Problems

For assigned homework and other learning materials, go to MasteringPhysics®. MP

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q24.1 Equation (24.2) shows that the capacitance of a parallel-plate capacitor becomes larger as the plate separation d decreases. However, there is a practical limit to how small d can be made, which places limits on how large C can be. Explain what sets the limit on d. (*Hint:* What happens to the magnitude of the electric field as $d \rightarrow 0$?)

Q24.2 Suppose several different parallel-plate capacitors are charged up by a constant-voltage source. Thinking of the actual movement and position of the charges on an atomic level, why does it make sense that the capacitances are proportional to the surface areas of the plates? Why does it make sense that the capacitances are *inversely* proportional to the distance between the plates?

Q24.3 Suppose the two plates of a capacitor have different areas. When the capacitor is charged by connecting it to a battery, do the charges on the two plates have equal magnitude, or may they be different? Explain your reasoning.

Q24.4 To store the maximum amount of energy in a parallel-plate capacitor with a given battery (voltage source), would it be better to have the plates far apart or close together?

Q24.5 In the parallel-plate capacitor of Fig. 24.2, suppose the plates are pulled apart so that the separation d is much larger than the size of the plates. (a) Is it still accurate to say that the electric field between the plates is uniform? Why or why not? (b) In the situation shown in Fig. 24.2, the potential difference between the plates is $V_{ab} = Qd/\epsilon_0 A$. If the plates are pulled apart as described above, is V_{ab} more or less than this formula would indicate? Explain your reasoning. (c) With the plates pulled apart as described above, is the capacitance more than, less than, or the same as that given by Eq. (24.2)? Explain your reasoning.

Q24.6 A parallel-plate capacitor is charged by being connected to a battery and is kept connected to the battery. The separation between the plates is then doubled. What is the change in the electric field? The charge on the plates? The total energy? Explain.

Q24.7 A physics student is given a parallel-plate capacitor to conduct an experiment. But he inadvertently changes the separation of the plates which leads to an increase in the distance between the plates. What happens to the original capacitance?

Q24.8 The same physics student is now asked to attain the original value of the capacitance without changing the distance between the capacitor plates. However, the physical dimensions of the capacitors cannot be changed. What is the solution?

Q24.9 The same careless student pours water on the device and everything gets wet. What value of the capacitance will the student measure now as compared to the original situation?

Q24.10 A capacitor loses half its charge every second. If its charge is Q after five seconds, how much charge does it contain initially?

Q24.11 As shown in Table 24.1, water has a very large dielectric constant $K = 80.4$. Why do you think water is not commonly used as a dielectric in capacitors?

Q24.12 Is dielectric strength the same thing as dielectric constant? Explain any differences between the two quantities. Is there a simple relationship between dielectric strength and dielectric constant (see Table 24.2)?

Q24.13 When a key is pressed on a keyboard, it directly interact with a capacitor inside. How does interacting with the capacitor lead to the computer recognizing the key being pressed?

Q24.14 Suppose you bring a slab of dielectric close to the gap between the plates of a charged capacitor, preparing to slide it between the plates. What force will you feel? What does this force tell you about the energy stored between the plates once the dielectric is in place, compared to before the dielectric is in place?

Q24.15 The freshness of fish can be measured by placing a fish between the plates of a capacitor and measuring the capacitance. How does this work? (*Hint:* As time passes, the fish dries out. See Table 24.1.)

Q24.16 *Electrolytic* capacitors use as their dielectric an extremely thin layer of nonconducting oxide between a metal plate and a conducting solution. Discuss the advantage of such a capacitor over one constructed using a solid dielectric between the metal plates.

Q24.17 In terms of the dielectric constant K, what happens to the electric flux through the Gaussian surface shown in Fig. 24.22 when the dielectric is inserted into the previously empty space between the plates? Explain.

Q24.18 A parallel-plate capacitor is connected to a power supply that maintains a fixed potential difference between the plates. (a) If a sheet of dielectric is then slid between the plates, what happens to (i) the electric field between the plates, (ii) the magnitude of charge on each plate, and (iii) the energy stored in the capacitor? (b) Now suppose that before the dielectric is inserted, the charged capacitor is disconnected from the power supply. In this case, what happens to (i) the electric field between the plates, (ii) the magnitude of charge on each plate, and (iii) the energy stored in the capacitor? Explain any differences between the two situations.

Q24.19 Inside a capacitor, there are equipotential lines. What do they represent and what is their most important characteristic?

Q24.20 A conductor is an extreme case of a dielectric, since if an electric field is applied to a conductor, charges are free to move within the conductor to set up "induced charges." What is the dielectric constant of a perfect conductor? Is it $K = 0$, $K \rightarrow \infty$, or something in between? Explain your reasoning.

Q24.21 The two plates of a capacitor are given charges $\pm Q$. The capacitor is then disconnected from the charging device so that the charges on the plates can't change, and the capacitor is immersed in a tank of oil. Does the electric field between the plates increase, decrease, or stay the same? Explain your reasoning. How can this field be measured?

EXERCISES

Section 24.1 Capacitors and Capacitance

24.1 • The plates of a parallel-plate capacitor are 3.50 mm apart, and each carries a charge of magnitude 85.0 nC. The plates are in vacuum. The electric field between the plates has a magnitude of 5.00×10^6 V/m. What is (a) the potential difference between the plates; (b) the area of each plate; (c) the capacitance?

24.2 • The plates of a parallel-plate capacitor are 3.46 mm apart, and each has an area of 11.2 cm^2. Each plate carries a charge of magnitude 6.80×10^{-8} C. The plates are in vacuum. What is (a) the capacitance; (b) the potential difference between the plates; (c) the magnitude of the electric field between the plates?

24.3 • A parallel-plate air capacitor of capacitance 246 pF has a charge of magnitude 0.149 μC on each plate. The plates are 0.371 mm apart. (a) What is the potential difference between the plates? (b) What is the area of each plate? (c) What is the electric-field magnitude between the plates? (d) What is the surface charge density on each plate?

24.4 •• Cathode-ray tube oscilloscopes have parallel metal plates inside them to deflect the electron beam. These plates are called the *deflecting plates*. Typically, they are squares 3.0 cm on a side and separated by 5.0 mm, with vacuum in between. What is the capacitance of these deflecting plates and hence of the oscilloscope? (*Note:* This capacitance can sometimes have an effect on the circuit you are trying to study and must be taken into consideration in your calculations.)

24.5 • A 11.9-μF parallel-plate capacitor with circular plates is connected to a 11.8-V battery. (a) What is the charge on each plate? (b) How much charge would be on the plates if their separation were doubled while the capacitor remained connected to the battery? (c) How much charge would be on the plates if the capacitor were connected to the 11.8-V battery after the radius of each plate was doubled without changing their separation?

24.6 • A 5.00-μF parallel-plate capacitor is connected to a 12.0-V battery. After the capacitor is fully charged, the battery is disconnected without loss of any of the charge on the plates. (a) A voltmeter is connected across the two plates without discharging them. What does it read? (b) What would the voltmeter read if (i) the plate separation were doubled; (ii) the radius of each plate were doubled but their separation was unchanged?

24.7 • A parallel-plate air capacitor is to store charge of magnitude 290 pC on each plate when the potential difference between the plates is 43.0 V. (a) If the area of each plate is 6.80 cm^2, what is the separation between the plates? (b) If the separation between the two plates is double the value calculated in part (a), what potential difference is required for the capacitor to store charge of magnitude 290 pC on each plate?

