(1) Find $\int \underbrace{x \underset{u}{\underline{\sin^{-1} x}} \, dx}_{dv}$

$$= \frac{x^2}{2}\sin^{-1}x - \int \frac{x^2}{2}\left(\frac{1}{\sqrt{1-x^2}}\right) dx$$

$$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}\, dx \longrightarrow \begin{array}{l} x = \sin\theta \\ dx = \cos\theta \, d\theta \end{array}$$

$$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\int \frac{\sin^2\theta\,(\cos\theta\, d\theta)}{\sqrt{1-\sin^2\theta}}$$

$$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\int \frac{\sin^2\theta\,(\cos\theta)\, d\theta}{\cos\theta} = \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\int \frac{1}{2}(1-\cos 2\theta)\, d\theta$$

6

$$\frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\left(\frac{1}{2}\right)\left(\theta - \frac{\overset{2\sin\theta\cos\theta}{\sin 2\theta}}{2}\right) + C$$

$$\frac{x^2}{2}\sin^{-1}x - \frac{1}{4}\left(\sin^{-1}x - x\sqrt{1-x^2}\right) + C$$

(2) Find (if exists) $\int_0^1 \frac{dx}{\sqrt{x}\ \sqrt{x+1}}$

$$\int_0^1 \frac{dx}{\sqrt{x}\,\sqrt{x+1}} = \lim_{a\to 0^+}\int_a^1 \frac{dx}{\sqrt{x}\ \sqrt{x+1}} \longrightarrow \begin{array}{l} \sqrt{x} = y \\ x = y^2 \\ dx = 2y\, dy \end{array}$$

$$= \lim_{a\to 0^+}\int_{\sqrt{a}}^1 \frac{2y\, dy}{y\sqrt{y^2+1}}$$

$$\lim_{a\to 0^+}\int_{\sqrt{a}}^1 \frac{2\, dy}{\sqrt{y^2+1}} \longrightarrow \begin{array}{l} y = \tan\theta \\ dy = \sec^2\theta\, d\theta \end{array}$$

6

$$\lim_{a\to 0^+}\int_{\tan^{-1}\sqrt{a}}^{\frac{\pi}{4}} \frac{2\sec^2\theta\, d\theta}{\sqrt{\tan^2\theta+1}} = \lim_{a\to 0^+}\int_{\tan^{-1}\sqrt{a}}^{\frac{\pi}{4}} \frac{2\sec^2\theta\, d\theta}{\sec\theta}$$

$$= \lim_{a\to 0^+}\int_{\tan^{-1}\sqrt{a}}^{\frac{\pi}{4}} 2\sec\theta\, d\theta$$

$$= \lim_{a\to 0^+} 2\ln|\sec\theta + \tan\theta| \Big]_{\tan^{-1}\sqrt{a}}^{\frac{\pi}{4}} = \lim_{a\to 0^+}\left(2\ln|\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right) - \left(2\ln|\sec(\tan^{-1}\sqrt{a}) + \overset{?}{\cancel{\tan(\tan^{-1}\sqrt{a})}}\right)$$

$\tan(\tan^{-1}\sqrt{a})$

([illegible]) In the back

1) $\int x \sin^{-1}(x) dx$

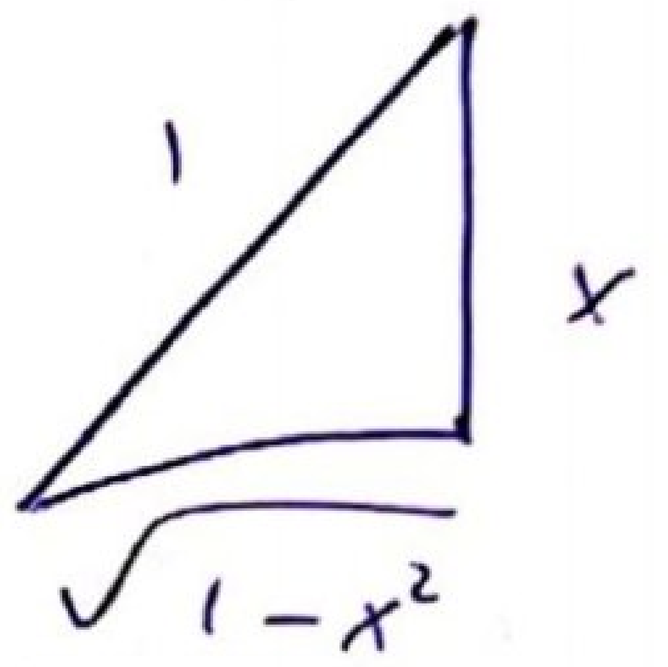
$u = \sin^{-1}(x) \qquad dv = x dx$

$du = \frac{1}{\sqrt{1-x^2}} \qquad v = x^2/2$

$= \frac{x^2}{2}\sin^{-1}(x) - \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}} dx$

$* \int \frac{x^2}{\sqrt{1-x^2}} dx \Longrightarrow \qquad x = \sin\theta$

$dx = \cos\theta d\theta$



$= \int \frac{\sin^2\theta \cdot \cancel{\cos\theta} d\theta}{\cancel{\cos\theta}}$

$2\sin\theta\cos\theta$

$= \int \frac{1}{2}[1-\cos 2\theta] d\theta = \frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right] + c$

$= \frac{1}{2}\left[\sin^{-1}(x) - x\sqrt{1-x^2}\right] + c$

$\Longrightarrow \frac{x^2}{2}\sin^{-1}(x) - \frac{1}{2}\cdot\frac{1}{2}\left[\sin^{-1}(x) - x\sqrt{1-x^2}\right] + c$

---

2) $\int_0^1 \frac{dx}{\sqrt{x}\sqrt{x+1}} = \lim_{a\to0^+}\int_a^1 \frac{dx}{\sqrt{x}\cdot\sqrt{x+1}}$ , $y = \sqrt{x}$

$y^2 = x$

$2y dy = dx$

$= \lim_{a\to0^+}\int_{\sqrt a}^1 \frac{2\cancel{y} dy}{\cancel{y}\sqrt{y^2+1}} = \lim_{a\to0^+}\int_{\sqrt a}^1 \frac{2dy}{\sqrt{y^2+1}}$

$y = \tan\theta$

$dy = \sec^2\theta d\theta$

$= \lim_{a\to0^+}\int_{\tan^{-1}\sqrt a}^{\pi/4} \frac{2\sec^{\cancel{2}}\theta d\theta}{\cancel{\sec\theta}} = \lim_{a\to0^+}\int_{\tan^{-1}\sqrt a}^{\pi/4} 2\sec\theta d\theta$

$= \lim_{a\to0^+} 2\ln|\sec\theta + \tan\theta\Big]_{\tan^{-1}\sqrt a}^{\pi/4}$

$\sec(0) = 1 \qquad \sqrt{a} = 0$

$= \lim_{a\to0^+} 2\ln|\sec\pi/4 + \tan\pi/4| - 2\ln|\sec\tan^{-1}\sqrt a + \tan\tan^{-1}\sqrt a$

$= 2\ln|\sec\pi/4 + \tan\pi/4| - \overset{0}{2\ln(1)}$

$= 2\ln|\sqrt2 + 1|$ (con)

$$x^2+4x = x^2+4x+4-4$$
$$= (x+2)^2-4$$

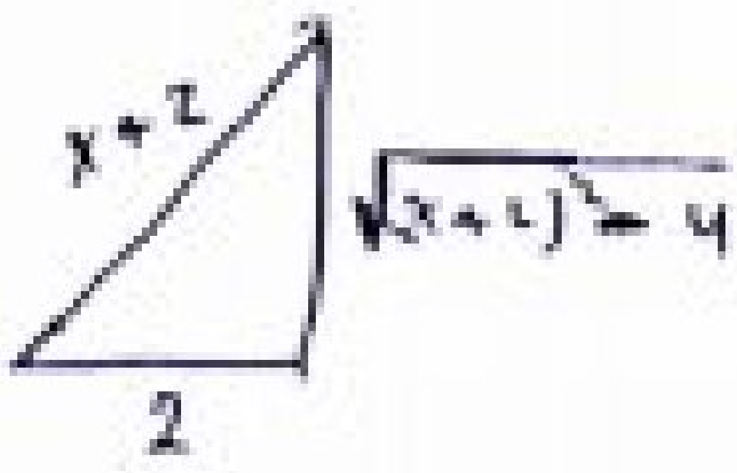
(3) Find $\int \frac{x^2dx}{\sqrt{x^2+4x}}$

$$= \int \frac{x^2\,dx}{\sqrt{(x+2)^2-4}}$$

$x+2 = 2\sec\theta \rightarrow x = 2\sec\theta - 2$

$dx = 2\sec\theta\tan\theta\, d\theta$

$$= \int \frac{(2\sec\theta-2)^2(2)\sec\theta\tan\theta\, d\theta}{\sqrt{4\sec^2\theta-4}}$$



$$= \int \frac{(2\sec\theta-2)^2(2)\sec\theta\tan\theta\, d\theta}{2\tan\theta}$$

