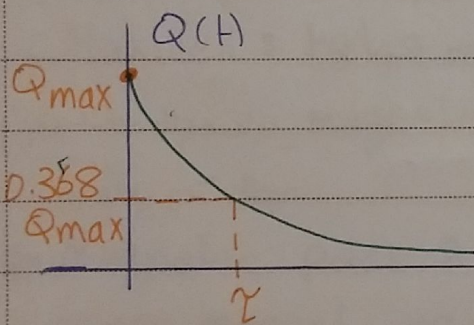


$$Q(t) = Q_{max} e^{-t/\tau}$$

disch

$$I(t) = I_0 e^{-t/\tau}$$

disch



$$|I_{ch}| = |I_{disch}|$$

$$\Delta V_C = \frac{q}{c} = \frac{Q_{max} e^{-t/\tau}}{c} = \frac{\mathcal{E} c e^{-t/\tau}}{c} = \mathcal{E} e^{-t/\tau}$$

and so on...

⇒ Notes :

- 1) initially uncharged C behaves as a short circuit (مثل سلك مقاومته صفر)
- 2) Fully charged C behaves (the circuit is open circuit) (القطع السلك) (تيار = صفر)

# Chapter 29 : Magnetic Field

## ⇒ Historical review :-

- The compass needle was invented by Arabs.
- Greek First observed that the stone magnetite ( $Fe_3O_4$ ) attracts pieces of iron.

\* Electricity and magnetism are related phenomena.

→ Orsted : a compass needle is deflected when placed near an electric current.

→ Faraday : observed that, an electric current can be produced by changing mag. field → moving a magnet relative to loop. / changing current in a nearby circuit.

→ Maxwell : he proved that changing electric field produces magnetic field ( $\vec{B}$ )

## ⇒ Magnetic poles :-

- every magnet regardless of its shape has 2 poles : north (N) and south (S).

- magnetic poles are always found in pairs (single magnetic pole (monopole) has never been isolated yet).

- like mag. poles repel each other, and opposite poles attract

- mag. poles are named according to the way the magnet behaves

→ north mag. pole → its north seeking pole (it points toward north geographic pole) (south mag. pole of earth).

→ south mag. pole → south seeking pole (south geographic) (north magnetic pole of earth).

القطب الباعث عن الشمال الجغرافي (بؤسب الجنوب) والعكس صحيح

الكهربائية  
توافقية  
الكهربائية

→ Magnetism of earth (still under investigation)

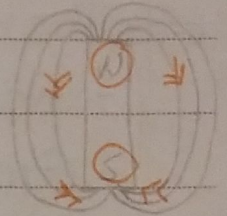
3 hypotheses :

1. iron core

2. convection currents

3. earth's rotation.

$$B_{\text{earth}} \approx 0.5 \text{ Gauss} = 0.5 \times 10^{-4} \text{ Tesla}$$



## 29.1 Analysis model: particle in a field (magnetic)

### magnetic forces and fields

#### ⇒ Magnetic field $\vec{B}$

$[B] = \text{Tesla in SI unit} = T$  Gauss =  $10^{-4} T$   $1G = 10^{-4} T$   
 surrounding a bar magnet or in space surrounding moving electric charge.

#### ⇒ Magnetic force $\vec{F}_B$

\* Properties of  $\vec{F}_B$  observed by experiment (e charge  $q$  moving at velocity  $\vec{v}$  in  $\vec{B}$ ) :-

1.  $|\vec{F}_B| \propto |\vec{v}| \neq |\vec{B}|$

2.  $|\vec{F}_B| \propto q$

3.  $\vec{F}_B$  on +ve  $q$  is opposite to  $\vec{F}_B$  on -ve charge.

4.  $\vec{F}_B = 0$  if  $\vec{v} \parallel \vec{B}$

5. if  $v$  makes an angle  $\theta$  with  $B \rightarrow |\vec{F}_B| \propto \sin\theta$   
 not magnitude?  $\theta \propto |\vec{F}_B|$

$$\vec{F}_B = q \vec{v} \times \vec{B} \quad v: \text{velocity of the moving charge}$$

#### \* Direction :-

$\vec{B} \rightarrow$  its direction is the direction of deflection of magnetic compass (needle).

$\vec{F}_B \rightarrow \perp \vec{v} \neq \vec{B}$  and its direction is the same direction as  $\vec{v} \times \vec{B}$  if  $q$  is +ve, opposite if  $q$  is -ve.

#### \* $\vec{v} \times \vec{B}$ (right hand rule)

align your 4 fingers along  $\vec{v}$  with your palm facing  $\vec{B}$  and then rotate  $\vec{v}$  towards  $\vec{B} \rightarrow$  your thumb along  $\vec{v} \times \vec{B}$

$\hookrightarrow +q \rightarrow F_B \parallel \vec{v} \times \vec{B}$

$\hookrightarrow -q \rightarrow \text{opposite}$

$$|\vec{F}_B| = |q| |\vec{v}| |\vec{B}| \sin \theta$$

$\theta$ : the smaller angle between  $\vec{v}$  &  $\vec{B}$   
(head to head / tail to tail)

$$[F_B] = \text{Newton} = N$$

$$[B] = \frac{[F]}{[q][v]}$$

$$T = \frac{N}{\text{Amp} \cdot \frac{C}{s}} = \frac{N}{\text{Amp} \cdot m}$$

⇒ Differences between electric & magnetic forces :-

- 1.  $\vec{F}_e$  acts on electric charges while moving or at rest.  
 $\vec{F}_B$  acts on electric charges only when moving.
- 2.  $\vec{F}_e$  is always along  $\vec{E}$  (parallel or antiparallel)  
 $\vec{F}_B$  is always  $\perp \vec{B}$
- 3.  $\vec{F}_e$  does work when displacing electric charges.  
 $\vec{F}_B$  does zero work when steering electric charges.

$$(\vec{F}_B + \vec{v}) \quad W = \vec{F} \cdot \Delta \vec{r}$$

$$dW = \vec{F} \cdot d\vec{r} = \vec{F} \cdot \vec{v} dt \quad \theta = 90 \quad \cos 90 = 0$$

$\vec{F}_e \rightarrow$  تسريع الشحنات       $\vec{F}_B \rightarrow$  توجيه الشحنات دون كسر/تغيير طاقتها

التيار

يزيدوا بزيادة المجال، بزيادة الشحنة، القوة على الموجة في وسط انكسار النسبي

\* Direction of  $\vec{B}$  :-

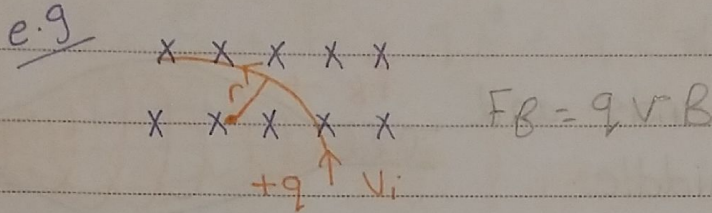
x into page    · out of page    ↗ in the plane of the Page

29.2 Motion of a charged particle in uniform magnetic field  
constant in "magnitude & direction"

$$\vec{F}_B = q \vec{v} \times \vec{B}$$

special case  $\vec{v}_i \perp \vec{B}$

$$\vec{F}_B = q \vec{v} \times \vec{B} \sin \theta \quad \theta = 90 \quad \sin 90 = 1 \quad \vec{F}_B = q \vec{v} \times \vec{B} \quad \text{max value}$$



it leads to uniform circular motion

$\vec{F}_B$  is  $F$  centripetal  $F_c = \frac{mv^2}{r} \quad qvB = \frac{mv^2}{r}$

$r = \frac{mv}{qB}$

$r$  depends on  $v$   
 $\propto mv \quad \propto \frac{1}{qB}$

angular Frequency  $\equiv$  angular velocity  $= \omega = \frac{v}{r}$

$\omega = \frac{qB}{m}$

independent of  $v$  &  $r$   
 depends on  $q, B,$  and  $m$ .

period  $T$

$T = \frac{2\pi m}{qB}$

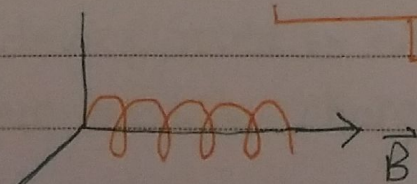
$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$  الزمن اللازم لفرة واحدة  
 independent of  $v$  &  $r$  depends on  $q, B, m$