24.8 • A 5.00-pF, parallel-plate, air-filled capacitor with circular plates is to be used in a circuit in which it will be subjected to potentials of up to 1.00×10^2 V. The electric field between the plates is to be no greater than 1.00×10^4 N/C. As a budding electrical engineer for Live-Wire Electronics, your tasks are to (a) design the capacitor by finding what its physical dimensions and separation must be; (b) find the maximum charge these plates can hold.

24.9 • A capacitor is made from two hollow, coaxial, iron cylinders, one inside the other. The inner cylinder is negatively charged and the outer is positively charged; the magnitude of the charge on each is 17.5 pC. The inner cylinder has radius 0.450 mm, the outer one has radius 6.20 mm, and the length of each cylinder is 21.0 cm. (a) What is the capacitance? (b) What applied potential difference is necessary to produce these charges on the cylinders?

24.10 • A cylindrical capacitor consists of a solid inner conducting core with radius 0.250 cm, surrounded by an outer hollow conducting tube. The two conductors are separated by air, and the length of the cylinder is 10.0 cm. The capacitance is 37.0 pF. (a) Calculate the inner radius of the hollow tube. (b) When the capacitor is charged to 150 V, what is the charge per unit length λ on the capacitor?

24.11 •• A spherical capacitor contains a charge of 3.10 nC when connected to a potential difference of 240 V. If its plates are separated by vacuum and the inner radius of the outer shell is 4.40 cm, calculate: (a) the capacitance; (b) the radius of the inner sphere; (c) the electric field just outside the surface of the inner sphere.

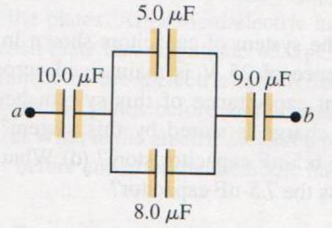
24.12 •• A cylindrical capacitor has an inner conductor of radius 1.1 mm and an outer conductor of radius 3.7 mm. The two conductors are separated by vacuum, and the entire capacitor is 2.4 m long. (a) What is the capacitance per unit length? (b) The potential of the inner conductor is 400 mV higher than that of the outer conductor. Find the charge (magnitude and sign) on both conductors.

24.13 • A spherical capacitor is formed from two concentric, spherical, conducting shells separated by vacuum. The inner sphere has radius 15.0 cm and the capacitance is 116 pF. (a) What is the radius of the outer sphere? (b) If the potential difference between the two spheres is 220 V, what is the magnitude of charge on each sphere?

Section 24.2 Capacitors in Series and Parallel

24.14 • **Figure E24.14** shows a system of four capacitors, where the potential difference across *ab* is 50.0 V. (a) Find the equivalent capacitance of this system between *a* and *b*. (b) How much charge is stored by this combination of capacitors? (c) How much charge is stored in each of the 10.0-μF and the 9.0-μF capacitors?

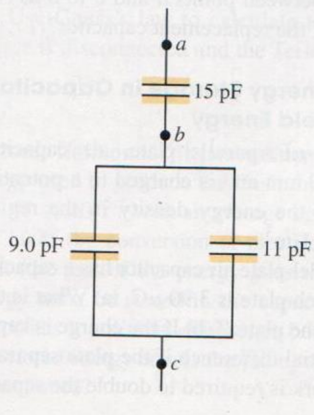
Figure **E24.14**



24.15 • **BIO Electric Eels.** Electric eels and electric fish generate large potential differences that are used to stun enemies and prey. These potentials are produced by cells that each can generate 0.10 V. We can plausibly model such cells as charged capacitors. (a) How should these cells be connected (in series or in parallel) to produce a total potential of more than 0.10 V? (b) Using the connection in part (a), how many cells must be connected together to produce the 600-V surge of the electric eel?

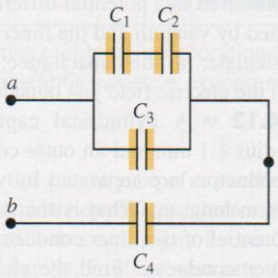
24.16 • For the system of capacitors shown in **Fig. E24.16**, find the equivalent capacitance (a) between *b* and *c*, and (b) between *a* and *c*.

Figure **E24.16**



24.17 • In **Fig. E24.17**, each capacitor has $C = 4.80\ \mu F$ and $V_{ab} = +30.0\ V$. Calculate (a) the charge on each capacitor; (b) the potential difference across each capacitor; (c) the potential difference between points a and d.

Figure **E24.17**

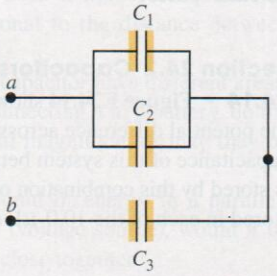


24.18 • In Fig. 24.8a, let $C_1 = 3.00\ \mu F$, $C_2 = 5.50\ \mu F$, and $V_{ab} = +55.0\ V$. Calculate (a) the charge on each capacitor and (b) the potential difference across each capacitor.

24.19 • In Fig. 24.9a, let $C_1 = 3.10\ \mu F$, $C_2 = 5.30\ \mu F$, and $V_{ab} = +50.0\ V$. Calculate (a) the charge on each capacitor and (b) the potential difference across each capacitor.

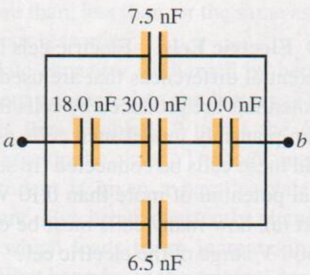
24.20 • In **Fig. E24.20**, $C_1 = 6.00\ \mu F$, $C_2 = 3.00\ \mu F$, and $C_3 = 5.00\ \mu F$. The capacitor network is connected to an applied potential V_{ab}. After the charges on the capacitors have reached their final values, the charge on C_2 is $30.0\ \mu C$. (a) What are the charges on capacitors C_1 and C_3? (b) What is the applied voltage V_{ab}?

Figure **E24.20**



24.21 •• For the system of capacitors shown in **Fig. E24.21**, a potential difference of 25 V is maintained across ab. (a) What is the equivalent capacitance of this system between a and b? (b) How much charge is stored by this system? (c) How much charge does the 6.5-nF capacitor store? (d) What is the potential difference across the 7.5-nF capacitor?

Figure **E24.21**



24.22 •• Suppose the 3-μF capacitor in Fig. 24.10a were removed and replaced by a different one, and that this changed the equivalent capacitance between points a and b to 8 μF. What would be the capacitance of the replacement capacitor?

Section 24.3 Energy Storage in Capacitors and Electric-Field Energy

24.23 • A 6.90-μF, parallel-plate, air capacitor has a plate separation of 3.30 mm and is charged to a potential difference of 380 V. Calculate the energy density in the region between the plates, in units of J/m^3.

24.24 • A parallel-plate air capacitor has a capacitance of 920 pF. The charge on each plate is 3.90 μC. (a) What is the potential difference between the plates? (b) If the charge is kept constant, what will be the potential difference if the plate separation is doubled? (c) How much work is required to double the separation?

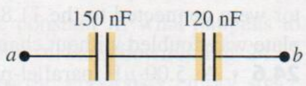
24.25 • An air capacitor is made from two flat parallel plates 1.50 mm apart. The magnitude of charge on each plate is 0.0180 μC when the potential difference is 200 V. (a) What is the capacitance? (b) What is the area of each plate? (c) What maximum voltage can be applied without dielectric breakdown? (Dielectric breakdown for air occurs at an electric-field strength of $3.0 \times 10^6\ V/m$.) (d) When the charge is 0.0180 μC, what total energy is stored?