6

$$= \int (4\sec^2\theta - 8\sec\theta + 4)\sec\theta\, d\theta$$

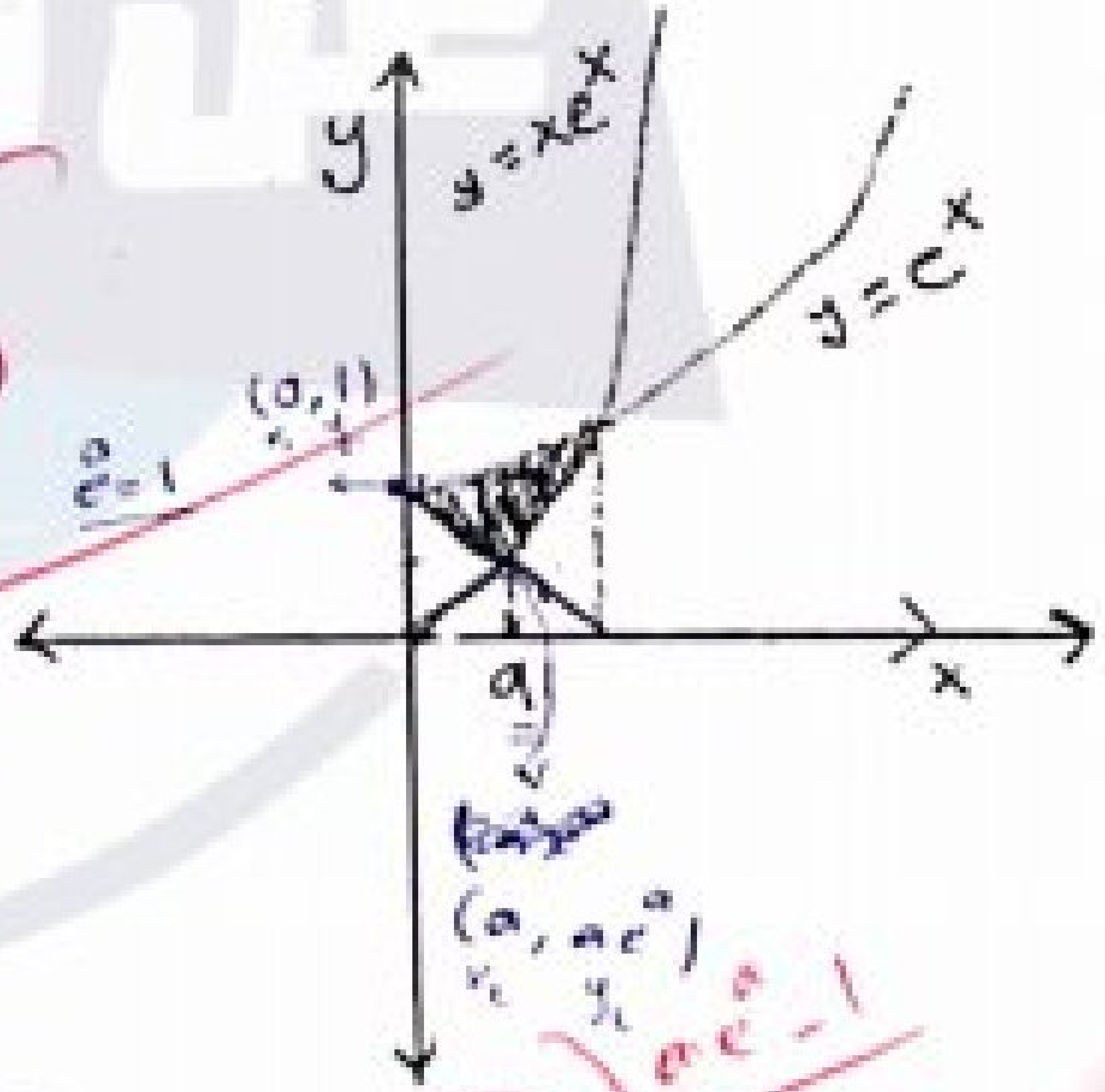
$$= \int (4\sec^3\theta - 8\sec^2\theta + 4\sec\theta)\, d\theta = \int 4\sec^3\theta\, d\theta - \int 8\sec^2\theta\, d\theta + \int 4\sec\theta\, d\theta$$

$4(\frac{1}{2}(\tan\theta\sec\theta + \ln|\tan\theta+\sec\theta|) - 8\tan\theta + 4\ln|\sec\theta+\tan\theta| + C$

$2\left(\frac{\sqrt{(x+2)^2-4}}{2}\left(\frac{x+2}{2}\right) + \ln\left|\frac{\sqrt{(x+2)^2-4}}{2} + \frac{x+2}{2}\right| - 8\frac{\sqrt{(x+2)^2-4}}{2} + 4\ln\left|\frac{x+2}{2} + \frac{\sqrt{(x+2)^2-4}}{2}\right| + C.$

(a بدلالة ع)

(4) Find the area of the shaded region in terms of $a^2$



$$\text{area} = \int_0^a \left(e^x - \left(\frac{a}{ae^a-1}x+1\right)\right)dx + \int_a^1 (e^x - xe^x)\, dx$$

5

$$= e^x - \frac{1}{2}\frac{a}{ae^a-1}x^2 + x\Big]_0^a + \int_a^1 e^x\,dx - \int_a^1 \underset{u}{x}\underset{dv}{e^x}\,dx$$

$$= e^x - \frac{a}{2ae^a-1}x^2 + x\Big]_0^a + e^x\Big]_a^1 - (xe^x)\Big]_a^1 - \int_a^1 e^x\,dx$$

$$= e^x - \frac{a}{2ae^a-1}x^2 + x\Big]_0^a + e^x\Big]_a^1 - xe^x\Big]_a^1 - e^x\Big]_a^1$$

$e^a - \frac{a^3}{2ae^a-1} + a - (1-0) + (e-e^a) - (e-ae^a)$

$-(e-e^a)$

$xe^x = e^x$

$xe^x - e^x = 0$

$e^x(x-1) = 0$

$\cancel{e^x = 0}$

$\boxed{x=1}$

$$\text{slope} = \frac{a-0}{ae^a-1} = \frac{a}{ae^a-1}$$

$\frac{ae^a-1}{a}$

$y-y_1 = m(x-x_1)$

$y-1 = \frac{a}{ae^a-1}(x-0)$

$$\boxed{y = \frac{a}{ae^a-1}x+1}$$

معادلة الخط المستقيم.

3) $\int \frac{x^2 dx}{\sqrt{x^2+4x}} = \int \frac{x^2 dx}{\sqrt{x^2+4x+4-4}} = \int \frac{x^2 dx}{\sqrt{(x+2)^2-4}}$

$= \int \frac{4(\sec\theta-1)^2 . 2\sec\theta \tan\theta \, d\theta}{2\cancel{\tan\theta}}$ | $x+2 = 2\sec\theta$
$dx = 2\sec\theta \tan\theta \, d\theta$

$= 4 \int (\sec^2\theta - 2\sec\theta + 1) \sec\theta \, d\theta$

$= 4 \int (\sec^3\theta - 2\sec^2\theta + \sec\theta) \, d\theta$

$= 4 \left[ \frac{1}{2} (\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right] - 2\tan\theta + \ln|\sec\theta + \tan\theta| \big]$

( put the answer in term of $x$ )

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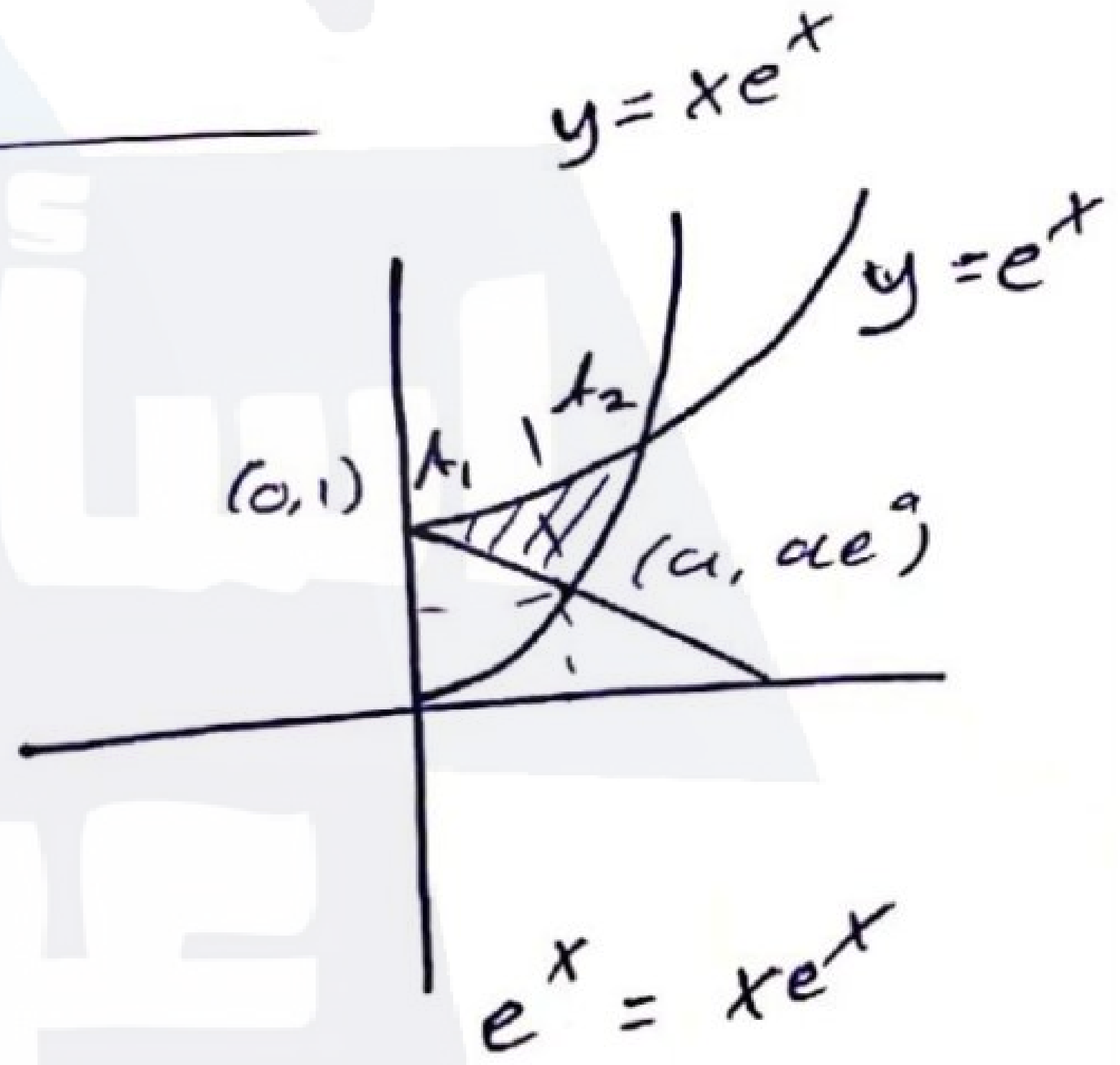
4) $\text{slope} = \frac{ae^a - 1}{a}$ , $y = mx + b$

$= \frac{ae^a - 1}{a} x + 1$



$e^x = xe^x$

$\boxed{x=1}$

$A_1 = \int_0^a \left(e^x - \frac{ae^a - 1}{a} x - 1\right) dx$

$= e^x - \frac{ae^a - 1}{a} \frac{x^2}{2} - x \Big]_0^a$

$A_1 = e^a - 1 - \frac{a^2 e^a}{2} + \frac{a}{2} - a$

$A_2 = \int_a^1 (e^x - xe^x) \, dx = \int_a^1 e^x (1-x) \, dx$

$u = 1-x \qquad dv = e^x dx$
$du = -dx \qquad v = e^x$

$= (1-x) e^x \Big]_a^1 + \int_a^1 e^x dx$

$= -e^a + ae^a + e - \cancel{e^a} + \cancel{e^a} - \frac{a^2 e^a}{2} - \frac{a}{2}$