IF  $v$  is not  $\perp \vec{B}$  ( $v$  makes an angle  $\theta$  with  $\vec{B}$ )

$$\vec{v} = \vec{v}_{\perp} + \vec{v}_{\parallel}$$

affected by  $\vec{F}_B$   $\rightarrow$  will not be affected by  $\vec{F}_B$   
 circular motion  $\rightarrow$  translational (linear) motion

$\rightarrow$  helical motion (axis along  $\vec{B}$ )



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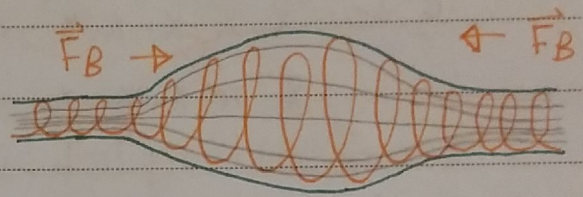
NO. 29.2

e.g if  $\vec{B} = B \hat{i}$   $F_x = 0$  definitely  $\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$   
 $v_{\perp} = \sqrt{v_y^2 + v_z^2}$

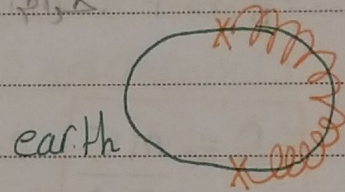
\*\* if  $q$  moves at  $\vec{v}$  but in non-uniform  $\vec{B} \rightarrow$  motion is more complicated

[1] magnetic bottle

$\vec{B}$  is strong at ends and weak in the middle.



[2] Van Allen Radiation Belt  $\text{قوس الجسيمات}$   
isolation around earth



### 29.3 Applications involving charged moving in a uniform magnetic field

#### ⇒ Lorenz Force

total Force acting on a charged particle  $q$  moving at velocity  $\vec{v}$  in magnetic and electric fields

$$\vec{F} = \vec{F}_e + \vec{F}_B = q\vec{E} + q\vec{v} \times \vec{B} \quad \boxed{\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}}$$

vector equation

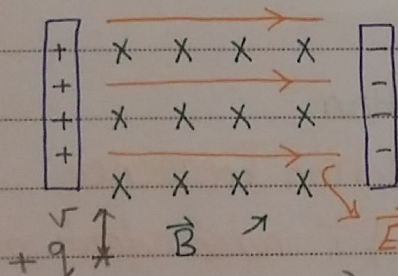
if  $\vec{F} = 0$   $|\vec{F}_e| = |\vec{F}_B|$  & opposite in direction.

#### 1] Velocity selector :-

a device allows particles of specific speeds only to pass.

$$\boxed{v = \frac{E}{B}}$$

$\vec{E}$  &  $\vec{B}$  crossed  $\rightarrow$  uniform & mutually  $\perp$  on other.   
 *متوازيين*   
 *parallel plots*



$$\begin{aligned} \vec{F}_B &= q\vec{v} \times \vec{B} & F_e &= q\vec{E} \\ &= qvB(-\hat{i}) & &= qE(\hat{i}) \\ &\text{to left} & &\text{to right} \end{aligned}$$

$$\begin{aligned} \vec{F} &= \vec{F}_e + \vec{F}_B = qE(\hat{i}) + qvB(-\hat{i}) \\ &= qE\hat{i} - qvB\hat{i} \end{aligned}$$

if  $F_{tot} = 0$   $|\vec{F}_e| = |\vec{F}_B|$   $qE = qvB$

$$\boxed{v = \frac{E}{B}}$$

# *السرعة*   
 *of the particle*

$\rightarrow$  only those charged particles will pass

$\rightarrow$  others with velocity  $< v$   $F_e > F_B$  will not pass *right*

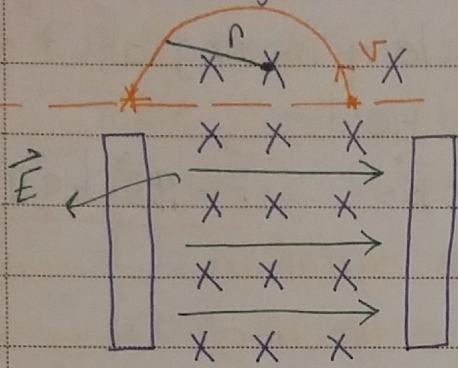
$\rightarrow$  others with velocity  $> v$   $F_B > F_e$  will not pass *left*

#### 2] Mass spectrometer

a device used to separate charged particles according to their mass to charge ratio ( $m/q$ )



→ velocity selector Followed by a region of uniform  $B_0$  (magnetic field) only in the same direction as that of velocity selector ( $\vec{E} = 0$ ).



the selected particle continuous in uniform circular motion.

$$v = \frac{E}{B}$$

$$F_c = \frac{mv^2}{r}$$

$$qvB_0 = \frac{mv^2}{r}$$

$$qB_0 = \frac{m(v^2)}{r} \frac{E}{B}$$

$$qB_0 = \frac{mE}{rB}$$

$$\boxed{\frac{m}{q} = \frac{rB_0}{E}}$$

useful to separate the isotopes (different mass no.) of a certain ion.

$\frac{q}{m}$  will

3 Cyclotron

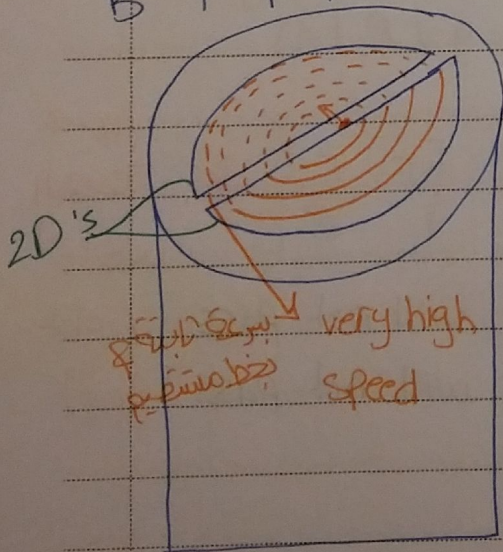
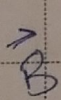
$\vec{v}_i + \vec{B} \rightarrow$  uniform circular motion

$$r = \frac{mv}{qB}$$

$$\boxed{\omega = \frac{qB}{m}}$$

cyclotron Frequency

$\omega$  &  $T$  independent of  $r$  &  $v$



- a device used to accelerate charged particles to very high speeds

- alternating  $\Delta V$  A.C

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

- consists of 2D's placed at the north pole of a huge magnet.

- the particle takes time  $T$  to couple one revolution, but after  $t = \frac{T}{2}$  the polarity  $\Delta V$  is reversed (its K.E increases by  $q\Delta V$ ).

2/4/2016

NO. 29.3

its  $v \uparrow$   $r \uparrow$  so it inters D with greater  $r$  & so on.

$$qvB = \frac{mv^2}{r} \quad v = \frac{rqB}{m}$$

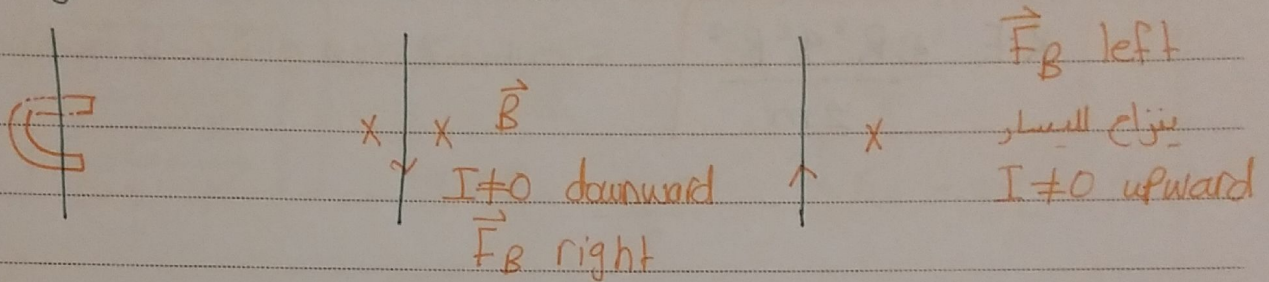
$$K.E = \frac{1}{2}mv^2 = \frac{1}{2} \frac{m r^2 q^2 B^2}{m^2} \quad (r = R \text{ أكبر نصف قطر مسار الجسيم})$$

$$K.E = \frac{R^2 q^2 B^2}{2m}$$

من أين نصف قطر المسار الجزيء في المسألة

29.4 magnetic force acting on a current carrying conductor is a collection of e. charges in motion (affected by  $\vec{F}_B$  when placed in  $\vec{B}$ ).