24.26 •• A parallel-plate vacuum capacitor has 9.36 J of energy stored in it. The separation between the plates is 2.70 mm. If the separation is decreased to 1.85 mm, what is the energy stored (a) if the capacitor is disconnected from the potential source so the charge on the plates remains constant, and (b) if the capacitor remains connected to the potential source so the potential difference between the plates remains constant?

24.27 • You have two identical capacitors and an external potential source. (a) Compare the total energy stored in the capacitors when they are connected to the applied potential in series and in parallel. (b) Compare the maximum amount of charge stored in each case. (c) Energy storage in a capacitor can be limited by the maximum electric field between the plates. What is the ratio of the electric field for the series and parallel combinations?

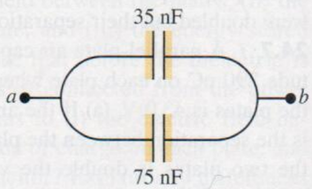
24.28 • For the capacitor network shown in **Fig. E24.28**, the potential difference across ab is 48 V. Find (a) the total charge stored in this network; (b) the charge on each capacitor; (c) the total energy stored in the network; (d) the energy stored in each capacitor; (e) the potential differences across each capacitor.

Figure **E24.28**



24.29 • For the capacitor network shown in **Fig. E24.29**, the potential difference across ab is 220 V. Find (a) the total charge stored in this network; (b) the charge on each capacitor; (c) the total energy stored in the network; (d) the energy stored in each capacitor; (e) the potential difference across each capacitor.

Figure **E24.29**



24.30 • A 0.340-m-long cylindrical capacitor consists of a solid conducting core with a radius of 1.30 mm and an outer hollow conducting tube with an inner radius of 2.25 mm. The two conductors are separated by air and charged to a potential difference of 6.20 V. Calculate (a) the charge per length for the capacitor; (b) the total charge on the capacitor; (c) the capacitance; (d) the energy stored in the capacitor when fully charged.

24.31 • A cylindrical air capacitor of length 13.2 m stores 2.50×10^{-9} J of energy when the potential difference between the two conductors is 4.30 V. (a) Calculate the magnitude of the charge on each conductor. (b) Calculate the ratio of the radii of the inner and outer conductors.

24.32 •• A capacitor is formed from two concentric spherical conducting shells separated by vacuum. The inner sphere has radius 10.5 cm, and the outer sphere has radius 16.5 cm. A potential difference of 150 V is applied to the capacitor. (a) What is the energy density at $r = 10.6$ cm, just outside the inner sphere? (b) What is the energy density at $r = 16.4$ cm, just inside the outer sphere? (c) For a parallel-plate capacitor the energy density is uniform in the region between the plates, except near the edges of the plates. Is this also true for a spherical capacitor?

Section 24.4 Dielectrics

24.33 • A 12.0-μF capacitor is connected to a power supply that keeps a constant potential difference of 26.0 V across the plates. A piece of material having a dielectric constant of 3.65 is placed between the plates, completely filling the space between them. (a) How much energy is stored in the capacitor before and after the dielectric is inserted? (b) By how much did the energy change during the insertion? Did it increase or decrease?

24.34 • A parallel-plate capacitor has capacitance $C_0 = 5.20$ pF when there is air between the plates. The separation between the plates is 1.70 mm. (a) What is the maximum magnitude of charge Q that can be placed on each plate if the electric field in the region between the plates is not to exceed 3.00×10^4 V/m? (b) A dielectric with $K = 3.10$ is inserted between the plates of the capacitor, completely filling the volume between the plates. Now what is the maximum magnitude of charge on each plate if the electric field between the plates is not to exceed 3.00×10^4 V/m?

24.35 • Two parallel plates have equal and opposite charges. When the space between the plates is evacuated, the electric field is $E = 3.30 \times 10^5$ V/m. When the space is filled with dielectric, the electric field is $E = 2.50 \times 10^5$ V/m. (a) What is the charge density on each surface of the dielectric? (b) What is the dielectric constant?

24.36 • A budding electronics hobbyist wants to make a simple 1.5-nF capacitor for tuning her crystal radio, using two sheets of aluminum foil as plates, with a few sheets of paper between them as a dielectric. The paper has a dielectric constant of 4.8, and the thickness of one sheet of it is 0.15 mm. (a) If the sheets of paper measure 30×38 cm and she cuts the aluminum foil to the same dimensions, how many sheets of paper should she use between her plates to get the proper capacitance? (b) Suppose for convenience she wants to use a single sheet of posterboard, with the same dielectric constant but a thickness of 11.00 mm, instead of the paper. What area of aluminum foil will she need for her plates to get her 1.5 nF of capacitance? (c) Suppose she goes high-tech and finds a sheet of Teflon of the same thickness as the posterboard to use as a dielectric. Will she need a larger or smaller area of Teflon than of posterboard? Explain.

24.37 • The dielectric to be used in a parallel-plate capacitor has a dielectric constant of 3.20 and a dielectric strength of 1.50×10^7 V/m. The capacitor is to have a capacitance of 1.45×10^{-9} F and must be able to withstand a maximum potential difference of 5200 V. What is the minimum area the plates of the capacitor may have?

24.38 •• BIO **Potential in Human Cells.** Some cell walls in the human body have a layer of negative charge on the inside surface and a layer of positive charge of equal magnitude on the outside surface. Suppose that the charge density on either surface is $\pm 0.50 \times 10^{-3}$ C/m^2, the cell wall is 5.0 nm thick, and the cell-wall material is air. (a) Find the magnitude of $\vec{E}$ in the wall between the two layers of charge. (b) Find the potential difference between the inside and the outside of the cell. Which is at the higher potential? (c) A typical cell in the human body has a volume of 10^{-16} m^3. Estimate the total electric-field energy stored in the wall of a cell of this size. (*Hint:* Assume that the cell is spherical, and calculate the volume of the cell wall.) (d) In reality, the cell wall is made up, not of air, but of tissue with a dielectric constant of 5.4. Repeat parts (a) and (b) in this case.

24.39 • A constant potential difference of 12 V is maintained between the terminals of a 0.25-μF, parallel-plate, air capacitor. (a) A sheet of Mylar is inserted between the plates of the capacitor, completely filling the space between the plates. When this is done, how much additional charge flows onto the positive plate of the capacitor (see Table 24.1)? (b) What is the total induced charge on either face of the Mylar sheet? (c) What effect does the Mylar sheet have on the electric field between the plates? Explain how you can reconcile this with the increase in charge on the plates, which acts to *increase* the electric field.

24.40 •• Polystyrene has dielectric constant 2.6 and dielectric strength 2.0×10^7 V/m. A piece of polystyrene is used as a dielectric in a parallel-plate capacitor, filling the volume between the plates. (a) When the electric field between the plates is 80% of the dielectric strength, what is the energy density of the stored energy? (b) When the capacitor is connected to a battery with voltage 500.0 V, the electric field between the plates is 80% of the dielectric strength. What is the area of each plate if the capacitor stores 0.200 mJ of energy under these conditions?

24.41 • When a 360-nF air capacitor ($1 \text{ nF} = 10^{-9}$ F) is connected to a power supply, the energy stored in the capacitor is 1.85×10^{-5} J. While the capacitor is kept connected to the power supply, a slab of dielectric is inserted that completely fills the space between the plates. This increases the stored energy by 2.32×10^{-5} J. (a) What is the potential difference between the capacitor plates? (b) What is the dielectric constant of the slab?

