BRIDGING PROBLEM A POINT CHARGE AND A LINE OF CHARGE

SOLUTION

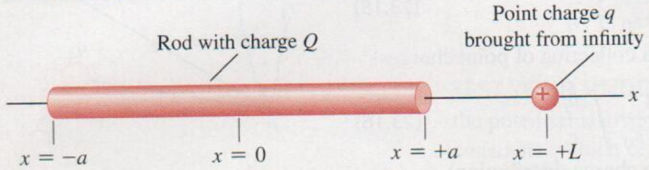
Positive electric charge Q is distributed uniformly along a thin rod of length $2a$. The rod lies along the x-axis between $x = -a$ and $x = +a$ (**Fig. 23.27**). Calculate how much work you must do to bring a positive point charge q from infinity to the point $x = +L$ on the x-axis, where $L > a$.

SOLUTION GUIDE

IDENTIFY and SET UP

1. In this problem you must first calculate the potential V at $x = +L$ due to the charged rod. You can then find the change in potential energy involved in bringing the point charge q from infinity (where $V = 0$) to $x = +L$.

23.27 How much work must you do to bring point charge q in from infinity?



2. To find V, divide the rod into infinitesimal segments of length dx'. How much charge is on such a segment? Consider one such segment located at $x = x'$, where $-a \le x' \le a$. What is the potential dV at $x = +L$ due to this segment?
3. The total potential at $x = +L$ is the integral of dV, including contributions from all of the segments for x' from $-a$ to $+a$. Set up this integral.

EXECUTE

4. Integrate your expression from step 3 to find the potential V at $x = +L$. A simple, standard substitution will do the trick; use a table of integrals only as a last resort.
5. Use your result from step 4 to find the potential energy for a point charge q placed at $x = +L$.
6. Use your result from step 5 to find the work you must do to bring the point charge from infinity to $x = +L$.

EVALUATE

7. What does your result from step 5 become in the limit $a \to 0$? Does this make sense?
8. Suppose the point charge q were negative rather than positive. How would this affect your result in step 4? In step 5?

Problems

For assigned homework and other learning materials, go to MasteringPhysics®.

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q23.1 A student asked, "Since electrical potential is always proportional to potential energy, why bother with the concept of potential at all?" How would you respond?

Q23.2 The potential (relative to a point at infinity) midway between two charges of equal magnitude and opposite sign is zero. Is it possible to bring a test charge from infinity to this midpoint in such a way that no work is done in any part of the displacement? If so, describe how it can be done. If it is not possible, explain why.

Q23.3 Is it possible to have an arrangement of two point charges separated by a finite distance such that the electric potential energy of the arrangement is the same as if the two charges were infinitely far apart? Why or why not? What if there are three charges? Explain.

Q23.4 Since potential can have any value you want depending on the choice of the reference level of zero potential, how does a voltmeter know what to read when you connect it between two points?

Q23.5 If a potential difference of V exists between the two ends of a wire, would the work done in moving a charge q be dependent on the length of the wire, L? What would be the work done in this case?

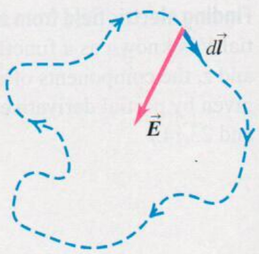
Q23.6 If $\vec{E}$ is zero throughout a certain region of space, is the potential necessarily also zero in this region? Why or why not? If not, what *can* be said about the potential?

Q23.7 Which way do electric field lines point, from high to low potential or from low to high? Explain.

Q23.8 (a) If the potential (relative to infinity) is zero at a point, is the electric field necessarily zero at that point? (b) If the electric field is zero at a point, is the potential (relative to infinity) necessarily zero there? Prove your answers, using simple examples.

Q23.9 If you carry out the integral of the electric field $\int \vec{E} \cdot d\vec{l}$ for a *closed* path like that shown in **Fig. Q23.9**, the integral will *always* be equal to zero, independent of the shape of the path and independent of where charges may be located relative to the path. Explain why.

Figure **Q23.9**



Q23.10 There are two spheres, placed far apart. The ratio of their radii is 10:1. A charge Q is placed on the big sphere. They are now connected by a long conducting wire. After this connection, which sphere will have more charge and what would be the ratio of the charges on both spheres?

Q23.11 It is easy to produce a potential difference of several thousand volts between your body and the floor by scuffing your shoes across a nylon carpet. When you touch a metal doorknob, you get a mild shock. Yet contact with a power line of comparable voltage would probably be fatal. Why is there a difference?

Q23.12 If the electric potential at a single point is known, can $\vec{E}$ at that point be determined? If so, how? If not, why not?

Q23.13 Because electric field lines and equipotential surfaces are always perpendicular, two equipotential surfaces can never cross; if they did, the direction of $\vec{E}$ would be ambiguous at the crossing points. Yet two equipotential surfaces appear to cross at the center of Fig. 23.23c. Explain why there is no ambiguity about the direction of $\vec{E}$ in this particular case.

Q23.14 A uniform electric field is directed due east. Point *B* is 2.00 m west of point *A*, point *C* is 2.00 m east of point *A*, and point *D* is 2.00 m south of *A*. For each point, *B*, *C*, and *D*, is the potential at that point larger, smaller, or the same as at point *A*? Give the reasoning behind your answers.

Q23.15 We often say that if point *A* is at a higher potential than point *B*, *A* is at positive potential and *B* is at negative potential. Does it necessarily follow that a point at positive potential is positively charged, or that a point at negative potential is negatively charged? Illustrate your answers with clear, simple examples.

Q23.16 A conducting sphere is to be charged by bringing in positive charge a little at a time until the total charge is *Q*. The total work required for this process is alleged to be proportional to Q^2. Is this correct? Why or why not?

Q23.17 In electronics it is customary to define the potential of ground (thinking of the earth as a large conductor) as zero. Is this consistent with the fact that the earth has a net electric charge that is not zero? (Refer to Exercise 21.28.)

Q23.18 A conducting sphere is placed between two charged parallel plates such as those shown in Fig. 23.2. Does the electric field inside the sphere depend on precisely where between the plates the sphere is placed? What about the electric potential inside the sphere? Do the answers to these questions depend on whether or not there is a net charge on the sphere? Explain your reasoning.

Q23.19 A conductor that carries a net charge *Q* has a hollow, empty cavity in its interior. Does the potential vary from point to point within the material of the conductor? What about within the cavity? How does the potential inside the cavity compare to the potential within the material of the conductor?

Q23.20 A high-voltage dc power line falls on a car, so the entire metal body of the car is at a potential of 10,000 V with respect to the ground. What happens to the occupants (a) when they are sitting in the car and (b) when they step out of the car? Explain your reasoning.

Q23.21 When a thunderstorm is approaching, sailors at sea sometimes observe a phenomenon called "St. Elmo's fire," a bluish flickering light at the tips of masts. What causes this? Why does it occur at the tips of masts? Why is the effect most pronounced when the masts are wet? (*Hint:* Seawater is a good conductor of electricity.)

Q23.22 A positive point charge is placed near a very large conducting plane. A professor of physics asserted that the field caused by this configuration is the same as would be obtained by removing the plane and placing a negative point charge of equal magnitude in the mirror-image position behind the initial position of the plane. Is this correct? Why or why not? (*Hint:* Inspect Fig. 23.23b.)

EXERCISES

Section 23.1 Electric Potential Energy

23.1 •• A point charge $q_1 = +2.10\ \mu C$ is held stationary at the origin. A second point charge $q_2 = -4.60\ \mu C$ moves from the point $x = 0.150$ m, $y = 0$ to the point $x = 0.250$ m, $y = 0.270$ m. How much work is done by the electric force on q_2?

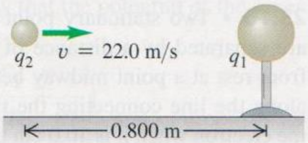
23.2 • A point charge q_1 is held stationary at the origin. A second charge q_2 is placed at point *a*, and the electric potential energy of the pair of charges is $+5.4 \times 10^{-8}$ J. When the second charge is moved to point *b*, the electric force on the charge does -1.9×10^{-8} J of work. What is the electric potential energy of the pair of charges when the second charge is at point *b*?