⇒ Demonstration of  $\vec{F}_B$  acting on a current carrying conductor straight wire of current I



→  $F_B$  on a straight current carrying conductor (wire of current I, length l)

$x \ x \ x \ x \ x \ x \ x \ \vec{B}$   
  
 $x \ x \ x \ x \ x \ x \ x$

$$\vec{F}_B \text{ on wire} = \sum \vec{F}_B \text{ on } q$$

$$= F_B \text{ on } q * \text{no. of charge carriers}$$

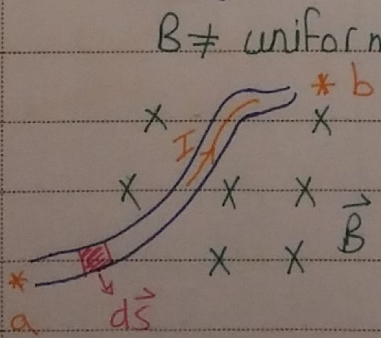
$$= q \vec{v}_d * \vec{B} * \text{no. of charge carriers}$$

no. = density \* volume  
= n \* volume  
= n l A

$\vec{F}_B = q \vec{v}_d B \sin 90 n l A = q \vec{v}_d B n l A$        $\vec{F}_B$  on a straight wire of length l, I placed in uniform  $\vec{B}$

but ...  $I = ne \vec{v}_d A$        $F_B = I l B$        $\vec{F}_B = I \vec{l} \times \vec{B}$

\* More generalized case :-



$B \neq$  uniform & wire of arbitrary shape  
take an element of this wire

$$d\vec{F}_B = I d\vec{s} \times \vec{B}$$

constant

$$\vec{F}_{B \text{ tot}} = \int_a^b I d\vec{s} \times \vec{B} = I \int_a^b (d\vec{s} \times \vec{B})$$

$B$  function of s

$$\vec{F}_{B \text{ tot}} = I \int_a^b (d\vec{s} \times \vec{B})$$

→ Special cases :-

1) if  $\vec{B}$  is uniform but the wire is not straight (arbitrarily shaped)

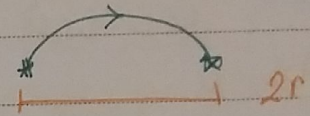
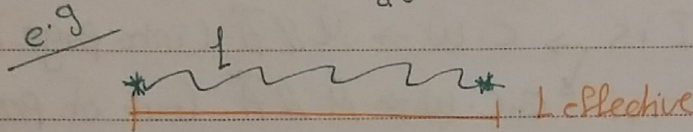
$$\vec{F}_B = I \left[ \int_a^b d\vec{s} \right] \times \vec{B} = I L_{\text{effective}} \times \vec{B}$$

↑ take B outside integral but still inside cross product

- what is  $L_{\text{effective}}$ ??

$$L_{\text{effective}} = \int_a^b d\vec{s}$$

المسافة العمودية على المجال



2) straight line (discussed before)

$$L_{\text{effective}} = l \quad \vec{F}_B = I l \times \vec{B}$$

3)  $\vec{B}$  = uniform but the current carrying conductor forms a closed loop

$$\vec{F}_B = I \left[ \int_a^b d\vec{s} \right] \times \vec{B} \quad \text{for a closed loop } \Rightarrow b=a$$

$$= I \left( \int_a^a d\vec{s} \right) \times \vec{B} = 0$$

بنيوي أن المجال مغلق

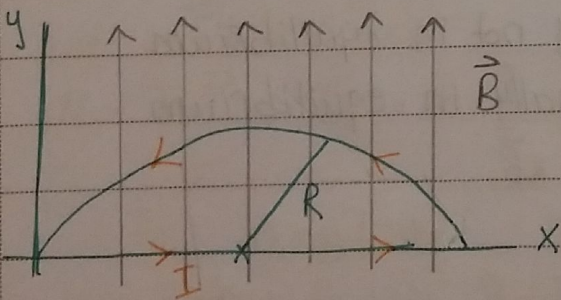
Example 29.4 :-

$$\vec{B} = B \hat{j}$$

$$F_B \rightarrow ?$$

$$F_B \nwarrow ?$$

$$F_B \nearrow ?$$



zero because B is uniform

$$F_B \rightarrow = I \vec{l} \times \vec{B} \quad \text{انجاز I نفس اتجاه}$$

$$= I l \hat{i} B \hat{j} = I l B \hat{k} = 2RIB \hat{k}$$

$$F_B \nwarrow = I L_{\text{effective}} \times \vec{B} = I 2R * -\hat{i} * B \hat{j} = -2RIB \hat{k}$$

or

$$dF_B = I d\vec{s} \times \vec{B} \quad s=r\theta \quad ds = R d\theta$$

$$= IR d\theta \sin\theta B$$

$$\nwarrow F_B = IRB \int_0^\pi \sin\theta d\theta = -2IRB \hat{k}$$

$$-\cos\theta \Big|_0^\pi = 1 - (-1) = 2$$

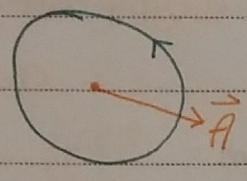
29.5 Torque on a current loop in uniform  $\vec{B}$

$\vec{p}$ : electric dipole moment =  $q\vec{d}$

$\tau = p \times E$     $U_E = -\vec{p} \cdot \vec{E}$

⇒ Magnetic dipole moment for a current loop  $\equiv \vec{\mu}$

$\vec{\mu} = I * \vec{A}$



$A_{\perp}$  to the surface of the loop determined by right hand rule    $|\vec{A}| = \text{area of loop}$

if  $I$  is  $\rightarrow$  C.W  $\rightarrow \vec{\mu} \parallel \vec{A}$  (into page)  
 $\downarrow$  C.C.W  $\rightarrow \vec{\mu} \parallel \vec{A}$  (out of page).

$[M] = [A][I] = \text{Amp} \cdot \text{m}^2$

⇒ For a coil (loop) of  $N$  turns

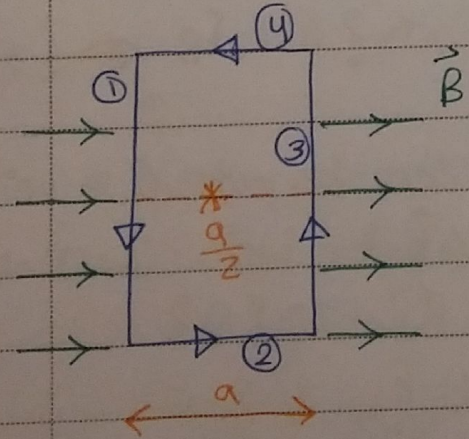
$\vec{\mu} = N I \vec{A}$

⇒  $\vec{\mu} = I \vec{A}$     $\vec{\tau} = \vec{\mu} \times \vec{B}$  torque exerted on current loop of  $\vec{\mu}$  placed in uniform  $\vec{B}$

$U_B = \int \tau d\theta = -\vec{\mu} \cdot \vec{B}$

$U_B = -\vec{\mu} \cdot \vec{B}$  energy stored in magnetic dipole moment  $\curvearrowright$  current loop and  $\vec{B}$  system

e.g rectangular current loop placed in uniform  $\vec{B}$  (to page)  $\left( \vec{\tau} = \vec{\mu} \times \vec{B} \right)$   
 Perpendicular



rotationally not in equilibrium  
 translationally in equilibrium

$F_B = I \vec{l} \times \vec{B}$

②  $F_B = 0$    ④  $F_B = 0$

③  $F_B = I b B (-\hat{k})$  (into page)

①  $F_B = I b B (+\hat{k})$  (out of page)