$= -e^a + ae^a + e - \frac{a^2 e^a}{2} - \frac{a}{2}$ #

②

(5) Find $\int \frac{dx}{x(x^2+1)^2}$

$\frac{1}{x(x^2+1)}$

$\frac{1}{x\,x^2} = \frac{A}{x} + \frac{Bx+C}{x^2} + \frac{Dx+E}{\ }$

$$5)\ \int \frac{dx}{x(x^2+1)^2} = \int \frac{A}{x}dx + \int \frac{Bx+C}{x^2+1}dx + \int \frac{Dx+E}{(x^2+1)^2}$$

$$1 = A(x^2+1)^2 + (Bx+C)x(x^2+1) + (Dx+E)x$$

$$1 = A(x^4+2x^2+1) + (Bx+C)(x^3+x) + (Dx^2+Ex)$$

$$1 = Ax^4 + 2Ax^2 + A + Bx^4 + Bx^2 + Cx^3 + Cx + Dx^2 + Ex$$

$x^4 :\ 0 = A + B$ $\quad$ $x^2 :\ 0 = 2A + B + D$

$x^3 :\ \boxed{0 = C}$ $\quad$ $x :\ 0 = C + E$ $\quad \boxed{E = 0}$

$c :\ \boxed{1 = A}$ $\quad$ $0 = 2 - 1 + D$

$\boxed{B = -1}$ $\quad$ $\boxed{D = -1}$

$$\Rightarrow \int \frac{1}{x}dx + \int \frac{-x}{x^2+1}dx + \int \frac{-x}{(x^2+1)^2}dx$$

$\downarrow$ ln $\quad$ $\downarrow$ u-sup $\quad$ $\downarrow$ u-sup

1. Choose the correct answer:

1. $\int 2x \ln x \, dx =$

[✓ A.] $x^2 \ln x - \frac{x^2}{2} + c$ B. $x^3 \ln x - \frac{x^3}{3} + c$ C. $x\frac{(\ln x)^3}{3} - \frac{(\ln x)^3}{2} + c$ D. $x\frac{(\ln x)^2}{6} - \frac{(\ln x)^4}{4} + c.$

2. $\int \sin^3 x \, dx =$

[✓ A.] $\frac{\cos^3 x}{3} - \cos x + c$ B. $\sin x - \frac{\sin^3 x}{3} + c$ C. $\cos x - \frac{\cos^3 x}{3} + c$ D. $\cos x + \frac{\cos^3 x}{3} + c.$

3. $\int \sin 2x \sin 5x \, dx =$

A. $\frac{\sin 3x}{3} - \frac{\sin 7x}{7} + c$ B. $-\frac{\cos 3x}{6} - \frac{\cos 7x}{14} + c$ [✓ C.] $\frac{\sin 3x}{6} - \frac{\sin 7x}{14} + c$ D. $\frac{\sin 3x}{3} + \frac{\sin 7x}{7} + c.$

4. The partial fraction decomposition of $\frac{2x^2+1}{(x^2-x-2)(x^2+x+1)}$ is

A. $\frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{x^2+x+1}$ [✓ B.] $\frac{A}{x-2} + \frac{B}{x+1} + \frac{Cx+D}{x^2+x+1}$

C. $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2-x+2}$ D. None of the above.

5. A suitable substitution to evaluate the integral $\int \frac{\sqrt{4x^2+3}}{x^2} dx$ is

[✓ A.] $x = \frac{\sqrt{3}}{2}\tan\theta.$ B. $x = \frac{\sqrt{3}}{\sqrt{2}}\tan\theta.$ C. $x = \frac{\sqrt{3}}{2}\sec\theta.$ D. $x = \frac{\sqrt{3}}{\sqrt{2}}\tan 2\theta..$

6. $\int \tan^{10} x \sec^4 x \, dx =$

A. $\frac{\sec^{13} x}{13} - \frac{\sec^{11} x}{11} + c$ B. $\frac{\sec^{11} x}{11} + \frac{\sec^{13} x}{13} + c$ [✓ C.] $\frac{\tan^{11} x}{11} + \frac{\tan^{13} x}{13} + c.$ D. $\frac{\sec^{11} x}{11} + \frac{\sec^{13} x}{13} + c.$

7. $\int_0^{\frac{\pi}{2}} \sec x \, dx =$

[✓ A.] $\infty.$ B. $-\infty.$ C. $\frac{1}{2}.$ D. 0.

8. $\int_1^{\infty} \frac{2+\sin x}{x^2} dx$ is

[✓ A.] Divergent. B. Convergent. C. Does not exist. D. None of the above

$\lim_{n \to \infty} \int_1^a \frac{2+\sin x}{x^2}$

$u = 2+\sin x \rightarrow du = \cos x$

$dv = x^{-2} dx \rightarrow v = \frac{-1}{x}$

$\left( \frac{-2-\sin x}{x} \right]_1^a + \int_1^a \frac{\cos x}{x^2} dx \right)$

~~$u = \cos x \rightarrow$~~

~~$dv = \frac{1}{x} dx \rightarrow$~~

$dv = \frac{1}{x}$

1) $\int 2x \ln x \, dx$ $\quad u = \ln x \quad dv = 2x\,dx$

$du = \frac{dx}{x} \quad v = x^2$

$= x^2 \ln x - \int x\,dx = x^2 \ln x - \frac{x^2}{2}$

2) $\int \sin^3 x\,dx = \int \sin x(1-\cos^2 x)\,dx$ $\quad u = \cos x$
$du = -\sin x\,dx$

$= -\int \frac{\cancel{\sin x}(1-u^2)\,du}{\cancel{\sin x}} = \int (u^2-1)\,du$

$= u^3/3 - u + c = \frac{\cos^3 x}{3} - \cos x + c$

3) $\int \sin 2x \sin 5x\,dx = \int -\frac{1}{2}(\cos 7x - \cos 3x)\,dx$

$= \frac{\sin 3x}{6} - \frac{\sin 7x}{14} + c$

4) $\int \frac{2x^2+1}{(x^2-x-2)(x^2+x-1)}\,dx = \int \frac{A}{x-2}\,dx + \int \frac{B}{x+1}\,dx + \int \frac{Cx+E}{x^2+x+1}\,dx$

5) $\int \frac{\sqrt{4(x^2+3/4)}}{x^2} \Rightarrow x = \frac{\sqrt{3}}{2}\tan\theta$

6) $\int \tan x^{10} \sec^4 x\,dx =$ ~~let $a = \sec^2 x$~~

let $u = \tan x$

$du = \sec^2 x\,dx$

$a^2 = \sec^2 - 1$

$- \quad = \int \frac{u^{10} \sec^4 x \cdot du}{\sec^2 x}$

$= \int u^{10} \sec^2 x\,du$

$= \int u^{10}(u^2+1)\,du = u^{13}/13 + \frac{u^{11}}{11} + c = \frac{\tan x^{13}}{13} + \frac{\tan^{11} x}{11} + c$

7) $\int_0^{\pi/2} \sec x\,dx = \lim_{a \to \pi/2^-} \int_0^a \sec x\,dx$

$= \lim_{a \to \pi/2^-} \ln|\sec x + \tan x| \Big]_0^a = \lim_{a \to \pi/2^-} \ln|\sec a + \tan a| - \ln|\sec 0 + \tan 0|$

$= \ln|\infty + \infty| = \underline{\underline{\infty}}$

8) Not included!!

④

2. Evaluate the integral $\int \frac{\sqrt{1+\sqrt{x}}}{x}\,dx$

$t = \sqrt{1+\sqrt{x}}$

$t^2 = 1+\sqrt{x}$

$2t\,dt = \frac{dx}{2\sqrt{x}}$

$4t\sqrt{x}\,dt = dx$

$\Rightarrow \int \frac{t \cdot 4t\sqrt{x}}{\sqrt{x}\sqrt{x}}\,dt$

$\sqrt{x} = t^2-1$

$\int \frac{4t^2\,dt}{t^2-1}$

$$t^2-1 \,\overline{)\, t^2}$$
$$\frac{-(t^2-1)}{1} \quad \text{quotient } 1$$

$4\left[\int dt + \int \frac{1}{(t-1)(t+1)}\,dt\right]$

$\frac{1}{(t-1)(t+1)} = \frac{A}{t-1} + \frac{B}{t+1}$

$1 = A(t+1) + B(t-1)$

$\boxed{t=-1} \Rightarrow 1 = -2B \quad \boxed{B = -\frac{1}{2}}$

$\boxed{t=1} \Rightarrow 1 = 2A \quad \boxed{A = \frac{1}{2}}$

$\int \frac{\frac{1}{2}}{t-1}\,dt + \int \frac{-\frac{1}{2}}{t+1}$

$\frac{1}{2}\ln|t-1| - \frac{1}{2}\ln|t+1|$

$\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|$

$4\left[t + \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|\right] + c$

$\int \frac{\sqrt{1+\sqrt{x}}}{x}\,dx = 4\sqrt{1+\sqrt{x}} + 2\ln\left|\frac{\sqrt{1+\sqrt{x}}-1}{\sqrt{1+\sqrt{x}}+1}\right| + c$

7

$$\int \frac{\sqrt{1+\sqrt{z}}}{z}\,dz \qquad \text{let } u = \sqrt{1+\sqrt{z}}$$

$$u^2 = 1+\sqrt{z}$$

$$\int \frac{u \cdot 2u \cdot 2\sqrt{z}\,du}{z} \qquad 2u\,du = \frac{dz}{2\sqrt{z}}$$

$$= \int \frac{4u^2\,du}{\sqrt{z}} = \int \frac{4u^2\,du}{u^2-1}$$

$$\begin{array}{r} 1 \\ u^2-1 \enclose{longdiv}{u^2} \\ -u^2 \pm 1 \\ \hline 1 \end{array}$$

$$= 4\left[\int du + \int \frac{1}{u^2-1}\,du\right]$$

$$* \int \frac{1}{u^2-1}\,du = \frac{A}{u-1} + \frac{B}{u+1}$$

$$1 = A(u+1) + B(u-1)$$

$u=1 \qquad 1 = 2A$

$\qquad\quad A = 1/2$

$u=-1 \qquad 1 = -2B$

$\qquad\quad B = -1/2$

$$= \int \frac{1/2}{u-1} - \int \frac{1/2}{u+1}$$

$$= \frac{1}{2}\ln|u-1| - \frac{1}{2}\ln|u+1|$$

$$4\left[u + \frac{1}{2}\ln|u-1| - \frac{1}{2}\ln|u+1|\right]$$

$\Longrightarrow$

(put the answer
in terms of Z)

$x^{\frac{2}{2}-\frac{1}{2}} = \frac{x^{\frac{1}{2}}}{1}$

$x = \frac{x^2}{2}$

3. Evaluate the integral $\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx$, and determine whether it is convergent or [✓ divergent].

