21

ELECTRIC CHARGE AND ELECTRIC FIELD

21.1. **(a) IDENTIFY and SET UP:** Use the charge of one electron $(-1.602\times10^{-19}\ \text{C})$ to find the number of electrons required to produce the net charge.

EXECUTE: The number of excess electrons needed to produce net charge q is

$$\frac{q}{-e}=\frac{-3.20\times10^{-9}\ \text{C}}{-1.602\times10^{-19}\ \text{C/electron}}=2.00\times10^{10}\ \text{electrons.}$$

(b) IDENTIFY and SET UP: Use the atomic mass of lead to find the number of lead atoms in 8.00×10^{-3} kg of lead. From this and the total number of excess electrons, find the number of excess electrons per lead atom.

EXECUTE: The atomic mass of lead is 207×10^{-3} kg/mol, so the number of moles in 8.00×10^{-3} kg is

$n=\dfrac{m_{\text{tot}}}{M}=\dfrac{8.00\times10^{-3}\ \text{kg}}{207\times10^{-3}\ \text{kg/mol}}=0.03865\ \text{mol.}$ N_A (Avogadro's number) is the number of atoms in 1 mole,

so the number of lead atoms is $N=nN_A=(0.03865\ \text{mol})(6.022\times10^{23}\ \text{atoms/mol})=2.328\times10^{22}$ atoms.

The number of excess electrons per lead atom is $\dfrac{2.00\times10^{10}\ \text{electrons}}{2.328\times10^{22}\ \text{atoms}}=8.59\times10^{-13}.$

EVALUATE: Even this small net charge corresponds to a large number of excess electrons. But the number of atoms in the sphere is much larger still, so the number of excess electrons per lead atom is very small.

21.2. **IDENTIFY:** The charge that flows is the rate of charge flow times the duration of the time interval.

SET UP: The charge of one electron has magnitude $e=1.60\times10^{-19}$ C.

EXECUTE: The rate of charge flow is $20{,}000$ C/s and $t=100\ \mu\text{s}=1.00\times10^{-4}$ s.

$Q=(20{,}000\ \text{C/s})(1.00\times10^{-4}\ \text{s})=2.00\ \text{C}.$ The number of electrons is $n_e=\dfrac{Q}{1.60\times10^{-19}\ \text{C}}=1.25\times10^{19}.$

EVALUATE: This is a very large amount of charge and a large number of electrons.

21.3. **IDENTIFY and SET UP:** A proton has charge $+e$ and an electron has charge $-e$, with $e=1.60\times10^{-19}$ C.

The force between them has magnitude $F=k\dfrac{|q_1q_2|}{r^2}=k\dfrac{e^2}{r^2}$ and is attractive since the charges have opposite sign. A proton has mass $m_p=1.67\times10^{-27}$ kg and an electron has mass 9.11×10^{-31} kg. The acceleration is related to the net force $\vec{F}$ by $\vec{F}=m\vec{a}$.

EXECUTE: $F=k\dfrac{e^2}{r^2}=(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times10^{-19}\ \text{C})^2}{(2.0\times10^{-10}\ \text{m})^2}=5.75\times10^{-9}\ \text{N}.$

proton: $a_p=\dfrac{F}{m_p}=\dfrac{5.75\times10^{-9}\ \text{N}}{1.67\times10^{-27}\ \text{kg}}=3.4\times10^{18}\ \text{m/s}^2.$

electron: $a_e = \dfrac{F}{m_e} = \dfrac{5.75\times10^{-9}\ \text{N}}{9.11\times10^{-31}\ \text{kg}} = 6.3\times10^{21}\ \text{m/s}^2$

The proton has an initial acceleration of $3.4\times10^{18}\ \text{m/s}^2$ toward the electron and the electron has an initial acceleration of $6.3\times10^{21}\ \text{m/s}^2$ toward the proton.

EVALUATE: The force the electron exerts on the proton is equal in magnitude to the force the proton exerts on the electron, but the accelerations of the two particles are very different because their masses are very different.

21.4. **IDENTIFY:** Use the mass m of the ring and the atomic mass M of gold to calculate the number of gold atoms. Each atom has 79 protons and an equal number of electrons.

SET UP: $N_A = 6.02\times10^{23}$ atoms/mol. A proton has charge $+e$.

EXECUTE: The mass of gold is 10.8 g and the atomic weight of gold is 197 g/mol. So the number of atoms is

$N_A n = (6.02\times10^{23}\ \text{atoms/mol})\left(\dfrac{10.8\ \text{g}}{197\ \text{g/mol}}\right) = 3.300\times10^{22}$ atoms. The number of protons is

$n_p = (79\ \text{protons/atom})(3.300\times10^{22}\ \text{atoms}) = 2.61\times10^{24}$ protons.

$Q = (n_p)(1.60\times10^{-19}\ \text{C/proton}) = 4.18\times10^{5}$ C.

(b) The number of electrons is $n_e = n_p = 2.61\times10^{24}$.

EVALUATE: The total amount of positive charge in the ring is very large, but there is an equal amount of negative charge.

21.5. **IDENTIFY:** Each ion carries charge as it enters the axon.

SET UP: The total charge Q is the number N of ions times the charge of each one, which is e. So $Q = Ne$, where $e = 1.60\times10^{-19}$ C.

EXECUTE: The number N of ions is $N = (5.6\times10^{11}\ \text{ions/m})(1.5\times10^{-2}\ \text{m}) = 8.4\times10^{9}$ ions. The total charge Q carried by these ions is $Q = Ne = (8.4\times10^{9})(1.60\times10^{-19}\ \text{C}) = 1.3\times10^{-9}\ \text{C} = 1.3$ nC.

EVALUATE: The amount of charge is small, but these charges are close enough together to exert large forces on nearby charges.

21.6. **IDENTIFY:** Apply Coulomb's law and calculate the net charge q on each sphere.

SET UP: The magnitude of the charge of an electron is $e = 1.60\times10^{-19}$ C.

EXECUTE: $F = k\dfrac{|q_1 q_2|}{r^2}$ gives

$|q| = \sqrt{4\pi\epsilon_0 F r^2} = \sqrt{4\pi\epsilon_0 (3.33\times10^{-21}\ \text{N})(0.200\ \text{m})^2} = 1.217\times10^{-16}$ C. Therefore, the total number of electrons required is $n = |q|/e = (1.217\times10^{-16}\ \text{C})/(1.60\times10^{-19}\ \text{C/electron}) = 760$ electrons.

EVALUATE: Each sphere has 760 excess electrons and each sphere has a net negative charge. The two like charges repel.

21.7. **IDENTIFY:** Apply $F = \dfrac{k|q_1 q_2|}{r^2}$ and solve for r.

SET UP: $F = 650$ N.

EXECUTE: $r = \sqrt{\dfrac{k|q_1 q_2|}{F}} = \sqrt{\dfrac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.0\ \text{C})^2}{650\ \text{N}}} = 3.7\times10^{3}\ \text{m} = 3.7$ km

EVALUATE: Charged objects typically have net charges much less than 1 C.

21.8. **IDENTIFY:** Use the mass of a sphere and the atomic mass of aluminum to find the number of aluminum atoms in one sphere. Each atom has 13 electrons. Apply Coulomb's law and calculate the magnitude of charge $|q|$ on each sphere.

SET UP: $N_A = 6.02\times10^{23}$ atoms/mol. $|q| = n'_e e$, where n'_e is the number of electrons removed from one sphere and added to the other.

EXECUTE: **(a)** The total number of electrons on each sphere equals the number of protons.

$$n_e = n_p = (13)(N_A)\left(\frac{0.0250\text{ kg}}{0.026982\text{ kg/mol}}\right) = 7.25\times10^{24}\text{ electrons.}$$

(b) For a force of 1.00×10^4 N to act between the spheres, $F = 1.00\times10^4\text{ N} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$. This gives

$|q| = \sqrt{4\pi\varepsilon_0(1.00\times10^4\text{ N})(0.800\text{ m})^2} = 8.43\times10^{-4}$ C. The number of electrons removed from one sphere and added to the other is $n'_e = |q|/e = 5.27\times10^{15}$ electrons.

(c) $n'_e/n_e = 7.27\times10^{-10}$.

EVALUATE: When ordinary objects receive a net charge, the fractional change in the total number of electrons in the object is very small.

21.9. IDENTIFY: Apply Coulomb's law.

SET UP: Consider the force on one of the spheres.

EXECUTE: **(a)** $q_1 = q_2 = q$ and $F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1q_2|}{r^2} = \frac{q^2}{4\pi\varepsilon_0 r^2}$, so

$$q = r\sqrt{\frac{F}{(1/4\pi\varepsilon_0)}} = 0.150\text{ m}\sqrt{\frac{0.220\text{ N}}{8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2}} = 7.42\times10^{-7}\text{ C (on each).}$$

(b) $q_2 = 4q_1$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1q_2|}{r^2} = \frac{4q_1^2}{4\pi\varepsilon_0 r^2}\ \text{so}\ q_1 = r\sqrt{\frac{F}{4(1/4\pi\varepsilon_0)}} = \frac{1}{2}r\sqrt{\frac{F}{(1/4\pi\varepsilon_0)}} = \frac{1}{2}(7.42\times10^{-7}\text{ C}) = 3.71\times10^{-7}\text{ C.}$$

And then $q_2 = 4q_1 = 1.48\times10^{-6}$ C.

EVALUATE: The force on one sphere is the same magnitude as the force on the other sphere, whether the spheres have equal charges or not.

21.10. IDENTIFY: We need to determine the number of protons in each box and then use Coulomb's law to calculate the force each box would exert on the other.

SET UP: The mass of a proton is 1.67×10^{-27} kg and the charge of a proton is 1.60×10^{-19} C. The distance from the earth to the moon is 3.84×10^8 m. The electrical force has magnitude $F_e = k\frac{|q_1q_2|}{r^2}$, where $k = 8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2$. The gravitational force has magnitude $F_{\text{grav}} = G\frac{m_1m_2}{r^2}$, where $G = 6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2$.

EXECUTE: **(a)** The number of protons in each box is $N = \frac{1.0\times10^{-3}\text{ kg}}{1.67\times10^{-27}\text{ kg}} = 5.99\times10^{23}$. The total charge of each box is $q = Ne = (5.99\times10^{23})(1.60\times10^{-19}\text{ C}) = 9.58\times10^4$ C. The electrical force on each box is

$F_e = k\frac{q^2}{r^2} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(9.58\times10^4\text{ C})^2}{(3.84\times10^8\text{ m})^2} = 560\text{ N} = 130\text{ lb}$. The tension in the string must equal this repulsive electrical force. The weight of the box on earth is $w = mg = 9.8\times10^{-3}$ N and the weight of the box on the moon is even less, since g is less on the moon. The gravitational forces exerted on the boxes by the earth and by the moon are much less than the electrical force and can be neglected.

(b) $F_{\text{grav}} = G\dfrac{m_1 m_2}{r^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(1.0\times10^{-3}\ \text{kg})^2}{(3.84\times10^8\ \text{m})^2} = 4.5\times10^{-34}\ \text{N}.$

EVALUATE: Both the electrical force and the gravitational force are proportional to $1/r^2$. But in SI units the coefficient k in the electrical force is much greater than the coefficient G in the gravitational force. And a small mass of protons contains a large amount of charge. It would be impossible to put 1.0 g of protons into a small box, because of the very large repulsive electrical forces the protons would exert on each other.

21.11. IDENTIFY: In a space satellite, the only force accelerating the free proton is the electrical repulsion of the other proton.

SET UP: Coulomb's law gives the force, and Newton's second law gives the acceleration: $a = F/m = (1/4\pi\varepsilon_0)(e^2/r^2)/m.$

EXECUTE:

(a) $a = (9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2/[(0.00250\ \text{m})^2(1.67\times10^{-27}\ \text{kg})] = 2.21\times10^4\ \text{m/s}^2.$

(b) The graphs are sketched in Figure 21.11.

EVALUATE: The electrical force of a single stationary proton gives the moving proton an initial acceleration about 20,000 times as great as the acceleration caused by the gravity of the entire earth. As the protons move farther apart, the electrical force gets weaker, so the acceleration decreases. Since the protons continue to repel, the velocity keeps increasing, but at a decreasing rate.

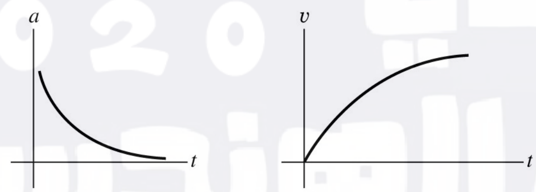


Figure 21.11

21.12. IDENTIFY: Apply Coulomb's law.

SET UP: Like charges repel and unlike charges attract.

EXECUTE: **(a)** $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q_1 q_2|}{r^2}$ gives $0.600\ \text{N} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(0.550\times10^{-6}\ \text{C})|q_2|}{(0.30\ \text{m})^2}$ and $|q_2| = +1.09\times10^{-5}\ \text{C} =$ 10.9 μC. The force is attractive and $q_1 < 0$, so $q_2 = +1.09\times10^{-5}\ \text{C} = +10.9\mu\text{C}$.

(b) $F = 0.600$ N. The force is attractive, so is downward.

EVALUATE: The forces between the two charges obey Newton's third law.

21.13. IDENTIFY: Apply Coulomb's law. The two forces on q_3 must have equal magnitudes and opposite directions.

SET UP: Like charges repel and unlike charges attract.

EXECUTE: The force $\vec{F}_2$ that q_2 exerts on q_3 has magnitude $F_2 = k\dfrac{|q_2 q_3|}{r_2^2}$ and is in the $+x$-direction.

$\vec{F}_1$ must be in the $-x$-direction, so q_1 must be positive. $F_1 = F_2$ gives $k\dfrac{|q_1||q_3|}{r_1^2} = k\dfrac{|q_2||q_3|}{r_2^2}$.

$$|q_1| = |q_2|\left(\frac{r_1}{r_2}\right)^2 = (3.00\ \text{nC})\left(\frac{2.00\ \text{cm}}{4.00\ \text{cm}}\right)^2 = 0.750\ \text{nC}.$$

EVALUATE: The result for the magnitude of q_1 doesn't depend on the magnitude of q_3.

21.14. IDENTIFY: Apply Coulomb's law and find the vector sum of the two forces on Q.

SET UP: The force that q_1 exerts on Q is repulsive, as in Example 21.4, but now the force that q_2 exerts is attractive.

EXECUTE: The x-components cancel. We only need the y-components, and each charge contributes equally. $F_{1y} = F_{2y} = -\frac{1}{4\pi\epsilon_0}\frac{(2.0\times10^{-6}\text{ C})(4.0\times10^{-6}\text{ C})}{(0.500\text{ m})^2}\sin\alpha = -0.173\text{ N}$ (since $\sin\alpha = 0.600$). Therefore, the total force is $2F = 0.35\text{ N}$, in the $-y$-direction.

EVALUATE: If q_1 is $-2.0\,\mu\text{C}$ and q_2 is $+2.0\,\mu\text{C}$, then the net force is in the $+y$-direction.

21.15. **IDENTIFY:** Apply Coulomb's law and find the vector sum of the two forces on q_1.

SET UP: Like charges repel and unlike charges attract, so $\vec{F}_2$ and $\vec{F}_3$ are both in the $+x$-direction.

EXECUTE: $F_2 = k\frac{|q_1q_2|}{r_{12}^2} = 6.749\times10^{-5}\text{ N}, F_3 = k\frac{|q_1q_3|}{r_{13}^2} = 1.124\times10^{-4}\text{ N}.\ F = F_2 + F_3 = 1.8\times10^{-4}\text{ N}.$

$F = 1.8\times10^{-4}$ N and is in the $+x$-direction.

EVALUATE: Comparing our results to those in Example 21.3, we see that $\vec{F}_{1\text{ on }3} = -\vec{F}_{3\text{ on }1}$, as required by Newton's third law.

21.16. **IDENTIFY:** Apply Coulomb's law and find the vector sum of the two forces on q_2.

SET UP: $\vec{F}_{2\text{ on }1}$ is in the $+y$-direction.

EXECUTE: $F_{2\text{ on }1} = \frac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.0\times10^{-6}\text{ C})(2.0\times10^{-6}\text{ C})}{(0.60\text{ m})^2} = 0.100\text{ N}.$ $(F_{2\text{ on }1})_x = 0$ and $(F_{2\text{ on }1})_y = +0.100\text{ N}$. $F_{Q\text{ on }1}$ is equal and opposite to $F_{1\text{ on }Q}$ (Example 21.4), so $(F_{Q\text{ on }1})_x = -0.23\text{ N}$ and $(F_{Q\text{ on }1})_y = 0.17\text{ N}$. $F_x = (F_{2\text{ on }1})_x + (F_{Q\text{ on }1})_x = -0.23\text{ N}$.

$F_y = (F_{2\text{ on }1})_y + (F_{Q\text{ on }1})_y = 0.100\text{ N} + 0.17\text{ N} = 0.27\text{ N}$. The magnitude of the total force is

$F = \sqrt{(0.23\text{ N})^2 + (0.27\text{ N})^2} = 0.35\text{ N}$. $\tan^{-1}\frac{0.23}{0.27} = 40°$, so $\vec{F}$ is $40°$ counterclockwise from the $+y$-axis, or $130°$ counterclockwise from the $+x$-axis.

EVALUATE: Both forces on q_1 are repulsive and are directed away from the charges that exert them.

21.17. **IDENTIFY and SET UP:** Apply Coulomb's law to calculate the force exerted by q_2 and q_3 on q_1. Add these forces as vectors to get the net force. The target variable is the x-coordinate of q_3.

EXECUTE: $\vec{F}_2$ is in the x-direction.

$F_2 = k\frac{|q_1q_2|}{r_{12}^2} = 3.37\text{ N, so }F_{2x} = +3.37\text{ N}$

$F_x = F_{2x} + F_{3x}$ and $F_x = -7.00\text{ N}$

$F_{3x} = F_x - F_{2x} = -7.00\text{ N} - 3.37\text{ N} = -10.37\text{ N}$

For F_{3x} to be negative, q_3 must be on the $-x$-axis.

$F_3 = k\frac{|q_1q_3|}{x^2}$, so $|x| = \sqrt{\frac{k|q_1q_3|}{F_3}} = 0.144\text{ m, so }x = -0.144\text{ m}$

EVALUATE: q_2 attracts q_1 in the $+x$-direction so q_3 must attract q_1 in the $-x$-direction, and q_3 is at negative x.

21.18. **IDENTIFY:** Apply Coulomb's law.

SET UP: Like charges repel and unlike charges attract. Let $\vec{F}_{21}$ be the force that q_2 exerts on q_1 and let $\vec{F}_{31}$ be the force that q_3 exerts on q_1.

EXECUTE: The charge q_3 must be to the right of the origin; otherwise both q_2 and q_3 would exert forces in the $+x$-direction. Calculating the two forces:

$F_{21} = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r_{12}^2} = \frac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-6}\text{ C})(5.00\times10^{-6}\text{ C})}{(0.200\text{ m})^2} = 3.375\text{ N}$, in the $+x$-direction.

$$F_{31} = \frac{(9\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times 10^{-6}\ \text{C})(8.00\times 10^{-6}\ \text{C})}{r_{13}^2} = \frac{0.216\ \text{N}\cdot\text{m}^2}{r_{13}^2},$$ in the $-x$-direction.

We need $F_x = F_{21} - F_{31} = -7.00\ \text{N}$, so $3.375\ \text{N} - \dfrac{0.216\ \text{N}\cdot\text{m}^2}{r_{13}^2} = -7.00\ \text{N}.$

$r_{13} = \sqrt{\dfrac{0.216\ \text{N}\cdot\text{m}^2}{3.375\ \text{N} + 7.00\ \text{N}}} = 0.144\ \text{m}.$ q_3 is at $x = 0.144\ \text{m}.$

EVALUATE: $F_{31} = 10.4\ \text{N}$. F_{31} is larger than F_{21}, because $|q_3|$ is larger than $|q_2|$ and also because r_{13} is less than r_{12}.

21.19. **IDENTIFY:** Apply Coulomb's law to calculate the force each of the two charges exerts on the third charge. Add these forces as vectors.

SET UP: The three charges are placed as shown in Figure 21.19a.

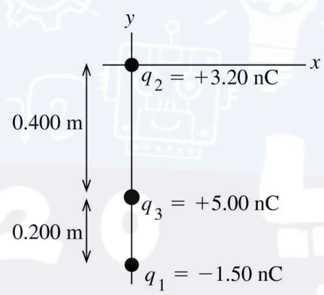
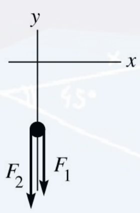


Figure 21.19a

EXECUTE: Like charges repel and unlike attract, so the free-body diagram for q_3 is as shown in Figure 21.19b.



$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1q_3|}{r_{13}^2}$$

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{|q_2q_3|}{r_{23}^2}$$

Figure 21.19b

$$F_1 = (8.988\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.50\times 10^{-9}\ \text{C})(5.00\times 10^{-9}\ \text{C})}{(0.200\ \text{m})^2} = 1.685\times 10^{-6}\ \text{N}$$

$$F_2 = (8.988\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.20\times 10^{-9}\ \text{C})(5.00\times 10^{-9}\ \text{C})}{(0.400\ \text{m})^2} = 8.988\times 10^{-7}\ \text{N}$$

The resultant force is $\vec{\boldsymbol{R}} = \vec{\boldsymbol{F}}_1 + \vec{\boldsymbol{F}}_2$.

$R_x = 0.$

$R_y = -(F_1 + F_2) = -(1.685\times 10^{-6}\ \text{N} + 8.988\times 10^{-7}\ \text{N}) = -2.58\times 10^{-6}\ \text{N}.$

The resultant force has magnitude $2.58\times 10^{-6}\ \text{N}$ and is in the $-y$-direction.

EVALUATE: The force between q_1 and q_3 is attractive and the force between q_2 and q_3 is replusive.

21.20. IDENTIFY: Apply $F = k\dfrac{|qq'|}{r^2}$ to each pair of charges. The net force is the vector sum of the forces due to q_1 and q_2.

SET UP: Like charges repel and unlike charges attract. The charges and their forces on q_3 are shown in Figure 21.20.

EXECUTE: $F_1 = k\dfrac{|q_1q_3|}{r_1^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(4.00\times10^{-9}\ \text{C})(6.00\times10^{-9}\ \text{C})}{(0.200\ \text{m})^2} = 5.394\times10^{-6}\ \text{N}.$

$F_2 = k\dfrac{|q_2q_3|}{r_2^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(5.00\times10^{-9}\ \text{C})(6.00\times10^{-9}\ \text{C})}{(0.300\ \text{m})^2} = 2.997\times10^{-6}\ \text{N}.$

$F_x = F_{1x} + F_{2x} = +F_1 - F_2 = 2.40\times10^{-6}\ \text{N}.$ The net force has magnitude $2.40\times10^{-6}\ \text{N}$ and is in the $+x$-direction.

EVALUATE: Each force is attractive, but the forces are in opposite directions because of the placement of the charges. Since the forces are in opposite directions, the net force is obtained by subtracting their magnitudes.

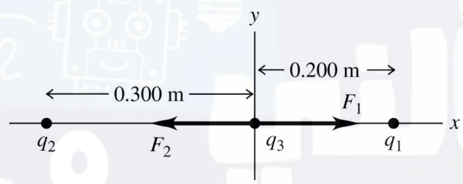


Figure 21.20

21.21. IDENTIFY: We use Coulomb's law to find each electrical force and combine these forces to find the net force.

SET UP: In the O-H-N combination the O^- is 0.170 nm from the H^+ and 0.280 nm from the N^-. In the N-H-N combination the N^- is 0.190 nm from the H^+ and 0.300 nm from the other N^-. Like charges repel and unlike charges attract. The net force is the vector sum of the individual forces. The force due to each pair of charges is $F = k\dfrac{|q_1q_2|}{r^2} = k\dfrac{e^2}{r^2}$.

EXECUTE: **(a)** $F = k\dfrac{|q_1q_2|}{r^2} = k\dfrac{e^2}{r^2}$.

O-H-N:

O^- - H^+: $F = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times10^{-19}\ \text{C})^2}{(0.170\times10^{-9}\ \text{m})^2} = 7.96\times10^{-9}\ \text{N}$, attractive

O^- - N^-: $F = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times10^{-19}\ \text{C})^2}{(0.280\times10^{-9}\ \text{m})^2} = 2.94\times10^{-9}\ \text{N}$, repulsive

N-H-N:

N^- - H^+: $F = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times10^{-19}\ \text{C})^2}{(0.190\times10^{-9}\ \text{m})^2} = 6.38\times10^{-9}\ \text{N}$, attractive

N^- - N^-: $F = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times10^{-19}\ \text{C})^2}{(0.300\times10^{-9}\ \text{m})^2} = 2.56\times10^{-9}\ \text{N}$, repulsive

The total attractive force is $1.43\times10^{-8}\ \text{N}$ and the total repulsive force is $5.50\times10^{-9}\ \text{N}$. The net force is attractive and has magnitude $1.43\times10^{-8}\ \text{N} - 5.50\times10^{-9}\ \text{N} = 8.80\times10^{-9}\ \text{N}$.

(b) $F = k\dfrac{e^2}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times10^{-19}\ \text{C})^2}{(0.0529\times10^{-9}\ \text{m})^2} = 8.22\times10^{-8}\ \text{N}.$

EVALUATE: The bonding force of the electron in the hydrogen atom is a factor of 10 larger than the bonding force of the adenine-thymine molecules.

21.22. IDENTIFY: We use Coulomb's law to find each electrical force and combine these forces to find the net force.

SET UP: In the O-H-O combination the O^- is 0.180 nm from the H^+ and 0.290 nm from the other O^-. In the N-H-N combination the N^- is 0.190 nm from the H^+ and 0.300 nm from the other N^-. In the O-H-N combination the O^- is 0.180 nm from the H^+ and 0.290 nm from the other N^-. Like charges repel and unlike charges attract. The net force is the vector sum of the individual forces. The force due to each pair of charges is $F = k\frac{|q_1 q_2|}{r^2} = k\frac{e^2}{r^2}$.

EXECUTE: Using $F = k\frac{|q_1 q_2|}{r^2} = k\frac{e^2}{r^2}$, we find that the attractive forces are: O^- - H^+, 7.10×10^{-9} N; N^- - H^+, 6.37×10^{-9} N; O^- - H^+, 7.10×10^{-9} N. The total attractive force is 2.06×10^{-8} N. The repulsive forces are: O^- - O^-, 2.74×10^{-9} N; N^- - N^-, 2.56×10^{-9} N; O^- - N^-, 2.74×10^{-9} N. The total repulsive force is 8.04×10^{-9} N. The net force is attractive and has magnitude 1.26×10^{-8} N.

EVALUATE: The net force is attractive, as it should be if the molecule is to stay together.

21.23. IDENTIFY: $F = |q|E$. Since the field is uniform, the force and acceleration are constant and we can use a constant acceleration equation to find the final speed.

SET UP: A proton has charge $+e$ and mass 1.67×10^{-27} kg.

EXECUTE: **(a)** $F = (1.60\times10^{-19}\text{ C})(2.75\times10^3\text{ N/C}) = 4.40\times10^{-16}\text{ N}$.

(b) $a = \frac{F}{m} = \frac{4.40\times10^{-16}\text{ N}}{1.67\times10^{-27}\text{ kg}} = 2.63\times10^{11}\text{ m/s}^2$.

(c) $v_x = v_{0x} + a_x t$ gives $v = (2.63\times10^{11}\text{ m/s}^2)(1.00\times10^{-6}\text{ s}) = 2.63\times10^5\text{ m/s}$.

EVALUATE: The acceleration is very large and the gravity force on the proton can be ignored.

21.24. IDENTIFY: For a point charge, $E = k\frac{|q|}{r^2}$.

SET UP: $\vec{E}$ is toward a negative charge and away from a positive charge.

EXECUTE: **(a)** The field is toward the negative charge so is downward.

$$E = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\text{ C}}{(0.250\text{ m})^2} = 719\text{ N/C}.$$

(b) $r = \sqrt{\frac{k|q|}{E}} = \sqrt{\frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-9}\text{ C})}{12.0\text{ N/C}}} = 1.94\text{ m}$.

EVALUATE: At different points the electric field has different directions, but it is always directed toward the negative point charge.

21.25. IDENTIFY: The acceleration that stops the charge is produced by the force that the electric field exerts on it. Since the field and the acceleration are constant, we can use the standard kinematics formulas to find acceleration and time.

(a) SET UP: First use kinematics to find the proton's acceleration. $v_x = 0$ when it stops. Then find the electric field needed to cause this acceleration using the fact that $F = qE$.

EXECUTE: $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$. $0 = (4.50\times10^6\text{ m/s})^2 + 2a(0.0320\text{ m})$ and $a = 3.16\times10^{14}\text{ m/s}^2$. Now find the electric field, with $q = e$. $eE = ma$ and

$$E = ma/e = (1.67\times10^{-27}\text{ kg})(3.16\times10^{14}\text{ m/s}^2)/(1.60\times10^{-19}\text{ C}) = 3.30\times10^6\text{ N/C},$$ to the left.

(b) SET UP: Kinematics gives $v = v_0 + at$, and $v = 0$ when the electron stops, so $t = v_0/a$.

EXECUTE: $t = v_0/a = (4.50\times10^6\text{ m/s})/(3.16\times10^{14}\text{ m/s}^2) = 1.42\times10^{-8}\text{ s} = 14.2\text{ ns}$.

(c) SET UP: In part (a) we saw that the electric field is proportional to m, so we can use the ratio of the electric fields. $E_e/E_p = m_e/m_p$ and $E_e = (m_e/m_p)E_p$.

EXECUTE: $E_e = [(9.11\times10^{-31}\text{ kg})/(1.67\times10^{-27}\text{ kg})](3.30\times10^6\text{ N/C}) = 1.80\times10^3\text{ N/C}$, to the right.

EVALUATE: Even a modest electric field, such as the ones in this situation, can produce enormous accelerations for electrons and protons.

21.26. IDENTIFY: Use constant acceleration equations to calculate the upward acceleration a and then apply $\vec{F} = q\vec{E}$ to calculate the electric field.

SET UP: Let $+y$ be upward. An electron has charge $q = -e$.

EXECUTE: **(a)** $v_{0y} = 0$ and $a_y = a$, so $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $y - y_0 = \frac{1}{2}at^2$. Then

$$a = \frac{2(y-y_0)}{t^2} = \frac{2(4.50\text{ m})}{(3.00\times10^{-6}\text{ s})^2} = 1.00\times10^{12}\text{ m/s}^2.$$

$$E = \frac{F}{q} = \frac{ma}{q} = \frac{(9.11\times10^{-31}\text{ kg})(1.00\times10^{12}\text{ m/s}^2)}{1.60\times10^{-19}\text{ C}} = 5.69\text{ N/C}$$

The force is up, so the electric field must be *downward* since the electron has negative charge.

(b) The electron's acceleration is $\sim 10^{11}g$, so gravity must be negligibly small compared to the electrical force.

EVALUATE: Since the electric field is uniform, the force it exerts is constant and the electron moves with constant acceleration.

21.27. IDENTIFY: The equation $\vec{F} = q\vec{E}$ relates the electric field, charge of the particle, and the force on the particle. If the particle is to remain stationary the net force on it must be zero.

SET UP: The free-body diagram for the particle is sketched in Figure 21.27. The weight is mg, downward. For the net force to be zero the force exerted by the electric field must be upward. The electric field is downward. Since the electric field and the electric force are in opposite directions the charge of the particle is negative.

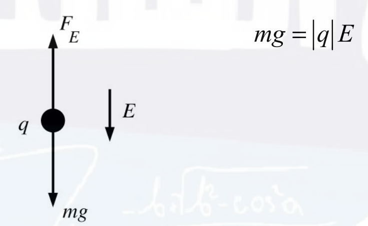


Figure 21.27

EXECUTE: **(a)** $|q| = \dfrac{mg}{E} = \dfrac{(1.45\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)}{650\text{ N/C}} = 2.19\times10^{-5}\text{ C}$ and $q = -21.9\ \mu\text{C}$.

(b) SET UP: The electrical force has magnitude $F_E = |q|E = eE$. The weight of a proton is $w = mg$. $F_E = w$ so $eE = mg$.

EXECUTE: $E = \dfrac{mg}{e} = \dfrac{(1.673\times10^{-27}\text{ kg})(9.80\text{ m/s}^2)}{1.602\times10^{-19}\text{ C}} = 1.02\times10^{-7}\text{ N/C}.$

This is a very small electric field.

EVALUATE: In both cases $|q|E = mg$ and $E = (m/|q|)g$. In part (b) the $m/|q|$ ratio is much smaller $(\sim10^{-8})$ than in part (a) $(\sim10^2)$ so E is much smaller in (b). For subatomic particles gravity can usually be ignored compared to electric forces.

21.28. IDENTIFY: The electric force is $\vec{F} = q\vec{E}$.

SET UP: The gravity force (weight) has magnitude $w = mg$ and is downward.

EXECUTE: **(a)** To balance the weight the electric force must be upward. The electric field is downward, so for an upward force the charge q of the person must be negative. $w = F$ gives $mg = |q|E$ and

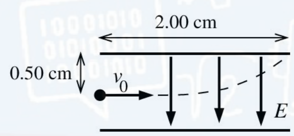
$$|q| = \frac{mg}{E} = \frac{(60\text{ kg})(9.80\text{ m/s}^2)}{150\text{ N/C}} = 3.9\text{ C}.$$

(b) $F = k\dfrac{|qq'|}{r^2} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{(3.9\text{ C})^2}{(100\text{ m})^2} = 1.4\times10^7\text{ N}.$ The repulsive force is immense and this is not a feasible means of flight.

EVALUATE: The net charge of charged objects is typically much less than 1 C.

21.29. IDENTIFY: The equation $\vec{F} = q\vec{E}$ gives the force on the particle in terms of its charge and the electric field between the plates. The force is constant and produces a constant acceleration. The motion is similar to projectile motion; use constant acceleration equations for the horizontal and vertical components of the motion.

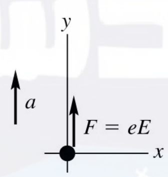
SET UP: The motion is sketched in Figure 21.29a.



For an electron $q = -e$.

Figure 21.29a

$\vec{F} = q\vec{E}$ and q negative gives that $\vec{F}$ and $\vec{E}$ are in opposite directions, so $\vec{F}$ is upward. The free-body diagram for the electron is given in Figure 21.29b.



EXECUTE: **(a)** $\sum F_y = ma_y$

$eE = ma$

Figure 21.29b

Solve the kinematics to find the acceleration of the electron: Just misses upper plate says that $x - x_0 = 2.00\text{ cm}$ when $y - y_0 = +0.500\text{ cm}$.

<u>x-component</u>:

$v_{0x} = v_0 = 1.60\times10^6\text{ m/s},\ a_x = 0,\ x - x_0 = 0.0200\text{ m},\ t = ?$

$$x - x_0 = v_{0x}t + \tfrac{1}{2}a_xt^2$$

$$t = \frac{x - x_0}{v_{0x}} = \frac{0.0200\text{ m}}{1.60\times10^6\text{ m/s}} = 1.25\times10^{-8}\text{ s}$$

In this same time t the electron travels 0.0050 m vertically.

<u>y-component</u>:

$t = 1.25\times10^{-8}\text{ s},\ v_{0y} = 0,\ y - y_0 = +0.0050\text{ m},\ a_y = ?$

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_yt^2$$

$$a_y = \frac{2(y - y_0)}{t^2} = \frac{2(0.0050\text{ m})}{(1.25\times10^{-8}\text{ s})^2} = 6.40\times10^{13}\text{ m/s}^2.$$

(This analysis is very similar to that used in Chapter 3 for projectile motion, except that here the acceleration is upward rather than downward.) This acceleration must be produced by the electric-field force: $eE = ma$.

$$E = \frac{ma}{e} = \frac{(9.109\times10^{-31}\text{ kg})(6.40\times10^{13}\text{ m/s}^2)}{1.602\times10^{-19}\text{ C}} = 364\text{ N/C}$$

Note that the acceleration produced by the electric field is <u>much</u> larger than g, the acceleration produced by gravity, so it is perfectly ok to neglect the gravity force on the electron in this problem.

(b) $a = \dfrac{eE}{m_\text{p}} = \dfrac{(1.602\times10^{-19}\text{ C})(364\text{ N/C})}{1.673\times10^{-27}\text{ kg}} = 3.49\times10^{10}\text{ m/s}^2.$

This is much less than the acceleration of the electron in part (a) so the vertical deflection is less and the proton won't hit the plates. The proton has the same initial speed, so the proton takes the same time $t = 1.25\times10^{-8}\text{ s}$ to travel horizontally the length of the plates. The force on the proton is downward (in the same direction as $\vec{E}$, since q is positive), so the acceleration is downward and $a_y = -3.49\times10^{10}\text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(-3.49\times10^{10}\text{ m/s}^2)(1.25\times10^{-8}\text{ s})^2 = -2.73\times10^{-6}\text{ m}$. The displacement is $2.73\times10^{-6}\text{ m}$, downward.

Evaluate: **(c)** The displacements are in opposite directions because the electron has negative charge and the proton has positive charge. The electron and proton have the same magnitude of charge, so the force the electric field exerts has the same magnitude for each charge. But the proton has a mass larger by a factor of 1836 so its acceleration and its vertical displacement are smaller by this factor.

(d) In each case $a \gg g$ and it is reasonable to ignore the effects of gravity.

21.30. **Identify:** Use the components of $\vec{E}$ from Example 21.6 to calculate the magnitude and direction of $\vec{E}$. Use $\vec{F} = q\vec{E}$ to calculate the force on the -2.5 nC charge and use Newton's third law for the force on the -8.0 nC charge.

Set Up: From Example 21.6, $\vec{E} = (-11\text{ N/C})\hat{i} + (14\text{ N/C})\hat{j}$.

Execute: **(a)** $E = \sqrt{E_x^2 + E_y^2} = \sqrt{(-11\text{ N/C})^2 + (14\text{ N/C})^2} = 17.8\text{ N/C}$.

$\tan^{-1}\left(\dfrac{|E_y|}{|E_x|}\right) = \tan^{-1}(14/11) = 51.8°$, so $\theta = 128°$ counterclockwise from the $+x$-axis.

(b) (i) $\vec{F} = \vec{E}q$ so $F = (17.8\text{ N/C})(2.5\times10^{-9}\text{ C}) = 4.45\times10^{-8}\text{ N}$, at $52°$ below the $+x$-axis.

(ii) 4.45×10^{-8} N at $128°$ counterclockwise from the $+x$-axis.

Evaluate: The forces in part (b) are repulsive so they are along the line connecting the two charges and in each case the force is directed away from the charge that exerts it.

21.31. **Identify:** Apply constant acceleration equations to the motion of the electron.

Set Up: Let $+x$ be to the right and let $+y$ be downward. The electron moves 2.00 cm to the right and 0.50 cm downward.

Execute: Use the horizontal motion to find the time when the electron emerges from the field. $x - x_0 = 0.0200\text{ m}$, $a_x = 0$, $v_{0x} = 1.60\times10^6\text{ m/s}$. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = 1.25\times10^{-8}$ s. Since $a_x = 0$, $v_x = 1.60\times10^6\text{ m/s}$. $y - y_0 = 0.0050\text{ m}$, $v_{0y} = 0$, $t = 1.25\times10^{-8}$ s. $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $v_y = 8.00\times10^5\text{ m/s}$. Then $v = \sqrt{v_x^2 + v_y^2} = 1.79\times10^6\text{ m/s}$.

Evaluate: $v_y = v_{0y} + a_y t$ gives $a_y = 6.4\times10^{13}\text{ m/s}^2$. The electric field between the plates is $E = \dfrac{ma_y}{e} = \dfrac{(9.11\times10^{-31}\text{ kg})(6.4\times10^{13}\text{ m/s}^2)}{1.60\times10^{-19}\text{ C}} = 364\text{ N/C}$. This is not a very large field.

21.32. **Identify:** Apply constant acceleration equations to the motion of the proton. $E = F/|q|$.

Set Up: A proton has mass $m_\text{p} = 1.67\times10^{-27}$ kg and charge $+e$. Let $+x$ be in the direction of motion of the proton.

EXECUTE: **(a)** $v_{0x} = 0$. $a = \dfrac{eE}{m_\text{p}}$. $x - x_0 = v_{0x}t + \dfrac{1}{2}a_x t^2$ gives $x - x_0 = \dfrac{1}{2}a_x t^2 = \dfrac{1}{2}\dfrac{eE}{m_\text{p}}t^2$. Solving for E gives

$$E = \frac{2(0.0160\text{ m})(1.67\times10^{-27}\text{ kg})}{(1.60\times10^{-19}\text{ C})(3.20\times10^{-6}\text{ s})^2} = 32.6\text{ N/C}.$$

(b) $v_x = v_{0x} + a_x t = \dfrac{eE}{m_\text{p}}t = \dfrac{e}{m_\text{p}}\left(\dfrac{2(x-x_0)m_\text{p}}{et^2}\right)t = \dfrac{2(x-x_0)}{t} = \dfrac{2(0.0160\text{ m})}{3.20\times10^{-6}\text{ s}} = 1.00\times10^4\text{ m/s}.$

EVALUATE: The electric field is directed from the positively charged plate toward the negatively charged plate and the force on the proton is also in this direction.

21.33. **IDENTIFY:** Find the angle θ that $\hat{\boldsymbol{r}}$ makes with the +x-axis. Then $\hat{\boldsymbol{r}} = (\cos\theta)\hat{\boldsymbol{i}} + (\sin\theta)\hat{\boldsymbol{j}}$.

SET UP: $\tan\theta = y/x$.

EXECUTE: **(a)** $\tan^{-1}\left(\dfrac{-1.35}{0}\right) = -\dfrac{\pi}{2}$ rad. $\hat{\boldsymbol{r}} = -\hat{\boldsymbol{j}}$.

(b) $\tan^{-1}\left(\dfrac{12}{12}\right) = \dfrac{\pi}{4}$ rad. $\hat{\boldsymbol{r}} = \dfrac{\sqrt{2}}{2}\hat{\boldsymbol{i}} + \dfrac{\sqrt{2}}{2}\hat{\boldsymbol{j}}$.

(c) $\tan^{-1}\left(\dfrac{2.6}{+1.10}\right) = 1.97\text{ rad} = 112.9°$. $\hat{\boldsymbol{r}} = -0.39\hat{\boldsymbol{i}} + 0.92\hat{\boldsymbol{j}}$ (Second quadrant).

EVALUATE: In each case we can verify that $\hat{\boldsymbol{r}}$ is a unit vector, because $\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{r}} = 1$.

21.34. **IDENTIFY:** The net force on each charge must be zero.

SET UP: The force diagram for the $-6.50\ \mu\text{C}$ charge is given in Figure 21.34. FE is the force exerted on the charge by the uniform electric field. The charge is negative and the field is to the right, so the force exerted by the field is to the left. F_q is the force exerted by the other point charge. The two charges have opposite signs, so the force is attractive. Take the +x-axis to be to the right, as shown in the figure.

EXECUTE: **(a)** $F_E = |q|E = (6.50\times10^{-6}\text{ C})(1.85\times10^8\text{ N/C}) = 1.20\times10^3\text{ N}$

$$F_q = k\frac{|q_1q_2|}{r^2} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(6.50\times10^{26}\text{ C})(8.75\times10^{-6}\text{ C})}{(0.0250\text{ m})^2} = 8.18\times10^2\text{ N}$$

$\sum F_x = 0$ gives $T + F_q - F_E = 0$ and $T = F_E - F_q = 382$ N.

(b) Now F_q is to the left, since like charges repel.

$\sum F_x = 0$ gives $T - F_q - F_E = 0$ and $T = F_E + F_q = 2.02\times10^3$ N.

EVALUATE: The tension is much larger when both charges have the same sign, so the force one charge exerts on the other is repulsive.

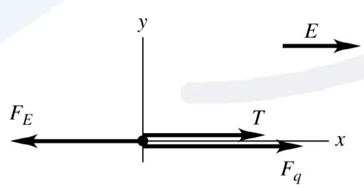


Figure 21.34

21.35. **IDENTIFY** and **SET UP:** Use $\vec{E}$ in $\vec{E} = \dfrac{\vec{F}_0}{q_0}$ to calculate $\vec{F}$, $\vec{F} = m\vec{a}$ to calculate $\vec{a}$, and a constant acceleration equation to calculate the final velocity. Let +x be east.

(a) EXECUTE: $F_x = |q|E = (1.602\times10^{-19}\text{ C})(1.50\text{ N/C}) = 2.403\times10^{-19}$ N.

$a_x = F_x/m = (2.403\times10^{-19}\text{ N})/(9.109\times10^{-31}\text{ kg}) = +2.638\times10^{11}\text{ m/s}^2$.

$v_{0x} = +4.50\times10^5$ m/s, $a_x = +2.638\times10^{11}\text{ m/s}^2$, $x - x_0 = 0.375$ m, $v_x = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = 6.33\times10^5$ m/s.

EVALUATE: $\vec{E}$ is west and q is negative, so $\vec{F}$ is east and the electron speeds up.

(b) EXECUTE: $F_x = -|q|E = -(1.602\times10^{-19}\text{ C})(1.50\text{ N/C}) = -2.403\times10^{-19}$ N.

$a_x = F_x/m = (-2.403\times10^{-19}\text{ N})/(1.673\times10^{-27}\text{ kg}) = -1.436\times10^8\text{ m/s}^2$.

$v_{0x} = +1.90\times10^4$ m/s, $a_x = -1.436\times10^8\text{ m/s}^2$, $x - x_0 = 0.375$ m, $v_x = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = 1.59\times10^4$ m/s.

EVALUATE: $q > 0$ so $\vec{F}$ is west and the proton slows down.

21.36. IDENTIFY: The net electric field is the vector sum of the fields due to the individual charges.

SET UP: The electric field points toward negative charge and away from positive charge.

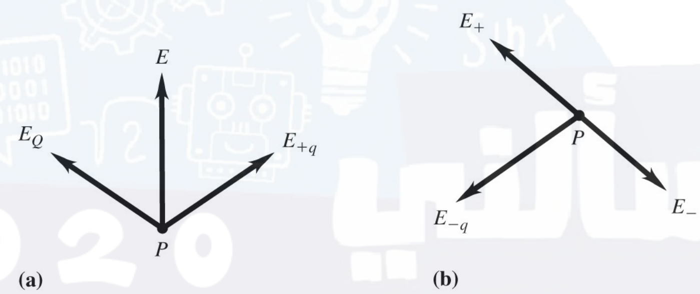


Figure 21.36

EXECUTE: **(a)** Figure 21.36a shows $\vec{E}_Q$ and $\vec{E}_{+q}$ at point P. $\vec{E}_Q$ must have the direction shown, to produce a resultant field in the specified direction. $\vec{E}_Q$ is toward Q, so Q is negative. In order for the horizontal components of the two fields to cancel, Q and q must have the same magnitude.

(b) No. If the lower charge were negative, its field would be in the direction shown in Figure 21.36b. The two possible directions for the field of the upper charge, when it is positive ($\vec{E}_+$) or negative ($\vec{E}_-$), are shown. In neither case is the resultant field in the direction shown in the figure in the problem.

EVALUATE: When combining electric fields, it is always essential to pay attention to their directions.

21.37. IDENTIFY: Calculate the electric field due to each charge and find the vector sum of these two fields.

SET UP: At points on the x-axis only the x-component of each field is nonzero. The electric field of a point charge points away from the charge if it is positive and toward it if it is negative.

EXECUTE: **(a)** Halfway between the two charges, $E = 0$.

(b) For $|x| < a$, $E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q}{(a+x)^2} - \dfrac{q}{(a-x)^2}\right) = -\dfrac{4q}{4\pi\epsilon_0}\dfrac{ax}{(x^2-a^2)^2}$.

For $x > a$, $E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q}{(a+x)^2} + \dfrac{q}{(a-x)^2}\right) = \dfrac{2q}{4\pi\epsilon_0}\dfrac{x^2+a^2}{(x^2-a^2)^2}$.

For $x < -a$, $E_x = \dfrac{-1}{4\pi\epsilon_0}\left(\dfrac{q}{(a+x)^2} + \dfrac{q}{(a-x)^2}\right) = -\dfrac{2q}{4\pi\epsilon_0}\dfrac{x^2+a^2}{(x^2-a^2)^2}$.

The graph of E_x versus x is sketched in Figure 21.37 (next page).

EVALUATE: The magnitude of the field approaches infinity at the location of one of the point charges.

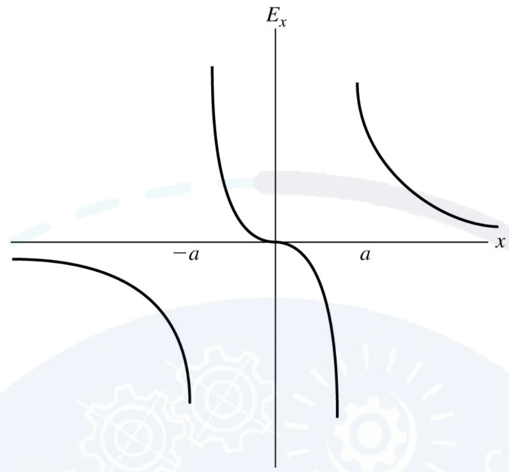


Figure 21.37

21.38. **IDENTIFY:** Add the individual electric fields to obtain the net field.
SET UP: The electric field points away from positive charge and toward negative charge. The electric fields $\vec{E}_1$ and $\vec{E}_2$ add to form the net field $\vec{E}$.

EXECUTE: **(a)** The electric field is toward A at points B and C and the field is zero at A.
(b) The electric field is away from A at B and C. The field is zero at A.
(c) The field is horizontal and to the right at points A, B, and C.
EVALUATE: Compare your results to the field lines shown in Figure 21.28a and b in the textbook.

21.39. **IDENTIFY:** $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q|}{r^2}$ gives the electric field of each point charge. Use the principle of superposition and add the electric field vectors. In part (b) use $\vec{E} = \dfrac{\vec{F}_0}{q_0}$ to calculate the force, using the electric field calculated in part (a).
SET UP: The placement of charges is sketched in Figure 21.39a.

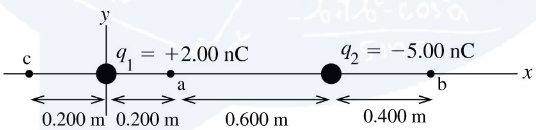


Figure 21.39a

The electric field of a point charge is directed away from the point charge if the charge is positive and toward the point charge if the charge is negative. The magnitude of the electric field is $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q|}{r^2}$, where r is the distance between the point where the field is calculated and the point charge.
(a) EXECUTE: (i) At point a the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.39b.

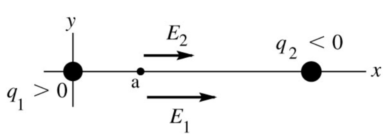


Figure 21.39b

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{2.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 449.4\ \text{N/C}.$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\ \text{C}}{(0.600\ \text{m})^2} = 124.8\ \text{N/C}.$$

$E_{1x} = 449.4$ N/C, $E_{1y} = 0$.

$E_{2x} = 124.8$ N/C, $E_{2y} = 0$.

$E_x = E_{1x} + E_{2x} = +449.4\ \text{N/C} + 124.8\ \text{N/C} = +574.2\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point a has magnitude 574 N/C and is in the $+x$-direction.

(ii) At point b the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.39c.

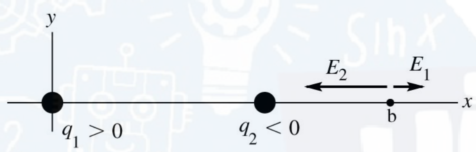


Figure 21.39c

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{2.00\times10^{-9}\ \text{C}}{(1.20\ \text{m})^2} = 12.5\ \text{N/C}.$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\ \text{C}}{(0.400\ \text{m})^2} = 280.9\ \text{N/C}.$$

$E_{1x} = 12.5$ N/C, $E_{1y} = 0$.

$E_{2x} = -280.9$ N/C, $E_{2y} = 0$.

$E_x = E_{1x} + E_{2x} = +12.5\ \text{N/C} - 280.9\ \text{N/C} = -268.4\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point b has magnitude 268 N/C and is in the $-x$-direction.

(iii) At point c the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.39d.

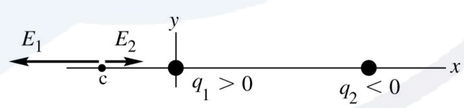


Figure 21.39d

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{2.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 449.4\ \text{N/C}.$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\ \text{C}}{(1.00\ \text{m})^2} = 44.9\ \text{N/C}.$$

$E_{1x} = -449.4$ N / C, $E_{1y} = 0$.

$E_{2x} = +44.9$ N / C, $E_{2y} = 0$.

$E_x = E_{1x} + E_{2x} = -449.4\ \text{N/C} + 44.9\ \text{N/C} = -404.5\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point b has magnitude 404 N/C and is in the $-x$-direction.

(b) SET UP: Since we have calculated $\vec{E}$ at each point the simplest way to get the force is to use $\vec{F} = -e\vec{E}$.

EXECUTE: (i) $F = (1.602\times10^{-19}\text{ C})(574.2\text{ N/C}) = 9.20\times10^{-17}\text{ N}$, $-x$-direction.

(ii) $F = (1.602\times10^{-19}\text{ C})(268.4\text{ N/C}) = 4.30\times10^{-17}\text{ N}$, $+x$-direction.

(iii) $F = (1.602\times10^{-19}\text{ C})(404.5\text{ N/C}) = 6.48\times10^{-17}\text{ N}$, $+x$-direction.

EVALUATE: The general rule for electric field direction is away from positive charge and toward negative charge. Whether the field is in the $+x$- or $-x$-direction depends on where the field point is relative to the charge that produces the field. In part (a), for (i) the field magnitudes were added because the fields were in the same direction and in (ii) and (iii) the field magnitudes were subtracted because the two fields were in opposite directions. In part (b) we could have used Coulomb's law to find the forces on the electron due to the two charges and then added these force vectors, but using the resultant electric field is much easier.

21.40. IDENTIFY: $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ gives the electric field of each point charge. Use the principle of superposition and add the electric field vectors. In part (b) use $\vec{E} = \dfrac{\vec{F}_0}{q_0}$ to calculate the force, using the electric field calculated in part (a).

(a) SET UP: The placement of charges is sketched in Figure 21.40a.

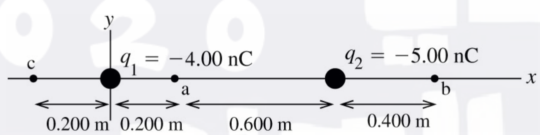


Figure 21.40a

The electric field of a point charge is directed away from the point charge if the charge is positive and toward the point charge if the charge is negative. The magnitude of the electric field is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$, where r is the distance between the point where the field is calculated and the point charge.

(i) At point a the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.40b.

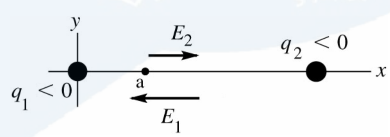


Figure 21.40b

EXECUTE: $E_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{4.00\times10^{-9}\text{ C}}{(0.200\text{ m})^2} = 898.8\text{ N/C}.$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\text{ C}}{(0.600\text{ m})^2} = 124.8\text{ N/C}.$$

$E_{1x} = 898.8\text{ N/C}, E_{1y} = 0.$

$E_{2x} = 124.8\text{ N/C}, E_{2y} = 0.$

$E_x = E_{1x} + E_{2x} = -898.8\text{ N/C} + 124.8\text{ N/C} = -774\text{ N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point a has magnitude 774 N/C and is in the $-x$-direction.

(ii) **SET UP:** At point b the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.40c.

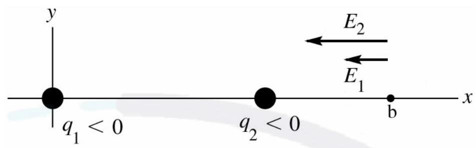


Figure 21.40c

EXECUTE: $E_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{4.00\times10^{-9}\ \text{C}}{(1.20\ \text{m})^2} = 24.97\ \text{N/C}.$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\ \text{C}}{(0.400\ \text{m})^2} = 280.9\ \text{N/C}.$$

$E_{1x} = -24.97\ \text{N/C}, E_{1y} = 0.$

$E_{2x} = -280.9\ \text{N/C}, E_{2y} = 0.$

$E_x = E_{1x} + E_{2x} = -24.97\ \text{N/C} - 280.9\ \text{N/C} = -305.9\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point b has magnitude 306 N/C and is in the $-x$-direction.

(iii) **SET UP:** At point c the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.40d.

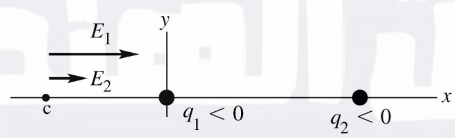


Figure 21.40d

EXECUTE: $E_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{4.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 898.8\ \text{N/C}.$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\ \text{C}}{(1.00\ \text{m})^2} = 44.9\ \text{N/C}.$$

$E_{1x} = +898.8\ \text{N/C}, E_{1y} = 0.$

$E_{2x} = +44.9\ \text{N/C}, E_{2y} = 0.$

$E_x = E_{1x} + E_{2x} = +898.8\ \text{N/C} + 44.9\ \text{N/C} = +943.7\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point b has magnitude 944 N/C and is in the $+x$-direction.

(b) SET UP: Since we have calculated $\vec{E}$ at each point the simplest way to get the force is to use $\vec{F} = -e\vec{E}$.

EXECUTE: (i) $F = (1.602\times10^{-19}\ \text{C})(774\ \text{N/C}) = 1.24\times10^{-16}\ \text{N}$, $+x$-direction.

(ii) $F = (1.602\times10^{-19}\ \text{C})(305.9\ \text{N/C}) = 4.90\times10^{-17}\ \text{N}$, $+x$-direction.

(iii) $F = (1.602\times10^{-19}\ \text{C})(943.7\ \text{N/C}) = 1.51\times10^{-16}\ \text{N}$, $-x$-direction.

EVALUATE: The general rule for electric field direction is away from positive charge and toward negative charge. Whether the field is in the $+x$- or $-x$-direction depends on where the field point is relative to the charge that produces the field. In part (a), for (i) the field magnitudes were subtracted because the fields were in opposite directions and in (ii) and (iii) the field magnitudes were added because the two fields were in the same direction. In part (b) we could have used Coulomb's law to find the forces on the electron due to the two charges and then added these force vectors, but using the resultant electric field is much easier.

21.41. IDENTIFY: $E = k\dfrac{|q|}{r^2}$. The net field is the vector sum of the fields due to each charge.

SET UP: The electric field of a negative charge is directed toward the charge. Label the charges q_1, q_2, and q_3, as shown in Figure 21.41a. This figure also shows additional distances and angles. The electric fields at point *P* are shown in Figure 21.41b. This figure also shows the *xy*-coordinates we will use and the *x*- and *y*-components of the fields $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$.

EXECUTE: $E_1 = E_3 = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{5.00\times10^{-6}\ \text{C}}{(0.100\ \text{m})^2} = 4.49\times10^6\ \text{N/C}.$

$E_2 = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{2.00\times10^{-6}\ \text{C}}{(0.0600\ \text{m})^2} = 4.99\times10^6\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} + E_{3y} = 0$ and $E_x = E_{1x} + E_{2x} + E_{3x} = E_2 + 2E_1\cos 53.1° = 1.04\times10^7\ \text{N/C}.$

$E = 1.04\times10^7\ \text{N/C}$, toward the $-2.00\ \mu\text{C}$ charge.

EVALUATE: The *x*-components of the fields of all three charges are in the same direction.

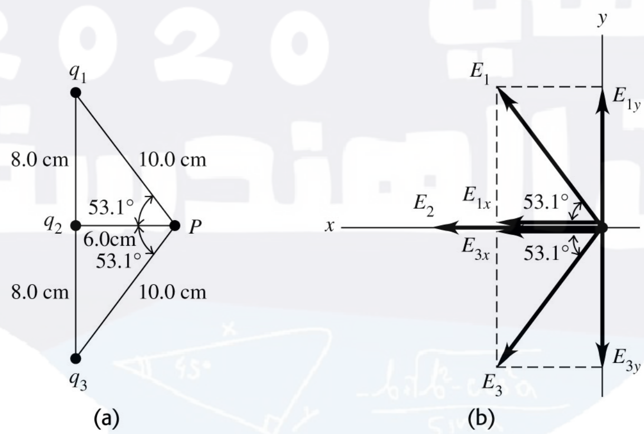


Figure 21.41

21.42. IDENTIFY: The net electric field is the vector sum of the individual fields.

SET UP: The distance from a corner to the center of the square is $r = \sqrt{(a/2)^2 + (a/2)^2} = a/\sqrt{2}$. The magnitude of the electric field due to each charge is the same and equal to $E_q = \dfrac{kq}{r^2} = 2\dfrac{kq}{a^2}$. All four *y*-components add and the *x*-components cancel.

EXECUTE: Each *y*-component is equal to $E_{qy} = -E_q\cos 45° = -\dfrac{E_q}{\sqrt{2}} = \dfrac{-2kq}{\sqrt{2}a^2} = -\dfrac{\sqrt{2}kq}{a^2}$. The resultant field is $\dfrac{4\sqrt{2}kq}{a^2}$, in the $-y$-direction.

EVALUATE: We must add the *y*-components of the fields, not their magnitudes.

21.43. IDENTIFY: For a point charge, $E = k\dfrac{|q|}{r^2}$. The net field is the vector sum of the fields produced by each charge. A charge *q* in an electric field $\vec{E}$ experiences a force $\vec{F} = q\vec{E}$.

SET UP: The electric field of a negative charge is directed toward the charge. Point A is 0.100 m from q_2 and 0.150 m from q_1. Point B is 0.100 m from q_1 and 0.350 m from q_2.

EXECUTE: **(a)** The electric fields at point A due to the charges are shown in Figure 21.43a.

$$E_1 = k\frac{|q_1|}{r_{A1}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.25\times10^{-9}\ \text{C}}{(0.150\ \text{m})^2} = 2.50\times10^3\ \text{N/C}.$$

$$E_2 = k\frac{|q_2|}{r_{A2}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12.5\times10^{-9}\ \text{C}}{(0.100\ \text{m})^2} = 1.124\times10^4\ \text{N/C}.$$

Since the two fields are in opposite directions, we subtract their magnitudes to find the net field.

$E = E_2 - E_1 = 8.74\times10^3\ \text{N/C}$, to the right.

(b) The electric fields at point B are shown in Figure 21.43b.

$$E_1 = k\frac{|q_1|}{r_{B1}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.25\times10^{-9}\ \text{C}}{(0.100\ \text{m})^2} = 5.619\times10^3\ \text{N/C}.$$

$$E_2 = k\frac{|q_2|}{r_{B2}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12.5\times10^{-9}\ \text{C}}{(0.350\ \text{m})^2} = 9.17\times10^2\ \text{N/C}.$$

Since the fields are in the same direction, we add their magnitudes to find the net field.

$E = E_1 + E_2 = 6.54\times10^3\ \text{N/C}$, to the right.

(c) At A, $E = 8.74\times10^3\ \text{N/C}$, to the right. The force on a proton placed at this point would be

$F = qE = (1.60\times10^{-19}\ \text{C})(8.74\times10^3\ \text{N/C}) = 1.40\times10^{-15}\ \text{N}$, to the right.

EVALUATE: A proton has positive charge so the force that an electric field exerts on it is in the same direction as the field.

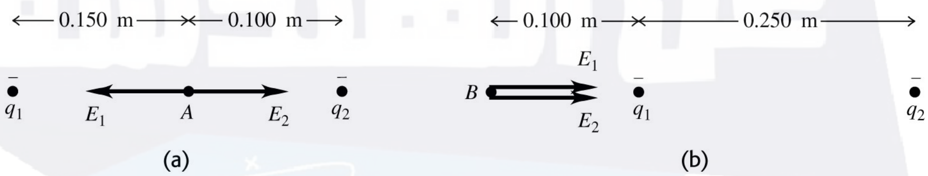


Figure 21.43

21.44. IDENTIFY: Apply $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ to calculate the electric field due to each charge and add the two field vectors to find the resultant field.

SET UP: For q_1, $\hat{r} = \hat{j}$. For q_2, $\hat{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$, where θ is the angle between $\vec{E}_2$ and the $+x$-axis.

EXECUTE: **(a)** $\vec{E}_1 = \dfrac{q_1}{4\pi\epsilon_0 r_1^2}\hat{j} = \dfrac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-5.00\times10^{-9}\ \text{C})}{(0.0400\ \text{m})^2}\hat{j} = (-2.813\times10^4\ \text{N/C})\ \hat{j}.$

$\left|\vec{E}_2\right| = \dfrac{q_2}{4\pi\epsilon_0 r_2^2} = \dfrac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-9}\ \text{C})}{(0.0300\ \text{m})^2 + (0.0400\ \text{m})^2} = 1.080\times10^4\ \text{N/C}.$ The angle of $\vec{E}_2$, measured from the x-axis, is $180° - \tan^{-1}\left(\dfrac{4.00\ \text{cm}}{3.00\ \text{cm}}\right) = 126.9°$ Thus

$$\vec{E}_2 = (1.080\times10^4\ \text{N/C})(\hat{i}\cos126.9° + \hat{j}\sin126.9°) = (-6.485\times10^3\ \text{N/C})\hat{i} + (8.64\times10^3\ \text{N/C})\hat{j}.$$

(b) The resultant field is $\vec{E}_1 + \vec{E}_2 = (-6.485\times10^3\ \text{N/C})\hat{i} + (-2.813\times10^4\ \text{N/C} + 8.64\times10^3\ \text{N/C})\hat{j}.$

$$\vec{E}_1 + \vec{E}_2 = (-6.485\times10^3\ \text{N/C})\hat{i} - (1.95\times10^4\ \text{N/C})\hat{j}.$$

EVALUATE: $\vec{E}_1$ is toward q_1 since q_1 is negative. $\vec{E}_2$ is directed away from q_2, since q_2 is positive.

21.45. IDENTIFY: The forces the charges exert on each other are given by Coulomb's law. The net force on the proton is the vector sum of the forces due to the electrons.

SET UP: $q_e = -1.60\times10^{-19}$ C. $q_p = +1.60\times10^{-19}$ C. The net force is the vector sum of the forces exerted by each electron. Each force has magnitude $F = k\dfrac{|q_1q_2|}{r^2} = k\dfrac{e^2}{r^2}$ and is attractive so is directed toward the electron that exerts it.

EXECUTE: Each force has magnitude

$F_1 = F_2 = k\dfrac{|q_1q_2|}{r^2} = k\dfrac{e^2}{r^2} = \dfrac{(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(1.50\times10^{-10}\ \text{m})^2} = 1.023\times10^{-8}$ N. The vector force diagram is shown in Figure 21.45.

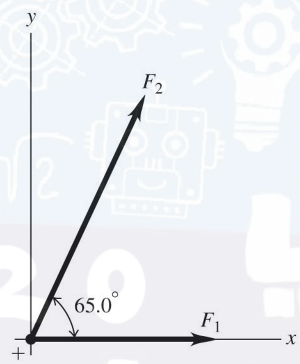


Figure 21. 45

Taking components, we get $F_{1x} = 1.023\times10^{-8}$ N; $F_{1y} = 0$. $F_{2x} = F_2\cos65.0° = 4.32\times10^{-9}$ N; $F_{2y} = F_2\sin65.0° = 9.27\times10^{-9}$ N. $F_x = F_{1x} + F_{2x} = 1.46\times10^{-8}$ N; $F_y = F_{1y} + F_{2y} = 9.27\times10^{-9}$ N.

$F = \sqrt{F_x^2 + F_y^2} = 1.73\times10^{-8}$ N. $\tan\theta = \dfrac{F_y}{F_x} = \dfrac{9.27\times10^{-9}\ \text{N}}{1.46\times10^{-8}\ \text{N}} = 0.6349$ which gives

$\theta = 32.4°$. The net force is 1.73×10^{-8} N and is directed toward a point midway between the two electrons.

EVALUATE: Note that the net force is less than the algebraic sum of the individual forces.

21.46. IDENTIFY: We can model a segment of the axon as a point charge.

SET UP: If the axon segment is modeled as a point charge, its electric field is $E = k\dfrac{q}{r^2}$. The electric field of a point charge is directed away from the charge if it is positive.

EXECUTE: **(a)** 5.6×10^{11} Na^+ ions enter per meter so in a 0.10 mm $= 1.0\times10^{-4}$ m section, 5.6×10^7 Na^+ ions enter. This number of ions has charge $q = (5.6\times10^7)(1.60\times10^{-19}\ \text{C}) = 9.0\times10^{-12}$ C.

(b) $E = k\dfrac{|q|}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{9.0\times10^{-12}\ \text{C}}{(5.00\times10^{-2}\ \text{m})^2} = 32$ N/C, directed away from the axon.

(c) $r = \sqrt{\dfrac{k|q|}{E}} = \sqrt{\dfrac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(9.0\times10^{-12}\ \text{C})}{1.0\times10^{-6}\ \text{N/C}}} = 280$ m.

EVALUATE: The field in (b) is considerably smaller than ordinary laboratory electric fields.

21.47. IDENTIFY: The electric field of a positive charge is directed radially outward from the charge and has magnitude $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$. The resultant electric field is the vector sum of the fields of the individual charges.

SET UP: The placement of the charges is shown in Figure 21.47a.

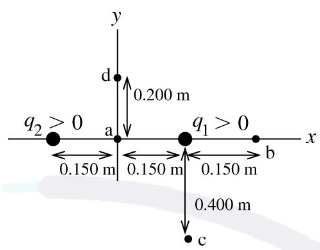
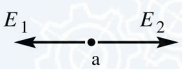


Figure 21.47a

EXECUTE: **(a)** The directions of the two fields are shown in Figure 21.47b.

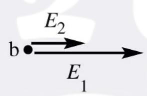


$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} \text{ with } r = 0.150 \text{ m}.$$

$$E = E_2 - E_1 = 0;\ E_x = 0,\ E_y = 0.$$

Figure 21. 47b

(b) The two fields have the directions shown in Figure 21.47c.



$E = E_1 + E_2$, in the $+x$-direction.

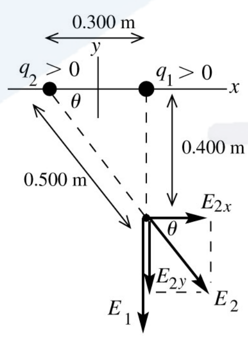
Figure 21. 47c

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\text{ C}}{(0.150\text{ m})^2} = 2396.8 \text{ N/C}.$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\text{ C}}{(0.450\text{ m})^2} = 266.3 \text{ N/C}.$$

$$E = E_1 + E_2 = 2396.8 \text{ N/C} + 266.3 \text{ N/C} = 2660 \text{ N/C};\ E_x = +2660 \text{ N/C},\ E_y = 0.$$

(c) The two fields have the directions shown in Figure 21.47d.



$$\sin\theta = \frac{0.400\text{ m}}{0.500\text{ m}} = 0.800.$$

$$\cos\theta = \frac{0.300\text{ m}}{0.500\text{ m}} = 0.600.$$

Figure 21. 47d

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\text{ C}}{(0.400\text{ m})^2} = 337.1 \text{ N/C}.$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.500\ \text{m})^2} = 215.7\ \text{N/C}.$$

$E_{1x} = 0,\ E_{1y} = -E_1 = -337.1$ N/C.

$E_{2x} = +E_2\cos\theta = +(215.7\ \text{N/C})(0.600) = +129.4$ N/C.

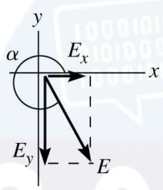
$E_{2y} = -E_2\sin\theta = -(215.7\ \text{N/C})(0.800) = -172.6$ N/C.

$E_x = E_{1x} + E_{2x} = +129$ N/C.

$E_y = E_{1y} + E_{2y} = -337.1\ \text{N/C} - 172.6\ \text{N/C} = -510$ N/C.

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(129\ \text{N/C})^2 + (-510\ \text{N/C})^2} = 526\ \text{N/C}.$$

$\vec{E}$ and its components are shown in Figure 21.47e.



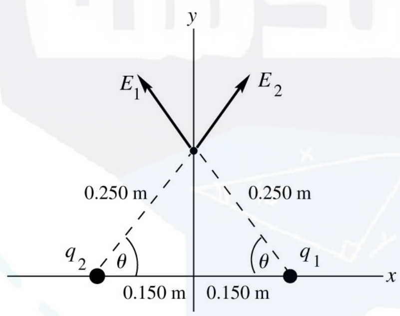
$$\tan\alpha = \frac{E_y}{E_x}.$$

$$\tan\alpha = \frac{-510\ \text{N/C}}{+129\ \text{N/C}} = -3.953.$$

$\alpha = 284°$, counterclockwise from $+x$-axis.

Figure 21. 47e

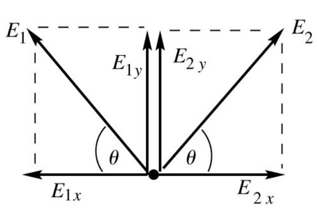
(d) The two fields have the directions shown in Figure 21.47f.



$$\sin\theta = \frac{0.200\ \text{m}}{0.250\ \text{m}} = 0.800.$$

Figure 21. 47f

The components of the two fields are shown in Figure 21.47g.



$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}.$$

$$E_1 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.250\ \text{m})^2}.$$

$E_1 = E_2 = 862.8$ N/C.

Figure 21. 47g

$E_{1x} = -E_1\cos\theta,\ E_{2x} = +E_2\cos\theta.$

$E_x = E_{1x} + E_{2x} = 0.$

$E_{1y} = +E_1\sin\theta,\ E_{2y} = +E_2\sin\theta.$

$E_y = E_{1y} + E_{2y} = 2E_{1y} = 2E_1 \sin\theta = 2(862.8 \text{ N/C})(0.800) = 1380 \text{ N/C}.$

$E = 1380$ N/C, in the $+y$-direction.

EVALUATE: Point *a* is symmetrically placed between identical charges, so symmetry tells us the electric field must be zero. Point *b* is to the right of both charges and both electric fields are in the +*x*-direction and the resultant field is in this direction. At point *c* both fields have a downward component and the field of q_2 has a component to the right, so the net $\vec{E}$ is in the fourth quadrant. At point *d* both fields have an upward component but by symmetry they have equal and opposite *x*-components so the net field is in the +*y*-direction. We can use this sort of reasoning to deduce the general direction of the net field before doing any calculations.

21.48. IDENTIFY: Apply $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ to calculate the field due to each charge and then calculate the vector sum of those fields.

SET UP: The fields due to q_1 and to q_2 are sketched in Figure 21.48.

EXECUTE: $\vec{E}_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{(6.00\times10^{-9}\text{ C})}{(0.6\text{ m})^2}(-\hat{i}) = -150\hat{i}$ N/C.

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}(4.00\times10^{-9}\text{ C})\left(\frac{1}{(1.00\text{ m})^2}(0.600)\hat{i} + \frac{1}{(1.00\text{ m})^2}(0.800)\hat{j}\right) = (21.6\hat{i} + 28.8\hat{j})\text{N/C}.$$

$\vec{E} = \vec{E}_1 + \vec{E}_2 = (-128.4\text{ N/C})\hat{i} + (28.8\text{ N/C})\hat{j}$. $E = \sqrt{(128.4\text{ N/C})^2 + (28.8\text{ N/C})^2} = 131.6$ N/C at $\theta = \tan^{-1}\left(\dfrac{28.8}{128.4}\right) = 12.6°$ above the $-x$-axis and therefore $167.4°$ counterclockwise from the +*x*-axis.

EVALUATE: $\vec{E}_1$ is directed toward q_1 because q_1 is negative and $\vec{E}_2$ is directed away from q_2 because q_2 is positive.

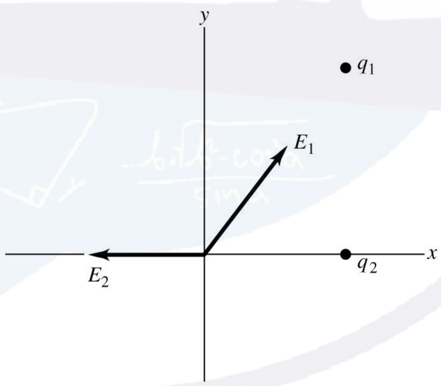


Figure 21.48

21.49. IDENTIFY: We must use the appropriate electric field formula: a uniform disk in (a), a ring in (b) because all the charge is along the rim of the disk, and a point-charge in (c).

(a) SET UP: First find the surface charge density (*Q*/*A*), then use the formula for the field due to a disk of charge, $E_x = \dfrac{\sigma}{2\epsilon_0}\left[1 - \dfrac{1}{\sqrt{(R/x)^2 + 1}}\right]$.

EXECUTE: The surface charge density is $\sigma = \dfrac{Q}{A} = \dfrac{Q}{\pi r^2} = \dfrac{6.50\times10^{-9}\text{ C}}{\pi(0.0125\text{ m})^2} = 1.324\times10^{-5}\text{ C/m}^2$.

The electric field is

$$E_x = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{(R/x)^2+1}}\right] = \frac{1.324\times10^{-5}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\left[1 - \frac{1}{\sqrt{\left(\frac{1.25\ \text{cm}}{2.00\ \text{cm}}\right)^2+1}}\right]$$

$$E_x = 1.14\times10^5\ \text{N/C, toward the center of the disk.}$$

(b) SET UP: For a ring of charge, the field is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}$.

EXECUTE: Substituting into the electric field formula gives

$$E = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}} = \frac{(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.50\times10^{-9}\ \text{C})(0.0200\ \text{m})}{[(0.0200\ \text{m})^2+(0.0125\ \text{m})^2]^{3/2}}$$

$$E = 8.92\times10^4\ \text{N/C, toward the center of the disk.}$$

(c) SET UP: For a point charge, $E = (1/4\pi\epsilon_0)q/r^2$.

EXECUTE: $E = (9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.50\times10^{-9}\ \text{C})/(0.0200\ \text{m})^2 = 1.46\times10^5\ \text{N/C}$.

(d) EVALUATE: With the ring, more of the charge is farther from P than with the disk. Also with the ring the component of the electric field parallel to the plane of the ring is greater than with the disk, and this component cancels. With the point charge in (c), all the field vectors add with no cancellation, and all the charge is closer to point P than in the other two cases.

21.50. IDENTIFY: For a long straight wire, $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$.

SET UP: $\dfrac{1}{2\pi\epsilon_0} = 1.80\times10^{10}\ \text{N}\cdot\text{m}^2/\text{C}^2$.

EXECUTE: Solve $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$ for r: $r = \dfrac{3.20\times10^{-10}\ \text{C/m}}{2\pi\epsilon_0(2.50\ \text{N/C})} = 2.30\ \text{m}$.

EVALUATE: For a point charge, E is proportional to $1/r^2$. For a long straight line of charge, E is proportional to $1/r$.

21.51. IDENTIFY: For a ring of charge, the magnitude of the electric field is given by $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}$.

Use $\vec{F} = q\vec{E}$. In part (b) use Newton's third law to relate the force on the ring to the force exerted by the ring.

SET UP: $Q = 0.125\times10^{-9}$ C, $a = 0.025$ m and $x = 0.400$ m.

EXECUTE: **(a)** $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}\hat{i} = (7.0\ \text{N/C})\hat{i}$.

(b) $\vec{F}_{\text{on ring}} = -\vec{F}_{\text{on q}} = -q\vec{E} = -(-2.50\times10^{-6}\ \text{C})(7.0\ \text{N/C})\hat{i} = (1.75\times10^{-5}\ \text{N})\hat{i}$.

EVALUATE: Charges q and Q have opposite sign, so the force that q exerts on the ring is attractive.

21.52. (a) IDENTIFY: The field is caused by a finite uniformly charged wire.

SET UP: The field for such a wire a distance x from its midpoint is

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{x\sqrt{(x/a)^2+1}} = 2\left(\frac{1}{4\pi\epsilon_0}\right)\frac{\lambda}{x\sqrt{(x/a)^2+1}}.$$

EXECUTE: $$E = \frac{(18.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(175\times10^{-9}\ \text{C/m})}{(0.0600\ \text{m})\sqrt{\left(\frac{6.00\ \text{cm}}{4.25\ \text{cm}}\right)^2+1}} = 3.03\times10^4\ \text{N/C, directed upward.}$$

(b) IDENTIFY: The field is caused by a uniformly charged circular wire.

SET UP: The field for such a wire a distance x from its midpoint is $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}$. We first find the radius a of the circle using $2\pi a = l$.

EXECUTE: Solving for a gives $a = l/2\pi = (8.50 \text{ cm})/2\pi = 1.353 \text{ cm}$.

The charge on this circle is $Q = \lambda l = (175 \text{ nC/m})(0.0850 \text{ m}) = 14.88 \text{ nC}$.

The electric field is

$$E = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}} = \frac{(9.00\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(14.88\times10^{-9} \text{ C/m})(0.0600 \text{ m})}{\left[(0.0600 \text{ m})^2 + (0.01353 \text{ m})^2\right]^{3/2}}$$

$$E = 3.45\times10^4 \text{ N/C, upward.}$$

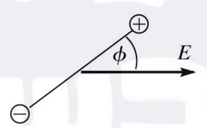
EVALUATE: In both cases, the fields are of the same order of magnitude, but the values are different because the charge has been bent into different shapes.

21.53. **(a) IDENTIFY and SET UP:** Use $p = qd$ to relate the dipole moment to the charge magnitude and the separation d of the two charges. The direction is from the negative charge toward the positive charge.

EXECUTE: $p = qd = (4.5\times10^{-9} \text{ C})(3.1\times10^{-3} \text{ m}) = 1.4\times10^{-11} \text{ C}\cdot\text{m}$. The direction of $\vec{p}$ is from q_1 toward q_2.

(b) IDENTIFY and SET UP: Use $\tau = pE\sin\phi$ to relate the magnitudes of the torque and field.

EXECUTE: $\tau = pE\sin\phi$, with ϕ as defined in Figure 21.53, so



$$E = \frac{\tau}{p\sin\phi}.$$

$$E = \frac{7.2\times10^{-9} \text{ N}\cdot\text{m}}{(1.4\times10^{-11} \text{ C}\cdot\text{m})\sin 36.9^\circ} = 860 \text{ N/C}.$$

Figure 21. 53

EVALUATE: The equation $\tau = pE\sin\phi$ gives the torque about an axis through the center of the dipole. But the forces on the two charges form a couple and the torque is the same for any axis parallel to this one. The force on each charge is $|q|E$ and the maximum moment arm for an axis at the center is $d/2$, so the maximum torque is $2(|q|E)(d/2) = 1.2\times10^{-8} \text{ N}\cdot\text{m}$. The torque for the orientation of the dipole in the problem is less than this maximum.

21.54. **(a) IDENTIFY:** The potential energy is given by $U(\phi) = -\vec{p}\cdot\vec{E} = -pE\cos\phi$.

SET UP: $U(\phi) = -\vec{p}\cdot\vec{E} = -pE\cos\phi$, where ϕ is the angle between $\vec{p}$ and $\vec{E}$.

EXECUTE: parallel: $\phi = 0$ and $U(0^\circ) = -pE$.

perpendicular: $\phi = 90^\circ$ and $U(90^\circ) = 0$.

$\Delta U = U(90^\circ) - U(0^\circ) = pE = (5.0\times10^{-30} \text{ C}\cdot\text{m})(1.6\times10^6 \text{ N/C}) = 8.0\times10^{-24} \text{ J}$.

(b) $\frac{3}{2}kT = \Delta U$ so $T = \dfrac{2\Delta U}{3k} = \dfrac{2(8.0\times10^{-24} \text{ J})}{3(1.381\times10^{-23} \text{ J/K})} = 0.39 \text{ K}$.

EVALUATE: Only at very low temperatures are the dipoles of the molecules aligned by a field of this strength. A much larger field would be required for alignment at room temperature.

21.55. **IDENTIFY:** The torque on a dipole in an electric field is given by $\vec{\tau} = \vec{p}\times\vec{E}$.

SET UP: $\tau = pE\sin\phi$, where ϕ is the angle between the direction of $\vec{p}$ and the direction of $\vec{E}$.

EXECUTE: **(a)** The torque is zero when $\vec{p}$ is aligned either in the *same* direction as $\vec{E}$ or in the *opposite* direction, as shown in Figure 21.55a (next page).

(b) The stable orientation is when $\vec{p}$ is aligned in the *same* direction as $\vec{E}$. In this case a small rotation of the dipole results in a torque directed so as to bring $\vec{p}$ back into alignment with $\vec{E}$. When $\vec{p}$ is directed opposite to $\vec{E}$, a small displacement results in a torque that takes $\vec{p}$ farther from alignment with $\vec{E}$.
(c) Field lines for E_{dipole} in the stable orientation are sketched in Figure 21.55b.
EVALUATE: The field of the dipole is directed from the + charge toward the − charge.

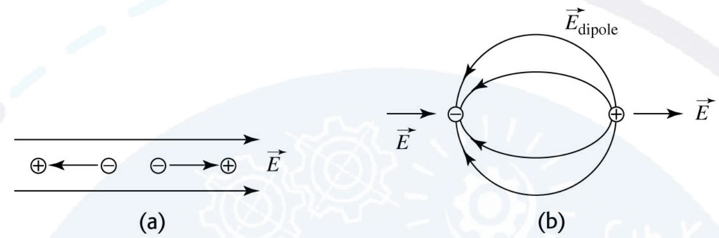


Figure 21. 55

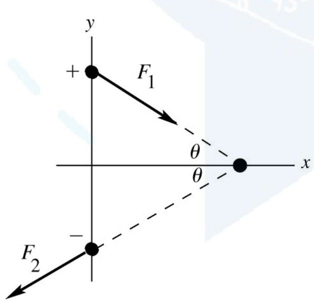
21.56. IDENTIFY: Calculate the electric field due to the dipole and then apply $\vec{F} = q\vec{E}$.

SET UP: The field of a dipole is $E_{\text{dipole}}(x) = \dfrac{p}{2\pi\epsilon_0 x^3}$.

EXECUTE: $E_{\text{dipole}} = \dfrac{6.17\times10^{-30}\ \text{C}\cdot\text{m}}{2\pi\epsilon_0(3.0\times10^{-9}\ \text{m})^3} = 4.11\times10^6\ \text{N/C}$. The electric force is

$F = qE = (1.60\times10^{-19}\ \text{C})(4.11\times10^6\ \text{N/C}) = 6.58\times10^{-13}\ \text{N}$ and is toward the water molecule (negative x-direction).
EVALUATE: $\vec{E}_{\text{dipole}}$ is in the direction of $\vec{p}$, so is in the $+x$-direction. The charge q of the ion is negative, so $\vec{F}$ is directed opposite to $\vec{E}$ and is therefore in the $-x$-direction.

21.57. (a) IDENTIFY: Use Coulomb's law to calculate each force and then add them as vectors to obtain the net force. Torque is force times moment arm.
SET UP: The two forces on each charge in the dipole are shown in Figure 21.57a.



$\sin\theta = 1.50/2.00$ so $\theta = 48.6°$.

Opposite charges attract and like charges repel.

$F_x = F_{1x} + F_{2x} = 0$.

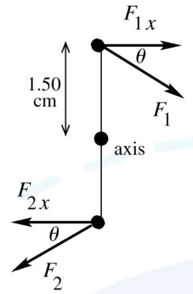
Figure 21. 57a

EXECUTE: $F_1 = k\dfrac{|qq'|}{r^2} = k\dfrac{(5.00\times10^{-6}\ \text{C})(10.0\times10^{-6}\ \text{C})}{(0.0200\ \text{m})^2} = 1.124\times10^3\ \text{N}$.

$F_{1y} = -F_1\sin\theta = -842.6\ \text{N}$.

$F_{2y} = -842.6\ \text{N}$ so $F_y = F_{1y} + F_{2y} = -1680\ \text{N}$ (in the direction from the $+5.00\text{-}\mu\text{C}$ charge toward the $-5.00\text{-}\mu\text{C}$ charge).
EVALUATE: The x-components cancel and the y-components add.

(b) SET UP: Refer to Figure 21.57b.



The y-components have zero moment arm and therefore zero torque.
F_{1x} and F_{2x} both produce clockwise torques.

Figure 21. 57b

EXECUTE: $F_{1x} = F_1 \cos\theta = 743.1$ N.

$\tau = 2(F_{1x})(0.0150\text{ m}) = 22.3\text{ N}\cdot\text{m}$, clockwise.

EVALUATE: The electric field produced by the $-10.00\ \mu$C charge is not uniform so $\tau = pE\sin\phi$ does not apply.

21.58. **IDENTIFY:** Find the vector sum of the fields due to each charge in the dipole.

SET UP: A point on the x-axis with coordinate x is a distance $r = \sqrt{(d/2)^2 + x^2}$ from each charge.

EXECUTE: **(a)** The magnitude of the field due to each charge is $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2} = \dfrac{q}{4\pi\varepsilon_0}\left(\dfrac{1}{(d/2)^2 + x^2}\right)$, where d is the distance between the two charges. The x-components of the forces due to the two charges are equal and oppositely directed and so cancel each other. The two fields have equal y-components, so $E = 2E_y = \dfrac{2q}{4\pi\varepsilon_0}\left(\dfrac{1}{(d/2)^2 + x^2}\right)\sin\theta$, where θ is the angle below the x-axis for both fields.

$\sin\theta = \dfrac{d/2}{\sqrt{(d/2)^2 + x^2}}$ and $E_{\text{dipole}} = \left(\dfrac{2q}{4\pi\varepsilon_0}\right)\left(\dfrac{1}{(d/2)^2 + x^2}\right)\left(\dfrac{d/2}{\sqrt{(d/2)^2 + x^2}}\right) = \dfrac{qd}{4\pi\varepsilon_0\left[(d/2)^2 + x^2\right]^{3/2}}$. The field is the $-y$-direction.

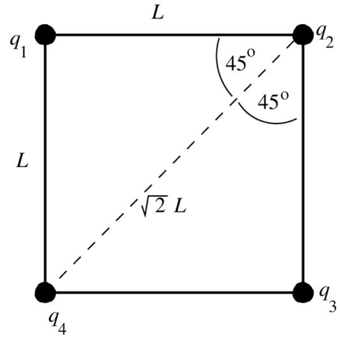
(b) At large x, $x^2 \gg (d/2)^2$, so the expression in part (a) reduces to the approximation $E_{\text{dipole}} \approx \dfrac{qd}{4\pi\varepsilon_0 x^3}$.

EVALUATE: Example 21.14 shows that at points on the $+y$-axis far from the dipole, $E_{\text{dipole}} \approx \dfrac{qd}{2\pi\varepsilon_0 y^3}$.

The expression in part (b) for points on the x-axis has a similar form.

21.59. **IDENTIFY:** Apply Coulomb's law to calculate the force exerted on one of the charges by each of the other three and then add these forces as vectors.

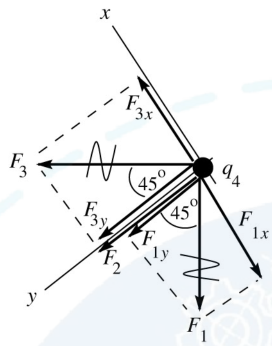
SET UP: The charges are placed as shown in Figure 21.59a.



$q_1 = q_2 = q_3 = q_4 = Q$

Figure 21.59a

Consider forces on q_4. The free-body diagram is given in Figure 21.59b. Take the y-axis to be parallel to the diagonal between q_2 and q_4 and let $+y$ be in the direction away from q_2. Then $\vec{F}_2$ is in the $+y$-direction.



EXECUTE: **(a)** $F_3 = F_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q^2}{L^2}$.

$F_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q^2}{2L^2}$.

$F_{1x} = -F_1 \sin 45° = -F_1/\sqrt{2}$.

$F_{1y} = +F_1 \cos 45° = +F_1/\sqrt{2}$.

$F_{3x} = +F_3 \sin 45° = +F_3/\sqrt{2}$.

$F_{3y} = +F_3 \cos 45° = +F_3/\sqrt{2}$.

$F_{2x} = 0, F_{2y} = F_2$.

Figure 21.59b

(b) $R_x = F_{1x} + F_{2x} + F_{3x} = 0$.

$$R_y = F_{1y} + F_{2y} + F_{3y} = (2/\sqrt{2})\frac{1}{4\pi\epsilon_0}\frac{Q^2}{L^2} + \frac{1}{4\pi\epsilon_0}\frac{Q^2}{2L^2} = \frac{Q^2}{8\pi\epsilon_0 L^2}(1+2\sqrt{2}).$$

$R = \dfrac{Q^2}{8\pi\epsilon_0 L^2}(1+2\sqrt{2})$. Same for all four charges.

EVALUATE: In general the resultant force on one of the charges is directed away from the opposite corner. The forces are all repulsive since the charges are all the same. By symmetry the net force on one charge can have no component perpendicular to the diagonal of the square.

21.60. **IDENTIFY:** Apply $F = \dfrac{k|qq'|}{r^2}$ to find the force of each charge on $+q$. The net force is the vector sum of the individual forces.

SET UP: Let $q_1 = +2.50\ \mu\text{C}$ and $q_2 = -3.50\ \mu\text{C}$. The charge $+q$ must be to the left of q_1 or to the right of q_2 in order for the two forces to be in opposite directions. But for the two forces to have equal magnitudes, $+q$ must be closer to the charge q_1, since this charge has the smaller magnitude. Therefore, the two forces can combine to give zero net force only in the region to the left of q_1. Let $+q$ be a distance d to the left of q_1, so it is a distance $d + 0.600$ m from q_2.

EXECUTE: $F_1 = F_2$ gives $\dfrac{kq|q_1|}{d^2} = \dfrac{kq|q_2|}{(d+0.600\text{ m})^2}$. $d = \pm\sqrt{\dfrac{|q_1|}{|q_2|}}(d+0.600\text{ m}) = \pm(0.8452)(d+0.600\text{ m})$.

d must be positive, so $d = \dfrac{(0.8452)(0.600\text{ m})}{1-0.8452} = 3.27$ m. The net force would be zero when $+q$ is at $x = -3.27$ m.

EVALUATE: When $+q$ is at $x = -3.27$ m, $\vec{F}_1$ is in the $-x$-direction and $\vec{F}_2$ is in the $+x$-direction.

21.61. **IDENTIFY:** Apply $F = k\dfrac{|qq'|}{r^2}$ for each pair of charges and find the vector sum of the forces that q_1 and q_2 exert on q_3.

SET UP: Like charges repel and unlike charges attract. The three charges and the forces on q_3 are shown in Figure 21.61.

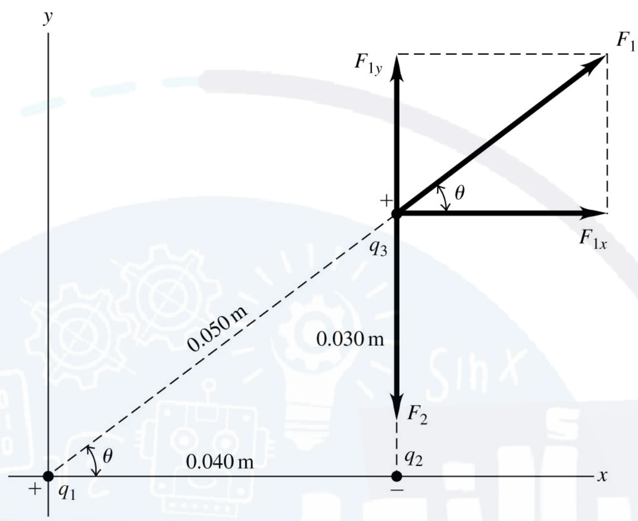


Figure 21.61

EXECUTE: **(a)** $F_1 = k\dfrac{|q_1 q_3|}{r_1^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(5.00\times10^{-9}\ \text{C})(6.00\times10^{-9}\ \text{C})}{(0.0500\ \text{m})^2} = 1.079\times10^{-4}\ \text{N}.$

$\theta = 36.9°$. $F_{1x} = +F_1\cos\theta = 8.63\times10^{-5}$ N. $F_{1y} = +F_1\sin\theta = 6.48\times10^{-5}$ N.

$$F_2 = k\frac{|q_2 q_3|}{r_2^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(2.00\times10^{-9}\ \text{C})(6.00\times10^{-9}\ \text{C})}{(0.0300\ \text{m})^2} = 1.20\times10^{-4}\ \text{N}.$$

$F_{2x} = 0$, $F_{2y} = -F_2 = -1.20\times10^{-4}$ N. $F_x = F_{1x} + F_{2x} = 8.63\times10^{-5}$ N.

$F_y = F_{1y} + F_{2y} = 6.48\times10^{-5}\ \text{N} + (-1.20\times10^{-4}\ \text{N}) = -5.52\times10^{-5}$ N.

(b) $F = \sqrt{F_x^2 + F_y^2} = 1.02\times10^{-4}$ N. $\tan\phi = \left|\dfrac{F_y}{F_x}\right| = 0.640$. $\phi = 32.6°$, below the +*x*-axis.

EVALUATE: The individual forces on q_3 are computed from Coulomb's law and then added as vectors, using components.

21.62. **IDENTIFY:** Apply $\sum F_x = 0$ and $\sum F_y = 0$ to one of the spheres.

SET UP: The free-body diagram is sketched in Figure 21.62 (next page). F_e is the repulsive Coulomb force between the spheres. For small θ, $\sin\theta \approx \tan\theta$.

EXECUTE: $\sum F_x = T\sin\theta - F_e = 0$ and $\sum F_y = T\cos\theta - mg = 0$. So $\dfrac{mg\sin\theta}{\cos\theta} = F_e = \dfrac{kq^2}{d^2}$. But

$\tan\theta \approx \sin\theta = \dfrac{d}{2L}$, so $d^3 = \dfrac{2kq^2L}{mg}$ and $d = \left(\dfrac{q^2L}{2\pi\varepsilon_0 mg}\right)^{1/3}$.

EVALUATE: d increases when q increases.

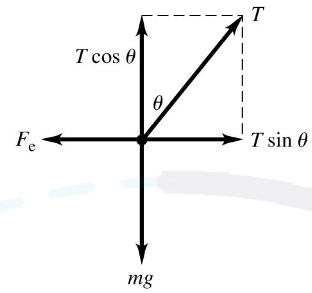


Figure 21.62

21.63. **Identify:** Use Coulomb's law for the force that one sphere exerts on the other and apply the first condition of equilibrium to one of the spheres.
Set Up: The placement of the spheres is sketched in Figure 21.63a.

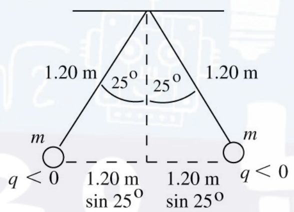


Figure 21.63a

Execute: **(a)** The free-body diagrams for each sphere are given in Figure 21.63b.

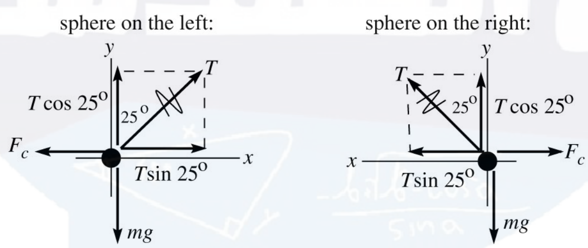


Figure 21.63b

F_c is the repulsive Coulomb force exerted by one sphere on the other.
(b) From either force diagram in part (a): $\sum F_y = ma_y$.

$T\cos 25.0° - mg = 0$ and $T = \dfrac{mg}{\cos 25.0°}$.

$\sum F_x = ma_x$.

$T\sin 25.0° - F_c = 0$ and $F_c = T\sin 25.0°$.

Use the first equation to eliminate T in the second: $F_c = (mg/\cos 25.0°)(\sin 25.0°) = mg\tan 25.0°$.

$$F_c = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{[2(1.20\text{ m})\sin 25.0°]^2}.$$

Combine this with $F_c = mg\tan 25.0°$ and get $mg\tan 25.0° = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{[2(1.20\text{ m})\sin 25.0°]^2}$.

$q = (2.40\ \text{m})\sin 25.0° \sqrt{\dfrac{mg\tan 25.0°}{(1/4\pi\varepsilon_0)}}.$

$q = (2.40\ \text{m})\sin 25.0° \sqrt{\dfrac{(15.0\times 10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)\tan 25.0°}{8.988\times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}} = 2.80\times 10^{-6}\ \text{C}.$

(c) The separation between the two spheres is given by $2L\sin\theta$. $q = 2.80\,\mu\text{C}$ as found in part (b).

$F_c = (1/4\pi\varepsilon_0)q^2/(2L\sin\theta)^2$ and $F_c = mg\tan\theta$. Thus $(1/4\pi\varepsilon_0)q^2/(2L\sin\theta)^2 = mg\tan\theta$.

$(\sin\theta)^2\tan\theta = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{4L^2mg} = (8.988\times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(2.80\times 10^{-6}\ \text{C})^2}{4(0.600\ \text{m})^2(15.0\times 10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)} = 0.3328.$

Solve this equation by trial and error. This will go quicker if we can make a good estimate of the value of θ that solves the equation. For θ small, $\tan\theta \approx \sin\theta$. With this approximation the equation becomes $\sin^3\theta = 0.3328$ and $\sin\theta = 0.6930$, so $\theta = 43.9°$. Now refine this guess:

θ	$\sin^2\theta\tan\theta$	
45.0°	0.5000	
40.0°	0.3467	
39.6°	0.3361	
39.5°	0.3335	
39.4°	0.3309	so $\theta = 39.5°$.

EVALUATE: The expression in part (c) says $\theta \to 0$ as $L \to \infty$ and $\theta \to 90°$ as $L \to 0$. When L is decreased from the value in part (a), θ increases.

21.64. **IDENTIFY:** Apply $\sum F_x = 0$ and $\sum F_y = 0$ to each sphere.

SET UP: **(a)** Free body diagrams are given in Figure 21.64 (next page). F_e is the repulsive electric force that one sphere exerts on the other.

EXECUTE: **(b)** $T = mg/\cos 20° = 0.0834\ \text{N}$, so $F_e = T\sin 20° = 0.0285\ \text{N} = \dfrac{kq_1q_2}{r_1^2}$.

(Note: $r_1 = 2(0.500\ \text{m})\sin 20° = 0.342\ \text{m}$.)

(c) From part (b), $q_1q_2 = 3.71\times 10^{-13}\ \text{C}^2$.

(d) The charges on the spheres are made equal by connecting them with a wire, but we still have

$F_e = mg\tan\theta = 0.0453\ \text{N} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q^2}{r_2^2}$, where $Q = \dfrac{q_1+q_2}{2}$. But the separation r_2 is known:

$r_2 = 2(0.500\ \text{m})\sin 30° = 0.500\ \text{m}$. Hence: $Q = \dfrac{q_1+q_2}{2} = \sqrt{4\pi\varepsilon_0 F_e r_2^2} = 1.12\times 10^{-6}\ \text{C}$. This equation, along with that from part (c), gives us two equations in q_1 and q_2: $q_1 + q_2 = 2.24\times 10^{-6}\ \text{C}$ and $q_1q_2 = 3.71\times 10^{-13}\ \text{C}^2$. By elimination, substitution and after solving the resulting quadratic equation, we find: $q_1 = 2.06\times 10^{-6}\ \text{C}$ and $q_2 = 1.80\times 10^{-7}\ \text{C}$.

EVALUATE: After the spheres are connected by the wire, the charge on sphere 1 decreases and the charge on sphere 2 increases. The product of the charges on the sphere increases and the thread makes a larger angle with the vertical.

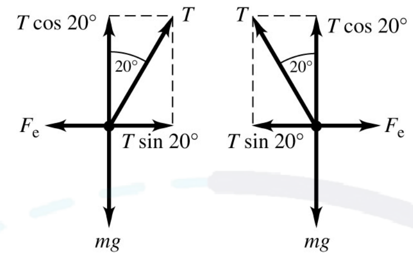


Figure 21.64

21.65. **IDENTIFY:** The electric field exerts a horizontal force away from the wall on the ball. When the ball hangs at rest, the forces on it (gravity, the tension in the string, and the electric force due to the field) add to zero.
SET UP: The ball is in equilibrium, so for it $\sum F_x = 0$ and $\sum F_y = 0$. The force diagram for the ball is given in Figure 21.65. F_E is the force exerted by the electric field. $\vec{F} = q\vec{E}$. Since the electric field is horizontal, $\vec{F}_E$ is horizontal. Use the coordinates shown in the figure. The tension in the string has been replaced by its x- and y-components.

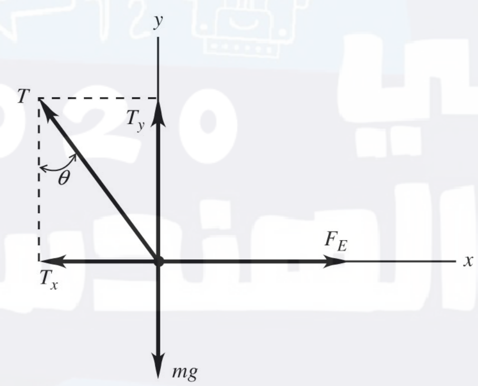


Figure 21.65

EXECUTE: $\sum F_y = 0$ gives $T_y - mg = 0$. $T\cos\theta - mg = 0$ and $T = \dfrac{mg}{\cos\theta}$. $\sum F_x = 0$ gives $F_E - T_x = 0$.
$F_E - T\sin\theta = 0$. Combing the equations and solving for F_E gives

$F_E = \left(\dfrac{mg}{\cos\theta}\right)\sin\theta = mg\tan\theta = (12.3\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)(\tan 17.4°) = 3.78\times10^{-2}\text{ N}$. $F_E = |q|E$ so

$E = \dfrac{F_E}{|q|} = \dfrac{3.78\times10^{-2}\text{ N}}{1.11\times10^{-6}\text{ C}} = 3.41\times10^4$ N/C. Since q is negative and $\vec{F}_E$ is to the right, $\vec{E}$ is to the left in the figure.

EVALUATE: The larger the electric field E the greater the angle the string makes with the wall.

21.66. **IDENTIFY:** The net force on q_3 is the vector sum of the individual forces. Coulomb's law gives the force between any two point-charges.

SET UP: Use $F = k\dfrac{|q_1 q_2|}{r^2}$. The force on q_3 due to q_1 is in the $-x$-direction, so q_2 must be negative to make the net force on q_3 in the $+x$-direction. We know that the x-component of the net force on q_3 is $F_{3x} = +6.00$ N.

(a) EXECUTE: The net force on q_3 is the sum of the two forces: $F_{3x} = F_{1x} + F_{2x} = +6.00$ N. Applying Coulomb's law gives
$6.00\text{ N} = k[-(6.00\ \mu\text{C})(3.00\ \mu\text{C})/(0.200\text{ m})^2 + (3.00\ \mu\text{C})q_2/(0.400\text{ m})^2]$, $q_2 = -5.96\times10^{-6}\text{ C} = -59.6\ \mu\text{C}$.

(b) Now $F_{3x} = -6.00$ N. In this case, assume that q_2 is positive, so the x-components all add. Using the same approach as in (a), we have
$-6.00\text{ N} = k[-(6.00\ \mu\text{C})(3.00\ \mu\text{C})/(0.200\text{ m})^2 - (3.00\ \mu\text{C})q_2/(0.400\text{ m})^2] = +1.16\times10^{-5}\text{ C} = +11.6\ \mu\text{C}$.

EVALUATE: Is is tempting to think that the answer to (b) should be just the negative of the answer to (a), but that is not the case. In (a) the two forces on q_3 were in opposite directions, but in (b) they are in the same direction.

21.67. IDENTIFY: For a point charge, $E = k\dfrac{|q|}{r^2}$. For the net electric field to be zero, $\vec{E}_1$ and $\vec{E}_2$ must have equal magnitudes and opposite directions.

SET UP: Let $q_1 = +0.500\text{ nC}$ and $q_2 = +8.00\text{ nC}$. $\vec{E}$ is toward a negative charge and away from a positive charge.

EXECUTE: The two charges and the directions of their electric fields in three regions are shown in Figure 21.67. Only in region II are the two electric fields in opposite directions. Consider a point a distance x from q_1 so a distance $1.20\text{ m} - x$ from q_2. $E_1 = E_2$ gives $k\dfrac{0.500\text{ nC}}{x^2} = k\dfrac{8.00\text{ nC}}{(1.20\text{ m} - x)^2}$. $16x^2 = (1.20\text{ m} - x)^2$. $4x = \pm(1.20\text{ m} - x)$ and $x = 0.24\text{ m}$ is the positive solution. The electric field is zero at a point between the two charges, 0.24 m from the 0.500 nC charge and 0.96 m from the 8.00 nC charge.

EVALUATE: There is only one point along the line connecting the two charges where the net electric field is zero. This point is closer to the charge that has the smaller magnitude.

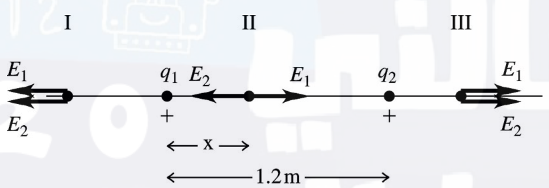


Figure 21.67

21.68. IDENTIFY: The net electric field at the origin is the vector sum of the fields due to the two charges.

SET UP: $E = k\dfrac{|q|}{r^2}$. $\vec{E}$ is toward a negative charge and away from a positive charge. At the origin, $\vec{E}_1$ due to the -3.00 nC charge is in the $+x$-direction, toward the charge.

EXECUTE: **(a)** $E_1 = (8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(3.00\times 10^{-9}\ \text{C})}{(1.20\ \text{m})^2} = 18.73\text{ N/C}$, so $E_{1x} = +18.73\text{ N/C}$.

$E_x = E_{1x} + E_{2x}$. $E_x = +45.0\text{ N/C}$, so $E_{2x} = E_x - E_{1x} = +45.0\text{ N/C} - 18.73\text{ N/C} = 26.27\text{ N/C}$. $\vec{E}$ is away from Q so Q is positive. Using $E_2 = k\dfrac{|Q|}{r^2}$ gives

$|Q| = \dfrac{E_2 r^2}{k} = \dfrac{(26.27\text{ N/C})(0.600\text{ m})^2}{8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 1.05\times 10^{-9}\ \text{C} = 1.05\text{ nC}$. Since Q is positive, $Q = +1.05\text{ nC}$.

(b) $E_x = -45.0\text{ N/C}$, so $E_{2x} = E_x - E_{1x} = -45.0\text{ N/C} - 18.73\text{ N/C} = -63.73\text{ N/C}$. $\vec{E}$ is toward Q so Q is negative. $|Q| = \dfrac{E_2 r^2}{k} = \dfrac{(63.73\text{ N/C})(0.600\text{ m})^2}{8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 2.55\times 10^{-9}\ \text{C} = 2.55\text{ nC}$. Since Q is negative, we have $Q = -2.55\text{ nC}$.

EVALUATE: The equation $E = k\dfrac{|q|}{r^2}$ gives only the *magnitude* of the electric field. When combining fields, you still must figure out whether to add or subtract the magnitudes depending on the direction in which the fields point.

21.69. **IDENTIFY:** For equilibrium, the forces must balance. The electrical force is given by Coulomb's law.

SET UP: Set up axes so that the charge $+Q$ is located at $x = 0$, the charge $+4Q$ is located at $x = d$, and the unknown charge that is required to produce equilibrium, q, is located at a position $x = a$. Apply $F = k\dfrac{|q_1 q_2|}{r^2}$ to each pair of charges to obtain eqilibrium.

EXECUTE: For a charge q to be in equilbrium, it must be placed between the two given positive charges $(0 < a < d)$ and the magnitude of the force between q and $+Q$ must be equal to the magnitude of the force between q and $+4Q$: $k\dfrac{|q|Q}{a^2} = k\dfrac{4|q|Q}{(d-a)^2}$. Solving for a we obtain $(d-a) = \pm 4a$, which has $a = \dfrac{d}{3}$ as its only root in the required interval $(0 < a < d)$. Furthermore, to conteract the repulsive force between $+Q$ and $+4Q$ the charge q must be negative $(q = -|q|)$. The condition that $+Q$ is in equilibrium gives us $k\dfrac{-qQ}{(d/3)^2} = k\dfrac{4Q^2}{d^2}$. Solving for q we obtain $q = -\dfrac{4}{9}Q$.

EVALUATE: We have shown that both q and $+Q$ are in equilibrium provided that $a = \dfrac{d}{3}$ and $q = -\dfrac{4}{9}Q$. To make sure that the problem is well posed, we should check that these conditions also place the charge $+4Q$ is in equilbrium. We can do this by showing that $k\dfrac{-4qQ}{(d-a)^2}$ is equal to $k\dfrac{4Q^2}{d^2}$ when the given values for both a and q are substituted.

21.70. **IDENTIFY** and **SET UP:** Like charges repel and unlike charges attract, and Coulomb's law applies. The positions of the three charges are sketched in Figure 21.70a, and each force acting on q_3 is shown. The distance between q_1 and q_3 is 5.00 cm.

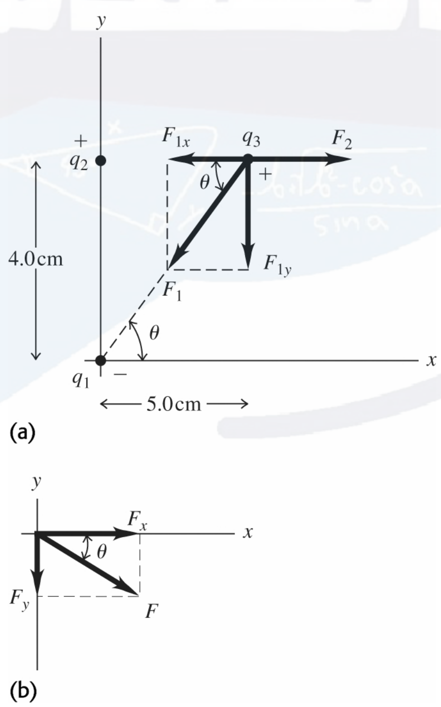


Figure 21.70

EXECUTE: **(a)** $F_1 = k\dfrac{|q_1q_3|}{r_1^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(3.00\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(5.00\times10^{-2}\ \text{m})^2} = 5.394\times10^{-5}\ \text{N}.$

$F_{1x} = -F_1\cos\theta = -(5.394\times10^{-5}\ \text{N})(0.600) = -3.236\times10^{-5}\ \text{N}.$

$F_{1y} = -F_1\sin\theta = -(5.394\times10^{-5}\ \text{N})(0.800) = -4.315\times10^{-5}\ \text{N}.$

$F_2 = k\dfrac{|q_2q_3|}{r_2^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(2.00\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(3.00\times10^{-2}\ \text{m})^2} = 9.989\times10^{-5}\ \text{N}.$

$F_{2x} = 9.989\times10^{-5}\ \text{N};\ F_{2y} = 0.$

$F_x = F_{1x} + F_{2x} = 9.989\times10^{-5}\ \text{N} + (-3.236\times10^{-5}\ \text{N}) = 6.75\times10^{-5}\ \text{N};$

$F_y = F_{1y} + F_{2y} = -4.32\times10^{-5}\ \text{N}.$

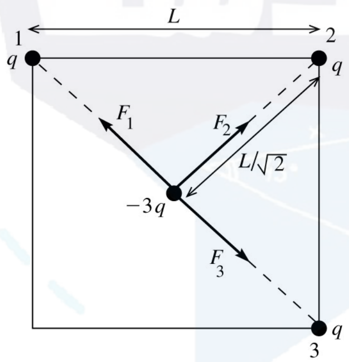
(b) $\vec{F}$ and its components are shown in Figure 21.70b.

$F = \sqrt{F_x^2 + F_y^2} = 8.01\times10^{-5}\ \text{N}.$ $\tan\theta = \left|\dfrac{F_y}{F_x}\right| = 0.640$ and $\theta = 32.6°$. $\vec{F}$ is 327° counterclockwise from the $+x$-axis.

EVALUATE: The equation $F = k\dfrac{|q_1q_2|}{r^2}$ gives only the magnitude of the force. We must find the direction by deciding if the force between the charges is attractive or repulsive.

21.71. IDENTIFY: Use Coulomb's law to calculate the forces between pairs of charges and sum these forces as vectors to find the net charge.

(a) SET UP: The forces are sketched in Figure 21.71a.

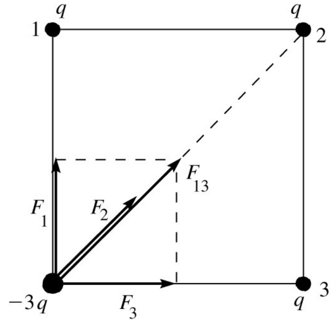


EXECUTE: $\vec{F}_1 + \vec{F}_3 = 0$, so the net force is $\vec{F} = \vec{F}_2$.

$F = \dfrac{1}{4\pi\epsilon_0}\dfrac{q(3q)}{(L/\sqrt{2})^2} = \dfrac{6q^2}{4\pi\epsilon_0 L^2}$, away from the vacant corner.

Figure 21. 71a

(b) SET UP: The forces are sketched in Figure 21.71b.



EXECUTE: $F_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q(3q)}{(\sqrt{2}L)^2} = \dfrac{3q^2}{4\pi\epsilon_0(2L^2)}.$

$F_1 = F_3 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q(3q)}{L^2} = \dfrac{3q^2}{4\pi\epsilon_0 L^2}.$

The vector sum of F_1 and F_3 is $F_{13} = \sqrt{F_1^2 + F_3^2}$.

Figure 21. 71b

$F_{13} = \sqrt{2}F_1 = \dfrac{3\sqrt{2}q^2}{4\pi\epsilon_0 L^2}$; $\vec{F}_{13}$ and $\vec{F}_2$ are in the same direction.

$F = F_{13} + F_2 = \dfrac{3q^2}{4\pi\epsilon_0 L^2}\left(\sqrt{2} + \dfrac{1}{2}\right)$, and is directed toward the center of the square.

EVALUATE: By symmetry the net force is along the diagonal of the square. The net force is only slightly larger when the $-3q$ charge is at the center. Here it is closer to the charge at point 2 but the other two forces cancel.

21.72. IDENTIFY: For the acceleration (and hence the force) on Q to be upward, as indicated, the forces due to q_1 and q_2 must have equal strengths, so q_1 and q_2 must have equal magnitudes. Furthermore, for the force to be upward, q_1 must be positive and q_2 must be negative.

SET UP: Since we know the acceleration of Q, Newton's second law gives us the magnitude of the force on it. We can then add the force components using $F = F_{Qq_1}\cos\theta + F_{Qq_2}\cos\theta = 2F_{Qq_1}\cos\theta$. The electrical force on Q is given by Coulomb's law, $F_{Qq_1} = \dfrac{1}{4\pi\epsilon_0}\dfrac{|Qq_1|}{r^2}$ (for q_1) and likewise for q_2.

EXECUTE: First find the net force: $F = ma =$ (0.00500 kg)(324 m/s^2) $= 1.62$ N. Now add the force components, calling θ the angle between the line connecting q_1 and q_2 and the line connecting q_1 and Q.

$F = F_{Qq_1}\cos\theta + F_{Qq_2}\cos\theta = 2F_{Qq_1}\cos\theta$ and $F_{Qq_1} = \dfrac{F}{2\cos\theta} = \dfrac{1.62\ \text{N}}{2\left(\dfrac{2.25\ \text{cm}}{3.00\ \text{cm}}\right)} = 1.08$ N. Now find the charges by solving for q_1 in Coulomb's law and use the fact that q_1 and q_2 have equal magnitudes but opposite signs. $F_{Qq_1} = \dfrac{1}{4\pi\epsilon_0}\dfrac{|Q|q_1}{r^2}$ and $q_1 = \dfrac{r^2 F_{Qq_1}}{\dfrac{1}{4\pi\epsilon_0}|Q|} = \dfrac{(0.0300\ \text{m})^2(1.08\ \text{N})}{(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.75\times10^{-6}\ \text{C})} = 6.17\times10^{-8}$ C.

$q_2 = -q_1 = -6.17\times10^{-8}$ C.

EVALUATE: Simple reasoning allows us first to conclude that q_1 and q_2 must have equal magnitudes but opposite signs, which makes the equations much easier to set up than if we had tried to solve the problem in the general case. As Q accelerates and hence moves upward, the magnitude of the acceleration vector will change in a complicated way.

21.73. IDENTIFY: The small bags of protons behave like point-masses and point-charges since they are extremely far apart.

SET UP: For point-particles, we use Newton's formula for universal gravitation $(F = Gm_1m_2/r^2)$ and Coulomb's law. The number of protons is the mass of protons in the bag divided by the mass of a single proton.

EXECUTE: **(a)** $(0.0010\ \text{kg})/(1.67\times10^{-27}\ \text{kg}) = 6.0\times10^{23}$ protons.

(b) Using Coulomb's law, where the separation is twice the radius of the earth, we have

$F_{\text{electrical}} = (9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.0\times10^{23}\times1.60\times10^{-19}\ \text{C})^2/(2\times6.37\times10^6\ \text{m})^2 = 5.1\times10^5$ N.

$F_{\text{grav}} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(0.0010\ \text{kg})^2/(2\times6.37\times10^6\ \text{m})^2 = 4.1\times10^{-31}$ N.

EVALUATE: **(c)** The electrical force (≈200,000 lb!) is certainly large enough to feel, but the gravitational force clearly is not since it is about 10^{36} times weaker.

21.74. IDENTIFY: The positive sphere will be deflected in the direction of the electric field but the negative sphere will be deflected in the direction opposite to the electric field. Since the spheres hang at rest, they are in equilibrium so the forces on them must balance. The external forces on each sphere are gravity, the tension in the string, the force due to the uniform electric field and the electric force due to the other sphere.

SET UP: The electric force on one sphere due to the other is $F_C = k\dfrac{|q^2|}{r^2}$ in the horizontal direction, the force on it due to the uniform electric field is $F_E = qE$ in the horizontal direction, the gravitational force is mg vertically downward and the force due to the string is T directed along the string. For equilibrium $\sum F_x = 0$ and $\sum F_y = 0$.

EXECUTE: **(a)** The positive sphere is deflected in the same direction as the electric field, so the one that is deflected to the left is positive.

(b) The separation between the two spheres is $2(0.530\text{ m})\sin 29.0^\circ = 0.5139\text{ m}$.

$F_C = k\dfrac{|q^2|}{r^2} = \dfrac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(72.0\times10^{-9}\text{ C})^2}{(0.5139\text{ m})^2} = 1.765\times10^{-4}\text{ N}.$ $F_E = qE.$ $\sum F_y = 0$ gives

$T\cos 29.0^\circ - mg = 0$ so $T = \dfrac{mg}{\cos 29.0^\circ}.$ $\sum F_x = 0$ gives $T\sin 29.0^\circ + F_C - F_E = 0.$

$mg\tan 29.0^\circ + F_C = qE.$ Combining the equations and solving for E gives

$$E = \frac{mg\tan 29.0^\circ + F_C}{q} = \frac{(6.80\times10^{-6}\text{ kg})(9.80\text{ m/s}^2)\tan 29.0^\circ + 1.765\times10^{-4}\text{ N}}{72.0\times10^{-9}\text{ C}} = 2.96\times10^3\text{ N/C}.$$

EVALUATE: Since the charges have opposite signs, they attract each other, which tends to reduce the angle between the strings. Therefore if their charges were negligibly small, the angle between the strings would be greater than 58.0°.

21.75. **IDENTIFY:** The only external force acting on the electron is the electrical attraction of the proton, and its acceleration is toward the center of its circular path (that is, toward the proton). Newton's second law applies to the electron and Coulomb's law gives the electrical force on it due to the proton.

SET UP: Newton's second law gives $F_C = m\dfrac{v^2}{r}$. Using the electrical force for F_C gives $k\dfrac{e^2}{r^2} = m\dfrac{v^2}{r}$.

EXECUTE: Solving for v gives $v = \sqrt{\dfrac{ke^2}{mr}} = \sqrt{\dfrac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{(9.109\times10^{-31}\text{ kg})(5.29\times10^{-11}\text{ m})}} = 2.19\times10^6\text{ m/s}.$

EVALUATE: This speed is less than 1% the speed of light, so it is reasonably safe to use Newtonian physics.

21.76. **IDENTIFY:** To be suspended, the electric force on the raindrop due to the earth's electric field must be equal to the weight of the drop.

SET UP: The weight of the raindrop is $w = mg$ and is downward. We can calculate the mass of the raindrop from the known density of water: $m = \rho V$, where $\rho = 10^3\text{ kg/m}^3$ and $V = \frac{4}{3}\pi r^3$. The electric force is $\vec{F} = q\vec{E}$, where $E = 150\text{ N/C}$.

EXECUTE: To balance the weight of the raindrop the electric force must be upward. Since the electric field is downward the net charge on the raindrop must be negative. For equilibrium we must have $w = mg = |q|E$. Therefore

$$|q| = \frac{mg}{E} = \left(\frac{4}{3}\pi r^3\rho g\right)/E = \frac{4}{3}\pi(1.0\times10^{-5}\text{ m})^3(10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)/(150\text{ N/C}) = 2.7\times10^{-13}\text{ C}.$$

The number of excess electrons is $\dfrac{|q|}{e} = \dfrac{2.7\times10^{-13}\text{ C}}{1.60\times10^{-19}\text{ C}} = 1.7\times10^6.$

EVALUATE: Although this may appear to be a large number in absolute terms, the excess number of electrons represents only about 10^{-7}% of the total number of electrons in the raindrop.

21.77. IDENTIFY: $\vec{E} = \dfrac{\vec{F}_0}{q_0}$ gives the force exerted by the electric field. This force is constant since the electric field is uniform and gives the proton a constant acceleration. Apply the constant acceleration equations for the x- and y-components of the motion, just as for projectile motion.

SET UP: The electric field is upward so the electric force on the positively charged proton is upward and has magnitude $F = eE$. Use coordinates where positive y is downward. Then applying $\sum \vec{F} = m\vec{a}$ to the proton gives that $a_x = 0$ and $a_y = -eE/m$. In these coordinates the initial velocity has components $v_x = +v_0 \cos\alpha$ and $v_y = +v_0 \sin\alpha$, as shown in Figure 21.77a.

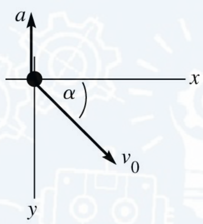


Figure 21.77a

EXECUTE: **(a)** Finding h_{max}: At $y = h_{max}$ the y-component of the velocity is zero.

$v_y = 0, v_{0y} = v_0 \sin\alpha, a_y = -eE/m, y - y_0 = h_{max} = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0).$

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y}.$$

$$h_{max} = \frac{-v_0^2 \sin^2\alpha}{2(-eE/m)} = \frac{mv_0^2 \sin^2\alpha}{2eE}.$$

(b) Use the vertical motion to find the time t: $y - y_0 = 0, v_{0y} = v_0 \sin\alpha, a_y = -eE/m, t = ?$

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2.$$

With $y - y_0 = 0$ this gives $t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(v_0 \sin\alpha)}{-eE/m} = \dfrac{2mv_0 \sin\alpha}{eE}.$

Then use the x-component motion to find d: $a_x = 0, v_{0x} = v_0 \cos\alpha, t = 2mv_0 \sin\alpha/eE, x - x_0 = d = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $d = v_0 \cos\alpha\left(\dfrac{2mv_0 \sin\alpha}{eE}\right) = \dfrac{mv_0^2 2\sin\alpha\cos\alpha}{eE} = \dfrac{mv_0^2 \sin 2\alpha}{eE}.$

(c) The trajectory of the proton is sketched in Figure 21.77b.

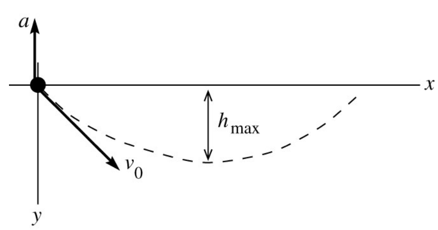


Figure 21.77b

(d) Use the expression in part (a): $h_{max} = \dfrac{[(4.00\times10^5\ \text{m/s})(\sin 30.0°)]^2(1.673\times10^{-27}\ \text{kg})}{2(1.602\times10^{-19}\ \text{C})(500\ \text{N/C})} = 0.418\ \text{m}.$

Use the expression in part (b): $d = \dfrac{(1.673\times10^{-27}\ \text{kg})(4.00\times10^{5}\ \text{m/s})^2\sin 60.0^\circ}{(1.602\times10^{-19}\ \text{C})(500\ \text{N/C})} = 2.89\ \text{m}.$

EVALUATE: In part (a), $a_y = -eE/m = -4.8\times10^{10}\ \text{m/s}^2$. This is much larger in magnitude than g, the acceleration due to gravity, so it is reasonable to ignore gravity. The motion is just like projectile motion, except that the acceleration is upward rather than downward and has a much different magnitude. h_{max} and d increase when α or v_0 increase and decrease when E increases.

21.78. **IDENTIFY:** The electric field is vertically downward and the charged object is deflected downward, so it must be positively charged. While the object is between the plates, it is accelerated downward by the electric field. Once it is past the plates, it moves downward with a constant vertical velocity which is the same downward velocity it acquired while between the plates. Its horizontal velocity remains constant at v_0 throughout its motion. The forces on the object are all constant, so its acceleration is constant; therefore we can use the standard kinematics equations. Newton's second law applies to the object.

SET UP: Call the x-axis positive to the right and the y-axis positive downward. The equations $\vec{E} = \dfrac{\vec{F}_0}{q_0}$,

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$, $v_y = v_{0y} + a_y t$, $x = v_x t$, and $\Sigma F_y = ma_y$ all apply. $v_x = v_0 =$ constant.

EXECUTE: *Time through the plates:* $t = x/v_x = x/v_0 = (0.260\ \text{m})/(5000\ \text{m/s}) = 5.20\times10^{-5}\ \text{s}$.

Vertical deflection between the plates: $\Delta y_1 = y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}ayt^2 = \frac{1}{2}(qE/m)t^2$

$\Delta y_1 = \frac{1}{2}(800\ \text{N/C})(5.20\times10^{-5}\ \text{s})^2(q/m) = (1.0816\times10^{-6}\ \text{kg}\cdot\text{m/C})\,(q/m)$.

v_y as the object just emerges from the plates:

$v_y = v_{0y} + a_y t = (qE/m)t = (q/m)(800\ \text{N/C})(5.20\times10^{-5}\ \text{s}) = (0.04160\ \text{kg}\cdot\text{m/C}\cdot\text{s})(q/m)$. (This is the initial vertical velocity for the next step.)

Time to travel 56.0 cm: $t = x/v_x = (0.560\ \text{m})/(5000\ \text{m/s}) = 1.120\times10^{-4}\ \text{s}$.

Vertical deflection after leaving the plates:

$\Delta y_2 = v_{0y}\,t = (0.04160\ \text{kg}\cdot\text{m/C}\cdot\text{s})\,(q/m)(1.120\times10^{-4}\ \text{s}) = (4.6592\times10^{-6}\ \text{kg}\cdot\text{m/C})\,(q/m)$.

Total vertical deflection:

$d = \Delta y_1 + \Delta y_2$.

$1.25\ \text{cm} = 0.0125\ \text{m} = (1.0816\times10^{-6}\ \text{kg}\cdot\text{m/C})\,(q/m) + (4.6592\times10^{-6}\ \text{kg}\cdot\text{m/C})\,(q/m)$.

$q/m = 2180\ \text{C/kg}$.

EVALUATE: The charge on 1.0 kg is so huge that it could not be dealt with in a laboratory. But this is a tiny object, more likely with a mass in the range of 1.0 μg, so its charge would be $(2180\ \text{C/kg})(10^{-9}\ \text{kg}) = 2.18\times10^{-6}\ \text{C} \approx 2\ \mu\text{C}$. That amount of charge could be used in an experiment.

21.79. **IDENTIFY:** Divide the charge distribution into infinitesimal segments of length dx'. Calculate E_x and E_y due to a segment and integrate to find the total field.

SET UP: The charge dQ of a segment of length dx' is $dQ = (Q/a)dx'$. The distance between a segment at x' and a point at x on the x-axis is $x - x'$ since $x > a$.

EXECUTE: **(a)** $dE_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{dQ}{(x-x')^2} = \dfrac{1}{4\pi\epsilon_0}\dfrac{(Q/a)dx'}{(x-x')^2}$. Integrating with respect to x' over the length of the charge distribution gives

$$E_x = \frac{1}{4\pi\epsilon_0}\int_0^a \frac{(Q/a)dx'}{(x-x')^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\left(\frac{1}{x-a}-\frac{1}{x}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\frac{a}{x(x-a)} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x(x-a)}.\ \ Ey = 0.$$

(b) At the location of the charge, $x = r + a$, so $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{(r+a)(r+a-a)} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r(r+a)}$.

Using $\vec{F} = q\vec{E}$, we have $\vec{F} = q\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{qQ}{r(r+a)}\hat{\boldsymbol{i}}$.

EVALUATE: **(c)** For $r \gg a$, $r + a \to r$, so the magnitude of the force becomes $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{r^2}$. The charge distribution looks like a point charge from far away, so the force takes the form of the force between a pair of point charges.

21.80. **IDENTIFY:** The electric field is upward, but whether it exerts an upward or downward force on the object depends on the sign of the charge on the object, so we should first find the sign of this charge. Then apply Newton's second law. The forces (gravity and the electric force) are both constant, so the acceleration is constant. Therefore the standard kinematics formulas apply.

SET UP: Call the +y-axis upward. The equations $\vec{E} = \dfrac{\vec{F}_0}{q_0}$, $\Sigma F_y = ma_y$, $y - y_0 = v_{0y}t + \dfrac{1}{2}a_y t^2$ all apply.

EXECUTE: First find the sign of the charge of the object. If no electric field were present, only gravity would be acting, so the distance the object would travel in 0.200 s would be

$y - y_0 = v_{0y}t + \dfrac{1}{2}a_y t^2 = (1.92 \text{ m/s})(0.200 \text{ s}) - \dfrac{1}{2}(9.80 \text{ m/s}^2)(0.200 \text{ s})^2 = 0.1880 \text{ m} = 18.8 \text{ cm}.$

Since the object travels only 6.98 cm in 0.200 s, the force due to the electric field must be opposing its motion, so this force must be downward. Since the electric field is upward, the charge must be negative. Now look at the motion with the electric field present. Newton's second law gives
$\Sigma F_y = ma_y$: $mg + qE = ma_y$. We get a_y using kinematics.

$y - y_0 = v_{0y}t + \dfrac{1}{2}a_y t^2$: $0.0698 \text{ m} = (1.92 \text{ m/s})(0.200 \text{ s}) + \dfrac{1}{2}a_y(0.200 \text{ s})^2$.