23.3 •• **Energy of the Nucleus.** How much work is needed to assemble an atomic nucleus containing three protons (such as Li) if we model it as an equilateral triangle of side 2.00×10^{-15} m with a proton at each vertex? Assume the protons started from very far away.

23.4 •• (a) How much work would it take to push two protons very slowly from a separation of 2.00×10^{-10} m (a typical atomic distance) to 3.00×10^{-15} m (a typical nuclear distance)? (b) If the protons are both released from rest at the closer distance in part (a), how fast are they moving when they reach their original separation?

23.5 •• A small metal sphere, carrying a net charge of $q_1 = -2.60\ \mu C$, is held in a stationary position by insulating supports. A second small metal sphere, with a net charge of $q_2 = -7.50\ \mu C$ and mass 1.50 g, is projected toward q_1. When the two spheres are 0.800 m apart, q_2, is moving toward q_1 with speed 22.0 m/s (**Fig. E23.5**). Assume that the two spheres can be treated as point charges. You can ignore the force of gravity. (a) What is the speed of q_2 when the spheres are 0.420 m apart? (b) How close does q_2 get to q_1?

Figure **E23.5**



23.6 •• BIO **Energy of DNA Base Pairing.** (See Exercise 21.21.) (a) Calculate the electric potential energy of the adenine–thymine bond, using the same combinations of molecules (O–H–N and N–H–N) as in Exercise 21.21. (b) Compare this energy with the potential energy of the proton–electron pair in the hydrogen atom.

23.7 •• Two protons, starting several meters apart, are aimed directly at each other with speeds of 1500 m/s, measured relative to the earth. Find the maximum electric force that these protons will exert on each other.

23.8 •• Three equal 1.40-μC point charges are placed at the corners of an equilateral triangle with sides 0.300 m long. What is the potential energy of the system? (Take as zero the potential energy of the three charges when they are infinitely far apart.)

23.9 •• Two protons are released from rest when they are 0.750 nm apart. (a) What is the maximum speed they will reach? When does this speed occur? (b) What is the maximum acceleration they will achieve? When does this acceleration occur?

23.10 •• Four electrons are located at the corners of a square 10.0 nm on a side, with an alpha particle at its midpoint. How much work is needed to move the alpha particle to the midpoint of one of the sides of the square?

23.11 •• Three point charges, which initially are infinitely far apart, are placed at the corners of an equilateral triangle with sides *d*. Two of the point charges are identical and have charge *q*. If zero net work is required to place the three charges at the corners of the triangle, what must the value of the third charge be?

Section 23.2 Electric Potential

23.12 • An object with charge $q = -6.00 \times 10^{-9}$ C is placed in a region of uniform electric field and is released from rest at point A. After the charge has moved to point B, 0.500 m to the right, it has kinetic energy 3.00×10^{-7} J. (a) If the electric potential at point A is +30.0 V, what is the electric potential at point B? (b) What are the magnitude and direction of the electric field?

23.13 • A small particle has charge $-5.70\ \mu$C and mass 2.70×10^{-4} kg. It moves from point A, where the electric potential is $V_A = +270$ V, to point B, where the electric potential is $V_B = +830$ V. The electric force is the only force acting on the particle. The particle has speed 5.90 m/s at point A. What is its speed at point B? Is it moving faster or slower at B than at A? Explain.

23.14 • A particle with charge +4.20 nC is in a uniform electric field $\vec{E}$ directed to the left. The charge is released from rest and moves to the left; after it has moved 6.00 cm, its kinetic energy is $+2.20 \times 10^{-6}$ J. What are (a) the work done by the electric force, (b) the potential of the starting point with respect to the end point, and (c) the magnitude of $\vec{E}$?

23.15 • A charge of 30.0 nC is placed in a uniform electric field that is directed vertically upward and has a magnitude of 3.60×10^4 V/m. What work is done by the electric force when the charge moves (a) 0.490 m to the right; (b) 0.700 m upward; (c) 2.80 m at an angle of 45.0° downward from the horizontal?

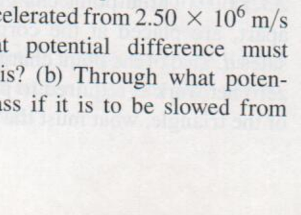
23.16 • Two stationary point charges +3.00 nC and +2.00 nC are separated by a distance of 50.0 cm. An electron is released from rest at a point midway between the two charges and moves along the line connecting the two charges. What is the speed of the electron when it is 10.0 cm from the +3.00-nC charge?

23.17 •• Point charges $q_1 = +2.00\ \mu$C and $q_2 = -2.00\ \mu$C are placed at adjacent corners of a square for which the length of each side is 5.00 cm. Point a is at the center of the square, and point b is at the empty corner closest to q_2. Take the electric potential to be zero at a distance far from both charges. (a) What is the electric potential at point a due to q_1 and q_2? (b) What is the electric potential at point b? (c) A point charge $q_3 = -6.00\ \mu$C moves from point a to point b. How much work is done on q_3 by the electric forces exerted by q_1 and q_2? Is this work positive or negative?

23.18 • Two point charges of equal magnitude Q are held a distance d apart. Consider only points on the line passing through both charges. (a) If the two charges have the same sign, find the location of all points (if there are any) at which (i) the potential (relative to infinity) is zero (is the electric field zero at these points?), and (ii) the electric field is zero (is the potential zero at these points?). (b) Repeat part (a) for two point charges having opposite signs.

23.19 • Two point charges $q_1 = +2.00$ nC and $q_2 = -6.10$ nC are 0.100 m apart. Point A is midway between them; point B is 0.080 m from q_1 and 0.060 m from q_2 (**Fig. E23.19**). Take the electric potential to be zero at infinity. Find (a) the potential at point A; (b) the potential at point B; (c) the work done by the electric field on a charge of 3.00 nC that travels from point B to point A.

Figure **E23.19**



23.20 •• (a) An electron is to be accelerated from 2.50×10^6 m/s to 8.50×10^6 m/s. Through what potential difference must the electron pass to accomplish this? (b) Through what potential difference must the electron pass if it is to be slowed from 8.50×10^6 m/s to a halt?

23.21 •• A positive charge q is fixed at the point $x = 0$, $y = 0$, and a negative charge $-2q$ is fixed at the point $x = a$, $y = 0$. (a) Show the positions of the charges in a diagram. (b) Derive an expression for the potential V at points on the x-axis as a function of the coordinate x. Take V to be zero at an infinite distance from the charges. (c) At which positions on the x-axis is $V = 0$? (d) Graph V at points on the x-axis as a function of x in the range from $x = -2a$ to $x = +2a$. (e) What does the answer to part (b) become when $x \gg a$? Explain why this result is obtained.

23.22 • At a certain distance from a point charge, the potential and electric-field magnitude due to that charge are 4.98 V and 16.2 V/m, respectively. (Take $V = 0$ at infinity.) (a) What is the distance to the point charge? (b) What is the magnitude of the charge? (c) Is the electric field directed toward or away from the point charge?

23.23 • A uniform electric field has magnitude E and is directed in the negative x-direction. The potential difference between point a (at $x = 0.60$ m) and point b (at $x = 0.85$ m) is 270 V. (a) Which point, a or b, is at the higher potential? (b) Calculate the value of E. (c) A negative point charge $q = -0.200\ \mu$C is moved from b to a. Calculate the work done on the point charge by the electric field.

23.24 • For each of the following arrangements of two point charges, find all the points along the line passing through both charges for which the electric potential V is zero (take $V = 0$ infinitely far from the charges) and for which the electric field E is zero: (a) charges $+Q$ and $+2Q$ separated by a distance d, and (b) charges $-Q$ and $+2Q$ separated by a distance d. (c) Are both V and E zero at the same places? Explain.

Section 23.3 Calculating Electric Potential

23.25 •• A thin spherical shell with radius $R_1 = 4.00$ cm is concentric with a larger thin spherical shell with radius $R_2 = 8.00$ cm. Both shells are made of insulating material. The smaller shell has charge $q_1 = +6.00$ nC distributed uniformly over its surface, and the larger shell has charge $q_2 = -9.00$ nC distributed uniformly over its surface. Take the electric potential to be zero at an infinite distance from both shells. (a) What is the electric potential due to the two shells at the following distance from their common center: (i) $r = 0$; (ii) $r = 5.00$ cm; (iii) $r = 9.00$ cm? (b) What is the magnitude of the potential difference between the surfaces of the two shells? Which shell is at higher potential: the inner shell or the outer shell?

