BRIDGING PROBLEM MAGNETIC TORQUE ON A CURRENT-CARRYING RING

SOLUTION

A circular ring with area 4.45 cm^2 is carrying a current of 12.5 A. The ring, initially at rest, is immersed in a region of uniform magnetic field given by $\vec{B} = (1.15 \times 10^{-2}\ \text{T})(12\hat{\imath} + 3\hat{\jmath} - 4\hat{k})$. The ring is positioned initially such that its magnetic moment is given by $\vec{\mu}_{\text{i}} = \mu(-0.800\hat{\imath} + 0.600\hat{\jmath})$, where μ is the (positive) magnitude of the magnetic moment. (a) Find the initial magnetic torque on the ring. (b) The ring (which is free to rotate around one diameter) is released and turns through an angle of 90.0°, at which point its magnetic moment is given by $\vec{\mu}_{\text{f}} = -\mu\hat{k}$. Determine the decrease in potential energy. (c) If the moment of inertia of the ring about a diameter is $8.50 \times 10^{-7}\ \text{kg} \cdot \text{m}^2$, determine the angular speed of the ring as it passes through the second position.

SOLUTION GUIDE

IDENTIFY and SET UP

1. The current-carrying ring acts as a magnetic dipole, so you can use the equations for a magnetic dipole in a uniform magnetic field.
2. There are no nonconservative forces acting on the ring as it rotates, so the sum of its rotational kinetic energy (discussed in Section 9.4) and the potential energy is conserved.

EXECUTE

3. Use the vector expression for the torque on a magnetic dipole to find the answer to part (a). (*Hint:* Review Section 1.10.)
4. Find the change in potential energy from the first orientation of the ring to the second orientation.
5. Use your result from step 4 to find the rotational kinetic energy of the ring when it is in the second orientation.
6. Use your result from step 5 to find the ring's angular speed when it is in the second orientation.

EVALUATE

7. If the ring were free to rotate around *any* diameter, in what direction would the magnetic moment point when the ring is in a state of stable equilibrium?

Problems

For assigned homework and other learning materials, go to MasteringPhysics®. MP

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q27.1 Can a charged particle move through a magnetic field without experiencing any force? If so, how? If not, why not?

Q27.2 At any point in space, the electric field $\vec{E}$ is defined to be in the direction of the electric force on a positively charged particle at that point. Why don't we similarly define the magnetic field $\vec{B}$ to be in the direction of the magnetic force on a moving, positively charged particle?

Q27.3 Section 27.2 describes a procedure for finding the direction of the magnetic force using your right hand. If you use the same procedure, but with your left hand, will you get the correct direction for the force? Explain.

Q27.4 The magnetic force on a moving charged particle is always perpendicular to the magnetic field $\vec{B}$. Is the trajectory of a moving charged particle always perpendicular to the magnetic field lines? Explain your reasoning.

Q27.5 A charged particle is fired into a cubical region of space where there is a uniform magnetic field. Outside this region, there is no magnetic field. Is it possible that the particle will remain inside the cubical region? Why or why not?

Q27.6 If the magnetic force does no work on a charged particle, how can it have any effect on the particle's motion? Are there other examples of forces that do no work but have a significant effect on a particle's motion?

Q27.7 A charged particle moves through a region of space with constant velocity (magnitude and direction). If the external magnetic field is zero in this region, can you conclude that the external electric field in the region is also zero? Explain. (By "external" we mean fields other than those produced by the charged particle.) If the external electric field is zero in the region, can you conclude that the external magnetic field in the region is also zero?

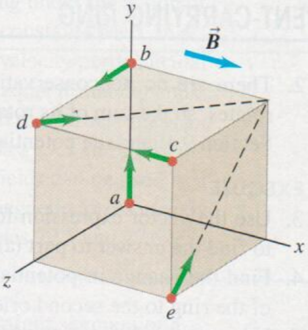
Q27.8 Do you think a sensitive device which measures minute amounts of current, such as a galvanometer, would be affected by earth's magnetic field? Why?

Q27.9 How does a magnetic field produced by a solenoid change if the solenoid is doubled both diametrically and lengthwise while keeping the number of loops constant?

Q27.10 A loose, floppy loop of wire is carrying current I. The loop of wire is placed on a horizontal table in a uniform magnetic field $\vec{B}$ perpendicular to the plane of the table. This causes the loop of wire to expand into a circular shape while still lying on the table. In a diagram, show all possible orientations of the current I and magnetic field $\vec{B}$ that could cause this to occur. Explain your reasoning.

Q27.11 Several charges enter a uniform magnetic field directed into the page. (a) What path would a positive charge q moving with a velocity of magnitude v follow through the field? (b) What path would a positive charge q moving with a velocity of magnitude $2v$ follow through the field? (c) What path would a negative charge $-q$ moving with a velocity of magnitude v follow through the field? (d) What path would a neutral particle follow through the field?

Figure **Q27.12**



Q27.12 Each of the lettered points at the corners of the cube in **Fig. Q27.12** represents a positive charge q moving with a velocity of magnitude v in the direction indicated. The region in the figure is in a uniform magnetic field $\vec{B}$, parallel to the x-axis and directed toward the right. Which charges experience a force due to $\vec{B}$? What is the direction of the force on each charge?

Q27.13 A student claims that if lightning strikes a metal flagpole, the force exerted by the earth's magnetic field on the current in the pole can be large enough to bend it. Typical lightning currents are of the order of 10^4 to 10^5 A. Is the student's opinion justified? Explain your reasoning.

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EXERCISES

Section 27.2 Magnetic Field

27.1 • A particle with a charge of -1.24×10^{-8} C is moving with instantaneous velocity $\vec{v} = (4.19 \times 10^4 \text{ m/s})\hat{\imath} + (-3.85 \times 10^4 \text{ m/s})\hat{\jmath}$. What is the force exerted on this particle by a magnetic field (a) $\vec{B} = (1.40 \text{ T})\hat{\imath}$ and (b) $\vec{B} = (1.40 \text{ T})\hat{k}$?

27.2 • A particle of mass 0.190 g carries a charge of -3.60×10^{-8} C. The particle is given an initial horizontal velocity that is due north and has magnitude 3.57×10^4 m/s. What are the magnitude and direction of the minimum magnetic field that will keep the particle moving in the earth's gravitational field in the same horizontal, northward direction?

27.3 • In a 1.27-T magnetic field directed vertically upward, a particle having a charge of magnitude 9.00 μC and initially moving northward at 4.78 km/s is deflected toward the east. (a) What is the sign of the charge of this particle? Make a sketch to illustrate how you found your answer. (b) Find the magnetic force on the particle.

27.4 • A particle with mass 1.81×10^{-3} kg and a charge of 1.22×10^{-8} C has, at a given instant, a velocity $\vec{v} = (3.00 \times 10^4 \text{ m/s})\hat{\jmath}$. What are the magnitude and direction of the particle's acceleration produced by a uniform magnetic field $\vec{B} = (1.63 \text{ T})\hat{\imath} + (0.980 \text{ T})\hat{\jmath}$?

27.5 • An electron experiences a magnetic force of magnitude 4.10×10^{-15} N when moving at an angle of 56.0° with respect to a magnetic field of magnitude 3.30×10^{-3} T. Find the speed of the electron.

27.6 • An electron moves at 2.10×10^6 m/s through a region in which there is a magnetic field of unspecified direction and magnitude 7.60×10^{-2} T. (a) What are the largest and smallest possible magnitudes of the acceleration of the electron due to the magnetic field? (b) If the actual acceleration of the electron is one-fourth of the largest magnitude in part (a), what is the angle between the electron velocity and the magnetic field?

27.7 •• **CP** A particle with charge 7.60 μC is moving with velocity $\vec{v} = -(3.40 \times 10^3 \text{m/s})\hat{\jmath}$. The magnetic force on the particle is measured to be $\vec{F} = +(7.60 \times 10^{-3} \text{ N})\hat{\imath} - (5.40 \times 10^{-3} \text{ N})\hat{k}$. (a) Calculate all the components of the magnetic field you can from this information. (b) Are there components of the magnetic field that are not determined by the measurement of the force? Explain. (c) Calculate the scalar product $\vec{B} \cdot \vec{F}$. What is the angle between $\vec{B}$ and $\vec{F}$?

27.8 •• **CP** A particle with charge -5.90 nC is moving in a uniform magnetic field $\vec{B} = -(1.30 \text{ T})\hat{k}$. The magnetic force on the particle is measured to be $\vec{F} = -(4.00 \times 10^{-7} \text{N})\hat{\imath} + (7.60 \times 10^{-7} \text{ N})\hat{\jmath}$. (a) Calculate all the components of the velocity of the particle that you can from this information. (b) Are there components of the velocity that are not determined by the measurement of the force? Explain. (c) Calculate the scalar product $\vec{v} \cdot \vec{F}$. What is the angle between $\vec{v}$ and $\vec{F}$?

27.9 •• A group of particles is traveling in a magnetic field of unknown magnitude and direction. You observe that a proton moving at 1.50 km/s in the $+x$-direction experiences a force of 2.06×10^{-16} N in the $+y$-direction, and an electron moving at 4.40 km/s in the $-z$-direction experiences a force of 8.30×10^{-16} N in the $+y$-direction. (a) What are the magnitude and direction of the magnetic field? (b) What are the magnitude and direction of the magnetic force on an electron moving in the $-y$-direction at 3.60 km/s?

Section 27.3 Magnetic Field Lines and Magnetic Flux

27.10 • A flat, square surface with side length 4.20 cm is in the xy-plane at $z = 0$. Calculate the magnitude of the flux through this surface produced by a magnetic field $\vec{B} = (0.250 \text{ T})\hat{\imath} + (0.300 \text{ T})\hat{\jmath} - (0.450 \text{ T})\hat{k}$.