$a_y = -15.71 \text{ m/s}^2$, with the minus sign telling us it is downward. Now use this value in Newton's second law. Solve $mg + qE = ma_y$ for q/m:
$q/m = (a_y - g)/E = (15.71 \text{ m/s}^2 - 9.80 \text{ m/s}^2)/(3.60\times10^4 \text{ N/C}) = 1.64\times10^{-4}$ C/kg.
EVALUATE: A kilogram of the material of this object would have a charge of 1.64×10^{-4} C = 164 μC.

21.81. **IDENTIFY:** $E_x = E_{1x} + E_{2x}$. Use $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q|}{r^2}$ for the electric field due to each point charge.

SET UP: $\vec{E}$ is directed away from positive charges and toward negative charges.

EXECUTE: **(a)** $E_x = +50.0$ N/C. $E_{1x} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q_1|}{r_1^2} = (8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{4.00\times10^{-9}\text{ C}}{(0.60\text{ m})^2} = +99.9$ N/C.

$E_x = E_{1x} + E_{2x}$, so $E_{2x} = E_x - E_{1x} = +50.0 \text{ N/C} - 99.9 \text{ N/C} = -49.9$ N/C. Since E_{2x} is negative, q_2 must be negative. $|q_2| = \dfrac{|E_{2x}|r_2^2}{(1/4\pi\varepsilon_0)} = \dfrac{(49.9 \text{ N/C})(1.20 \text{ m})^2}{8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} = 7.99\times10^{-9}$ C. $q_2 = -7.99\times10^{-9}$ C.

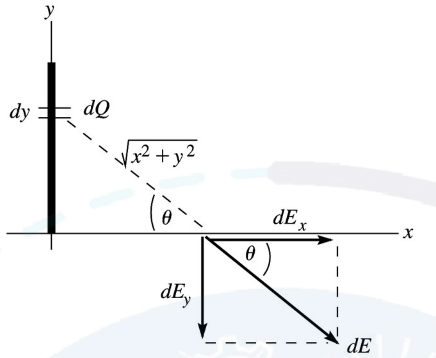
(b) $E_x = -50.0$ N/C. $E_{1x} = +99.9$ N/C, as in part (a). $E_{2x} = E_x - E_{1x} = -149.9$ N/C. q_2 is negative.

$|q_2| = \dfrac{|E_{2x}|r_2^2}{(1/4\pi\varepsilon_0)} = \dfrac{(149.9 \text{ N/C})(1.20 \text{ m})^2}{8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} = 2.40\times10^{-8}$ C. $q_2 = -2.40\times10^{-8}$ C.

EVALUATE: q_2 would be positive if E_{2x} were positive.

21.82. **IDENTIFY:** Use $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q|}{r^2}$ to calculate the electric field due to a small slice of the line of charge and integrate as in Example 21.10. Use $\vec{E} = \dfrac{\vec{F}}{q_0}$ to calculate $\vec{F}$.

SET UP: The electric field due to an infinitesimal segment of the line of charge is sketched in Figure 21.82.



$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}}.$$

$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}.$$

Figure 21.82

Slice the charge distribution up into small pieces of length dy. The charge dQ in each slice is $dQ = Q(dy/a)$. The electric field this produces at a distance x along the x-axis is dE. Calculate the components of $d\vec{E}$ and then integrate over the charge distribution to find the components of the total field.

EXECUTE: $dE = \frac{1}{4\pi\varepsilon_0}\left(\frac{dQ}{x^2+y^2}\right) = \frac{Q}{4\pi\varepsilon_0 a}\left(\frac{dy}{x^2+y^2}\right).$

$$dE_x = dE\cos\theta = \frac{Qx}{4\pi\varepsilon_0 a}\left(\frac{dy}{(x^2+y^2)^{3/2}}\right).$$

$$dE_y = -dE\sin\theta = -\frac{Q}{4\pi\varepsilon_0 a}\left(\frac{ydy}{(x^2+y^2)^{3/2}}\right).$$

$$E_x = \int dE_x = -\frac{Qx}{4\pi\varepsilon_0 a}\int_0^a \frac{dy}{(x^2+y^2)^{3/2}} = \frac{Qx}{4\pi\varepsilon_0 a}\left[\frac{1}{x^2}\frac{y}{\sqrt{x^2+y^2}}\right]_0^a = \frac{Q}{4\pi\varepsilon_0 x}\frac{1}{\sqrt{x^2+a^2}}.$$

$$E_y = \int dEy = -\frac{Q}{4\pi\varepsilon_0 a}\int_0^a \frac{ydy}{(x^2+y^2)^{3/2}} = -\frac{Q}{4\pi\varepsilon_0 a}\left[-\frac{1}{\sqrt{x^2+y^2}}\right]_0^a = -\frac{Q}{4\pi\varepsilon_0 a}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+a^2}}\right).$$

(b) $\vec{F} = q_0\vec{E}.$

$$F_x = -qE_x = \frac{-qQ}{4\pi\varepsilon_0 x}\frac{1}{\sqrt{x^2+a^2}};\; F_y = -qE_y = \frac{qQ}{4\pi\varepsilon_0 a}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+a^2}}\right).$$

(c) For $x \gg a$, $\frac{1}{\sqrt{x^2+a^2}} = \frac{1}{x}\left(1+\frac{a^2}{x^2}\right)^{-1/2} = \frac{1}{x}\left(1-\frac{a^2}{2x^2}\right) = \frac{1}{x} - \frac{a^2}{2x^3}.$

$$F_x \approx -\frac{qQ}{4\pi\varepsilon_0 x^2},\; F_y \approx \frac{qQ}{4\pi\varepsilon_0 a}\left(\frac{1}{x} - \frac{1}{x} + \frac{a^2}{2x^3}\right) = \frac{qQa}{8\pi\varepsilon_0 x^3}.$$

EVALUATE: For $x \gg a$, $F_y \ll F_x$ and $F \approx |F_x| = \frac{qQ}{4\pi\varepsilon_0 x^2}$ and $\vec{F}$ is in the $-x$-direction. For $x \gg a$ the charge distribution Q acts like a point charge.

21.83. IDENTIFY: Apply $E = \frac{\sigma}{2\varepsilon_0}[1-(R^2/x^2+1)^{-1/2}].$

SET UP: $\sigma = Q/A = Q/\pi R^2$. $(1+y^2)^{-1/2} \approx 1 - y^2/2$, when $y^2 \ll 1$.

EXECUTE: **(a)** $E=\dfrac{\sigma}{2\varepsilon_0}[1-(R^2/x^2+1)^{-1/2}]$ gives

$$E=\frac{7.00\text{ pC}/\pi(0.025\text{ m})^2}{2\varepsilon_0}\left[1-\left(\frac{(0.025\text{ m})^2}{(0.200\text{ m})^2}+1\right)^{-1/2}\right]=1.56\text{ N/C, in the }+x\text{-direction.}$$

(b) For $x\gg R$, $E=\dfrac{\sigma}{2\varepsilon_0}[1-(1-R^2/2x^2+\cdots)]\approx\dfrac{\sigma}{2\varepsilon_0}\dfrac{R^2}{2x^2}=\dfrac{\sigma\pi R^2}{4\pi\varepsilon_0x^2}=\dfrac{Q}{4\pi\varepsilon_0x^2}$.

(c) The electric field of (a) is less than that of the point charge (0.90 N/C) since the first correction term to the point charge result is negative.

(d) For $x=0.200$ m, the percent difference is $\dfrac{(1.58-1.56)}{1.56}=0.01=1\%$. For $x=0.100$ m,

$E_{\text{disk}}=6.00$ N/C and $E_{\text{point}}=6.30$ N/C, so the percent difference is $\dfrac{(6.30-6.00)}{6.30}=0.047\approx5\%$.

EVALUATE: The field of a disk becomes closer to the field of a point charge as the distance from the disk increases. At $x=10.0$ cm, $R/x=25\%$ and the percent difference between the field of the disk and the field of a point charge is 5%.

21.84. **IDENTIFY:** Apply $\sum F_x=0$ and $\sum F_y=0$ to the sphere, with x horizontal and y vertical.

SET UP: The free-body diagram for the sphere is given in Figure 21.84. The electric field $\vec{E}$ of the sheet is directed away from the sheet and has magnitude $E=\dfrac{\sigma}{2\varepsilon_0}$.

EXECUTE: $\sum F_y=0$ gives $T\cos\alpha=mg$ and $T=\dfrac{mg}{\cos\alpha}$. $\sum F_x=0$ gives $T\sin\alpha=\dfrac{q\sigma}{2\varepsilon_0}$ and

$T=\dfrac{q\sigma}{2\varepsilon_0\sin\alpha}$. Combining these two equations we have $\dfrac{mg}{\cos\alpha}=\dfrac{q\sigma}{2\varepsilon_0\sin\alpha}$ and $\tan\alpha=\dfrac{q\sigma}{2\varepsilon_0 mg}$. Therefore,

$\alpha=\arctan\left(\dfrac{q\sigma}{2\varepsilon_0 mg}\right)$.

EVALUATE: The electric field of the sheet, and hence the force it exerts on the sphere, is independent of the distance of the sphere from the sheet.

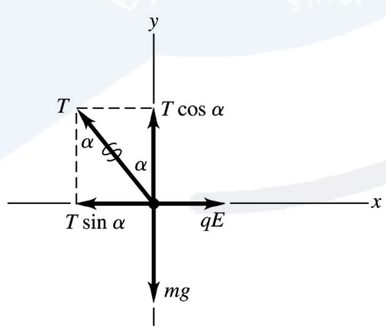


Figure 21.84

21.85. **IDENTIFY:** Divide the charge distribution into small segments, use the point charge formula for the electric field due to each small segment and integrate over the charge distribution to find the x- and y-components of the total field.

SET UP: Consider the small segment shown in Figure 21.85a.

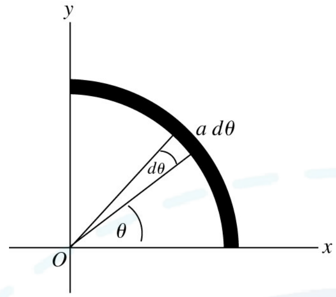
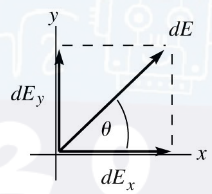


Figure 21.85a

EXECUTE: A small segment that subtends angle $d\theta$ has length $a\,d\theta$ and contains charge $dQ = \left(\dfrac{ad\theta}{\frac{1}{2}\pi a}\right)Q = \dfrac{2Q}{\pi}d\theta.$ ($\frac{1}{2}\pi a$ is the total length of the charge distribution.)

The charge is negative, so the field at the origin is directed toward the small segment. The small segment is located at angle θ as shown in the sketch. The electric field due to dQ is shown in Figure 21.85b, along with its components.



$$dE = \frac{1}{4\pi\epsilon_0}\frac{|dQ|}{a^2}.$$

$$dE = \frac{Q}{2\pi^2\epsilon_0 a^2}d\theta.$$

Figure 21.85b

$$dE_x = dE\cos\theta = (Q/2\pi^2\epsilon_0 a^2)\cos\theta d\theta.$$

$$E_x = \int dE_x = \frac{Q}{2\pi^2\epsilon_0 a^2}\int_0^{\pi/2}\cos\theta d\theta = \frac{Q}{2\pi^2\epsilon_0 a^2}(\sin\theta\Big|_0^{\pi/2}) = \frac{Q}{2\pi^2\epsilon_0 a^2}.$$

$$dE_y = dE\sin\theta = (Q/2\pi^2\epsilon_0 a^2)\sin\theta d\theta.$$

$$E_y = \int dE_y = \frac{Q}{2\pi^2\epsilon_0 a^2}\int_0^{\pi/2}\sin\theta d\theta = \frac{Q}{2\pi^2\epsilon_0 a^2}(-\cos\theta\Big|_0^{\pi/2}) = \frac{Q}{2\pi^2\epsilon_0 a^2}.$$

EVALUATE: Note that $E_x = E_y$, as expected from symmetry.

21.86. IDENTIFY: We must add the electric field components of the positive half and the negative half.

SET UP: From Problem 21.85, the electric field due to the quarter-circle section of positive charge has components $E_x = +\dfrac{Q}{2\pi^2\epsilon_0 a^2}$, $E_y = -\dfrac{Q}{2\pi^2\epsilon_0 a^2}$. The field due to the quarter-circle section of negative charge has components $E_x = +\dfrac{Q}{2\pi^2\epsilon_0 a^2}$, $E_y = +\dfrac{Q}{2\pi^2\epsilon_0 a^2}$.

EXECUTE: The components of the resultant field is the sum of the x- and y-components of the fields due to each half of the semicircle. The y-components cancel, but the x-components add, giving

$E_x = +\dfrac{Q}{\pi^2\epsilon_0 a^2}$, in the $+x$-direction.

EVALUATE: Even though the net charge on the semicircle is zero, the field it produces is *not* zero because of the way the charge is arranged.

21.87. **IDENTIFY:** Each wire produces an electric field at P due to a finite wire. These fields add by vector addition.

SET UP: Each field has magnitude $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{x\sqrt{x^2+a^2}}$. The field due to the negative wire points to the left, while the field due to the positive wire points downward, making the two fields perpendicular to each other and of equal magnitude. The net field is the vector sum of these two, which is

$E_{\text{net}} = 2E_1\cos 45° = 2\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{x\sqrt{x^2+a^2}}\cos 45°$. In part (b), the electrical force on an electron at P is eE.

EXECUTE: **(a)** The net field is $E_{\text{net}} = 2\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{x\sqrt{x^2+a^2}}\cos 45°$.

$$E_{\text{net}} = \frac{2(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.50\times10^{-6}\ \text{C})\cos 45°}{(0.600\ \text{m})\sqrt{(0.600\ \text{m})^2+(0.600\ \text{m})^2}} = 6.25\times10^4\ \text{N/C}.$$

The direction is 225° counterclockwise from an axis pointing to the right at point P.

(b) $F = eE = (1.60\times10^{-19}\ \text{C})(6.25\times10^4\ \text{N/C}) = 1.00\times10^{-14}\ \text{N}$, opposite to the direction of the electric field, since the electron has negative charge.

EVALUATE: Since the electric fields due to the two wires have equal magnitudes and are perpendicular to each other, we only have to calculate one of them in the solution.

21.88. **IDENTIFY:** Each sheet produces an electric field that is independent of the distance from the sheet. The net field is the vector sum of the two fields.

SET UP: The formula for each field is $E = \sigma/2\varepsilon_0$, and the net field is the vector sum of these,

$E_{\text{net}} = \dfrac{\sigma_B}{2\varepsilon_0} \pm \dfrac{\sigma_A}{2\varepsilon_0} = \dfrac{\sigma_B \pm \sigma_A}{2\varepsilon_0}$, where we use the + or – sign depending on whether the fields are in the same or opposite directions and σ_B and σ_A are the magnitudes of the surface charges.

EXECUTE: **(a)** The two fields oppose and the field of B is stronger than that of A, so

$$E_{\text{net}} = \frac{\sigma_B}{2\varepsilon_0} - \frac{\sigma_A}{2\varepsilon_0} = \frac{\sigma_B - \sigma_A}{2\varepsilon_0} = \frac{11.6\ \mu\text{C/m}^2 - 8.80\ \mu\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 1.58\times10^5\ \text{N/C},\ \text{to the right.}$$

(b) The fields are now in the same direction, so their magnitudes add.

$$E_{\text{net}} = (11.6\ \mu\text{C/m}^2 + 8.80\ \mu\text{C/m}^2)/2\varepsilon_0 = 1.15\times10^6\ \text{N/C},\ \text{to the right.}$$

(c) The fields add but now point to the left, so $E_{\text{net}} = 1.15\times10^6$ N/C, to the left.

EVALUATE: We can simplify the calculations by sketching the fields and doing an algebraic solution first.

21.89. **IDENTIFY:** Each sheet produces an electric field that is independent of the distance from the sheet. The net field is the vector sum of the two fields.

SET UP: The formula for each field is $E = \sigma/2\varepsilon_0$, and the net field is the vector sum of these.

$E_{\text{net}} = \dfrac{\sigma_B}{2\varepsilon_0} \pm \dfrac{\sigma_A}{2\varepsilon_0} = \dfrac{\sigma_B \pm \sigma_A}{2\varepsilon_0}$, where we use the + or – sign depending on whether the fields are in the same or opposite directions and σ_B and σ_A are the magnitudes of the surface charges.

EXECUTE: **(a)** The fields add and point to the left, giving $E_{\text{net}} = 1.15\times10^6$ N/C.

(b) The fields oppose and point to the left, so $E_{\text{net}} = 1.58\times10^5$ N/C.

(c) The fields oppose but now point to the right, giving $E_{\text{net}} = 1.58\times10^5$ N/C.

EVALUATE: We can simplify the calculations by sketching the fields and doing an algebraic solution first.

21.90. **IDENTIFY:** The sheets produce an electric field in the region between them which is the vector sum of the fields from the two sheets.

SET UP: The force on the negative oil droplet must be upward to balance gravity. The net electric field between the sheets is $E = \sigma/\varepsilon_0$, and the electrical force on the droplet must balance gravity, so $qE = mg$.

EXECUTE: **(a)** The electrical force on the drop must be upward, so the field should point downward since the drop is negative.

(b) The charge of the drop is $5e$, so $qE = mg$. $(5e)(\sigma/\varepsilon_0) = mg$ and

$$\sigma = \frac{mg\varepsilon_0}{5e} = \frac{(486\times10^{-9}\text{ kg})(9.80\text{ m/s}^2)(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}{5(1.60\times10^{-19}\text{ C})} = 52.7\text{ C/m}^2.$$

EVALUATE: Balancing oil droplets between plates was the basis of the Milliken Oil-Drop Experiment which produced the first measurement of the mass of an electron.

21.91. **IDENTIFY:** Apply the formula for the electric field of a disk. The hole can be described by adding a disk of charge density $-\sigma$ and radius R_1 to a solid disk of charge density $+\sigma$ and radius R_2.

SET UP: The area of the annulus is $\pi(R_2^2 - R_1^2)\sigma$. The electric field of a disk is

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - 1/\sqrt{(R/x)^2+1}\right].$$

EXECUTE: **(a)** $Q = A\sigma = \pi(R_2^2 - R_1^2)\sigma.$

(b) $\vec{E}(x) = \dfrac{\sigma}{2\varepsilon_0}\left(\left[1 - 1/\sqrt{(R_2/x)^2+1}\right] - \left[1 - 1/\sqrt{(R_1/x)^2+1}\right]\right)\dfrac{|x|}{x}\hat{\boldsymbol{i}}.$

$\vec{E}(x) = \dfrac{\sigma}{2\varepsilon_0}\left(1/\sqrt{(R_1/x)^2+1} - 1/\sqrt{(R_2/x)^2+1}\right)\dfrac{|x|}{x}\hat{\boldsymbol{i}}$. The electric field is in the $+x$-direction at points above the disk and in the $-x$-direction at points below the disk, and the factor $\dfrac{|x|}{x}\hat{\boldsymbol{i}}$ specifies these directions.

(c) Note that $1/\sqrt{(R_1/x)^2+1} = \dfrac{|x|}{R_1}(1+(x/R_1)^2)^{-1/2} \approx \dfrac{|x|}{R_1}$. This gives

$\vec{E}(x) = \dfrac{\sigma}{2\varepsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)\dfrac{|x|^2}{x}\hat{\boldsymbol{i}} = \dfrac{\sigma}{2\varepsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)x\hat{\boldsymbol{i}}$. Sufficiently close means that $(x/R_1)^2 \ll 1$.

(d) $F_x = -qE_x = -\dfrac{q\sigma}{2\varepsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)x$. The force is in the form of Hooke's law: $F_x = -kx$, with

$$k = \frac{q\sigma}{2\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right).\quad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{q\sigma}{2\varepsilon_0 m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}.$$

EVALUATE: The frequency is independent of the initial position of the particle, so long as this position is sufficiently close to the center of the annulus for $(x/R_1)^2$ to be small.

21.92. **IDENTIFY:** Apply constant acceleration equations to a drop to find the acceleration. Then use $F = ma$ to find the force and $F = |q|E$ to find $|q|$.

SET UP: Let $D = 2.0$ cm be the horizontal distance the drop travels and $d = 0.30$ mm be its vertical displacement. Let $+x$ be horizontal and in the direction from the nozzle toward the paper and let $+y$ be vertical, in the direction of the deflection of the drop. $a_x = 0$ and call $a_y = a$.

EXECUTE: **(a)** Find the time of flight: $t = D/v = (0.020\text{ m})/(50\text{ m/s}) = 4.00\times10^{-4}$ s. $d = \dfrac{1}{2}at^2$.

$a = \dfrac{2d}{t^2} = \dfrac{2(3.00\times10^{-4}\text{ m})}{(4.00\times10^{-4}\text{ s})^2} = 3750\text{ m/s}^2$. Then $a = F/m = qE/m$ gives

$q = ma/E = \dfrac{(1.4\times10^{-11}\text{ kg})(3750\text{ m/s}^2)}{8.00\times10^4\text{ N/C}} = 6.56\times10^{-13}$ C, which rounds to 6.6×10^{-13} s.

(b) Use the equations and calculations above: if $v \to v/2$, then $t \to 2t$, so $a \to a/4$, which means that $q \to q/4$, so $q = (6.56\times10^{-13}\text{ s})/4 = 1.64\times10^{-13}$ s, which rounds to 1.6×10^{-13} s.

EVALUATE: Since q is positive the vertical deflection is in the direction of the electric field.

21.93 **IDENTIFY:** The net force on the third sphere is the vector sum of the forces due to the other two charges. Coulomb's law gives the forces.

SET UP: $F = k\dfrac{|q_1 q_2|}{r^2}$.

EXECUTE: **(a)** Between the two fixed charges, the electric forces on the third sphere q_3 are in opposite directions and have magnitude 4.50 N in the $+x$-direction. Applying Coulomb's law gives
4.50 N = $k[q_1(4.00\ \mu\text{C})/(0.200\ \text{m})^2 - q_2(4.00\ \mu\text{C})/(0.200\ \text{m})^2]$.
Simplifying gives $q_1 - q_2 = 5.00\ \mu\text{C}$.
With q_3 at $x = +0.600$ m, the electric forces on q_3 are all in the $+x$-direction and add to 3.50 N. As before, Coulomb's law gives
3.50 N = $k[q_1(4.00\ \mu\text{C})/(0.600\ \text{m})^2 + q_2(4.00\ \mu\text{C})/(0.200\ \text{m})^2]$.
Simplifying gives $q_1 + 9q_2 = 35.0\ \mu\text{C}$.
Solving the two equations simultaneously gives $q_1 = 8.00\ \mu\text{C}$ and $q_2 = 3.00\ \mu\text{C}$.
(b) Both forces on q_3 are in the $-x$-direction, so their magnitudes add. Factoring out common factors and using the values for q_1 and q_2 we just found, Coulomb's law gives
$F_{\text{net}} = kq_3\ [q_1/(0.200\ \text{m})^2 + q_2/(0.600\ \text{m})^2]$.
$F_{\text{net}} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\ [(8.00\ \mu\text{C})/(0.200\ \text{m})^2 + (3.00\ \mu\text{C})/(0.600\ \text{m})^2] = 7.49$ N, and it is in the $-x$-direction.
(c) The forces are in opposite direction and add to zero, so
$0 = kq_1q_3/x^2 - kq_2q_3/(0.400\ \text{m} - x)^2$.
$(0.400\ \text{m} - x)^2 = (q_2/q_1)x^2$.
Taking square roots of both sides gives
$0.400\ \text{m} - x = \pm x\sqrt{q_2/q_1} = \pm 0.6124x$.
Solving for x, we get two values: $x = 0.248$ m and $x = 1.03$ m. The charge q_3 must be between the other two charges for the forces on it to balance. Only the first value is between the two charges, so it is the correct one: $x = 0.248$ m.

EVALUATE: Check the answers in part (a) by substituting these values back into the original equations. $8.00\ \mu\text{C} - 3.00\ \mu\text{C} = 5.00\ \mu\text{C}$ and $8.00\ \mu\text{C} + 9(3.00\ \mu\text{C}) = 35.0\ \mu\text{C}$, so the answers check in both equations. In part (c), the second root, $x = 1.03$ m, has some meaning. The condition we imposed to solve the problem was that the magnitudes of the two forces were equal. This happens at $x = 0.248$ mn, but it also happens at $x = 1.03$ m. However at the second root the forces are both in the $+x$-direction and therefore cannot cancel.

21.94. **IDENTIFY** and **SET UP:** The electric field E_x produced by a uniform ring of charge, for points on an axis perpendicular to the plane of the ring at its center, is $E_x = \dfrac{kQx}{(x^2 + a^2)^{3/2}}$, where a is the radius of the ring, x is the distance from its center along the axis, and Q is the total charge on the ring.

EXECUTE: **(a)** Far from the ring, at large values of x, the ring can be considered as a point-charge, so its electric field would be $E = kQ/x^2$. Therefore $Ex^2 = kQ$, which is a constant. From the graph (a) in the problem, we read off that at large distances $Ex^2 = 45\ \text{N}\cdot\text{m}^2/\text{C}$, which is equal to kQ, so
$Q = (45\ \text{N}\cdot\text{m}^2/\text{C})/k = 5.0\times10^{-9}\ \text{C} = 5.0$ nC.
(b) The electric field along the axis a distance x from the ring is $E_x = \dfrac{kQx}{(x^2 + a^2)^{3/2}}$. Very close to the ring, $x^2 << a^2$, so the formula becomes $Ex = kQx/a^3$. Therefore $E/x = kQ/a^3$, which is a constant. From graph (b) in the problem, E/x approaches $700\ \text{N/C}\cdot\text{m}$ as x approaches zero. So $kQ/a^3 = 700\ \text{N/C}\cdot\text{m}$, which gives
$a = [kQ/(700\ \text{N/C}\cdot\text{m})]^{1/3} = [(45\ \text{N}\cdot\text{m}^2/\text{C})/(700\ \text{N/C}\cdot\text{m})]^{1/3} = 0.40\ \text{m} = 40$ cm.

EVALUATE: It is physically reasonable that a ring 40 cm in radius could carry 5.0 nC of charge.

21.95. **IDENTIFY:** Apply Coulomb's law to calculate the forces that q_1 and q_2 exert on q_3, and add these force vectors to get the net force.

SET UP: Like charges repel and unlike charges attract. Let $+x$ be to the right and $+y$ be toward the top of the page.

EXECUTE: **(a)** The four possible force diagrams are sketched in Figure 21.95a.

Only the last picture can result in a net force in the $-x$-direction.

(b) $q_1 = -2.00\ \mu\text{C}, q_3 = +4.00\ \mu\text{C}$, and $q_2 > 0$.

(c) The forces $\vec{F}_1$ and $\vec{F}_2$ and their components are sketched in Figure 21.95b.

$F_y = 0 = -\dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1||q_3|}{(0.0400\ \text{m})^2}\sin\theta_1 + \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2||q_3|}{(0.0300\ \text{m})^2}\sin\theta_2$. This gives

$$q_2 = \frac{9}{16}|q_1|\frac{\sin\theta_1}{\sin\theta_2} = \frac{9}{16}|q_1|\frac{3/5}{4/5} = \frac{27}{64}|q_1| = 0.843\ \mu\text{C}.$$

(d) $F_x = F_{1x} + F_{2x}$ and $F_y = 0$, so $F = |q_3|\dfrac{1}{4\pi\epsilon_0}\left(\dfrac{|q_1|}{(0.0400\ \text{m})^2}\dfrac{4}{5} + \dfrac{|q_2|}{(0.0300\ \text{m})^2}\dfrac{3}{5}\right) = 56.2\ \text{N}$.

EVALUATE: The net force $\vec{F}$ on q_3 is in the same direction as the resultant electric field at the location of q_3 due to q_1 and q_2.

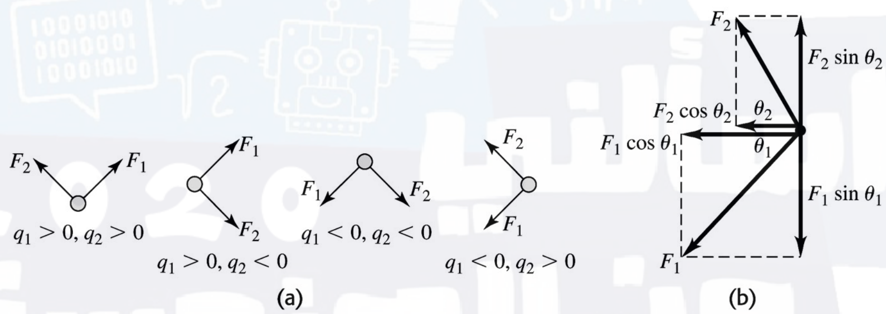


Figure 21.95

21.96. IDENTIFY: Calculate the electric field at P due to each charge and add these field vectors to get the net field.

SET UP: The electric field of a point charge is directed away from a positive charge and toward a negative charge. Let $+x$ be to the right and let $+y$ be toward the top of the page.

EXECUTE: **(a)** The four possible diagrams are sketched in Figure 21.96a (next page).
The first diagram is the only one in which the electric field must point in the negative y-direction.

(b) $q_1 = -3.00\ \mu\text{C}$, and $q_2 < 0$.

(c) The electric fields $\vec{E}_1$ and $\vec{E}_2$ and their components are sketched in Figure 21.96b. $\cos\theta_1 = \dfrac{5}{13}$,
$\sin\theta_1 = \dfrac{12}{13}$, $\cos\theta_2 = \dfrac{12}{13}$ and $\sin\theta_2 = \dfrac{5}{13}$. $E_x = 0 = -\dfrac{k|q_1|}{(0.050\ \text{m})^2}\dfrac{5}{13} + \dfrac{k|q_2|}{(0.120\ \text{m})^2}\dfrac{12}{13}$. This gives
$\dfrac{k|q_2|}{(0.120\ \text{m})^2} = \dfrac{k|q_1|}{(0.050\ \text{m})^2}\dfrac{5}{12}$. Solving for $|q_2|$ gives $|q_2| = 7.2\ \mu\text{C}$, so $q_2 = -7.2\ \mu\text{C}$. Then
$E_y = -\dfrac{k|q_1|}{(0.050\ \text{m})^2}\dfrac{12}{13} - \dfrac{kq_2}{(0.120\ \text{m})^2}\dfrac{5}{13} = -1.17\times10^7\ \text{N/C}$. $E = 1.17\times10^7\ \text{N/C}$.

EVALUATE: With q_1 known, specifying the direction of $\vec{E}$ determines both q_2 and E.

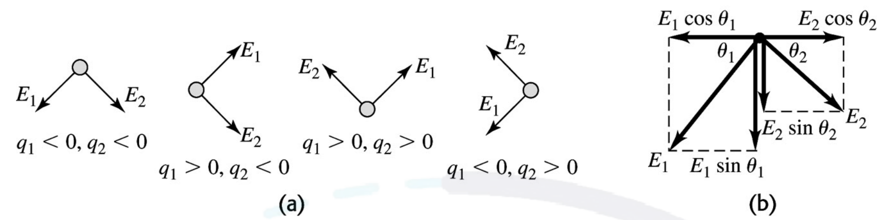


Figure 21.96

21.97. **IDENTIFY:** To find the electric field due to the second rod, divide that rod into infinitesimal segments of length dx, calculate the field dE due to each segment and integrate over the length of the rod to find the total field due to the rod. Use $d\vec{F} = dq\,\vec{E}$ to find the force the electric field of the second rod exerts on each infinitesimal segment of the first rod.

SET UP: An infinitesimal segment of the second rod is sketched in Figure 21.97. $dQ = (Q/L)dx'$.

EXECUTE: **(a)** $dE = \dfrac{k\,dQ}{(x+a/2+L-x')^2} = \dfrac{kQ}{L}\dfrac{dx'}{(x+a/2+L-x')^2}$.

$$E_x = \int_0^L dE_x = \frac{kQ}{L}\int_0^L \frac{dx'}{(x+a/2+L-x')^2} = \frac{kQ}{L}\left[\frac{1}{x+a/2+L-x'}\right]_0^L = \frac{kQ}{L}\left(\frac{1}{x+a/2} - \frac{1}{x+a/2+L}\right).$$

$$E_x = \frac{2kQ}{L}\left(\frac{1}{2x+a} - \frac{1}{2L+2x+a}\right).$$

(b) Now consider the force that the field of the second rod exerts on an infinitesimal segment dq of the first rod. This force is in the $+x$-direction. $dF = dq\,E$.

$$F = \int E\,dq = \int_{a/2}^{L+a/2}\frac{EQ}{L}dx = \frac{2kQ^2}{L^2}\int_{a/2}^{L+a/2}\left(\frac{1}{2x+a} - \frac{1}{2L+2x+a}\right)dx.$$

$$F = \frac{2kQ^2}{L^2}\frac{1}{2}\left(\left[\ln(a+2x)\right]_{a/2}^{L+a/2} - \left[\ln(2L+2x+a)\right]_{a/2}^{L+a/2}\right) = \frac{kQ^2}{L^2}\ln\left[\left(\frac{a+2L+a}{2a}\right)\left(\frac{2L+2a}{4L+2a}\right)\right].$$

$$F = \frac{kQ^2}{L^2}\ln\left(\frac{(a+L)^2}{a(a+2L)}\right).$$

(c) For $a \gg L$, $F = \dfrac{kQ^2}{L^2}\ln\left(\dfrac{a^2(1+L/a)^2}{a^2(1+2L/a)}\right) = \dfrac{kQ^2}{L^2}(2\ln(1+L/a) - \ln(1+2L/a))$.

For small z, $\ln(1+z) \approx z - \dfrac{z^2}{2}$. Therefore, for $a \gg L$,

$$F \approx \frac{kQ^2}{L^2}\left[2\left(\frac{L}{a} - \frac{L^2}{2a^2} + \cdots\right) - \left(\frac{2L}{a} - \frac{2L^2}{a^2} + \cdots\right)\right] \approx \frac{kQ^2}{a^2}.$$

EVALUATE: The distance between adjacent ends of the rods is a. When $a \gg L$ the distance between the rods is much greater than their lengths and they interact as point charges.

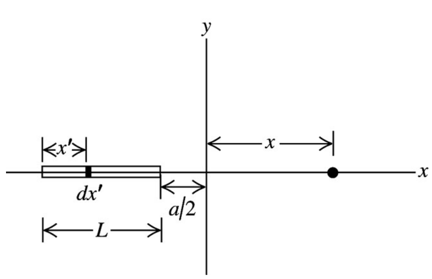


Figure 21.97

21.98. **IDENTIFY** and **SET UP:** The charge of n electrons is ne.

EXECUTE: The charge on the bee is $Q = ne$, so the number of missing electrons is $n = Q/e = (30\text{ pC})/e = (30\times10^{-12}\text{ C})/(1.60\times10^{-19}\text{ C}) = 1.88\times10^{8} \approx 1.9\times10^{8}$ electrons, which makes choice (a) correct.

EVALUATE: This charge is due to around 190 million electrons.

21.99. **IDENTIFY** and **SET UP:** One charge exerts a force on another charge without being in contact.

EXECUTE: Even though the bee does not touch the stem, the positive charges on the bee attract negative charges (electrons normally) in the stem. This pulls electrons toward the bee, leaving positive charge at the opposite end of the stem, which polarizes it. Thus choice (c) is correct.

EVALUATE: Choice (b) cannot be correct because the bee is positive and would therefore not attract the positive charges in the stem.

21.100. **IDENTIFY** and **SET UP:** Electric field lines begin on positive charges and end on negative charges.

EXECUTE: The flower and bee are both positive, so no field lines can end on either of them. This makes the figure in choice (c) the correct one.

EVALUATE: The net electric field is the vector sum of the field due to the bee and the field due to the flower. Somewhere between the bee and flower the fields cancel, depending on the relative amounts of charge on the bee and flower.

21.101. **IDENTIFY** and **SET UP:** Assume that the charge remains at the end of the stem and that the bees approach to 15 cm from this end of the stem. The electric field is $E = k\dfrac{|q|}{r^2}$.

EXECUTE: Using the numbers given, we have

$E = k\dfrac{|q|}{r^2} = (8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(40\times10^{-12}\text{ C})/(0.15\text{ m})^2 = 16\text{ N/C}$, which is choice (b).

EVALUATE: Even if the charge spread out a bit over the stem, the result would be in the neighborhood of the value we calculated.

22

GAUSS'S LAW

22.1. **IDENTIFY and SET UP:** $\Phi_E = \int E\cos\phi\, dA,$ where ϕ is the angle between the normal to the sheet $\hat{\boldsymbol{n}}$ and the electric field $\vec{\boldsymbol{E}}$.

(a) EXECUTE: In this problem E and $\cos\phi$ are constant over the surface so

$\Phi_E = E\cos\phi\int dA = E\cos\phi A = (14\ \text{N/C})(\cos 60°)(0.250\ \text{m}^2) = 1.8\ \text{N}\cdot\text{m}^2/\text{C}.$

EVALUATE: **(b)** Φ_E is independent of the shape of the sheet as long as ϕ and E are constant at all points on the sheet.

(c) EXECUTE: (i) $\Phi_E = E\cos\phi A.$ Φ_E is largest for $\phi = 0°$, so $\cos\phi = 1$ and $\Phi_E = EA.$

(ii) Φ_E is smallest for $\phi = 90°$, so $\cos\phi = 0$ and $\Phi_E = 0.$

EVALUATE: Φ_E is 0 when the surface is parallel to the field so no electric field lines pass through the surface.

22.2. **IDENTIFY:** The field is uniform and the surface is flat, so use $\Phi_E = EA\cos\phi.$

SET UP: ϕ is the angle between the normal to the surface and the direction of $\vec{\boldsymbol{E}}$, so $\phi = 70°.$

EXECUTE: $\Phi_E = (90.0\ \text{N/C})(0.400\ \text{m})(0.600\ \text{m})\cos 70° = 7.39\ \text{N}\cdot\text{m}^2/\text{C}.$

EVALUATE: If the field were perpendicular to the surface the flux would be $\Phi_E = EA = 21.6\ \text{N}\cdot\text{m}^2/\text{C}.$ The flux in this problem is much less than this because only the component of $\vec{\boldsymbol{E}}$ perpendicular to the surface contributes to the flux.

22.3. **IDENTIFY:** The electric flux through an area is defined as the product of the component of the electric field perpendicular to the area times the area.

(a) SET UP: In this case, the electric field is perpendicular to the surface of the sphere, so $\Phi_E = EA = E(4\pi r^2).$

EXECUTE: Substituting in the numbers gives

$$\Phi_E = (1.25\times10^6\ \text{N/C})4\pi(0.150\ \text{m})^2 = 3.53\times10^5\ \text{N}\cdot\text{m}^2/\text{C}.$$

(b) IDENTIFY: We use the electric field due to a point charge.

SET UP: $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q|}{r^2}$

EXECUTE: Solving for q and substituting the numbers gives

$$q = 4\pi\varepsilon_0 r^2 E = \frac{1}{9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}(0.150\ \text{m})^2(1.25\times10^6\ \text{N/C}) = 3.13\times10^{-6}\ \text{C}.$$

EVALUATE: The flux would be the same no matter how large the sphere, since the area is proportional to r^2 while the electric field is proportional to $1/r^2$.

22.4. IDENTIFY: Use $\Phi_E = \int \vec{E} \cdot d\vec{A} = \int E\cos\phi dA$ to calculate the flux through the surface of the cylinder.

SET UP: The line of charge and the cylinder are sketched in Figure 22.4.

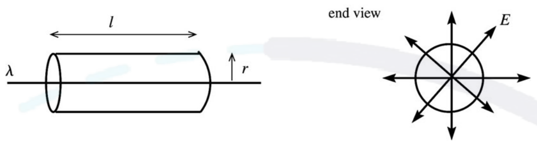


Figure 22.4

EXECUTE: **(a)** The area of the curved part of the cylinder is $A = 2\pi rl$.

The electric field is parallel to the end caps of the cylinder, so $\vec{E} \cdot \vec{A} = 0$ for the ends and the flux through the cylinder end caps is zero.

The electric field is normal to the curved surface of the cylinder and has the same magnitude $E = \lambda/2\pi\epsilon_0 r$ at all points on this surface. Thus $\phi = 0°$ and

$$\Phi_E = EA\cos\phi = EA = (\lambda/2\pi\epsilon_0 r)(2\pi rl) = \frac{\lambda l}{\epsilon_0} = \frac{(3.00\times10^{-6}\ \text{C/m})(0.400\ \text{m})}{8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 1.36\times10^5\ \text{N}\cdot\text{m}^2/\text{C}.$$

(b) In the calculation in part (a) the radius r of the cylinder divided out, so the flux remains the same, $\Phi_E = 1.36\times10^5\ \text{N}\cdot\text{m}^2/\text{C}$.

(c) $\Phi_E = \dfrac{\lambda l}{\epsilon_0} = \dfrac{(3.00\times10^{-6}\ \text{C/m})(0.800\ \text{m})}{8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 2.71\times10^5\ \text{N}\cdot\text{m}^2/\text{C}$, which is twice the flux calculated in parts (a) and (b).

EVALUATE: The flux depends on the number of field lines that pass through the surface of the cylinder.

22.5. IDENTIFY: The flux through the curved upper half of the hemisphere is the same as the flux through the flat circle defined by the bottom of the hemisphere because every electric field line that passes through the flat circle also must pass through the curved surface of the hemisphere.

SET UP: The electric field is perpendicular to the flat circle, so the flux is simply the product of E and the area of the flat circle of radius r.

EXECUTE: $\Phi_E = EA = E(\pi r^2) = \pi r^2 E$

EVALUATE: The flux would be the same if the hemisphere were replaced by any other surface bounded by the flat circle.

22.6. IDENTIFY: Use $\Phi_E = \vec{E} \cdot \vec{A}$ to calculate the flux for each surface.

SET UP: $\Phi = \vec{E} \cdot \vec{A} = EA\cos\phi$ where $\vec{A} = A\hat{n}$.

EXECUTE: **(a)** $\hat{n}_{S_1} = -\hat{j}$ (left). $\Phi_{S_1} = -(4\times10^3\ \text{N/C})(0.10\ \text{m})^2\cos(90° - 53.1°) = -32\ \text{N}\cdot\text{m}^2/\text{C}$.

$\hat{n}_{S_2} = +\hat{k}$ (top). $\Phi_{S_2} = -(4\times10^3\ \text{N/C})(0.10\ \text{m})^2\cos 90° = 0$.

$\hat{n}_{S_3} = +\hat{j}$ (right). $\Phi_{S_3} = +(4\times10^3\ \text{N/C})(0.10\ \text{m})^2\cos(90° - 53.1°) = +32\ \text{N}\cdot\text{m}^2/\text{C}$.

$\hat{n}_{S_4} = -\hat{k}$ (bottom). $\Phi_{S_4} = (4\times10^3\ \text{N/C})(0.10\ \text{m})^2\cos 90° = 0$.

$\hat{n}_{S_5} = +\hat{i}$ (front). $\Phi_{S_5} = +(4\times10^3\ \text{N/C})(0.10\ \text{m})^2\cos 53.1° = 24\ \text{N}\cdot\text{m}^2/\text{C}$.

$\hat{n}_{S_6} = -\hat{i}$ (back). $\Phi_{S_6} = -(4\times10^3\ \text{N/C})(0.10\ \text{m})^2\cos 53.1° = -24\ \text{N}\cdot\text{m}^2/\text{C}$.

EVALUATE: **(b)** The total flux through the cube must be zero; any flux entering the cube must also leave it, since the field is uniform. Our calculation gives the result; the sum of the fluxes calculated in part (a) is zero.

22.7. **IDENTIFY:** Apply Gauss's law to a Gaussian surface that coincides with the cell boundary.

SET UP: $\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$.

EXECUTE: $\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0} = \dfrac{-8.65\times10^{-12}\ \text{C}}{8.854\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)} = -0.977\ \text{N}\cdot\text{m}^2/\text{C}$. Q_{encl} is negative, so the flux is inward.

EVALUATE: If the cell were positive, the field would point outward, so the flux would be positive.

22.8. **IDENTIFY:** Apply Gauss's law to each surface.

SET UP: Q_{encl} is the algebraic sum of the charges enclosed by each surface. Flux out of the volume is positive and flux into the enclosed volume is negative.

EXECUTE: **(a)** $\Phi_{S_1} = q_1/\varepsilon_0 = (4.00\times10^{-9}\ \text{C})/\varepsilon_0 = 452\ \text{N}\cdot\text{m}^2/\text{C}$.

(b) $\Phi_{S_2} = q_2/\varepsilon_0 = (-7.80\times10^{-9}\ \text{C})/\varepsilon_0 = -881\ \text{N}\cdot\text{m}^2/\text{C}$.

(c) $\Phi_{S_3} = (q_1+q_2)/\varepsilon_0 = ((4.00-7.80)\times10^{-9}\ \text{C})/\varepsilon_0 = -429\ \text{N}\cdot\text{m}^2/\text{C}$.

(d) $\Phi_{S_4} = (q_1+q_3)/\varepsilon_0 = [(4.00+2.40)\times10^{-9}\ \text{C}]/\varepsilon_0 = 723\ \text{N}\cdot\text{m}^2/\text{C}$.

(e) $\Phi_{S_5} = (q_1+q_2+q_3)/\varepsilon_0 = ((4.00-7.80+2.40)\times10^{-9}\ \text{C})/\varepsilon_0 = -158\ \text{N}\cdot\text{m}^2/\text{C}$.

EVALUATE: **(f)** All that matters for Gauss's law is the total amount of charge enclosed by the surface, not its distribution within the surface.

22.9. **IDENTIFY:** Apply the results in Example 22.5 for the field of a spherical shell of charge.

SET UP: Example 22.5 shows that $E=0$ inside a uniform spherical shell and that $E = k\dfrac{|q|}{r^2}$ outside the shell.

EXECUTE: **(a)** $E=0$.

(b) $r = 0.060\ \text{m}$ and $E = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{49.0\times10^{-6}\ \text{C}}{(0.060\ \text{m})^2} = 1.22\times10^8\ \text{N/C}$.

(c) $r = 0.110\ \text{m}$ and $E = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{49.0\times10^{-6}\ \text{C}}{(0.110\ \text{m})^2} = 3.64\times10^7\ \text{N/C}$.

EVALUATE: Outside the shell the electric field is the same as if all the charge were concentrated at the center of the shell. But inside the shell the field is not the same as for a point charge at the center of the shell, inside the shell the electric field is zero.

22.10. **IDENTIFY:** Apply Gauss's law to the spherical surface.

SET UP: Q_{encl} is the algebraic sum of the charges enclosed by the sphere.

EXECUTE: **(a)** No charge enclosed so $\Phi_E = 0$.

(b) $\Phi_E = \dfrac{q_2}{\varepsilon_0} = \dfrac{-6.00\times10^{-9}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = -678\ \text{N}\cdot\text{m}^2/\text{C}$.

(c) $\Phi_E = \dfrac{q_1+q_2}{\varepsilon_0} = \dfrac{(4.00-6.00)\times10^{-9}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = -226\ \text{N}\cdot\text{m}^2/\text{C}$.

EVALUATE: Negative flux corresponds to flux directed into the enclosed volume. The net flux depends only on the net charge enclosed by the surface and is not affected by any charges outside the enclosed volume.

22.11. **(a) IDENTIFY and SET UP:** It is rather difficult to calculate the flux directly from $\Phi_E = \int \vec{E} \cdot d\vec{A}$ since the magnitude of $\vec{E}$ and its angle with $d\vec{A}$ varies over the surface of the cube. A much easier approach is to use Gauss's law to calculate the total flux through the cube. Let the cube be the Gaussian surface. The charge enclosed is the point charge. $\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$.

EXECUTE: $\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0} = \dfrac{6.20 \times 10^{-6}\ \text{C}}{8.854 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2} = 7.002 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}.$ By symmetry the flux is the same through each of the six faces, so the flux through one face is $\frac{1}{6}(7.002 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}) = 1.17 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}.$

(b) EVALUATE: In part (a) the size of the cube did not enter into the calculations. The flux through one face depends only on the amount of charge at the center of the cube. So the answer to (a) would not change if the size of the cube were changed.

22.12. **IDENTIFY:** Apply the results of Examples 22.9 and 22.10.

SET UP: $E = k\dfrac{|q|}{r^2}$ outside the sphere. A proton has charge $+e$.

EXECUTE: **(a)** $E = k\dfrac{|q|}{r^2} = (8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)\dfrac{92(1.60 \times 10^{-19}\ \text{C})}{(7.4 \times 10^{-15}\ \text{m})^2} = 2.4 \times 10^{21}\ \text{N/C}.$

(b) For $r = 1.0 \times 10^{-10}\ \text{m},\ \ E = (2.4 \times 10^{21}\ \text{N/C})\left(\dfrac{7.4 \times 10^{-15}\ \text{m}}{1.0 \times 10^{-10}\ \text{m}}\right)^2 = 1.3 \times 10^{13}\ \text{N/C}.$

(c) $E = 0,$ inside a spherical shell.

EVALUATE: The electric field in an atom is very large.

22.13. **IDENTIFY:** Each line lies in the electric field of the other line, and therefore each line experiences a force due to the other line.

SET UP: The field of one line at the location of the other is $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$. For charge $dq = \lambda dx$ on one line, the force on it due to the other line is $dF = Edq$. The total force is $F = \int Edq = E\int dq = Eq$.

EXECUTE: $E = \dfrac{\lambda}{2\pi\varepsilon_0 r} = \dfrac{5.20 \times 10^{-6}\ \text{C/m}}{2\pi(8.854 \times 10^{-12}\ \text{C}^2/(\text{N} \cdot \text{m}^2))(0.300\ \text{m})} = 3.116 \times 10^5\ \text{N/C}.$ The force on one line due to the other is $F = Eq$, where $q = \lambda(0.0500\ \text{m}) = 2.60 \times 10^{-7}\ \text{C}.$ The net force is $F = Eq = (3.116 \times 10^5\ \text{N/C})(2.60 \times 10^{-7}\ \text{C}) = 0.0810\ \text{N}.$

EVALUATE: Since the electric field at each line due to the other line is uniform, each segment of line experiences the same force, so all we need to use is $F = Eq$, even though the line is *not* a point charge.

22.14. **IDENTIFY:** Apply the results of Example 22.5.

SET UP: At a point 0.100 m outside the surface, $r = 0.550$ m.

EXECUTE: **(a)** $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(2.50 \times 10^{-10}\ \text{C})}{(0.550\ \text{m})^2} = 7.44\ \text{N/C}.$

(b) $E = 0$ inside of a conductor or else free charges would move under the influence of forces, violating our electrostatic assumptions (i.e., that charges aren't moving).

EVALUATE: Outside the sphere its electric field is the same as would be produced by a point charge at its center, with the same charge.

22.15. **IDENTIFY and SET UP:** Example 22.5 derived that the electric field just outside the surface of a spherical conductor that has net charge $|q|$ is $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q|}{R^2}$. Calculate $|q|$ and from this the number of excess electrons.

EXECUTE: $|q| = \dfrac{R^2E}{(1/4\pi\epsilon_0)} = \dfrac{(0.130\text{ m})^2(1150\text{ N/C})}{8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 2.162\times10^{-9}\text{ C}.$

Each electron has a charge of magnitude $e = 1.602\times10^{-19}$ C, so the number of excess electrons needed is $\dfrac{2.162\times10^{-9}\text{ C}}{1.602\times10^{-19}\text{ C}} = 1.35\times10^{10}.$

EVALUATE: The result we obtained for q is a typical value for the charge of an object. Such net charges correspond to a large number of excess electrons since the charge of each electron is very small.

22.16. IDENTIFY: According to the problem, Mars's flux is negative, so its electric field must point toward the center of Mars. Therefore the charge on Mars must be negative. We use Gauss's law to relate the electric flux to the charge causing it.

SET UP: Gauss's law is $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$. The enclosed charge is negative, so the electric flux must also be negative. The flux is $\Phi_E = EA\cos\phi = -EA$ since $\phi = 180°$ and E is the magnitude of the electric field, which is positive.

EXECUTE: **(a)** Solving Gauss's law for q, putting in the numbers, and recalling that q is negative, gives $q = \epsilon_0\Phi_E = (-3.63\times10^{16}\text{ N}\cdot\text{m}^2/\text{C})(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2) = -3.21\times10^5\text{ C}.$

(b) Use the definition of electric flux to find the electric field. The area to use is the surface area of Mars.

$$E = \frac{\Phi_E}{A} = \frac{3.63\times10^{16}\text{ N}\cdot\text{m}^2/\text{C}}{4\pi(3.39\times10^6\text{ m})^2} = 2.51\times10^2\text{ N/C}.$$

(c) The surface charge density on Mars is therefore $\sigma = \dfrac{q}{A_{\text{Mars}}} = \dfrac{-3.21\times10^5\text{ C}}{4\pi(3.39\times10^6\text{ m})^2} = -2.22\times10^{-9}\text{ C/m}^2.$

EVALUATE: Even though the charge on Mars is very large, it is spread over a large area, giving a small surface charge density.

22.17. IDENTIFY: Add the vector electric fields due to each line of charge. $E(r)$ for a line of charge is given by Example 22.6 and is directed toward a negative line of charge and away from a positive line.

SET UP: The two lines of charge are shown in Figure 22.17. $E = \dfrac{1}{2\pi\epsilon_0}\dfrac{\lambda}{r}.$

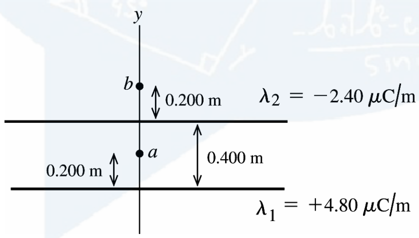


Figure 22.17

EXECUTE: **(a)** At point a, $\vec{E}_1$ and $\vec{E}_2$ are in the $+y$-direction (toward negative charge, away from positive charge).

$E_1 = (1/2\pi\epsilon_0)[(4.80\times10^{-6}\text{ C/m})/(0.200\text{ m})] = 4.314\times10^5\text{ N/C}.$

$E_2 = (1/2\pi\epsilon_0)[(2.40\times10^{-6}\text{ C/m})/(0.200\text{ m})] = 2.157\times10^5\text{ N/C}.$

$E = E_1 + E_2 = 6.47\times10^5$ N/C, in the y-direction.

(b) At point b, $\vec{E}_1$ is in the $+y$-direction and $\vec{E}_2$ is in the $-y$-direction.

$E_1 = (1/2\pi\epsilon_0)[(4.80\times10^{-6}\text{ C/m})/(0.600\text{ m})] = 1.438\times10^5\text{ N/C}.$

$E_2 = (1/2\pi\epsilon_0)[(2.40\times10^{-6}\ \text{C/m})/(0.200\ \text{m})] = 2.157\times10^5\ \text{N/C}.$

$E = E_2 - E_1 = 7.2\times10^4\ \text{N/C},$ in the $-y$-direction.

EVALUATE: At point *a* the two fields are in the same direction and the magnitudes add. At point *b* the two fields are in opposite directions and the magnitudes subtract.

22.18. **IDENTIFY:** Apply Gauss's law.

SET UP: Draw a cylindrical Gaussian surface with the line of charge as its axis. The cylinder has radius 0.400 m and is 0.0200 m long. The electric field is then 840 N/C at every point on the cylindrical surface and is directed perpendicular to the surface.

EXECUTE: $\oint \vec{E}\cdot d\vec{A} = EA_{\text{cylinder}} = E(2\pi rL) = (840\ \text{N/C})(2\pi)(0.400\ \text{m})(0.0200\ \text{m}) = 42.2\ \text{N}\cdot\text{m}^2/\text{C}.$

The field is parallel to the end caps of the cylinder, so for them $\oint \vec{E}\cdot d\vec{A} = 0.$ From Gauss's law,

$q = \epsilon_0\Phi_E = (8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(42.2\ \text{N}\cdot\text{m}^2/\text{C}) = 3.74\times10^{-10}\ \text{C}.$

EVALUATE: We could have applied the result in Example 22.6 and solved for λ. Then $q = \lambda L.$

22.19. **IDENTIFY:** The electric field inside the conductor is zero, and all of its initial charge lies on its outer surface. The introduction of charge into the cavity induces charge onto the surface of the cavity, which induces an equal but opposite charge on the outer surface of the conductor. The net charge on the outer surface of the conductor is the sum of the positive charge initially there and the additional negative charge due to the introduction of the negative charge into the cavity.

(a) SET UP: First find the initial positive charge on the outer surface of the conductor using $q_{\text{i}} = \sigma A,$ where A is the area of its outer surface. Then find the net charge on the surface after the negative charge has been introduced into the cavity. Finally, use the definition of surface charge density.

EXECUTE: The original positive charge on the outer surface is

$$q_{\text{i}} = \sigma A = \sigma(4\pi r^2) = (6.37\times10^{-6}\ \text{C/m}^2)4\pi(0.250\ \text{m})^2 = 5.00\times10^{-6}\ \text{C}.$$

After the introduction of $-0.500\ \mu\text{C}$ into the cavity, the outer charge is now

$$5.00\ \mu\text{C} - 0.500\ \mu\text{C} = 4.50\ \mu\text{C}.$$

The surface charge density is now $\sigma = \dfrac{q}{A} = \dfrac{q}{4\pi r^2} = \dfrac{4.50\times10^{-6}\ \text{C}}{4\pi(0.250\ \text{m})^2} = 5.73\times10^{-6}\ \text{C/m}^2.$

(b) SET UP: Using Gauss's law, the electric field is $E = \dfrac{\Phi_E}{A} = \dfrac{q}{\epsilon_0 A} = \dfrac{q}{\epsilon_0 4\pi r^2}.$

EXECUTE: Substituting numbers gives

$$E = \frac{4.50\times10^{-6}\ \text{C}}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi)(0.250\ \text{m})^2} = 6.47\times10^5\ \text{N/C}.$$

(c) SET UP: We use Gauss's law again to find the flux. $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}.$

EXECUTE: Substituting numbers gives

$$\Phi_E = \frac{-0.500\times10^{-6}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = -5.65\times10^4\ \text{N}\cdot\text{m}^2/\text{C}.$$

EVALUATE: The excess charge on the conductor is still $+5.00\ \mu\text{C},$ as it originally was. The introduction of the $-0.500\ \mu\text{C}$ inside the cavity merely induced equal but opposite charges (for a net of zero) on the surfaces of the conductor.

22.20. **IDENTIFY:** Apply the results of Examples 22.5, 22.6, and 22.7.

SET UP: Gauss's law can be used to show that the field outside a long conducting cylinder is the same as for a line of charge along the axis of the cylinder.

EXECUTE: **(a)** For points outside a uniform spherical charge distribution, all the charge can be considered to be concentrated at the center of the sphere. The field outside the sphere is thus inversely proportional to the square of the distance from the center. In this case,

$$E = (480\text{ N/C})\left(\frac{0.200\text{ cm}}{0.600\text{ cm}}\right)^2 = 53\text{ N/C}.$$

(b) For points outside a long cylindrically symmetrical charge distribution, the field is identical to that of a long line of charge: $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$, that is, inversely proportional to the distance from the axis of the cylinder. In this case $E = (480\text{ N/C})\left(\dfrac{0.200\text{ cm}}{0.600\text{ cm}}\right) = 160\text{ N/C}.$

(c) The field of an infinite sheet of charge is $E = \sigma/2\varepsilon_0$; i.e., it is independent of the distance from the sheet. Thus in this case $E = 480\text{ N/C}.$

EVALUATE: For each of these three distributions of charge the electric field has a different dependence on distance.

22.21. IDENTIFY: The magnitude of the electric field is constant at any given distance from the center because the charge density is uniform inside the sphere. We can use Gauss's law to relate the field to the charge causing it.

(a) SET UP: Gauss's law tells us that $EA = \dfrac{q}{\varepsilon_0}$, and the charge density is given by $\rho = \dfrac{q}{V} = \dfrac{q}{(4/3)\pi R^3}.$

EXECUTE: Solving for q and substituting numbers gives $q = EA\varepsilon_0 = E(4\pi r^2)\varepsilon_0 = (1750\text{ N/C})(4\pi)(0.500\text{ m})^2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2) = 4.866\times10^{-8}\text{ C}.$ Using the formula for charge density we get $\rho = \dfrac{q}{V} = \dfrac{q}{(4/3)\pi R^3} = \dfrac{4.866\times10^{-8}\text{ C}}{(4/3)\pi(0.355\text{ m})^3} = 2.60\times10^{-7}\text{ C/m}^3.$

(b) SET UP: Take a Gaussian surface of radius $r = 0.200\text{ m},$ concentric with the insulating sphere. The charge enclosed within this surface is $q_{\text{encl}} = \rho V = \rho\left(\dfrac{4}{3}\pi r^3\right)$, and we can treat this charge as a point-charge, using Coulomb's law $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_{\text{encl}}}{r^2}$. The charge beyond $r = 0.200\text{ m}$ makes no contribution to the electric field.

EXECUTE: First find the enclosed charge:

$$q_{\text{encl}} = \rho\left(\frac{4}{3}\pi r^3\right) = (2.60\times10^{-7}\text{ C/m}^3)\left[\frac{4}{3}\pi(0.200\text{ m})^3\right] = 8.70\times10^{-9}\text{ C}$$

Now treat this charge as a point-charge and use Coulomb's law to find the field:

$$E = (9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{8.70\times10^{-9}\text{ C}}{(0.200\text{ m})^2} = 1.96\times10^3\text{ N/C}$$

EVALUATE: Outside this sphere, it behaves like a point-charge located at its center. Inside of it, at a distance r from the center, the field is due only to the charge between the center and r.

22.22. IDENTIFY: We apply Gauss's law, taking the Gaussian surface beyond the cavity but inside the solid.

SET UP: Because of the symmetry of the charge, Gauss's law gives us $E_1 = \dfrac{q_{\text{total}}}{\varepsilon_0 A}$, where A is the surface area of a sphere of radius $R = 9.50\text{ cm}$ centered on the point-charge, and q_{total} is the total charge contained within that sphere. This charge is the sum of the $-3.00\,\mu\text{C}$ point charge at the center of the cavity plus the charge within the solid between $r = 6.50\text{ cm}$ and $R = 9.50\text{ cm}.$ The charge within the solid is $q_{\text{solid}} = \rho V = \rho[(4/3)\pi R^3 - (4/3)\pi r^3] = (4\pi/3)\rho(R^3 - r^3).$

EXECUTE: First find the charge within the solid between $r = 6.50\text{ cm}$ and $R = 9.50\text{ cm}$:

$q_{\text{solid}} = \dfrac{4\pi}{3}(7.35\times10^{-4}\text{ C/m}^3)[(0.0950\text{ m})^3 - (0.0650\text{ m})^3] = 1.794\times10^{-6}\text{ C}.$

Now find the total charge within the Gaussian surface:

$$q_{\text{total}} = q_{\text{solid}} + q_{\text{point}} = -3.00\ \mu\text{C} + 1.794\ \mu\text{C} = -1.206\ \mu\text{C}.$$

Now find the magnitude of the electric field from Gauss's law:

$$E = \frac{|q|}{\varepsilon_0 A} = \frac{|q|}{\varepsilon_0 4\pi r^2} = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.206\times10^{-6}\ \text{C})}{(0.0950\ \text{m})^2} = 1.20\times10^6\ \text{N/C}.$$

The fact that the charge is negative means that the electric field points radially inward.

EVALUATE: Because of the uniformity of the charge distribution, the charge beyond 9.50 cm does not contribute to the electric field.

22.23. **IDENTIFY:** The charged sheet exerts a force on the electron and therefore does work on it.

SET UP: The electric field is uniform so the force on the electron is constant during the displacement. The electric field due to the sheet is $E = \dfrac{\sigma}{2\varepsilon_0}$ and the magnitude of the force the sheet exerts on the electron is $F = qE$. The work the force does on the electron is $W = Fs$. In (b) we can use the work-energy theorem, $W_{\text{tot}} = \Delta K = K_2 - K_1$.

EXECUTE: **(a)** $W = Fs$, where $s = 0.250$ m. $F = Eq$, where

$E = \dfrac{\sigma}{2\varepsilon_0} = \dfrac{2.90\times10^{-12}\ \text{C/m}^2}{2(8.854\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2))} = 0.1638\ \text{N/C}.$ Therefore the force is

$F = (0.1638\ \text{N/C})(1.602\times10^{-19}\ \text{C}) = 2.624\times10^{-20}\ \text{N}.$ The work this force does is $W = Fs = 6.56\times10^{-21}$ J.

(b) Use the work-energy theorem: $W_{\text{tot}} = \Delta K = K_2 - K_1$. $K_1 = 0$. $K_2 = \frac{1}{2}mv_2^2$. So, $\frac{1}{2}mv_2^2 = W$, which

gives $v_2 = \sqrt{\dfrac{2W}{m}} = \sqrt{\dfrac{2(6.559\times10^{-21}\ \text{J})}{9.109\times10^{-31}\ \text{kg}}} = 1.2\times10^5$ m/s.

EVALUATE: If the field were not constant, we would have to integrate in (a), but we could still use the work-energy theorem in (b).

22.24. **IDENTIFY:** The charge distribution is uniform, so we can readily apply Gauss's law. Outside a spherically symmetric charge distribution, the electric field is equivalent to that of a point-charge at the center of the sphere.

SET UP: Gauss's law: $\oint \vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$, $E = k\dfrac{|q|}{r^2}$ outside the sphere.

EXECUTE: **(a)** Outside the sphere, $E = k\dfrac{|q|}{r^2}$, so $Q = Er^2/k$, which gives

$Q = (940\ \text{N/C})(0.0800\ \text{m})^2/(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) = 6.692\times10^{-10}$ C. The volume charge density is

$\rho = \dfrac{Q}{V} = \dfrac{Q}{\frac{4}{3}\pi R^3} = (6.692\times10^{-10}\ \text{C})/(4\pi/3)(0.0400\ \text{m})^3 = 2.50\times10^{-6}\ \text{C/m}^3.$

(b) Apply Gauss's law: $\oint \vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$, with the Gaussian surface being a sphere of radius $r = 0.0200$ m centered on the sphere of charge. This gives

$E(4\pi r^2) = Q_{\text{enc}}/\varepsilon_0$, where $Q_{\text{encl}} = 4/3\ \pi r^3\rho$. Solving for E and simplifying gives

$E = r\rho/3\,\varepsilon_0 = (0.0200\ \text{m})(2.50\times10^{-6}\ \text{C/m}^3)/[3(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)] = 1880\ \text{N/C}.$

EVALUATE: Outside the sphere of charge, the electric field obeys an inverse-square law, but inside the field is proportional to the distance from the center of the sphere.

22.25. **IDENTIFY:** Apply Gauss's law and conservation of charge.

SET UP: Use a Gaussian surface that lies wholly within the conducting material.

EXECUTE: **(a)** Positive charge is attracted to the inner surface of the conductor by the charge in the cavity. Its magnitude is the same as the cavity charge: $q_{\text{inner}} = +6.00 \text{ nC}$, since $E = 0$ inside a conductor and a Gaussian surface that lies wholly within the conductor must enclose zero net charge.
(b) On the outer surface the charge is a combination of the net charge on the conductor and the charge "left behind" when the $+6.00 \text{ nC}$ moved to the inner surface:

$$q_{\text{tot}} = q_{\text{inner}} + q_{\text{outer}} \Rightarrow q_{\text{outer}} = q_{\text{tot}} - q_{\text{inner}} = 5.00 \text{ nC} - 6.00 \text{ nC} = -1.00 \text{ nC}.$$

EVALUATE: The electric field outside the conductor is due to the charge on its surface.

22.26. **IDENTIFY:** If the sphere is to remain motionless, the downward force of gravity must be balanced by the upward electric force due to the sheet. The nonconducting sheet produces a uniform electric field that is perpendicular to the sheet and independent of the distance from the sheet.

SET UP: $\Sigma F_y = 0$, $E = \dfrac{\sigma}{2\epsilon_0}$ for a large nonconducting sheet, $\vec{F} = q\vec{E}$.

EXECUTE: **(a)** $\Sigma F_y = 0 : qE - mg = 0$. Solving for q and using $E = \dfrac{\sigma}{2\epsilon_0}$ gives

$$q = \frac{mg}{E} = \frac{mg}{\dfrac{\sigma}{2\epsilon_0}} = \frac{2\epsilon_0 mg}{\sigma} = 2(8.854\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(8.00\times10^{-6} \text{ kg})(9.80 \text{ m/s}^2)/(5.00\times10^{-6} \text{ C/m}^2).$$

$q = 2.78\times10^{-10}$ C.
(b) The electric field does not depend on the distance from the sheet, so the field, and therefore the charge, would be the same as in (a).
EVALUATE: If the object were to be very far from the sheet, the field would not be uniform. And if the object were extremely far away compared to the dimensions of the sheet, the sheet would resemble a point charge.

22.27. **IDENTIFY:** Apply Gauss's law to each surface.
SET UP: The field is zero within the plates. By symmetry the field is perpendicular to a plate outside the plate and can depend only on the distance from the plate. Flux into the enclosed volume is positive.
EXECUTE: S_2 and S_3 enclose no charge, so the flux is zero, and electric field outside the plates is zero.
Between the plates, S_4 shows that $-EA = -q/\epsilon_0 = -\sigma A/\epsilon_0$ and $E = \sigma/\epsilon_0$.
EVALUATE: Our result for the field between the plates agrees with the result stated in Example 22.8.

22.28. **IDENTIFY:** Close to a finite sheet the field is the same as for an infinite sheet. Very far from a finite sheet the field is that of a point charge.

SET UP: For an infinite sheet, $E = \dfrac{\sigma}{2\epsilon_0}$. For a positive point charge, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$.

EXECUTE: **(a)** At a distance of 0.100 mm from the center, the sheet appears "infinite," so

$$E \approx \frac{\sigma}{2\epsilon_0} = \frac{q}{2\epsilon_0 A} = \frac{4.50\times10^{-9} \text{ C}}{2\epsilon_0(0.800 \text{ m})^2} = 397 \text{ N/C}.$$

(b) At a distance of 100 m from the center, the sheet looks like a point, so:

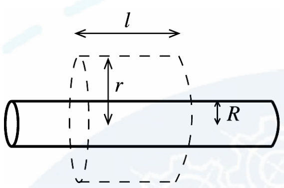
$$E \approx \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{(4.50\times10^{-9} \text{ C})}{(100 \text{ m})^2} = 4.05\times10^{-3} \text{ N/C}.$$

(c) There would be no difference if the sheet was a conductor. The charge would automatically spread out evenly over both faces, giving it half the charge density on either face as the insulator but the same electric field. Far away, they both look like points with the same charge.
EVALUATE: The sheet can be treated as infinite at points where the distance to the sheet is much less than the distance to the edge of the sheet. The sheet can be treated as a point charge at points for which the distance to the sheet is much greater than the dimensions of the sheet.

22.29. **IDENTIFY:** Apply Gauss's law to a Gaussian surface and calculate E.

(a) SET UP and EXECUTE: Consider the charge on a length l of the cylinder. This can be expressed as $q = \lambda l$. But since the surface area is $2\pi Rl$ it can also be expressed as $q = \sigma 2\pi Rl$. These two expressions must be equal, so $\lambda l = \sigma 2\pi Rl$ and $\lambda = 2\pi R\sigma$.

(b) SET UP: Apply Gauss's law to a Gaussian surface that is a cylinder of length l, radius r, and whose axis coincides with the axis of the charge distribution, as shown in Figure 22.29.



EXECUTE:

$Q_{\text{encl}} = \sigma(2\pi Rl)$

$\Phi_E = 2\pi rlE$

Figure 22.29

$\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$ gives $2\pi rlE = \dfrac{\sigma(2\pi Rl)}{\varepsilon_0}$, so $E = \dfrac{\sigma R}{\varepsilon_0 r}$.

EVALUATE: **(c)** Example 22.6 shows that the electric field of an infinite line of charge is $E = \lambda/2\pi\varepsilon_0 r$.

$\sigma = \dfrac{\lambda}{2\pi R}$, so $E = \dfrac{\sigma R}{\varepsilon_0 r} = \dfrac{R}{\varepsilon_0 r}\left(\dfrac{\lambda}{2\pi R}\right) = \dfrac{\lambda}{2\pi\varepsilon_0 r}$, the same as for an infinite line of charge that is along the axis of the cylinder.

22.30. **IDENTIFY:** The net electric field is the vector sum of the fields due to each of the four sheets of charge.

SET UP: The electric field of a large sheet of charge is $E = \sigma/2\varepsilon_0$. The field is directed away from a positive sheet and toward a negative sheet.

EXECUTE: **(a)** At A: $E_A = \dfrac{|\sigma_2|}{2\varepsilon_0} + \dfrac{|\sigma_3|}{2\varepsilon_0} + \dfrac{|\sigma_4|}{2\varepsilon_0} - \dfrac{|\sigma_1|}{2\varepsilon_0} = \dfrac{|\sigma_2| + |\sigma_3| + |\sigma_4| - |\sigma_1|}{2\varepsilon_0}$.

$$E_A = \frac{1}{2\varepsilon_0}(5\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 6\ \mu\text{C/m}^2) = 2.82 \times 10^5\ \text{N/C to the left.}$$

(b) $E_B = \dfrac{|\sigma_1|}{2\varepsilon_0} + \dfrac{|\sigma_3|}{2\varepsilon_0} + \dfrac{|\sigma_4|}{2\varepsilon_0} - \dfrac{|\sigma_2|}{2\varepsilon_0} = \dfrac{|\sigma_1| + |\sigma_3| + |\sigma_4| - |\sigma_2|}{2\varepsilon_0}$.

$$E_B = \frac{1}{2\varepsilon_0}(6\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 5\ \mu\text{C/m}^2) = 3.95 \times 10^5\ \text{N/C to the left.}$$

(c) $E_C = \dfrac{|\sigma_4|}{2\varepsilon_0} + \dfrac{|\sigma_1|}{2\varepsilon_0} - \dfrac{|\sigma_2|}{2\varepsilon_0} - \dfrac{|\sigma_3|}{2\varepsilon_0} = \dfrac{|\sigma_4| + |\sigma_1| - |\sigma_2| - |\sigma_3|}{2\varepsilon_0}$.

$$E_C = \frac{1}{2\varepsilon_0}(4\mu\text{C/m}^2 + 6\mu\text{C/m}^2 - 5\mu\text{C/m}^2 - 2\mu\text{C/m}^2) = 1.69 \times 10^5\ \text{N/C to the left.}$$

EVALUATE: The field at C is not zero. The pieces of plastic are not conductors.

22.31. **IDENTIFY:** The uniform electric field of the sheet exerts a constant force on the proton perpendicular to the sheet, and therefore does not change the parallel component of its velocity. Newton's second law allows us to calculate the proton's acceleration perpendicular to the sheet, and uniform-acceleration kinematics allows us to determine its perpendicular velocity component.

SET UP: Let $+x$ be the direction of the initial velocity and let $+y$ be the direction perpendicular to the sheet and pointing away from it. $a_x = 0$ so $v_x = v_{0x} = 9.70 \times 10^2$ m/s. The electric field due to the sheet is $E = \dfrac{\sigma}{2\varepsilon_0}$ and the magnitude of the force the sheet exerts on the proton is $F = eE$.

EXECUTE: $E = \dfrac{\sigma}{2\varepsilon_0} = \dfrac{2.34\times10^{-9}\ \text{C/m}^2}{2(8.854\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2))} = 132.1\ \text{N/C}.$ Newton's second law gives

$a_y = \dfrac{Eq}{m} = \dfrac{(132.1\ \text{N/C})(1.602\times10^{-19}\ \text{C})}{1.673\times10^{-27}\ \text{kg}} = 1.265\times10^{10}\ \text{m/s}^2.$ Kinematics gives

$v_y = v_{0y} + a_y y = (1.265\times10^{10}\ \text{m/s}^2)(5.00\times10^{-8}\ \text{s}) = 632.7\ \text{m/s}.$ The speed of the proton is the magnitude of its velocity, so $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(9.70\times10^2\ \text{m/s})^2 + (632.7\ \text{m/s})^2} = 1.16\times10^3\ \text{m/s}.$

EVALUATE: We can use the constant-acceleration kinematics equations because the uniform electric field of the sheet exerts a constant force on the proton, giving it a constant acceleration. We could *not* use this approach if the sheet were replaced with a sphere, for example.

22.32. IDENTIFY: The sheet repels the charge electrically, slowing it down and eventually stopping it at its closest approach.

SET UP: Let $+y$ be in the direction toward the sheet. The electric field due to the sheet is $E = \dfrac{\sigma}{2\varepsilon_0}$ and the magnitude of the force the sheet exerts on the object is $F = qE.$ Newton's second law, and the constant-acceleration kinematics formulas, apply to the object as it is slowing down.

EXECUTE: $E = \dfrac{\sigma}{2\varepsilon_0} = \dfrac{5.90\times10^{-8}\ \text{C/m}^2}{2[8.854\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)]} = 3.332\times10^3\ \text{N/C}.$

$a_y = -\dfrac{F}{m} = -\dfrac{Eq}{m} = -\dfrac{(3.332\times10^3\ \text{N/C})(6.50\times10^{-9}\ \text{C})}{8.20\times10^{-9}\ \text{kg}} = -2.641\times10^3\ \text{m/s}^2.$ Using $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_{0y} = \sqrt{-2a_y(y-y_0)} = \sqrt{-2(-2.64\times10^3\ \text{m/s}^2)(0.300\ \text{m})} = 39.8\ \text{m/s}.$

EVALUATE: We can use the constant-acceleration kinematics equations because the uniform electric field of the sheet exerts a constant force on the object, giving it a constant acceleration. We could *not* use this approach if the sheet were replaced with a sphere, for example.

22.33. IDENTIFY: First make a free-body diagram of the sphere. The electric force acts to the left on it since the electric field due to the sheet is horizontal. Since it hangs at rest, the sphere is in equilibrium so the forces on it add to zero, by Newton's first law. Balance horizontal and vertical force components separately.

SET UP: Call T the tension in the thread and E the electric field. Balancing horizontal forces gives $T\sin\theta = qE.$ Balancing vertical forces we get $T\cos\theta = mg.$ Combining these equations gives $\tan\theta = qE/mg,$ which means that $\theta = \arctan(qE/mg).$ The electric field for a sheet of charge is $E = \sigma/2\varepsilon_0.$

EXECUTE: Substituting the numbers gives us

$E = \dfrac{\sigma}{2\varepsilon_0} = \dfrac{2.50\times10^{-9}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 1.41\times10^2\ \text{N/C}.$ Then

$$\theta = \arctan\left[\frac{(5.00\times10^{-8}\ \text{C})(1.41\times10^2\ \text{N/C})}{(4.00\times10^{-6}\ \text{kg})(9.80\ \text{m/s}^2)}\right] = 10.2^\circ.$$

EVALUATE: Increasing the field, or decreasing the mass of the sphere, would cause the sphere to hang at a larger angle.

22.34. IDENTIFY: Use $\Phi_E = \vec{E}\cdot\vec{A}$ to calculate the flux for each surface. Use $\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$ to calculate the total enclosed charge.

SET UP: $\vec{E} = (-5.00\ \text{N/C}\cdot\text{m})x\hat{\boldsymbol{i}} + (3.00\ \text{N/C}\cdot\text{m})z\hat{\boldsymbol{k}}.$ The area of each face is $L^2,$ where $L = 0.300\ \text{m}.$

EXECUTE: **(a)** $\hat{\boldsymbol{n}}_{S_1} = -\hat{\boldsymbol{j}} \Rightarrow \Phi_1 = \vec{E}\cdot\hat{\boldsymbol{n}}_{S_1}A = 0.$

$\hat{\boldsymbol{n}}_{S_2} = +\hat{\boldsymbol{k}} \Rightarrow \Phi_2 = \vec{\boldsymbol{E}} \cdot \hat{\boldsymbol{n}}_{S_2} A = (3.00\ \text{N/C} \cdot \text{m})(0.300\ \text{m})^2 z = (0.27\ (\text{N/C}) \cdot \text{m})z.$

$\Phi_2 = (0.27\ (\text{N/C}) \cdot \text{m})(0.300\ \text{m}) = 0.081\ (\text{N/C}) \cdot \text{m}^2.$

$\hat{\boldsymbol{n}}_{S_3} = +\hat{\boldsymbol{j}} \Rightarrow \Phi_3 = \vec{\boldsymbol{E}} \cdot \hat{\boldsymbol{n}}_{S_3} A = 0.$

$\hat{\boldsymbol{n}}_{S_4} = -\hat{\boldsymbol{k}} \Rightarrow \Phi_4 = \vec{\boldsymbol{E}} \cdot \hat{\boldsymbol{n}}_{S_4} A = -(0.27\ (\text{N/C}) \cdot \text{m})z = 0$ (since $z = 0$).

$\hat{\boldsymbol{n}}_{S_5} = +\hat{\boldsymbol{i}} \Rightarrow \Phi_5 = \vec{\boldsymbol{E}} \cdot \hat{\boldsymbol{n}}_{S_5} A = (-5.00\ \text{N/C} \cdot \text{m})(0.300\ \text{m})^2 x = -(0.45\ (\text{N/C}) \cdot \text{m})x.$

$\Phi_5 = -(0.45\ (\text{N/C}) \cdot \text{m})(0.300\ \text{m}) = -(0.135\ (\text{N/C}) \cdot \text{m}^2).$

$\hat{\boldsymbol{n}}_{S_6} = -\hat{\boldsymbol{i}} \Rightarrow \Phi_6 = \vec{\boldsymbol{E}} \cdot \hat{\boldsymbol{n}}_{S_6} A = +(0.45\ (\text{N/C}) \cdot \text{m})x = 0$ (since $x = 0$).

(b) Total flux: $\Phi = \Phi_2 + \Phi_5 = (0.081 - 0.135)(\text{N/C}) \cdot \text{m}^2 = -0.054\ \text{N} \cdot \text{m}^2/\text{C}$. Therefore,

$q = \varepsilon_0 \Phi = -4.78 \times 10^{-13}$ C.

EVALUATE: Flux is positive when $\vec{\boldsymbol{E}}$ is directed out of the volume and negative when it is directed into the volume.

22.35. IDENTIFY: Use $\Phi_E = \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{A}}$ to calculate the flux through each surface and use Gauss's law to relate the net flux to the enclosed charge.

SET UP: Flux into the enclosed volume is negative and flux out of the volume is positive.

EXECUTE: **(a)** $\Phi = EA = (125\ \text{N/C})(6.0\ \text{m}^2) = 750\ \text{N} \cdot \text{m}^2/\text{C}$.

(b) Since the field is parallel to the surface, $\Phi = 0$.

(c) Choose the Gaussian surface to equal the volume's surface. Then $750\ \text{N} \cdot \text{m}^2/\text{C} - EA = q/\varepsilon_0$ and

$E = \dfrac{1}{6.0\ \text{m}^2}(2.40 \times 10^{-8}\ \text{C}/\varepsilon_0 + 750\ \text{N} \cdot \text{m}^2/\text{C}) = 577\ \text{N/C}$, in the positive x-direction. Since $q < 0$ we must have some net flux flowing *in* so the flux is $-|EA|$ on second face.

EVALUATE: **(d)** $q < 0$ but we have E pointing *away* from face I. This is due to an external field that does not affect the flux but affects the value of E. The electric field is produced by charges both inside and outside the slab.

22.36. IDENTIFY: The electric field is perpendicular to the square but varies in magnitude over the surface of the square, so we will need to integrate to find the flux.

SET UP and EXECUTE: $\vec{\boldsymbol{E}} = (964\ \text{N/C} \cdot \text{m})x\hat{\boldsymbol{k}}$. Consider a thin rectangular slice parallel to the y-axis and at coordinate x with width dx. $d\vec{\boldsymbol{A}} = (Ldx)\hat{\boldsymbol{k}}$. $d\Phi_E = \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = (964\ \text{N/C} \cdot \text{m})Lxdx$.

$$\Phi_E = \int_0^L d\Phi_E = (964\ \text{N/C} \cdot \text{m})L\int_0^L xdx = (964\ \text{N/C} \cdot \text{m})L\left(\frac{L^2}{2}\right).$$

$$\Phi_E = \frac{1}{2}(964\ \text{N/C} \cdot \text{m})(0.350\ \text{m})^3 = 20.7\ \text{N} \cdot \text{m}^2/\text{C}.$$

EVALUATE: To set up the integral, we take rectangular slices parallel to the y-axis (and not the x-axis) because the electric field is constant over such a slice. It would not be constant over a slice parallel to the x-axis.

22.37. IDENTIFY: Find the net flux through the parallelepiped surface and then use that in Gauss's law to find the net charge within. Flux out of the surface is positive and flux into the surface is negative.

(a) SET UP: $\vec{\boldsymbol{E}}_1$ gives flux out of the surface. See Figure 22.37a.

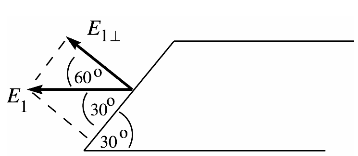


Figure 22.37a

EXECUTE: $\Phi_1 = +E_{1\perp}A$.

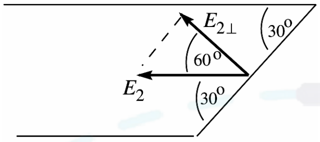
$A = (0.0600\ \text{m})(0.0500\ \text{m}) = 3.00 \times 10^{-3}\ \text{m}^2.$

$E_{1\perp} = E_1 \cos 60° = (2.50 \times 10^4\ \text{N/C})\cos 60°.$

$E_{1\perp} = 1.25 \times 10^4\ \text{N/C}.$

$\Phi_{E_1} = +E_{1\perp}A = +(1.25 \times 10^4\ \text{N/C})(3.00 \times 10^{-3}\ \text{m}^2) = 37.5\ \text{N} \cdot \text{m}^2/\text{C}.$

SET UP: $\vec{E}_2$ gives flux into the surface. See Figure 22.37b.



EXECUTE: $\Phi_2 = -E_{2\perp}A.$

$A = (0.0600 \text{ m})(0.0500 \text{ m}) = 3.00\times10^{-3} \text{ m}^2.$

$E_{2\perp} = E_2 \cos 60° = (7.00\times10^4 \text{ N/C})\cos 60°.$

$E_{2\perp} = 3.50\times10^4 \text{ N/C}.$

Figure 22.37b

$\Phi_{E_2} = -E_{2\perp}A = -(3.50\times10^4 \text{ N/C})(3.00\times10^{-3} \text{ m}^2) = -105.0 \text{ N}\cdot\text{m}^2/\text{C}.$

The net flux is $\Phi_E = \Phi_{E_1} + \Phi_{E_2} = +37.5 \text{ N}\cdot\text{m}^2/\text{C} - 105.0 \text{ N}\cdot\text{m}^2/\text{C} = -67.5 \text{ N}\cdot\text{m}^2/\text{C}.$

The net flux is negative (inward), so the net charge enclosed is negative.

Apply Gauss's law: $\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$

$Q_{\text{encl}} = \Phi_E\varepsilon_0 = (-67.5 \text{ N}\cdot\text{m}^2/\text{C})(8.854\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2) = -5.98\times10^{-10} \text{ C}.$

EVALUATE: **(b)** If there were no charge within the parallelepiped the net flux would be zero. This is not the case, so there is charge inside. The electric field lines that pass out through the surface of the parallelepiped must terminate on charges, so there also must be charges outside the parallelepiped.

22.38. **IDENTIFY:** The α particle feels no force where the net electric field due to the two distributions of charge is zero.

SET UP: The fields can cancel only in the regions *A* and *B* shown in Figure 22.38, because only in these two regions are the two fields in opposite directions.

EXECUTE: $E_{\text{line}} = E_{\text{sheet}}$ gives $\dfrac{\lambda}{2\pi\varepsilon_0 r} = \dfrac{\sigma}{2\varepsilon_0}$ and $r = \lambda/\pi\sigma = \dfrac{50\ \mu\text{C/m}}{\pi(100\ \mu\text{C/m}^2)} = 0.16 \text{ m} = 16 \text{ cm}.$

The fields cancel 16 cm from the line in regions *A* and *B*.

EVALUATE: The result is independent of the distance between the line and the sheet. The electric field of an infinite sheet of charge is uniform, independent of the distance from the sheet.

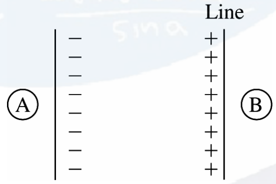
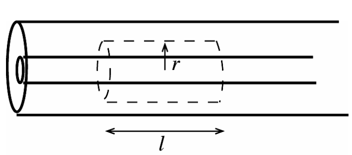


Figure 22.38

22.39. **(a) IDENTIFY:** Apply Gauss's law to a Gaussian cylinder of length *l* and radius *r*, where $a < r < b$, and calculate *E* on the surface of the cylinder.

SET UP: The Gaussian surface is sketched in Figure 22.39a.



EXECUTE: $\Phi_E = E(2\pi rl)$

$Q_{\text{encl}} = \lambda l$ (the charge on the length *l* of the inner conductor that is inside the Gaussian surface).

Figure 22.39a

$\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$ gives $E(2\pi rl) = \dfrac{\lambda l}{\varepsilon_0}$.

$E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$. The enclosed charge is positive so the direction of $\vec{E}$ is radially outward.

(b) IDENTIFY and **SET UP:** Apply Gauss's law to a Gaussian cylinder of length l and radius r, where $r > c$, as shown in Figure 22.39b.

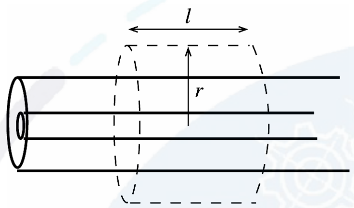


Figure 22.39b

EXECUTE: $\Phi_E = E(2\pi rl)$.

$Q_{\text{encl}} = \lambda l$ (the charge on the length l of the inner conductor that is inside the Gaussian surface; the outer conductor carries no net charge).

$\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$ gives $E(2\pi rl) = \dfrac{\lambda l}{\varepsilon_0}$.

$E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$. The enclosed charge is positive so the direction of $\vec{E}$ is radially outward.

(c) IDENTIFY and **EXECUTE:** $E = 0$ within a conductor. Thus $E = 0$ for $r < a$;

$E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$ for $a < r < b; E = 0$ for $b < r < c$;

$E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$ for $r > c$. The graph of E versus r is sketched in Figure 22.39c.

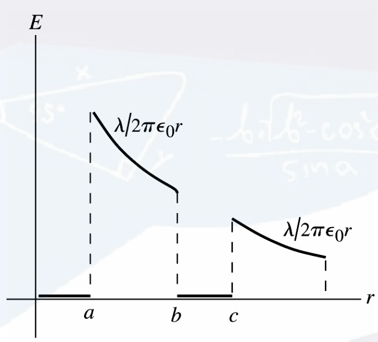


Figure 22.39c

EVALUATE: Inside either conductor $E = 0$. Between the conductors and outside both conductors the electric field is the same as for a line of charge with linear charge density λ lying along the axis of the inner conductor.

(d) IDENTIFY and **SET UP:** <u>inner surface:</u> Apply Gauss's law to a Gaussian cylinder with radius r, where $b < r < c$. We know E on this surface; calculate Q_{encl}.

EXECUTE: This surface lies within the conductor of the outer cylinder, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses charge λl on the inner conductor, so it must enclose charge $-\lambda l$ on the inner surface of the outer conductor. The charge per unit length on the inner surface of the outer cylinder is $-\lambda$.

<u>outer surface:</u> The outer cylinder carries no net charge. So if there is charge per unit length $-\lambda$ on its inner surface there must be charge per unit length $+\lambda$ on the outer surface.

EVALUATE: The electric field lines between the conductors originate on the surface charge on the outer surface of the inner conductor and terminate on the surface charges on the inner surface of the outer conductor.

These surface charges are equal in magnitude (per unit length) and opposite in sign. The electric field lines outside the outer conductor originate from the surface charge on the outer surface of the outer conductor.

22.40. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a cylinder of radius r, length l and that has the line of charge along its axis. The charge on a length l of the line of charge or of the tube is $q = \alpha l$.

EXECUTE: **(a)** (i) For $r < a$, Gauss's law gives $E(2\pi rl) = \dfrac{Q_{\text{encl}}}{\varepsilon_0} = \dfrac{\alpha l}{\varepsilon_0}$ and $E = \dfrac{\alpha}{2\pi\varepsilon_0 r}$.

(ii) The electric field is zero because these points are within the conducting material.

(iii) For $r > b$, Gauss's law gives $E(2\pi rl) = \dfrac{Q_{\text{encl}}}{\varepsilon_0} = \dfrac{2\alpha l}{\varepsilon_0}$ and $E = \dfrac{\alpha}{\pi\varepsilon_0 r}$.

The graph of E versus r is sketched in Figure 22.40.

(b) (i) The Gaussian cylinder with radius r, for $a < r < b$, must enclose zero net charge, so the charge per unit length on the inner surface is $-\alpha$. (ii) Since the net charge per length for the tube is $+\alpha$ and there is $-\alpha$ on the inner surface, the charge per unit length on the outer surface must be $+2\alpha$.

EVALUATE: For $r > b$ the electric field is due to the charge on the outer surface of the tube.

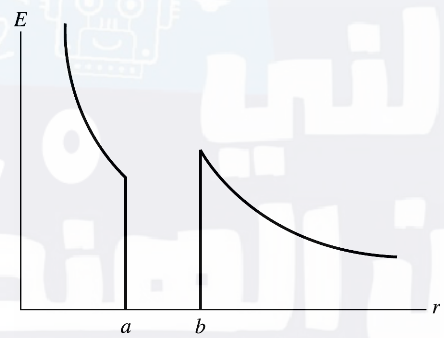


Figure 22.40

22.41. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a cylinder of radius r and length l, and that is coaxial with the cylindrical charge distributions. The volume of the Gaussian cylinder is $\pi r^2 l$ and the area of its curved surface is $2\pi rl$. The charge on a length l of the charge distribution is $q = \lambda l$, where $\lambda = \rho\pi R^2$.

EXECUTE: **(a)** For $r < R$, $Q_{\text{encl}} = \rho\pi r^2 l$ and Gauss's law gives $E(2\pi rl) = \dfrac{Q_{\text{encl}}}{\varepsilon_0} = \dfrac{\rho\pi r^2 l}{\varepsilon_0}$ and $E = \dfrac{\rho r}{2\varepsilon_0}$, radially outward.

(b) For $r > R$, $Q_{\text{encl}} = \lambda l = \rho\pi R^2 l$ and Gauss's law gives $E(2\pi rl) = \dfrac{Q_{\text{encl}}}{\varepsilon_0} = \dfrac{\rho\pi R^2 l}{\varepsilon_0}$ and

$$E = \frac{\rho R^2}{2\varepsilon_0 r} = \frac{\lambda}{2\pi\varepsilon_0 r}, \text{ radially outward.}$$

(c) At $r = R$, the electric field for *both* regions is $E = \dfrac{\rho R}{2\varepsilon_0}$, so they are consistent.

(d) The graph of E versus r is sketched in Figure 22.41 (next page).

EVALUATE: For $r > R$ the field is the same as for a line of charge along the axis of the cylinder.

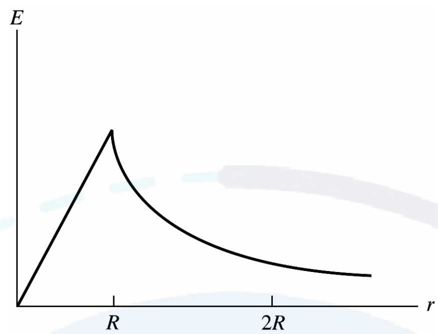


Figure 22.41

22.42. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the conducting spheres.

EXECUTE: **(a)** For $r < a, E = 0,$ since these points are within the conducting material.

For $a < r < b, E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2},$ since there is $+q$ inside a radius r.

For $b < r < c, E = 0,$ since these points are within the conducting material.

For $r > c, E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2},$ since again the total charge enclosed is $+q$.

(b) The graph of E versus r is sketched in Figure 22.42a.

(c) Since the Gaussian sphere of radius r, for $b < r < c,$ must enclose zero net charge, the charge on the inner shell surface is $-q$.

(d) Since the hollow sphere has no net charge and has charge $-q$ on its inner surface, the charge on the outer shell surface is $+q$.

(e) The field lines are sketched in Figure 22.42b. Where the field is nonzero, it is radially outward.

EVALUATE: The net charge on the inner solid conducting sphere is on the surface of that sphere. The presence of the hollow sphere does not affect the electric field in the region $r < b$.

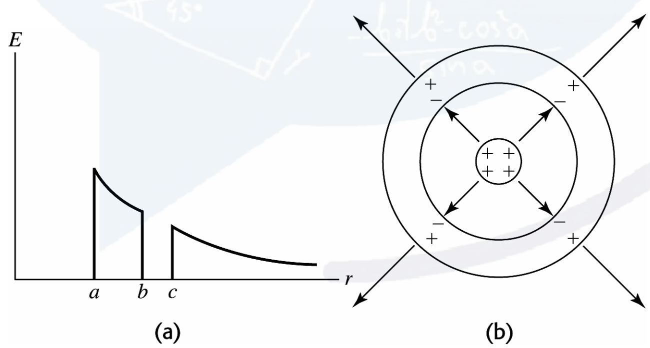


Figure 22.42

22.43. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the charge distributions.

EXECUTE: **(a)** For $r < R, E = 0,$ since these points are within the conducting material. For $R < r < 2R,$ $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r^2},$ since the charge enclosed is Q. The field is radially outward. For $r > 2R,$ $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{2Q}{r^2}$ since the charge enclosed is $2Q$. The field is radially outward.

(b) The graph of E versus r is sketched in Figure 22.43.

EVALUATE: For $r < 2R$ the electric field is unaffected by the presence of the charged shell.

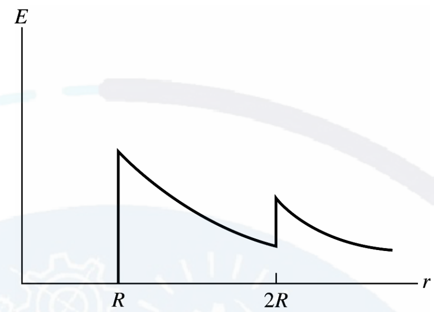


Figure 22.43

22.44. **IDENTIFY:** Apply Gauss's law and conservation of charge.

SET UP: Use a Gaussian surface that is a sphere of radius r and that has the point charge at its center.

EXECUTE: **(a)** For $r < a$, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r^2}$, radially outward, since the charge enclosed is Q, the charge of the point charge. For $a < r < b$, $E = 0$ since these points are within the conducting material. For $r > b$, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{2Q}{r^2}$, radially inward, since the total enclosed charge is $-2Q$.

(b) Since a Gaussian surface with radius r, for $a < r < b$, must enclose zero net charge because $E = 0$ inside the conductor, the total charge on the inner surface is $-Q$ and the surface charge density on the inner surface is $\sigma = -\dfrac{Q}{4\pi a^2}$.

(c) Since the net charge on the shell is $-3Q$ and there is $-Q$ on the inner surface, there must be $-2Q$ on the outer surface. The surface charge density on the outer surface is $\sigma = -\dfrac{2Q}{4\pi b^2}$.

(d) The field lines and the locations of the charges are sketched in Figure 22.44a.

(e) The graph of E versus r is sketched in Figure 22.44b.

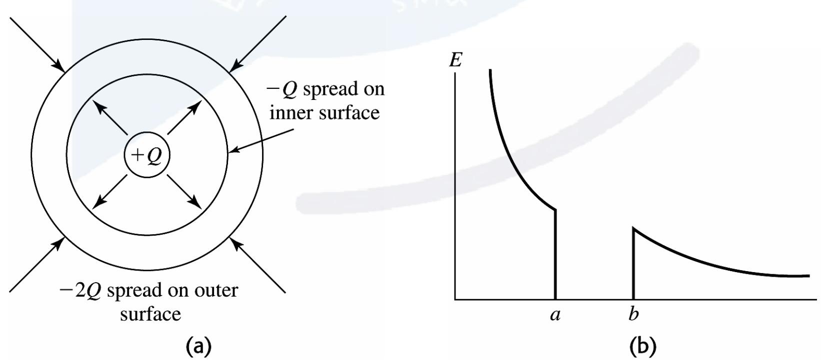
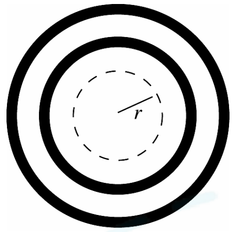


Figure 22.44

EVALUATE: For $r < a$ the electric field is due solely to the point charge Q. For $r > b$ the electric field is due to the charge $-2Q$ that is on the outer surface of the shell.

22.45. **IDENTIFY:** Apply Gauss's law to a spherical Gaussian surface with radius r. Calculate the electric field at the surface of the Gaussian sphere.

(a) SET UP: (i) $r < a$: The Gaussian surface is sketched in Figure 22.45a (next page).



EXECUTE: $\Phi_E = EA = E(4\pi r^2)$.

$Q_{\text{encl}} = 0$; no charge is enclosed.

$\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$ says

$E(4\pi r^2) = 0$ and $E = 0$.

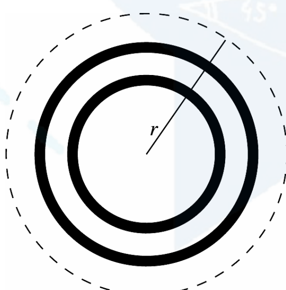
Figure 22.45a

(ii) $a < r < b$: Points in this region are in the conductor of the small shell, so $E = 0$.
(iii) **SET UP:** $b < r < c$: The Gaussian surface is sketched in Figure 22.45b.
Apply Gauss's law to a spherical Gaussian surface with radius $b < r < c$.

EXECUTE: $\Phi_E = EA = E(4\pi r^2)$.
The Gaussian surface encloses all of the small shell and none of the large shell, so $Q_{\text{encl}} = +2q$.

Figure 22.45b

$\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$ gives $E(4\pi r^2) = \dfrac{2q}{\varepsilon_0}$ so $E = \dfrac{2q}{4\pi\varepsilon_0 r^2}$. Since the enclosed charge is positive the electric field is radially outward.
(iv) $c < r < d$: Points in this region are in the conductor of the large shell, so $E = 0$.
(v) **SET UP:** $r > d$: Apply Gauss's law to a spherical Gaussian surface with radius $r > d$, as shown in Figure 22.45c.



EXECUTE: $\Phi_E = EA = E(4\pi r^2)$.
The Gaussian surface encloses all of the small shell and all of the large shell, so $Q_{\text{encl}} = +2q + 4q = 6q$.

Figure 22.45c

$\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$ gives $E(4\pi r^2) = \dfrac{6q}{\varepsilon_0}$.

$E = \dfrac{6q}{4\pi\varepsilon_0 r^2}$. Since the enclosed charge is positive the electric field is radially outward.

The graph of E versus r is sketched in Figure 22.45d.

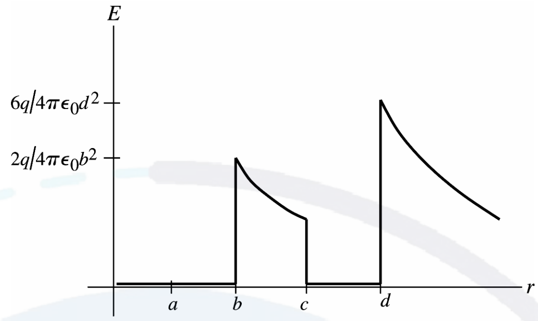


Figure 22.45d

(b) IDENTIFY and **SET UP:** Apply Gauss's law to a sphere that lies outside the surface of the shell for which we want to find the surface charge.

EXECUTE: (i) charge on inner surface of the small shell: Apply Gauss's law to a spherical Gaussian surface with radius $a < r < b$. This surface lies within the conductor of the small shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$, so there is zero charge on the inner surface of the small shell.

(ii) charge on outer surface of the small shell: The total charge on the small shell is $+2q$. We found in part (i) that there is zero charge on the inner surface of the shell, so all $+2q$ must reside on the outer surface.

(iii) charge on inner surface of large shell: Apply Gauss's law to a spherical Gaussian surface with radius $c < r < d$. The surface lies within the conductor of the large shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses the $+2q$ on the small shell so there must be charge $-2q$ on the inner surface of the large shell to make the total enclosed charge zero.

(iv) charge on outer surface of large shell: The total charge on the large shell is $+4q$. We showed in part (iii) that the charge on the inner surface is $-2q$, so there must be $+6q$ on the outer surface.

EVALUATE: The electric field lines for $b < r < c$ originate from the surface charge on the outer surface of the inner shell and all terminate on the surface charge on the inner surface of the outer shell. These surface charges have equal magnitude and opposite sign. The electric field lines for $r > d$ originate from the surface charge on the outer surface of the outer sphere.

22.46. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the charged shells.

EXECUTE: **(a)** (i) For $r < a$, $E = 0$, since the charge enclosed is zero. (ii) For $a < r < b$, $E = 0$, since the points are within the conducting material. (iii) For $b < r < c$, $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2q}{r^2}$, outward, since the charge enclosed is $+2q$. (iv) For $c < r < d$, $E = 0$, since the points are within the conducting material. (v) For $r > d$, $E = 0$, since the net charge enclosed is zero. The graph of E versus r is sketched in Figure 22.46 (next page).

(b) (i) small shell inner surface: Since a Gaussian surface with radius r, for $a < r < b$, must enclose zero net charge, the charge on this surface is zero. (ii) small shell outer surface: $+2q$. (iii) large shell inner surface: Since a Gaussian surface with radius r, for $c < r < d$, must enclose zero net charge, the charge on this surface is $-2q$. (iv) large shell outer surface: Since there is $-2q$ on the inner surface and the total charge on this conductor is $-2q$, the charge on this surface is zero.

EVALUATE: The outer shell has no effect on the electric field for $r < c$. For $r > d$ the electric field is due only to the charge on the outer surface of the larger shell.

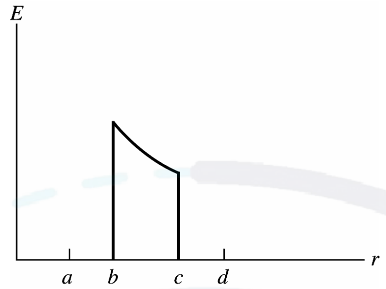
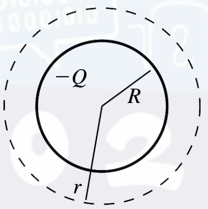


Figure 22.46

22.47. **IDENTIFY:** Use Gauss's law to find the electric field $\vec{E}$ produced by the shell for $r < R$ and $r > R$ and then use $\vec{F} = q\vec{E}$ to find the force the shell exerts on the point charge.

(a) SET UP: Apply Gauss's law to a spherical Gaussian surface that has radius $r > R$ and that is concentric with the shell, as sketched in Figure 22.47a.



EXECUTE: $\Phi_E = -E(4\pi r^2)$.

$Q_{\text{encl}} = -Q$.

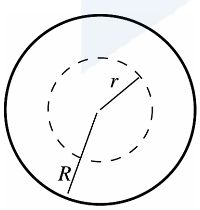
Figure 22.47a

$\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$ gives $-E(4\pi r^2) = \dfrac{-Q}{\varepsilon_0}$.

The magnitude of the field is $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ and it is directed toward the center of the shell. Then

$F = qE = \dfrac{qQ}{4\pi\varepsilon_0 r^2}$, directed toward the center of the shell. (Since q is positive, $\vec{E}$ and $\vec{F}$ are in the same direction.)

(b) SET UP: Apply Gauss's law to a spherical Gaussian surface that has radius $r < R$ and that is concentric with the shell, as sketched in Figure 22.47b.



EXECUTE: $\Phi_E = E(4\pi r^2)$.

$Q_{\text{encl}} = 0$.

Figure 22.47b

$\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$ gives $E(4\pi r^2) = 0$.

Then $E = 0$ so $F = 0$.

EVALUATE: Outside the shell the electric field and the force it exerts is the same as for a point charge $-Q$ located at the center of the shell. Inside the shell $E = 0$ and there is no force.

22.48. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the sphere and shell. The volume of the insulating shell is $V = \frac{4}{3}\pi[(2R)^3 - R^3] = \frac{28\pi}{3}R^3.$

EXECUTE: **(a)** Zero net charge requires that $-Q = \frac{28\pi\rho R^3}{3}$, so $\rho = -\frac{3Q}{28\pi R^3}.$

(b) For $r < R, E = 0$ since this region is within the conducting sphere. For $r > 2R, E = 0,$ since the net charge enclosed by the Gaussian surface with this radius is zero. For $R < r < 2R,$ Gauss's law gives $E(4\pi r^2) = \frac{Q}{\epsilon_0} + \frac{4\pi\rho}{3\epsilon_0}(r^3 - R^3)$ and $E = \frac{Q}{4\pi\epsilon_0 r^2} + \frac{\rho}{3\epsilon_0 r^2}(r^3 - R^3).$ Substituting ρ from part (a) gives $E = \frac{2}{7\pi\epsilon_0}\frac{Q}{r^2} - \frac{Qr}{28\pi\epsilon_0 R^3}.$ The net enclosed charge for each r in this range is positive and the electric field is outward.

(c) The graph is sketched in Figure 22.48. We see a discontinuity in going from the conducting sphere to the insulator due to the thin surface charge of the conducting sphere. But we see a smooth transition from the uniform insulator to the surrounding space.

EVALUATE: The expression for E within the insulator gives $E = 0$ at $r = 2R.$

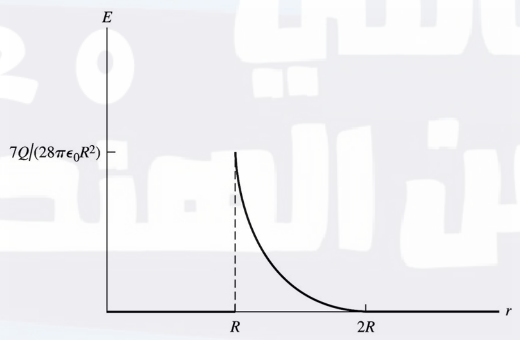


Figure 22.48

22.49. **IDENTIFY:** We apply Gauss's law in (a) and take a spherical Gaussian surface because of the spherical symmetry of the charge distribution. In (b), the net field is the vector sum of the field due to q and the field due to the sphere.

(a) SET UP: $\rho(r) = \frac{\alpha}{r}$, $dV = 4\pi r^2 dr$, and $Q = \int_a^r \rho(r')dV.$

EXECUTE: For a Gaussian sphere of radius r, $Q_{\text{encl}} = \int_a^r \rho(r')dV = 4\pi\alpha\int_a^r r'dr' = 4\pi\alpha\frac{1}{2}(r^2 - a^2).$ Gauss's law says that $E(4\pi r^2) = \frac{2\pi\alpha(r^2 - a^2)}{\epsilon_0}$, which gives $E = \frac{\alpha}{2\epsilon_0}\left(1 - \frac{a^2}{r^2}\right).$

(b) SET UP and EXECUTE: The electric field of the point charge is $E_q = \frac{q}{4\pi\epsilon_0 r^2}.$ The total electric field is $E_{\text{total}} = \frac{\alpha}{2\epsilon_0} - \frac{\alpha}{2\epsilon_0}\frac{a^2}{r^2} + \frac{q}{4\pi\epsilon_0 r^2}.$ For E_{total} to be constant, $-\frac{\alpha a^2}{2\epsilon_0} + \frac{q}{4\pi\epsilon_0} = 0$ and $q = 2\pi\alpha a^2.$ The constant electric field is $\frac{\alpha}{2\epsilon_0}.$

EVALUATE: The net field is constant, but not zero.

22.50. **IDENTIFY:** Example 22.9 gives the expression for the electric field both inside and outside a uniformly charged sphere. Use $\vec{F} = -e\vec{E}$ to calculate the force on the electron.

SET UP: The sphere has charge $Q = +e$.

EXECUTE: **(a)** Only at $r = 0$ is $E = 0$ for the uniformly charged sphere.

(b) At points inside the sphere, $E_r = \dfrac{er}{4\pi\epsilon_0 R^3}$. The field is radially outward. $F_r = -eE = -\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2 r}{R^3}$. The minus sign denotes that F_r is radially inward. For simple harmonic motion, $F_r = -kr = -m\omega^2 r$, where $\omega = \sqrt{k/m} = 2\pi f$. $F_r = -m\omega^2 r = -\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2 r}{R^3}$ so $\omega = \sqrt{\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{mR^3}}$ and $f = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{mR^3}}$.

(c) If $f = 4.57 \times 10^{14}\ \text{Hz} = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{mR^3}}$ then

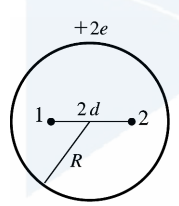
$R = \sqrt[3]{\dfrac{1}{4\pi\epsilon_0}\dfrac{(1.60\times 10^{-19}\ \text{C})^2}{4\pi^2(9.11\times 10^{-31}\ \text{kg})(4.57\times 10^{14}\ \text{Hz})^2}} = 3.13\times 10^{-10}\ \text{m}$. The atom radius in this model is the correct order of magnitude.

(d) If $r > R$, $E_r = \dfrac{e}{4\pi\epsilon_0 r^2}$ and $F_r = -\dfrac{e^2}{4\pi\epsilon_0 r^2}$. The electron would still oscillate because the force is directed toward the equilibrium position at $r = 0$. But the motion would not be simple harmonic, since F_r is proportional to $1/r^2$ and simple harmonic motion requires that the restoring force be proportional to the displacement from equilibrium.

EVALUATE: As long as the initial displacement is less than R the frequency of the motion is independent of the initial displacement.

22.51. **IDENTIFY:** There is a force on each electron due to the other electron and a force due to the sphere of charge. Use Coulomb's law for the force between the electrons. Example 22.9 gives E inside a uniform sphere and $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1 q_2|}{r^2}$ gives the force.

SET UP: The positions of the electrons are sketched in Figure 22.51a.



If the electrons are in equilibrium the net force on each one is zero.

Figure 22.51a

EXECUTE: Consider the forces on electron 2. There is a repulsive force F_1 due to the other electron, electron 1.

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{(2d)^2}$$

The electric field inside the uniform distribution of positive charge is $E = \dfrac{Qr}{4\pi\epsilon_0 R^3}$ (Example 22.9), where $Q = +2e$. At the position of electron 2, $r = d$. The force F_{cd} exerted by the positive charge distribution is $F_{cd} = eE = \dfrac{e(2e)d}{4\pi\epsilon_0 R^3}$ and is attractive.

The force diagram for electron 2 is given in Figure 22.51b.

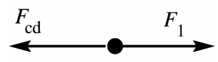


Figure 22.51b

Net force equals zero implies $F_1 = F_{cd}$ and $\dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{4d^2} = \dfrac{2e^2 d}{4\pi\varepsilon_0 R^3}$.

Thus $(1/4d^2) = 2d/R^3$, so $d^3 = R^3/8$ and $d = R/2$.

EVALUATE: The electric field of the sphere is radially outward; it is zero at the center of the sphere and increases with distance from the center. The force this field exerts on one of the electrons is radially inward and increases as the electron is farther from the center. The force from the other electron is radially outward, is infinite when $d = 0$ and decreases as d increases. It is reasonable therefore for there to be a value of d for which these forces balance.

22.52. IDENTIFY: The method of Example 22.9 shows that the electric field outside the sphere is the same as for a point charge of the same charge located at the center of the sphere.

SET UP: The charge of an electron has magnitude $e = 1.60\times10^{-19}$ C.

EXECUTE: **(a)** $E = k\dfrac{|q|}{r^2}$. For $r = R = 0.150$ m, $E = 1390$ N/C so

$|q| = \dfrac{Er^2}{k} = \dfrac{(1390 \text{ N/C})(0.150 \text{ m})^2}{8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} = 3.479\times10^{-9}$ C. The number of excess electrons is

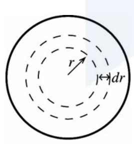
$\dfrac{3.479\times10^{-9} \text{ C}}{1.60\times10^{-19} \text{ C/electron}} = 2.17\times10^{10}$ electrons.

(b) $r = R + 0.100 \text{ m} = 0.250$ m. $E = k\dfrac{|q|}{r^2} = (8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{3.479\times10^{-9} \text{ C}}{(0.250 \text{ m})^2} = 5.00\times10^2$ N/C.

EVALUATE: The magnitude of the electric field decreases according to the square of the distance from the center of the sphere.

22.53. (a) IDENTIFY: The charge density varies with r inside the spherical volume. Divide the volume up into thin concentric shells, of radius r and thickness dr. Find the charge dq in each shell and integrate to find the total charge.

SET UP: $\rho(r) = \rho_0(1 - r/R)$ for $r \le R$ where $\rho_0 = 3Q/\pi R^3$. $\rho(r) = 0$ for $r \ge R$. The thin shell is sketched in Figure 22.53a.



EXECUTE: The volume of such a shell is $dV = 4\pi r^2 dr$.
The charge contained within the shell is $dq = \rho(r)dV = 4\pi r^2 \rho_0(1 - r/R)dr$.

Figure 22.53a

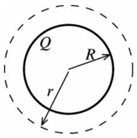
The total charge Q_{tot} in the charge distribution is obtained by integrating dq over all such shells into which the sphere can be subdivided:

$$Q_{tot} = \int dq = \int_0^R 4\pi r^2 \rho_0(1 - r/R)dr = 4\pi\rho_0 \int_0^R (r^2 - r^3/R)dr$$

$Q_{tot} = 4\pi\rho_0\left[\dfrac{r^3}{3} - \dfrac{r^4}{4R}\right]_0^R = 4\pi\rho_0\left(\dfrac{R^3}{3} - \dfrac{R^4}{4R}\right) = 4\pi\rho_0(R^3/12) = 4\pi(3Q/\pi R^3)(R^3/12) = Q$, as was to be shown.

(b) IDENTIFY: Apply Gauss's law to a spherical surface of radius r, where $r > R$.

SET UP: The Gaussian surface is shown in Figure 22.53b.



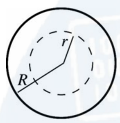
EXECUTE: $\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$.

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}.$$

Figure 22.53b

$E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$; same as for point charge of charge Q.

(c) IDENTIFY: Apply Gauss's law to a spherical surface of radius r, where $r < R$.
SET UP: The Gaussian surface is shown in Figure 22.53c.



EXECUTE: $\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$.

$$\Phi_E = E(4\pi r^2).$$

Figure 22.53c

To calculate the enclosed charge Q_{encl} use the same technique as in part (a), except integrate dq out to r rather than R. (We want the charge that is inside radius r.)

$$Q_{\text{encl}} = \int_0^r 4\pi r'^2 \rho_0 \left(1 - \frac{r'}{R}\right) dr' = 4\pi\rho_0 \int_0^r \left(r'^2 - \frac{r'^3}{R}\right) dr'.$$

$$Q_{\text{encl}} = 4\pi\rho_0 \left[\frac{r'^3}{3} - \frac{r'^4}{4R}\right]_0^r = 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^4}{4R}\right) = 4\pi\rho_0 r^3 \left(\frac{1}{3} - \frac{r}{4R}\right).$$

$$\rho_0 = \frac{3Q}{\pi R^3} \text{ so } Q_{\text{encl}} = 12Q\frac{r^3}{R^3}\left(\frac{1}{3} - \frac{r}{4R}\right) = Q\left(\frac{r^3}{R^3}\right)\left(4 - 3\frac{r}{R}\right).$$

Thus Gauss's law gives $E(4\pi r^2) = \dfrac{Q}{\varepsilon_0}\left(\dfrac{r^3}{R^3}\right)\left(4 - 3\dfrac{r}{R}\right)$.

$$E = \frac{Qr}{4\pi\varepsilon_0 R^3}\left(4 - \frac{3r}{R}\right), r \le R.$$

(d) The graph of E versus r is sketched in Figure 22.53d.

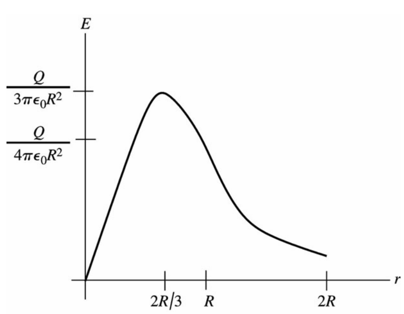


Figure 22.53d

(e) Where the electric field is a maximum, $\frac{dE}{dr} = 0$. Thus $\frac{d}{dr}\left(4r - \frac{3r^2}{R}\right) = 0$ so $4 - 6r/R = 0$ and $r = 2R/3$.

At this value of r, $E = \frac{Q}{4\pi\varepsilon_0 R^3}\left(\frac{2R}{3}\right)\left(4 - \frac{3}{R}\frac{2R}{3}\right) = \frac{Q}{3\pi\varepsilon_0 R^2}$.

EVALUATE: Our expressions for $E(r)$ for $r < R$ and for $r > R$ agree at $r = R$. The results of part (e) for the value of r where $E(r)$ is a maximum agrees with the graph in part (d).

22.54. IDENTIFY: Use Gauss's law to find the electric field both inside and outside the slab.

SET UP: Use a Gaussian surface that has one face of area A in the yz plane at $x = 0$, and the other face at a general value x. The volume enclosed by such a Gaussian surface is Ax.

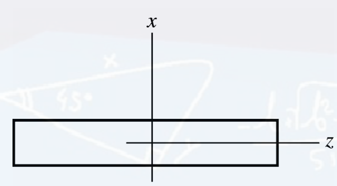
EXECUTE: **(a)** The electric field of the slab must be zero by symmetry. There is no preferred direction in the yz plane, so the electric field can only point in the x-direction. But at the origin, neither the positive nor negative x-directions should be singled out as special, and so the field must be zero.

(b) For $|x| \le d$, Gauss's law gives $EA = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\rho A|x|}{\varepsilon_0}$ and $E = \frac{\rho|x|}{\varepsilon_0}$, with direction given by $\frac{x}{|x|}\hat{\boldsymbol{i}}$ (away from the center of the slab). Note that this expression does give $E = 0$ at $x = 0$. Outside the slab, the enclosed charge does not depend on x and is equal to $\rho A d$. For $|x| \ge d$, Gauss's law gives

$EA = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\rho A d}{\varepsilon_0}$ and $E = \frac{\rho d}{\varepsilon_0}$, again with direction given by $\frac{x}{|x|}\hat{\boldsymbol{i}}$.

EVALUATE: At the surfaces of the slab, $x = \pm d$. For these values of x the two expressions for E (for inside and outside the slab) give the same result. The charge per unit area σ of the slab is given by $\sigma A = \rho A(2d)$ and $\rho d = \sigma/2$. The result for E outside the slab can therefore be written as $E = \sigma/2\varepsilon_0$ and is the same as for a thin sheet of charge.

22.55. (a) IDENTIFY and **SET UP:** Consider the direction of the field for x slightly greater than and slightly less than zero. The slab is sketched in Figure 22.55a.



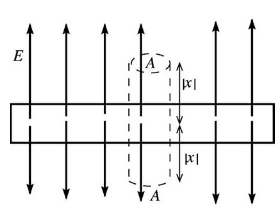
$\rho(x) = \rho_0(x/d)^2$.

Figure 22.55a

EXECUTE: The charge distribution is symmetric about $x = 0$, so by symmetry $E(x) = E(-x)$. But for $x > 0$ the field is in the $+x$-direction and for $x < 0$ the field is in the $-x$-direction. At $x = 0$ the field can't be both in the $+x$- and $-x$-directions so must be zero. That is, $E_x(x) = -E_x(-x)$. At point $x = 0$ this gives $E_x(0) = -E_x(0)$ and this equation is satisfied only for $E_x(0) = 0$.

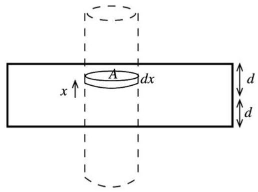
(b) IDENTIFY and **SET UP:** $|x| > d$ (outside the slab).

Apply Gauss's law to a cylindrical Gaussian surface whose axis is perpendicular to the slab and whose end caps have area A and are the same distance $|x| > d$ from $x = 0$, as shown in Figure 22.55b.



EXECUTE: $\Phi_E = 2EA$.

Figure 22.55b



To find Q_{encl} consider a thin disk at coordinate x and with thickness dx, as shown in Figure 22.55c.
The charge within this disk is
$dq = \rho dV = \rho A dx = (\rho_0 A/d^2)\, x^2 dx.$

Figure 22.55c

The total charge enclosed by the Gaussian cylinder is

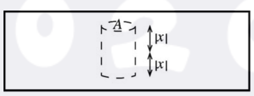
$$Q_{\text{encl}} = 2\int_0^d dq = (2\rho_0 A/d^2)\int_0^d x^2 dx = (2\rho_0 A/d^2)(d^3/3) = \tfrac{2}{3}\rho_0 A d.$$

Then $\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$ gives $2EA = 2\rho_0 A d/3\varepsilon_0$. This gives $E = \rho_0 d/3\varepsilon_0$.

$\vec{E}$ is directed away from $x = 0$, so $\vec{E} = (\rho_0 d/3\varepsilon_0)(x/|x|)\hat{\boldsymbol{i}}$.

(c) IDENTIFY and SET UP: $|x| < d$ (inside the slab).

Apply Gauss's law to a cylindrical Gaussian surface whose axis is perpendicular to the slab and whose end caps have area A and are the same distance $|x| < d$ from $x = 0$, as shown in Figure 22.55d.



EXECUTE: $\Phi_E = 2EA.$

Figure 22.55d

Q_{encl} is found as above, but now the integral on dx is only from 0 to x instead of 0 do d.

$$Q_{\text{encl}} = 2\int_0^x dq = (2\rho_0 A/d^2)\int_0^x x^2 dx = (2\rho_0 A/d^2)(x^3/3).$$

Then $\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$ gives $2EA = 2\rho_0 A x^3/3\varepsilon_0 d^2$. This gives $E = \rho_0 x^3/3\varepsilon_0 d^2$.

$\vec{E}$ is directed away from $x = 0$, so $\vec{E} = (\rho_0 x^3/3\varepsilon_0 d^2)\hat{\boldsymbol{i}}$.

EVALUATE: Note that $E = 0$ at $x = 0$ as stated in part (a). Note also that the expressions for $|x| > d$ and $|x| < d$ agree for $x = d$.

22.56. **IDENTIFY:** Apply Gauss's law.
SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the spherical distribution of charge. The volume of a thin spherical shell of radius r and thickness dr is $dV = 4\pi r^2 dr$.

EXECUTE: **(a)** $Q = \int \rho(r) dV = 4\pi\int_0^\infty \rho(r) r^2 dr = 4\pi\rho_0 \int_0^R \left(1 - \dfrac{4r}{3R}\right) r^2 dr = 4\pi\rho_0 \left[\int_0^R r^2 dr - \dfrac{4}{3R}\int_0^R r^3 dr\right].$

$Q = 4\pi\rho_0 \left[\dfrac{R^3}{3} - \dfrac{4}{3R}\dfrac{R^4}{4}\right] = 0.$ The total charge is zero.

(b) For $r \ge R$, $\oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\varepsilon_0} = 0,$ so $E = 0.$

(c) For $r \le R$, $\oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\varepsilon_0} = \dfrac{4\pi}{\varepsilon_0}\int_0^r \rho(r') r'^2 dr'.$ $E 4\pi r^2 = \dfrac{4\pi\rho_0}{\varepsilon_0}\left[\int_0^r r'^2 dr' - \dfrac{4}{3R}\int_0^r r'^3 dr'\right]$ and

$$E = \frac{\rho_0}{\varepsilon_0}\frac{1}{r^2}\left[\frac{r^3}{3} - \frac{r^4}{3R}\right] = \frac{\rho_0}{3\varepsilon_0} r\left[1 - \frac{r}{R}\right].$$

(d) The graph of E versus r is sketched in Figure 22.56.

(e) Where E is a maximum, $\dfrac{dE}{dr}=0$. This gives $\dfrac{\rho_0}{3\varepsilon_0}-\dfrac{2\rho_0 r_{\max}}{3\varepsilon_0 R}=0$ and $r_{\max}=\dfrac{R}{2}$. At this r,

$$E=\frac{\rho_0}{3\varepsilon_0}\frac{R}{2}\left[1-\frac{1}{2}\right]=\frac{\rho_0 R}{12\varepsilon_0}.$$

EVALUATE: The result in part (b) for $r \le R$ gives $E=0$ at $r=R$; the field is continuous at the surface of the charge distribution.

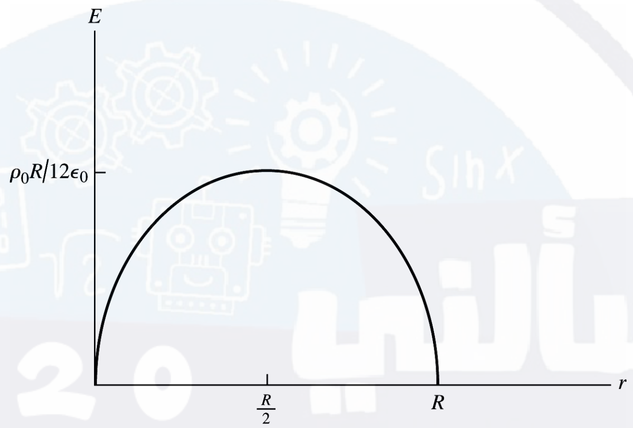


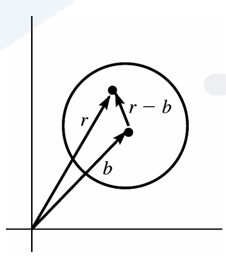
Figure 22.56

22.57. **(a) IDENTIFY:** Use $\vec{E}(\vec{r})$ from Example (22.9) (inside the sphere) and relate the position vector of a point inside the sphere measured from the origin to that measured from the center of the sphere.

SET UP: For an insulating sphere of uniform charge density ρ and centered at the origin, the electric field inside the sphere is given by $E=Qr'/4\pi\varepsilon_0 R^3$ (Example 22.9), where $\vec{r}\,'$ is the vector from the center of the sphere to the point where E is calculated.

But $\rho=3Q/4\pi R^3$ so this may be written as $E=\rho r/3\varepsilon_0$. And $\vec{E}$ is radially outward, in the direction of $\vec{r}\,'$, so $\vec{E}=\rho\vec{r}\,'/3\varepsilon_0$.

For a sphere whose center is located by vector $\vec{b}$, a point inside the sphere and located by $\vec{r}$ is located by the vector $\vec{r}\,'=\vec{r}-\vec{b}$ relative to the center of the sphere, as shown in Figure 22.57.



EXECUTE: Thus $\vec{E}=\dfrac{\rho(\vec{r}-\vec{b})}{3\varepsilon_0}$.

Figure 22.57

EVALUATE: When $b=0$ this reduces to the result of Example 22.9. When $\vec{r}=\vec{b}$, this gives $E=0$, which is correct since we know that $E=0$ at the center of the sphere.

(b) IDENTIFY: The charge distribution can be represented as a uniform sphere with charge density ρ and centered at the origin added to a uniform sphere with charge density $-\rho$ and centered at $\vec{r}=\vec{b}$.

SET UP: $\vec{E}=\vec{E}_{\text{uniform}}+\vec{E}_{\text{hole}}$, where $\vec{E}_{\text{uniform}}$ is the field of a uniformly charged sphere with charge density ρ and $\vec{E}_{\text{hole}}$ is the field of a sphere located at the hole and with charge density $-\rho$. (Within the spherical hole the net charge density is $+\rho-\rho=0$.)

EXECUTE: $\vec{E}_{\text{uniform}}=\dfrac{\rho\vec{r}}{3\epsilon_0}$, where $\vec{r}$ is a vector from the center of the sphere.

$\vec{E}_{\text{hole}}=\dfrac{-\rho(\vec{r}-\vec{b})}{3\epsilon_0}$, at points inside the hole. Then $\vec{E}=\dfrac{\rho\vec{r}}{3\epsilon_0}+\left(\dfrac{-\rho(\vec{r}-\vec{b})}{3\epsilon_0}\right)=\dfrac{\rho\vec{b}}{3\epsilon_0}$.

EVALUATE: $\vec{E}$ is independent of $\vec{r}$ so is uniform inside the hole. The direction of $\vec{E}$ inside the hole is in the direction of the vector $\vec{b}$, the direction from the center of the insulating sphere to the center of the hole.

22.58. IDENTIFY: We first find the field of a cylinder off-axis, then the electric field in a hole in a cylinder is the difference between two electric fields: that of a solid cylinder on-axis, and one off-axis, at the location of the hole.

SET UP: Let $\vec{r}$ locate a point within the hole, relative to the axis of the cylinder and let $\vec{r}'$ locate this point relative to the axis of the hole. Let $\vec{b}$ locate the axis of the hole relative to the axis of the cylinder. As shown in Figure 22.58, $\vec{r}'=\vec{r}-\vec{b}$. Problem 22.41 shows that at points within a long insulating cylinder, $\vec{E}=\dfrac{\rho\vec{r}}{2\epsilon_0}$.

EXECUTE: $\vec{E}_{\text{off-axis}}=\dfrac{\rho\vec{r}'}{2\epsilon_0}=\dfrac{\rho(\vec{r}-\vec{b})}{2\epsilon_0}$. $\vec{E}_{\text{hole}}=\vec{E}_{\text{cylinder}}-\vec{E}_{\text{off-axis}}=\dfrac{\rho\vec{r}}{2\epsilon_0}-\dfrac{\rho(\vec{r}-\vec{b})}{2\epsilon_0}=\dfrac{\rho\vec{b}}{2\epsilon_0}$.

Note that $\vec{E}$ is uniform.

EVALUATE: If the hole is coaxial with the cylinder, $b=0$ and $E_{\text{hole}}=0$.

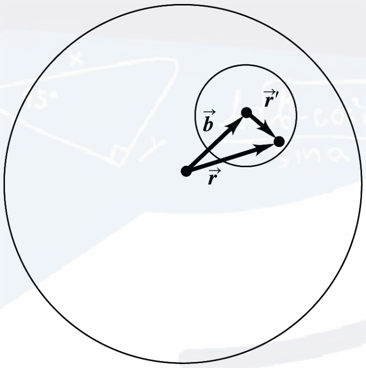


Figure 22.58

22.59. IDENTIFY and SET UP: For a uniformly charged sphere, $E=k\dfrac{|Q|}{r^2}$, so $Er^2=k|Q|=$ constant. For a long uniform line of charge, $E=\dfrac{\lambda}{2\pi\epsilon_0 r}$, so $Er=\dfrac{\lambda}{2\pi\epsilon_0}=$ constant.

EXECUTE: **(a)** Figure 22.59a shows the graphs for data set A. We see that the graph of Er versus r is a horizontal line, which means that Er = constant. Therefore data set A is for a uniform straight line of charge.