24.42 • A parallel-plate capacitor has capacitance $C = 12.5$ pF when the volume between the plates is filled with air. The plates are circular, with radius 3.00 cm. The capacitor is connected to a battery, and a charge of magnitude 25.0 pC goes onto each plate. With the capacitor still connected to the battery, a slab of dielectric is inserted between the plates, completely filling the space between the plates. After the dielectric has been inserted, the charge on each plate has magnitude 45.0 pC. (a) What is the dielectric constant K of the dielectric? (b) What is the potential difference between the plates before and after the dielectric has been inserted? (c) What is the electric field at a point midway between the plates before and after the dielectric has been inserted?

Section 24.6 Gauss's Law in Dielectrics

24.43 • A parallel-plate capacitor has the volume between its plates filled with plastic with dielectric constant K. The magnitude of the charge on each plate is Q. Each plate has area A, and the distance between the plates is d. (a) Use Gauss's law as stated in Eq. (24.23) to calculate the magnitude of the electric field in the dielectric. (b) Use the electric field determined in part (a) to calculate the potential difference between the two plates. (c) Use the result of part (b) to determine the capacitance of the capacitor. Compare your result to Eq. (24.12).

24.44 • A parallel-plate capacitor has plates with area 0.0210 m^2 separated by 1.80 mm of Teflon. (a) Calculate the charge on the plates when they are charged to a potential difference of 15.0 V. (b) Use Gauss's law (Eq. 24.23) to calculate the electric field inside the Teflon. (c) Use Gauss's law to calculate the electric field if the voltage source is disconnected and the Teflon is removed.

PROBLEMS

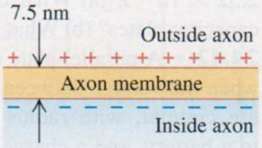
24.45 • Electronic flash units for cameras contain a capacitor for storing the energy used to produce the flash. In one such unit, the flash lasts for 1.43×10^{-3} s with an average light power output of 2.90×10^5 W. (a) If the conversion of electrical energy to light is 96% efficient (the rest of the energy goes to thermal energy), how much energy must be stored in the capacitor for one flash? (b) The capacitor has a potential difference between its plates of 110 V when the stored energy equals the value calculated in part (a). What is the capacitance?

24.46 • A parallel-plate air capacitor is made by using two plates 12 cm square, spaced 3.7 mm apart. It is connected to a 12-V battery. (a) What is the capacitance? (b) What is the charge on each plate? (c) What is the electric field between the plates? (d) What is the energy stored in the capacitor? (e) If the battery is disconnected and then the plates are pulled apart to a separation of 7.4 mm, what are the answers to parts (a)–(d)?

24.47 ••• In one type of computer keyboard, each key holds a small metal plate that serves as one plate of a parallel-plate, air-filled capacitor. When the key is depressed, the plate separation decreases and the capacitance increases. Electronic circuitry detects the change in capacitance and thus detects that the key has been pressed. In one particular keyboard, the area of each metal plate is 51.0 mm^2, and the separation between the plates is 0.740 mm before the key is depressed. (a) Calculate the capacitance before the key is depressed. (b) If the circuitry can detect a change in capacitance of 0.250 pF, how far must the key be depressed before the circuitry detects its depression?

24.48 ••• **BIO Cell Membranes.** Cell membranes (the walled enclosure around a cell) are typically about 7.5 nm thick. They are partially permeable to allow charged material to pass in and out, as needed. Equal but opposite charge densities build up on the inside and outside faces of such a membrane, and these charges prevent additional charges from passing through the cell wall. We can model a cell membrane as a parallel-plate capacitor, with the membrane itself containing proteins embedded in an organic material to give the membrane a dielectric constant of about 10. (See **Fig. P24.48.**) (a) What is the capacitance per square centimeter of such a cell wall? (b) In its normal resting state, a cell has a potential difference of 85 mV across its membrane. What is the electric field inside this membrane?

Figure **P24.48**

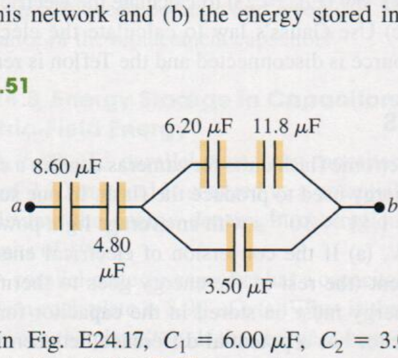


24.49 •• A 25.0-μF capacitor is charged to a potential difference of 850 V. The terminals of the charged capacitor are then connected to those of an uncharged 9.0-μF capacitor. Compute (a) the original charge of the system, (b) the final potential difference across each capacitor, (c) the final energy of the system, and (d) the decrease in energy when the capacitors are connected.

24.50 •• In Fig. 24.9a, let $C_1 = 9.5\ \mu F$, $C_2 = 4.3\ \mu F$, and $V_{ab} = 28$ V. Suppose the charged capacitors are disconnected from the source and from each other, and then reconnected to each other with plates of *opposite* sign together. By how much does the energy of the system decrease?

24.51 • For the capacitor network shown in **Fig. P24.51**, the potential difference across *ab* is 12.0 V. Find (a) the total energy stored in this network and (b) the energy stored in the 4.80-μF capacitor.

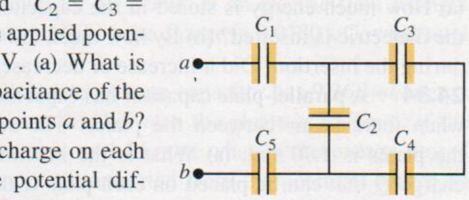
Figure **P24.51**



24.52 •• In Fig. E24.17, $C_1 = 6.00\ \mu F$, $C_2 = 3.00\ \mu F$, $C_3 = 4.00\ \mu F$, and $C_4 = 8.00\ \mu F$. The capacitor network is connected to an applied potential difference V_{ab}. After the charges on the capacitors have reached their final values, the voltage across C_3 is 40.0 V. What are (a) the voltages across C_1 and C_2, (b) the voltage across C_4, and (c) the voltage V_{ab} applied to the network?

24.53 • In **Fig. P24.53**, $C_1 = C_5 = 8.1\ \mu F$ and $C_2 = C_3 = C_4 = 4.5\ \mu F$. The applied potential is $V_{ab} = 220$ V. (a) What is the equivalent capacitance of the network between points *a* and *b*? (b) Calculate the charge on each capacitor and the potential difference across each capacitor.

Figure **P24.53**

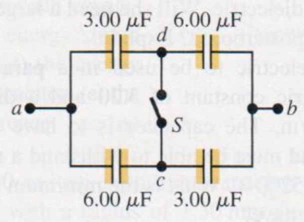


24.54 •• Current materials-science technology allows engineers to construct capacitors with much higher values of *C* than were previously possible. A capacitor has $C = 3000$ F and is rated to withstand a maximum potential difference of 2.7 V. The cylindrical capacitor has diameter 6.0 cm and length 13.5 cm. (a) Find the maximum electric potential energy that can be stored in this capacitor. (b) Does your value in part (a) agree with the 3.0-Wh value printed on the capacitor? (c) What is the maximum attainable energy density in this capacitor? (d) Compare this maximum energy density to the maximum possible energy density for polyester (see Table 24.2).

24.55 •• In Fig. E24.20, $C_1 = 3.00\ \mu F$ and $V_{ab} = 150$ V. The charge on capacitor C_1 is 150 μC and the charge on C_3 is 450 μC. What are the values of the capacitances of C_2 and C_3?

24.56 • The capacitors in **Fig. P24.56** are initially uncharged and are connected, as in the diagram, with switch *S* open. The applied potential difference is $V_{ab} = +210$ V. (a) What is the potential difference V_{cd}? (b) What is the potential difference across each capacitor after switch *S* is closed? (c) How much charge flowed through the switch when it was closed?