$\sum F_B = \vec{F}_{B1} + \vec{F}_{B2} + \vec{F}_{B3} + \vec{F}_{B4} = 0 \rightarrow$  no translational motion

2/4/2016

NO. 29.5

But ...  $F_1$  &  $F_2 \rightarrow$  net torque

$$\tau = r \times \vec{F} = rF \sin \theta \quad \theta = 90^\circ \quad \tau = rF$$

$$\Sigma \vec{\tau} = I b B \frac{a}{2} \sin 90^\circ + I b B \frac{a}{2} \sin 90^\circ = I b B a$$

but  $A = ba$        $\tau = I A B = \mu B$        $\vec{\tau} = \vec{\mu} \times \vec{B} \neq$

المتجه  $\vec{\tau}$  هو حاصل الضرب المتجهي للمتجه  $\vec{\mu}$  والمتجه  $\vec{B}$

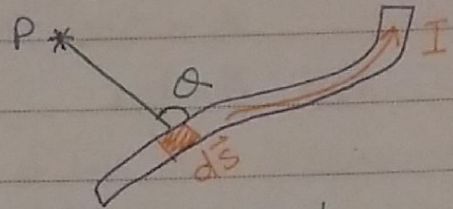
$\rightarrow$  See example 29.5 & 29.6

## No. Chapter 30

## Chapter 30 : Sources of magnetic field

30.1 The Biot - Savart law (experimental) used to find the magnetic field produced by a segment of current.

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$



$I$ : the amount of current in this segment.

$|d\vec{s}|$ : element of length  $d\vec{s} \parallel \vec{I}$  (direction of the current)

$r$ : distance between the current segment & observation point  $P$ .

$\hat{r}$ : position vector between  $d\vec{s}$  &  $P$

$\theta$ : head to head or tail to tail.

$\mu_0$ :  $4\pi \times 10^{-7}$  <sup>نفاذية</sup> permeability of free space.

$$[\mu_0] = \text{T.m/Amp}$$

$$|d\vec{B}| \propto \frac{1}{r^2} \quad |d\vec{B}| \propto I$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

over the whole distribution of the current.

⇒ Quick comparison between coulomb & Biot-Savart law

Biot - Savart law Vs Coulomb law

1.  $e^-$  charges produce  $\vec{E}$

electric current produce  $\vec{B}$

2.  $\vec{E}$  can be produced by isolated point charges

$\vec{B}$  can be produced by current must be a part of a complete circuit.

3.  $|\vec{E}| \propto \frac{1}{r^2}$

$|\vec{B}| \propto \frac{1}{r^2}$

4.  $\vec{E}$  is always radial

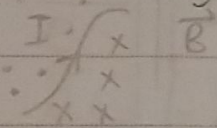
$$E = K_e \int \frac{dq}{r^2} \hat{r}$$

$\vec{B}$  always  $\perp$  to  $d\vec{s} \& \hat{r}$

$$d\vec{B} \propto \frac{d\vec{s} \times \hat{r}}{r^2}$$

→ use right hand rule for direction of  $\vec{B}$  produced by a segment of current.

grasp the wire with your right hand, if your thumb is along  $I$ , then your four fingers wrapped with  $\vec{B}$



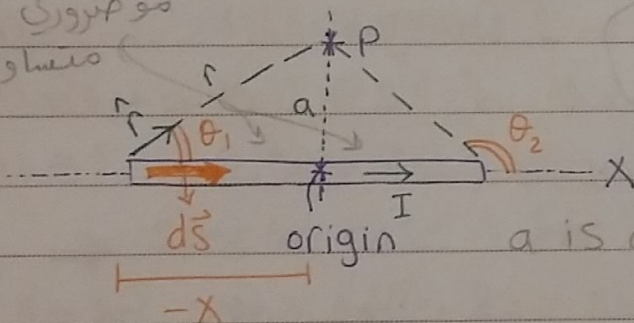
12/4/2016

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

**Case 1**  $\vec{B}$  due a thin straight current carrying wire (Finite wire)

السلك جزء من البارية

مساويين  
موازي



$$|d\vec{s} \times \hat{r}| = |d\vec{s}| |\hat{r}| \sin\theta = d \sin\theta$$

$$d\vec{s} \times \hat{r} = dx \sin\theta \hat{k}$$

a is constant

$$\sin\theta = \frac{a}{r}$$

$$dB = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \frac{dx \sin\theta}{r^2}$$

$$r = \frac{a}{\sin\theta} = a \csc\theta$$

$$dB = \frac{\mu_0 I}{4\pi} \frac{a \csc^2\theta \sin\theta}{a^2 \csc^2\theta}$$

$$\tan\theta = \frac{a}{-x} \quad -x = \frac{a}{\tan\theta}$$

$$B = \frac{\mu_0 I}{4\pi a} \int_{\theta_1}^{\theta_2} \sin\theta d\theta = \frac{-\mu_0 I}{4\pi a} \cos\theta \Big|_{\theta_1}^{\theta_2}$$

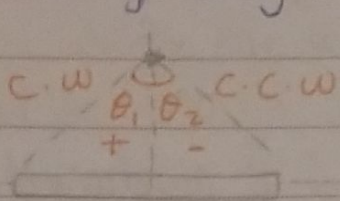
$$-x = a \cot\theta \quad dx = a \csc^2\theta d\theta$$

$$B = \frac{\mu_0 I}{4\pi a} (\cos\theta_1 - \cos\theta_2)$$

due a straight wire at point p a distance a  $\perp$  to the wire



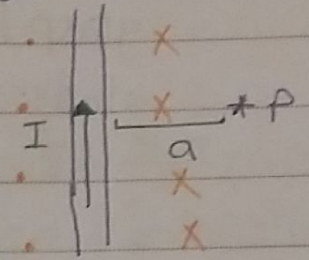
serway way:



$$B = \frac{\mu_0 I}{4\pi a} [\sin\theta_1 - \sin\theta_2]$$

→ special case: for infinitely long straight wire  
 $\theta_1$  decrease  $\theta_2 = \pi$

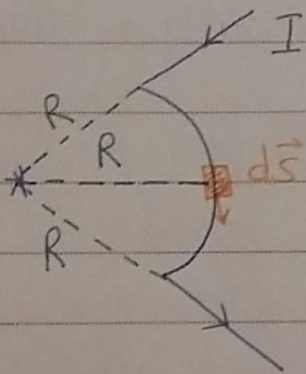
$$B = \frac{\mu_0 I}{2\pi a} \text{ For infinitely long wire}$$



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**Case 2**

B due a curved wire segment



$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

$d\vec{B}$  due to 2 straight parts is zero

$$d\vec{s} \times \hat{r} = 0 \quad d\vec{s} \parallel \hat{r} \quad (\theta = 0 \rightarrow \sin\theta = 0)$$

For the curved part  $d\vec{s} \perp \hat{r} \quad (\theta = 90^\circ)$

$$|d\vec{s} \times \hat{r}| = ds$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{ds}{r^2} = \frac{\mu_0 I}{4\pi} \frac{ds}{R^2} \quad R \text{ is the same for all current element } d\vec{s}$$

$$B = \frac{\mu_0 I}{4\pi R^2} \int ds \quad \text{or} \quad s = r\theta \quad ds = r d\theta \quad (\theta \text{ in radian})$$

$$B = \frac{\mu_0 I s}{4\pi R^2}$$

$$\text{or} \quad B = \frac{\mu_0 I \theta}{4\pi R}$$

due to curved wire circuit. ( $\theta$  in radian)