23.26 • A total electric charge of 1.50 nC is distributed uniformly over the surface of a metal sphere with a radius of 24.0 cm. If the potential is zero at a point at infinity, find the value of the potential at the following distances from the center of the sphere: (a) 55.0 cm; (b) 24.0 cm; (c) 13.0 cm.

23.27 •• A uniformly charged, thin ring has radius 15.0 cm and total charge +21.5 nC. An electron is placed on the ring's axis a distance 32.0 cm from the center of the ring and is constrained to stay on the axis of the ring. The electron is then released from rest. (a) Describe the subsequent motion of the electron. (b) Find the speed of the electron when it reaches the center of the ring.

23.28 • A solid conducting sphere has net positive charge and radius $R = 0.400$ m. At a point 1.20 m from the center of the sphere, the electric potential due to the charge on the sphere is 24.0 V. Assume that $V = 0$ at an infinite distance from the sphere. What is the electric potential at the center of the sphere?

23.29 • Charge $Q = 8.00\ \mu C$ is distributed uniformly over the volume of an insulating sphere that has radius $R = 12.0$ cm. A small sphere with charge $q = +1.00\ \mu C$ and mass 6.00×10^{-5} kg is projected toward the center of the large sphere from an initial large distance. The large sphere is held at a fixed position and the small sphere can be treated as a point charge. What minimum speed must the small sphere have in order to come within 7.00 cm of the surface of the large sphere?

23.30 •• An infinitely long line of charge has linear charge density 5.50×10^{-12} C/m. A proton (mass 1.67×10^{-27} kg, charge $+1.60 \times 10^{-19}$ C) is 16.0 cm from the line and moving directly toward the line at 2.50×10^3 m/s. (a) Calculate the proton's initial kinetic energy. (b) How close does the proton get to the line of charge?

23.31 • A very long wire carries a uniform linear charge density λ. Using a voltmeter to measure potential difference, you find that when one probe of the meter is placed 3.00 cm from the wire and the other probe is 1.10 cm farther from the wire, the meter reads 600 V. (a) What is λ? (b) If you now place one probe at 4.00 cm from the wire and the other probe 1.10 cm farther away, will the voltmeter read 600 V? If not, will it read more or less than 600 V? Why? (c) If you place both probes 4.00 cm from the wire but 17.0 cm from each other, what will the voltmeter read?

23.32 •• A very long insulating cylinder of charge of radius 2.50 cm carries a uniform linear density of 18.0 nC/m. If you put one probe of a voltmeter at the surface, how far from the surface must the other probe be placed so that the voltmeter reads 190 V?

23.33 •• A very long insulating cylindrical shell of radius 6.70 cm carries charge of linear density 9.00 μC/m spread uniformly over its outer surface. What would a voltmeter read if it were connected between (a) the surface of the cylinder and a point 4.70 cm above the surface, and (b) the surface and a point 1.00 cm from the central axis of the cylinder?

23.34 • A ring of diameter 7.70 cm is fixed in place and carries a charge of $+5.00\ \mu C$ uniformly spread over its circumference. (a) How much work does it take to move a tiny $+3.40$-μC charged ball of mass 2.50 g from very far away to the center of the ring? (b) Is it necessary to take a path along the axis of the ring? Why? (c) If the ball is slightly displaced from the center of the ring, what will it do and what is the maximum speed it will reach?

23.35 •• A very small sphere with positive charge $q = +7.00\ \mu C$ is released from rest at a point 1.70 cm from a very long line of uniform linear charge density $\lambda = +1.00\ \mu C/m$. What is the kinetic energy of the sphere when it is 3.50 cm from the line of charge if the only force on it is the force exerted by the line of charge?

23.36 • CP Two large, parallel conducting plates carrying opposite charges of equal magnitude are separated by 2.20 cm. (a) If the surface charge density for each plate has magnitude 47.0 nC/m², what is the magnitude of $\vec{E}$ in the region between the plates? (b) What is the potential difference between the two plates? (c) If the separation between the plates is doubled while the surface charge density is kept constant at the value in part (a), what happens to the magnitude of the electric field and to the potential difference?

23.37 • Two large, parallel, metal plates carry opposite charges of equal magnitude. They are separated by 50.0 mm, and the potential difference between them is 365 V. (a) What is the magnitude of the electric field (assumed to be uniform) in the region between the plates? (b) What is the magnitude of the force this field exerts on a particle with charge +2.50 nC? (c) Use the results of part (b) to compute the work done by the field on the particle as it moves from the higher-potential plate to the lower. (d) Compare the result of part (c) to the change of potential energy of the same charge, computed from the electric potential.

23.38 • BIO **Electrical Sensitivity of Sharks.** Certain sharks can detect an electric field as weak as 1.0 μV/m. To grasp how weak this field is, if you wanted to produce it between two parallel metal plates by connecting an ordinary 1.5-V AA battery across these plates, how far apart would the plates have to be?

23.39 • The electric field at the surface of a charged, solid, copper sphere with radius 0.210 m is 3900 N/C, directed toward the center of the sphere. What is the potential at the center of the sphere, if we take the potential to be zero infinitely far from the sphere?

23.40 •• (a) How much excess charge must be placed on a copper sphere 25.0 cm in diameter so that the potential of its center, relative to infinity, is 3.75 kV? (b) What is the potential of the sphere's surface relative to infinity?

Section 23.4 Equipotential Surfaces and Section 23.5 Potential Gradient

23.41 •• CALC A metal sphere with radius r_a is supported on an insulating stand at the center of a hollow, metal, spherical shell with radius r_b. There is charge $+q$ on the inner sphere and charge $-q$ on the outer spherical shell. (a) Calculate the potential $V(r)$ for (i) $r < r_a$; (ii) $r_a < r < r_b$; (iii) $r > r_b$. (*Hint:* The net potential is the sum of the potentials due to the individual spheres.) Take V to be zero when r is infinite. (b) Show that the potential of the inner sphere with respect to the outer is

$$V_{ab} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

(c) Use Eq. (23.23) and the result from part (a) to show that the electric field at any point between the spheres has magnitude

$$E(r) = \frac{V_{ab}}{(1/r_a - 1/r_b)}\frac{1}{r^2}$$

(d) Use Eq. (23.23) and the result from part (a) to find the electric field at a point outside the larger sphere at a distance r from the center, where $r > r_b$. (e) Suppose the charge on the outer sphere is not $-q$ but a negative charge of different magnitude, say $-Q$. Show that the answers for parts (b) and (c) are the same as before but the answer for part (d) is different.

23.42 • A very large plastic sheet carries a uniform charge density of -6.00 nC/m² on one face. (a) As you move away from the sheet along a line perpendicular to it, does the potential increase or decrease? How do you know, without doing any calculations? Does your answer depend on where you choose the reference point for potential? (b) Find the spacing between equipotential surfaces that differ from each other by 1.00 V. What type of surfaces are these?

23.43 • CALC In a certain region of space, the electric potential is $V(x, y, z) = Axy - Bx^2 + Cy$, where A, B, and C are positive constants. (a) Calculate the x-, y-, and z-components of the electric field. (b) At which points is the electric field equal to zero?

23.44 • CALC In a certain region of space the electric potential is given by $V = +Ax^2y - Bxy^2$, where $A = 5.00$ V/m³ and $B = 8.00$ V/m³. Calculate the magnitude and direction of the electric field at the point in the region that has coordinates $x = 2.00$ m, $y = 0.400$ m, and $z = 0$.

23.45 • A metal sphere with radius $r_a = 1.20$ cm is supported on an insulating stand at the center of a hollow, metal, spherical shell with radius $r_b = 9.60$ cm. Charge $+q$ is put on the inner sphere and charge $-q$ on the outer spherical shell. The magnitude of q is chosen to make the potential difference between the spheres 500 V, with the inner sphere at higher potential. (a) Use the result of Exercise 23.41(b) to calculate q. (b) With the help of the result of Exercise 23.41(a), sketch the equipotential surfaces that correspond to 500, 400, 300, 200, 100, and 0 V. (c) In your sketch, show the electric field lines. Are the electric field lines and equipotential surfaces mutually perpendicular? Are the equipotential surfaces closer together when the magnitude of $\vec{E}$ is largest?