Figure **P24.56**

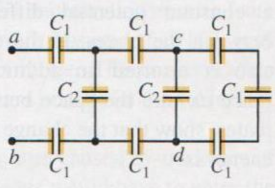


24.57 •• Three capacitors having capacitances of 8.4, 8.1, and 4.9 μF are connected in series across a 40-V potential difference. (a) What is the charge on the 4.9-μF capacitor? (b) What is the total energy stored in all three capacitors? (c) The capacitors are disconnected from the potential difference without allowing them to discharge. They are then reconnected in parallel with each other, with the positively charged plates connected together. What is the voltage across each capacitor in the parallel combination? (d) What is the total energy now stored in the capacitors?

24.58 • **Capacitance of a Thundercloud.** The charge center of a thundercloud, drifting 3.0 km above the earth's surface, contains 20 C of negative charge. Assuming the charge center has a radius of 1.0 km, and modeling the charge center and the earth's surface as parallel plates, calculate: (a) the capacitance of the system; (b) the potential difference between charge center and ground; (c) the average strength of the electric field between cloud and ground; (d) the electrical energy stored in the system.

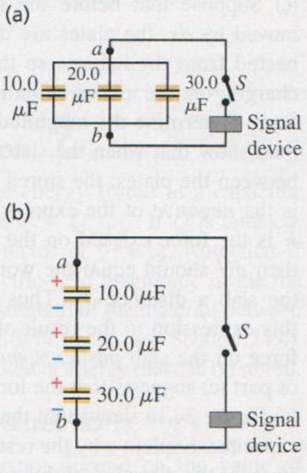
24.59 •• In **Fig. P24.59**, each capacitance C_1 is 6.6 μF, and each capacitance C_2 is 4.4 μF. (a) Compute the equivalent capacitance of the network between points a and b. (b) Compute the charge on each of the three capacitors nearest a and b when $V_{ab} = 470$ V. (c) With 470 V across a and b, compute V_{cd}.

Figure **P24.59**



24.60 • Each combination of capacitors between points a and b in **Fig. P24.60** is first connected across a 120-V battery, charging the combination to 120 V. These combinations are then connected to make the circuits shown. When the switch S is thrown, a surge of charge for the discharging capacitors flows to trigger the signal device. How much charge flows through the signal device in each case?

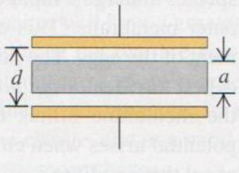
Figure **P24.60**



24.61 • A parallel-plate capacitor with only air between the plates is charged by connecting it to a battery. The capacitor is then disconnected from the battery, without any of the charge leaving the plates. (a) A voltmeter reads 50 V when placed across the capacitor. When a dielectric is inserted between the plates, completely filling the space, the voltmeter reads 10.0 V. What is the dielectric constant of this material? (b) What will the voltmeter read if the dielectric is now pulled partway out so it fills only one-third of the space between the plates?

24.62 •• An air capacitor is made by using two flat plates, each with area A, separated by a distance d. Then a metal slab having thickness a (less than d) and the same shape and size as the plates is inserted between them, parallel to the plates and not touching either plate (**Fig. P24.62**). (a) What is the capacitance of this arrangement? (b) Express the capacitance as a multiple of the capacitance C_0 when the metal slab is not present. (c) Discuss what happens to the capacitance in the limits $a \rightarrow 0$ and $a \rightarrow d$.

Figure **P24.62**



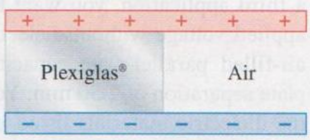
24.63 •• A potential difference $V_{ab} = 48.0$ V is applied across the capacitor network of Fig. E24.17. If $C_1 = C_2 = 4.00$ μF and $C_4 = 8.00$ μF, what must the capacitance C_3 be if the network is to store 2.90×10^{-3} J of electrical energy?

24.64 • **CALC** The inner cylinder of a long, cylindrical capacitor has radius r_a and linear charge density $+\lambda$. It is surrounded by a coaxial cylindrical conducting shell with inner radius r_b and linear charge density $-\lambda$ (see Fig. 24.6). (a) What is the energy density in the region between the conductors at a distance r from the axis? (b) Integrate the energy density calculated in part (a) over the volume between the conductors in a length L of the capacitor to obtain the total electric-field energy per unit length. (c) Use Eq. (24.9) and the capacitance per unit length calculated in Example 24.4 (Section 24.1) to calculate U/L. Does your result agree with that obtained in part (b)?

24.65 •• A parallel-plate capacitor has square plates that are 5.00 cm on each side and 3.50 mm apart. The space between the plates is completely filled with two square slabs of dielectric, each 5.00 cm on a side and 1.75 mm thick. One slab is Pyrex glass and the other is polystyrene. If the potential difference between the plates is 86.0 V, how much electrical energy is stored in the capacitor?

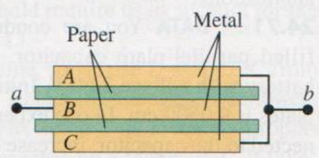
24.66 •• A parallel-plate capacitor is made from two plates 12.0 cm on each side and 4.50 mm apart. Half of the space between these plates contains only air, but the other half is filled with Plexiglas® of dielectric constant 3.40 (**Fig. P24.66**). An 18.0-V battery is connected across the plates. (a) What is the capacitance of this combination? (*Hint:* Can you think of this capacitor as equivalent to two capacitors in parallel?) (b) How much energy is stored in the capacitor? (c) If we remove the Plexiglas® but change nothing else, how much energy will be stored in the capacitor?

Figure **P24.66**



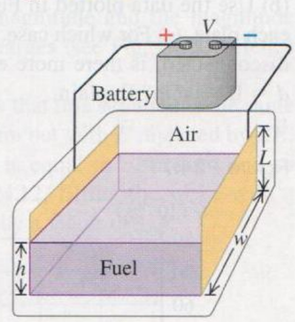
24.67 •• Three square metal plates A, B, and C, each 12.0 cm on a side and 1.50 mm thick, are arranged as in **Fig. P24.67**. The plates are separated by sheets of paper 0.40 mm thick and with dielectric constant 4.4. The outer plates are connected together and connected to point b. The inner plate is connected to point a. (a) Copy the diagram and show by plus and minus signs the charge distribution on the plates when point a is maintained at a positive potential relative to point b. (b) What is the capacitance between points a and b?

Figure **P24.67**



24.68 •• A fuel gauge uses a capacitor to determine the height of the fuel in a tank. The effective dielectric constant K_{eff} changes from a value of 1 when the tank is empty to a value of K, the dielectric constant of the fuel, when the tank is full. The appropriate electronic circuitry can determine the effective dielectric constant of the combined air and fuel between the capacitor plates. Each of the two rectangular plates has a width w and a length L (**Fig. P24.68**). The height of the fuel between the plates is h. You can ignore any fringing effects. (a) Derive an expression for K_{eff} as a function of h. (b) What is the effective dielectric constant for a tank $\frac{1}{4}$ full, $\frac{1}{2}$ full, and $\frac{3}{4}$ full if the fuel is gasoline ($K = 1.95$)? (c) Repeat part (b) for methanol ($K = 33.0$). (d) For which fuel is this fuel gauge more practical?