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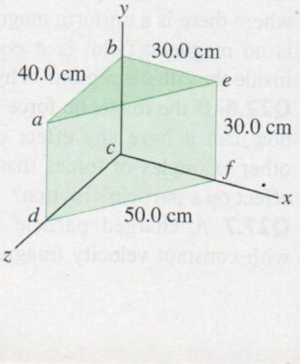
27.12 • A horizontal rectangular surface has dimensions 2.90 cm by 3.00 cm and is in a uniform magnetic field that is directed at an angle of 27.5° above the horizontal. What must the magnitude of the magnetic field be to produce a flux of 4.50×10^{-4} Wb through the surface?

27.13 •• An open plastic soda bottle with an opening diameter of 2.8 cm is placed on a table. A uniform 2.00-T magnetic field directed upward and oriented 24° from vertical encompasses the bottle. What is the total magnetic flux through the plastic of the soda bottle?

27.14 •• The magnetic field $\vec{B}$ in a certain region is 0.128 T, and its direction is that of the $+z$-axis in **Fig. E27.14**. (a) What is the

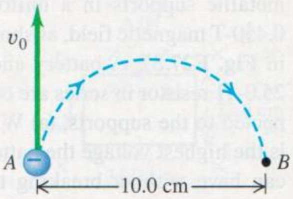
Figure **E27.14**



magnetic flux across the surface *abcd* in the figure? (b) What is the magnetic flux across the surface *befc*? (c) What is the magnetic flux across the surface *aefd*? (d) What is the net flux through all five surfaces that enclose the shaded volume?

Section 27.4 Motion of Charged Particles in a Magnetic Field

Figure **E27.15**



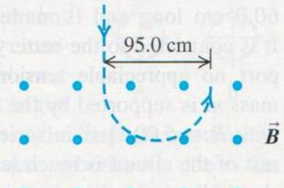
27.15 •• An electron at point *A* in **Fig. E27.15** has a speed v_0 of 1.41×10^6 m/s. Find (a) the magnitude and direction of the magnetic field that will cause the electron to follow the semicircular path from *A* to *B*, and (b) the time required for the electron to move from *A* to *B*.

27.16 •• Repeat Exercise 27.15 for the case in which the particle is a proton rather than an electron.

27.17 • **CP** A 140-g ball containing 4.10×10^8 excess electrons is dropped into a 110-m vertical shaft. At the bottom of the shaft, the ball suddenly enters a uniform horizontal magnetic field that has magnitude 0.300 T and direction from east to west. If air resistance is negligibly small, find the magnitude and direction of the force that this magnetic field exerts on the ball just as it enters the field.

27.18 • An alpha particle (a He nucleus, containing two protons and two neutrons and having a mass of 6.64×10^{-27} kg) traveling horizontally at 35.6 km/s enters a uniform, vertical, 1.80-T magnetic field. (a) What is the diameter of the path followed by this alpha particle? (b) What effect does the magnetic field have on the speed of the particle? (c) What are the magnitude and direction of the acceleration of the alpha particle while it is in the magnetic field? (d) Explain why the speed of the particle does not change even though an unbalanced external force acts on it.

Figure **E27.19**



27.19 • In an experiment with cosmic rays, a vertical beam of particles that have charge of magnitude 3*e* and mass 12 times the proton mass enters a uniform horizontal magnetic field of 0.250 T and is bent in a semicircle of diameter 95.0 cm, as shown in **Fig. E27.19**. (a) Find the speed of the particles and the sign of their charge. (b) Is it reasonable to ignore the gravity force on the particles? (c) How does the speed of the particles as they enter the field compare to their speed as they exit the field?

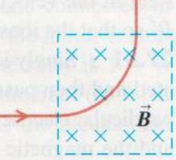
27.20 • **BIO** Cyclotrons are widely used in nuclear medicine for producing short-lived radioactive isotopes. These cyclotrons typically accelerate H^- (the *hydride* ion, which has one proton and two electrons) to an energy of 5 MeV to 20 MeV. This ion has a mass very close to that of a proton because the electron mass is negligible—about $\frac{1}{2000}$ of the proton's mass. A typical magnetic field in such cyclotrons is 1.9 T. (a) What is the speed of a 5.0-MeV H^-? (b) If the H^- has energy 5.0 MeV and $B = 1.9$ T, what is the radius of this ion's circular orbit?

27.21 • A deuteron (the nucleus of an isotope of hydrogen) has a mass of 3.34×10^{-27} kg and a charge of $+e$. The deuteron travels in a circular path with a radius of 7.40 mm in a magnetic field with magnitude 2.40 T. (a) Find the speed of the deuteron. (b) Find the time required for it to make half a revolution. (c) Through what potential difference would the deuteron have to be accelerated to acquire this speed?

27.22 •• In a cyclotron, the orbital radius of protons with energy 300 keV is 16.0 cm. You are redesigning the cyclotron to be used instead for alpha particles with energy 300 keV. An alpha particle has charge $q = +2e$ and mass $m = 6.64 \times 10^{-27}$ kg. If the magnetic field isn't changed, what will be the orbital radius of the alpha particles?

27.23 • An electron in the beam of a cathode-ray tube is accelerated by a potential difference of 1.99 kV. Then it passes through a region of transverse magnetic field, where it moves in a circular arc with radius 0.181 m. What is the magnitude of the field?

Figure **E27.24**



27.24 •• A beam of protons traveling at 1.20 km/s enters a uniform magnetic field, traveling perpendicular to the field. The beam exits the magnetic field, leaving the field in a direction perpendicular to its original direction (**Fig. E27.24**). The beam travels a distance of 1.10 cm *while in the field*. What is the magnitude of the magnetic field?

27.25 •• A proton ($q = 1.60 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) moves in a uniform magnetic field $\vec{B} = (0.550\text{ T})\hat{\imath}$. At $t = 0$ the proton has velocity components $v_x = 1.80 \times 10^5$ m/s, $v_y = 0$, and $v_z = 1.50 \times 10^5$ m/s (see Example 27.4). (a) What are the magnitude and direction of the magnetic force acting on the proton? In addition to the magnetic field there is a uniform electric field in the $+x$-direction, $\vec{E} = (+1.80 \times 10^4\text{ V/m})\hat{\imath}$. (b) Will the proton have a component of acceleration in the direction of the electric field? (c) Describe the path of the proton. Does the electric field affect the radius of the helix? Explain. (d) At $t = T/2$, where *T* is the period of the circular motion of the proton, what is the *x*-component of the displacement of the proton from its position at $t = 0$?

27.26 • A singly charged ion of ^{7}Li (an isotope of lithium) has a mass of 1.16×10^{-26} kg. It is accelerated through a potential difference of 300 V and then enters a magnetic field with magnitude 0.728 T perpendicular to the path of the ion. What is the radius of the ion's path in the magnetic field?

Section 27.5 Applications of Motion of Charged Particles

27.27 • **Crossed $\vec{E}$ and $\vec{B}$ Fields.** A particle with initial velocity $\vec{v}_0 = (5.85 \times 10^3\text{m/s})\hat{\jmath}$ enters a region of uniform electric and magnetic fields. The magnetic field in the region is $\vec{B} = -(1.35\text{ T})\hat{k}$. Calculate the magnitude and direction of the electric field in the region if the particle is to pass through undeflected, for a particle of charge (a) +0.640 nC and (b) −0.320 nC. You can ignore the weight of the particle.

27.28 • (a) What is the speed of a beam of electrons when the simultaneous influence of an electric field of 1.56×10^4 V/m and a magnetic field of 4.62×10^{-3} T, with both fields normal to the beam and to each other, produces no deflection of the electrons? (b) In a diagram, show the relative orientation of the vectors $\vec{v}$, $\vec{E}$, and $\vec{B}$. (c) When the electric field is removed, what is the radius of the electron orbit? What is the period of the orbit?

27.29 •• A 150-V battery is connected across two parallel metal plates of area 28.5 cm^2 and separation 8.50 mm. A beam of alpha particles (charge $+2e$, mass 6.64×10^{-27} kg) is accelerated from rest through a potential difference of 1.80 kV and enters the region between the plates perpendicular to the electric field, as shown in

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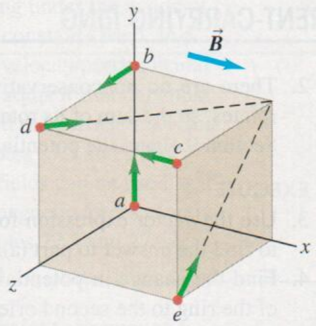

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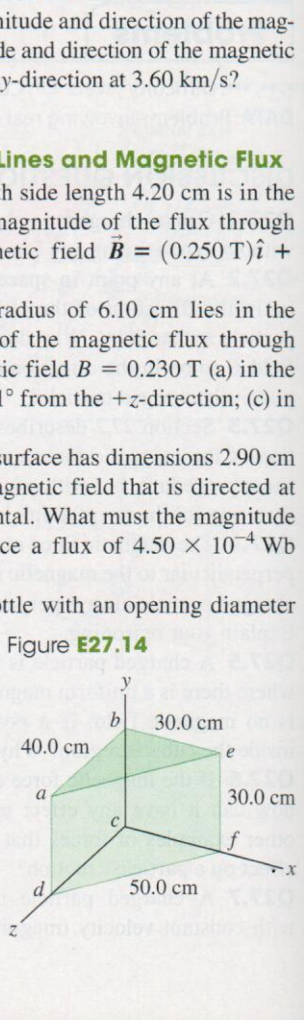
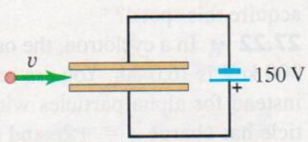


Fig. E27.29. What magnitude and direction of magnetic field are needed so that the alpha particles emerge undeflected from between the plates?