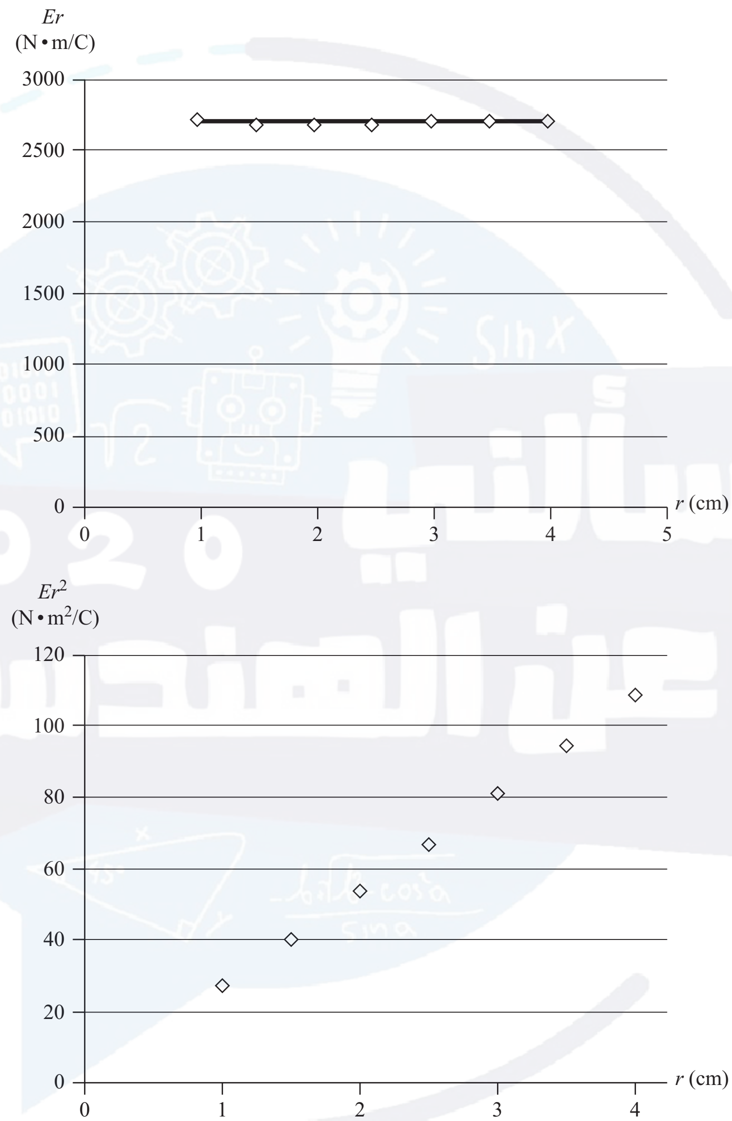


Figure 22.59a

Figure 22.59b shows the graphs for data set B. We see that the graph of Er^2 versus r is a horizontal line, so Er^2 = constant. Thus data set B is for a uniformly charged sphere.

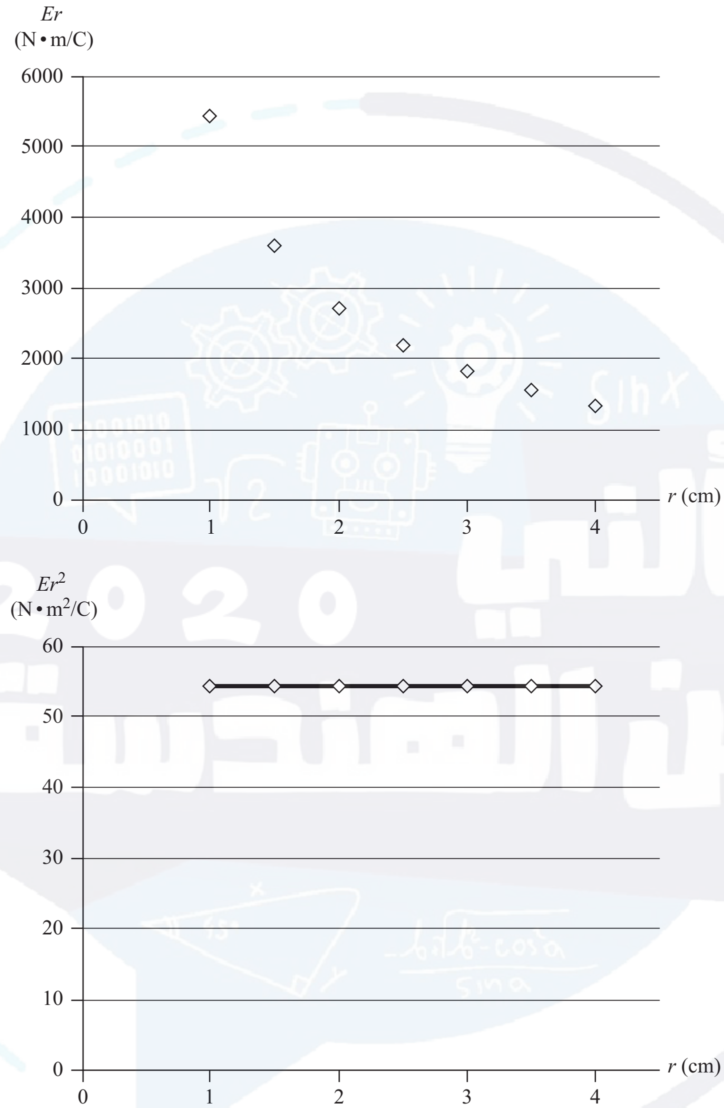


Figure 22.59b

(b) For A: $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$, so $\lambda = 2\pi\epsilon_0 Er$. From our graph in Figure 22.59a, Er = constant = $2690\ \text{N}\cdot\text{m/C}$.

Therefore

$\lambda = 2\pi\epsilon_0 Er = 2\pi(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2690\ \text{N}\cdot\text{m/C}) = 1.50\times10^{-7}\ \text{C/m} = 0.150\ \mu\text{C/m}.$

For B: $E = k\dfrac{|Q|}{r^2}$, so $kQ = Er^2$ = constant, which means that Q = (constant)/k. From our graph in Figure 22.59b, Er^2 = constant = $54.1\ \text{N}\cdot\text{m}^2/\text{C}$. Therefore

$Q = (54.1\ \text{N}\cdot\text{m}^2/\text{C})/(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) = 6.0175\times10^{-9}\ \text{C}.$

The charge density ρ is $\rho = \dfrac{Q}{V} = \dfrac{Q}{\frac{4}{3}\pi R^3}$ $= (6.0175\times10^{-9}\ \text{C})/[(4\pi/3)(0.00800\ \text{m})^3 = 2.81\times10^{-3}\ \text{C/m}^3.$

EVALUATE: A linear charge density of 0.150 C/m and a volume charge density of 2.81×10^{-3} C/m^3 are both physically reasonable and could be achieved in a normal laboratory.

22.60. **IDENTIFY** and **SET UP:** The electric field inside a uniform sphere of charge does not follow an inverse-square law. Apply Gauss's law, $\oint \vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$, to find the field.

SET UP: Apply $\oint \vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$. As the Gaussian surface, use a sphere of radius r that is centered on the given sphere.

EXECUTE: Gauss's law gives $E(4\pi r^2) = \dfrac{\rho\left(\frac{4}{3}\pi r^3\right)}{\varepsilon_0}$, from which we get $E = \dfrac{\rho}{3\varepsilon_0}r$. Therefore in a graph of E versus r, the slope is $\dfrac{\rho}{3\varepsilon_0}$. From the graph in the problem, the slope is

slope $= \dfrac{(6-3)\times10^4\ \text{N/C}}{(8-4)\times10^{-3}\ \text{m}} = 7.5\times10^6\ \text{N/m}\cdot\text{C}$. Solving for ρ gives

ρ = (slope)(3 ε_0) = $(7.5\times10^6\ \text{N/m}\cdot\text{C})\,(3)\,(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = 1.99\times10^{-4}$ C/m^3.

EVALUATE: A sphere of volume 1.0 m^3 would have only 199 μC of charge, which is physically realistic.

22.61. **IDENTIFY** and **SET UP:** Apply Gauss's law, $\oint \vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$. The enclosed charge is $Q_{\text{encl}} = \rho V$, where $V = \dfrac{4}{3}\pi r^3$ for a sphere of radius r. Read the charge densities from the graph in the problem.

EXECUTE: Apply Gauss's law $\oint \vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$. As a Gaussian surface, use a sphere of radius r centered on the given sphere. This gives $E(4\pi r^2) = Q_{\text{encl}}/\varepsilon_0$, so $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_{\text{encl}}}{r^2} = k\dfrac{Q_{\text{encl}}}{r^2}$. In each case, we must first use $Q_{\text{encl}} = \rho V$ to calculate Q_{encl} and then use that result to calculate E.

(i) First find Q_{encl}: $Q_{\text{encl}} = \rho V = (10.0\times10^{-6}\ \text{C/m}^3)(4\pi/3)(0.00100\ \text{m})^3 = 4.19\times10^{-14}$ C.

Now calculate E: $E = k\dfrac{Q_{\text{encl}}}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.19\times10^{-14}\ \text{C})/(0.00100\ \text{m})^2 = 377$ N/C.

(ii) $Q_{\text{encl}} = (10.0\times10^{-6}\ \text{C/m}^3)(4\pi/3)(0.00200\ \text{m})^3 + (4.0\times10^{-6}\ \text{C/m}^3)(4\pi/3)[(0.00300\ \text{m})^3 - (0.00200\ \text{m})^3]$
$Q_{\text{encl}} = 6.534\times10^{-13}$ C.

$E = k\dfrac{Q_{\text{encl}}}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.534\times10^{-13}\ \text{C})/(0.00300\ \text{m})^2 = 653$ N/C.

(iii) $Q_{\text{encl}} = (10.0\times10^{-6}\ \text{C/m}^3)(4\pi/3)(0.00200\ \text{m})^3 + (4.0\times10^{-6}\ \text{C/m}^3)(4\pi/3)[(0.00400\ \text{m})^3 - (0.00200\ \text{m})^3]$ $+ (-2.0\times10^{-6}\ \text{C/m}^3)(4\pi/3)[(0.00500\ \text{m})^3 - (0.00400\ \text{m})^3]$.
$Q_{\text{encl}} = 7.624\times10^{-13}$ C.

$E = k\dfrac{Q_{\text{encl}}}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\,(7.624\times10^{-13}\ \text{C})/(0.00500\ \text{m})^2 = 274$ N/C.

(iv) $Q_{\text{encl}} = 7.624\times10^{-13}\ \text{C} + (-2.0\times10^{-6}\ \text{C/m}^3)(4\pi/3)[(0.00600\ \text{m})^3 - (0.00500\ \text{m})^3] = 0$, so $E = 0$.

EVALUATE: We found that $E = 0$ at $r = 7.00$ mm, but E is also zero at all points beyond $r = 6.00$ mm because the enclosed charge is zero for any Gaussian surface having a radius $r > 6.00$ mm.

22.62. **IDENTIFY:** The charge in a spherical shell of radius r and thickness dr is $dQ = \rho(r)4\pi r^2\,dr$. Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r. Let Q_i be the charge in the region $r \le R/2$ and let Q_0 be the charge in the region where $R/2 \le r \le R$.

EXECUTE: **(a)** The total charge is $Q = Q_i + Q_0$, where $Q_i = 4\pi\int_0^{R/2}\frac{3\alpha r^3}{2R}dr = \frac{6\pi\alpha}{R}\frac{1}{4}\frac{R^4}{16} = \frac{3}{32}\pi\alpha R^3$ and

$Q_0 = 4\pi\alpha\int_{R/2}^{R}(1-(r/R)^2)r^2\,dr = 4\pi\alpha R^3\left(\frac{7}{24}-\frac{31}{160}\right) = \frac{47}{120}\pi\alpha R^3$. Therefore,

$Q = \left(\frac{3}{32}+\frac{47}{120}\right)\pi\alpha R^3 = \frac{233}{480}\pi\alpha R^3$ and $\alpha = \frac{480Q}{233\pi R^3}$.

(b) For $r \le R/2$, Gauss's law gives $E4\pi r^2 = \frac{4\pi}{\epsilon_0}\int_0^r\frac{3\alpha r'^3}{2R}dr' = \frac{3\pi\alpha r^4}{2\epsilon_0 R}$ and $E = \frac{6\alpha r^2}{16\epsilon_0 R} = \frac{180Qr^2}{233\pi\epsilon_0 R^4}$.

For $R/2 \le r \le R$, $E4\pi r^2 = \frac{Q_i}{\epsilon_0} + \frac{4\pi\alpha}{\epsilon_0}\int_{R/2}^{r}(1-(r'/R)^2)r'^2\,dr' = \frac{Q_i}{\epsilon_0} + \frac{4\pi\alpha}{\epsilon_0}\left(\frac{r^3}{3}-\frac{R^3}{24}-\frac{r^5}{5R^2}+\frac{R^3}{160}\right)$.

$E4\pi r^2 = \frac{3}{128}\frac{4\pi\alpha R^3}{\epsilon_0} + \frac{4\pi\alpha R^3}{\epsilon_0}\left(\frac{1}{3}\left(\frac{r}{R}\right)^3 - \frac{1}{5}\left(\frac{r}{R}\right)^5 - \frac{17}{480}\right)$ and $E = \frac{480Q}{233\pi\epsilon_0 r^2}\left(\frac{1}{3}\left(\frac{r}{R}\right)^3 - \frac{1}{5}\left(\frac{r}{R}\right)^5 - \frac{23}{1920}\right)$.

For $r \ge R$, $E = \frac{Q}{4\pi\epsilon_0 r^2}$, since all the charge is enclosed.

(c) The fraction of Q between $R/2 \le r \le R$ is $\frac{Q_0}{Q} = \frac{47}{120}\frac{480}{233} = 0.807$.

(d) $E = \frac{180}{233}\frac{Q}{4\pi\epsilon_0 R^2}$ using either of the electric field expressions above, evaluated at $r = R/2$.

EVALUATE: **(e)** The force an electron would feel never is proportional to $-r$ which is necessary for simple harmonic oscillations. It is oscillatory since the force is always attractive, but it has the wrong power of r to be simple harmonic motion.

22.63. **IDENTIFY** and **SET UP:** Treat the sphere as a point-charge, so $E = k\frac{|q|}{r^2}$, so $|q| = Er^2/k$.

EXECUTE: $|q| = Er^2/k = (1\times10^6\ \text{N/C})(25\ \text{m})^2/(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) = 0.0695\ \text{C} \approx 0.07\ \text{C}$. The charge must be negative since the field is intended to repel negative electrons. Choice (a) is correct.
EVALUATE: 0.07 C is quite a large amount of charge, much larger than normally encountered in typical college physics laboratories.

22.64. **IDENTIFY** and **SET UP:** Treat the sphere as a point-charge, so $E = k\frac{|q|}{r^2}$. Use the result from the previous problem for the charge on the sphere.

EXECUTE: $E = k\frac{|q|}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.0695\ \text{C})/(2.5\ \text{m})^2 = 1.0\times10^8\ \text{N/C}$, choice (d).

EVALUATE: The field strength at 2.5 m is 100 times what it is at 25 m. This is reasonable since the field strength obeys an inverse-square law. At 25 m, which is a distance 10 times as far as 2.5 m, the field strength is $[(2.5\ \text{m})/(25\ \text{m})]^2(1\times10^6\ \text{N/C}) = 1\times10^6$ N/C, which was given in the previous problem.

22.65. **IDENTIFY** and **SET UP:** Electric field lines point away from positive charges and toward negative charges. For a point-charge, the lines radiated from (or to) the charge. For a uniform sphere of charge, the field lines look the same as those for a point-charge for points outside the sphere.
EXECUTE: The sphere is negative and equivalent to a negative point-charge, so at its surface the field lines are perpendicular to it and pointing inward, which is choice (b).
EVALUATE: The sphere behaves like a point-charge at or above its surface.

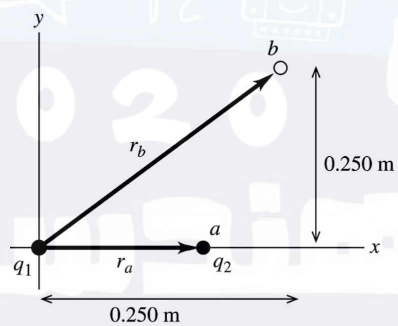
22.66. **IDENTIFY** and **SET UP:** All the charge is on the surface of a spherical shell.
EXECUTE: The field inside the sphere comes from any charge that is inside, but there is none. So the field is zero, choice (c).
EVALUATE: This result is true only if the surface of the sphere is uniformly charged.

23

ELECTRIC POTENTIAL

23.1. **IDENTIFY:** Apply $W_{a\to b} = U_a - U_b$ to calculate the work. The electric potential energy of a pair of point charges is given by $U = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1q_2}{r}$.

SET UP: Let the initial position of q_2 be point a and the final position be point b, as shown in Figure 23.1.



$r_a = 0.150$ m.

$r_b = \sqrt{(0.250\text{ m})^2 + (0.250\text{ m})^2}$.

$r_b = 0.3536$ m.

Figure 23.1

EXECUTE: $W_{a\to b} = U_a - U_b$.

$$U_a = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_a} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(+2.40\times10^{-6}\text{ C})(-4.30\times10^{-6}\text{ C})}{0.150\text{ m}}.$$

$U_a = -0.6184$ J.

$$U_b = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_b} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(+2.40\times10^{-6}\text{ C})(-4.30\times10^{-6}\text{ C})}{0.3536\text{ m}}.$$

$U_b = -0.2623$ J.

$$W_{a\to b} = U_a - U_b = -0.6184\text{ J} - (-0.2623\text{ J}) = -0.356\text{ J}.$$

EVALUATE: The attractive force on q_2 is toward the origin, so it does negative work on q_2 when q_2 moves to larger r.

23.2. **IDENTIFY:** Apply $W_{a\to b} = U_a - U_b$.

SET UP: $U_a = +5.4\times10^{-8}$ J. Solve for U_b.

EXECUTE: $W_{a\to b} = -1.9\times10^{-8}\text{ J} = U_a - U_b$. $U_b = U_a - W_{a\to b} = +5.4\times10^{-8}\text{ J} - (-1.9\times10^{-8}\text{ J}) = 7.3\times10^{-8}$ J.

EVALUATE: When the electric force does negative work the electrical potential energy increases.

23.3. **IDENTIFY:** The work needed to assemble the nucleus is the sum of the electrical potential energies of the protons in the nucleus, relative to infinity.

SET UP: The total potential energy is the scalar sum of all the individual potential energies, where each potential energy is $U = (1/4\pi\epsilon_0)(qq_0/r)$. Each charge is e and the charges are equidistant from each other, so the total potential energy is $U = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{e^2}{r}+\dfrac{e^2}{r}+\dfrac{e^2}{r}\right) = \dfrac{3e^2}{4\pi\epsilon_0 r}$.

EXECUTE: Adding the potential energies gives

$$U = \frac{3e^2}{4\pi\epsilon_0 r} = \frac{3(1.60\times10^{-19}\ \text{C})^2(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)}{2.00\times10^{-15}\ \text{m}} = 3.46\times10^{-13}\ \text{J} = 2.16\ \text{MeV}.$$

EVALUATE: This is a small amount of energy on a macroscopic scale, but on the scale of atoms, 2 MeV is quite a lot of energy.

23.4. IDENTIFY: The work required is the change in electrical potential energy. The protons gain speed after being released because their potential energy is converted into kinetic energy.

(a) SET UP: Using the potential energy of a pair of point charges relative to infinity, $U = (1/4\pi\epsilon_0)(qq_0/r)$, we have $W = \Delta U = U_2 - U_1 = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{e^2}{r_2}-\dfrac{e^2}{r_1}\right)$.

EXECUTE: Factoring out the e^2 and substituting numbers gives

$$W = (9.00\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2\left(\frac{1}{3.00\times10^{-15}\ \text{m}}-\frac{1}{2.00\times10^{-10}\ \text{m}}\right) = 7.68\times10^{-14}\ \text{J}$$

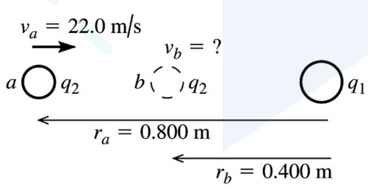
(b) SET UP: The protons have equal momentum, and since they have equal masses, they will have equal speeds and hence equal kinetic energy. $\Delta U = K_1 + K_2 = 2K = 2\left(\frac{1}{2}mv^2\right) = mv^2$.

EXECUTE: Solving for v gives $v = \sqrt{\dfrac{\Delta U}{m}} = \sqrt{\dfrac{7.68\times10^{-14}\ \text{J}}{1.67\times10^{-27}\ \text{kg}}} = 6.78\times10^6\ \text{m/s}$.

EVALUATE: The potential energy may seem small (compared to macroscopic energies), but it is enough to give each proton a speed of nearly 7 million m/s.

23.5. (a) IDENTIFY: Use conservation of energy: $K_a + U_a + W_{\text{other}} = K_b + U_b$. U for the pair of point charges is given by $U = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1q_2}{r}$.

SET UP:



Let point a be where q_2 is 0.800 m from q_1 and point b be where q_2 is 0.400 m from q_1, as shown in Figure 23.5a.

Figure 23.5a

EXECUTE: Only the electric force does work, so $W_{\text{other}} = 0$ and $U = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1q_2}{r}$.

$$K_a = \tfrac{1}{2}mv_a^2 = \tfrac{1}{2}(1.50\times10^{-3}\ \text{kg})(22.0\ \text{m/s})^2 = 0.3630\ \text{J}.$$

$$U_a = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_a} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6}\ \text{C})(-7.80\times10^{-6}\ \text{C})}{0.800\ \text{m}} = +0.2454\ \text{J}.$$

$$K_b = \tfrac{1}{2}mv_b^2.$$

$$U_b = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_b} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6}\ \text{C})(-7.80\times10^{-6}\ \text{C})}{0.400\ \text{m}} = +0.4907\ \text{J}.$$

The conservation of energy equation then gives $K_b = K_a + (U_a - U_b)$.

$$\tfrac{1}{2}mv_b^2 = +0.3630\ \text{J} + (0.2454\ \text{J} - 0.4907\ \text{J}) = 0.1177\ \text{J}.$$

$$v_b = \sqrt{\frac{2(0.1177\ \text{J})}{1.50\times10^{-3}\ \text{kg}}} = 12.5\ \text{m/s}.$$

EVALUATE: The potential energy increases when the two positively charged spheres get closer together, so the kinetic energy and speed decrease.

(b) IDENTIFY: Let point c be where q_2 has its speed momentarily reduced to zero. Apply conservation of energy to points a and c: $K_a + U_a + W_{\text{other}} = K_c + U_c$.

SET UP: Points a and c are shown in Figure 23.5b.

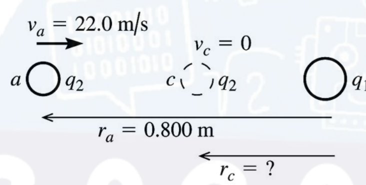


Figure 23.5b

EXECUTE: $K_a = +0.3630\ \text{J}$ (from part (a)).

$U_a = +0.2454\ \text{J}$ (from part (a)).

$K_c = 0$ (at distance of closest approach the speed is zero).

$$U_c = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_c}.$$

Thus conservation of energy $K_a + U_a = U_c$ gives $\dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r_c} = +0.3630\ \text{J} + 0.2454\ \text{J} = 0.6084\ \text{J}.$

$$r_c = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{0.6084\ \text{J}} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6}\ \text{C})(-7.80\times10^{-6}\ \text{C})}{+0.6084\ \text{J}} = 0.323\ \text{m}.$$

EVALUATE: $U \to \infty$ as $r \to 0$ so q_2 will stop no matter what its initial speed is.

23.6. IDENTIFY: The total potential energy is the scalar sum of the individual potential energies of each pair of charges.

SET UP: For a pair of point charges the electrical potential energy is $U = k\dfrac{qq'}{r}$. In the O-H-N combination the O^- is 0.170 nm from the H^+ and 0.280 nm from the N^-. In the N-H-N combination the N^- is 0.190 nm from the H^+ and 0.300 nm from the other N^-. U is positive for like charges and negative for unlike charges.

EXECUTE: **(a)** O-H-N:

$$O^-\text{-}H^+: U = -(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{0.170\times10^{-9}\ \text{m}} = -1.35\times10^{-18}\ \text{J}.$$

$$O^-\text{-}N^-: U = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{0.280\times10^{-9}\ \text{m}} = +8.22\times10^{-19}\ \text{J}.$$

N-H-N:

$$N^-\text{-}H^+: U = -(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19}\ \text{C})^2}{0.190\times10^{-9}\ \text{m}} = -1.21\times10^{-18}\ \text{J}.$$

$$\text{N}^- \text{-N}^- : U = (8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19} \text{ C})^2}{0.300\times10^{-9} \text{ m}} = +7.67\times10^{-19} \text{ J.}$$

The total potential energy is

$U_{\text{tot}} = -1.35\times10^{-18} \text{ J} + 8.22\times10^{-19} \text{ J} - 1.21\times10^{-18} \text{ J} + 7.67\times10^{-19} \text{ J} = -9.71\times10^{-19} \text{ J.}$

(b) In the hydrogen atom the electron is 0.0529 nm from the proton.

$$U = -(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19} \text{ C})^2}{0.0529\times10^{-9} \text{ m}} = -4.35\times10^{-18} \text{ J.}$$

EVALUATE: The magnitude of the potential energy in the hydrogen atom is about a factor of 4 larger than what it is for the adenine-thymine bond.

23.7. IDENTIFY: Use conservation of energy $U_a + K_a = U_b + K_b$ to find the distance of closest approach r_b. The maximum force is at the distance of closest approach, $F = k\dfrac{|q_1 q_2|}{r_b^2}$.

SET UP: $K_b = 0$. Initially the two protons are far apart, so $U_a = 0$. A proton has mass 1.67×10^{-27} kg and charge $q = +e = +1.60\times10^{-19}$ C.

EXECUTE: $K_a = U_b$. $2(\frac{1}{2}mv_a^2) = k\dfrac{q_1 q_2}{r_b}$. $mv_a^2 = k\dfrac{e^2}{r_b}$ and

$$r_b = \frac{ke^2}{mv_a^2} = \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19} \text{ C})^2}{(1.67\times10^{-27} \text{ kg})(2.00\times10^5 \text{ m/s})^2} = 3.45\times10^{-12} \text{ m.}$$

$$F = k\frac{e^2}{r_b^2} = (8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19} \text{ C})^2}{(3.445\times10^{-12} \text{ m})^2} = 1.94\times10^{-5} \text{ N.}$$

EVALUATE: The acceleration $a = F/m$ of each proton produced by this force is extremely large.

23.8. IDENTIFY: Call the three charges 1, 2, and 3. $U = U_{12} + U_{13} + U_{23}$.

SET UP: $U_{12} = U_{23} = U_{13}$ because the charges are equal and each pair of charges has the same separation, 0.400 m.

EXECUTE: $U = \dfrac{3kq^2}{0.400 \text{ m}} = \dfrac{3k(1.2\times10^{-6} \text{ C})^2}{0.400 \text{ m}} = 0.0971$ J.

EVALUATE: When the three charges are brought in from infinity to the corners of the triangle, the repulsive electrical forces between each pair of charges do negative work and electrical potential energy is stored.

23.9. IDENTIFY: The protons repel each other and therefore accelerate away from one another. As they get farther and farther away, their kinetic energy gets greater and greater but their acceleration keeps decreasing. Conservation of energy and Newton's laws apply to these protons.

SET UP: Let a be the point when they are 0.750 nm apart and b be the point when they are very far apart. A proton has charge $+e$ and mass 1.67×10^{-27} kg. As they move apart the protons have equal kinetic energies and speeds. Their potential energy is $U = ke^2/r$ and $K = \frac{1}{2}mv^2$. $K_a + U_a = K_b + U_b$.

EXECUTE: **(a)** They have maximum speed when they are far apart and all their initial electrical potential energy has been converted to kinetic energy. $K_a + U_a = K_b + U_b$.

$K_a = 0$ and $U_b = 0$, so

$$K_b = U_a = k\frac{e^2}{r_a} = (8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times10^{-19} \text{ C})^2}{0.750\times10^{-9} \text{ m}} = 3.07\times10^{-19} \text{ J.}$$

$K_b = \frac{1}{2}mv_b^2 + \frac{1}{2}mv_b^2$, so $K_b = mv_b^2$ and $v_b = \sqrt{\dfrac{K_b}{m}} = \sqrt{\dfrac{3.07\times10^{-19} \text{ J}}{1.67\times10^{-27} \text{ kg}}} = 1.36\times10^4$ m/s.

(b) Their acceleration is largest when the force between them is largest and this occurs at $r = 0.750$ nm, when they are closest.

$$F = k\frac{e^2}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{1.60\times10^{-19}\ \text{C}}{0.750\times10^{-9}\ \text{m}}\right)^2 = 4.09\times10^{-10}\ \text{N}.$$

$$a = \frac{F}{m} = \frac{4.09\times10^{-10}\ \text{N}}{1.67\times10^{-27}\ \text{kg}} = 2.45\times10^{17}\ \text{m/s}^2.$$

EVALUATE: The acceleration of the protons decreases as they move farther apart, but the force between them is repulsive so they continue to increase their speeds and hence their kinetic energies.

23.10. IDENTIFY: The work done on the alpha particle is equal to the difference in its potential energy when it is moved from the midpoint of the square to the midpoint of one of the sides.

SET UP: We apply the formula $W_{a\to b} = U_a - U_b$. In this case, a is the center of the square and b is the midpoint of one of the sides. Therefore $W_{\text{center}\to\text{side}} = U_{\text{center}} - U_{\text{side}}$ is the work done by the Coulomb force. There are 4 electrons, so the potential energy at the center of the square is 4 times the potential energy of a single alpha-electron pair. At the center of the square, the alpha particle is a distance $r_1 = \sqrt{50}\ \text{nm}$ from each electron. At the midpoint of the side, the alpha is a distance $r_2 = 5.00\ \text{nm}$ from the two nearest electrons and a distance $r_3 = \sqrt{125}\ \text{nm}$ from the two most distant electrons. Using the formula for the potential energy (relative to infinity) of two point charges, $U = (1/4\pi\varepsilon_0)(qq_0/r)$, the total work done by the Coulomb force is

$$W_{\text{center}\to\text{side}} = U_{\text{center}} - U_{\text{side}} = 4\frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_\text{e}}{r_1} - \left(2\frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_\text{e}}{r_2} + 2\frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_\text{e}}{r_3}\right).$$

Substituting $q_\text{e} = -e$ and $q_\alpha = 2e$ and simplifying gives

$$W_{\text{center}\to\text{side}} = -4e^2\frac{1}{4\pi\varepsilon_0}\left[\frac{2}{r_1} - \left(\frac{1}{r_2} + \frac{1}{r_3}\right)\right].$$

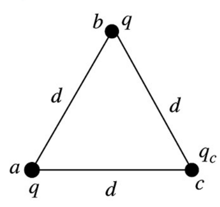
EXECUTE: Substituting the numerical values into the equation for the work gives

$$W = -4(1.60\times10^{-19}\ \text{C})^2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left[\frac{2}{\sqrt{50}\ \text{nm}} - \left(\frac{1}{5.00\ \text{nm}} + \frac{1}{\sqrt{125}\ \text{nm}}\right)\right] = 6.08\times10^{-21}\ \text{J}.$$

EVALUATE: Since the work done by the Coulomb force is positive, the system has more potential energy with the alpha particle at the center of the square than it does with it at the midpoint of a side. To move the alpha particle to the midpoint of a side and leave it there at rest an external force must do $-6.08\times10^{-21}\ \text{J}$ of work.

23.11. IDENTIFY: Apply $W_{a\to b} = U_a - U_b$. The net work to bring the charges in from infinity is equal to the change in potential energy. The total potential energy is the sum of the potential energies of each pair of charges, calculated from $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$.

SET UP: Let 1 be where all the charges are infinitely far apart. Let 2 be where the charges are at the corners of the triangle, as shown in Figure 23.11.



Let q_c be the third, unknown charge.

Figure 23.11

EXECUTE: $W = -\Delta U = -(U_2 - U_1)$, where W is the work done by the Coulomb force.

$$U_1 = 0$$

$$U_2 = U_{ab} + U_{ac} + U_{bc} = \frac{1}{4\pi\varepsilon_0 d}(q^2 + 2qq_c).$$

We want $W = 0$, so $W = -(U_2 - U_1)$ gives $0 = -U_2$.

$$0 = \frac{1}{4\pi\varepsilon_0 d}(q^2 + 2qq_c).$$

$q^2 + 2qq_c = 0$ and $q_c = -q/2$.

EVALUATE: The potential energy for the two charges q is positive and for each q with q_c it is negative. There are two of the q, q_c terms so must have $q_c < q$.

23.12. IDENTIFY: Work is done on the object by the electric field, and this changes its kinetic energy, so we can use the work-energy theorem.

SET UP: $W_{A\to B} = \Delta K$ and $W_{A\to B} = q(V_A - V_B)$.

EXECUTE: **(a)** Applying the two equations above gives $W_{A\to B} = q(V_A - V_B) = K_B - 0 = K_B$.

$V_B = V_A - K_B/q = 30.0 \text{ V} - (3.00\times10^{-7} \text{ J})/(-6.00\times10^{-9} \text{ C}) = 80.0 \text{ V}$.

(b) The negative charge accelerates from A to B, so the electric field must point from B toward A. Since the field is uniform, we have $E = \dfrac{\Delta V}{\Delta x}$ = (50.0 V)/(0.500 m) = 100 V/m.

EVALUATE: A positive charge is accelerated from high to low potential, but a negative charge (as we have here) is accelerated from low to high potential.

23.13. IDENTIFY and **SET UP:** Apply conservation of energy to points A and B.

EXECUTE: $K_A + U_A = K_B + U_B$.

$U = qV$, so $K_A + qV_A = K_B + qV_B$.

$K_B = K_A + q(V_A - V_B) = 0.00250 \text{ J} + (-5.00\times10^{-6} \text{ C})(200 \text{ V} - 800 \text{ V}) = 0.00550 \text{ J}$.

$v_B = \sqrt{2K_B/m} = 7.42 \text{ m/s}$.

EVALUATE: It is faster at B; a negative charge gains speed when it moves to higher potential.

23.14. IDENTIFY: The work-energy theorem says $W_{a\to b} = K_b - K_a$. $\dfrac{W_{a\to b}}{q} = V_a - V_b$.

SET UP: Point a is the starting point and point b is the ending point. Since the field is uniform, $W_{a\to b} = Fs\cos\phi = E|q|s\cos\phi$. The field is to the left so the force on the positive charge is to the left. The particle moves to the left so $\phi = 0°$ and the work $W_{a\to b}$ is positive.

EXECUTE: **(a)** $W_{a\to b} = K_b - K_a = 2.20\times10^{-6} \text{ J} - 0 = 2.20\times10^{-6} \text{ J}$.

(b) $V_a - V_b = \dfrac{W_{a\to b}}{q} = \dfrac{2.20\times10^{-6} \text{ J}}{4.20\times10^{-9} \text{ C}} = 524 \text{ V}$. Point a is at higher potential than point b.

(c) $E|q|s = W_{a\to b}$, so $E = \dfrac{W_{a\to b}}{|q|s} = \dfrac{V_a - V_b}{s} = \dfrac{524 \text{ V}}{6.00\times10^{-2} \text{ m}} = 8.73\times10^3 \text{ V/m}$.

EVALUATE: A positive charge gains kinetic energy when it moves to lower potential; $V_b < V_a$.

23.15. IDENTIFY: Apply $W_{a\to b} = q'\int_a^b \vec{E}\cdot d\vec{l}$. Use coordinates where $+y$ is upward and $+x$ is to the right. Then $\vec{E} = E\hat{j}$ with $E = 4.00\times10^4$ N/C.

SET UP: **(a)** The path is sketched in Figure 23.15a.

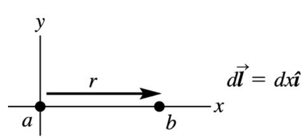
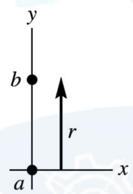


Figure 23.15a

EXECUTE: $\vec{E} \cdot d\vec{l} = (E\hat{j}) \cdot (dx\hat{i}) = 0$ so $W_{a \to b} = q' \int_a^b \vec{E} \cdot d\vec{l} = 0.$

EVALUATE: The electric force on the positive charge is upward (in the direction of the electric field) and does no work for a horizontal displacement of the charge.

(b) SET UP: The path is sketched in Figure 23.15b.



$d\vec{l} = dy\hat{j}.$

Figure 23.15b

EXECUTE: $\vec{E} \cdot d\vec{l} = (E\hat{j}) \cdot (dy\hat{j}) = E\,dy.$

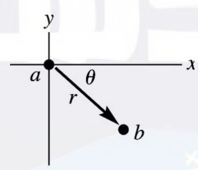
$$W_{a \to b} = q' \int_a^b \vec{E} \cdot d\vec{l} = q'E \int_a^b dy = q'E(y_b - y_a).$$

$y_b - y_a = +0.670$ m; it is positive since the displacement is upward and we have taken $+y$ to be upward.

$$W_{a \to b} = q'E(y_b - y_a) = (+28.0 \times 10^{-9}\ \text{C})(4.00 \times 10^{4}\ \text{N/C})(+0.670\ \text{m}) = +7.50 \times 10^{-4}\ \text{J}.$$

EVALUATE: The electric force on the positive charge is upward so it does positive work for an upward displacement of the charge.

(c) SET UP: The path is sketched in Figure 23.15c.



$y_a = 0.$

$y_b = -r\sin\theta = -(2.60\ \text{m})\sin 45° = -1.838\ \text{m}.$

The vertical component of the 2.60 m displacement is 1.838 m downward.

Figure 23.15c

EXECUTE: $d\vec{l} = dx\hat{i} + dy\hat{j}$ (The displacement has both horizontal and vertical components.)

$\vec{E} \cdot d\vec{l} = (E\hat{j}) \cdot (dx\hat{i} + dy\hat{j}) = E\,dy$ (Only the vertical component of the displacement contributes to the work.)

$$W_{a \to b} = q' \int_a^b \vec{E} \cdot d\vec{l} = q'E \int_a^b dy = q'E(y_b - y_a).$$

$$W_{a \to b} = q'E(y_b - y_a) = (+28.0 \times 10^{-9}\text{C})(4.00 \times 10^{4}\ \text{N/C})(-1.838\ \text{m}) = -2.06 \times 10^{-3}\ \text{J}.$$

EVALUATE: The electric force on the positive charge is upward so it does negative work for a displacement of the charge that has a downward component.

23.16. IDENTIFY: Apply $K_a + U_a = K_b + U_b$.

SET UP: Let $q_1 = +3.00$ nC and $q_2 = +2.00$ nC. At point a, $r_{1a} = r_{2a} = 0.250$ m. At point b, $r_{1b} = 0.100$ m and $r_{2b} = 0.400$ m. The electron has $q = -e$ and $m_e = 9.11 \times 10^{-31}$ kg. $K_a = 0$ since the electron is released from rest.

EXECUTE: $-\dfrac{keq_1}{r_{1a}} - \dfrac{keq_2}{r_{2a}} = -\dfrac{keq_1}{r_{1b}} - \dfrac{keq_2}{r_{2b}} + \dfrac{1}{2}m_e v_b^2.$

$$E_a = K_a + U_a = k(-1.60\times10^{-19}\text{ C})\left(\frac{(3.00\times10^{-9}\text{ C})}{0.250\text{ m}} + \frac{(2.00\times10^{-9}\text{ C})}{0.250\text{ m}}\right) = -2.88\times10^{-17}\text{ J}.$$

$$E_b = K_b + U_b = k(-1.60\times10^{-19}\text{ C})\left(\frac{(3.00\times10^{-9}\text{ C})}{0.100\text{ m}} + \frac{(2.00\times10^{-9}\text{ C})}{0.400\text{ m}}\right) + \frac{1}{2}m_e v_b^2 = -5.04\times10^{-17}\text{ J} + \frac{1}{2}m_e v_b^2.$$

Setting $E_a = E_b$ gives $v_b = \sqrt{\dfrac{2}{9.11\times10^{-31}\text{ kg}}(5.04\times10^{-17}\text{ J} - 2.88\times10^{-17}\text{ J})} = 6.89\times10^{6}\text{ m/s}.$

EVALUATE: $V_a = V_{1a} + V_{2a} = 180\text{ V}$. $V_b = V_{1b} + V_{2b} = 315\text{ V}$. $V_b > V_a$. The negatively charged electron gains kinetic energy when it moves to higher potential.

23.17. IDENTIFY: The potential at any point is the scalar sum of the potentials due to individual charges.

SET UP: $V = kq/r$ and $W_{ab} = q(V_a - V_b)$.

EXECUTE: **(a)** $r_{a1} = r_{a2} = \dfrac{1}{2}\sqrt{(0.0300\text{ m})^2 + (0.0300\text{ m})^2} = 0.0212\text{ m}$. $V_a = k\left(\dfrac{q_1}{r_{a1}} + \dfrac{q_2}{r_{a2}}\right) = 0.$

(b) $r_{b1} = 0.0424\text{ m}$, $r_{b2} = 0.0300\text{ m}$.

$$V_b = k\left(\frac{q_1}{r_{b1}} + \frac{q_2}{r_{b2}}\right) = (8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)\left(\frac{+2.00\times10^{-6}\text{ C}}{0.0424\text{ m}} + \frac{-2.00\times10^{-6}\text{ C}}{0.0300\text{ m}}\right) = -1.75\times10^{5}\text{ V}.$$

(c) $W_{ab} = q_3(V_a - V_b) = (-5.00\times10^{-6}\text{ C})[0 - (-1.75\times10^{5}\text{ V})] = -0.875\text{ J}.$

EVALUATE: Since $V_b < V_a$, a positive charge would be pulled by the existing charges from *a* to *b*, so they would do positive work on this charge. But they would repel a negative charge and hence do negative work on it, as we found in part (c).

23.18. IDENTIFY: The total potential is the *scalar* sum of the individual potentials, but the net electric field is the *vector* sum of the two fields.

SET UP: The net potential can only be zero if one charge is positive and the other is negative, since it is a scalar. The electric field can only be zero if the two fields point in opposite directions.

EXECUTE: **(a)** (i) Since both charges have the same sign, there are no points for which the potential is zero.
(ii) The two electric fields are in opposite directions only between the two charges, and midway between them the fields have equal magnitudes. So $E = 0$ midway between the charges, but V is never zero.

(b) (i) The two potentials have equal magnitude but opposite sign midway between the charges, so $V = 0$ midway between the charges, but $E \neq 0$ there since the fields point in the same direction.
(ii) Between the two charges, the fields point in the same direction, so E cannot be zero there. In the other two regions, the field due to the nearer charge is always greater than the field due to the more distant charge, so they cannot cancel. Hence E is not zero anywhere.

EVALUATE: It does *not* follow that the electric field is zero where the potential is zero, or that the potential is zero where the electric field is zero.

23.19. IDENTIFY: Apply $V = \dfrac{1}{4\pi\epsilon_0}\sum_i \dfrac{q_i}{r_i}$.

SET UP: The locations of the charges and points *A* and *B* are sketched in Figure 23.19.

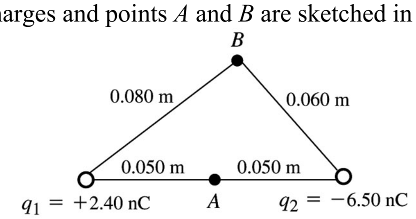


Figure 23.19

EXECUTE: **(a)** $V_A = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q_1}{r_{A1}} + \dfrac{q_2}{r_{A2}}\right).$

$$V_A = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{+2.40\times10^{-9}\ \text{C}}{0.050\ \text{m}} + \frac{-6.50\times10^{-9}\ \text{C}}{0.050\ \text{m}}\right) = -737\ \text{V}.$$

(b) $V_B = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q_1}{r_{B1}} + \dfrac{q_2}{r_{B2}}\right).$

$$V_B = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{+2.40\times10^{-9}\ \text{C}}{0.080\ \text{m}} + \frac{-6.50\times10^{-9}\ \text{C}}{0.060\ \text{m}}\right) = -704\ \text{V}.$$

(c) IDENTIFY and **SET UP:** Use $W_{a\to b} = q(V_a - V_b)$ and the results of parts (a) and (b) to calculate W.

EXECUTE: $W_{B\to A} = q(V_B - V_A) = (2.50\times10^{-9}\ \text{C})[-704\ \text{V} - (-737\ \text{V})] = +8.2\times10^{-8}\ \text{J}.$

EVALUATE: The electric force does positive work on the positive charge when it moves from higher potential (point B) to lower potential (point A).

23.20. IDENTIFY and **SET UP:** Apply conservation of energy: $K_a + U_a = K_b + U_b$. Use $V = U/q_0$ to express U in terms of V.

(a) EXECUTE: $K_1 + qV_1 = K_2 + qV_2$, $q(V_2 - V_1) = K_1 - K_2$; $q = -1.602\times10^{-19}$ C.

$K_1 = \frac{1}{2}m_e v_1^2 = 4.099\times10^{-18}$ J; $K_2 = \frac{1}{2}m_e v_2^2 = 2.915\times10^{-17}$ J. $\Delta V = V_2 - V_1 = \dfrac{K_1 - K_2}{q} = 156$ V.

EVALUATE: The electron gains kinetic energy when it moves to higher potential.

(b) EXECUTE: Now $K_1 = 2.915\times10^{-17}$ J, $K_2 = 0$. $V_2 - V_1 = \dfrac{K_1 - K_2}{q} = -182$ V.

EVALUATE: The electron loses kinetic energy when it moves to lower potential.

23.21. IDENTIFY: For a point charge, $V = \dfrac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

SET UP: **(a)** The positions of the two charges are shown in Figure 23.21a.

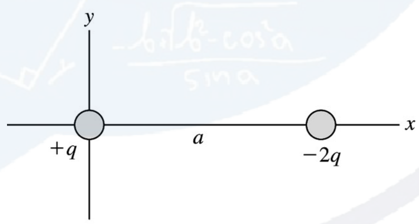


Figure 23.21a

(b) $x > a: V = \dfrac{kq}{x} - \dfrac{2kq}{x-a} = \dfrac{-kq(x+a)}{x(x-a)}$. $0 < x < a: V = \dfrac{kq}{x} - \dfrac{2kq}{a-x} = \dfrac{kq(3x-a)}{x(x-a)}$.

$x < 0: V = \dfrac{-kq}{x} + \dfrac{2kq}{x-a} = \dfrac{kq(x+a)}{x(x-a)}$. A general expression valid for any y is $V = k\left(\dfrac{q}{|x|} - \dfrac{2q}{|x-a|}\right)$.

(c) The potential is zero at $x = -a$ and $a/3$.

(d) The graph of V versus x is sketched in Figure 23.21b (next page).

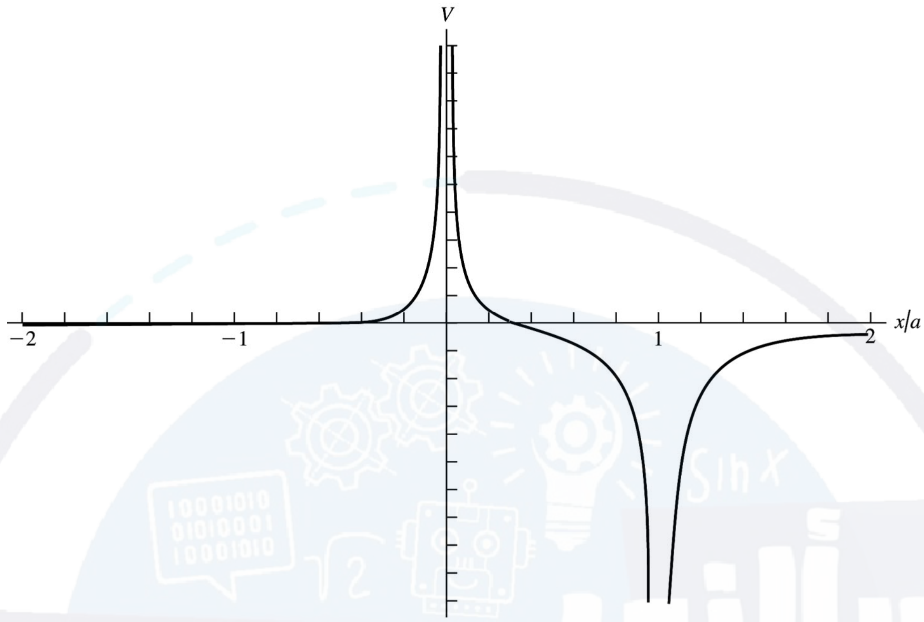


Figure 23.21b

EVALUATE: **(e)** For $x >> a: V \approx \dfrac{-kqx}{x^2} = \dfrac{-kq}{x}$, which is the same as the potential of a point charge $-q$.

Far from the two charges they appear to be a point charge with a charge that is the algebraic sum of their two charges.

23.22. **IDENTIFY:** For a point charge, $E = \dfrac{k|q|}{r^2}$ and $V = \dfrac{kq}{r}$.

SET UP: The electric field is directed toward a negative charge and away from a positive charge.

EXECUTE: **(a)** $V > 0$ so $q > 0$. $\dfrac{V}{E} = \dfrac{kq/r}{k|q|/r^2} = \left(\dfrac{kq}{r}\right)\left(\dfrac{r^2}{kq}\right) = r$. $r = \dfrac{4.98 \text{ V}}{16.2 \text{ V/m}} = 0.307$ m.

(b) $q = \dfrac{rV}{k} = \dfrac{(0.307 \text{ m})(4.98 \text{V})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 1.70 \times 10^{-10}$ C.

(c) $q > 0$, so the electric field is directed away from the charge.

EVALUATE: The ratio of V to E due to a point charge increases as the distance r from the charge increases, because E falls off as $1/r^2$ and V falls off as $1/r$.

23.23. **(a) IDENTIFY** and **EXECUTE:** The direction of $\vec{E}$ is always from high potential to low potential so point b is at higher potential.

(b) IDENTIFY and **SET UP:** Apply $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$ to relate $V_b - V_a$ to E.

EXECUTE: $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E\,dx = E(x_b - x_a)$.

$$E = \frac{V_b - V_a}{x_b - x_a} = \frac{+240 \text{ V}}{0.90 \text{ m} - 0.60 \text{ m}} = 800 \text{ V/m}$$

(c) SET UP and **EXECUTE:** $W_{b \to a} = q(V_b - V_a) = (-0.200 \times 10^{-6} \text{ C})(+240 \text{ V}) = -4.80 \times 10^{-5}$ J.

EVALUATE: The electric force does negative work on a negative charge when the negative charge moves from high potential (point b) to low potential (point a).

24.24. **IDENTIFY:** For a point charge, $V = \dfrac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges. For a point charge, $E = \dfrac{k|q|}{r^2}$. The net electric field is the vector sum of the electric fields of the two charges.

SET UP: $\vec{E}$ produced by a point charge is directed away from the point charge if it is positive and toward the charge if it is negative.

EXECUTE: **(a)** $V = V_Q + V_{2Q} > 0$, so V is zero nowhere except for infinitely far from the charges. The fields can cancel only between the charges, because only there are the fields of the two charges in opposite directions. Consider a point a distance x from Q and $d - x$ from $2Q$, as shown in Figure 23.24a.

$E_Q = E_{2Q} \rightarrow \dfrac{kQ}{x^2} = \dfrac{k(2Q)}{(d-x)^2} \rightarrow (d-x)^2 = 2x^2$. $x = \dfrac{d}{1+\sqrt{2}}$. The other root, $x = \dfrac{d}{1-\sqrt{2}}$, does not lie between the charges.

(b) V can be zero in 2 places, A and B, as shown in Figure 23.24b. Point A is a distance x from $-Q$ and $d - x$ from $2Q$. B is a distance y from $-Q$ and $d + y$ from $2Q$. At A: $\dfrac{k(-Q)}{x} + \dfrac{k(2Q)}{d-x} = 0 \rightarrow x = d/3$.

At B: $\dfrac{k(-Q)}{y} + \dfrac{k(2Q)}{d+y} = 0 \rightarrow y = d$.

The two electric fields are in opposite directions to the left of $-Q$ or to the right of $2Q$ in Figure 23.24c. But for the magnitudes to be equal, the point must be closer to the charge with smaller magnitude of charge. This can be the case only in the region to the left of $-Q$. $E_Q = E_{2Q}$ gives $\dfrac{kQ}{x^2} = \dfrac{k(2Q)}{(d+x)^2}$ and

$x = \dfrac{d}{\sqrt{2}-1}$.

EVALUATE: **(d)** E and V are not zero at the same places. $\vec{E}$ is a vector and V is a scalar. E is proportional to $1/r^2$ and V is proportional to $1/r$. $\vec{E}$ is related to the force on a test charge and ΔV is related to the work done on a test charge when it moves from one point to another.

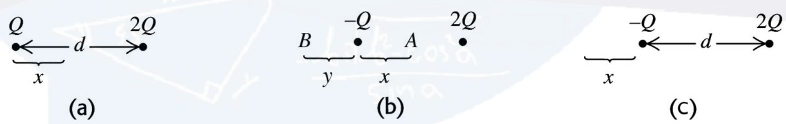


Figure 23.24

23.25. **IDENTIFY:** The potential at any point is the scalar sum of the potential due to each shell.

SET UP: $V = \dfrac{kq}{R}$ for $r \le R$ and $V = \dfrac{kq}{r}$ for $r > R$.

EXECUTE: **(a)** (i) $r = 0$. This point is inside both shells so

$$V = k\left(\frac{q_1}{R_1} + \frac{q_2}{R_2}\right) = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{6.00\times10^{-9}\ \text{C}}{0.0300\ \text{m}} + \frac{-9.00\times10^{-9}\ \text{C}}{0.0500\ \text{m}}\right).$$

$V = +1.798\times10^3\ \text{V} + (-1.618\times10^3\ \text{V}) = 180\ \text{V}$.

(ii) $r = 4.00$ cm. This point is outside shell 1 and inside shell 2.

$$V = k\left(\frac{q_1}{r} + \frac{q_2}{R_2}\right) = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{6.00\times10^{-9}\ \text{C}}{0.0400\ \text{m}} + \frac{-9.00\times10^{-9}\ \text{C}}{0.0500\ \text{m}}\right).$$

$V = +1.348\times10^3\ \text{V} + (-1.618\times10^3\ \text{V}) = -270\ \text{V}$.

(iii) $r = 6.00$ cm. This point is outside both shells.

$$V = k\left(\frac{q_1}{r} + \frac{q_2}{r}\right) = \frac{k}{r}(q_1 + q_2) = \frac{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}{0.0600\ \text{m}}\left[6.00\times10^{-9}\ \text{C} + (-9.00\times10^{-9}\ \text{C})\right].\ V = -450\ \text{V}.$$

(b) At the surface of the inner shell, $r = R_1 = 3.00$ cm. This point is inside the larger shell, so $V_1 = k\left(\frac{q_1}{R_1} + \frac{q_2}{R_2}\right) = 180$ V. At the surface of the outer shell, $r = R_2 = 5.00$ cm. This point is outside the smaller shell, so

$$V = k\left(\frac{q_1}{r} + \frac{q_2}{R_2}\right) = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{6.00\times10^{-9}\ \text{C}}{0.0500\ \text{m}} + \frac{-9.00\times10^{-9}\ \text{C}}{0.0500\ \text{m}}\right).$$

$V_2 = +1.079\times10^3\ \text{V} + (-1.618\times10^3\ \text{V}) = -539$ V. The potential difference is $V_1 - V_2 = 719$ V. The inner shell is at higher potential. The potential difference is due entirely to the charge on the inner shell.

EVALUATE: Inside a uniform spherical shell, the electric field is zero so the potential is constant (but *not* necessarily zero).

23.26. IDENTIFY and SET UP: Outside a solid conducting sphere $V = k\frac{q}{r}$. Inside the sphere the potential is constant because $E = 0$, and it has the same value as at the surface of the sphere.

EXECUTE: **(a)** This is outside the sphere, so $V = \frac{kq}{r} = \frac{k(3.50\times10^{-9}\ \text{C})}{0.480\ \text{m}} = 65.6$ V.

(b) This is at the surface of the sphere, so $V = \frac{k(3.50\times10^{-9}\ \text{C})}{0.240\ \text{m}} = 131$ V.

(c) This is inside the sphere. The potential has the same value as at the surface, 131 V.

EVALUATE: All points of a conductor are at the same potential.

23.27. (a) IDENTIFY and SET UP: The electric field on the ring's axis is given by $E_x = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}}$. The magnitude of the force on the electron exerted by this field is given by $F = eE$.

EXECUTE: When the electron is on either side of the center of the ring, the ring exerts an attractive force directed toward the center of the ring. This restoring force produces oscillatory motion of the electron along the axis of the ring, with amplitude 30.0 cm. The force on the electron is *not* of the form $F = -kx$ so the oscillatory motion is not simple harmonic motion.

(b) IDENTIFY: Apply conservation of energy to the motion of the electron.

SET UP: $K_a + U_a = K_b + U_b$ with a at the initial position of the electron and b at the center of the ring.

From Example 23.11, $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x^2+a^2}}$, where a is the radius of the ring.

EXECUTE: $x_a = 30.0$ cm, $x_b = 0$.

$K_a = 0$ (released from rest), $K_b = \frac{1}{2}mv^2$.

Thus $\frac{1}{2}mv^2 = U_a - U_b$.

And $U = qV = -eV$ so $v = \sqrt{\frac{2e(V_b - V_a)}{m}}$.

$$V_a = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x_a^2+a^2}} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{24.0\times10^{-9}\ \text{C}}{\sqrt{(0.300\ \text{m})^2 + (0.150\ \text{m})^2}}.$$

$$V_a = 643\ \text{V}.$$

$$V_b = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x_b^2+a^2}} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{24.0\times10^{-9}\ \text{C}}{0.150\ \text{m}} = 1438\ \text{V}.$$

$$v=\sqrt{\frac{2e(V_b-V_a)}{m}}=\sqrt{\frac{2(1.602\times10^{-19}\ \text{C})(1438\ \text{V}-643\ \text{V})}{9.109\times10^{-31}\ \text{kg}}}=1.67\times10^{7}\ \text{m/s}.$$

EVALUATE: The positively charged ring attracts the negatively charged electron and accelerates it. The electron has its maximum speed at this point. When the electron moves past the center of the ring the force on it is opposite to its motion and it slows down.

23.28. **IDENTIFY:** For an isolated conducting sphere, all the excess charge is on its outer surface. For points outside the sphere, it behaves like a point-charge at its center, and the electric field is zero inside the sphere.

SET UP: Use V at 1.20 m to find V at the surface. $V=k\dfrac{q}{r}$. We don't know the charge on the sphere, but we know the potential 1.20 m from its center.

EXECUTE: Take the ratio of the potentials: $\dfrac{V_{\text{surface}}}{V_{1.20\ \text{m}}}=\dfrac{kq/(0.400\ \text{m})}{kq(1.20\ \text{m})}=\dfrac{1.20}{0.400}=3.00,$ so

$V_{\text{surface}}=(3.00)(24.0\ \text{V})=72.0\ \text{V}$.

The electric field is zero inside the sphere, so the potential inside is constant and equal to the potential at the surface. So at the center $V=72.0$ V.

EVALUATE: An alternative approach would be to use the given information to find the charge on the sphere. Then use that charge to calculate the potential at the surface. The potential is 72.0 V at *all* points inside the sphere, not just at the center. Careful! Just because the electric field inside the sphere is zero, it does not follow that the potential is zero there.

23.29. **IDENTIFY:** If the small sphere is to have its *minimum* speed, it must just stop at 8.00 cm from the surface of the large sphere. In that case, the initial kinetic energy of the small sphere is all converted to electrical potential energy at its point of closest approach.

SET UP: $K_1+U_1=K_2+U_2$. $K_2=0$. $U_1=0$. Therefore, $K_1=U_2$. Outside a spherical charge distribution the potential is the same as for a point charge at the location of the center of the sphere, so $U=kqQ/r$. $K=\frac{1}{2}mv^2$.

EXECUTE: $U_2=\dfrac{kqQ}{r_2}$, with $r_2=12.0\ \text{cm}+8.0\ \text{cm}=0.200\ \text{m}$. $\dfrac{1}{2}mv_1^2=\dfrac{kqQ}{r_2}$.

$$v_1=\sqrt{\frac{2kqQ}{mr_2}}=\sqrt{\frac{2(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-6}\ \text{C})(5.00\times10^{-6}\ \text{C})}{(6.00\times10^{-5}\ \text{kg})(0.200\ \text{m})}}=150\ \text{m/s}.$$

EVALUATE: If the small sphere had enough initial speed to actually penetrate the surface of the large sphere, we could no longer treat the large sphere as a point charge once the small sphere was inside.

23.30. **IDENTIFY:** For a line of charge, $V_a-V_b=\dfrac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a)$. Apply conservation of energy to the motion of the proton.

SET UP: Let point a be 18.0 cm from the line and let point b be at the distance of closest approach, where $K_b=0$.

EXECUTE: **(a)** $K_a=\frac{1}{2}mv^2=\frac{1}{2}(1.67\times10^{-27}\ \text{kg})(3.50\times10^{3}\ \text{m/s})^2=1.02\times10^{-20}\ \text{J}$.

(b) $K_a+qV_a=K_b+qV_b$. $V_a-V_b=\dfrac{K_b-K_a}{q}=\dfrac{-1.02\times10^{-20}\ \text{J}}{1.60\times10^{-19}\ \text{C}}=-0.06397\ \text{V}$.

$$\ln(r_b/r_a)=\left(\frac{2\pi\epsilon_0}{\lambda}\right)(-0.06397\ \text{V}).$$

$$r_b=r_a\exp\left(\frac{2\pi\epsilon_0(-0.06397\ \text{V})}{\lambda}\right)=(0.180\ \text{m})\exp\left(-\frac{2\pi\epsilon_0(0.06397\ \text{V})}{5.00\times10^{-12}\ \text{C/m}}\right)=0.0883\ \text{m}=8.83\ \text{cm}.$$

EVALUATE: The potential increases with decreasing distance from the line of charge. As the positively charged proton approaches the line of charge it gains electrical potential energy and loses kinetic energy.

23.31. IDENTIFY: The voltmeter measures the potential difference between the two points. We must relate this quantity to the linear charge density on the wire.
SET UP: For a very long (infinite) wire, the potential difference between two points is given by
$\Delta V = \dfrac{\lambda}{2\pi\varepsilon_0}\ln(r_b/r_a)$.
EXECUTE: **(a)** Solving for λ gives

$$\lambda = \frac{(\Delta V)2\pi\varepsilon_0}{\ln(r_b/r_a)} = \frac{575\text{ V}}{(18\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\ln\left(\dfrac{3.50\text{ cm}}{2.50\text{ cm}}\right)} = 9.49\times10^{-8}\text{ C/m}.$$

(b) The meter will read less than 575 V because the electric field is weaker over this 1.00-cm distance than it was over the 1.00-cm distance in part (a).
(c) The potential difference is zero because both probes are at the same distance from the wire, and hence at the same potential.
EVALUATE: Since a voltmeter measures potential difference, we are actually given ΔV, even though that is not stated explicitly in the problem.

23.32. IDENTIFY: The voltmeter reads the potential difference between the two points where the probes are placed. Therefore we must relate the potential difference to the distances of these points from the center of the cylinder. For points outside the cylinder, its electric field behaves like that of a line of charge.
SET UP: Using $\Delta V = \dfrac{\lambda}{2\pi\varepsilon_0}\ln(r_b/r_a)$ and solving for r_b, we have $r_b = r_a e^{2\pi\varepsilon_0\Delta V/\lambda}$.
EXECUTE: The exponent is $\dfrac{\left(\dfrac{1}{2\times8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}\right)(175\text{ V})}{15.0\times10^{-9}\text{ C/m}} = 0.648$, which gives

$$r_b = (2.50\text{ cm})\,e^{0.648} = 4.78\text{ cm}.$$

The distance above the *surface* is $4.78\text{ cm} - 2.50\text{ cm} = 2.28\text{ cm}$.
EVALUATE: Since a voltmeter measures potential difference, we are actually given ΔV, even though that is not stated explicitly in the problem. We must also be careful when using the formula for the potential difference because each *r* is the distance from the *center* of the cylinder, not from the surface.

23.33. IDENTIFY: For points outside the cylinder, its electric field behaves like that of a line of charge. Since a voltmeter reads potential difference, that is what we need to calculate.
SET UP: The potential difference is $\Delta V = \dfrac{\lambda}{2\pi\varepsilon_0}\ln(r_b/r_a)$.
EXECUTE: **(a)** Substituting numbers gives

$$\Delta V = \frac{\lambda}{2\pi\varepsilon_0}\ln(r_b/r_a) = (8.50\times10^{-6}\text{ C/m})(2\times9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\ln\left(\frac{10.0\text{ cm}}{6.00\text{ cm}}\right).$$

$$\Delta V = 7.82\times10^4\text{ V} = 78{,}200\text{ V} = 78.2\text{ kV}.$$

(b) $E = 0$ inside the cylinder, so the potential is constant there, meaning that the voltmeter reads zero.
EVALUATE: Caution! The fact that the voltmeter reads zero in part (b) does not mean that $V = 0$ inside the cylinder. The electric field is zero, but the potential is constant and equal to the potential at the surface.

23.34. IDENTIFY: The work required is equal to the change in the electrical potential energy of the charge-ring system. We need only look at the beginning and ending points, since the potential difference is independent of path for a conservative field.
SET UP: **(a)** $W = \Delta U = q\Delta V = q(V_{\text{center}} - V_\infty) = q\left(\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{a} - 0\right)$.
EXECUTE: Substituting numbers gives

$$\Delta U = (3.00\times10^{-6}\text{ C})(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-6}\text{ C})/(0.0400\text{ m}) = 3.38\text{ J}.$$

(b) We can take any path since the potential is independent of path.

(c) SET UP: The net force is away from the ring, so the ball will accelerate away. Energy conservation gives $U_0 = K_{\text{max}} = \frac{1}{2}mv^2.$

EXECUTE: Solving for v gives

$$v = \sqrt{\frac{2U_0}{m}} = \sqrt{\frac{2(3.38\text{ J})}{0.00150\text{ kg}}} = 67.1\text{ m/s}.$$

EVALUATE: Direct calculation of the work from the electric field would be extremely difficult, and we would need to know the path followed by the charge. But, since the electric field is conservative, we can bypass all this calculation just by looking at the end points (infinity and the center of the ring) using the potential.

23.35. **IDENTIFY:** The electric field of the line of charge does work on the sphere, increasing its kinetic energy.

SET UP: $K_1 + U_1 = K_2 + U_2$ and $K_1 = 0$. $U = qV$ so $qV_1 = K_2 + qV_2$. $V = \dfrac{\lambda}{2\pi\varepsilon_0}\ln\left(\dfrac{r_0}{r}\right)$.

EXECUTE: $V_1 = \dfrac{\lambda}{2\pi\varepsilon_0}\ln\left(\dfrac{r_0}{r_1}\right)$. $V_2 = \dfrac{\lambda}{2\pi\varepsilon_0}\ln\left(\dfrac{r_0}{r_2}\right)$.

$$K_2 = q(V_1 - V_2) = \frac{q\lambda}{2\pi\varepsilon_0}\left(\ln\left(\frac{r_0}{r_1}\right) - \ln\left(\frac{r_0}{r_2}\right)\right) = \frac{\lambda q}{2\pi\varepsilon_0}(\ln r_2 - \ln r_1) = \frac{\lambda q}{2\pi\varepsilon_0}\ln\left(\frac{r_2}{r_1}\right).$$

$$K_2 = \frac{(3.00\times10^{-6}\text{ C/m})(8.00\times10^{-6}\text{ C})}{2\pi(8.854\times10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2))}\ln\left(\frac{4.50}{1.50}\right) = 0.474\text{ J}.$$

EVALUATE: The potential due to the line of charge does *not* go to zero at infinity but is defined to be zero at an arbitrary distance r_0 from the line.

23.36. **IDENTIFY** and **SET UP:** For oppositely charged parallel plates, $E = \sigma/\varepsilon_0$ between the plates and the potential difference between the plates is $V = Ed$.

EXECUTE: **(a)** $E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{47.0\times10^{-9}\text{ C/m}^2}{\varepsilon_0} = 5310\text{ N/C}.$

(b) $V = Ed = (5310\text{ N/C})(0.0220\text{ m}) = 117\text{ V}.$

(c) The electric field stays the same if the separation of the plates doubles. The potential difference between the plates doubles.

EVALUATE: The electric field of an infinite sheet of charge is uniform, independent of distance from the sheet. The force on a test charge between the two plates is constant because the electric field is constant. The potential difference is the work per unit charge on a test charge when it moves from one plate to the other. When the distance doubles, the work, which is force times distance, doubles and the potential difference doubles.

23.37. **IDENTIFY** and **SET UP:** Use $\Delta V = Ed$ to relate the electric field between the plates to the potential difference between them and their separation. The magnitude of the force this field exerts on the particle is given by $F = qE$. Use $W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l}$ to calculate the work.

EXECUTE: **(a)** Using $\Delta V = Ed$ gives $E = \dfrac{V_{ab}}{d} = \dfrac{360\text{ V}}{0.0450\text{ m}} = 8000\text{ V/m}.$

(b) $F = |q|E = (2.40\times10^{-9}\text{ C})(8000\text{ V/m}) = +1.92\times10^{-5}\text{ N}.$

(c) The electric field between the plates is shown in Figure 23.37.

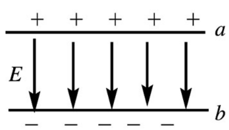


Figure 22.37

The plate with positive charge (plate a) is at higher potential. The electric field is directed from high potential toward low potential (or, $\vec{E}$ is from + charge toward − charge), so $\vec{E}$ points from a to b. Hence the force that $\vec{E}$ exerts on the positive charge is from a to b, so it does positive work.

$W = \int_a^b \vec{F} \cdot d\vec{l} = Fd$, where d is the separation between the plates.

$W = Fd = (1.92\times10^{-5}\ \text{N})(0.0450\ \text{m}) = +8.64\times10^{-7}\ \text{J}.$

(d) $V_a - V_b = +360\ \text{V}$ (plate a is at higher potential).

$\Delta U = U_b - U_a = q(V_b - V_a) = (2.40\times10^{-9}\ \text{C})(-360\ \text{V}) = -8.64\times10^{-7}\ \text{J}.$

EVALUATE: We see that $W_{a\to b} = -(U_b - U_a) = U_a - U_b$.

23.38. **IDENTIFY** and **SET UP:** $V_{ab} = Ed$ for parallel plates.

EXECUTE: $d = \dfrac{V_{ab}}{E} = \dfrac{1.5\ \text{V}}{1.0\times10^{-6}\ \text{V/m}} = 1.5\times10^{6}\ \text{m} = 1.5\times10^{3}\ \text{km}.$

EVALUATE: The plates would have to be nearly a thousand miles apart with only a AA battery across them! This is a small field!

23.39. **IDENTIFY:** The potential of a solid conducting sphere is the same at every point inside the sphere because $E = 0$ inside, and this potential has the value $V = q/4\pi\epsilon_0 R$ at the surface. Use the given value of E to find q.

SET UP: For negative charge the electric field is directed toward the charge.
For points outside this spherical charge distribution the field is the same as if all the charge were concentrated at the center.

EXECUTE: $E = \dfrac{|q|}{4\pi\epsilon_0 r^2}$ and $|q| = 4\pi\epsilon_0 E r^2 = \dfrac{(3800\ \text{N/C})(0.200\ \text{m})^2}{8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2} = 1.69\times10^{-8}\ \text{C}.$

Since the field is directed inward, the charge must be negative. The potential of a point charge, taking ∞ as zero, is $V = \dfrac{q}{4\pi\epsilon_0 r} = \dfrac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(-1.69\times10^{-8}\ \text{C})}{0.200\ \text{m}} = -760\ \text{V}$ at the surface of the sphere.

Since the charge all resides on the surface of a conductor, the field inside the sphere due to this symmetrical distribution is zero. No work is therefore done in moving a test charge from just inside the surface to the center, and the potential at the center must also be –760 V.

EVALUATE: Inside the sphere the electric field is zero and the potential is constant.

23.40. **IDENTIFY:** The electric field is zero inside the sphere, so the potential is constant there. Thus the potential at the center must be the same as at the surface, where it is equivalent to that of a point-charge.

SET UP: At the surface, and hence also at the center of the sphere, the potential is that of a point-charge, $V = Q/(4\pi\epsilon_0 R)$.

EXECUTE: **(a)** Solving for Q and substituting the numbers gives

$$Q = 4\pi\epsilon_0 RV = (0.125\ \text{m})(3750\ \text{V})/(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2) = 5.21\times10^{-8}\ \text{C} = 52.1\ \text{nC}.$$

(b) Since the potential is constant inside the sphere, its value at the surface must be the same as at the center, 3.75 kV.

EVALUATE: The electric field inside the sphere is zero, so the potential is constant but is not zero.

23.41. **IDENTIFY** and **SET UP:** For a solid metal sphere or for a spherical shell, $V = \dfrac{kq}{r}$ outside the sphere and $V = \dfrac{kq}{R}$ at all points inside the sphere, where R is the radius of the sphere. When the electric field is radial, $E = -\dfrac{\partial V}{\partial r}$.

EXECUTE: **(a)** (i) $r < r_a$: This region is inside both spheres. $V = \dfrac{kq}{r_a} - \dfrac{kq}{r_b} = kq\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)$.

(ii) $r_a < r < r_b$: This region is outside the inner shell and inside the outer shell. $V = \dfrac{kq}{r} - \dfrac{kq}{r_b} = kq\left(\dfrac{1}{r} - \dfrac{1}{r_b}\right)$.

(iii) $r > r_b$: This region is outside both spheres and $V = 0$ since outside a sphere the potential is the same as for a point charge. Therefore the potential is the same as for two oppositely charged point charges at the same location. These potentials cancel.

(b) $V_a = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{r_a} - \dfrac{q}{r_b}\right)$ and $V_b = 0$, so $V_{ab} = \dfrac{1}{4\pi\varepsilon_0} q\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)$.

(c) Between the spheres $r_a < r < r_b$ and $V = kq\left(\dfrac{1}{r} - \dfrac{1}{r_b}\right)$.

$$E_r = -\frac{\partial V}{\partial r} = -\frac{q}{4\pi\varepsilon_0}\frac{\partial}{\partial r}\left(\frac{1}{r} - \frac{1}{r_b}\right) = +\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{V_{ab}}{\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)}\frac{1}{r^2}.$$

(d) Since $E_r = -\dfrac{\partial V}{\partial r}$, $E = 0$, since V is constant (zero) outside the spheres.

(e) If the outer charge is different, then outside the outer sphere the potential is no longer zero but is $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r} - \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(q-Q)}{r}$. All potentials inside the outer shell are just shifted by an amount $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r_b}$. Therefore relative potentials within the shells are not affected. Thus (b) and (c) do not change. However, now that the potential does vary outside the spheres, there is an electric field there:

$$E = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{kq}{r} + \frac{-kQ}{r}\right) = \frac{kq}{r^2}\left(1 - \frac{Q}{q}\right) = \frac{k}{r^2}(q - Q).$$

EVALUATE: In part (a) the potential is greater than zero for all $r < r_b$.

23.42. IDENTIFY: By the definition of electric potential, if a positive charge gains potential along a path, then the potential along that path must have increased. The electric field produced by a very large sheet of charge is uniform and is independent of the distance from the sheet.

(a) SET UP: No matter what the reference point, we must do work on a positive charge to move it away from the negative sheet.

EXECUTE: Since we must do work on the positive charge, it gains potential energy, so the potential increases.

(b) SET UP: Since the electric field is uniform and is equal to $\sigma/2\varepsilon_0$, we have $\Delta V = Ed = \dfrac{\sigma}{2\varepsilon_0}d$.

EXECUTE: Solving for d gives

$$d = \frac{2\varepsilon_0 \Delta V}{\sigma} = \frac{2(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(1.00\ \mathrm{V})}{6.00\times10^{-9}\ \mathrm{C/m^2}} = 0.00295\ \mathrm{m} = 2.95\ \mathrm{mm}.$$

EVALUATE: Since the spacing of the equipotential surfaces (d = 2.95 mm) is independent of the distance from the sheet, the equipotential surfaces are planes parallel to the sheet and spaced 2.95 mm apart.

23.43. IDENTIFY and **SET UP:** Use $E_x = -\dfrac{\partial V}{\partial x}$, $E_y = -\dfrac{\partial V}{\partial y}$, and $E_z = \dfrac{\partial V}{\partial z}$ to calculate the components of $\vec{E}$.

EXECUTE: $V = Axy - Bx^2 + Cy$.

(a) $E_x = -\dfrac{\partial V}{\partial x} = -Ay + 2Bx$.

$E_y = -\dfrac{\partial V}{\partial y} = -Ax - C$.

$E_z = \dfrac{\partial V}{\partial z} = 0.$

(b) $E = 0$ requires that $E_x = E_y = E_z = 0.$

$E_z = 0$ everywhere.

$E_y = 0$ at $x = -C/A.$

And E_x is also equal to zero for this x, any value of z and $y = 2Bx/A = (2B/A)(-C/A) = -2BC/A^2.$

EVALUATE: V doesn't depend on z so $E_z = 0$ everywhere.

23.44. IDENTIFY: Apply $E_x = -\dfrac{\partial V}{\partial x}$ and $E_y = -\dfrac{\partial V}{\partial y}$ to find the components of $\vec{E}$, then use them to find its magnitude and direction. $V(x, y) = Ax^2y - Bxy^2$.

SET UP: $E = \sqrt{E_x^2 + E_y^2}$ and $\tan\theta = E_y/E_x$.

EXECUTE: First find the components of $\vec{E}$: $E_x = -\dfrac{\partial V}{\partial x} = -\dfrac{\partial}{\partial x}(Ax^2y - Bxy^2) = -(2Axy - By^2).$

Now evaluate this result at the point x = 2.00 m, y = 0.400 m using the given values for A and B.

$E_x = -[2(5.00\ \text{V/m}^3)(2.00\ \text{m})(0.400\ \text{m}) - (8.00\ \text{V/m}^3)(0.400\ \text{m})^2] = -6.72\ \text{V/m}.$

$E_y = -\dfrac{\partial V}{\partial y} = -\dfrac{\partial}{\partial y}(Ax^2y - Bxy^2) = -(Ax^2 - 2Bxy).$ At the point (2.00 m, 0.400 m), this is

$E_y = -[(5.00\ \text{V/m}^3)(2.00\ \text{m})^2 - 2(8.00\ \text{V/m}^3)(2.00\ \text{m})(0.400\ \text{m})] = -7.20\ \text{V/m}.$

Now use the components to find the magnitude and direction of $\vec{E}$.

$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(-6.72\ \text{V/m})^2 + (-7.20\ \text{V/m})^2} = 9.85\ \text{V/m}.$

$\tan\theta = E_y/E_x$ = (–7.20 V/m)/(–6.72 V/m), which gives θ = 47.0°. Since both components are negative, the vector lies in the third quadrant in the xy-plane and makes an angle of 47.0° + 180.0° = 227.0° with the +x-axis.

EVALUATE: V is a scalar but $\vec{E}$ is a vector and has components.

23.45. IDENTIFY: Exercise 23.41 shows that $V = kq\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)$ for $r < r_a$, $V = kq\left(\dfrac{1}{r} - \dfrac{1}{r_b}\right)$ for $r_a < r < r_b$ and

$V_{ab} = kq\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right).$

SET UP: $E = \dfrac{kq}{r^2}$, radially outward, for $r_a \le r \le r_b$.

EXECUTE: **(a)** $V_{ab} = kq\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right) = 500\ \text{V}$ gives $q = \dfrac{500\ \text{V}}{k\left(\dfrac{1}{0.012\ \text{m}} - \dfrac{1}{0.096\ \text{m}}\right)} = 7.62\times10^{-10}\ \text{C} = 0.762\ \text{nC}.$

(b) $V_b = 0$ so $V_a = 500$ V. The inner metal sphere is an equipotential with $V = 500$ V. $\dfrac{1}{r} = \dfrac{1}{r_a} + \dfrac{V}{kq}.$

$V = 400$ V at $r = 1.45$ cm, $V = 300$ V at $r = 1.85$ cm, $V = 200$ V at $r = 2.53$ cm, $V = 100$ V at $r = 4.00$ cm, $V = 0$ at $r = 9.60$ cm. The equipotential surfaces are sketched in Figure 23.45.

EVALUATE: **(c)** The equipotential surfaces are concentric spheres and the electric field lines are radial, so the field lines and equipotential surfaces are mutually perpendicular. The equipotentials are closest at smaller r, where the electric field is largest.

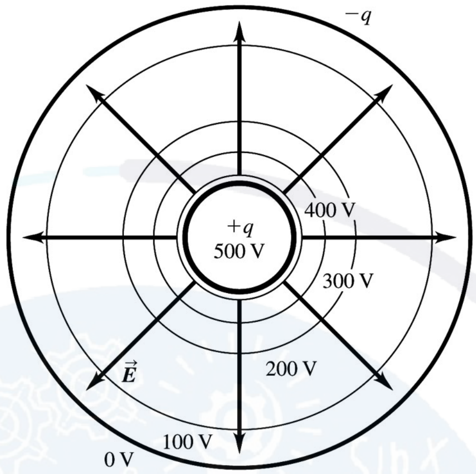


Figure 23.45

23.46. **IDENTIFY:** As the sphere approaches the point charge, the speed of the sphere decreases because it loses kinetic energy, but its acceleration increases because the electric force on it increases. Its mechanical energy is conserved during the motion, and Newton's second law and Coulomb's law both apply.

SET UP: $K_a + U_a = K_b + U_b$, $K = \frac{1}{2}mv^2$, $U = kq_1q_2/r$, $F = kq_1q_2/r^2$, and $F = ma$.

EXECUTE: Find the distance between the two charges when $v_2 = 25.0$ m/s.

$K_a + U_a = K_b + U_b$.

$$K_a = \frac{1}{2}mv_a^2 = \frac{1}{2}(4.00\times10^{-3}\text{ kg})(40.0\text{ m/s})^2 = 3.20\text{ J}.$$

$$K_b = \frac{1}{2}mv_b^2 = \frac{1}{2}(4.00\times10^{-3}\text{ kg})(25.0\text{ m/s})^2 = 1.25\text{ J}.$$

$$U_a = k\frac{q_1q_2}{r_a} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-6}\text{ C})(2.00\times10^{-6}\text{ C})}{0.0600\text{ m}} = 1.498\text{ J}.$$

$U_b = K_a + U_a - K_b = 3.20\text{ J} + 1.498\text{ J} - 1.25\text{ J} = 3.448\text{ J}$. $U_b = k\dfrac{q_1q_2}{r_b}$ and

$$r_b = \frac{kq_1q_2}{U_b} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-6}\text{ C})(2.00\times10^{-6}\text{ C})}{3.448\text{ J}} = 0.02607\text{ m}.$$

$$F_b = \frac{kq_1q_2}{r_b^2} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-6}\text{ C})(2.00\times10^{-6}\text{ C})}{(0.02607\text{ m})^2} = 132.3\text{ N}.$$

$$a = \frac{F}{m} = \frac{132.3\text{ N}}{4.00\times10^{-3}\text{ kg}} = 3.31\times10^4\text{ m/s}^2.$$

EVALUATE: As the sphere approaches the point charge, its speed decreases but its acceleration keeps increasing because the electric force on it keeps increasing.

23.47. **IDENTIFY:** $U = k\left(\dfrac{q_1q_2}{r_{12}} + \dfrac{q_1q_3}{r_{13}} + \dfrac{q_2q_3}{r_{23}}\right)$.

SET UP: In part (a), $r_{12} = 0.200$ m, $r_{23} = 0.100$ m and $r_{13} = 0.100$ m. In part (b) let particle 3 have coordinate *x*, so $r_{12} = 0.200$ m, $r_{13} = x$ and $r_{23} = 0.200\text{ m} - x$.

EXECUTE: **(a)** $U = k\left(\dfrac{(4.00\text{ nC})(-3.00\text{ nC})}{(0.200\text{ m})} + \dfrac{(4.00\text{ nC})(2.00\text{ nC})}{(0.100\text{ m})} + \dfrac{(-3.00\text{ nC})(2.00\text{ nC})}{(0.100\text{ m})}\right) = -3.60\times10^{-7}\text{ J}.$

(b) If $U=0$, then $0=k\left(\frac{q_1q_2}{r_{12}}+\frac{q_1q_3}{x}+\frac{q_2q_3}{r_{12}-x}\right)$. Solving for x we find:

$0=-60+\frac{8}{x}-\frac{6}{0.2-x}\Rightarrow 60x^2-26x+1.6=0\Rightarrow x=0.074\text{ m}, 0.360\text{ m}$. Therefore, $x=0.074\text{ m}$ since it is the only value between the two charges.

Evaluate: U_{13} is positive and both U_{23} and U_{12} are negative. If $U=0$, then $|U_{13}|=|U_{23}|+|U_{12}|$. For $x=0.074\text{ m}$, $U_{13}=+9.7\times10^{-7}\text{ J}$, $U_{23}=-4.3\times10^{-7}\text{ J}$ and $U_{12}=-5.4\times10^{-7}\text{ J}$. It is true that $U=0$ at this x.

23.48. **Identify:** The electric field of the fixed charge does work on the charged object and therefore changes it kinetic energy. We apply the work-energy theorem.

Set Up: $W_{a\to b}=\Delta K$ and $W_{a\to b}=q(V_a-V_b)$, $V=k\frac{q}{r}$.

Execute: $W_{a\to b}=\Delta K=K_b-K_a=q_2(V_a-V_b)$, which gives $K_b=K_a+q_2(V_a-V_b)$.

$K_b=\frac{1}{2}mv_a^2+q_2\left(\frac{kq_1}{r_a}-\frac{kq_1}{r_b}\right)=\frac{1}{2}mv_a^2+kq_1q_2\left(\frac{1}{r_a}-\frac{1}{r_b}\right)$.

Putting in the numbers gives

$K_b=\frac{1}{2}(0.00400\text{ kg})(800\text{ m/s})^2+(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-4}\text{ C})(-3.00\times10^{-4}\text{ C})\times$ $[1/(0.400\text{ m})-1/(0.200\text{ m})]$.

$K_b = 4651$ J.

$v_b=(2K_b/m)^{1/2}=[2(4651\text{ J})/(0.00400\text{ kg})]^{1/2}=1520\text{ m/s}$.

Evaluate: The negatively charged small object gains kinetic energy because it is attracted by the positive charge q_1, which does positive work on the object, so $v_b > v_a$.

23.49. **Identify** and **Set Up:** Treat the gold nucleus as a point charge so that $V=k\frac{q}{r}$. According to conservation of energy we have $K_1+U_1=K_2+U_2$, where $U=qV$.

Execute: Assume that the alpha particle is at rest before it is accelerated and that it momentarily stops when it arrives at its closest approach to the surface of the gold nucleus. Thus we have $K_1=K_2=0$, which implies that $U_1=U_2$. Since $U=qV$ we conclude that the accelerating voltage must be equal to the voltage at its point of closest approach to the surface of the gold nucleus. Therefore

$$V_a=V_b=k\frac{q}{r}=(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{79(1.60\times10^{-19}\text{ C})}{(7.3\times10^{-15}\text{ m}+2.0\times10^{-14}\text{ m})}=4.2\times10^6\text{ V}.$$

Evaluate: Although the alpha particle has kinetic energy as it approaches the gold nucleus this is irrelevant to our solution since energy is conserved for the whole process.

23.50. **Identify:** Two forces do work on the sphere as it falls: gravity and the electrical force due to the sheet. The energy of the sphere is conserved.

Set Up: The gravity force is mg, downward. The electric field of the sheet is $E=\frac{\sigma}{2\epsilon_0}$ upward, and the force it exerts on the sphere is $F=qE$. The sphere gains kinetic energy $K=\frac{1}{2}mv^2$ as it falls.

Execute: $mg=4.90\times10^{-6}\text{ N}$. $E=\frac{\sigma}{2\epsilon_0}=\frac{8.00\times10^{-12}\text{ C/m}^2}{2(8.854\times10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2))}=0.4518\text{ N/C}$. The electric force is $qE=(7.00\times10^{-6}\text{ C})(0.4518\text{ N/C})=3.1626\times10^{-6}\text{ N}$, upward. The net force is downward, so the sphere moves downward when released. Let $y=0$ at the sheet. $U_{\text{grav}}=mgy$. For the electric force, $\frac{W_{a\to b}}{q}=V_a-V_b$. Let point a be at the sheet and let point b be a distance y above the sheet. Take $V_a=0$.

The force on q is qE, upward, so $\dfrac{W_{a\to b}}{q} = Ey$ and $V_b = -Ey$. $U_b = -Eyq$. $K_1 + U_1 = K_2 + U_2$. $K_1 = 0$.
$y_1 = 0.400\text{ m}$, $y_2 = 0.100\text{ m}$. $K_2 = U_1 - U_2 = mg(y_1 - y_2) - E(y_1 - y_2)q$.
$K_2 = (5.00\times10^{-7}\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m}) - (0.4518\text{ N/C})(0.300\text{ m})(7.00\times10^{-6}\text{ C})$.
$K_2 = 1.470\times10^{-6}\text{ J} - 0.94878\times10^{-6}\text{ J} = 0.52122\times10^{-6}\text{ J}$. $K_2 = \frac{1}{2}mv_2^2$ so

$$v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(0.52122\times10^{-6}\text{ J})}{5.00\times10^{-7}\text{ kg}}} = 1.44\text{ m/s}.$$

EVALUATE: Because the weight is greater than the electric force, the sphere will accelerate downward, but if it were light enough the electric force would exceed the weight. In that case it would never get closer to the sheet after being released. We could also solve this problem using Newton's second law and the constant-acceleration kinematics formulas. $a = F/m = (mg - qE)/m$ gives the acceleration. Then we use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ with $v_{0x} = 0$ to find v.

23.51. **IDENTIFY:** The remaining nucleus (radium minus the ejected alpha particle) repels the alpha particle, giving it 4.79 MeV of kinetic energy when it is far from the nucleus. The mechanical energy of the system is conserved.

SET UP: $U = k\dfrac{q_1q_2}{r}$. $U_a + K_a = U_b + K_b$. The charge of the alpha particle is $+2e$ and the charge of the radon nucleus is $+86e$.

EXECUTE: **(a)** The final energy of the alpha particle, 4.79 MeV, equals the electrical potential energy of the alpha-radon combination just before the decay. $U = 4.79\text{ MeV} = 7.66\times10^{-13}\text{ J}$.

(b) $r = \dfrac{kq_1q_2}{U} = \dfrac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2)(86)(1.60\times10^{-19}\text{ C})^2}{7.66\times10^{-13}\text{ J}} = 5.17\times10^{-14}\text{ m}.$

EVALUATE: Although we have made some simplifying assumptions (such as treating the atomic nucleus as a spherically symmetric charge, even when very close to it), this result gives a fairly reasonable estimate for the size of a nucleus.

23.52. **IDENTIFY:** The charged particles repel each other and therefore accelerate away from one another, causing their speeds and kinetic energies to continue to increase. They do not have equal speeds because they have different masses. The mechanical energy and momentum of the system are conserved.

SET UP: The proton has charge $q_\text{p} = +e$ and mass $m_\text{p} = 1.67\times10^{-27}\text{ kg}$. The alpha particle has charge $q_\text{a} = +2e$ and mass $m_\text{a} = 4m_\text{p} = 6.68\times10^{-27}\text{ kg}$. We can apply both conservation of energy and conservation of linear momentum to the system. $a = \dfrac{F}{m}$, where $F = k\dfrac{|q_1q_2|}{r^2}$.

EXECUTE: *Acceleration:* The maximum force and hence the maximum acceleration occurs just after they are released, when $r = 0.225\text{ nm}$. $F = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{(2)(1.60\times10^{-19}\text{ C})^2}{(0.225\times10^{-9}\text{ m})^2} = 9.09\times10^{-9}\text{ N}$.

$a_\text{p} = \dfrac{F}{m_\text{p}} = \dfrac{9.09\times10^{-9}\text{ N}}{1.67\times10^{-27}\text{ kg}} = 5.44\times10^{18}\text{ m/s}^2$; $a_\text{a} = \dfrac{F}{m_\text{a}} = \dfrac{9.09\times10^{-9}\text{ N}}{6.68\times10^{-27}\text{ kg}} = 1.36\times10^{18}\text{ m/s}^2$. The acceleration of the proton is larger by a factor of m_a/m_p.

Speed: Conservation of energy says $U_1 + K_1 = U_2 + K_2$. $K_1 = 0$ and $U_2 = 0$, so $K_2 = U_1$.

$U_1 = k\dfrac{q_1q_2}{r} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{(2)(1.60\times10^{-19}\text{ C})^2}{0.225\times10^{-9}\text{ m}} = 2.05\times10^{-18}\text{ J}$, so the total kinetic energy of the two particles when they are far apart is $K_2 = 2.05\times10^{-18}$ J. Conservation of linear momentum says how this energy is divided between the proton and alpha particle. $p_1 = p_2$. $0 = m_\text{p}v_\text{p} - m_\text{a}v_\text{a}$ so $v_\text{a} = \left(\dfrac{m_\text{p}}{m_\text{a}}\right)v_\text{p}$.

$$K_2 = \tfrac{1}{2}m_\text{p}v_\text{p}^2 + \tfrac{1}{2}m_\text{a}v_\text{a}^2 = \tfrac{1}{2}m_\text{p}v_\text{p}^2 + \tfrac{1}{2}m_\text{a}\left(\frac{m_\text{p}}{m_\text{a}}\right)^2 v_\text{p}^2 = \tfrac{1}{2}m_\text{p}v_\text{p}^2\left(1+\frac{m_\text{p}}{m_\text{a}}\right).$$

$$v_\text{p} = \sqrt{\frac{2K_2}{m_\text{p}(1+(m_\text{p}/m_\text{a}))}} = \sqrt{\frac{2(2.05\times10^{-18}\ \text{J})}{(1.67\times10^{-27}\ \text{kg})(1+\frac{1}{4})}} = 4.43\times10^4\ \text{m/s}.$$

$v_\text{a} = \left(\dfrac{m_\text{p}}{m_\text{a}}\right)v_\text{p} = \frac{1}{4}(4.43\times10^4\ \text{m/s}) = 1.11\times10^4\ \text{m/s}$. The maximum acceleration occurs just after they are released. The maximum speed occurs after a long time.

EVALUATE: The proton and alpha particle have equal momenum, but proton has a greater acceleration and more kinetic energy.

23.53. **(a) IDENTIFY:** Apply the work-energy theorem.

SET UP: Points a and b are shown in Figure 23.53a.

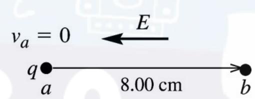


Figure 23.53a

EXECUTE: $W_\text{tot} = \Delta K = K_b - K_a = K_b = 4.35\times10^{-5}$ J.

The electric force F_E and the additional force F both do work, so that $W_\text{tot} = W_{F_E} + W_F$.

$$W_{F_E} = W_\text{tot} - W_F = 4.35\times10^{-5}\ \text{J} - 6.50\times10^{-5}\ \text{J} = -2.15\times10^{-5}\ \text{J}.$$

EVALUATE: The forces on the charged particle are shown in Figure 23.53b.

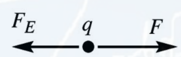


Figure 23.53b

The electric force is to the left (in the direction of the electric field since the particle has positive charge). The displacement is to the right, so the electric force does negative work. The additional force F is in the direction of the displacement, so it does positive work.

(b) IDENTIFY and SET UP: For the work done by the electric force, $W_{a\to b} = q(V_a - V_b)$.

EXECUTE: $V_a - V_b = \dfrac{W_{a\to b}}{q} = \dfrac{-2.15\times10^{-5}\ \text{J}}{7.60\times10^{-9}\ \text{C}} = -2.83\times10^3\ \text{V}.$

EVALUATE The starting point (point a) is at 2.83×10^3 V lower potential than the ending point (point b). We know that $V_b > V_a$ because the electric field always points from high potential toward low potential.

(c) IDENTIFY: Calculate E from $V_a - V_b$ and the separation d between the two points.

SET UP: Since the electric field is uniform and directed opposite to the displacement $W_{a\to b} = -F_E d = -qEd$, where $d = 8.00$ cm is the displacement of the particle.

EXECUTE: $E = -\dfrac{W_{a\to b}}{qd} = -\dfrac{V_a - V_b}{d} = -\dfrac{-2.83\times10^3\ \text{V}}{0.0800\ \text{m}} = 3.54\times10^4\ \text{V/m}.$

EVALUATE: In part (a), W_tot is the total work done by both forces. In parts (b) and (c) $W_{a\to b}$ is the work done just by the electric force.

23.54. IDENTIFY: The net force on q_0 is the vector sum of the forces due to the two charges. Coulomb's law applies.

SET UP: $F = k\dfrac{|q_1 q_2|}{r^2}$, $W_{a\to b} = q(V_a - V_b)$, $V = k\dfrac{q}{r}$.

EXECUTE: **(a)** The magnitude of the force on q_0 due to each of the two charges at opposite corners of the square is $F = k\dfrac{|q_1 q_2|}{r^2} = k(5.00\ \mu\text{C})(3.00\ \mu\text{C})/(0.0800\ \text{m})^2 = 21.07$ N. Adding the two forces vectorially gives the net force $F_{\text{net}} = (21.07\ \text{N})\sqrt{2} = 29.8$ N. The direction is from A to B since both charges attract q_0. Figure 23.54 shows this force.

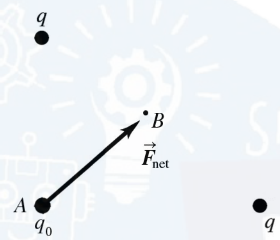


Figure 23.54

(b) At point B the two forces on q_0 are in opposite directions and have equal magnitudes, so they add to zero: $F_{\text{net}} = 0$.

(c) For each charge, $W_{A\to B} = q(V_A - V_B)$, so for both we must double this. Using $V = k\dfrac{q}{r}$ and simplifying we get $W_{A\to B} = 2q(V_A - V_B) = 2kqq_0\left(\dfrac{1}{r_A} - \dfrac{1}{r_B}\right)$. Putting in $q_0 = -3.00\ \mu$C, $q = 5.00\ \mu$C, $r_A = 0.0800$ m, and $r_b = 0.0400\sqrt{2}$ m, we get $W_{A\to B} = +1.40$ J. The work done on q_0 by the electric field is positive since it this charge moves from A to B in the direction of the force. The charge loses potential energy as it gains kinetic energy. But since q_0 is negative, it moves to a point of higher potential.

EVALUATE: Positive charges accelerate toward lower potential, but negative charges accelerate toward higher potential.

23.55. IDENTIFY and SET UP: Calculate the components of $\vec{E}$ using $E_x = -\dfrac{\partial V}{\partial x}$, $E_y = -\dfrac{\partial V}{\partial y}$, and $E_z = -\dfrac{\partial V}{\partial z}$, and use $\vec{F} = q\vec{E}$.

EXECUTE: **(a)** $V = Cx^{4/3}$.

$$C = V/x^{4/3} = 240\ \text{V}/(13.0\times10^{-3}\ \text{m})^{4/3} = 7.85\times10^4\ \text{V/m}^{4/3}.$$

(b) $E_x(x) = -\dfrac{\partial V}{\partial x} = -\dfrac{4}{3}Cx^{1/3} = -(1.05\times10^5\ \text{V/m}^{4/3})x^{1/3}$.

The minus sign means that E_x is in the $-x$-direction, which says that $\vec{E}$ points from the positive anode toward the negative cathode.

(c) $\vec{F} = q\vec{E}$ so $F_x = -eE_x = \frac{4}{3}eCx^{1/3}$.

Halfway between the electrodes means $x = 6.50\times10^{-3}$ m.

$$F_x = \tfrac{4}{3}(1.602\times10^{-19}\ \text{C})(7.85\times10^4\ \text{V/m}^{4/3})(6.50\times10^{-3}\ \text{m})^{1/3} = 3.13\times10^{-15}\ \text{N}.$$

F_x is positive, so the force is directed toward the positive anode.

EVALUATE: V depends only on x, so $E_y = E_z = 0$. $\vec{E}$ is directed from high potential (anode) to low potential (cathode). The electron has negative charge, so the force on it is directed opposite to the electric field.

23.56. IDENTIFY: At each point (a and b), the potential is the sum of the potentials due to *both* spheres. The voltmeter reads the difference between these two potentials. The spheres behave like point charges since the meter is connected to the surface of each one.

SET UP: **(a)** Call a the point on the surface of one sphere and b the point on the surface of the other sphere, call r the radius of each sphere and call d the center-to-center distance between the spheres. The potential difference V_{ba} between points a and b is then

$$V_b - V_a = V_{ba} = \frac{1}{4\pi\varepsilon_0}\left[\frac{-q}{r} + \frac{q}{d-r} - \left(\frac{q}{r} + \frac{-q}{d-r}\right)\right] = \frac{2q}{4\pi\varepsilon_0}\left(\frac{1}{d-r} - \frac{1}{r}\right).$$

EXECUTE: Substituting the numbers gives

$V_b - V_a = 2(250\ \mu\text{C})\ (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{1}{0.750\ \text{m}} - \frac{1}{0.250\ \text{m}}\right) = -12.0\times10^6\ \text{V} = -12.0\ \text{MV}$. The meter reads 12.0 MV.

(b) Since $V_b - V_a$ is negative, $V_a > V_b$, so point a is at the higher potential.

EVALUATE: An easy way to see that the potential at a is higher than the potential at b is that it would require positive work to move a positive test charge from b to a since this charge would be attracted by the negative sphere and repelled by the positive sphere.

23.57. IDENTIFY: $U = \dfrac{kq_1q_2}{r}$.

SET UP: Eight charges means there are $8(8-1)/2 = 28$ pairs. There are 12 pairs of q and $-q$ separated by d, 12 pairs of equal charges separated by $\sqrt{2}d$ and 4 pairs of q and $-q$ separated by $\sqrt{3}d$.

EXECUTE: **(a)** $U = kq^2\left(-\dfrac{12}{d} + \dfrac{12}{\sqrt{2}d} - \dfrac{4}{\sqrt{3}d}\right) = -\dfrac{12kq^2}{d}\left(1 - \dfrac{1}{\sqrt{2}} + \dfrac{1}{3\sqrt{3}}\right) = -1.46q^2/\pi\varepsilon_0 d.$

EVALUATE: **(b)** The fact that the electric potential energy is less than zero means that it is energetically favorable for the crystal ions to be together.

23.58. IDENTIFY: For two small spheres, $U = \dfrac{kq_1q_2}{r}$. For part (b) apply conservation of energy.

SET UP: Let $q_1 = 2.00\ \mu\text{C}$ and $q_2 = -3.50\ \mu\text{C}$. Let $r_a = 0.180$ m and $r_b \to \infty$.

EXECUTE: **(a)** $U = \dfrac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\ \text{C})(-3.50\times10^{-6}\ \text{C})}{0.180\ \text{m}} = -0.350\ \text{J}.$

(b) $K_b = 0$. $U_b = 0$. $U_a = -0.350$ J. $K_a + U_a = K_b + U_b$ gives $K_a = 0.350$ J. $K_a = \frac{1}{2}mv_a^2$, so

$$v_a = \sqrt{\frac{2K_a}{m}} = \sqrt{\frac{2(0.350\ \text{J})}{1.50\times10^{-3}\ \text{kg}}} = 21.6\ \text{m/s}.$$

EVALUATE: As the sphere moves away, the attractive electrical force exerted by the other sphere does negative work and removes all the kinetic energy it initially had.

23.59. IDENTIFY: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the sphere. The electric force on the sphere is $F_e = qE$. The potential difference between the plates is $V = Ed$.

SET UP: The free-body diagram for the sphere is given in Figure 23.59.

EXECUTE: $T\cos\theta = mg$ and $T\sin\theta = F_e$ gives

$$F_e = mg\tan\theta = (1.50\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)\tan(30^\circ) = 0.0085\ \text{N}.$$

$$F_e = Eq = \frac{Vq}{d} \text{ and } V = \frac{Fd}{q} = \frac{(0.0085\ \text{N})(0.0500\ \text{m})}{8.90\times10^{-6}\ \text{C}} = 47.8\ \text{V}.$$

EVALUATE: $E = V/d = 956$ V/m. $E = \sigma/\varepsilon_0$ and $\sigma = E\varepsilon_0 = 8.46\times10^{-9}\ \text{C/m}^2$.

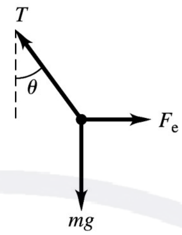


Figure 23.59

23.60. **IDENTIFY:** Outside a uniform spherical shell of charge, the electric field and potential are the same as for a point-charge at the center. Inside the shell, the electric field is zero so the potential is constant and equal to its value at the surface of the shell. The net potential is the scalar sum of the individual potentials.

SET UP: $V = k\dfrac{q}{r}$. Call V_1 the potential due to the inner shell and V_2 the potential due to the outer shell. $V_{net} = V_1 + V_2$.

EXECUTE: **(a)** At $r = 2.50$ cm, we are inside both shells. V_1 is the potential at the surface of the inner shell, so $V_1 = kq_1/R_1$; and V_2 is the potential at the surface of the outer shell, so $V_2 = kq_2/R_2$. The net potential is

$V_{net} = kq_1/R_1 + kq_2/R_2 = k(q_1/R_1 + q_2/R_2)$.

$V_{net} = (8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)[(3.00\mu\text{C})/(0.0500 \text{ m}) + (-5.00\,\mu\text{C})/(0.150 \text{ m})] = 2.40\times 10^5 \text{ V} = 240 \text{ kV}.$

(b) At $r = 10.0$ cm, we are outside the inner shell but still inside the outer shell. The inner shell now is equivalent to a point-charge at its center, so the net potential is

$V_{net} = kq_1/r + kq_2/R_2 = k(q_1/r + q_2/R_2)$.

$V_{net} = k[(3.00\,\mu\text{C})/(0.100 \text{ m}) + (-5.00\,\mu\text{C})/(0.150 \text{ m})] = -30.0$ kV.

(c) At $r = 20.0$ cm, we are outside both shells, so both are equivalent to point-charges at their center. So $V_{net} = kq_1/r + kq_2/r = k(q_1 + q_2)/r = k(-2.00\,\mu\text{C})/(0.200 \text{ m}) = -89.9$ kV.

EVALUATE: $E = 0$ inside a spherically symmetric shell, but that does not necessarily mean that $V = 0$ there. It only means that $V_a - V_b = 0$ for any two points in side the shell, so V is constant.

23.61. **(a) IDENTIFY:** The potential at any point is the sum of the potentials due to each of the two charged conductors.

SET UP: For a conducting cylinder with charge per unit length λ the potential outside the cylinder is given by $V = (\lambda/2\pi\epsilon_0)\ln(r_0/r)$ where r is the distance from the cylinder axis and r_0 is the distance from the axis for which we take $V = 0$. Inside the cylinder the potential has the same value as on the cylinder surface. The electric field is the same for a solid conducting cylinder or for a hollow conducting tube so this expression for V applies to both. This problem says to take $r_0 = b$.

EXECUTE: For the hollow tube of radius b and charge per unit length $-\lambda$: outside $V = -(\lambda/2\pi\epsilon_0)\ln(b/r)$; inside $V = 0$ since $V = 0$ at $r = b$.

For the metal cylinder of radius a and charge per unit length λ:

outside $V = (\lambda/2\pi\epsilon_0)\ln(b/r)$, inside $V = (\lambda/2\pi\epsilon_0)\ln(b/a)$, the value at $r = a$.

(i) $r < a$; inside both $V = (\lambda/2\pi\epsilon_0)\ln(b/a)$.

(ii) $a < r < b$; outside cylinder, inside tube $V = (\lambda/2\pi\epsilon_0)\ln(b/r)$.

(iii) $r > b$; outside both the potentials are equal in magnitude and opposite in sign so $V = 0$.

(b) For $r = a$, $V_a = (\lambda/2\pi\epsilon_0)\ln(b/a)$.

For $r = b$, $V_b = 0$.

Thus $V_{ab} = V_a - V_b = (\lambda/2\pi\epsilon_0)\ln(b/a)$.

(c) IDENTIFY and **SET UP:** Use $E_r = -\dfrac{\partial V}{\partial r}$ to calculate E.

EXECUTE: $E=-\dfrac{\partial V}{\partial r}=-\dfrac{\lambda}{2\pi\varepsilon_0}\dfrac{\partial}{\partial r}\ln\left(\dfrac{b}{r}\right)=-\dfrac{\lambda}{2\pi\varepsilon_0}\left(\dfrac{r}{b}\right)\left(-\dfrac{b}{r^2}\right)=\dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}.$

(**d**) The electric field between the cylinders is due only to the inner cylinder, so V_{ab} is not changed, $V_{ab}=(\lambda/2\pi\varepsilon_0)\ln(b/a)$.

EVALUATE: The electric field is not uniform between the cylinders, so $V_{ab}\neq E(b-a)$.

23.62. IDENTIFY: The wire and hollow cylinder form coaxial cylinders. Problem 23.61 gives $E(r)=\dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}$.

SET UP: $a=145\times10^{-6}$ m, $b=0.0180$ m.

EXECUTE: $E=\dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}$ and

$V_{ab}=E\ln(b/a)r=(2.00\times10^4\text{ N/C})(\ln(0.018\text{ m}/145\times10^{-6}\text{ m}))0.012\text{ m}=1157\text{ V}.$

EVALUATE: The electric field at any *r* is directly proportional to the potential difference between the wire and the cylinder.

23.63. IDENTIFY and **SET UP:** Use $\vec{F}=q\vec{E}$ to calculate $\vec{F}$ and then $\vec{F}=m\vec{a}$ gives $\vec{a}$. $E=V/d$.

EXECUTE: **(a)** $\vec{F}_E=q\vec{E}$. Since $q=-e$ is negative $\vec{F}_E$ and $\vec{E}$ are in opposite directions; $\vec{E}$ is upward so $\vec{F}_E$ is downward. The magnitude of *E* is $E=\dfrac{V}{d}=\dfrac{22.0\text{ V}}{0.0200\text{ m}}=1.10\times10^3\text{ V/m}=1.10\times10^3\text{ N/C}.$ The magnitude of F_E is $F_E=|q|E=eE=(1.602\times10^{-19}\text{ C})(1.10\times10^3\text{ N/C})=1.76\times10^{-16}\text{ N}.$

(b) Calculate the acceleration of the electron produced by the electric force:

$$a=\frac{F}{m}=\frac{1.76\times10^{-16}\text{ N}}{9.109\times10^{-31}\text{ kg}}=1.93\times10^{14}\text{ m/s}^2.$$

EVALUATE: This acceleration is much larger than $g=9.80\text{ m/s}^2$, so the gravity force on the electron can be neglected. $\vec{F}_E$ is downward, so $\vec{a}$ is downward.

(c) IDENTIFY and **SET UP:** The acceleration is constant and downward, so the motion is like that of a projectile. Use the horizontal motion to find the time and then use the time to find the vertical displacement.

EXECUTE: x-component: $v_{0x}=6.50\times10^6$ m/s; $a_x=0$; $x-x_0=0.060$ m; $t=?$

$x-x_0=v_{0x}t+\frac{1}{2}a_xt^2$ and the a_x term is zero, so $t=\dfrac{x-x_0}{v_{0x}}=\dfrac{0.060\text{ m}}{6.50\times10^6\text{ m/s}}=9.231\times10^{-9}\text{ s}.$

y-component: $v_{0y}=0$; $a_y=1.93\times10^{14}\text{ m/s}^2$; $t=9.231\times10^{-9}$ m/s; $y-y_0=?$

$y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$. $y-y_0=\frac{1}{2}(1.93\times10^{14}\text{ m/s}^2)(9.231\times10^{-9}\text{ s})^2=0.00822\text{ m}=0.822\text{ cm}.$

(d) IDENTIFY and **SET UP:** The velocity and its components as the electron leaves the plates are sketched in Figure 23.63.

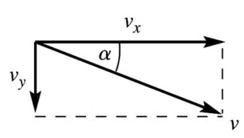


Figure 23.63

EXECUTE:

$v_x=v_{0x}=6.50\times10^6$ m/s (since $a_x=0$).

$v_y=v_{0y}+a_yt.$

$v_y=0+(1.93\times10^{14}\text{ m/s}^2)(9.231\times10^{-9}\text{ s}).$

$v_y=1.782\times10^6$ m/s.

$$\tan\alpha=\frac{v_y}{v_x}=\frac{1.782\times10^6\text{ m/s}}{6.50\times10^6\text{ m/s}}=0.2742\quad\text{so}\quad\alpha=15.3°.$$

EVALUATE: The greater the electric field or the smaller the initial speed the greater the downward deflection.
(e) IDENTIFY and **SET UP:** Consider the motion of the electron after it leaves the region between the plates. Outside the plates there is no electric field, so $a = 0$. (Gravity can still be neglected since the electron is traveling at such high speed and the times are small.) Use the horizontal motion to find the time it takes the electron to travel 0.120 m horizontally to the screen. From this time find the distance downward that the electron travels.

EXECUTE: x-component: $v_{0x} = 6.50\times10^6$ m/s; $a_x = 0$; $x - x_0 = 0.120$ m; $t = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ and the a_x term is term is zero, so $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{0.120\text{ m}}{6.50\times10^6\text{ m/s}} = 1.846\times10^{-8}$ s.

y-component: $v_{0y} = 1.782\times10^6$ m/s (from part (b)); $a_y = 0$; $t = 1.846\times10^{-8}$ m/s; $y - y_0 = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (1.782\times10^6\text{ m/s})(1.846\times10^{-8}\text{ s}) = 0.0329\text{ m} = 3.29\text{ cm}.$

EVALUATE: The electron travels downward a distance 0.822 cm while it is between the plates and a distance 3.29 cm while traveling from the edge of the plates to the screen. The total downward deflection is 0.822 cm + 3.29 cm = 4.11 cm. The horizontal distance between the plates is half the horizontal distance the electron travels after it leaves the plates. And the vertical velocity of the electron increases as it travels between the plates, so it makes sense for it to have greater downward displacement during the motion after it leaves the plates.

23.64. IDENTIFY: The charge on the plates and the electric field between them depend on the potential difference across the plates.

SET UP: For two parallel plates, the potential difference between them is $V = Ed = \dfrac{\sigma}{\epsilon_0}d = \dfrac{Qd}{\epsilon_0 A}$.

EXECUTE: **(a)** Solving for Q gives $Q = \epsilon_0 AV/d = \dfrac{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.030\text{ m})^2(25.0\text{ V})}{0.0050\text{ m}}$.

$Q = 3.98\times10^{-11}\text{C} = 39.8\text{ pC}.$

(b) $E = V/d = (25.0\text{ V})/(0.0050\text{ m}) = 5.00\times10^3$ V/m.

(c) SET UP: Energy conservation gives $\frac{1}{2}mv^2 = eV$.

EXECUTE: Solving for v gives $v = \sqrt{\dfrac{2eV}{m}} = \sqrt{\dfrac{2(1.60\times10^{-19}\text{C})(25.0\text{ V})}{9.11\times10^{-31}\text{ kg}}} = 2.96\times10^6\text{m/s}.$

EVALUATE: Typical voltages in student laboratory work run up to around 25 V, so typical reasonable values for the charge on the plates is about 40 pC and a reasonable value for the electric field is about 5000 V/m, as we found here. The electron speed would be about 3 million m/s.

23.65. (a) IDENTIFY and **SET UP:** Problem 23.61 derived that $E = \dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}$, where a is the radius of the inner cylinder (wire) and b is the radius of the outer hollow cylinder. The potential difference between the two cylinders is V_{ab}. Use this expression to calculate E at the specified r.

EXECUTE: Midway between the wire and the cylinder wall is at a radius of

$r = (a + b)/2 = (90.0\times10^{-6}\text{ m} + 0.140\text{ m})/2 = 0.07004\text{ m}.$

$$E = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r} = \frac{50.0\times10^3\text{ V}}{\ln(0.140\text{ m}/90.0\times10^{-6}\text{ m})(0.07004\text{ m})} = 9.71\times10^4\text{ V/m}.$$

(b) IDENTIFY and **SET UP:** The magnitude of the electric force is given by $F = |q|E$. Set this equal to ten times the weight of the particle and solve for $|q|$, the magnitude of the charge on the particle.

EXECUTE: $F_E = 10mg$.

$$|q|E = 10mg \text{ and } |q| = \frac{10mg}{E} = \frac{10(30.0\times10^{-9}\text{ kg})(9.80\text{ m/s}^2)}{9.71\times10^4\text{ V/m}} = 3.03\times10^{-11}\text{ C}.$$

EVALUATE: It requires only this modest net charge for the electric force to be much larger than the weight.

23.66. **(a) IDENTIFY:** Calculate the potential due to each thin ring and integrate over the disk to find the potential. V is a scalar so no components are involved.

SET UP: Consider a thin ring of radius y and width dy. The ring has area $2\pi y\,dy$ so the charge on the ring is $dq = \sigma(2\pi y\,dy)$.

EXECUTE: The result of Example 23.11 then says that the potential due to this thin ring at the point on the axis at a distance x from the ring is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{x^2+y^2}} = \frac{2\pi\sigma}{4\pi\varepsilon_0}\frac{y\,dy}{\sqrt{x^2+y^2}}.$$

$$V = \int dV = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{y\,dy}{\sqrt{x^2+y^2}} = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2+y^2}\right]_0^R = \frac{\sigma}{2\varepsilon_0}(\sqrt{x^2+R^2}-x).$$

EVALUATE: For $x \gg R$ this result should reduce to the potential of a point charge with $Q = \sigma\pi R^2$.

$$\sqrt{x^2+R^2} = x(1+R^2/x^2)^{1/2} \approx x(1+R^2/2x^2) \text{ so } \sqrt{x^2+R^2}-x \approx R^2/2x.$$

Then $V \approx \dfrac{\sigma}{2\varepsilon_0}\dfrac{R^2}{2x} = \dfrac{\sigma\pi R^2}{4\pi\varepsilon_0 x} = \dfrac{Q}{4\pi\varepsilon_0 x}$, as expected.

(b) IDENTIFY and **SET UP:** Use $E_x = -\dfrac{\partial V}{\partial x}$ to calculate E_x.

EXECUTE: $E_x = -\dfrac{\partial V}{\partial x} = -\dfrac{\sigma}{2\varepsilon_0}\left(\dfrac{x}{\sqrt{x^2+R^2}}-1\right) = \dfrac{\sigma x}{2\varepsilon_0}\left(\dfrac{1}{x}-\dfrac{1}{\sqrt{x^2+R^2}}\right).$

EVALUATE: Our result agrees with the results of Example 21.11.

23.67. **IDENTIFY:** We must integrate to find the total energy because the energy to bring in more charge depends on the charge already present.

SET UP: If ρ is the uniform volume charge density, the charge of a spherical shell of radius r and thickness dr is $dq = \rho 4\pi r^2\,dr$, and $\rho = Q/(4/3\ \pi R^3)$. The charge already present in a sphere of radius r is $q = \rho(4/3\ \pi r^3)$. The energy to bring the charge dq to the surface of the charge q is Vdq, where V is the potential due to q, which is $q/4\pi\varepsilon_0 r$.

EXECUTE: The total energy to assemble the entire sphere of radius R and charge Q is sum (integral) of the tiny increments of energy.

$$U = \int V dq = \int \frac{q}{4\pi\varepsilon_0 r}\,dq = \int_0^R \frac{\rho\frac{4}{3}\pi r^3}{4\pi\varepsilon_0 r}(\rho 4\pi r^2 dr) = \frac{3}{5}\left(\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{R}\right)$$

where we have substituted $\rho = Q/(4/3\ \pi R^3)$ and simplified the result.

EVALUATE: For a point charge, $R \to 0$ so $U \to \infty$, which means that a point charge should have infinite self-energy. This suggests that either point charges are impossible, or that our present treatment of physics is not adequate at the extremely small scale, or both.

23.68. **IDENTIFY:** Divide the rod into infinitesimal segments with charge dq. The potential dV due to the segment is $dV = \dfrac{1}{4\pi\varepsilon_0}\dfrac{dq}{r}$. Integrate over the rod to find the total potential.

SET UP: $dq = \lambda dl$, with $\lambda = Q/\pi a$ and $dl = a\,d\theta$.

EXECUTE: $dV = \dfrac{1}{4\pi\varepsilon_0}\dfrac{dq}{r} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\lambda\,dl}{a} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{\pi a}\dfrac{dl}{a} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q\,d\theta}{\pi a}$. $V = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\int_0^\pi \dfrac{Q\,d\theta}{\pi a} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{a}$.

EVALUATE: All the charge of the ring is the same distance a from the center of curvature.

23.69. **IDENTIFY** and **SET UP:** The sphere no longer behaves as a point charge because we are inside of it. We know how the electric field varies with distance from the center of the sphere and want to use this to find the potential difference between the center and surface, which requires integration. $V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$. The electric field is radially outward, so $\vec{E} \cdot d\vec{l} = E\,dr$.

EXECUTE: For $r < R$: $E = \dfrac{kQr}{R^3}$. Integrating gives

$V = -\int_\infty^R \vec{E} \cdot d\vec{r}' - \int_R^r \vec{E} \cdot d\vec{r}' = \dfrac{kQ}{R} - \dfrac{kQ}{R^3}\int_R^r r'dr' = \dfrac{kQ}{R} - \dfrac{kQ}{R^3}\dfrac{1}{2}r'^2\Big|_R^r = \dfrac{kQ}{R} + \dfrac{kQ}{2R} - \dfrac{kQr^2}{2R^3} = \dfrac{kQ}{2R}\left[3 - \dfrac{r^2}{R^2}\right]$. At the center of the sphere, $r = 0$ and $V_1 = \dfrac{3kQ}{2R}$. At the surface of the sphere, $r = R$ and $V_2 = \dfrac{kQ}{R}$. The potential difference is $V_1 - V_2 = \dfrac{kQ}{2R} = \dfrac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.00\times10^{-6}\ \text{C})}{2(0.0500\ \text{m})} = 3.60\times10^5\ \text{V}$.

EVALUATE: To check our answer, we could actually do the integration. We can use the fact that $E = \dfrac{kQr}{R^3}$ so $V_1 - V_2 = \int_0^R E\,dr = \dfrac{kQ}{R^3}\int_0^R r\,dr = \dfrac{kQ}{R^3}\left(\dfrac{R^2}{2}\right) = \dfrac{kQ}{2R}$.

23.70. IDENTIFY: For $r < c$, $E = 0$ and the potential is constant. For $r > c$, E is the same as for a point charge and $V = \dfrac{kq}{r}$.

SET UP: $V_\infty = 0$.

EXECUTE: **(a)** Points $a, b,$ and c are all at the same potential, so $V_a - V_b = V_b - V_c = V_a - V_c = 0$.

$V_c - V_\infty = \dfrac{kq}{R} = \dfrac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(150\times10^{-6}\ \text{C})}{0.60\ \text{m}} = 2.25\times10^6\ \text{V}$.

(b) They are all at the same potential.

(c) Only $V_c - V_\infty$ would change; it would be -2.25×10^6 V.

EVALUATE: The voltmeter reads the potential difference between the two points to which it is connected.

23.71. IDENTIFY: Apply Newton's second law to calculate the acceleration. Apply conservation of energy and conservation of momentum to the motions of the spheres.

SET UP: Since the spheres behave as though all the charge were at their centers, we have $F = k\dfrac{|q_1q_2|}{r^2}$ and $U = \dfrac{kq_1q_2}{r}$, where q_1 and q_2 are the charges of the objects and r is the distance between their centers.

EXECUTE: Maximum speed occurs when the spheres are very far apart. Energy conservation gives $\dfrac{kq_1q_2}{r} = \dfrac{1}{2}m_{50}v_{50}^2 + \dfrac{1}{2}m_{150}v_{150}^2$. Momentum conservation gives $m_{50}v_{50} = m_{150}v_{150}$ and $v_{50} = 3v_{150}$. $r = 0.50$ m. Solve for v_{50} and v_{150}: $v_{50} = 12.7$ m/s, $v_{150} = 4.24$ m/s. Maximum acceleration occurs just after spheres are released. $\sum F = ma$ gives $\dfrac{kq_1q_2}{r^2} = m_{150}a_{150}$.

$\dfrac{(9\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(10^{-5}\ \text{C})(3\times10^{-5}\ \text{C})}{(0.50\ \text{m})^2} = (0.15\ \text{kg})a_{150}$. $a_{150} = 72.0\ \text{m/s}^2$ and $a_{50} = 3a_{150} = 216\ \text{m/s}^2$.

EVALUATE: The more massive sphere has a smaller acceleration and a smaller final speed.

23.72. IDENTIFY: The potential at the surface of a uniformly charged sphere is $V = \dfrac{kQ}{R}$.

SET UP: For a sphere, $V = \dfrac{4}{3}\pi R^3$. When the raindrops merge, the total charge and volume are conserved.

EXECUTE: **(a)** $V = \dfrac{kQ}{R} = \dfrac{k(-3.60\times10^{-12}\text{ C})}{6.50\times10^{-4}\text{ m}} = -49.8\text{ V}.$

(b) The volume doubles, so the radius increases by the cube root of two: $R_{\text{new}} = \sqrt[3]{2}\,R = 8.19\times10^{-4}\text{ m}$ and the new charge is $Q_{\text{new}} = 2Q = -7.20\times10^{-12}\text{ C}$. The new potential is

$$V_{\text{new}} = \frac{kQ_{\text{new}}}{R_{\text{new}}} = \frac{k(-7.20\times10^{-12}\text{ C})}{8.19\times10^{-4}\text{ m}} = -79.0\text{ V}.$$

EVALUATE: The charge doubles but the radius also increases and the potential at the surface increases by only a factor of $\dfrac{2}{2^{1/3}} = 2^{2/3} \approx 1.6$.

23.73. **IDENTIFY:** Slice the rod into thin slices and use $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r}$ to calculate the potential due to each slice. Integrate over the length of the rod to find the total potential at each point.

(a) SET UP: An infinitesimal slice of the rod and its distance from point *P* are shown in Figure 23.73a.

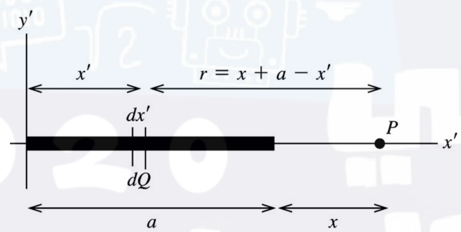


Figure 23.73a

Use coordinates with the origin at the left-hand end of the rod and one axis along the rod. Call the axes x' and y' so as not to confuse them with the distance *x* given in the problem.

EXECUTE: Slice the charged rod up into thin slices of width dx'. Each slice has charge $dQ = Q(dx'/a)$ and a distance $r = x + a - x'$ from point *P*. The potential at *P* due to the small slice dQ is

$$dV = \frac{1}{4\pi\epsilon_0}\left(\frac{dQ}{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\left(\frac{dx'}{x+a-x'}\right).$$

Compute the total *V* at *P* due to the entire rod by integrating *dV* over the length of the rod $(x' = 0 \text{ to } x' = a)$:

$$V = \int dV = \frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{dx'}{(x+a-x')} = \frac{Q}{4\pi\epsilon_0 a}[-\ln(x+a-x')]_0^a = \frac{Q}{4\pi\epsilon_0 a}\ln\left(\frac{x+a}{x}\right).$$

EVALUATE: As $x \to \infty$, $V \to \dfrac{Q}{4\pi\epsilon_0 a}\ln\left(\dfrac{x}{x}\right) = 0.$

(b) SET UP: An infinitesimal slice of the rod and its distance from point *R* are shown in Figure 23.73b.

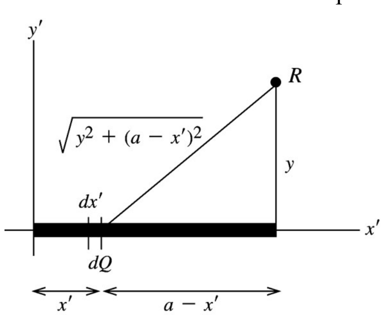


Figure 23.73b

$dQ = (Q/a)dx'$ as in part (a).

Each slice dQ is a distance $r = \sqrt{y^2 + (a - x')^2}$ from point R.

EXECUTE: The potential dV at R due to the small slice dQ is

$$dV = \frac{1}{4\pi\varepsilon_0}\left(\frac{dQ}{r}\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a}\frac{dx'}{\sqrt{y^2+(a-x')^2}}.$$

$$V = \int dV = \frac{Q}{4\pi\varepsilon_0 a}\int_0^a \frac{dx'}{\sqrt{y^2+(a-x')^2}}.$$

In the integral make the change of variable $u = a - x'; du = -dx'$

$$V = -\frac{Q}{4\pi\varepsilon_0 a}\int_a^0 \frac{du}{\sqrt{y^2+u^2}} = -\frac{Q}{4\pi\varepsilon_0 a}\left[\ln\left(u+\sqrt{y^2+u^2}\right)\right]_a^0.$$

$$V = -\frac{Q}{4\pi\varepsilon_0 a}\left[\ln y - \ln(a+\sqrt{y^2+a^2})\right] = \frac{Q}{4\pi\varepsilon_0 a}\left[\ln\left(\frac{a+\sqrt{a^2+y^2}}{y}\right)\right].$$

(The expression for the integral was found in Appendix B.)

EVALUATE: As $y \to \infty$, $V \to \frac{Q}{4\pi\varepsilon_0 a}\ln\left(\frac{y}{y}\right) = 0.$

(c) SET UP: *part (a):* $V = \frac{Q}{4\pi\varepsilon_0 a}\ln\left(\frac{x+a}{x}\right) = \frac{Q}{4\pi\varepsilon_0 a}\ln\left(1+\frac{a}{x}\right).$

From Appendix B, $\ln(1+u) = u - u^2/2\ldots,$ so $\ln(1+a/x) = a/x - a^2/2x^2$ and this becomes a/x when x is large.

EXECUTE: Thus $V \to \frac{Q}{4\pi\varepsilon_0 a}\left(\frac{a}{x}\right) = \frac{Q}{4\pi\varepsilon_0 x}.$ For large x, V becomes the potential of a point charge.

part (b): $V = \frac{Q}{4\pi\varepsilon_0 a}\left[\ln\left(\frac{a+\sqrt{a^2+y^2}}{y}\right)\right] = \frac{Q}{4\pi\varepsilon_0 a}\ln\left(\frac{a}{y}+\sqrt{1+\frac{a^2}{y^2}}\right).$

From Appendix B, $\sqrt{1+a^2/y^2} = (1+a^2/y^2)^{1/2} = 1 + a^2/2y^2 + \ldots$

Thus $a/y + \sqrt{1+a^2/y^2} \to 1 + a/y + a^2/2y^2 + \ldots \to 1 + a/y.$ And then using $\ln(1+u) \approx u$ gives

$$V \to \frac{Q}{4\pi\varepsilon_0 a}\ln(1+a/y) \to \frac{Q}{4\pi\varepsilon_0 a}\left(\frac{a}{y}\right) = \frac{Q}{4\pi\varepsilon_0 y}.$$

EVALUATE: For large y, V becomes the potential of a point charge.

23.74. IDENTIFY: Apply conservation of energy, $K_a + U_a = K_b + U_b$.

SET UP: Assume the particles initially are far apart, so $U_a = 0$. The alpha particle has zero speed at the distance of closest approach, so $K_b = 0$. $1\text{ eV} = 1.60\times10^{-19}$ J. The alpha particle has charge $+2e$ and the lead nucleus has charge $+82e$.

EXECUTE: Set the alpha particle's kinetic energy equal to its potential energy: $K_a = U_b$ gives

$$9.50\text{ MeV} = \frac{k(2e)(82e)}{r} \text{ and } r = \frac{k(164)(1.60\times10^{-19}\text{ C})^2}{(9.50\times10^6\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 2.48\times10^{-14}\text{ m}.$$

EVALUATE: The calculation assumes that at the distance of closest approach the alpha particle is outside the radius of the lead nucleus.

23.75. **(a) IDENTIFY and SET UP:** The potential at the surface of a charged conducting sphere is: $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R}$.

For spheres A and B this gives $V_A = \dfrac{Q_A}{4\pi\varepsilon_0 R_A}$ and $V_B = \dfrac{Q_B}{4\pi\varepsilon_0 R_B}$.

EXECUTE: $V_A = V_B$ gives $Q_A/4\pi\varepsilon_0 R_A = Q_B/4\pi\varepsilon_0 R_B$ and $Q_B/Q_A = R_B/R_A$. And then $R_A = 3R_B$ implies $Q_B/Q_A = 1/3$.

(b) IDENTIFY and SET UP: The electric field at the surface of a charged conducting sphere is $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q|}{R^2}$.

EXECUTE: For spheres A and B this gives $E_A = \dfrac{|Q_A|}{4\pi\varepsilon_0 R_A^2}$ and $E_B = \dfrac{|Q_B|}{4\pi\varepsilon_0 R_B^2}$.

$$\frac{E_B}{E_A} = \left(\frac{|Q_B|}{4\pi\varepsilon_0 R_B^2}\right)\left(\frac{4\pi\varepsilon_0 R_A^2}{|Q_A|}\right) = |Q_B/Q_A|(R_A/R_B)^2 = (1/3)(3)^2 = 3.$$

EVALUATE: The sphere with the larger radius needs more net charge to produce the same potential. We can write $E = V/R$ for a sphere, so with equal potentials the sphere with the smaller R has the larger E.

23.76. **IDENTIFY and SET UP:** For points outside of them, the spheres behave as though all the charge were concentrated at their centers. The charge initially on sphere 1 spreads between the two spheres such as to bring them to the same potential.

EXECUTE: **(a)** $E_1 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_1}{R_1^2}$, $V_1 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_1}{R_1} = R_1 E_1$.

(b) Two conditions must be met:

1) Let q_1 and q_2 be the final charges of each sphere. Then $q_1 + q_2 = Q_1$ (charge conservation).

2) Let V_1 and V_2 be the final potentials of each sphere. All points of a conductor are at the same potential, so $V_1 = V_2$.

$V_1 = V_2$ requires that $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1}{R_1} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_2}{R_2}$ and then $q_1/R_1 = q_2/R_2$.

$$q_1 R_2 = q_2 R_1 = (Q_1 - q_1)R_1.$$

This gives $q_1 = (R_1/[R_1 + R_2])Q_1$ and $q_2 = Q_1 - q_1 = Q_1(1 - R_1/[R_1 + R_2]) = Q_1(R_2/[R_1 + R_2])$.

(c) $V_1 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1}{R_1} = \dfrac{Q_1}{4\pi\varepsilon_0(R_1 + R_2)}$ and $V_2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_2}{R_2} = \dfrac{Q_1}{4\pi\varepsilon_0(R_1 + R_2)}$, which equals V_1 as it should.

(d) $E_1 = \dfrac{V_1}{R_1} = \dfrac{Q_1}{4\pi\varepsilon_0 R_1(R_1 + R_2)}$. $E_2 = \dfrac{V_2}{R_2} = \dfrac{Q_1}{4\pi\varepsilon_0 R_2(R_1 + R_2)}$.