Figure **P24.68**

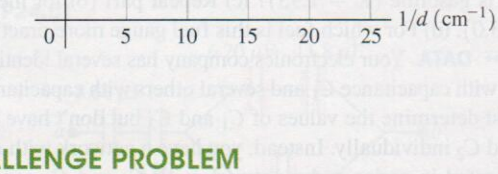


24.69 •• **DATA** Your electronics company has several identical capacitors with capacitance C_1 and several others with capacitance C_2. You must determine the values of C_1 and C_2 but don't have access to C_1 and C_2 individually. Instead, you have a network with C_1 and C_2 connected in series and a network with C_1 and C_2 connected in parallel. You have a 200.0-V battery and instrumentation that measures the total energy supplied by the battery when it is connected to the network. When the parallel combination is connected to the battery, 0.180 J of energy is stored in the network. When the series combination is connected, 0.0400 J of energy is stored. You are told that C_1 is greater than C_2. (a) Calculate C_1 and C_2. (b) For the series combination, does C_1 or C_2 store more charge, or are the values equal? Does C_1 or C_2 store more energy, or are the values equal? (c) Repeat part (b) for the parallel combination.

24.70 •• DATA You are designing capacitors for various applications. For one application, you want the maximum possible stored energy. For another, you want the maximum stored charge. For a third application, you want the capacitor to withstand a large applied voltage without dielectric breakdown. You start with an air-filled parallel-plate capacitor that has $C_0 = 6.00$ pF and a plate separation of 2.50 mm. You then consider the use of each of the dielectric materials listed in Table 24.2. In each application, the dielectric will fill the volume between the plates, and the electric field between the plates will be 50% of the dielectric strength given in the table. (a) For each of the five materials given in the table, calculate the energy stored in the capacitor. Which dielectric allows the maximum stored energy? (b) For each material, what is the charge Q stored on each plate of the capacitor? (c) For each material, what is the voltage applied across the capacitor? (d) Is one dielectric material in the table your best choice for all three applications?

24.71 •• DATA You are conducting experiments with an air-filled parallel-plate capacitor. You connect the capacitor to a battery with voltage 24.0 V. Initially the separation d between the plates is 0.0500 cm. In one experiment, you leave the battery connected to the capacitor, increase the separation between the plates, and measure the energy stored in the capacitor for each value of d. In a second experiment, you make the same measurements but disconnect the battery before you change the plate separation. One set of your data is given in **Fig. P24.71**, where you have plotted the stored energy U versus $1/d$. (a) For which experiment does this data set apply: the first (battery remains connected) or the second (battery disconnected before d is changed)? Explain. (b) Use the data plotted in Fig. P24.71 to calculate the area A of each plate. (c) For which case, the battery connected or the battery disconnected, is there more energy stored in the capacitor when $d = 0.400$ cm? Explain.

Figure **P24.71**

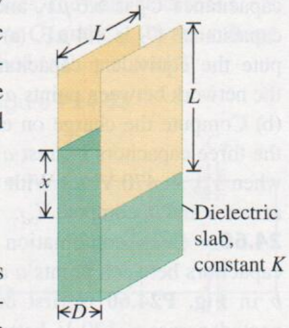


CHALLENGE PROBLEM

24.72 ••• Two square conducting plates with sides of length L are separated by a distance D. A dielectric slab with constant K with dimensions $L \times L \times D$ is inserted a distance x into the space between the plates, as shown in **Fig. P24.72**. (a) Find the capacitance C of this system. (b) Suppose that the capacitor is connected to a battery that maintains a constant potential difference V between the plates. If the dielectric slab is inserted an additional distance dx into the space between the plates, show that the change in stored energy is

$$dU = +\frac{(K-1)\epsilon_0 V^2 L}{2D}dx$$

Figure **P24.72**



(c) Suppose that before the slab is moved by dx, the plates are disconnected from the battery, so that the charges on the plates remain constant. Determine the magnitude of the charge on each plate, and then show that when the slab is moved dx farther into the space between the plates, the stored energy changes by an amount that is the *negative* of the expression for dU given in part (b). (d) If F is the force exerted on the slab by the charges on the plates, then dU should equal the work done *against* this force to move the slab a distance dx. Thus $dU = -F\,dx$. Show that applying this expression to the result of part (b) suggests that the electric force on the slab pushes it *out* of the capacitor, while the result of part (c) suggests that the force pulls the slab *into* the capacitor. (e) Figure 24.16 shows that the force in fact pulls the slab into the capacitor. Explain why the result of part (b) gives an incorrect answer for the direction of this force, and calculate the magnitude of the force. (This method does not require knowledge of the nature of the fringing field.)

PASSAGE PROBLEMS

BIO THE ELECTRIC EGG. Upon fertilization, the eggs of many species undergo a rapid change in potential difference across their outer membrane. This change affects the physiological development of the eggs. The potential difference across the membrane is called the *membrane potential,* V_m, which is the potential inside the membrane minus the potential outside it. The membrane potential arises when enzymes use the energy available in ATP to expel three sodium ions (Na^+) actively and accumulate two potassium ions (K^+) inside the membrane—making the interior less positively charged than the exterior. For a sea urchin egg, V_m is about −70 mV; that is, the potential inside is 70 mV less than that outside. The egg membrane behaves as a capacitor with a capacitance of about 1 $\mu F/cm^2$. The membrane of the unfertilized egg is *selectively permeable* to K^+; that is, K^+ can readily pass through certain channels in the membrane, but other ions cannot. When a sea urchin egg is fertilized, Na^+ channels in the membrane open, Na^+ enters the egg, and V_m rapidly increases to +30 mV, where it remains for several minutes. The concentration of Na^+ is about 30 mmol/L in the egg's interior but 450 mmol/L in the surrounding seawater. The K^+ concentration is about 200 mmol/L inside but 10 mmol/L outside. A useful constant that connects electrical and chemical units is the *Faraday number,* which has a value of approximately 10^5 C/mol; that is, Avogadro's number (a mole) of monovalent ions, such as Na^+ or K^+, carries a charge of 10^5 C.

24.73 How many moles of Na^+ must m ove per unit area of membrane to change V_m from −70 mV to +30 mV, if we assume that the membrane behaves purely as a capacitor? (a) 10^{-4} mol/cm^2; (b) 10^{-9} mol/cm^2; (c) 10^{-12} mol/cm^2; (d) 10^{-14} mol/cm^2.

24.74 Suppose that the egg has a diameter of 200 μm. What fractional change in the internal Na^+ concentration results from the fertilization-induced change in V_m? Assume that Na^+ ions are distributed throughout the cell volume. The concentration increases by (a) 1 part in 10^4; (b) 1 part in 10^5; (c) 1 part in 10^6; (d) 1 part in 10^7.

24.75 Suppose that the change in V_m was caused by the entry of Ca^{2+} instead of Na^+. How many Ca^{2+} ions would have to enter the cell per unit membrane to produce the change? (a) Half as many as for Na^+; (b) the same as for Na^+; (c) twice as many as for Na^+; (d) cannot say without knowing the inside and outside concentrations of Ca^{2+}.

24.76 What is the minimum amount of work that must be done by the cell to restore V_m to -70 mV? (a) 3 mJ; (b) 3 μJ; (c) 3 nJ; (d) 3 pJ.

Answers

Chapter Opening Question ?

(iv) Equation (24.9) shows that the energy stored in a capacitor with capacitance C and charge Q is $U = Q^2/2C$. If Q is doubled, the stored energy increases by a factor of $2^2 = 4$. Note that if the value of Q is too great, the electric-field magnitude inside the capacitor will exceed the dielectric strength of the material between the plates and dielectric breakdown will occur (see Section 24.4). This puts a practical limit on the amount of energy that can be stored.