$\lim_{a\to 0^+} \int_a^1 \frac{\ln x}{\sqrt{x}}\,dx$

$\int_a^1 \frac{\ln x}{\sqrt{x}}\,dx$

$u = \ln x \rightarrow du = \frac{1}{x}\,dx$

$dv = x^{-\frac{1}{2}}\,dx \rightarrow v = \frac{2x^{\frac{1}{2}}}{1} = 2\sqrt{x}$

$2\sqrt{x}\ln x\Big|_a^1 - \int_a^1 \frac{2\sqrt{x}}{\sqrt{x}\sqrt{x}}\,dx \rightarrow x^{-\frac{1}{2}} = \frac{2x^{\frac{1}{2}}}{1}$

$2\sqrt{x}\ln x\Big|_a^1 - 2.2\sqrt{x}\Big|_a^1$

$\lim_{a\to 0^+}\left(2\sqrt{x}\ln x\Big|_a^1 - 4\sqrt{x}\Big|_a^1\right)$

$\left((2.\sqrt{1}\ln 1 - 2\sqrt{a}\ln a) - (4\sqrt{1} - 4\sqrt{a})\right)$

$\left(\overset{0}{0} - \boxed{2\sqrt{0}\ln 0^+} - (4 - 4\sqrt{0})\right)$

$\boxed{-\infty}$ conv

$\left(\overset{0}{\cancel{-\infty}} - 4 + 0\right)$

DNE conv

div

div ± conv = divergent

4

$$\int_0^1 \frac{\ln z}{\sqrt{z}}\,dz$$

$$\Rightarrow \quad u = \ln z \qquad dv = z^{-1/2}\,dz$$

$$du = \frac{dz}{z} \qquad v = 2z^{1/2}$$

$$= \lim_{a\to 0^+} 2\sqrt{z}\ln z \Big]_a^1 - \int_a^1 \frac{2\sqrt{z}}{z}\,dz$$

$$= \lim_{a\to 0^+} -2\sqrt{a}\ln a - 4z^{1/2}\Big]_a^1$$

$$= \lim_{a\to 0^+} \underbrace{-2\sqrt{a}\ln a} - 4 + \overset{0}{4\sqrt{a}}$$

$\underline{\underline{0\cdot\infty}}$ $\Delta$

$$= 0 - 4 + 0$$

$$\underline{\underline{= -4}}$$

$$\lim_{a\to 0^+} \sqrt{a}\ln a =$$

$$\lim_{a\to 0^+} \frac{\ln a}{\frac{1}{\sqrt{a}}} =$$

$$\lim_{a\to 0^+} \frac{\frac{1}{\cancel{a}}}{\frac{\frac{-1}{2\sqrt{a}}}{\cancel{a}}} = -2\sqrt{0} = \underline{\underline{0}}$$

V.good

(1) Find $\int \frac{\tan^{-1} x}{x^2}\,dx$

$\int \frac{1}{x^2} \tan^{-1}(x)\,dx$ (parts)

$u = \tan^{-1}(x)$

$du = \frac{1}{1+x^2}\,dx$

$dv = \frac{1}{x^2}\,dx$

$v = -\frac{1}{x}$

$$= \frac{-\tan^{-1}(x)}{x} + \int \frac{1}{x} \cdot \frac{1}{(1+x^2)}\,dx$$

$$\frac{1}{x(1+x^2)} = \frac{A}{x} + \frac{Bx+c}{x^2+1}$$

$1 = A(x^2+1) + (Bx+c)(x)$

$1 = Ax^2 + A + Bx^2 + cx$

$\boxed{A=1}$ $\boxed{c=0}$ →

$A + B = 0$

$1 + B = 0$

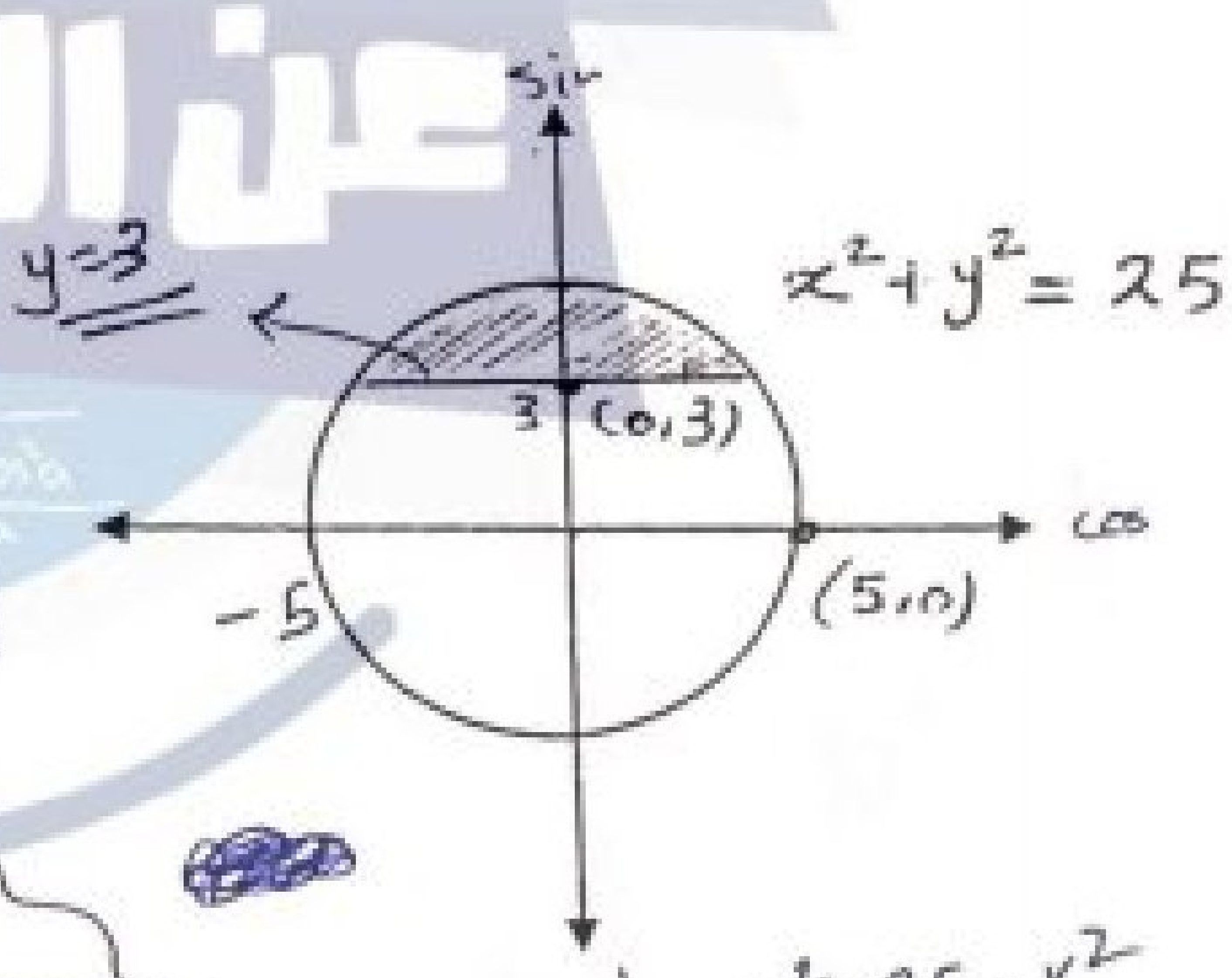
$\boxed{B = -1}$

$$\int \frac{1}{x(x^2+1)}\,dx = \int \frac{1}{x} - \frac{x}{x^2+1}\,dx$$

$$\ln|x| - \frac{\ln|x^2+1|}{2} + c$$ ✓

6

(2) Find the area of the shaded region



$$\text{Area} = \int_{-5}^{0} +\sqrt{25-x^2} - 3\,dx$$

$$+ \int_{0}^{5} \sqrt{25-x^2} - 3\,dx$$

$$\text{Area} = \int_{-5}^{5} \sqrt{25-x^2} - 3\,dx$$

$$\text{Area} = \int_{-5}^{5} \sqrt{25-x^2}\,dx - \boxed{\int_{-5}^{5} 3\,dx}$$

$3x\Big]_{-5}^{5}$

$= 15 + 15 = \boxed{30}$

$\therefore$ Area = [illegible]

$$-\frac{25\pi}{2} - 30$$ ✓

5

$y^2 = 25 - x^2$

$y = \sqrt{25-x^2}$

$x = 5\sin\theta$

$dx = 5\cos\theta\,d\theta$

$x = 5 \to \theta = 90° = \frac{\pi}{2}$ →

$x = -5 \to \theta = +\frac{3\pi}{2}$

$$\therefore \int_{+\frac{3\pi}{2}}^{\frac{\pi}{2}} 5\cos\theta \cdot 5\cos\theta\,d\theta$$

$$= \int_{\frac{3\pi}{2}}^{\frac{\pi}{2}} 25\cos^2\theta\,d\theta \text{ (even)}$$

$$25\int_{\frac{3\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}(1+\cos(2\theta))\,d\theta$$

$$\to \frac{25}{2}\left(\theta + \frac{\sin(2\theta)}{2}\right)\Big]_{\frac{3\pi}{2}}^{\frac{\pi}{2}} = -\frac{25\pi}{2}$$

$\int \frac{\tan^{-1}x}{x^2}dx \qquad u = \tan^{-1}x \qquad dv = x^{-2}dx$

$du = \frac{1}{x^2+1}dx \qquad v = \frac{-1}{x}$

$$\Rightarrow -\frac{\tan^{-1}x}{x} + \int \frac{1\cancel{x}}{x(x^2+1)}dx$$

$$= -\frac{\tan^{-1}x}{x} + \int \frac{A}{x}dx + \int \frac{Bx+c}{x^2+1}dx$$

$$= -\frac{\tan^{-1}x}{x} + \int \frac{dx}{x} \overset{-}{\cancel{+}} \int \frac{x}{x^2+1}dx$$