PROBLEMS

23.46 • **CP** A point charge $q_1 = +15.00\ \mu C$ is held fixed in space. From a horizontal distance of 4.00 cm, a small sphere with mass 4.00×10^{-3} kg and charge $q_2 = +2.00\ \mu C$ is fired toward the fixed charge with an initial speed of 44.0 m/s. Gravity can be neglected. What is the acceleration of the sphere at the instant when its speed is 20.0 m/s?

23.47 ••• A point charge $q_1 = 4.05$ nC is placed at the origin, and a second point charge $q_2 = -2.95$ nC is placed on the x-axis at $x = +21.0$ cm. A third point charge $q_3 = 2.05$ nC is to be placed on the x-axis between q_1 and q_2. (Take as zero the potential energy of the three charges when they are infinitely far apart.) (a) What is the potential energy of the system of the three charges if q_3 is placed at $x = +11.0$ cm? (b) Where should q_3 be placed to make the potential energy of the system equal to zero?

23.48 •• A positive point charge $q_1 = +5.00 \times 10^{-4}$ C is held at a fixed position. A small object with mass 4.00×10^{-3} kg and charge $q_2 = -3.00 \times 10^{-4}$ C is projected directly at q_1. Ignore gravity. When q_2 is 0.400 m away, its speed is 800 m/s. What is its speed when it is 0.200 m from q_1?

23.49 •• A gold nucleus has a radius of 7.3×10^{-15} m and a charge of $+79e$. Through what voltage must an alpha particle, with charge $+2e$, be accelerated so that it has just enough energy to reach a distance of 2.0×10^{-14} m from the surface of a gold nucleus? (Assume that the gold nucleus remains stationary and can be treated as a point charge.)

23.50 ••• A small sphere with mass 5.00×10^{-7} kg and charge $+3.00\ \mu C$ is released from rest a distance of 0.500 m above a large horizontal insulating sheet of charge that has uniform surface charge density $\sigma = +8.00\ pC/m^2$. Using energy methods, calculate the speed of the sphere when it is 0.200 m above the sheet.

23.51 •• **Determining the Size of the Nucleus.** When radium-226 decays radioactively, it emits an alpha particle (the nucleus of helium), and the end product is radon-222. We can model this decay by thinking of the radium-226 as consisting of an alpha particle emitted from the surface of the spherically symmetric radon-222 nucleus, and we can treat the alpha particle as a point charge. The energy of the alpha particle has been measured in the laboratory and has been found to be 4.79 MeV when the alpha particle is essentially infinitely far from the nucleus. Since radon is much heavier than the alpha particle, we can assume that there is no appreciable recoil of the radon after the decay. The radon nucleus contains 86 protons, while the alpha particle has 2 protons and the radium nucleus has 88 protons. (a) What was the electric potential energy of the alpha–radon combination just before the decay, in MeV and in joules? (b) Use your result from part (a) to calculate the radius of the radon nucleus.

23.52 •• **CP** A proton and an alpha particle are released from rest when they are 0.225 nm apart. The alpha particle (a helium nucleus) has essentially four times the mass and two times the charge of a proton. Find the maximum *speed* and maximum *acceleration* of each of these particles. When do these maxima occur: just following the release of the particles or after a very long time?

23.53 • A particle with charge +7.80 nC is in a uniform electric field directed to the left. Another force, in addition to the electric force, acts on the particle so that when it is released from rest, it moves to the right. After it has moved 7.50 cm, the additional force has done 7.10×10^{-5} J of work and the particle has 3.55×10^{-5} J of kinetic energy. (a) What work was done by the electric force? (b) What is the potential of the starting point with respect to the end point? (c) What is the magnitude of the electric field?

23.54 •• Identical charges $q = +5.00\ \mu C$ are placed at opposite corners of a square that has sides of length 8.00 cm. Point A is at one of the empty corners, and point B is at the center of the square. A charge $q_0 = -3.00\ \mu C$ is placed at point A and moves along the diagonal of the square to point B. (a) What is the magnitude of the net electric force on q_0 when it is at point A? Sketch the placement of the charges and the direction of the net force. (b) What is the magnitude of the net electric force on q_0 when it is at point B? (c) How much work does the electric force do on q_0 during its motion from A to B? Is this work positive or negative? When it goes from A to B, does q_0 move to higher potential or to lower potential?

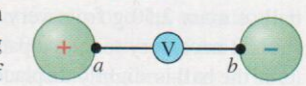
23.55 •• **CALC** A vacuum tube diode consists of concentric cylindrical electrodes, the negative cathode and the positive anode. Because of the accumulation of charge near the cathode, the electric potential between the electrodes is given by

$$V(x) = Cx^{4/3}$$

where x is the distance from the cathode and C is a constant, characteristic of a particular diode and operating conditions. Assume that the distance between the cathode and anode is 11.0 mm and the potential difference between electrodes is 260 V. (a) Determine the value of C. (b) Obtain a formula for the electric field between the electrodes as a function of x. (c) Determine the force on an electron when the electron is halfway between the electrodes.

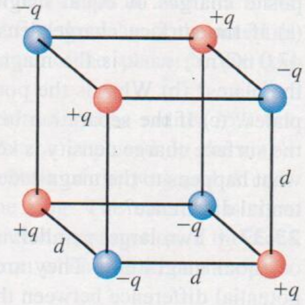
23.56 •• Two oppositely charged, identical insulating spheres, each 53.0 cm in diameter and carrying a uniformly distributed charge of magnitude 190 μC, are placed 1.20 m apart center to center (**Fig. P23.56**). (a) If a voltmeter is connected between the nearest points (a and b) on their surfaces, what will it read? (b) Which point, a or b, is at the higher potential? How can you know this without any calculations?

Figure **P23.56**



23.57 •• **An Ionic Crystal.** **Figure P23.57** shows eight point charges arranged at the corners of a cube with sides of length d. The values of the charges are $+q$ and $-q$, as shown. This is a model of one cell of a cubic ionic crystal. In sodium chloride (NaCl), for instance, the positive ions are Na^+ and the negative ions are Cl^-. (a) Calculate the potential energy U of this arrangement. (Take as zero the potential energy of the eight charges when they are infinitely far apart.) (b) In part (a), you should have found that $U < 0$. Explain the relationship between this result and the observation that such ionic crystals exist in nature.

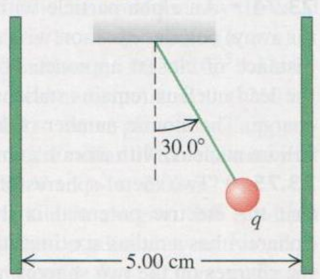
Figure **P23.57**



23.58 • (a) Calculate the potential energy of a system of two small spheres, one carrying a charge of 2.50 μC and the other a charge of -4.50 μC, with their centers separated by a distance of 0.200 m. Assume that $U = 0$ when the charges are infinitely separated. (b) Suppose that one sphere is held in place; the other sphere, with mass 2.40 g, is shot away from it. What minimum initial speed would the moving sphere need to escape completely from the attraction of the fixed sphere? (To escape, the moving sphere would have to reach a velocity of zero when it is infinitely far from the fixed sphere.)

23.59 •• **CP** A small sphere with mass 2.50 g hangs by a thread between two very large parallel vertical plates 5.00 cm apart (**Fig. P23.59**). The plates are insulating and have uniform surface charge densities $+\sigma$ and $-\sigma$. The charge on the sphere is $q = 7.10 \times 10^{-6}$ C. What potential difference between the plates will cause the thread to assume an angle of 30.0° with the vertical?

Figure P23.59



23.60 •• Two spherical shells have a common center. The inner shell has radius $R_1 = 5.00$ cm and charge $q_1 = +3.00 \times 10^{-6}$ C; the outer shell has radius $R_2 = 15.0$ cm and charge $q_2 = -5.00 \times 10^{-6}$ C. Both charges are spread uniformly over the shell surface. What is the electric potential due to the two shells at the following distances from their common center: (a) $r = 2.50$ cm; (b) $r = 10.0$ cm; (c) $r = 20.0$ cm? Take $V = 0$ at a large distance from the shells.