EVALUATE: Part (a) says $q_2 = q_1(R_2/R_1)$. The sphere with the larger radius needs more charge to produce the same potential at its surface. When $R_1 = R_2$, $q_1 = q_2 = Q_1/2$. The sphere with the larger radius has the smaller electric field at its surface.

23.77. **IDENTIFY:** Apply conservation of energy: $E_1 = E_2$.

SET UP: In the collision the initial kinetic energy of the two particles is converted into potential energy at the distance of closest approach.

EXECUTE: **(a)** The two protons must approach to a distance of $2r_p$, where r_p is the radius of a proton.

$E_1 = E_2$ gives $2\left[\dfrac{1}{2}m_p v^2\right] = \dfrac{ke^2}{2r_p}$ and $v = \sqrt{\dfrac{k(1.60\times10^{-19}\ \text{C})^2}{2(1.2\times10^{-15}\ \text{m})(1.67\times10^{-27}\ \text{kg})}} = 7.58\times10^6$ m/s.

(b) For a helium-helium collision, the charges and masses change from (a) and

$$v=\sqrt{\frac{k(2(1.60\times10^{-19}\text{ C}))^2}{(3.5\times10^{-15}\text{ m})(2.99)(1.67\times10^{-27}\text{ kg})}}=7.26\times10^6\text{ m/s}.$$

(c) $K=\dfrac{3kT}{2}=\dfrac{mv^2}{2}$. $T_{\text{p}}=\dfrac{m_{\text{p}}v^2}{3k}=\dfrac{(1.67\times10^{-27}\text{kg})(7.58\times10^6\text{m/s})^2}{3(1.38\times10^{-23}\text{J/K})}=2.3\times10^9\text{K}.$

$$T_{\text{He}}=\frac{m_{\text{He}}v^2}{3k}=\frac{(2.99)(1.67\times10^{-27}\text{ kg})(7.26\times10^6\text{ m/s})^2}{3(1.38\times10^{-23}\text{ J/K})}=6.4\times10^9\text{ K}.$$

(d) These calculations were based on the particles' average speed. The distribution of speeds ensures that there is always a certain percentage with a speed greater than the average speed, and these particles can undergo the necessary reactions in the sun's core.

EVALUATE: The kinetic energies required for fusion correspond to very high temperatures.

23.78. IDENTIFY and **SET UP:** Apply $\vec{E}=-\left(\dfrac{\partial V}{\partial x}\hat{i}+\dfrac{\partial V}{\partial y}\hat{j}+\dfrac{\partial V}{\partial z}\hat{k}\right)$. $\dfrac{W_{a\to b}}{q_0}=V_a-V_b$ and $V_a-V_b=\int_a^b\vec{E}\cdot d\vec{l}$.

EXECUTE: **(a)** $\vec{E}=-\dfrac{\partial V}{\partial x}\hat{i}-\dfrac{\partial V}{\partial y}\hat{j}-\dfrac{\partial V}{\partial z}\hat{k}=-2Ax\hat{i}+6Ay\hat{j}-2Az\hat{k}.$

(b) A charge is moved in along the z-axis. The work done is given by

$W=q\int_{z_0}^0\vec{E}\cdot\hat{k}dz=q\int_{z_0}^0(-2Az)dz=+(Aq)z_0^2$. Therefore, $A=\dfrac{W_{a\to b}}{qz_0^2}=\dfrac{6.00\times10^{-5}\text{ J}}{(1.5\times10^{-6}\text{ C})(0.250\text{ m})^2}=640\text{ V/m}^2.$

(c) $\vec{E}(0,0,0.250)=-2(640\text{ V/m}^2)(0.250\text{ m})\hat{k}=-(320\text{ V/m})\hat{k}.$

(d) In every plane parallel to the xz-plane, y is constant, so $V(x,y,z)=Ax^2+Az^2-C$, where $C=3Ay^2$.

$x^2+z^2=\dfrac{V+C}{A}=R^2$, which is the equation for a circle since R is constant as long as we have constant potential on those planes.

(e) $V=1280\text{ V}$ and $y=2.00\text{ m}$, so $x^2+z^2=\dfrac{1280\text{ V}+3(640\text{ V/m}^2)(2.00\text{ m})^2}{640\text{ V/m}^2}=14.0\text{ m}^2$ and the radius of the circle is 3.74 m.

EVALUATE: In any plane parallel to the xz-plane, $\vec{E}$ projected onto the plane is radial and hence perpendicular to the equipotential circles.

23.79. IDENTIFY and **SET UP:** We know that the potential is of the mathematical form $V(x,y,z)=Ax^l+By^m+Cz^n+D$. We also know that $E_x=-\dfrac{\partial V}{\partial x}$, $E_y=-\dfrac{\partial V}{\partial y}$, and $E_z=-\dfrac{\partial V}{\partial z}$. Various measurements are given in the table with the problem in the text.

EXECUTE: **(a)** First get A, B, C, and D using data from the table in the problem.

$V(0, 0, 0) = 10.0\text{ V} = 0 + 0 + 0 + D$, so $D = 10.0$ V.

$V(1.00, 0, 0) = A(1.00\text{ m})^l + 0 + 0 + 10.0\text{ V} = 4.00\text{ V}$, so $A = -6.0\text{ V}\cdot\text{m}^{-l}$.

$V(0, 1.00, 0) = B(1.00\text{ m})^m + 10.0\text{ V} = 6.0\text{ V}$, so $B = -4.0\text{ V}\cdot\text{m}^{-m}$.

$V(0, 0, 1.00\text{ m}) = C(1.00\text{ m})^n + 10.0\text{ V} = 8.0\text{ V}$, so $C = -2.0\text{ V}\cdot\text{m}^{-n}$.

Now get l, m, and n.

$E_x=-\dfrac{\partial V}{\partial x}=-lAx^{l-1}$, and from the table we know that $E_x(1.00, 0, 0) = 12.0$ V/m. Therefore

$-l(-6.0\text{ V}\cdot\text{m}^{-l})(1.00\text{ m})^{l-1} = 12.0\text{ V/m}.$

$l(6.0\text{ V}\cdot\text{m}^{-1}) = 12.0\text{ V/m}.$

$l = 2.0.$

$E_y=-\dfrac{\partial V}{\partial y}=-mBy^{m-1}.$

$E_y(0, 1.00, 0) = -m(-4.0\text{ V}\cdot\text{m}^{-m})(1.00\text{ m})^{m-1} = 12.0\text{ V/m}.$

$m = 3.0.$

$E_z = -\dfrac{\partial V}{\partial z} = -nCz^{n-1}.$

$E_z(0, 0, 1.00) = -n(-2.0\ \text{V}\cdot\text{m}^{-n})(1.00\ \text{m})^{n-1} = 12.0\ \text{V/m}.$

$n = 6.0.$

Now that we have l, m, and n, we see the units of A, B, and C, so

$A = -6.0\ \text{V/m}^2.$

$B = -4.0\ \text{V/m}^3.$

$C = -2.0\ \text{V/m}^6.$

Therefore the equation for $V(x,y,z)$ is

$V = (-6.0\ \text{V/m}^2)x^2 + (-4.0\ \text{V/m}^3)y^3 + (-2.0\ \text{V/m}^6)z^6 + 10.0\ \text{V}.$

(b) At (0, 0, 0): $V = 0$ and $E = 0$ (from the table with the problem).

At (0.50 m, 0.50 m, 0.50 m):

$V = (-6.0\ \text{V/m}^2)(0.50\ \text{m})^2 + (-4.0\ \text{V/m}^3)(0.50\ \text{m})^3 + (-2.0\ \text{V/m}^6)(0.50\ \text{m})^6 + 10.0\ \text{V} = 8.0\ \text{V}.$

$E_x = -\dfrac{\partial V}{\partial x} = -(-12.0\ \text{V/m}^2)x = (12.0\ \text{V/m}^2)(0.50\ \text{m}) = 6.0\ \text{V/m}.$

$E_y = -\dfrac{\partial V}{\partial y} = -3(-4.0\ \text{V/m}^3)y^2 = (12\ \text{V/m}^3)(0.50\ \text{m})^2 = 3.0\ \text{V/m}.$

$E_z = -\dfrac{\partial V}{\partial z} = -(-12.0\ \text{V/m}^6)z^5 = (12.0\ \text{V/m}^6)(0.50\ \text{m})^5 = 0.375\ \text{V/m}.$

$E = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(6.0\ \text{V/m})^2 + (3.0\ \text{V/m})^2 + (0.375\ \text{V/m})^2} = 6.7\ \text{V/m}.$

At (1.00 m, 1.00 m, 1.00 m):

Follow the same procedure as above. The results are $V = -2.0$ V, $E = 21$ V/m.

EVALUATE: We know that l, m, and n must be greater that 1 because the components of the electric field are all zero at (0, 0, 0).

23.80. IDENTIFY and SET UP: Energy is conserved and the potential energy is $U = k\dfrac{q_1q_2}{r}$. $K_1 + U_1 = K_2 + U_2$.

EXECUTE: **(a)** Energy conservation gives $K_1 + 0 = K_2 + U_2$.

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + k\frac{qQ}{x} \qquad \rightarrow \qquad v^2 = v_0^2 - \frac{2kqQ}{m}\cdot\frac{1}{x}.$$

On a graph of v_2 versus $1/x$, the graph of this equation will be a straight line with y-intercept equal to v_0^2 and slope equal to $-\dfrac{2kqQ}{m}$.

(b) With the given equation of the line in the problem, we have $v^2 = 400\ \text{m}^2/\text{s}^2 - (15.75\ \text{m}^3/\text{s}^2)\dfrac{1}{x}$. As x gets very large, $1/x$ approaches zero, so $v_0 = \sqrt{400\,\text{m}^2/\text{s}^2} = 20$ m/s.

(c) The slope is $-\dfrac{2kqQ}{m} = -15.75\ \text{m}^3/\text{s}^2$, which gives

$Q = -m(\text{slope})/2kq = -(4.00\times10^{-4}\ \text{kg})(-15.75\ \text{m}^3/\text{s}^2)/[2k(5.00\times10^{-8}\ \text{C})] = +7.01\times10^{-6}\ \text{C} = +7.01\ \mu\text{C}.$

(d) The particle is closest when its speed is zero, so

$v^2 = 400\ \text{m}^2/\text{s}^2 - (15.75\ \text{m}^3/\text{s}^2)\dfrac{1}{x} = 0$, which gives $x = 3.94\times10^{-2}\ \text{m} = 3.94\ \text{cm}.$

EVALUATE: From the graph in the problem, we see that v^2 decreases as $1/x$ increases, so v^2 decreases as x decreases. This means that the positively charged particle is slowing down as it gets closer to the sphere, so the sphere is repelling it. Therefore the sphere must be positively charged, as we found.

23.81. IDENTIFY: When the oil drop is at rest, the upward force $|q|E$ from the electric field equals the downward weight of the drop. When the drop is falling at its terminal speed, the upward viscous force equals the downward weight of the drop.

SET UP: The volume of the drop is related to its radius r by $V = \dfrac{4}{3}\pi r^3$.

EXECUTE: **(a)** $F_g = mg = \dfrac{4\pi r^3}{3}\rho g$. $F_e = |q|E = |q|V_{AB}/d$. $F_e = F_g$ gives $|q| = \dfrac{4\pi}{3}\dfrac{\rho r^3 gd}{V_{AB}}$.

(b) $\dfrac{4\pi r^3}{3}\rho g = 6\pi\eta r v_t$ gives $r = \sqrt{\dfrac{9\eta v_t}{2\rho g}}$. Using this result to replace r in the expression in part (a) gives

$$|q| = \frac{4\pi}{3}\frac{\rho g d}{V_{AB}}\left[\sqrt{\frac{9\eta v_t}{2\rho g}}\right]^3 = 18\pi\frac{d}{V_{AB}}\sqrt{\frac{\eta^3 v_t^3}{2\rho g}}.$$

(c) We use the values for V_{AB} and v_t given in the table in the problem and the formula $|q| = 18\pi\dfrac{d}{V_{AB}}\sqrt{\dfrac{\eta^3 v_t^3}{2\rho g}}$ from (c). For example, for drop 1 we get

$|q| = 18\pi\dfrac{1.00\times10^{-3}\text{ m}}{9.16\text{ V}}\sqrt{\dfrac{(1.81\times10^{-5}\text{ N}\cdot\text{s/m}^2)^3(2.54\times10^{-5}\text{ m/s})^3}{2(824\text{ kg/m}^3)(9.80\text{ m/s}^2)}} = 4.79\times10^{-19}\text{ C}$. Similar calculations for the remaining drops gives the following results:

Drop 1: 4.79×10^{-19} C
Drop 2: 1.59×10^{-19} C
Drop 3: 8.09×10^{-19} C
Drop 4: 3.23×10^{-19} C

(d) Use $n = q/e_2$ to find the number of excess electrons on each drop. Since all quantities have a power of 10^{-19} C, this factor will cancel, so all we need to do is divide the coefficients of 10^{-19} C. This gives

Drop 1: $n = q_1/q_2 = 4.79/1.59 = 3$ excess electrons
Drop 2: $n = q_2/q_2 = 1$ excess electron
Drop 3: $n = q_3/q_2 = 8.09/1.59 = 5$ excess electrons
Drop 4: $n = q_4/q_2 = 3.23/1.59 = 2$ excess electrons

(e) Using $q = -ne$ gives $e = -q/n$. All the charges are negative, so e will come out positive. Thus we get

Drop 1: $e_1 = q_1/n_1 = (4.79\times10^{-19}\text{ C})/3 = 1.60\times10^{-19}$ C
Drop 2: $e_2 = q_2/n_2 = (1.59\times10^{-19}\text{ C})/1 = 1.59\times10^{-19}$ C
Drop 3: $e_3 = q_3/n_3 = (8.09\times10^{-19}\text{ C})/5 = 1.62\times10^{-19}$ C
Drop 4: $e_4 = q_4/n_4 = (3.23\times10^{-19}\text{ C})/2 = 1.61\times10^{-19}$ C

The average is

$e_{av} = (e_1 + e_2 + e_3 + e_4)/4 = [(1.60 + 1.59 + 1.62 + 1.61)\times10^{-19}\text{ C}]/4 = 1.61\times10^{-19}$ C.

EVALUATE: The result $e = 1.61\times10^{-19}$ C is very close to the well-established value of 1.60×10^{-19} C.

23.82. **IDENTIFY:** Consider the potential due to an infinitesimal slice of the cylinder and integrate over the length of the cylinder to find the total potential. The electric field is along the axis of the tube and is given by $E = -\dfrac{\partial V}{\partial x}$.

SET UP: Use the expression from Example 23.11 for the potential due to each infinitesimal slice. Let the slice be at coordinate z along the x-axis, relative to the center of the tube.

EXECUTE: **(a)** For an infinitesimal slice of the finite cylinder, we have the potential

$dV = \dfrac{k\,dQ}{\sqrt{(x-z)^2 + R^2}} = \dfrac{kQ}{L}\dfrac{dz}{\sqrt{(x-z)^2 + R^2}}$. Integrating gives

$V = \dfrac{kQ}{L}\displaystyle\int_{-L/2}^{L/2}\dfrac{dz}{\sqrt{(x-z)^2 + R^2}} = \dfrac{kQ}{L}\int_{-L/2-x}^{L/2-x}\dfrac{du}{\sqrt{u^2 + R^2}}$ where $u = x - z$. Therefore,

$V = \dfrac{kQ}{L}\ln\left[\dfrac{\sqrt{(L/2-x)^2 + R^2} + (L/2-x)}{\sqrt{(L/2+x)^2 + R^2} - L/2 - x}\right]$ on the cylinder axis.

(b) For $L << R$, $V \approx \dfrac{kQ}{L}\ln\left[\dfrac{\sqrt{(L/2-x)^2 + R^2} + L/2 - x}{\sqrt{(L/2+x)^2 + R^2} - L/2 - x}\right] \approx \dfrac{kQ}{L}\ln\left[\dfrac{\sqrt{x^2 - xL + R^2} + L/2 - x}{\sqrt{x^2 + xL + R^2} - L/2 - x}\right]$.

$$V \approx \frac{kQ}{L}\ln\left[\frac{\sqrt{1-xL/(R^2+x^2)}+(L/2-x)/\sqrt{R^2+x^2}}{\sqrt{1+xL/(R^2+x^2)}+(-L/2-x)/\sqrt{R^2+x^2}}\right] = \frac{kQ}{L}\ln\left[\frac{1-xL/2(R^2+x^2)+(L/2-x)/\sqrt{R^2+x^2}}{1+xL/2(R^2+x^2)+(-L/2-x)/\sqrt{R^2+x^2}}\right].$$

$$V \approx \frac{kQ}{L}\ln\left[\frac{1+L/2\sqrt{R^2+x^2}}{1-L/2\sqrt{R^2+x^2}}\right] = \frac{kQ}{L}\left(\ln\left[1+\frac{L}{2\sqrt{R^2+x^2}}\right]-\ln\left[1-\frac{L}{2\sqrt{R^2+x^2}}\right]\right).$$

$V \approx \dfrac{kQ}{L}\dfrac{2L}{2\sqrt{x^2+R^2}} = \dfrac{kQ}{\sqrt{x^2+R^2}}$, which is the same as for a ring.

(c) $E_x = -\dfrac{\partial V}{\partial x} = \dfrac{2kQ\left(\sqrt{(L-2x)^2+4R^2}-\sqrt{(L+2x)^2+4R^2}\right)}{\sqrt{(L-2x)^2+4R^2}\sqrt{(L+2x)^2+4R^2}}.$

EVALUATE: For $L << R$ the expression for E_x reduces to that for a ring of charge, $E_x = \dfrac{kQx}{(x^2+a^2)^{3/2}}$, as shown in Example 23.14.

23.83. **IDENTIFY:** Angular momentum and energy must be conserved.

SET UP: At the distance of closest approach the speed is not zero. $E = K + U$. $q_1 = 2e$, $q_2 = 82e$.

EXECUTE: $mv_1b = mv_2r_2$. $E_1 = E_2$ gives $E_1 = \frac{1}{2}mv_2{}^2 + \dfrac{kq_1q_2}{r_2}$. $E_1 = 11\text{ MeV} = 1.76\times10^{-12}$ J. r_2 is the distance of closest approach. Substituting in for $v_2 = v_1\left(\dfrac{b}{r_2}\right)$ we find $E_1 = E_1\dfrac{b^2}{r_2{}^2} + \dfrac{kq_1q_2}{r_2}$.

$(E_1)r_2^2 - (kq_1q_2)r_2 - E_1b^2 = 0$. For $b = 10^{-12}$ m, $r_2 = 1.01\times10^{-12}$ m. For $b = 10^{-13}$ m, $r_2 = 1.11\times10^{-13}$ m. And for $b = 10^{-14}$ m, $r_2 = 2.54\times10^{-14}$ m.

EVALUATE: As b decreases the collision is closer to being head-on and the distance of closest approach decreases. Problem 23.74 shows that the distance of closest approach is 2.48×10^{-14} m when $b = 0$, which is very close to our value.

23.84. **IDENTIFY** and **SET UP:** The He ions are first accelerated toward the center and then accelerated away from the center, but always in the same direction. During the first acceleration, their charge is –e, and during the second acceleration it is +2e. The work-energy theorem gives $\Delta K = q\Delta V$. Call V the voltage at the center.

EXECUTE: **(a)** Toward the center: $\Delta K = q\Delta V = eV$.

Away from the center: $\Delta K = q\Delta V = 2eV$.

The ions gain 3.0 MeV of kinetic energy, so $eV + 2eV$ = 3.0 MeV.
3eV = 3.0 MeV.
V = +1.0 MV, since the e cancels. This is choice (d).

EVALUATE: The negative He^- ions are accelerating to higher potential, and the positive He^{++} ions are accelerating toward lower potential.

23.85. **IDENTIFY** and **SET UP:** Conservation of energy gives $K = U_{\text{electric}} = k\dfrac{q_1q_2}{r}$.

EXECUTE: Solve for Q: $Q = rK/kq = (10\times10^{-15}\text{ m})(3.0\text{ MeV})/(2ek) = 1.67\times10^{-18}$ C. In terms of e, this is $Q = (1.67\times10^{-18}\text{ C})/(1.60\times10^{-19}\text{ C}) = 10.4e \approx 11e$, so choice (b) is best.

EVALUATE: If $Q = 11e$, the atom is sodium (Na), which has an atomic mass of 23, compared to 4 for He. So it is reasonable to assume that the nucleus does not move appreciably, since it is about 6 times more massive than the He.

23.86. **IDENTIFY** and **SET UP:** The potential changes by 6.0 MV over a distance of 12 m. $E_{\text{av}} = \dfrac{\Delta V}{\Delta x}$.

EXECUTE: $E_{\text{av}} = \dfrac{\Delta V}{\Delta x} = (6.0\text{ MV})/(12\text{ m}) = 0.50\times10^6\text{ V/m} = 500{,}000\text{ V/m}$, which is choice (c).

EVALUATE: The actual variation of the field may be somewhat complicated, but the average value gives a good idea of a typical electric field in such apparatus.

24

CAPACITANCE AND DIELECTRICS

24.1. IDENTIFY: The capacitance depends on the geometry (area and plate separation) of the plates.

SET UP: For a parallel-plate capacitor, $V_{ab} = Ed$, $E = \dfrac{Q}{\varepsilon_0 A}$, and $C = \dfrac{Q}{V_{ab}}$.

EXECUTE: **(a)** $V_{ab} = Ed = (4.00\times10^6 \text{ V/m})(2.50\times10^{-3} \text{ m}) = 1.00\times10^4 \text{ V}$.

(b) Solving for the area gives

$$A = \frac{Q}{E\varepsilon_0} = \frac{80.0\times10^{-9} \text{ C}}{(4.00\times10^6 \text{ V/m})[8.854\times10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)]} = 2.26\times10^{-3} \text{ m}^2 = 22.6 \text{ cm}^2.$$

(c) $C = \dfrac{Q}{V_{ab}} = \dfrac{80.0\times10^{-9} \text{ C}}{1.00\times10^4 \text{ V}} = 8.00\times10^{-12} \text{ F} = 8.00 \text{ pF}$.

EVALUATE: The capacitance is reasonable for laboratory capacitors, but the area is rather large.

24.2. IDENTIFY and SET UP: $C = \dfrac{\varepsilon_0 A}{d}$, $C = \dfrac{Q}{V}$ and $V = Ed$.

EXECUTE: **(a)** $C = \varepsilon_0 \dfrac{A}{d} = \varepsilon_0 \dfrac{0.000982 \text{ m}^2}{0.00328 \text{ m}} = 2.65 \text{ pF}$.

(b) $V = \dfrac{Q}{C} = \dfrac{4.35\times10^{-8} \text{ C}}{2.65\times10^{-12} \text{ F}} = 16.4 \text{ kV}$.

(c) $E = \dfrac{V}{d} = \dfrac{16.4\times10^3 \text{ V}}{0.00328 \text{ m}} = 5.00\times10^6 \text{ V/m}$.

EVALUATE: The electric field is uniform between the plates, at points that aren't close to the edges.

24.3. IDENTIFY and SET UP: It is a parallel-plate air capacitor, so we can apply the equations of Section 24.1.

EXECUTE: **(a)** $C = \dfrac{Q}{V_{ab}}$ so $V_{ab} = \dfrac{Q}{C} = \dfrac{0.148\times10^{-6} \text{ C}}{245\times10^{-12} \text{ F}} = 604 \text{ V}$.

(b) $C = \dfrac{\varepsilon_0 A}{d}$ so $A = \dfrac{Cd}{\varepsilon_0} = \dfrac{(245\times10^{-12} \text{ F})(0.328\times10^{-3} \text{ m})}{8.854\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2} = 9.08\times10^{-3} \text{ m}^2 = 90.8 \text{ cm}^2$.

(c) $V_{ab} = Ed$ so $E = \dfrac{V_{ab}}{d} = \dfrac{604 \text{ V}}{0.328\times10^{-3} \text{ m}} = 1.84\times10^6 \text{ V/m}$.

(d) $E = \dfrac{\sigma}{\varepsilon_0}$ so $\sigma = E\varepsilon_0 = (1.84\times10^6 \text{ V/m})(8.854\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2) = 1.63\times10^{-5} \text{ C/m}^2$.

EVALUATE: We could also calculate σ directly as Q/A. $\sigma = \dfrac{Q}{A} = \dfrac{0.148\times10^{-6} \text{ C}}{9.08\times10^{-3} \text{ m}^2} = 1.63\times10^{-5} \text{ C/m}^2$, which checks.

24.4. **IDENTIFY:** $C=\epsilon_0\dfrac{A}{d}$ when there is air between the plates.

SET UP: $A=(3.0\times10^{-2}\text{ m})^2$ is the area of each plate.

EXECUTE: $C=\dfrac{(8.854\times10^{-12}\text{ F/m})(3.0\times10^{-2}\text{ m})^2}{5.0\times10^{-3}\text{ m}}=1.59\times10^{-12}\text{ F}=1.59\text{ pF}.$

EVALUATE: C increases when A increases and C increases when d decreases.

24.5. **IDENTIFY:** $C=\dfrac{Q}{V_{ab}}$. $C=\dfrac{\epsilon_0 A}{d}$.

SET UP: When the capacitor is connected to the battery, $V_{ab}=12.0\text{ V}$.

EXECUTE: **(a)** $Q=CV_{ab}=(10.0\times10^{-6}\text{ F})(12.0\text{ V})=1.20\times10^{-4}\text{ C}=120\ \mu\text{C}$.

(b) When d is doubled C is halved, so Q is halved. $Q=60\ \mu\text{C}$.

(c) If r is doubled, A increases by a factor of 4. C increases by a factor of 4 and Q increases by a factor of 4. $Q=480\ \mu\text{C}$.

EVALUATE: When the plates are moved apart, less charge on the plates is required to produce the same potential difference. With the separation of the plates constant, the electric field must remain constant to produce the same potential difference. The electric field depends on the surface charge density, σ. To produce the same σ, more charge is required when the area increases.

24.6. **IDENTIFY:** $C=\dfrac{Q}{V_{ab}}$. $C=\dfrac{\epsilon_0 A}{d}$.

SET UP: When the capacitor is connected to the battery, enough charge flows onto the plates to make $V_{ab}=12.0\text{ V}$.

EXECUTE: **(a)** 12.0 V.

(b) (i) When d is doubled, C is halved. $V_{ab}=\dfrac{Q}{C}$ and Q is constant, so V doubles. $V=24.0\text{ V}$.

(ii) When r is doubled, A increases by a factor of 4. V decreases by a factor of 4 and $V=3.0\text{ V}$.

EVALUATE: The electric field between the plates is $E=\dfrac{\sigma}{\epsilon_0}=\dfrac{Q}{\epsilon_0 A}$. $V_{ab}=Ed$. When d is doubled E is unchanged and V doubles. When A is increased by a factor of 4, E decreases by a factor of 4 so V decreases by a factor of 4.

24.7. **IDENTIFY:** The energy stored in a capacitor depends on its capacitance, which in turn depends on its geometry.

SET UP: $C=Q/V$ for any capacitor, and $C=\dfrac{\epsilon_0 A}{d}$ for a parallel-plate capacitor.

EXECUTE: **(a)** $C=\dfrac{Q}{V}=\dfrac{2.40\times10^{-10}\text{ C}}{42.0\text{ V}}=5.714\times10^{-12}\text{ F}$. Using $C=\dfrac{\epsilon_0 A}{d}$ gives

$$d=\frac{\epsilon_0 A}{C}=\frac{[8.854\times10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2)](6.80\times10^{-4}\text{ m}^2)}{5.714\times10^{-12}\text{ F}}=1.05\text{ mm}.$$

(b) $d=2.10\times10^{-3}\text{ m}$. $C=\dfrac{\epsilon_0 A}{d}=\dfrac{5.714\times10^{-12}\text{ F}}{2}=2.857\times10^{-12}\text{ F}$. $V=\dfrac{Q}{C}$, so $V=2(42.0\text{ V})=84.0\text{ V}$.

EVALUATE: Doubling the plate separation halves the capacitance, so twice the potential difference is required to keep the same charge on the plates.

24.8. **IDENTIFY:** $C=\dfrac{Q}{V_{ab}}$. $V_{ab}=Ed$. $C=\dfrac{\epsilon_0 A}{d}$.

SET UP: We want $E=1.00\times10^4\text{ N/C}$ when $V=100\text{ V}$.

EXECUTE: **(a)** $d = \dfrac{V_{ab}}{E} = \dfrac{1.00\times10^{2}\ \text{V}}{1.00\times10^{4}\ \text{N/C}} = 1.00\times10^{-2}\ \text{m} = 1.00\ \text{cm}.$

$A = \dfrac{Cd}{\varepsilon_0} = \dfrac{(5.00\times10^{-12}\ \text{F})(1.00\times10^{-2}\ \text{m})}{8.854\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)} = 5.65\times10^{-3}\ \text{m}^2.$ $A = \pi r^2$ so

$r = \sqrt{\dfrac{A}{\pi}} = 4.24\times10^{-2}\ \text{m} = 4.24\ \text{cm}.$

(b) $Q = CV_{ab} = (5.00\times10^{-12}\ \text{F})(1.00\times10^{2}\ \text{V}) = 5.00\times10^{-10}\ \text{C} = 500\ \text{pC}.$

EVALUATE: $C = \dfrac{\varepsilon_0 A}{d}$. We could have a larger d, along with a larger A, and still achieve the required C without exceeding the maximum allowed E.

24.9. **IDENTIFY:** Apply the results of Example 24.4. $C = Q/V$.

SET UP: $r_a = 0.50\ \text{mm}$, $r_b = 5.00\ \text{mm}$.

EXECUTE: **(a)** $C = \dfrac{L2\pi\varepsilon_0}{\ln(r_b/r_a)} = \dfrac{(0.180\ \text{m})2\pi\varepsilon_0}{\ln(5.00/0.50)} = 4.35\times10^{-12}\ \text{F}.$

(b) $V = Q/C = (10.0\times10^{-12}\ \text{C})/(4.35\times10^{-12}\ \text{F}) = 2.30\ \text{V}.$

EVALUATE: $\dfrac{C}{L} = 24.2\ \text{pF}$. This value is similar to those in Example 24.4. The capacitance is determined entirely by the dimensions of the cylinders.

24.10. **IDENTIFY:** Capacitance depends on the geometry of the object.

(a) SET UP: The capacitance of a cylindrical capacitor is $C = \dfrac{2\pi\varepsilon_0 L}{\ln(r_b/r_a)}$. Solving for r_b gives $r_b = r_a e^{2\pi\varepsilon_0 L/C}$.

EXECUTE: Substituting in the numbers for the exponent gives

$$\frac{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.120\ \text{m})}{3.67\times10^{-11}\ \text{F}} = 0.182.$$

Now use this value to calculate r_b: $r_b = r_a e^{0.182} = (0.250\ \text{cm})e^{0.182} = 0.300\ \text{cm}.$

(b) SET UP: For any capacitor, $C = Q/V$ and $\lambda = Q/L$. Combining these equations and substituting the numbers gives $\lambda = Q/L = CV/L$.

EXECUTE: Numerically we get

$$\lambda = \frac{CV}{L} = \frac{(3.67\times10^{-11}\ \text{F})(125\ \text{V})}{0.120\ \text{m}} = 3.82\times10^{-8}\ \text{C/m} = 38.2\ \text{nC/m}.$$

EVALUATE: The distance between the surfaces of the two cylinders would be only 0.050 cm, which is just 0.50 mm. These cylinders would have to be carefully constructed.

24.11. **IDENTIFY:** We can use the definition of capacitance to find the capacitance of the capacitor, and then relate the capacitance to geometry to find the inner radius.

(a) SET UP: By the definition of capacitance, $C = Q/V$.

EXECUTE: $C = \dfrac{Q}{V} = \dfrac{3.30\times10^{-9}\ \text{C}}{2.20\times10^{2}\ \text{V}} = 1.50\times10^{-11}\ \text{F} = 15.0\ \text{pF}.$

(b) SET UP: The capacitance of a spherical capacitor is $C = 4\pi\varepsilon_0 \dfrac{r_a r_b}{r_b - r_a}$.

EXECUTE: Solve for r_a and evaluate using $C = 15.0\ \text{pF}$ and $r_b = 4.00\ \text{cm}$, giving $r_a = 3.09\ \text{cm}$.

(c) SET UP: We can treat the inner sphere as a point charge located at its center and use Coulomb's law, $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$.

EXECUTE: $E = \dfrac{(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.30\times 10^{-9}\ \text{C})}{(0.0309\ \text{m})^2} = 3.12\times 10^4\ \text{N/C}$.

EVALUATE: Outside the capacitor, the electric field is zero because the charges on the spheres are equal in magnitude but opposite in sign.

24.12. IDENTIFY and SET UP: Use $\dfrac{C}{L} = \dfrac{2\pi\varepsilon_0}{\ln(r_b/r_a)}$ which was derived in Example 24.4. Then use $Q = CV$ to calculate Q.

EXECUTE: **(a)** Using $\dfrac{C}{L} = \dfrac{2\pi\varepsilon_0}{\ln(r_b/r_a)}$ gives

$\dfrac{C}{L} = \dfrac{2\pi(8.854\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}{\ln[(3.5\ \text{mm})/(2.2\ \text{mm})]} = 1.2\times 10^{-10}\ \text{F/m} = 120\ \text{pF/m}$.

(b) $C = (1.20\times 10^{-10}\ \text{F/m})(2.8\ \text{m}) = 3.355\times 10^{-10}\ \text{F}$.

$Q = CV = (3.355\times 10^{-10}\ \text{F})(350\times 10^{-3}\ \text{V}) = 1.2\times 10^{-10}\ \text{C} = 120\ \text{pC}$.

The conductor at higher potential has the positive charge, so there is +120 pC on the inner conductor and −120 pC on the outer conductor.

EVALUATE: C depends only on the dimensions of the capacitor. Q and V are proportional.

24.13. IDENTIFY: Apply the results of Example 24.3. $C = Q/V$.

SET UP: $r_a = 15.0$ cm. Solve for r_b.

EXECUTE: **(a)** For two concentric spherical shells, the capacitance is $C = \dfrac{1}{k}\left(\dfrac{r_a r_b}{r_b - r_a}\right)$. $kCr_b - kCr_a = r_a r_b$

and $r_b = \dfrac{kCr_a}{kC - r_a} = \dfrac{k(116\times 10^{-12}\ \text{F})(0.150\ \text{m})}{k(116\times 10^{-12}\ \text{F}) - 0.150\ \text{m}} = 0.175\ \text{m} = 17.5\ \text{cm}$.

(b) $V = 220\ \text{V}$ and $Q = CV = (116\times 10^{-12}\ \text{F})(220\ \text{V}) = 2.55\times 10^{-8}\ \text{C} = 25.5\ \text{nC}$.

EVALUATE: A parallel-plate capacitor with $A = 4\pi r_a r_b = 0.33\ \text{m}^2$ and $d = r_b - r_a = 2.5\times 10^{-2}\ \text{m}$ has $C = \dfrac{\varepsilon_0 A}{d} = 117\ \text{pF}$, in excellent agreement with the value of C for the spherical capacitor.

24.14. IDENTIFY: Simplify the network by replacing series and parallel combinations of capacitors by their equivalents.

SET UP: For capacitors in series the voltages add and the charges are the same; $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$ For capacitors in parallel the voltages are the same and the charges add; $C_{\text{eq}} = C_1 + C_2 + \ldots$ $C = \dfrac{Q}{V}$.

EXECUTE: **(a)** The equivalent capacitance of the $5.0\ \mu\text{F}$ and $8.0\ \mu\text{F}$ capacitors in parallel is $13.0\ \mu\text{F}$. When these two capacitors are replaced by their equivalent we get the network sketched in Figure 24.14. The equivalent capacitance of these three capacitors in series is $3.47\ \mu\text{F}$.

(b) $Q_{\text{tot}} = C_{\text{tot}}V = (3.47\ \mu\text{F})(50.0\ \text{V}) = 174\ \mu\text{C}$.

(c) Q_{tot} is the same as Q for each of the capacitors in the series combination shown in Figure 24.22, so Q for each of the capacitors is $174\ \mu\text{C}$.

EVALUATE: The voltages across each capacitor in Figure 24.14 are $V_{10} = \dfrac{Q_{\text{tot}}}{C_{10}} = 17.4\ \text{V}$,

$V_{13} = \dfrac{Q_{\text{tot}}}{C_{13}} = 13.4\ \text{V}$, and $V_9 = \dfrac{Q_{\text{tot}}}{C_9} = 19.3\ \text{V}$. $V_{10} + V_{13} + V_9 = 17.4\ \text{V} + 13.4\ \text{V} + 19.3\ \text{V} = 50.1\ \text{V}$. The sum of the voltages equals the applied voltage, apart from a small difference due to rounding.

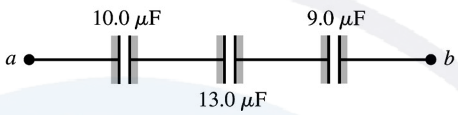


Figure 24.14

24.15. IDENTIFY: For capacitors in series the voltage across the combination equals the sum of the voltages in the individual capacitors. For capacitors in parallel the voltage across the combination is the same as the voltage across each individual capacitor.

SET UP and EXECUTE: **(a)** Connect the capacitors in series so their voltages will add.

(b) $V = V_1 + V_2 + V_3 + \ldots = NV_1$, where N is the number of capacitors in the series combination, since the capacitors are identical. $N = \dfrac{V}{V_1} = \dfrac{500\ \text{V}}{0.10\ \text{V}} = 5000$.

EVALUATE: It requires many small cells to produce a large voltage surge.

24.16. IDENTIFY: The capacitors between b and c are in parallel. This combination is in series with the 15 pF capacitor.

SET UP: Let $C_1 = 15\ \text{pF}$, $C_2 = 9.0\ \text{pF}$ and $C_3 = 11\ \text{pF}$.

EXECUTE: **(a)** For capacitors in parallel, $C_{\text{eq}} = C_1 + C_2 + \ldots$ so $C_{23} = C_2 + C_3 = 20\ \text{pF}$.

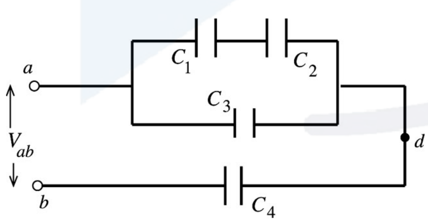
(b) $C_1 = 15\ \text{pF}$ is in series with $C_{23} = 20\ \text{pF}$. For capacitors in series, $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$ so

$\dfrac{1}{C_{123}} = \dfrac{1}{C_1} + \dfrac{1}{C_{23}}$ and $C_{123} = \dfrac{C_1 C_{23}}{C_1 + C_{23}} = \dfrac{(15\ \text{pF})(20\ \text{pF})}{15\ \text{pF} + 20\ \text{pF}} = 8.6\ \text{pF}$.

EVALUATE: For capacitors in parallel the equivalent capacitance is larger than any of the individual capacitors. For capacitors in series the equivalent capacitance is smaller than any of the individual capacitors.

24.17. IDENTIFY: Replace series and parallel combinations of capacitors by their equivalents. In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.

SET UP: Do parts (a) and (b) together. The capacitor network is drawn in Figure 24.17a.

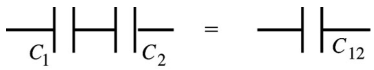


$C_1 = C_2 = C_3 = C_4 = 4.00\ \mu\text{F}$.

$V_{ab} = 28.0\ \text{V}$.

Figure 24.17a

EXECUTE: Simplify the circuit by replacing the capacitor combinations by their equivalents: C_1 and C_2 are in series and are equivalent to C_{12} (Figure 24.17b).

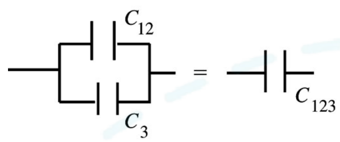


$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2}.$$

Figure 24.17b

$$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(4.00\times10^{-6}\text{ F})(4.00\times10^{-6}\text{ F})}{4.00\times10^{-6}\text{ F} + 4.00\times10^{-6}\text{ F}} = 2.00\times10^{-6}\text{ F}.$$

C_{12} and C_3 are in parallel and are equivalent to C_{123} (Figure 24.17c).



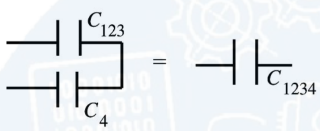
$$C_{123} = C_{12} + C_3.$$

$$C_{123} = 2.00\times10^{-6}\text{ F} + 4.00\times10^{-6}\text{ F}.$$

$$C_{123} = 6.00\times10^{-6}\text{ F}.$$

Figure 24.17c

C_{123} and C_4 are in series and are equivalent to C_{1234} (Figure 24.17d).

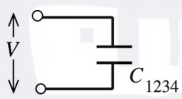


$$\frac{1}{C_{1234}} = \frac{1}{C_{123}} + \frac{1}{C_4}.$$

Figure 24.17d

$$C_{1234} = \frac{C_{123} C_4}{C_{123} + C_4} = \frac{(6.00\times10^{-6}\text{ F})(4.00\times10^{-6}\text{ F})}{6.00\times10^{-6}\text{ F} + 4.00\times10^{-6}\text{ F}} = 2.40\times10^{-6}\text{ F}.$$

The circuit is equivalent to the circuit shown in Figure 24.17e.

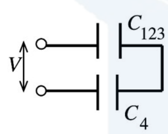


$$V_{1234} = V = 28.0\text{ V}.$$

$$Q_{1234} = C_{1234}V = (2.40\times10^{-6}\text{ F})(28.0\text{ V}) = 67.2\ \mu\text{C}.$$

Figure 24.17e

Now build back up the original circuit, step by step. C_{1234} represents C_{123} and C_4 in series (Figure 24.17f).



$$Q_{123} = Q_4 = Q_{1234} = 67.2\ \mu\text{C}$$

(charge same for capacitors in series).

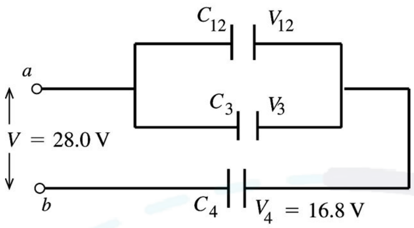
Figure 24.17f

Then $V_{123} = \dfrac{Q_{123}}{C_{123}} = \dfrac{67.2\ \mu\text{C}}{6.00\ \mu\text{F}} = 11.2\text{ V}$.

$$V_4 = \frac{Q_4}{C_4} = \frac{67.2\ \mu\text{C}}{4.00\ \mu\text{F}} = 16.8\text{ V}.$$

Note that $V_4 + V_{123} = 16.8\text{ V} + 11.2\text{ V} = 28.0\text{ V}$, as it should.

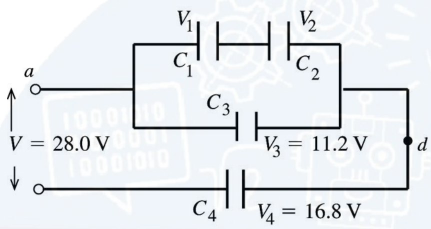
Next consider the circuit as written in Figure 24.17g (next page).



$V_3 = V_{12} = 28.0\text{ V} - V_4$.
$V_3 = 11.2$ V.
$Q_3 = C_3V_3 = (4.00\ \mu\text{F})(11.2\text{ V})$.
$Q_3 = 44.8\ \mu\text{C}$.
$Q_{12} = C_{12}V_{12} = (2.00\ \mu\text{F})(11.2\text{ V})$.
$Q_{12} = 22.4\ \mu\text{C}$.

Figure 24.17g

Finally, consider the original circuit, as shown in Figure 24.17h.



$Q_1 = Q_2 = Q_{12} = 22.4\ \mu\text{C}$
(charge same for capacitors in series).

$$V_1 = \frac{Q_1}{C_1} = \frac{22.4\ \mu\text{C}}{4.00\ \mu\text{F}} = 5.6\text{ V}.$$

$$V_2 = \frac{Q_2}{C_2} = \frac{22.4\ \mu\text{C}}{4.00\ \mu\text{F}} = 5.6\text{ V}.$$

Figure 24.17h

Note that $V_1 + V_2 = 11.2$ V, which equals V_3 as it should.

Summary: $Q_1 = 22.4\ \mu\text{C}$, $V_1 = 5.6$ V.

$Q_2 = 22.4\ \mu\text{C}$, $V_2 = 5.6$ V.

$Q_3 = 44.8\ \mu\text{C}$, $V_3 = 11.2$ V.

$Q_4 = 67.2\ \mu\text{C}$, $V_4 = 16.8$ V.

(c) $V_{ad} = V_3 = 11.2$ V.

EVALUATE: $V_1 + V_2 + V_4 = V$, or $V_3 + V_4 = V$. $Q_1 = Q_2, Q_1 + Q_3 = Q_4$ and $Q_4 = Q_{1234}$.

24.18. IDENTIFY: The two capacitors are in series. The equivalent capacitance is given by $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$.

SET UP: For capacitors in series the charges are the same and the potentials add to give the potential across the network.

EXECUTE: **(a)** $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{1}{(3.00\times10^{-6}\text{ F})} + \dfrac{1}{(5.00\times10^{-6}\text{ F})}$, so $C_{\text{eq}} = 1.875\times10^{-6}$ F. Then

$Q = VC_{\text{eq}} = (64.0\text{ V})(1.875\times10^{-6}\text{ F}) = 1.20\times10^{-4}\text{ C} = 120\ \mu\text{C}$. Each capacitor has a charge of

$1.20\times10^{-4}\text{ C} = 120\ \mu\text{C}$.

(b) $V_1 = Q/C_1 = (1.20\times10^{-4}\text{ C})/(3.0\times10^{-6}\text{ F}) = 40.0$ V.

$V_2 = Q/C_2 = (1.20\times10^{-4}\text{ C})/(5.0\times10^{-6}\text{ F}) = 24.0$ V.

EVALUATE: $V_1 + V_2 = 64.0$ V, which is equal to the applied potential V_{ab}. The capacitor with the smaller C has the larger V.

24.19. IDENTIFY: The two capacitors are in parallel so the voltage is the same on each, and equal to the applied voltage V_{ab}.

SET UP: Do parts (a) and (b) together. The network is sketched in Figure 24.19 (next page).

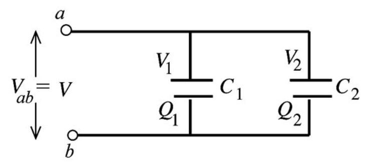


Figure 24.19

EXECUTE: $V_1 = V_2 = V$.
$V_1 = 52.0$ V.
$V_2 = 52.0$ V.

$C = Q/V$ so $Q = CV$.

$Q_1 = C_1V_1 = (3.00\ \mu\text{F})(52.0\ \text{V}) = 156\ \mu\text{C}$. $Q_2 = C_2V_2 = (5.00\ \mu\text{F})(52.0\ \text{V}) = 260\ \mu\text{C}$.

EVALUATE: To produce the same potential difference, the capacitor with the larger C has the larger Q.

24.20. **IDENTIFY:** For capacitors in parallel the voltages are the same and the charges add. For capacitors in series, the charges are the same and the voltages add. $C = Q/V$.

SET UP: C_1 and C_2 are in parallel and C_3 is in series with the parallel combination of C_1 and C_2.

EXECUTE: **(a)** C_1 and C_2 are in parallel and so have the same potential across them:

$V_1 = V_2 = \dfrac{Q_2}{C_2} = \dfrac{30.0\times 10^{-6}\ \text{C}}{3.00\times 10^{-6}\ \text{F}} = 10.0$ V. Therefore, $Q_1 = V_1C_1 = (10.0\ \text{V})(6.00\times 10^{-6}\ \text{F}) = 60.0\times 10^{-6}$ C.

Since C_3 is in series with the parallel combination of C_1 and C_2, its charge must be equal to their combined charge: $Q_3 = 30.0\times 10^{-6}\ \text{C} + 60.0\times 10^{-6}\ \text{C} = 90.0\times 10^{-6}$ C.

(b) The total capacitance is found from $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_{12}} + \dfrac{1}{C_3} = \dfrac{1}{9.00\times 10^{-6}\ \text{F}} + \dfrac{1}{5.00\times 10^{-6}\ \text{F}}$ and

$C_{\text{eq}} = 3.21\mu\text{F}$. $V_{ab} = \dfrac{Q_{\text{tot}}}{C_{\text{eq}}} = \dfrac{90.0\times 10^{-6}\ \text{C}}{3.21\times 10^{-6}\ \text{F}} = 28.0$ V.

EVALUATE: $V_3 = \dfrac{Q_3}{C_3} = \dfrac{90.0\times 10^{-6}\ \text{C}}{5.00\times 10^{-6}\ \text{F}} = 18.0$ V. $V_{ab} = V_1 + V_3 = 10.0\ \text{V} + 18.0\ \text{V} = 28.0$ V, as we just found.

24.21. **IDENTIFY:** Three of the capacitors are in series, and this combination is in parallel with the other two capacitors.

SET UP: For capacitors in series the voltages add and the charges are the same;

$\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$. For capacitors in parallel the voltages are the same and the charges add;

$C_{\text{eq}} = C_1 + C_2 + \ldots$. $C = \dfrac{Q}{V}$.

EXECUTE: **(a)** The equivalent capacitance of the 18.0 nF, 30.0 nF and 10.0 nF capacitors in series is 5.29 nF. When these capacitors are replaced by their equivalent we get the network sketched in Figure 24.21. The equivalent capacitance of these three capacitors in parallel is 19.3 nF, and this is the equivalent capacitance of the original network.

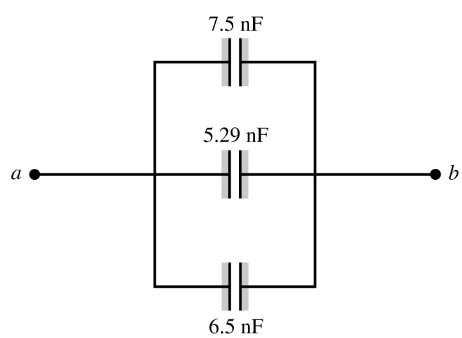


Figure 24.21

(b) $Q_{\text{tot}} = C_{\text{eq}}V = (19.3\text{ nF})(25\text{ V}) = 482\text{ nC}.$

(c) The potential across each capacitor in the parallel network of Figure 24.21 is 25 V. $Q_{6.5} = C_{6.5}V_{6.5} = (6.5\text{ nF})(25\text{ V}) = 162\text{ nC}.$

(d) 25 V.

EVALUATE: As with most circuits, we must go through a series of steps to simplify it as we solve for the unknowns.

24.22. **IDENTIFY:** Refer to Figure 24.10b in the textbook. For capacitors in parallel, $C_{\text{eq}} = C_1 + C_2 + \ldots$. For capacitors in series, $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$.

SET UP: The $11\,\mu\text{F}$, $4\,\mu\text{F}$ and replacement capacitor are in parallel and this combination is in series with the $9.0\,\mu\text{F}$ capacitor.

EXECUTE: $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{8.0\,\mu\text{F}} = \left(\dfrac{1}{(11+4.0+x)\mu\text{F}} + \dfrac{1}{9.0\,\mu\text{F}}\right)$. $(15+x)\mu\text{F} = 72\,\mu\text{F}$ and $x = 57\,\mu\text{F}$.

EVALUATE: Increasing the capacitance of the one capacitor by a large amount makes a small increase in the equivalent capacitance of the network.

24.23. **IDENTIFY** and **SET UP:** The energy density is given by $u = \frac{1}{2}\epsilon_0 E^2$. Use $V = Ed$ to solve for E.

EXECUTE: Calculate E: $E = \dfrac{V}{d} = \dfrac{400\text{ V}}{5.00\times10^{-3}\text{ m}} = 8.00\times10^{4}\text{ V/m}.$

Then $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}(8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(8.00\times10^4\text{ V/m})^2 = 0.0283\text{ J/m}^3.$

EVALUATE: E is smaller than the value in Example 24.8 by about a factor of 6 so u is smaller by about a factor of $6^2 = 36$.

24.24. **IDENTIFY:** Apply $C = Q/V$. $C = \dfrac{\epsilon_0 A}{d}$. The work done to double the separation equals the change in the stored energy.

SET UP: $U = \dfrac{1}{2}CV^2 = \dfrac{Q^2}{2C}$.

EXECUTE: **(a)** $V = Q/C = (3.90\,\mu\text{C})/(920\times10^{-12}\text{ F}) = 4240\text{ V} = 4.24\text{ kV}.$

(b) $C = \dfrac{\epsilon_0 A}{d}$ says that since the charge is kept constant while the separation doubles, that means that the capacitance halves and the voltage doubles to 8480 V = 8.48 kV.

(c) $U_i = \dfrac{Q^2}{2C} = \dfrac{(3.90\times10^{-6}\text{ C})^2}{2(920\times10^{-12}\text{ F})} = 8.27\times10^{-3}\text{ J} = 8.27\text{ mJ}.$ If the separation is doubled while Q stays the same, the capacitance halves, and the energy stored doubles to $2U_i$. The amount of work done to move the plates equals the difference in energy stored in the capacitor, so $\Delta U = U_f - U_i = 2U_i - U_i = U_i = 8.27\text{ mJ}.$

EVALUATE: The oppositely charged plates attract each other so positive work must be done by an external force to pull them farther apart.

24.25. **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$. $C = \dfrac{\epsilon_0 A}{d}$. $V_{ab} = Ed$. The stored energy is $\frac{1}{2}QV$.

SET UP: $d = 1.50\times10^{-3}\text{ m}$. $1\,\mu\text{C} = 10^{-6}\text{ C}$

EXECUTE: **(a)** $C = \dfrac{0.0180\times10^{-6}\text{ C}}{200\text{ V}} = 9.00\times10^{-11}\text{ F} = 90.0\text{ pF}.$

(b) $C = \dfrac{\varepsilon_0 A}{d}$ so $A = \dfrac{Cd}{\varepsilon_0} = \dfrac{(9.00 \times 10^{-11}\ \text{F})(1.50 \times 10^{-3}\ \text{m})}{8.854 \times 10^{-12}\ \text{C}^2/(\text{N} \cdot \text{m}^2)} = 0.0152\ \text{m}^2.$

(c) $V = Ed = (3.0 \times 10^6\ \text{V/m})(1.50 \times 10^{-3}\ \text{m}) = 4.5 \times 10^3\ \text{V} = 4.5\ \text{kV}.$

(d) Energy $= \frac{1}{2}QV = \frac{1}{2}(0.0180 \times 10^{-6}\ \text{C})(200\ \text{V}) = 1.80 \times 10^{-6}\ \text{J} = 1.80\ \mu\text{J}.$

EVALUATE: We could also calculate the stored energy as $\dfrac{Q^2}{2C} = \dfrac{(0.0180 \times 10^{-6}\ \text{C})^2}{2(9.00 \times 10^{-11}\ \text{F})} = 1.80\ \mu\text{J}.$

24.26. **IDENTIFY:** $C = \dfrac{\varepsilon_0 A}{d}$. The stored energy can be expressed either as $\dfrac{Q^2}{2C}$ or as $\dfrac{CV^2}{2}$, whichever is more convenient for the calculation.

SET UP: Since d is halved, C doubles.

EXECUTE: **(a)** If the separation distance is halved while the charge is kept fixed, then the capacitance increases and the stored energy, which was 8.38 J, decreases since $U = Q^2/2C$. Therefore the new energy is 4.19 J.

(b) If the voltage is kept fixed while the separation is decreased by one half, then the doubling of the capacitance leads to a doubling of the stored energy to 16.8 J, using $U = CV^2/2$, when V is held constant throughout.

EVALUATE: When the capacitor is disconnected, the stored energy decreases because of the positive work done by the attractive force between the plates. When the capacitor remains connected to the battery, $Q = CV$ tells us that the charge on the plates increases. The increased stored energy comes from the battery when it puts more charge onto the plates.

24.27. **IDENTIFY:** Use the rules for series and for parallel capacitors to express the voltage for each capacitor in terms of the applied voltage. Express U, Q, and E in terms of the capacitor voltage.

SET UP: Let the applied voltage be V. Let each capacitor have capacitance C. $U = \frac{1}{2}CV^2$ for a single capacitor with voltage V.

EXECUTE: **(a)** Series: The voltage across each capacitor is $V/2$. The total energy stored is $U_\text{s} = 2(\frac{1}{2}C(V/2)^2) = \frac{1}{4}CV^2.$

Parallel: The voltage across each capacitor is V. The total energy stored is

$U_\text{p} = 2(\frac{1}{2}CV^2) = CV^2 \quad \to \quad U_\text{p} = 4U_\text{s}.$

(b) $Q = CV$ for a single capacitor with voltage V. $Q_\text{s} = 2[C(V/2)] = CV$; $Q_\text{p} = 2(CV) = 2CV$; $Q_\text{p} = 2Q_\text{s}$.

(c) $E = V/d$ for a capacitor with voltage V. $E_\text{s} = V/2d$; $E_\text{p} = V/d$; $E_\text{p} = 2E_\text{s}$.

EVALUATE: The parallel combination stores more energy and more charge since the voltage for each capacitor is larger for parallel. More energy stored and larger voltage for parallel means larger electric field in the parallel case.

24.28. **IDENTIFY:** The two capacitors are in series. $\dfrac{1}{C_\text{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$, $C = \dfrac{Q}{V}$, and $U = \frac{1}{2}CV^2$.

SET UP: For capacitors in series the voltages add and the charges are the same.

EXECUTE: **(a)** $\dfrac{1}{C_\text{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$ so $C_\text{eq} = \dfrac{C_1 C_2}{C_1 + C_2} = \dfrac{(150\ \text{nF})(120\ \text{nF})}{150\ \text{nF} + 120\ \text{nF}} = 66.7\ \text{nF}.$

$Q = CV = (66.7\ \text{nF})(48\ \text{V}) = 3.2 \times 10^{-6}\ \text{C} = 3.2\ \mu\text{C}.$

(b) $Q = 3.2\ \mu\text{C}$ for each capacitor.

(c) $U = \frac{1}{2}C_\text{eq}V^2 = \frac{1}{2}(66.7 \times 10^{-9}\ \text{F})(48\ \text{V})^2 = 77\ \mu\text{J}.$

(d) We know C and Q for each capacitor so rewrite U in terms of these quantities.

$U = \frac{1}{2}CV^2 = \frac{1}{2}C(Q/C)^2 = Q^2/2C.$

150 nF: $U=\dfrac{(3.2\times10^{-6}\ \text{C})^2}{2(150\times10^{-9}\ \text{F})}=34\ \mu\text{J}.$

120 nF: $U=\dfrac{(3.2\times10^{-6}\ \text{C})^2}{2(120\times10^{-9}\ \text{F})}=43\ \mu\text{J}.$

Note that $34\ \mu\text{J}+43\ \mu\text{J}=77\ \mu\text{J},$ the total stored energy calculated in part (c).

(e) 150 nF: $V=\dfrac{Q}{C}=\dfrac{3.2\times10^{-6}\ \text{C}}{150\times10^{-9}\ \text{F}}=21\ \text{V}.$

120 nF: $V=\dfrac{Q}{C}=\dfrac{3.2\times10^{-6}\ \text{C}}{120\times10^{-9}\ \text{F}}=27\ \text{V}.$

Note that these two voltages sum to 48 V, the voltage applied across the network.

EVALUATE: Since Q is the same, the capacitor with smaller C stores more energy $(U=Q^2/2C)$ and has a larger voltage $(V=Q/C)$.

24.29. IDENTIFY: The two capacitors are in parallel. $C_{\text{eq}}=C_1+C_2$. $C=\dfrac{Q}{V}$. $U=\tfrac{1}{2}CV^2$.

SET UP: For capacitors in parallel, the voltages are the same and the charges add.

EXECUTE: **(a)** $C_{\text{eq}}=C_1+C_2=35\ \text{nF}+75\ \text{nF}=110\ \text{nF}$. $Q_{\text{tot}}=C_{\text{eq}}V=(110\times10^{-9}\ \text{F})(220\ \text{V})=24.2\ \mu\text{C}$

(b) $V=220\ \text{V}$ for each capacitor.

35 nF: $Q_{35}=C_{35}V=(35\times10^{-9}\ \text{F})(220\ \text{V})=7.7\ \mu\text{C}$; 75 nF: $Q_{75}=C_{75}V=(75\times10^{-9}\ \text{F})(220\ \text{V})=16.5\ \mu\text{C}$.

Note that $Q_{35}+Q_{75}=Q_{\text{tot}}$.

(c) $U_{\text{tot}}=\tfrac{1}{2}C_{\text{eq}}V^2=\tfrac{1}{2}(110\times10^{-9}\ \text{F})(220\ \text{V})^2=2.66\ \text{mJ}.$

(d) 35 nF: $U_{35}=\tfrac{1}{2}C_{35}V^2=\tfrac{1}{2}(35\times10^{-9}\ \text{F})(220\ \text{V})^2=0.85\ \text{mJ};$

75 nF: $U_{75}=\tfrac{1}{2}C_{75}V^2=\tfrac{1}{2}(75\times10^{-9}\ \text{F})(220\ \text{V})^2=1.81\ \text{mJ}.$ Since V is the same the capacitor with larger C stores more energy.

(e) 220 V for each capacitor.

EVALUATE: The capacitor with the larger C has the larger Q.

24.30. IDENTIFY: Capacitance depends on the geometry of the object.

(a) SET UP: The potential difference between the core and tube is $V=\dfrac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a)$. Solving for the linear charge density gives $\lambda=\dfrac{2\pi\epsilon_0 V}{\ln(r_b/r_a)}=\dfrac{4\pi\epsilon_0 V}{2\ln(r_b/r_a)}$.

EXECUTE: Using the given values gives $\lambda=\dfrac{6.00\ \text{V}}{2(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\ln\left(\dfrac{2.00}{1.20}\right)}=6.53\times10^{-10}\ \text{C/m}.$

(b) SET UP: $Q=\lambda L.$

EXECUTE: $Q=(6.53\times10^{-10}\ \text{C/m})(0.350\ \text{m})=2.29\times10^{-10}\ \text{C}.$

(c) SET UP: The definition of capacitance is $C=Q/V$.

EXECUTE: $C=\dfrac{2.29\times10^{-10}\ \text{C}}{6.00\ \text{V}}=3.81\times10^{-11}\ \text{F}.$

(d) SET UP: The energy stored in a capacitor is $U=\tfrac{1}{2}CV^2$.

EXECUTE: $U=\tfrac{1}{2}(3.81\times10^{-11}\ \text{F})(6.00\ \text{V})^2=6.85\times10^{-10}\ \text{J}.$

EVALUATE: The stored energy could be converted to heat or other forms of energy.

24.31. **IDENTIFY:** $U = \frac{1}{2}QV$. Solve for Q. $C = Q/V$.

SET UP: Example 24.4 shows that for a cylindrical capacitor, $\dfrac{C}{L} = \dfrac{2\pi\varepsilon_0}{\ln(r_b/r_a)}$.

EXECUTE: **(a)** $U = \frac{1}{2}QV$ gives $Q = \dfrac{2U}{V} = \dfrac{2(3.20\times10^{-9}\ \text{J})}{4.00\ \text{V}} = 1.60\times10^{-9}\ \text{C}$.

(b) $\dfrac{C}{L} = \dfrac{2\pi\varepsilon_0}{\ln(r_b/r_a)}$. Solving for r_b/r_a gives

$$\frac{r_b}{r_a} = \exp(2\pi\varepsilon_0 L/C) = \exp(2\pi\varepsilon_0\ LV/Q) = \exp[2\pi\varepsilon_0(15.0\ \text{m})(4.00\ \text{V})/(1.60\times10^{-9}\ \text{C})] = 8.05.$$

The radius of the outer conductor is 8.05 times the radius of the inner conductor.

EVALUATE: When the ratio r_b/r_a increases, C/L decreases and less charge is stored for a given potential difference.

24.32. **IDENTIFY:** Apply $u = \frac{1}{2}\varepsilon_0 E^2$.

SET UP: Example 24.3 shows that $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ between the conducting shells and that

$$\frac{Q}{4\pi\varepsilon_0} = \left(\frac{r_a r_b}{r_b - r_a}\right)V_{ab}.$$

EXECUTE: $E = \left(\dfrac{r_a r_b}{r_b - r_a}\right)\dfrac{V_{ab}}{r^2} = \left(\dfrac{(0.125\ \text{m})(0.148\ \text{m})}{0.148\ \text{m} - 0.125\ \text{m}}\right)\dfrac{120\ \text{V}}{r^2} = \dfrac{96.5\ \text{V}\cdot\text{m}}{r^2}$.

(a) For $r = 0.126$ m, $E = 6.08\times10^3$ V/m. $u = \frac{1}{2}\varepsilon_0 E^2 = 1.64\times10^{-4}\ \text{J/m}^3$.

(b) For $r = 0.147$ m, $E = 4.47\times10^3$ V/m. $u = \frac{1}{2}\varepsilon_0 E^2 = 8.85\times10^{-5}\ \text{J/m}^3$.

EVALUATE: **(c)** No, the results of parts (a) and (b) show that the energy density is not uniform in the region between the plates. E decreases as r increases, so u decreases also.

25.33. **IDENTIFY:** $C = KC_0$. $U = \frac{1}{2}CV^2$.

SET UP: $C_0 = 12.5\ \mu\text{F}$ is the value of the capacitance without the dielectric present.

EXECUTE: **(a)** With the dielectric, $C = (3.75)(12.5\ \mu\text{F}) = 46.9\ \mu\text{F}$.

Before: $U = \frac{1}{2}C_0V^2 = \frac{1}{2}(12.5\times10^{-6}\ \text{F})(24.0\ \text{V})^2 = 3.60\ \text{mJ}$.

After: $U = \frac{1}{2}CV^2 = \frac{1}{2}(46.9\times10^{-6}\ \text{F})(24.0\ \text{V})^2 = 13.5\ \text{mJ}$.

(b) $\Delta U = 13.5\ \text{mJ} - 3.6\ \text{mJ} = 9.9\ \text{mJ}$. The energy increased.

EVALUATE: The power supply must put additional charge on the plates to maintain the same potential difference when the dielectric is inserted. $U = \frac{1}{2}QV$, so the stored energy increases.

24.34. **IDENTIFY:** $V = Ed$ and $C = Q/V$. With the dielectric present, $C = KC_0$.

SET UP: $V = Ed$ holds both with and without the dielectric.

EXECUTE: **(a)** $V = Ed = (3.00\times10^4\ \text{V/m})(1.50\times10^{-3}\ \text{m}) = 45.0\ \text{V}$.

$Q = C_0V = (8.00\times10^{-12}\ \text{F})(45.0\ \text{V}) = 3.60\times10^{-10}\ \text{C} = 360\ \text{pC}$.

(b) With the dielectric, $C = KC_0 = (2.70)(8.00\ \text{pF}) = 21.6\ \text{pF}$. V is still 45.0 V, so

$Q = CV = (21.6\times10^{-12}\ \text{F})(45.0\ \text{V}) = 9.72\times10^{-10}\ \text{C} = 972\ \text{pC}$.

EVALUATE: The presence of the dielectric increases the amount of charge that can be stored for a given potential difference and electric field between the plates. Q increases by a factor of K.

24.35. **IDENTIFY** and **SET UP:** Q is constant so we can apply Eq. (24.14). The charge density on each surface of the dielectric is given by $\sigma_i = \sigma(1-1/K)$.

EXECUTE: $E = \dfrac{E_0}{K}$ so $K = \dfrac{E_0}{E} = \dfrac{3.20\times 10^5 \text{ V/m}}{2.50\times 10^5 \text{ V/m}} = 1.28.$

(a) $\sigma_i = \sigma(1-1/K)$.

$\sigma = \epsilon_0 E_0 = (8.854\times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(3.20\times 10^5 \text{ N/C}) = 2.833\times 10^{-6} \text{ C/m}^2.$

$\sigma_i = (2.833\times 10^{-6} \text{ C/m}^2)(1-1/1.28) = 6.20\times 10^{-7} \text{ C/m}^2.$

(b) As calculated above, $K = 1.28$.

EVALUATE: The surface charges on the dielectric produce an electric field that partially cancels the electric field produced by the charges on the capacitor plates.

24.36. **IDENTIFY:** Capacitance depends on geometry, and the introduction of a dielectric increases the capacitance.

SET UP: For a parallel-plate capacitor with dielectric, $C = K\epsilon_0 A/d$.

EXECUTE: **(a)** Solving for d gives

$$d = \frac{K\epsilon_0 A}{C} = \frac{(3.0)(8.85\times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(0.22 \text{ m})(0.28 \text{ m})}{1.0\times 10^{-9} \text{ F}} = 1.64\times 10^{-3} \text{ m} = 1.64 \text{ mm}.$$

Dividing this result by the thickness of a sheet of paper gives $\dfrac{1.64 \text{ mm}}{0.20 \text{ mm/sheet}} \approx 8$ sheets.

(b) Solving for the area of the plates gives $A = \dfrac{Cd}{K\epsilon_0} = \dfrac{(1.0\times 10^{-9} \text{ F})(0.012 \text{ m})}{(3.0)(8.85\times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)} = 0.45 \text{ m}^2.$

(c) Teflon has a smaller dielectric constant (2.1) than the posterboard, so she will need more area to achieve the same capacitance.

EVALUATE: The use of dielectric makes it possible to construct reasonable-sized capacitors since the dielectric increases the capacitance by a factor of K.

24.37. **IDENTIFY** and **SET UP:** For a parallel-plate capacitor with a dielectric we can use the equation $C = K\epsilon_0 A/d$. Minimum A means smallest possible d. d is limited by the requirement that E be less than 1.60×10^7 V/m when V is as large as 5500 V.

EXECUTE: $V = Ed$ so $d = \dfrac{V}{E} = \dfrac{5500 \text{ V}}{1.60\times 10^7 \text{ V/m}} = 3.44\times 10^{-4} \text{ m}.$

Then $A = \dfrac{Cd}{K\epsilon_0} = \dfrac{(1.25\times 10^{-9} \text{ F})(3.44\times 10^{-4} \text{ m})}{(3.60)(8.854\times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)} = 0.0135 \text{ m}^2.$

EVALUATE: The relation $V = Ed$ applies with or without a dielectric present. A would have to be larger if there were no dielectric.

24.38. **IDENTIFY:** We can model the cell wall as a large sheet carrying equal but opposite charges, which makes it equivalent to a parallel-plate capacitor.

SET UP: With air between the layers, $E_0 = \dfrac{Q}{\epsilon_0 A} = \dfrac{\sigma}{\epsilon_0}$ and $V_0 = E_0 d$. The energy density in the electric field is $u = \frac{1}{2}\epsilon_0 E^2$. The volume of a shell of thickness t and average radius R is $4\pi R^2 t$. The volume of a solid sphere of radius R is $\frac{4}{3}\pi R^3$. With the dielectric present, $E = \dfrac{E_0}{K}$ and $V = \dfrac{V_0}{K}$.

EXECUTE: **(a)** $E_0 = \dfrac{\sigma}{\epsilon_0} = \dfrac{0.50\times 10^{-3} \text{ C/m}^2}{8.854\times 10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)} = 5.6\times 10^7 \text{ V/m}.$

(b) $V_0 = E_0 d = (5.6\times 10^7 \text{ V/m})(5.0\times 10^{-9} \text{ m}) = 0.28 \text{ V}.$ The outer wall of the cell is at higher potential, since it has positive charge.

(c) For the cell, $V_{\text{cell}} = \frac{4}{3}\pi R^3$, which gives $R = \left(\frac{3V_{\text{cell}}}{4\pi}\right)^{1/3} = \left(\frac{3(10^{-16}\text{ m}^3)}{4\pi}\right)^{1/3} = 2.9\times10^{-6}$ m. The volume of the cell wall is $V_{\text{wall}} = 4\pi R^2 t = 4\pi(2.9\times10^{-6}\text{ m})^2(5.0\times10^{-9}\text{ m}) = 5.3\times10^{-19}\text{ m}^3$. The energy density in the cell wall is $u_0 = \frac{1}{2}\epsilon_0 E_0^2 = \frac{1}{2}[8.854\times10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2)](5.6\times10^7\text{ V/m})^2 = 1.39\times10^4\text{ J/m}^3$. The total electric-field energy in the cell wall is $(1.39\times10^4\text{ J/m}^3)(5.3\times10^{-19}\text{ m}^3) = 7\times10^{-15}$ J.

(d) $E = \frac{E_0}{K} = \frac{5.6\times10^7\text{ V/m}}{5.4} = 1.0\times10^7$ V/m and $V = \frac{V_0}{K} = \frac{0.28\text{ V}}{5.4} = 0.052$ V.

EVALUATE: To a first approximation, many biological structures can be modeled as basic circuit elements.

24.39. **IDENTIFY:** $C = Q/V$. $C = KC_0$. $V = Ed$.

SET UP: Table 24.1 gives $K = 3.1$ for mylar.

EXECUTE: **(a)** $\Delta Q = Q - Q_0 = (K-1)Q_0 = (K-1)C_0V_0 = (2.1)(2.5\times10^{-7}\text{ F})(12\text{ V}) = 6.3\times10^{-6}$ C.

(b) $\sigma_{\text{i}} = \sigma(1-1/K)$ so $Q_{\text{i}} = Q(1-1/K) = (9.3\times10^{-6}\text{ C})(1-1/3.1) = 6.3\times10^{-6}$ C.

(c) The addition of the mylar doesn't affect the electric field since the induced charge cancels the additional charge drawn to the plates.

EVALUATE: $E = V/d$ and V is constant so E doesn't change when the dielectric is inserted.

24.40. **IDENTIFY** and **SET UP:** The energy density is due to the electric field in the dielectric. $u = \frac{1}{2}\epsilon E^2$, where $\epsilon = K\epsilon_0$. $V = Ed$. In this case, $E = 0.800E_{\text{m}}$.

EXECUTE: **(a)** Using $u = \frac{1}{2}\epsilon E^2$ with $\epsilon = K\epsilon_0$, we have

$u = (1/2)(2.6)(8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)[(0.800)(2.0\times10^7\text{ V/m})]^2 = 2945\text{ J/m}^3$, which rounds to 2900 J/m^3.

(b) First get the plate separation d: $V = Ed$ gives

$d = V/E = (500\,V)/[(0.800)(2.0\times10^7\text{ V/m})] = 3.125\times10^{-5}$ m.

The stored energy is $U = u\times\text{volume} = uAd$, so

$A = U/ud = (0.200\times10^{-3}\text{ J})/[(2945\text{ J/m}^3)(3.125\times10^{-5}\text{m})] = 2.2\times10^{-3}\text{ m}^2 = 22\text{ cm}^2$.

EVALUATE: If this capacitor has square plates, their dimensions would be $x = (22\text{ cm}^2)^{1/2} = 4.7$ cm on each side. This is considerably larger than ordinary laboratory capacitors used in circuits.

24.41. **(a) IDENTIFY** and **SET UP:** Since the capacitor remains connected to the power supply the potential difference doesn't change when the dielectric is inserted. Use $U = \frac{1}{2}CV^2$ to calculate V and combine it with $K = C/C_0$ to obtain a relation between the stored energies and the dielectric constant and use this to calculate K.

EXECUTE: Before the dielectric is inserted $U_0 = \frac{1}{2}C_0V^2$ so $V = \sqrt{\frac{2U_0}{C_0}} = \sqrt{\frac{2(1.85\times10^{-5}\text{ J})}{360\times10^{-9}\text{ F}}} = 10.1$ V.

(b) $K = C/C_0$.

$U_0 = \frac{1}{2}C_0V^2$, $U = \frac{1}{2}CV^2$ so $C/C_0 = U/U_0$.

$$K = \frac{U}{U_0} = \frac{1.85\times10^{-5}\text{ J} + 2.32\times10^{-5}\text{ J}}{1.85\times10^{-5}\text{ J}} = 2.25.$$

EVALUATE: K increases the capacitance and then from $U = \frac{1}{2}CV^2$, with V constant an increase in C gives an increase in U.

24.42. **IDENTIFY:** $C = KC_0$. $C = Q/V$. $V = Ed$.

SET UP: Since the capacitor remains connected to the battery the potential between the plates of the capacitor doesn't change.

EXECUTE: **(a)** The capacitance changes by a factor of K when the dielectric is inserted. Since V is unchanged (the battery is still connected), $\dfrac{C_{\text{after}}}{C_{\text{before}}} = \dfrac{Q_{\text{after}}}{Q_{\text{before}}} = \dfrac{45.0\ \text{pC}}{25.0\ \text{pC}} = K = 1.80.$

(b) The area of the plates is $\pi r^2 = \pi(0.0300\ \text{m})^2 = 2.827\times10^{-3}\ \text{m}^2$ and the separation between them is thus $d = \dfrac{\varepsilon_0 A}{C} = \dfrac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\ \text{m}^2)}{12.5\times10^{-12}\ \text{F}} = 2.00\times10^{-3}\ \text{m}.$ Before the dielectric is inserted, $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{Q}{V}$ and $V = \dfrac{Qd}{\varepsilon_0 A} = \dfrac{(25.0\times10^{-12}\ \text{C})(2.00\times10^{-3}\ \text{m})}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\ \text{m}^2)} = 2.00\ \text{V}.$ The battery remains connected, so the potential difference is unchanged after the dielectric is inserted.

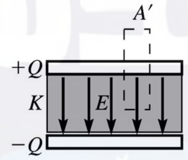
(c) Before the dielectric is inserted, $E = \dfrac{Q}{\varepsilon_0 A} = \dfrac{25.0\times10^{-12}\ \text{C}}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\ \text{m}^2)} = 1000\ \text{N/C}.$

Again, since the voltage is unchanged after the dielectric is inserted, the electric field is also unchanged.

EVALUATE: $E = \dfrac{V}{d} = \dfrac{2.00\ \text{V}}{2.00\times10^{-3}\ \text{m}} = 1000\ \text{N/C},$ whether or not the dielectric is present. This agrees with the result in part (c). The electric field has this value at any point between the plates. We need d to calculate E because V is the potential difference between points separated by distance d.

24.43. **IDENTIFY:** Apply $\oint K\vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl-free}}}{\varepsilon_0}$ to calculate E. $V = Ed$ and $C = Q/V$ apply whether there is a dielectric between the plates or not.

(a) SET UP: Apply $\oint K\vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl-free}}}{\varepsilon_0}$ to the dashed surface in Figure 24.43.



EXECUTE: $\oint K\vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl-free}}}{\varepsilon_0}.$

$\oint K\vec{E}\cdot d\vec{A} = KEA'.$

since $E = 0$ outside the plates

$Q_{\text{encl-free}} = \sigma A' = (Q/A)A'.$

Figure 24.43

Thus $KEA' = \dfrac{(Q/A)A'}{\varepsilon_0}$ and $E = \dfrac{Q}{\varepsilon_0 AK}.$

SET UP and **EXECUTE:** **(b)** $V = Ed = \dfrac{Qd}{\varepsilon_0 AK}.$

(c) $C = \dfrac{Q}{V} = \dfrac{Q}{Qd/\varepsilon_0 AK} = K\dfrac{\varepsilon_0 A}{d} = KC_0.$

EVALUATE: Our result shows that $K = C/C_0$, which is Eq. (24.12).

24.44. **IDENTIFY:** Gauss's law in dielectrics has the same form as in vacuum except that the electric field is multiplied by a factor of K and the charge enclosed by the Gaussian surface is the free charge. The capacitance of an object depends on its geometry.

(a) SET UP: The capacitance of a parallel-plate capacitor is $C = K\varepsilon_0 A/d$ and the charge on its plates is $Q = CV$.

EXECUTE: First find the capacitance:

$$C = \frac{K\varepsilon_0 A}{d} = \frac{(2.1)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0225\ \text{m}^2)}{1.00\times10^{-3}\ \text{m}} = 4.18\times10^{-10}\ \text{F}.$$

Now find the charge on the plates: $Q = CV = (4.18\times10^{-10}\ \text{F})(12.0\ \text{V}) = 5.02\times10^{-9}\ \text{C}.$

(b) Set Up: Gauss's law within the dielectric gives $KEA = Q_{\text{free}}/\varepsilon_0$.

Execute: Solving for E gives

$$E = \frac{Q_{\text{free}}}{KA\varepsilon_0} = \frac{5.02\times10^{-9}\ \text{C}}{(2.1)(0.0225\ \text{m}^2)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 1.20\times10^4\ \text{N/C}.$$

(c) Set Up: Without the Teflon and the voltage source, the charge is unchanged but the potential increases, so $C = \varepsilon_0 A/d$ and Gauss's law now gives $EA = Q/\varepsilon_0$.

Execute: First find the capacitance:

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0225\ \text{m}^2)}{1.00\times10^{-3}\ \text{m}} = 1.99\times10^{-10}\ \text{F}.$$

The potential difference is $V = \dfrac{Q}{C} = \dfrac{5.02\times10^{-9}\ \text{C}}{1.99\times10^{-10}\ \text{F}} = 25.2\ \text{V}$. From Gauss's law, the electric field is

$$E = \frac{Q}{\varepsilon_0 A} = \frac{5.02\times10^{-9}\ \text{C}}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0225\ \text{m}^2)} = 2.52\times10^4\ \text{N/C}.$$

Evaluate: The dielectric reduces the electric field inside the capacitor because the electric field due to the dipoles of the dielectric is opposite to the external field due to the free charge on the plates.

24.45. Identify: $P = E/t$, where E is the total light energy output. The energy stored in the capacitor is $U = \frac{1}{2}CV^2$.

Set Up: $E = 0.95U$.

Execute: **(a)** The power output is 2.70×10^5 W, and 95% of the original energy is converted, so

$E = Pt = (2.70\times10^5\ \text{W})(1.48\times10^{-3}\ \text{s}) = 400\ \text{J}$. $U = \dfrac{400\ \text{J}}{0.95} = 421\ \text{J}$.

(b) $U = \frac{1}{2}CV^2$ so $C = \dfrac{2U}{V^2} = \dfrac{2(421\ \text{J})}{(125\ \text{V})^2} = 0.054\ \text{F}$.

Evaluate: For a given V, the stored energy increases linearly with C.

24.46. Identify and Set Up: $C = \dfrac{\varepsilon_0 A}{d}$. $C = Q/V$. $V = Ed$. $U = \frac{1}{2}CV^2$. With the battery disconnected, Q is constant. When the separation d is doubled, C is halved.

Execute: **(a)** $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{\varepsilon_0(0.12\ \text{m})^2}{3.7\times10^{-3}\ \text{m}} = 3.446\times10^{-11}\ \text{F}$, which rounds to 34 pF.

(b) $Q = CV = (3.446\times10^{-11}\ \text{F})(12\ \text{V}) = 4.135\times10^{-10}\ \text{C}$, which rounds to 410 pC.

(c) $E = V/d = (12\ \text{V})/(3.7\times10^{-3}\ \text{m}) = 3200\ \text{V/m}$.

(d) $U = \frac{1}{2}CV^2 = \frac{1}{2}(3.446\times10^{-11}\ \text{F})(12\ \text{V})^2 = 2.48\times10^{-9}\ \text{J}$, which rounds to 2.5 nJ.

(e) If the battery is disconnected, so the charge remains constant, and the plates are pulled farther apart to 0.0074 m, then the calculations above can be carried out just as before, and we find:

(a) $C = 1.7\times10^{-11}\ \text{F} = 17\ \text{pF}$.

(b) $Q = 4.1\times10^{-10}\ \text{C} = 410\ \text{pC}$.

(c) $E = 3200\ \text{V/m}$.

(d) $U = \dfrac{Q^2}{2C} = \dfrac{(4.1\times10^{-10}\ \text{C})^2}{2(1.7\times10^{-11}\ \text{F})} = 5.0\times10^{-9}\ \text{J} = 5.0\ \text{nJ}$.

Evaluate: Q is unchanged. $E = \dfrac{Q}{\varepsilon_0 A}$ so E is therefore unchanged. U doubles because C is halved with Q unchanged. The additional stored energy comes from the work done by the force that pulled the plates apart.

24.47. **IDENTIFY:** $C = \dfrac{\varepsilon_0 A}{d}$.

SET UP: $A = 4.2\times10^{-5}\ \text{m}^2$. The original separation between the plates is $d = 0.700\times10^{-3}$ m. d' is the separation between the plates at the new value of C.

EXECUTE: $C_0 = \dfrac{A\varepsilon_0}{d} = \dfrac{(4.20\times10^{-5}\ \text{m}^2)\varepsilon_0}{7.00\times10^{-4}\ \text{m}} = 5.31\times10^{-13}$ F. The new value of C is

$C = C_0 + 0.25\ \text{pF} = 7.81\times10^{-13}$ F. But $C = \dfrac{A\varepsilon_0}{d'}$, so $d' = \dfrac{A\varepsilon_0}{C} = \dfrac{(4.20\times10^{-5}\ \text{m}^2)\varepsilon_0}{7.81\times10^{-13}\ \text{F}} = 4.76\times10^{-4}$m.

Therefore the key must be depressed by a distance of $7.00\times10^{-4}\ \text{m} - 4.76\times10^{-4}\ \text{m} = 0.224$ mm.

EVALUATE: When the key is depressed, d decreases and C increases.

24.48. **IDENTIFY:** $C = KC_0 = K\varepsilon_0 \dfrac{A}{d}$. $V = Ed$ for a parallel plate capacitor; this equation applies whether or not a dielectric is present.

SET UP: $A = 1.0\ \text{cm}^2 = 1.0\times10^{-4}\ \text{m}^2$.

EXECUTE: **(a)** $C = (10)\dfrac{(8.85\times10^{-12}\ \text{F/m})(1.0\times10^{-4}\ \text{m}^2)}{7.5\times10^{-9}\ \text{m}} = 1.18\ \mu\text{F per cm}^2$.

(b) $E = \dfrac{V}{d} = \dfrac{85\ \text{mV}}{7.5\times10^{-9}\ \text{m}} = 1.13\times10^{7}$ V/m.

EVALUATE: The dielectric material increases the capacitance. If the dielectric were not present, the same charge density on the faces of the membrane would produce a larger potential difference across the membrane.

24.49. **IDENTIFY:** Some of the charge from the original capacitor flows onto the uncharged capacitor until the potential differences across the two capacitors are the same.

SET UP: $C = \dfrac{Q}{V_{ab}}$. Let $C_1 = 20.0\ \mu\text{F}$ and $C_2 = 10.0\ \mu\text{F}$. The energy stored in a capacitor is

$\frac{1}{2}QV_{ab} = \frac{1}{2}CV_{ab}^2 = \dfrac{Q^2}{2C}$.

EXECUTE: **(a)** The initial charge on the $20.0\ \mu\text{F}$ capacitor is

$Q = C_1(800\ \text{V}) = (20.0\times10^{-6}\ \text{F})(800\ \text{V}) = 0.0160$ C.

(b) In the final circuit, charge Q is distributed between the two capacitors and $Q_1 + Q_2 = Q$. The final circuit contains only the two capacitors, so the voltage across each is the same, $V_1 = V_2$. $V = \dfrac{Q}{C}$ so $V_1 = V_2$ gives $\dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2}$. $Q_1 = \dfrac{C_1}{C_2}Q_2 = 2Q_2$. Using this in $Q_1 + Q_2 = 0.0160$ C gives $3Q_2 = 0.0160$ C and

$Q_2 = 5.33\times10^{-3}$ C. $Q = 2Q_2 = 1.066\times10^{-2}$ C. $V_1 = \dfrac{Q_1}{C_1} = \dfrac{1.066\times10^{-2}\ \text{C}}{20.0\times10^{-6}\ \text{F}} = 533$ V.

$V_2 = \dfrac{Q_2}{C_2} = \dfrac{5.33\times10^{23}\ \text{C}}{10.0\times10^{26}\ \text{F}} = 533$ V. The potential differences across the capacitors are the same, as they should be.

(c) $\text{Energy} = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}(C_1 + C_2)V^2$ gives

$\text{Energy} = \frac{1}{2}(20.0\times10^{-6}\ \text{F} + 10.0\times10^{-6}\ \text{F})(533\ \text{V})^2 = 4.26$ J.

(d) The $20.0\ \mu\text{F}$ capacitor initially has $\text{energy} = \frac{1}{2}C_1V^2 = \frac{1}{2}(20.0\times10^{-6}\ \text{F})(800\ \text{V})^2 = 6.40$ J. The decrease in stored energy that occurs when the capacitors are connected is $6.40\ \text{J} - 4.26\ \text{J} = 2.14$ J.

EVALUATE: The decrease in stored energy is because of conversion of electrical energy to other forms during the motion of the charge when it becomes distributed between the two capacitors. Thermal energy is generated by the current in the wires and energy is emitted in electromagnetic waves.

24.50. IDENTIFY: Initially the capacitors are connected in parallel to the source and we can calculate the charges Q_1 and Q_2 on each. After they are reconnected to each other the total charge is $Q = Q_2 - Q_1$. $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$.

SET UP: After they are reconnected, the charges add and the voltages are the same, so $C_{eq} = C_1 + C_2$, as for capacitors in parallel.

EXECUTE: Originally $Q_1 = C_1V_1 = (9.0\ \mu\text{F})(64\ \text{V}) = 5.8\times10^{-4}\ \text{C} = 580\ \mu\text{C}$, and $Q_2 = C_2V_2 = (4.0\ \mu\text{F})(64\ \text{V}) = 2.6\times10^{-4}\ \text{C} = 260\ \mu\text{C}$. $C_{eq} = C_1 + C_2 = 13.0\ \mu\text{F}$. The original energy stored is $U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}(13.0\times10^{-6}\ \text{F})(64\ \text{V})^2 = 2.662\times10^{-2}\ \text{J}$. Disconnect and flip the capacitors, so now the total charge is $Q = Q_2 - Q_1 = 3.20\times10^{-4}\ \text{C}$ and the equivalent capacitance is still the same, $C_{eq} = 13.0\ \mu\text{F}$.

The new energy stored is $U = \frac{Q^2}{2C_{eq}} = \frac{(3.20\times10^{-4}\ \text{C})^2}{2(13.0\times10^{-6}\ \text{F})} = 3.983\times10^{-3}\ \text{J}$. The change in stored energy is

$\Delta U = 3.983\times10^{-3}\ \text{J} - 2.662\times10^{-2}\ \text{J} = -2.3\times10^{-2}\ \text{J} = -0.023\ \text{J}$.

EVALUATE: When they are reconnected, charge flows and thermal energy is generated and energy is radiated as electromagnetic waves.

24.51. IDENTIFY: Simplify the network by replacing series and parallel combinations by their equivalent. The stored energy in a capacitor is $U = \frac{1}{2}CV^2$.

SET UP: For capacitors in series the voltages add and the charges are the same; $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$. For capacitors in parallel the voltages are the same and the charges add; $C_{eq} = C_1 + C_2 + \ldots$ $C = \frac{Q}{V}$. $U = \frac{1}{2}CV^2$.

EXECUTE: **(a)** Find C_{eq} for the network by replacing each series or parallel combination by its equivalent. The successive simplified circuits are shown in Figure 24.51.

$U_{tot} = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}(2.19\times10^{-6}\ \text{F})(12.0\ \text{V})^2 = 1.58\times10^{-4}\ \text{J} = 158\ \mu\text{J}$.

(b) From Figure 24.51c, $Q_{tot} = C_{eq}V = (2.19\times10^{-6}\ \text{F})(12.0\ \text{V}) = 2.63\times10^{-5}\ \text{C}$. From Figure 24.51b, $Q_{4.8} = 2.63\times10^{-5}\ \text{C}$. $V_{4.8} = \frac{Q_{4.8}}{C_{4.8}} = \frac{2.63\times10^{-5}\ \text{C}}{4.80\times10^{-6}\ \text{F}} = 5.48\ \text{V}$.

$U_{4.8} = \frac{1}{2}CV^2 = \frac{1}{2}(4.80\times10^{-6}\ \text{F})(5.48\ \text{V})^2 = 7.21\times10^{-5}\ \text{J} = 72.1\ \mu\text{J}$.

This one capacitor stores nearly half the total stored energy.

EVALUATE: $U = \frac{Q^2}{2C}$. For capacitors in series the capacitor with the smallest C stores the greatest amount of energy.

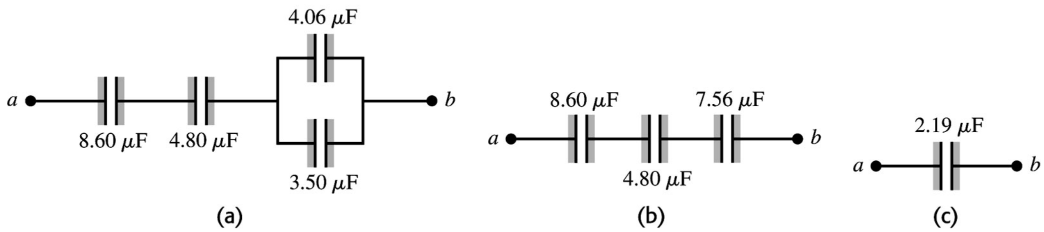


Figure 24.51

24.52. **Identify** and **Set Up:** The charge Q is the same on capacitors in series, and the potential V is the same for capacitors in parallel. C_1 is in series with C_2, and that combination is in parallel with C_3. The C_1-C_2-C_3 combination is in series with C_4. $V = Q/C$.

Execute: **(a)** Since C_1 and C_2 are in series, and that combination is in parallel with C_3, the potential difference across the C_1-C_2 combination is the same as the potential difference across C_3, which is 40.0 V. Also, $Q_1 = Q_2 = Q$.

$V_1 + V_2 = 40.0$ V.

$Q/C_1 + Q/C_2 = 40.0$ V.

$Q/(6.00\ \mu\text{F}) + Q/(3.00\ \mu\text{F}) = 40.0$ V.

$Q = 80.0\ \mu$C.

Therefore

$V_1 = Q/C_1 = (80.0\ \mu\text{C})/(6.00\ \mu\text{F}) = 13.3$ V.

$V_2 = Q/C_2 = (80.0\ \mu\text{C})/(3.00\ \mu\text{F}) = 26.7$ V.

(b) First get the charge Q_4 on C_4. We know that $Q_1 = Q_2 = Q = 80.0\ \mu$C. We also have

$Q_3 = C_3V_3 = (4.00\ \mu\text{F})(40.0\ \text{V}) = 160\ \mu$C.

$Q_4 = Q + Q_3 = 80.0\ \mu\text{C} + 160\ \mu\text{C} = 240\ \mu$C.

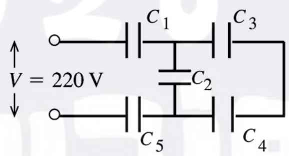
$V_4 = Q_4/C_4 = (240\ \mu\text{C})/(8.00\ \mu\text{F}) = 30.0$ V.

(c) $V_{ab} = V_3 + V_4 = 40.0\ \text{V} + 30.0\ \text{V} = 70.0$ V.

Evaluate: C_3 and C_4 are *not* in parallel, so $V_3 \neq V_4$.

24.53. **(a) Identify:** Replace series and parallel combinations of capacitors by their equivalents.

Set Up: The network is sketched in Figure 24.53a.

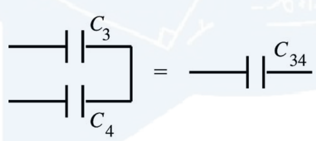


$C_1 = C_5 = 8.4\ \mu\text{F}.$

$C_2 = C_3 = C_4 = 4.2\ \mu\text{F}.$

Figure 24.53a

Execute: Simplify the circuit by replacing the capacitor combinations by their equivalents: C_3 and C_4 are in series and can be replaced by C_{34} (Figure 24.53b):



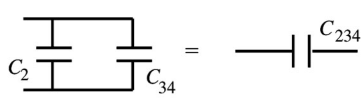
$$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4}.$$

$$\frac{1}{C_{34}} = \frac{C_3 + C_4}{C_3C_4}.$$

Figure 24.53b

$$C_{34} = \frac{C_3C_4}{C_3 + C_4} = \frac{(4.2\ \mu\text{F})(4.2\ \mu\text{F})}{4.2\ \mu\text{F} + 4.2\ \mu\text{F}} = 2.1\ \mu\text{F}.$$

C_2 and C_{34} are in parallel and can be replaced by their equivalent (Figure 24.53c):



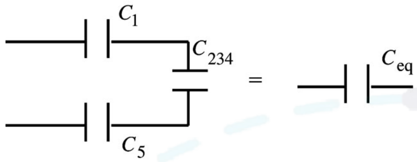
$C_{234} = C_2 + C_{34}.$

$C_{234} = 4.2\ \mu\text{F} + 2.1\ \mu\text{F}.$

$C_{234} = 6.3\ \mu\text{F}.$

Figure 24.53c

C_1, C_5, and C_{234} are in series and can be replaced by C_{eq} (Figure 24.53d):



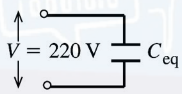
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_5} + \frac{1}{C_{234}}.$$

$$\frac{1}{C_{eq}} = \frac{2}{8.4\ \mu\text{F}} + \frac{1}{6.3\ \mu\text{F}}.$$

$$C_{eq} = 2.5\ \mu\text{F}.$$

Figure 24.53d

EVALUATE: For capacitors in series the equivalent capacitor is smaller than any of those in series. For capacitors in parallel the equivalent capacitance is larger than any of those in parallel.
(b) IDENTIFY and **SET UP:** In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.
EXECUTE: The equivalent circuit is drawn in Figure 24.53e.



$$Q_{eq} = C_{eq}V.$$

$$Q_{eq} = (2.5\ \mu\text{F})(220\ \text{V}) = 550\ \mu\text{C}.$$

Figure 24.53e

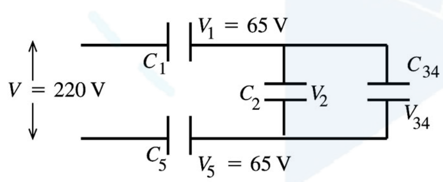
$Q_1 = Q_5 = Q_{234} = 550\ \mu\text{C}$ (capacitors in series have same charge).

$$V_1 = \frac{Q_1}{C_1} = \frac{550\ \mu\text{C}}{8.4\ \mu\text{F}} = 65\ \text{V}.$$

$$V_5 = \frac{Q_5}{C_5} = \frac{550\ \mu\text{C}}{8.4\ \mu\text{F}} = 65\ \text{V}.$$

$$V_{234} = \frac{Q_{234}}{C_{234}} = \frac{550\ \mu\text{C}}{6.3\ \mu\text{F}} = 87\ \text{V}.$$

Now draw the network as in Figure 24.53f.



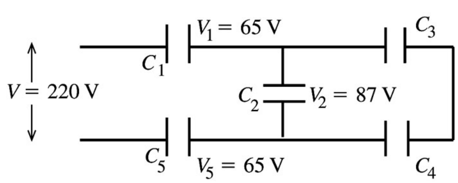
$V_2 = V_{34} = V_{234} = 87$ V
capacitors in parallel have the same potential.

Figure 24.53f

$Q_2 = C_2V_2 = (4.2\ \mu\text{F})(87\ \text{V}) = 370\ \mu\text{C}.$

$Q_{34} = C_{34}V_{34} = (2.1\ \mu\text{F})(87\ \text{V}) = 180\ \mu\text{C}.$

Finally, consider the original circuit (Figure 24.53g).



$Q_3 = Q_4 = Q_{34} = 180\ \mu\text{C}$
capacitors in series have the same charge.

Figure 24.53g

$V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu\text{C}}{4.2\ \mu\text{F}} = 43\ \text{V}.$

$V_4 = \frac{Q_4}{C_4} = \frac{180\ \mu\text{C}}{4.2\ \mu\text{F}} = 43\ \text{V}.$

Summary: $Q_1 = 550\ \mu\text{C},\ V_1 = 65\ \text{V}.$

$Q_2 = 370\ \mu\text{C},\ V_2 = 87\ \text{V}.$

$Q_3 = 180\ \mu\text{C},\ V_3 = 43\ \text{V}.$

$Q_4 = 180\ \mu\text{C},\ V_4 = 43\ \text{V}.$

$Q_5 = 550\ \mu\text{C},\ V_5 = 65\ \text{V}.$

EVALUATE: $V_3 + V_4 = V_2$ and $V_1 + V_2 + V_5 = 220\ \text{V}$ (apart from some small rounding error)

$Q_1 = Q_2 + Q_3$ and $Q_5 = Q_2 + Q_4.$

24.54. **IDENTIFY** and **SET UP:** The total stored energy is $U = \frac{1}{2}CV^2$, and the energy density is $u = U/(\text{volume})$. The volume of a cylinder is $\pi r^2 l$. $u = \frac{1}{2}\varepsilon E^2$, where $\varepsilon = K\varepsilon_0$.

EXECUTE: **(a)** $U = \frac{1}{2}CV^2 = (1/2)(3000\ \text{F})(2.7\ \text{V})^2 = 1.09\times10^4\ \text{J}$, which rounds to 1.1×10^4 J.

(b) $3.0\ \text{Wh} = (3.0\ \text{J/s})(3600\ \text{s}) = 1.1\times10^4\ \text{J}$, which agrees with our result in (a) within the accuracy of the given numbers.

(c) $u = U/(\text{volume}) = U/(\pi r^2 l) = (1.09\times10^4\ \text{J})/[\pi(0.030\ \text{m})^2(0.135\ \text{m})] = 2.9\times10^7\ \text{J/m}^3$.

(d) For polyester, $K = 3.3$ and $E_\text{m} = 6\times10^7\ \text{V/m}$, so

$u = \frac{1}{2}K\varepsilon_0 E^2 = (1/2)(3.3)(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(6\times10^7\ \text{V/m})^2 = 5.3\times10^4\ \text{J/m}^3.$

$u/u_\text{polyester} = (2.9\times10^7\ \text{J/m}^3)/(5.3\times10^4\ \text{J/m}^3) = 540$, so this capacitor can have over 500 times the energy density of a polyester capacitor.

EVALUATE: It requires only 2.7 V to give this capacitor a stored energy of 1.1×10^4 J. For a typical 1.0-μF capacitor, the voltage would be $V = (2U/C)^{1/2} = [2(1.11\times0^4\ \text{J})/(1.0\times10^{-6}\ \text{F})] = 1.5\times10^5\ \text{V} =$ 150 kV. That's quite a difference from 2.7 V!

24.55. **IDENTIFY:** Capacitors in series carry the same charge, while capacitors in parallel have the same potential difference across them.

SET UP: $V_{ab} = 150\ \text{V},\ Q_1 = 150\ \mu\text{C},\ Q_3 = 450\ \mu\text{C}$, and $V = Q/C$.

EXECUTE: $C_1 = 3.00\ \mu\text{F}$ so $V_1 = \frac{Q_1}{C_1} = \frac{150\ \mu\text{C}}{3.00\ \mu\text{F}} = 50.0\ \text{V}$ and $V_1 = V_2 = 50.0\ \text{V}$. $V_1 + V_3 = V_{ab}$ so

$V_3 = 100\ \text{V}$. $C_3 = \frac{Q_3}{V_3} = \frac{450\ \mu\text{C}}{100\ \text{V}} = 4.50\ \mu\text{F}$. $Q_1 + Q_2 = Q_3$ so $Q_2 = Q_3 - Q_1 = 450\ \mu\text{C} - 150\ \mu\text{C} = 300\ \mu\text{C}$

and $C_2 = \frac{Q_2}{V_2} = \frac{300\ \mu\text{C}}{50.0\ \text{V}} = 6.00\ \mu\text{F}$.

EVALUATE: Capacitors in parallel only carry the same charge if they have the same capacitance.

24.56. **IDENTIFY:** Apply the rules for combining capacitors in series and in parallel.

SET UP: With the switch open, each pair of $3.00\ \mu\text{F}$ and $6.00\ \mu\text{F}$ capacitors are in series with each other and each pair is in parallel with the other pair. When the switch is closed, each pair of $3.00\ \mu\text{F}$ and $6.00\ \mu\text{F}$ capacitors are in parallel with each other and the two pairs are in series.

EXECUTE: **(a)** With the switch open $C_\text{eq} = \left(\left(\frac{1}{3\ \mu\text{F}} + \frac{1}{6\ \mu\text{F}}\right)^{-1} + \left(\frac{1}{3\ \mu\text{F}} + \frac{1}{6\ \mu\text{F}}\right)^{-1}\right) = 4.00\ \mu\text{F}.$

$Q_\text{total} = C_\text{eq}V = (4.00\ \mu\text{F})(210\ \text{V}) = 8.40\times10^{-4}\ \text{C}$. By symmetry, each capacitor carries 4.20×10^{-4} C. The voltages are then calculated via $V = Q/C$. This gives $V_{ad} = Q/C_3 = 140\text{ V}$ and $V_{ac} = Q/C_6 = 70\text{ V}$. $V_{cd} = V_{ad} - V_{ac} = 70\text{ V}$.

(b) When the switch is closed, the points *c* and *d* must be at the same potential, so the equivalent capacitance is $C_{\text{eq}} = \left(\dfrac{1}{(3.00+6.00)\ \mu\text{F}} + \dfrac{1}{(3.00+6.00)\ \mu\text{F}}\right)^{-1} = 4.5\ \mu\text{F}.$

$Q_{\text{total}} = C_{\text{eq}}V = (4.50\ \mu\text{F})(210\text{ V}) = 9.5\times10^{-4}\text{ C}$, and each capacitor has the same potential difference of 105 V (again, by symmetry).

(c) Consider the $C_3 = 3.00\ \mu\text{F}$ and $C_6 = 6.00\ \mu\text{F}$ capacitors in the upper branch of the network. The only way for the net charge Q_{net} on the negative plate of C_3 and the positive plate of C_6 to change is by charge to flow through the switch. With the switch open all four capacitors have the same charge and $Q_{\text{net}} = 0$. With the switch closed the charge on C_3 is $Q_3 = (3.00\ \mu\text{F})(105\text{ V}) = 315\ \mu\text{C}$ and the charge on C_6 is $Q_6 = (6.00\ \mu\text{F})(105\text{ V}) = 630\ \mu\text{C}$ and $Q_{\text{net}} = Q_2 - Q_1 = 315\ \mu\text{C}$. Therefore, the change in Q_{net} is $315\ \mu\text{C}$ and this is the amount of charge that flowed through the switch when it was closed.

EVALUATE: When the switch is closed the charge must redistribute to make points *c* and *d* be at the same potential.

24.57. **(a) IDENTIFY:** Replace the three capacitors in series by their equivalent. The charge on the equivalent capacitor equals the charge on each of the original capacitors.

SET UP: The three capacitors can be replaced by their equivalent as shown in Figure 24.57a.

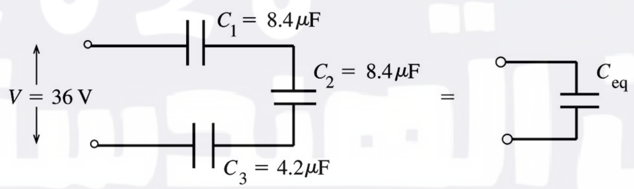


Figure 24.57a

EXECUTE: $C_3 = C_1/2$ so $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} = \dfrac{4}{8.4\ \mu\text{F}}$ and $C_{\text{eq}} = 8.4\ \mu\text{F}/4 = 2.1\ \mu\text{F}$.

$Q = C_{\text{eq}}V = (2.1\ \mu\text{F})(36\text{ V}) = 76\ \mu\text{C}$.

The three capacitors are in series so they each have the same charge: $Q_1 = Q_2 = Q_3 = 76\ \mu\text{C}$.

EVALUATE: The equivalent capacitance for capacitors in series is smaller than each of the original capacitors.

(b) IDENTIFY and **SET UP:** Use $U = \frac{1}{2}QV$. We know each Q and we know that $V_1 + V_2 + V_3 = 36\text{ V}$.

EXECUTE: $U = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2V_2 + \frac{1}{2}Q_3V_3$.

But $Q_1 = Q_2 = Q_3 = Q$ so $U = \frac{1}{2}Q(V_1 + V_2 + V_3)$.

But also $V_1 + V_2 + V_3 = V = 36\text{ V}$, so $U = \frac{1}{2}QV = \frac{1}{2}(76\ \mu\text{C})(36\text{ V}) = 1.4\times10^{-3}\text{ J}$.

EVALUATE: We could also use $U = Q^2/2C$ and calculate U for each capacitor.

(c) IDENTIFY: The charges on the plates redistribute to make the potentials across each capacitor the same.

SET UP: The capacitors before and after they are connected are sketched in Figure 24.57b.

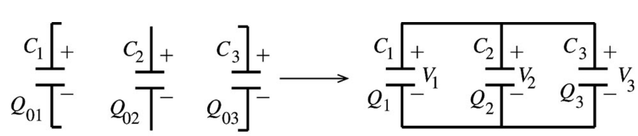


Figure 24.57b

EXECUTE: The total positive charge that is available to be distributed on the upper plates of the three capacitors is $Q_0 = Q_{01} + Q_{02} + Q_{03} = 3(76\ \mu\text{C}) = 228\ \mu\text{C}$. Thus $Q_1 + Q_2 + Q_3 = 228\ \mu\text{C}$. After the circuit is completed the charge distributes to make $V_1 = V_2 = V_3$. $V = Q/C$ and $V_1 = V_2$ so $Q_1/C_1 = Q_2/C_2$ and then $C_1 = C_2$ says $Q_1 = Q_2$. $V_1 = V_3$ says $Q_1/C_1 = Q_3/C_3$ and $Q_1 = Q_3(C_1/C_3) = Q_3(8.4\ \mu\text{F}/4.2\ \mu\text{F}) = 2Q_3$.
Using $Q_2 = Q_1$ and $Q_1 = 2Q_3$ in the above equation gives $2Q_3 + 2Q_3 + Q_3 = 228\ \mu\text{C}$.
$5Q_3 = 228\ \mu\text{C}$ and $Q_3 = 45.6\ \mu\text{C}$, $Q_1 = Q_2 = 91.2\ \mu\text{C}$

Then $V_1 = \dfrac{Q_1}{C_1} = \dfrac{91.2\ \mu\text{C}}{8.4\ \mu\text{F}} = 11\ \text{V}$, $V_2 = \dfrac{Q_2}{C_2} = \dfrac{91.2\ \mu\text{C}}{8.4\ \mu\text{F}} = 11\ \text{V}$, and $V_3 = \dfrac{Q_3}{C_3} = \dfrac{45.6\ \mu\text{C}}{4.2\ \mu\text{F}} = 11\ \text{V}$.

The voltage across each capacitor in the parallel combination is 11 V.
(d) $U = \tfrac{1}{2}Q_1V_1 + \tfrac{1}{2}Q_2V_2 + \tfrac{1}{2}Q_3V_3$.
But $V_1 = V_2 = V_3$ so $U = \tfrac{1}{2}V_1(Q_1 + Q_2 + Q_3) = \tfrac{1}{2}(11\ \text{V})(228\ \mu\text{C}) = 1.3\times10^{-3}\ \text{J}$.

EVALUATE: This is less than the original energy of 1.4×10^{-3} J. The stored energy has decreased, as in Example 24.7.

24.58. **IDENTIFY:** $C = \dfrac{\epsilon_0 A}{d}$. $C = \dfrac{Q}{V}$. $V = Ed$. $U = \tfrac{1}{2}QV$.

SET UP: $d = 3.0\times10^3$ m. $A = \pi r^2$, with $r = 1.0\times10^3$ m.

EXECUTE: **(a)** $C = \dfrac{\epsilon_0 A}{d} = \dfrac{(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\pi(1.0\times10^3\ \text{m})^2}{3.0\times10^3\ \text{m}} = 9.3\times10^{-9}\ \text{F}$.

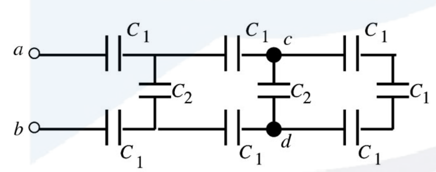
(b) $V = \dfrac{Q}{C} = \dfrac{20\ \text{C}}{9.3\times10^{-9}\ \text{F}} = 2.2\times10^9\ \text{V}$.

(c) $E = \dfrac{V}{d} = \dfrac{2.2\times10^9\ \text{V}}{3.0\times10^3\ \text{m}} = 7.3\times10^5\ \text{V/m}$.

(d) $U = \tfrac{1}{2}QV = \tfrac{1}{2}(20\ \text{C})(2.2\times10^9\ \text{V}) = 2.2\times10^{10}\ \text{J}$.

EVALUATE: Thunderclouds involve very large potential differences and large amounts of stored energy.

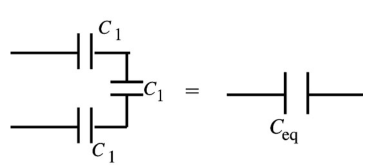
24.59. **IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents. In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.
(a) SET UP: The network is sketched in Figure 24.59a.



$C_1 = 6.9\ \mu\text{F}$.
$C_2 = 4.6\ \mu\text{F}$.

Figure 24.59a

EXECUTE: Simplify the network by replacing the capacitor combinations by their equivalents. Make the replacement shown in Figure 24.59b.

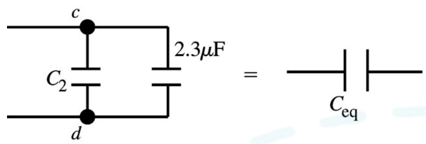


$\dfrac{1}{C_{eq}} = \dfrac{3}{C_1}$.

$C_{eq} = \dfrac{C_1}{3} = \dfrac{6.9\ \mu\text{F}}{3} = 2.3\ \mu\text{F}$.

Figure 24.59b

Next make the replacement shown in Figure 24.59c.

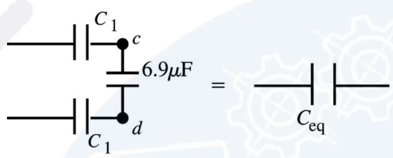


$$C_{\text{eq}} = 2.3\ \mu\text{F} + C_2.$$

$$C_{\text{eq}} = 2.3\ \mu\text{F} + 4.6\ \mu\text{F} = 6.9\ \mu\text{F}.$$

Figure 24.59c

Make the replacement shown in Figure 24.59d.

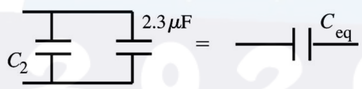


$$\frac{1}{C_{\text{eq}}} = \frac{2}{C_1} + \frac{1}{6.9\ \mu\text{F}} = \frac{3}{6.9\ \mu\text{F}}.$$

$$C_{\text{eq}} = 2.3\ \mu\text{F}.$$

Figure 24.59d

Make the replacement shown in Figure 24.59e.

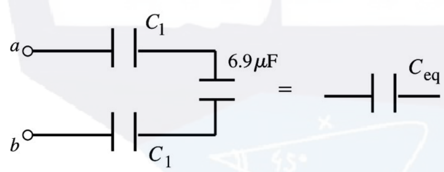


$$C_{\text{eq}} = C_2 + 2.3\ \mu\text{F} = 4.6\ \mu\text{F} + 2.3\ \mu\text{F}.$$

$$C_{\text{eq}} = 6.9\ \mu\text{F}.$$

Figure 24.59e

Make the replacement shown in Figure 24.59f.

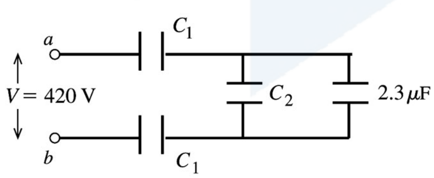


$$\frac{1}{C_{\text{eq}}} = \frac{2}{C_1} + \frac{1}{6.9\ \mu\text{F}} = \frac{3}{6.9\ \mu\text{F}}.$$

$$C_{\text{eq}} = 2.3\ \mu\text{F}.$$

Figure 24.59f

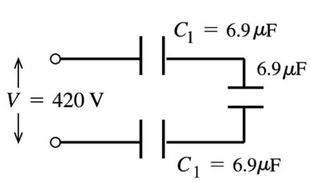
(b) SET UP and EXECUTE: Consider the network as drawn in Figure 24.59g.



From part (a) $2.3\ \mu\text{F}$ is the equivalent capacitance of the rest of the network.

Figure 24.59g

The equivalent network is shown in Figure 24.59h.

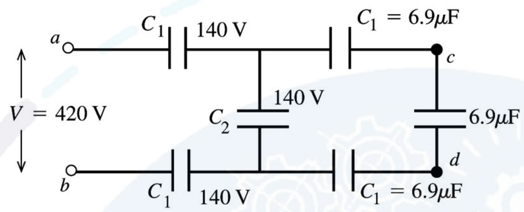


The capacitors are in series, so all three capacitors have the same Q.

Figure 24.59h

But here all three have the same C, so by $V = Q/C$ all three must have the same V. The three voltages must add to 420 V, so each capacitor has $V = 140$ V. The 6.9 μF to the right is the equivalent of C_2 and the 2.3 μF capacitor in parallel, so $V_2 = 140$ V. (Capacitors in parallel have the same potential difference.) Hence $Q_1 = C_1V_1 = (6.9\ \mu\text{F})(140\ \text{V}) = 9.7\times10^{-4}$ C and $Q_2 = C_2V_2 = (4.6\ \mu\text{F})(140\ \text{V}) = 6.4\times10^{-4}$ C.

(c) From the potentials deduced in part (b) we have the situation shown in Figure 24.59i.



From part (a) 6.9 μF is the equivalent capacitance of the rest of the network.

Figure 24.59i

The three right-most capacitors are in series and therefore have the same charge. But their capacitances are also equal, so by $V = Q/C$ they each have the same potential difference. Their potentials must sum to 140 V, so the potential across each is 47 V and $V_{cd} = 47$ V.

EVALUATE: In each capacitor network the rules for combining V for capacitors in series and parallel are obeyed. Note that $V_{cd} < V$, in fact $V - 2(140\ \text{V}) - 2(47\ \text{V}) = V_{cd}$.

24.60. IDENTIFY: Find the total charge on the capacitor network when it is connected to the battery. This is the amount of charge that flows through the signal device when the switch is closed.

Circuit (a):

SET UP: For capacitors in parallel, $C_{\text{eq}} = C_1 + C_2 + C_3 + \dots$

EXECUTE: $C_{\text{equiv}} = C_1 + C_2 + C_3 = 60.0\ \mu\text{F}$. $Q = CV = (60.0\ \mu\text{F})(120\ \text{V}) = 7200\ \mu\text{C}$.

EVALUATE: More charge is stored by the three capacitors in parallel than would be stored in each capacitor used alone.

Circuit (b):

SET UP: $C_{\text{equiv}} = \left(\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}\right)^{-1}$.

EXECUTE: $C_{\text{equiv}} = 5.45\ \mu\text{F}$. $Q = (5.45\ \mu\text{F})(120\text{V}) = 654\ \mu\text{C}$.

EVALUATE: Less charge is stored by the three capacitors in series than would be stored in each capacitor used alone.

24.61. (a) IDENTIFY and **SET UP:** Q is constant. $C = KC_0$; use $C = Q/V_{ab}$ to relate the dielectric constant K to the ratio of the voltages without and with the dielectric.

EXECUTE: With the dielectric: $V = Q/C = Q/(KC_0)$.

without the dielectric: $V_0 = Q/C_0$.

$V_0/V = K$, so $K = (45.0\ \text{V})/(11.5\ \text{V}) = 3.91$.

EVALUATE: Our analysis agrees with Eq. (24.13).

(b) IDENTIFY: The capacitor can be treated as equivalent to two capacitors C_1 and C_2 in parallel, one with area $2A/3$ and air between the plates and one with area $A/3$ and dielectric between the plates.

SET UP: The equivalent network is shown in Figure 24.61 (next page).

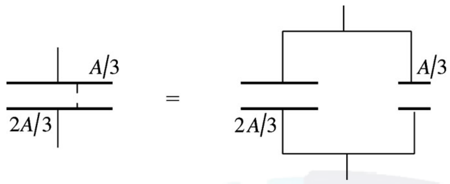


Figure 24.61

EXECUTE: Let $C_0 = \epsilon_0 A/d$ be the capacitance with only air between the plates. $C_1 = KC_0/3, C_2 = 2C_0/3;$ $C_{eq} = C_1 + C_2 = (C_0/3)(K+2).$

$$V = \frac{Q}{C_{eq}} = \frac{Q}{C_0}\left(\frac{3}{K+2}\right) = V_0\left(\frac{3}{K+2}\right) = (45.0 \text{ V})\left(\frac{3}{5.91}\right) = 22.8 \text{ V}.$$

EVALUATE: The voltage is reduced by the dielectric. The voltage reduction is less when the dielectric doesn't completely fill the volume between the plates.

24.62. **IDENTIFY:** This situation is analogous to having two capacitors C_1 in series, each with separation $\frac{1}{2}(d-a)$.

SET UP: For capacitors in series, $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}.$

EXECUTE: **(a)** $C = \left(\dfrac{1}{C_1} + \dfrac{1}{C_1}\right)^{-1} = \frac{1}{2}C_1 = \frac{1}{2}\dfrac{\epsilon_0 A}{(d-a)/2} = \dfrac{\epsilon_0 A}{d-a}.$

(b) $C = \dfrac{\epsilon_0 A}{d-a} = \dfrac{\epsilon_0 A}{d}\dfrac{d}{d-a} = C_0\dfrac{d}{d-a}.$

EVALUATE: **(c)** As $a \to 0$, $C \to C_0$. The metal slab has no effect if it is very thin. And as $a \to d$, $C \to \infty$. $V = Q/C$. $V = Ey$ is the potential difference between two points separated by a distance y parallel to a uniform electric field. When the distance is very small, it takes a very large field and hence a large Q on the plates for a given potential difference. Since $Q = CV$ this corresponds to a very large C.

24.63. **IDENTIFY:** Capacitors in series carry the same charge, but capacitors in parallel have the same potential difference across them.

SET UP: $V_{ab} = 48.0$ V. $C = Q/V$ and $U = \dfrac{1}{2}CV^2$. For capacitors in parallel, $C = C_1 + C_2$, and for capacitors in series, $1/C = 1/C_1 + 1/C_2$.

EXECUTE: Using $U = \dfrac{1}{2}CV^2$ gives $C = \dfrac{2U}{V^2} = \dfrac{2(2.90\times10^{-3}\text{ J})}{(48.0\text{ V})^2} = 2.517\times10^{-6}$ F, which is the equivalent capacitance of the network. The equivalent capacitance for C_1 and C_2 in series is $C_{12} = \dfrac{1}{2}(4.00\,\mu\text{F}) = 2.00\,\mu$F. If C_{123} is the equivalent capacitance for C_{12} and C_3 in parallel, then $\dfrac{1}{C_{123}} + \dfrac{1}{C_4} = \dfrac{1}{C}$. Solving for C_{123} gives

$$\frac{1}{C_{123}} = \frac{1}{C} - \frac{1}{C_4} = \frac{1}{2.517\times10^{-6}\text{ F}} - \frac{1}{8.00\times10^{-6}\text{ F}} = 2.722\times10^{5}\text{ F}^{-1}, \text{ so } C_{123} = 3.673\times10^{-6}\text{ F}.$$

$C_{12} + C_3 = C_{123}$. $C_3 = C_{123} - C_{12} = 3.673\,\mu\text{F} - 2.00\,\mu\text{F} = 1.67\,\mu\text{F}.$

EVALUATE: As with most circuits, it is necessary to solve them in a series of steps rather than using a single step.

24.64. **IDENTIFY:** The electric field energy density is $u = \frac{1}{2}\epsilon_0 E^2$. $U = \dfrac{Q^2}{2C}$.

SET UP: For this charge distribution, $E = 0$ for $r < r_a$, $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$ for $r_a < r < r_b$ and $E = 0$ for $r > r_b$.

Example 24.4 shows that $\dfrac{C}{L} = \dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$ for a cylindrical capacitor.

EXECUTE: **(a)** $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0\left(\dfrac{\lambda}{2\pi\epsilon_0 r}\right)^2 = \dfrac{\lambda^2}{8\pi^2\epsilon_0 r^2}$.

(b) $U = \int u\,dV = 2\pi L\int u r\,dr = \dfrac{L\lambda^2}{4\pi\epsilon_0}\int_{r_a}^{r_b}\dfrac{dr}{r}$ and $\dfrac{U}{L} = \dfrac{\lambda^2}{4\pi\epsilon_0}\ln(r_b/r_a)$.

(c) $U = \dfrac{Q^2}{2C} = \dfrac{Q^2}{4\pi\epsilon_0 L}\ln(r_b/r_a) = \dfrac{\lambda^2 L}{4\pi\epsilon_0}\ln(r_b/r_a)$. This agrees with the result of part (b).

EVALUATE: We could have used the results of part (b) and $U = \dfrac{Q^2}{2C}$ to calculate C/L and would obtain the same result as in Example 24.4.

24.65. **IDENTIFY:** The two slabs of dielectric are in series with each other.

SET UP: The capacitor is equivalent to C_1 and C_2 in series, so $\dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{1}{C}$, which gives $C = \dfrac{C_1 C_2}{C_1 + C_2}$.

EXECUTE: With $d = 1.90$ mm, $C_1 = \dfrac{K_1\epsilon_0 A}{d}$ and $C_2 = \dfrac{K_2\epsilon_0 A}{d}$.

$$C = \left(\frac{K_1 K_2}{K_1 + K_2}\right)\frac{\epsilon_0 A}{d} = \left(\frac{(4.7)(2.6)}{4.7 + 2.6}\right)\frac{(8.854\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(0.0800\ \mathrm{m})^2}{1.90\times10^{-3}\ \mathrm{m}} = 4.992\times10^{-11}\ \mathrm{F}.$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(4.992\times10^{-11}\ \mathrm{F})(86.0\ \mathrm{V})^2 = 1.85\times10^{-7}\ \mathrm{J}.$$

EVALUATE: The dielectrics increase the capacitance, allowing the capacitor to store more energy than if it were air-filled.

24.66. **IDENTIFY:** The capacitor is equivalent to two capacitors in parallel, as shown in Figure 24.66.

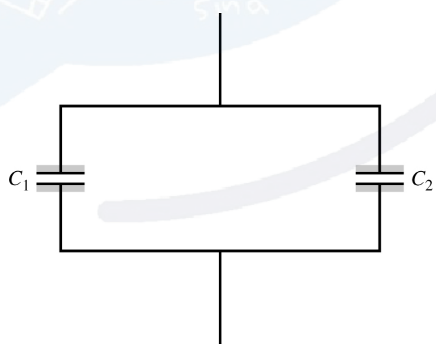


Figure 24.66

SET UP: Each of these two capacitors have plates that are 12.0 cm by 6.0 cm. For a parallel-plate capacitor with dielectric filling the volume between the plates, $C = K\epsilon_0\dfrac{A}{d}$. For two capacitors in parallel, $C = C_1 + C_2$. The energy stored in a capacitor is $U = \frac{1}{2}CV^2$.

EXECUTE: **(a)** $C = C_1 + C_2$.

$$C_2 = \epsilon_0 \frac{A}{d} = \frac{(8.854\times10^{-12}\ \text{F/m})(0.120\ \text{m})(0.060\ \text{m})}{4.50\times10^{-3}\ \text{m}} = 1.42\times10^{-11}\ \text{F}.$$

$C_1 = KC_2 = (3.40)(1.42\times10^{-11}\ \text{F}) = 4.83\times10^{-11}\ \text{F}$. $C = C_1 + C_2 = 6.25\times10^{-11}\ \text{F} = 62.5\ \text{pF}$.

(b) $U = \frac{1}{2}CV^2 = \frac{1}{2}(6.25\times10^{-11}\ \text{F})(18.0\ \text{V})^2 = 1.01\times10^{-8}\ \text{J}$.

(c) Now $C_1 = C_2$ and $C = 2(1.42\times10^{-11}\ \text{F}) = 2.84\times10^{-11}\ \text{F}$.

$U = \frac{1}{2}CV^2 = \frac{1}{2}(2.84\times10^{-11}\ \text{F})(18.0\ \text{V})^2 = 4.60\times10^{-9}\ \text{J}$.

EVALUATE: The plexiglass increases the capacitance and that increases the energy stored for the same voltage across the capacitor.

24.67. IDENTIFY: The object is equivalent to two identical capacitors in parallel, where each has the same area *A*, plate separation *d* and dielectric with dielectric constant *K*.

SET UP: For each capacitor in the parallel combination, $C = \frac{\epsilon_0 A}{d}$.

EXECUTE: **(a)** The charge distribution on the plates is shown in Figure 24.67.

(b) $C = 2\left(\frac{\epsilon_0 A}{d}\right) = \frac{2(4.2)\epsilon_0(0.120\ \text{m})^2}{4.5\times10^{-4}\ \text{m}} = 2.38\times10^{-9}\ \text{F}$.

EVALUATE: If two of the plates are separated by both sheets of paper to form a capacitor,

$C = \frac{\epsilon_0 A}{2d} = \frac{2.38\times10^{-9}\ \text{F}}{4}$, smaller by a factor of 4 compared to the capacitor in the problem.

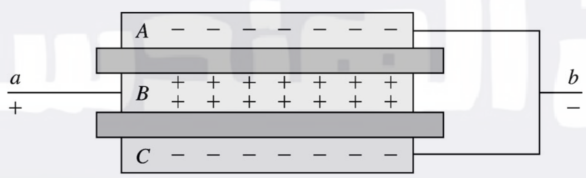


Figure 24.67

24.68. IDENTIFY: The system is equivalent to two capacitors in parallel. One of the capacitors has plate separation *d*, plate area $w(L-h)$ and air between the plates. The other has the same plate separation *d*, plate area *wh* and dielectric constant *K*.

SET UP: Define K_{eff} by $C_{\text{eq}} = \frac{K_{\text{eff}}\epsilon_0 A}{d}$, where $A = wL$. For two capacitors in parallel, $C_{\text{eq}} = C_1 + C_2$.

EXECUTE: **(a)** The capacitors are in parallel, so $C = \frac{\epsilon_0 w(L-h)}{d} + \frac{K\epsilon_0 wh}{d} = \frac{\epsilon_0 wL}{d}\left(1 + \frac{Kh}{L} - \frac{h}{L}\right)$. This gives $K_{\text{eff}} = \left(1 + \frac{Kh}{L} - \frac{h}{L}\right)$.

(b) For gasoline, with $K = 1.95$: $\frac{1}{4}$ full: $K_{\text{eff}}\left(h = \frac{L}{4}\right) = 1.24$; $\frac{1}{2}$ full: $K_{\text{eff}}\left(h = \frac{L}{2}\right) = 1.48$;

$\frac{3}{4}$ full: $K_{\text{eff}}\left(h = \frac{3L}{4}\right) = 1.71$.

(c) For methanol, with $K = 33$: $\frac{1}{4}$ full: $K_{\text{eff}}\left(h = \frac{L}{4}\right) = 9$; $\frac{1}{2}$ full: $K_{\text{eff}}\left(h = \frac{L}{2}\right) = 17$;

$\frac{3}{4}$ full: $K_{\text{eff}}\left(h = \frac{3L}{4}\right) = 25$.

(d) This kind of fuel tank sensor will work best for methanol since it has the greater range of K_{eff} values.

EVALUATE: When $h=0$, $K_{\text{eff}}=1$. When $h=L$, $K_{\text{eff}}=K$.

24.69. IDENTIFY and SET UP: For two capacitors in series, $\dfrac{1}{C_1}+\dfrac{1}{C_2}=\dfrac{1}{C_{\text{eq}}}$, which gives $C_{\text{eq}}=\dfrac{C_1C_2}{C_1+C_2}$. For two capacitors in parallel, $C_{\text{eq}} = C_1 + C_2$. $C = Q/V$. The stored energy can be written as $U=\frac{1}{2}CV^2$ or $U=\dfrac{Q^2}{2C}$.

EXECUTE: **(a)** When connected in series, the stored energy is 0.0400 J, so

$U=\frac{1}{2}CV^2 = \frac{1}{2}\left(\dfrac{C_1C_2}{C_1+C_2}\right)V^2 = \frac{1}{2}\left(\dfrac{C_1C_2}{C_1+C_2}\right)(200.0\text{ V})^2 = 0.0400\text{ J}$, which gives

$\dfrac{C_1C_2}{C_1+C_2}=2.00\ \mu\text{F}$.

When connected in parallel, the stored energy is 0.180 J, so

$U=\frac{1}{2}CV^2 = \frac{1}{2}(C_1+C_2)V^2 = \frac{1}{2}(C_1+C_2)(200.0\text{ V})^2 = 0.180\text{ J}$.

$C_1 + C_2 = 9.00\ \mu\text{F}$.

Solving the two equations for C_1 and C_2 gives $C_1 = 6.00\ \mu\text{F}$ and $C_2 = 3.00\ \mu\text{F}$.

(b) When the capacitors are in series, both have the same charge. The stored energy is $U=\dfrac{Q^2}{2C}$, so the capacitor with the *smaller* capacitance stores more energy, which is C_2.

(c) When the capacitors are in parallel, the potential across them is the same. The stored energy is $U=\frac{1}{2}CV^2$, so the capacitor with the *larger* capacitance stores the most energy, which is C_1.

EVALUATE: When the two capacitors are connected in parallel, they can store considerably more energy than when in series.

24.70. IDENTIFY and SET UP: The presence of the dielectric affects the charge and energy in the capacitor for a given potential difference. $V = Ed$, $Q = CV$, $K = C/C_0$, $U=\frac{1}{2}CV^2$. We use the values for K and E_{m} from Table 24.2. In this case, $E = 0.500E_{\text{m}}$ and $d = 2.50\text{ mm} = 0.00250\text{ m}$.

EXECUTE: **(a)** Using $U=\frac{1}{2}CV^2$, $C = KC_0$, $V = Ed$, and $E = 0.500E_{\text{m}}$, the stored energy is

$U=\frac{1}{2}CV^2 = \frac{1}{2}KC_0(Ed)^2 = \frac{1}{2}KC_0(0.500E_{\text{m}}d)^2$.

For polycarbonate, $K = 2.8$ and $E_{\text{m}}=3\times10^7$ V/m. Therefore the stored energy is

$U=(1/2)[(2.8)(6.00\times10^{-12}\text{ F})][(0.500)(3\times10^7\text{ V/m})(0.00250\text{ m})]^2=1.18\times10^{-2}\text{ J}$, which rounds to 12 mJ.

Using similar calculations for the other materials, the results for U are:

12 mJ (polycarbonate)
56 mJ (polyester)
51 mJ (polypropylene)
4.9 mJ (polystyrene)
2.2 mJ (pyrex)

(b) $Q = CV = KC_0(Ed) = KC_0(0.500E_{\text{m}})d$.

For polycarbonate we have

$Q=(2.8)(6.00\times10^{-12}\text{ F})(0.500)(3\times10^7\text{ V/m})(0.00250\text{ m})=6.3\times10^{-7}\text{ C}=0.63\ \mu\text{C}$.