$$= -\frac{\tan^{-1}x}{x} + \ln x \cancel{+} - \ln|x^2+1| + c$$

$1 = A(x^2+1) + x(Bx+c)$

$x=0 \Rightarrow \boxed{A=1}$

$x=1 \Rightarrow B+c=-1$

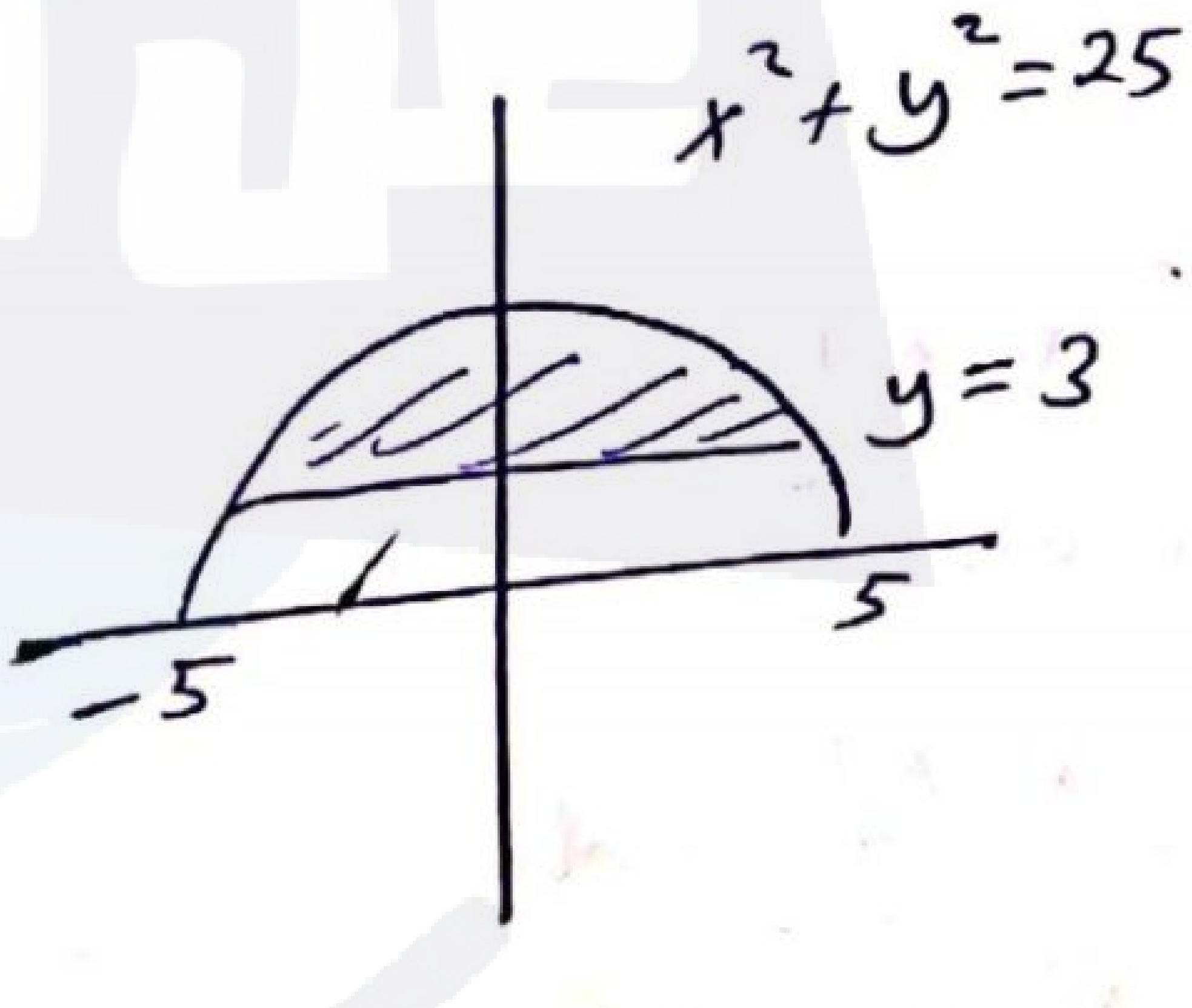
$x=-1 \Rightarrow B-c=-1$

solve for $B, c$

$B=-1, \quad c=0$

---

$$A = \int_{-5}^{5} (\sqrt{25-x^2} - 3)\,dx$$



$$= \int_{-5}^{5} \sqrt{25-x^2} - \int_{-5}^{5} 3\,dx \quad \hookrightarrow \underline{\underline{= 30}}$$

$x = 5\sin\theta$

$dx = 5\cos\theta\, d\theta$

$$\Rightarrow \int_{-\pi/2}^{\pi/2} 25\cos^2\theta\, d\theta = \int_{-\pi/2}^{\pi/2} \frac{25}{2}\left[1+\cos 2\theta\right] d\theta$$

$$= \frac{25}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{25}{2}\left[\frac{\pi}{2} + 0 + \frac{\pi}{2} \cancel{+} 0\right] = \frac{25\pi}{2}$$

$$\boxed{A = \frac{25\pi}{2} - 30}$$

**(3)** Find (if exists) $\int_1^{\infty}(1-x)e^{-x}dx$

$\lim_{a\to\infty}\int_1^a \frac{(1-x)}{e^x}dx$

$\int_1^a \frac{(1-x)}{e^x}dx$ (parts)

$u = 1-x \qquad dv = e^{-x}$

$du = -dx \qquad v = -e^{-x}$

$= -(1-x)e^{-x} - \int e^{-x}dx$

$= \frac{(x-1)}{e^x} + e^{-x}\Big]_1^a$

$= \left(\frac{(a-1)}{e^a} + \frac{1}{e^a}\right) - \left(0 + e^{-1}\right)$

$= \left(\frac{a}{e^a}\right) - \frac{1}{e}$

$\lim_{a\to\infty}\left(\frac{a}{e^a}\right) - \frac{1}{e}$

$\frac{\infty}{e^{\infty}} - \frac{1}{e}$

$\frac{\infty}{\infty} \to$ L.H

$\frac{a}{e^a} \overset{L.H}{\Longrightarrow} \frac{1}{e^a}$

$\therefore \lim_{a\to\infty}\left(\frac{1}{e^a}\right) - \frac{1}{e} = 0 - \frac{1}{e}$

$\boxed{= -e^{-1}}$ conv ✓

6

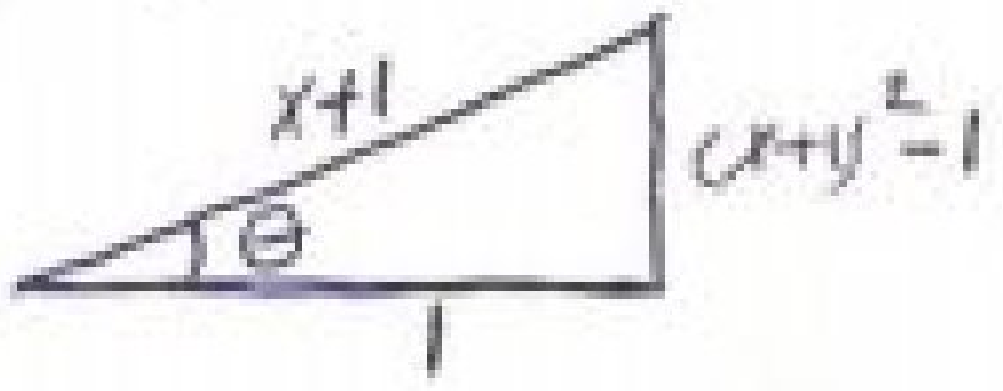
**(4)** Find $\int \frac{x^2+1}{\sqrt{x^2+2x}}dx$

$x^2 + 2x + 1 - 1$ (إكمال مربع)

$= (x+1)^2 - 1$

$x + 1 = 1.\sec\theta$ ( Trigono. sub )

$dx = \sec\theta\tan\theta\, d\theta$



$x^2 = (\sec\theta - 1)^2$

$x^2 = \sec^2\theta - 2\sec\theta + 1$

$\int \frac{x^2+1}{\sqrt{(x+1)^2-1}}dx$

$\therefore \int \frac{x^2+1}{\sqrt{\sec^2\theta - 1}}dx$

$= \int \frac{x^2+1}{\tan\theta} \cdot \sec\theta . \tan\theta\, d\theta$

$= \int \sec\theta \cdot (x^2+1)\, d\theta$

~~$u = \sec^2\theta + 1$~~ ~~$dv = \sec\theta$~~

~~$du = 2x\,dx$~~ ~~$v = \ln|\sec\theta + \tan\theta|$~~

5

$= \int \sec\theta \cdot (\sec^2\theta - 2\sec\theta + 2)\, d\theta$

$= \int \sec^3\theta\, d\theta - 2\int \sec^2\theta\, d\theta + 2\int \sec\theta\, d\theta$

$= \int \sec\theta \sec^2\theta\, d\theta - 2\int \tan^2\theta + 1\, d\theta + 2\ln|\sec\theta + \tan\theta| + c$

تكملة الإجابة بالخلف

$\rightarrow$ to be continued in the back

③ $\int_1^\infty (1-x)e^{-x}dx = \lim_{a\to\infty}\int_1^a (1-x)e^{-x}dx$

By parts $\Rightarrow$ $u = 1-x$ $\quad dv = e^{-x}dx$

$du = -dx$ $\quad v = -e^{-x}$

$= \lim_{a\to\infty}\left[ -(1-x)e^{-x}\Big]_1^a - \int_1^a e^{-x}dx \right]$

$= \lim_{a\to\infty}\left[ (x-1)e^{-x}\Big|_1^a - \int e^{-x}\Big]_1^a \right]$

$= \lim_{a\to\infty} (a-1)e^{-a} + e^{-a} - e^{-1}$

$= \lim_{a\to\infty} ae^{-a} \cancel{-e^{-a}} \cancel{+e^{-a}} - e^{-1} = \lim_{a\to\infty} ae^{-a} - e^{-1}$

$= \lim_{a\to\infty} \frac{a}{e^a} - e^{-1} = \lim_{a\to a} \frac{1}{e^a} - e^{-1} = 0 - e^{-1}$

$= \underline{\underline{\frac{-1}{e}}}$

---

4) $\int \frac{x^2+1}{\sqrt{x^2+2x}}dx = \int \frac{x^2+1}{\sqrt{x^2+2x+1-1}}dx = \int \frac{x^2+1}{\sqrt{(x+1)^2-1}}dx$

let $x+1 = \sec\theta$

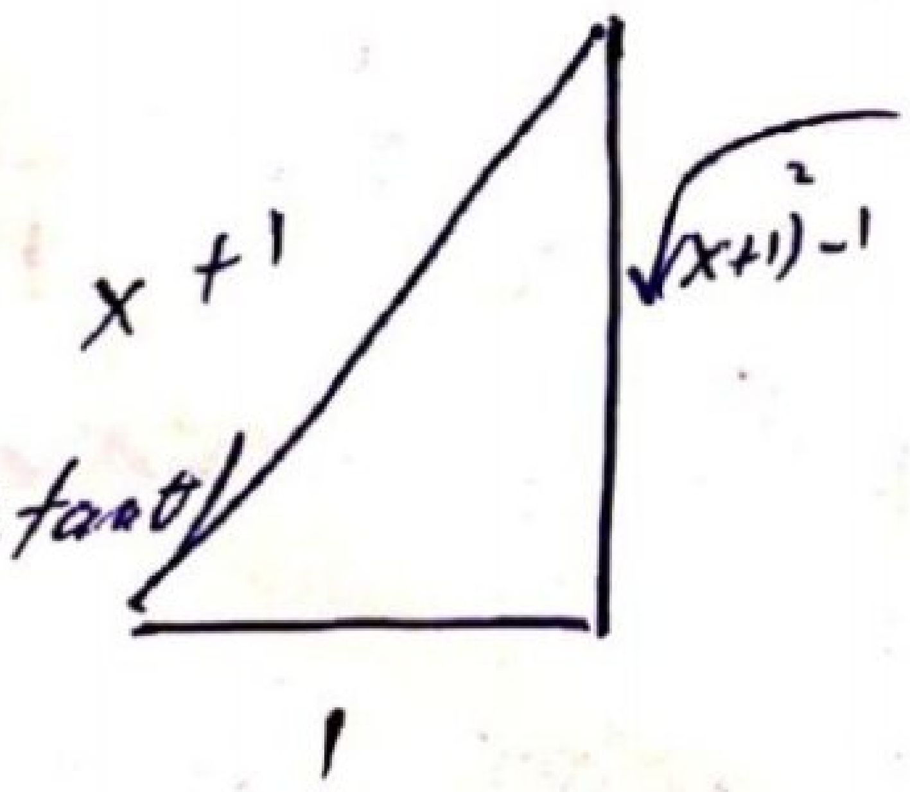
$dx = \sec\theta\tan\theta\,d\theta$

$x = \sec\theta - 1$

$= \int \frac{(\sec\theta-1)^2+1}{\cancel{\tan\theta}} \cdot \sec\theta\,\cancel{\tan\theta}\,d\theta$

$= \int (\sec^2\theta - 2\sec\theta + 1 + 1)\sec\theta\,d\theta$

$= \int (\sec^3\theta - 2\sec^2\theta + 2\sec\theta)\,d\theta$



$= \frac{1}{2}\tan\theta\sec\theta + \frac{1}{2}\ln|\sec\theta\tan\theta| - 2\tan\theta + 2\ln|\sec\theta+\tan\theta| + C$

↳ put the answer in terms of $\underline{\underline{x}}$

(5) Find $\int \frac{\sqrt{1+\sqrt{x}}}{x}\, dx$

$\sqrt{1+\sqrt{x}} = u$

[redacted]

= [redacted]

= $-2$ [redacted]

= [redacted]

= [redacted]

= [redacted] $-C$

✓

②

$\frac{25}{30}$

0188582

The University of Jordan | Mathematics Department

*Math* (102) / *First Exam*
Date: 27/06/2019

Name: زيد ناصر بني صخر Student Number: علاء ابو محفوظ Section:.........