23.61 • **CALC Coaxial Cylinders.** A long metal cylinder with radius a is supported on an insulating stand on the axis of a long, hollow, metal tube with radius b. The positive charge per unit length on the inner cylinder is λ, and there is an equal negative charge per unit length on the outer cylinder. (a) Calculate the potential $V(r)$ for (i) $r < a$; (ii) $a < r < b$; (iii) $r > b$. (*Hint:* The net potential is the sum of the potentials due to the individual conductors.) Take $V = 0$ at $r = b$. (b) Show that the potential of the inner cylinder with respect to the outer is

$$V_{ab} = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{b}{a}$$

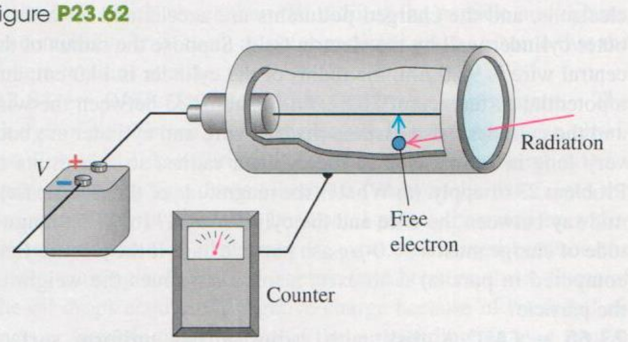
(c) Use Eq. (23.23) and the result from part (a) to show that the electric field at any point between the cylinders has magnitude

$$E(r) = \frac{V_{ab}}{\ln(b/a)} \frac{1}{r}$$

(d) What is the potential difference between the two cylinders if the outer cylinder has no net charge?

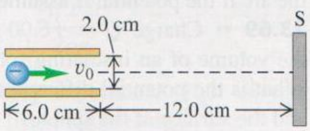
23.62 •• A *Geiger counter* detects radiation such as alpha particles by using the fact that the radiation ionizes the air along its path. A thin wire lies on the axis of a hollow metal cylinder and is insulated from it (**Fig. P23.62**). A large potential difference is established between the wire and the outer cylinder, with the wire at higher potential; this sets up a strong electric field directed radially outward. When ionizing radiation enters the device, it ionizes a few air molecules. The free electrons produced are accelerated by the electric field toward the wire and, on the way there, ionize many more air molecules. Thus a current pulse is produced that can be detected by appropriate electronic circuitry and converted to an audible "click." Suppose the radius of the central wire is 145 μm and the radius of the hollow cylinder is 1.80 cm. What potential difference between the wire and the cylinder produces an electric field of 2.00×10^4 V/m at a distance of 1.20 cm from the axis of the wire? (The wire and cylinder are both very long in comparison to their radii, so the results of Problem 23.61 apply.)

Figure P23.62



23.63 • **CP Deflection in a CRT.** Cathode-ray tubes (CRTs) were often found in oscilloscopes and computer monitors. In **Fig. P23.63** an electron with an initial speed of 6.40×10^6 m/s is projected along the axis midway between the deflection plates of a cathode-ray tube. The potential difference between the two plates is 26.0 V and the lower plate is the one at higher potential. (a) What is the force (magnitude and direction) on the electron when it is between the plates? (b) What is the acceleration of the electron (magnitude and direction) when acted on by the force in part (a)? (c) How far below the axis has the electron moved when it reaches the end of the plates? (d) At what angle with the axis is it moving as it leaves the plates? (e) How far below the axis will it strike the fluorescent screen S?

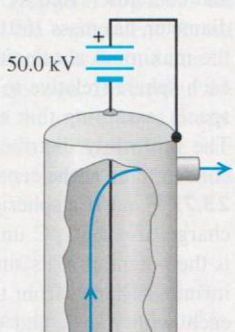
Figure P23.63



23.64 •• **CP Deflecting Plates of an Oscilloscope.** The vertical deflecting plates of a typical classroom oscilloscope are a pair of parallel square metal plates carrying equal but opposite charges. Typical dimensions are about 3.3 cm on a side, with a separation of about 5.3 mm. The potential difference between the plates is 25.0 V. The plates are close enough that we can ignore fringing at the ends. Under these conditions: (a) how much charge is on each plate, and (b) how strong is the electric field between the plates? (c) If an electron is ejected at rest from the negative plate, how fast is it moving when it reaches the positive plate?

23.65 •• *Electrostatic precipitators* use electric forces to remove pollutant particles from smoke, in particular in the smokestacks of coal-burning power plants. One form of precipitator consists of a vertical, hollow, metal cylinder with a thin wire, insulated from the cylinder, running along its axis (**Fig. P23.65**). A large potential difference is established between the wire and the outer cylinder, with the wire at lower potential. This sets up a strong radial electric field directed inward. The field produces a region of ionized air near the wire. Smoke enters the precipitator at the bottom, ash and dust in it pick up

Figure P23.65



electrons, and the charged pollutants are accelerated toward the outer cylinder wall by the electric field. Suppose the radius of the central wire is 90.0 μm, the radius of the cylinder is 14.0 cm, and a potential difference of 50.0 kV is established between the wire and the cylinder. Also assume that the wire and cylinder are both very long in comparison to the cylinder radius, so the results of Problem 23.61 apply. (a) What is the magnitude of the electric field midway between the wire and the cylinder wall? (b) What magnitude of charge must a 30.0-μg ash particle have if the electric field computed in part (a) is to exert a force ten times the weight of the particle?

23.66 •• **CALC** A disk with radius R has uniform surface charge density σ. (a) By regarding the disk as a series of thin concentric rings, calculate the electric potential V at a point on the disk's axis a distance x from the center of the disk. Assume that the potential is zero at infinity. (*Hint:* Use the result of Example 23.11 in Section 23.3.) (b) Calculate $-\partial V/\partial x$. Show that the result agrees with the expression for E_x calculated in Example 21.11 (Section 21.5).

23.67 ••• **CALC** **Self-Energy of a Sphere of Charge.** A solid sphere of radius R contains a total charge Q distributed uniformly throughout its volume. Find the energy needed to assemble this charge by bringing infinitesimal charges from far away. This energy is called the "self-energy" of the charge distribution. (*Hint:* After you have assembled a charge q in a sphere of radius r, how much energy would it take to add a spherical shell of thickness dr having charge dq? Then integrate to get the total energy.)

23.68 • **CALC** A thin insulating rod is bent into a semicircular arc of radius a, and a total electric charge Q is distributed uniformly along the rod. Calculate the potential at the center of curvature of the arc if the potential is assumed to be zero at infinity.

23.69 •• Charge $Q = +6.00\ \mu\text{C}$ is distributed uniformly over the volume of an insulating sphere that has radius $R = 3.00$ cm. What is the potential difference between the center of the sphere and the surface of the sphere?

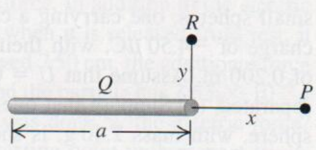
23.70 • An insulating spherical shell with inner radius 25.0 cm and outer radius 60.0 cm carries a charge of $+150.0\ \mu\text{C}$ uniformly distributed over its outer surface. Point a is at the center of the shell, point b is on the inner surface, and point c is on the outer surface. (a) What will a voltmeter read if it is connected between the following points: (i) a and b; (ii) b and c; (iii) c and infinity; (iv) a and c? (b) Which is at higher potential: (i) a or b; (ii) b or c; (iii) a or c? (c) Which, if any, of the answers would change sign if the charge were $-150\ \mu\text{C}$?

23.71 •• **CP** Two plastic spheres, each carrying charge uniformly distributed throughout its interior, are initially placed in contact and then released. One sphere is 65.0 cm in diameter, has mass 50.0 g, and contains $-10.0\ \mu\text{C}$ of charge. The other sphere is 42.0 cm in diameter, has mass 160.0 g, and contains $-25.0\ \mu\text{C}$ of charge. Find the maximum acceleration and the maximum speed achieved by each sphere (relative to the fixed point of their initial location in space), assuming that no other forces are acting on them. (*Hint:* The uniformly distributed charges behave as though they were concentrated at the centers of the two spheres.)

23.72 • (a) If a spherical raindrop of radius 0.550 mm carries a charge of -3.80 pC uniformly distributed over its volume, what is the potential at its surface? (Take the potential to be zero at an infinite distance from the raindrop.) (b) Two identical raindrops, each with radius and charge specified in part (a), collide and merge into one larger raindrop. What is the radius of this larger drop, and what is the potential at its surface, if its charge is uniformly distributed over its volume?