Figure **E27.29**



27.30 • A singly ionized (one electron removed) ^{40}K atom passes through a velocity selector consisting of uniform perpendicular electric and magnetic fields. The selector is adjusted to allow ions having a speed of 4.50 km/s to pass through undeflected when the magnetic field is 0.0250 T. The ions next enter a second uniform magnetic field (B') oriented at right angles to their velocity. ^{40}K contains 19 protons and 21 neutrons and has a mass of 6.64×10^{-26} kg. (a) What is the magnitude of the electric field in the velocity selector? (b) What must be the magnitude of B' so that the ions will be bent into a semicircle of radius 12.5 cm?

27.31 • Singly ionized (one electron removed) atoms are accelerated and then passed through a velocity selector consisting of perpendicular electric and magnetic fields. The electric field is 160 V/m and the magnetic field is 0.0313 T. The ions next enter a uniform magnetic field of magnitude 0.0176 T that is oriented perpendicular to their velocity. (a) How fast are the ions moving when they emerge from the velocity selector? (b) If the radius of the path of the ions in the second magnetic field is 17.1 cm, what is their mass?

27.32 • In the Bainbridge mass spectrometer (see Fig. 27.24), the magnetic-field magnitude in the velocity selector is 0.510 T, and ions having a speed of 1.82×10^6 m/s pass through undeflected. (a) What is the electric-field magnitude in the velocity selector? (b) If the separation of the plates is 5.20 mm, what is the potential difference between the plates?

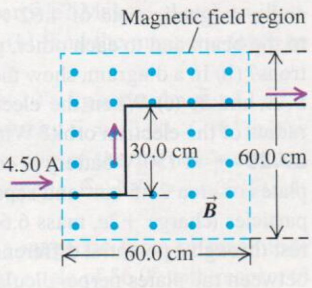
27.33 •• BIO **Ancient Meat Eating.** The amount of meat in prehistoric diets can be determined by measuring the ratio of the isotopes ^{15}N to ^{14}N in bone from human remains. Carnivores concentrate ^{15}N, so this ratio tells archaeologists how much meat was consumed. For a mass spectrometer that has a path radius of 12.5 cm for ^{12}C ions (mass 1.99×10^{-26} kg), find the separation of the ^{14}N (mass 2.32×10^{-26} kg) and ^{15}N (mass 2.49×10^{-26} kg) isotopes at the detector.

Section 27.6 Magnetic Force on a Current-Carrying Conductor

27.34 • A straight, 2.2-m wire carries a typical household current of 1.5 A (in one direction) at a location where the earth's magnetic field is 0.55 gauss from south to north. Find the magnitude and direction of the force that our planet's magnetic field exerts on this wire if it is oriented so that the current in it is running (a) from west to east, (b) vertically upward, (c) from north to south. (d) Is the magnetic force ever large enough to cause significant effects under normal household conditions?

27.35 •• A long wire carrying 4.50 A of current makes two 90° bends, as shown in **Fig. E27.35**. The bent part of the wire passes through a uniform 0.240-T magnetic field directed as shown in the figure and confined to a limited region of space. Find the magnitude and direction of the force that the magnetic field exerts on the wire.

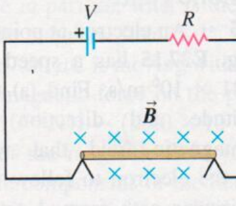
Figure **E27.35**



27.36 •• An electromagnet produces a magnetic field of 0.600 T in a cylindrical region of radius 2.60 cm between its poles. A straight wire carrying a current of 11.0 A passes through the center of this region and is perpendicular to both the axis of the cylindrical region and the magnetic field. What magnitude of force does this field exert on the wire?

27.37 • A thin, 50.0-cm-long metal bar with mass 750 g rests on, but is not attached to, two metallic supports in a uniform 0.450-T magnetic field, as shown in **Fig. E27.37**. A battery and a 25.0-Ω resistor in series are connected to the supports. (a) What is the highest voltage the battery can have without breaking the circuit at the supports? (b) The battery voltage has the maximum value calculated in part (a). If the resistor suddenly gets partially short-circuited, decreasing its resistance to 2.0 Ω, find the initial acceleration of the bar.

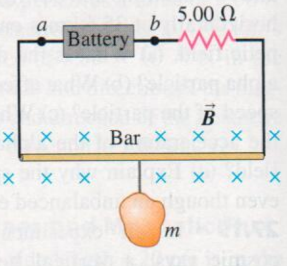
Figure **E27.37**



27.38 • A straight, vertical wire carries a current of 1.23 A downward in a region between the poles of a large superconducting electromagnet, where the magnetic field has magnitude $B = 0.591$ T and is horizontal. What are the magnitude and direction of the magnetic force on a 1.00-cm section of the wire that is in this uniform magnetic field, if the magnetic field direction is (a) east; (b) south; (c) 25.0° south of west?

27.39 • **Magnetic Balance.** The circuit shown in **Fig. E27.39** is used to make a magnetic balance to weigh objects. The mass m to be measured is hung from the center of the bar that is in a uniform magnetic field of 1.50 T, directed into the plane of the figure. The battery voltage can be adjusted to vary the current in the circuit. The horizontal bar is 60.0 cm long and is made of extremely light-weight material. It is connected to the battery by thin vertical wires that can support no appreciable tension; all the weight of the suspended mass m is supported by the magnetic force on the bar. A resistor with $R = 5.00$ Ω is in series with the bar; the resistance of the rest of the circuit is much less than this. (a) Which point, a or b, should be the positive terminal of the battery? (b) If the maximum terminal voltage of the battery is 175 V, what is the greatest mass m that this instrument can measure?

Figure **E27.39**

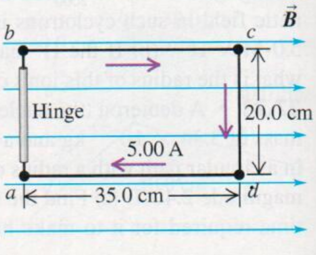


Section 27.7 Force and Torque on a Current Loop

27.40 •• The plane of a 5.0 cm × 8.0 cm rectangular loop of wire is parallel to a 0.24-T magnetic field. The loop carries a current of 6.6 A. (a) What torque acts on the loop? (b) What is the magnetic moment of the loop? (c) What is the maximum torque that can be obtained with the same total length of wire carrying the same current in this magnetic field?

27.41 • The 20.0 cm × 35.0 cm rectangular circuit shown in **Fig. E27.41** is hinged along side ab. It carries a clockwise

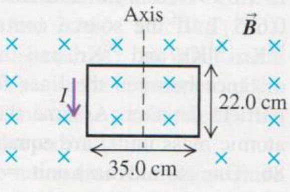
Figure **E27.41**



5.00-A current and is located in a uniform 1.20-T magnetic field oriented perpendicular to two of its sides, as shown. (a) Draw a clear diagram showing the direction of the force that the magnetic field exerts on each segment of the circuit (*ab*, *bc*, etc.). (b) Of the four forces you drew in part (a), decide which ones exert a torque about the hinge *ab*. Then calculate only those forces that exert this torque. (c) Use your results from part (b) to calculate the torque that the magnetic field exerts on the circuit about the hinge axis *ab*.

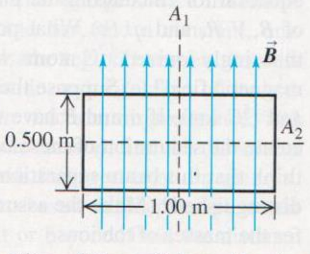
27.42 • A rectangular coil of wire, 22.0 cm by 35.0 cm and carrying a current of 1.95 A, is oriented with the plane of its loop perpendicular to a uniform 1.50-T magnetic field (**Fig. E27.42**). (a) Calculate the net force and torque that the magnetic field exerts on the coil. (b) The coil is rotated through a 30.0° angle about the axis shown, with the left side coming out of the plane of the figure and the right side going into the plane. Calculate the net force and torque that the magnetic field now exerts on the coil. (*Hint:* To visualize this three-dimensional problem, make a careful drawing of the coil as viewed along the rotation axis.)

Figure **E27.42**



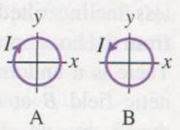
27.43 • **CP** A uniform rectangular coil of total mass 270 g and dimensions 0.500 m × 1.00 m is oriented with its plane parallel to a uniform 4.00-T magnetic field (**Fig. E27.43**). A current of 2.70 A is suddenly started in the coil. (a) About which axis (A_1 or A_2) will the coil begin to rotate? Why? (b) Find the initial angular acceleration of the coil just after the current is started.

Figure **E27.43**



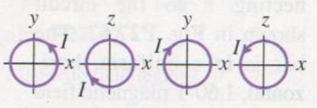
27.44 • Both circular coils A and B (**Fig. E27.44**) have area A and N turns. They are free to rotate about a diameter that coincides with the x-axis. Current I circulates in each coil in the direction shown. There is a uniform magnetic field $\vec{B}$ in the $+z$-direction. (a) What is the direction of the magnetic moment $\vec{\mu}$ for each coil? (b) Explain why the torque on both coils due to the magnetic field is zero, so the coil is in rotational equilibrium. (c) Use Eq. (27.27) to calculate the potential energy for each coil. (d) For each coil, is the equilibrium stable or unstable? Explain.