Test Your Understanding Questions

24.1 (iii) The capacitance does not depend on the value of the charge Q. Doubling Q causes the potential difference V_{ab} to double, so the capacitance $C = Q/V_{ab}$ remains the same. These statements are true no matter what the geometry of the capacitor.

24.2 (a) (i), (b) (iv) In a series connection the two capacitors carry the same charge Q but have different potential differences $V_{ab} = Q/C$; the capacitor with the smaller capacitance C has the greater potential difference. In a parallel connection the two capacitors have the same potential difference V_{ab} but carry different charges $Q = CV_{ab}$; the capacitor with the larger capacitance C has the greater charge. Hence a 4-μF capacitor will have a greater potential difference than an 8-μF capacitor if the two are connected in series. The 4-μF capacitor cannot carry more charge than the 8-μF capacitor no matter how they are connected: In a series connection they will carry the same charge, and in a parallel connection the 8-μF capacitor will carry more charge.

24.3 (i) Capacitors connected in series carry the same charge Q. To compare the amount of energy stored, we use the expression $U = Q^2/2C$ from Eq. (24.9); it shows that the capacitor with the *smaller* capacitance ($C = 4\ \mu$F) has more stored energy in a series combination. By contrast, capacitors in parallel have the same potential difference V, so to compare them we use $U = \frac{1}{2}CV^2$ from Eq. (24.9). It shows that in a parallel combination, the capacitor with the *larger* capacitance ($C = 8\ \mu$F) has more stored energy. (If we had instead used $U = \frac{1}{2}CV^2$ to analyze the series combination, we would have to account for the different potential differences across the two capacitors. Likewise, using $U = Q^2/2C$ to study the parallel combination would require us to account for the different charges on the capacitors.)

24.4 (i) Here Q remains the same, so we use $U = Q^2/2C$ from Eq. (24.9) for the stored energy. Removing the dielectric lowers the capacitance by a factor of $1/K$; since U is inversely proportional to C, the stored energy *increases* by a factor of K. It takes work to pull the dielectric slab out of the capacitor because the fringing field tries to pull the slab back in (Fig. 24.16). The work that you do goes into the energy stored in the capacitor.

24.5 (i), (iii), (ii) Equation (24.14) says that if E_0 is the initial electric-field magnitude (before the dielectric slab is inserted), then the resultant field magnitude after the slab is inserted is $E_0/K = E_0/3$. The magnitude of the resultant field equals the difference between the initial field magnitude and the magnitude E_i of the field due to the bound charges (see Fig. 24.20). Hence $E_0 - E_i = E_0/3$ and $E_i = 2E_0/3$.

24.6 (iii) Equation (24.23) shows that this situation is the same as an isolated point charge in vacuum but with $\vec{E}$ replaced by $K\vec{E}$. Hence KE at the point of interest is equal to $q/4\pi\epsilon_0 r^2$, and so $E = q/4\pi K\epsilon_0 r^2$. As in Example 24.12, filling the space with a dielectric reduces the electric field by a factor of $1/K$.

Bridging Problem

(a) 0 **(b)** $Q^2/32\pi^2\epsilon_0 r^4$ **(c)** $Q^2/8\pi\epsilon_0 R$ **(d)** $Q^2/8\pi\epsilon_0 R$ **(e)** $C = 4\pi\epsilon_0 R$

The University of Jordan
physics Department

General Physics (102) / Sample Solutions
Chapter 24 / Capacitance / Sears and Zemansky
Prof. Mahmoud Jaghoub (أ.د. محمود الجاغوب)

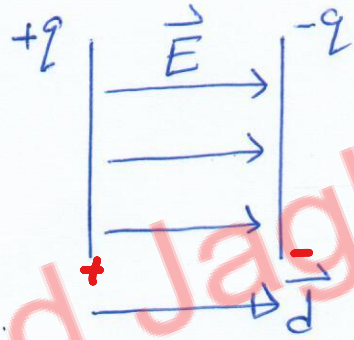
Q1]

$q = 85\ nC$

$d = 3.5\ mm$

$V = Ed \Rightarrow E = \frac{V}{d}$

$E = 5\times10^{6}\ \frac{V}{m}\ \left(\text{or } \frac{N}{C}\right)$



a) $\Delta V = -\int_{+}^{-} \vec{E}\cdot d\vec{s} = -\vec{E}\cdot\vec{d}$

$\Delta V = -Ed\cos(0) = -Ed$

$V = |\Delta V| = Ed = 5\times10^{6} \times 3.5\times10^{-3} = 17.5\times10^{3}$ Volt

b) $C = \frac{\epsilon_0 A}{d} = \frac{q}{V} \Rightarrow A = \left(\frac{q}{V}\right)\frac{d}{\epsilon_0}$

$A = \frac{85\times10^{-9}}{17.5\times10^{3}} \times \frac{3.5\times10^{-3}}{8.85\times10^{-12}} \approx 1.92\times10^{-3}\ m^2$
$= 19.2\ cm^2$

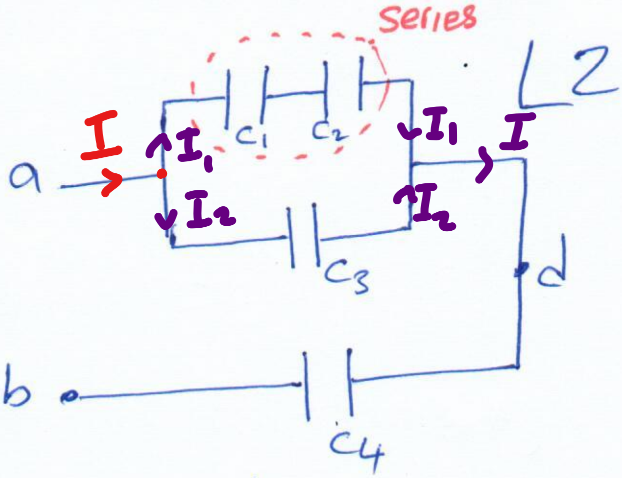
c) $C = \frac{q}{V} = \frac{85\times10^{-9}}{17.5\times10^{3}} \approx 4.86\times10^{-12} F = 4.86 PF$

equivalently $C = \frac{\epsilon_0 A}{d}$

Q17]

L2

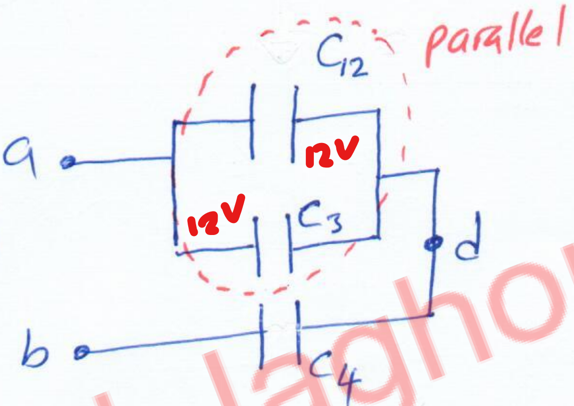
(i)



$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}$$

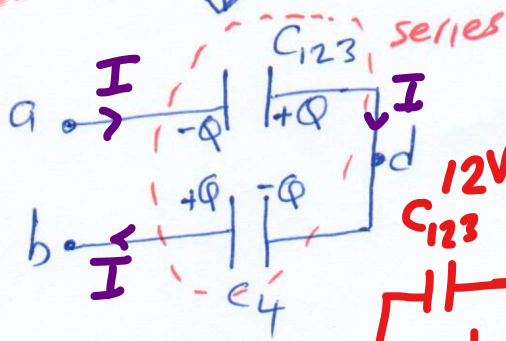
$$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{C^2}{2C} = \frac{C}{2} = 2.4\,\mu F$$