23.73 • **CALC** Electric charge is distributed uniformly along a thin rod of length a, with total charge Q. Take the potential to be zero at infinity. Find the potential at the following points (**Fig. P23.73**): (a) point P, a distance x to the right of the rod, and (b) point R, a distance y above the right-hand end of the rod. (c) In parts (a) and (b), what does your result reduce to as x or y becomes much larger than a?

Figure **P23.73**



23.74 • An alpha particle with kinetic energy 9.50 MeV (when far away) collides head-on with a lead nucleus at rest. What is the distance of closest approach of the two particles? (Assume that the lead nucleus remains stationary and may be treated as a point charge. The atomic number of lead is 82. The alpha particle is a helium nucleus, with atomic number 2.)

23.75 •• Two metal spheres of different sizes are charged such that the electric potential is the same at the surface of each. Sphere A has a radius six times that of sphere B. Let Q_A and Q_B be the charges on the two spheres, and let E_A and E_B be the electric-field magnitudes at the surfaces of the two spheres. What are (a) the ratio Q_B/Q_A and (b) the ratio E_B/E_A?

23.76 • A metal sphere with radius R_1 has a charge Q_1. Take the electric potential to be zero at an infinite distance from the sphere. (a) What are the electric field and electric potential at the surface of the sphere? This sphere is now connected by a long, thin conducting wire to another sphere of radius R_2 that is several meters from the first sphere. Before the connection is made, this second sphere is uncharged. After electrostatic equilibrium has been reached, what are (b) the total charge on each sphere; (c) the electric potential at the surface of each sphere; (d) the electric field at the surface of each sphere? Assume that the amount of charge on the wire is much less than the charge on each sphere.

23.77 •• **CP** **Nuclear Fusion in the Sun.** The source of the sun's energy is a sequence of nuclear reactions that occur in its core. The first of these reactions involves the collision of two protons, which fuse together to form a heavier nucleus and release energy. For this process, called *nuclear fusion,* to occur, the two protons must first approach until their surfaces are essentially in contact. (a) Assume both protons are moving with the same speed and they collide head-on. If the radius of the proton is 1.2×10^{-15} m, what is the minimum speed that will allow fusion to occur? The charge distribution within a proton is spherically symmetric, so the electric field and potential outside a proton are the same as if it were a point charge. The mass of the proton is 1.67×10^{-27} kg. (b) Another nuclear fusion reaction that occurs in the sun's core involves a collision between two helium nuclei, each of which has 2.99 times the mass of the proton, charge $+2e$, and radius 1.7×10^{-15} m. Assuming the same collision geometry as in part (a), what minimum speed is required for this fusion reaction to take place if the nuclei must approach a center-to-center distance of about 3.5×10^{-15} m? As for the proton, the charge of the helium nucleus is uniformly distributed throughout its volume. (c) In Section 18.3 it was shown that the average translational kinetic energy of a particle with mass m in a gas at absolute temperature T is $\frac{3}{2}kT$, where k is the Boltzmann constant (given in Appendix F). For two protons with kinetic energy equal to this average value to be able to undergo the process described in part (a), what absolute temperature is required? What absolute temperature is required for two average helium nuclei to be able to undergo the process described in part (b)? (At these temperatures, atoms are completely ionized, so nuclei and electrons move separately.) (d) The temperature in the sun's core is about 1.5×10^7 K. How does this compare to the temperatures calculated in part (c)? How can the reactions described in parts (a) and (b) occur at all in the interior of the sun? (*Hint:* See the discussion of the distribution of molecular speeds in Section 18.5.)

23.78 • **CALC** The electric potential V in a region of space is given by

$$V(x, y, z) = A(x^2 - 3y^2 + z^2)$$

where A is a constant. (a) Derive an expression for the electric field $\vec{E}$ at any point in this region. (b) The work done by the field when a 1.50-μC test charge moves from the point $(x, y, z) = (0, 0, 0.250\text{ m})$ to the origin is measured to be 6.00×10^{-5} J. Determine A. (c) Determine the electric field at the point (0, 0, 0.250 m). (d) Show that in every plane parallel to the xz-plane the equipotential contours are circles. (e) What is the radius of the equipotential contour corresponding to $V = 1280$ V and $y = 2.00$ m?

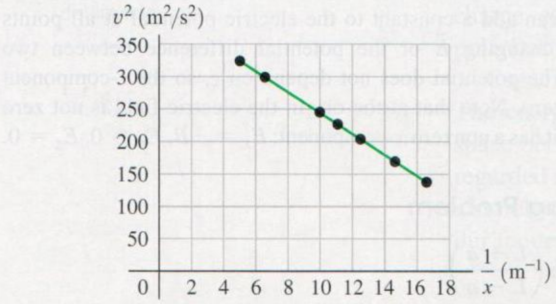
23.79 •• **DATA** The electric potential in a region that is within 2.00 m of the origin of a rectangular coordinate system is given by $V = Ax^l + By^m + Cz^n + D$, where A, B, C, D, l, m, and n are constants. The units of A, B, C, and D are such that if x, y, and z are in meters, then V is in volts. You measure V and each component of the electric field at four points and obtain these results:

Point	(x, y, z) (m)	V (V)	E_x (V/m)	E_y (V/m)	E_z (V/m)
1	(0, 0, 0)	10.0	0	0	0
2	(1.00, 0, 0)	4.0	12.0	0	0
3	(0, 1.00, 0)	6.0	0	12.0	0
4	(0, 0, 1.00)	8.0	0	0	12.0

(a) Use the data in the table to calculate A, B, C, D, l, m, and n. (b) What are V and the magnitude of E at the points (0, 0, 0), (0.50 m, 0.50 m, 0.50 m), and (1.00 m, 1.00 m, 1.00 m)?

23.80 •• **DATA** A small, stationary sphere carries a net charge Q. You perform the following experiment to measure Q: From a large distance you fire a small particle with mass $m = 4.00 \times 10^{-4}$ kg and charge $q = 5.00 \times 10^{-8}$ C directly at the center of the sphere. The apparatus you are using measures the particle's speed v as a function of the distance x from the sphere. The sphere's mass is much greater than the mass of the projectile particle, so you assume that the sphere remains at rest. All of the measured values of x are much larger than the radius of either object, so you treat both objects as point particles. You plot your data on a graph of v^2 versus $(1/x)$ (**Fig. P23.80**). The straight line $v^2 = 400\text{ m}^2/\text{s}^2 - [(15.75\text{ m}^3/\text{s}^2)/x]$ gives a good fit to the data points. (a) Explain why the graph is a straight line. (b) What is the initial speed v_0 of the particle when it is very far from the sphere? (c) What is Q? (d) How close does the particle get to the sphere? Assume that this distance is much larger than the radii of the particle and sphere, so continue to treat them as point particles and to assume that the sphere remains at rest.

Figure **P23.80**



23.81 ••• **DATA** **The Millikan Oil-Drop Experiment.** The charge of an electron was first measured by the American physicist Robert Millikan during 1909–1913. In his experiment, oil was sprayed in very fine drops (about 10^{-4} mm in diameter) into the space between two parallel horizontal plates separated by a distance d. A potential difference V_{AB} was maintained between the plates, causing a downward electric field between them. Some of the oil drops acquired a negative charge because of frictional effects or because of ionization of the surrounding air by x rays or radioactivity. The drops were observed through a microscope. (a) Show that an oil drop of radius r at rest between the plates remained at rest if the magnitude of its charge was

$$q = \frac{4\pi}{3}\frac{\rho r^3 g d}{V_{AB}}$$

where ρ is oil's density. (Ignore the buoyant force of the air.) By adjusting V_{AB} to keep a given drop at rest, Millikan determined the charge on that drop, provided its radius r was known. (b) Millikan's oil drops were much too small to measure their radii directly. Instead, Millikan determined r by cutting off the electric field and measuring the *terminal speed* v_t of the drop as it fell. (We discussed terminal speed in Section 5.3.) The viscous force F on a sphere of radius r moving at speed v through a fluid with viscosity η is given by Stokes's law: $F = 6\pi\eta r v$. When a drop fell at v_t, the viscous force just balanced the drop's weight $w = mg$. Show that the magnitude of the charge on the drop was

$$q = 18\pi\frac{d}{V_{AB}}\sqrt{\frac{\eta^3 v_t^3}{2\rho g}}$$

(c) You repeat the Millikan oil-drop experiment. Four of your measured values of V_{AB} and v_t are listed in the table:

Drop	1	2	3	4
V_{AB} (V)	9.16	4.57	12.32	6.28
v_t (10^{-5} m/s)	2.54	0.767	4.39	1.52

In your apparatus, the separation d between the horizontal plates is 1.00 mm. The density of the oil you use is 824 kg/m³. For the viscosity η of air, use the value $1.81 \times 10^{-5}\text{ N}\cdot\text{s/m}^2$. Assume that $g = 9.80\text{ m/s}^2$. Calculate the charge q of each drop. (d) If electric charge is *quantized* (that is, exists in multiples of the magnitude of the charge of an electron), then the charge on each drop is $-ne$, where n is the number of excess electrons on each drop. (All four drops in your table have negative charge.) Drop 2 has the smallest magnitude of charge observed in the experiment, for all 300 drops on which measurements were made, so assume that its charge is due to an excess charge of one electron. Determine the number of excess electrons n for each of the other three drops. (e) Use $q = -ne$ to calculate e from the data for each of the four drops, and average these four values to get your best experimental value of e.