→ special case: For a circular loop  $\theta = 2\pi$

$$B = \frac{\mu_0 I 2\pi}{4\pi R}$$

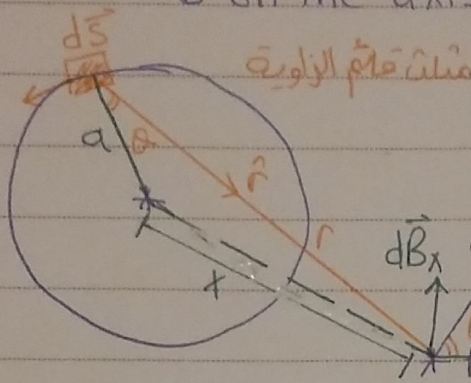
$$B = \frac{\mu_0 I}{2R}$$

at the center of circular current loop

Direction by R.H.R

case 3

$\vec{B}$  on the axis of a circular current loop



مقطع الزاوية

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

$$|d\vec{s} \times \hat{r}| = ds (\theta = 90)$$

$$r^2 = x^2 + a^2 \quad d\vec{B} \perp \text{plane of } d\vec{s} \text{ \& } \hat{r}$$

$$\int d\vec{B}_x \quad \leftarrow \int d\vec{B}_y$$

$d\vec{B}_x \cos\theta$  zero by symmetry

$$\vec{B}_{tot} = \int d\vec{B}_x = \frac{\mu_0 I}{4\pi} \int \frac{ds \cos\theta}{x^2 + a^2} \quad \text{but } \cos\theta = \frac{a}{r} = \frac{a}{\sqrt{x^2 + a^2}}$$

$$\vec{B}_{tot} = \frac{\mu_0 I a}{4\pi} \int \frac{ds}{(x^2 + a^2)^{3/2}} \quad a, x \text{ are the same for all element}$$

$$\vec{B}_{tot} = \frac{\mu_0 I a}{4\pi (x^2 + a^2)^{3/2}} \int ds = \frac{\mu_0 I a S}{4\pi (x^2 + a^2)^{3/2}} \quad \text{but } S = 2\pi a$$

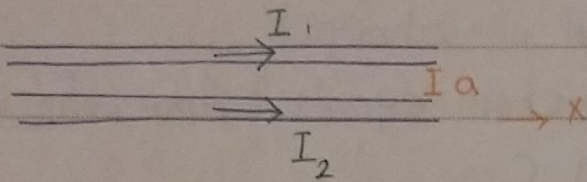
$$\vec{B}_{tot} = \frac{\mu_0 I a^2 2\pi}{4\pi (x^2 + a^2)^{3/2}} \quad \boxed{\vec{B} = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}}$$

Special case : For  $x=0$  (the point at the center)

$$B = \frac{\mu_0 I}{2a} \quad \text{Case 2 نفس المثال في}$$

### 30.2 Magnetic Force between 2 parallel current carrying conductors

every current wire carrying conductor produces its own  $\vec{B}$  & is affected by external  $\vec{B}$  if exist.



infinitely long wires

the two wires exert  $\vec{B}$  on each other

$$B \text{ due to very long wire} = \frac{\mu_0 I}{2\pi a}$$

$$\vec{F}_B \text{ exerted on a current wire by external } \vec{B} = I \vec{L} \times \vec{B}$$

- wire 1 produces  $B_1$  (into page)
- wire 2 produces  $B_2$  (out page)
- affected wire 2
- affected wire 1
- $F_{B_1}$  on 2 is upward.
- $F_{B_2}$  on 1 is downward

$$\vec{F}_{2 \text{ on } 1} = -\vec{F}_{1 \text{ on } 2}$$

$$F_{B_1 \text{ on } 2} = |I_2 \vec{L}_2 \times \vec{B}_1| = \frac{I_2 L \mu_0 I_1}{2\pi a}$$

$$F_{B_2 \text{ on } 1} = |I_1 \vec{L}_1 \times \vec{B}_2| = \frac{I_1 L \mu_0 I_2}{2\pi a}$$

$$F_B = \frac{\mu_0 I_1 I_2 L}{2\pi a}$$

between 2 parallel current wires too infinitely long separated by a distance  $a$

Direction

- if the two currents have the same direction  $\rightarrow$  attractive force
- opposite direction  $\rightarrow$  repulsive force.

$\rightarrow$  Force per unit length

$$\frac{F_B}{L} = \frac{\mu_0 I_1 I_2}{2\pi a} \propto I_1 \& I_2 \propto \frac{1}{a}$$

\* Ampere (the SI unit of current)

when the  $F_B$  per unit length exerted between two parallel wires separated by 1m equals  $2 \times 10^{-7} \frac{N}{m}$ , the current in each wire is 1A.

\* Coulomb C (the SI unit of charge)

when a current in a conductor equals 1 Amp, the charge that flows in 1 second is 1 C.

used for highly symmetric current

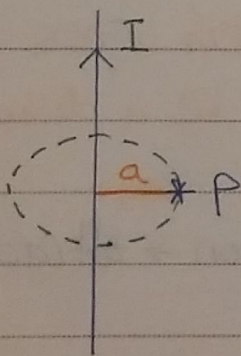
30.3 Ampere's law  $\rightarrow$  distribution

The path integral of  $\vec{B} \cdot d\vec{s}$  (element of length) over any closed path is equal to  $\mu_0 I$ , I is the total steady current

$\rightarrow$  Amperian loop: closed path of any shape (mathematical construction)

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{net} \quad B \parallel \text{or} \perp$$

□  $\vec{B}$  due to infinitely long straight current carrying wire & thin <sup>ليس لها</sup> <sub>أبعاد</sub>



$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{net}$  but B is constant

$B \cdot \oint d\vec{s} = \mu_0 I$  but  $\theta = 0$   $B \parallel d\vec{s}$

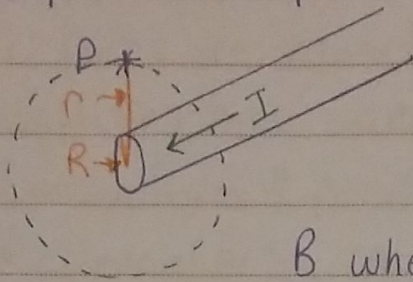
$B \oint d\vec{s} = \mu_0 I \quad B 2\pi a = \mu_0 I$

here...  $I_{net} = I_{wire}$

$$B = \frac{\mu_0 I}{2\pi a}$$

see example 30.4

2 example 30.5 p.913



$I$  is uniformly distributed

$$J = \frac{I}{A} = \text{constant}$$

$B$  when a)  $r \gg R$       b)  $r \leq R$

a) For  $r \gg R$  (outside the wire)

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{net}} \quad \text{but } \theta = 0 \text{ \& } B \text{ is constant}$$

$$B \oint ds = \mu_0 I_{\text{net}} \quad B 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{when } r \gg R \quad \propto \frac{1}{r}$$

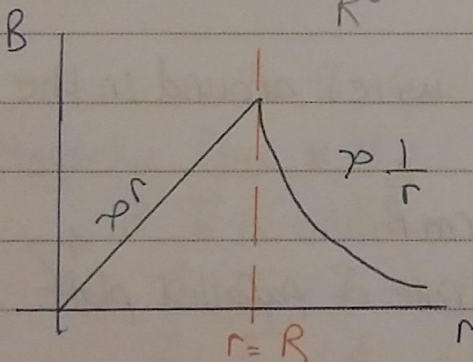
b) For  $r \leq R$  (inside the wire)

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{net}} \quad B 2\pi r = \mu_0 I_{\text{net}}$$

$$\text{but } J = \text{constant} = \frac{I}{A} \quad I_{\text{net}} = J A_{\text{inside}} = \frac{I}{A_{\text{outside}}} * A_{\text{inside}}$$

$$B 2\pi r = \frac{\mu_0 I r^2}{R^2}$$

$$B = \frac{\mu_0 I r}{2\pi R^2} \quad \propto r \quad \text{when } r \leq R$$



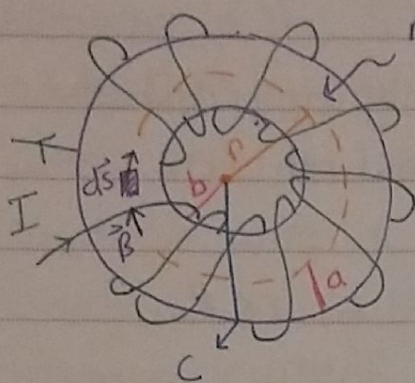
17/4/2016

\*\*  $\vec{B}$  produced by a Toroid

a device consists of conducting wire wrapped around a ring (torus) of non-conducting material (solid, air)

- used to produce a strong  $\vec{B}$  (almost uniform) within a certain region.