14

1. Choose the correct answer:

1. $\int \sin^3 x\,dx =$

[✓ A. $\frac{\cos^3 x}{3} - \cos x + c$] B. $\sin x - \frac{\sin^3 x}{3} + c$ C. $\cos x - \frac{\cos^3 x}{3} + c$ D. $\cos x + \frac{\cos^3 x}{3} + c$.

2. $\int \ln x\,dx =$

A. $x^2 \ln x - \frac{x^2}{2} + c$ B. $x^3 \ln x - \frac{x^3}{3} + c$ [✓ C. $x \ln x - x + c$] D. $x\frac{(\ln x)^2}{8} - \frac{(\ln x)^4}{4} + c$.

3. The partial fraction decomposition of $\frac{2x^2+1}{(x^2-x-2)(x^2+x+1)}$ is

A. $\frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{x^2+x+1}$ [✓ B. $\frac{A}{x-2} + \frac{B}{x+1} + \frac{Cx+D}{x^2+x+1}$]

C. $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2-x+2}$ D. None of the above.

4. $\int 2\sin 2x \sin 7x\,dx =$

A. $\frac{\sin 5x}{5} - \frac{\sin 7x}{7} + c$ B. $-\frac{\cos 5x}{5} - \frac{\cos 7x}{14} + c$ [✓ C. $\frac{\sin 5x}{5} - \frac{\sin 7x}{14} + c$] D. $\frac{\sin 5x}{5} + \frac{\sin 7x}{7} + c$.

5. $\int \tan^{10} x \sec^4 x\,dx =$

A. $\frac{\sec^{13} x}{13} - \frac{\sec^{11} x}{11} + c$ B. $\frac{\sec^{11} x}{11} + \frac{\sec^{13} x}{13} + c$ [✓ C. $\frac{\tan^{11} x}{11} + \frac{\tan^{13} x}{13} + c$] D. $\frac{\sec^{11} x}{11} + \frac{\sec^{13} x}{13} + c$.

6. $\int_0^{\frac{\pi}{2}} \csc x\,dx =$

A. $\infty$. [✓ B. $-\infty$.] C. $\frac{1}{2}$. D. 0.

7. A suitable substitution to evaluate the integral $\int \frac{\sqrt{2x^2+3}}{x^2}\,dx$ is

A. $x = \frac{\sqrt{3}}{2}\tan\theta$. [✓ B. $x = \frac{\sqrt{3}}{\sqrt{2}}\tan\theta$.] C. $x = \frac{\sqrt{3}}{2}\sec\theta$. D. $x = \frac{\sqrt{3}}{\sqrt{2}}\tan 2\theta$.

8. $\int_1^{\infty} \frac{2+\cos x}{x^2}\,dx$ is

A. Divergent. [✓ B. Convergent.] C. Does not exist. D. None of the above.

1) $\int \sin^3 x \, dx = \int \sin x (1-\cos^2 x) \, dx$

$u = \cos x$

$du = -\sin x \, dx$

$= -\int \frac{\sin x (1-u^2) du}{\sin x} = \int (u^2-1) du$

$= \frac{u^3}{3} - u + c = \frac{\cos^3 x}{3} - \cos x + c$

2) $\int \ln x \, dx \Rightarrow \quad u = \ln x \quad dv = dx$

$du = \frac{dx}{x} \quad v = x$

$= x \ln x - \int dx = x \ln x - x + c$

3) $\frac{2x^2+1}{(x^2-x-2)(x^2+x+1)} = \frac{2x^2+1}{(x-2)(x+1)(x^2+x+1)} = \frac{A}{x-2} + \frac{B}{x+1} + \frac{Cx+E}{x^2+x+1}$

4) $\int 2 \sin 2x \sin 5x \, dx = 2 \int -\frac{1}{2} (\cos 7x - \cos 3x) dx$

$= -\frac{\sin 7x}{7} + \frac{\sin 3x}{3} + c$

5) $\int \tan^{10} x \sec^4 x \, dx \quad u = \tan x \longrightarrow u^2 = \sec^2 x - 1$

$du = \sec^2 x \, dx$

$\int \frac{u^{10} \sec^4 x \, du}{\sec^2 x} = \int u^{10} (u^2+1) du = \int (u^{12} + u^{10}) du$

$= \frac{u^{13}}{13} + \frac{u^{11}}{11} + c = \frac{\tan^{13} x}{13} + \frac{\tan^{4} x}{11} + c$

6) $\int_0^{\pi/2} \csc x \, dx = \lim_{a \to 0} \int_a^{\pi/2} \csc x \, dx = \lim_{a \to 0} -\ln|\csc x + \cot x| \Big]_a^{\pi/2}$

$= \lim_{a \to 0} -\left[ \overset{0}{\ln 1} - \ln|\csc a + \cot a| \right]$

$= -\lim_{a \to 0} \ln|\csc a + \cot a| = -\ln(\infty) = -\infty$

7) $x = \sqrt{\frac{3}{2}} \tan \theta$ 8) Not included

⑩

2. Evaluate the integral $\int \frac{\sqrt{1+\sqrt{x}}}{x}\,dx$

$u = \sqrt{x} \rightarrow u^2 = x$

$2u \cdot du = dx$

$= \int \frac{\sqrt{1+u}}{\cancel{u^2}} \cdot 2\cancel{u} \cdot du$

$= 2\int \frac{\sqrt{1+u}}{u} \cdot du \quad = 2\int u^{-1}\,(1+u)^{\frac{1}{2}} \cdot du$

$2\left[\begin{array}{l} (1+u)^{\frac{1}{2}} \longrightarrow \frac{1}{2}(1+u)^{-\frac{1}{2}} \\ \frac{1}{u}du \longrightarrow \ln u \end{array}\right.$

$2\left(\ln u\sqrt{1+u} - \frac{1}{2}\int (1+u)^{-\frac{1}{2}} \cdot \ln u \cdot du\right]$

$u = 1+\sqrt{x}$

$\boxed{u-1 = \sqrt{x}} \leftarrow du = \frac{1}{2\sqrt{x}} \cdot dx$

~~$2\sqrt{x}\,du$~~

$dx = 2\sqrt{x}\,du$

$\int \frac{\sqrt{u}}{\sqrt{x}\,x} \cdot 2\sqrt{x}\,du$

$2\int \frac{\sqrt{u}}{u-1} \cdot du$

$u = \sqrt{1+\sqrt{x}}$

$u^2 = 1+\sqrt{x}$

$u^2 - 1 = \sqrt{x}$

$2u \cdot du = \frac{1}{2\sqrt{x}} \cdot dx$

$dx = 4u\sqrt{x} \cdot du$

$\int \frac{u}{\sqrt{x}} \cdot 4u\sqrt{x} \cdot du$

$4\int \frac{u^2}{\sqrt{x}} \cdot du \rightarrow 4\int \frac{u^2}{u^2-1} \cdot du$

$$u^2 - 1 \,\overline{\big)\, u^2}\quad 1$$

$$\underline{-u^2 + 1}$$

$$1$$

$= 4\int 1 + \frac{1}{u^2-1} \cdot du$

$\frac{A}{(u-1)} + \frac{B}{(u+1)}$ ل $\longrightarrow A(u+1) + B(u-1) = 1$

$u = -1$

$u = 1$

$-2B = 1 \leftrightarrow \boxed{B = -\frac{1}{2}}$

$2A = 1 \rightarrow \boxed{A = \frac{1}{2}}$

تبع كل

3. Evaluate the integral $\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx$, and determine whether it is convergent or divergent.

$$\int_a^1 \frac{\ln x}{\sqrt{x}} = \int_a^1 \ln x \; x^{-\frac{1}{2}} . dx$$

$$= * \ln x \longrightarrow \frac{1}{x} . dx$$

$$x^{-\frac{1}{2}} \longrightarrow 2 x^{\frac{1}{2}}$$

$$2\sqrt{x} \ln x \Big]_a^1 - 2\int_a^1 \frac{\sqrt{x}}{x}$$

$$= 2\sqrt{x} \ln x \Big]_a^1 - 2\int_a^1 \frac{1}{\sqrt{x}} . dx$$

$$= 2\sqrt{x} \ln x \Big]_a^1 - 2\int_a^1 x^{-\frac{1}{2}} . dx$$

$$= 2\sqrt{x} \ln x \Big]_a^1 - 2 x^{\frac{1}{2}} . 2 \Big]_a^1$$

$$48\sqrt{x}$$

$$(0 - 2\sqrt{a} \ln a) - (4 - 4\sqrt{a})$$

$$\lim_{a \to 0^+} (-2\sqrt{a} \ln a) - (4 - 4\sqrt{a})$$

$$= (-2\sqrt{0^+} \ln 0^+) - (4 - 4\sqrt{0^+})$$

$$= -0 * \infty - 4 + 0 = \boxed{-\infty} \text{ Dne}$$

Divergent

4

Excellent $\frac{29}{30}$

Jordan University
Mathematics Department
Calculus II, Second Exam, 12/8/2017

Student's Name: سجود عمر مطلق اليوسف . Student's Number: [redacted]

Instructor's Name: د. عبد الرحمن يوسف . Lecture Time: 9:30 - 11 .