Similar calculations for the other materials yield:

0.63 μC (polycarbonate)
1.5 μC (polyester)
1.2 μC (polypropylene)
0.39 μC (polystyrene)
0.35 μC (pyrex)

(c) $V = Ed = 0.500E_m d$. For polycarbonate this gives
$V = (0.500)(3\times10^7 \text{ V/m})(0.00250 \text{ m}) = 3.8\times10^4 \text{ V} = 38 \text{ kV}$.
Similar calculations for the other materials yield:
38 kV (polycarbonate)
75 kV (polyester)
88 kV (polypropylene)
25 kV (polystyrene)
13 kV (pyrex)
EVALUATE: **(d)** Polyester is best for maximum energy storage and maximum charge, but polypropylene is best for maximum voltage. No single material is best for all three categories. As so often occurs, the choice of materials is a trade-off.

24.71. **IDENTIFY** and **SET UP:** For a parallel-plate capacitor, $C = \dfrac{\varepsilon_0 A}{d}$. The stored energy can be expressed as

$$U = \tfrac{1}{2}CV^2 \text{ or } U = \frac{Q^2}{2C}.$$

EXECUTE: **(a)** If the battery remains connected, V remains constant, so it is useful to write the energy in terms of V and C:

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)V^2 = \frac{\varepsilon_0 A V^2}{2}\cdot\frac{1}{d}.$$

If the battery is disconnected, Q remains constant, so it is useful to write the energy in terms of Q and C:

$$U = \frac{Q^2}{2C} = \frac{Q^2}{2\left(\dfrac{\varepsilon_0 A}{d}\right)} = \left(\frac{Q^2}{2\varepsilon_0 A}\right)d.$$

The graph shows a linear relationship between U and $1/d$, so it must represent the case where the battery remains connected to the capacitor.

(b) In a graph of U versus $1/d$ for the equation $U = \dfrac{\varepsilon_0 A V^2}{2}\cdot\dfrac{1}{d}$, the slope should be equal to $\dfrac{\varepsilon_0 A V^2}{2}$.

Choosing points on the graph in the problem, the slope is $\dfrac{(73-18)\times10^{-9} \text{ J}}{20.0 \text{ cm}^{-1} - 5.0 \text{ cm}^{-1}} = 3.67\times10^{-11} \text{ J}\cdot\text{m}$.

Solving for A gives
$A = 2(\text{slope})/\varepsilon_0 V^2 = 2(3.67\times10^{-11} \text{ J}\cdot\text{m})/(8.854\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(24.0 \text{ V})^2] = 0.014 \text{ m}^2 = 144 \text{ cm}^2$.

(c) With the battery connected: $U = \dfrac{\varepsilon_0 A V^2}{2}\cdot\dfrac{1}{d}$, so as we increase d from 0.0500 cm to 0.400 cm, the energy *decreases* since V remains constant.

With the battery disconnected: $U = \left(\dfrac{Q^2}{2\varepsilon_0 A}\right)d$, so as we increase d, the energy *increases* since Q does not change. Therefore there is more energy stored with the battery *disconnected* as d is increased.

EVALUATE: If this capacitor were square, its plates would be $12 \text{ cm}\times12 \text{ cm}$. This is a reasonable size for a piece of apparatus for use in a laboratory and could easily be manufactured.

24.72. **IDENTIFY:** The system can be considered to be two capacitors in parallel, one with plate area $L(L-x)$ and air between the plates and one with area Lx and dielectric filling the space between the plates.

SET UP: $C = \dfrac{K\varepsilon_0 A}{d}$ for a parallel-plate capacitor with plate area A.

EXECUTE: **(a)** $C = \dfrac{\varepsilon_0}{D}\left[(L-x)L + xKL\right] = \dfrac{\varepsilon_0 L}{D}\left[L + (K-1)x\right].$

(b) $dU = \frac{1}{2}(dC)V^2$, where $C = C_0 + \frac{\varepsilon_0 L}{D}(-dx + dxK)$, with $C_0 = \frac{\varepsilon_0 L}{D}\left[L + (K-1)x\right]$. This gives

$$dU = \frac{1}{2}\left(\frac{\varepsilon_0 L dx}{D}(K-1)\right)V^2 = \frac{(K-1)\varepsilon_0 V^2 L}{2D}dx.$$

(c) If the charge is kept constant on the plates, then $Q = \frac{\varepsilon_0 LV}{D}\left[L + (K-1)x\right]$ and

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}C_0 V^2\left(\frac{C}{C_0}\right).\ U \approx \frac{C_0 V^2}{2}\left(1 - \frac{\varepsilon_0 L}{DC_0}(K-1)dx\right) \text{ and } \Delta U = U - U_0 = -\frac{(K-1)\varepsilon_0 V^2 L}{2D}dx.$$

(d) Since $dU = -Fdx = -\frac{(K-1)\varepsilon_0 V^2 L}{2D}dx$, the force is in the opposite direction to the motion dx, meaning that the slab feels a force pushing it out.

EVALUATE: **(e)** When the plates are connected to the battery, the plates plus slab are not an isolated system. In addition to the work done on the slab by the charges on the plates, energy is also transferred between the battery and the plates. Comparing the results for dU in part (c) to $dU = -Fdx$ gives

$$F = \frac{(K-1)\varepsilon_0 V^2 L}{2D}.$$

24.73. **IDENTIFY** and **SET UP:** The potential difference is $V = 30$ mV – (–70 mV) = 100 mV, and $Q = CV$.

EXECUTE: $Q = CV$ gives $Q/\text{cm}^2 = (C/\text{cm}^2)V = (1\ \mu\text{F/cm}^2)(100\ \text{mV})(1\ \text{mol}/10^5\ \text{C}) = 10^{-12}\ \text{mol/cm}^2$, which is choice (c).

EVALUATE: This charge produces a potential difference of 100 mV = 0.1 V, which is certainly measurable using ordinary laboratory meters.

24.74. **IDENTIFY** and **SET UP:** The change in concentration of Na^+ ions is equal to the added charge divided by the volume of the spherical egg. The original concentration of ions is given as 30 mmol/L. We use the answer from Problem 24.73 to get the added charge.

EXECUTE: The added charge is $(10^{-12}\ \text{mol/cm}^2)$(surface area of egg) = $(10^{-12}\ \text{mol/cm}^2)(4\pi R^2)$, and the original volume of the egg is $(4\pi/3)R^3$. Therefore the change in concentration is

$$(10^{-12}\ \text{mol/cm}^2)(4\pi R^2)/\left[(4\pi/3)R^3\right] = 3(10^{-12}\ \text{mol/cm}^2)/R = 3(10^{-12}\ \text{mol/cm}^2)/(100\times10^{-4}\text{cm}) =$$

$$3\times10^{10}\ \text{mol/cm}^3 = 3\times10^{-5}\ \text{mmol/L}.$$

The fractional change in the concentrations is $(3\times10^{-5}\ \text{mmol/L})/(30\ \text{mmol/L}) = 10^{-5}$, which is 1 part in 10^5. Therefore choice (b) is correct.

EVALUATE: As a percent, this change is $10^{-3}\% = 0.001\%$, which is quite small yet certainly important for the organism.

24.75. **IDENTIFY** and **SET UP:** The calcium Ca^{2+} ions carry twice the charge of the Na^+ ions.

EXECUTE: The charge to produce the given voltage change would be the same as with Na^+, so we would need only half as many Ca^{2+} ions to accomplish this. Thus choice (a) is correct.

EVALUATE: Ca^{+2} ions are nearly twice as heavy as Na^+ ions, so they may not move as readily as the sodium ions.

24.76. **IDENTIFY** and **SET UP:** The energy is needed to change the potential from 30 mV to –70 mV. $U = \frac{1}{2}CV^2$. The capacitance is $(1\ \mu\text{F/cm}^2)$(surface area of egg).

EXECUTE: For a spherical egg, the surface area is $4\pi R^2$, so the capacitance is

$$C = (1\ \mu\text{F/cm}^2)(4\pi R^2) = (1\ \mu\text{F/cm}^2)(4\pi)(100\times10^{-4}\ \text{cm})^2 = 1.26\times10^{-9}\ \text{F}.$$

The change in stored energy is

$$\Delta U = \tfrac{1}{2}CV_2^2 - \tfrac{1}{2}CV_1^2 = \tfrac{1}{2}C(V_2^2 - V_1^2).$$

$\Delta U = (1/2)(1.26\times10^{-9}\ \text{F})[(-70\times10^{-3}\ \text{V})^2 - (30\times10^{-3}\ \text{V})^2] = 2.5\times10^{-12}\ \text{J} = 2.5\ \text{pJ} \approx 3\ \text{pJ}$, which makes choice (d) the correct one.

EVALUATE: The actual energy required would probably be greater than 2.5 pJ, depending on the process by which the charging is accomplished, but our value is the minimum energy needed.

25

CURRENT, RESISTANCE, AND ELECTROMOTIVE FORCE

25.1. IDENTIFY and SET UP: The lightning is a current that lasts for a brief time. $I = \dfrac{\Delta Q}{\Delta t}$.

EXECUTE: $\Delta Q = I\Delta t = (25{,}000\ \text{A})(40\times10^{-6}\ \text{s}) = 1.0\ \text{C}$.

EVALUATE: Even though it lasts for only 40 μs, the lightning carries a huge amount of charge since it is an enormous current.

25.2. IDENTIFY: $I = Q/t$. Use $I = n|q|v_\text{d}A$ to calculate the drift velocity v_d.

SET UP: $n = 5.8\times10^{28}\ \text{m}^{-3}$. $|q| = 1.60\times10^{-19}\ \text{C}$.

EXECUTE: **(a)** $I = \dfrac{Q}{t} = \dfrac{420\ \text{C}}{80(60\ \text{s})} = 8.75\times10^{-2}\ \text{A}$.

(b) $I = n|q|v_\text{d}A$. This gives $v_\text{d} = \dfrac{I}{n|q|A} = \dfrac{8.75\times10^{-2}\ \text{A}}{(5.8\times10^{28})(1.60\times10^{-19}\ \text{C})(\pi(1.3\times10^{-3}\ \text{m})^2)} = 1.78\times10^{-6}\ \text{m/s}$.

EVALUATE: v_d is smaller than in Example 25.1, because I is smaller in this problem.

25.3. IDENTIFY: $I = Q/t$. $J = I/A$. $J = n|q|v_\text{d}$.

SET UP: $A = (\pi/4)D^2$, with $D = 2.05\times10^{-3}\ \text{m}$. The charge of an electron has magnitude $+e = 1.60\times10^{-19}\ \text{C}$.

EXECUTE: **(a)** $Q = It = (5.00\ \text{A})(1.00\ \text{s}) = 5.00\ \text{C}$. The number of electrons is $\dfrac{Q}{e} = 3.12\times10^{19}$.

(b) $J = \dfrac{I}{(\pi/4)D^2} = \dfrac{5.00\ \text{A}}{(\pi/4)(2.05\times10^{-3}\ \text{m})^2} = 1.51\times10^6\ \text{A/m}^2$.

(c) $v_\text{d} = \dfrac{J}{n|q|} = \dfrac{1.51\times10^6\ \text{A/m}^2}{(8.5\times10^{28}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})} = 1.11\times10^{-4}\ \text{m/s} = 0.111\ \text{mm/s}$.

EVALUATE: **(d)** If I is the same, $J = I/A$ would decrease and v_d would decrease. The number of electrons passing through the light bulb in 1.00 s would not change.

25.4. **(a) IDENTIFY:** By definition, $J = I/A$ and radius is one-half the diameter.

SET UP: Solve for the current: $I = JA = J\pi(D/2)^2$

EXECUTE: $I = (3.20\times10^6\ \text{A/m}^2)(\pi)[(0.00102\ \text{m})/2]^2 = 2.61\ \text{A}$.

EVALUATE: This is a realistic current.

(b) IDENTIFY: The current density is $J = n|q|v_\text{d}$.

SET UP: Solve for the drift velocity: $v_\text{d} = J/n|q|$

EXECUTE: We use the value of n for copper, giving

$v_\text{d} = (3.20\times10^6\ \text{A/m}^2)/[(8.5\times10^{28}/\text{m}^3)(1.60\times10^{-19}\ \text{C})] = 2.4\times10^{-4}\ \text{m/s} = 0.24\ \text{mm/s}$.

EVALUATE: This is a typical drift velocity for ordinary currents and wires.

25.5. IDENTIFY and SET UP: Use $J = n|q|v_{\text{d}}$ to calculate the drift speed and then use that to find the time to travel the length of the wire.

EXECUTE: **(a)** Calculate the drift speed v_{d}:

$$J = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{4.85\text{ A}}{\pi(1.025\times10^{-3}\text{ m})^2} = 1.469\times10^6\text{ A/m}^2.$$

$$v_{\text{d}} = \frac{J}{n|q|} = \frac{1.469\times10^6\text{ A/m}^2}{(8.5\times10^{28}/\text{m}^3)(1.602\times10^{-19}\text{ C})} = 1.079\times10^{-4}\text{ m/s}.$$

$$t = \frac{L}{v_{\text{d}}} = \frac{0.710\text{ m}}{1.079\times10^{-4}\text{ m/s}} = 6.58\times10^3\text{ s} = 110\text{ min}.$$

(b) $v_{\text{d}} = \dfrac{I}{\pi r^2 n|q|}$.

$$t = \frac{L}{v_{\text{d}}} = \frac{\pi r^2 n|q|L}{I}.$$

t is proportional to r^2 and hence to d^2 where $d = 2r$ is the wire diameter.

$$t = (6.58\times10^3\text{ s})\left(\frac{4.12\text{ mm}}{2.05\text{ mm}}\right)^2 = 2.66\times10^4\text{ s} = 440\text{ min}.$$

(c) EVALUATE: The drift speed is proportional to the current density and therefore it is inversely proportional to the square of the diameter of the wire. Increasing the diameter by some factor decreases the drift speed by the square of that factor.

25.6. IDENTIFY: The resistance depends on the length, cross-sectional area, and material of the wires.

SET UP: $R = \dfrac{\rho L}{A}$, $A = \pi r^2 = d^2/4$. The resistivities come from Table 25.1.

EXECUTE: **(a)** Combining $R = \dfrac{\rho L}{A}$ and $A = \pi d^2/4$, gives $R = \dfrac{\rho L}{\frac{\pi}{4}d^2} = \dfrac{4\rho L}{\pi d^2}$. Solving for L gives

$L = \dfrac{R\pi d^2}{4\rho}$. Using this formula gives the length of each type of metal.

Gold: $L = \dfrac{(1.00\,\Omega)\pi(1.00\times10^{-3}\text{ m})^2}{4(2.44\times10^{-8}\ \Omega\cdot\text{m})} = 32.2\text{ m}.$

Copper: Using $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$ we get L = 45.7 m.

Aluminum: Using $\rho = 2.75\times10^{-8}\ \Omega\cdot\text{m}$, we get L = 28.6 m.

(b) The mass of the gold is the product of its mass density and its volume, so

$$m = (density)(\pi d^2/4)L = (1.93\times10^4\text{ kg/m}^3)\pi(1.00\times10^{-3}\text{ m})^2(32.2\text{ m})/4 = 0.488\text{ kg} = 488\ g.$$

If gold is currently worth \$40 per gram, the cost of the gold wire would be (\$40/g)(488 g) = \$19,500. At this price, you wouldn't want to wire your house with gold wires!

EVALUATE: The resistivities of the three metals are all fairly close to each other, so it is reasonable to expect that the lengths of the wires would also be fairly close to each other, which is just what we find.

25.7. IDENTIFY and SET UP: Apply $I = \dfrac{dQ}{dt}$ to find the charge dQ in time dt. Integrate to find the total charge in the whole time interval.

EXECUTE: **(a)** $dQ = I\,dt$.

$$Q = \int_0^{8.0\text{s}}(55\text{ A} - (0.65\text{ A/s}^2)t^2)dt = \Big[(55\text{ A})t - (0.217\text{ A/s}^2)t^3\Big]_0^{8.0\text{ s}}.$$

$$Q = (55\text{ A})(8.0\text{ s}) - (0.217\text{ A/s}^2)(8.0\text{ s})^3 = 330\text{ C}.$$

(b) $I = \dfrac{Q}{t} = \dfrac{330\text{ C}}{8.0\text{ s}} = 41\text{ A}.$

EVALUATE: The current decreases from 55 A to 13.4 A during the interval. The decrease is not linear and the average current is not equal to $(55\text{A} + 13.4\text{ A})/2$.

25.8. IDENTIFY: $I = Q/t$. Positive charge flowing in one direction is equivalent to negative charge flowing in the opposite direction, so the two currents due to Cl^- and Na^+ are in the same direction and add.

SET UP: Na^+ and Cl^- each have magnitude of charge $|q| = +e$.

EXECUTE: **(a)** $Q_{\text{total}} = (n_{\text{Cl}} + n_{\text{Na}})e = (3.92\times10^{16} + 2.68\times10^{16})(1.60\times10^{-19}\text{ C}) = 0.0106\text{ C}$. Then

$I = \dfrac{Q_{\text{total}}}{t} = \dfrac{0.0106\text{ C}}{1.00\text{ s}} = 0.0106\text{ A} = 10.6\text{ mA}.$

(b) Current flows, by convention, in the direction of positive charge. Thus, current flows with Na^+ toward the negative electrode.

EVALUATE: The Cl^- ions have negative charge and move in the direction opposite to the conventional current direction.

25.9. IDENTIFY and **SET UP:** The number of ions that enter gives the charge that enters the axon in the specified time. $I = \dfrac{\Delta Q}{\Delta t}$.

EXECUTE: $\Delta Q = (5.6\times10^{11}\text{ ions})(1.60\times10^{-19}\text{ C/ion}) = 9.0\times10^{-8}\text{ C}$. $I = \dfrac{\Delta Q}{\Delta t} = \dfrac{9.0\times10^{-8}\text{ C}}{10\times10^{-3}\text{ s}} = 9.0\ \mu\text{A}.$

EVALUATE: This current is much smaller than household currents but are comparable to many currents in electronic equipment.

25.10. (a) IDENTIFY: Start with the definition of resistivity and solve for E.

SET UP: $E = \rho J = \rho I/\pi r^2$.

EXECUTE: $E = (1.72\times10^{-8}\ \Omega\cdot\text{m})(4.50\text{ A})/[\pi(0.001025\text{ m})^2] = 2.345\times10^{-2}\text{ V/m}$, which rounds to 0.0235 V/m.

EVALUATE: The field is quite weak, since the potential would drop only a volt in 43 m of wire.

(b) IDENTIFY: Take the ratio of the field in silver to the field in copper.

SET UP: Take the ratio and solve for the field in silver: $E_{\text{S}} = E_{\text{C}}(\rho_{\text{S}}/\rho_{\text{C}})$.

EXECUTE: $E_{\text{S}} = (0.02345\text{ V/m})[(1.47)/(1.72)] = 2.00\times10^{-2}\text{ V/m}$.

EVALUATE: Since silver is a better conductor than copper, the field in silver is smaller than the field in copper.

25.11. IDENTIFY: First use Ohm's law to find the resistance at 20.0°C; then calculate the resistivity from the resistance. Finally use the dependence of resistance on temperature to calculate the temperature coefficient of resistance.

SET UP: Ohm's law is $R = V/I$, $R = \rho L/A$, $R = R_0[1 + \alpha(T - T_0)]$, and the radius is one-half the diameter.

EXECUTE: **(a)** At 20.0°C, $R = V/I = (15.0\text{ V})/(18.5\text{ A}) = 0.811\ \Omega$. Using $R = \rho L/A$ and solving for ρ gives $\rho = RA/L = R\pi(D/2)^2/L = (0.811\ \Omega)\pi[(0.00500\text{ m})/2]^2/(1.50\text{ m}) = 1.06\times10^{-5}\ \Omega\cdot\text{m}$.

(b) At 92.0°C, $R = V/I = (15.0\text{ V})/(17.2\text{ A}) = 0.872\ \Omega$. Using $R = R_0[1 + \alpha(T - T_0)]$ with T_0 taken as 20.0°C, we have $0.872\ \Omega = (0.811\ \Omega)[1 + \alpha(92.0°\text{C} - 20.0°\text{C})]$. This gives $\alpha = 0.00105\ (\text{C}°)^{-1}$.

EVALUATE: The results are typical of ordinary metals.

25.12. IDENTIFY: $E = \rho J$, where $J = I/A$. The drift velocity is given by $I = n|q|v_{\text{d}}A$.

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$. $n = 8.5\times10^{28}/\text{m}^3$.

EXECUTE: **(a)** $J = \dfrac{I}{A} = \dfrac{3.6\text{ A}}{(2.3\times10^{-3}\text{ m})^2} = 6.81\times10^5\text{ A/m}^2.$

(b) $E = \rho J = (1.72\times10^{-8}\ \Omega\cdot\text{m})(6.81\times10^{5}\ \text{A/m}^2) = 0.012\ \text{V/m}.$

(c) The time to travel the wire's length l is

$$t = \frac{l}{v_{\text{d}}} = \frac{ln|q|A}{I} = \frac{(4.0\ \text{m})(8.5\times10^{28}/\text{m}^3)(1.6\times10^{-19}\ \text{C})(2.3\times10^{-3}\ \text{m})^2}{3.6\ \text{A}} = 8.0\times10^4\ \text{s}.$$

$t = 1333\ \text{min} \approx 22\ \text{hrs!}$

EVALUATE: The currents propagate very quickly along the wire but the individual electrons travel very slowly.

25.13. IDENTIFY: Knowing the resistivity of a metal, its geometry and the current through it, we can use Ohm's law to find the potential difference across it.

SET UP: $V = IR$. For copper, Table 25.1 gives that $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$ and for silver, $\rho = 1.47\times10^{-8}\ \Omega\cdot\text{m}$. $R = \dfrac{\rho L}{A}$.

EXECUTE: **(a)** $R = \dfrac{\rho L}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(2.00\ \text{m})}{\pi(0.814\times10^{-3}\ \text{m})^2} = 1.65\times10^{-2}\ \Omega.$

$V = (12.5\times10^{-3}\ \text{A})(1.65\times10^{-2}\ \Omega) = 2.06\times10^{-4}\ \text{V}.$

(b) $V = \dfrac{I\rho L}{A}$. $\dfrac{V}{\rho} = \dfrac{IL}{A} = \text{constant}$, so $\dfrac{V_{\text{s}}}{\rho_{\text{s}}} = \dfrac{V_{\text{c}}}{\rho_{\text{c}}}$.

$$V_{\text{s}} = V_{\text{c}}\left(\frac{\rho_{\text{s}}}{\rho_{\text{c}}}\right) = (2.06\times10^{-4}\ \text{V})\left(\frac{1.47\times10^{-8}\ \Omega\cdot\text{m}}{1.72\times10^{-8}\ \Omega\cdot\text{m}}\right) = 1.76\times10^{-4}\ \text{V}.$$

EVALUATE: The potential difference across a 2-m length of wire is less than 0.2 mV, so normally we do not need to worry about these potential drops in laboratory circuits.

25.14. IDENTIFY: The resistivity of the wire should identify what the material is.

SET UP: $R = \rho L/A$ and the radius of the wire is half its diameter.

EXECUTE: Solve for ρ and substitute the numerical values.

$$\rho = AR/L = \pi(D/2)^2 R/L = \frac{\pi([0.00205\ \text{m}]/2)^2(0.0290\ \Omega)}{6.50\ \text{m}} = 1.47\times10^{-8}\ \Omega\cdot\text{m}$$

EVALUATE: This result is the same as the resistivity of silver, which implies that the material is silver.

25.15. (a) IDENTIFY: Start with the definition of resistivity and use its dependence on temperature to find the electric field.

SET UP: $E = \rho J = \rho_{20}[1+\alpha(T-T_0)]\dfrac{I}{\pi r^2}$.

EXECUTE: $E = (5.25\times10^{-8}\ \Omega\cdot\text{m})[1+(0.0045/\text{C}°)(120°\text{C}-20°\text{C})](12.5\ \text{A})/[\pi(0.000500\ \text{m})^2] = 1.21\ \text{V/m}.$

(Note that the resistivity at 120°C turns out to be $7.61\times10^{-8}\ \Omega\cdot\text{m}$.)

EVALUATE: This result is fairly large because tungsten has a larger resistivity than copper.

(b) IDENTIFY: Relate resistance and resistivity.

SET UP: $R = \rho L/A = \rho L/\pi r^2$.

EXECUTE: $R = (7.61\times10^{-8}\ \Omega\cdot\text{m})(0.150\ \text{m})/[\pi(0.000500\ \text{m})^2] = 0.0145\ \Omega.$

EVALUATE: Most metals have very low resistance.

(c) IDENTIFY: The potential difference is proportional to the length of wire.

SET UP: $V = EL$.

EXECUTE: $V = (1.21\ \text{V/m})(0.150\ \text{m}) = 0.182\ \text{V}.$

EVALUATE: We could also calculate $V = IR = (12.5\ \text{A})(0.0145\ \Omega) = 0.181\ \text{V}$, in agreement with part (c).

25.16. IDENTIFY: The geometry of the wire is changed, so its resistance will also change.

SET UP: $R = \dfrac{\rho L}{A}$. $L_{\text{new}} = 3L$. The volume of the wire remains the same when it is stretched.

EXECUTE: Volume $= LA$ so $LA = L_{\text{new}} A_{\text{new}}$. $A_{\text{new}} = \dfrac{L}{L_{\text{new}}} A = \dfrac{A}{3}$.

$$R_{\text{new}} = \frac{\rho L_{\text{new}}}{A_{\text{new}}} = \frac{\rho(3L)}{A/3} = 9\frac{\rho L}{A} = 9R.$$

EVALUATE: When the length increases the resistance increases and when the area decreases the resistance increases.

25.17. **IDENTIFY:** $R = \dfrac{\rho L}{A}$.

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$. $A = \pi r^2$.

EXECUTE: $R = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(24.0\ \text{m})}{\pi(1.025\times10^{-3}\ \text{m})^2} = 0.125\ \Omega.$

EVALUATE: The resistance is proportional to the length of the piece of wire.

25.18. **IDENTIFY:** $R = \dfrac{\rho L}{A} = \dfrac{\rho L}{\pi d^2/4}$.

SET UP: For aluminum, $\rho_{\text{al}} = 2.75\times10^{-8}\ \Omega\cdot\text{m}$. For copper, $\rho_{\text{c}} = 1.72\times10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: $\dfrac{\rho}{d^2} = \dfrac{R\pi}{4L} =$ constant, so $\dfrac{\rho_{\text{al}}}{d_{\text{al}}^2} = \dfrac{\rho_{\text{c}}}{d_{\text{c}}^2}$. $d_{\text{c}} = d_{\text{al}}\sqrt{\dfrac{\rho_{\text{c}}}{\rho_{\text{al}}}} = (2.14\ \text{mm})\sqrt{\dfrac{1.72\times10^{-8}\ \Omega\cdot\text{m}}{2.75\times10^{-8}\ \Omega\cdot\text{m}}} = 1.69\ \text{mm}.$

EVALUATE: Copper has a smaller resistivity, so the copper wire has a smaller diameter in order to have the same resistance as the aluminum wire.

25.19. **IDENTIFY and SET UP:** Apply $R = \dfrac{\rho L}{A}$ to determine the effect of increasing A and L.

EXECUTE: **(a)** If 120 strands of wire are placed side by side, we are effectively increasing the area of the current carrier by 120. So the resistance is smaller by that factor: $R = (5.60\times10^{-6}\ \Omega)/120 = 4.67\times10^{-8}\ \Omega$.

(b) If 120 strands of wire are placed end to end, we are effectively increasing the length of the wire by 120, and so $R = (5.60\times10^{-6}\ \Omega)(120) = 6.72\times10^{-4}\ \Omega$.

EVALUATE: Placing the strands side by side decreases the resistance and placing them end to end increases the resistance.

25.20. **IDENTIFY:** Apply $R = \dfrac{\rho L}{A}$ and $V = IR$.

SET UP: $A = \pi r^2$.

EXECUTE: $\rho = \dfrac{RA}{L} = \dfrac{VA}{IL} = \dfrac{(4.50\ \text{V})\pi(6.54\times10^{-4}\ \text{m})^2}{(17.6\ \text{A})(2.50\ \text{m})} = 1.37\times10^{-7}\ \Omega\cdot\text{m}.$

EVALUATE: Our result for ρ shows that the wire is made of a metal with resistivity greater than that of good metallic conductors such as copper and aluminum.

25.21. **IDENTIFY and SET UP:** The equation $\rho = E/J$ relates the electric field that is given to the current density. $V = EL$ gives the potential difference across a length L of wire and $V = IR$ allows us to calculate R.

EXECUTE: **(a)** $\rho = E/J$ so $J = E/\rho$.

From Table 25.1 the resistivity for gold is $2.44\times10^{-8}\ \Omega\cdot\text{m}$.

$$J = \frac{E}{\rho} = \frac{0.49\ \text{V/m}}{2.44\times10^{-8}\ \Omega\cdot\text{m}} = 2.008\times10^7\ \text{A/m}^2.$$

$$I = JA = J\pi r^2 = (2.008\times10^7\ \text{A/m}^2)\pi(0.42\times10^{-3}\ \text{m})^2 = 11\ \text{A}.$$

(b) $V = EL = (0.49\ \text{V/m})(6.4\ \text{m}) = 3.1\ \text{V}$.

(c) We can use Ohm's law: $V = IR$.

$$R = \frac{V}{I} = \frac{3.1\ \text{V}}{11\ \text{A}} = 0.28\ \Omega.$$

EVALUATE: We can also calculate R from the resistivity and the dimensions of the wire:

$$R=\frac{\rho L}{A}=\frac{\rho L}{\pi r^2}=\frac{(2.44\times10^{-8}\ \Omega\cdot\text{m})(6.4\ \text{m})}{\pi(0.42\times10^{-3}\ \text{m})^2}=0.28\ \Omega,\ \text{which checks.}$$

25.22. IDENTIFY: When the ohmmeter is connected between the opposite faces, the current flows along its length, but when the meter is connected between the inner and outer surfaces, the current flows radially outward.

(a) SET UP: For a hollow cylinder, $R=\rho L/A$, where $A=\pi(b^2-a^2)$.

EXECUTE: $R=\rho L/A=\dfrac{\rho L}{\pi(b^2-a^2)}=\dfrac{(2.75\times10^{-8}\ \Omega\cdot\text{m})(2.50\ \text{m})}{\pi[(0.0460\ \text{m})^2-(0.0275\ \text{m})^2]}=1.61\times10^{-5}\ \Omega.$

(b) SET UP: For a thin cylindrical shell of inner radius r and thickness dr, the resistance is $dR=\dfrac{\rho dr}{2\pi rL}$.

For radial current flow from $r=a$ to $r=b$, $R=\int dR=\dfrac{\rho}{2\pi L}\int_a^b\dfrac{1}{r}dr=(\rho/2\pi L)\ln(b/a)$.

EXECUTE: $R=\dfrac{\rho}{2\pi L}\ln(b/a)=\dfrac{2.75\times10^{-8}\ \Omega\cdot\text{m}}{2\pi(2.50\ \text{m})}\ln\left(\dfrac{4.60\ \text{cm}}{2.75\ \text{cm}}\right)=9.01\times10^{-10}\ \Omega.$

EVALUATE: The resistance is much smaller for the radial flow because the current flows through a much smaller distance and the area through which it flows is much larger.

25.23. IDENTIFY: Apply $R=R_0[1+\alpha(T-T_0)]$ to calculate the resistance at the second temperature.

(a) SET UP: $\alpha=0.0004\ (\text{C}°)^{-1}$ (Table 25.2). Let T_0 be 0.0°C and T be 11.5°C.

EXECUTE: $R_0=\dfrac{R}{1+\alpha(T-T_0)}=\dfrac{100.0\ \Omega}{1+(0.0004\ (\text{C}°)^{-1}(11.5\ \text{C}°))}=99.54\ \Omega.$

(b) SET UP: $\alpha=-0.0005\ (\text{C}°)^{-1}$ (Table 25.2). Let $T_0=0.0°\text{C}$ and $T=25.8°\text{C}$.

EXECUTE: $R=R_0[1+\alpha(T-T_0)]=0.0160\ \Omega[1+(-0.0005\ (\text{C}°)^{-1})(25.8\ \text{C}°)]=0.0158\ \Omega.$

EVALUATE: Nichrome, like most metallic conductors, has a positive α and its resistance increases with temperature. For carbon, α is negative and its resistance decreases as T increases.

25.24. IDENTIFY: $R_T=R_0[1+\alpha(T-T_0)]$.

SET UP: $R_0=217.3\ \Omega$. $R_T=215.8\ \Omega$. For carbon, $\alpha=-0.00050(\text{C}°)^{-1}$.

EXECUTE: $T-T_0=\dfrac{(R_T/R_0)-1}{\alpha}=\dfrac{(215.8\ \Omega/217.3\ \Omega)-1}{-0.00050\ (\text{C}°)^{-1}}=13.8\ \text{C}°.$ $T=13.8\ \text{C}°+4.0°\text{C}=17.8°\text{C}.$

EVALUATE: For carbon, α is negative so R decreases as T increases.

25.25. IDENTIFY: Use $R=\dfrac{\rho L}{A}$ to calculate R and then apply $V=IR$. $P=VI$ and energy $=Pt$.

SET UP: For copper, $\rho=1.72\times10^{-8}\ \Omega\cdot\text{m}$. $A=\pi r^2$, where $r=0.050$ m.

EXECUTE: **(a)** $R=\dfrac{\rho L}{A}=\dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(100\times10^3\text{m})}{\pi(0.050\ \text{m})^2}=0.219\ \Omega.$ $V=IR=(125\ \text{A})(0.219\ \Omega)=27.4\ \text{V}.$

(b) $P=VI=(27.4\ \text{V})(125\ \text{A})=3422\ \text{W}=3422\ \text{J/s}$ and energy $=Pt=(3422\ \text{J/s})(3600\ \text{s})=1.23\times10^7\ \text{J}.$

EVALUATE: The rate of electrical energy loss in the cable is large, over 3 kW.

25.26. IDENTIFY: When current passes through a battery in the direction from the $-$ terminal toward the $+$ terminal, the terminal voltage V_{ab} of the battery is $V_{ab}=\varepsilon-Ir$. Also, $V_{ab}=IR$, the potential across the circuit resistor.

SET UP: $\varepsilon=24.0\ \text{V}$. $I=4.00\ \text{A}$.

EXECUTE: **(a)** $V_{ab}=\varepsilon-Ir$ gives $r=\dfrac{\varepsilon-V_{ab}}{I}=\dfrac{24.0\ \text{V}-21.2\ \text{V}}{4.00\ \text{A}}=0.700\ \Omega.$

(b) $V_{ab} - IR = 0$ so $R = \frac{V_{ab}}{I} = \frac{21.2\text{ V}}{4.00\text{ A}} = 5.30\,\Omega.$

EVALUATE: The voltage drop across the internal resistance of the battery causes the terminal voltage of the battery to be less than its emf. The total resistance in the circuit is $R + r = 6.00\,\Omega.$

$I = \frac{24.0\text{ V}}{6.00\,\Omega} = 4.00\text{ A},$ which agrees with the value specified in the problem.

25.27. IDENTIFY: The terminal voltage of the battery is $V_{ab} = \mathcal{E} - Ir.$ The voltmeter reads the potential difference between its terminals.

SET UP: An ideal voltmeter has infinite resistance.

EXECUTE: **(a)** Since an ideal voltmeter has infinite resistance, so there would be NO current through the $2.0\,\Omega$ resistor.

(b) $V_{ab} = \mathcal{E} = 5.0\text{ V};$ Since there is no current there is no voltage lost over the internal resistance.

(c) The voltmeter reading is therefore 5.0 V since with no current flowing there is no voltage drop across either resistor.

EVALUATE: This not the proper way to connect a voltmeter. If we wish to measure the terminal voltage of the battery in a circuit that does not include the voltmeter, then connect the voltmeter across the terminals of the battery.

25.28. IDENTIFY: The *idealized* ammeter has no resistance so there is no potential drop across it. Therefore it acts like a short circuit across the terminals of the battery and removes the $4.00\text{-}\Omega$ resistor from the circuit. Thus the only resistance in the circuit is the $2.00\text{-}\Omega$ internal resistance of the battery.

SET UP: Use Ohm's law: $I = \mathcal{E}/r.$

EXECUTE: **(a)** $I = (10.0\text{ V})/(2.00\,\Omega) = 5.00\text{ A}.$

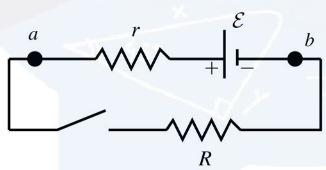
(b) The zero-resistance ammeter is in parallel with the $4.00\text{-}\Omega$ resistor, so all the current goes through the ammeter. If no current goes through the $4.00\text{-}\Omega$ resistor, the potential drop across it must be zero.

(c) The terminal voltage is zero since there is no potential drop across the ammeter.

EVALUATE: An ammeter should *never* be connected this way because it would seriously alter the circuit!

25.29. IDENTIFY: The voltmeter reads the potential difference V_{ab} between the terminals of the battery.

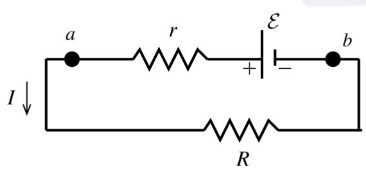
SET UP: <u>open circuit:</u> $I = 0.$ The circuit is sketched in Figure 25.29a.



EXECUTE: $V_{ab} = \mathcal{E} = 3.08\text{ V}.$

Figure 25.29a

SET UP: <u>switch closed:</u> The circuit is sketched in Figure 25.29b.



EXECUTE: $V_{ab} = \mathcal{E} - Ir = 2.97\text{ V}.$

$r = \frac{\mathcal{E} - 2.97\text{ V}}{I}.$

$r = \frac{3.08\text{ V} - 2.97\text{ V}}{1.65\text{ A}} = 0.067\,\Omega.$

Figure 25.29b

And $V_{ab} = IR$ so $R = \frac{V_{ab}}{I} = \frac{2.97\text{ V}}{1.65\text{ A}} = 1.80\,\Omega.$

EVALUATE: When current flows through the battery there is a voltage drop across its internal resistance and its terminal voltage V is less than its emf.

25.30. **IDENTIFY:** The sum of the potential changes around the circuit loop is zero. Potential decreases by IR when going through a resistor in the direction of the current and increases by ε when passing through an emf in the direction from the $-$ to $+$ terminal.

SET UP: The current is counterclockwise, because the 16-V battery determines the direction of current flow.

EXECUTE: $+16.0\text{ V} - 8.0\text{ V} - I(1.6\,\Omega + 5.0\,\Omega + 1.4\,\Omega + 9.0\,\Omega) = 0.$

$$I = \frac{16.0\text{ V} - 8.0\text{ V}}{1.6\,\Omega + 5.0\,\Omega + 1.4\,\Omega + 9.0\,\Omega} = 0.47\text{ A}.$$

(b) $V_b + 16.0\text{ V} - I(1.6\,\Omega) = V_a$, so $V_a - V_b = V_{ab} = 16.0\text{ V} - (1.6\,\Omega)(0.47\text{ A}) = 15.2\text{ V}.$

(c) $V_c + 8.0\text{ V} + I(1.4\,\Omega + 5.0\,\Omega) = V_a$ so $V_{ac} = (5.0\,\Omega)(0.47\text{ A}) + (1.4\,\Omega)(0.47\text{ A}) + 8.0\text{ V} = 11.0\text{ V}.$

(d) The graph is sketched in Figure 25.30.

EVALUATE: $V_{cb} = (0.47\text{ A})(9.0\,\Omega) = 4.2\text{ V}.$ The potential at point b is 15.2 V below the potential at point a and the potential at point c is 11.0 V below the potential at point a, so the potential of point c is $15.2\text{ V} - 11.0\text{ V} = 4.2\text{ V}$ above the potential of point b.

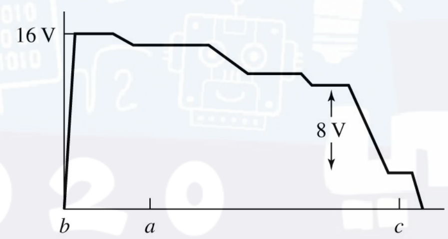


Figure 25.30

25.31. **(a) IDENTIFY** and **SET UP:** Assume that the current is clockwise. The circuit is sketched in Figure 25.31a.

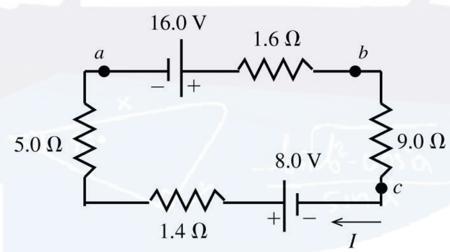


Figure 25.31a

Add up the potential rises and drops as travel clockwise around the circuit.

EXECUTE: $16.0\text{ V} - I(1.6\,\Omega) - I(9.0\,\Omega) + 8.0\text{ V} - I(1.4\,\Omega) - I(5.0\,\Omega) = 0.$

$$I = \frac{16.0\text{ V} + 8.0\text{ V}}{9.0\,\Omega + 1.4\,\Omega + 5.0\,\Omega + 1.6\,\Omega} = \frac{24.0\text{ V}}{17.0\,\Omega} = 1.41\text{ A, clockwise.}$$

EVALUATE: The 16.0-V battery and the 8.0-V battery both drive the current in the same direction.

(b) IDENTIFY and **SET UP:** Start at point a and travel through the battery to point b, keeping track of the potential changes. At point b the potential is V_b.

EXECUTE: $V_a + 16.0\text{ V} - I(1.6\,\Omega) = V_b.$

$V_a - V_b = -16.0\text{ V} + (1.41\text{ A})(1.6\,\Omega).$

$V_{ab} = -16.0\text{ V} + 2.3\text{ V} = -13.7\text{ V}$ (point a is at lower potential; it is the negative terminal). Therefore, $V_{ba} = 13.7\text{V}.$

EVALUATE: Could also go counterclockwise from a to b:

$V_a + (1.41\text{ A})(5.0\ \Omega) + (1.41\text{ A})(1.4\ \Omega) - 8.0\text{ V} + (1.41\text{ A})(9.0\ \Omega) = V_b$.

$V_{ab} = -13.7$ V, which checks.

(c) IDENTIFY and SET UP: Start at point a and travel through the battery to point c, keeping track of the potential changes.

EXECUTE: $V_a + 16.0\text{ V} - I(1.6\ \Omega) - I(9.0\ \Omega) = V_c$.

$V_a - V_c = -16.0\text{ V} + (1.41\text{ A})(1.6\ \Omega + 9.0\ \Omega)$.

$V_{ac} = -16.0\text{ V} + 15.0\text{ V} = -1.0\text{ V}$ (point a is at lower potential than point c).

EVALUATE: Could also go counterclockwise from a to c:

$V_a + (1.41\text{ A})(5.0\ \Omega) + (1.41\text{ A})(1.4\ \Omega) - 8.0\text{ V} = V_c$.

$V_{ac} = -1.0$ V, which checks.

(d) Call the potential zero at point a. Travel clockwise around the circuit. The graph is sketched in Figure 25.31b.

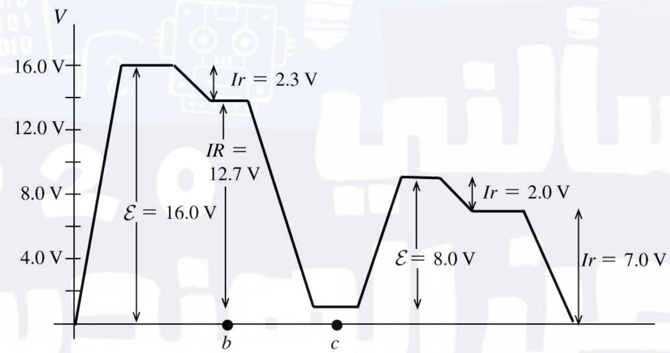


Figure 25.31b

25.32. IDENTIFY: The sum of the potential changes around the loop is zero.

SET UP: The voltmeter reads the IR voltage across the 9.0-Ω resistor. The current in the circuit is counterclockwise because the 16-V battery determines the direction of the current flow.

EXECUTE: **(a)** $V_{bc} = 1.9$ V gives $I = V_{bc}/R_{bc} = 1.9\text{ V}/9.0\ \Omega = 0.21$ A.

(b) $16.0\text{ V} - 8.0\text{ V} = (1.6\ \Omega + 9.0\ \Omega + 1.4\ \Omega + R)(0.21\text{ A})$ and $R = \dfrac{5.48\text{ V}}{0.21\text{ A}} = 26.1\ \Omega$.

(c) The graph is sketched in Figure 25.32.

EVALUATE: In Exercise 25.30 the current is 0.47 A. When the 5.0-Ω resistor is replaced by the 26.1-Ω resistor the current decreases to 0.21 A.

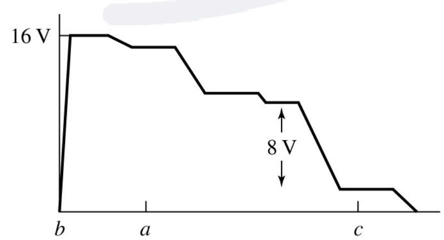


Figure 25.32

25.33. IDENTIFY and SET UP: There is a single current path so the current is the same at all points in the circuit. Assume the current is counterclockwise and apply Kirchhoff's loop rule.

EXECUTE: **(a)** Apply the loop rule, traveling around the circuit in the direction of the current.

$+16.0\text{ V} - I(1.6\ \Omega + 5.0\ \Omega + 1.4\ \Omega + 9.0\ \Omega) - 8.0\text{ V} = 0.$ $I = \dfrac{16.0\text{ V} - 8.0\text{ V}}{17.0\ \Omega} = 0.471\text{ A}.$ Our calculated I is positive so I is counterclockwise, as we assumed.

(b) $V_b + 16.0\text{ V} - I(1.6\ \Omega) = V_a.$ $V_{ab} = 16.0\text{ V} - (0.471\text{ A})(1.6\ \Omega) = 15.2\text{ V}.$

EVALUATE: If we traveled around the circuit in the direction opposite to the current, the final answers would be the same.

25.34. **IDENTIFY and SET UP:** The resistance is the same in both cases, and $P = V^2/R$.

EXECUTE: **(a)** Solving $P = V^2/R$ for R, gives $R = V^2/P$. Since the resistance is the same in both cases, we have $\dfrac{V_1^2}{P_1} = \dfrac{V_2^2}{P_2}$. Solving for P_2 gives $P_2 = P_1(V_2/V_1)^2 = (0.0625\text{ W})[(12.5\text{ V})/(1.50\text{ V})]^2 = 4.41\text{ W}$.

(b) Solving for V_2 gives $V_2 = V_1\sqrt{\dfrac{P_2}{P_1}} = (1.50\text{ V})\sqrt{\dfrac{5.00\text{ W}}{0.0625\text{ W}}} = 13.4\text{ V}.$

EVALUATE: These calculations are correct assuming that the resistor obeys Ohm's law throughout the range of currents involved.

25.35. **IDENTIFY:** The bulbs are each connected across a 120-V potential difference.

SET UP: Use $P = V^2/R$ to solve for R and Ohm's law $(I = V/R)$ to find the current.

EXECUTE: **(a)** $R = V^2/P = (120\text{ V})^2/(100\text{ W}) = 144\ \Omega.$

(b) $R = V^2/P = (120\text{ V})^2/(60\text{ W}) = 240\ \Omega.$

(c) For the 100-W bulb: $I = V/R = (120\text{ V})/(144\ \Omega) = 0.833\text{ A}.$

For the 60-W bulb: $I = (120\text{ V})/(240\ \Omega) = 0.500\text{ A}.$

EVALUATE: The 60-W bulb has *more* resistance than the 100-W bulb, so it draws less current.

25.36. **IDENTIFY:** Across 120 V, a 75-W bulb dissipates 75 W. Use this fact to find its resistance, and then find the power the bulb dissipates across 220 V.

SET UP: $P = V^2/R$, so $R = V^2/P$.

EXECUTE: Across 120 V: $R = (120\text{ V})^2/(75\text{ W}) = 192\ \Omega.$ Across a 220-V line, its power will be $P = V^2/R = (220\text{ V})^2/(192\ \Omega) = 252\text{ W}.$

EVALUATE: The bulb dissipates much more power across 220 V, so it would likely blow out at the higher voltage. An alternative solution to the problem is to take the ratio of the powers.

$\dfrac{P_{220}}{P_{120}} = \dfrac{V_{220}^2/R}{V_{120}^2/R} = \left(\dfrac{V_{220}}{V_{120}}\right)^2 = \left(\dfrac{220}{120}\right)^2$. This gives $P_{220} = (75\text{ W})\left(\dfrac{220}{120}\right)^2 = 252\text{ W}.$

25.37. **IDENTIFY:** A "100-W" European bulb dissipates 100 W when used across 220 V.

(a) SET UP: Take the ratio of the power in the U.S. to the power in Europe, as in the alternative method for Problem 25.36, using $P = V^2/R$.

EXECUTE: $\dfrac{P_{\text{US}}}{P_{\text{E}}} = \dfrac{V_{\text{US}}^2/R}{V_{\text{E}}^2/R} = \left(\dfrac{V_{\text{US}}}{V_{\text{E}}}\right)^2 = \left(\dfrac{120\text{ V}}{220\text{ V}}\right)^2$. This gives $P_{\text{US}} = (100\text{ W})\left(\dfrac{120\text{ V}}{220\text{ V}}\right)^2 = 29.8\text{ W}.$

(b) SET UP: Use $P = IV$ to find the current.

EXECUTE: $I = P/V = (29.8\text{ W})/(120\text{ V}) = 0.248\text{ A}.$

EVALUATE: The bulb draws considerably less power in the U.S., so it would be much dimmer than in Europe.

25.38. **IDENTIFY:** $P = VI$. Energy $= Pt$.

SET UP: $P = (9.0\text{ V})(0.13\text{ A}) = 1.17\text{ W}.$

EXECUTE: Energy $= (1.17\text{ W})(30\text{ min})(60\text{ s/min}) = 2100\text{ J}.$

EVALUATE: The energy consumed is proportional to the voltage, to the current and to the time.

25.39. **IDENTIFY:** Calculate the current in the circuit. The power output of a battery is its terminal voltage times the current through it. The power dissipated in a resistor is I^2R.

SET UP: The sum of the potential changes around the circuit is zero.

EXECUTE: **(a)** $I = \dfrac{8.0\ \text{V}}{17\ \Omega} = 0.47\ \text{A}$. Then $P_{5\Omega} = I^2R = (0.47\ \text{A})^2(5.0\ \Omega) = 1.1\ \text{W}$ and

$P_{9\Omega} = I^2R = (0.47\ \text{A})^2(9.0\ \Omega) = 2.0\ \text{W}$, so the total is 3.1 W.

(b) $P_{16\text{V}} = \varepsilon I - I^2 r = (16\ \text{V})(0.47\ \text{A}) - (0.47\ \text{A})^2(1.6\ \Omega) = 7.2\ \text{W}$.

(c) $P_{8\text{V}} = \varepsilon I + Ir^2 = (8.0\ \text{V})(0.47\ \text{A}) + (0.47\ \text{A})^2(1.4\ \Omega) = 4.1\ \text{W}$.

EVALUATE: **(d)** (b) = (a) + (c). The rate at which the 16.0-V battery delivers electrical energy to the circuit equals the rate at which it is consumed in the 8.0-V battery and the 5.0-Ω and 9.0-Ω resistors.

25.40. **IDENTIFY:** Knowing the current and potential difference, we can find the power.

SET UP: $P = VI$ and energy is the product of power and time.

EXECUTE: $P = (500\ \text{V})(80\times10^{-3}\ \text{A}) = 40\ \text{W}$.

$\text{Energy} = Pt = (40\ \text{W})(10\times10^{-3}\ \text{s}) = 0.40\ \text{J}$.

EVALUATE: The energy delivered depends not only on the voltage and current but also on the length of the pulse. The pulse is short but the voltage is large.

25.41. **IDENTIFY:** We know the current, voltage and time the current lasts, so we can calculate the power and the energy delivered.

SET UP: Power is energy per unit time. The power delivered by a voltage source is $P = V_{ab}I$.

EXECUTE: **(a)** $P = (25\ \text{V})(12\ \text{A}) = 300\ \text{W}$.

(b) $\text{Energy} = Pt = (300\ \text{W})(3.0\times10^{-3}\ \text{s}) = 0.90\ \text{J}$.

EVALUATE: The energy is not very great, but it is delivered in a short time (3 ms) so the power is large, which produces a short shock.

25.42. **IDENTIFY** and **SET UP:** The average power delivered by the battery can be calculated in two different ways: $P = \dfrac{\text{energy}}{\text{time}}$ or $P = VI$. The time is 5.25 h, which in seconds is

$5.25\ \text{h} = (5.25\ \text{h})(3600\ \text{s/h}) = 1.89\times10^4\ \text{s}$.

EXECUTE: The average power delivered by the battery is $P = \dfrac{\text{energy}}{\text{time}} = \dfrac{3.15\times10^4\ \text{J}}{1.89\times10^4\ \text{s}} = 1.6667\ \text{W}$. Thus, the current must be $I = \dfrac{P}{V} = \dfrac{1.6667\ \text{W}}{3.70\ \text{V}} = 0.450\ \text{A}$.

EVALUATE: The energy stored in the battery can be expressed in joules or watt-hours. The energy is equal to Pt, so we can express the stored energy as either $3.15\times10^4\ \text{J}$ or (1.6667 W)(5.25 h) = $8.75\ \text{W}\cdot\text{h}$.

25.43. **(a) IDENTIFY** and **SET UP:** $P = VI$ and energy = (power) × (time).

EXECUTE: $P = VI = (12\ \text{V})(60\ \text{A}) = 720\ \text{W}$.

The battery can provide this for 1.0 h, so the energy the battery has stored is

$U = Pt = (720\ \text{W})(3600\ \text{s}) = 2.6\times10^6\ \text{J}$.

(b) IDENTIFY and **SET UP:** For gasoline the heat of combustion is $L_c = 46\times10^6\ \text{J/kg}$. Solve for the mass m required to supply the energy calculated in part (a) and use density $\rho = m/V$ to calculate V.

EXECUTE: The mass of gasoline that supplies 2.6×10^6 J is $m = \dfrac{2.6\times10^6\ \text{J}}{46\times10^6\ \text{J/kg}} = 0.0565\ \text{kg}$.

The volume of this mass of gasoline is

$$V = \frac{m}{\rho} = \frac{0.0565\ \text{kg}}{900\ \text{kg/m}^3} = 6.3\times10^{-5}\ \text{m}^3\left(\frac{1000\ \text{L}}{1\ \text{m}^3}\right) = 0.063\ \text{L}.$$

(c) IDENTIFY and **SET UP:** Energy = (power) × (time); the energy is that calculated in part (a).

EXECUTE: $U = Pt,\ t = \dfrac{U}{P} = \dfrac{2.6\times10^6\ \text{J}}{450\ \text{W}} = 5800\ \text{s} = 97\ \text{min} = 1.6\ \text{h}$.

EVALUATE: The battery discharges at a rate of 720 W (for 1.0 h) and is charged at a rate of 450 W (for 1.6 h), so it takes longer to charge than to discharge.

25.44. **IDENTIFY:** The voltmeter reads the terminal voltage of the battery, which is the potential difference across the appliance. The terminal voltage is less than 15.0 V because some potential is lost across the internal resistance of the battery.

(a) SET UP: $P = V^2/R$ gives the power dissipated by the appliance.

EXECUTE: $P = (11.9\text{ V})^2/(75.0\ \Omega) = 1.888\text{ W},$ which rounds to 1.89 W.

(b) SET UP: The drop in terminal voltage $(\varepsilon - V_{ab})$ is due to the potential drop across the internal resistance *r*. Use $Ir = \varepsilon - V_{ab}$ to find the internal resistance *r*, but first find the current using $P = IV$.

EXECUTE: $I = P/V = (1.888\text{ W})/(11.9\text{ V}) = 0.1587\text{ A}.$ Then $Ir = \varepsilon - V_{ab}$ gives $(0.1587\text{ A})r = 15.0\text{ V} - 11.9\text{ V}$ and $r = 19.5\ \Omega.$

EVALUATE: The full 15.0-V of the battery would be available only when no current (or a very small current) is flowing in the circuit. This would be the case if the appliance had a resistance much greater than 19.5 Ω.

25.45. **IDENTIFY:** Some of the power generated by the internal emf of the battery is dissipated across the battery's internal resistance, so it is not available to the bulb.

SET UP: Use $P = I^2R$ and take the ratio of the power dissipated in the internal resistance *r* to the total power.

EXECUTE: $\dfrac{P_r}{P_{\text{Total}}} = \dfrac{I^2r}{I^2(r+R)} = \dfrac{r}{r+R} = \dfrac{3.5\ \Omega}{28.5\ \Omega} = 0.123 = 12.3\%.$

EVALUATE: About 88% of the power of the battery goes to the bulb. The rest appears as heat in the internal resistance.

25.46. **IDENTIFY:** The power delivered to the bulb is $I^2R.$ Energy $= Pt.$

SET UP: The circuit is sketched in Figure 25.46. r_{total} is the combined internal resistance of both batteries.

EXECUTE: **(a)** $r_{\text{total}} = 0.$ The sum of the potential changes around the circuit is zero, so $1.5\text{ V} + 1.5\text{ V} - I(17\ \Omega) = 0.$ $I = 0.1765\text{ A}.$ $P = I^2R = (0.1765\text{ A})^2(17\ \Omega) = 0.530\text{ W}.$ This is also $(3.0\text{ V})(0.1765\text{ A}).$

(b) Energy $= (0.530\text{ W})(5.0\text{ h})(3600\text{ s/h}) = 9540\text{ J}.$

(c) $P = \dfrac{0.530\text{ W}}{2} = 0.265\text{ W}.$ $P = I^2R$ so $I = \sqrt{\dfrac{P}{R}} = \sqrt{\dfrac{0.265\text{ W}}{17\ \Omega}} = 0.125\text{ A}.$

The sum of the potential changes around the circuit is zero, so $1.5\text{ V} + 1.5\text{ V} - IR - Ir_{\text{total}} = 0.$

$r_{\text{total}} = \dfrac{3.0\text{ V} - (0.125\text{ A})(17\ \Omega)}{0.125\text{ A}} = 7.0\ \Omega.$

EVALUATE: When the power to the bulb has decreased to half its initial value, the total internal resistance of the two batteries is nearly half the resistance of the bulb. Compared to a single battery, using two identical batteries in series doubles the emf but also doubles the total internal resistance.

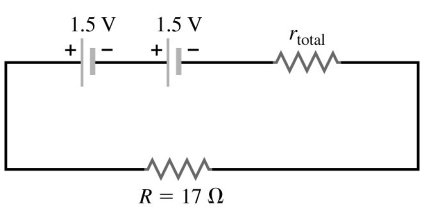


Figure 25.46

25.47. **IDENTIFY:** Solve for the current *I* in the circuit. Apply $P = VI = I^2R$ to the specified circuit elements to find the rates of energy conversion.

SET UP: The circuit is sketched in Figure 25.47 (next page).

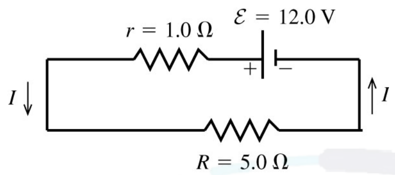


Figure 25.47

EXECUTE: Compute I:

$\varepsilon - Ir - IR = 0.$

$$I = \frac{\varepsilon}{r+R} = \frac{12.0\text{ V}}{1.0\ \Omega + 5.0\ \Omega} = 2.00\text{ A.}$$

(a) The rate of conversion of chemical energy to electrical energy in the emf of the battery is $P = \varepsilon I = (12.0\text{ V})(2.00\text{ A}) = 24.0\text{ W}.$

(b) The rate of dissipation of electrical energy in the internal resistance of the battery is $P = I^2 r = (2.00\text{ A})^2(1.0\ \Omega) = 4.0\text{ W}.$

(c) The rate of dissipation of electrical energy in the external resistor R is $P = I^2 R = (2.00\text{ A})^2(5.0\ \Omega) = 20.0\text{ W}.$

EVALUATE: The rate of production of electrical energy in the circuit is 24.0 W. The total rate of consumption of electrical energy in the circuit is $4.00\text{ W} + 20.0\text{ W} = 24.0\text{ W}.$ Equal rates of production and consumption of electrical energy are required by energy conservation.

25.48. IDENTIFY: $P = I^2 R = \dfrac{V^2}{R} = VI.$ $V = IR.$

SET UP: The heater consumes 540 W when $V = 120$ V. Energy $= Pt$.

EXECUTE: **(a)** $P = \dfrac{V^2}{R}$ so $R = \dfrac{V^2}{P} = \dfrac{(120\text{ V})^2}{540\text{ W}} = 26.7\ \Omega.$

(b) $P = VI$ so $I = \dfrac{P}{V} = \dfrac{540\text{ W}}{120\text{ V}} = 4.50\text{ A}.$

(c) Assuming that R remains $26.7\ \Omega$, $P = \dfrac{V^2}{R} = \dfrac{(110\text{ V})^2}{26.7\ \Omega} = 453\text{ W}.$ P is smaller by a factor of $(110/120)^2$.

EVALUATE: **(d)** With the lower line voltage the current will decrease and the operating temperature will decrease. R will be less than $26.7\ \Omega$ and the power consumed will be greater than the value calculated in part (c).

25.49. IDENTIFY: The resistivity is $\rho = \dfrac{m}{ne^2\tau}.$

SET UP: For silicon, $\rho = 2300\ \Omega \cdot \text{m}.$

EXECUTE: **(a)** $\tau = \dfrac{m}{ne^2\rho} = \dfrac{9.11\times10^{-31}\text{ kg}}{(1.0\times10^{16}\text{ m}^{-3})(1.60\times10^{-19}\text{ C})^2(2300\ \Omega\cdot\text{m})} = 1.55\times10^{-12}\text{ s}.$

EVALUATE: **(b)** The number of free electrons in copper $(8.5\times10^{28}\text{ m}^{-3})$ is much larger than in pure silicon $(1.0\times10^{16}\text{ m}^{-3})$. A smaller density of current carriers means a higher resistivity.

25.50. IDENTIFY: Negative charge moving from A to B is equivalent to an equal magnitude of positive charge going from B to A.

SET UP: $I = \dfrac{\Delta Q}{\Delta t}.$ The current direction is the direction of flow of positive charge.

EXECUTE: The total positive charge moving from B to A is

$\Delta Q = [5.11\times10^{18} + 2(3.24\times10^{18})](1.60\times10^{-19}\text{ C}) = 1.85\text{ C}.$ $I = \dfrac{\Delta Q}{\Delta t} = \dfrac{1.85\text{ C}}{30\text{ s}} = 62\text{ mA}.$ Positive charge flows from B to A so the current is in this direction.

EVALUATE: The charges flowing in opposite directions do not cancel each other out because one is positive and the other is negative.

25.51. **(a) IDENTIFY and SET UP:** Use $R = \dfrac{\rho L}{A}$.

EXECUTE: $\rho = \dfrac{RA}{L} = \dfrac{(0.104\ \Omega)\pi(1.25\times10^{-3}\ \text{m})^2}{14.0\ \text{m}} = 3.65\times10^{-8}\ \Omega\cdot\text{m}.$

EVALUATE: This value is similar to that for good metallic conductors in Table 25.1.

(b) IDENTIFY and SET UP: Use $V = EL$ to calculate E and then Ohm's law gives I.

EXECUTE: $V = EL = (1.28\ \text{V/m})(14.0\ \text{m}) = 17.9\ \text{V}.$

$$I = \frac{V}{R} = \frac{17.9\ \text{V}}{0.104\ \Omega} = 172\ \text{A}.$$

EVALUATE: We could do the calculation another way:

$$E = \rho J \text{ so } J = \frac{E}{\rho} = \frac{1.28\ \text{V/m}}{3.65\times10^{-8}\ \Omega\cdot\text{m}} = 3.51\times10^{7}\ \text{A/m}^2.$$

$I = JA = (3.51\times10^{7}\ \text{A/m}^2)\pi(1.25\times10^{-3}\ \text{m})^2 = 172\ \text{A},$ which checks.

(c) IDENTIFY and SET UP: Calculate $J = I/A$ or $J = E/\rho$ and then use Eq. (25.3) for the target variable v_d.

EXECUTE: $J = n|q|v_d = nev_d.$

$$v_d = \frac{J}{ne} = \frac{3.51\times10^{7}\ \text{A/m}^2}{(8.5\times10^{28}\ \text{m}^{-3})(1.602\times10^{-19}\ \text{C})} = 2.58\times10^{-3}\ \text{m/s} = 2.58\ \text{mm/s}.$$

EVALUATE: Even for this very large current the drift speed is small.

25.52. **IDENTIFY and SET UP:** Use $R = \dfrac{\rho L}{A}$ and $V = RI$. Call x the distance from point A to the short. The distance from B to the short is 2000 m – x. V is the same in both measurements since we use the same 9.00-V battery.

EXECUTE: Since V is the same in both measurements, $V = R_1I_1 = R_2I_2$. Also $R_1 = \dfrac{\rho x}{A}$ and $R_2 = \dfrac{\rho(2000\ \text{m} - x)}{A}$. Combining these two conditions gives $\dfrac{\rho x}{A}I_1 = \dfrac{\rho(2000\ \text{m} - x)}{A}I_2$. This gives (2.86 A)$x$ = (1.65 A)(2000 m – x), so x = 732 m from point A.

EVALUATE: Our result assumes that the wire has uniform thickness with no kinks in it. These would affect the cross-sectional area and hence the resistance.

25.53. **IDENTIFY and SET UP:** With the voltmeter connected across the terminals of the battery there is no current through the battery and the voltmeter reading is the battery emf; $\varepsilon = 12.6$ V.

With a wire of resistance R connected to the battery current I flows and $\varepsilon - Ir - IR = 0$, where r is the internal resistance of the battery. Apply this equation to each piece of wire to get two equations in the two unknowns.

EXECUTE: Call the resistance of the 20.0-m piece R_1; then the resistance of the 40.0-m piece is $R_2 = 2R_1$.

$\varepsilon - I_1r - I_1R_1 = 0;\quad 12.6\ \text{V} - (7.00\ \text{A})r - (7.00\ \text{A})R_1 = 0.$

$\varepsilon - I_2r - I_2(2R_2) = 0;\quad 12.6\ \text{V} - (4.20\ \text{A})r - (4.20\ \text{A})(2R_1) = 0.$

Solving these two equations in two unknowns gives $R_1 = 1.20\ \Omega$. This is the resistance of 20.0 m, so the resistance of one meter is $[1.20\ \Omega/(20.0\ \text{m})](1.00\ \text{m}) = 0.060\ \Omega.$

EVALUATE: We can also solve for r and we get $r = 0.600\ \Omega$. When measuring small resistances, the internal resistance of the battery has a large effect.

25.54. **IDENTIFY:** Conservation of charge requires that the current is the same in both sections. The voltage drops across each section add, so $R = R_{Cu} + R_{Ag}$. The total resistance is the sum of the resistances of each section. The electric field in a conductor is $E = \dfrac{V}{L} = \dfrac{IR}{L}$, where R is the resistance of a section and L is its length.

SET UP: For copper, $\rho_{Cu} = 1.72\times10^{-8}\ \Omega\cdot\text{m}$. For silver, $\rho_{Ag} = 1.47\times10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: (a) $I = \dfrac{V}{R} = \dfrac{V}{R_{\text{Cu}} + R_{\text{Ag}}}$. $R_{\text{Cu}} = \dfrac{\rho_{\text{Cu}} L_{\text{Cu}}}{A_{\text{Cu}}} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(0.8\ \text{m})}{(\pi/4)(6.0\times10^{-4}\text{m})^2} = 0.049\ \Omega$ and

$R_{\text{Ag}} = \dfrac{\rho_{\text{Ag}} L_{\text{Ag}}}{A_{\text{Ag}}} = \dfrac{(1.47\times10^{-8}\ \Omega\cdot\text{m})(1.2\ \text{m})}{(\pi/4)(6.0\times10^{-4}\ \text{m})^2} = 0.062\ \Omega$. This gives $I = \dfrac{9.0\ \text{V}}{0.049\ \Omega + 0.062\ \Omega} = 81.1\ \text{A}$, which rounds to 81 A, so the current in the copper wire is 81 A.

(b) The current in the silver wire is 81.1 A, the same as that in the copper wire or else charge would build up at their interface.

(c) $E_{\text{Cu}} = \dfrac{V_{\text{Cu}}}{L_{\text{Cu}}} = \dfrac{IR_{\text{Cu}}}{L_{\text{Cu}}} = \dfrac{(81.1\ \text{A})(0.049\ \Omega)}{0.80\ \text{m}} = 4.97\ \text{V/m}$, which rounds to 5.0 V/m.

(d) $E_{\text{Ag}} = \dfrac{V_{\text{Ag}}}{L_{\text{Ag}}} = \dfrac{IR_{\text{Ag}}}{L_{\text{Ag}}} = \dfrac{(81.1\ \text{A})(0.062\ \Omega)}{1.2\ \text{m}} = 4.19\ \text{V/m}$, which rounds to 4.2 V/m.

(e) $V_{\text{Ag}} = IR_{\text{Ag}} = (81.1\ \text{A})(0.062\ \Omega) = 5.03\ \text{V}$, which rounds to 5.0 V.

EVALUATE: For the copper section, $V_{\text{Cu}} = IR_{\text{Cu}} = (81.1\ \text{A})(0.049\ \Omega) = 3.97\ \text{V}$. Note that $V_{\text{Cu}} + V_{\text{Ag}} = 3.97\ \text{V} + 5.03\ \text{V} = 9.0\ \text{V}$, the voltage applied across the ends of the composite wire.

25.55. IDENTIFY: Conservation of charge requires that the current be the same in both sections of the wire.

$E = \rho J = \dfrac{\rho I}{A}$. For each section, $V = IR = JAR = \left(\dfrac{EA}{\rho}\right)\left(\dfrac{\rho L}{A}\right) = EL$. The voltages across each section add.

SET UP: $A = (\pi/4)D^2$, where D is the diameter.

EXECUTE: (a) The current must be the same in both sections of the wire, so the current in the thin end is 2.5 mA.

(b) $E_{1.6\text{mm}} = \rho J = \dfrac{\rho I}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(2.5\times10^{-3}\ \text{A})}{(\pi/4)(1.6\times10^{-3}\ \text{m})^2} = 2.14\times10^{-5}\ \text{V/m}$.

(c) $E_{0.8\text{mm}} = \rho J = \dfrac{\rho I}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(2.5\times10^{-3}\ \text{A})}{(\pi/4)(0.80\times10^{-3}\ \text{m})^2} = 8.55\times10^{-5}\ \text{V/m}$. This is $4E_{1.6\text{mm}}$.

(d) $V = E_{1.6\text{mm}}L_{1.6\ \text{mm}} + E_{0.8\ \text{mm}}L_{0.8\ \text{mm}}$. $V = (2.14\times10^{-5}\ \text{V/m})(1.20\ \text{m}) + (8.55\times10^{-5}\ \text{V/m})(1.80\ \text{m}) = 1.80\times10^{-4}\ \text{V}$.

EVALUATE: The currents are the same but the current density is larger in the thinner section and the electric field is larger there.

25.56. IDENTIFY and SET UP: The voltage is the same at both temperatures since the same battery is used. The power is $P = V^2/R$ and $R = R_0(1 + \alpha\Delta T)$.

EXECUTE: Since the voltage is the same, we have $V^2 = P_{80}R_{80} = P_{150}R_{150}$. Therefore $P_{80}R_0[1 + \alpha(T_{80} - T_0)] = P_{150}R_0[1 + \alpha(T_{150} - T_0)]$. Solving for P_{150} and putting in the numbers gives

$$P_{150} = P_{80}\frac{1 + \alpha(T_{80} - T_0)}{1 + \alpha(T_{150} - T_0)} = (480\ \text{W})\frac{1 + (0.0045\ \text{K}^{-1})(80°\text{C} - 20°\text{C})}{1 + (0.0045\ \text{K}^{-1})(150°\text{C} - 20°\text{C})} = 385\ \text{W}.$$

EVALUATE: This result assumes that α is the same at all the temperatures.

25.57. IDENTIFY: Knowing the current and the time for which it lasts, plus the resistance of the body, we can calculate the energy delivered.

SET UP: Electric energy is deposited in his body at the rate $P = I^2R$. Heat energy Q produces a temperature change ΔT according to $Q = mc\Delta T$, where $c = 4190\ \text{J/kg}\cdot\text{C}°$.

EXECUTE: (a) $P = I^2R = (25{,}000\ \text{A})^2(1.0\ \text{k}\Omega) = 6.25\times10^{11}\ \text{W}$. The energy deposited is $Pt = (6.15\times10^{11}\ \text{W})(40\times10^{-6}\ \text{s}) = 2.5\times10^7\ \text{J}$. Find ΔT when $Q = 2.5\times10^7\ \text{J}$.

$$\Delta T = \frac{Q}{mc} = \frac{2.5\times10^7\ \text{J}}{(75\ \text{kg})(4190\ \text{J/kg}\cdot\text{C}°)} = 80\ \text{C}°.$$

(b) An increase of only $63\,\text{C}°$ brings the water in the body to the boiling point; part of the person's body will be vaporized.

EVALUATE: Even this approximate calculation shows that being hit by lightning is very dangerous.

25.58. **IDENTIFY:** The current in the circuit depends on R and on the internal resistance of the battery, as well as the emf of the battery. It is only the current in R that dissipates energy in the resistor R.

SET UP: $I = \dfrac{\varepsilon}{R+r}$, where ε is the emf of the battery, and $P = I^2R$.

EXECUTE: $P = I^2R = \dfrac{\varepsilon^2}{(R+r)^2}R$, which gives $\varepsilon^2 R = (R^2 + 2Rr + r^2)P$.

$$R^2 + \left(2r - \frac{\varepsilon^2}{P}\right)R + r^2 = 0. \quad R = \frac{1}{2}\left[\left(\frac{\varepsilon^2}{P} - 2r\right) \pm \sqrt{\left(\frac{\varepsilon^2}{P} - 2r\right)^2 - 4r^2}\right].$$

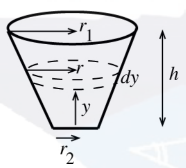
$$R = \frac{1}{2}\left[\left(\frac{(12.0\text{ V})^2}{80.0\text{ W}} - 2(0.40\,\Omega)\right) \pm \sqrt{\left(\frac{(12.0\text{ V})^2}{80.0\text{ W}} - 2(0.40\,\Omega)\right)^2 - 4(0.40\,\Omega)^2}\right].$$

$R = 0.50\,\Omega \pm 0.30\,\Omega$. $R = 0.20\,\Omega$ and $R = 0.80\,\Omega$.

EVALUATE: There are two values for R because there are two ways for the power dissipated in R to be 80 W. The power is $P = I^2R$, so we can have a small $R(0.20\,\Omega)$ and large current, or a larger $R(0.80\,\Omega)$ and a smaller current.

25.59. **(a) IDENTIFY:** Apply $R = \dfrac{\rho L}{A}$ to calculate the resistance of each thin disk and then integrate over the truncated cone to find the total resistance.

SET UP:



EXECUTE: The radius of a truncated cone a distance y above the bottom is given by $r = r_2 + (y/h)(r_1 - r_2) = r_2 + y\beta$ with $\beta = (r_1 - r_2)/h$.

Figure 25.59

Consider a thin slice a distance y above the bottom. The slice has thickness dy and radius r (see Figure 25.59.) The resistance of the slice is $dR = \dfrac{\rho\,dy}{A} = \dfrac{\rho\,dy}{\pi r^2} = \dfrac{\rho\,dy}{\pi(r_2 + \beta y)^2}$.

The total resistance of the cone if obtained by integrating over these thin slices:

$$R = \int dR = \frac{\rho}{\pi}\int_0^h \frac{dy}{(r_2 + \beta y)^2} = \frac{\rho}{\pi}\left[-\frac{1}{\beta}(r_2 + y\beta)^{-1}\right]_0^h = -\frac{\rho}{\pi\beta}\left[\frac{1}{r_2 + h\beta} - \frac{1}{r_2}\right].$$

But $r_2 + h\beta = r_1$.

$$R = \frac{\rho}{\pi\beta}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] = \frac{\rho}{\pi}\left(\frac{h}{r_1 - r_2}\right)\left(\frac{r_1 - r_2}{r_1 r_2}\right) = \frac{\rho h}{\pi r_1 r_2}.$$

(b) EVALUATE: Let $r_1 = r_2 = r$. Then $R = \rho h/\pi r^2 = \rho L/A$ where $A = \pi r^2$ and $L = h$. This agrees with $R = \dfrac{\rho L}{A}$.

25.60. **IDENTIFY:** Divide the region into thin spherical shells of radius r and thickness dr. The total resistance is the sum of the resistances of the thin shells and can be obtained by integration.

SET UP: $I = V/R$ and $J = I/4\pi r^2$, where $4\pi r^2$ is the surface area of a shell of radius r.

EXECUTE: **(a)** $dR = \dfrac{\rho dr}{4\pi r^2} \Rightarrow R = \dfrac{\rho}{4\pi}\int_a^b \dfrac{dr}{r^2} = -\dfrac{\rho}{4\pi}\dfrac{1}{r}\Big|_a^b = \dfrac{\rho}{4\pi}\left(\dfrac{1}{a}-\dfrac{1}{b}\right) = \dfrac{\rho}{4\pi}\left(\dfrac{b-a}{ab}\right).$

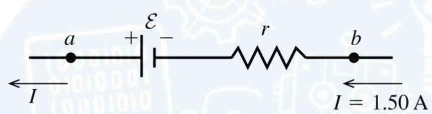
(b) $I = \dfrac{V_{ab}}{R} = \dfrac{V_{ab}4\pi ab}{\rho(b-a)}$ and $J = \dfrac{I}{A} = \dfrac{V_{ab}4\pi ab}{\rho(b-a)4\pi r^2} = \dfrac{V_{ab}ab}{\rho(b-a)r^2}.$

(c) If the thickness of the shells is small, then $4\pi ab \approx 4\pi a^2$ is the surface area of the conducting material. $R = \dfrac{\rho}{4\pi}\left(\dfrac{1}{a}-\dfrac{1}{b}\right) = \dfrac{\rho(b-a)}{4\pi ab} \approx \dfrac{\rho L}{4\pi a^2} = \dfrac{\rho L}{A}$, where $L = b-a$.

EVALUATE: The current density in the material is proportional to $1/r^2$.

25.61. **IDENTIFY:** In each case write the terminal voltage in terms of ε, I, and r. Since I is known, this gives two equations in the two unknowns ε and r.

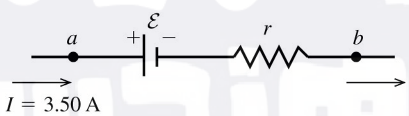
SET UP: The battery with the 1.50-A current is sketched in Figure 25.61a.



$V_{ab} = 8.40$ V.
$V_{ab} = \varepsilon - Ir.$
$\varepsilon - (1.50\text{ A})r = 8.40$ V.

Figure 25.61a

The battery with the 3.50-A current is sketched in Figure 25.61b.



$V_{ab} = 10.2$ V.
$V_{ab} = \varepsilon + Ir.$
$\varepsilon + (3.50\text{ A})r = 10.2$ V.

Figure 25.61b

EXECUTE: **(a)** Solve the first equation for ε and use that result in the second equation:

$\varepsilon = 8.40\text{ V} + (1.50\text{ A})r.$

$8.40\text{ V} + (1.50\text{ A})r + (3.50\text{ A})r = 10.2\text{ V}.$

$(5.00\text{ A})r = 1.8\text{ V}$ so $r = \dfrac{1.8\text{ V}}{5.00\text{ A}} = 0.36\ \Omega.$

(b) Then $\varepsilon = 8.40\text{ V} + (1.50\text{ A})r = 8.40\text{ V} + (1.50\text{ A})(0.36\ \Omega) = 8.94\text{ V}.$

EVALUATE: When the current passes through the emf in the direction from $-$ to $+$, the terminal voltage is less than the emf and when it passes through from $+$ to $-$, the terminal voltage is greater than the emf.

25.62. **IDENTIFY:** Consider the potential changes around the circuit. For a complete loop the sum of the potential changes is zero.

SET UP: There is a potential drop of IR when you pass through a resistor in the direction of the current.

EXECUTE: **(a)** $I = \dfrac{8.0\text{ V} - 4.0\text{ V}}{24.0\ \Omega} = 0.167\text{ A}.$ $V_d + 8.00\text{ V} - I(0.50\ \Omega + 8.00\ \Omega) = V_a$, so

$V_{ad} = 8.00\text{ V} - (0.167\text{ A})(8.50\ \Omega) = 6.58\text{ V}.$

(b) The terminal voltage is $V_{bc} = V_b - V_c$. $V_c + 4.00\text{ V} + I(0.50\ \Omega) = V_b$ and

$V_{bc} = +4.00\text{ V} + (0.167\text{ A})(0.50\ \Omega) = +4.08\text{ V}.$

(c) Adding another battery at point d in the opposite sense to the 8.0-V battery produces a counterclockwise current with magnitude $I = \dfrac{10.3\text{ V} - 8.0\text{ V} + 4.0\text{ V}}{24.5\ \Omega} = 0.257\text{ A}.$ Then $V_c + 4.00\text{ V} - I(0.50\ \Omega) = V_b$ and

$V_{bc} = 4.00\text{ V} - (0.257\text{ A})\,(0.50\ \Omega) = 3.87\text{ V}.$

EVALUATE: When current enters the battery at its negative terminal, as in part (c), the terminal voltage is less than its emf. When current enters the battery at the positive terminal, as in part (b), the terminal voltage is greater than its emf.

25.63. IDENTIFY: $R = \dfrac{\rho L}{A}$. $V = IR$. $P = I^2R$.

SET UP: The area of the end of a cylinder of radius r is πr^2.

EXECUTE: **(a)** $R = \dfrac{(5.0\ \Omega\cdot\text{m})(1.6\ \text{m})}{\pi(0.050\ \text{m})^2} = 1.0\times10^3\ \Omega.$

(b) $V = IR = (100\times10^{-3}\ \text{A})(1.0\times10^3\ \Omega) = 100\ \text{V}.$

(c) $P = I^2R = (100\times10^{-3}\ \text{A})^2(1.0\times10^3\ \Omega) = 10\ \text{W}.$

EVALUATE: The resistance between the hands when the skin is wet is about a factor of ten less than when the skin is dry (Problem 25.64).

25.64. IDENTIFY: $V = IR$. $P = I^2R$.

SET UP: The total resistance is the resistance of the person plus the internal resistance of the power supply.

EXECUTE: **(a)** $I = \dfrac{V}{R_{\text{tot}}} = \dfrac{14\times10^3\ \text{V}}{10\times10^3\ \Omega + 2000\ \Omega} = 1.17\ \text{A}.$

(b) $P = I^2R = (1.17\ \text{A})^2(10\times10^3\ \Omega) = 1.37\times10^4\ \text{J} = 13.7\ \text{kJ}.$

(c) $R_{\text{tot}} = \dfrac{V}{I} = \dfrac{14\times10^3\ \text{V}}{1.00\times10^{-3}\ \text{A}} = 14\times10^6\ \Omega.$ The resistance of the power supply would need to be $14\times10^6\ \Omega - 10\times10^3\ \Omega = 14\times10^6\ \Omega = 14\ \text{M}\Omega.$

EVALUATE: The current through the body in part (a) is large enough to be fatal.

25.65. IDENTIFY: The cost of operating an appliance is proportional to the amount of energy consumed. The energy depends on the power the item consumes and the length of time for which it is operated.

SET UP: At a constant power, the energy is equal to Pt, and the total cost is the cost per kilowatt-hour (kWh) times the energy (in kWh).

EXECUTE: **(a)** Use the fact that $1.00\ \text{k Wh} = (1000\ \text{J/s})(3600\ \text{s}) = 3.60\times10^6\ \text{J},$ and one year contains $3.156\times10^7\ \text{s}.$

$$(75\ \text{J/s})\left(\frac{3.156\times10^7\ \text{s}}{1\ \text{yr}}\right)\left(\frac{\$0.120}{3.60\times10^6\ \text{J}}\right) = \$78.90.$$

(b) At $8\ \text{h/day},$ the refrigerator runs for 1/3 of a year. Using the same procedure as above gives

$$(400\ \text{J/s})\left(\frac{1}{3}\right)\left(\frac{3.156\times10^7\ \text{s}}{1\ \text{yr}}\right)\left(\frac{\$0.120}{3.60\times10^6\ \text{J}}\right) = \$140.27.$$

EVALUATE: Electric lights can be a substantial part of the cost of electricity in the home if they are left on for a long time!

25.66. IDENTIFY: As the resistance R varies, the current in the circuit also varies, which causes the potential drop across the internal resistance of the battery to vary. The largest current will occur when $R = 0,$ and the smallest current will occur when $R \to \infty$. The largest terminal voltage will occur when the current is zero ($R \to \infty$) and the smallest terminal voltage will be when the current is a maximum ($R = 0$).

SET UP: If ε is the internal emf of the battery and r is its internal resistance, then $V_{ab} = \varepsilon - rI.$

EXECUTE: **(a)** As $R \to \infty,$ $I \to 0,$ so $V_{ab} \to \varepsilon = 15.0\ \text{V},$ which is the largest reading of the voltmeter. When $R = 0,$ the current is largest at $(15.0\ \text{V})/(4.00\ \Omega) = 3.75\ \text{A},$ so the smallest terminal voltage is $V_{ab} = \varepsilon - rI = 15.0\ \text{V} - (4.00\ \Omega)(3.75\ \text{A}) = 0.$

(b) From part (a), the maximum current is $3.75\ \text{A}$ when $R = 0,$ and the minimum current is $0.00\ \text{A}$ when $R \to \infty.$

(c) The graphs are sketched in the Figure 25.66.
EVALUATE: Increasing the resistance R increases the terminal voltage, but at the same time it decreases the current in the circuit.

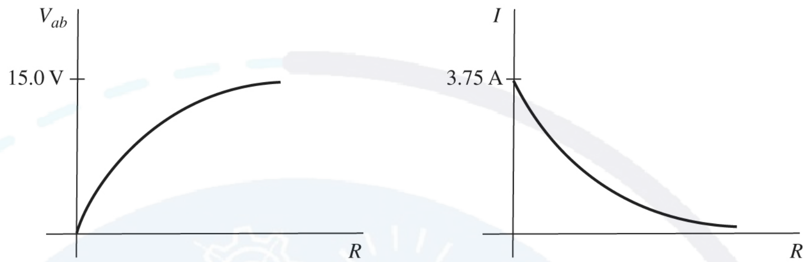
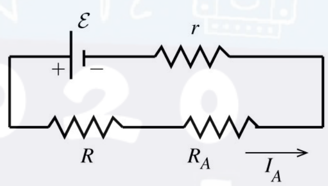


Figure 25.66

25.67. **IDENTIFY:** The ammeter acts as a resistance in the circuit loop. Set the sum of the potential rises and drops around the circuit equal to zero.
(a) SET UP: The circuit with the ammeter is sketched in Figure 25.67a.

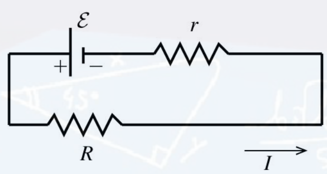


EXECUTE: $I_A = \dfrac{\varepsilon}{r+R+R_A}$.

$\varepsilon = I_A(r+R+R_A)$.

Figure 25.67a

SET UP: The circuit with the ammeter removed is sketched in Figure 25.67b.



EXECUTE: $I = \dfrac{\varepsilon}{R+r}$.

Figure 25.67b

Combining the two equations gives

$$I = \left(\frac{1}{R+r}\right) I_A(r+R+R_A) = I_A\left(1+\frac{R_A}{r+R}\right).$$

(b) Want $I_A = 0.990I$. Use this in the result for part (a).

$$I = 0.990I\left(1+\frac{R_A}{r+R}\right).$$

$$0.010 = 0.990\left(\frac{R_A}{r+R}\right).$$

$$R_A = (r+R)(0.010/0.990) = (0.45\ \Omega + 3.80\ \Omega)(0.010/0.990) = 0.0429\ \Omega.$$

(c) $I - I_A = \dfrac{\varepsilon}{r+R} - \dfrac{\varepsilon}{r+R+R_A}$.

$$I - I_A = \varepsilon\left(\frac{r+R+R_A-r-R}{(r+R)(r+R+R_A)}\right) = \frac{\varepsilon R_A}{(r+R)(r+R+R_A)}.$$

EVALUATE: The difference between I and I_A increases as R_A increases. If R_A is larger than the value calculated in part (b) then I_A differs from I by more than 1.0%.

25.68. **(a) IDENTIFY:** The rate of heating (power) in the cable depends on the potential difference across the cable and the resistance of the cable.

SET UP: The power is $P = V^2/R$ and the resistance is $R = \rho L/A$. The diameter D of the cable is twice its radius. $P = \dfrac{V^2}{R} = \dfrac{V^2}{(\rho L/A)} = \dfrac{AV^2}{\rho L} = \dfrac{\pi r^2 V^2}{\rho L}$. The electric field in the cable is equal to the potential difference across its ends divided by the length of the cable: $E = V/L$.

EXECUTE: Solving for r and using the resistivity of copper gives

$$r = \sqrt{\frac{P\rho L}{\pi V^2}} = \sqrt{\frac{(90.0\ \text{W})(1.72\times10^{-8}\ \Omega\cdot\text{m})(1500\ \text{m})}{\pi(220.0\ \text{V})^2}} = 1.236\times10^{-4}\ \text{m} = 0.1236\ \text{mm}. \quad D = 2r = 0.247\ \text{mm}.$$

(b) IDENTIFY and **SET UP:** $E = V/L$.

EXECUTE: $E = (220\ \text{V})/(1500\ \text{m}) = 0.147\ \text{V/m}$.

EVALUATE: This would be an extremely thin (and hence fragile) cable.

25.69. **(a) IDENTIFY:** Since the resistivity is a function of the position along the length of the cylinder, we must integrate to find the resistance.

SET UP: The resistance of a cross-section of thickness dx is $dR = \rho dx/A$.

EXECUTE: Using the given function for the resistivity and integrating gives

$$R = \int \frac{\rho dx}{A} = \int_0^L \frac{(a+bx^2)dx}{\pi r^2} = \frac{aL + bL^3/3}{\pi r^2}.$$

Now get the constants a and b: $\rho(0) = a = 2.25\times10^{-8}\ \Omega\cdot\text{m}$ and $\rho(L) = a + bL^2$ gives $8.50\times10^{-8}\ \Omega\cdot\text{m} = 2.25\times10^{-8}\ \Omega\cdot\text{m} + b(1.50\ \text{m})^2$ which gives $b = 2.78\times10^{-8}\ \Omega/\text{m}$. Now use the above result to find R.

$$R = \frac{(2.25\times10^{-8}\ \Omega\cdot\text{m})(1.50\ \text{m}) + (2.78\times10^{-8}\ \Omega/\text{m})(1.50\ \text{m})^3/3}{\pi(0.0110\ \text{m})^2} = 1.71\times10^{-4}\ \Omega = 171\ \mu\Omega.$$

(b) IDENTIFY: Use the definition of resistivity to find the electric field at the midpoint of the cylinder, where $x = L/2$.

SET UP: $E = \rho J$. Evaluate the resistivity, using the given formula, for $x = L/2$.

EXECUTE: At the midpoint, $x = L/2$, giving $E = \dfrac{\rho I}{\pi r^2} = \dfrac{[a + b(L/2)^2]I}{\pi r^2}$.

$$E = \frac{[2.25\times10^{-8}\ \Omega\cdot\text{m} + (2.78\times10^{-8}\ \Omega/\text{m})(0.750\ \text{m})^2](1.75\ \text{A})}{\pi(0.0110\ \text{m})^2} = 1.76\times10^{-4}\ \text{V/m} = 176\ \mu\text{V/m}$$

(c) IDENTIFY: For the first segment, the result is the same as in part (a) except that the upper limit of the integral is $L/2$ instead of L.

SET UP: Integrating using the upper limit of $L/2$ gives $R_1 = \dfrac{a(L/2) + (b/3)(L^3/8)}{\pi r^2}$.

EXECUTE: Substituting the numbers gives

$$R_1 = \frac{(2.25\times10^{-8}\ \Omega\cdot\text{m})(0.750\ \text{m}) + (2.78\times10^{-8}\ \Omega/\text{m})/3((1.50\ \text{m})^3/8)}{\pi(0.0110\ \text{m})^2} = 5.47\times10^{-5}\ \Omega = 54.7\ \mu\Omega.$$

The resistance R_2 of the second half is equal to the total resistance minus the resistance of the first half.

$$R_2 = R - R_1 = 1.71\times10^{-4}\ \Omega - 5.47\times10^{-5}\ \Omega = 1.16\times10^{-4}\ \Omega = 116\ \mu\Omega.$$

EVALUATE: The second half has a greater resistance than the first half because the resistance increases with distance along the cylinder.

25.70. **IDENTIFY:** Compact fluorescent bulbs draw much less power than incandescent bulbs and last much longer. Hence they cost less to operate.

SET UP: A kWh is power of 1 kW for a time of 1 h. $P = \dfrac{V^2}{R}$.

EXECUTE: **(a)** In 3.0 yr the bulbs are on for $(3.0 \text{ yr})(365.24 \text{ days/yr})(4.0 \text{ h/day}) = 4.38\times10^3 \text{ h}$.

Compact bulb: The energy used is $(23 \text{ W})(4.38\times10^3 \text{ h}) = 1.01\times10^5 \text{ Wh} = 101 \text{ kWh}$. The cost of this energy is $(\$0.080/\text{kWh})(101 \text{ kWh}) = \$8.08$. One bulb will last longer than this. The bulb cost is \$11.00, so the total cost is \$19.08.

Incandescent bulb: The energy used is $(100 \text{ W})(4.38\times10^3 \text{ h}) = 4.38\times10^5 \text{ Wh} = 438 \text{ kWh}$. The cost of this energy is $(\$0.080/\text{kWh})(438 \text{ kWh}) = \$35.04$. Six bulbs will be used during this time and the bulb cost will be \$4.50. The total cost will be \$39.54.

(b) The compact bulb will save $\$39.54 - \$19.08 = \$20.46$.

(c) $R = \dfrac{V^2}{P} = \dfrac{(120 \text{ V})^2}{23 \text{ W}} = 626\ \Omega$.

EVALUATE: The initial cost of the bulb is much greater for the compact fluorescent bulb but the savings soon repay the cost of the bulb. The compact bulb should last for over six years, so over a 6-year period the savings per year will be even greater. The cost of compact fluorescent bulbs has come down dramatically, so the savings today would be considerably greater than indicated here.

25.71. **IDENTIFY:** Apply $R = \dfrac{\rho L}{A}$ for each material. The total resistance is the sum of the resistances of the rod and the wire. The rate at which energy is dissipated is $I^2 R$.

SET UP: For steel, $\rho = 2.0\times10^{-7}\ \Omega\cdot\text{m}$. For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: **(a)** $R_{\text{steel}} = \dfrac{\rho L}{A} = \dfrac{(2.0\times10^{-7}\ \Omega\cdot\text{m})(2.0 \text{ m})}{(\pi/4)(0.018 \text{ m})^2} = 1.57\times10^{-3}\ \Omega$ and

$R_{\text{Cu}} = \dfrac{\rho L}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(35 \text{ m})}{(\pi/4)(0.008 \text{ m})^2} = 0.012\ \Omega$. This gives

$V = IR = I(R_{\text{steel}} + R_{\text{Cu}}) = (15000 \text{ A})(1.57\times10^{-3}\ \Omega + 0.012\ \Omega) = 204 \text{ V}$.

(b) $E = Pt = I^2 Rt = (15000 \text{ A})^2 (0.0136\ \Omega)(65\times10^{-6} \text{ s}) = 199 \text{ J}$.

EVALUATE: $I^2 R$ is large but t is very small, so the energy deposited is small. The wire and rod each have a mass of about 1 kg, so their temperature rise due to the deposited energy will be small.

25.72. **IDENTIFY:** No current flows to the capacitors when they are fully charged.

SET UP: $V_R = RI$ and $V_C = Q/C$.

EXECUTE: **(a)** $V_{C_1} = \dfrac{Q_1}{C_1} = \dfrac{18.0\ \mu\text{C}}{3.00\ \mu\text{F}} = 6.00 \text{ V}$. $V_{C_2} = V_{C_1} = 6.00 \text{ V}$.

$Q_2 = C_2 V_{C_2} = (6.00\ \mu\text{F})(6.00 \text{ V}) = 36.0\ \mu\text{C}$.

(b) No current flows to the capacitors when they are fully charged, so $\varepsilon = IR_1 + IR_2$.

$V_{R_2} = V_{C_1} = 6.00 \text{ V}$. $I = \dfrac{V_{R_2}}{R_2} = \dfrac{6.00 \text{ V}}{2.00\ \Omega} = 3.00 \text{ A}$.

$R_1 = \dfrac{\varepsilon - IR_2}{I} = \dfrac{72.0 \text{ V} - 6.00 \text{ V}}{3.00 \text{ A}} = 22.0\ \Omega$.

EVALUATE: When a capacitor is fully charged, it acts like an open circuit and prevents any current from flowing though it.

25.73. **IDENTIFY:** No current flows through the capacitor when it is fully charged.

SET UP: With the capacitor fully charged, $I = \dfrac{\varepsilon}{R_1 + R_2}$. $V_R = IR$ and $V_C = Q/C$.

Execute: $V_C = \dfrac{Q}{C} = \dfrac{36.0\ \mu\text{C}}{9.00\ \mu\text{F}} = 4.00\ \text{V}$. $V_{R_1} = V_C = 4.00\ \text{V}$ and $I = \dfrac{V_{R_1}}{R_1} = \dfrac{4.00\ \text{V}}{6.00\ \Omega} = 0.667\ \text{A}$.

$V_{R_2} = IR_2 = (0.667\ \text{A})(4.00\ \Omega) = 2.668\ \text{V}$. $\varepsilon = V_{R_1} + V_{R_2} = 4.00\ \text{V} + 2.668\ \text{V} = 6.67\ \text{V}$.

Evaluate: When a capacitor is fully charged, it acts like an open circuit and prevents any current from flowing though it.

25.74. **Identify** and **Set Up:** Ohm's law applies. The terminal voltage V_{ab} is less than the internal emf ε due to voltage losses in the internal resistance r of the battery when current I is flowing in the circuit. $V_{ab} = \varepsilon - rI$.

Execute: **(a)** The equation $V_{ab} = \varepsilon - rI$ applies to this circuit, so a graph of V_{ab} versus I should be a straight line with a slope equal to $-r$ and a y-intercept equal to ε. Using points where the graph crosses grid lines, the slope is: $\text{slope} = \dfrac{22.0\ \text{V} - 30.0\ \text{V}}{7.00\ \text{A} - 3.00\ \text{A}} = -2.00\ \text{V/A}$. Therefore $r = -(-2.00\ \text{V/A}) = 2.00\ \Omega$.

The equation of the graph is $V_{ab} = \varepsilon - rI$, so we can solve for ε and use a point on the graph to calculate ε. This gives

$\varepsilon = V_{ab} + rI = 30.0\ \text{V} + (2.00\ \Omega)(3.00\ \text{A}) = 36.0\ \text{V}$.

(b) $R = V_{ab}/I$ and $I = \dfrac{\varepsilon - V_{ab}}{r}$, so $R = \dfrac{V_{ab}}{\dfrac{\varepsilon - V_{ab}}{r}} = \dfrac{rV_{ab}}{\varepsilon - V_{ab}}$. Putting in the numbers gives

$R = (2.00\ \Omega)(0.800)(36.0\ \text{V})/[36.0\ \text{V} - (0.800)(36.0\ \text{V})] = 8.00\ \Omega$.

Evaluate: For large currents, the terminal voltage can be much less than the internal emf, as shown by the graph with the problem.

25.75. **Identify:** According to Ohm's law, $R = \dfrac{V_{ab}}{I}$ = constant, and a graph of V_{ab} versus I will be a straight line with positive slope passing through the origin.

Set Up and **Execute:** **(a)** Figure 25.75a shows the graphs of V_{ab} versus I and R versus I for resistor A. Figure 25.75b shows these graphs for resistor B.

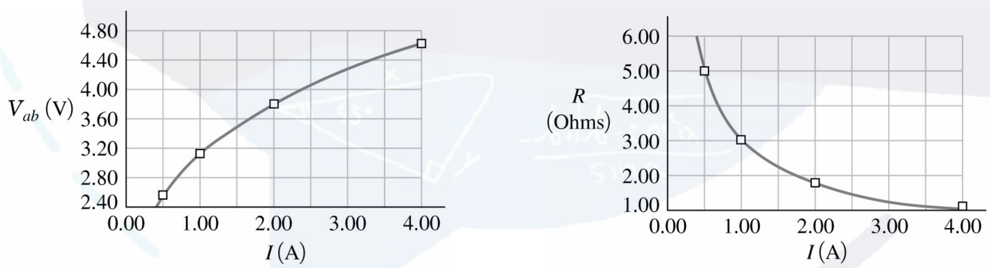


Figure 25.75a

(b) In Figure 25.75a, the graph of V_{ab} versus I is not a straight line so resistor A does not obey Ohm's law. In the graph of R versus I, R is not constant; it decreases as I increases.

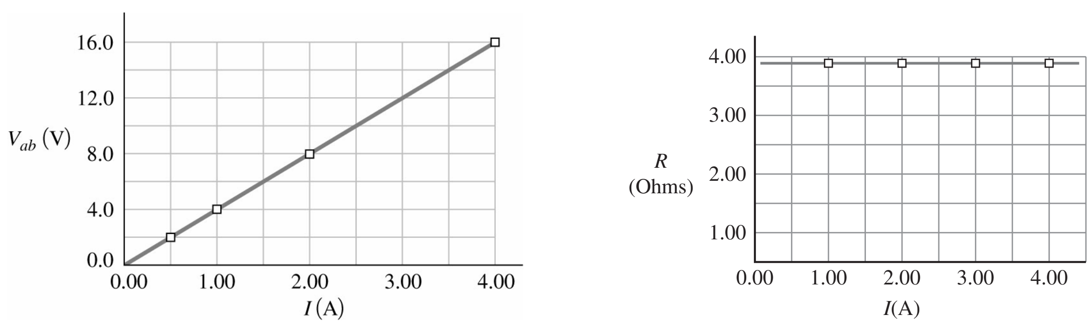


Figure 25.75b

(c) In Figure 25.75b, the graph of V_{ab} versus I is a straight line with positive slope passing through the origin, so resistor B obeys Ohm's law. The graph of R versus I is a horizontal line. This means that R is constant, which is consistent with Ohm's law.

(d) We use $P = IV$. From the graph of V_{ab} versus I in Figure 25.75a, we read that $I = 2.35$ A when $V = 4.00$ V. Therefore $P = IV = (2.35\text{ A})(4.00\text{ V}) = 9.40$ W.

(e) We use $P = V^2/R$. From the graph of R versus I in Figure 25.75b, we find that $R = 3.88\ \Omega$. Thus $P = V^2/R = (4.00\text{ V})^2/(3.88\ \Omega) = 4.12\text{ W}$.

EVALUATE: Since resistor B obeys Ohm's law $V_{ab} = RI$, R is the slope of the graph of V_{ab} versus I in Figure 25.75b. The given data points lie on the line, so we use them to calculate the slope.

slope = $R = \dfrac{15.52\text{ V} - 1.94\text{ V}}{4.00\text{ A} - 0.50\text{ A}} = 3.88\ \Omega.$ This value is the same as the one we got from the graph of R versus I in Figure 25.75b, so our results agree.

25.76. **IDENTIFY:** The power supplied to the house is $P = VI$. The rate at which electrical energy is dissipated in the wires is I^2R, where $R = \dfrac{\rho L}{A}$.

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: **(a)** The line voltage, current to be drawn, and wire diameter are what must be considered in household wiring.

(b) $P = VI$ gives $I = \dfrac{P}{V} = \dfrac{4200\text{ W}}{120\text{ V}} = 35\text{ A},$ so the 8-gauge wire is necessary, since it can carry up to 40 A.

(c) $P = I^2R = \dfrac{I^2\rho L}{A} = \dfrac{(35\text{ A})^2\,(1.72\times10^{-8}\ \Omega\cdot\text{m})(42.0\text{ m})}{(\pi/4)(0.00326\text{ m})^2} = 106\text{ W}.$

(d) If 6-gauge wire is used, $P = \dfrac{I^2\rho L}{A} = \dfrac{(35\text{ A})^2\,(1.72\times10^{-8}\ \Omega\cdot\text{m})\,(42\text{ m})}{(\pi/4)\,(0.00412\text{ m})^2} = 66\text{ W}.$ The decrease in energy consumption is $\Delta E = \Delta Pt = (40\text{ W})(365\text{ days/yr})\,(12\text{ h/day}) = 175\text{ kWh/yr}$ and the savings is $(175\text{ kWh/yr})(\$0.11/\text{kWh}) = \$19.25$ per year.

EVALUATE: The cost of the 4200 W used by the appliances is \$2020. The savings is about 1%.

25.77. **IDENTIFY:** Apply $R = \dfrac{\rho L}{A}$ to find the resistance of a thin slice of the rod and integrate to find the total R. $V = IR$. Also find $R(x)$, the resistance of a length x of the rod.

SET UP: $E(x) = \rho(x)J$

EXECUTE: **(a)** $dR = \dfrac{\rho dx}{A} = \dfrac{\rho_0 \exp[-x/L]\,dx}{A}$ so

$R = \dfrac{\rho_0}{A}\int_0^L \exp\left[-x/L\right]dx = \dfrac{\rho_0}{A}[-L\exp(-x/L)]_0^L = \dfrac{\rho_0 L}{A}(1-e^{-1})$ and $I = \dfrac{V_0}{R} = \dfrac{V_0 A}{\rho_0 L(1-e^{-1})}$. With an upper limit of x rather than L in the integration, $R(x) = \dfrac{\rho_0 L}{A}(1-e^{-x/L})$.

(b) $E(x) = \rho(x)J = \dfrac{I\,\rho_0 e^{-x/L}}{A} = \dfrac{V_0 e^{-x/L}}{L(1-e^{-1})}$.

(c) $V = V_0 - IR(x)$. $V = V_0 - \left(\dfrac{V_0 A}{\rho_0 L[1-e^{-1}]}\right)\left(\dfrac{\rho_0 L}{A}\right)(1-e^{-x/L}) = V_0\dfrac{(e^{-x/L}-e^{-1})}{(1-e^{-1})}$.

(d) Graphs of resistivity, electric field, and potential from $x = 0$ to L are given in Figure 25.77 (next page). Each quantity is given in terms of the indicated unit.

EVALUATE: The current is the same at all points in the rod. Where the resistivity is larger the electric field must be larger, in order to produce the same current density.

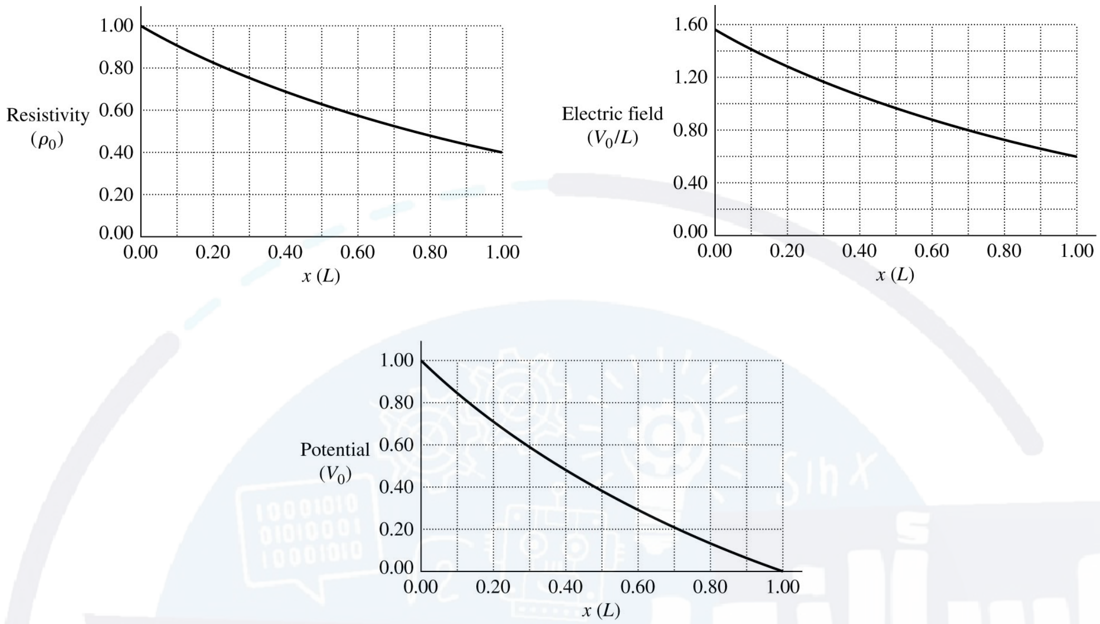


Figure 25.77

25.78. **IDENTIFY** and **SET UP:** The power output P of the source is the power delivered to the resistor R, so P is the power output of the internal emf ε minus the power consumed by the internal resistance r. Therefore $P = \varepsilon I - I^2 r$. For the entire circuit, $\varepsilon = (R + r)I$.

EXECUTE: **(a)** Combining $P = I^2/R$ and $\varepsilon = (R + r)I$ gives $P = \left(\dfrac{\varepsilon}{R+r}\right)^2 R = \dfrac{\varepsilon^2 R}{(R+r)^2}$. From this result, we can see that as $R \to 0, \quad P \to 0$.

(b) Using the same equation as in (a), we see that as $R \to \infty, \quad P \to \dfrac{\varepsilon^2}{R} \to 0$.

(c) In (a) we showed that $P = \dfrac{\varepsilon^2 R}{(R+r)^2}$. For maximum power, $dP/dR = 0$.

$$\frac{dP}{dR} = \varepsilon^2\left[-\frac{2R}{(R+r)^3} + \frac{1}{(R+r)^2}\right] = 0 \quad \to \qquad \frac{2R}{R+r} = 1 \qquad \to \qquad R = r.$$

The maximum power is therefore

$$P_{\max} = \left.\frac{R\varepsilon^2}{(R+r)^2}\right|_{R=r} = \frac{r\varepsilon^2}{(2r)^2} = \frac{\varepsilon^2}{4r}.$$

(d) Use $P = \dfrac{\varepsilon^2 R}{(R+r)^2}$ to calculate P.

<u>For $R = 2.00\ \Omega$</u>: $P_2 = (64.0\ \text{V})^2(2.00\ \Omega)/(6.00\ \Omega)^2 = 228$ W.
<u>For $R = 4.00\ \Omega$</u>: $P_4 = (64.0\ \text{V})^2(4.00\ \Omega)/(8.00\ \Omega)^2 = 256$ W.
<u>For $R = 6.00\ \Omega$</u>: $P_6 = (64.0\ \text{V})^2(6.00\ \Omega)/(10.0\ \Omega)^2 = 246$ W.

EVALUATE: The maximum power in (d) occurred when $R = r = 4.00\ \Omega$, so it is consistent with the result from (c). The equation we found, $P_{\max} = \dfrac{\varepsilon^2}{4r}$, gives $P_{\max} = (64.0\ \text{V})^2/[4(4.00\ \Omega)] = 256$ W, which agrees with our calculation in (d). When R is smaller than r, I is large and the $I^2 r$ losses in the battery are large. When R is larger than r, I is small and the power output εI of the battery emf is small.

25.79. IDENTIFY and SET UP: $R=\dfrac{\rho L}{A}.$

EXECUTE: From the equation $R=\dfrac{\rho L}{A},$ if we double the length of a resistor and change nothing else, the resistance will double. But from the data table given in the problem, we see that doubling the length of the thread causes its resistance to do much more than double. For example, at 5 mm the resistance is $9\times10^9\ \Omega$ and at 11 mm (approximately double) the resistance is $63\times10^9\ \Omega,$ which is much more than twice the resistance at 5 mm. Therefore as the thread stretches, its coating gets thinner, which decreases its cross-sectional area. This decreased area contributes significantly to the increase in resistance. Therefore choice (c) is correct.

EVALUATE: The cross-sectional area of the coating depends on the square of the radius of the thread, so a decrease in the radius has a very large effect on the resistance.

25.80. IDENTIFY and SET UP: Use data from the table for 5 mm and 13 mm to compare the resistance. $R=\dfrac{\rho L}{A}.$

EXECUTE: $\dfrac{R_{13}}{R_5}=\dfrac{102}{9}=\dfrac{\dfrac{\rho(13\text{ mm})}{A_5}}{\dfrac{\rho(5\text{ mm})}{A_{13}}}=\dfrac{13A_5}{5A_{13}}.$ Solving for A_{13} gives

$A_{13}=A_5\left(\dfrac{13}{5}\right)\left(\dfrac{9}{102}\right)=0.23\approx\dfrac{1}{4},$ which is choice (b).

EVALUATE: It is reasonable that $A_{13}<A_5$ because the thread and its coating stretch out and get thinner.

25.81. IDENTIFY and SET UP: Apply Ohm's law, $V=RI$. The minimum resistance will give the maximum current. Get data from the table in the problem.

EXECUTE: $I_{\text{max}}=V/R_{\text{min}}=(9\text{ V})/(9\times10^9\ \Omega)=1\times10^{-9}\text{ A}=1\text{ nA}$, which is choice (d).

EVALUATE: This is a very small current, but the thread of a spider web is very thin.

25.82. IDENTIFY and SET UP: An electrically neutral conductor contains equal amounts of positive and negative charge, and these charges can move if a charged object comes near to them.

EXECUTE: If a positively charged object comes near to the web, it attracts negative charges in the web. The attraction between these negative charges in the web and the positive charges in the charged object pull the web toward the object. If a negatively charged object comes near the web, it repels negative charges in the web, leaving the web positively charged near the object. The attraction between the negatively charged object and the positive side of the web pulls the web toward the object. This is best explained by choice (d).

EVALUATE: This is similar to the principle of charging by induction. The amounts of charge are small, but the web is moved because it is extremely light.

26

DIRECT-CURRENT CIRCUITS

26.1. IDENTIFY: The newly-formed wire is a combination of series and parallel resistors.
SET UP: Each of the three linear segments has resistance $R/3$. The circle is two $R/6$ resistors in parallel.
EXECUTE: The resistance of the circle is $R/12$ since it consists of two $R/6$ resistors in parallel. The equivalent resistance is two $R/3$ resistors in series with an $R/12$ resistor, giving $R_{\text{equiv}} = R/3 + R/3 + R/12 = 3R/4.$
EVALUATE: The equivalent resistance of the original wire has been reduced because the circle's resistance is less than it was as a linear wire.

26.2. IDENTIFY: It may appear that the meter measures X directly. But note that X is in parallel with three other resistors, so the meter measures the equivalent parallel resistance between *ab.*
SET UP: We use the formula for resistors in parallel.
EXECUTE: $1/(2.00\ \Omega) = 1/X + 1/(15.0\ \Omega) + 1/(5.0\ \Omega) + 1/(10.0\ \Omega)$, so $X = 7.5\ \Omega$.
EVALUATE: X is *greater* than the equivalent parallel resistance of $2.00\ \Omega$.

26.3. IDENTIFY: The emf of the battery remains constant, but changing the resistance across it changes its power output.
SET UP: The power consumption in a resistor is $P = \dfrac{V^2}{R}$.
EXECUTE: With just R_1, $P_1 = \dfrac{V^2}{R_1}$ and $V = \sqrt{P_1 R_1} = \sqrt{(36.0\ \text{W})(25.0\ \Omega)} = 30.0\ \text{V}$ is the battery voltage.

With R_2 added, $R_{\text{tot}} = 40.0\ \Omega$. $P = \dfrac{V^2}{R_{\text{tot}}} = \dfrac{(30.0\ \text{V})^2}{40.0\ \Omega} = 22.5\ \text{W}$.

EVALUATE: The two resistors in series dissipate electrical energy at a smaller rate than R_1 alone.

26.4. IDENTIFY: For resistors in parallel the voltages are the same and equal to the voltage across the equivalent resistance.
SET UP: $V = IR$. $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$.

EXECUTE: **(a)** $R_{\text{eq}} = \left(\dfrac{1}{42\ \Omega} + \dfrac{1}{20\ \Omega}\right)^{-1} = 13.548\ \Omega$, which rounds to $13\ \Omega$.

(b) $I = \dfrac{V}{R_{\text{eq}}} = \dfrac{240\ \text{V}}{13.548\ \Omega} = 17.7\ \text{A}$, which rounds to 18 A.

(c) $I_{42\Omega} = \dfrac{V}{R} = \dfrac{240\ \text{V}}{42\ \Omega} = 5.7\ \text{A}$; $I_{20\Omega} = \dfrac{V}{R} = \dfrac{240\ \text{V}}{20\ \Omega} = 12\ \text{A}$.

EVALUATE: More current flows through the resistor that has the smaller R.

26.5. IDENTIFY: The equivalent resistance will vary for the different connections because the series-parallel combinations vary, and hence the current will vary.
SET UP: First calculate the equivalent resistance using the series-parallel formulas, then use Ohm's law $(V = RI)$ to find the current.

EXECUTE: **(a)** $1/R = 1/(15.0\ \Omega) + 1/(30.0\ \Omega)$ gives $R = 10.0\ \Omega$. $I = V/R = (35.0\ \text{V})/(10.0\ \Omega) = 3.50$ A.

(b) $1/R = 1/(10.0\ \Omega) + 1/(35.0\ \Omega)$ gives $R = 7.78\ \Omega$. $I = (35.0\ \text{V})/(7.78\ \Omega) = 4.50$ A.

(c) $1/R = 1/(20.0\ \Omega) + 1/(25.0\ \Omega)$ gives $R = 11.11\ \Omega$, so $I = (35.0\ \text{V})/(11.11\ \Omega) = 3.15$ A.

(d) From part (b), the resistance of the triangle alone is 7.78 Ω. Adding the 3.00-Ω internal resistance of the battery gives an equivalent resistance for the circuit of 10.78 Ω. Therefore the current is $I = (35.0\text{V})/(10.78\ \Omega) = 3.25$ A.

EVALUATE: It makes a big difference how the triangle is connected to the battery.

26.6. **IDENTIFY:** The potential drop is the same across the resistors in parallel, and the current into the parallel combination is the same as the current through the 45.0-Ω resistor.

(a) SET UP: Apply Ohm's law in the parallel branch to find the current through the 45.0-Ω resistor. Then apply Ohm's law to the 45.0-Ω resistor to find the potential drop across it.

EXECUTE: The potential drop across the 25.0-Ω resistor is $V_{25} = (25.0\ \Omega)(1.25\ \text{A}) = 31.25$ V. The potential drop across each of the parallel branches is 31.25 V. For the 15.0-Ω resistor: $I_{15} = (31.25\text{V})/(15.0\ \Omega) = 2.083$ A. The resistance of the $10.0\text{-}\Omega + 15.0\text{-}\Omega$ combination is $25.0\ \Omega$, so the current through it must be the same as the current through the upper 25.0-Ω resistor: $I_{10+15} = 1.25$ A. The sum of currents in the parallel branch will be the current through the 45.0-Ω resistor.

$$I_{\text{Total}} = 1.25\ \text{A} + 2.083\ \text{A} + 1.25\ \text{A} = 4.58\ \text{A}.$$

Apply Ohm's law to the 45.0-Ω resistor: $V_{45} = (4.58\ \text{A})(45.0\ \Omega) = 206$ V.

(b) SET UP: First find the equivalent resistance of the circuit and then apply Ohm's law to it.

EXECUTE: The resistance of the parallel branch is $1/R = 1/(25.0\ \Omega) + 1/(15.0\ \Omega) + 1/(25.0\ \Omega)$, so $R = 6.82\ \Omega$. The equivalent resistance of the circuit is $6.82\ \Omega + 45.0\ \Omega + 35.00\ \Omega = 86.82\ \Omega$. Ohm's law gives $V_{\text{Bat}} = (86.62\ \Omega)(4.58\ \text{A}) = 398$ V.

EVALUATE: The emf of the battery is the sum of the potential drops across each of the three segments (parallel branch and two series resistors).

26.7. **IDENTIFY:** First do as much series-parallel reduction as possible.

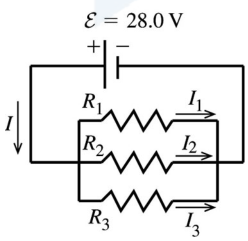
SET UP: The 45.0-Ω and 15.0-Ω resistors are in parallel, so first reduce them to a single equivalent resistance. Then find the equivalent series resistance of the circuit.

EXECUTE: $1/R_\text{p} = 1/(45.0\ \Omega) + 1/(15.0\ \Omega)$ and $R_\text{p} = 11.25\ \Omega$. The total equivalent resistance is $18.0\ \Omega + 11.25\ \Omega + 3.26\ \Omega = 32.5\ \Omega$. Ohm's law gives $I = (25.0\ \text{V})/(32.5\ \Omega) = 0.769$ A.

EVALUATE: The circuit appears complicated until we realize that the 45.0-Ω and 15.0-Ω resistors are in parallel.

26.8. **IDENTIFY:** The equivalent resistance of the resistors in parallel is given by $\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$. For resistors in parallel, the voltages are the same and the currents add.

SET UP: The circuit is sketched in Figure 26.8a.



EXECUTE: **(a)** parallel

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{1.60\ \Omega} + \frac{1}{2.40\ \Omega} + \frac{1}{4.80\ \Omega}.$$

$$R_{\text{eq}} = 0.800\ \Omega.$$

Figure 26.8a

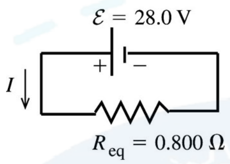
(b) For resistors in parallel the voltage is the same across each and equal to the applied voltage; $V_1 = V_2 = V_3 = \mathcal{E} = 28.0$ V.

$V = IR$ so $I_1 = \dfrac{V_1}{R_1} = \dfrac{28.0\ \text{V}}{1.60\ \Omega} = 17.5$ A.

$I_2 = \dfrac{V_2}{R_2} = \dfrac{28.0 \text{ V}}{2.40\ \Omega} = 11.7 \text{ A}$ and $I_3 = \dfrac{V_3}{R_3} = \dfrac{28.0 \text{ V}}{4.8\ \Omega} = 5.8 \text{ A}.$

(c) The currents through the resistors add to give the current through the battery:
$I = I_1 + I_2 + I_3 = 17.5 \text{ A} + 11.7 \text{ A} + 5.8 \text{ A} = 35.0 \text{ A}.$

EVALUATE: Alternatively, we can use the equivalent resistance R_{eq} as shown in Figure 26.8b.



$\mathcal{E} - IR_{eq} = 0.$

$I = \dfrac{\mathcal{E}}{R_{eq}} = \dfrac{28.0 \text{ V}}{0.800\ \Omega} = 35.0 \text{ A},$ which checks.

Figure 26.8b

(d) As shown in part (b), the voltage across each resistor is 28.0 V.

(e) IDENTIFY and **SET UP:** We can use any of the three expressions for $P: P = VI = I^2R = V^2/R.$ They will all give the same results, if we keep enough significant figures in intermediate calculations.

EXECUTE: Using $P = V^2/R$, $P_1 = V_1^2/R_1 = \dfrac{(28.0 \text{ V})^2}{1.60\ \Omega} = 490 \text{ W}$, $P_2 = V_2^2/R_2 = \dfrac{(28.0 \text{ V})^2}{2.40\ \Omega} = 327 \text{ W}$, and

$P_3 = V_3^2/R_3 = \dfrac{(28.0 \text{ V})^2}{4.80\ \Omega} = 163 \text{W}.$

(f) $P = V^2/R.$ The resistors in parallel each have the same voltage, so the power P is largest for the one with the least resistance.

EVALUATE: The total power dissipated is $P_{out} = P_1 + P_2 + P_3 = 980 \text{ W}.$ This is the same as the power $P_{in} = \mathcal{E}I = (28.0 \text{ V})(35.0 \text{ A}) = 980 \text{ W}$ delivered by the battery.

26.9. IDENTIFY: For a series network, the current is the same in each resistor and the sum of voltages for each resistor equals the battery voltage. The equivalent resistance is $R_{eq} = R_1 + R_2 + R_3.$ $P = I^2R.$

SET UP: Let $R_1 = 1.60\ \Omega$, $R_2 = 2.40\ \Omega$, $R_3 = 4.80\ \Omega.$

EXECUTE: **(a)** $R_{eq} = 1.60\ \Omega + 2.40\ \Omega + 4.80\ \Omega = 8.80\ \Omega.$

(b) $I = \dfrac{V}{R_{eq}} = \dfrac{28.0 \text{ V}}{8.80\ \Omega} = 3.18 \text{ A}.$

(c) $I = 3.18 \text{ A}$, the same as for each resistor.

(d) $V_1 = IR_1 = (3.18 \text{ A})(1.60\ \Omega) = 5.09 \text{ V}.$ $V_2 = IR_2 = (3.18 \text{ A})(2.40\ \Omega) = 7.63 \text{ V}.$
$V_3 = IR_3 = (3.18 \text{ A})(4.80\ \Omega) = 15.3 \text{ V}.$ Note that $V_1 + V_2 + V_3 = 28.0 \text{ V}.$

(e) $P_1 = I^2R_1 = (3.18 \text{ A})^2(1.60\ \Omega) = 16.2 \text{ W}.$ $P_2 = I^2R_2 = (3.18 \text{ A})^2(2.40\ \Omega) = 24.3 \text{ W}.$
$P_3 = I^2R_3 = (3.18 \text{ A})^2(4.80\ \Omega) = 48.5 \text{ W}.$

(f) Since $P = I^2R$ and the current is the same for each resistor, the resistor with the greatest R dissipates the greatest power.

EVALUATE: When resistors are connected in parallel, the resistor with the smallest R dissipates the greatest power.

26.10. IDENTIFY: The current, and hence the power, depends on the potential difference across the resistor.

SET UP: $P = V^2/R.$

EXECUTE: **(a)** $V = \sqrt{PR} = \sqrt{(5.0 \text{ W})(15{,}000\ \Omega)} = 274 \text{ V}.$

(b) $P = V^2/R = (120 \text{ V})^2/(9{,}000\ \Omega) = 1.6 \text{ W}.$

(c) **SET UP:** If the larger resistor generates 2.00 W, the smaller one will generate less and hence will be safe. Therefore the maximum power in the larger resistor must be 2.00 W. Use $P = I^2R$ to find the maximum current through the series combination and use Ohm's law to find the potential difference across the combination.

EXECUTE: $P = I^2R$ gives $I = \sqrt{P/R} = \sqrt{(2.00\text{ W})/(150\ \Omega)} = 0.115\text{ A}$. The same current flows through both resistors, and their equivalent resistance is $250\ \Omega$. Ohm's law gives

$V = IR = (0.115\text{ A})(250\ \Omega) = 28.8\text{ V}$. Therefore $P_{150} = 2.00\text{ W}$ and

$P_{100} = I^2R = (0.115\text{ A})^2(100\ \Omega) = 1.32\text{ W}$.

EVALUATE: If the resistors in a series combination all have the same power rating, it is the *largest* resistance that limits the amount of current.

26.11. **IDENTIFY** and **SET UP:** Ohm's law applies to the resistors, the potential drop across resistors in parallel is the same for each of them, and at a junction the currents in must equal the currents out.

EXECUTE: **(a)** $V_2 = I_2R_2 = (4.00\text{ A})(6.00\ \Omega) = 24.0\text{ V}$. $V_1 = V_2 = 24.0\text{ V}$.

$$I_1 = \frac{V_1}{R_1} = \frac{24.0\text{ V}}{3.00\ \Omega} = 8.00\text{ A}. \quad I_3 = I_1 + I_2 = 4.00\text{ A} + 8.00\text{ A} = 12.0\text{ A}.$$

(b) $V_3 = I_3R_3 = (12.0\text{ A})(5.00\ \Omega) = 60.0\text{ V}$. $\varepsilon = V_1 + V_3 = 24.0\text{ V} + 60.0\text{ V} = 84.0\text{ V}$.

EVALUATE: Series/parallel reduction was not necessary in this case.

26.12. **IDENTIFY** and **SET UP:** Ohm's law applies to the resistors, and at a junction the currents in must equal the currents out.

EXECUTE: $V_1 = I_1R_1 = (1.50\text{ A})(5.00\ \Omega) = 7.50\text{ V}$. $V_2 = 7.50\text{ V}$. $I_1 + I_2 = I_3$ so

$$I_2 = I_3 - I_1 = 4.50\text{ A} - 1.50\text{ A} = 3.00\text{ A}. \quad R_2 = \frac{V_2}{I_2} = \frac{7.50\text{ V}}{3.00\text{ A}} = 2.50\ \Omega.$$

$$V_3 = \varepsilon - V_1 = 35.0\text{ V} - 7.50\text{ V} = 27.5\text{ V}. \quad R_3 = \frac{V_3}{I_3} = \frac{27.5\text{ V}}{4.50\text{ A}} = 6.11\ \Omega.$$

EVALUATE: Series/parallel reduction was not necessary in this case.

26.13. **IDENTIFY:** For resistors in parallel, the voltages are the same and the currents add. $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ so

$R_{eq} = \dfrac{R_1R_2}{R_1 + R_2}$, For resistors in series, the currents are the same and the voltages add. $R_{eq} = R_1 + R_2$.

SET UP: The rules for combining resistors in series and parallel lead to the sequences of equivalent circuits shown in Figure 26.13.

EXECUTE: $R_{eq} = 5.00\ \Omega$. In Figure 26.13c, $I = \dfrac{60.0\text{ V}}{5.00\ \Omega} = 12.0\text{ A}$. This is the current through each of the resistors in Figure 26.13b. $V_{12} = IR_{12} = (12.0\text{ A})(2.00\ \Omega) = 24.0\text{ V}$.

$V_{34} = IR_{34} = (12.0\text{ A})(3.00\ \Omega) = 36.0\text{ V}$. Note that $V_{12} + V_{34} = 60.0\text{ V}$. V_{12} is the voltage across R_1 and across R_2, so $I_1 = \dfrac{V_{12}}{R_1} = \dfrac{24.0\text{ V}}{3.00\ \Omega} = 8.00\text{ A}$ and $I_2 = \dfrac{V_{12}}{R_2} = \dfrac{24.0\text{ V}}{6.00\ \Omega} = 4.00\text{ A}$. V_{34} is the voltage across R_3 and across R_4, so $I_3 = \dfrac{V_{34}}{R_3} = \dfrac{36.0\text{ V}}{12.0\ \Omega} = 3.00\text{ A}$ and $I_4 = \dfrac{V_{34}}{R_4} = \dfrac{36.0\text{ V}}{4.00\ \Omega} = 9.00\text{ A}$.

EVALUATE: Note that $I_1 + I_2 = I_3 + I_4$.

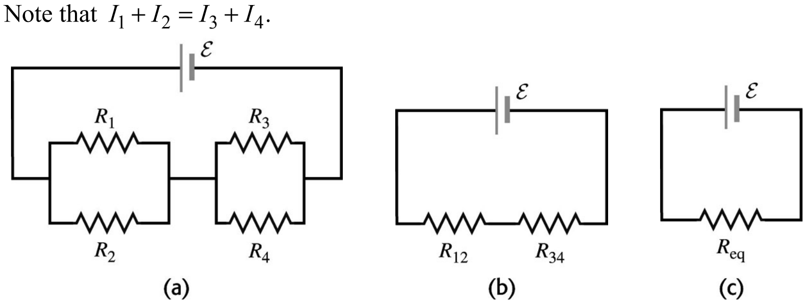


Figure 26.13

26.14. **IDENTIFY:** Replace the series combinations of resistors by their equivalents. In the resulting parallel network the battery voltage is the voltage across each resistor.

SET UP: The circuit is sketched in Figure 26.14a.

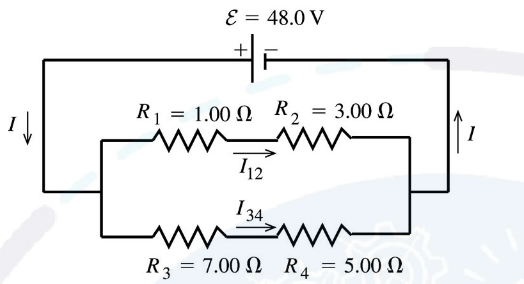


Figure 26.14a

EXECUTE: R_1 and R_2 in series have an equivalent resistance of $R_{12} = R_1 + R_2 = 4.00\ \Omega$. R_3 and R_4 in series have an equivalent resistance of $R_{34} = R_3 + R_4 = 12.0\ \Omega$.

The circuit is equivalent to the circuit sketched in Figure 26.14b.

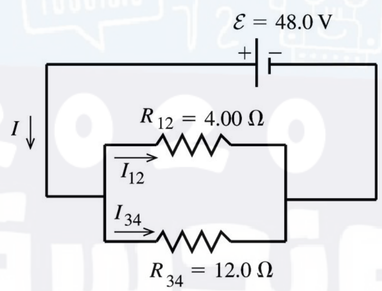


Figure 26.14b

R_{12} and R_{34} in parallel are equivalent to R_{eq} given by $\dfrac{1}{R_{eq}} = \dfrac{1}{R_{12}} + \dfrac{1}{R_{34}} = \dfrac{R_{12} + R_{34}}{R_{12}R_{34}}$.

$$R_{eq} = \frac{R_{12}R_{34}}{R_{12} + R_{34}}.$$

$$R_{eq} = \frac{(4.00\ \Omega)(12.0\ \Omega)}{4.00\ \Omega + 12.0\ \Omega} = 3.00\ \Omega.$$

The voltage across each branch of the parallel combination is ε, so $\varepsilon - I_{12}R_{12} = 0$.

$$I_{12} = \frac{\varepsilon}{R_{12}} = \frac{48.0\ \text{V}}{4.00\ \Omega} = 12.0\ \text{A}.$$

$$\varepsilon - I_{34}R_{34} = 0 \text{ so } I_{34} = \frac{\varepsilon}{R_{34}} = \frac{48.0\ \text{V}}{12.0\ \Omega} = 4.0\ \text{A}.$$

The current is 12.0 A through the $1.00\text{-}\Omega$ and $3.00\text{-}\Omega$ resistors, and it is 4.0 A through the $7.00\text{-}\Omega$ and $5.00\text{-}\Omega$ resistors.

EVALUATE: The current through the battery is $I = I_{12} + I_{34} = 12.0\ \text{A} + 4.0\ \text{A} = 16.0\ \text{A}$, and this is equal to $\varepsilon/R_{eq} = 48.0\ \text{V}/3.00\ \Omega = 16.0\ \text{A}$.

26.15. **IDENTIFY:** In both circuits, with and without R_4, replace series and parallel combinations of resistors by their equivalents. Calculate the currents and voltages in the equivalent circuit and infer from this the currents and voltages in the original circuit. Use $P = I^2R$ to calculate the power dissipated in each bulb.

(a) SET UP: The circuit is sketched in Figure 26.15a.