Figure **E27.44**



27.45 • A circular coil with area A and N turns is free to rotate about a diameter that coincides with the x-axis. Current I is circulating in the coil. There is a uniform magnetic field $\vec{B}$ in the positive y-direction. Calculate the magnitude and direction of the torque $\vec{\tau}$ and the value of the potential energy U, as given in Eq. (27.27), when the coil is oriented as shown in parts (a) through (d) of Fig. E27.45.

Figure **E27.45**

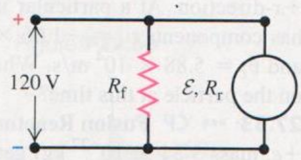


27.46 •• A coil with magnetic moment 1.46 A · m² is oriented initially with its magnetic moment antiparallel to a uniform 0.805-T magnetic field. What is the change in potential energy of the coil when it is rotated 180° so that its magnetic moment is parallel to the field?

Section 27.8 The Direct-Current Motor

27.47 •• In a shunt-wound dc motor with the field coils and rotor connected in parallel (**Fig. E27.47**), the resistance R_f of the field coils is 110 Ω, and the resistance R_r of the rotor is 5.5 Ω. When a potential difference of 120 V is applied to the brushes and the motor is running at full speed delivering mechanical power, the current supplied to it is 4.90 A. (a) What is the current in the field coils? (b) What is the current in the rotor? (c) What is the induced emf developed by the motor? (d) How much mechanical power is developed by this motor?

Figure **E27.47**

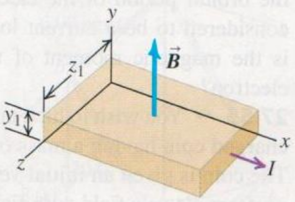


27.48 • A dc motor with its rotor and field coils connected in series has an internal resistance of 3.2 Ω. When the motor is running at full load on a 120-V line, the emf in the rotor is 105 V. (a) What is the current drawn by the motor from the line? (b) What is the power delivered to the motor? (c) What is the mechanical power developed by the motor?

Section 27.9 The Hall Effect

27.49 • Figure E27.49 shows a portion of a silver ribbon with $z_1 = 14.0$ mm and $y_1 = 0.24$ mm, carrying a current of 140 A in the $+x$-direction. The ribbon lies in a uniform magnetic field, in the y-direction, with magnitude 0.94 T. Apply the simplified model of the Hall effect presented in Section 27.9. If there are 5.90×10^{28} free electrons per cubic meter, find (a) the magnitude of the drift velocity of the electrons in the x-direction; (b) the magnitude and direction of the electric field in the z-direction due to the Hall effect; (c) the Hall emf.

Figure **E27.49**

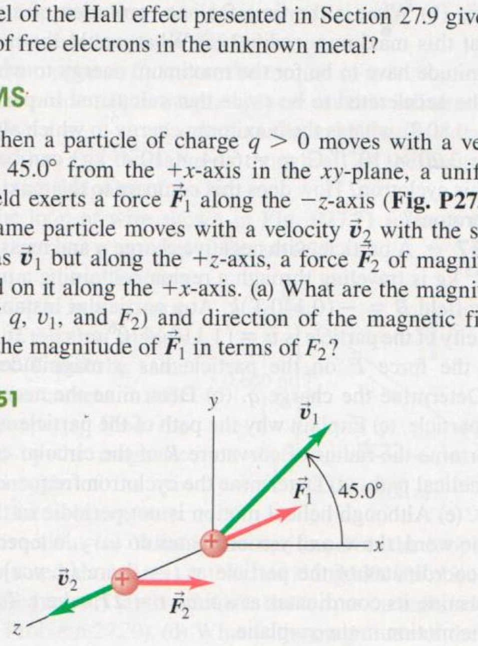


27.50 • Let Fig. E27.49 represent a strip of an unknown metal of the same dimensions as those of the silver ribbon in Exercise 27.49. When the magnetic field is 2.29 T and the current is 78.0 A, the Hall emf is found to be 131 μV. What does the simplified model of the Hall effect presented in Section 27.9 give for the density of free electrons in the unknown metal?

PROBLEMS

27.51 • When a particle of charge $q > 0$ moves with a velocity of $\vec{v}_1$ at 45.0° from the $+x$-axis in the xy-plane, a uniform magnetic field exerts a force $\vec{F}_1$ along the $-z$-axis (**Fig. P27.51**). When the same particle moves with a velocity $\vec{v}_2$ with the same magnitude as $\vec{v}_1$ but along the $+z$-axis, a force $\vec{F}_2$ of magnitude F_2 is exerted on it along the $+x$-axis. (a) What are the magnitude (in terms of q, v_1, and F_2) and direction of the magnetic field? (b) What is the magnitude of $\vec{F}_1$ in terms of F_2?

Figure **P27.51**



27.52 • A particle with charge 9.41×10^{-8} C is moving in a region where there is a uniform 0.660-T magnetic field in the $+x$-direction. At a particular instant, the velocity of the particle has components $v_x = -1.66 \times 10^4$ m/s, $v_y = -3.14 \times 10^4$ m/s, and $v_z = 5.88 \times 10^4$ m/s. What are the components of the force on the particle at this time?

27.53 ••• CP **Fusion Reactor.** If two deuterium nuclei (charge $+e$, mass 3.34×10^{-27} kg) get close enough together, the attraction of the strong nuclear force will fuse them to make an isotope of helium, releasing vast amounts of energy. The range of this force is about 10^{-15} m. This is the principle behind the fusion reactor. The deuterium nuclei are moving much too fast to be contained by physical walls, so they are confined magnetically. (a) How fast would two nuclei have to move so that in a head-on collision they would get close enough to fuse? (Assume their speeds are equal. Treat the nuclei as point charges, and assume that a separation of 1.0×10^{-15} is required for fusion.) (b) What strength magnetic field is needed to make deuterium nuclei with this speed travel in a circle of diameter 2.60 m?

27.54 •• **Magnetic Moment of the Hydrogen Atom.** In the Bohr model of the hydrogen atom (see Section 39.3), in the lowest energy state the electron orbits the proton at a speed of 2.2×10^6 m/s in a circular orbit of radius 5.3×10^{-11} m. (a) What is the orbital period of the electron? (b) If the orbiting electron is considered to be a current loop, what is the current I? (c) What is the magnetic moment of the atom due to the motion of the electron?

27.55 •• You wish to hit a target from several meters away with a charged coin having a mass of 4.90 g and a charge of $+2000\ \mu$C. The coin is given an initial velocity of 14.0 m/s, and a downward, uniform electric field with field strength 25.0 N/C exists throughout the region. If you aim directly at the target and fire the coin horizontally, what magnitude and direction of uniform magnetic field are needed in the region for the coin to hit the target?

27.56 • The magnetic poles of a small cyclotron produce a magnetic field with magnitude 0.80 T. The poles have a radius of 0.45 m, which is the maximum radius of the orbits of the accelerated particles. (a) What is the maximum energy to which protons ($q = 1.60 \times 10^{-19}$C, $m = 1.67 \times 10^{-27}$ kg) can be accelerated by this cyclotron? Give your answer in electron volts and in joules. (b) What is the time for one revolution of a proton orbiting at this maximum radius? (c) What would the magnetic-field magnitude have to be for the maximum energy to which a proton can be accelerated to be twice that calculated in part (a)? (d) For $B = 0.80$ T, what is the maximum energy to which alpha particles ($q = 3.20 \times 10^{-19}$ C, $m = 6.64 \times 10^{-27}$ kg) can be accelerated by this cyclotron? How does this compare to the maximum energy for protons?

27.57 •• A particle with negative charge q and mass $m = 2.65 \times 10^{-15}$ kg is traveling through a region containing a uniform magnetic field $\vec{B} = -(0.140\text{ T})\hat{k}$. At a particular instant of time the velocity of the particle is $\vec{v} = (1.110 \times 10^6\text{ m/s})(-3\hat{\imath} + 4\hat{\jmath} + 12\hat{k})$ and the force $\vec{F}$ on the particle has a magnitude of 2.45 N. (a) Determine the charge q. (b) Determine the acceleration $\vec{a}$ of the particle. (c) Explain why the path of the particle is a helix, and determine the radius of curvature R of the circular component of the helical path. (d) Determine the cyclotron frequency of the particle. (e) Although helical motion is not periodic in the full sense of the word, the x- and y-coordinates do vary in a periodic way. If the coordinates of the particle at $t = 0$ are $(x, y, z) = (R, 0, 0)$, determine its coordinates at a time $t = 2T$, where T is the period of the motion in the xy-plane.

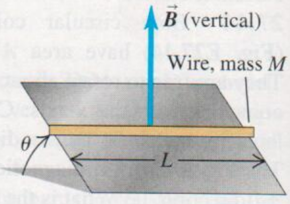
27.58 •• A particle of charge $q > 0$ is moving at speed v in the $+z$-direction through a region of uniform magnetic field $\vec{B}$. The magnetic force on the particle is $\vec{F} = F_0(3\hat{\imath} + 4\hat{\jmath})$, where F_0 is a positive constant. (a) Determine the components B_x, B_y, and B_z, or at least as many of the three components as is possible from the information given. (b) If it is given in addition that the magnetic field has magnitude $6F_0/qv$, determine as much as you can about the remaining components of $\vec{B}$.

27.59 •• Suppose the electric field between the plates in Fig. 27.24 is 1.89×10^4 V/m and the magnetic field in both regions is 0.675 T. If the source contains the three isotopes of krypton, ^{82}Kr, ^{84}Kr, and ^{86}Kr, and the ions are singly charged, find the distance between the lines formed by the three isotopes on the particle detector. Assume the atomic masses of the isotopes (in atomic mass units) are equal to their mass numbers, 82, 84, and 86. (One atomic mass unit = 1 u = 1.66×10^{-27} kg.)