1) Find the following integrals

a) ( 4 points ) $\int 5\sqrt{2x-x^2}\,dx$

4

$2x - x^2 = -x^2 + 2x$
$= -(x^2 - 2x + 1 - 1)$
$= -((x-1)^2 - 1)$
$= (1 - (x-1)^2)$

$\int 5\sqrt{1-(x-1)^2}\,dx$

let $x - 1 = \sin\theta$ , $(-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}) \Rightarrow \theta = \sin^{-1}(x-1)$
$dx = \cos\theta\, d\theta$

$\int 5\sqrt{1-\sin^2\theta} \cdot \cos\theta\, d\theta$ $\quad \cos^2\theta$

$\int 5 . \cos\theta . \cos\theta\, d\theta$

$\int 5\cos^2\theta\, d\theta$ ✓

$5\int (\frac{1}{2} + \frac{1}{2}\cos 2\theta)\,d\theta$

$5(\frac{\theta}{2} + \frac{\sin 2\theta}{4}) + C$

$5\left(\frac{\sin^{-1}(x-1)}{2} + \frac{2.(x-1)\sqrt{1-(x-1)^2}}{4}\right) + C$

$\frac{5}{2}\sin^{-1}(x-1) + \frac{5}{2}(x-1)\sqrt{1-(x-1)^2}$
$+ C$

$\sin 2\theta = 2. \sin\theta . \cos\theta$
$= 2.(x-1) . \sqrt{1-(x-1)^2}$

b) ( 4 points ) $\int \frac{2\,dx}{2+\cos(x)}$

let $t = \tan(\frac{x}{2})$ , $[-\pi < x < \pi]$

$dx = \frac{2}{1+t^2}dt$ , $\cos(x) = \frac{1-t^2}{1+t^2}$

$\int \frac{2dx}{2+\cos(x)} = \int \frac{2}{2 + \frac{1-t^2}{1+t^2}} \cdot \frac{2}{1+t^2}dt$

$= \int \frac{4}{\frac{2(1+t^2)+(1-t^2)}{\cancel{1(1+t^2)}}} \cdot \frac{dt}{\cancel{(1+t^2)}}$

$= \int \frac{4\ dt}{2 + 2t^2 + 1 - t^2}$

$\int \frac{4\ dt}{t^2 + 3}$

$4\int \frac{dt}{t^2 + 3}$ ✓

$4 . \frac{1}{\sqrt{3}} \tan^{-1}(\frac{t}{\sqrt{3}}) + C$

$\frac{4}{\sqrt{3}} \tan^{-1}\left(\frac{\tan(\frac{x}{2})}{\sqrt{3}}\right) + C$ ✓

4

1) a) $\int 5\sqrt{2x-x^2}\,dx =$

$\int 5\sqrt{-(x^2-2x)}\,dx =$

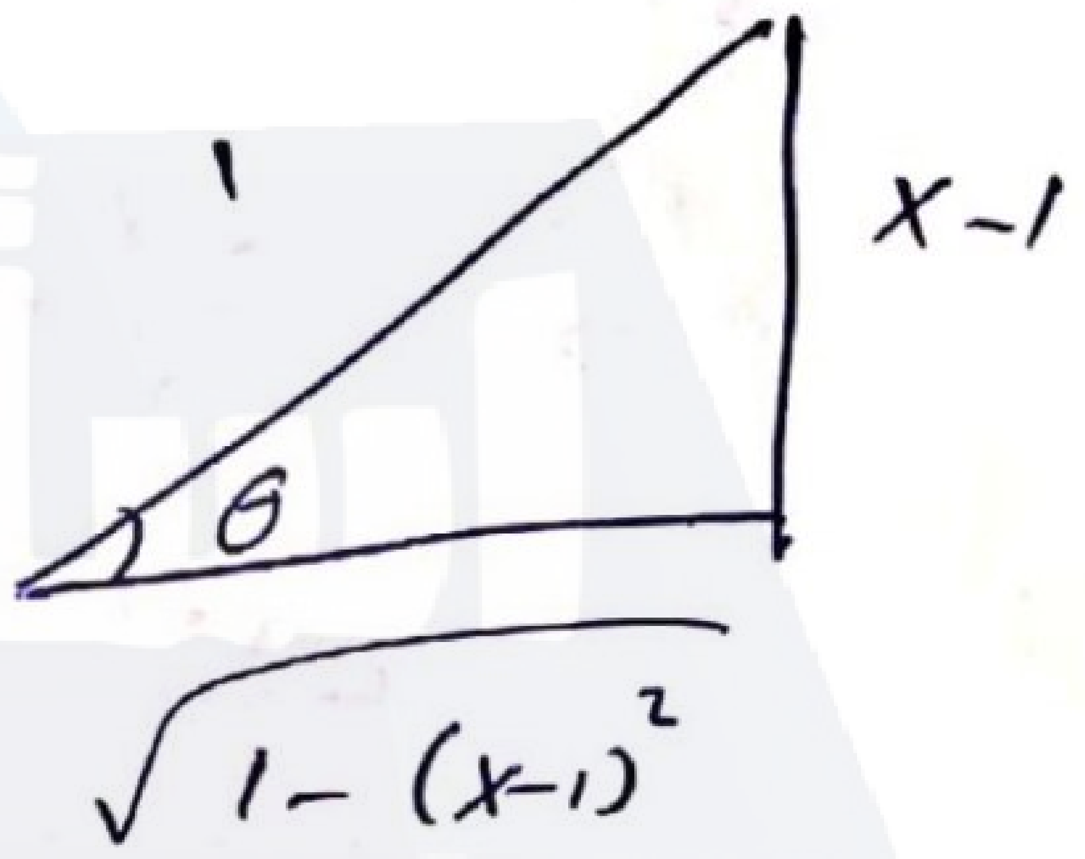
$\int 5\sqrt{-[[x^2-2x+1-1]}\,dx =$

$\int 5\sqrt{-[(x-1)^2-1]}\,dx =$ $\qquad x-1=\sin\theta$

$\int 5\sqrt{1-(x-1)^2}\,dx =$ $\qquad dx=\cos\theta\,d\theta$

$\int 5\cos^2\theta\,d\theta = \frac{5}{2}\int(1+\cos 2\theta)\,d\theta$

$= \frac{5}{2}\left[\theta+\frac{\sin 2\theta}{2}\right]+c$



$= \frac{5}{2}\left[\theta+\sin\theta\cos\theta\right]$

$= \frac{5}{2}\left[\sin^{-1}(x-1)+(x-1)\sqrt{1-(x-1)^2}+c\right.$

b) $\int\frac{2dx}{2+\cos x}$ $\qquad$ let $u=\tan\frac{x}{2}$, $dx=\frac{2du}{1+u^2}$, $\cos x=\frac{1-u^2}{1+u^2}$

$= \int\frac{2}{2+\frac{1-u^2}{1+u^2}}\cdot\frac{2du}{1+u^2} = \int\frac{4du}{\frac{2(1+u^2)+1-u^2}{\cancel{1+u^2}}}\frac{1}{(\cancel{1+u^2})}$

$= \int\frac{4du}{u^2+3} = \frac{4}{\sqrt{3}}\tan^{-1}\left(\frac{u}{\sqrt{3}}\right)$

$= \frac{4}{\sqrt{3}}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{\sqrt{3}}\right)+c$

2) ( 3 points ) Write out the form of the partial fraction decomposition: ( (**Do not evaluate the coefficients**).

$$\frac{x^2+7}{(x^3-1)(x^2+x+1)} = \frac{x^2+7}{(x-1)(x^2+x+1)^2}$$

$$= \frac{a}{(x-1)} + \frac{bx+C}{(x^2+x+1)} + \frac{dx+e}{(x^2+x+1)^2}$$ ✓

3

3) ( 4 points ) Evaluate the integral or show that it is divergent

$$\int_3^5 \frac{dx}{x^2-3x} = \int_3^5 \frac{dx}{x(x-3)}$$

$x(x-3)$

$x=0$ , $\boxed{x=3}$

$$\int \frac{dx}{x(x-3)} = \int \frac{1\,dx}{3x} - \int \frac{1}{3(x-3)}$$

$$\frac{1}{x(x-3)} = \frac{a}{x} + \frac{b}{x-3}$$

$$1 = ax + b(x-3)$$

let $x=0 \Rightarrow \boxed{b=-\frac{1}{3}}$

let $x=3 \Rightarrow \boxed{a=\frac{1}{3}}$

$$\int \frac{dx}{x(x-3)} = \frac{1}{3}\ln|x| - \frac{1}{3}\ln|x-3|$$

$$= \lim_{u\to 3^+} \int_u^5 \frac{dx}{x(x-3)}$$ ✓

$$= \lim_{u\to 3} \frac{1}{3}\ln|x| - \frac{1}{3}\ln|x-3| \Big|_u^5$$

$$= \lim_{u\to 3} \left[\frac{1}{3}\ln 5 - \frac{1}{3}\ln 2\right] - \left[\frac{1}{3}\ln|u| - \frac{1}{3}\ln|u-3|\right.$$

$$= \frac{1}{3}(\ln 5 - \ln 2) - \left[\frac{1}{3}\ln 3 - \frac{1}{3}\ln|0|\right]$$

$$\frac{1}{3}\ln\left(\frac{5}{2}\right) - \frac{1}{3}\ln 3 + \infty = \infty \quad \therefore \text{divergent}$$ ✓

4

4) ( 4 points ) Set up the integral that finds the area of the region bounded by $x + y^2 = 3y$ and $y - x = 3$. (Do not evaluate the integral).