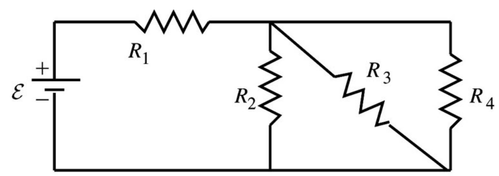
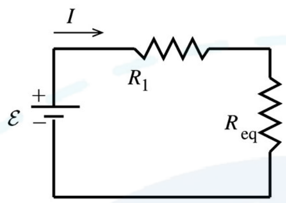


Figure 26.15a

EXECUTE: R_2, R_3, and R_4 are in parallel, so their equivalent resistance R_{eq} is given by $\dfrac{1}{R_{eq}} = \dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R_4}$.

$$\frac{1}{R_{\text{eq}}}=\frac{3}{4.50\ \Omega} \text{ and } R_{\text{eq}}=1.50\ \Omega.$$

The equivalent circuit is drawn in Figure 26.15b.



$$\mathcal{E}-I(R_1+R_{\text{eq}})=0.$$

$$I=\frac{\mathcal{E}}{R_1+R_{\text{eq}}}.$$

Figure 26.15b

$$I=\frac{9.00\ \text{V}}{4.50\ \Omega+1.50\ \Omega}=1.50\ \text{A and } I_1=1.50\ \text{A}.$$

Then $V_1=I_1R_1=(1.50\ \text{A})(4.50\ \Omega)=6.75\ \text{V}.$

$I_{\text{eq}}=1.50\ \text{A},\ V_{\text{eq}}=I_{\text{eq}}R_{\text{eq}}=(1.50\ \text{A})(1.50\ \Omega)=2.25\ \text{V}.$

For resistors in parallel the voltages are equal and are the same as the voltage across the equivalent resistor, so $V_2=V_3=V_4=2.25\ \text{V}.$

$$I_2=\frac{V_2}{R_2}=\frac{2.25\ \text{V}}{4.50\ \Omega}=0.500\ \text{A},\ I_3=\frac{V_3}{R_3}=0.500\ \text{A},\ I_4=\frac{V_4}{R_4}=0.500\ \text{A}.$$

EVALUATE: Note that $I_2+I_3+I_4=1.50\ \text{A},$ which is $I_{\text{eq}}.$ For resistors in parallel the currents add and their sum is the current through the equivalent resistor.

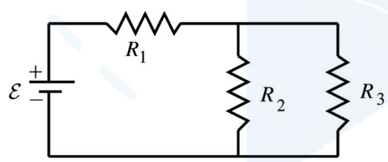
(b) SET UP: $P=I^2R.$

EXECUTE: $P_1=(1.50\ \text{A})^2(4.50\ \Omega)=10.1\ \text{W}.$

$P_2=P_3=P_4=(0.500\ \text{A})^2(4.50\ \Omega)=1.125\ \text{W},$ which rounds to 1.12 W. R_1 glows brightest.

EVALUATE: Note that $P_2+P_3+P_4=3.37\ \text{W}.$ This equals $P_{\text{eq}}=I_{\text{eq}}^2R_{\text{eq}}=(1.50\ \text{A})^2(1.50\ \Omega)=3.37\ \text{W},$ the power dissipated in the equivalent resistor.

(c) SET UP: With R_4 removed the circuit becomes the circuit in Figure 26.15c.

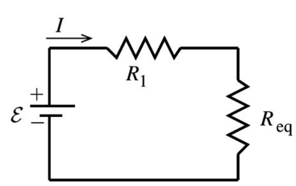


EXECUTE: R_2 and R_3 are in parallel and their equivalent resistance R_{eq} is given by

$$\frac{1}{R_{\text{eq}}}=\frac{1}{R_2}+\frac{1}{R_3}=\frac{2}{4.50\ \Omega} \text{ and } R_{\text{eq}}=2.25\ \Omega.$$

Figure 26.15c

The equivalent circuit is shown in Figure 26.15d.



$$\mathcal{E}-I(R_1+R_{\text{eq}})=0.$$

$$I=\frac{\mathcal{E}}{R_1+R_{\text{eq}}}.$$

$$I=\frac{9.00\ \text{V}}{4.50\ \Omega+2.25\ \Omega}=1.333\ \text{A}.$$

Figure 26.15d

$I_1=1.33\ \text{A},\ V_1=I_1R_1=(1.333\ \text{A})(4.50\ \Omega)=6.00\ \text{V}.$

$I_{\text{eq}}=1.33\ \text{A},\ V_{\text{eq}}=I_{\text{eq}}R_{\text{eq}}=(1.333\ \text{A})(2.25\ \Omega)=3.00\ \text{V}$ and $V_2=V_3=3.00\ \text{V}.$

$I_2 = \dfrac{V_2}{R_2} = \dfrac{3.00\text{ V}}{4.50\ \Omega} = 0.667\text{ A}, I_3 = \dfrac{V_3}{R_3} = 0.667\text{ A}.$

(d) SET UP: $P = I^2 R.$

EXECUTE: $P_1 = (1.333\text{ A})^2(4.50\ \Omega) = 8.00\text{ W}.$

$P_2 = P_3 = (0.667\text{ A})^2(4.50\ \Omega) = 2.00\text{ W}.$

EVALUATE: **(e)** When R_4 is removed, P_1 decreases and P_2 and P_3 increase. Bulb R_1 glows less brightly and bulbs R_2 and R_3 glow more brightly. When R_4 is removed the equivalent resistance of the circuit increases and the current through R_1 decreases. But in the parallel combination this current divides into two equal currents rather than three, so the currents through R_2 and R_3 increase. Can also see this by noting that with R_4 removed and less current through R_1 the voltage drop across R_1 is less so the voltage drop across R_2 and across R_3 must become larger.

26.16. IDENTIFY: Apply Ohm's law to each resistor.

SET UP: For resistors in parallel the voltages are the same and the currents add. For resistors in series the currents are the same and the voltages add.

EXECUTE: From Ohm's law, the voltage drop across the $6.00\text{-}\Omega$ resistor is $V = IR = (4.00\text{ A})(6.00\ \Omega) = 24.0\text{ V}.$ The voltage drop across the $8.00\text{-}\Omega$ resistor is the same, since these two resistors are wired in parallel. The current through the $8.00\text{-}\Omega$ resistor is then $I = V/R = 24.0\text{ V}/8.00\ \Omega = 3.00\text{ A}.$ The current through the $25.0\text{-}\Omega$ resistor is the sum of the current through these two resistors: 7.00 A. The voltage drop across the $25.0\text{-}\Omega$ resistor is $V = IR = (7.00\text{ A})(25.0\ \Omega) = 175\text{ V},$ and total voltage drop across the top branch of the circuit is $175\text{ V} + 24.0\text{ V} = 199\text{ V},$ which is also the voltage drop across the $20.0\text{-}\Omega$ resistor. The current through the $20.0\text{-}\Omega$ resistor is then $I = V/R = 199\text{ V}/20\ \Omega = 9.95\text{ A}.$

EVALUATE: The total current through the battery is $7.00\text{ A} + 9.95\text{ A} = 16.95\text{ A}.$ Note that we did not need to calculate the emf of the battery.

26.17. IDENTIFY: Apply Ohm's law to each resistor.

SET UP: For resistors in parallel the voltages are the same and the currents add. For resistors in series the currents are the same and the voltages add.

EXECUTE: The current through the $2.00\text{-}\Omega$ resistor is 6.00 A. Current through the $1.00\text{-}\Omega$ resistor also is 6.00 A and the voltage is 6.00 V. Voltage across the $6.00\text{-}\Omega$ resistor is $12.0\text{ V} + 6.0\text{ V} = 18.0\text{ V}.$ Current through the $6.00\text{-}\Omega$ resistor is $(18.0\text{ V})/(6.00\ \Omega) = 3.00\text{ A}.$ The battery emf is 18.0 V.

EVALUATE: The current through the battery is $6.00\text{ A} + 3.00\text{ A} = 9.00\text{ A}.$ The equivalent resistor of the resistor network is $2.00\ \Omega,$ and this equals $(18.0\text{ V})/(9.00\text{ A}).$

26.18. IDENTIFY: Ohm's law applies to each resistor. In one case, the resistors are connected in series, and in the other case they are in parallel.

SET UP: $V = RI,$ $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots$ (in parallel), $R_{eq} = R_1 + R_2 + \ldots$ (in series). Figure 26.18 shows the equivalent circuit when *S* is open and when *S* is closed.

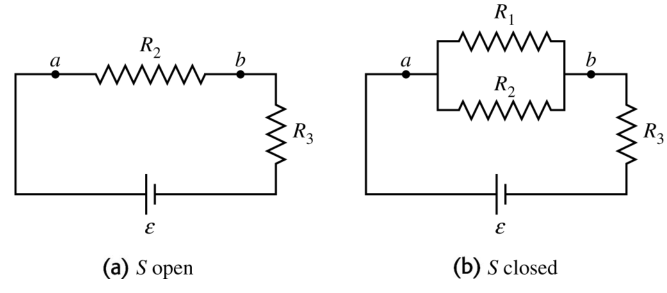


Figure 26.18

EXECUTE: **(a)** S open: We use the circuit in Figure 26.18a. R_2 and R_3 are in series. Ohm's law gives $\varepsilon = (R_2 + R_3)I$.

$I = \varepsilon/(R_2 + R_3) = (36.0\ V)/(9.00\ \Omega) = 4.00$ A.

$V_{ab} = R_2 I = (6.00\ \Omega)(4.00\ \text{A}) = 24.0$ V.

S closed: We use the circuit in Figure 26.18b. R_1 and R_2 are in parallel, and this combination is in series with R_3. For the parallel branch $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots = 1/(4.00\ \Omega) + 1/(6.00\ \Omega)$, which gives $R_{eq} = 2.40\ \Omega$. The equivalent resistance R of the circuit is $2.40\ \Omega + 3.00\ \Omega = 5.40\ \Omega$. The current is $I = \mathcal{E}/R = (36.0\ \text{V})/(5.40\ \Omega) = 6.667$ A. Therefore $V_{ab} = IR_{eq} = (6.667\ \text{A})(2.40\ \Omega) = 16.0$ V.

(b) S open: From part (a), we know that $I_2 = 4.00$ A through R_2. Since S is open, no current can flow through R_1, so $I_1 = 0$, $I_2 = I_3 = 4.00$ A.

S closed: $I_1 = V_{ab}/R_1 = (16.0\ \text{V})/(4.00\ \Omega) = 4.00$ A. $I_2 = V_{ab}/R_2 = (16.0\ \text{V})/(6.00\ \Omega) = 2.67$ A.

$I_3 = I_1 + I_2 = 4.00\ \text{A} + 2.67\ \text{A} = 6.67$ A.

I_1 increased from 0 to 4.00 A.

I_2 decreased from 4.00 A to 2.67 A.

I_3 increased from 4.00 A to 6.67 A.

EVALUATE: With S closed, $V_{ab} + V_3 = 16.0\ \text{V} + (3.00\ \Omega)(6.67\ \text{A}) = 36.0$ V, which is equal to ε, as it should be.

26.19. **IDENTIFY** and **SET UP:** Replace series and parallel combinations of resistors by their equivalents until the circuit is reduced to a single loop. Use the loop equation to find the current through the 20.0-Ω resistor. Set $P = I^2R$ for the 20.0-Ω resistor equal to the rate Q/t at which heat goes into the water and set $Q = mc\Delta T$.

EXECUTE: Replace the network by the equivalent resistor, as shown in Figure 26.19.

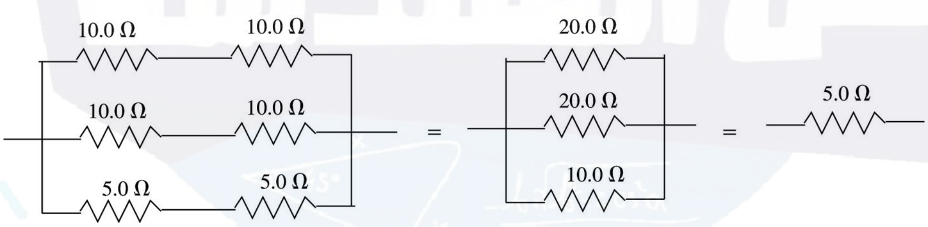


Figure 26.19

$30.0\ \text{V} - I(20.0\ \Omega + 5.0\ \Omega + 5.0\ \Omega) = 0;\ I = 1.00$ A.

For the 20.0-Ω resistor thermal energy is generated at the rate $P = I^2R = 20.0$ W. $Q = Pt$ and $Q = mc\Delta T$ gives $t = \frac{mc\Delta T}{P} = \frac{(0.100\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})(48.0\ \text{C°})}{20.0\ \text{W}} = 1.01\times10^3$ s.

EVALUATE: The battery is supplying heat at the rate $P = \varepsilon I = 30.0$ W. In the series circuit, more energy is dissipated in the larger resistor $(20.0\ \Omega)$ than in the smaller ones $(5.00\ \Omega)$.

26.20. **IDENTIFY:** $P = I^2R$ determines R_1. $R_1, R_2,$ and the 10.0-Ω resistor are all in parallel so have the same voltage. Apply the junction rule to find the current through R_2.

SET UP: $P = I^2R$ for a resistor and $P = \varepsilon I$ for an emf. The emf inputs electrical energy into the circuit and electrical energy is removed in the resistors.

EXECUTE: **(a)** $P_1 = I_1^2R_1$. $15.0\ \text{W} = (2.00\ \text{A})^2 R_1$ so $R_1 = 3.75\ \Omega$. R_1 and $10.0\ \Omega$ are in parallel, so $(10.0\ \Omega)I_{10} = (3.75\ \Omega)(2.00\ \text{A})$ so $I_{10} = 0.750$ A. So $I_2 = 3.50\ \text{A} - I_1 - I_{10} = 3.50\ \text{A} - 2.00\ \text{A} - 0.750\ \text{A} = 0.750$ A. R_1 and R_2 are in parallel, so $(0.750\ \text{A})R_2 = (2.00\ \text{A})(3.75\ \Omega)$ which gives $R_2 = 10.0\ \Omega$.

(b) $\varepsilon = V_1 = (2.00\text{ A})(3.75\ \Omega) = 7.50\text{ V}.$

(c) From part (a), $I_2 = 0.750\text{ A}$, $I_{10} = 0.750\text{ A}.$

(d) $P_1 = 15.0\text{ W}$ (given). $P_2 = I_2^2 R_2 = (0.750\text{ A})^2(10.0\ \Omega) = 5.625\text{ W},$ which rounds to 5.63 W.

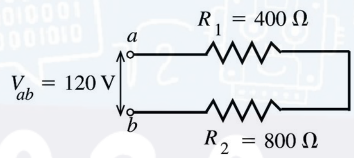
$P_{10} = I_{10}^2 R_{10} = (0.750\text{ A})^2(10.0\ \Omega) = 5.625\text{ W}.$ The total rate at which the resistors remove electrical energy is $P_{\text{Resist}} = 15.0\text{ W} + 5.625\text{ W} + 5.625\text{ W} = 26.25\text{ W},$ which rounds to 26.3 W.

The total rate at which the battery inputs electrical energy is $P_{\text{Battery}} = I\varepsilon = (3.50\text{ A})(7.50\text{ V}) = 26.3\text{ W}\cdot$ Therefore $P_{\text{Resist}} = P_{\text{Battery}},$ which agrees with conservation of energy.

EVALUATE: The three resistors are in parallel, so the voltage for each is the battery voltage, 7.50 V. The currents in the three resistors add to give the current in the battery.

26.21. IDENTIFY: For resistors in series, the voltages add and the current is the same. For resistors in parallel, the voltages are the same and the currents add. $P = I^2 R.$

(a) SET UP: The circuit is sketched in Figure 26.21a.



For resistors in series the current is the same through each.

Figure 26.21a

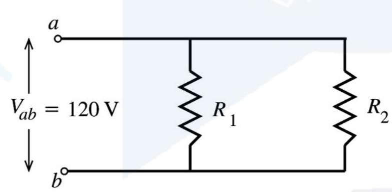
EXECUTE: $R_{\text{eq}} = R_1 + R_2 = 1200\ \Omega.$ $I = \dfrac{V}{R_{\text{eq}}} = \dfrac{120\text{ V}}{1200\ \Omega} = 0.100\text{ A}.$ This is the current drawn from the line.

(b) $P_1 = I_1^2 R_1 = (0.100\text{ A})^2(400\ \Omega) = 4.0\text{ W}.$

$P_2 = I_2^2 R_2 = (0.100\text{ A})^2(800\ \Omega) = 8.0\text{ W}.$

(c) $P_{\text{out}} = P_1 + P_2 = 12.0\text{ W},$ the total power dissipated in both bulbs. Note that $P_{\text{in}} = V_{ab} I = (120\text{ V})(0.100\text{ A}) = 12.0\text{ W},$ the power delivered by the potential source, equals $P_{\text{out}}.$

(d) SET UP: The circuit is sketched in Figure 26.21b.



For resistors in parallel the voltage across each resistor is the same.

Figure 26.21b

EXECUTE: $I_1 = \dfrac{V_1}{R_1} = \dfrac{120\text{ V}}{400\ \Omega} = 0.300\text{ A},\ I_2 = \dfrac{V_2}{R_2} = \dfrac{120\text{ V}}{800\ \Omega} = 0.150\text{ A}.$

EVALUATE: Note that each current is larger than the current when the resistors are connected in series.

EXECUTE: **(e)** $P_1 = I_1^2 R_1 = (0.300\text{ A})^2(400\ \Omega) = 36.0\text{ W}.$

$P_2 = I_2^2 R_2 = (0.150\text{ A})^2(800\ \Omega) = 18.0\text{ W}.$

(f) $P_{\text{out}} = P_1 + P_2 = 54.0\text{ W}.$

EVALUATE: Note that the total current drawn from the line is $I = I_1 + I_2 = 0.450\text{ A}.$ The power input from the line is $P_{\text{in}} = V_{ab} I = (120\text{ V})(0.450\text{ A}) = 54.0\text{ W},$ which equals the total power dissipated by the bulbs.

(g) The bulb that is dissipating the most power glows most brightly. For the series connection the currents are the same and by $P = I^2R$ the bulb with the larger *R* has the larger *P*; the $800\text{-}\Omega$ bulb glows more brightly. For the parallel combination the voltages are the same and by $P = V^2/R$ the bulb with the smaller *R* has the larger *P*; the $400\text{-}\Omega$ bulb glows more brightly.

(h) The total power output P_{out} equals $P_{\text{in}} = V_{ab}I$, so P_{out} is larger for the parallel connection where the current drawn from the line is larger (because the equivalent resistance is smaller.)

26.22. **IDENTIFY:** Use $P = V^2/R$ with $V = 120\text{ V}$ and the wattage for each bulb to calculate the resistance of each bulb. When connected in series the voltage across each bulb will not be 120 V and the power for each bulb will be different.

SET UP: For resistors in series the currents are the same and $R_{\text{eq}} = R_1 + R_2$.

EXECUTE: **(a)** $R_{60\text{W}} = \dfrac{V^2}{P} = \dfrac{(120\text{ V})^2}{60\text{ W}} = 240\ \Omega$; $R_{200\text{W}} = \dfrac{V^2}{P} = \dfrac{(120\text{ V})^2}{200\text{ W}} = 72\ \Omega.$

Therefore, $I_{60\text{W}} = I_{200\text{W}} = \dfrac{\varepsilon}{R} = \dfrac{240\text{ V}}{(240\ \Omega + 72\ \Omega)} = 0.769\text{ A}.$

(b) $P_{60\text{W}} = I^2R = (0.769\text{ A})^2(240\ \Omega) = 142\text{ W}$; $P_{200\text{W}} = I^2R = (0.769\text{ A})^2(72\ \Omega) = 42.6\text{ W}.$

(c) The 60 W bulb burns out quickly because the power it delivers (142 W) is 2.4 times its rated value.

EVALUATE: In series the largest resistance dissipates the greatest power.

26.23. **IDENTIFY:** Apply Kirchhoff's rules.

SET UP: Figure 26.23 shows the loops taken. When we go around loop (1) in the direction shown there is a potential rise across the 200.0 V battery, so there must be a drop across *R* and the current in *R* must be in the direction shown in the figure. Similar analysis of loops (2) and (3) tell us that currents I_2 and I_5 must be in the directions shown. The junction rule has been used to label the currents in all the other branches of the circuit.

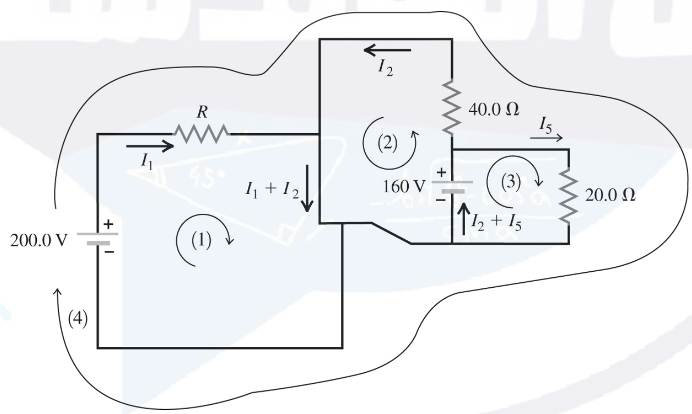


Figure 26.23

EXECUTE: **(a)** Apply the Kirchhoff loop rule to loop (1): $+200.0\text{ V} - I_1R = 0.$ Solving for *R* gives

$$R = \frac{+200.0\text{ V}}{I_1} = \frac{+200.0\text{ V}}{10.0\text{ A}} = 20.0\ \Omega.$$

(b) Loop (2): $+160.0\text{ V} - I_2(40.0\ \Omega) = 0.$ $I_2 = \dfrac{160.0\text{ V}}{40.0\ \Omega} = 4.00\text{ A}.$

Loop (3): $+160.0\text{ V} - I_5(20.0\ \Omega) = 0.$ $I_5 = \dfrac{160.0\text{ V}}{20.0\ \Omega} = 8.00\text{ A}.$

A_2 reads $I_2 = 4.00\text{ A}$. A_3 reads $I_2 + I_5 = 12.0\text{ A}$. A_4 reads $I_1 + I_2 = 14.0\text{ A}$. A_5 reads $I_5 = 8.00\text{ A}$.

EVALUATE: The sum of potential changes around the outer loop (4) is

$$+200.0\text{ V} - I_1 R + I_2(40.0\ \Omega) - I_5(20.0\ \Omega) = 200.0\text{ V} - (10.0\text{ A})(20.0\ \Omega) + (4.00\text{ A})(40.0\ \Omega) - (8.00\text{ A})(20.0\ \Omega) = 200.0\text{ V} - 200.0\text{ V} - 160.0\text{ V} = 0.$$

The loop rule is satisfied for loop (4) and this is a good check of our calculations.

26.24. **IDENTIFY:** This circuit cannot be reduced using series/parallel combinations, so we apply Kirchhoff's rules. The target variables are the currents in each segment.

SET UP: Assume the unknown currents have the directions shown in Figure 26.24. We have used the junction rule to write the current through the 10.0 V battery as $I_1 + I_2$. There are two unknowns, I_1 and I_2, so we will need two equations. Three possible circuit loops are shown in the figure.

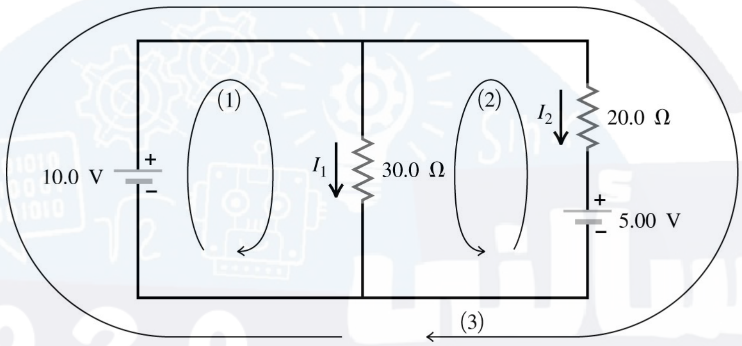


Figure 26.24

EXECUTE: **(a)** Apply the loop rule to loop (1), going around the loop in the direction shown: $+10.0\text{ V} - (30.0\ \Omega)I_1 = 0$ and $I_1 = 0.333\text{ A}$.

(b) Apply the loop rule to loop (3): $+10.0\text{ V} - (20.0\ \Omega)I_2 - 5.00\text{ V} = 0$ and $I_2 = 0.250\text{ A}$.

(c) $I_1 + I_2 = 0.333\text{ A} + 0.250\text{ A} = 0.583\text{ A}$.

EVALUATE: For loop (2) we get

$+5.00\text{ V} + I_2(20.0\ \Omega) - I_1(30.0\ \Omega) = 5.00\text{ V} + (0.250\text{ A})(20.0\ \Omega) - (0.333\text{ A})(30.0\ \Omega) =$

$5.00\text{ V} + 5.00\text{ V} - 10.0\text{ V} = 0$, so that with the currents we have calculated the loop rule is satisfied for this third loop.

26.25. **IDENTIFY:** Apply Kirchhoff's junction rule at point *a* to find the current through *R*. Apply Kirchhoff's loop rule to loops (1) and (2) shown in Figure 26.25a to calculate *R* and $\mathcal{E}$. Travel around each loop in the direction shown.

SET UP:

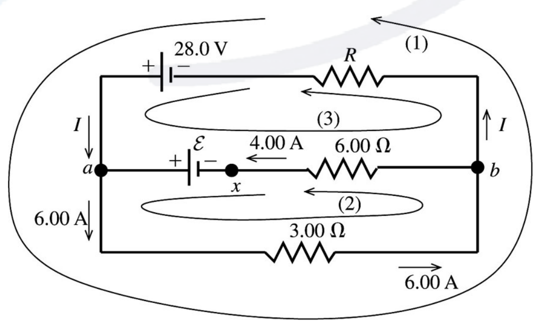


Figure 26.25a

EXECUTE: **(a)** Apply Kirchhoff's junction rule to point *a*: $\sum I = 0$ so $I + 4.00\text{ A} - 6.00\text{ A} = 0$

$I = 2.00\text{ A}$ (in the direction shown in the diagram).

(b) Apply Kirchhoff's loop rule to loop (1): $-(6.00\text{ A})(3.00\,\Omega)-(2.00\text{ A})R+28.0\text{ V}=0$

$-18.0\text{ V}-(2.00\,\Omega)R+28.0\text{ V}=0.$

$$R=\frac{28.0\text{ V}-18.0\text{ V}}{2.00\text{ A}}=5.00\,\Omega.$$

(c) Apply Kirchhoff's loop rule to loop (2): $-(6.00\text{ A})(3.00\,\Omega)-(4.00\text{ A})(6.00\,\Omega)+\varepsilon=0.$

$\varepsilon=18.0\text{ V}+24.0\text{ V}=42.0\text{ V}.$

EVALUATE: We can check that the loop rule is satisfied for loop (3), as a check of our work:

$28.0\text{ V}-\varepsilon+(4.00\text{ A})(6.00\,\Omega)-(2.00\text{ A})R=0.$

$28.0\text{ V}-42.0\text{ V}+24.0\text{ V}-(2.00\text{ A})(5.00\,\Omega)=0.$

$52.0\text{ V}=42.0\text{ V}+10.0\text{ V}.$

$52.0\text{ V}=52.0\text{ V},$ so the loop rule is satisfied for this loop.

(d) IDENTIFY: If the circuit is broken at point *x* there can be no current in the 6.00-Ω resistor. There is now only a single current path and we can apply the loop rule to this path.

SET UP: The circuit is sketched in Figure 26.25b.

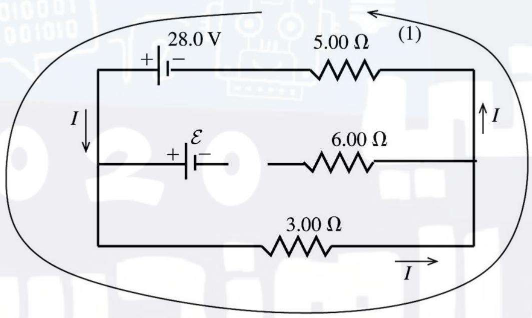


Figure 26.25b

EXECUTE: $+28.0\text{ V}-(3.00\,\Omega)I-(5.00\,\Omega)I=0.$

$$I=\frac{28.0\text{ V}}{8.00\,\Omega}=3.50\text{ A}.$$

EVALUATE: Breaking the circuit at *x* removes the 42.0-V emf from the circuit and the current through the 3.00-Ω resistor is reduced.

26.26. IDENTIFY: Apply Kirchhoff's loop rule and junction rule.

SET UP: The circuit diagram is given in Figure 26.26. The junction rule has been used to find the magnitude and direction of the current in the middle branch of the circuit. There are no remaining unknown currents.

EXECUTE: The loop rule applied to loop (1) gives:

$+20.0\text{V}-(1.00\text{ A})(1.00\,\Omega)+(1.00\text{ A})(4.00\,\Omega)+(1.00\text{ A})(1.00\,\Omega)-\varepsilon_1-(1.00\text{ A})(6.00\,\Omega)=0.$

$\varepsilon_1=20.0\text{ V}-1.00\text{ V}+4.00\text{ V}+1.00\text{ V}-6.00\text{ V}=18.0\text{ V}.$ The loop rule applied to loop (2) gives:

$+20.0\text{ V}-(1.00\text{ A})(1.00\,\Omega)-(2.00\text{ A})(1.00\,\Omega)-\varepsilon_2-(2.00\text{ A})(2.00\,\Omega)-(1.00\text{ A})(6.00\,\Omega)=0.$

$\varepsilon_2=20.0\text{ V}-1.00\text{ V}-2.00\text{ V}-4.00\text{ V}-6.00\text{ V}=7.0\text{ V}.$ Going from *b* to *a* along the lower branch,

$V_b+(2.00\text{ A})(2.00\,\Omega)+7.0\text{ V}+(2.00\text{ A})(1.00\,\Omega)=V_a\cdot V_b-V_a=-13.0\text{ V};$ point *b* is at 13.0 V lower potential than point *a*.

EVALUATE: We can also calculate V_b-V_a by going from *b* to *a* along the upper branch of the circuit.

$V_b-(1.00\text{ A})(6.00\,\Omega)+20.0\text{ V}-(1.00\text{ A})(1.00\,\Omega)=V_a$ and $V_b-V_a=-13.0\text{ V}.$ This agrees with V_b-V_a calculated along a different path between *b* and *a*.

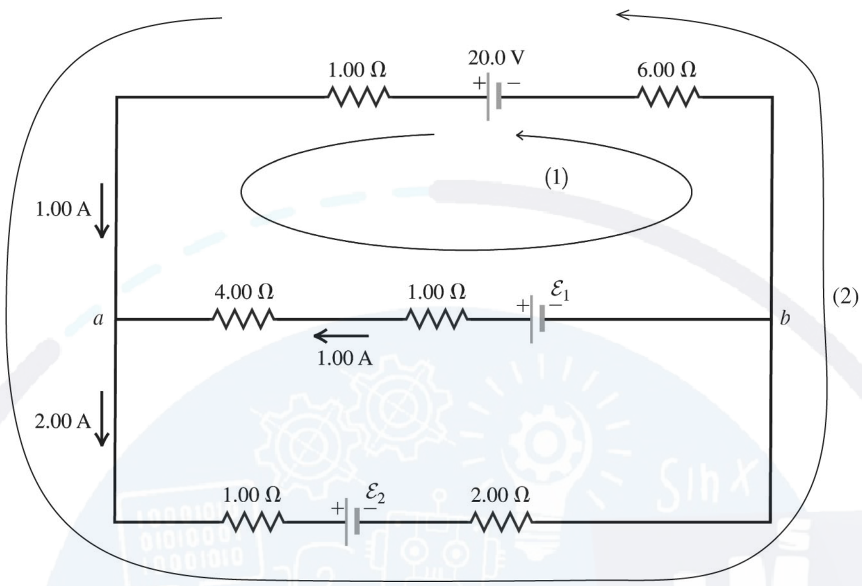


Figure 26.26

26.27. **IDENTIFY:** Apply Kirchhoff's junction rule at points *a*, *b*, *c*, and *d* to calculate the unknown currents. Then apply the loop rule to three loops to calculate $\varepsilon_1, \varepsilon_2$, and *R*.

SET UP: The circuit is sketched in Figure 26.27.

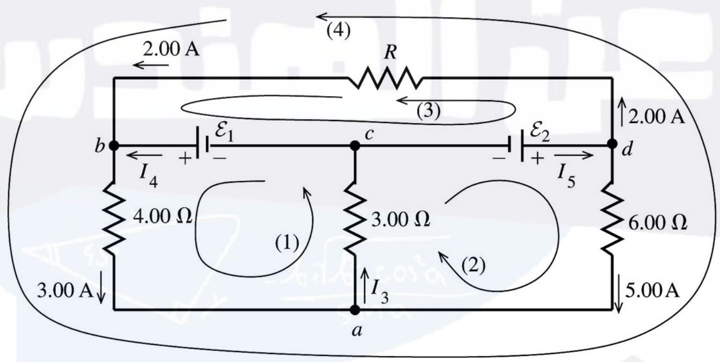


Figure 26.27

(a) EXECUTE: Apply the junction rule to point *a*: $3.00\text{ A} + 5.00\text{ A} - I_3 = 0$.

$I_3 = 8.00\text{ A}$.

Apply the junction rule to point *b*: $2.00\text{ A} + I_4 - 3.00\text{ A} = 0$.

$I_4 = 1.00\text{ A}$.

Apply the junction rule to point *c*: $I_3 - I_4 - I_5 = 0$.

$I_5 = I_3 - I_4 = 8.00\text{ A} - 1.00\text{ A} = 7.00\text{ A}$.

EVALUATE: As a check, apply the junction rule to point *d*: $I_5 - 2.00\text{ A} - 5.00\text{ A} = 0$.

$I_5 = 7.00\text{ A}$.

(b) EXECUTE: Apply the loop rule to loop (1): $\varepsilon_1 - (3.00\text{ A})(4.00\,\Omega) - I_3(3.00\,\Omega) = 0$.

$\varepsilon_1 = 12.0\text{ V} + (8.00\text{ A})(3.00\,\Omega) = 36.0\text{ V}$.

Apply the loop rule to loop (2): $\varepsilon_2 - (5.00\text{ A})(6.00\,\Omega) - I_3(3.00\,\Omega) = 0$.

$\varepsilon_2 = 30.0\text{ V} + (8.00\text{ A})(3.00\,\Omega) = 54.0\text{ V}$.

(c) EXECUTE: Apply the loop rule to loop (3): $-(2.00\ \text{A})R - \varepsilon_1 + \varepsilon_2 = 0.$

$$R = \frac{\varepsilon_2 - \varepsilon_1}{2.00\ \text{A}} = \frac{54.0\ \text{V} - 36.0\ \text{V}}{2.00\ \text{A}} = 9.00\ \Omega.$$

EVALUATE: Apply the loop rule to loop (4) as a check of our calculations:

$-(2.00\ \text{A})R - (3.00\ \text{A})(4.00\ \Omega) + (5.00\ \text{A})(6.00\ \Omega) = 0.$

$-(2.00\ \text{A})(9.00\ \Omega) - 12.0\ \text{V} + 30.0\ \text{V} = 0.$

$-18.0\ \text{V} + 18.0\ \text{V} = 0.$

26.28. IDENTIFY: Use Kirchhoff's rules to find the currents.

SET UP: Since the 10.0-V battery has the larger voltage, assume I_1 is to the left through the 10-V battery, I_2 is to the right through the 5-V battery, and I_3 is to the right through the 10-Ω resistor. Go around each loop in the counterclockwise direction.

EXECUTE: (a) Upper loop: $10.0\ \text{V} - (2.00\ \Omega + 3.00\ \Omega)I_1 - (1.00\ \Omega + 4.00\ \Omega)I_2 - 5.00\ \text{V} = 0.$ This gives $5.0\ \text{V} - (5.00\ \Omega)I_1 - (5.00\ \Omega)I_2 = 0,$ and $\Rightarrow I_1 + I_2 = 1.00\ \text{A}.$

Lower loop: $5.00\ \text{V} + (1.00\ \Omega + 4.00\ \Omega)I_2 - (10.0\ \Omega)I_3 = 0.$ This gives $5.00\ \text{V} + (5.00\ \Omega)I_2 - (10.0\ \Omega)I_3 = 0,$ and $I_2 - 2I_3 = -1.00\ \text{A}.$

Along with $I_1 = I_2 + I_3,$ we can solve for the three currents and find:

$$I_1 = 0.800\ \text{A},\ I_2 = 0.200\ \text{A},\ I_3 = 0.600\ \text{A}.$$

(b) $V_{ab} = -(0.200\ \text{A})(4.00\ \Omega) - (0.800\ \text{A})(3.00\ \Omega) = -3.20\ \text{V}.$

EVALUATE: Traveling from *b* to *a* through the 4.00-Ω and 3.00-Ω resistors you pass through the resistors in the direction of the current and the potential decreases. Therefore point *b* is at higher potential than point *a*.

26.29. IDENTIFY: Apply the junction rule to reduce the number of unknown currents. Apply the loop rule to two loops to obtain two equations for the unknown currents I_1 and I_2.

(a) SET UP: The circuit is sketched in Figure 26.29.

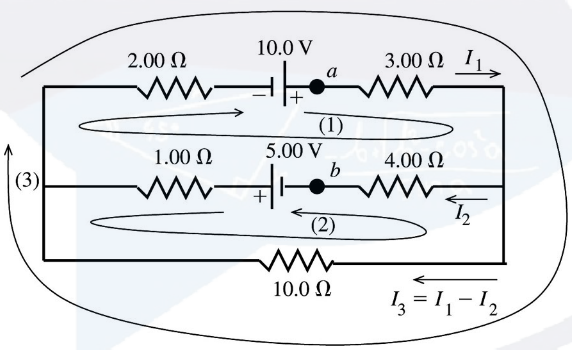


Figure 26.29

Let I_1 be the current in the 3.00-Ω resistor and I_2 be the current in the 4.00-Ω resistor and assume that these currents are in the directions shown. Then the current in the 10.0-Ω resistor is $I_3 = I_1 - I_2,$ in the direction shown, where we have used Kirchhoff's junction rule to relate I_3 to I_1 and I_2. If we get a negative answer for any of these currents we know the current is actually in the opposite direction to what we have assumed. Three loops and directions to travel around the loops are shown in the circuit diagram in Figure 26.29. Apply Kirchhoff's loop rule to each loop.

EXECUTE: Loop (1):

$+10.0\ \text{V} - I_1(3.00\ \Omega) - I_2(4.00\ \Omega) + 5.00\ \text{V} - I_2(1.00\ \Omega) - I_1(2.00\ \Omega) = 0.$

$15.00\ \text{V} - (5.00\ \Omega)I_1 - (5.00\ \Omega)I_2 = 0.$

$3.00\ \text{A} - I_1 - I_2 = 0.$

<u>Loop (2)</u>:

$+5.00\text{ V} - I_2(1.00\,\Omega) + (I_1 - I_2)10.0\,\Omega - I_2(4.00\,\Omega) = 0.$

$5.00\text{ V} + (10.0\,\Omega)I_1 - (15.0\,\Omega)I_2 = 0.$

$1.00\text{ A} + 2.00I_1 - 3.00I_2 = 0.$

The first equation says $I_2 = 3.00\text{ A} - I_1$.

Use this in the second equation: $1.00\text{ A} + 2.00I_1 - 9.00\text{ A} + 3.00I_1 = 0.$

$5.00I_1 = 8.00\text{ A}, I_1 = 1.60\text{ A}.$

Then $I_2 = 3.00\text{ A} - I_1 = 3.00\text{ A} - 1.60\text{ A} = 1.40\text{ A}.$

$I_3 = I_1 - I_2 = 1.60\text{ A} - 1.40\text{ A} = 0.20\text{ A}.$

EVALUATE: Loop (3) can be used as a check.

$+10.0\text{ V} - (1.60\text{ A})(3.00\,\Omega) - (0.20\text{ A})(10.00\,\Omega) - (1.60\text{ A})(2.00\,\Omega) = 0.$

$10.0\text{ V} = 4.8\text{ V} + 2.0\text{ V} + 3.2\text{ V}.$

$10.0\text{ V} = 10.0\text{ V}.$

We find that with our calculated currents the loop rule is satisfied for loop (3). Also, all the currents came out to be positive, so the current directions in the circuit diagram are correct.

(b) IDENTIFY and **SET UP:** To find $V_{ab} = V_a - V_b$ start at point *b* and travel to point *a*. Many different routes can be taken from *b* to *a* and all must yield the same result for V_{ab}.

EXECUTE: Travel through the $4.00\text{-}\Omega$ resistor and then through the $3.00\text{-}\Omega$ resistor:

$V_b + I_2(4.00\,\Omega) + I_1(3.00\,\Omega) = V_a.$

$V_a - V_b = (1.40\text{ A})(4.00\,\Omega) + (1.60\text{ A})(3.00\,\Omega) = 5.60\text{ V} + 4.8\text{ V} = 10.4\text{ V}$ (point *a* is at higher potential than point *b*).

EVALUATE: Alternatively, travel through the 5.00-V emf, the $1.00\text{-}\Omega$ resistor, the $2.00\text{-}\Omega$ resistor, and the 10.0-V emf.

$V_b + 5.00\text{ V} - I_2(1.00\,\Omega) - I_1(2.00\,\Omega) + 10.0\text{ V} = V_a.$

$V_a - V_b = 15.0\text{ V} - (1.40\text{ A})(1.00\,\Omega) - (1.60\text{ A})(2.00\,\Omega) = 15.0\text{ V} - 1.40\text{ V} - 3.20\text{ V} = 10.4\text{ V},$ the same as before.

26.30. **IDENTIFY:** Use Kirchhoff's rules to find the currents.

SET UP: Since the 15.0-V battery has the largest voltage, assume I_1 is to the right through the 10.0-V battery, I_2 is to the left through the 15.0-V battery, and I_3 is to the right through the $10.00\text{-}\Omega$ resistor. Go around each loop in the counterclockwise direction.

EXECUTE: **(a)** <u>Upper loop:</u> $10.0\text{ V} + (2.00\,\Omega + 3.00\,\Omega)I_1 + (1.00\,\Omega + 4.00\,\Omega)I_2 - 15.00\text{ V} = 0.$

$-5.00\text{ V} + (5.00\,\Omega)I_1 + (5.00\,\Omega)I_2 = 0,$ so $I_1 + I_2 = +1.00\text{A}.$

<u>Lower loop:</u> $15.00\text{ V} - (1.00\,\Omega + 4.00\,\Omega)I_2 - (10.0\,\Omega)I_3 = 0.$

$15.00\text{ V} - (5.00\,\Omega)I_2 - (10.0\,\Omega)I_3 = 0,$ so $I_2 + 2I_3 = 3.00\text{ A}.$

Along with $I_2 = I_1 + I_3,$ we can solve for the three currents and find

$I_1 = 0.00\text{ A}, I_2 = +1.00\text{ A}$ (to the left), $I_3 = +1.00\text{ A}$ (to the right).

(b) $V_{ab} = I_2(4.00\Omega) + I_1(3.00\Omega) = (1.00\text{ A})(4.00\Omega) + (0.00\text{ A})(3.00\Omega) = 4.00\text{ V}.$

EVALUATE: Traveling from *b* to *a* through the $4.00\text{-}\Omega$ and $3.00\text{-}\Omega$ resistors you pass through each resistor opposite to the direction of the current and the potential increases; point *a* is at higher potential than point *b*.

26.31. **(a) IDENTIFY:** With the switch open, the circuit can be solved using series-parallel reduction.

SET UP: Find the current through the unknown battery using Ohm's law. Then use the equivalent resistance of the circuit to find the emf of the battery.

EXECUTE: The $30.0\text{-}\Omega$ and $50.0\text{-}\Omega$ resistors are in series, and hence have the same current. Using Ohm's law $I_{50} = (15.0\text{ V})/(50.0\,\Omega) = 0.300\text{ A} = I_{30}$. The potential drop across the $75.0\text{-}\Omega$ resistor is the same as the potential drop across the $80.0\text{-}\Omega$ series combination. We can use this fact to find the current through the $75.0\text{-}\Omega$ resistor using Ohm's law: $V_{75} = V_{80} = (0.300\text{ A})(80.0\text{ }\Omega) = 24.0\text{ V}$ and $I_{75} = (24.0\text{ V})/(75.0\text{ }\Omega) = 0.320\text{ A}$.

The current through the unknown battery is the sum of the two currents we just found:

$$I_{\text{Total}} = 0.300\text{ A} + 0.320\text{ A} = 0.620\text{ A}.$$

The equivalent resistance of the resistors in parallel is $1/R_{\text{p}} = 1/(75.0\text{ }\Omega) + 1/(80.0\text{ }\Omega)$. This gives $R_{\text{p}} = 38.7\text{ }\Omega$. The equivalent resistance "seen" by the battery is $R_{\text{equiv}} = 20.0\text{ }\Omega + 38.7\text{ }\Omega = 58.7\text{ }\Omega$. Applying Ohm's law to the battery gives $\varepsilon = R_{\text{equiv}} I_{\text{Total}} = (58.7\text{ }\Omega)(0.620\text{ A}) = 36.4\text{ V}$.

(b) IDENTIFY: With the switch closed, the 25.0-V battery is connected across the $50.0\text{-}\Omega$ resistor.
SET UP: Take a loop around the right part of the circuit.
EXECUTE: Ohm's law gives $I = (25.0\text{ V})/(50.0\text{ }\Omega) = 0.500\text{ A}$.
EVALUATE: The current through the $50.0\text{-}\Omega$ resistor, and the rest of the circuit, depends on whether or not the switch is open.

26.32. IDENTIFY: We need to use Kirchhoff's rules.
SET UP: Take a loop around the outside of the circuit, apply the junction rule at the upper junction, and then take a loop around the right side of the circuit.
EXECUTE: The outside loop gives $75.0\text{ V} - (12.0\text{ }\Omega)(1.50\text{ A}) - (48.0\text{ }\Omega)I_{48} = 0$, so $I_{48} = 1.188\text{ A}$. At a junction we have $1.50\text{A} = I_\varepsilon + 1.188\text{ A}$, and $I_\varepsilon = 0.313\text{ A}$. A loop around the right part of the circuit gives $\varepsilon - (48\text{ }\Omega)(1.188\text{ A}) + (15.0\text{ }\Omega)(0.313\text{ A})$. $\varepsilon = 52.3\text{ V}$, with the polarity shown in the figure in the problem.
EVALUATE: The unknown battery has a smaller emf than the known one, so the current through it goes against its polarity.

26.33. (a) IDENTIFY: With the switch open, we have a series circuit with two batteries.
SET UP: Take a loop to find the current, then use Ohm's law to find the potential difference between *a* and *b*.
EXECUTE: Taking the loop: $I = (40.0\text{ V})/(175\text{ }\Omega) = 0.229\text{ A}$. The potential difference between *a* and *b* is $V_b - V_a = +15.0\text{ V} - (75.0\text{ }\Omega)(0.229\text{ A}) = -2.14\text{ V}$.
EVALUATE: The minus sign means that *a* is at a higher potential than *b*.
(b) IDENTIFY: With the switch closed, the ammeter part of the circuit divides the original circuit into two circuits. We can apply Kirchhoff's rules to both parts.
SET UP: Take loops around the left and right parts of the circuit, and then look at the current at the junction.
EXECUTE: The left-hand loop gives $I_{100} = (25.0\text{ V})/(100.0\text{ }\Omega) = 0.250\text{ A}$. The right-hand loop gives $I_{75} = (15.0\text{ V})/(75.0\text{ }\Omega) = 0.200\text{ A}$. At the junction just above the switch we have $I_{100} = 0.250\text{ A}$ (in) and $I_{75} = 0.200\text{ A}$ (out), so $I_{\text{A}} = 0.250\text{ A} - 0.200\text{ A} = 0.050\text{ A}$, downward. The voltmeter reads zero because the potential difference across it is zero with the switch closed.
EVALUATE: The ideal ammeter acts like a short circuit, making *a* and *b* at the same potential. Hence the voltmeter reads zero.

26.34. IDENTIFY: We first reduce the parallel combination of the $20.0\text{-}\Omega$ resistors and then apply Kirchhoff's rules.
SET UP: $P = I^2R$ so the power consumption of the $6.0\text{-}\Omega$ resistor allows us to calculate the current through it. Unknown currents I_1, I_2, and I_3 are shown in Figure 26.34. The junction rule says that $I_1 = I_2 + I_3$. In Figure 26.34 the two $20.0\text{-}\Omega$ resistors in parallel have been replaced by their equivalent $(10.0\text{ }\Omega)$.

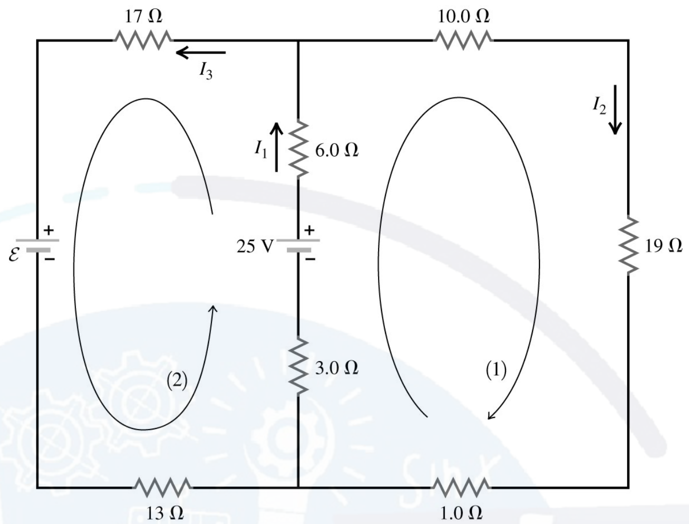


Figure 26.34

EXECUTE: **(a)** $P = I^2R$ gives $I_1 = \sqrt{\dfrac{P}{R}} = \sqrt{\dfrac{24\text{ J/s}}{6.0\ \Omega}} = 2.0\text{ A}$. The loop rule applied to loop (1) gives:

$-(2.0\text{ A})(3.0\ \Omega) - (2.0\text{ A})(6.0\ \Omega) + 25\text{ V} - I_2(10.0\ \Omega + 19.0\ \Omega + 1.0\ \Omega) = 0.$ $I_2 = \dfrac{25\text{ V} - 18\text{ V}}{30.0\ \Omega} = 0.233\text{ A}.$

(b) $I_3 = I_1 - I_2 = 2.0\text{ A} - 0.233\text{ A} = 1.77\text{ A}$. The loop rule applied to loop (2) gives:

$-(2.0\text{ A})(3.0\ \Omega + 6.0\ \Omega) + 25\text{ V} - (1.77\text{ A})(17\ \Omega) - \mathcal{E} - (1.77\text{ A})(13\ \Omega) = 0.$

$\mathcal{E} = 25\text{ V} - 18\text{ V} - 53.1\text{ V} = -46.1\text{ V}$. The emf is 46.1 V.

EVALUATE: Because of the minus sign for the emf, the polarity of the battery is opposite to what is shown in the figure in the problem; the + terminal is adjacent to the 13-Ω resistor.

26.35. **IDENTIFY:** To construct an ammeter, add a shunt resistor in parallel with the galvanometer coil. To construct a voltmeter, add a resistor in series with the galvanometer coil.

SET UP: The full-scale deflection current is $500\ \mu\text{A}$ and the coil resistance is $25.0\ \Omega$.

EXECUTE: **(a)** For a 20-mA ammeter, the two resistances are in parallel and the voltages across each are the same. $V_c = V_s$ gives $I_cR_c = I_sR_s$. $(500\times 10^{-6}\text{ A})(25.0\ \Omega) = (20\times 10^{-3}\text{ A} - 500\times 10^{-6}\text{ A})R_s$ and $R_s = 0.641\ \Omega$.

(b) For a 500-mV voltmeter, the resistances are in series and the current is the same through each:

$V_{ab} = I(R_c + R_s)$ and $R_s = \dfrac{V_{ab}}{I} - R_c = \dfrac{500\times 10^{-3}\text{ V}}{500\times 10^{-6}\text{ A}} - 25.0\ \Omega = 975\ \Omega.$

EVALUATE: The equivalent resistance of the voltmeter is $R_{eq} = R_s + R_c = 1000\ \Omega$. The equivalent resistance of the ammeter is given by $\dfrac{1}{R_{eq}} = \dfrac{1}{R_{sh}} + \dfrac{1}{R_c}$ and $R_{eq} = 0.625\ \Omega$. The voltmeter is a high-resistance device and the ammeter is a low-resistance device.

26.36. **IDENTIFY:** The galvanometer is represented in the circuit as a resistance R_G. Use the junction rule to relate the current through the galvanometer and the current through the shunt resistor. The voltage drop across each parallel path is the same; use this to write an equation for the resistance R.

SET UP: The circuit is sketched in Figure 26.36.

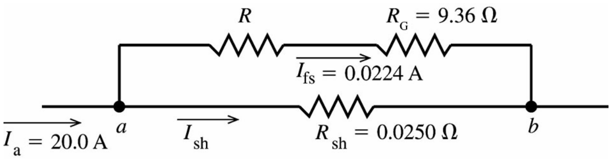


Figure 26.36

We want that $I_a = 20.0\text{ A}$ in the external circuit to produce $I_{fs} = 0.0224\text{ A}$ through the galvanometer coil.

Execute: Applying the junction rule to point *a* gives $I_a - I_{fs} - I_{sh} = 0$.

$I_{sh} = I_a - I_{fs} = 20.0\text{ A} - 0.0224\text{ A} = 19.98\text{ A}$.

The potential difference V_{ab} between points *a* and *b* must be the same for both paths between these two points: $I_{fs}(R + R_G) = I_{sh}R_{sh}$.

$$R = \frac{I_{sh}R_{sh}}{I_{fs}} - R_G = \frac{(19.98\text{ A})(0.0250\,\Omega)}{0.0224\text{ A}} - 9.36\,\Omega = 22.30\,\Omega - 9.36\,\Omega = 12.9\,\Omega.$$

Evaluate: $R_{sh} << R + R_G$; most of the current goes through the shunt. Adding *R* decreases the fraction of the current that goes through R_G.

26.37. Identify: The meter introduces resistance into the circuit, which affects the current through the $5.00\text{-k}\Omega$ resistor and hence the potential drop across it.

Set Up: Use Ohm's law to find the current through the $5.00\text{-k}\Omega$ resistor and then the potential drop across it.

Execute: **(a)** The parallel resistance with the voltmeter is $3.33\text{ k}\Omega$, so the total equivalent resistance across the battery is $9.33\text{ k}\Omega$, giving $I = (50.0\text{ V})/(9.33\text{ k}\Omega) = 5.36\text{ mA}$. Ohm's law gives the potential drop across the $5.00\text{-k}\Omega$ resistor: $V_{5\text{ k}\Omega} = (3.33\text{ k}\Omega)(5.36\text{ mA}) = 17.9\text{ V}$.

(b) The current in the circuit is now $I = (50.0\text{ V})/(11.0\text{ k}\Omega) = 4.55\text{ mA}$.

$V_{5\text{ k}\Omega} = (5.00\text{ k}\Omega)(4.55\text{ mA}) = 22.7\text{ V}$.

(c) % error $= (22.7\text{ V} - 17.9\text{ V})/(22.7\text{ V}) = 0.214 = 21.4\%$. (We carried extra decimal places for accuracy since we had to subtract our answers.)

Evaluate: The presence of the meter made a very large percent error in the reading of the "true" potential across the resistor.

26.38. Identify: The resistance of the galvanometer can alter the resistance in a circuit.

Set Up: The shunt is in parallel with the galvanometer, so we find the parallel resistance of the ammeter. Then use Ohm's law to find the current in the circuit.

Execute: **(a)** The resistance of the ammeter is given by $1/R_A = 1/(1.00\ \Omega) + 1/(25.0\ \Omega)$, so $R_A = 0.962\ \Omega$. The current through the ammeter, and hence the current it measures, is $I = V/R = (25.0\text{ V})/(15.96\ \Omega) = 1.57\text{ A}$.

(b) Now there is no meter in the circuit, so the total resistance is only 15.0 Ω. $I = (25.0\text{ V})/(15.0\ \Omega) = 1.67\text{ A}$.

(c) $(1.67\text{ A} - 1.57\text{ A})/(1.67\text{ A}) = 0.060 = 6.0\%$.

Evaluate: A $1\text{-}\Omega$ shunt can introduce noticeable error in the measurement of an ammeter.

26.39. Identify: The capacitor discharges exponentially through the voltmeter. Since the potential difference across the capacitor is directly proportional to the charge on the plates, the voltage across the plates decreases exponentially with the same time constant as the charge.

Set Up: The reading of the voltmeter obeys the equation $V = V_{0e}{}^{-t/RC}$, where *RC* is the time constant.

Execute: **(a)** Solving for *C* and evaluating the result when $t = 4.00\text{s}$ gives

$$C = \frac{t}{R\ln(V/V_0)} = \frac{4.00\text{ s}}{(3.40\times10^6\ \Omega)\ln\left(\dfrac{12.0\text{ V}}{3.00\text{ V}}\right)} = 8.49\times10^{-7}\text{F}.$$

(b) $\tau = RC = (3.40\times10^6\ \Omega)(8.49\times10^{-7}\text{ F}) = 2.89\text{ s}$.

Evaluate: In most laboratory circuits, time constants are much shorter than this one.

26.40. **IDENTIFY:** When S is closed, charge starts to flow and charge the capacitor until the potential difference across the capacitor is equal to the emf of the battery.

SET UP: $V_R = RI$, $V_C = \varepsilon(1 - e^{-t/RC})$, and $U_C = Q^2/2C$.

EXECUTE: **(a)** Kirchhoff's loop rule gives $V_C + V_R = \varepsilon$, so $I = (\varepsilon - V_C)/R = (36.0 \text{ V} - 8.00 \text{ V})/(120\ \Omega) = 0.2333$ A, which rounds to 0.233 A.

(b) From $V_C = \varepsilon(1 - e^{-t/RC})$, we get $e^{-t/RC} = 1 - V_C/\varepsilon$. Taking logs gives $-t/RC = \ln(1 - V_C/\varepsilon)$. Solving for t gives

$t = -(120\ \Omega)(5.00\ \mu\text{F}) \ln[1 - (8.00 \text{ V})/(36.0 \text{ V})] = 151\ \mu\text{s}$.

(c) $U_C = Q^2/2C$, so $P_C = dU_C/dt = (Q/C)\, dQ/dt = V_C I = (8.00 \text{ V})(0.2333 \text{ A}) = 1.87$ W.

EVALUATE: $P_C + P_R = P_C + I^2R = 1.87 \text{ W} + (0.2333 \text{ A})^2(120\ \Omega) = 8.40$ W. $P_\varepsilon = I\varepsilon = (0.2333 \text{ A})(36.0 \text{ V}) = 8.40$ W. These results for the power agree, as they should by conservation of energy.

26.41. **IDENTIFY:** An uncharged capacitor is placed into a circuit. Apply the loop rule at each time.

SET UP: The voltage across a capacitor is $V_C = q/C$.

EXECUTE: **(a)** At the instant the circuit is completed, there is no voltage across the capacitor, since it has no charge stored.

(b) Since $V_C = 0$, the full battery voltage appears across the resistor $V_R = \varepsilon = 245$ V.

(c) There is no charge on the capacitor.

(d) The current through the resistor is $i = \dfrac{\varepsilon}{R_{\text{total}}} = \dfrac{245 \text{ V}}{7500\Omega} = 0.0327 \text{ A} = 32.7$ mA.

(e) After a long time has passed the full battery voltage is across the capacitor and $i = 0$. The voltage across the capacitor balances the emf: $V_C = 245$ V. The voltage across the resistor is zero. The capacitor's charge is $q = CV_C = (4.60 \times 10^{-6} \text{ F})(245 \text{ V}) = 1.13 \times 10^{-3}$ C. The current in the circuit is zero.

EVALUATE: The current in the circuit starts at 0.0327 A and decays to zero. The charge on the capacitor starts at zero and rises to $q = 1.13 \times 10^{-3}$ C.

26.42. **IDENTIFY:** Once the switch S is closed, current starts to flow and charge the capacitor.

SET UP: $P = IV$, $V_R = RI$, $U_C = Q^2/2C$, $Q = C\varepsilon(1 - e^{-t/RC})$, $(1 - e^{-t/RC})$, and $I = (\varepsilon/R)\, e^{-t/RC}$.

EXECUTE: **(a)** $\varepsilon = V_R + V_C = IR + Q/C = (3.00 \text{ A})(12.0\ \Omega) + (40.0\ \mu\text{C})/(5.00\ \mu\text{F}) = 44.0$ V.

(b) The current is $I = (\varepsilon/R)\, e^{-t/RC}$. The current is 3.00 A when $Q = 40.0\ \mu$C, so

$3.00 \text{ A} = [(44.0\ V)/(12.0\ \Omega)]e^{-t/RC}$. Taking logs and solving for t gives

$-t/RC = \ln(36.0/44.0)$.

$t = -(12.0\ \Omega)(5.00\ \mu\text{F}) \ln(36.0/44.0) = 12.0\ \mu\text{s}$.

(c) (i) The power in the capacitor is $P_C = dU/dt = d(Q^2/2C)/dt = (Q/C)\, dQ/dt = QI/C$, so

$P_C = (40.0\ \mu C)(3.00\ A)/(5.00\ \mu F) = 24.0$ W.

(ii) $P_\varepsilon = I\varepsilon = (3.00 \text{ A})(44.0 \text{ V}) = 132$ W.

EVALUATE: In (c), when $I = 3.00$ A, $P_R = I^2R = (3.00\ A)^2(12.0\ \Omega) = 108\ W$. Therefore $P_R + P_C = 108$ W + 24.0 W = 132 W, which is equal to P_ε, as it should be by energy conservation. In (b), we can use the equation $Q = C\varepsilon(1 - e^{-t/RC})$ to calculate Q when $t = 12.0\ \mu$s; it should be 40.0 μC. We have

$Q = (44.0 \text{ V})(5.00\ \mu\text{F})(1 - e^{-(12.0\ \mu s)/[(12.0\ \Omega)(5.00\ \mu F)]}) = 40.0\ \mu\text{C}$, as expected.

26.43. **IDENTIFY:** The capacitors, which are in parallel, will discharge exponentially through the resistors.

SET UP: Since V is proportional to Q, V must obey the same exponential equation as Q, $V = V_0 e^{-t/RC}$. The current is $I = (V_0/R)\, e^{-t/RC}$.

EXECUTE: **(a)** Solve for time when the potential across each capacitor is 10.0 V:

$$t = -RC \ln(V/V_0) = -(80.0\ \Omega)(35.0\ \mu\text{F}) \ln(10/45) = 4210\ \mu\text{s} = 4.21 \text{ ms}.$$

(b) $I = (V_0/R)\, e^{-t/RC}$. Using the above values, with $V_0 = 45.0$ V, gives $I = 0.125$ A.

EVALUATE: Since the current and the potential both obey the same exponential equation, they are both reduced by the same factor (0.222) in 4.21 ms.

26.44. IDENTIFY: For a charging capacitor $q(t) = C\varepsilon(1 - e^{-t/\tau})$ and $i(t) = \dfrac{\varepsilon}{R}e^{-t/\tau}$.

SET UP: The time constant is $RC = (0.895 \times 10^6\ \Omega)\,(12.4 \times 10^{-6}\ \text{F}) = 11.1\ \text{s}$.

EXECUTE: **(a)** At $t = 0$ s: $q = C\varepsilon(1 - e^{-t/RC}) = 0$.

At $t = 5$ s: $q = C\varepsilon(1 - e^{-t/RC}) = (12.4 \times 10^{-6}\ \text{F})(60.0\ \text{V})(1 - e^{-(5.0\ \text{s})/(11.1\ \text{s})}) = 2.70 \times 10^{-4}\ \text{C}$.

At $t = 10$ s: $q = C\varepsilon(1 - e^{-t/RC}) = (12.4 \times 10^{-6}\ \text{F})(60.0\ \text{V})(1 - e^{-(10.0\ \text{s})/(11.1\ \text{s})}) = 4.42 \times 10^{-4}\ \text{C}$.

At $t = 20$ s: $q = C\varepsilon(1 - e^{-t/RC}) = (12.4 \times 10^{-6}\ \text{F})(60.0\ \text{V})(1 - e^{-(20.0\ \text{s})/(11.1\ \text{s})}) = 6.21 \times 10^{-4}\ \text{C}$.

At $t = 100$ s: $q = C\varepsilon(1 - e^{-t/RC}) = (12.4 \times 10^{-6}\ \text{F})(60.0\ \text{V})(1 - e^{-(100\ \text{s})/(11.1\ \text{s})}) = 7.44 \times 10^{-4}\ \text{C}$.

(b) The current at time *t* is given by: $i = \dfrac{\varepsilon}{R}e^{-t/RC}$.

At $t = 0$ s: $i = \dfrac{60.0\ \text{V}}{8.95 \times 10^5\ \Omega}e^{-0/11.1} = 6.70 \times 10^{-5}\ \text{A}$.

At $t = 5$ s: $i = \dfrac{60.0\ \text{V}}{8.95 \times 10^5\ \Omega}e^{-5/11.1} = 4.27 \times 10^{-5}\ \text{A}$.

At $t = 10$ s: $i = \dfrac{60.0\ \text{V}}{8.95 \times 10^5\ \Omega}e^{-10/11.1} = 2.72 \times 10^{-5}\ \text{A}$.

At $t = 20$ s: $i = \dfrac{60.0\ \text{V}}{8.95 \times 10^5\ \Omega}e^{-20/11.1} = 1.11 \times 10^{-5}\ \text{A}$.

At $t = 100$ s: $i = \dfrac{60.0\ \text{V}}{8.95 \times 10^5\ \Omega}e^{-100/11.1} = 8.20 \times 10^{-9}\ \text{A}$.

(c) The graphs of $q(t)$ and $i(t)$ are given in Figure 26.44a and b.

EVALUATE: The charge on the capacitor increases in time as the current decreases.

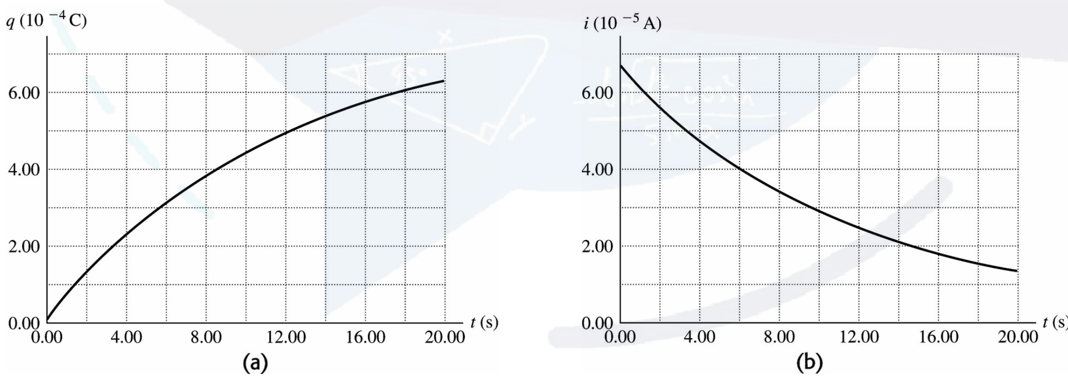


Figure 26.44

26.45. IDENTIFY and SET UP: Apply Kirchhoff's loop rule. The voltage across the resistor depends on the current through it and the voltage across the capacitor depends on the charge on its plates.

EXECUTE: $\varepsilon - V_R - V_C = 0$.

$\varepsilon = 120\ \text{V}$, $V_R = IR = (0.900\ \text{A})(80.0\ \Omega) = 72\ \text{V}$, so $V_C = 48\ \text{V}$.

$Q = CV = (4.00 \times 10^{-6}\ \text{F})(48\ \text{V}) = 192\ \mu\text{C}$.

EVALUATE: The initial charge is zero and the final charge is $C\varepsilon = 480\ \mu\text{C}$. Since current is flowing at the instant considered in the problem the capacitor is still being charged and its charge has not reached its final value.

26.46. IDENTIFY: In $\tau = RC$ use the equivalent capacitance of the two capacitors.

SET UP: For capacitors in series, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$. For capacitors in parallel, $C_{eq} = C_1 + C_2$. Originally, $\tau = RC = 0.780$ s.

EXECUTE: **(a)** The combined capacitance of the two identical capacitors in series is given by $\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$, so $C_{eq} = \frac{C}{2}$. The new time constant is thus $R(C/2) = \frac{0.780 \text{ s}}{2} = 0.390$ s.

(b) With the two capacitors in parallel the new total capacitance is simply $2C$. Thus the time constant is $R(2C) = 2(0.780 \text{ s}) = 1.56$ s.

EVALUATE: The time constant is proportional to C_{eq}. For capacitors in series the capacitance is decreased and for capacitors in parallel the capacitance is increased.

26.47. IDENTIFY: The stored energy is proportional to the square of the charge on the capacitor, so it will obey an exponential equation, but not the same equation as the charge.

SET UP: The energy stored in the capacitor is $U = Q^2/2C$ and the charge on the plates is $Q_0 e^{-t/RC}$. The current is $I = I_0 e^{-t/RC}$.

EXECUTE: $U = Q^2/2C = (Q_0 e^{-t/RC})^2/2C = U_0 e^{-2t/RC}$. When the capacitor has lost 80% of its stored energy, the energy is 20% of the initial energy, which is $U_0/5. U_0/5 = U_0\ e^{-2t/RC}$ gives $t = (RC/2) \ln 5 = (25.0\ \Omega)(4.62 \text{ pF})(\ln 5)/2 = 92.9$ ps.

At this time, the current is $I = I_0\ e^{-t/RC} = (Q_0/RC)\ e^{-t/RC}$, so

$$I = (3.5 \text{ nC})/[(25.0\ \Omega)(4.62 \text{ pF})]\ e^{-(92.9 \text{ ps})/[(25.0\ \Omega)(4.62 \text{ pF})]} = 13.6 \text{ A}.$$

EVALUATE: When the energy is reduced by 80%, neither the current nor the charge are reduced by that percent.

26.48. IDENTIFY: The charge is increasing while the current is decreasing. Both obey exponential equations, but they are not the same equation.

SET UP: The charge obeys the equation $Q = Q_{max}(1 - e^{-t/RC})$, but the equation for the current is $I = I_{max} e^{-t/RC}$.

EXECUTE: When the charge has reached $\frac{1}{4}$ of its maximum value, we have $Q_{max}/4 = Q_{max}(1 - e^{-t/RC})$, which says that the exponential term has the value $e^{-t/RC} = \frac{3}{4}$. The current at this time is $I = I_{max} e^{-t/RC} = I_{max}(3/4) = (3/4)[(10.0 \text{ V})/(12.0\ \Omega)] = 0.625$ A.

EVALUATE: Notice that the current will be $\frac{3}{4}$, not $\frac{1}{4}$, of its maximum value when the charge is $\frac{1}{4}$ of its maximum. Although current and charge both obey exponential equations, the equations have different forms for a charging capacitor.

26.49. IDENTIFY: In both cases, simplify the complicated circuit by eliminating the appropriate circuit elements. The potential across an uncharged capacitor is initially zero, so it behaves like a short circuit. A fully charged capacitor allows no current to flow through it.

(a) SET UP: Just after closing the switch, the uncharged capacitors all behave like short circuits, so any resistors in parallel with them are eliminated from the circuit.

EXECUTE: The equivalent circuit consists of $50\ \Omega$ and $25\ \Omega$ in parallel, with this combination in series with $75\ \Omega$, $15\ \Omega$, and the 100-V battery. The equivalent resistance is $90\ \Omega + 16.7\ \Omega = 106.7\ \Omega$, which gives $I = (100 \text{ V})/(106.7\ \Omega) = 0.937$ A.

(b) SET UP: Long after closing the switch, the capacitors are essentially charged up and behave like open circuits since no charge can flow through them. They effectively eliminate any resistors in series with them since no current can flow through these resistors.

EXECUTE: The equivalent circuit consists of resistances of $75\ \Omega$, $15\ \Omega$, and three $25\text{-}\Omega$ resistors, all in series with the 100-V battery, for a total resistance of $165\ \Omega$. Therefore $I = (100 \text{V})/(165\ \Omega) = 0.606$ A.

EVALUATE: The initial and final behavior of the circuit can be calculated quite easily using simple series-parallel circuit analysis. Intermediate times would require much more difficult calculations!

26.50. IDENTIFY: Both the charge and energy decay exponentially, but not with the same time constant since the energy is proportional to the *square* of the charge.

SET UP: The charge obeys the equation $Q = Q_0 e^{-t/RC}$ but the energy obeys the equation $U = Q^2/2C = (Q_0 e^{-t/RC})^2/2\mathrm{C} = U_0 e^{-2t/RC}$.

EXECUTE: **(a)** The charge is reduced by half: $Q_0/2 = Q_0 e^{-t/RC}$. This gives $t = RC \ln 2 = (225\ \Omega)(12.0\ \mu\text{F})(\ln 2) = 1.871\ \text{ms}$, which rounds to 1.87 ms.

(b) The energy is reduced by half: $U_0/2 = U_0 e^{-2t/RC}$. This gives $t = (RC \ln 2)/2 = (1.871\ \text{ms})/2 = 0.936\ \text{ms}$.

EVALUATE: The energy decreases faster than the charge because it is proportional to the square of the charge.

26.51. IDENTIFY: When the capacitor is fully charged the voltage V across the capacitor equals the battery emf and $Q = CV$. For a charging capacitor, $q = Q(1 - e^{-t/RC})$.

SET UP: $\ln e^x = x$.

EXECUTE: **(a)** $Q = CV = (5.90 \times 10^{-6}\ \text{F})(28.0\ \text{V}) = 1.65 \times 10^{-4}\ \text{C} = 165\ \mu\text{C}$.

(b) $q = Q(1 - e^{-t/RC})$, so $e^{-t/RC} = 1 - \dfrac{q}{Q}$ and $R = \dfrac{-t}{C \ln(1 - q/Q)}$. After

$$t = 3 \times 10^{-3}\ \text{s: } R = \frac{-3 \times 10^{-3}\ \text{s}}{(5.90 \times 10^{-6}\ \text{F})(\ln(1 - 110/165))} = 463\ \Omega.$$

(c) If the charge is to be 99% of final value: $\dfrac{q}{Q} = (1 - e^{-t/RC})$ gives

$$t = -RC \ln(1 - q/Q) = -(463\ \Omega)(5.90 \times 10^{-6}\ \text{F}) \ln(0.01) = 0.0126\ \text{s} = 12.6\ \text{ms}.$$

EVALUATE: The time constant is $\tau = RC = 2.73\ \text{ms}$. The time in part (b) is a bit more than one time constant and the time in part (c) is about 4.6 time constants.

26.52. IDENTIFY: $P = VI = I^2 R$

SET UP: Problem 25.76 says that for 12-gauge wire the maximum safe current is 25 A.

EXECUTE: **(a)** $I = \dfrac{P}{V} = \dfrac{4100\ \text{W}}{240\ \text{V}} = 17.1\ \text{A}$. So we need at least 14-gauge wire (good up to 18 A). 12-gauge is also ok (good up to 25 A).

(b) $P = \dfrac{V^2}{R}$ and $R = \dfrac{V^2}{P} = \dfrac{(240\ \text{V})^2}{4100\ \text{W}} = 14\ \Omega$.

(c) At 11¢ per kWh, for 1 hour the cost is $(11¢/\text{kWh})(1\ \text{h})(4.1\ \text{kW}) = 45¢$.

EVALUATE: The cost to operate the device is proportional to its power consumption.

26.53. IDENTIFY and SET UP: The heater and hair dryer are in parallel so the voltage across each is 120 V and the current through the fuse is the sum of the currents through each appliance. As the power consumed by the dryer increases, the current through it increases. The maximum power setting is the highest one for which the current through the fuse is less than 20 A.

EXECUTE: Find the current through the heater. $P = VI$ so $I = P/V = (1500\ \text{W})/(120\ \text{V}) = 12.5\ \text{A}$. The maximum total current allowed is 20 A, so the current through the dryer must be less than $20\ \text{A} - 12.5\ \text{A} = 7.5\ \text{A}$. The power dissipated by the dryer if the current has this value is $P = VI = (120\ \text{V})(7.5\ \text{A}) = 900\ \text{W}$. For P at this value or larger the circuit breaker trips.

EVALUATE: $P = V^2/R$ and for the dryer V is a constant 120 V. The higher power settings correspond to a smaller resistance R and larger current through the device.

26.54. **IDENTIFY:** We need to do series/parallel reduction to solve this circuit.

SET UP: $P = \dfrac{\varepsilon^2}{R}$, where R is the equivalent resistance of the network. For resistors in series, $R_{eq} = R_1 + R_2$, and for resistors in parallel $1/R_P = 1/R_1 + 1/R_2$.

EXECUTE: $R = \dfrac{\varepsilon^2}{P} = \dfrac{(48.0\text{ V})^2}{295\text{ W}} = 7.810\,\Omega$. $R_{12} = R_1 + R_2 = 8.00\,\Omega$. $R = R_{123} + R_4$.

$R_{123} = R - R_4 = 7.810\,\Omega - 3.00\,\Omega = 4.810\,\Omega$. $\dfrac{1}{R_{12}} + \dfrac{1}{R_3} = \dfrac{1}{R_{123}}$. $\dfrac{1}{R_3} = \dfrac{1}{R_{123}} - \dfrac{1}{R_{12}} = \dfrac{R_{12} - R_{123}}{R_{123}R_{12}}$.

$$R_3 = \frac{R_{123}R_{12}}{R_{12} - R_{123}} = \frac{(4.810\,\Omega)(8.00\,\Omega)}{8.00\,\Omega - 4.810\,\Omega} = 12.1\,\Omega.$$

EVALUATE: The resistance R_3 is greater than R, since the equivalent parallel resistance is less than any of the resistors in parallel.

26.55. **IDENTIFY:** The terminal voltage of the battery depends on the current through it and therefore on the equivalent resistance connected to it. The power delivered to each bulb is $P = I^2R$, where I is the current through it.

SET UP: The terminal voltage of the source is $\varepsilon - Ir$.

EXECUTE: **(a)** The equivalent resistance of the two bulbs is $1.0\,\Omega$. This equivalent resistance is in series with the internal resistance of the source, so the current through the battery is

$I = \dfrac{V}{R_{total}} = \dfrac{8.0\text{ V}}{1.0\,\Omega + 0.80\,\Omega} = 4.4\text{ A}$. and the current through each bulb is 2.2 A. The voltage applied to each bulb is $\varepsilon - Ir = 8.0\text{ V} - (4.4\text{ A})(0.80\,\Omega) = 4.4\text{ V}$. Therefore, $P_{bulb} = I^2R = (2.2\text{ A})^2(2.0\,\Omega) = 9.7\text{ W}$.

(b) If one bulb burns out, then $I = \dfrac{V}{R_{total}} = \dfrac{8.0\text{ V}}{2.0\,\Omega + 0.80\,\Omega} = 2.9\text{ A}$. The current through the remaining bulb is 2.9 A, and $P = I^2R = (2.9\text{ A})^2(2.0\,\Omega) = 16.3\text{ W}$. The remaining bulb is brighter than before, because it is consuming more power.

EVALUATE: In Example 26.2 the internal resistance of the source is negligible and the brightness of the remaining bulb doesn't change when one burns out.

26.56. **IDENTIFY:** Half the current flows through each parallel resistor and the full current flows through the third resistor, that is in series with the parallel combination. Therefore, only the series resistor will be at its maximum power.

SET UP: $P = I^2R$.

EXECUTE: The maximum allowed power is when the total current is the maximum allowed value of $I = \sqrt{P/R} = \sqrt{(48\text{ W})/(2.4\,\Omega)} = 4.47\text{ A}$. Then half the current flows through the parallel resistors and the maximum power is $P_{max} = (I/2)^2R + (I/2)^2R + I^2R = \frac{3}{2}I^2R = \frac{3}{2}(4.47\text{ A})^2(2.4\,\Omega) = 72\text{ W}$.

EVALUATE: If all three resistors were in series or all three were in parallel, then the maximum power would be $3(48\text{ W}) = 144\text{ W}$. For the network in this problem, the maximum power is half this value.

26.57. **(a) IDENTIFY:** Break the circuit between points a and b means no current in the middle branch that contains the 3.00-Ω resistor and the 10.0-V battery. The circuit therefore has a single current path. Find the current, so that potential drops across the resistors can be calculated. Calculate V_{ab} by traveling from a to b, keeping track of the potential changes along the path taken.

SET UP: The circuit is sketched in Figure 26.57a.

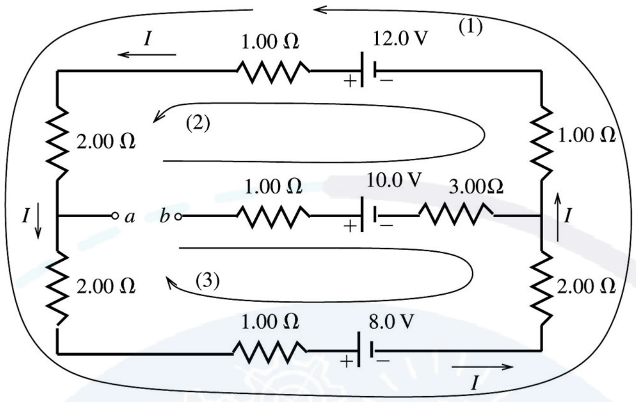


Figure 26.57a

EXECUTE: Apply Kirchhoff's loop rule to loop (1).
$+12.0\text{ V} - I(1.00\,\Omega + 2.00\,\Omega + 2.00\,\Omega + 1.00\,\Omega) - 8.0\text{ V} - I(2.00\,\Omega + 1.00\,\Omega) = 0.$

$$I = \frac{12.0\text{ V} - 8.0\text{ V}}{9.00\,\Omega} = 0.4444\text{ A}.$$

To find V_{ab} start at point b and travel to a, adding up the potential rises and drops. Travel on path (2) shown on the diagram. The $1.00\text{-}\Omega$ and $3.00\text{-}\Omega$ resistors in the middle branch have no current through them and hence no voltage across them. Therefore,
$V_b - 10.0\text{ V} + 12.0\text{ V} - I(1.00\,\Omega + 1.00\,\Omega + 2.00\,\Omega) = V_a$; thus
$V_a - V_b = 2.0\text{ V} - (0.4444\text{ A})(4.00\,\Omega) = +0.22\text{ V}$ (point a is at higher potential).

EVALUATE: As a check on this calculation we also compute V_{ab} by traveling from b to a on path (3).
$V_b - 10.0\text{ V} + 8.0\text{ V} + I(2.00\,\Omega + 1.00\,\Omega + 2.00\,\Omega) = V_a$.
$V_{ab} = -2.00\text{ V} + (0.4444\text{ A})(5.00\,\Omega) = +0.22\text{ V}$, which checks.

(b) IDENTIFY and SET UP: With points a and b connected by a wire there are three current branches, as shown in Figure 26.57b.

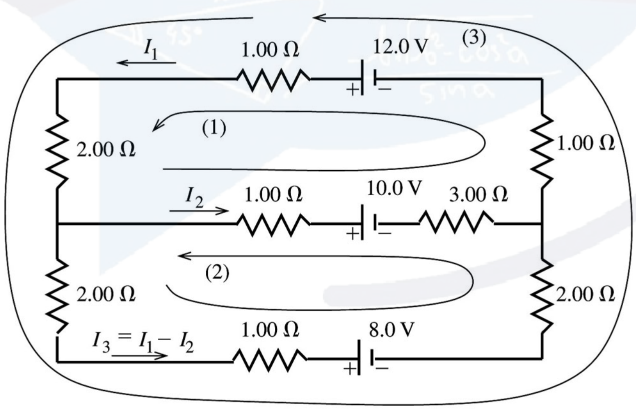


Figure 26.57b

The junction rule has been used to write the third current (in the 8.0-V battery) in terms of the other currents. Apply the loop rule to loops (1) and (2) to obtain two equations for the two unknowns I_1 and I_2.

EXECUTE: Apply the loop rule to loop (1).
$12.0\text{ V} - I_1(1.00\,\Omega) - I_1(2.00\,\Omega) - I_2(1.00\,\Omega) - 10.0\text{ V} - I_2(3.00\,\Omega) - I_1(1.00\,\Omega) = 0$
$2.0\text{ V} - I_1(4.00\,\Omega) - I_2(4.00\,\Omega) = 0$
$(2.00\,\Omega)I_1 + (2.00\,\Omega)I_2 = 1.0\text{ V}$ eq. (1)

Apply the loop rule to loop (2).

$-(I_1 - I_2)(2.00\,\Omega) - (I_1 - I_2)(1.00\,\Omega) - 8.0\text{ V} - (I_1 - I_2)(2.00\,\Omega) + I_2(3.00\,\Omega) + 10.0\text{ V} + I_2(1.00\,\Omega) = 0$

$2.0\text{ V} - (5.00\,\Omega)I_1 + (9.00\,\Omega)I_2 = 0$ eq. (2)

Solve eq. (1) for I_2 and use this to replace I_2 in eq. (2).

$I_2 = 0.50\text{ A} - I_1$

$2.0\text{ V} - (5.00\,\Omega)I_1 + (9.00\,\Omega)(0.50\text{ A} - I_1) = 0$

$(14.0\,\Omega)I_1 = 6.50\text{ V}$ so $I_1 = (6.50\text{ V})/(14.0\,\Omega) = 0.464\text{ A}$

$I_2 = 0.500\text{ A} - 0.464\text{ A} = 0.036\text{ A}.$

The current in the 12.0-V battery is $I_1 = 0.464\text{ A}$

EVALUATE: We can apply the loop rule to loop (3) as a check.

$+12.0\text{ V} - I_1(1.00\,\Omega + 2.00\,\Omega + 1.00\,\Omega) - (I_1 - I_2)(2.00\,\Omega + 1.00\,\Omega + 2.00\,\Omega) - 8.0\text{ V} = 4.0\text{ V} - 1.86\text{ V} - 2.14\text{ V} = 0,$ as it should.

26.58. **IDENTIFY:** Heat, which is generated in the resistor, melts the ice.

SET UP: Find the rate at which heat is generated in the $20.0\text{-}\Omega$ resistor using $P = V^2/R$. Then use the heat of fusion of ice to find the rate at which the ice melts. The heat dH to melt a mass of ice dm is $dH = L_F\ dm$, where L_F is the latent heat of fusion. The rate at which heat enters the ice, dH/dt, is the power P in the resistor, so $P = L_F\ dm/dt$. Therefore the rate of melting of the ice is $dm/dt = P/L_F$.

EXECUTE: The equivalent resistance of the parallel branch is $5.00\ \Omega$, so the total resistance in the circuit is $35.0\,\Omega$. Therefore the total current in the circuit is $I_{\text{Total}} = (45.0\text{ V})/(35.0\ \Omega) = 1.286\text{ A}$. The potential difference across the $20.0\text{-}\Omega$ resistor in the ice is the same as the potential difference across the parallel branch: $V_{\text{ice}} = I_{\text{Total}}R_p = (1.286\text{ A})(5.00\ \Omega) = 6.429\text{ V}$. The rate of heating of the ice is $P_{\text{ice}} = V_{\text{ice}}^2/R = (6.429\text{ V})^2/(20.0\ \Omega) = 2.066\text{ W}$. This power goes into to heat to melt the ice, so

$$dm/dt = P/L_F = (2.066\text{ W})/(3.34\times10^5\text{ J/kg}) = 6.19\times10^{-6}\text{ kg/s} = 6.19\times10^{-3}\text{ g/s}.$$

EVALUATE: The melt rate is about 6 mg/s, which is not much. It would take 1000 s to melt just 6 g of ice.

25.59. **IDENTIFY:** Apply Kirchhoff's junction rule to express the currents through the $5.00\text{-}\Omega$ and $8.00\text{-}\Omega$ resistors in terms of $I_1, I_2,$ and I_3. Apply the loop rule to three loops to get three equations in the three unknown currents.

SET UP: The circuit is sketched in Figure 26.59.

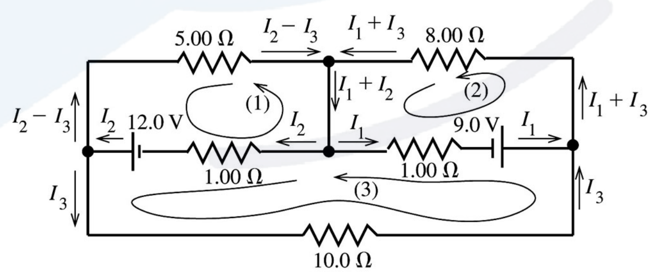


Figure 26.59

The current in each branch has been written in terms of $I_1, I_2,$ and I_3 such that the junction rule is satisfied at each junction point.

EXECUTE: Apply the loop rule to loop (1).

$-12.0\text{ V} + I_2(1.00\,\Omega) + (I_2 - I_3)(5.00\,\Omega) = 0$

$I_2(6.00\,\Omega) - I_3(5.00\,\Omega) = 12.0\text{ V}$ eq. (1)

Apply the loop rule to loop (2).

$$-I_1(1.00\ \Omega)+9.00\ \text{V}-(I_1+I_3)(8.00\ \Omega)=0$$

$$I_1(9.00\ \Omega)+I_3(8.00\ \Omega)=9.00\ \text{V} \qquad \text{eq. (2)}$$

Apply the loop rule to loop (3).

$$-I_3(10.0\ \Omega)-9.00\ \text{V}+I_1(1.00\ \Omega)-I_2(1.00\ \Omega)+12.0\ \text{V}=0$$

$$-I_1(1.00\ \Omega)+I_2(1.00\ \Omega)+I_3(10.0\ \Omega)=3.00\ \text{V} \qquad \text{eq. (3)}$$

Eq. (1) gives $I_2=2.00\ \text{A}+\frac{5}{6}I_3$; eq. (2) gives $I_1=1.00\ \text{A}-\frac{8}{9}I_3$.

Using these results in eq. (3) gives

$$-(1.00\ \text{A}-\tfrac{8}{9}I_3)(1.00\ \Omega)+(2.00\ \text{A}+\tfrac{5}{6}I_3)(1.00\ \Omega)+I_3(10.0\ \Omega)=3.00\ \text{V}.$$

$$(\tfrac{16+15+180}{18})I_3=2.00\ \text{A};\ I_3=\tfrac{18}{211}(2.00\ \text{A})=0.171\ \text{A}.$$

Then $I_2=2.00\ \text{A}+\frac{5}{6}I_3=2.00\ \text{A}+\frac{5}{6}(0.171\ \text{A})=2.14\ \text{A}$ and

$I_1=1.00\ \text{A}-\frac{8}{9}I_3=1.00\ \text{A}-\frac{8}{9}(0.171\ \text{A})=0.848\ \text{A}.$

EVALUATE: We could check that the loop rule is satisfied for a loop that goes through the $5.00\text{-}\Omega$, $8.00\text{-}\Omega$ and $10.0\text{-}\Omega$ resistors. Going around the loop clockwise: $-(I_2-I_3)(5.00\ \Omega)+(I_1+I_3)(8.00\ \Omega)+I_3(10.0\ \Omega)=-9.85\ \text{V}+8.15\ \text{V}+1.71\ \text{V}$, which does equal zero, apart from rounding.

26.60. IDENTIFY: Apply the junction rule and the loop rule to the circuit.

SET UP: Because of the polarity of each emf, the current in the $7.00\text{-}\Omega$ resistor must be in the direction shown in Figure 26.60a. Let I be the current in the 24.0-V battery.

EXECUTE: The loop rule applied to loop (1) gives: $+24.0\ \text{V}-(1.80\ \text{A})(7.00\ \Omega)-I(3.00\ \Omega)=0.$ $I=3.80\ \text{A}.$ The junction rule then says that the current in the middle branch is 2.00 A, as shown in Figure 26.64b. The loop rule applied to loop (2) gives: $+\mathcal{E}-(1.80\ \text{A})(7.00\ \Omega)+(2.00\ \text{A})(2.00\ \Omega)=0$ and $\mathcal{E}=8.6\ \text{V}.$

EVALUATE: We can check our results by applying the loop rule to loop (3) in Figure 26.60b: $+24.0\ \text{V}-\mathcal{E}-(2.00\ \text{A})(2.00\ \Omega)-(3.80\ \text{A})(3.00\ \Omega)=0$ and $\mathcal{E}=24.0\ \text{V}-4.0\ \text{V}-11.4\ \text{V}=8.6\ \text{V}$, which agrees with our result from loop (2).

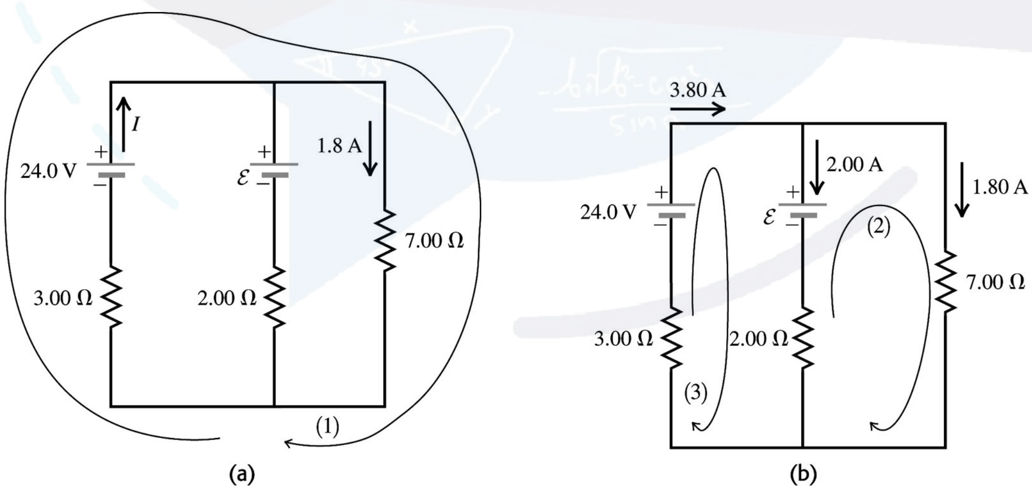
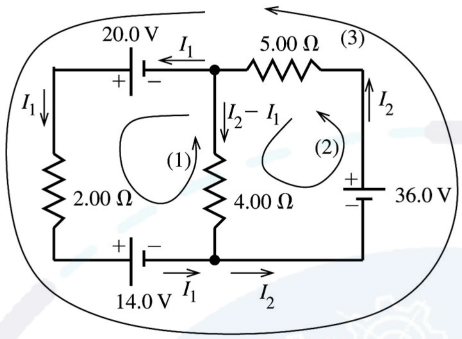


Figure 26.60

26.61. IDENTIFY and SET UP: The circuit is sketched in Figure 26.61.



Two unknown currents I_1 (through the 2.00-Ω resistor) and I_2 (through the 5.00-Ω resistor) are labeled on the circuit diagram. The current through the 4.00-Ω resistor has been written as $I_2 - I_1$ using the junction rule.

Figure 26.61

Apply Kirchhoff's loop rule to loops (1) and (2) to get two equations for the unknown currents, I_1 and I_2. Loop (3) can then be used to check the results.

EXECUTE: Loop (1): $+20.0\text{ V} - I_1(2.00\,\Omega) - 14.0\text{ V} + (I_2 - I_1)(4.00\,\Omega) = 0$

$6.00I_1 - 4.00I_2 = 6.00\text{ A}$

$3.00I_1 - 2.00I_2 = 3.00\text{ A}$ eq. (1)

Loop (2): $+36.0\text{ V} - I_2(5.00\,\Omega) - (I_2 - I_1)(4.00\,\Omega) = 0$

$-4.00I_1 + 9.00I_2 = 36.0\text{ A}$ eq. (2)

Solving eq. (1) for I_1 gives $I_1 = 1.00\text{ A} + \frac{2}{3}I_2$.

Using this in eq. (2) gives $-4.00(1.00\text{ A} + \frac{2}{3}I_2) + 9.00I_2 = 36.0\text{ A}$.

$\left(-\frac{8}{3} + 9.00\right)I_2 = 40.0\text{ A}$ and $I_2 = 6.32\text{ A}$.

Then $I_1 = 1.00\text{ A} + \frac{2}{3}I_2 = 1.00\text{ A} + \frac{2}{3}(6.32\text{ A}) = 5.21\text{ A}$.

In summary then

Current through the 2.00-Ω resistor: $I_1 = 5.21\text{ A}$.

Current through the 5.00-Ω resistor: $I_2 = 6.32\text{ A}$.

Current through the 4.00-Ω resistor: $I_2 - I_1 = 6.32\text{ A} - 5.21\text{ A} = 1.11\text{ A}$.

EVALUATE: Use loop (3) to check. $+20.0\text{ V} - I_1(2.00\,\Omega) - 14.0\text{ V} + 36.0\text{ V} - I_2(5.00\,\Omega) = 0$.

$(5.21\text{ A})(2.00\,\Omega) + (6.32\text{ A})(5.00\,\Omega) = 42.0\text{ V}$.

$10.4\text{ V} + 31.6\text{ V} = 42.0\text{ V}$, so the loop rule is satisfied for this loop.

26.62. IDENTIFY: Apply the loop and junction rules.

SET UP: Use the currents as defined on the circuit diagram in Figure 26.62 and obtain three equations to solve for the currents.

EXECUTE: (a) Left loop: $14 - I_1 - 2(I_1 - I_2) = 0$ and $3I_1 - 2I_2 = 14$.

Top loop: $-2(I - I_1) + I_2 + I_1 = 0$ and $-2I + 3I_1 + I_2 = 0$.

Bottom loop: $-(I - I_1 + I_2) + 2(I_1 - I_2) - I_2 = 0$ and $-I + 3I_1 - 4I_2 = 0$.

Solving these equations for the currents we find: $I = I_{\text{battery}} = 10.0\text{ A}$; $I_1 = I_{R_1} = 6.0\text{ A}$; $I_2 = I_{R_3} = 2.0\text{ A}$.

So the other currents are: $I_{R_2} = I - I_1 = 4.0\text{ A}$; $I_{R_4} = I_1 - I_2 = 4.0\text{ A}$; $I_{R_5} = I - I_1 + I_2 = 6.0\text{ A}$.

(b) $R_{\text{eq}} = \dfrac{V}{I} = \dfrac{14.0\text{ V}}{10.0\text{ A}} = 1.40\,\Omega$.

EVALUATE: It isn't possible to simplify the resistor network using the rules for resistors in series and parallel. But the equivalent resistance is still defined by $V = IR_{\text{eq}}$.

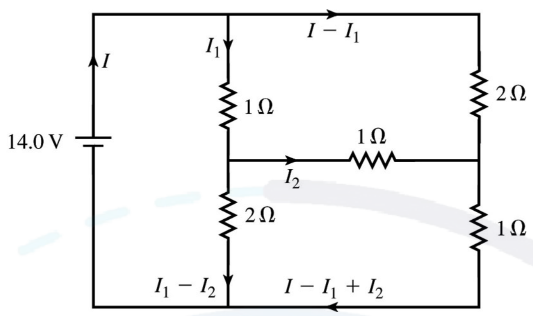


Figure 26.62

26.63. **IDENTIFY:** Simplify the resistor networks as much as possible using the rule for series and parallel combinations of resistors. Then apply Kirchhoff's laws.

SET UP: First do the series/parallel reduction. This gives the circuit in Figure 26.63. The rate at which the $10.0\text{-}\Omega$ resistor generates thermal energy is $P = I^2R$.

EXECUTE: **(a)** Apply Kirchhoff's laws and solve for ε. $\Delta V_{\text{adefa}} = 0$: $-(20\ \Omega)(2\ \text{A}) - 5\ \text{V} - (20\ \Omega)I_2 = 0$. This gives $I_2 = -2.25$ A. Then $I_1 + I_2 = 2$ A gives $I_1 = 2\ \text{A} - (-2.25\ \text{A}) = 4.25$ A. $\Delta V_{\text{abcdefa}} = 0$: $(15\ \Omega)(4.25\ \text{A}) + \varepsilon - (20\ \Omega)(-2.25\ \text{A}) = 0$. This gives $\varepsilon = -109$ V. Since ε is calculated to be negative, its polarity should be reversed.

(b) The parallel network that contains the $10.0\text{-}\Omega$ resistor in one branch has an equivalent resistance of $10\ \Omega$. The voltage across each branch of the parallel network is $V_{\text{par}} = RI = (10\ \Omega)(2\text{A}) = 20$ V. The current in the upper branch is $I = \dfrac{V}{R} = \dfrac{20\ \text{V}}{30\ \Omega} = \dfrac{2}{3}$ A. $Pt = E$, so $I^2\,Rt = E$, where $E = 60.0$ J. $\left(\tfrac{2}{3}\text{A}\right)^2(10\ \Omega)t = 60$ J, and $t = 13.5$ s.

EVALUATE: For the $10.0\text{-}\Omega$ resistor, $P = I^2R = 4.44$ W. The total rate at which electrical energy is inputted to the circuit in the emf is $(5.0\ \text{V})(2.0\ \text{A}) + (109\ \text{V})(4.25\ \text{A}) = 473$ J. Only a small fraction of the energy is dissipated in the $10.0\text{-}\Omega$ resistor.

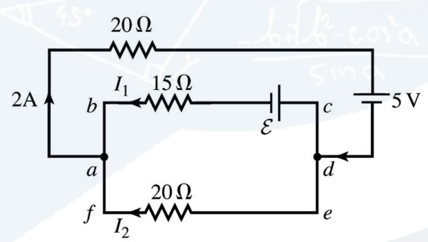


Figure 26.63

26.64. **IDENTIFY:** The resistor R_2 can vary between $3.00\ \Omega$ and $24.0\ \Omega$. R_2 is in parallel with R_1, so as R_2 is changed it affects the current in R_1 and hence the power dissipated in R_1. Ohm's law and Kirchhoff's rules apply.

SET UP: $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots$, $V_R = IR$, $P_R = I^2R$.

EXECUTE: $P_1 = V_1^2/R_1$, so P_1 is largest when V_1 is largest. By Kirchhoff's loop rule, $\varepsilon - V_1 - V_3 = 0$, so $V_1 = \varepsilon - V_3$, which means that V_1 is largest when V_3 is smallest.

$V_3 = IR_3 = \varepsilon/(R_{eq} + R_3)$, where R_{eq} is the equivalent resistance of the R_1-R_2 combination. Since they are in parallel, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$, which gives $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$. The smallest V_3 is for the smallest I, which occurs for the largest $R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1}{\frac{R_1}{R_2} + 1}$.

As we can see, the largest R_{eq} occurs when R_2 is largest, which is $R_2 = 24.0\ \Omega$.
The equivalent parallel resistance is then

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = (6.00\ \Omega)(24.0\ \Omega)/(6.00\ \Omega + 24.0\ \Omega) = 4.80\ \Omega.$$

The current I is then
$I = \varepsilon/(R_{eq} + R_3) = (24.0\text{ V})/(4.80\ \Omega + 12.0\ \Omega) = 1.429\text{ A}$.
$V_3 = IR_3 = (1.429\text{ A})(12.0\ \Omega) = 17.148\text{ V}$.
The potential difference across R_1 is
$V_1 = \varepsilon - V_3 = 24.0\text{ V} - 17.148\text{ V} = 6.852\text{ V}$.
The power dissipated in R_1 is
$P_1 = V_1^2/R_1 = (6.852\text{ V})^2/(6.00\ \Omega) = 7.83\text{ W}$.

EVALUATE: Since all the circuit elements except for R_2 are fixed, varying R_2 affects the current in the circuit as well as the current through R_1.

26.65. **IDENTIFY** and **SET UP:** Simplify the circuit by replacing the parallel networks of resistors by their equivalents. In this simplified circuit apply the loop and junction rules to find the current in each branch.
EXECUTE: The 20.0-Ω and 30.0-Ω resistors are in parallel and have equivalent resistance 12.0 Ω. The two resistors R are in parallel and have equivalent resistance $R/2$. The circuit is equivalent to the circuit sketched in Figure 26.65.

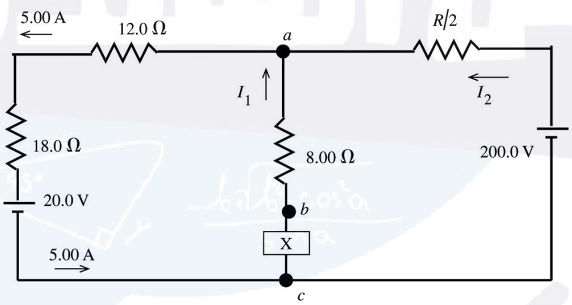


Figure 26.65

(a) Calculate V_{ca} by traveling along the branch that contains the 20.0-V battery, since we know the current in that branch.

$V_a - (5.00\text{ A})(12.0\ \Omega) - (5.00\text{ A})(18.0\ \Omega) - 20.0\text{ V} = V_c.$

$V_a - V_c = 20.0\text{ V} + 90.0\text{ V} + 60.0\text{ V} = 170.0\text{ V}.$

$V_b - V_a = V_{ab} = 16.0\text{ V}.$

$X - V_{ba} = 170.0\text{ V}$ so $X = 186.0\text{ V}$, with the upper terminal +.

(b) $I_1 = (16.0\text{ V})/(8.0\ \Omega) = 2.00\text{ A}.$

The junction rule applied to point a gives $I_2 + I_1 = 5.00\text{ A}$, so $I_2 = 3.00\text{ A}$. The current through the 200.0-V battery is in the direction from the – to the + terminal, as shown in the diagram.

(c) $200.0\text{ V} - I_2(R/2) = 170.0\text{ V}.$

$(3.00\text{ A})(R/2) = 30.0\text{ V}$ so $R = 20.0\ \Omega.$

EVALUATE: We can check the loop rule by going clockwise around the outer circuit loop. This gives $+20.0\text{ V}+(5.00\text{ A})(18.0\,\Omega+12.0\,\Omega)+(3.00\text{ A})(10.0\,\Omega)-200.0\text{ V}=20.0\text{ V}+150.0\text{ V}+30.0\text{ V}-200.0\text{ V}$, which does equal zero.

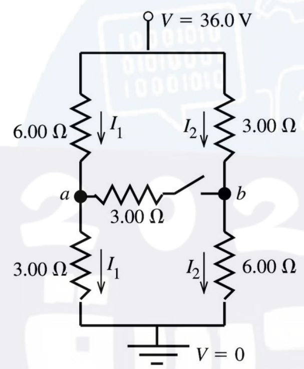
26.66. IDENTIFY: The current through the 40.0-Ω resistor equals the current through the emf, and the current through each of the other resistors is less than or equal to this current. So, set $P_{40}=2.00\text{ W}$, and use this to solve for the current I through the emf. If $P_{40}=2.00\text{ W}$, then P for each of the other resistors is less than 2.00 W.

SET UP: Use the equivalent resistance for series and parallel combinations to simplify the circuit.

EXECUTE: $I^2R=P$ gives $I^2(40\,\Omega)=2.00\text{ W}$, and $I=0.2236\text{ A}$. Now use series/parallel reduction to simplify the circuit. The upper parallel branch is $6.38\,\Omega$ and the lower one is $25\,\Omega$. The series sum is now $126\,\Omega$. Ohm's law gives $\varepsilon=(126\,\Omega)(0.2236\text{ A})=28.2\text{ V}$.

EVALUATE: The power input from the emf is $\varepsilon I=6.30\text{ W}$, so nearly one-third of the total power is dissipated in the 40.0-Ω resistor.

26.67. (a) IDENTIFY and SET UP: The circuit is sketched in Figure 26.67a.



With the switch open there is no current through it and there are only the two currents I_1 and I_2 indicated in the sketch.

Figure 26.67a

The potential drop across each parallel branch is 36.0 V. Use this fact to calculate I_1 and I_2. Then travel from point *a* to point *b* and keep track of the potential rises and drops in order to calculate V_{ab}.

EXECUTE: $-I_1(6.00\,\Omega+3.00\,\Omega)+36.0\text{ V}=0.$

$$I_1=\frac{36.0\text{ V}}{6.00\,\Omega+3.00\,\Omega}=4.00\text{ A}.$$

$-I_2(3.00\,\Omega+6.00\,\Omega)+36.0\text{ V}=0.$

$$I_2=\frac{36.0\text{ V}}{3.00\,\Omega+6.00\,\Omega}=4.00\text{ A}.$$

To calculate $V_{ab}=V_a-V_b$ start at point *b* and travel to point *a*, adding up all the potential rises and drops along the way. We can do this by going from *b* up through the 3.00-Ω resistor:

$V_b+I_2(3.00\,\Omega)-I_1(6.00\,\Omega)=V_a.$

$V_a-V_b=(4.00\text{ A})(3.00\,\Omega)-(4.00\text{ A})(6.00\,\Omega)=12.0\text{ V}-24.0\text{ V}=-12.0\text{ V}.$

$V_{ab}=-12.0\text{ V}$ (point *a* is 12.0 V lower in potential than point *b*).

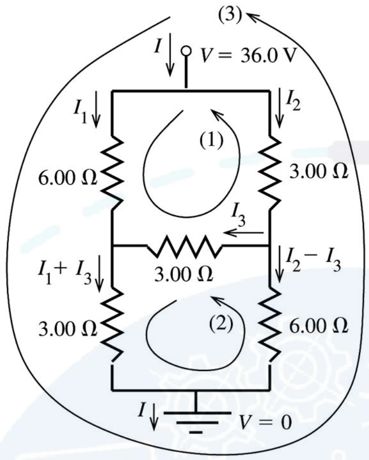
EVALUATE: Alternatively, we can go from point *b* down through the 6.00-Ω resistor.

$V_b-I_2(6.00\,\Omega)+I_1(3.00\,\Omega)=V_a.$

$V_a-V_b=-(4.00\text{ A})(6.00\,\Omega)+(4.00\text{ A})(3.00\,\Omega)=-24.0\text{ V}+12.0\text{ V}=-12.0\text{ V}$, which checks.

(b) IDENTIFY: Now there are multiple current paths, as shown in Figure 26.67b. Use the junction rule to write the current in each branch in terms of three unknown currents I_1, I_2, and I_3. Apply the loop rule to three loops to get three equations for the three unknowns. The target variable is I_3, the current through the switch. R_{eq} is calculated from $V=IR_{eq}$, where I is the total current that passes through the network.

SET UP:



The three unknown currents I_1, I_2, and I_3 are labeled on Figure 26.67b.

Figure 26.67b

EXECUTE: Apply the loop rule to loops (1), (2) and (3).

<u>Loop (1)</u>: $-I_1(6.00\ \Omega) + I_3(3.00\ \Omega) + I_2(3.00\ \Omega) = 0$

$I_2 = 2I_1 - I_3$ eq. (1)

<u>Loop (2)</u>: $-(I_1 + I_3)(3.00\ \Omega) + (I_2 - I_3)(6.00\ \Omega) - I_3(3.00\ \Omega) = 0$

$6I_2 - 12I_3 - 3I_1 = 0$ so $2I_2 - 4I_3 - I_1 = 0$

Use eq (1) to replace I_2:

$4I_1 - 2I_3 - 4I_3 - I_1 = 0$

$3I_1 = 6I_3$ and $I_1 = 2I_3$ eq. (2)

<u>Loop (3)</u>: This loop is completed through the battery (not shown), in the direction from the $-$ to the $+$ terminal.

$-I_1(6.00\ \Omega) - (I_1 + I_3)(3.00\ \Omega) + 36.0\text{ V} = 0$

$9I_1 + 3I_3 = 36.0\text{ A}$ and $3I_1 + I_3 = 12.0\text{ A}$ eq. (3)

Use eq. (2) in eq. (3) to replace I_1:

$3(2I_3) + I_3 = 12.0\text{ A}$

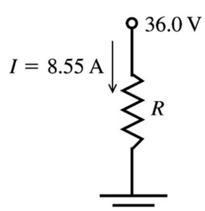
$I_3 = 12.0\text{ A}/7 = 1.71\text{ A}$

$I_1 = 2I_3 = 3.42\text{ A}$

$I_2 = 2I_1 - I_3 = 2(3.42\text{ A}) - 1.71\text{ A} = 5.13\text{ A}$

The current through the switch is $I_3 = 1.71\text{ A}$.

(c) SET UP and **EXECUTE:** From the results in part (a) the current through the battery is $I = I_1 + I_2 = 3.42\text{ A} + 5.13\text{ A} = 8.55\text{ A}$. The equivalent circuit is a single resistor that produces the same current through the 36.0-V battery, as shown in Figure 26.67c.



$-IR + 36.0\text{ V} = 0.$

$$R = \frac{36.0\text{ V}}{I} = \frac{36.0\text{ V}}{8.55\text{ A}} = 4.21\ \Omega.$$

Figure 26.67c

EVALUATE: With the switch open (part a), point b is at higher potential than point a, so when the switch is closed the current flows in the direction from b to a. With the switch closed the circuit cannot be simplified using series and parallel combinations but there is still an equivalent resistance that represents the network.

26.68. IDENTIFY: $P_{\text{tot}} = \dfrac{V^2}{R_{\text{eq}}}$.

SET UP: Let R be the resistance of each resistor.

EXECUTE: When the resistors are in series, $R_{\text{eq}} = 3R$ and $P_s = \dfrac{V^2}{3R}$. When the resistors are in parallel, $R_{\text{eq}} = R/3$. $P_p = \dfrac{V^2}{R/3} = 3\dfrac{V^2}{R} = 9P_s = 9(45.0\text{ W}) = 405\text{ W}$.

EVALUATE: In parallel, the voltage across each resistor is the full applied voltage V. In series, the voltage across each resistor is $V/3$ and each resistor dissipates less power.

26.69. IDENTIFY and SET UP: For part (a) use that the full emf is across each resistor. In part (b), calculate the power dissipated by the equivalent resistance, and in this expression express R_1 and R_2 in terms of P_1, P_2, and ε.

EXECUTE: $P_1 = \varepsilon^2/R_1$ so $R_1 = \varepsilon^2/P_1$.

$P_2 = \varepsilon^2/R_2$ so $R_2 = \varepsilon^2/P_2$.

(a) When the resistors are connected in parallel to the emf, the voltage across each resistor is ε and the power dissipated by each resistor is the same as if only the one resistor were connected. $P_{\text{tot}} = P_1 + P_2$.

(b) When the resistors are connected in series the equivalent resistance is $R_{\text{eq}} = R_1 + R_2$.

$$P_{\text{tot}} = \frac{\varepsilon^2}{R_1 + R_2} = \frac{\varepsilon^2}{\varepsilon^2/P_1 + \varepsilon^2/P_2} = \frac{P_1 P_2}{P_1 + P_2}.$$

EVALUATE: The result in part (b) can be written as $\dfrac{1}{P_{\text{tot}}} = \dfrac{1}{P_1} + \dfrac{1}{P_2}$. Our results are that for parallel the powers add and that for series the reciprocals of the power add. This is opposite the result for combining resistance. Since $P = \varepsilon^2/R$ tells us that P is proportional to $1/R$, this makes sense.

26.70. IDENTIFY and SET UP: Just after the switch is closed the charge on the capacitor is zero, the voltage across the capacitor is zero and the capacitor can be replaced by a wire in analyzing the circuit. After a long time the current to the capacitor is zero, so the current through R_3 is zero. After a long time the capacitor can be replaced by a break in the circuit.

EXECUTE: **(a)** Ignoring the capacitor for the moment, the equivalent resistance of the two parallel resistors is $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{6.00\ \Omega} + \dfrac{1}{3.00\ \Omega} = \dfrac{3}{6.00\ \Omega}$; $R_{\text{eq}} = 2.00\ \Omega$. In the absence of the capacitor, the total current in the circuit (the current through the 8.00-Ω resistor) would be $i = \dfrac{\varepsilon}{R} = \dfrac{42.0\text{ V}}{8.00\ \Omega + 2.00\ \Omega} = 4.20\text{ A}$, of which 2/3, or 2.80 A, would go through the 3.00-Ω resistor and 1/3, or 1.40 A, would go through the 6.00-Ω resistor. Since the current through the capacitor is given by $i = \dfrac{V}{R}e^{-t/RC}$, at the instant $t = 0$ the circuit behaves as through the capacitor were not present, so the currents through the various resistors are as calculated above.

(b) Once the capacitor is fully charged, no current flows through that part of the circuit. The 8.00-Ω and the 6.00-Ω resistors are now in series, and the current through them is $i = \varepsilon/R = (42.0\text{ V})/(8.00\ \Omega + 6.00\ \Omega) = 3.00\text{ A}$. The voltage drop across both the 6.00-Ω resistor and the capacitor is thus $V = iR = (3.00\text{ A})(6.00\ \Omega) = 18.0\text{ V}$. (There is no current through the 3.00-Ω resistor and so no voltage drop across it.) The charge on the capacitor is $Q = CV = (4.00 \times 10^{-6}\text{ F})(18.0\text{ V}) = 7.2 \times 10^{-5}\text{ C}$.

EVALUATE: The equivalent resistance of R_2 and R_3 in parallel is less than R_3, so initially the current through R_1 is larger than its value after a long time has elapsed.

26.71. **IDENTIFY:** An initially uncharged capacitor is charged up by an emf source. The current in the circuit and the charge on the capacitor both obey exponential equations.

SET UP: $U_C = \dfrac{q^2}{2C}$, $P_R = i^2R$, $q = Q_\text{f}(1-e^{-t/RC})$, and $i = I_0 e^{-t/RC}$.

EXECUTE: **(a)** Initially, $q = 0$ so $V_R = \varepsilon$ and $I = \dfrac{\varepsilon}{R} = \dfrac{90.0\text{ V}}{6.00\times 10^3\ \Omega} = 0.0150\text{ A}$. $P_R = I^2R = 1.35\text{ W}$.

(b) $U_C = \dfrac{q^2}{2C}$. $P_C = \dfrac{dU_C}{dt} = \dfrac{qi}{C}$. $P_R = i^2R$. $P_C = P_R$ gives $\dfrac{qi}{C} = i^2R$. $\dfrac{q}{RC} = i$.

$q = Q_\text{f}(1-e^{-t/RC}) = \varepsilon C(1-e^{-t/RC})$. $i = I_0 e^{-t/RC} = \dfrac{\varepsilon}{R}e^{-t/RC}$. $i = \dfrac{q}{RC}$ gives

$\dfrac{\varepsilon}{R}e^{-t/RC} = \dfrac{\varepsilon C}{RC}(1-e^{-t/RC})$. $e^{-t/RC} = 1-e^{-t/RC}$ and $e^{t/RC} = 2$.

$t = RC\ln 2 = (6.00\times 10^3\ \Omega)(2.00\times 10^{-6}\text{ F})\ln 2 = 8.31\times 10^{-3}\text{ s} = 8.31\text{ ms}$.

(c) $i = \dfrac{\varepsilon}{R}e^{-t/RC} = \dfrac{90.0\text{ V}}{6.00\times 10^3\ \Omega}e^{-(8.318\times 10^{-3}\text{ s})/[(6.00\times 10^3\ \Omega)(2.00\times 10^{-6}\text{ F})]} = 7.50\times 10^{-3}\text{ A}$.

$P_R = i^2R = (7.50\times 10^{-3}\text{ A})^2(6.00\times 10^3\ \Omega) = 0.337\text{ W}$.

EVALUATE: Initially energy is dissipated in the resistor at a higher rate because the current is high, but as time goes by the current deceases, as does the power dissipated in the resistor.

26.72. **IDENTIFY** and **SET UP:** $P_R = i^2R$, $\varepsilon - iR - \dfrac{q}{C} = 0$, and $U_C = \dfrac{q^2}{2C}$.

EXECUTE: $P_R = i^2R$ so $i = \sqrt{\dfrac{P_R}{R}} = \sqrt{\dfrac{300\text{ W}}{5.00\ \Omega}} = 7.746\text{ A}$. $\varepsilon - iR - \dfrac{q}{C} = 0$ so

$q = C(\varepsilon - iR) = (6.00\times 10^{-6}\text{ F})[50.0\text{ V} - (7.746\text{ A})(5.00\ \Omega)] = 6.762\times 10^{-5}\text{ C}$.

$U_C = \dfrac{q^2}{2C} = \dfrac{(6.762\times 10^{-5}\text{ C})^2}{2(6.00\times 10^{-6}\text{ F})} = 3.81\times 10^{-4}\text{ J}$.

EVALUATE: The energy stored in the capacitor can be returned to a circuit as current, but the energy dissipated in a resistor cannot.

26.73. **IDENTIFY:** Connecting the voltmeter between point *b* and ground gives a resistor network and we can solve for the current through each resistor. The voltmeter reading equals the potential drop across the 200-kΩ resistor.

SET UP: For two resistors in parallel, $\dfrac{1}{R_\text{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$. For two resistors in series, $R_\text{eq} = R_1 + R_2$.

EXECUTE: **(a)** $R_\text{eq} = 100\text{ k}\Omega + \left(\dfrac{1}{200\text{ k}\Omega} + \dfrac{1}{50\text{ k}\Omega}\right)^{-1} = 140\text{ k}\Omega$. The total current is

$I = \dfrac{0.400\text{ kV}}{140\text{ k}\Omega} = 2.86\times 10^{-3}\text{ A}$. The voltage across the $200\text{-k}\Omega$ resistor is

$V_{200\text{ k}\Omega} = IR = (2.86\times 10^{-3}\text{ A})\left(\dfrac{1}{200\text{ k}\Omega} + \dfrac{1}{50\text{ k}\Omega}\right)^{-1} = 114.4\text{ V}$.

(b) If the resistance of the voltmeter is $5.00\times 10^6\ \Omega$, then we carry out the same calculations as above to find $R_\text{eq} = 292\text{ k}\Omega$, $I = 1.37\times 10^{-3}\text{ A}$ and $V_{200\text{ k}\Omega} = 263\text{ V}$.

(c) If the resistance of the voltmeter is infinite, then we find $R_\text{eq} = 300\text{ k}\Omega$, $I = 1.33\times 10^{-3}\text{ A}$ and $V_{200\text{k}\Omega} = 266\text{ V}$.

EVALUATE: When a voltmeter of finite resistance is connected to a circuit, current flows through the voltmeter and the presence of the voltmeter alters the currents and voltages in the original circuit. The effect of the voltmeter on the circuit decreases as the resistance of the voltmeter increases.

26.74. **IDENTIFY** and **SET UP:** Zero current through the galvanometer means the current I_1 through N is also the current through M and the current I_2 through P is the same as the current through X. And it means that points b and c are at the same potential, so $I_1N = I_2P$.

EXECUTE: **(a)** The voltage between points a and d is ε, so $I_1 = \dfrac{\varepsilon}{N+M}$ and $I_2 = \dfrac{\varepsilon}{P+X}$. Using these expressions in $I_1N = I_2P$ gives $\dfrac{\varepsilon}{N+M}N = \dfrac{\varepsilon}{P+X}P$. $N(P+X) = P(N+M)$. $NX = PM$ and $X = MP/N$.

(b) $X = \dfrac{MP}{N} = \dfrac{(850.0\,\Omega)(33.48\,\Omega)}{15.00\,\Omega} = 1897\,\Omega$

EVALUATE: The measurement of X does not require that we know the value of the emf.

26.75. **IDENTIFY:** With S open and after equilibrium has been reached, no current flows and the voltage across each capacitor is 18.0 V. When S is closed, current I flows through the $6.00\text{-}\Omega$ and $3.00\text{-}\Omega$ resistors.

SET UP: With the switch closed, a and b are at the same potential and the voltage across the $6.00\text{-}\Omega$ resistor equals the voltage across the $6.00\text{-}\mu\text{F}$ capacitor and the voltage is the same across the $3.00\text{-}\mu\text{F}$ capacitor and $3.00\text{-}\Omega$ resistor.

EXECUTE: **(a)** With an open switch: $V_{ab} = \varepsilon = 18.0\text{ V}$.

(b) Point a is at a higher potential since it is directly connected to the positive terminal of the battery.

(c) When the switch is closed $18.0\text{ V} = I(6.00\,\Omega + 3.00\,\Omega)$. $I = 2.00\text{ A}$ and $V_b = (2.00\text{ A})(3.00\,\Omega) = 6.00\text{ V}$.

(d) Initially the capacitor's charges were $Q_3 = CV = (3.00\times10^{-6}\text{ F})(18.0\text{ V}) = 5.40\times10^{-5}\text{ C}$ and $Q_6 = CV = (6.00\times10^{-6}\text{ F})(18.0\text{ V}) = 1.08\times10^{-4}\text{ C}$. After the switch is closed $Q_3 = CV = (3.00\times10^{-6}\text{ F})(18.0\text{ V} - 12.0\text{ V}) = 1.80\times10^{-5}\text{ C}$ and $Q_6 = CV = (6.00\times10^{-6}\text{ F})(18.0\text{ V} - 6.0\text{ V}) = 7.20\times10^{-5}\text{ C}$. Both capacitors lose $3.60\times10^{-5}\text{ C} = 36.0\,\mu\text{C}$.

EVALUATE: The voltage across each capacitor decreases when the switch is closed, because there is then current through each resistor and therefore a potential drop across each resistor.

26.76. **IDENTIFY:** Just after the connection is made, $q = 0$ and the voltage across the capacitor is zero. After a long time $i = 0$.

SET UP: The rate at which the resistor dissipates electrical energy is $P_R = V^2/R$, where V is the voltage across the resistor. The energy stored in the capacitor is $q^2/2C$. The power output of the source is $P_\varepsilon = \varepsilon i$.

EXECUTE: **(a)** (i) $P_R = \dfrac{V^2}{R} = \dfrac{(120\text{ V})^2}{5.86\,\Omega} = 2460\text{ W}$.

(ii) $P_C = \dfrac{dU}{dt} = \dfrac{1}{2C}\dfrac{d(q^2)}{dt} = \dfrac{iq}{C} = 0$.

(iii) $P_\varepsilon = \varepsilon I = (120\text{ V})\dfrac{120\text{ V}}{5.86\,\Omega} = 2460\text{ W}$.

The power output of the source is the sum of the power dissipated in the resistor and the power stored in the capacitor.

(b) After a long time, $i = 0$, so $P_R = 0, P_C = 0, P_\varepsilon = 0$.

(c) (i) Since $q = q_{\max}(1 - e^{-t/RC})$, when $q = q_{\max}/2$, $e^{-t/RC} = \frac{1}{2}$. $P_R = i^2R$, so

$$P_R = (i_0e^{-t/RC})^2R = i_0^2R(e^{-t/RC})^2 = (i_0^2R)\left(\frac{1}{2}\right)^2 = \frac{i_0^2R}{4} = \frac{(\varepsilon/R)^2R}{4} = \frac{\varepsilon^2}{4R},$$ which gives

$$P_R = \frac{(120\text{ V})^2}{4(5.86\,\Omega)} = 614\text{ W}.$$

(ii) $\dfrac{dU_C}{dt} = \dfrac{d}{dt}\left[\dfrac{q_{\max}^2}{2C}(1-e^{-t/RC})^2\right] = \dfrac{\varepsilon^2}{4R} = 614 \text{ W}.$

(iii) $P_\varepsilon = \varepsilon i = \varepsilon(i_0 e^{-t/RC}) = (120 \text{ V})\left(\dfrac{120 \text{ V}}{5.86\ \Omega}\right)\left(\dfrac{1}{2}\right) = 1230 \text{ W}.$

The power output of the source is the sum of the power dissipated in the resistor and the power stored in the capacitor.

EVALUATE: Initially all the power output of the source is dissipated in the resistor. After a long time energy is stored in the capacitor but the amount stored isn't changing. For intermediate times, part of the energy of the power source is dissipated in the resistor and part of it is stored in the capacitor. Conservation of energy tells us that the power output of the source should be equal to the power dissipated in the resistor plus the power stored in the capacitor, which is exactly what we have found in part (iii).

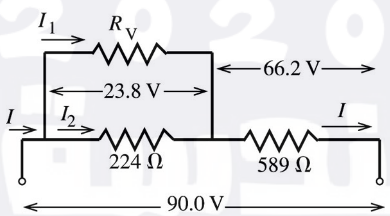
26.77. **IDENTIFY** and **SET UP:** Without the meter, the circuit consists of the two resistors in series. When the meter is connected, its resistance is added to the circuit in parallel with the resistor it is connected across.

(a) EXECUTE: $I = I_1 = I_2$.

$$I = \frac{90.0 \text{ V}}{R_1 + R_2} = \frac{90.0 \text{ V}}{224\ \Omega + 589\ \Omega} = 0.1107 \text{ A}.$$

$V_1 = I_1 R_1 = (0.1107 \text{ A})(224\ \Omega) = 24.8 \text{ V}$; $V_2 = I_2 R_2 = (0.1107 \text{ A})(589\ \Omega) = 65.2 \text{ V}$.

(b) SET UP: The resistor network is sketched in Figure 26.77a.



The voltmeter reads the potential difference across its terminals, which is 23.8 V. If we can find the current I_1 through the voltmeter then we can use Ohm's law to find its resistance.

Figure 26.77a

EXECUTE: The voltage drop across the 589-Ω resistor is $90.0 \text{ V} - 23.8 \text{ V} = 66.2 \text{ V}$, so

$I = \dfrac{V}{R} = \dfrac{66.2 \text{ V}}{589\ \Omega} = 0.1124 \text{ A}.$ The voltage drop across the 224-Ω resistor is 23.8 V, so

$I_2 = \dfrac{V}{R} = \dfrac{23.8 \text{ V}}{224\ \Omega} = 0.1062 \text{ A}.$ Then $I = I_1 + I_2$ gives $I_1 = I - I_2 = 0.1124 \text{ A} - 0.1062 \text{ A} = 0.0062 \text{ A}.$

$$R_V = \frac{V}{I_1} = \frac{23.8 \text{ V}}{0.0062\ A} = 3840\ \Omega.$$

(c) SET UP: The circuit with the voltmeter connected is sketched in Figure 26.77b.

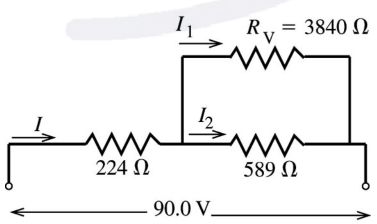
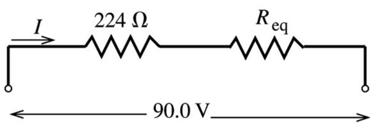


Figure 26.77b

EXECUTE: Replace the two resistors in parallel by their equivalent, as shown in Figure 26.77c.



$$\frac{1}{R_{eq}} = \frac{1}{3840\ \Omega} + \frac{1}{589\ \Omega};$$

$$R_{eq} = \frac{(3840\ \Omega)(589\ \Omega)}{3840\ \Omega + 589\ \Omega} = 510.7\ \Omega.$$

Figure 26.77c

$$I = \frac{90.0\text{ V}}{224\ \Omega + 510.7\ \Omega} = 0.1225\text{ A}.$$

The potential drop across the 224-Ω resistor then is $IR = (0.1225\text{ A})(224\ \Omega) = 27.4\text{ V},$ so the potential drop across the 589-Ω resistor and across the voltmeter (what the voltmeter reads) is $90.0\text{ V} - 27.4\text{ V} = 62.6\text{ V}.$

EVALUATE: **(d)** No, any real voltmeter will draw some current and thereby reduce the current through the resistance whose voltage is being measured. Thus the presence of the voltmeter connected in parallel with the resistance lowers the voltage drop across that resistance. The resistance of the voltmeter in this problem is only about a factor of ten larger than the resistances in the circuit, so the voltmeter has a noticeable effect on the circuit.

26.78. **IDENTIFY:** The energy stored in a capacitor is $U = q^2/2C$. The electrical power dissipated in the resistor is $P = i^2R$.

SET UP: For a discharging capacitor, $i = -\dfrac{q}{RC}$.

EXECUTE: **(a)** $U_0 = \dfrac{Q_0^{\ 2}}{2C} = \dfrac{(0.0069\text{ C})^2}{2(4.62\times10^{-6}\text{ F})} = 5.15\text{ J}.$

(b) $P_0 = I_0^{\ 2}R = \left(\dfrac{Q_0}{RC}\right)^2 R = \dfrac{(0.0069\text{ C})^2}{(850\ \Omega)(4.62\times10^{-6}\text{ F})^2} = 2620\text{ W}.$

(c) Since $U = q^2/2C$, when $U \to U_0/2$, $q \to Q_0/\sqrt{2}$. Since $q = Q_0e^{-t/RC}$, this means that $e^{-t/RC} = 1/\sqrt{2}$. Therefore the current is $i = i_0e^{-t/RC} = i_0/\sqrt{2}$. Therefore

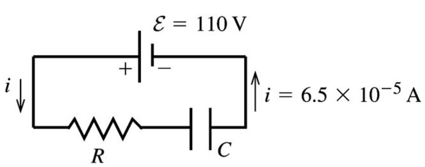
$$P_R = \left(\frac{i_0}{\sqrt{2}}\right)^2 R = \frac{1}{2}\left(\frac{V_0}{R}\right)^2 R = \frac{1}{2}\left(\frac{Q_0}{RC}\right)^2 R = \frac{1}{RC}\left(\frac{Q_0^2}{2C}\right) = \frac{U_0}{RC}.$$ Putting in the numbers gives

$$P_R = \frac{5.15\text{ J}}{(850\ \Omega)(4.62\ \mu\text{F})} = 1310\text{ W}.$$

EVALUATE: All the energy originally stored in the capacitor is eventually dissipated as current flows through the resistor.

26.79. **IDENTIFY:** Apply the loop rule to the circuit. The initial current determines *R*. We can then use the time constant to calculate *C*.

SET UP: The circuit is sketched in Figure 26.79.



Initially, the charge of the capacitor is zero, so by $V = q/C$ the voltage across the capacitor is zero.

Figure 26.79

EXECUTE: The loop rule therefore gives $\varepsilon - iR = 0$ and $R = \dfrac{\varepsilon}{i} = \dfrac{110\text{ V}}{6.5\times10^{-5}\text{ A}} = 1.7\times10^6\ \Omega.$

The time constant is given by $\tau = RC$, so $C = \dfrac{\tau}{R} = \dfrac{5.2\text{ s}}{1.7\times10^6\ \Omega} = 3.1\ \mu\text{F}.$

EVALUATE: The resistance is large so the initial current is small and the time constant is large.

26.80. **IDENTIFY** and **SET UP:** When the switch S is closed, current begins to flow as the capacitor plates discharge. The current in the circuit is $i = (Q_0/RC)e^{-t/RC}$.

EXECUTE: **(a)** Taking logs of the equation for i gives $\ln(i) = \ln(Q_0/RC) - t/RC$. A graph of $\ln(i)$ versus t will be a straight line with slope equal to $-1/RC$.

(b) Using the points (1.50 ms, –3.0) and (3.00 ms, –4.0) on the graph in the problem, the slope is

$\text{slope} = \dfrac{-4.0-(-3.0)}{3.00\text{ ms} - 1.50\text{ ms}} = -0.667\ (\text{ms})^{-1} = -667\text{ s}^{-1}.$ Therefore

$-1/RC = -667\text{ s}^{-1}$.

$C = 1/[(196\ \Omega)(667\text{ s}^{-1})] = 7.65\times10^{-6}$ F, which rounds to 7.7 μF.