27.60 •• **Mass Spectrograph.** A mass spectrograph is used to measure the masses of ions, or to separate ions of different masses (see Section 27.5). In one design for such an instrument, ions with mass m and charge q are accelerated through a potential difference V. They then enter a uniform magnetic field that is perpendicular to their velocity, and they are deflected in a semicircular path of radius R. A detector measures where the ions complete the semicircle and from this it is easy to calculate R. (a) Derive the equation for calculating the mass of the ion from measurements of B, V, R, and q. (b) What potential difference V is needed so that singly ionized ^{12}C atoms will have $R = 50.0$ cm in a 0.150-T magnetic field? (c) Suppose the beam consists of a mixture of ^{12}C and ^{14}C ions. If v and B have the same values as in part (b), calculate the separation of these two isotopes at the detector. Do you think that this beam separation is sufficient for the two ions to be distinguished? (Make the assumption described in Problem 27.59 for the masses of the ions.)

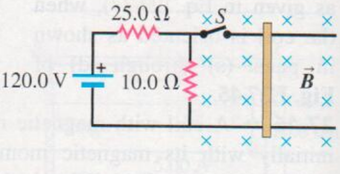
27.61 •• A straight piece of conducting wire with mass M and length L is placed on a frictionless incline tilted at an angle θ from the horizontal (**Fig. P27.61**). There is a uniform, vertical magnetic field $\vec{B}$ at all points (produced by an arrangement of magnets not shown in the figure). To keep the wire from sliding down the incline, a voltage source is attached to the ends of the wire. When just the right amount of current flows through the wire, the wire remains at rest. Determine the magnitude and direction of the current in the wire that will cause the wire to remain at rest. Copy the figure and draw the direction of the current on your copy. In addition, show in a free-body diagram all the forces that act on the wire.

Figure **P27.61**



27.62 •• CP A 2.60-N metal bar, 0.850 m long and having a resistance of 10.0 Ω, rests horizontally on conducting wires connecting it to the circuit shown in **Fig. P27.62**. The bar is in a uniform, horizontal, 1.60-T magnetic field and is not attached to the wires in the circuit. What is the acceleration of the bar just after the switch S is closed?

Figure **P27.62**

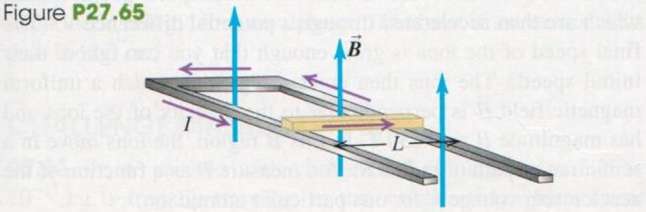


27.63 •• BIO **Determining Diet.** One method for determining the amount of corn in early Native American diets is the *stable* *isotope ratio analysis* (SIRA) technique. As corn photosynthesizes, it concentrates the isotope carbon-13, whereas most other plants concentrate carbon-12. Overreliance on corn consumption can then be correlated with certain diseases, because corn lacks the essential amino acid lysine. Archaeologists use a mass spectrometer to separate the ^{12}C and ^{13}C isotopes in samples of human remains. Suppose you use a velocity selector to obtain singly ionized (missing one electron) atoms of speed 8.50 km/s, and you want to bend them within a uniform magnetic field in a semicircle of diameter 25.0 cm for the ^{12}C. The measured masses of these isotopes are 1.99×10^{-26} kg (^{12}C) and 2.16×10^{-26} kg (^{13}C). (a) What strength of magnetic field is required? (b) What is the diameter of the ^{13}C semicircle? (c) What is the separation of the ^{12}C and ^{13}C ions at the detector at the end of the semicircle? Is this distance large enough to be easily observed?

27.64 •• **CP** A plastic circular loop has radius R, and a positive charge q is distributed uniformly around the circumference of the loop. The loop is then rotated around its central axis, perpendicular to the plane of the loop, with angular speed ω. If the loop is in a region where there is a uniform magnetic field $\vec{B}$ directed parallel to the plane of the loop, calculate the magnitude of the magnetic torque on the loop.

27.65 •• **CP An Electromagnetic Rail Gun.** A conducting bar with mass m and length L slides over horizontal rails that are connected to a voltage source. The voltage source maintains a constant current I in the rails and bar, and a constant, uniform, vertical magnetic field $\vec{B}$ fills the region between the rails (**Fig. P27.65**). (a) Find the magnitude and direction of the net force on the conducting bar. Ignore friction, air resistance, and electrical resistance. (b) If the bar has mass m, find the distance d that the bar must move along the rails from rest to attain speed v. (c) It has been suggested that rail guns based on this principle could accelerate payloads into earth orbit or beyond. Find the distance the bar must travel along the rails if it is to reach the escape speed for the earth (11.2 km/s). Let $B = 0.82$ T, $I = 2.4 \times 10^3$ A, $m = 30$ kg, and $L = 51$ cm. For simplicity assume the net force on the object is equal to the magnetic force, as in parts (a) and (b), even though gravity plays an important role in an actual launch in space.

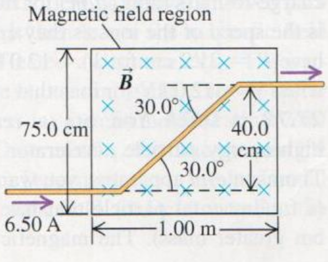
Figure **P27.65**



27.66 • A wire 30.0 cm long lies along the z-axis and carries a current of 6.60 A in the $+z$-direction. The magnetic field is uniform and has components $B_x = -0.201$ T, $B_y = -0.982$ T, and $B_z = -0.317$ T. (a) Find the components of the magnetic force on the wire. (b) What is the magnitude of the net magnetic force on the wire?

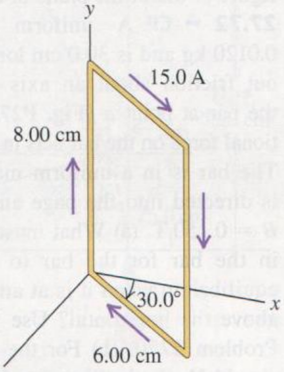
27.67 •• A long wire carrying 6.50 A of current makes two bends, as shown in **Fig. P27.67**. The bent part of the wire passes through a uniform 0.280-T magnetic field directed as shown and confined to a limited region of space. Find the magnitude and direction of the force that the magnetic field exerts on the wire.

Figure **P27.67**



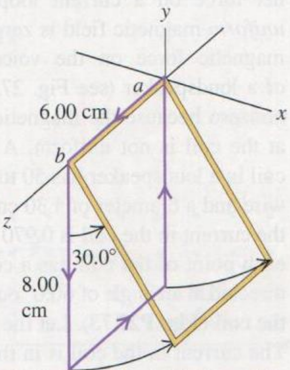
27.68 •• The rectangular loop shown in **Fig. P27.68** is pivoted about the y-axis and carries a current of 15.0 A in the direction indicated. (a) If the loop is in a uniform magnetic field with magnitude 0.48 T in the $+x$-direction, find the magnitude and direction of the torque required to hold the loop in the position shown. (b) Repeat part (a) for the case in which the field is in the $-z$-direction. (c) For each of the above magnetic fields, what torque would be required if the loop were pivoted about an axis through its center, parallel to the y-axis?

Figure **P27.68**



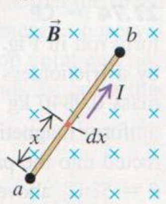
27.69 •• **CP** The rectangular loop of wire shown in **Fig. P27.69** has a mass of 0.16 g per centimeter of length and is pivoted about side ab on a frictionless axis. The current in the wire is 8.9 A in the direction shown. Find the magnitude and direction of the magnetic field parallel to the y-axis that will cause the loop to swing up until its plane makes an angle of 30.0° with the yz-plane.

Figure **P27.69**



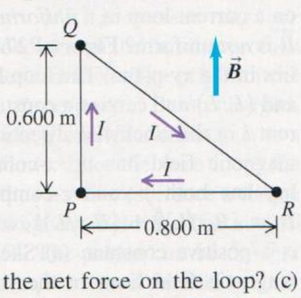
27.70 •• **CALC** A uniform bar of length L carries a current I in the direction from point a to point b (**Fig. P27.70**). The bar is in a uniform magnetic field that is directed into the page. Consider the torque about an axis perpendicular to the bar at point a that is due to the force that the magnetic field exerts on the bar. (a) Suppose that an infinitesimal section of the bar has length dx and is located a distance x from point a. Calculate the torque $d\tau$ about point a due to the magnetic force on this infinitesimal section. (b) Use $\tau = \int_a^b d\tau$ to calculate the total torque τ on the bar. (c) Show that τ is the same as though all of the magnetic force acted at the midpoint of the bar.

Figure **P27.70**



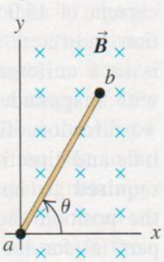
27.71 •• The loop of wire shown in **Fig. P27.71** forms a right triangle and carries a current $I = 5.00$ A in the direction shown. The loop is in a uniform magnetic field that has magnitude $B = 3.00$ T and the same direction as the current in side PQ of the loop. (a) Find the force exerted by the magnetic field on each side of the triangle. If the force is not zero, specify its direction. (b) What is the net force on the loop? (c) The loop is pivoted about an axis that lies along side PR. Use the forces calculated in part (a) to calculate the torque on each side of the loop (see Problem 27.70). (d) What is the magnitude of the net

Figure **P27.71**



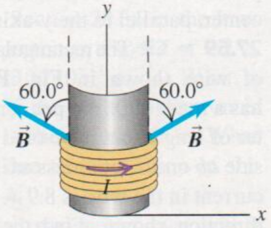
(see Problem 27.70). (d) What is the magnitude of the net torque on the loop? Calculate the net torque from the torques calculated in part (c) and also from Eq. (27.28). Do these two results agree? (e) Is the net torque directed to rotate point Q into the plane of the figure or out of the plane of the figure?