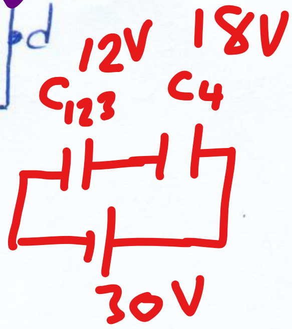
(ii)



$$C_{123} = C_{12} + C_3$$
$$= \frac{C}{2} + C = \frac{3}{2}C$$
$$= 7.2\,\mu F$$

(iii)

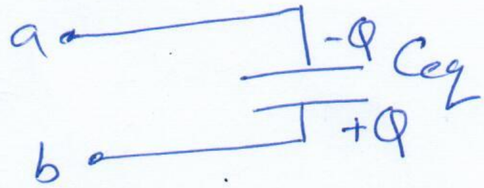




$$\frac{1}{C_{eq}} = \frac{1}{C_{123}} + \frac{1}{C_4} = \frac{C_4 + C_{123}}{C_4 C_{123}}$$

$$C_{eq} = \frac{C_4 C_{123}}{C_4 + C_{123}}$$
$$= \frac{(C)(\frac{3}{2}C)}{C + \frac{3}{2}C}$$
$$= \frac{\frac{3}{2}C}{1 + \frac{3}{2}} = \frac{3}{5}C$$
$$= 2.88\,\mu F.$$

(iv)



$C_{eq} = \frac{Q}{V_{ab}} \Rightarrow Q = C_{eq} V_{ab} = (2.88\times10^{-6})(30)$

$Q = 86.4\mu C$

now work backwards.

C_{123} and C_4 each has the same charge Q as C_{eq} since they are in series.

Diagram (iii) $V_4 = \frac{Q}{C_4} = \frac{86.4\times10^{-6}}{4.8\times10^{-6}} = 18V$

$V_{123} = \frac{Q}{C_{123}} = \frac{86.4\times10^{-6}}{7.2\times10^{-6}} = 12V$

NOTE: $V_4 + V_{123} = 30V = V_{ab}$ as expected.

Diagram (ii) $V_{12} = V_3 = V_{123} = 12V$.

$Q_{12} = C_{12}V_{12} = (2.4\times10^{-6})(12) = 28.8\mu C$

$Q_3 = C_3 V_3 = (4.8\times10^{-6})(12) = 57.6\mu C$

NOTE: $Q_{12} + Q_3 = 86.4\mu C = Q$ as expected.

$Q_1 = Q_2 = Q_{12} = 28.8\mu C$

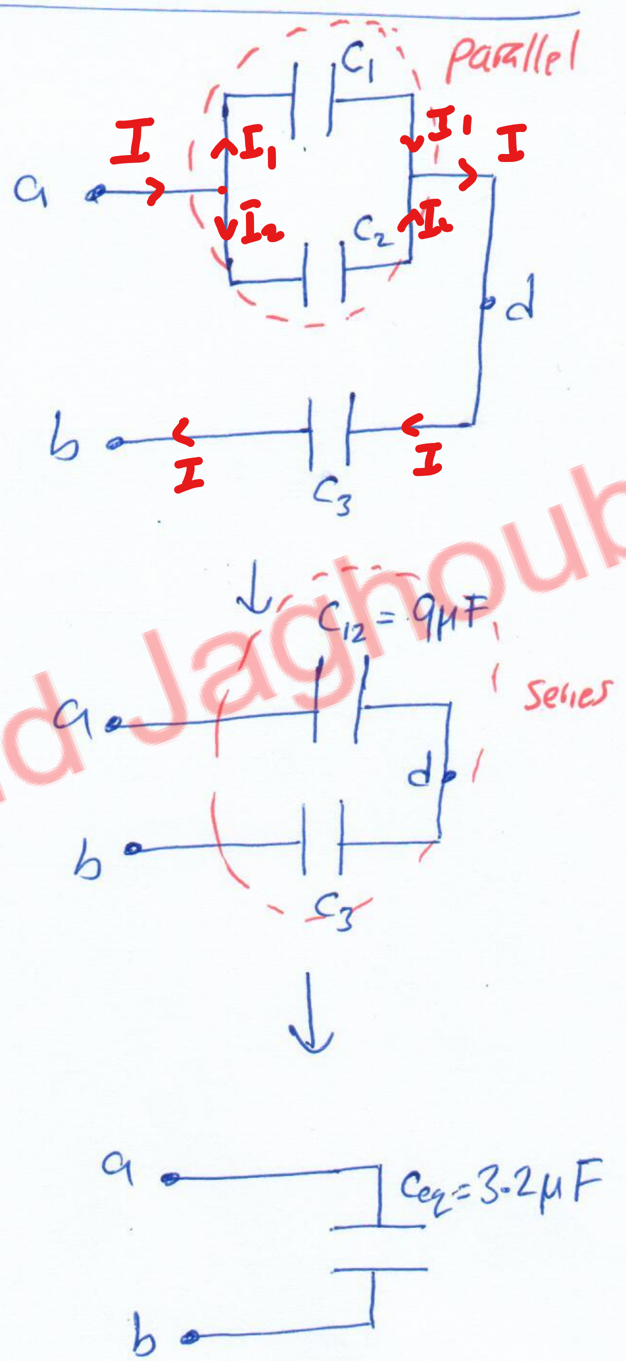
$V_1 = \frac{Q_1}{C_1} = 6V$, $V_2 = \frac{Q_2}{C_2} = 6V$

NOTE: $V_1 + V_2 = 12V = V_{12}$ as expected.

c) $V_{ad} = V_3 = 12\,V$

Also $V_{ad} = V_1 + V_2 = 12\,V$

Q20] $C_1 = 6\mu F$, $C_2 = 3\mu F$
$C_3 = 5\mu F$



$Q_2 = 30\mu C$

$V_2 = \frac{Q_2}{C_2} = 10\,V$

$\Rightarrow V_1 = 10\,V$ since C_1 and C_2 are in parallel.

$Q_1 = C_1 V_1 = 60\mu C$

$\Rightarrow Q_{12} = Q_1 + Q_2 = 90\mu C$

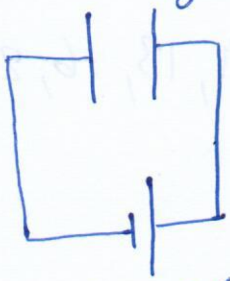
$Q_3 = Q_{12} = 90\mu C$ since C_{12} and C_3 are in series.

$Q_{eq} = Q_3 = Q_{12} = 90\mu C$

$V_{ab} = V_{eq} = \frac{Q_{eq}}{C_{eq}} = 28.125\,V$

L5

Q33] dielectric is inserted while battery is connected to the capacitor.



before inserting dielectric

$$U_i = \frac{1}{2} C_0 V^2 = \frac{1}{2}(12\times10^{-6})(26)^2 = 4056\,\mu J$$

after inserting dielectric

$$U_f = \frac{1}{2} C V^2 = \frac{1}{2}(K C_0) V^2 = K\,U_i$$

$$= \frac{1}{2}(3.65 \times 12\times10^{-6})(26)^2 =$$

$$= 3.65\,(4056\,\mu J) = 14804.4\,\mu J$$

$$\Delta U = U_f - U_i = 10748.4\,\mu J$$

It increased as the battery provides the energy after inserting the dielectric since the battery maintains a potential difference of 26 V accross the capacitor.