Using point (1.50 ms, –3.0) on the graph, the equation of the graph gives

$-3.0 = \ln(Q_0/RC) - (1.50\text{ ms})/RC$.

Simplifying and rearranging gives

$-2.0 = \ln(Q_0/RC)$.

$Q_0 = RC\ e^{-2.0} = (196\ \Omega)(7.65\ \mu F)\ e^{-2.0} = 203\ \mu C,$ which rounds to 200 μC.

(c) Taking a loop around the circuit gives

$V_R + V_C = 0$.

$-IR + Q/C = 0$.

$Q = RCI = (196\ \Omega)(7.65\ \mu\text{F})(0.0500\text{ A}) = 75\ \mu$C.

(d) From (c), we have $Q = RCI$, so $I = Q/RC = (500\ \mu\text{C})/[(196\ \Omega)(7.65\ \mu\text{F})] = 0.33$ A.

EVALUATE: The accuracy of the answers depends on how well we can get information from the graph with the problem, so answers may differ slightly from those given here.

26.81. **IDENTIFY** and **SET UP:** Kirchhoff's rules apply to the circuit. Taking a loop around the circuit gives $\varepsilon - Ri - q/C = 0$.

EXECUTE: **(a)** Solving the loop equation for q gives $q =\ q = \varepsilon C - RCi.$ A graph of q as a function of i should be a straight line with slope equal to $-RC$ and y-intercept equal to εC. Figure 26.81 shows this graph.

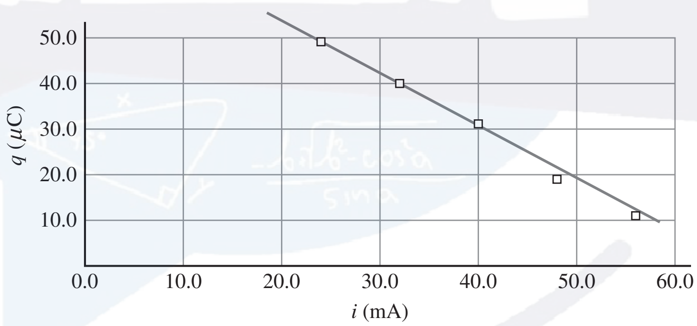


Figure 26.81

The best-fit slope of this graph is -1.233×10^{-3} C/A, and the y-intercept is 7.054×10^{-5} C.

(b) $RC = -\text{slope} = -(-1.233\times10^{-3}\text{ C/A})$, which gives

$R = (-1.233\times10^{-3}\text{ C/A})/(5.00\times10^{-6}\text{ F}) = 246.6\ \Omega$, which rounds to 247 Ω.

The y-intercept is εC, so

$7.054\times10^{-5}\text{ C} = \varepsilon\,(5.00\times10^{-6}\text{ F})$.

$\varepsilon = 15.9$ V.

(c) $V_C = \varepsilon(1-e^{-t/RC})$.

$V_C/\varepsilon = 1-e^{-t/RC} = (10.0\text{ V})(15.9\text{ V})$.

Solving for t gives

$t = (247\ \Omega)(5.00\ \mu\text{F})\ln(0.3714) = 1223\ \mu$s, which rounds to 1.22 ms.

(d) $V_R = \varepsilon - V_C = 15.9\text{ V} - 4.00\text{ V} = 11.9$ V.

EVALUATE: As time increases, the potential difference across the capacitor increases as it gets charged, but the potential difference across the resistor decreases as the current decreases.

26.82. IDENTIFY and SET UP: When connected in series across a 48.0-V battery, R_1 and R_2 dissipate 48.0 W of power, and when in parallel across the same battery, they dissipate a total of 256 W. $PR = I^2R = V^2/R$.

EXECUTE: **(a)** In series: $I = \varepsilon/(R_1 + R_2)$.

$P_s = I^2(R_1 + R_2) = [\varepsilon/(R_1 + R_2)]2(R_1 + R_2) = \varepsilon^2/(R_1 + R_2)$.

$48.0 \text{ W} = (48.0 \text{ V})^2/(R_1 + R_2)$.

$R_1 + R_2 = 48.0\ \Omega$.

In parallel: $P_\text{p} = I_1^2R_1 + I_2^2R_2 = \dfrac{\varepsilon^2}{R_1^2}R_1 + \dfrac{\varepsilon^2}{R_2^2}R_2 = \varepsilon^2\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right) = \varepsilon^2\left(\dfrac{R_1 + R_2}{R_1R_2}\right) = 256 \text{ W}$.

Therefore $(48.0 \text{ V})^2\left(\dfrac{R_1R_2}{R_1 + R_2}\right) = 256 \text{ W}$. Using $R_1 + R_2 = 48.0\ \Omega$, this becomes $R_1R_2 = 432\ \Omega^2$.

Solving the two equations for R_1 and R_2 simultaneously, we get two sets of answers: $R_1 = 36.0\ \Omega$, $R_2 = 12.0\ \Omega$ and $R_1 = 12.0\ \Omega$, $R_2 = 36.0\ \Omega$. But we are told that that $R_1 > R_2$, so the solution to use is $R_1 = 36.0\ \Omega$, $R_2 = 12.0\ \Omega$.

(b) In series, both resistors have the same current. $P = I^2R$, so the larger resistor, which is R_1, consumes more power.

(c) In parallel, the potential difference across both resistors is the same. $P = V^2R$, so the smaller resistor, which is R_2, consumes more power.

EVALUATE: If we did not know which resistor was larger, we would know that one resistor was 12.0 Ω and the other was 36.0 Ω, but we would not know which one was the larger of the two.

26.83. IDENTIFY: Consider one segment of the network attached to the rest of the network.

SET UP: We can re-draw the circuit as shown in Figure 26.83.

EXECUTE: $R_T = 2R_1 + \left(\dfrac{1}{R_2} + \dfrac{1}{R_T}\right)^{-1} = 2R_1 + \dfrac{R_2R_T}{R_2 + R_T}$. $R_T^{\,2} - 2R_1R_T - 2R_1R_2 = 0$.

$R_T = R_1 \pm \sqrt{R_1^{\,2} + 2R_1R_2}$. $R_T > 0$, so $R_T = R_1 + \sqrt{R_1^{\,2} + 2R_1R_2}$.

EVALUATE: Even though there are an infinite number of resistors, the equivalent resistance of the network is finite.

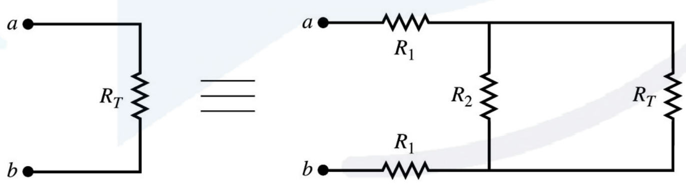


Figure 26.83

26.84. IDENTIFY: Assume a voltage V applied between points a and b and consider the currents that flow along each path between a and b.

SET UP: The currents are shown in Figure 26.84.

EXECUTE: Let current I enter at a and exit at b. At a there are three equivalent branches, so current is $I/3$ in each. At the next junction point there are two equivalent branches so each gets current $I/6$. Then at b there are three equivalent branches with current $I/3$ in each. The voltage drop from a to b then is

$V = \left(\dfrac{I}{3}\right)R + \left(\dfrac{I}{6}\right)R + \left(\dfrac{I}{3}\right)R = \frac{5}{6}IR$. This must be the same as $V = IR_\text{eq}$, so $R_\text{eq} = \frac{5}{6}R$.

EVALUATE: The equivalent resistance is less than R, even though there are 12 resistors in the network.

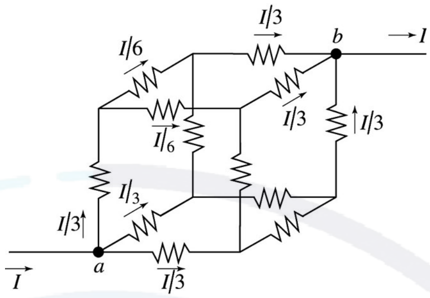


Figure 26.84

26.85. **IDENTIFY:** The network is the same as the one in Challenge Problem 26.83, and that problem shows that the equivalent resistance of the network is $R_T = \sqrt{R_1^2 + 2R_1R_2}$.

SET UP: The circuit can be redrawn as shown in Figure 26.85.

EXECUTE: **(a)** $V_{cd} = V_{ab}\dfrac{R_{\text{eq}}}{2R_1 + R_{\text{eq}}} = V_{ab}\dfrac{1}{2R_1/R_{\text{eq}} + 1}$ and $R_{\text{eq}} = \dfrac{R_2 R_T}{R_2 + R_T}$. But $\beta = \dfrac{2R_1(R_T + R_2)}{R_T R_2} = \dfrac{2R_1}{R_{\text{eq}}}$, so $V_{cd} = V_{ab}\dfrac{1}{1+\beta}$.

(b) $V_1 = \dfrac{V_0}{(1+\beta)} \Rightarrow V_2 = \dfrac{V_1}{(1+\beta)} = \dfrac{V_0}{(1+\beta)^2} \Rightarrow V_n = \dfrac{V_{n-1}}{(1+\beta)} = \dfrac{V_0}{(1+\beta)^n}$.

If $R_1 = R_2$, then $R_T = R_1 + \sqrt{R_1{}^2 + 2R_1R_1} = R_1(1+\sqrt{3})$ and $\beta = \dfrac{2(2+\sqrt{3})}{1+\sqrt{3}} = 2.73$. So, for the nth segment to have 1% of the original voltage, we need: $\dfrac{1}{(1+\beta)^n} = \dfrac{1}{(1+2.73)^n} \le 0.01$. This says $n = 4$, and then $V_4 = 0.005V_0$.

(c) $R_T = R_1 + \sqrt{R_1{}^2 + 2R_1R_2}$ gives $R_T = 6400\ \Omega + \sqrt{(6400\ \Omega)^2 + 2(6400\ \Omega)(8.0\times10^8\ \Omega)} = 3.2\times10^6\ \Omega$ and $\beta = \dfrac{2(6400\ \Omega)(3.2\times10^6\ \Omega + 8.0\times10^8\ \Omega)}{(3.2\times10^6\ \Omega)(8.0\times10^8\ \Omega)} = 4.0\times10^{-3}$.

(d) Along a length of 2.0 mm of axon, there are 2000 segments each 1.0 μm long. The voltage therefore attenuates by $V_{2000} = \dfrac{V_0}{(1+\beta)^{2000}}$, so $\dfrac{V_{2000}}{V_0} = \dfrac{1}{(1+4.0\times10^{-3})^{2000}} = 3.4\times10^{-4}$.

(e) If $R_2 = 3.3\times10^{12}\ \Omega$, then $R_T = 2.1\times10^8\ \Omega$ and $\beta = 6.2\times10^{-5}$. This gives $\dfrac{V_{2000}}{V_0} = \dfrac{1}{(1+6.2\times10^{-5})^{2000}} = 0.88$.

EVALUATE: As R_2 increases, β decreases and the potential difference decrease from one section to the next is less.

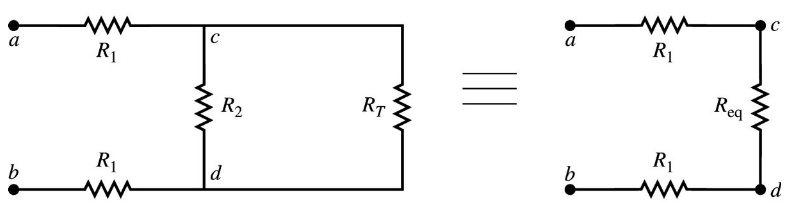


Figure 26.85

26.86. **IDENTIFY and SET UP:** $R = \dfrac{\rho L}{A}$.

EXECUTE: Solve for ρ: $\rho = \dfrac{AR}{L} = \dfrac{\pi r^2 R}{L} = \dfrac{\pi(0.3 \text{ nm})^2(1\times10^{11}\ \Omega)}{12 \text{ nm}} = 2.4\ \Omega\cdot\text{m}\ \Omega \approx 2\ \Omega\cdot\text{m}$, which is choice (c).

EVALUATE: According to the information in Table 25.1, this resistivity is much greater than that of conductors but much less than that of insulators. It is closer to that of semiconductors.

26.87. **IDENTIFY and SET UP:** The channels are all in parallel. For n identical resistors R in parallel,

$\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots = \dfrac{1}{R} + \dfrac{1}{R} + \ldots = \dfrac{n}{R}$, so $R_{\text{eq}} = R/n$. $I = jA$.

EXECUTE: $I = jA = V/\text{R}_{\text{eq}} = V/(R/n) = nV/R$.

$jR/V = n/A = (5 \text{ mA/cm}^2)(10^{11}\ \Omega)/(50 \text{ mV}) = 10^{10}/\text{cm}^2 = 100/\mu\text{m}^2$, which is choice (d).

EVALUATE: A density of 100 per μm^2 seems plausible, since these are microscopic structures.

26.88. **IDENTIFY and SET UP:** $\tau = RC$. The resistance is $1\times10^{11}\ \Omega$. C is the capacitance per area divided by the number density of channels, which is $100/\mu\text{m}^2$ from Problem 26.87.

EXECUTE: $C = (1\ \mu\text{F/cm}^2)/(100/\mu\text{m}^2) = 10^{-16}$ F. The time constant is

$\tau = RC = (1\times10^{11}\ \Omega)(10^{-16}\text{ F}) = 1\times10^{-5}\text{ s} = 10\ \mu s$, which is choice (b).

EVALUATE: This time constant is comparable to that of typical laboratory RC circuits.

27

MAGNETIC FIELD AND MAGNETIC FORCES

27.1. **IDENTIFY** and **SET UP:** Apply $\vec{F} = q\vec{v}\times\vec{B}$ to calculate $\vec{F}$. Use the cross products of unit vectors from Chapter 1. $\vec{v} = (+4.19\times10^4 \text{ m/s})\hat{i} + (-3.85\times10^4 \text{ m/s})\hat{j}$.

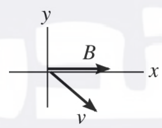
(a) EXECUTE: $\vec{B} = (1.40 \text{ T})\hat{i}$.

$\vec{F} = q\vec{v}\times\vec{B} = (-1.24\times10^{-8} \text{ C})(1.40 \text{ T})[(4.19\times10^4 \text{ m/s})\hat{i}\times\hat{i} - (3.85\times10^4 \text{ m/s})\hat{j}\times\hat{i}]$.

$\hat{i}\times\hat{i} = 0,\ \hat{j}\times\hat{i} = -\hat{k}$.

$\vec{F} = (-1.24\times10^{-8} \text{ C})(1.40 \text{ T})(-3.85\times10^4 \text{ m/s})(-\hat{k}) = (-6.68\times10^{-4} \text{ N})\hat{k}$.

EVALUATE: The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1a.



The right-hand rule gives that $\vec{v}\times\vec{B}$ is directed out of the paper $(+z\text{-direction})$. The charge is negative so $\vec{F}$ is opposite to $\vec{v}\times\vec{B}$.

Figure 27.1a

$\vec{F}$ is in the $-z$-direction. This agrees with the direction calculated with unit vectors.

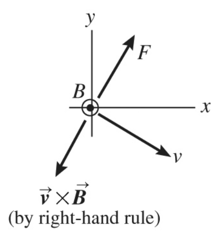
(b) EXECUTE: $\vec{B} = (1.40 \text{ T})\hat{k}$.

$\vec{F} = q\vec{v}\times\vec{B} = (-1.24\times10^{-8} \text{ C})(1.40 \text{ T})[(+4.19\times10^4 \text{ m/s})\hat{i}\times\hat{k} - (3.85\times10^4 \text{ m/s})\hat{j}\times\hat{k}]$.

$\hat{i}\times\hat{k} = -\hat{j},\ \hat{j}\times\hat{k} = \hat{i}$.

$\vec{F} = (-7.27\times10^{-4} \text{ N})(-\hat{j}) + (6.68\times10^{-4} \text{ N})\hat{i} = [(6.68\times10^{-4} \text{ N})\hat{i} + (7.27\times10^{-4} \text{ N})\hat{j}]$.

EVALUATE: The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1b.



The direction of $\vec{F}$ is opposite to $\vec{v}\times\vec{B}$ since q is negative. The direction of $\vec{F}$ computed from the right-hand rule agrees qualitatively with the direction calculated with unit vectors.

Figure 27.1b

27.2. **IDENTIFY:** The net force must be zero, so the magnetic and gravity forces must be equal in magnitude and opposite in direction.

SET UP: The gravity force is downward so the force from the magnetic field must be upward. The charge's velocity and the forces are shown in Figure 27.2. Since the charge is negative, the magnetic force is opposite to the right-hand rule direction. The minimum magnetic field is when the field is perpendicular to $\vec{v}$. The force is also perpendicular to $\vec{B}$, so $\vec{B}$ is either eastward or westward.

EXECUTE: If $\vec{B}$ is eastward, the right-hand rule direction is into the page and $\vec{F}_B$ is out of the page, as required. Therefore, $\vec{B}$ is eastward. $mg = |q|vB\sin\phi$. $\phi = 90°$ and

$$B = \frac{mg}{v|q|} = \frac{(0.195\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{(4.00\times10^{4}\ \text{m/s})(2.50\times10^{-8}\ \text{C})} = 1.91\ \text{T}.$$

EVALUATE: The magnetic field could also have a component along the north-south direction, that would not contribute to the force, but then the field wouldn't have minimum magnitude.

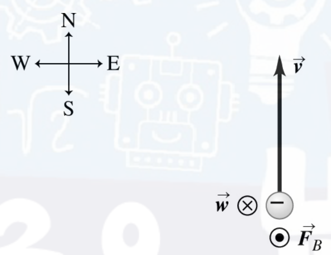


Figure 27.2

27.3. **IDENTIFY:** The force $\vec{F}$ on the particle is in the direction of the deflection of the particle. Apply the right-hand rule to the directions of $\vec{v}$ and $\vec{B}$. See if your thumb is in the direction of $\vec{F}$, or opposite to that direction. Use $F = |q|vB\sin\phi$ with $\phi = 90°$ to calculate F.

SET UP: The directions of $\vec{v}$, $\vec{B}$, and $\vec{F}$ are shown in Figure 27.3.

EXECUTE: **(a)** When you apply the right-hand rule to $\vec{v}$ and $\vec{B}$, your thumb points east. $\vec{F}$ is in this direction, so the charge is positive.

(b) $F = |q|vB\sin\phi = (8.50\times10^{-6}\ \text{C})(4.75\times10^{3}\ \text{m/s})(1.25\ \text{T})\sin 90° = 0.0505\ \text{N}$

EVALUATE: If the particle had negative charge and $\vec{v}$ and $\vec{B}$ are unchanged, the particle would be deflected toward the west.

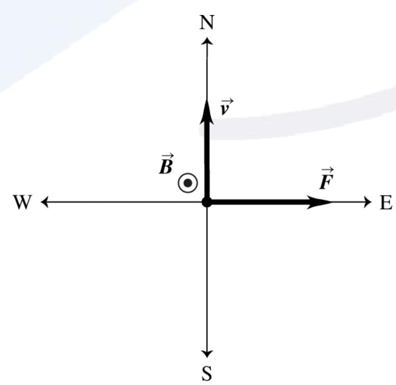


Figure 27.3

27.4. **IDENTIFY:** Apply Newton's second law, with the force being the magnetic force.

SET UP: $\hat{j}\times\hat{i} = -\hat{k}$.

EXECUTE: $\vec{F} = m\vec{a} = q\vec{v}\times\vec{B}$ gives $\vec{a} = \dfrac{q\vec{v}\times\vec{B}}{m}$ and

$$\vec{a} = \frac{(1.22\times10^{-8}\ \text{C})(3.0\times10^{4}\ \text{m/s})(1.63\ \text{T})(\hat{\boldsymbol{j}}\times\hat{\boldsymbol{i}})}{1.81\times10^{-3}\ \text{kg}} = -(0.330\ \text{m/s}^2)\hat{\boldsymbol{k}}.$$

EVALUATE: The acceleration is in the $-z$-direction and is perpendicular to both $\vec{v}$ and $\vec{B}$.

27.5. IDENTIFY: Apply $F = |q|vB\sin\phi$ and solve for v.

SET UP: An electron has $q = -1.60\times10^{-19}$ C.

EXECUTE: $v = \dfrac{F}{|q|B\sin\phi} = \dfrac{4.60\times10^{-15}\ \text{N}}{(1.6\times10^{-19}\ \text{C})(3.5\times10^{-3}\ \text{T})\sin 60^\circ} = 9.49\times10^{6}\,\text{m/s}.$

EVALUATE: Only the component $B\sin\phi$ of the magnetic field perpendicular to the velocity contributes to the force.

27.6. IDENTIFY: Apply Newton's second law and $F = |q|vB\sin\phi$.

SET UP: ϕ is the angle between the direction of $\vec{v}$ and the direction of $\vec{B}$.

EXECUTE: **(a)** The smallest possible acceleration is zero, when the motion is parallel to the magnetic field. The greatest acceleration is when the velocity and magnetic field are at right angles:

$$a = \frac{|q|vB}{m} = \frac{(1.6\times10^{-19}\ \text{C})(1.40\times10^{6}\ \text{m/s})(7.4\times10^{-2}\ \text{T})}{(9.11\times10^{-31}\ \text{kg})} = 1.82\times10^{16}\ \text{m/s}^2.$$

(b) If $a = \frac{1}{4}(1.82\times10^{16}\ \text{m/s}^2) = \dfrac{|q|vB\sin\phi}{m}$, then $\sin\phi = 0.25$ and $\phi = 14.5^\circ$.

EVALUATE: The force and acceleration decrease as the angle ϕ approaches zero.

27.7. IDENTIFY: Apply $\vec{F} = q\vec{v}\times\vec{B}$.

SET UP: $\vec{v} = v_y\hat{\boldsymbol{j}}$, with $v_y = -3.80\times10^{3}$ m/s. $F_x = +7.60\times10^{-3}$ N, $F_y = 0$, and $F_z = -5.20\times10^{-3}$ N.

EXECUTE: **(a)** $F_x = q(v_yB_z - v_zB_y) = qv_yB_z$.

$B_z = F_x/qv_y = (7.60\times10^{-3}\ \text{N})/[(7.80\times10^{-6}\ \text{C})(-3.80\times10^{3}\ \text{m/s})] = -0.256\ \text{T}.$

$F_y = q(v_zB_x - v_xB_z) = 0$, which is consistent with $\vec{F}$ as given in the problem. There is no force component along the direction of the velocity.

$F_z = q(v_xB_y - v_yB_x) = -qv_yB_x$. $B_x = -F_z/qv_y = -0.175\ \text{T}.$

(b) B_y is not determined. No force due to this component of $\vec{B}$ along $\vec{v}$; measurement of the force tells us nothing about B_y.

(c) $\vec{B}\cdot\vec{F} = B_xF_x + B_yF_y + B_zF_z = (-0.175\ \text{T})(+7.60\times10^{-3}\ \text{N}) + (-0.256\ \text{T})(-5.20\times10^{-3}\ \text{N})$

$\vec{B}\cdot\vec{F} = 0$. $\vec{B}$ and $\vec{F}$ are perpendicular (angle is 90°).

EVALUATE: The force is perpendicular to both $\vec{v}$ and $\vec{B}$, so $\vec{v}\cdot\vec{F}$ is also zero.

27.8. IDENTIFY and **SET UP:** $\vec{F} = q\vec{v}\times\vec{B} = qB_z[v_x(\hat{\boldsymbol{i}}\times\hat{\boldsymbol{k}}) + v_y(\hat{\boldsymbol{j}}\times\hat{\boldsymbol{k}}) + v_z(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{k}})] = qB_z[v_x(-\hat{\boldsymbol{j}}) + v_y(\hat{\boldsymbol{i}})].$

EXECUTE: **(a)** Set the expression for $\vec{F}$ equal to the given value of $\vec{F}$ to obtain:

$$v_x = \frac{F_y}{-qB_z} = \frac{(7.40\times10^{-7}\ \text{N})}{-(-5.60\times10^{-9}\ \text{C})(-1.25\ \text{T})} = -106\ \text{m/s}.$$

$$v_y = \frac{F_x}{qB_z} = \frac{-(3.40\times10^{-7}\ \text{N})}{(-5.60\times10^{-9}\ \text{C})(-1.25\ \text{T})} = -48.6\ \text{m/s}.$$

(b) v_z does not contribute to the force, so is not determined by a measurement of $\vec{F}$.

(c) $\vec{v}\cdot\vec{F}=v_xF_x+v_yF_y+v_zF_z=\dfrac{F_y}{-qB_z}F_x+\dfrac{F_x}{qB_z}F_y=0;\ \theta=90°.$

EVALUATE: The force is perpendicular to both $\vec{v}$ and $\vec{B}$, so $\vec{B}\cdot\vec{F}$ is also zero.

27.9. IDENTIFY: Apply $\vec{F}=q\vec{v}\times\vec{B}$ to the force on the proton and to the force on the electron. Solve for the components of $\vec{B}$ and use them to find its magnitude and direction.

SET UP: $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$. Since the force on the proton is in the $+y$-direction, $B_y=0$ and $\vec{B}=B_x\hat{i}+B_z\hat{k}$. For the proton, $\vec{v}_\text{p}=(1.50\text{ km/s})\hat{i}=v_\text{p}\hat{i}$ and $\vec{F}_\text{p}=(2.25\times10^{-16}\text{ N})\hat{j}=F_\text{p}\hat{j}$. For the electron, $\vec{v}_\text{e}=-(4.75\text{ km/s})\hat{k}=-v_\text{e}\hat{k}$ and $\vec{F}_\text{e}=(8.50\times10^{-16}\text{ N})\hat{j}=F_\text{e}\hat{j}$. The magnetic force is $\vec{F}=q\vec{v}\times\vec{B}$.

EXECUTE: **(a)** For the proton, $\vec{F}_\text{p}=q\vec{v}_\text{p}\times\vec{B}$ gives $F_\text{p}\hat{j}=ev_\text{p}\hat{i}\times(B_x\hat{i}+B_z\hat{k})=-ev_\text{p}B_z\hat{j}$. Solving for B_z gives $B_z=-\dfrac{F_\text{p}}{ev_\text{p}}=-\dfrac{2.25\times10^{-16}\text{ N}}{(1.60\times10^{-19}\text{ C})(1500\text{ m/s})}=-0.9375\text{ T}$. For the electron, $\vec{F}_\text{e}=-e\vec{v}_\text{e}\times\vec{B}$, which gives $F_\text{e}\hat{j}=(-e)(-v_\text{e}\hat{k})\times(B_x\hat{i}+B_z\hat{k})=ev_\text{e}B_x\hat{j}$. Solving for B_x gives $B_x=\dfrac{F_\text{e}}{ev_\text{e}}=\dfrac{8.50\times10^{-16}\text{ N}}{(1.60\times10^{-19}\text{ C})(4750\text{ m/s})}=1.118\text{ T}$. Therefore $\vec{B}=1.118\text{ T}\hat{i}-0.9375\text{ T}\hat{k}$. The magnitude of the field is $B=\sqrt{B_x^2+B_z^2}=\sqrt{(1.118\text{ T})^2+(-0.9375\text{ T})^2}=1.46\text{ T}$. Calling θ the angle that the magnetic field makes with the $+x$-axis, we have $\tan\theta=\dfrac{B_z}{B_x}=\dfrac{-0.9375\text{ T}}{1.118\text{ T}}=-0.8386$, so $\theta=-40.0°$. Therefore the magnetic field is in the xz-plane directed at 40.0° from the $+x$-axis toward the $-z$-axis, having a magnitude of 1.46 T.

(b) $\vec{B}=B_x\hat{i}+B_z\hat{k}$ and $\vec{v}=(3.2\text{ km/s})(-\hat{j})$.

$\vec{F}=q\vec{v}\times\vec{B}=(-e)(3.2\text{ km/s})(-\hat{j})\times(B_x\hat{i}+B_z\hat{k})=e(3.2\times10^3\text{ m/s})[B_x(-\hat{k})+B_z\hat{i}]$.

$\vec{F}=e(3.2\times10^3\text{ m/s})(-1.118\text{ T}\hat{k}-0.9375\text{ T}\hat{i})=-4.80\times10^{-16}\text{ N}\hat{i}-5.724\times10^{-16}\text{ N}\hat{k}$.

$F=\sqrt{F_x^2+F_z^2}=7.47\times10^{-16}\text{ N}$. Calling θ the angle that the force makes with the $-x$-axis, we have $\tan\theta=\dfrac{F_z}{F_x}=\dfrac{-5.724\times10^{-16}\text{ N}}{-4.800\times10^{-16}\text{ N}}$, which gives $\theta=50.0°$. The force is in the xz-plane and is directed at 50.0° from the $-x$-axis toward either the $-z$-axis.

EVALUATE: The force on the electrons in parts (a) and (b) are comparable in magnitude because the electron speeds are comparable in both cases.

27.10. IDENTIFY: Knowing the area of a surface and the magnetic field it is in, we want to calculate the flux through it.

SET UP: $d\vec{A}=dA\hat{k}$, so $d\Phi_B=\vec{B}\cdot d\vec{A}=B_z dA$.

EXECUTE: $\Phi_B=B_zA=(-0.500\text{ T})(0.0340\text{ m})^2=-5.78\times10^{-4}\text{ T}\cdot\text{m}^2$. $|\Phi_B|=5.78\times10^{-4}\text{ Wb}$.

EVALUATE: Since the field is uniform over the surface, it is not necessary to integrate to find the flux.

27.11. IDENTIFY and **SET UP:** $\Phi_B=\int\vec{B}\cdot d\vec{A}$.

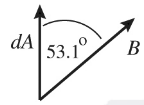
Circular area in the xy-plane, so $A=\pi r^2=\pi(0.0650\text{ m})^2=0.01327\text{ m}^2$ and $d\vec{A}$ is in the z-direction. Use Eq. (1.18) to calculate the scalar product.

EXECUTE: **(a)** $\vec{B}=(0.230\text{ T})\hat{k}$; $\vec{B}$ and $d\vec{A}$ are parallel $(\phi=0°)$ so $\vec{B}\cdot d\vec{A}=B\,dA$.

B is constant over the circular area so

$\Phi_B=\int\vec{B}\cdot d\vec{A}=\int B\,dA=B\int dA=BA=(0.230\text{ T})(0.01327\text{ m}^2)=3.05\times10^{-3}\text{ Wb}$.

(b) The directions of $\vec{B}$ and $d\vec{A}$ are shown in Figure 27.11a.



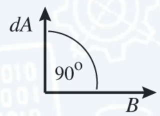
$\vec{B} \cdot d\vec{A} = B\cos\phi dA$
with $\phi = 53.1°$.

Figure 27.11a

B and ϕ are constant over the circular area so $\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B\cos\phi dA = B\cos\phi \int dA = B\cos\phi A$

$\Phi_B = (0.230 \text{ T})\cos 53.1°(0.01327 \text{ m}^2) = 1.83 \times 10^{-3}$ Wb.

(c) The directions of $\vec{B}$ and $d\vec{A}$ are shown in Figure 27.11b.



$\vec{B} \cdot d\vec{A} = 0$ since $d\vec{A}$ and $\vec{B}$ are perpendicular ($\phi = 90°$).
$\Phi_B = \int \vec{B} \cdot d\vec{A} = 0$.

Figure 27.11b

EVALUATE: Magnetic flux is a measure of how many magnetic field lines pass through the surface. It is maximum when $\vec{B}$ is perpendicular to the plane of the loop (part a) and is zero when $\vec{B}$ is parallel to the plane of the loop (part c).

27.12. **IDENTIFY:** Knowing the area of a surface and the magnetic flux through it, we want to find the magnetic field needed to produce this flux.

SET UP: $\Phi_B = BA\cos\phi$ where $\phi = 60.0°$.

EXECUTE: Solving $\Phi_B = BA\cos\phi$ for B gives $B = \dfrac{\Phi_B}{A\cos\phi} = \dfrac{3.10 \times 10^{-4} \text{ Wb}}{(0.0280 \text{ m})(0.0320 \text{ m})\cos 60.0°} = 0.692$ T.

EVALUATE: This is a fairly strong magnetic field, but not impossible to achieve in modern laboratories.

27.13. **IDENTIFY:** The total flux through the bottle is zero because it is a closed surface.

SET UP: The total flux through the bottle is the flux through the plastic plus the flux through the open cap, so the sum of these must be zero. $\Phi_{\text{plastic}} + \Phi_{\text{cap}} = 0$.

$\Phi_{\text{plastic}} = -\Phi_{\text{cap}} = -BA\cos\phi = -B(\pi r^2)\cos\phi$.

EXECUTE: Substituting the numbers gives $\Phi_{\text{plastic}} = -(1.75 \text{ T})\pi(0.0125 \text{ m})^2 \cos 25° = -7.8 \times 10^{-4}$ Wb.

EVALUATE: It would be very difficult to calculate the flux through the plastic directly because of the complicated shape of the bottle, but with a little thought we can find this flux through a simple calculation.

27.14. **IDENTIFY:** When $\vec{B}$ is uniform across the surface, $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\phi$.

SET UP: $\vec{A}$ is normal to the surface and is directed outward from the enclosed volume. For surface *abcd*, $\vec{A} = -A\hat{\imath}$. For surface *befc*, $\vec{A} = -A\hat{k}$. For surface *aefd*, $\cos\phi = 3/5$ and the flux is positive.

EXECUTE: **(a)** $\Phi_B(abcd) = \vec{B} \cdot \vec{A} = 0$.

(b) $\Phi_B(befc) = \vec{B} \cdot \vec{A} = -(0.128 \text{ T})(0.300 \text{ m})(0.300 \text{ m}) = -0.0115$ Wb.

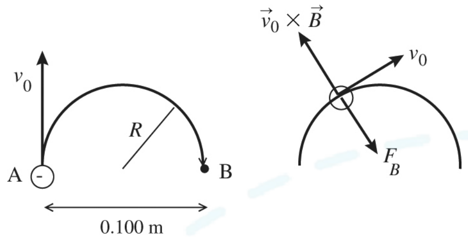
(c) $\Phi_B(aefd) = \vec{B} \cdot \vec{A} = BA\cos\phi = \frac{3}{5}(0.128 \text{ T})(0.500 \text{ m})(0.300 \text{ m}) = +0.0115$ Wb.

(d) The net flux through the rest of the surfaces is zero since they are parallel to the x-axis. The total flux is the sum of all parts above, which is zero.

EVALUATE: The total flux through any closed surface, that encloses a volume, is zero.

27.15. **(a) IDENTIFY:** Apply $\vec{F} = q\vec{v} \times \vec{B}$ to relate the magnetic force $\vec{F}$ to the directions of $\vec{v}$ and $\vec{B}$. The electron has negative charge so $\vec{F}$ is opposite to the direction of $\vec{v} \times \vec{B}$. For motion in an arc of a circle the acceleration is toward the center of the arc so $\vec{F}$ must be in this direction. $a = v^2/R$.

SET UP:



As the electron moves in the semicircle, its velocity is tangent to the circular path. The direction of $\vec{v}_0 \times \vec{B}$ at a point along the path is shown in Figure 27.15.

Figure 27.15

EXECUTE: For circular motion the acceleration of the electron $\vec{a}_{\text{rad}}$ is directed in toward the center of the circle. Thus the force $\vec{F}_B$ exerted by the magnetic field, since it is the only force on the electron, must be radially inward. Since q is negative, $\vec{F}_B$ is opposite to the direction given by the right-hand rule for $\vec{v}_0 \times \vec{B}$. Thus $\vec{B}$ is directed into the page. Apply Newton's second law to calculate the magnitude of $\vec{B}$:

$\sum \vec{F} = m\vec{a}$ gives $\sum F_{\text{rad}} = ma \quad F_B = m(v^2/R)$.

$F_B = |q|vB\sin\phi = |q|vB$, so $|q|vB = m(v^2/R)$.

$$B = \frac{mv}{|q|R} = \frac{(9.109\times10^{-31}\text{ kg})(1.41\times10^{6}\text{ m/s})}{(1.602\times10^{-19}\text{ C})(0.050\text{ m})} = 1.60\times10^{-4}\text{ T}.$$

(b) IDENTIFY and **SET UP:** The speed of the electron as it moves along the path is constant. ($\vec{F}_B$ changes the direction of $\vec{v}$ but not its magnitude.) The time is given by the distance divided by v_0.

EXECUTE: The distance along the semicircular path is πR, so $t = \dfrac{\pi R}{v_0} = \dfrac{\pi(0.050\text{ m})}{1.41\times10^{6}\text{ m/s}} = 1.11\times10^{-7}\text{ s}.$

EVALUATE: The magnetic field required increases when v increases or R decreases and also depends on the mass to charge ratio of the particle.

27.16. IDENTIFY: Newton's second law gives $|q|vB = mv^2/R$. The speed v is constant and equals v_0. The direction of the magnetic force must be in the direction of the acceleration and is toward the center of the semicircular path.

SET UP: A proton has $q = +1.60\times10^{-19}$ C and $m = 1.67\times10^{-27}$ kg. The direction of the magnetic force is given by the right-hand rule.

EXECUTE: **(a)** $B = \dfrac{mv}{qR} = \dfrac{(1.67\times10^{-27}\text{ kg})(1.41\times10^{6}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.0500\text{ m})} = 0.294\text{ T}.$

The direction of the magnetic field is out of the page (the charge is positive), in order for $\vec{F}$ to be directed to the right at point A.

(b) The time to complete half a circle is $t = \pi R/v_0 = 1.11\times10^{-7}$ s.

EVALUATE: The magnetic field required to produce this path for a proton has a different magnitude (because of the different mass) and opposite direction (because of opposite sign of the charge) than the field required to produce the path for an electron.

27.17. IDENTIFY and **SET UP:** Use conservation of energy to find the speed of the ball when it reaches the bottom of the shaft. The right-hand rule gives the direction of $\vec{F}$ and $F = |q|v\,B\sin\phi$ gives its magnitude. The number of excess electrons determines the charge of the ball.

EXECUTE: $q = (4.00\times10^{8})(-1.602\times10^{-19}\text{ C}) = -6.408\times10^{-11}$ C.

speed at bottom of shaft: $\frac{1}{2}mv^2 = mgy$; $v = \sqrt{2gy} = 49.5$ m/s.

$\vec{v}$ is downward and $\vec{B}$ is west, so $\vec{v}\times\vec{B}$ is north. Since $q < 0$, $\vec{F}$ is south.

$F = |q|vB\sin\theta = (6.408\times10^{-11}\text{ C})(49.5\text{ m/s})(0.250\text{ T})\sin 90° = 7.93\times10^{-10}$ N.

EVALUATE: Both the charge and speed of the ball are relatively small so the magnetic force is small, much less than the gravity force of 1.5 N.

27.18. **IDENTIFY:** Since the particle moves perpendicular to the uniform magnetic field, the radius of its path is $R = \dfrac{mv}{|q|B}$. The magnetic force is perpendicular to both $\vec{v}$ and $\vec{B}$.

SET UP: The alpha particle has charge $q = +2e = 3.20\times10^{-19}$ C.

EXECUTE: **(a)** $R = \dfrac{(6.64\times10^{-27}\text{ kg})(35.6\times10^{3}\text{ m/s})}{(3.20\times10^{-19}\text{ C})(1.80\text{ T})} = 4.104\times10^{-4}\text{ m} = 0.4104\text{ mm}.$ The alpha particle moves in a circular arc of diameter $2R = 2(0.4104\text{ mm}) = 0.821\text{ mm}$.

(b) For a very short time interval the displacement of the particle is in the direction of the velocity. The magnetic force is always perpendicular to this direction so it does no work. The work-energy theorem therefore says that the kinetic energy of the particle, and hence its speed, is constant.

(c) The acceleration is

$a = \dfrac{F_B}{m} = \dfrac{|q|vB\sin\phi}{m} = \dfrac{(3.20\times10^{-19}\text{ C})(35.6\times10^{3}\text{ m/s})(1.80\text{ T})\sin 90°}{6.64\times10^{-27}\text{ kg}} = 3.09\times10^{12}\text{ m/s}^2.$ We can also use $a = \dfrac{v^2}{R}$ and the result of part (a) to calculate $a = \dfrac{(35.6\times10^{3}\text{ m/s})^2}{4.104\times10^{-4}\text{ m}} = 3.09\times10^{12}\text{ m/s}^2,$ the same result. The acceleration is perpendicular to $\vec{v}$ and $\vec{B}$ and so is horizontal, toward the center of curvature of the particle's path.

EVALUATE: **(d)** The unbalanced force $(\vec{F}_B)$ is perpendicular to $\vec{v}$, so it changes the direction of $\vec{v}$ but not its magnitude, which is the speed.

27.19. **IDENTIFY:** For motion in an arc of a circle, $a = \dfrac{v^2}{R}$ and the net force is radially inward, toward the center of the circle.

SET UP: The direction of the force is shown in Figure 27.19. The mass of a proton is 1.67×10^{-27} kg.

EXECUTE: **(a)** $\vec{F}$ is opposite to the right-hand rule direction, so the charge is negative. $\vec{F} = m\vec{a}$ gives $|q|vB\sin\phi = m\dfrac{v^2}{R}$. $\phi = 90°$ and $v = \dfrac{|q|BR}{m} = \dfrac{3(1.60\times10^{-19}\text{ C})(0.250\text{ T})(0.475\text{ m})}{12(1.67\times10^{-27}\text{ kg})} = 2.84\times10^{6}\text{ m/s}.$

(b) $F_B = |q|vB\sin\phi = 3(1.60\times10^{-19}\text{ C})(2.84\times10^{6}\text{ m/s})(0.250\text{ T})\sin 90° = 3.41\times10^{-13}\text{ N}.$

$w = mg = 12(1.67\times10^{-27}\text{ kg})(9.80\text{ m/s}^2) = 1.96\times10^{-25}\text{ N}.$ The magnetic force is much larger than the weight of the particle, so it is a very good approximation to neglect gravity.

EVALUATE: **(c)** The magnetic force is always perpendicular to the path and does no work. The particles move with constant speed.

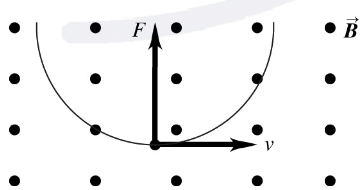


Figure 27.19

27.20. **IDENTIFY:** The magnetic field acts perpendicular to the velocity, causing the ion to move in a circular path but not changing its speed.

SET UP: $R = \dfrac{mv}{|q|B}$ and $K = \frac{1}{2}mv^2$. $K = 5.0\text{ MeV} = 8.0\times10^{-13}$ J.

EXECUTE: **(a)** Solving $K = \frac{1}{2}mv^2$ for v gives $v = \sqrt{2K/m}$.

$v = [2(8.0\times10^{-13}\text{ J})/(1.67\times10^{-27}\text{ kg})]^{1/2} = 3.095\times10^7$ m/s, which rounds to 3.1×10^7 m/s.

(b) Using $R = \dfrac{mv}{|q|B}$ = $(1.67\times10^{-27}\text{ kg})(3.095\times10^7\text{ m/s})/[(1.602\times10^{-19}\text{ C})(1.9\text{ T})] = 0.17\text{ m} = 17\text{ cm}$.

EVALUATE: If the hydride ions were accelerated to 20 MeV, which is 4 times the value used here, their speed would be twice as great, so the radius of their path would also be twice as great.

27.21. **(a) IDENTIFY and SET UP:** Apply Newton's second law, with $a = v^2/R$ since the path of the particle is circular.

EXECUTE: $\sum \vec{F} = m\vec{a}$ says $|q|vB = m(v^2/R)$.

$$v = \frac{|q|BR}{m} = \frac{(1.602\times10^{-19}\text{ C})(2.50\text{ T})(6.96\times10^{-3}\text{ m})}{3.34\times10^{-27}\text{ kg}} = 8.35\times10^5\text{ m/s}.$$

(b) IDENTIFY and SET UP: The speed is constant so $t = \text{distance}/v$.

EXECUTE: $t = \dfrac{\pi R}{v} = \dfrac{\pi(6.96\times10^{-3}\text{ m})}{8.35\times10^5\text{ m/s}} = 2.62\times10^{-8}\text{ s}.$

(c) IDENTIFY and SET UP: kinetic energy gained = electric potential energy lost.

EXECUTE: $\frac{1}{2}mv^2 = |q|V$.

$$V = \frac{mv^2}{2|q|} = \frac{(3.34\times10^{-27}\text{ kg})(8.35\times10^5\text{ m/s})^2}{2(1.602\times10^{-19}\text{ C})} = 7.27\times10^3\text{ V} = 7.27\text{ kV}.$$

EVALUATE: The deuteron has a much larger mass to charge ratio than an electron so a much larger B is required for the same v and R. The deutron has positive charge so gains kinetic energy when it goes from high potential to low potential.

27.22. **IDENTIFY:** An alpha particle has twice as much charge and about 4 times as much mass as a proton.

SET UP: $R = \dfrac{mv}{|q|B}$ and $K = \frac{1}{2}mv^2$. $K = (mv)^2/2m = p^2/2m$, so $mv = \sqrt{2mK}$.

EXECUTE: The kinetic energy is the same in both cases, so express the radius in terms of it.

$R = \dfrac{mv}{|q|B} = \dfrac{\sqrt{2mK}}{|q|B}$. Now take ratios of the radii for an alpha particle and a proton.

$$\frac{R_\alpha}{R_p} = \frac{\dfrac{\sqrt{2m_\alpha K}}{2eB}}{\dfrac{\sqrt{2m_p K}}{eB}} = \frac{1}{2}\sqrt{\frac{m_\alpha}{m_p}} = \frac{1}{2}\sqrt{\frac{6.64}{1.67}} = 0.997, \text{ which gives}$$

$R_\alpha = 0.997R_p = (0.997)(16.0\text{ cm}) = 16.0$ cm, which is the same as for the proton.

EVALUATE: The radius is proportional to $\dfrac{\sqrt{m}}{|q|}$. The alpha particle has twice the charge of the proton and about 4 times its mass, so the result is the same for both particles.

27.23. **IDENTIFY:** When a particle of charge $-e$ is accelerated through a potential difference of magnitude V, it gains kinetic energy eV. When it moves in a circular path of radius R, its acceleration is $\dfrac{v^2}{R}$.

SET UP: An electron has charge $q = -e = -1.60\times10^{-19}$ C and mass 9.11×10^{-31} kg.

EXECUTE: $\frac{1}{2}mv^2 = eV$ and $v = \sqrt{\dfrac{2eV}{m}} = \sqrt{\dfrac{2(1.60\times10^{-19}\text{ C})(2.00\times10^3\text{ V})}{9.11\times10^{-31}\text{ kg}}} = 2.65\times10^7$ m/s. $\vec{F} = m\vec{a}$ gives $|q|vB\sin\phi = m\dfrac{v^2}{R}$. $\phi = 90°$ and $B = \dfrac{mv}{|q|R} = \dfrac{(9.11\times10^{-31}\text{ kg})(2.65\times10^7\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.180\text{ m})} = 8.38\times10^{-4}$ T.

EVALUATE: The smaller the radius of the circular path, the larger the magnitude of the magnetic field that is required.

27.24. IDENTIFY: The magnetic force on the beam bends it through a quarter circle.

SET UP: The distance that particles in the beam travel is $s = R\theta$, and the radius of the quarter circle is $R = mv/qB$.

EXECUTE: Solving for R gives $R = s/\theta = s/(\pi/2) = 1.18\text{ cm}/(\pi/2) = 0.751\text{ cm}$. Solving for the magnetic field: $B = mv/qR = (1.67\times10^{-27}\text{ kg})(1200\text{ m/s})/[(1.60\times10^{-19}\text{ C})(0.00751\text{ m})] = 1.67\times10^{-3}\text{ T}$.

EVALUATE: This field is about 10 times stronger than the earth's magnetic field, but much weaker than many laboratory fields.

27.25. IDENTIFY and **SET UP:** $\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$ gives the total force on the proton. At $t = 0$,

$\vec{F} = q\vec{v}\times\vec{B} = q(v_x\hat{i} + v_z\hat{k})\times B_x\hat{i} = qv_zB_x\hat{j}$. **c)**

EXECUTE: (a) $\vec{F} = (1.60\times10^{-19}\text{ C})(2.00\times10^{5}\text{ m/s})(0.500\text{ T})\hat{j} = (1.60\times10^{-14}\text{ N})\hat{j}$.

(b) Yes. The electric field exerts a force in the direction of the electric field, since the charge of the proton is positive, and there is a component of acceleration in this direction.

(c) In the plane perpendicular to $\vec{B}$ (the yz-plane) the motion is circular. But there is a velocity component in the direction of $\vec{B}$, so the motion is a helix. The electric field in the $+\hat{i}$-direction exerts a force in the $+\hat{i}$-direction. This force produces an acceleration in the $+\hat{i}$-direction and this causes the pitch of the helix to vary. The force does not affect the circular motion in the yz-plane, so the electric field does not affect the radius of the helix.

(d) IDENTIFY and **SET UP:** Use $\omega = |q|B/m$ and $T = 2\pi/\omega$ to calculate the period of the motion. Calculate a_x produced by the electric force and use a constant acceleration equation to calculate the displacement in the x-direction in time $T/2$.

EXECUTE: Calculate the period T: $\omega = |q|B/m$.

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{|q|B} = \frac{2\pi(1.67\times10^{-27}\text{ kg})}{(1.60\times10^{-19}\text{ C})(0.500\text{ T})} = 1.312\times10^{-7}\text{ s. Then } t = T/2 = 6.56\times10^{-8}\text{ s.}$$

$v_{0x} = 1.50\times10^{5}\text{ m/s}$.

$$a_x = \frac{F_x}{m} = \frac{(1.60\times10^{-19}\text{ C})(2.00\times10^{4}\text{ V/m})}{1.67\times10^{-27}\text{ kg}} = +1.916\times10^{12}\text{ m/s}^2.$$

$$x - x_0 = v_{0x}t + \tfrac{1}{2}a_xt^2.$$

$$x - x_0 = (1.50\times10^{5}\text{ m/s})(6.56\times10^{-8}\text{ s}) + \tfrac{1}{2}(1.916\times10^{12}\text{ m/s}^2)(6.56\times10^{-8}\text{ s})^2 = 1.40\text{ cm}.$$

EVALUATE: The electric and magnetic fields are in the same direction but produce forces that are in perpendicular directions to each other.

27.26. IDENTIFY: After being accelerated through a potential difference V the ion has kinetic energy qV. The acceleration in the circular path is v^2/R.

SET UP: The ion has charge $q = +e$.

EXECUTE: $K = qV = +eV$. $\tfrac{1}{2}mv^2 = eV$ and $v = \sqrt{\dfrac{2eV}{m}} = \sqrt{\dfrac{2(1.60\times10^{-19}\text{ C})(220\text{ V})}{1.16\times10^{-26}\text{ kg}}} = 7.79\times10^{4}\text{ m/s}$.

$F_B = |q|vB\sin\phi$. $\phi = 90°$. $\vec{F} = m\vec{a}$ gives $|q|vB = m\dfrac{v^2}{R}$.

$$R = \frac{mv}{|q|B} = \frac{(1.16\times10^{-26}\text{ kg})(7.79\times10^{4}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.874\text{ T})} = 6.46\times10^{-3}\text{ m} = 6.46\text{ mm}.$$

EVALUATE: The larger the accelerating voltage, the larger the speed of the particle and the larger the radius of its path in the magnetic field.

27.27. IDENTIFY: For no deflection the magnetic and electric forces must be equal in magnitude and opposite in direction.

SET UP: $v = E/B$ for no deflection.

EXECUTE: To pass undeflected in both cases, $E = vB = (5.85\times10^3\text{ m/s})(1.35\text{ T}) = 7898\text{ N/C}$.

(a) If $q = 0.640\times10^{-9}\text{ C}$, the electric field direction is given by $-(\hat{\boldsymbol{j}}\times(-\hat{\boldsymbol{k}})) = \hat{\boldsymbol{i}}$, since it must point in the opposite direction to the magnetic force.

(b) If $q = -0.320\times10^{-9}\text{ C}$, the electric field direction is given by $((-\hat{\boldsymbol{j}})\times(-\hat{\boldsymbol{k}})) = \hat{\boldsymbol{i}}$, since the electric force must point in the opposite direction as the magnetic force. Since the particle has negative charge, the electric force is opposite to the direction of the electric field and the magnetic force is opposite to the direction it has in part (a).

EVALUATE: The same configuration of electric and magnetic fields works as a velocity selector for both positively and negatively charged particles.

27.28. **IDENTIFY:** For no deflection the magnetic and electric forces must be equal in magnitude and opposite in direction.

SET UP: $v = E/B$ for no deflection. With only the magnetic force, $|q|vB = mv^2/R$.

EXECUTE: **(a)** $v = E/B = (1.56\times10^4\text{ V/m})/(4.62\times10^{-3}\text{ T}) = 3.38\times10^6\text{ m/s}$.

(b) The directions of the three vectors $\vec{\boldsymbol{v}}$, $\vec{\boldsymbol{E}}$, and $\vec{\boldsymbol{B}}$ are sketched in Figure 27.28.

(c) $R = \dfrac{mv}{|q|B} = \dfrac{(9.11\times10^{-31}\text{ kg})(3.38\times10^6\text{ m/s})}{(1.60\times10^{-19}\text{ C})(4.62\times10^{-3}\text{ T})} = 4.17\times10^{-3}\text{ m}$.

$T = \dfrac{2\pi m}{|q|B} = \dfrac{2\pi R}{v} = \dfrac{2\pi(4.17\times10^{-3}\text{ m})}{(3.38\times10^6\text{ m/s})} = 7.74\times10^{-9}\text{ s}$.

EVALUATE: For the field directions shown in Figure 27.28, the electric force is toward the top of the page and the magnetic force is toward the bottom of the page.

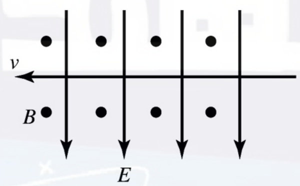


Figure 27.28

27.29. **IDENTIFY:** For the alpha particles to emerge from the plates undeflected, the magnetic force on them must exactly cancel the electric force. The battery produces an electric field between the plates, which acts on the alpha particles.

SET UP: First use energy conservation to find the speed of the alpha particles as they enter the region between the plates: $qV = 1/2\, mv^2$. The electric field between the plates due to the battery is $E = V_b/d$. For the alpha particles not to be deflected, the magnetic force must cancel the electric force, so $qvB = qE$, giving $B = E/v$.

EXECUTE: Solve for the speed of the alpha particles just as they enter the region between the plates. Their charge is $2e$.

$$v_\alpha = \sqrt{\frac{2(2e)V}{m}} = \sqrt{\frac{4(1.60\times10^{-19}\text{ C})(1750\text{V})}{6.64\times10^{-27}\text{ kg}}} = 4.11\times10^5\text{ m/s}.$$

The electric field between the plates, produced by the battery, is

$$E = V_b/d = (150\text{ V})/(0.00820\text{ m}) = 18{,}300\text{ V/m}.$$

The magnetic force must cancel the electric force:

$$B = E/v_\alpha = (18{,}300\text{ V/m})/(4.11\times10^5\text{ m/s}) = 0.0445\text{ T}.$$

The magnetic field is perpendicular to the electric field. If the charges are moving to the right and the electric field points upward, the magnetic field is out of the page.

EVALUATE: The sign of the charge of the alpha particle does not enter the problem, so negative charges of the same magnitude would also not be deflected.

27.30. IDENTIFY: The velocity selector eliminates all ions not having the desired velocity. Then the magnetic field bends the ions in a circular arc.

SET UP: In a velocity selector, $E = vB$. For motion in a circular arc in a magnetic field of magnitude B', $R = \dfrac{mv}{|q|B'}$. The ion has charge $+e$.

EXECUTE: **(a)** $E = vB = (4.50\times10^3\ \text{m/s})(0.0250\ \text{T}) = 112\ \text{V/m}$.

(b) $B' = \dfrac{mv}{|q|R} = \dfrac{(6.64\times10^{-26}\ \text{kg})(4.50\times10^3\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.125\ \text{m})} = 1.49\times10^{-2}\ \text{T}$.

EVALUATE: By laboratory standards, both the electric field and the magnetic field are rather weak and should easily be achievable.

27.31. IDENTIFY: The velocity selector eliminates all ions not having the desired velocity. Then the magnetic field bends the ions in a circular arc.

SET UP: In a velocity selector, $E = vB$. For motion in a circular arc in a magnetic field of magnitude B, $R = \dfrac{mv}{|q|B}$. The ion has charge $+e$.

EXECUTE: **(a)** $v = \dfrac{E}{B} = \dfrac{155\ \text{V/m}}{0.0315\ \text{T}} = 4.92\times10^3\ \text{m/s}$.

(b) $m = \dfrac{R|q|B}{v} = \dfrac{(0.175\ \text{m})(1.60\times10^{-19}\ \text{C})(0.0175\ \text{T})}{4.92\times10^3\ \text{m/s}} = 9.96\times10^{-26}\ \text{kg}$.

EVALUATE: Ions with larger ratio $\dfrac{m}{|q|}$ will move in a path of larger radius.

27.32. IDENTIFY and SET UP: For a velocity selector, $E = vB$. For parallel plates with opposite charge, $V = Ed$.

EXECUTE: **(a)** $E = vB = (1.82\times10^6\ \text{m/s})(0.510\ \text{T}) = 9.28\times10^5\ \text{V/m}$.

(b) $V = Ed = (9.28\times10^5\ \text{V/m})(5.20\times10^{-3}\ \text{m}) = 4.83\ \text{kV}$.

EVALUATE: Any charged particle with $v = 1.82\times10^6\ \text{m/s}$ will pass through undeflected, regardless of the sign and magnitude of its charge.

27.33. IDENTIFY: A mass spectrometer separates ions by mass. Since ^{14}N and ^{15}N have different masses they will be separated and the relative amounts of these isotopes can be determined.

SET UP: $R = \dfrac{mv}{|q|B}$. For $m = 1.99\times10^{-26}\ \text{kg}$ (^{12}C), $R_{12} = 12.5\ \text{cm}$. The separation of the isotopes at the detector is $2(R_{15} - R_{14})$.

EXECUTE: Since $R = \dfrac{mv}{|q|B}$, $\dfrac{R}{m} = \dfrac{v}{|q|B} = \text{constant}$. Therefore $\dfrac{R_{14}}{m_{14}} = \dfrac{R_{12}}{m_{12}}$ which gives

$$R_{14} = R_{12}\left(\frac{m_{14}}{m_{12}}\right) = (12.5\ \text{cm})\left(\frac{2.32\times10^{-26}\ \text{kg}}{1.99\times10^{-26}\ \text{kg}}\right) = 14.6\ \text{cm}$$ and

$$R_{15} = R_{12}\left(\frac{m_{15}}{m_{12}}\right) = (12.5\ \text{cm})\left(\frac{2.49\times10^{-26}\ \text{kg}}{1.99\times10^{-26}\ \text{kg}}\right) = 15.6\ \text{cm}.$$ The separation of the isotopes at the detector is $2(R_{15} - R_{14}) = 2(15.6\ \text{cm} - 14.6\ \text{cm}) = 2.0\ \text{cm}$.

EVALUATE: The separation is large enough to be easily detectable. Since the diameter of the ion path is large, about 30 cm, the uniform magnetic field within the instrument must extend over a large area.

27.34. **IDENTIFY:** The earth's magnetic field exerts a force on the moving charges in the wire.

SET UP: $F = IlB\sin\phi$. The direction of $\vec{F}$ is determined by applying the right-hand rule to the directions of I and $\vec{B}$. $1\text{ gauss} = 10^{-4}\text{ T}$.

EXECUTE: **(a)** The directions of I and $\vec{B}$ are sketched in Figure 27.34a. $\phi = 90°$ so $F = (1.5\text{ A})(2.5\text{ m})(0.55\times10^{-4}\text{ T}) = 2.1\times10^{-4}\text{ N}$. The right-hand rule says that $\vec{F}$ is directed out of the page, so it is upward.

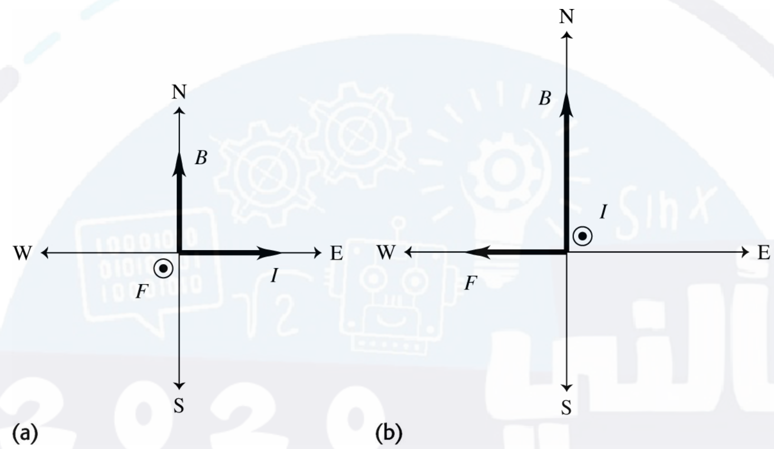


Figure 27.34

(b) The directions of I and $\vec{B}$ are sketched in Figure 27.34b. $\phi = 90°$ and $F = 2.1\times10^{-4}\text{ N}$. $\vec{F}$ is directed east to west.

(c) $\vec{B}$ and the direction of the current are antiparallel. $\phi = 180°$ so $F = 0$.

(d) The magnetic force of $2.1\times10^{-4}\text{ N}$ is not large enough to cause significant effects.

EVALUATE: The magnetic force is a maximum when the directions of I and $\vec{B}$ are perpendicular and it is zero when the current and magnetic field are either parallel or antiparallel.

27.35. **IDENTIFY:** Apply $F = IlB\sin\phi$.

SET UP: Label the three segments in the field as a, b, and c. Let x be the length of segment a. Segment b has length 0.300 m and segment c has length $0.600\text{ m} - x$. Figure 27.35a shows the direction of the force on each segment. For each segment, $\phi = 90°$. The total force on the wire is the vector sum of the forces on each segment.

EXECUTE: $F_a = IlB = (4.50\text{ A})x(0.240\text{ T})$. $F_c = (4.50\text{ A})(0.600\text{ m} - x)(0.240\text{ T})$. Since $\vec{F}_a$ and $\vec{F}_c$ are in the same direction their vector sum has magnitude

$F_{ac} = F_a + F_c = (4.50\text{ A})(0.600\text{ m})(0.240\text{ T}) = 0.648\text{ N}$ and is directed toward the bottom of the page in Figure 27.35a. $F_b = (4.50\text{ A})(0.300\text{ m})(0.240\text{ T}) = 0.324\text{ N}$ and is directed to the right. The vector addition diagram for $\vec{F}_{ac}$ and $\vec{F}_b$ is given in Figure 27.35b.

$F = \sqrt{F_{ac}^2 + F_b^2} = \sqrt{(0.648\text{ N})^2 + (0.324\text{ N})^2} = 0.724\text{ N}$. $\tan\theta = \dfrac{F_{ac}}{F_b} = \dfrac{0.648\text{ N}}{0.324\text{ N}}$ and $\theta = 63.4°$. The net force has magnitude 0.724 N and its direction is specified by $\theta = 63.4°$ in Figure 27.35b.

EVALUATE: All three current segments are perpendicular to the magnetic field, so $\phi = 90°$ for each in the force equation. The direction of the force on a segment depends on the direction of the current for that segment.

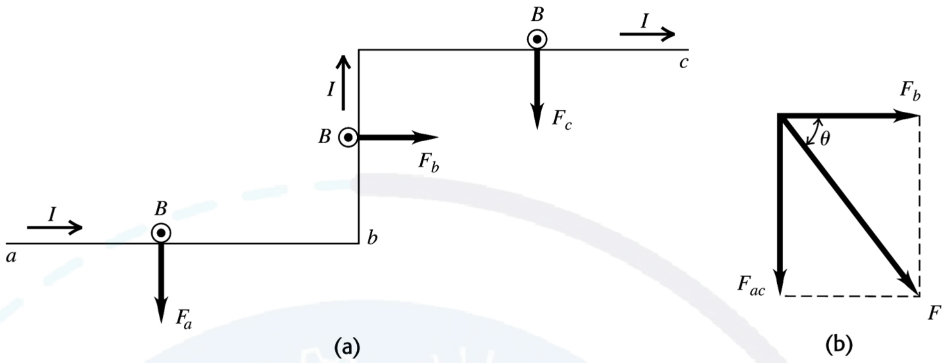


Figure 27.35

27.36. **IDENTIFY:** Apply $F = IlB\sin\phi$.

SET UP: $l = 0.0500$ m is the length of wire in the magnetic field. Since the wire is perpendicular to $\vec{B}$, $\phi = 90°$.

EXECUTE: $F = IlB = (10.8\text{ A})(0.0500\text{ m})(0.550\text{ T}) = 0.297\text{ N}$.

EVALUATE: The force per unit length of wire is proportional to both B and I.

27.37. **IDENTIFY** and **SET UP:** The magnetic force is given by $F = IlB\sin\phi$. $F_I = mg$ when the bar is just ready to levitate. When I becomes larger, $F_I > mg$ and $F_I - mg$ is the net force that accelerates the bar upward. Use Newton's second law to find the acceleration.

EXECUTE: **(a)** $IlB = mg,\ I = \dfrac{mg}{lB} = \dfrac{(0.750\text{ kg})(9.80\text{ m/s}^2)}{(0.500\text{ m})(0.450\text{ T})} = 32.67\text{ A}$.

$V = IR = (32.67\text{ A})(25.0\ \Omega) = 817\text{ V}$.

(b) $R = 2.0\ \Omega, I = \varepsilon/R = (816.7\text{ V})/(2.0\ \Omega) = 408\text{ A}$.

$F_I = IlB = 92\text{ N}$.

$a = (F_I - mg)/m = 113\text{ m/s}^2$.

EVALUATE: I increases by over an order of magnitude when R changes to $F_I >> mg$ and a is an order of magnitude larger than g.

27.38. **IDENTIFY** and **SET UP:** $F = IlB\sin\phi$. The direction of $\vec{F}$ is given by applying the right-hand rule to the directions of I and $\vec{B}$.

EXECUTE: **(a)** The current and field directions are shown in Figure 27.38a (next page). The right-hand rule gives that $\vec{F}$ is directed to the south, as shown. $\phi = 90°$ and

$F = (2.60\text{ A})(1.00 \times 10^{-2}\text{ m})(0.588\text{ T}) = 0.0153\text{ N}$.

(b) The right-hand rule gives that $\vec{F}$ is directed to the west, as shown in Figure 27.38b. $\phi = 90°$ and $F = 0.0153\text{ N}$, the same as in part (a).

(c) The current and field directions are shown in Figure 27.38c. The right-hand rule gives that $\vec{F}$ is $60.0°$ north of west. $\phi = 90°$ so $F = 0.0153\text{ N}$, the same as in part (a).

EVALUATE: In each case the current direction is perpendicular to the magnetic field. The magnitude of the magnetic force is the same in each case but its direction depends on the direction of the magnetic field.

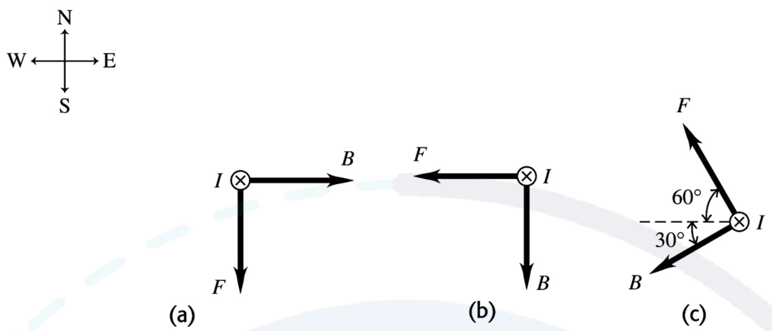


Figure 27.38

27.39. IDENTIFY: The magnetic force $\vec{F}_B$ must be upward and equal to mg. The direction of $\vec{F}_B$ is determined by the direction of I in the circuit.

SET UP: $F_B = IlB\sin\phi$, with $\phi = 90°$. $I = \dfrac{V}{R}$, where V is the battery voltage.

EXECUTE: **(a)** The forces are shown in Figure 27.39. The current I in the bar must be to the right to produce $\vec{F}_B$ upward. To produce current in this direction, point a must be the positive terminal of the battery.

(b) $F_B = mg$. $IlB = mg$. $m = \dfrac{IlB}{g} = \dfrac{VlB}{Rg} = \dfrac{(175\text{ V})(0.600\text{ m})(1.50\text{ T})}{(5.00\,\Omega)(9.80\text{ m/s}^2)} = 3.21\text{ kg}$.

EVALUATE: If the battery had opposite polarity, with point a as the negative terminal, then the current would be clockwise and the magnetic force would be downward.

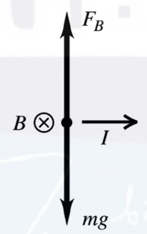


Figure 27.39

27.40. IDENTIFY: $\tau = IAB\sin\phi$. The magnetic moment of the loop is $\mu = IA$.

SET UP: Since the plane of the loop is parallel to the field, the field is perpendicular to the normal to the loop and $\phi = 90°$.

EXECUTE: **(a)** $\tau = IAB = (6.2\text{ A})(0.050\text{ m})(0.080\text{ m})(0.19\text{ T}) = 4.7\times10^{-3}\text{ N}\cdot\text{m}$.

(b) $\mu = IA = (6.2\text{ A})(0.050\text{ m})(0.080\text{ m}) = 0.025\text{ A}\cdot\text{m}^2$.

(c) Maximum area is when the loop is circular. $R = \dfrac{0.050\text{ m} + 0.080\text{ m}}{\pi} = 0.0414\text{ m}$.

$A = \pi R^2 = 5.38\times10^{-3}\text{ m}^2$ and $\tau = (6.2\text{ A})(5.38\times10^{-3}\text{ m}^2)(0.19\text{ T}) = 6.34\times10^{-3}\text{ N}\cdot\text{m}$.

EVALUATE: The torque is a maximum when the field is in the plane of the loop and $\phi = 90°$.

27.41. IDENTIFY: The wire segments carry a current in an external magnetic field. Only segments ab and cd will experience a magnetic force since the other two segments carry a current parallel (and antiparallel) to the magnetic field. Only the force on segment cd will produce a torque about the hinge.

SET UP: $F = IlB\sin\phi$. The direction of the magnetic force is given by the right-hand rule applied to the directions of I and $\vec{B}$. The torque due to a force equals the force times the moment arm, the perpendicular distance between the axis and the line of action of the force.

EXECUTE: **(a)** The direction of the magnetic force on each segment of the circuit is shown in Figure 27.41. For segments *bc* and *da* the current is parallel or antiparallel to the field and the force on these segments is zero.

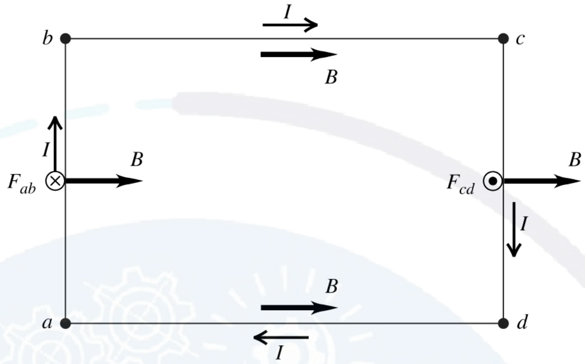


Figure 27.41

(b) $\vec{F}_{ab}$ acts at the hinge and therefore produces no torque. $\vec{F}_{cd}$ tends to rotate the loop about the hinge so it does produce a torque about this axis. $F_{cd} = IlB\sin\phi = (5.00\text{ A})(0.200\text{ m})(1.20\text{ T})\sin 90° = 1.20\text{ N}$

(c) $\tau = Fl = (1.20\text{ N})(0.350\text{ m}) = 0.420\text{ N}\cdot\text{m}.$

EVALUATE: The torque is directed so as to rotate side *cd* out of the plane of the page in Figure 27.41.

27.42. **IDENTIFY:** $\tau = IAB\sin\phi$, where ϕ is the angle between $\vec{B}$ and the normal to the loop.

SET UP: The coil as viewed along the axis of rotation is shown in Figure 27.42a for its original position and in Figure 27.42b after it has rotated $30.0°$.

EXECUTE: **(a)** The forces on each side of the coil are shown in Figure 27.42a. $\vec{F}_1 + \vec{F}_2 = 0$ and $\vec{F}_3 + \vec{F}_4 = 0.$ The net force on the coil is zero. $\phi = 0°$ and $\sin\phi = 0,$ so $\tau = 0.$ The forces on the coil produce no torque.

(b) The net force is still zero. $\phi = 30.0°$ and the net torque is $\tau = (1)(1.95\text{ A})(0.220\text{ m})(0.350\text{ m})(1.50\text{ T})\sin 30.0° = 0.113\text{ N}\cdot\text{m}.$ The net torque is clockwise in Figure 27.42b and is directed so as to increase the angle ϕ.

EVALUATE: For any current loop in a uniform magnetic field the net force on the loop is zero. The torque on the loop depends on the orientation of the plane of the loop relative to the magnetic field direction.

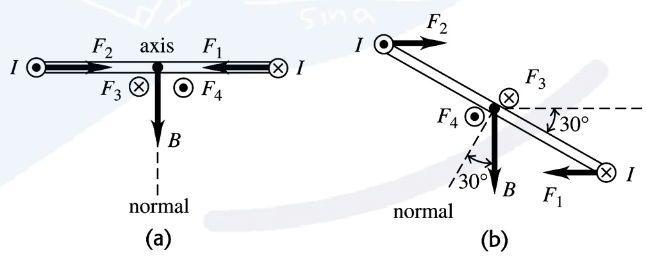


Figure 27.42

27.43. **IDENTIFY:** The magnetic field exerts a torque on the current-carrying coil, which causes it to turn. We can use the rotational form of Newton's second law to find the angular acceleration of the coil.

SET UP: The magnetic torque is given by $\vec{\tau} = \vec{\mu} \times \vec{B}$, and the rotational form of Newton's second law is $\sum\tau = I\alpha.$ The magnetic field is parallel to the plane of the loop.

EXECUTE: **(a)** The coil rotates about axis A_2 because the only torque is along top and bottom sides of the coil.

(b) To find the moment of inertia of the coil, treat the two 1.00-m segments as point-masses (since all the points in them are 0.250 m from the rotation axis) and the two 0.500-m segments as thin uniform bars rotated about their centers. Since the coil is uniform, the mass of each segment is proportional to its fraction of the total perimeter of the coil. Each 1.00-m segment is 1/3 of the total perimeter, so its mass is

(1/3)(210 g) = 70 g = 0.070 kg. The mass of each 0.500-m segment is half this amount, or 0.035 kg. The result is

$$I = 2(0.070 \text{ kg})(0.250 \text{ m})^2 + 2\tfrac{1}{12}(0.035 \text{ kg})(0.500 \text{ m})^2 = 0.0102 \text{ kg}\cdot\text{m}^2.$$

The torque is

$$|\vec{\tau}| = |\vec{\mu}\times\vec{B}| = IAB\sin 90° = (2.00\text{A})(0.500\text{m})(1.00\text{m})(3.00\text{T}) = 3.00 \text{ N}\cdot\text{m}.$$

Using the above values, the rotational form of Newton's second law gives

$$\alpha = \frac{\tau}{I} = 290 \text{ rad/s}^2.$$

EVALUATE: This angular acceleration will not continue because the torque changes as the coil turns.

27.44. IDENTIFY and SET UP: Both coils A and B have the same area A and N turns, but they carry current in opposite directions in a magnetic field. The torque is $\vec{\tau} = \vec{\mu}\times\vec{B}$ and the potential energy is $U = -\mu B\cos\phi$. The magnetic moment is $\vec{\mu} = I\vec{A}$.

EXECUTE: **(a)** Using the right-hand rule for the magnetic moment, $\vec{\mu}$ points in the $-z$-direction (into the page) for coil A and in the $+z$-direction (out of the page) for coil B.

(b) The torque is $\vec{\tau} = \vec{\mu}\times\vec{B}$ which has magnitude $\tau = \mu B\sin\phi$. For coil A, ϕ = 180°, and for coil B, ϕ = 0°. In both cases, sin ϕ = 0, making the torque zero.

(c) For coil A: $U_A = -\mu B\cos\phi = -NIAB\cos 180° = NIAB.$

For coil B: $U_B = -\mu B\cos\phi = -NIAB\cos 0° = -NIAB.$

(d) If coil A is rotated slightly from its equilibrium position, the magnetic field will flip it 180°, so its equilibrium is unstable. But if the same thing it done to coil B, the magnetic field will return it to its original equilibrium position, which makes its equilibrium stable.

EVALUATE: For the stable equilibrium (coil B), its potential energy is a minimum, while for the unstable equilibrium (coil A), its potential energy is a maximum.

27.45. IDENTIFY: $\vec{\tau} = \vec{\mu}\times\vec{B}$ and $U = -\mu B\cos\phi$, where $\mu = NIA$. $\tau = \mu B\sin\phi$.

SET UP: ϕ is the angle between $\vec{B}$ and the normal to the plane of the loop.

EXECUTE: **(a)** $\phi = 90°$. $\tau = NIAB\sin(90°) = NIAB$, direction $\hat{k}\times\hat{j} = -\hat{i}$. $U = -\mu B\cos\phi = 0$.

(b) $\phi = 0$. $\tau = NIAB\sin(0) = 0$, no direction. $U = -\mu B\cos\phi = -NIAB$.

(c) $\phi = 90°$. $\tau = NIAB\sin(90°) = NIAB$, direction $-\hat{k}\times\hat{j} = \hat{i}$. $U = -\mu B\cos\phi = 0$.

(d) $\phi = 180°$: $\tau = NIAB\sin(180°) = 0$, no direction, $U = -\mu B\cos(180°) = NIAB$.

EVALUATE: When τ is maximum, $U = 0$. When $|U|$ is maximum, $\tau = 0$.

27.46. IDENTIFY and SET UP: The potential energy is given by $U = -\vec{\mu}\cdot\vec{B}$. The scalar product depends on the angle between $\vec{\mu}$ and $\vec{B}$.

EXECUTE: For $\vec{\mu}$ and $\vec{B}$ parallel, $\phi = 0°$ and $\vec{\mu}\cdot\vec{B} = \mu B\cos\phi = \mu B$. For $\vec{\mu}$ and $\vec{B}$ antiparallel, $\phi = 180°$ and $\vec{\mu}\cdot\vec{B} = \mu B\cos\phi = -\mu B$.

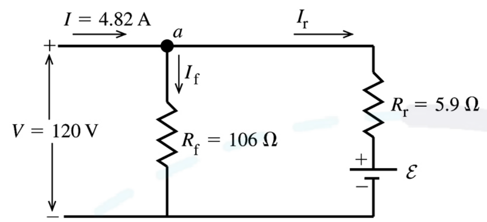
$U_1 = +\mu B$, $U_2 = -\mu B$.

$\Delta U = U_2 - U_1 = -2\mu B = -2(1.45 \text{ A}\cdot\text{m}^2)(0.835 \text{ T}) = -2.42$ J.

EVALUATE: U is maximum when $\vec{\mu}$ and $\vec{B}$ are antiparallel and minimum when they are parallel. When the coil is rotated as specified its magnetic potential energy decreases.

27.47. IDENTIFY: The circuit consists of two parallel branches with the potential difference of 120 V applied across each. One branch is the rotor, represented by a resistance R_r and an induced emf that opposes the applied potential. Apply the loop rule to each parallel branch and use the junction rule to relate the currents through the field coil and through the rotor to the 4.82 A supplied to the motor.

SET UP: The circuit is sketched in Figure 27.47.



ε is the induced emf developed by the motor. It is directed so as to oppose the current through the rotor.

Figure 27.47

EXECUTE: **(a)** The field coils and the rotor are in parallel with the applied potential difference V, so $V = I_f R_f$. $I_f = \dfrac{V}{R_f} = \dfrac{120\text{ V}}{106\ \Omega} = 1.13\text{ A}$.

(b) Applying the junction rule to point a in the circuit diagram gives $I - I_f - I_r = 0$.

$I_r = I - I_f = 4.82\text{ A} - 1.13\text{ A} = 3.69\text{ A}$.

(c) The potential drop across the rotor, $I_r R_r + \varepsilon$, must equal the applied potential difference V: $V = I_r R_r + \varepsilon$

$\varepsilon = V - I_r R_r = 120\text{ V} - (3.69\text{ A})(5.9\ \Omega) = 98.2\text{ V}$

(d) The mechanical power output is the electrical power input minus the rate of dissipation of electrical energy in the resistance of the motor:

electrical power input to the motor

$P_{in} = IV = (4.82\text{ A})(120\text{ V}) = 578\text{ W}$.

electrical power loss in the two resistances

$P_{loss} = I_f^2 R_f + I_r^2 R_r = (1.13\text{ A})^2(106\ \Omega) + (3.69\text{ A})^2(5.9\ \Omega) = 216\text{ W}$.

mechanical power output

$P_{out} = P_{in} - P_{loss} = 578\text{ W} - 216\text{ W} = 362\text{ W}$.

The mechanical power output is the power associated with the induced emf ε.

$P_{out} = P_\varepsilon = \varepsilon I_r = (98.2\text{ V})(3.69\text{ A}) = 362\text{ W}$, which agrees with the above calculation.

EVALUATE: The induced emf reduces the amount of current that flows through the rotor. This motor differs from the one described in Example 27.11. In that example the rotor and field coils are connected in series and in this problem they are in parallel.

27.48. **IDENTIFY:** Apply $V_{ab} = \varepsilon + Ir$ in order to calculate I. The power drawn from the line is $P_{supplied} = IV_{ab}$.

The mechanical power is the power supplied minus the $I^2 r$ electrical power loss in the internal resistance of the motor.

SET UP: $V_{ab} = 120\text{ V}$, $\varepsilon = 105\text{ V}$, and $r = 3.2\ \Omega$.

EXECUTE: **(a)** $V_{ab} = \varepsilon + Ir \Rightarrow I = \dfrac{V_{ab} - \varepsilon}{r} = \dfrac{120\text{ V} - 105\text{ V}}{3.2\ \Omega} = 4.7\text{ A}$.

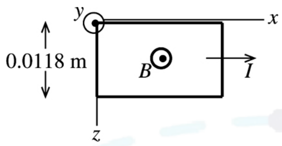
(b) $P_{supplied} = IV_{ab} = (4.7\text{ A})(120\text{ V}) = 564\text{ W}$.

(c) $P_{mech} = IV_{ab} - I^2 r = 564\text{ W} - (4.7\text{ A})^2(3.2\ \Omega) = 493\text{ W}$.

EVALUATE: If the rotor isn't turning, when the motor is first turned on or if the rotor bearings fail, then $\varepsilon = 0$ and $I = \dfrac{120\text{V}}{3.2\ \Omega} = 37.5\text{ A}$. This large current causes large $I^2 r$ heating and can trip the circuit breaker.

27.49. **IDENTIFY:** The drift velocity is related to the current density by $J_x = n|q|v_d$. The electric field is determined by the requirement that the electric and magnetic forces on the current-carrying charges are equal in magnitude and opposite in direction.

SET UP and EXECUTE: **(a)** The section of the silver ribbon is sketched in Figure 27.49a.



$$J_x = n|q|v_{\mathrm{d}}.$$

$$\text{so } v_{\mathrm{d}} = \frac{J_x}{n|q|}.$$

Figure 27.49a

EXECUTE: $J_x = \dfrac{I}{A} = \dfrac{I}{y_1 z_1} = \dfrac{120\text{ A}}{(0.23\times10^{-3}\text{ m})(0.0118\text{ m})} = 4.42\times10^{7}\text{ A/m}^2.$

$$v_{\mathrm{d}} = \frac{J_x}{n|q|} = \frac{4.42\times10^{7}\text{ A/m}^2}{(5.85\times10^{28}/\text{m}^3)(1.602\times10^{-19}\text{ C})} = 4.7\times10^{-3}\text{ m/s} = 4.7\text{ mm/s}.$$

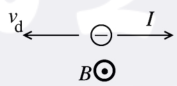
(b) magnitude of $\vec{E}$:

$|q|E_z = |q|v_{\mathrm{d}}B_y.$

$E_z = v_{\mathrm{d}}B_y = (4.7\times10^{-3}\text{ m/s})(0.95\text{ T}) = 4.5\times10^{-3}\text{ V/m}.$

direction of $\vec{E}$:

The drift velocity of the electrons is in the opposite direction to the current, as shown in Figure 27.49b.

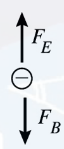


$\vec{v}\times\vec{B}\uparrow.$

$\vec{F}_B = q\vec{v}\times\vec{B} = -e\vec{v}\times\vec{B}\downarrow.$

Figure 27.49b

The directions of the electric and magnetic forces on an electron in the ribbon are shown in Figure 27.49c.



$\vec{F}_E$ must oppose $\vec{F}_B$ so $\vec{F}_E$ is in the $-z$-direction.

Figure 27.49c

$\vec{F}_E = q\vec{E} = -e\vec{E}$ so $\vec{E}$ is opposite to the direction of $\vec{F}_E$ and thus $\vec{E}$ is in the $+z$-direction.

(c) The Hall emf is the potential difference between the two edges of the strip (at $z = 0$ and $z = z_1$) that results from the electric field calculated in part (b). $\varepsilon_{\text{Hall}} = Ez_1 = (4.5\times10^{-3}\text{ V/m})(0.0118\text{ m}) = 53\,\mu\text{V}.$

EVALUATE: Even though the current is quite large the Hall emf is very small. Our calculated Hall emf is more than an order of magnitude larger than in Example 27.12. In this problem the magnetic field and current density are larger than in the example, and this leads to a larger Hall emf.

27.50. **IDENTIFY:** Apply $qn = \dfrac{-J_x B_y}{E_z}.$

SET UP: $A = y_1 z_1.$ $E = \varepsilon/z_1.$ $|q| = e.$

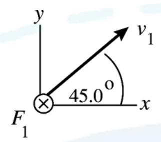
EXECUTE: $n = \dfrac{J_x B_y}{|q|E_z} = \dfrac{IB_y}{A|q|E_z} = \dfrac{IB_y z_1}{A|q|\varepsilon} = \dfrac{IB_y}{y_1|q|\varepsilon}.$

$$n = \frac{(78.0\text{ A})(2.29\text{ T})}{(2.3\times10^{-4}\text{ m})(1.6\times10^{-19}\text{ C})(1.31\times10^{-4}\text{ V})} = 3.7\times10^{28}\text{ electrons/m}^3.$$

EVALUATE: The value of n for this metal is about one-third the value of n calculated in Example 27.12 for copper.

27.51. IDENTIFY: Use $\vec{F} = q\vec{v}\times\vec{B}$ to relate $\vec{v}$, $\vec{B}$, and $\vec{F}$.

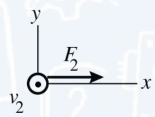
SET UP: The directions of $\vec{v}_1$ and $\vec{F}_1$ are shown in Figure 27.51a.



$\vec{F} = q\vec{v}\times\vec{B}$ says that $\vec{F}$ is perpendicular to $\vec{v}$ and $\vec{B}$. The information given here means that $\vec{B}$ can have no z-component.

Figure 27.51a

The directions of $\vec{v}_2$ and $\vec{F}_2$ are shown in Figure 27.51b.



$\vec{F}$ is perpendicular to $\vec{v}$ and $\vec{B}$, so $\vec{B}$ can have no x-component.

Figure 27.51b

Both pieces of information taken together say that $\vec{B}$ is in the y-direction; $\vec{B} = B_y\hat{j}$.

EXECUTE: **(a)** Use the information given about $\vec{F}_2$ to calculate B_y: $\vec{F}_2 = F_2\hat{i}, \vec{v}_2 = v_2\hat{k}, \vec{B} = B_y\hat{j}$.

$\vec{F}_2 = q\vec{v}_2\times\vec{B}$ says $F_2\hat{i} = qv_2B_y\hat{k}\times\hat{j} = qv_2B_y(-\hat{i})$ and $F_2 = -qv_2B_y$.

$B_y = -F_2/(qv_2) = -F_2/(qv_1)$. $\vec{B}$ has the magnitude $F_2/(qv_1)$ and is in the $-y$-direction.

(b) $F_1 = qvB\sin\phi = qv_1\left|B_y\right|/\sqrt{2} = F_2/\sqrt{2}$.

EVALUATE: $v_1 = v_2$. $\vec{v}_2$ is perpendicular to $\vec{B}$ whereas only the component of $\vec{v}_1$ perpendicular to $\vec{B}$ contributes to the force, so it is expected that $F_2 > F_1$, as we found.

27.52. IDENTIFY: Apply $\vec{F} = q\vec{v}\times\vec{B}$.

SET UP: $B_x = 0.650$ T. $B_y = 0$ and $B_z = 0$.

EXECUTE: $F_x = q(v_yB_z - v_zB_y) = 0$.

$F_y = q(v_zB_x - v_xB_z) = (7.26\times10^{-8}\text{ C})(5.85\times10^{4}\text{ m/s})(0.650\text{ T}) = 2.76\times10^{-3}\text{ N}$.

$F_z = q(v_xB_y - v_yB_x) = -(7.26\times10^{-8}\text{ C})(-3.11\times10^{4}\text{ m/s})(0.650\text{ T}) = 1.47\times10^{-3}\text{ N}$.

EVALUATE: $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$. We can verify that $\vec{F}\cdot\vec{v} = 0$. Since $\vec{B}$ is along the x-axis, v_x does not affect the force components.

27.53. IDENTIFY: In part (a), apply conservation of energy to the motion of the two nuclei. In part (b) apply $|q|vB = mv^2/R$.

SET UP: In part (a), let point 1 be when the two nuclei are far apart and let point 2 be when they are at their closest separation.

EXECUTE: **(a)** $K_1 + U_1 = K_2 + U_2$. $U_1 = K_2 = 0$, so $K_1 = U_2$. There are two nuclei having equal kinetic energy, so $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = ke^2/r$. Solving for v gives

$$v = e\sqrt{\frac{k}{mr}} = (1.602\times10^{-19}\text{ C})\sqrt{\frac{8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2}{(3.34\times10^{-27}\text{ kg})(1.0\times10^{-15}\text{ m})}} = 8.3\times10^{6}\text{ m/s}.$$

(b) $\sum \vec{F} = m\vec{a}$ gives $qvB = mv^2/r$. $B = \dfrac{mv}{qr} = \dfrac{(3.34\times10^{-27}\text{ kg})(8.3\times10^{6}\text{ m/s})}{(1.602\times10^{-19}\text{ C})(1.25\text{ m})} = 0.14\text{ T}$.

EVALUATE: The speed calculated in part (a) is large, nearly 3% of the speed of light.

27.54. **IDENTIFY:** The period is $T = 2\pi r/v$, the current is Q/t and the magnetic moment is $\mu = IA$.

SET UP: The electron has charge $-e$. The area enclosed by the orbit is πr^2.

EXECUTE: **(a)** $T = 2\pi r/v = 1.5\times10^{-16}$ s.

(b) Charge $-e$ passes a point on the orbit once during each period, so $I = Q/t = e/t = 1.1$ mA.

(c) $\mu = IA = I\pi r^2 = 9.3\times10^{-24}\text{ A}\cdot\text{m}^2$.

EVALUATE: Since the electron has negative charge, the direction of the current is opposite to the direction of motion of the electron.

27.55. **IDENTIFY:** The sum of the magnetic, electrical and gravitational forces must be zero to aim at and hit the target.

SET UP: The magnetic field must point to the left when viewed in the direction of the target for no net force. The net force is zero, so $\sum F = F_B - F_E - mg = 0$ and $qvB - qE - mg = 0$.

EXECUTE: Solving for B gives

$$B = \frac{qE + mg}{qv} = \frac{(2500\times10^{-6}\text{ C})(27.5\text{ N/C}) + (0.00425\text{ kg})(9.80\text{ m/s}^2)}{(2500\times10^{-6}\text{ C})(12.8\text{ m/s})} = 3.45\text{ T}.$$

The direction should be perpendicular to the initial velocity of the coin.

EVALUATE: This is a very strong magnetic field, but achievable in some labs.

27.56. **IDENTIFY** and **SET UP:** The maximum radius of the orbit determines the maximum speed v of the protons. Use Newton's second law and $a_{\text{rad}} = v^2/R$ for circular motion to relate the variables. The energy of the particle is the kinetic energy $K = \frac{1}{2}mv^2$.

EXECUTE: **(a)** $\sum \vec{F} = m\vec{a}$ gives $|q|vB = m(v^2/R)$.

$v = \dfrac{|q|BR}{m} = \dfrac{(1.60\times10^{-19}\text{ C})(0.85\text{ T})(0.40\text{ m})}{1.67\times10^{-27}\text{ kg}} = 3.257\times10^{7}\text{ m/s}$. The kinetic energy of a proton moving with this speed is $K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67\times10^{-27}\text{ kg})(3.257\times10^{7}\text{ m/s})^2 = 8.9\times10^{-13}\text{ J} = 5.5\text{ MeV}$.

(b) The time for one revolution is the period $T = \dfrac{2\pi R}{v} = \dfrac{2\pi(0.40\text{ m})}{3.257\times10^{7}\text{ m/s}} = 7.7\times10^{-8}$ s.

(c) $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\dfrac{|q|BR}{m}\right)^2 = \frac{1}{2}\dfrac{|q|^2B^2R^2}{m}$. Or, $B = \dfrac{\sqrt{2Km}}{|q|R}$. B is proportional to $\sqrt{K}$, so if K is increased by a factor of 2 then B must be increased by a factor of $\sqrt{2}$. $B = \sqrt{2}(0.85\text{ T}) = 1.2\text{ T}$.

(d) $v = \dfrac{|q|BR}{m} = \dfrac{(3.20\times10^{-19}\text{ C})(0.85\text{ T})(0.40\text{ m})}{6.65\times10^{-27}\text{ kg}} = 1.636\times10^{7}\text{ m/s}$

$K = \frac{1}{2}mv^2 = \frac{1}{2}(6.65\times10^{-27}\text{ kg})(1.636\times10^{7}\text{ m/s})^2 = 8.9\times10^{-13}\text{ J} = 5.5\text{ MeV}$, the same as the maximum energy for protons.

EVALUATE: We can see that the maximum energy must be approximately the same as follows: From part (c), $K = \frac{1}{2}m\left(\dfrac{|q|BR}{m}\right)^2$. For alpha particles $|q|$ is larger by a factor of 2 and m is larger by a factor of 4 (approximately). Thus $|q|^2/m$ is unchanged and K is the same.

27.57. **IDENTIFY** and **SET UP:** Use $\vec{F} = q\vec{v}\times\vec{B}$ to relate q, $\vec{v}$, $\vec{B}$ and $\vec{F}$. The force $\vec{F}$ and $\vec{a}$ are related by Newton's second law. $\vec{B} = -(0.120\text{ T})\hat{k}$, $\vec{v} = (1.05\times10^{6}\text{ m/s})(-3\hat{i} + 4\hat{j} + 12\hat{k})$, $F = 2.45$ N.

EXECUTE: **(a)** $\vec{F} = q\vec{v}\times\vec{B}$. $\vec{F} = q(-0.120\text{ T})(1.05\times10^{6}\text{ m/s})(-3\hat{i}\times\hat{k} + 4\hat{j}\times\hat{k} + 12\hat{k}\times\hat{k})$.

$\hat{\boldsymbol{i}}\times\hat{\boldsymbol{k}}=-\hat{\boldsymbol{j}},\ \hat{\boldsymbol{j}}\times\hat{\boldsymbol{k}}=\hat{\boldsymbol{i}},\ \hat{\boldsymbol{k}}\times\hat{\boldsymbol{k}}=0.$ $\vec{\boldsymbol{F}}=-q(1.26\times10^5\text{ N/C})(+3\hat{\boldsymbol{j}}+4\hat{\boldsymbol{i}})=-q(1.26\times10^5\text{ N/C})(+4\hat{\boldsymbol{i}}+3\hat{\boldsymbol{j}})$. The magnitude of the vector $+4\hat{\boldsymbol{i}}+3\hat{\boldsymbol{j}}$ is $\sqrt{3^2+4^2}=5$. Thus $F=-q(1.26\times10^5\text{ N/C})(5)$.

$$q=-\frac{F}{5(1.26\times10^5\text{ N/C})}=-\frac{2.45\text{ N}}{5(1.26\times10^5\text{ N/C})}=-3.89\times10^{-6}\text{ C}.$$

(b) $\Sigma\vec{\boldsymbol{F}}=m\vec{\boldsymbol{a}}$ so $\vec{\boldsymbol{a}}=\vec{\boldsymbol{F}}/m$.

$\vec{\boldsymbol{F}}=-q(1.26\times10^5\text{ N/C})(+4\hat{\boldsymbol{i}}+3\hat{\boldsymbol{j}})=-(-3.89\times10^{-6}\text{ C})(1.26\times10^5\text{ N/C})(+4\hat{\boldsymbol{i}}+3\hat{\boldsymbol{j}})=+0.490\text{ N}(+4\hat{\boldsymbol{i}}+3\hat{\boldsymbol{j}})$.

Then

$$\vec{\boldsymbol{a}}=\vec{\boldsymbol{F}}/m=\left(\frac{0.490\text{ N}}{2.58\times10^{-15}\text{ kg}}\right)(+4\hat{\boldsymbol{i}}+3\hat{\boldsymbol{j}})=(1.90\times10^{14}\text{ m/s}^2)(+4\hat{\boldsymbol{i}}+3\hat{\boldsymbol{j}})=7.60\times10^{14}\text{ m/s}^2\hat{\boldsymbol{i}}+5.70\times10^{14}\text{ m/s}^2\hat{\boldsymbol{j}}.$$

(c) IDENTIFY and SET UP: $\vec{\boldsymbol{F}}$ is in the *xy*-plane, so in the *z*-direction the particle moves with constant speed 12.6×10^6 m/s. In the *xy*-plane the force $\vec{\boldsymbol{F}}$ causes the particle to move in a circle, with $\vec{\boldsymbol{F}}$ directed in toward the center of the circle.

EXECUTE: $\sum\vec{\boldsymbol{F}}=m\vec{\boldsymbol{a}}$ gives $F=m(v^2/R)$ and $R=mv^2/F$.

$$v^2=v_x^2+v_y^2=(-3.15\times10^6\text{ m/s})^2+(+4.20\times10^6\text{ m/s})^2=2.756\times10^{13}\text{ m}^2/\text{s}^2.$$

$$F=\sqrt{F_x^2+F_y^2}=(0.490\text{ N})\sqrt{4^2+3^2}=2.45\text{ N}.$$

$$R=\frac{mv^2}{F}=\frac{(2.58\times10^{-15}\text{ kg})(2.756\times10^{13}\text{ m}^2/\text{s}^2)}{2.45\text{ N}}=0.0290\text{ m}=2.90\text{ cm}.$$

(d) IDENTIFY and SET UP: The cyclotron frequency is $f=\omega/2\pi=v/2\pi R$.

EXECUTE: The circular motion is in the *xy*-plane, so $v=\sqrt{v_x^2+v_y^2}=5.25\times10^6$ m/s.

$$f=\frac{v}{2\pi R}=\frac{5.25\times10^6\text{ m/s}}{2\pi(0.0290\text{ m})}=2.88\times10^7\text{ Hz, and }\omega=2\pi f=1.81\times10^8\text{ rad/s}.$$

(e) IDENTIFY and SET UP: Compare *t* to the period *T* of the circular motion in the *xy*-plane to find the *x*- and *y*-coordinates at this *t*. In the *z*-direction the particle moves with constant speed, so $z=z_0+v_zt$.

EXECUTE: The period of the motion in the *xy*-plane is given by $T=\dfrac{1}{f}=\dfrac{1}{2.88\times10^7\text{ Hz}}=3.47\times10^{-8}$ s. In $t=2T$ the particle has returned to the same *x*- and *y*-coordinates. The *z*-component of the motion is motion with a constant velocity of $v_z=+12.6\times10^6$ m/s. Thus

$z=z_0+v_zt=0+(12.6\times10^6\text{ m/s})(2)(3.47\times10^{-8}\text{ s})=+0.874$ m. The coordinates at $t=2T$ are $x=R=0.0290$ m, $y=0$, $z=+0.874$ m.

EVALUATE: The circular motion is in the plane perpendicular to $\vec{\boldsymbol{B}}$. The radius of this motion gets smaller when *B* increases and it gets larger when *v* increases. There is no magnetic force in the direction of $\vec{\boldsymbol{B}}$ so the particle moves with constant velocity in that direction. The superposition of circular motion in the *xy*-plane and constant speed motion in the *z*-direction is a helical path.

27.58. IDENTIFY: Apply $\vec{\boldsymbol{F}}=q\vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}}$.

SET UP: $\vec{\boldsymbol{v}}=v\hat{\boldsymbol{k}}$.

EXECUTE: **(a)** $\vec{\boldsymbol{F}}=-qvB_y\hat{\boldsymbol{i}}+qvB_x\hat{\boldsymbol{j}}$. But $\vec{\boldsymbol{F}}=3F_0\hat{\boldsymbol{i}}+4F_0\hat{\boldsymbol{j}}$, so $3F_0=-qvB_y$ and $4F_0=qvB_x$.

Therefore, $B_y=-\dfrac{3F_0}{qv}$, $B_x=\dfrac{4F_0}{qv}$ and B_z is undetermined.

(b) $B=\dfrac{6F_0}{qv}=\sqrt{B_x^2+B_y^2+B_z^2}=\dfrac{F_0}{qv}\sqrt{9+16+\left(\dfrac{qv}{F_0}\right)^2B_z^2}=\dfrac{F_0}{qv}\sqrt{25+\left(\dfrac{qv}{F_0}\right)^2B_z^2}$, so $B_z=\pm\dfrac{\sqrt{11}F_0}{qv}$.

EVALUATE: The force doesn't depend on B_z, since $\vec{\boldsymbol{v}}$ is along the *z*-direction.

27.59. **IDENTIFY:** For the velocity selector, $E = vB$. For circular motion in the field B', $R = \dfrac{mv}{|q|B'}$.

SET UP: $B = B' = 0.682$ T.

EXECUTE: $v = \dfrac{E}{B} = \dfrac{1.88\times10^4\ \text{N/C}}{0.682\ \text{T}} = 2.757\times10^4$ m/s. $R = \dfrac{mv}{qB'}$, so

$$R_{82} = \frac{82(1.66\times10^{-27}\ \text{kg})(2.757\times10^4\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.682\ \text{T})} = 0.0344\ \text{m} = 3.44\ \text{cm}.$$

$$R_{84} = \frac{84(1.66\times10^{-27}\ \text{kg})(2.757\times10^4\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.682\ \text{T})} = 0.0352\ \text{m} = 3.52\ \text{cm}.$$

$$R_{86} = \frac{86(1.66\times10^{-27}\ \text{kg})(2.757\times10^4\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.682\ \text{T})} = 0.0361\ \text{m} = 3.61\ \text{cm}.$$

The distance between two adjacent lines is $2\Delta R = 2(3.52\ \text{cm} - 3.44\ \text{cm}) = 0.16\ \text{cm} = 1.6\ \text{mm}$.

EVALUATE: The distance between the ^{82}Kr line and the ^{84}Kr line is 1.6 mm and the distance between the ^{84}Kr line and the ^{86}Kr line is 1.6 mm. Adjacent lines are equally spaced since the ^{82}Kr versus ^{84}Kr and ^{84}Kr versus ^{86}Kr mass differences are the same.

27.60. **IDENTIFY:** Apply conservation of energy to the acceleration of the ions and Newton's second law to their motion in the magnetic field.

SET UP: The singly ionized ions have $q = +e$. A ^{12}C ion has mass 12 u and a ^{14}C ion has mass 14 u, where $1\ \text{u} = 1.66\times10^{-27}$ kg.

EXECUTE: **(a)** During acceleration of the ions, $qV = \frac{1}{2}mv^2$ and $v = \sqrt{\dfrac{2qV}{m}}$. In the magnetic field,

$$R = \frac{mv}{qB} = \frac{m\sqrt{2qV/m}}{qB} \text{ and } m = \frac{qB^2R^2}{2V}.$$

(b) $V = \dfrac{qB^2R^2}{2m} = \dfrac{(1.60\times10^{-19}\ \text{C})(0.150\ \text{T})^2(0.500\ \text{m})^2}{2(12)(1.66\times10^{-27}\ \text{kg})} = 2.26\times10^4$ V.

(c) The ions are separated by the differences in the diameters of their paths. $D = 2R = 2\sqrt{\dfrac{2Vm}{qB^2}}$, so

$$\Delta D = D_{14} - D_{12} = 2\sqrt{\frac{2Vm}{qB^2}}\Bigg|_{14} - 2\sqrt{\frac{2Vm}{qB^2}}\Bigg|_{12} = 2\sqrt{\frac{2V(1\ \text{u})}{qB^2}}(\sqrt{14} - \sqrt{12}).$$

$$\Delta D = 2\sqrt{\frac{2(2.26\times10^4\ \text{V})(1.66\times10^{-27}\ \text{kg})}{(1.6\times10^{-19}\ \text{C})(0.150\ \text{T})^2}}(\sqrt{14} - \sqrt{12}) = 8.01\times10^{-2}\ \text{m}.$$ This is about 8 cm and is easily distinguishable.

EVALUATE: The speed of the ^{12}C ion is $v = \sqrt{\dfrac{2(1.60\times10^{-19}\ \text{C})(2.26\times10^4\ \text{V})}{12(1.66\times10^{-27}\ \text{kg})}} = 6.0\times10^5$ m/s. This is very fast, but well below the speed of light, so relativistic mechanics is not needed.

27.61. **IDENTIFY:** The force exerted by the magnetic field is given by $F = IlB\sin\phi$. The net force on the wire must be zero.

SET UP: For the wire to remain at rest the force exerted on it by the magnetic field must have a component directed up the incline. To produce a force in this direction, the current in the wire must be directed from right to left in the figure with the problem in the textbook. Or, viewing the wire from its left-hand end the directions are shown in Figure 27.61a.

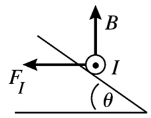


Figure 27.61a

The free-body diagram for the wire is given in Figure 27.61b.

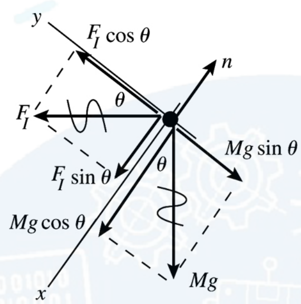


Figure 27.61b

EXECUTE: $\sum F_y = 0.$

$F_I \cos\theta - Mg\sin\theta = 0.$

$F_I = ILB\sin\phi.$

$\phi = 90°$ since $\vec{B}$ is perpendicular to the current direction.

Thus $(ILB)\,\cos\theta - Mg\sin\theta = 0$ and $I = \dfrac{Mg\tan\theta}{LB}$.

EVALUATE: The magnetic and gravitational forces are in perpendicular directions so their components parallel to the incline involve different trig functions. As the tilt angle θ increases there is a larger component of Mg down the incline and the component of F_I up the incline is smaller; I must increase with θ to compensate. As $\theta \to 0$, $I \to 0$ and as $\theta \to 90°$, $I \to \infty$.

27.62. IDENTIFY: In the figure shown with the problem in the text, the current in the bar is toward the bottom of the page, so the magnetic force is toward the right. Newton's second law gives the acceleration. The bar is in parallel with the 10.0-Ω resistor, so we must use circuit analysis to find the initial current through the bar.

SET UP: First find the current. The equivalent resistance across the battery is $30.0\ \Omega$, so the total current is $4.00\,\text{A}$, half of which goes through the bar. Applying Newton's second law to the bar gives $\sum F = ma = F_B = ILB.$

EXECUTE: Equivalent resistance of the $10.0\text{-}\Omega$ resistor and the bar is $5.0\,\Omega$. Current through the $25.0\text{-}\Omega$ resistor is $I_{\text{tot}} = \dfrac{120.0\ \text{V}}{30.0\,\Omega} = 4.00\ \text{A}.$ The current in the bar is 2.00 A, toward the bottom of the page. The force $\vec{F}_I$ that the magnetic field exerts on the bar has magnitude $F_I = IlB$ and is directed to the right. $a = \dfrac{F_I}{m} = \dfrac{IlB}{m} = \dfrac{(2.00\ \text{A})(0.850\ \text{m})(1.60\ \text{T})}{(2.60\ \text{N})/(9.80\ \text{m/s}^2)} = 10.3\ \text{m/s}^2.$ $\vec{a}$ is directed to the right.

EVALUATE: Once the bar has acquired a non-zero speed there will be an induced emf (Chapter 29) and the current and acceleration will start to decrease.

27.63. IDENTIFY: $R = \dfrac{mv}{|q|B}.$

SET UP: After completing one semicircle the separation between the ions is the difference in the diameters of their paths, or $2(R_{13} - R_{12})$. A singly ionized ion has charge $+e$.

EXECUTE: **(a)** $B = \dfrac{mv}{|q|R} = \dfrac{(1.99\times10^{-26}\ \text{kg})(8.50\times10^{3}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.125\ \text{m})} = 8.46\times10^{-3}\ \text{T}.$

(b) The only difference between the two isotopes is their masses. $\dfrac{R}{m}=\dfrac{v}{|q|B}=\text{constant}$ and $\dfrac{R_{12}}{m_{12}}=\dfrac{R_{13}}{m_{13}}$.

$R_{13}=R_{12}\left(\dfrac{m_{13}}{m_{12}}\right)=(12.5\text{ cm})\left(\dfrac{2.16\times10^{-26}\text{ kg}}{1.99\times10^{-26}\text{ kg}}\right)=13.6\text{ cm}.$ The diameter is 27.2 cm.

(c) The separation is $2(R_{13}-R_{12})=2(13.6\text{ cm}-12.5\text{ cm})=2.2\text{ cm}.$ This distance can be easily observed.

EVALUATE: Decreasing the magnetic field increases the separation between the two isotopes at the detector.

27.64. **IDENTIFY:** Turning the charged loop creates a current, and the external magnetic field exerts a torque on that current.

SET UP: The current is $I=q/T=q/(1/f)=qf=q(\omega/2\pi)=q\omega/2\pi.$ The torque is $\tau=\mu B\sin\phi.$

EXECUTE: In this case, $\phi=90°$ and $\mu=IA,$ giving $\tau=IAB.$ Combining the results for the torque and current and using $A=\pi r^2$ gives $\tau=\left(\dfrac{q\omega}{2\pi}\right)\pi r^2B=\frac{1}{2}q\omega r^2B.$

EVALUATE: Any moving charge is a current, so turning the loop creates a current causing a magnetic force.

27.65. **IDENTIFY:** The force exerted by the magnetic field is $F=ILB\sin\phi.$ $a=F/m$ and is constant. Apply a constant acceleration equation to relate v and d.

SET UP: $\phi=90°.$ The direction of $\vec{F}$ is given by the right-hand rule.

EXECUTE: **(a)** $F=ILB,$ to the right.

(b) $v_x^2=v_{0x}^2+2a_x(x-x_0)$ gives $v^2=2ad$ and $d=\dfrac{v^2}{2a}=\dfrac{v^2m}{2ILB}.$

(c) $d=\dfrac{(1.12\times10^4\text{ m/s})^2(25\text{ kg})}{2(2000\text{ A})(0.50\text{ m})(0.80\text{ T})}=1.96\times10^6\text{ m}=1960\text{ km}.$

EVALUATE: $a=\dfrac{ILB}{m}=\dfrac{(2.0\times10^3\text{ A})(0.50\text{ m})(0.80\text{ T})}{25\text{ kg}}=32\text{ m/s}^2.$ The acceleration due to gravity is not negligible. Since the bar would have to travel nearly 2000 km, this would not be a very effective launch mechanism using the numbers given.

27.66. **IDENTIFY:** Apply $\vec{F}=I\vec{l}\times\vec{B}.$

SET UP: $\vec{l}=l\hat{k}.$

EXECUTE: **(a)** $\vec{F}=I(l\hat{k})\times\vec{B}=Il[(-B_y)\hat{i}+(B_x)\hat{j}].$ This gives

$F_x=-IlB_y=-(7.40\text{ A})(0.250\text{ m})(-0.985\text{ T})=1.82\text{ N}$ and

$F_y=IlB_x=(7.40\text{ A})(0.250\text{ m})(-0.242\text{ T})=-0.448\text{ N}.$ $F_z=0,$ since the wire is in the z-direction.

(b) $F=\sqrt{F_x^2+F_y^2}=\sqrt{(1.82\text{ N})^2+(0.448\text{ N})^2}=1.88\text{ N}.$

EVALUATE: $\vec{F}$ must be perpendicular to the current direction, so $\vec{F}$ has no z-component.

27.67. **IDENTIFY:** The magnetic field exerts a force on each of the three segments of the wire due to the current in them. The net force on the wire is the vector sum of these three forces.

SET UP: Label the three segments in the magnetic field 1, 2, and 3, as shown in Figure 27.67. The force on a current carrying conductor is $F=IlB\sin\phi,$ where ϕ is the angle between the direction of the current and the direction of the magnetic field. The direction of the force on each segment is given by the right-hand rule and is shown in the figure. The sum of $\vec{F}_1$ and $\vec{F}_3$ is the same as the force $\vec{F}_{13}$ on a wire 0.307 m long. Section 2 has length 0.800 m. The current in each segment is perpendicular to the magnetic field, so $\phi=90°.$

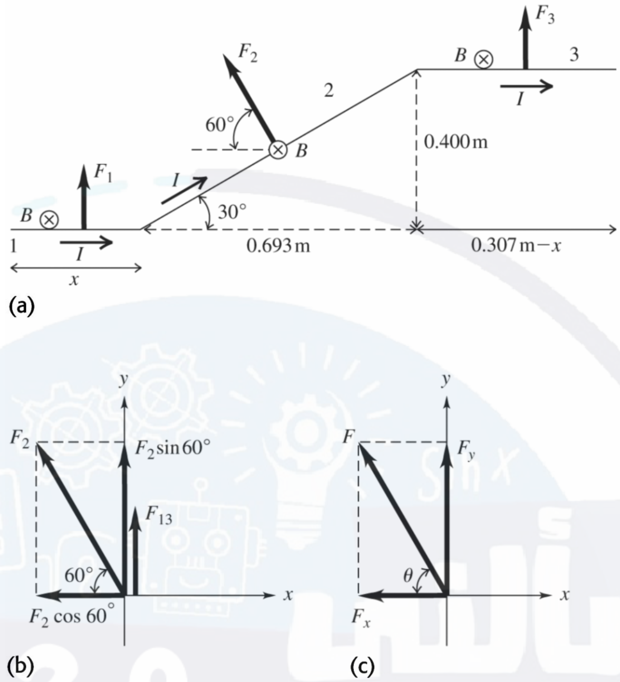


Figure 27.67

EXECUTE: $F_{13} = IlB\sin\phi = (6.50\text{ A})(0.307\text{ m})(0.280\text{ T})\sin 90° = 0.559\text{ N}.$
$F_2 = IlB\sin\phi = (6.50\text{ A})(0.800\text{ m})(0.280\text{ T})\sin 90° = 1.46\text{ N}.$ The forces and a coordinate system are shown in Figure 27.67b. $\vec{F}_2$ has been resolved into its x- and y-components.

$$F_x = F_{2x} + F_{13x} = -F_2\cos 60.0° = -(1.46\text{ N})(\cos 60.0°) = -0.730\text{ N}.$$

$$F_y = F_{2y} + F_{13y} = F_2 \sin 60.0° + F_{13} = +(1.46\text{ N})(\sin 60.0°) + 0.559\text{ N} = +1.83\text{ N}.$$

F_x, F_y, and the resultant total force $\vec{F}$ are shown in Figure 27.67c. The resultant force has magnitude 1.97 N and is at $68.3°$ clockwise from the left-hand straight segment.

EVALUATE: Even though all three segments are perpendicular to the magnetic field, the direction of the force on the segments is not the same. Therefore we must use vector addition to find the force on the wire.

27.68. **IDENTIFY:** The torque exerted by the magnetic field is $\vec{\tau} = \vec{\mu} \times \vec{B}$. The torque required to hold the loop in place is $-\vec{\tau}$.

SET UP: $\mu = IA$. $\vec{\mu}$ is normal to the plane of the loop, with a direction given by the right-hand rule that is illustrated in Figure 27.32 in the textbook. $\tau = IAB\sin\phi$, where ϕ is the angle between the normal to the loop and the direction of $\vec{B}$.

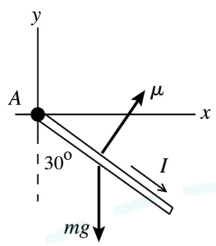
EXECUTE: **(a)** $\tau = IAB \sin 60° = (15.0\text{ A})(0.060\text{ m})(0.080\text{ m})(0.48\text{ T})\sin 60° = 0.030\text{ N}\cdot\text{m}$, in the $-\hat{j}$-direction. To keep the loop in place, you must provide a torque in the $+\hat{j}$-direction.

(b) $\tau = IAB \sin 30° = (15.0\text{ A})(0.60\text{ m})(0.080\text{ m})(0.48\text{ T})\sin 30° = 0.017\text{ N}\cdot\text{m}$, in the $+\hat{j}$-direction. You must provide a torque in the $-\hat{j}$-direction to keep the loop in place.

EVALUATE: **(c)** If the loop was pivoted through its center, then there would be a torque on both sides of the loop parallel to the rotation axis. However, the lever arm is only half as large, so the total torque in each case is identical to the values found in parts (a) and (b).

27.69. **IDENTIFY:** For the loop to be in equilibrium the net torque on it must be zero. Use $\vec{\tau} = \vec{\mu} \times \vec{B}$ to calculate the torque due to the magnetic field and $\tau_{mg} = mgr\sin\phi$ for the torque due to gravity.

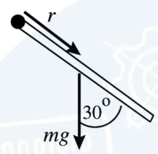
SET UP: See Figure 27.69a (next page).



Use $\sum \tau_A = 0$, where point A is at the origin.

Figure 27.69a

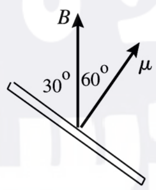
EXECUTE: See Figure 27.69b.



$\tau_{mg} = mgr\sin\phi = mg(0.400\text{ m})\sin 30.0°$.
The torque is clockwise; $\vec{\boldsymbol{\tau}}_{mg}$ is directed into the paper.

Figure 27.69b

For the loop to be in equilibrium the torque due to $\vec{B}$ must be counterclockwise (opposite to $\vec{\boldsymbol{\tau}}_{mg}$) and it must be that $\tau_B = \tau_{mg}$. See Figure 27.69c.



$\vec{\boldsymbol{\tau}}_B = \vec{\boldsymbol{\mu}} \times \vec{B}$. For this torque to be counterclockwise ($\vec{\boldsymbol{\tau}}_B$ directed out of the paper), $\vec{B}$ must be in the $+y$-direction.

Figure 27.69c

$\tau_B = \mu B \sin\phi = IAB\sin 60.0°$.

$\tau_B = \tau_{mg}$ gives $IAB\sin 60.0° = mg(0.0400\text{ m})\sin 30.0°$.

$m = (0.15\text{ g/cm})2(8.00\text{ cm} + 6.00\text{ cm}) = 4.2\text{ g} = 4.2\times10^{-3}\text{ kg}$.

$A = (0.0800\text{ m})(0.0600\text{ m}) = 4.80\times10^{-3}\text{ m}^2$.

$$B = \frac{mg(0.0400\text{ m})(\sin 30.0°)}{IA\sin 60.0°}.$$

$$B = \frac{(4.2\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)(0.0400\text{ m})\sin 30.0°}{(8.2\text{ A})(4.80\times10^{-3}\text{ m}^2)\sin 60.0°} = 0.024\text{ T}.$$

EVALUATE: As the loop swings up the torque due to $\vec{B}$ decreases to zero and the torque due to mg increases from zero, so there must be an orientation of the loop where the net torque is zero.

27.70. **IDENTIFY and SET UP:** The force on a current-carrying bar of length l is $F = IlB$ if the field is perpendicular to the bar. The torque is $\tau_z = \mu B \sin\phi$.

EXECUTE: **(a)** The force on the infinitesimal segment is $dF = IBdl = IBdx$. The torque about point a is $d\tau_z = xdF\sin\phi = xIBdx$. In this case, $\sin\phi = 1$ because the force is perpendicular to the bar.

(b) We integrate to get the total torque: $\tau_z = \int_0^L xIBdx = \frac{1}{2}IBL^2$.

(c) For $F = ILB$ at the center of the bar, the torque is $\tau_z = F\left(\frac{L}{2}\right) = ILB\left(\frac{L}{2}\right) = \frac{1}{2}IBL^2$, which is the same result we got by integrating.

EVALUATE: We can think of the magnetic force as all acting at the center of the bar because the magnetic field is uniform. This is the same reason we can think of gravity acting at the center of a uniform bar.

27.71. IDENTIFY: Apply $\vec{F} = I\vec{l} \times \vec{B}$ to calculate the force on each side of the loop.

SET UP: The net force is the vector sum of the forces on each side of the loop.

EXECUTE: **(a)** $F_{PQ} = (5.00\text{ A})(0.600\text{ m})(3.00\text{ T})\sin(0°) = 0\text{ N}$.

$F_{RP} = (5.00\text{ A})(0.800\text{ m})(3.00\text{ T})\sin(90°) = 12.0\text{ N}$, into the page.

$F_{QR} = (5.00\text{ A})(1.00\text{ m})(3.00\text{ T})(0.800/1.00) = 12.0\text{ N}$, out of the page.

(b) The net force on the triangular loop of wire is zero.

(c) For calculating torque on a straight wire we can assume that the force on a wire is applied at the wire's center. Also, note that we are finding the torque with respect to the *PR*-axis (not about a point), and consequently the lever arm will be the distance from the wire's center to the *x*-axis. $\tau = rF\sin\phi$ gives $\tau_{PQ} = r(0\text{ N}) = 0$, $\tau_{RP} = (0\text{ m})F\sin\phi = 0$ and $\tau_{QR} = (0.300\text{ m})(12.0\text{ N})\sin(90°) = 3.60\text{ N}\cdot\text{m}$. The net torque is $3.60\text{ N}\cdot\text{m}$.

(d) Using $\tau = NIAB\sin\phi$ gives

$\tau = NIAB\sin\phi = (1)(5.00\text{ A})\left(\frac{1}{2}\right)(0.600\text{ m})(0.800\text{ m})(3.00\text{ T})\sin(90°) = 3.60\text{ N}\cdot\text{m}$, which agrees with our result in part (c).

(e) Since F_{QR} is out of the page and since this is the force that produces the net torque, the point *Q* will be rotated out of the plane of the figure.

EVALUATE: In the expression $\tau = NIAB\sin\phi$, ϕ is the angle between the plane of the loop and the direction of $\vec{B}$. In this problem, $\phi = 90°$.

27.72. IDENTIFY: For rotational equilibrium, the torques due to gravity and the magnetic field must balance around point *a*.

SET UP: From Problem 27.70 we have $\tau_z = \frac{1}{2}IBL^2$.

EXECUTE: **(a)** Balancing the two torques gives: $mg\frac{L}{2}\cos\theta = \frac{1}{2}IBL^2$. Simplifying gives $ILB = mg\cos\theta$.

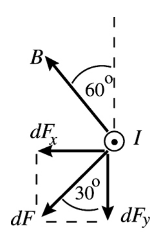
Putting in the numbers gives
$I(0.150\text{ T})(0.300\text{ m}) = (0.0120\text{ kg})(9.80\text{ m/s}^2)\cos(30.0°)$, so $I = 2.26\text{ A}$.

(b) Gravity tends to rotate the bar clockwise about point *a*, so the magnetic force must be upward and to the left to tend to rotate the bar clockwise. Therefore the current must flow from *a* to *b*.

EVALUATE: If the current were from *b* to *a*, the bar could not balance.

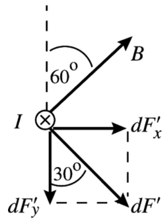
27.73. IDENTIFY: Use $dF = Idl\,B\sin\phi$ to calculate the force on a short segment of the coil and integrate over the entire coil to find the total force.

SET UP: See Figures 27.73a and 27.73b. The two sketches show that the *x*-components cancel and that the *y*-components add. This is true for all pairs of short segments on opposite sides of the coil. The net magnetic force on the coil is in the *y*-direction and its magnitude is given by $F = \int dF_y$.



Consider the force $d\vec{F}$ on a short segment *dl* at the left-hand side of the coil, as viewed in the figure with the problem in the textbook. The current at this point is directed out of the page. $d\vec{F}$ is perpendicular both to $\vec{B}$ and to the direction of *I*.

Figure 27.73a



Consider also the force $d\vec{F}'$ on a short segment on the opposite side of the coil, at the right-hand side of the coil in the figure with the problem in the textbook. The current at this point is directed into the page.

Figure 27.73b

EXECUTE: $dF = Idl\,B\sin\phi$. But $\vec{B}$ is perpendicular to the current direction so $\phi = 90°$.

$dF_y = dF\cos 30.0 = IB\cos 30.0°dl$.

$F = \int dF_y = IB\cos 30.0° \int dl$.

But $\int dl = N(2\pi r)$, the total length of wire in the coil.

$F = IB\cos 30.0° N(2\pi r) = (0.950\text{ A})(0.220\text{ T})(\cos 30.0°)(50)2\pi(0.0078\text{ m}) = 0.444\text{ N}$ and $\vec{F} = -(0.444\text{ N})\hat{\boldsymbol{j}}$

EVALUATE: The magnetic field makes a constant angle with the plane of the coil but has a different direction at different points around the circumference of the coil so is not uniform. The net force is proportional to the magnitude of the current and reverses direction when the current reverses direction.

27.74. **IDENTIFY** and **SET UP:** The rod is in rotational equilibrium, so the torques must balance. Take torques about point P and use $\tau_z = \frac{1}{2}IBL^2$ from Problem 27.70.

EXECUTE: Balancing torques gives $mg\frac{L}{2}\cos\theta + \frac{1}{2}IBL^2 = T\sin\theta L$, where L is the length of the bar and T is the tension in the string. Solving for T and putting in the numbers gives
$T = [(0.0840\text{ kg})(9.80\text{ m/s}^2)\cos(53.0°) + (12.0\text{ A})(0.120\text{ T})(0.180\text{ m})]/[2\sin(53.0°)] = 0.472\text{ N}$.
EVALUATE: If the current were reversed, the tension would be less than 0.472 N.

27.75. **IDENTIFY:** Apply $d\vec{F} = Id\vec{l} \times \vec{B}$ to each side of the loop.

SET UP: For each side of the loop, $d\vec{l}$ is parallel to that side of the loop and is in the direction of I. Since the loop is in the xy-plane, $z = 0$ at the loop and $B_y = 0$ at the loop.

EXECUTE: **(a)** The magnetic field lines in the yz-plane are sketched in Figure 27.75.

(b) Side 1, that runs from (0,0) to (0,L): $\vec{F} = \int_0^L Id\vec{l} \times \vec{B} = I\int_0^L \frac{B_0 y\,dy}{L}\hat{\boldsymbol{i}} = \frac{1}{2}B_0 LI\hat{\boldsymbol{i}}$.

Side 2, that runs from (0,L) to (L,L): $\vec{F} = \int_{0,y=L}^{L} Id\vec{l} \times \vec{B} = I\int_{0,y=L}^{L} \frac{B_0 y\,dx}{L}\hat{\boldsymbol{j}} = -IB_0 L\hat{\boldsymbol{j}}$.

Side 3, that runs from (L,L) to (L,0): $\vec{F} = \int_{L,x=L}^{0} Id\vec{l} \times \vec{B} = I\int_{L,x=L}^{0} \frac{B_0 y\,dy}{L}(-\hat{\boldsymbol{i}}) = -\frac{1}{2}IB_0 L\hat{\boldsymbol{i}}$.

Side 4, that runs from (L,0) to (0,0): $\vec{F} = \int_{L,y=0}^{0} Id\vec{l} \times \vec{B} = I\int_{L,y=0}^{0} \frac{B_0 y\,dx}{L}\hat{\boldsymbol{j}} = 0$.

(c) The sum of all forces is $\vec{F}_{\text{total}} = -IB_0 L\hat{\boldsymbol{j}}$.

EVALUATE: The net force on sides 1 and 3 is zero. The force on side 4 is zero, since $y = 0$ and $z = 0$ at that side and therefore $B = 0$ there. The net force on the loop equals the force on side 2.

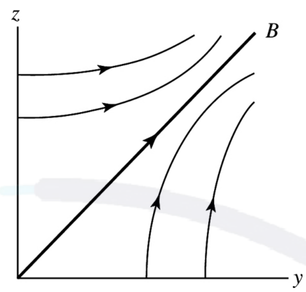


Figure 27.75

27.76. **IDENTIFY:** $I = \dfrac{\Delta q}{\Delta t}$ and $\mu = IA$.

SET UP: The direction of $\vec{\boldsymbol{\mu}}$ is given by the right-hand rule that is illustrated in Figure 27.32 in the textbook. I is in the direction of flow of positive charge and opposite to the direction of flow of negative charge.

EXECUTE: **(a)** $I_u = \dfrac{dq}{dt} = \dfrac{\Delta q}{\Delta t} = \dfrac{q_u v}{2\pi r} = \dfrac{ev}{3\pi r}$.

(b) $\mu_u = I_u A = \dfrac{ev}{3\pi r}\pi r^2 = \dfrac{evr}{3}$.

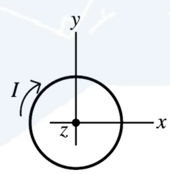
(c) Since there are two down quarks, each of half the charge of the up quark, $\mu_d = \mu_u = \dfrac{evr}{3}$. Therefore, $\mu_{\text{total}} = \dfrac{2evr}{3}$.

(d) $v = \dfrac{3\mu}{2er} = \dfrac{3(9.66\times10^{-27}\ \text{A}\cdot\text{m}^2)}{2(1.60\times10^{-19}\ \text{C})(1.20\times10^{-15}\text{m})} = 7.55\times10^{7}$ m/s.

EVALUATE: The speed calculated in part (d) is 25% of the speed of light.

27.77. **IDENTIFY:** Use $U = -\vec{\boldsymbol{\mu}}\cdot\vec{\boldsymbol{B}}$ to relate U, μ, and $\vec{\boldsymbol{B}}$ and use $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}}\times\vec{\boldsymbol{B}}$ to relate $\vec{\boldsymbol{\tau}}, \vec{\boldsymbol{\mu}}$, and $\vec{\boldsymbol{B}}$. We also know that $B_0^2 = B_x^2 + B_y^2 + B_z^2$. This gives three equations for the three components of $\vec{\boldsymbol{B}}$.

SET UP: The loop and current are shown in Figure 27.77.



$\vec{\boldsymbol{\mu}}$ is into the plane of the paper, in the $-z$-direction.

Figure 27.77

EXECUTE: **(a)** $\vec{\boldsymbol{\mu}} = -\mu\hat{\boldsymbol{k}} = -IA\hat{\boldsymbol{k}}$.

(b) $\vec{\boldsymbol{\tau}} = D(+4\hat{\boldsymbol{i}} - 3\hat{\boldsymbol{j}})$, where $D > 0$.

$\vec{\boldsymbol{\mu}} = -IA\hat{\boldsymbol{k}}, \vec{\boldsymbol{B}} = B_x\hat{\boldsymbol{i}} + B_y\hat{\boldsymbol{j}} + B_y\hat{\boldsymbol{k}}$.

$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}}\times\vec{\boldsymbol{B}} = (-IA)(B_x\hat{\boldsymbol{k}}\times\hat{\boldsymbol{i}} + B_y\hat{\boldsymbol{k}}\times\hat{\boldsymbol{j}} + B_z\hat{\boldsymbol{k}}\times\hat{\boldsymbol{k}}) = IAB_y\hat{\boldsymbol{i}} - IAB_x\hat{\boldsymbol{j}}$.

Compare this to the expression given for $\vec{\boldsymbol{\tau}}$: $IAB_y = 4D$ so $B_y = 4D/IA$ and $-IAB_x = -3D$ so $B_x = 3D/IA$.

B_z doesn't contribute to the torque since $\vec{\boldsymbol{\mu}}$ is along the z-direction. But $B = B_0$ and $B_x^2 + B_y^2 + B_z^2 = B_0^2$; with $B_0 = 13D/IA$. Thus $B_z = \pm\sqrt{B_0^2 - B_x^2 - B_y^2} = \pm(D/IA)\sqrt{169 - 9 - 16} = \pm12(D/IA)$.

That $U = -\vec{\boldsymbol{\mu}}\,.\,\vec{\boldsymbol{B}}$ is negative determines the sign of B_z: $U = -\vec{\boldsymbol{\mu}}\cdot\vec{\boldsymbol{B}} = -(-IA\hat{\boldsymbol{k}})\cdot(B_x\hat{\boldsymbol{i}} + B_y\hat{\boldsymbol{j}} + B_z\hat{\boldsymbol{k}}) = +IAB_z$.

So U negative says that B_z is negative, and thus $B_z = -12D/IA$.

EVALUATE: $\vec{\mu}$ is along the z-axis so only B_x and B_y contribute to the torque. B_x produces a y-component of $\vec{\tau}$ and B_y produces an x-component of $\vec{\tau}$. Only B_z affects U, and U is negative when $\vec{\mu}$ and $\vec{B}_z$ are parallel.

27.78. IDENTIFY: The ions are accelerated from rest. When they enter the magnetic field, they are bent into a circular path. Newton's second law applies to the ions in the magnetic field.

SET UP: $K = \frac{1}{2}mv^2 = qV$. $R = \dfrac{mv}{qB}$, where q is the magnitude of the charge.

EXECUTE: **(a)** As the ions are accelerated through the potential difference V, we have $K = \frac{1}{2}mv^2 = qV$, which gives $v = \sqrt{\dfrac{2qV}{m}}$. In the magnetic field, $R = \dfrac{mv}{qB}$. Using the v we just found gives $R = \dfrac{mv}{qB} = \dfrac{m}{qB}\sqrt{\dfrac{2qV}{m}} = \sqrt{\dfrac{m}{q}}\dfrac{\sqrt{2V}}{B} = \dfrac{1}{B}\sqrt{\dfrac{2m}{q}}\sqrt{V}$. From this result we see that a graph of R versus $\sqrt{V}$ should be a straight line with a slope equal to $\dfrac{1}{B}\sqrt{\dfrac{2m}{q}}$.

(b) The graph of R versus $\sqrt{V}$ is shown in Figure 27.78. The slope of the best-fit line is $(6.355\text{ cm})/\sqrt{\text{kV}} = (0.06355\text{ m})/\sqrt{1000\text{ V}} = 0.00201\text{ m}\cdot\text{V}^{-1/2}$. We know that $\dfrac{1}{B}\sqrt{\dfrac{2m}{q}} = \text{slope}$, so

$$\frac{q}{m} = \frac{2}{[B(\text{slope})]^2} = \frac{2}{[(0.250\text{ T})(0.00201\text{ m}\cdot\text{V}^{-1/2}]^2} = 7.924\times10^{-6}\text{ C/kg},$$ which rounds to 7.92×10^{6} C/kg.