Figure **P27.72**



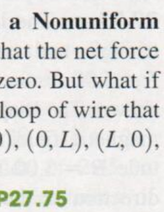
27.72 •• CP A uniform bar has mass 0.0120 kg and is 30.0 cm long. It pivots without friction about an axis perpendicular to the bar at point a (**Fig. P27.72**). The gravitational force on the bar acts in the $-y$-direction. The bar is in a uniform magnetic field that is directed into the page and has magnitude $B = 0.150$ T. (a) What must be the current I in the bar for the bar to be in rotational equilibrium when it is at an angle $\theta = 30.0°$ above the horizontal? Use your result from Problem 27.70. (b) For the bar to be in rotational equilibrium, should I be in the direction from a to b or b to a?

Figure **P27.73**



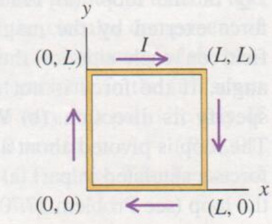
27.73 •• CALC A Voice Coil. It was shown in Section 27.7 that the net force on a current loop in a *uniform* magnetic field is zero. The magnetic force on the voice coil of a loudspeaker (see Fig. 27.28) is nonzero because the magnetic field at the coil is not uniform. A voice coil in a loudspeaker has 50 turns of wire and a diameter of 1.30 cm, and the current in the coil is 0.970 A. Assume that the magnetic field at each point of the coil has a constant magnitude of 0.250 T and is directed at an angle of 60.0° outward from the normal to the plane of the coil (**Fig. P27.73**). Let the axis of the coil be in the y-direction. The current in the coil is in the direction shown (counterclockwise as viewed from a point above the coil on the y-axis). Calculate the magnitude and direction of the net magnetic force on the coil.

Figure **P27.74**



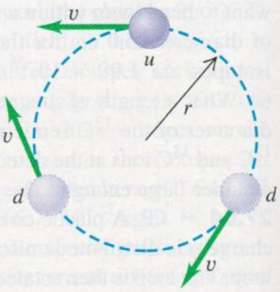
27.74 •• CP The lower end of the thin uniform rod in **Fig. P27.74** is attached to the floor by a frictionless hinge at point P. The rod has mass 0.0840 kg and length 18.0 cm and is in a uniform magnetic field $B = 0.120$ T that is directed into the page. The rod is held at an angle $\theta = 53.0°$ above the horizontal by a horizontal string that connects the top of the rod to the wall. The rod carries a current $I = 12.0$ A in the direction toward P. Calculate the tension in the string. Use your result from Problem 27.70 to calculate the torque due to the magnetic-field force.

Figure **P27.75**



27.75 •• CALC Force on a Current Loop in a Nonuniform Magnetic Field. It was shown in Section 27.7 that the net force on a current loop in a *uniform* magnetic field is zero. But what if $\vec{B}$ is *not* uniform? **Figure P27.75** shows a square loop of wire that lies in the xy-plane. The loop has corners at $(0, 0)$, $(0, L)$, $(L, 0)$, and (L, L) and carries a constant current I in the clockwise direction. The magnetic field has no x-component but has both y- and z-components: $\vec{B} = (B_0 z/L)\hat{\jmath} + (B_0 y/L)\hat{k}$, where B_0 is a positive constant. (a) Sketch the magnetic field lines in the yz-plane. (b) Find the magnitude and direction of the magnetic force exerted on each of the sides of the loop by integrating Eq. (27.20). (c) Find the magnitude and direction of the net magnetic force on the loop.

Figure **P27.76**



27.76 •• Quark Model of the Neutron. The neutron is a particle with zero charge. Nonetheless, it has a nonzero magnetic moment with z-component $9.66 \times 10^{-27}\ \text{A} \cdot \text{m}^2$. This can be explained by the internal structure of the neutron. A substantial body of evidence indicates that a neutron is composed of three fundamental particles called *quarks:* an "up" (u) quark, of charge $+2e/3$, and two "down" (d) quarks, each of charge $-e/3$. The combination of the three quarks produces a net charge of $\frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e = 0$. If the quarks are in motion, they can produce a nonzero magnetic moment. As a very simple model, suppose the u quark moves in a counterclockwise circular path and the d quarks move in a clockwise circular path, all of radius r and all with the same speed v (**Fig. P27.76**). (a) Determine the current due to the circulation of the u quark. (b) Determine the magnitude of the magnetic moment due to the circulating u quark. (c) Determine the magnitude of the magnetic moment of the three-quark system. (Be careful to use the correct magnetic moment directions.) (d) With what speed v must the quarks move if this model is to reproduce the magnetic moment of the neutron? Use $r = 1.20 \times 10^{-15}$ m (the radius of the neutron) for the radius of the orbits.

27.77 •• A circular loop of wire with area A lies in the xy-plane. As viewed along the z-axis looking in the $-z$-direction toward the origin, a current I is circulating clockwise around the loop. The torque produced by an external magnetic field $\vec{B}$ is given by $\vec{\tau} = D(4\hat{\imath} - 3\hat{\jmath})$, where D is a positive constant, and for this orientation of the loop the magnetic potential energy $U = -\vec{\mu} \cdot \vec{B}$ is negative. The magnitude of the magnetic field is $B_0 = 13D/IA$. (a) Determine the vector magnetic moment of the current loop. (b) Determine the components B_x, B_y, and B_z of $\vec{B}$.

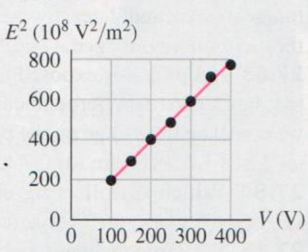
27.78 •• DATA You are using a type of mass spectrometer to measure charge-to-mass ratios of atomic ions. In the device, atoms are ionized with a beam of electrons to produce positive ions, which are then accelerated through a potential difference V. (The final speed of the ions is great enough that you can ignore their initial speed.) The ions then enter a region in which a uniform magnetic field $\vec{B}$ is perpendicular to the velocity of the ions and has magnitude $B = 0.250$ T. In this $\vec{B}$ region, the ions move in a semicircular path of radius R. You measure R as a function of the accelerating voltage V for one particular atomic ion:

V (kV)	10.0	12.0	14.0	16.0	18.0
R (cm)	19.9	21.8	23.6	25.2	26.8

(a) How can you plot the data points so that they will fall close to a straight line? Explain. (b) Construct the graph described in part (a). Use the slope of the best-fit straight line to calculate the charge-to-mass ratio (q/m) for the ion. (c) For $V = 20.0$ kV, what is the speed of the ions as they enter the $\vec{B}$ region? (d) If ions that have $R = 21.2$ cm for $V = 12.0$ kV are singly ionized, what is R when $V = 12.0$ kV for ions that are doubly ionized?

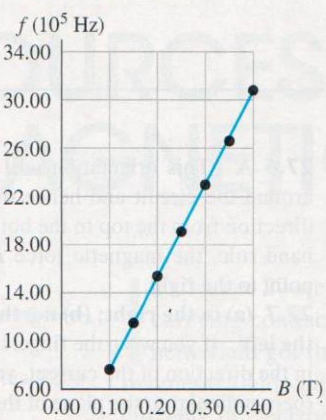
27.79 •• DATA You are a research scientist working on a high-energy particle accelerator. Using a modern version of the Thomson e/m apparatus, you want to measure the mass of a muon (a fundamental particle that has the same charge as an electron but greater mass). The magnetic field between the two charged plates is 0.340 T. You measure the electric field for zero particle deflection as a function of the accelerating potential V. This potential is large enough that you can assume the initial speed of the muons to be zero. **Figure P27.79** is an E^2-versus-V graph of your data. (a) Explain why the data points fall close to a straight line. (b) Use the graph in Fig. P27.79 to calculate the mass m of a muon. (c) If the two charged plates are separated by 6.00 mm, what must be the voltage between the plates in order for the electric field between the plates to be 2.00×10^5 V/m? Assume that the dimensions of the plates are much larger than the separation between them. (d) When $V = 400$ V, what is the speed of the muons as they enter the region between the plates?

Figure **P27.79**



27.80 •• DATA You are a technician testing the operation of a cyclotron. An alpha particle in the device moves in a circular path in a magnetic field $\vec{B}$ that is directed perpendicular to the path of the alpha particle. You measure the number of revolutions per second (the frequency f) of the alpha particle as a function of the magnetic field strength B. **Figure P27.80** shows your results and the best straight-line fit to your data. (a) Use the graph in Fig. P27.80 to calculate the charge-to-mass ratio of the alpha particle, which has charge $+2e$. On the basis of your data, what is the mass of an alpha particle? (b) With $B = 0.300$ T, what are the cyclotron frequencies f of a proton and of an electron? How do these f values compare to the frequency of an alpha particle? (c) With $B = 0.300$ T, what speed and kinetic energy does an alpha particle have if the radius of its path is 12.0 cm?