NO 30.3 & 30.4



non-conducting  
 $b$ : inner radius  
 $c$ : outer radius  
 $a$ : radius of cross section

The Amperian loop:  $b < r < c$

$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{net}$  but  $\theta = 0$  &  $B$  is almost uniform

$B \oint d\vec{s} = \mu_0 I_{net}$        $B(2\pi r) = \mu_0 N I$        $N$ : no. of turns

$B = \frac{\mu_0 N I}{2\pi r}$  by toroid  $\propto \frac{1}{r}$   $r_{\text{avg}}$

But if  $r \gg a$  (cross section)  $\rightarrow B$  inside toroid  $\approx$  constant  
 ( $B$  is almost uniform)

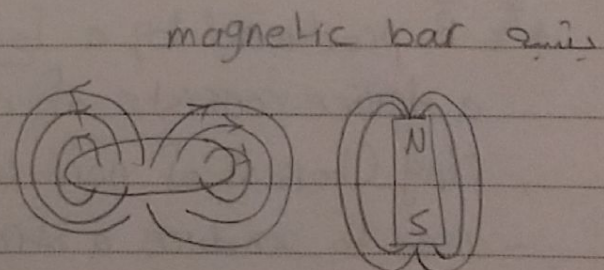
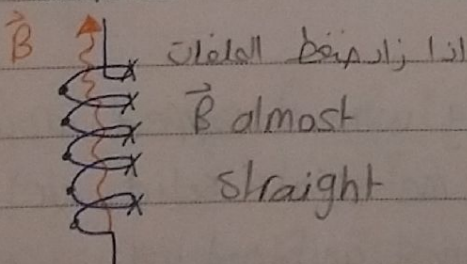
### 30.4 The magnetic Field of a solenoid

$\rightarrow$  Solenoid :

long conducting wire (carrying wire) around in the form of helix.

- used to produce a strong & uniform  $\vec{B}$

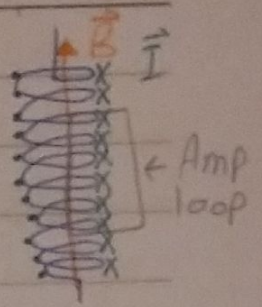
(solenoid in magnetism plays the role of parallel plate capacitor in electricity)



**\*\* ideal solenoid**

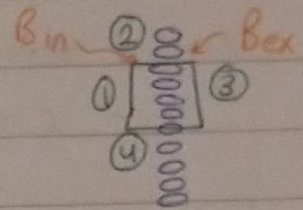
the turns are closely spaced and its length  $\gg$  cross sectional radius.

$\vec{B} \approx$  uniform & parallel to the axis of solenoid  
 $\vec{B}_{\text{external}} \approx 0$



By Ampere's law

$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{net}}$  remember  $B_{\text{ex}} = 0$



$\oint_1 \vec{B} \cdot d\vec{s} + \int_2 \vec{B} \cdot d\vec{s} + \int_3 \vec{B} \cdot d\vec{s} + \int_4 \vec{B} \cdot d\vec{s}$   
 $\theta = 90^\circ$     zero    zero    zero

( $B_{\text{in}} = 0$  &  $B_{\text{ex}} = 0$ )    ( $B_{\text{ex}} = 0$ )    ( $B_{\text{in}} = 0$  ( $\theta = 90^\circ$ ) &  $B_{\text{ex}} = 0$ )

$\int \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{net}}$  but  $B$  is uniform  $B \int ds = \mu_0 I_{\text{net}}$

$B l = \mu_0 N I$   $N$ : no. of turns

$B = \frac{\mu_0 N I}{l}$   $N$  &  $I$  &  $l$  depends on  $\int$   $n = \frac{N}{l}$  = no. of turns per unit length

$B = \mu_0 n I$  independent on geometry *لغيرها الطول و عدد اللفات*  
*نفسها n*

**30.5 Gauss's law in magnetism**

Magnetic Flux  $\equiv \Phi_B$   $\Phi_B = \int \vec{B} \cdot d\vec{A}$  scalar product

if  $\vec{B}$  is uniform  $\Phi_B = B \cdot \int d\vec{A} = \vec{B} \cdot \vec{A} = BA \cos \theta$

if  $\vec{B} \perp \vec{A}$  ( $\vec{B} \parallel$  surface)  $\cos 90 = 0$   $\Phi_B = 0$

$\vec{B} \parallel \vec{A}$  ( $\vec{B} \perp$  surface)  $\cos 0 = 1$   $\Phi_B = BA$  max

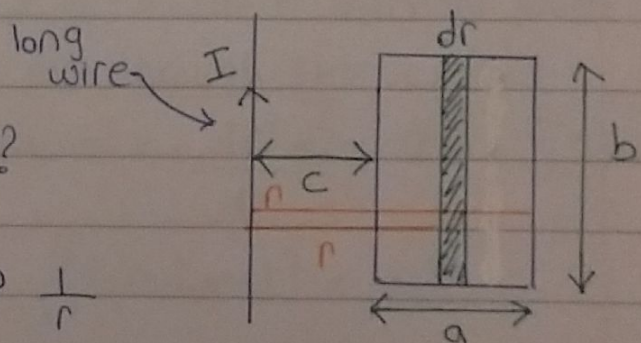
$[\Phi_B] = [B][A] = T \cdot m^2$  is SI = Weber = Wb

- example 30.7 p.917

what is  $\Phi_B$  through the loop?

$B$  is not uniform

$B$  due to long wire =  $\frac{\mu_0 I}{2\pi r} \propto \frac{1}{r}$



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NO. 30.5

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \frac{\mu_0 I}{2\pi} \int \frac{1}{r} dA \quad \text{but } A = r * b \text{ \& } b \text{ is cons.}$$

$$\Phi_B = \frac{\mu_0 I b}{2\pi} \int_c^{a+c} \frac{1}{r} dr = \frac{\mu_0 I b}{2\pi} \ln \left[ 1 + \frac{a}{c} \right]$$

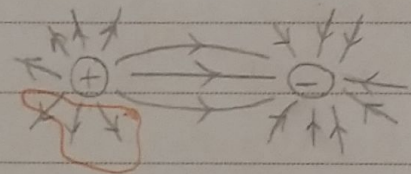
رقتور کئی اعداد loop  
ویر loop کے wire

Special case :

- $c \rightarrow 0 \rightarrow \Phi_B \rightarrow \infty$
- $c \rightarrow \infty \rightarrow \Phi_B \rightarrow 0$

→ Gauss law in electricity

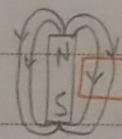
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{net, enc}}}{\epsilon_0}$$



net closed surface  $\neq 0$

→ Gauss law in magnetism

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$



net closed surface = 0

$\Phi_B \propto$  net number of lines (lines entering - lines leaving)  
 because  $\vec{B}$  lines are closed loops (no specific end or start)



Chapter 31 : Faraday's law

31.1 Faraday's law of induction توليد تيار كهربائي من حث مغناطيسي

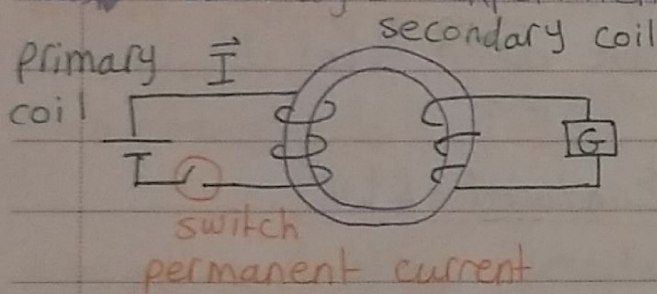
→ Demonstration of e.m induction :- مغناطيسي

- while moving the magnet into/away relative to the loop, the galvanometer deflects and returns to zero.
- if the magnet and loop are stationary relative to each other, the galvanometer reads zero.

I is induced due to change in magnetic Flux.

التغير في التدفق المغناطيسي (التغير في عدد خطوط المجال) هو الذي يولد التيار

→ Faraday's experiment



\* Switch is closed

$I = I_{\text{primary}} \neq 0 \quad I_{\text{induced}} \neq 0$

\* when switch is opened

$I_{\text{primary}} = 0 \quad I_{\text{induced}} \neq 0$

on  $\Rightarrow I_{\text{primary}} = 0$  } induced \* when  $I_{\text{primary}}$  is zero or steady

off  $\Rightarrow I_{\text{primary}} = I$  } equal zero  $I_{\text{induced}} = 0$

\* the current induced due to change in magnetic Flux

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

→  $\Phi_B$  can be changed by :-

مقدار واتجاه

1. changing  $\vec{B}$
2. changing A
3. changing angle between  $\vec{B}$  &  $\vec{A}$
4. any combination of 1-3

→ The induced e.m.f ( $\mathcal{E}_{\text{induced}}$ ) in a circuit is directly proportional to the time rate of change in  $\Phi_B$  through the circuit.