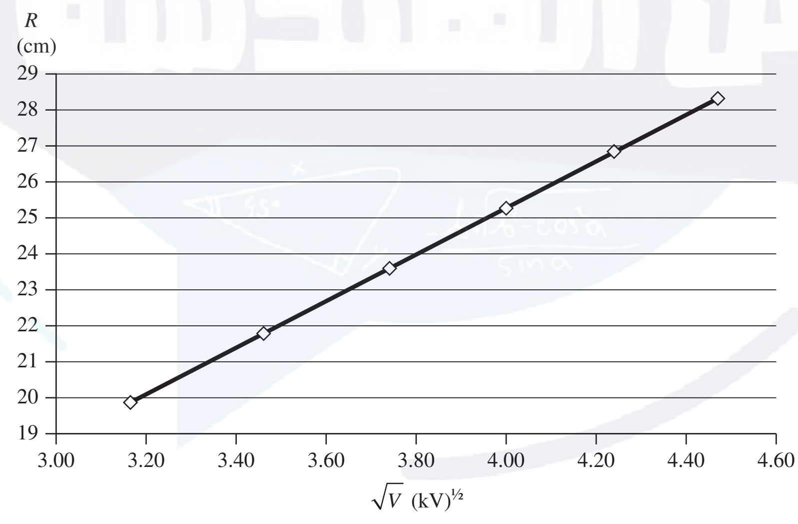


Figure 27.78

(c) Use our result for q/m: $v = \sqrt{\dfrac{2qV}{m}} = \sqrt{2(20.0\times10^3\text{ V})(7.924\times10^6\text{ C/kg})} = 5.63\times10^5$ m/s.

(d) Since $R = \dfrac{1}{B}\sqrt{\dfrac{2m}{q}}\sqrt{V}$, doubling q means that R is smaller by a factor of $\sqrt{2}$. Therefore $R = (21.1\text{ cm})/\sqrt{2} = 15.0\text{ cm}$.

EVALUATE: Besides the approach we have taken, the equation $R = \frac{1}{B}\sqrt{\frac{2m}{q}}\sqrt{V}$ can be graphed in other ways to obtain a straight line. For example, we could graph R^2 versus V, or even log$R$ versus log$V$. Ideally they should all give the same result for q/m. But differences can arise because we are dealing with less-than-ideal data points.

27.79. IDENTIFY and SET UP: The analysis in the text of the Thomson e/m experiment gives $\frac{e}{m} = \frac{E^2}{2VB^2}$. For a particle of charge e and mass m accelerated through a potential V, $eV = \frac{1}{2}mv^2$.

EXECUTE: **(a)** Solving the equation $\frac{e}{m} = \frac{E^2}{2VB^2}$ for E^2 gives $E^2 = 2\left(\frac{e}{m}\right)B^2V$. Therefore a graph of E^2 versus V should be a straight line with slope equal to $2(e/m)B^2$.

(b) We can find the slope using two easily-read points on the graph. Using (100, 200) and (300, 600), we get $\frac{600\times10^8\ \text{V}^2/\text{m}^2 - 200\times10^8\ \text{V}^2/\text{m}^2}{300\ \text{V} - 100\ \text{V}} = 2.00\times10^8\ \text{V/m}^2$ for the slope. This gives

e/m = (slope)/$2B^2$ = $(2.00\times10^8\ \text{V/m}^2)/[2(0.340\ \text{T})^2] = 8.65\times10^8$ C/kg, which gives $m = 1.85\times10^{-28}$ kg.

(c) $V = Ed = (2.00\times10^5\ \text{V/m})(0.00600\ \text{m}) = 1.20$ kV.

(d) Using $eV = \frac{1}{2}mv^2$ to find the muon speed gives

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{2(8.65\times10^8\ \text{C/kg})(400\ \text{V})} = 8.32\times10^5\ \text{m/s}.$$

EVALUATE: Results may vary due to inaccuracies in determining the slope of the graph.

27.80. IDENTIFY and SET UP: If q is the magnitude of the charge, the cyclotron frequency is $\omega = \frac{qB}{m}$, where $\omega = 2\pi f$, and $R = mv/qB$.

EXECUTE: **(a)** Combining $\omega = \frac{qB}{m}$ and $\omega = 2\pi f$ gives $f = \left(\frac{1}{2\pi}\frac{q}{m}\right)B$. Therefore a graph of f versus B should be a straight line having slope equal to $q/2\pi m = (2e)/2\pi m = e/\pi m$. Solving for m gives

$m = \frac{e}{\pi(\text{slope})}$. We use two points on the graph to calculate the slope, giving 7.667×10^6 Hz/T. Therefore

$$m = \frac{e}{\pi(\text{slope})} = e/[\pi(7.667\times10^6\ \text{Hz/T})] = 6.65\times10^{-27}\ \text{kg}.$$

(b) Apply $f = \left(\frac{1}{2\pi}\frac{q}{m}\right)B = qB/2\pi m$ to the electron and the proton.

Electron: $f_e = (1.602\times10^{-19}\ \text{C})(0.300\ \text{T})/[2\pi(9.11\times10^{-31}\ \text{kg})] = 8.40\times10^9\ \text{Hz} = 8.40\ \text{GHz}.$

Proton: $f_p = (1.602\times10^{-19}\ \text{C})(0.300\ \text{T})/[2\pi(1.67\times10^{-27}\ \text{kg})] = 4.58\times10^6\ \text{Hz} = 4.58\ \text{MHz}.$

For an alpha particle, $q = 2e$ and $m \approx 4m_p$, so q/m for an alpha particle is $(2e)/(4m_p) = \frac{1}{2}$ of what it is for a proton. Therefore $f_\alpha = \frac{1}{2}f_p = 2.3$ MHz.

For an alpha particle, $q = 2e$ and $m = 4(1836)m_e$, so q/m for an alpha particle is $2/[4(1836)] = 1/[2(1836)]$ what it is for an electron. Therefore $f_\alpha = \frac{1}{2(1836)}f_e = \frac{1}{3672}f_e = 2.3$ MHz.

(c) $R = mv/qB$ gives $v = RqB/m = (0.120\ \text{m})(3.2\times10^{-19}\ \text{C})(0.300\ \text{T})/(6.65\times10^{-27}\ \text{kg}) = 1.73\times10^6$ m/s.

$K = \frac{1}{2}mv^2 = (1/2)(6.65\times10^{-27}\ \text{kg})(1.73\times10^6\ \text{m/s})^2 = 1.0\times10^{-14}\ \text{J} = 6.25\times10^5\ \text{eV} = 625\ \text{keV} = 0.625\ \text{MeV}.$

EVALUATE: We could use $v = R\omega$ to find v in part (c), where $\omega = 2\pi f$.

27.81. **IDENTIFY** and **SET UP:** In the magnetic field, $R = \frac{mv}{qB}$. Once the particle exits the field it travels in a straight line. Throughout the motion the speed of the particle is constant.

EXECUTE: **(a)** $R = \frac{mv}{qB} = \frac{(3.20\times10^{-11}\text{ kg})(1.45\times10^{5}\text{ m/s})}{(2.15\times10^{-6}\text{ C})(0.420\text{ T})} = 5.14\text{ m}.$

(b) See Figure 27.81. The distance along the curve, d, is given by $d = R\theta$. $\sin\theta = \frac{0.25\text{ m}}{5.14\text{ m}}$, so $\theta = 2.79° = 0.0486\text{ rad}$. $d = R\theta = (5.14\text{ m})(0.0486\text{ rad}) = 0.25\text{ m}$. And

$t = \frac{d}{v} = \frac{0.25\text{ m}}{1.45\times10^{5}\text{ m/s}} = 1.72\times10^{-6}\text{s}.$

(c) $\Delta x_1 = d\tan(\theta/2) = (0.25\text{ m})\tan(2.79°/2) = 6.08\times10^{-3}\text{ m}.$

(d) $\Delta x = \Delta x_1 + \Delta x_2$, where Δx_2 is the horizontal displacement of the particle from where it exits the field region to where it hits the wall. $\Delta x_2 = (0.50\text{ m})\tan 2.79° = 0.0244\text{ m}$. Therefore, $\Delta x = 6.08\times10^{-3}\text{ m} + 0.0244\text{ m} = 0.0305\text{ m}.$

EVALUATE: d is much less than R, so the horizontal deflection of the particle is much smaller than the distance it travels in the y-direction.

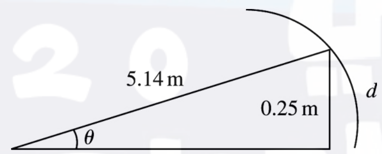


Figure 27.81

27.82. **IDENTIFY:** The electric and magnetic fields exert forces on the moving charge. The work done by the electric field equals the change in kinetic energy. At the top point, $a_y = \frac{v^2}{R}$ and this acceleration must correspond to the net force.

SET UP: The electric field is uniform so the work it does for a displacement y in the y-direction is $W = Fy = qEy$. At the top point, $\vec{F}_B$ is in the $-y$-direction and $\vec{F}_E$ is in the $+y$-direction.

EXECUTE: **(a)** The maximum speed occurs at the top of the cycloidal path, and hence the radius of curvature is greatest there. Once the motion is beyond the top, the particle is being slowed by the electric field. As it returns to $y = 0$, the speed decreases, leading to a smaller magnetic force, until the particle stops completely. Then the electric field again provides the acceleration in the y-direction of the particle, leading to the repeated motion.

(b) $W = qEy = \frac{1}{2}mv^2$ and $v = \sqrt{\frac{2qEy}{m}}$.

(c) At the top, $F_y = qE - qvB = -\frac{mv^2}{R} = -\frac{m}{2y}\frac{2qEy}{m} = -qE$. $2qE = qvB$ and $v = \frac{2E}{B}$.

EVALUATE: The speed at the top depends on B because B determines the y-displacement and the work done by the electric force depends on the y-displacement.

27.83. **IDENTIFY** and **SET UP:** The torque on a magnetic moment is $\tau = \mu B\sin\phi$.

EXECUTE: $\tau = \mu B\sin\phi = (1.4\times10^{-26}\text{ J/T})(2\text{ T})(\sin 90°) = 2.8\times10^{-26}\text{ N}\cdot\text{m}$, which is choice (c).

EVALUATE: The value we have found is the maximum torque. It could be less, depending on the orientation of the proton relative to the magnetic field.

27.84. **IDENTIFY** and **SET UP:** For the nucleus to have a net magnetic moment, it must have an odd number of protons and neutrons.

EXECUTE: Only $^{31}P_{15}$ has an odd number of protons and neutrons, so choice (d) is correct.

EVALUATE: All the other choices have an even number of protons and an even number of neutrons.

27.85. **IDENTIFY** and **SET UP:** Model the nerve as a current-carrying bar in a magnetic field. The resistance of the nerve is $R = \dfrac{\rho L}{A}$, the current through it is $I = V/R$ (by Ohm's law), and the maximum magnetic force on it is $F = ILB$.

EXECUTE: The resistance is $R = \dfrac{\rho L}{A} = (0.6\ \Omega \cdot \text{m})(0.001\ \text{m})/[\pi(0.0015/2\ \text{m})^2] = 340\ \Omega.$

The current is $I = V/R = (0.1\ \text{V})/(340\ \Omega) = 2.9 \times 10^{-4}$ A.

The maximum force is $F = ILB = (2.9 \times 10^{-4}\ \text{A})(0.001\ \text{m})(2\ \text{T}) = 5.9 \times 10^{-7}\ \text{N} \approx 6 \times 10^{-7}\ \text{N},$ which is choice (a).

EVALUATE: This is the force on a 1-mm segment of nerve. The force on the entire nerve would be somewhat larger, depending on the length of the nerve.

28

SOURCES OF MAGNETIC FIELD

28.1. **IDENTIFY and SET UP:** Use $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$ to calculate $\vec{B}$ at each point.

$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$, since $\hat{r} = \frac{\vec{r}}{r}$.

$\vec{v} = (8.00\times10^6\ \text{m/s})\hat{j}$ and $\vec{r}$ is the vector from the charge to the point where the field is calculated.

EXECUTE: **(a)** $\vec{r} = (0.500\ \text{m})\hat{i}, r = 0.500\ \text{m}$.

$\vec{v}\times\vec{r} = vr\hat{j}\times\hat{i} = -vr\hat{k}$.

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{qv}{r^2}\hat{k} = -(1\times10^{-7}\ \text{T}\cdot\text{m/A})\frac{(6.00\times10^{-6}\ \text{C})(8.00\times10^6\ \text{m/s})}{(0.500\ \text{m})^2}\hat{k}.$$

$\vec{B} = -(1.92\times10^{-5}\ \text{T})\hat{k}$.

(b) $\vec{r} = -(0.500\ \text{m})\hat{j}, r = 0.500\ \text{m}$.

$\vec{v}\times\vec{r} = -vr\hat{j}\times\hat{j} = 0$ and $\vec{B} = 0$.

(c) $\vec{r} = (0.500\ \text{m})\hat{k}, r = 0.500\ \text{m}$.

$\vec{v}\times\vec{r} = vr\hat{j}\times\hat{k} = vr\hat{i}$.

$$\vec{B} = (1\times10^{-7}\ \text{T}\cdot\text{m/A})\frac{(6.00\times10^{-6}\ \text{C})(8.00\times10^6\ \text{m/s})}{(0.500\ \text{m})^2}\hat{i} = +(1.92\times10^{-5}\ \text{T})\hat{i}.$$

(d) $\vec{r} = -(0.500\ \text{m})\hat{j} + (0.500\ \text{m})\hat{k}, r = \sqrt{(0.500\ \text{m})^2 + (0.500\ \text{m})^2} = 0.7071\ \text{m}$.

$\vec{v}\times\vec{r} = v(0.500\ \text{m})(-\hat{j}\times\hat{j} + \hat{j}\times\hat{k}) = (4.00\times10^6\ \text{m}^2/\text{s})\hat{i}$.

$$\vec{B} = (1\times10^{-7}\ \text{T}\cdot\text{m/A})\frac{(6.00\times10^{-6}\ \text{C})(4.00\times10^6\ \text{m}^2/\text{s})}{(0.7071\ \text{m})^3}\hat{i} = +(6.79\times10^{-6}\ \text{T})\hat{i}.$$

EVALUATE: At each point $\vec{B}$ is perpendicular to both $\vec{v}$ and $\vec{r}$. $B = 0$ along the direction of $\vec{v}$.

28.2. **IDENTIFY:** A moving charge creates a magnetic field as well as an electric field.

SET UP: The magnetic field caused by a moving charge is $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$, and its electric field is

$E = \frac{1}{4\pi\epsilon_0}\frac{e}{r^2}$ since $q = e$.

EXECUTE: Substitute the appropriate numbers into the above equations.

$$B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2} = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\frac{(1.60\times10^{-19}\text{C})(2.2\times10^6\text{m/s})\sin 90°}{(5.3\times10^{-11}\text{m})^2} = 13\ \text{T},\ \text{out of the page.}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{e}{r^2} = \frac{(9.00\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\,\text{C})}{(5.3\times10^{-11}\,\text{m})^2} = 5.1\times10^{11}\,\text{N/C},\ \text{toward the electron.}$$

EVALUATE: There are enormous fields within the atom!

28.3. **IDENTIFY:** A moving charge creates a magnetic field.

SET UP: The magnetic field due to a moving charge is $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$.

EXECUTE: Substituting numbers into the above equation gives

(a) $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2} = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\frac{(1.6\times10^{-19}\text{C})(3.0\times10^7\,\text{m/s})\sin30^\circ}{(2.00\times10^{-6}\text{m})^2}$.

$B = 6.00\times10^{-8}$ T, out of the paper, and it is the same at point B.

(b) $B = (1.00\times10^{-7}\ \text{T}\cdot\text{m/A})(1.60\times10^{-19}\ \text{C})(3.00\times10^7\ \text{m/s})/(2.00\times10^{-6}\ \text{m})^2$.

$B = 1.20\times10^{-7}$ T, out of the page.

(c) $B = 0$ T since $\sin(180^\circ) = 0$.

EVALUATE: Even at high speeds, these charges produce magnetic fields much less than the earth's magnetic field.

28.4. **IDENTIFY:** Both moving charges produce magnetic fields, and the net field is the vector sum of the two fields.

SET UP: Both fields point out of the paper, so their magnitudes add, giving

$$B = B_{\text{alpha}} + B_{\text{el}} = \frac{\mu_0 v}{4\pi r^2}(e\sin40^\circ + 2e\sin140^\circ).$$

EXECUTE: Factoring out an e and putting in the numbers gives

$$B = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\frac{(1.60\times10^{-19}\ \text{C})(2.50\times10^5\ \text{m/s})}{(8.65\times10^{-9}\ \text{m})^2}(\sin40^\circ + 2\sin140^\circ).$$

$$B = 1.03\times10^{-4}\ \text{T} = 0.103\ \text{mT, out of the page.}$$

EVALUATE: At distances very close to the charges, the magnetic field is strong enough to be important.

28.5. **IDENTIFY:** Apply $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$.

SET UP: Since the charge is at the origin, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

EXECUTE: **(a)** $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{i}$; $\vec{v}\times\vec{r} = 0$, $B = 0$.

(b) $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{j}$; $\vec{v}\times\vec{r} = vr\hat{k}$, $r = 0.500$ m.

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{|q|v}{r^2} = \frac{(1.0\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(6.80\times10^5\ \text{m/s})}{(0.500\ \text{m})^2} = 1.31\times10^{-6}\ \text{T}.$$

q is negative, so $\vec{B} = -(1.31\times10^{-6}\ \text{T})\hat{k}$.

(c) $\vec{v} = v\hat{i}$, $\vec{r} = (0.500\ \text{m})(\hat{i} + \hat{j})$; $\vec{v}\times\vec{r} = (0.500\ \text{m})v\hat{k}$, $r = 0.7071$ m.

$$B = \left(\frac{\mu_0}{4\pi}\right)\left(|q||\vec{v}\times\vec{r}|/r^3\right) = \frac{(1.0\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(0.500\ \text{m})(6.80\times10^5\ \text{m/s})}{(0.7071\ \text{m})^3}.$$

$B = 4.62\times10^{-7}$ T. $\vec{B} = -(4.62\times10^{-7}\ \text{T})\hat{k}$.

(d) $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{k}$; $\vec{v}\times\vec{r} = -vr\hat{j}$, $r = 0.500$ m.

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{|q|v}{r^2} = \frac{(1.0\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(6.80\times10^5\ \text{m/s})}{(0.500\ \text{m})^2} = 1.31\times10^{-6}\ \text{T}.$$

$\vec{B} = (1.31\times10^{-6}\ \text{T})\hat{j}$.

EVALUATE: In each case, $\vec{B}$ is perpendicular to both $\vec{r}$ and $\vec{v}$.

28.6. **IDENTIFY:** Apply $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$. For the magnetic force, apply the results of Example 28.1, except here the two charges and velocities are different.

SET UP: In part (a), $r = d$ and $\vec{r}$ is perpendicular to $\vec{v}$ in each case, so $\frac{|\vec{v}\times\vec{r}|}{r^3} = \frac{v}{d^2}$. For calculating the force between the charges, $r = 2d$.

EXECUTE: **(a)** $B_{\text{total}} = B + B' = \frac{\mu_0}{4\pi}\left(\frac{qv}{d^2} + \frac{q'v'}{d^2}\right)$.

$$B = \frac{\mu_0}{4\pi}\left(\frac{(8.0\times10^{-6}\ \text{C})(4.5\times10^{6}\ \text{m/s})}{(0.120\ \text{m})^2} + \frac{(3.0\times10^{-6}\ \text{C})(9.0\times10^{6}\ \text{m/s})}{(0.120\ \text{m})^2}\right) = 4.38\times10^{-4}\ \text{T}.$$

The direction of $\vec{B}$ is into the page.

(b) Following Example 28.1 we can find the magnetic force between the charges:

$$F_B = \frac{\mu_0}{4\pi}\frac{qq'vv'}{r^2} = (10^{-7}\ \text{T}\cdot\text{m/A})\frac{(8.00\times10^{-6}\ \text{C})(3.00\times10^{-6}\ \text{C})(4.50\times10^{6}\ \text{m/s})(9.00\times10^{6}\ \text{m/s})}{(0.240\ \text{m})^2}.$$

$F_B = 1.69\times10^{-3}$ N. The force on the upper charge points up and the force on the lower charge points down. The Coulomb force between the charges is

$F_C = k\frac{q_1q_2}{r^2} = (8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(8.0\times10^{-6}\ \text{C})(3.0\times10^{-6}\ \text{C})}{(0.240\ \text{m})^2} = 3.75\ \text{N}.$ The force on the upper charge points up and the force on the lower charge points down. The ratio of the Coulomb force to the magnetic force is $\frac{F_C}{F_B} = \frac{c^2}{v_1v_2} = \frac{3.75\ \text{N}}{1.69\times10^{-3}\ \text{N}} = 2.22\times10^{3}$; the Coulomb force is much larger.

(c) The magnetic forces are reversed in direction when the direction of only one velocity is reversed but the magnitude of the force is unchanged.

EVALUATE: When two charges have the same sign and move in opposite directions, the force between them is repulsive. When two charges of the same sign move in the same direction, the force between them is attractive.

28.7. **IDENTIFY:** A moving charge creates a magnetic field.

SET UP: Apply $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$. $\vec{r} = (0.200\ \text{m})\hat{i} + (-0.300\ \text{m})\hat{j}$, and $r = 0.3606$ m.

EXECUTE: $\vec{v}\times\vec{r} = [(7.50\times10^{4}\ \text{m/s})\hat{i} + (-4.90\times10^{4}\ \text{m/s})\hat{j}]\times[(0.200\ \text{m})\hat{i} + (-0.300\ \text{m})\hat{j}]$, which simplifies to $\vec{v}\times\vec{r} = (-2.25\times10^{4}\ \text{m}^2/\text{s})\hat{k} + (9.80\times10^{3}\ \text{m}^2/\text{s})\hat{k} = (-1.27\times10^{4}\ \text{m}^2/\text{s})\hat{k}$.

$$\vec{B} = (1.00\times10^{-7}\ \text{T}\cdot\text{m/A})\frac{(-3.00\times10^{-6}\ \text{C})(-1.27\times10^{4}\ \text{m}^2/\text{s})}{(0.3606\ \text{m})^3}\hat{k} = (9.75\times10^{-8}\ \text{T})\hat{k}.$$

EVALUATE: We can check the direction of the magnetic field using the right-hand rule, which shows that the field points in the $+z$-direction.

28.8. **IDENTIFY:** Both moving charges create magnetic fields, and the net field is the vector sum of the two. The magnetic force on a moving charge is $F_{\text{mag}} = qvB\sin\phi$ and the electrical force obeys Coulomb's law.

SET UP: The magnetic field due to a moving charge is $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$.

EXECUTE: **(a)** Both fields are into the page, so their magnitudes add, giving

$$B = B_e + B_p = \frac{\mu_0}{4\pi}\left(\frac{ev}{r_e^2} + \frac{ev}{r_p^2}\right)\sin 90°.$$

$$B = \frac{\mu_0}{4\pi}(1.60\times10^{-19}\text{ C})(735{,}000\text{ m/s})\left[\frac{1}{(5.00\times10^{-9}\text{ m})^2}+\frac{1}{(4.00\times10^{-9}\text{ m})^2}\right].$$

$$B = 1.21\times10^{-3}\text{ T} = 1.21\text{ mT, into the page.}$$

(b) Using $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$, where $r = \sqrt{41}$ nm and $\phi = 180° - \arctan(5/4) = 128.7°$, we get

$$B = \frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{4\pi}\frac{(1.6\times10^{-19}\text{ C})(735{,}000\text{ m/s})\sin128.7°}{(\sqrt{41}\times10^{-9}\text{ m})^2} = 2.24\times10^{-4}\text{ T, into the page.}$$

(c) $F_{\text{mag}} = qvB\sin90° = (1.60\times10^{-19}\text{ C})(735{,}000\text{ m/s})(2.24\times10^{-4}\text{ T}) = 2.63\times10^{-17}\text{ N}$, in the $+x$-direction.

$$F_{\text{elec}} = (1/4\pi\epsilon_0)e^2/r^2 = \frac{(9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{(\sqrt{41}\times10^{-9}\text{ m})^2} = 5.62\times10^{-12}\text{ N, at }129°$$

counterclockwise from the $+x$-axis.

EVALUATE: The electric force is over 200,000 times as strong as the magnetic force.

28.9. IDENTIFY: A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2}$.

EXECUTE: Applying the law of Biot and Savart gives

(a) $dB = \frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{4\pi}\frac{(10.0\text{ A})(0.00110\text{ m})\sin90°}{(0.0500\text{ m})^2} = 4.40\times10^{-7}\text{ T}$, out of the paper.

(b) The same as above, except $r = \sqrt{(5.00\text{ cm})^2+(14.0\text{ cm})^2}$ and $\phi = \arctan(5/14) = 19.65°$, giving $dB = 1.67\times10^{-8}\text{ T}$, out of the page.

(c) $dB = 0$ since $\phi = 0°$.

EVALUATE: This is a very small field, but it comes from a very small segment of current.

28.10. IDENTIFY: Apply the Biot-Savart law.

SET UP: Apply $d\vec{B} = \frac{\mu_0}{4\pi}\frac{q d\vec{l}\times\vec{r}}{r^3}$. $r = \sqrt{(-0.730\text{ m})^2+(0.390\text{ m})^2} = 0.8276\text{ m}$.

EXECUTE:

$d\vec{l}\times\vec{r} = [0.500\times10^{-3}\text{ m}]\hat{j}\times[(-0.730\text{ m})\hat{i}+(0.390\text{ m})\hat{k}] = (+3.65\times10^{-4}\text{ m}^2)\hat{k}+(+1.95\times10^{-4}\text{ m}^2)\hat{i}$.

$$d\vec{B} = (1.00\times10^{-7}\text{ T}\cdot\text{m/A})\frac{5.40\text{ A}}{(0.8276\text{ m})^3}[(3.65\times10^{-4}\text{ m}^2)\hat{k}+(1.95\times10^{-4}\text{ m}^2)\hat{i}].$$

$$d\vec{B} = (1.86\times10^{-10}\text{ T})\hat{i}+(3.48\times10^{-10}\text{ T})\hat{k}.$$

EVALUATE: The magnetic field lies in the xz-plane.

28.11. IDENTIFY and **SET UP:** The magnetic field produced by an infinitesimal current element is given by $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\vec{l}\times\hat{r}}{r^2}$.

As in Example 28.2, use $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\vec{l}\times\hat{r}}{r^2}$ for the finite 0.500-mm segment of wire since the $\Delta l = 0.500\text{-mm}$ length is much smaller than the distances to the field points.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l}\times\vec{r}}{r^3}$$

I is in the $+z$-direction, so $\Delta\vec{l} = (0.500\times10^{-3}\text{ m})\hat{k}$.

EXECUTE: **(a)** The field point is at $x = 2.00$ m, $y = 0$, $z = 0$ so the vector $\vec{r}$ from the source point (at the origin) to the field point is $\vec{r} = (2.00\text{ m})\hat{\boldsymbol{i}}$.

$\Delta\vec{l} \times \vec{r} = (0.500\times10^{-3}\text{ m})(2.00\text{ m})\hat{\boldsymbol{k}} \times \hat{\boldsymbol{i}} = +(1.00\times10^{-3}\text{ m}^2)\hat{\boldsymbol{j}}$.

$$\vec{B} = \frac{(1\times10^{-7}\text{ T}\cdot\text{m/A})(4.00\text{ A})(1.00\times10^{-3}\text{ m}^2)}{(2.00\text{ m})^3}\hat{\boldsymbol{j}} = (5.00\times10^{-11}\text{ T})\hat{\boldsymbol{j}}.$$

(b) $\vec{r} = (2.00\text{ m})\hat{\boldsymbol{j}}$, $r = 2.00$ m.

$\Delta\vec{l} \times \vec{r} = (0.500\times10^{-3}\text{ m})(2.00\text{ m})\hat{\boldsymbol{k}} \times \hat{\boldsymbol{j}} = -(1.00\times10^{-3}\text{ m}^2)\hat{\boldsymbol{i}}$.

$$\vec{B} = \frac{(1\times10^{-7}\text{ T}\cdot\text{m/A})(4.00\text{ A})(-1.00\times10^{-3}\text{ m}^2)}{(2.00\text{ m})^3}\hat{\boldsymbol{i}} = -(5.00\times10^{-11}\text{ T})\hat{\boldsymbol{i}}.$$

(c) $\vec{r} = (2.00\text{ m})(\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}})$, $r = \sqrt{2}(2.00\text{ m})$.

$\Delta\vec{l} \times \vec{r} = (0.500\times10^{-3}\text{ m})(2.00\text{ m})\hat{\boldsymbol{k}} \times (\hat{\boldsymbol{i}} + \hat{\boldsymbol{j}}) = (1.00\times10^{-3}\text{ m}^2)(\hat{\boldsymbol{j}} - \hat{\boldsymbol{i}})$.

$$\vec{B} = \frac{(1\times10^{-7}\text{ T}\cdot\text{m/A})(4.00\text{ A})(1.00\times10^{-3}\text{ m}^2)}{[\sqrt{2}(2.00\text{ m})]^3}(\hat{\boldsymbol{j}} - \hat{\boldsymbol{i}}) = (-1.77\times10^{-11}\text{ T})(\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}}).$$

(d) $\vec{r} = (2.00\text{ m})\hat{\boldsymbol{k}}$, $r = 2.00$ m.

$\Delta\vec{l} \times \vec{r} = (0.500\times10^{-3}\text{ m})(2.00\text{ m})\hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}} = 0;\ \vec{B} = 0$.

EVALUATE: At each point $\vec{B}$ is perpendicular to both $\vec{r}$ and $\Delta\vec{l}$. $B = 0$ along the length of the wire.

28.12. IDENTIFY: A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \dfrac{\mu_0}{4\pi}\dfrac{Idl\sin\phi}{r^2}$.

Both fields are into the page, so their magnitudes add.

EXECUTE: Applying the law of Biot and Savart for the 12.0-A current gives

$$dB = \frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{4\pi}\frac{(12.0\text{ A})(0.00150\text{ m})\left(\dfrac{2.50\text{ cm}}{8.00\text{ cm}}\right)}{(0.0800\text{ m})^2} = 8.79\times10^{-8}\text{ T}.$$

The field from the 24.0-A segment is twice this value, so the total field is 2.64×10^{-7} T, into the page.

EVALUATE: The rest of each wire also produces field at P. We have calculated just the field from the two segments that are indicated in the problem.

28.13. IDENTIFY: A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \dfrac{\mu_0}{4\pi}\dfrac{Idl\sin\phi}{r^2}$. Both fields are into the page, so their magnitudes add.

EXECUTE: Applying the Biot and Savart law, where $r = \frac{1}{2}\sqrt{(3.00\text{ cm})^2 + (3.00\text{ cm})^2} = 2.121\text{ cm}$, we have

$$dB = 2\frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{4\pi}\frac{(28.0\text{ A})(0.00200\text{ m})\sin 45.0^\circ}{(0.02121\text{ m})^2} = 1.76\times10^{-5}\text{ T, into the paper.}$$

EVALUATE: Even though the two wire segments are at right angles, the magnetic fields they create are in the same direction.

28.14. IDENTIFY: A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \dfrac{\mu_0}{4\pi}\dfrac{Idl\sin\phi}{r^2}$. All four fields are of equal magnitude and into the page, so their magnitudes add.

EXECUTE: $dB = 4\dfrac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\dfrac{(8.00\ \text{A})(0.00120\ \text{m})\ \sin 90°}{(0.0500\ \text{m})^2} = 1.54\times10^{-6}\ \text{T} = 1.54\ \mu\text{T},$ into the page.

EVALUATE: A small current element causes a small magnetic field.

28.15. **IDENTIFY:** We can model the lightning bolt and the household current as very long current-carrying wires.

SET UP: The magnetic field produced by a long wire is $B = \dfrac{\mu_0 I}{2\pi r}$.

EXECUTE: Substituting the numerical values gives

(a) $B = \dfrac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(20{,}000\ \text{A})}{2\pi(5.0\ \text{m})} = 8\times10^{-4}\ \text{T}.$

(b) $B = \dfrac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(10\ \text{A})}{2\pi(0.050\ \text{m})} = 4.0\times10^{-5}\ \text{T}.$

EVALUATE: The field from the lightning bolt is about 20 times as strong as the field from the household current.

28.16. **IDENTIFY:** The long current-carrying wire produces a magnetic field.

SET UP: The magnetic field due to a long wire is $B = \dfrac{\mu_0 I}{2\pi r}$.

EXECUTE: First find the current: $I = (8.20\times10^{18}\ \text{el/s})(1.60\times10^{-19}\ \text{C/el}) = 1.312\ \text{A}.$

Now find the magnetic field: $\dfrac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(1.312\ \text{A})}{2\pi(0.0400\ \text{m})} = 6.56\times10^{-6}\ \text{T} = 6.56\ \mu\text{T}.$

Since electrons are negative, the conventional current runs from east to west, so the magnetic field above the wire points toward the north.

EVALUATE: This magnetic field is much less than that of the earth, so any experiments involving such a current would have to be shielded from the earth's magnetic field, or at least would have to take it into consideration.

28.17. **IDENTIFY:** We can model the current in the heart as that of a long straight wire. It produces a magnetic field around it.

SET UP: For a long straight wire, $B = \dfrac{\mu_0 I}{2\pi r}$. $\mu_0 = 4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}$. 1 gauss $= 10^{-4}$ T.

EXECUTE: Solving for the current gives

$$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi(0.050\ \text{m})(1.0\times10^{-9}\ \text{T})}{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}} = 25\times10^{-5}\ \text{A} = 250\ \mu\text{A}.$$

EVALUATE: By household standards, this is a very small current. But the magnetic field around the heart $(\approx 10\ \mu\text{G})$ is also very small.

28.18. **IDENTIFY:** The current in the transmission line creates a magnetic field. If this field is greater than 5% of the earth's magnetic field, it will interfere with the navigation of the bacteria.

SET UP: $B = \dfrac{\mu_0 I}{2\pi r}$ due to a long straight wire.

EXECUTE: We know the field is $B = (0.05)(5\times10^{-5}\ \text{T}) = 2.5\times10^{-6}\ \text{T}.$ Solving $B = \dfrac{\mu_0 I}{2\pi r}$ for r gives

$$r = \frac{\mu_0 I}{2\pi B} = (2\times10^{-7}\ \text{T}\cdot\text{m/A})\frac{100\ \text{A}}{2.5\times10^{-6}\ \text{T}} = 8\ \text{m}.$$

EVALUATE: If the bacteria are within 8 m $(\approx 25\ \text{ft})$ of the cable, its magnetic field may be strong enough to affect their navigation.

28.19. **IDENTIFY:** The long current-carrying wire produces a magnetic field.

SET UP: The magnetic field due to a long wire is $B = \dfrac{\mu_0 I}{2\pi r}$.

EXECUTE: First solve for the current, then substitute the numbers using the above equation.

(a) Solving for the current gives

$$I = 2\pi rB/\mu_0 = 2\pi(0.0200\text{ m})(1.00\times10^{-4}\text{ T})/(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}) = 10.0\text{ A}.$$

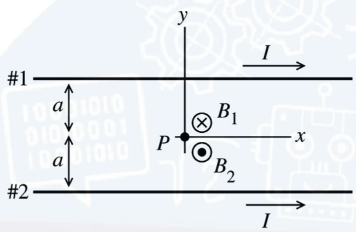
(b) The earth's horizontal field points northward, so at all points directly above the wire the field of the wire would point northward.

(c) At all points directly east of the wire, its field would point northward.

EVALUATE: Even though the earth's magnetic field is rather weak, it requires a fairly large current to cancel this field.

28.20. **IDENTIFY:** For each wire $B = \dfrac{\mu_0 I}{2\pi r}$, and the direction of $\vec{B}$ is given by the right-hand rule (Figure 28.6 in the textbook). Add the field vectors for each wire to calculate the total field.

(a) SET UP: The two fields at this point have the directions shown in Figure 28.20a.

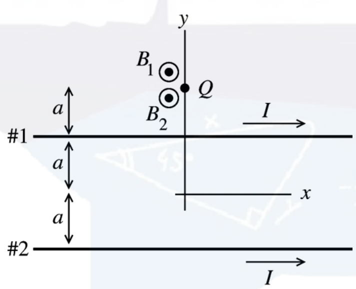


EXECUTE: At point P midway between the two wires the fields $\vec{B}_1$ and $\vec{B}_2$ due to the two currents are in opposite directions, so $B = B_2 - B_1$.

Figure 28.20a

But $B_1 = B_2 = \dfrac{\mu_0 I}{2\pi a}$, so $B = 0$.

(b) SET UP: The two fields at this point have the directions shown in Figure 28.20b.



EXECUTE: At point Q above the upper wire $\vec{B}_1$ and $\vec{B}_2$ are both directed out of the page ($+z$-direction), so $B = B_1 + B_2$.

Figure 28.20b

$$B_1 = \frac{\mu_0 I}{2\pi a},\ B_2 = \frac{\mu_0 I}{2\pi(3a)}.$$

$$B = \frac{\mu_0 I}{2\pi a}\left(1+\tfrac{1}{3}\right) = \frac{2\mu_0 I}{3\pi a};\ \vec{B} = \frac{2\mu_0 I}{3\pi a}\hat{k}.$$

(c) SET UP: The two fields at this point have the directions shown in Figure 28.20c (next page).

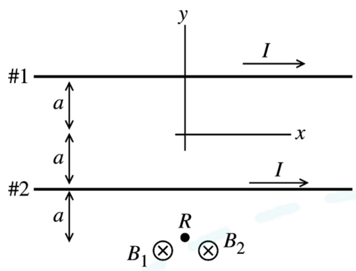


Figure 28.20c

EXECUTE: At point R below the lower wire $\vec{B}_1$ and $\vec{B}_2$ are both directed into the page ($-z$-direction), so $B = B_1 + B_2$.

$$B_1 = \frac{\mu_0 I}{2\pi(3a)},\ B_2 = \frac{\mu_0 I}{2\pi a}.$$

$$B_1 = \frac{\mu_0 I}{2\pi a}\left(1+\tfrac{1}{3}\right) = \frac{2\mu_0 I}{3\pi a};\ \vec{B} = -\frac{2\mu_0 I}{3\pi a}\hat{k}.$$

EVALUATE: In the figures we have drawn, $\vec{B}$ due to each wire is out of the page at points above the wire and into the page at points below the wire. If the two field vectors are in opposite directions the magnitudes subtract.

28.21. IDENTIFY: The total magnetic field is the vector sum of the constant magnetic field and the wire's magnetic field.

SET UP: For the wire, $B_{\text{wire}} = \dfrac{\mu_0 I}{2\pi r}$ and the direction of B_{wire} is given by the right-hand rule that is illustrated in Figure 28.6 in the textbook. $\vec{B}_0 = (1.50\times10^{-6}\ \text{T})\hat{i}$.

EXECUTE: **(a)** At (0, 0, 1 m), $\vec{B} = \vec{B}_0 - \dfrac{\mu_0 I}{2\pi r}\hat{i} = (1.50\times10^{-6}\ \text{T})\hat{i} - \dfrac{\mu_0(8.00\ \text{A})}{2\pi(1.00\ \text{m})}\hat{i} = -(1.0\times10^{-7}\ \text{T})\hat{i}$.

(b) At (1 m, 0, 0), $\vec{B} = \vec{B}_0 + \dfrac{\mu_0 I}{2\pi r}\hat{k} = (1.50\times10^{-6}\ \text{T})\hat{i} + \dfrac{\mu_0(8.00\ \text{A})}{2\pi(1.00\ \text{m})}\hat{k}$.

$\vec{B} = (1.50\times10^{-6}\ \text{T})\hat{i} + (1.6\times10^{-6}\ \text{T})\hat{k} = 2.19\times10^{-6}\ \text{T}$, at $\theta = 46.8°$ from x to z.

(c) At $(0, 0, -0.25\ \text{m})$, $\vec{B} = \vec{B}_0 + \dfrac{\mu_0 I}{2\pi r}\hat{i} = (1.50\times10^{-6}\ \text{T})\hat{i} + \dfrac{\mu_0(8.00\ \text{A})}{2\pi(0.25\ \text{m})}\hat{i} = (7.9\times10^{-6}\ \text{T})\hat{i}$.

EVALUATE: At point c the two fields are in the same direction and their magnitudes add. At point a they are in opposite directions and their magnitudes subtract. At point b the two fields are perpendicular.

28.22. IDENTIFY: The magnetic field is that of a long current-carrying wire.

SET UP: $B = \dfrac{\mu_0 I}{2\pi r}$.

EXECUTE: $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{(2.0\times10^{-7}\ \text{T}\cdot\text{m/A})(150\ \text{A})}{8.0\ \text{m}} = 3.8\times10^{-6}\ \text{T}$. This is 7.5% of the earth's field.

EVALUATE: Since this field is much smaller than the earth's magnetic field, it would be expected to have less effect than the earth's field.

28.23. IDENTIFY: $B = \dfrac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule.

SET UP: Call the wires a and b, as indicated in Figure 28.23. The magnetic fields of each wire at points P_1 and P_2 are shown in Figure 28.23a. The fields at point 3 are shown in Figure 28.23b.

EXECUTE: **(a)** At P_1, $B_a = B_b$ and the two fields are in opposite directions, so the net field is zero.

(b) $B_a = \dfrac{\mu_0 I}{2\pi r_a}$. $B_b = \dfrac{\mu_0 I}{2\pi r_b}$. $\vec{B}_a$ and $\vec{B}_b$ are in the same direction so

$$B = B_a + B_b = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r_a}+\frac{1}{r_b}\right) = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(4.00\ \text{A})}{2\pi}\left[\frac{1}{0.300\ \text{m}}+\frac{1}{0.200\ \text{m}}\right] = 6.67\times10^{-6}\ \text{T}.$$

$\vec{B}$ has magnitude $6.67\ \mu\text{T}$ and is directed toward the top of the page.

(c) In Figure 28.25b, $\vec{B}_a$ is perpendicular to $\vec{r}_a$ and $\vec{B}_b$ is perpendicular to $\vec{r}_b$. $\tan\theta = \frac{5\ \text{cm}}{20\ \text{cm}}$ and $\theta = 14.04°$. $r_a = r_b = \sqrt{(0.200\ \text{m})^2 + (0.050\ \text{m})^2} = 0.206\ \text{m}$ and $B_a = B_b$.

$$B = B_a\cos\theta + B_b\cos\theta = 2B_a\cos\theta = 2\left(\frac{\mu_0 I}{2\pi r_a}\right)\cos\theta = \frac{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(4.0\ \text{A})\cos14.04°}{2\pi(0.206\ \text{m})} = 7.54\ \mu\text{T}$$

B has magnitude $7.53\ \mu\text{T}$ and is directed to the left.

EVALUATE: At points directly to the left of both wires the net field is directed toward the bottom of the page.

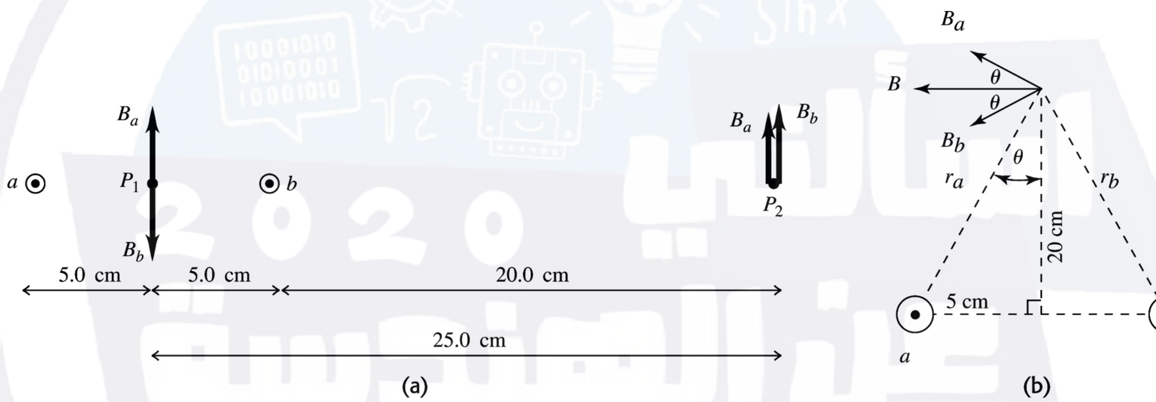


Figure 28.23

28.24. IDENTIFY: Each segment of the rectangular loop creates a magnetic field at the center of the loop, and all these fields are in the same direction.

SET UP: The field due to each segment is $B = \frac{\mu_0 I}{4\pi}\frac{2a}{x\sqrt{x^2+a^2}}$. $\vec{B}$ is into paper so I is clockwise around the loop.

EXECUTE: Long sides: $a = 4.75$ cm. $x = 2.10$ cm. For the two long sides,

$$B = 2(1.00\times10^{-7}\ \text{T}\cdot\text{m/A})I\frac{2(4.75\times10^{-2}\ \text{m})}{(2.10\times10^{-2}\ \text{m})\sqrt{(0.0210\ \text{m})^2+(0.0475\ \text{m})^2}} = (1.742\times10^{-5}\ \text{T/A})I.$$

Short sides: $a = 2.10$ cm. $x = 4.75$ cm. For the two short sides,

$$B = 2(1.00\times10^{-7}\ \text{T}\cdot\text{m/A})I\frac{2(2.10\times10^{-2}\ \text{m})}{(4.75\times10^{-2}\ \text{m})\sqrt{(0.0475\ \text{m})^2+(0.0210\ \text{m})^2}} = (3.405\times10^{-6}\ \text{T/A})I.$$

Using the known field, we have $B = (2.082\times10^{-5}\ \text{T/A})I = 5.50\times10^{-5}\ \text{T}$, which gives $I = 2.64$ A.

EVALUATE: This is a typical household current, yet it produces a magnetic field which is about the same as the earth's magnetic field.

28.25. IDENTIFY: The net magnetic field at the center of the square is the vector sum of the fields due to each wire.

SET UP: For each wire, $B = \frac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is given by the right-hand rule that is illustrated in Figure 28.6 in the textbook.

EXECUTE: **(a)** and **(b)** $B = 0$ since the magnetic fields due to currents at opposite corners of the square cancel.

(c) The fields due to each wire are sketched in Figure 28.25.

$$B = B_a \cos 45° + B_b \cos 45° + B_c \cos 45° + B_d \cos 45° = 4B_a \cos 45° = 4\left(\frac{\mu_0 I}{2\pi r}\right)\cos 45°.$$

$r = \sqrt{(10\text{ cm})^2 + (10\text{ cm})^2} = 10\sqrt{2}\text{ cm} = 0.10\sqrt{2}\text{ m},$ so

$$B = 4\frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(100\text{ A})}{2\pi(0.10\sqrt{2}\text{ m})}\cos 45° = 4.0\times10^{-4}\text{ T, to the left.}$$

EVALUATE: In part (c), if all four currents are reversed in direction, the net field at the center of the square would be to the right.

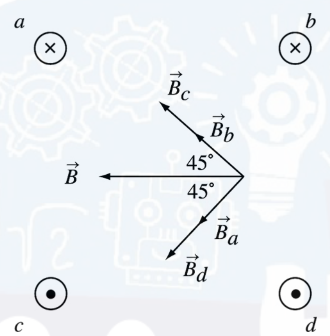
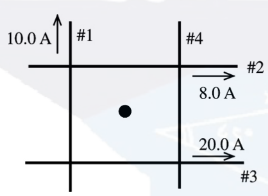


Figure 28.25

28.26. IDENTIFY: Use $B = \dfrac{\mu_0 I}{2\pi r}$ and the right-hand rule to determine the field due to each wire. Set the sum of the four fields equal to zero and use that equation to solve for the field and the current of the fourth wire.

SET UP: The three known currents are shown in Figure 28.26.



$\vec{B}_1 \otimes, \vec{B}_2 \otimes, \vec{B}_3 \odot$

$B = \dfrac{\mu_0 I}{2\pi r}$; $r = 0.200$ m for each wire.

Figure 28.26

EXECUTE: Let $\odot$ be the positive z-direction. $I_1 = 10.0\text{ A}, I_2 = 8.0\text{ A}, I_3 = 20.0\text{ A}$. Then

$B_1 = 1.00\times10^{-5}\text{ T},\ B_2 = 0.80\times10^{-5}\text{ T, and } B_3 = 2.00\times10^{-5}\text{ T}.$

$B_{1z} = -1.00\times10^{-5}\text{ T}, B_{2z} = -0.80\times10^{-5}\text{ T}, B_{3z} = +2.00\times10^{-5}\text{ T}.$

$B_{1z} + B_{2z} + B_{3z} + B_{4z} = 0.$

$B_{4z} = -(B_{1z} + B_{2z} + B_{3z}) = -2.0\times10^{-6}\text{ T}.$

To give $\vec{B}_4$ in the $\otimes$ direction the current in wire 4 must be toward the bottom of the page.

$$B_4 = \frac{\mu_0 I}{2\pi r} \text{ so } I_4 = \frac{rB_4}{(\mu_0/2\pi)} = \frac{(0.200\text{ m})(2.0\times10^{-6}\text{ T})}{(2\times10^{-7}\text{ T}\cdot\text{m/A})} = 2.0\text{ A}.$$

EVALUATE: The fields of wires #2 and #3 are in opposite directions and their net field is the same as due to a current $20.0\text{ A} - 8.0\text{ A} = 12.0\text{ A}$ in one wire. The field of wire #4 must be in the same direction as that of wire #1, and $10.0\text{ A} + I_4 = 12.0\text{ A}$.

28.27. **IDENTIFY:** The net magnetic field at any point is the vector sum of the magnetic fields of the two wires.

SET UP: For each wire $B = \dfrac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is determined by the right-hand rule described in the text. Let the wire with 12.0 A be wire 1 and the wire with 10.0 A be wire 2.

EXECUTE: **(a)** Point Q: $B_1 = \dfrac{\mu_0 I_1}{2\pi r_1} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(12.0\text{ A})}{2\pi(0.15\text{ m})} = 1.6\times10^{-5}$ T.

The direction of $\vec{B}_1$ is out of the page. $B_2 = \dfrac{\mu_0 I_2}{2\pi r_2} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(10.0\text{ A})}{2\pi(0.080\text{ m})} = 2.5\times10^{-5}$ T.

The direction of $\vec{B}_2$ is out of the page. Since $\vec{B}_1$ and $\vec{B}_2$ are in the same direction,

$B = B_1 + B_2 = 4.1\times10^{-5}$ T and $\vec{B}$ is directed out of the page.

Point P: $B_1 = 1.6\times10^{-5}$ T, directed into the page. $B_2 = 2.5\times10^{-5}$ T, directed into the page.

$B = B_1 + B_2 = 4.1\times10^{-5}$ T and $\vec{B}$ is directed into the page.

(b) $\vec{B}_1$ is the same as in part (a), out of the page at Q and into the page at P. The direction of $\vec{B}_2$ is reversed from what it was in (a) so is into the page at Q and out of the page at P.

Point Q: $\vec{B}_1$ and $\vec{B}_2$ are in opposite directions so $B = B_2 - B_1 = 2.5\times10^{-5}\text{ T} - 1.6\times10^{-5}\text{ T} = 9.0\times10^{-6}$ T and $\vec{B}$ is directed into the page.

Point P: $\vec{B}_1$ and $\vec{B}_2$ are in opposite directions so $B = B_2 - B_1 = 9.0\times10^{-6}$ T and $\vec{B}$ is directed out of the page.

EVALUATE: Points P and Q are the same distances from the two wires. The only difference is that the fields point in either the same direction or in opposite directions.

28.28. **IDENTIFY:** Apply $\dfrac{F}{L} = \dfrac{\mu_0 I'I}{2\pi r}$ for the force from each wire.

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: On the top wire $\dfrac{F}{L} = \dfrac{\mu_0 I^2}{2\pi}\left(\dfrac{1}{d} - \dfrac{1}{2d}\right) = \dfrac{\mu_0 I^2}{4\pi d}$, upward. On the middle wire, the magnetic forces cancel so the net force is zero. On the bottom wire $\dfrac{F}{L} = \dfrac{\mu_0 I^2}{2\pi}\left(\dfrac{1}{d} - \dfrac{1}{2d}\right) = \dfrac{\mu_0 I^2}{4\pi d}$, downward.

EVALUATE: The net force on the middle wire is zero because at the location of the middle wire the net magnetic field due to the other two wires is zero.

28.29. **IDENTIFY:** Apply $\dfrac{F}{L} = \dfrac{\mu_0 I'I}{2\pi r}$.

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: **(a)** $F = \dfrac{\mu_0 I_1 I_2 L}{2\pi r} = \dfrac{\mu_0(5.00\text{ A})(2.00\text{ A})(1.20\text{ m})}{2\pi(0.400\text{ m})} = 6.00\times10^{-6}$ N, and the force is repulsive since the currents are in opposite directions.

(b) Doubling the currents makes the force increase by a factor of four to $F = 2.40\times10^{-5}$ N.

EVALUATE: Doubling the current in a wire doubles the magnetic field of that wire. For fixed magnetic field, doubling the current in a wire doubles the force that the magnetic field exerts on the wire.

28.30. **IDENTIFY:** Apply $\dfrac{F}{L} = \dfrac{\mu_0 I'I}{2\pi r}$.

SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: **(a)** $\dfrac{F}{L} = \dfrac{\mu_0 I_1 I_2}{2\pi r}$ gives $I_2 = \dfrac{F}{L}\dfrac{2\pi r}{\mu_0 I_1} = (4.0\times10^{-5}\text{ N/m})\dfrac{2\pi(0.0250\text{ m})}{\mu_0(0.60\text{ A})} = 8.33$ A.

(b) The two wires repel so the currents are in opposite directions.

EVALUATE: The force between the two wires is proportional to the product of the currents in the wires.

28.31. IDENTIFY: The lamp cord wires are two parallel current-carrying wires, so they must exert a magnetic force on each other.

SET UP: First find the current in the cord. Since it is connected to a light bulb, the power consumed by the bulb is $P = IV$. Then find the force per unit length using $\frac{F}{L} = \frac{\mu_0 I'I}{2\pi r}$.

EXECUTE: For the light bulb, $100\text{ W} = I(120\text{ V})$ gives $I = 0.833\text{ A}$. The force per unit length is

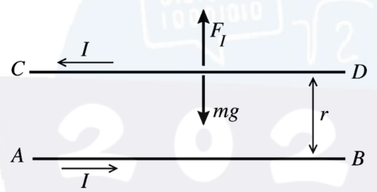
$$F/L = \frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{2\pi}\frac{(0.833\text{ A})^2}{0.003\text{ m}} = 4.6\times10^{-5}\text{ N/m}$$

Since the currents are in opposite directions, the force is repulsive.

EVALUATE: This force is too small to have an appreciable effect for an ordinary cord.

28.32. IDENTIFY: The wire CD rises until the upward force F_I due to the currents balances the downward force of gravity.

SET UP: The forces on wire CD are shown in Figure 28.32.



Currents in opposite directions so the force is repulsive and F_I is upward, as shown.

Figure 28.32

$\frac{F}{L} = \frac{\mu_0 I'I}{2\pi r}$ says $F_I = \frac{\mu_0 I^2 L}{2\pi h}$ where L is the length of wire CD and h is the distance between the wires.

EXECUTE: $mg = \lambda L g$.

Thus $F_I - mg = 0$ says $\frac{\mu_0 I^2 L}{2\pi h} = \lambda L g$ and $h = \frac{\mu_0 I^2}{2\pi g \lambda}$.

EVALUATE: The larger I is or the smaller λ is, the larger h will be.

28.33. IDENTIFY: We can model the current in the brain as a ring. Since we know the magnetic field at the center of the ring, we can calculate the current.

SET UP: At the center of a ring, $B = \frac{\mu_0 I}{2R}$. In this case, $R = 8$ cm. $1\text{ gauss} = 1\times10^{-4}$ T.

EXECUTE: Solving for I gives $I = \frac{2RB}{\mu_0} = \frac{2(8\times10^{-2}\text{ m})(3.0\times10^{-12}\text{ T})}{4\times10^{-7}\text{ T}\cdot\text{m/A}} = 3.8\times10^{-7}$ A.

EVALUATE: This current is about a third of a microamp, which is a very small current by household standards. However, the magnetic field in the brain is a very weak field, about a hundreth of the earth's magnetic field.

28.34. IDENTIFY: The magnetic field at the center of a circular loop is $B = \frac{\mu_0 I}{2R}$. By symmetry each segment of the loop that has length Δl contributes equally to the field, so the field at the center of a semicircle is $\frac{1}{2}$ that of a full loop.

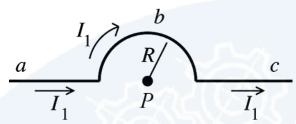
SET UP: Since the straight sections produce no field at P, the field at P is $B = \frac{\mu_0 I}{4R}$.

EXECUTE: $B = \dfrac{\mu_0 I}{4R}$. The direction of $\vec{B}$ is given by the right-hand rule: $\vec{B}$ is directed into the page.

EVALUATE: For a quarter-circle section of wire the magnetic field at its center of curvature is $B = \dfrac{\mu_0 I}{8R}$.

28.35. IDENTIFY: Calculate the magnetic field vector produced by each wire and add these fields to get the total field.

SET UP: First consider the field at P produced by the current I_1 in the upper semicircle of wire. See Figure 28.35a.

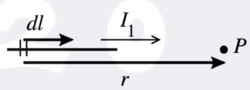


Consider the three parts of this wire:
a: long straight section
b: semicircle
c: long, straight section

Figure 28.35a

Apply the Biot-Savart law $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I d\vec{l} \times \hat{r}}{r^2} = \dfrac{\mu_0}{4\pi}\dfrac{I d\vec{l} \times \vec{r}}{r^3}$ to each piece.

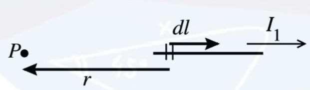
EXECUTE: <u>part a</u>: See Figure 28.35b.



$d\vec{l} \times \vec{r} = 0,$
so $dB = 0.$

Figure 28.35b

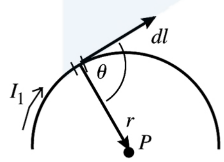
The same is true for all the infinitesimal segments that make up this piece of the wire, so $B = 0$ for this piece.
<u>part c</u>: See Figure 28.35c.



$d\vec{l} \times \vec{r} = 0,$
so $dB = 0$ and $B = 0$ for this piece.

Figure 28.35c

<u>part b</u>: See Figure 28.35d.



$d\vec{l} \times \vec{r}$ is directed into the paper for all infinitesimal segments that make up this semicircular piece, so $\vec{B}$ is directed into the paper and $B = \int dB$ (the vector sum of the $d\vec{B}$ is obtained by adding their magnitudes since they are in the same direction).

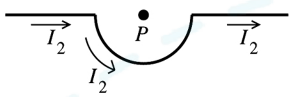
Figure 28.35d

$\left|d\vec{l} \times \vec{r}\right| = r\,dl \sin\theta$. The angle θ between $d\vec{l}$ and $\vec{r}$ is 90° and $r = R$, the radius of the semicircle. Thus $\left|d\vec{l} \times \vec{r}\right| = R\,dl.$

$$dB = \frac{\mu_0}{4\pi}\frac{I\left|d\vec{l} \times \vec{r}\right|}{r^3} = \frac{\mu_0 I_1}{4\pi}\frac{R}{R^3}dl = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)dl.$$

$$B=\int dB=\left(\frac{\mu_0 I_1}{4\pi R^2}\right)\int dl=\left(\frac{\mu_0 I_1}{4\pi R^2}\right)(\pi R)=\frac{\mu_0 I_1}{4R}.$$

(We used that $\int dl$ is equal to πR, the length of wire in the semicircle.) We have shown that the two straight sections make zero contribution to $\vec{B}$, so $B_1=\mu_0 I_1/4R$ and is directed into the page.



For current in the direction shown in Figure 28.35e, a similar analysis gives $B_2=\mu_0 I_2/4R$, out of the paper.

Figure 28.35e

$\vec{B}_1$ and $\vec{B}_2$ are in opposite directions, so the magnitude of the net field at P is $B=|B_1-B_2|=\dfrac{\mu_0|I_1-I_2|}{4R}.$

EVALUATE: When $I_1=I_2, B=0.$

28.36. **IDENTIFY:** Apply $B_x=\dfrac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}}.$

SET UP: At the center of the coil, $x=0$. a is the radius of the coil, 0.0240 m.

EXECUTE: **(a)** $B_x=\mu_0 NI/2a$, so $I=\dfrac{2aB_x}{\mu_0 N}=\dfrac{2(0.024\text{ m})\,(0.0770\text{ T})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(800)}=3.68\text{ A}.$

(b) At the center, $B_c=\mu_0 NI/2a$. At a distance x from the center,

$B_x=\dfrac{\mu_0 NIa^2}{2(x^2+a^2)^{3/2}}=\left(\dfrac{\mu_0 NI}{2a}\right)\left(\dfrac{a^3}{(x^2+a^2)^{3/2}}\right)=B_c\left(\dfrac{a^3}{(x^2+a^2)^{3/2}}\right)$. $B_x=\frac{1}{2}B_c$ says $\dfrac{a^3}{(x^2+a^2)^{3/2}}=\frac{1}{2}$, and $(x^2+a^2)^3=4a^6$. Since $a=0.024$ m, $x=0.0184\text{ m}=1.84\text{ cm}$.

EVALUATE: As shown in Figure 28.14 in the textbook, the field has its largest magnitude at the center of the coil and decreases with distance along the axis from the center.

28.37. **IDENTIFY:** We use the equation for the magnetic field at the center of a single circular loop and then use the equation for the magnetic field inside a solenoid.

SET UP: The magnetic field at the center of a circular loop is $B_{\text{loop}}=\dfrac{\mu_0 I}{2R}$. The magnetic field at the center of a solenoid is $B_{\text{solenoid}}=\mu_0 nI$, where $n=\dfrac{N}{L}$ is the number of turns per meter.

EXECUTE: **(a)** $B_{\text{loop}}=\dfrac{\mu_0 I}{2R}=\dfrac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(2.00\text{ A})}{2(0.050\text{ m})}=2.51\times10^{-5}\text{ T}.$

(b) $n=\dfrac{N}{L}=\dfrac{1000}{5.00\text{ m}}=200\text{ m}^{-1}.$

$B_{\text{solenoid}}=\mu_0 nI=(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(200\text{ m}^{-1})(2.00\text{ A})=5.03\times10^{-4}\text{ T}$. $B_{\text{solenoid}}=20B_{\text{loop}}$. The field at the center of a circular loop depends on the radius of the loop. The field at the center of a solenoid depends on the length of the solenoid, not on its radius.

EVALUATE: The equation $B=\mu_0 nI$ for the field at the center of a solenoid is only correct for a very long solenoid, one whose length L is much greater than its radius R. We cannot consider the limit that L gets small and expect the expression for the solenoid to go over to the expression for N circular loops.

28.38. **IDENTIFY** and **SET UP:** The magnetic field at a point on the axis of N circular loops is given by $B_x=\dfrac{\mu_0 NIa^2}{2(x^2+a^2)^{3/2}}$. Solve for N and set $x=0.0600$ m.

EXECUTE: $N = \dfrac{2B_x(x^2+a^2)^{3/2}}{\mu_0 I a^2} = \dfrac{2(6.39\times10^{-4}\ \text{T})[(0.0600\ \text{m})^2+(0.0600\ \text{m})^2]^{3/2}}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.50\ \text{A})(0.0600\ \text{m})^2} = 69.$

EVALUATE: At the center of the coil the field is $B_x = \dfrac{\mu_0 N I}{2a} = 1.8\times10^{-3}$ T. The field 6.00 cm from the center is a factor of $1/2^{3/2}$ times smaller.

28.39. **IDENTIFY:** The field at the center of the loops is the vector sum of the field due to each loop. They must be in opposite directions in order to add to zero.

SET UP: Let wire 1 be the inner wire with diameter 20.0 cm and let wire 2 be the outer wire with diameter 30.0 cm. To produce zero net field, the fields $\vec{B}_1$ and $\vec{B}_2$ of the two wires must have equal magnitudes and opposite directions. At the center of a wire loop $B = \dfrac{\mu_0 I}{2R}$. The direction of $\vec{B}$ is given by the right-hand rule applied to the current direction.

EXECUTE: $B_1 = \dfrac{\mu_0\, I}{2R_1}$, $B_2 = \dfrac{\mu_0\, I}{2R_2}$. $B_1 = B_2$ gives $\dfrac{\mu_0\, I_1}{2R_1} = \dfrac{\mu_0\, I_2}{2R_2}$. Solving for I_2 gives

$I_2 = \left(\dfrac{R_2}{R_1}\right) I_1 = \left(\dfrac{15.0\ \text{cm}}{10.0\ \text{cm}}\right)(12.0\ \text{A}) = 18.0\ \text{A}$. The directions of I_1 and of its field are shown in Figure 28.39.

Since $\vec{B}_1$ is directed into the page, $\vec{B}_2$ must be directed out of the page and I_2 is counterclockwise.

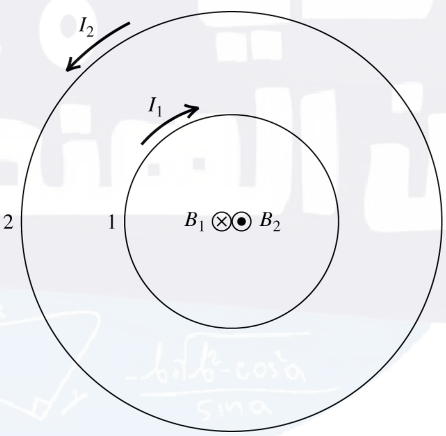


Figure 28.39

EVALUATE: The outer current, I_2, must be larger than the inner current, I_1, because the outer ring is larger than the inner ring, which makes the outer current farther from the center than the inner current is.

28.40. **IDENTIFY:** Apply Ampere's law.

SET UP: From the right-hand rule, when going around the path in a counterclockwise direction currents out of the page are positive and currents into the page are negative.

EXECUTE: Path *a*: $I_{\text{encl}} = 0 \Rightarrow \oint \vec{B}\cdot d\vec{l} = 0.$

Path *b*: $I_{\text{encl}} = -I_1 = -4.0\ \text{A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = -\mu_0(4.0\ \text{A}) = -5.03\times10^{-6}\ \text{T}\cdot\text{m}.$

Path *c*: $I_{\text{encl}} = -I_1 + I_2 = -4.0\ \text{A} + 6.0\ \text{A} = 2.0\ \text{A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0(2.0\ \text{A}) = 2.51\times10^{-6}\ \text{T}\cdot\text{m}$

Path *d*: $I_{\text{encl}} = -I_1 + I_2 + I_3 = 4.0\ \text{A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = +\mu_0(4.0\ \text{A}) = 5.03\times10^{-6}\ \text{T}\cdot\text{m}.$

EVALUATE: If we instead went around each path in the clockwise direction, the sign of the line integral would be reversed.

28.41. **IDENTIFY:** Apply Ampere's law.

SET UP: $\mu_0 = 4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}.$

Execute: **(a)** $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} = 3.83\times10^{-4}\ \text{T}\cdot\text{m}$ and $I_{\text{encl}} = 305\ \text{A}$.

(b) $-3.83\times10^{-4}\ \text{T}\cdot\text{m}$ since at each point on the curve the direction of $d\vec{l}$ is reversed.

Evaluate: The line integral $\oint \vec{B} \cdot d\vec{l}$ around a closed path is proportional to the net current that is enclosed by the path.

28.42. **Identify** and **Set Up:** At the center of a long solenoid $B = \mu_0 nI = \mu_0 \dfrac{N}{L} I$.

Execute: $I = \dfrac{BL}{\mu_0 N} = \dfrac{(0.150\ \text{T})(0.550\ \text{m})}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(4000)} = 16.4\ \text{A}$.

Evaluate: The magnetic field inside the solenoid is independent of the radius of the solenoid, if the radius is much less than the length, as is the case here.

28.43. **Identify:** Apply Ampere's law.

Set Up: To calculate the magnetic field at a distance r from the center of the cable, apply Ampere's law to a circular path of radius r. By symmetry, $\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$ for such a path.

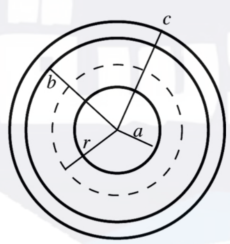
Execute: **(a)** For $a < r < b, I_{\text{encl}} = I \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 I \Rightarrow B2\pi r = \mu_0 I \Rightarrow B = \dfrac{\mu_0 I}{2\pi r}$.

(b) For $r > c,$ the enclosed current is zero, so the magnetic field is also zero.

Evaluate: A useful property of coaxial cables for many applications is that the current carried by the cable doesn't produce a magnetic field outside the cable.

28.44. **Identify:** Apply Ampere's law to calculate $\vec{B}$.

(a) Set Up: For $a < r < b$ the end view is shown in Figure 28.44a.



Apply Ampere's law to a circle of radius r, where $a < r < b$. Take currents I_1 and I_2 to be directed into the page. Take this direction to be positive, so go around the integration path in the clockwise direction.

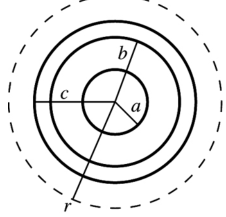
Figure 28.44a

Execute: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$.

$\oint \vec{B} \cdot d\vec{l} = B(2\pi r), I_{\text{encl}} = I_1$.

Thus $B(2\pi r) = \mu_0 I_1$ and $B = \dfrac{\mu_0 I_1}{2\pi r}$.

(b) Set Up: $\underline{r > c}$: See Figure 28.44b.



Apply Ampere's law to a circle of radius r, where $r > c$. Both currents are in the positive direction.

Figure 28.44b

Execute: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$.

$\oint \vec{B} \cdot d\vec{l} = B(2\pi r)$, $I_{\text{encl}} = I_1 + I_2$.

Thus $B(2\pi r) = \mu_0(I_1 + I_2)$ and $B = \dfrac{\mu_0(I_1 + I_2)}{2\pi r}$.

EVALUATE: For $a < r < b$ the field is due only to the current in the central conductor. For $r > c$ both currents contribute to the total field.

28.45. **IDENTIFY:** We treat the solenoid as being ideal.

SET UP: At the center of an ideal solenoid, $B_{\text{solenoid}} = \mu_0 nI = \mu_0 \dfrac{N}{L} I$. A distance r from a long straight wire, $B_{\text{wire}} = \dfrac{\mu_0 I}{2\pi r}$.

EXECUTE: **(a)** $B_{\text{solenoid}} = (4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})\left(\dfrac{450}{0.35\ \text{m}}\right)(1.75\ \text{A}) = 2.83 \times 10^{-3}\ \text{T}$.

(b) $B_{\text{wire}} = \dfrac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(1.75\ \text{A})}{2\pi(1.0 \times 10^{-2}\ \text{m})} = 3.50 \times 10^{-5}\ \text{T}$.

EVALUATE: The magnetic field due to the wire is much less than the field at the center of the solenoid. For the solenoid, the fields of all the wires add to give a much larger field.

28.46. **IDENTIFY:** $B = \mu_0 nI = \dfrac{\mu_0 NI}{L}$.

SET UP: $L = 0.150$ m.

EXECUTE: $B = \dfrac{\mu_0\,(600)(8.00\ \text{A})}{(0.150\ \text{m})} = 0.0402\ \text{T}$.

EVALUATE: The field near the center of the solenoid is independent of the radius of the solenoid, as long as the radius is much less than the length, as it is here.

28.47. **IDENTIFY** and **SET UP:** The magnetic field near the center of a long solenoid is given by $B = \mu_0 nI$.

EXECUTE: **(a)** Turns per unit length $n = \dfrac{B}{\mu_0 I} = \dfrac{0.0270\ \text{T}}{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(12.0\ \text{A})} = 1790$ turns/m.

(b) $N = nL = (1790\ \text{turns/m})(0.400\ \text{m}) = 716$ turns.

Each turn of radius R has a length $2\pi R$ of wire. The total length of wire required is

$N(2\pi R) = (716)(2\pi)(1.40\times 10^{-2}\ \text{m}) = 63.0\ \text{m}$.

EVALUATE: A large length of wire is required. Due to the length of wire the solenoid will have appreciable resistance.

28.48. **IDENTIFY:** Knowing the magnetic field at the center of the toroidal solenoid, we can find the current causing that field.

SET UP: $B = \dfrac{\mu_0 NI}{2\pi r}$. $r = 0.140$ m is the distance from the center of the torus to the point where B is to be calculated. This point must be between the inner and outer radii of the solenoid, but otherwise the field doesn't depend on those radii.

EXECUTE: Solving for N gives $N = \dfrac{2\pi rB}{\mu_0 I} = \dfrac{2\pi(0.140\ \text{m})(3.75\times 10^{-3}\ \text{T})}{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(1.50\ \text{A})} = 1750$ turns.

EVALUATE: With an outer radius of 15 cm, the outer circumference of the toroid is about 100 cm, or about a meter. It is reasonable that the toroid could have 1750 turns spread over a circumference of one meter.

28.49. **IDENTIFY** and **SET UP:** Use the appropriate expression for the magnetic field produced by each current configuration.

EXECUTE: **(a)** $B = \dfrac{\mu_0 I}{2\pi r}$ so $I = \dfrac{2\pi rB}{\mu_0} = \dfrac{2\pi(2.00\times 10^{-2}\ \text{m})(37.2\ \text{T})}{4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}} = 3.72\times 10^{6}\ \text{A} = 3.72\ \text{MA}$.

(b) $B = \dfrac{N\mu_0 I}{2R}$ so $I = \dfrac{2RB}{N\mu_0} = \dfrac{2(0.420\text{ m})(37.2\text{ T})}{(100)(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})} = 2.49\times10^5\text{ A} = 249\text{ kA}.$

(c) $B = \mu_0 \dfrac{N}{L} I$ so $I = \dfrac{BL}{\mu_0 N} = \dfrac{(37.2\text{ T})(0.320\text{ m})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(40{,}000)} = 237\text{ A}.$

EVALUATE: Much less current is needed for the solenoid, because of its large number of turns per unit length.

28.50. IDENTIFY: Outside an ideal toroidal solenoid there is no magnetic field and inside it the magnetic field is given by $B = \dfrac{\mu_0 NI}{2\pi r}.$

SET UP: The torus extends from $r_1 = 15.0\text{ cm}$ to $r_2 = 18.0\text{ cm}.$

EXECUTE: **(a)** $r = 0.12\text{ m},$ which is outside the torus, so $B = 0.$

(b) $r = 0.16\text{ m},$ so $B = \dfrac{\mu_0 NI}{2\pi r} = \dfrac{\mu_0(250)(8.50\text{ A})}{2\pi(0.160\text{ m})} = 2.66\times10^{-3}\text{ T}.$

(c) $r = 0.20\text{ m},$ which is outside the torus, so $B = 0.$

EVALUATE: The magnetic field inside the torus is proportional to $1/r,$ so it varies somewhat over the cross-section of the torus.

28.51. IDENTIFY: Inside an ideal toroidal solenoid, $B = \dfrac{\mu_0 NI}{2\pi r}.$

SET UP: $r = 0.070\text{ m}.$

EXECUTE: $B = \dfrac{\mu_0 NI}{2\pi r} = \dfrac{\mu_0(600)(0.650\text{ A})}{2\pi(0.070\text{ m})} = 1.11\times10^{-3}\text{ T}.$

EVALUATE: If the radial thickness of the torus is small compared to its mean diameter, B is approximately uniform inside its windings.

28.52. IDENTIFY: Use $B = \dfrac{\mu_0 NI}{2\pi r},$ with μ_0 replaced by $\mu = K_m\mu_0,$ with $K_m = 80.$

SET UP: The contribution from atomic currents is the difference between B calculated with μ and B calculated with $\mu_0.$

EXECUTE: **(a)** $B = \dfrac{\mu NI}{2\pi r} = \dfrac{K_m\mu_0 NI}{2\pi r} = \dfrac{\mu_0(80)(400)(0.25\text{ A})}{2\pi(0.060\text{ m})} = 0.0267\text{ T}.$

(b) The amount due to atomic currents is $B' = \dfrac{79}{80}B = \dfrac{79}{80}(0.0267\text{ T}) = 0.0263\text{ T}.$

EVALUATE: The presence of the core greatly enhances the magnetic field produced by the solenoid.

28.53. IDENTIFY: The magnetic field from the solenoid alone is $B_0 = \mu_0 nI.$ The total magnetic field is $B = K_m B_0.$ M is given by $\vec{B} = \vec{B}_0 + \mu_0\vec{M}.$

SET UP: $n = 6000\text{ turns/m}.$

EXECUTE: **(a)** (i) $B_0 = \mu_0 nI = \mu_0(6000\text{ m}^{-1})(0.15\text{ A}) = 1.13\times10^{-3}\text{ T}.$

(ii) $M = \dfrac{K_m - 1}{\mu_0}B_0 = \dfrac{5199}{\mu_0}(1.13\times10^{-3}\text{ T}) = 4.68\times10^6\text{ A/m}.$

(iii) $B = K_m B_0 = (5200)(1.13\times10^{-3}\text{ T}) = 5.88\text{ T}.$

(b) The directions of $\vec{B}$, $\vec{B}_0$ and $\vec{M}$ are shown in Figure 28.53. Silicon steel is paramagnetic and $\vec{B}_0$ and $\vec{M}$ are in the same direction.

EVALUATE: The total magnetic field is much larger than the field due to the solenoid current alone.

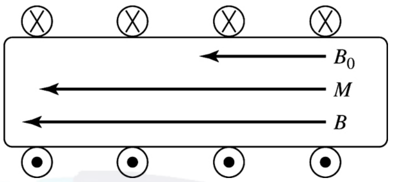


Figure 28.53

28.54. **IDENTIFY:** Apply $B = \dfrac{K_\text{m}\mu_0 NI}{2\pi r}$.

SET UP: K_m is the relative permeability and $\chi_\text{m} = K_\text{m} - 1$ is the magnetic susceptibility.

EXECUTE: **(a)** $K_\text{m} = \dfrac{2\pi rB}{\mu_0 NI} = \dfrac{2\pi(0.2500\text{ m})(1.940\text{ T})}{\mu_0(500)(2.400\text{ A})} = 2021.$

(b) $\chi_\text{m} = K_\text{m} - 1 = 2020.$

EVALUATE: Without the magnetic material the magnetic field inside the windings would be $B/2021 = 9.6\times 10^{-4}\text{ T}$. The presence of the magnetic material greatly enhances the magnetic field inside the windings.

28.55. **IDENTIFY:** Moving charges create magnetic fields. The net field is the vector sum of the two fields. A charge moving in an external magnetic field feels a force.

(a) SET UP: The magnitude of the magnetic field due to a moving charge is $B = \dfrac{\mu_0}{4\pi}\dfrac{|q|v\sin\phi}{r^2}$. Both fields are into the paper, so their magnitudes add, giving $B_\text{net} = B + B' = \dfrac{\mu_0}{4\pi}\left(\dfrac{|q|v\sin\phi}{r^2} + \dfrac{|q'|v'\sin\phi'}{r'^2}\right)$.

EXECUTE: Substituting numbers gives

$$B_\text{net} = \frac{\mu_0}{4\pi}\left[\frac{(8.00\,\mu\text{C})(9.00\times 10^4\text{ m/s})\sin 90^\circ}{(0.300\text{ m})^2} + \frac{(5.00\,\mu\text{C})(6.50\times 10^4\text{ m/s})\sin 90^\circ}{(0.400\text{ m})^2}\right].$$

$B_\text{net} = 1.00\times 10^{-6}\text{ T} = 1.00\,\mu\text{T}$, into the paper.

(b) SET UP: The magnetic force on a moving charge is $\vec{F} = q\vec{v}\times\vec{B}$, and the magnetic field of charge q' at the location of charge q is into the page. The force on q is

$$\vec{F} = q\vec{v}\times\vec{B}' = (qv)\hat{i}\times\frac{\mu_0}{4\pi}\frac{q\vec{v}'\times\hat{r}}{r^2} = (qv)\hat{i}\times\left(\frac{\mu_0}{4\pi}\frac{qv'\sin\phi}{r^2}\right)(-\hat{k}) = \left(\frac{\mu_0}{4\pi}\frac{qq'vv'\sin\phi}{r^2}\right)\hat{j}$$

where ϕ is the angle between $\vec{v}'$ and $\hat{r}'$.

EXECUTE: Substituting numbers gives

$$\vec{F} = \frac{\mu_0}{4\pi}\left[\frac{(8.00\times 10^{-6}\text{ C})(5.00\times 10^{-6}\text{ C})(9.00\times 10^4\text{ m/s})(6.50\times 10^4\text{ m/s})}{(0.500\text{ m})^2}\left(\frac{0.400}{0.500}\right)\right]\hat{j}.$$

$$\vec{F} = (7.49\times 10^{-8}\text{N})\hat{j}.$$

EVALUATE: These are small fields and small forces, but if the charge has small mass, the force can affect its motion.

28.56. **IDENTIFY:** Charge q_1 creates a magnetic field due to its motion. This field exerts a magnetic force on q_2, which is moving in that field.

SET UP: Find $\vec{B}_1$, the field produced by q_1 at the location of q_2. $\vec{B}_1 = \dfrac{\mu_0}{4\pi}\dfrac{q_1\vec{v}_1\times\vec{r}_{1\to 2}}{r_{1\to 2}^3}$, since $\hat{r} = \vec{r}/r$.

EXECUTE: $\vec{r}_{1\to 2} = (0.150\text{ m})\hat{\boldsymbol{i}} + (-0.250\text{ m})\hat{\boldsymbol{j}}$, so $r_{1\to 2} = 0.2915$ m.

$\vec{v}_1 \times \vec{r}_{1\to 2} = [(9.20\times 10^5\text{ m/s})\hat{\boldsymbol{i}}]\times[(0.150\text{ m})\hat{\boldsymbol{i}} + (-0.250\text{ m})\hat{\boldsymbol{j}}] = (9.20\times 10^5\text{ m/s})(-0.250\text{ m})\hat{\boldsymbol{k}}$.

$$\vec{B}_1 = (1.00\times 10^{-7}\text{ T}\cdot\text{m/A})\frac{(4.80\times 10^{-6}\text{ C})(9.20\times 10^5\text{ m/s})(-0.250\text{ m})}{(0.2915\text{ m})^3}\hat{\boldsymbol{k}} = -(4.457\times 10^{-6}\text{ T})\hat{\boldsymbol{k}}.$$

The force that $\vec{B}_1$ exerts on q_2 is

$$F_2 = q_2\vec{v}_2 \times \vec{B}_1 = (-2.90\times 10^{-6}\text{ C})(-5.30\times 10^5\text{ m/s})(-4.457\times 10^{-6}\text{ T})\hat{\boldsymbol{j}}\times\hat{\boldsymbol{k}} = -(6.85\times 10^{-6}\text{ N})\hat{\boldsymbol{i}}.$$

EVALUATE: If we think of the moving charge q_1 as a current, we can use the right-hand rule for the direction of the magnetic field due to a current to find the direction of the magnetic field it creates in the vicinity of q_2. Then we can use the cross product right-hand rule to find the direction of the force this field exerts on q_2, which is in the $-x$-direction, in agreement with our result.

28.57. IDENTIFY: Use $B = \dfrac{\mu_0 I}{2\pi r}$ and the right-hand rule to determine points where the fields of the two wires cancel.

(a) SET UP: The only place where the magnetic fields of the two wires are in opposite directions is between the wires, in the plane of the wires. Consider a point a distance x from the wire carrying $I_2 = 75.0$ A. B_{tot} will be zero where $B_1 = B_2$.

EXECUTE: $\dfrac{\mu_0 I_1}{2\pi(0.400\text{ m} - x)} = \dfrac{\mu_0 I_2}{2\pi x}$.

$I_2(0.400\text{ m} - x) = I_1 x$; $I_1 = 25.0$ A, $I_2 = 75.0$ A.

$x = 0.300$ m; $B_{\text{tot}} = 0$ along a line 0.300 m from the wire carrying 75.0 A and 0.100 m from the wire carrying current 25.0 A.

(b) SET UP: Let the wire with $I_1 = 25.0$ A be 0.400 m above the wire with $I_2 = 75.0$ A. The magnetic fields of the two wires are in opposite directions in the plane of the wires and at points above both wires or below both wires. But to have $B_1 = B_2$ must be closer to wire #1 since $I_1 < I_2$, so can have $B_{\text{tot}} = 0$ only at points above both wires. Consider a point a distance x from the wire carrying $I_1 = 25.0$ A. B_{tot} will be zero where $B_1 = B_2$.

EXECUTE: $\dfrac{\mu_0 I_1}{2\pi x} = \dfrac{\mu_0 I_2}{2\pi(0.400\text{ m} + x)}$.

$I_2 x = I_1(0.400\text{ m} + x)$; $x = 0.200$ m.

$B_{\text{tot}} = 0$ along a line 0.200 m from the wire carrying current 25.0 A and 0.600 m from the wire carrying current $I_2 = 75.0$ A.

EVALUATE: For parts (a) and (b) the locations of zero field are in different regions. In each case the points of zero field are closer to the wire that has the smaller current.

28.58. IDENTIFY: The wire creates a magnetic field near it, and the moving electron feels a force due to this field.

SET UP: The magnetic field due to the wire is $B = \dfrac{\mu_0 I}{2\pi r}$, and the force on a moving charge is $F = |q|vB\sin\phi$.

EXECUTE: $F = |q|vB\sin\phi = (ev\mu_0 I\sin\phi)/2\pi r$. Substituting numbers gives

$$F = (1.60\times 10^{-19}\text{ C})(6.00\times 10^4\text{ m/s})(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})(8.60\text{ A})(\sin 90°)/[2\pi(0.0450\text{ m})].$$

$F = 3.67\times 10^{-19}$ N. From the right-hand rule for the cross product, the direction of $\vec{v}\times\vec{B}$ is opposite to the current, but since the electron is negative, the force is in the same direction as the current.

EVALUATE: This force is small at an everyday level, but it would give the electron an acceleration of over 10^{11} m/s^2.

29.59. IDENTIFY: Find the force that the magnetic field of the wire exerts on the electron.

SET UP: The force on a moving charge has magnitude $F = |q|vB\sin\phi$ and direction given by the right-hand rule. For a long straight wire, $B = \dfrac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is given by the right-hand rule.

EXECUTE: **(a)** $a = \dfrac{F}{m} = \dfrac{|q|vB\sin\phi}{m} = \dfrac{ev}{m}\left(\dfrac{\mu_0 I}{2\pi r}\right)$. Substituting numbers gives

$$a = \frac{(1.6\times10^{-19}\,\text{C})(2.50\times10^{5}\ \text{m/s})(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(13.0\ \text{A})}{(9.11\times10^{-31}\ \text{kg})(2\pi)(0.0200\ \text{m})} = 5.7\times10^{12}\ \text{m/s}^2,$$ away from the wire.

(b) The electric force must balance the magnetic force. $eE = evB$, and

$$E = vB = v\frac{\mu_0 I}{2\pi r} = \frac{(250{,}000\ \text{m/s})(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(13.0\ \text{A})}{2\pi(0.0200\ \text{m})} = 32.5\ \text{N/C}.$$ The magnetic force is directed away from the wire so the force from the electric field must be toward the wire. Since the charge of the electron is negative, the electric field must be directed away from the wire to produce a force in the desired direction.

EVALUATE: **(c)** $mg = (9.11\times10^{-31}\ \text{kg})(9.8\ \text{m/s}^2) \approx 10^{-29}\ \text{N}.$

$F_{\text{el}} = eE = (1.6\times10^{-19}\ \text{C})(32.5\ \text{N/C}) \approx 5\times10^{-18}\ \text{N}.$ $F_{\text{el}} \approx 5\times10^{11}\ F_{\text{grav}}$, so we can neglect gravity.

28.60. IDENTIFY: The current in the wire creates a magnetic field, and that field exerts a force on the moving electron.

SET UP: The magnetic field due to the current in the wire is $B = \dfrac{\mu_0 I}{2\pi r}$. The force the field exerts on the electron is $\vec{F} = q\vec{v}\times\vec{B}$, where $q = -e$. The magnitude of a vector is $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$. The electron is on the $+y$-axis. The current is in the $-x$-direction so, by the right-hand rule, the magnetic field it produces at the location of the electron is in the $-z$-direction, so $\vec{B} = -\dfrac{\mu_0 I}{2\pi r}\,\hat{k}$.

EXECUTE: The magnitude of the magnetic field is $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{\mu_0(9.00\ \text{A})}{2\pi(0.200\ \text{m})} = 9.00\times10^{-6}\ \text{T}$, so

$\vec{B} = -9.00\times10^{-6}\ \text{T}\,\hat{k}$. The force on the electron is $\vec{F} = q\vec{v}\times\vec{B}$, so

$\vec{F} = q\vec{v}\times\vec{B} = -e(5.00\times10^{4}\ \text{m/s}\,\hat{i} - 3.00\times10^{4}\ \text{m/s}\,\hat{j})\times(-9.00\times10^{-6}\ \text{T}\,\hat{k}).$

Taking out common factors gives $\vec{F} = (9\times10^{-2}e\ \text{T}\cdot\text{m/s})(5\hat{i} - 3\hat{j})\times\hat{k}.$ Using the fact that $\boldsymbol{i}\times\boldsymbol{k} = -\boldsymbol{j}$ and $\boldsymbol{j}\times\boldsymbol{k} = \boldsymbol{i}$, we get $\vec{F} = (9\times10^{-2}e\ \text{T}\cdot\text{m/s})(-5\hat{j} - 3\hat{i})$. Using $e = 1.60\times10^{-19}\ \text{C}$ gives

$\vec{F} = -4.32\times10^{-20}\ \text{N}\,\hat{i} - 7.20\times10^{-20}\ \text{N}\,\hat{j}.$

The magnitude of this force is

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(-4.32\times10^{-20}\ \text{N})^2 + (-7.20\times10^{-20}\ \text{N})^2} = 8.40\times10^{-20}\ \text{N}.$$

EVALUATE: This is a small force on an everyday scale, but it would give the electron an acceleration of $a = F/m = (8.40\times10^{-20}\ \text{N})/(9.11\times10^{-31}\ \text{kg}) \approx 9\times10^{10}\ \text{m/s}^2$.

28.61. IDENTIFY and SET UP: The power input of the motor is 65 hp. We know that $1\ \text{hp} = 746\ \text{W}$. The relation between power, voltage, and current is $P = VI$. The attractive force between two parallel wires is $F = \dfrac{\mu_0 L I_1 I_2}{2\pi r}$.

EXECUTE: **(a)** We find the current from $I = \dfrac{P}{V} = \dfrac{(65\ \text{hp})(746\ \text{W/hp})}{600\ \text{V}} = 80.8\ \text{A}$, which rounds to 81 A.

(b) The attractive force between the wires per unit length is

$$F/L = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(80.8\ \text{A})^2}{2\pi(0.55\ \text{m})} = 2.4\times10^{-3}\ \text{N/m}.$$

EVALUATE: If the current from the cables is in the same direction, the force will be attractive; however, if the current runs in opposite directions the force will be repulsive.

28.62. IDENTIFY: Find the vector sum of the magnetic fields due to each wire.

SET UP: For a long straight wire $B = \dfrac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule and is perpendicular to the line from the wire to the point where the field is calculated.

EXECUTE: **(a)** The magnetic field vectors are shown in Figure 28.62a.

(b) At a position on the x-axis $B_{\text{net}} = 2\dfrac{\mu_0 I}{2\pi r}\sin\theta = \dfrac{\mu_0 I}{\pi\sqrt{x^2+a^2}}\dfrac{a}{\sqrt{x^2+a^2}} = \dfrac{\mu_0 I a}{\pi(x^2+a^2)}$, in the positive x-direction.

(c) The graph of B versus x/a is given in Figure 28.62b.

EVALUATE: **(d)** The magnetic field is a maximum at the origin, $x = 0$.

(e) When $x \gg a$, $B \approx \dfrac{\mu_0 I a}{\pi x^2}$.

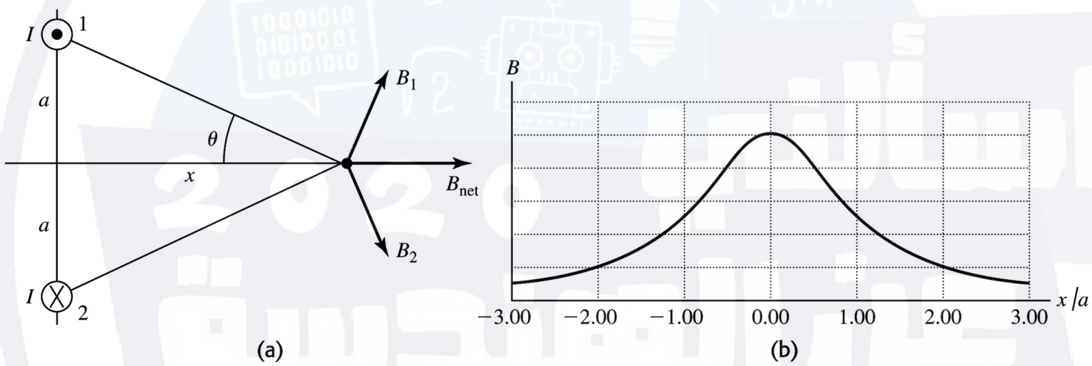
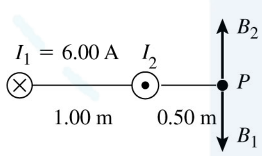


Figure 28.62

28.63. IDENTIFY: Use $B = \dfrac{\mu_0 I}{2\pi r}$ and the right-hand rule to calculate the magnitude and direction of the magnetic field at P produced by each wire. Add these two field vectors to find the net field.

(a) SET UP: The directions of the fields at point P due to the two wires are sketched in Figure 28.63a.



EXECUTE: $\vec{B}_1$ and $\vec{B}_2$ must be equal and opposite for the resultant field at P to be zero. $\vec{B}_2$ is to the upward so I_2 is out of the page.

Figure 28.63a

$$B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{\mu_0}{2\pi}\left(\frac{6.00\ \text{A}}{1.50\ \text{m}}\right) \qquad B_2 = \frac{\mu_0 I_2}{2\pi r_2} = \frac{\mu_0}{2\pi}\left(\frac{I_2}{0.50\ \text{m}}\right).$$

$$B_1 = B_2 \text{ says } \frac{\mu_0}{2\pi}\left(\frac{6.00\ \text{A}}{1.50\ \text{m}}\right) = \frac{\mu_0}{2\pi}\left(\frac{I_2}{0.50\ \text{m}}\right).$$

$$I_2 = \left(\frac{0.50\ \text{m}}{1.50\ \text{m}}\right)(6.00\ \text{A}) = 2.00\ \text{A}.$$

(b) SET UP: The directions of the fields at point Q are sketched in Figure 28.63b.

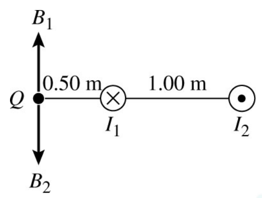


Figure 28.63b

EXECUTE: $B_1 = \dfrac{\mu_0 I_1}{2\pi r_1}$.

$$B_1 = (2\times10^{-7}\ \text{T}\cdot\text{m/A})\left(\frac{6.00\ \text{A}}{0.50\ \text{m}}\right) = 2.40\times10^{-6}\ \text{T}.$$

$$B_2 = \frac{\mu_0 I_2}{2\pi r_2}.$$

$$B_2 = (2\times10^{-7}\ \text{T}\cdot\text{m/A})\left(\frac{2.00\ \text{A}}{1.50\ \text{m}}\right) = 2.67\times10^{-7}\ \text{T}.$$

$\vec{B}_1$ and $\vec{B}_2$ are in opposite directions and $B_1 > B_2$ so

$B = B_1 - B_2 = 2.40\times10^{-6}\ \text{T} - 2.67\times10^{-7}\ \text{T} = 2.13\times10^{-6}\ \text{T}$, and $\vec{B}$ is upward.

(c) SET UP: The directions of the fields at point S are sketched in Figure 28.63c.

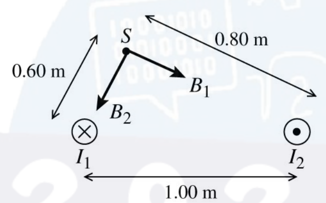


Figure 28.63c

EXECUTE: $B_1 = \dfrac{\mu_0 I_1}{2\pi r_1}$.

$$B_1 = (2\times10^{-7}\ \text{T}\cdot\text{m/A})\left(\frac{6.00\ \text{A}}{0.60\ \text{m}}\right) = 2.00\times10^{-6}\ \text{T}.$$

$$B_2 = \frac{\mu_0 I_2}{2\pi r_2}.$$

$$B_2 = (2\times10^{-7}\ \text{T}\cdot\text{m/A})\left(\frac{2.00\ \text{A}}{0.80\ \text{m}}\right) = 5.00\times10^{-7}\ \text{T}.$$

$\vec{B}_1$ and $\vec{B}_2$ are right angles to each other, so the magnitude of their resultant is given by

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(2.00\times10^{-6}\ \text{T})^2 + (5.00\times10^{-7}\ \text{T})^2} = 2.06\times10^{-6}\ \text{T}.$$

EVALUATE: The magnetic field lines for a long, straight wire are concentric circles with the wire at the center. The magnetic field at each point is tangent to the field line, so $\vec{B}$ is perpendicular to the line from the wire to the point where the field is calculated.

28.64. IDENTIFY: Consider the forces on each side of the loop.

SET UP: The forces on the left and right sides cancel. The forces on the top and bottom segments of the loop are in opposite directions, so the magnitudes subtract.

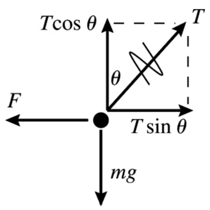
EXECUTE: $F = F_\text{t} - F_\text{b} = \left(\dfrac{\mu_0 I_\text{wire}}{2\pi}\right)\left(\dfrac{Il}{r_\text{t}} - \dfrac{Il}{r_\text{b}}\right) = \dfrac{\mu_0 I l I_\text{wire}}{2\pi}\left(\dfrac{1}{r_\text{t}} - \dfrac{1}{r_\text{b}}\right)$.

$F = \dfrac{\mu_0 (5.00\ \text{A})(0.200\ \text{m})(14.0\ \text{A})}{2\pi}\left(-\dfrac{1}{0.100\ \text{m}} + \dfrac{1}{0.026\ \text{m}}\right) = 7.97\times10^{-5}\ \text{N}.$ The force on the top segment is toward the wire, so the net force is toward the wire.

EVALUATE: The net force on a current loop in a uniform magnetic field is zero, but the magnetic field of the wire is not uniform; it is stronger closer to the wire.

28.65. IDENTIFY: Apply $\sum\vec{F} = 0$ to one of the wires. The force one wire exerts on the other depends on I so $\sum\vec{F} = 0$ gives two equations for the two unknowns T and I.

SET UP: The force diagram for one of the wires is given in Figure 28.65 (next page).



The force one wire exerts on the other is $F = \left(\dfrac{\mu_0 I^2}{2\pi r}\right)L,$ where $r = 2(0.040\text{ m})\sin\theta = 8.362\times10^{-3}\text{ m}$ is the distance between the two wires.

Figure 28.65

EXECUTE: $\sum F_y = 0$ gives $T\cos\theta = mg$ and $T = mg/\cos\theta$.
$\sum F_x = 0$ gives $F = T\sin\theta = (mg/\cos\theta)\sin\theta = mg\tan\theta$.
And $m = \lambda L$, so $F = \lambda Lg\tan\theta$.

$$\left(\frac{\mu_0 I^2}{2\pi r}\right)L = \lambda Lg\tan\theta.$$

$$I = \sqrt{\frac{\lambda gr\tan\theta}{(\mu_0/2\pi)}}.$$

$$I = \sqrt{\frac{(0.0125\text{ kg/m})(9.80\text{ m/s}^2)(\tan 6.00°)(8.362\times10^{-3}\text{ m})}{2\times10^{-7}\text{ T}\cdot\text{m/A}}} = 23.2\text{ A}.$$

EVALUATE: Since the currents are in opposite directions the wires repel. When I is increased, the angle θ from the vertical increases; a large current is required even for the small displacement specified in this problem.

29.66. **IDENTIFY:** Apply $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{l}\times\hat{r}}{r^2}$.

SET UP: The two straight segments produce zero field at P. The field at the center of a circular loop of radius R is $B = \dfrac{\mu_0 I}{2R}$, so the field at the center of curvature of a semicircular loop is $B = \dfrac{\mu_0 I}{4R}$.

EXECUTE: The semicircular loop of radius a produces field out of the page at P and the semicircular loop of radius b produces field into the page. Therefore, $B = B_a - B_b = \dfrac{1}{2}\left(\dfrac{\mu_0 I}{2}\right)\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = \dfrac{\mu_0 I}{4a}\left(1 - \dfrac{a}{b}\right)$, out of page.

EVALUATE: If $a = b$, $B = 0$.

28.67. **IDENTIFY:** Find the vector sum of the fields due to each loop.

SET UP: For a single loop $B_x = \dfrac{\mu_0 Ia^2}{2(x^2 + a^2)^{3/2}}$. Here we have two loops, each of N turns, and measuring the field along the x-axis from between them means that the "x" in the formula is different for each case.

EXECUTE: **(a)**

$$\underline{\text{Left coil:}}\ x \to x + \frac{a}{2} \Rightarrow B_{\text{l}} = \frac{\mu_0 NIa^2}{2[(x + a/2)^2 + a^2]^{3/2}}.$$

$$\underline{\text{Right coil:}}\ x \to x - \frac{a}{2} \Rightarrow B_{\text{r}} = \frac{\mu_0 NIa^2}{2[(x - a/2)^2 + a^2]^{3/2}}.$$

So, the total field at a point a distance x from the point between them is

$$B = \frac{\mu_0 NIa^2}{2}\left(\frac{1}{[(x + a/2)^2 + a^2]^{3/2}} + \frac{1}{[(x - a/2)^2 + a^2]^{3/2}}\right).$$

(b) B versus x is graphed in Figure 28.67. Figure 28.67a is the total field and Figure 28.67b is the field from the right-hand coil.

(c) At point P, $x = 0$ and $B = \dfrac{\mu_0 NIa^2}{2}\left(\dfrac{1}{[(a/2)^2 + a^2]^{3/2}} + \dfrac{1}{[(-a/2)^2 + a^2]^{3/2}}\right) = \dfrac{\mu_0 NIa^2}{(5a^2/4)^{3/2}} = \left(\dfrac{4}{5}\right)^{3/2}\dfrac{\mu_0 NI}{a}$.

(d) $B=\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 NI}{a}=\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0(300)(6.00\text{ A})}{(0.080\text{ m})}=0.0202\text{ T}.$

(e) $\frac{dB}{dx}=\frac{\mu_0 NIa^2}{2}\left(\frac{-3(x+a/2)}{[(x+a/2)^2+a^2]^{5/2}}+\frac{-3(x-a/2)}{[(x-a/2)^2+a^2]^{5/2}}\right)$. At $x=0$,

$$\left.\frac{dB}{dx}\right|_{x=0}=\frac{\mu_0 NIa^2}{2}\left(\frac{-3(a/2)}{[(a/2)^2+a^2]^{5/2}}+\frac{-3(-a/2)}{[(-a/2)^2+a^2]^{5/2}}\right)=0.$$

$$\frac{d^2B}{dx^2}=\frac{\mu_0 NIa^2}{2}\left(\frac{-3}{[(x+a/2)^2+a^2]^{5/2}}+\frac{6(x+a/2)^2(5/2)}{[(x+a/2)^2+a^2]^{7/2}}+\frac{-3}{[(x-a/2)^2+a^2]^{5/2}}+\frac{6(x-a/2)^2(5/2)}{[(x-a/2)^2+a^2]^{7/2}}\right).$$

At $x=0$,

$$\left.\frac{d^2B}{dx^2}\right|_{x=0}=\frac{\mu_0 NIa^2}{2}\left(\frac{-3}{[(a/2)^2+a^2]^{5/2}}+\frac{6(a/2)^2(5/2)}{[(a/2)^2+a^2]^{7/2}}+\frac{-3}{[(a/2)^2+a^2]^{5/2}}+\frac{6(-a/2)^2(5/2)}{[(a/2)^2+a^2]^{7/2}}\right)=0.$$

EVALUATE: Since both first and second derivatives are zero, the field can only be changing very slowly.

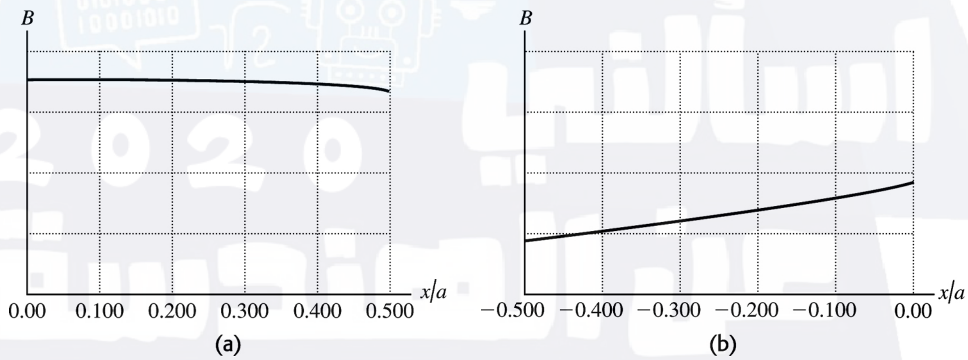


Figure 28.67

28.68. **IDENTIFY:** Both arcs produce magnetic fields at point *P* perpendicular to the plane of the page. The field due to arc *DA* points into the page, and the field due to arc *BC* points out of the page. The field due to *DA* has a greater magnitude than the field due to arc *BC*. The net field is the sum of these two fields.

SET UP: The magnitude field at the center of a circular loop of radius *a* is $B=\frac{\mu_0 I}{2\pi a}$. Each arc is 120°/360° = 1/3 of a complete loop, so the field due to each of them is $B=\frac{1}{3}\frac{\mu_0 I}{2\pi a}=\frac{\mu_0 I}{6\pi a}$.

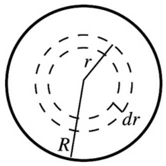
EXECUTE: The net field is

$B_{net}=B_{20}-B_{30}=\frac{\mu_0(12.0\text{ A})}{6\pi}\left(\frac{1}{0.200\text{ m}}-\frac{1}{0.300\text{ m}}\right)=4.19\times10^{-6}\text{ T}=4.19\ \mu\text{T}$. Since $B_{20}>B_{30}$, the net field points into the page at *P*.

EVALUATE: The current in segments *CD* and *AB* produces no magnetic field at *P* because its direction is directly toward (or away from) point *P*.

28.69. **(a) IDENTIFY:** Consider current density *J* for a small concentric ring and integrate to find the total current in terms of α and *R*.

SET UP: We can't say $I=JA=J\pi R^2$, since *J* varies across the cross section.



To integrate J over the cross section of the wire, divide the wire cross section up into thin concentric rings of radius r and width dr, as shown in Figure 28.69.

Figure 28.69

EXECUTE: The area of such a ring is dA, and the current through it is $dI = J\,dA$; $dA = 2\pi r dr$ and $dI = J\,dA = \alpha r(2\pi r\,dr) = 2\pi\alpha r^2 dr$.

$I = \int dI = 2\pi\alpha\int_0^R r^2 dr = 2\pi\alpha(R^3/3)$ so $\alpha = \dfrac{3I}{2\pi R^3}$.

(b) IDENTIFY and SET UP: (i) $r \le R$.

Apply Ampere's law to a circle of radius $r < R$. Use the method of part (a) to find the current enclosed by Ampere's law path.

EXECUTE: $\oint \vec{B}\cdot d\vec{l} = \oint B\,dl = B\oint dl = B(2\pi r)$, by the symmetry and direction of $\vec{B}$. The current passing through the path is $I_{\text{encl}} = \int dl$, where the integration is from 0 to r.

$I_{\text{encl}} = 2\pi\alpha\int_0^r r^2 dr = \dfrac{2\pi\alpha r^3}{3} = \dfrac{2\pi}{3}\left(\dfrac{3I}{2\pi R^3}\right)r^3 = \dfrac{Ir^3}{R^3}$. Thus $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives

$B(2\pi r) = \mu_0\left(\dfrac{Ir^3}{R^3}\right)$ and $B = \dfrac{\mu_0 I r^2}{2\pi R^3}$.

(ii) **IDENTIFY and SET UP:** $r \ge R$.

Apply Ampere's law to a circle of radius $r > R$.

EXECUTE: $\oint \vec{B}\cdot d\vec{l} = \oint B\,dl = B\oint dl = B(2\pi r)$.

$I_{\text{encl}} = I$; all the current in the wire passes through this path. Thus $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives $B(2\pi r) = \mu_0 I$ and $B = \dfrac{\mu_0 I}{2\pi r}$.

EVALUATE: Note that at $r = R$ the expression in (i) (for $r \le R$) gives $B = \dfrac{\mu_0 I}{2\pi R}$. At $r = R$ the expression in (ii) (for $r \ge R$) gives $B = \dfrac{\mu_0 I}{2\pi R}$, which is the same.

28.70. IDENTIFY: Apply $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I d\vec{l}\times\hat{r}}{r^2}$.

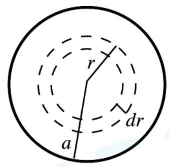
SET UP: The horizontal wire yields zero magnetic field since $d\vec{l}\times\vec{r} = 0$. The vertical current provides the magnetic field of half of an infinite wire. (The contributions from all infinitesimal pieces of the wire point in the same direction, so there is no vector addition or components to worry about.)

EXECUTE: $B = \frac{1}{2}\left(\dfrac{\mu_0 I}{2\pi a}\right) = \dfrac{\mu_0 I}{4\pi a}$ and is directed out of the page.

EVALUATE: In the equation preceding Eq. (28.8) the limits on the integration are 0 to a rather than $-a$ to a and this introduces a factor of $\frac{1}{2}$ into the expression for B.

28.71. IDENTIFY: Use the current density J to find dI through a concentric ring and integrate over the appropriate cross section to find the current through that cross section. Then use Ampere's law to find $\vec{B}$ at the specified distance from the center of the wire.

(a) SET UP:



Divide the cross section of the cylinder into thin concentric rings of radius r and width dr, as shown in Figure 28.71a. The current through each ring is $dI = J\,dA = J2\pi r\,dr$.

Figure 28.71a

EXECUTE: $dI = \dfrac{2I_0}{\pi a^2}[1-(r/a)^2]2\pi r\,dr = \dfrac{4I_0}{a^2}[1-(r/a)^2]r\,dr$. The total current I is obtained by integrating dI over the cross section $I = \int_0^a dI = \left(\dfrac{4I_0}{a^2}\right)\int_0^a (1-r^2/a^2)r\,dr = \left(\dfrac{4I_0}{a^2}\right)\left[\dfrac{1}{2}r^2 - \dfrac{1}{4}r^4/a^2\right]_0^a = I_0$, as was to be shown.

(b) SET UP: Apply Ampere's law to a path that is a circle of radius $r > a$, as shown in Figure 28.71b.

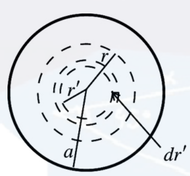
$\oint \vec{B}\cdot d\vec{l} = B(2\pi r)$.

$I_{\text{encl}} = I_0$ (the path encloses the entire cylinder).

Figure 28.71b

EXECUTE: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ says $B(2\pi r) = \mu_0 I_0$ and $B = \dfrac{\mu_0 I_0}{2\pi r}$.

(c) SET UP:



Divide the cross section of the cylinder into concentric rings of radius r' and width dr', as was done in part (a). See Figure 28.71c. The current dI through each ring is $dI = \dfrac{4I_0}{a^2}\left[1-\left(\dfrac{r'}{a}\right)^2\right]r'\,dr'$.

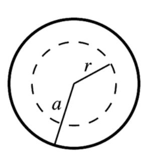
Figure 28.71c

EXECUTE: The current I is obtained by integrating dI from $r' = 0$ to $r' = r$:

$$I = \int dI = \frac{4I_0}{a^2}\int_0^r \left[1-\left(\frac{r'}{a}\right)^2\right]r'\,dr' = \frac{4I_0}{a^2}\left[\tfrac{1}{2}(r')^2 - \tfrac{1}{4}(r')^4/a^2\right]_0^r.$$

$$I = \frac{4I_0}{a^2}(r^2/2 - r^4/4a^2) = \frac{I_0 r^2}{a^2}\left(2-\frac{r^2}{a^2}\right).$$

(d) SET UP: Apply Ampere's law to a path that is a circle of radius $r < a$, as shown in Figure 28.71d.



$\oint \vec{B}\cdot d\vec{l} = B(2\pi r)$.

$I_{\text{encl}} = \dfrac{I_0 r^2}{a^2}\left(2-\dfrac{r^2}{a^2}\right)$ (from part (c)).

Figure 28.71d

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ says $B(2\pi r) = \mu_0 \dfrac{I_0 r^2}{a^2}(2 - r^2/a^2)$ and $B = \dfrac{\mu_0 I_0}{2\pi} \dfrac{r}{a^2}(2 - r^2/a^2)$.

EVALUATE: Result in part (b) evaluated at $r = a$: $B = \dfrac{\mu_0 I_0}{2\pi a}$. Result in part (d) evaluated at $r = a$: $B = \dfrac{\mu_0 I_0}{2\pi} \dfrac{a}{a^2}(2 - a^2/a^2) = \dfrac{\mu_0 I_0}{2\pi a}$. The two results, one for $r > a$ and the other for $r < a$, agree at $r = a$.

28.72. IDENTIFY: The net field is the vector sum of the fields due to the circular loop and to the long straight wire.

SET UP: For the long wire, $B = \dfrac{\mu_0 I_1}{2\pi D}$, and for the loop, $B = \dfrac{\mu_0 I_2}{2R}$.

EXECUTE: At the center of the circular loop the current I_2 generates a magnetic field that is into the page, so the current I_1 must point to the right. For complete cancellation the two fields must have the same magnitude: $\dfrac{\mu_0 I_1}{2\pi D} = \dfrac{\mu_0 I_2}{2R}$. Thus, $I_1 = \dfrac{\pi D}{R} I_2$.

EVALUATE: If I_1 is to the left the two fields add.

28.73. IDENTIFY: Use what we know about the magnetic field of a long, straight conductor to deduce the symmetry of the magnetic field. Then apply Ampere's law to calculate the magnetic field at a distance *a* above and below the current sheet.

SET UP: Do parts (a) and (b) together.

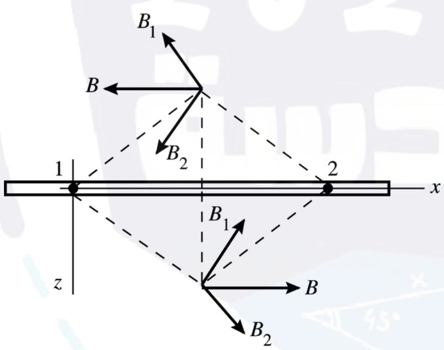


Figure 28.73a

Consider the individual currents in pairs, where the currents in each pair are equidistant on either side of the point where $\vec{B}$ is being calculated. Figure 28.73a shows that for each pair the *z*-components cancel, and that above the sheet the field is in the $-x$-direction and that below the sheet it is in the $+x$-direction.

Also, by symmetry the magnitude of $\vec{B}$ a distance *a* above the sheet must equal the magnitude of $\vec{B}$ a distance *a* below the sheet. Now that we have deduced the symmetry of $\vec{B}$, apply Ampere's law. Use a path that is a rectangle, as shown in Figure 28.73b.

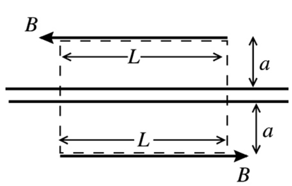


Figure 28.73b

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$.

I is directed out of the page, so for *I* to be positive the integral around the path is taken in the counterclockwise direction.

EXECUTE: Since $\vec{B}$ is parallel to the sheet, on the sides of the rectangle that have length $2a$, $\oint \vec{B} \cdot d\vec{l} = 0$. On the long sides of length L, $\vec{B}$ is parallel to the side, in the direction we are integrating around the path, and has the same magnitude, B, on each side. Thus $\oint \vec{B} \cdot d\vec{l} = 2BL.$ n conductors per unit length and current I out of the page in each conductor gives $I_{\text{encl}} = InL.$ Ampere's law then gives

$2BL = \mu_0 InL$ and $B = \frac{1}{2}\mu_0 In.$

EVALUATE: Note that B is independent of the distance a from the sheet. Compare this result to the electric field due to an infinite sheet of charge in Chapter 22.

28.74. IDENTIFY: Find the vector sum of the fields due to each sheet.

SET UP: Problem 28.73 shows that for an infinite sheet $B = \frac{1}{2}\mu_0 In.$ If I is out of the page, $\vec{B}$ is to the left above the sheet and to the right below the sheet. If I is into the page, $\vec{B}$ is to the right above the sheet and to the left below the sheet. B is independent of the distance from the sheet. The directions of the two fields at points P, R and S are shown in Figure 28.74.

EXECUTE: **(a)** Above the two sheets, the fields cancel (since there is no dependence upon the distance from the sheets).

(b) In between the sheets the two fields add up to yield $B = \mu_0 nI$, to the right.

(c) Below the two sheets, their fields again cancel (since there is no dependence upon the distance from the sheets).

EVALUATE: The two sheets with currents in opposite directions produce a uniform field between the sheets and zero field outside the two sheets. This is analogous to the electric field produced by large parallel sheets of charge of opposite sign.

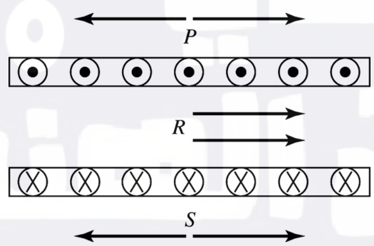


Figure 28.74

28.75. IDENTIFY: Apply Ampere's law to a circle of radius r.

SET UP: The current within a radius r is $I = \int \vec{J} \cdot d\vec{A}$, where the integration is over a disk of radius r.

EXECUTE: **(a)** $I_0 = \int \vec{J} \cdot d\vec{A} = \int \left(\frac{b}{r} e^{(r-a)/\delta}\right) r dr d\theta = 2\pi b \int_0^a e^{(r-a)/\delta} dr = 2\pi b\delta\, e^{(r-a)/\delta}\Big|_0^a = 2\pi b\delta(1 - e^{-a/\delta}).$

$I_0 = 2\pi(600 \text{ A/m})(0.025 \text{ m})(1 - e^{(0.050/0.025)}) = 81.5 \text{ A}.$

(b) For $r \ge a, \oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0$ and $B = \dfrac{\mu_0 I_0}{2\pi r}.$

(c) For $r \le a$, $I(r) = \int \vec{J} \cdot d\vec{A} = \int \left(\frac{b}{r'} e^{(r'-a)/\delta}\right) r' dr' d\theta = 2\pi b \int_0^r e^{(r-a)/\delta} dr = 2\pi b\delta e^{(r'-a)/\delta}\Big|_0^r.$

$I(r) = 2\pi b\delta(e^{(r-a)/\delta} - e^{-a/\delta}) = 2\pi b\delta e^{-a/\delta}(e^{r/\delta} - 1)$ and $I(r) = I_0 \dfrac{(e^{r/\delta} - 1)}{(e^{a/\delta} - 1)}.$

(d) For $r \le a$, $\oint \vec{B} \cdot d\vec{l} = B(r)2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0 \dfrac{(e^{r/\delta} - 1)}{(e^{a/\delta} - 1)}$ and $B = \dfrac{\mu_0 I_0 (e^{r/\delta} - 1)}{2\pi r(e^{a/\delta} - 1)}.$

(e) At $r = \delta = 0.025$ m, $B = \dfrac{\mu_0 I_0 (e - 1)}{2\pi\delta(e^{a/\delta} - 1)} = \dfrac{\mu_0 (81.5 \text{ A})}{2\pi(0.025 \text{ m})} \dfrac{(e - 1)}{(e^{0.050/0.025} - 1)} = 1.75 \times 10^{-4}$ T.

At $r = a = 0.050$ m, $B = \dfrac{\mu_0 I_0}{2\pi a} \dfrac{(e^{a/\delta} - 1)}{(e^{a/\delta} - 1)} = \dfrac{\mu_0 (81.5 \text{ A})}{2\pi(0.050 \text{ m})} = 3.26 \times 10^{-4}$ T.

At $r = 2a = 0.100$ m, $B = \dfrac{\mu_0 I_0}{2\pi r} = \dfrac{\mu_0 (81.5\text{ A})}{2\pi(0.100\text{ m})} = 1.63\times10^{-4}$ T.

EVALUATE: At points outside the cylinder, the magnetic field is the same as that due to a long wire running along the axis of the cylinder.

28.76. **IDENTIFY** and **SET UP:** We assume that both solenoids are ideal, in which case the field due to each one is given by $B = \mu_0 nI = \mu_0 \dfrac{N}{L} I$. The net field inside is the sum of both the fields.

EXECUTE: **(a)** The net field is $B = \mu_0 \dfrac{N_1}{L} I_1 + \mu_0 \dfrac{N_2}{L} I_2 = \dfrac{\mu_0}{L}[N_1 I_1 + N_2 I_2]$. For the numbers in this problem, we have $BL/\mu_0 = (0.00200\text{ A})N_1 + N_2 I_2$. Therefore a graph of BL/μ_0 versus I_2 should be a straight line with slope equal to N_2 and y-intercept equal to (0.00200 A)N_1.

(b) Using the graph given with the problem, we calculate the slope using the points (5.00 mA, 16.00 A) and (2.00 mA, 8.00 A), which gives slope = (16.00 A – 8.00 A)/(5.00 mA – 2.00 mA) = 2667. Therefore N_2 = 2667 turns, which rounds to 2670 turns. To find the y-intercept, we use the point (5.00 mA, 16.00 A) and the slope to deduce the equation of the line. This gives $\dfrac{y - 16.00\text{ A}}{x - 0.00500\text{ A}} = 2667$, which simplifies to $y = 2667x + 2.67$. When $x = 0$, y = 2.67 A. As we saw, the y-intercept is equal to (0.00200 A)N_1, so N_1 = (2.67 A)/(0.00200 A) = 1335 turns, which rounds to 1340 turns.

(c) Now the fields are in opposite directions, so $B = \mu_0 \dfrac{N_1}{L} I_1 - \mu_0 \dfrac{N_2}{L} I_2 = \dfrac{\mu_0}{L}[N_1 I_1 - N_2 I_2]$.

B = [(μ_0)/(0.400 m)][(0.00200 A)(1335) – (0.00500 A)(2667)] $= -3.35\times10^{-5}$ T. The minus sign just tells us that the field due to I_2 is stronger than the field due to I_1. So the magnitude of the net field is $B = 3.35\times0^{-5}\text{ T} = 33.5\ \mu\text{T}$.

EVALUATE: As a check for N_1 in part (b), we could use a ruler to extrapolate the graph in the textbook back to its intersection with the y-axis to find the y-intercept. This method is not particularly accurate, but it should give reasonable agreement with the result for N_1 from part (b).

28.77. **IDENTIFY** and **SET UP:** The magnitude of the magnetic a distance r from the center of a very long current-carrying wire is $B = \dfrac{\mu_0 I}{2\pi r}$. In this case, the measured quantity x is the distance from the *surface* of the cable, not from the center.

EXECUTE: **(a)** Multiplying the quantities given in the table in the problem, we get the following values for Bx in units of $\text{T}\cdot\text{cm}$, starting with the first pair: 0.812, 1.00, 1.09, 1.13, 1.16. As we can see, these values are not constant. However the last three values are nearly constant. Therefore Bx is not truly constant. The reason for this is that x is the distance from the *surface* of the cable, not from the center. In the formula $B = \dfrac{\mu_0 I}{2\pi r}$, r is the distance from the center of the cable. In that case, we would expect Br to be constant. For the last three points, it does appear that Bx is nearly constant. The reason for this is that the proper formula for the magnetic field for this cable is $B = \dfrac{\mu_0 I}{2\pi(R + x)}$, where R is the radius of the cable. As x gets large compared to R, $r \approx x$ and the magnitude approaches $\dfrac{\mu_0 I}{2\pi r}$.

(b) Using the equation appropriate for the cable and solving for x gives $x = (\mu_0 I/2\pi)\dfrac{1}{B} - R$. A graph of x versus $1/B$ should have a slope equal to $\mu_0 I/2\pi$ and a y-intercept equal to $-R$. Figure 28.77 shows the graph of x versus $1/B$.

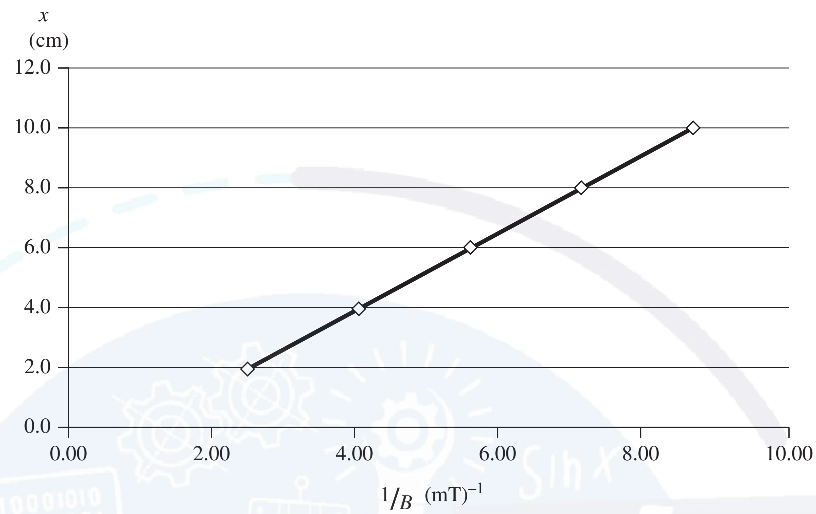


Figure 28.77

(c) The best-fit equation for this graph is $x = (1.2981\ \text{mT}\cdot\text{cm})\dfrac{1}{B} - 1.1914$ cm. The slope is $1.2981\ \text{mT}\cdot\text{cm} = 1.2981\times10^{-5}\ \text{T}\cdot\text{m}$. Since the slope is equal to $\mu_0 I/2\pi$, we have $\mu_0 I/2\pi$ = slope, which gives $I = 2\pi(\text{slope})/\mu_0 = 2\pi(1.2981\times10^{-5}\ \text{T}\cdot\text{m})/\mu_0 = 64.9\ \text{A}$, which rounds to 65 A. The y-intercept is $-R$, so $R = -(-1.1914\ \text{cm}) = 1.2$ cm.

EVALUATE: As we can see, the field within 2 cm or so of the surface of the cable would vary considerably from the value given by $B = \dfrac{\mu_0 I}{2\pi r}$.

28.78. IDENTIFY and SET UP: The wires repel each other since they carry currents in opposite directions, so the wires will move away from each other until the magnetic force is just balanced by the force due to the spring. The force per unit length between two parallel current-carrying wires of equal length and separation r is $\dfrac{F}{L} = \dfrac{\mu_0}{2\pi}\dfrac{I'I}{r}$. In this case, the currents are the same and the distance between the wires is $l_0 + x$, where x is the distance the spring stretches. Therefore the force is $F = \dfrac{\mu_0 I^2 L}{2\pi(l_0 + x)}$. The magnitude of the force that each spring exerts is $F = kx$, by Hooke's law. On each wire, $F_{\text{spr}} = F_{\text{mag}}$, and there are *two* spring forces on each wire. Therefore $\dfrac{\mu_0 I^2 L}{2\pi(l_0 + x)} = 2kx$.

EXECUTE: **(a)** We are given two cases with values for I and x, and each one leads to an equation involving l_0 and k. If we take the ratio of these two equations, common factors such as L will cancel. This gives us

$\dfrac{(13.1\ \text{A})^2(l_0 + 0.40\ \text{m})}{(8.05\ \text{A})^2(l_0 + 0.80\ \text{m})} = \dfrac{0.80\ \text{cm}}{0.40\ \text{cm}} = 2.0.$ Solving for l_0 gives $l_0 = 0.834$ cm, which rounds to 0.83 cm.

Now we can solve for k using this value for l_0 using $\dfrac{\mu_0 I^2 L}{2\pi(l_0 + x)} = 2kx.$

$\dfrac{\mu_0(13.1\ \text{A})^2(0.50\ \text{m})}{2\pi(0.0080\ \text{m} + 0.00834\ \text{m})} = 2k(0.0080\ \text{m}).$ $k = 0.0656$ N/m, which rounds to 0.066 N/m.

(b) For a 12.0-A current, we have $\dfrac{\mu_0(12.0\text{ A})^2(0.50\text{ m})}{2\pi(x+0.00834\text{ m})}=2(0.0656\text{ N/m})x.$ Carrying out the multiplication and division and simplifying we get the quadratic equation
$x^2+(0.00834\text{ m})x-1.097\times10^{-4}\text{ m}^2=0.$
Using the quadratic formula and taking the positive solution gives x = 0.0071 m = 0.71 cm.
(c) To stretch the spring by 1.00 cm, the current must satisfy the equation
$\dfrac{\mu_0 I^2(0.50\text{ m})}{2\pi(0.0100\text{ m}+0.00834\text{ m})}=2(0.0656\text{ N/m})(0.0100\text{ m}).$ This gives I = 15.5 A, which rounds to 16 A.

EVALUATE: The spring force in part (c) is kx =(0.0656 N/m)(0.0100 m) = 6.56×10^{-4} N. This is a very small force resulting from a rather large 16-A current. This tells us that magnetic forces between parallel wires, such as extension cords, are not very significant for typical household currents.

28.79. IDENTIFY: The current-carrying wires repel each other magnetically, causing them to accelerate horizontally. Since gravity is vertical, it plays no initial role.

SET UP: The magnetic force per unit length is $\dfrac{F}{L}=\dfrac{\mu_0}{2\pi}\dfrac{I^2}{d},$ and the acceleration obeys the equation $F/L=m/L\ a.$ The rms current over a short discharge time is $I_0/\sqrt{2}.$

EXECUTE: **(a)** First get the force per unit length:

$$\frac{F}{L}=\frac{\mu_0}{2\pi}\frac{I^2}{d}=\frac{\mu_0}{2\pi d}\left(\frac{I_0}{\sqrt{2}}\right)^2=\frac{\mu_0}{4\pi d}\left(\frac{V}{R}\right)^2=\frac{\mu_0}{4\pi d}\left(\frac{Q_0}{RC}\right)^2.$$

Now apply Newton's second law using the result above: $\dfrac{F}{L}=\dfrac{m}{L}a=\lambda a=\dfrac{\mu_0}{4\pi d}\left(\dfrac{Q_0}{RC}\right)^2.$ Solving for a gives

$a=\dfrac{\mu_0 Q_0^2}{4\pi\lambda R^2C^2 d}.$ From the kinematics equation $v_x=v_{0x}+a_x t,$ we have $v_0=at=aRC=\dfrac{\mu_0 Q_0^2}{4\pi\lambda RCd}.$

(b) Conservation of energy gives $\frac{1}{2}mv_0^2=mgh$ and $h=\dfrac{v_0^2}{2g}=\dfrac{\left(\dfrac{\mu_0 Q_0^2}{4\pi\lambda RCd}\right)^2}{2g}=\dfrac{1}{2g}\left(\dfrac{\mu_0 Q_0^2}{4\pi\lambda RCd}\right)^2.$

EVALUATE: Once the wires have swung apart, we would have to consider gravity in applying Newton's second law.

28.80. IDENTIFY: Approximate the moving belt as an infinite current sheet.

SET UP: Problem 28.73 shows that $B=\frac{1}{2}\mu_0 In$ for an infinite current sheet. Let L be the width of the sheet, so $n=1/L.$

EXECUTE: The amount of charge on a length Δx of the belt is $\Delta Q=L\Delta x\sigma,$ so $I=\dfrac{\Delta Q}{\Delta t}=L\dfrac{\Delta x}{\Delta t}\sigma=Lv\sigma.$

Approximating the belt as an infinite sheet $B=\dfrac{\mu_0 I}{2L}=\dfrac{\mu_0 v\sigma}{2}.$ $\vec{B}$ is directed out of the page, as shown in Figure 28.80.

EVALUATE: The field is uniform above the sheet, for points close enough to the sheet for it to be considered infinite.

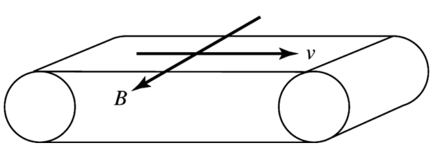


Figure 28.80

28.81. **IDENTIFY and SET UP:** This solenoid is not ideal since its width is fairly large compared to its length. But we can get a rough estimate using the ideal formula, $B = \mu_0 nI$.

EXECUTE: $B = \mu_0 nI = \mu_0 (1000\ \text{m}^{-1})I = 150\times10^{-6}\ \text{T}$, which gives $I = 0.12$ A, choice (b).

EVALUATE: This is a reasonable laboratory current of 120 mA.

28.82. **IDENTIFY and SET UP:** The magnetic field of an ideal solenoid is $B = \mu_0 nI$.

EXECUTE: Both solenoids have the same current, the same length, and the same number of turns, so the magnetic field inside both of them should be the same, which is choice (c).

EVALUATE: This answer is somewhat of an approximation. Even though both solenoids have the same current and same length and number of turns, the second (larger) solenoid is even farther from the ideal case than the first one. Therefore there would be some difference in the magnetic fields inside.

28.83. **IDENTIFY and SET UP:** The enclosure is no longer present to shield the solenoid from the earth's magnetic field of $50\ \mu\text{T}$, so net field inside is a sum of the solenoid field and the earth's field. Whether the earth's field adds or subtracts from the solenoid's field depends on the orientation of the solenoid. The magnetic field due to the solenoid is $150\ \mu\text{T}$.

EXECUTE: When the solenoid field is parallel to the earth's field, the net field is $150\ \mu\text{T} + 50\ \mu\text{T} = 200\ \mu\text{T}$. When the field's are antiparallel (opposite), the net field is $150\ \mu\text{T} - 50\ \mu\text{T} = 100\ \mu\text{T}$. So the field that the bacteria experience is between $100\ \mu\text{T}$ and $200\ \mu\text{T}$, which is choice (c).

EVALUATE: Since the earth's field is quite appreciable compared to the solenoid's field, it is important to shield the solenoid from external fields, such as that of the earth. The earth's field can make a difference of up to a factor of 2 in the field experienced by the bacteria.