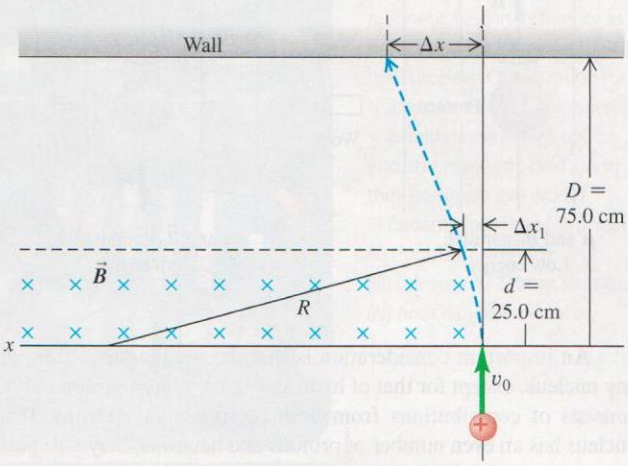
Figure **P27.80**



CHALLENGE PROBLEMS

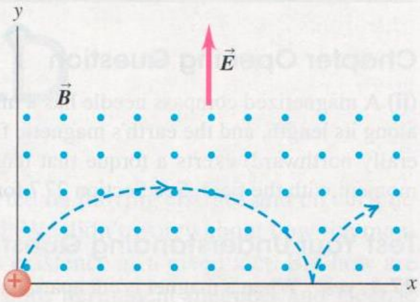
27.81 ••• A particle with charge 2.15 μC and mass 3.20×10^{-11} kg is initially traveling in the $+y$-direction with a speed $v_0 = 1.45 \times 10^5$ m/s. It then enters a region containing a uniform magnetic field that is directed into, and perpendicular to, the page in **Fig. P27.81**. The magnitude of the field is 0.420 T. The region extends a distance of 25.0 cm along the initial direction of travel; 75.0 cm from the point of entry into the magnetic field region is a wall. The length of the field-free region is thus 50.0 cm. When the charged particle enters the magnetic field, it follows a curved path whose radius of curvature is R. It then leaves the magnetic field after a time t_1, having been deflected a distance Δx_1. The particle then travels in the field-free region and strikes the wall after undergoing a total deflection Δx. (a) Determine the radius R of the curved part of the path. (b) Determine t_1, the time the particle spends in the magnetic field. (c) Determine Δx_1, the horizontal deflection at the point of exit from the field. (d) Determine Δx, the total horizontal deflection.

Figure **P27.81**



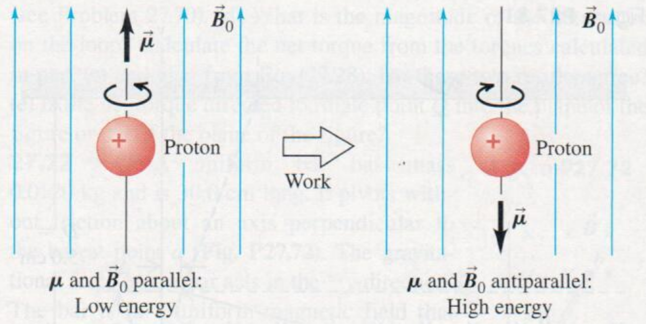
27.82 ••• CP A Cycloidal Path. A particle with mass m and positive charge q starts from rest at the origin shown in **Fig. P27.82**. There is a uniform electric field $\vec{E}$ in the $+y$-direction and a uniform magnetic field $\vec{B}$ directed out of the page. It is shown in more advanced books that the path is a *cycloid* whose radius of curvature at the top points is twice the y-coordinate at that level. (a) Explain why the path has this general shape and why it is repetitive. (b) Prove that the speed at any point is equal to $\sqrt{2qEy/m}$. (*Hint:* Use energy conservation.) (c) Applying Newton's second law at the top point and taking as given that the radius of curvature here equals $2y$, prove that the speed at this point is $2E/B$.

Figure **P27.82**



PASSAGE PROBLEMS

BIO MAGNETIC FIELDS AND MRI. *Magnetic resonance imaging* (MRI) is a powerful imaging method that, unlike x-ray imaging, allows sharp images of soft tissue to be made without exposing the patient to potentially damaging radiation. A rudimentary understanding of this method can be achieved by the relatively simple application of the classical (that is, non-quantum) physics of magnetism. The starting point for MRI is *nuclear magnetic resonance* (NMR), a technique that depends on the fact that protons in the atomic nucleus have a magnetic field $\vec{B}$. The origin of the proton's magnetic field is the spin of the proton. Being charged, the spinning proton constitutes an electric current analogous to a wire loop through which current flows. Like the wire loop, the proton has a magnetic moment $\vec{\mu}$; thus it will experience a torque when it is subjected to an external magnetic field $\vec{B}_0$. The magnitude of $\vec{\mu}$ is about 1.4×10^{-26} J/T. The proton can be thought of as being in one of two states, with $\vec{\mu}$ oriented parallel or antiparallel to the applied magnetic field, and work must be done to flip the proton from the low-energy state to the high-energy state, as the accompanying figure (next page) shows.



An important consideration is that the net magnetic field of any nucleus, except for that of hydrogen (which has a proton only), consists of contributions from both protons and neutrons. If a nucleus has an even number of protons and neutrons, they will pair in such a way that half of the protons have spins in one orientation and half have spins in the other orientation. Thus the net magnetic moment of the nucleus is zero. Only nuclei with a net magnetic moment are candidates for MRI. Hydrogen is the atom that is most commonly imaged.

27.83 If a proton is exposed to an external magnetic field of 2 T that has a direction perpendicular to the axis of the proton's spin, what will be the torque on the proton? (a) 0; (b) 1.4×10^{-26} N · m; (c) 2.8×10^{-26} N · m; (d) 0.7×10^{-26} N · m.

27.84 Which of following elements is a candidate for MRI? (a) ${}^{12}C_6$; (b) ${}^{16}O_8$; (c) ${}^{40}Ca_{20}$; (d) ${}^{31}P_{15}$.

27.85 The large magnetic fields used in MRI can produce forces on electric currents within the human body. This effect has been proposed as a possible method for imaging "biocurrents" flowing in the body, such as the current that flows in individual nerves. For a magnetic field strength of 2 T, estimate the magnitude of the maximum force on a 1-mm-long segment of a single cylindrical nerve that has a diameter of 1.5 mm. Assume that the entire nerve carries a current due to an applied voltage of 100 mV (that of a typical action potential). The resistivity of the nerve is 0.6 Ω · m. (a) 6×10^{-7} N; (b) 1×10^{-6} N; (c) 3×10^{-4} N; (d) 0.3 N.

Answers

Chapter Opening Question ?

(ii) A magnetized compass needle has a magnetic dipole moment along its length, and the earth's magnetic field (which points generally northward) exerts a torque that tends to align that dipole moment with the field. See Section 27.7 for details.

Test Your Understanding Questions

27.1 yes When a magnet is cut apart, each part has a north and south pole (see Fig. 27.4). Hence the small red part behaves much like the original, full-sized compass needle.

27.2 path 3 Applying the right-hand rule to the vectors $\vec{v}$ (which points to the right) and $\vec{B}$ (which points into the plane of the figure) says that the force $\vec{F} = q\vec{v} \times \vec{B}$ on a *positive* charge would point *upward*. Since the charge is *negative*, the force points *downward* and the particle follows a trajectory that curves downward.

27.3 (a) (ii), (b) no The magnitude of $\vec{B}$ would increase as you moved to the right, reaching a maximum as you passed through the plane of the loop. As you moved beyond the plane of the loop, the field magnitude would decrease. You can tell this from the spacing of the field lines: The closer the field lines, the stronger the field. The direction of the field would be to the right at all points along the path, since the path is along a field line and the direction of $\vec{B}$ at any point is tangent to the field line through that point.

27.4 (a) (ii), (b) (i) The radius of the orbit as given by Eq. (27.11) is directly proportional to the speed, so doubling the particle speed causes the radius to double as well. The particle has twice as far to travel to complete one orbit but is traveling at double the speed, so the time for one orbit is unchanged. This result also follows from Eq. (27.12), which states that the angular speed ω is independent of the linear speed v. Hence the time per orbit, $T = 2\pi/\omega$, likewise does not depend on v.

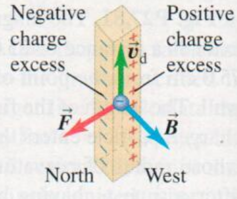
27.5 (iii) From Eq. (27.13), the speed $v = E/B$ at which particles travel straight through the velocity selector does not depend on the magnitude or sign of the charge or the mass of the particle. All that is required is that the particles (in this case, ions) have a nonzero charge.

27.6 A This orientation will cause current to flow clockwise around the circuit and hence through the conducting bar in the direction from the top to the bottom of the figure. From the right-hand rule, the magnetic force $\vec{F} = I\vec{l} \times \vec{B}$ on the bar will then point to the right.

27.7 (a) to the right; (b) north pole on the right, south pole on the left If you wrap the fingers of your right hand around the coil in the direction of the current, your right thumb points to the right (perpendicular to the plane of the coil). This is the direction of the magnetic moment $\vec{\mu}$. The magnetic moment points from the south pole to the north pole, so the right side of the loop is equivalent to a north pole and the left side is equivalent to a south pole.

27.8 no The rotor will not begin to turn when the switch is closed if the rotor is initially oriented as shown in Fig. 27.39b. In this case there is no current through the rotor and hence no magnetic torque. This situation can be remedied by using multiple rotor coils oriented at different angles around the rotation axis. With this arrangement, there is always a magnetic torque no matter what the orientation.