$\mathcal{E}_{\text{induced}} \propto \left| \frac{d\Phi_B}{dt} \right| \quad \mathcal{E}_{\text{induced}} = - \frac{d\Phi_B}{dt} \quad \Phi_B = \int \vec{B} \cdot d\vec{A}$

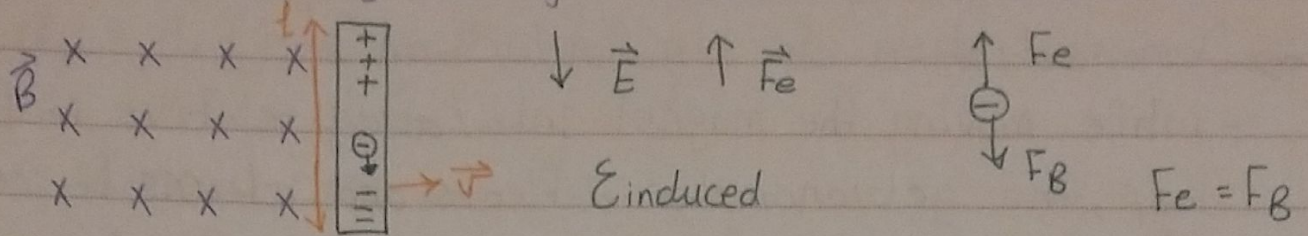
زيادة او نقصان المجال في وقت معين

$I_{\text{induced}} = \frac{|\mathcal{E}_{\text{induced}}|}{R}$

$[\mathcal{E}_{\text{induced}}] = \text{volt}$

31.2 Motional induced emf

induced by moving a conductor in uniform  $\vec{B}$



a slab of conductor is moving in uniform  $\vec{B}$

$|\mathcal{E}_{\text{induced}}| = \left| \frac{d\Phi_B}{dt} \right|$  by pulling the conductor to right  $F_B$  (on every charge carrier) =  $q\vec{v} \times \vec{B}$  (downward)

$\vec{F}_e = q\vec{E}$  (upward)

$F_B$  (on every charge carrier) =  $q\vec{v} \times \vec{B}$  (downward)

electrons will continue to move until  $|\vec{F}_e| = |\vec{F}_B|$

$qE = qvB$

$E = vB$  توقف الإلكترونات

$|\Delta V| = El$

$\mathcal{E}_{\text{induced}} = Blv$

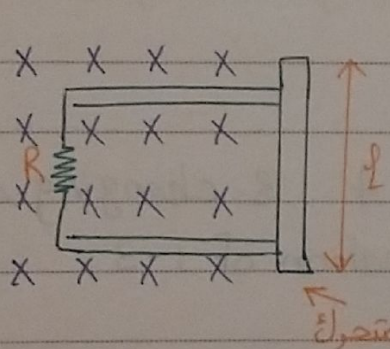
$\mathcal{E}_{\text{induced}}$  المحرك

$\mathcal{E}_{\text{motional}}$  إذا كان يتحرك عمودياً على  $\vec{B}$

if the conducting slab is a part of a circuit

of zero resistance

has R resistance



$\Phi_B = \vec{B} \cdot \vec{A}$      $\theta = 0$      $\Phi_B = B \times l$

$\mathcal{E}_{\text{induced}} = \left| \frac{d\Phi_B}{dt} \right|$

$\mathcal{E}_{\text{induced}} = Bl \frac{dx}{dt} = Blv$

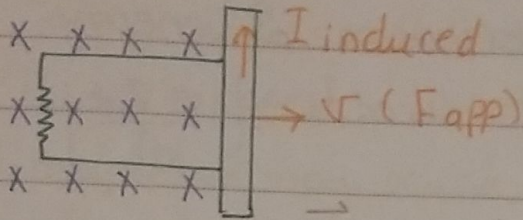
$I_{\text{induced}} = \frac{|\mathcal{E}_{\text{induced}}|}{R} = \frac{Blv}{R}$

$I_{\text{induced}} = \frac{Blv}{R}$

(حركة السلك) حركة السلك (+)

التيار الكهربائي في حركة السلك الكهربائي

31.2 motional e.m.f



$\mathcal{E}_{\text{motional induced}} = Blv$       $I_{\text{induced}} = \frac{Blv}{R}$

$|\vec{F}_B| = |\vec{F}_{\text{app}}|$

$\vec{F}_B = I \vec{L} \times \vec{B}$       $\vec{F}_B = \frac{B^2 l^2 v^2}{R}$

Power =  $I^2 R$

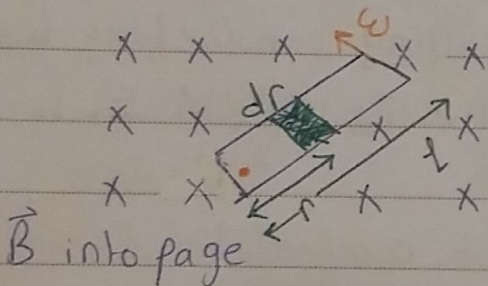
Power =  $\vec{F}_{\text{app}} \cdot \vec{v}$

→ example 31.1 p.942

$F_{\text{app}} = ma = m \frac{dv}{dt}$       $F_{\text{app}} = -F_B$       $v = v_i e^{\frac{t}{\tau}}$

→ example 31.4 p.943

rotational motion



take element of length → translationally  
For a conducting bar of length  $l$   
rotating about axis passing through one  
of it's end

$\mathcal{E}_{\text{induced motional}} = \frac{B^2 l^2 v^2}{R}$       $d\mathcal{E}_{\text{induced}} = Bv dr$

$\mathcal{E}_{\text{induced}} = B \int v dr$  but  $v = r\omega$       $\mathcal{E}_{\text{induced}} = B\omega \int_0^l r dr$

$\mathcal{E}_{\text{induced}} = \frac{1}{2} B\omega l^2$

### 13.3 Lenz's law (not new law)

it is a consequence of conservation of energy

$$\mathcal{E}_{\text{induced}} = \ominus \frac{d\Phi_B}{dt} \quad |\mathcal{E}_{\text{induced}}| \propto \left| \frac{d\Phi_B}{dt} \right|$$

منه  
سواء تكلم

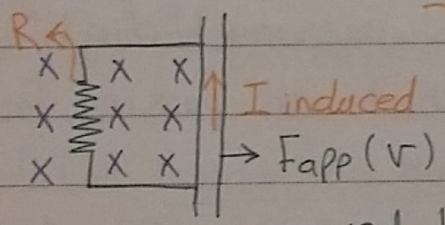
→ The induced current and induced  $\mathcal{E}$  in a loop flow in a direction that creates  $\vec{B}$  that opposes the change in  $\Phi_B$  through the area of loop.

اتجاه التيار الحثي و القوة الدافعة الكهربائية في ملامف (مسار مغلق) يكون باتجاه يولد مجالاً مغناطيسياً معاكساً للتغير في التدفق المغناطيسي الذي أنشأه.

→ The induced current flows in such a direction as to oppose the change that produced it (current)

يكون اتجاه التيار الحثي معاكساً للتغير الذي أنشأه

- Example: I induced in a moving conducting bar



↻ C.C.W  $\Delta \Phi_B \downarrow$  ما يزيد A  $\downarrow$  B ✓

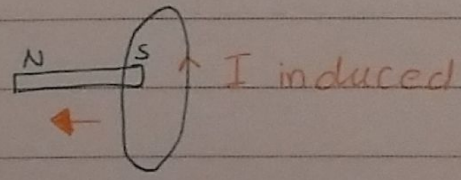
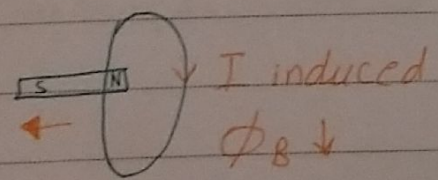
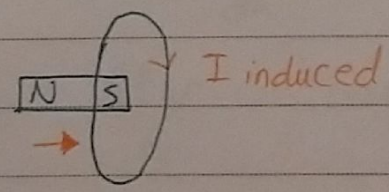
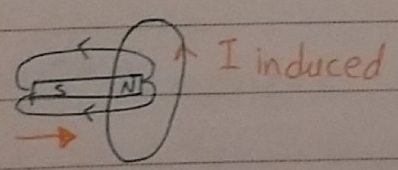
in order to compensate decrease in  $\Phi_B$

العكس صحيح لو  
outpage's

moved to right  $\rightarrow$  C.C.W (I)  $\Phi_B \uparrow$  AT  $\vec{B}$  out

moved to left  $\rightarrow$  C.W (I)  $\Phi_B \downarrow$  AT  $\vec{B}$  in

→ I induced in a stationary loop by moving a bar magnet into & away from the loop.