CHALLENGE PROBLEMS

23.82 ••• **CALC** A hollow, thin-walled insulating cylinder of radius R and length L (like the cardboard tube in a roll of toilet paper) has charge Q uniformly distributed over its surface. (a) Calculate the electric potential at all points along the axis of the tube. Take the origin to be at the center of the tube, and take the potential to be zero at infinity. (b) Show that if $L \ll R$, the result of part (a) reduces to the potential on the axis of a ring of charge of radius R. (See Example 23.11 in Section 23.3.) (c) Use the result of part (a) to find the electric field at all points along the axis of the tube.

23.83 ••• **CP** In experiments in which atomic nuclei collide, head-on collisions like that described in Problem 23.74 do happen, but "near misses" are more common. Suppose the alpha particle in that problem is not "aimed" at the center of the lead nucleus but has an initial nonzero angular momentum (with respect to the stationary lead nucleus) of magnitude $L = p_0 b$, where p_0 is the magnitude of the particle's initial momentum and $b = 1.00 \times 10^{-12}$ m. What is the distance of closest approach? Repeat for $b = 1.00 \times 10^{-13}$ m and $b = 1.00 \times 10^{-14}$ m.

PASSAGE PROBLEMS

MATERIALS ANALYSIS WITH IONS. *Rutherford backscattering spectrometry* (RBS) is a technique used to determine the structure and composition of materials. A beam of ions (typically helium ions) is accelerated to high energy and aimed at a sample. By analyzing the distribution and energy of the ions that are scattered from (that is, deflected by collisions with) the atoms in the sample, researchers can determine the sample's composition. To accelerate the ions to high energies, a *tandem electrostatic accelerator* may be used. In this device, negative ions (He^-) start at a potential $V = 0$ and are accelerated by a high positive voltage at the midpoint of the accelerator. The high voltage produces a constant electric field in the acceleration tube through which the ions move. When accelerated ions reach the midpoint, the electrons are stripped off, turning the negative ions into doubly positively charged ions (He^{++}). These positive ions are then repelled from the midpoint by the high positive voltage there and continue to accelerate to the far end of the accelerator, where again $V = 0$.

23.84 For a particular experiment, helium ions are to be given a kinetic energy of 3.0 MeV. What should the voltage at the center of the accelerator be, assuming that the ions start essentially at rest? (a) −3.0 MV; (b) +3.0 MV; (c) +1.5 MV; (d) +1.0 MV.

23.85 A helium ion (He^{++}) that comes within about 10 fm of the center of the nucleus of an atom in the sample may induce a nuclear reaction instead of simply scattering. Imagine a helium ion with a kinetic energy of 3.0 MeV heading straight toward an atom at rest in the sample. Assume that the atom stays fixed. What minimum charge can the nucleus of the atom have such that the helium ion gets no closer than 10 fm from the center of the atomic nucleus? (1 fm $= 1 \times 10^{-15}$ m, and e is the magnitude of the charge of an electron or a proton.) (a) $2e$; (b) $11e$; (c) $20e$; (d) $22e$.

23.86 The maximum voltage at the center of a typical tandem electrostatic accelerator is 6.0 MV. If the distance from one end of the acceleration tube to the midpoint is 12 m, what is the magnitude of the average electric field in the tube under these conditions? (a) 41,000 V/m; (b) 250,000 V/m; (c) 500,000 V/m; (d) 6,000,000 V/m.

Answers

Chapter Opening Question ?

(iii) A large, constant potential difference V_{ab} is maintained between the welding tool (a) and the metal pieces to be welded (b). For a given potential difference between two conductors a and b, the smaller the distance d separating the conductors, the greater is the magnitude E of the field between them. Hence d must be small in order for E to be large enough to ionize the gas between the conductors (see Section 23.3) and produce an arc through this gas.

Test Your Understanding Questions

23.1 (a) (i), (b) (ii) The three charges q_1, q_2, and q_3 are all positive, so all three of the terms in the sum in Eq. (23.11)—q_1q_2/r_{12}, q_1q_3/r_{13}, and q_2q_3/r_{23}—are positive. Hence the total electric potential energy U is positive. This means that it would take positive work to bring the three charges from infinity to the positions shown in Fig. 21.14, and hence *negative* work to move the three charges from these positions back to infinity.

23.2 no If $V = 0$ at a certain point, $\vec{E}$ does *not* have to be zero at that point. An example is point c in Figs. 21.23 and 23.13, for which there is an electric field in the $+x$-direction (see Example 21.9 in Section 21.5) even though $V = 0$ (see Example 23.4). This isn't a surprising result because V and $\vec{E}$ are quite different quantities: V is the net amount of work required to bring a unit charge from infinity to the point in question, whereas $\vec{E}$ is the electric force that acts on a unit charge when it arrives at that point.

23.3 no If $\vec{E} = \mathbf{0}$ at a certain point, V does *not* have to be zero at that point. An example is point O at the center of the charged ring in Figs. 21.23 and Fig. 23.21. From Example 21.9 (Section 21.5), the electric field is zero at O because the electric-field contributions from different parts of the ring completely cancel. From Example 23.11, however, the potential at O is *not* zero: This point corresponds to $x = 0$, so $V = (1/4\pi\epsilon_0)(Q/a)$. This value of V corresponds to the work that would have to be done to move a unit positive test charge along a path from infinity to point O; it is nonzero because the charged ring repels the test charge, so positive work must be done to move the test charge toward the ring.

23.4 no If the positive charges in Fig. 23.23 were replaced by negative charges, and vice versa, the equipotential surfaces would be the same but the sign of the potential would be reversed. For example, the surfaces in Fig. 23.23b with potential $V = +30$ V and $V = -50$ V would have potential $V = -30$ V and $V = +50$ V, respectively.

23.5 (iii) From Eqs. (23.19), the components of the electric field are $E_x = -\partial V/\partial x = -(B + Dy)$, $E_y = -\partial V/\partial y = -(3Cy^2 + Dx)$, and $E_z = -\partial V/\partial z = 0$. The value of A has no effect, which means that we can add a constant to the electric potential at all points without changing $\vec{E}$ or the potential difference between two points. The potential does not depend on z, so the z-component of $\vec{E}$ is zero. Note that at the origin the electric field is not zero because it has a nonzero x-component: $E_x = -B$, $E_y = 0$, $E_z = 0$.

Bridging Problem

$$\frac{qQ}{8\pi\epsilon_0 a}\ln\left(\frac{L+a}{L-a}\right)$$

The University of Jordan
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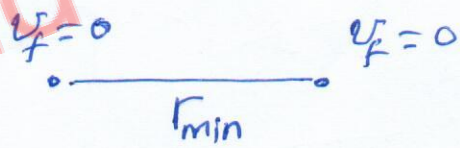
General Physics (102) / Sample Solutions
Chapter 23 / Electric Potential / Sears and Zemansky

Prof. Mahmoud Jaghoub (أ.د. محمود الجاغوب)

Q7] Initially two protons are far apart and all the energy is kinetic energy T_0

$$T_0 = \frac{1}{2} m v_0^2 \times 2 = m v_0^2$$

Finally, all kinetic energy has changed to electric potential energy when separation is r_{min}.



$$\Delta U = T_0 = m v_0^2$$

$$\therefore \frac{k q^2}{r_{min}} = m v_0^2 \Rightarrow$$

$$r_{min} = \frac{k q^2}{m v_0^2}$$

maximum Force is exerted when two protons are closest at r_{min}.