27.9 (ii) The mobile charge carriers in copper are negatively charged electrons, which move upward through the wire to give a downward current. From the right-hand rule, the force on a positively charged particle moving upward in a westward-pointing magnetic field would be to the south; hence the force on a negatively charged particle is to the north. The result is an excess of negative charge on the north side of the wire, leaving an excess of positive charge—and hence a higher electric potential—on the south side.



Bridging Problem

(a) $\tau_x = 1.54 \times 10^{-4}$ N · m,
$\tau_y = -2.05 \times 10^{-4}$ N · m,
$\tau_z = -6.14 \times 10^{-4}$ N · m
(b) -7.55×10^{-4} J (c) 42.1 rad/s

The University of Jordan / Physics Department

Magnetic Fields and Magnetic Forces / chapter 27

Sears and Zemansky / 14th edition

Sample Solutions

Prof. Mahmoud Jaghoub (أ.د. محمود الجاغوب)

94] $m = 1.81 \times 10^{-3}$ kg , $q = 1.22 \times 10^{-8}$ C

$\vec{v} = 3 \times 10^{4} \hat{j}$ (m/s)

$\vec{B} = 1.63 \hat{i} + 0.980 \hat{j}$ in Tesla.

$$\vec{F} = q \vec{v} \times \vec{B} = m\vec{a}$$

$$\vec{F} = q \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3\times10^{4} & 0 \\ 1.63 & 0.980 & 0 \end{vmatrix}$$

$$= q\left[\hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(0 - 1.63 \times 3\times10^{4})\right]$$

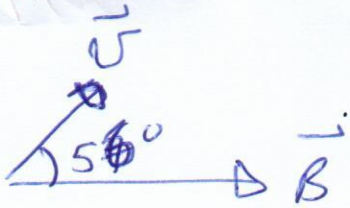
$$= -q(4.89)\times10^{4}\,\hat{k} = -(1.22\times10^{-8})(4.89\times10^{4})\,\hat{k}$$

$$= -5.97 \times 10^{-4}\,\hat{k} \text{ in N.}$$

$$\vec{a} = \frac{\vec{F}}{m} = -\frac{5.97\times10^{-4}}{1.81\times10^{-3}}\,\hat{k} = -0.33\,\hat{k} \text{ in m/s}^2$$

L2

Q5] $F = qvB\sin\theta$



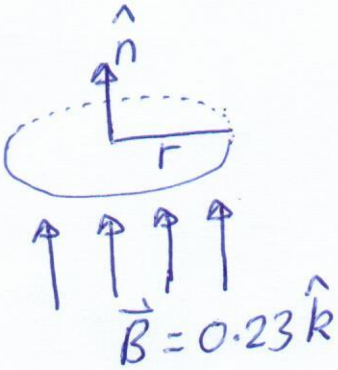
$$v = \frac{F}{qB\sin\theta}$$

$$= \frac{4.1\times10^{-15}}{(1.6\times10^{-19})(3.3\times10^{-3})\sin 56} \simeq 9.4\times10^{6}\ \text{m/s}$$

Q11]

$\vec{A} = \pi(6.1\times10^{-2})^2\,\hat{n} = \pi(6.1\times10^{-2})^2\,\hat{k}$

⑨



$\phi = \vec{B}\cdot\vec{A} = BA\cos(0)$

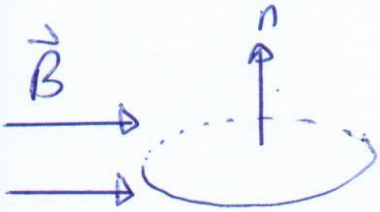
$= (0.23\hat{k})\cdot(\pi(6.1\times10^{-2})^2)\hat{k}$

$= 0.0027\ \text{m}^2\text{T}.$

$\phi = BA\cos 51.3°$

$= (0.23)(\pi(6.1\times10^{-2})^2)$

$= 0.0168\ \text{m}^2\text{T}.$



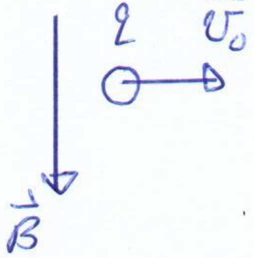
$\phi = BA\cos 90°$

$= 0.$

L3

27] $\vec{F}_m = q\vec{v}\times\vec{B}$

for q positive ⇒ Fm is into the page. For NO deflection the electric force must be out of the page and equals in magnitude to Fm.



$$\Rightarrow \quad qE = qvB \Rightarrow E = vB \quad \text{out of the page}$$

$$= (5.85\times10^{3})(1.35)$$

$$= 7897.5\ \frac{N}{C}\ (\text{or } V/m)$$

---

when the charge is negative the magnetic force is out of the page ⇒ the electric force must be into the page. Therefore, the electric field must be out of the page [$\vec{F} = q\vec{E}$]

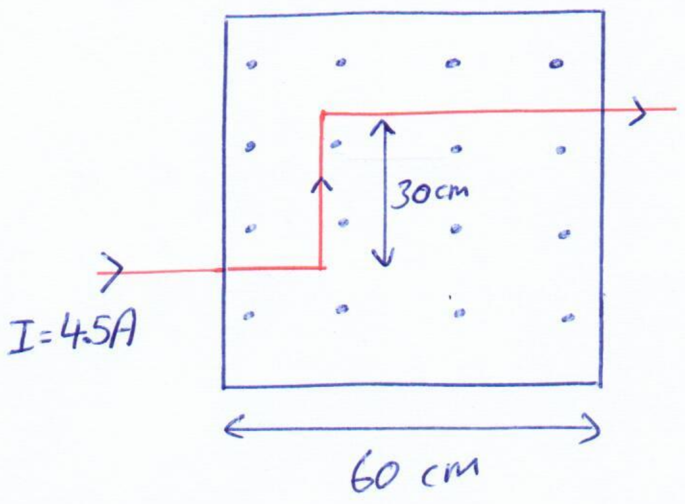
$$qE = qvB \Rightarrow E = vB \quad \text{as above.}$$

$$= 7897.5\ N/C \text{ or } V/m.$$

note $v = \frac{E}{B}$ independent of the charge

Q35]

L4



The resultant magnetic force acting on the horizontal wire is

$F_1 = I L B \sin 90^0$

vertically downwards.

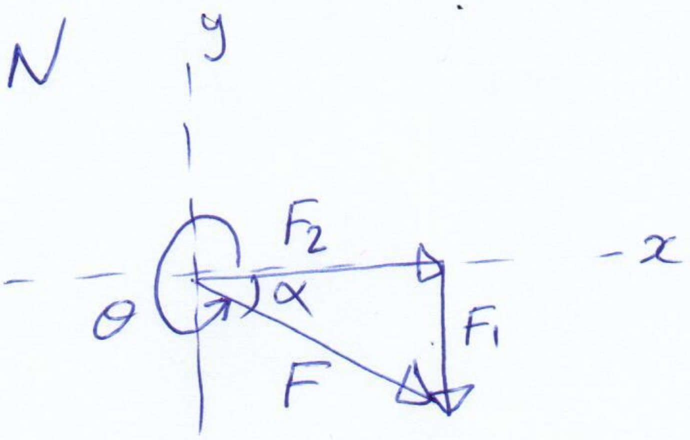
The magnetic force acting on the vertical wire is

$F_2 = I L' B \sin 90^0 = I L' B$ to the right.

$F_1 = I L B = 4.5(0.6)(0.24) = 0.648$ N vertically downwards.

$F_2 = I L' B = 4.5(0.3)(0.24) = 0.324$ N to the right

$F = \sqrt{F_1^2 + F_2^2} \simeq 0.724$ N



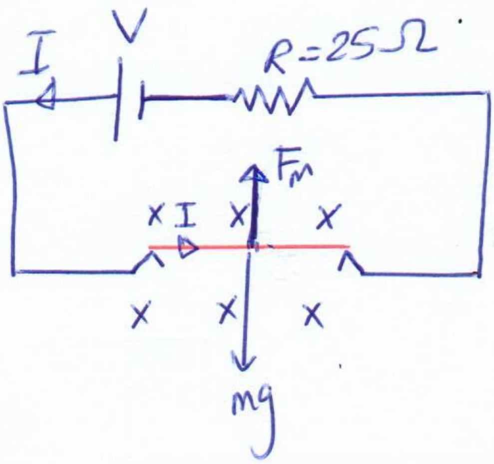
$\tan\alpha = \left|\frac{F_1}{F_2}\right|$

$\therefore \alpha = 63.4^0$

$\theta = 360 - \alpha = 296.6^0$.

Q37] $m = 0.75\,kg$, $L = 0.5\,m$
$B = 0.45\,T$

L5



Not breaking the circuit ⇒

ⓐ $F_M = mg$

$ILB \sin 90 = mg$

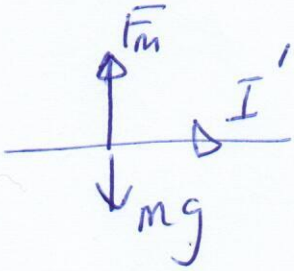
$I = \dfrac{mg}{LB} = 32.7\ A.$

$V = IR = 817.5$ Volts.

ⓑ $V = 817.5$ Volts.

$R \rightarrow 2\,\Omega$ what is the initial acceleration of the wire?

$817.5 = I'R' \Rightarrow I' = \dfrac{817.5}{2} \simeq 409\ A.$



$F = F_m - mg = I'LB - mg$

$= 409(0.5)(0.45) - 0.75g$

$\simeq 84.7$ N upwards.

$F = ma \Rightarrow a = \dfrac{F}{m} = \dfrac{84.7}{0.75} \simeq 112.9\ m/s^2$ upwards.