NO. Chapter 32

Chapter 32 % inductance

32.1 Self-inductor & inductance

Inductance :-

a major of how much opposition a loop (coil) offers to any change in current.

unit of inductance =  $\frac{V \cdot S}{A} = \text{Henry} = H$

Self inductance L :-

the opposition of a loop to any change in its current.

$\mathcal{E}_{\text{induced}} \propto -\frac{d\phi_B}{dt}$        $\phi_B \propto B$        $B \propto I$       So ...

$\mathcal{E}_{\text{induced}} \propto -\frac{dI}{dt}$        $\mathcal{E}_{\text{induced}} = -\text{constant} \frac{dI}{dt}$   
 independent of I & t = L

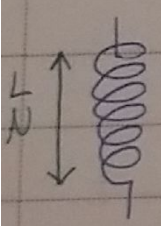
$\mathcal{E}_{\text{induced}} = -L \frac{dI}{dt}$        $L = -\frac{\mathcal{E}_{\text{induced}}}{dI/dt} = \frac{-\mathcal{E}_L}{dI/dt}$

مقاومة التغير في التيار  $\rightarrow L = -\frac{\mathcal{E}_L}{dI/dt}$  looks like  $R = \frac{\Delta V}{I}$       L is always +ve  
 مقاومة التيار

$[L] = \frac{V \cdot S}{A} = \text{Henry} = H$

But  $\mathcal{E}_{\text{induced}} = -N \frac{d\phi}{dt}$        $L = \frac{N\phi_B}{I}$

- example 32.1 p.972 [L of a solenoid]



$L = \frac{N\phi_B}{I}$       B is uniform       $\phi_B = \vec{B} \cdot \vec{A}$        $\theta = 0$   
 $\phi_B = BA$

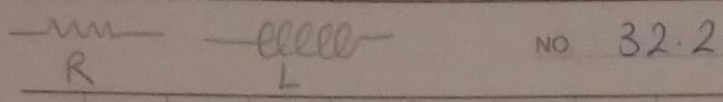
B for a solenoid  $B = \mu_0 n I$ ,  $n = \frac{N}{l}$        $\phi_B = \mu_0 n I A$  or  
 $L_{\text{solenoid}} = \frac{\mu_0 N^2 A}{l}$        $\phi_B = \mu_0 \frac{N}{l} I A$

$L_{\text{solenoid}} = \mu_0 n^2 (Al) = \mu_0 n^2 \text{Volume}$

Capacitor in Solenoid  
المكثف في الملف

Solenoid ← plus B  
ملف ← E متوازي

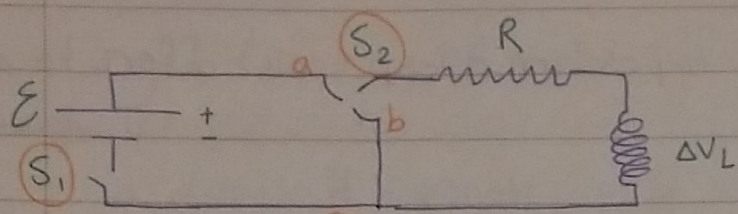
26/4/2016



### 32.2 RL circuit

circuit of resistor & inductor in series connected  
without battery with battery

- S<sub>2</sub> must be on either a or b.



→ at t=0 ① S<sub>1</sub> is closed & S<sub>2</sub> at a  
I(t=0) = 0    ε = ΔV<sub>L</sub>    ΔV<sub>R</sub> = 0

→ as time passes t > 0

$$\epsilon = \Delta V_L + \Delta V_R$$

$$+\epsilon - IR - L \frac{dI}{dt} = 0$$

$$I(t) = I_0 (1 - e^{-\frac{t}{\tau}})$$

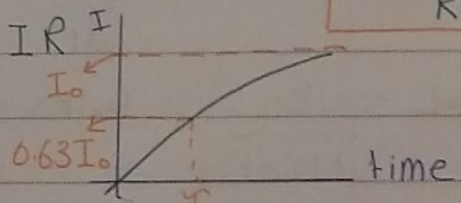
with battery

$$I_0 = \frac{\epsilon}{R}$$

τ: time constant of RL circuit

$$\tau = \frac{L}{R}$$

→ as t → ∞    ε = ΔV<sub>R</sub> = IR



inductor  
محث

$$[\tau] = H = \frac{V \cdot s}{A} = \frac{V \cdot s}{\frac{V}{\Omega}} = \Omega \cdot s = \frac{V \cdot s}{A} = \frac{V \cdot s}{\frac{V}{\Omega}} = \Omega \cdot s = \frac{V \cdot s}{A}$$

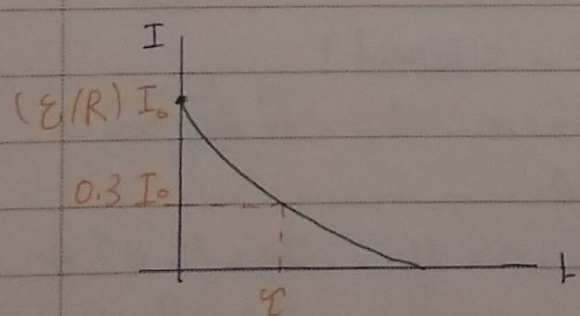
② if S<sub>2</sub> at b (after sufficient time being at a)

R-L circuit without battery

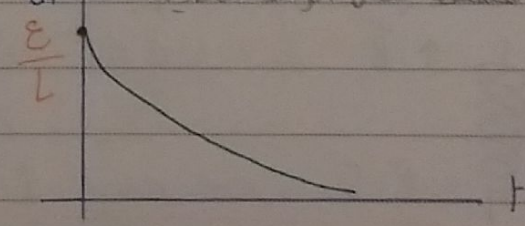
$$IR = L \frac{dI}{dt}$$

$$I = I_0 e^{-\frac{t}{\tau}}$$

$$\tau = \frac{L}{R} \quad I_0 = \frac{\epsilon}{R}$$



المعدل الزمني للتغير في التيار



$$\frac{\frac{\epsilon}{R}}{\frac{L}{R}} = \frac{\epsilon}{L}$$

$$\frac{dI}{dt} \propto e^{-\frac{t}{\tau}}$$

inductor ←  $\propto \frac{dI}{dt}$

## 32.3 Energy stored in magnetic field

→ RL circuit with battery

$$\mathcal{E} = \Delta V_R + \Delta V_L = IR + L \frac{dI}{dt} \times I$$

$$I\mathcal{E} = I^2R + IL \frac{dI}{dt}$$

Power produced by  $\mathcal{E}$       power stored in L  
 power dissipated (consumed) by R

- power of L =  $IL \frac{dI}{dt}$

$$\frac{dU_B}{dt} = LI \frac{dI}{dt}$$

$$\int dU_B = L \int I dI$$

$$U_B = \frac{1}{2} LI^2 \quad \text{For any } \vec{B} \quad [U_B] = \text{J}$$

→ For a solenoid

$$L \text{ solenoid} = \mu_0 n^2 \text{ Volume}$$

$$B \text{ solenoid} = \mu_0 n I \quad I = \frac{B}{\mu_0 n}$$

$$U_B = \frac{1}{2} \mu_0 n^2 \text{ Volume} \frac{B^2}{\mu_0^2 n^2} = \frac{B^2}{2\mu_0} \text{ Volume}$$

$$\text{energy density} = u_B = \frac{B^2}{2\mu_0} \quad [u_B] = \frac{\text{J}}{\text{m}^3}$$

small ↑