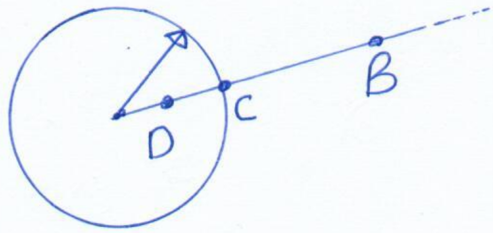
$$F_{max} = \frac{k q^2}{r_{min}^2} = \frac{k q^2}{\left(\frac{k q^2}{m v_0^2}\right)^2} = (m v_0^2)^2 \times \frac{1}{k q^2}$$

$$F_{max} = \frac{1}{k}\left(\frac{m v_0}{q}\right)^2 \simeq 6.12 \times 10^{-14}\,N$$

To get the answer in the book $v_0 = 1500\,km/s$ NOT $1500\,m/s$ as given in the question

Q3] $V_\infty = 0$

Charge is distributed uniformly over metal sphere.
$\Rightarrow$ outside sphere it behaves as if it were concentrated at the center of the sphere.



(i) Find V_B where $r_B > R$.

$$r > R \;, \quad E = \frac{kQ}{r^2}$$

$$V_B - V_\infty = -\int_\infty^{r_B} \frac{kQ}{r'^2}\,dr' = -\left[-\frac{kQ}{r'}\right]_\infty^{r_B}$$

$$\therefore \; V_B - 0 = \frac{kQ}{r_B} \Rightarrow V_B = \frac{kQ}{r_B}$$ for any point outside the sphere.

(ii) At point C where $r_C = R$ i.e on the surface of the sphere.

$$V_C - V_\infty = -\int_\infty^{r_C} \frac{kQ}{r'^2}\,dr' = -\left[-\frac{kQ}{r'}\right]_\infty^{r_C = R} = \frac{kQ}{R}$$

(iii) At point D where $r_D < R$; $V = \frac{kQ}{R^3} r$

$$V_D - V_C = -\int_{r_C = R}^{r_D} \frac{kQ}{R^3} r'\,dr' = -\frac{kQ}{R^3}\left[\frac{r'^2}{2}\right]_R^{r_D} = -\frac{1}{2}\frac{kQ}{R^3}\left[r_D^2 - R^2\right]$$

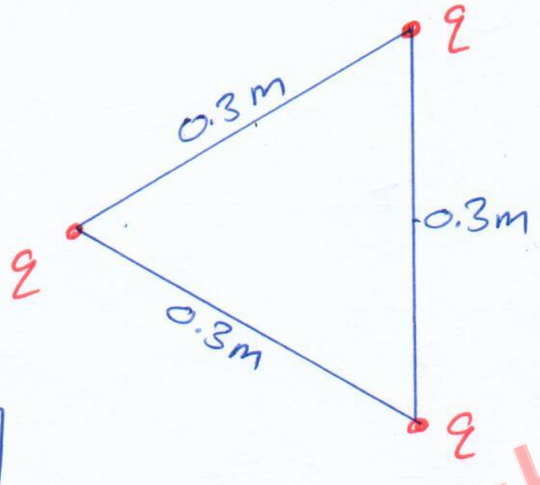
$$\therefore V_D = \frac{kQ}{R} - \frac{1}{2}\frac{kQ}{R^3}\left[r_D^2 - R^2\right] = \frac{kQ}{R} + \frac{1}{2}\frac{kQ}{R} - \frac{1}{2}\frac{kQ}{R^3} r_D^2$$

$$V_D = \frac{kQ}{2R}\left[3 - \frac{r_D^2}{R^2}\right]$$ for $r_D < r$ i.e inside the metal sphere.

Q8]

In General

$$U = k\sum_{i<j} \frac{q_i q_j}{r_{ij}}$$



j	i
2	1
3	1,2

$$\Rightarrow U = k\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

but $r_{12} = r_{13} = r_{23} = r = 0.3\ m$

$q_1 = q_2 = q_3 = 1.4\,\mu C = 1.4\times10^{-6}$

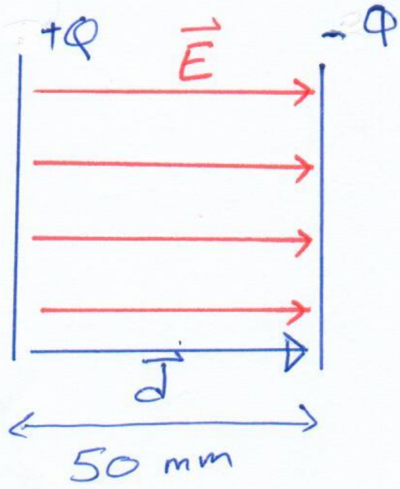
$$\therefore U = 3k\frac{q^2}{r} = 3\times 9\times10^{9}\,\frac{(1.4\times10^{-6})^2}{0.3}$$

$$U = 176.4\times10^{-3}\,J = 176.4\ mJ.$$

L4

Q37]

$\Delta V = 365\ V$



a) $\Delta V = -\int_{+}^{-} \vec{E} \cdot dr = -\vec{E} \cdot \vec{d}$

$\Delta V = -Ed\cos(0) = -Ed$

$-365 = -E(50 \times 10^{-3})$

$\therefore\ E = \frac{365}{50 \times 10^{-3}} = 7300\ \frac{N}{C}$.

b) $\vec{F} = q\vec{E} = 2.5 \times 10^{-9}(7300\,\hat{i}) = 1.825 \times 10^{-5}\,\hat{i}\ N$.

c) $W_F = \int_{+}^{-} \vec{F} \cdot d\vec{s} = +\vec{F} \cdot \vec{d} = (1.825 \times 10^{-5}\,\hat{i}) \cdot (50 \times 10^{-3}\,\hat{i})$

$= 9.125 \times 10^{-7}\ J$.

d) Alternatively

$W^{field}_{+\to-} = -\Delta U = -q\Delta V = -(2.5 \times 10^{-9})(-365)$

$= 9.125 \times 10^{-7}\ J$ As before in part (c)

Q43] $V(x,y,z) = Axy - Bx^2 + Cy$

(a) $\vec{E} = -\nabla V = -\hat{i}\frac{\partial}{\partial x}V - \hat{j}\frac{\partial}{\partial y}V - \hat{k}\frac{\partial}{\partial z}V$

$\vec{E} = -\hat{i}(Ay - 2Bx) - \hat{j}(Ay + C) - \hat{k}(0)$

$= \hat{i}(2Bx - Ay) - \hat{j}(Ay + C)$

(B) $E_x = 0$ when $2Bx = Ay$ — (1)

$E_y = 0$ when $-C = Ay$ — (2)

$\therefore y = -\frac{C}{A}$

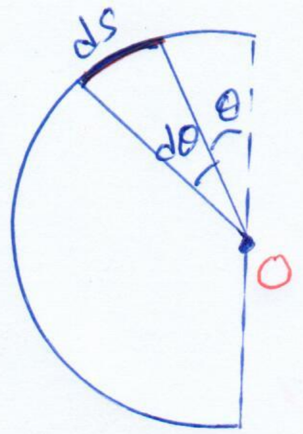
evaluate in (1) $\Rightarrow 2Bx = -C \Rightarrow x = -\frac{C}{2B}$

$\therefore$ electric field $E = 0$ at

$x = -\frac{C}{2B}$, $y = -\frac{C}{A}$.

Q68

$$\lambda = \frac{Q}{\pi R}$$



$$dV = \frac{k\,dq}{R} = k\,\frac{\lambda\,ds}{R}$$

$$dV = k\lambda\,\frac{R\,d\theta}{R} \quad , \; ds = R\,d\theta$$

$$dV = k\lambda\,d\theta$$

Integrate over the semicircle from $\theta = 0 \rightarrow \pi$

$$V = k\lambda\int_0^{\pi} d\theta = k\lambda\pi$$

$$V = k\left(\frac{Q}{\pi R}\right)\pi = \frac{kQ}{R}.$$