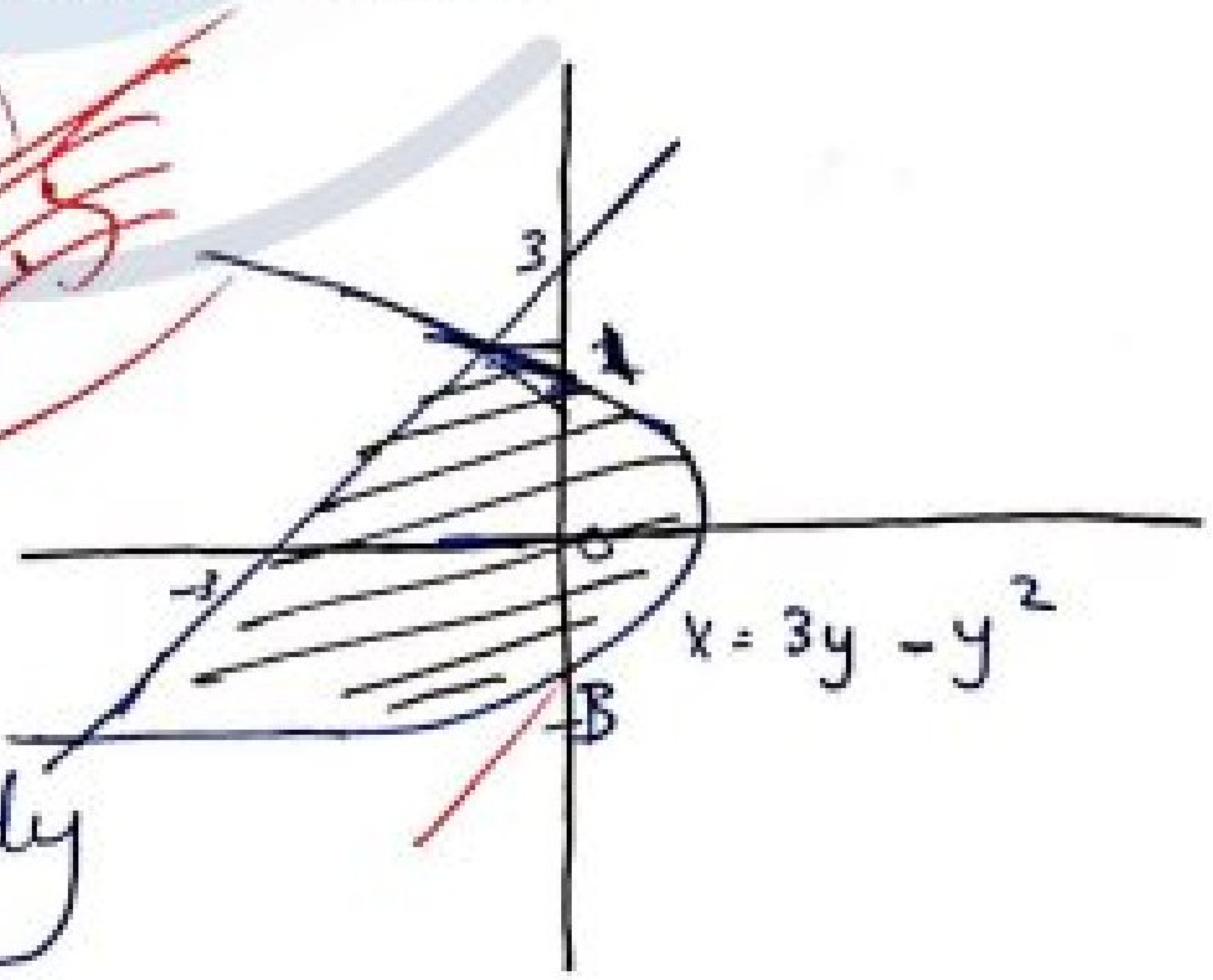
$A = A_1 + A_2$

~~$\int_3^0 (3y-y^2)\,d\,y + \int_0^1$~~

$$= \int_{-3}^{0} \left[(3y-y^2) - (y-3)\right] dy + \int_0^1 \left[(3y-y^2) - (y-3)\right] dy$$

~~2.5~~

3



2) $$\frac{x^2+7}{(x^3-1)(x^2+x+1)} = \frac{x^2+7}{(x-1)(x^2+x+1)^2} = \frac{A}{x-1} + \frac{Bx+c}{x^2+x+1} + \frac{Dx+E}{(x^2+x+1)^2}$$

3) $$\int_3^5 \frac{dx}{x^2-3x} = \lim_{a\to 3^+} \int_a^5 \frac{dx}{x^2-3x} = \lim_{a\to 3^+} \int_a^5 \frac{dx}{x(x-3)}$$

$$= \lim_{a\to 3^+} \int_a^5 \left(\frac{A}{x} + \frac{B}{x-3}\right) dx$$

$1 = A(x-3) + Bx$

$x=0 \Rightarrow A = -1/3$

$x=3 \Rightarrow B = 1/3$

$$= \lim_{a\to 3^+} \int_a^5 \left(\frac{-1/3}{x} dx\right) + \int_a^5 \frac{1/3}{x-3} dx$$

$$= \lim_{a\to 3^+} -1/3 \ln x \Big|_a^5 + 1/3 \ln|x-3| \Big|_a^5$$

$$= \lim_{a\to 3^+} \frac{-1}{3}\ln 5 + \overset{\frac{1}{3}\ln 3}{\cancel{\frac{1}{3}\ln a}} + \frac{1}{3}\ln 2 - \underline{\overset{-\infty}{\cancel{\frac{1}{3}\ln|a-3|}}}$$

$= \boxed{\infty}$ (divergent)

---

4) $x = y-3$ , $x = 3y-y^2$

$y-3 = 3y-y^2$

$y^2-2y-3=0$

$y=3$, $y=-1$

$$A = \int_{-1}^{3} (3y-y^2)-(y-3)\,dy$$

#

5) Set up the integral that gives the volume when the region enclosed by $y = e^x$, $y = 1$, and $x = 4$ is revolved about the line $x = -3$.

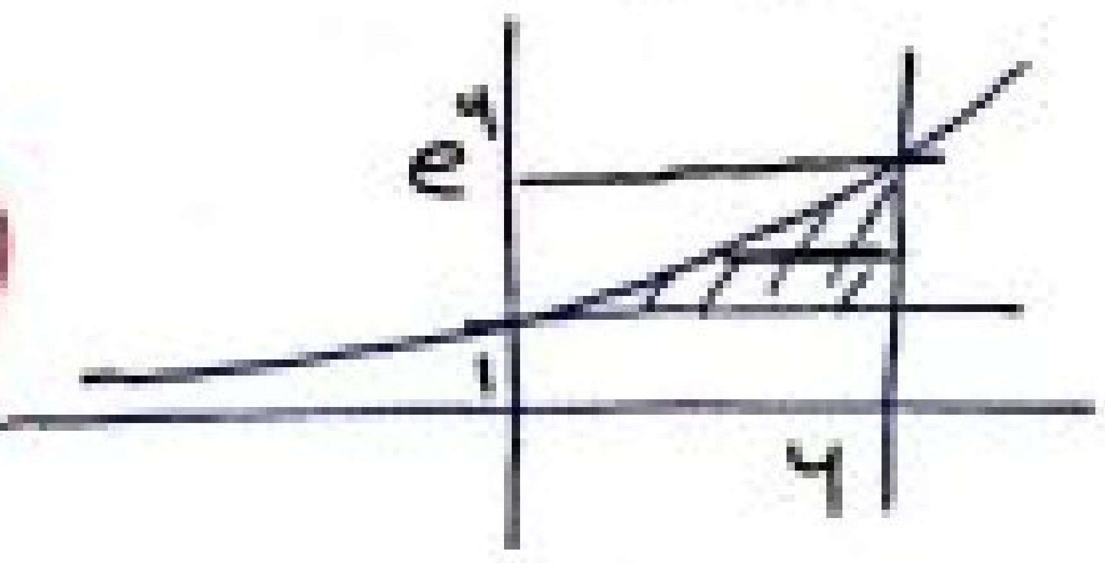
$\ln y = x$

a) ( 3 points ) Using disk (washers) method, (Do not evaluate the integral).

$$V = \pi \int_a^b \left( f(y)^2 - g(y)^2 \right) dy$$

$$= \pi \int_1^{e^4} (7)^2 - (\ln y + 3)^2 \, dy$$

✓ 3

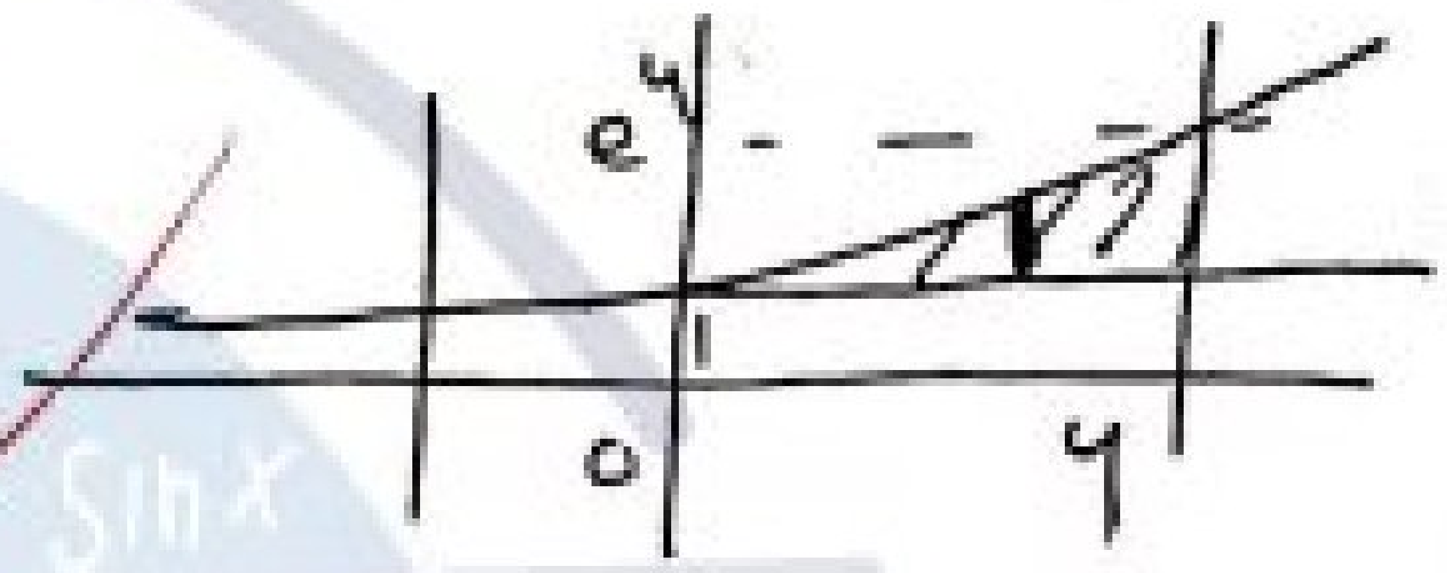


b) ( 3 points ) Using cylindrical shell method, (Do not evaluate the integral).

$$V = 2\pi \int_a^b (\text{radius})(\text{hight})$$

$$= 2\pi \int_0^4 (x+3)(e^x - 1) \, dx$$

✓ 3



6) ( 5 points ) Show that

$$\int \csc^n(x)\,dx = \frac{-1}{n-1}\csc^{n-2}(x)\cot(x) + \frac{n-2}{n-1}\int \csc^{n-2}(x)\,dx$$

$$\int \underline{\csc^{(n-2)} x} \cdot \csc^2 x \, dx$$

$u = \csc^{(n-2)} x \qquad dv = \csc^2 x$

$du = (n-2)\csc^{n-3} x \cdot -\csc x \cot x\,dx \qquad v = -\cot x$

$\int u \cdot dv = u \cdot v - \int v \cdot du$

✓ 5

$$\int \csc^{(n-2)} x = -\csc^{(n-2)} x \cot x - \int \underline{(n-2)\csc^{(n-2)} x} \cdot \cot^2 x \, dx$$

$$\int \csc^{(n-2)} = -\csc^{n-2} \cot x - (n-2) \int \underbrace{\csc^{(n-2)} x}_{dv} \cdot \underbrace{\cot^2 x}_{u} \, dx$$

$\cancel{u = \cot^2 x} \qquad \cancel{dv = \csc^{(n-2)} x}$

$\cancel{du = 2\cot x \cdot -\csc^2 x} \qquad \cancel{v = \int \csc^{n-2} x}$

$= \cancel{\csc^{n-2}}$

$\cancel{-\csc^{(n-2)} x \cot x - \left[\cot^2 \int \csc^{(n-2)} x \right.}$ [illegible]

⟹

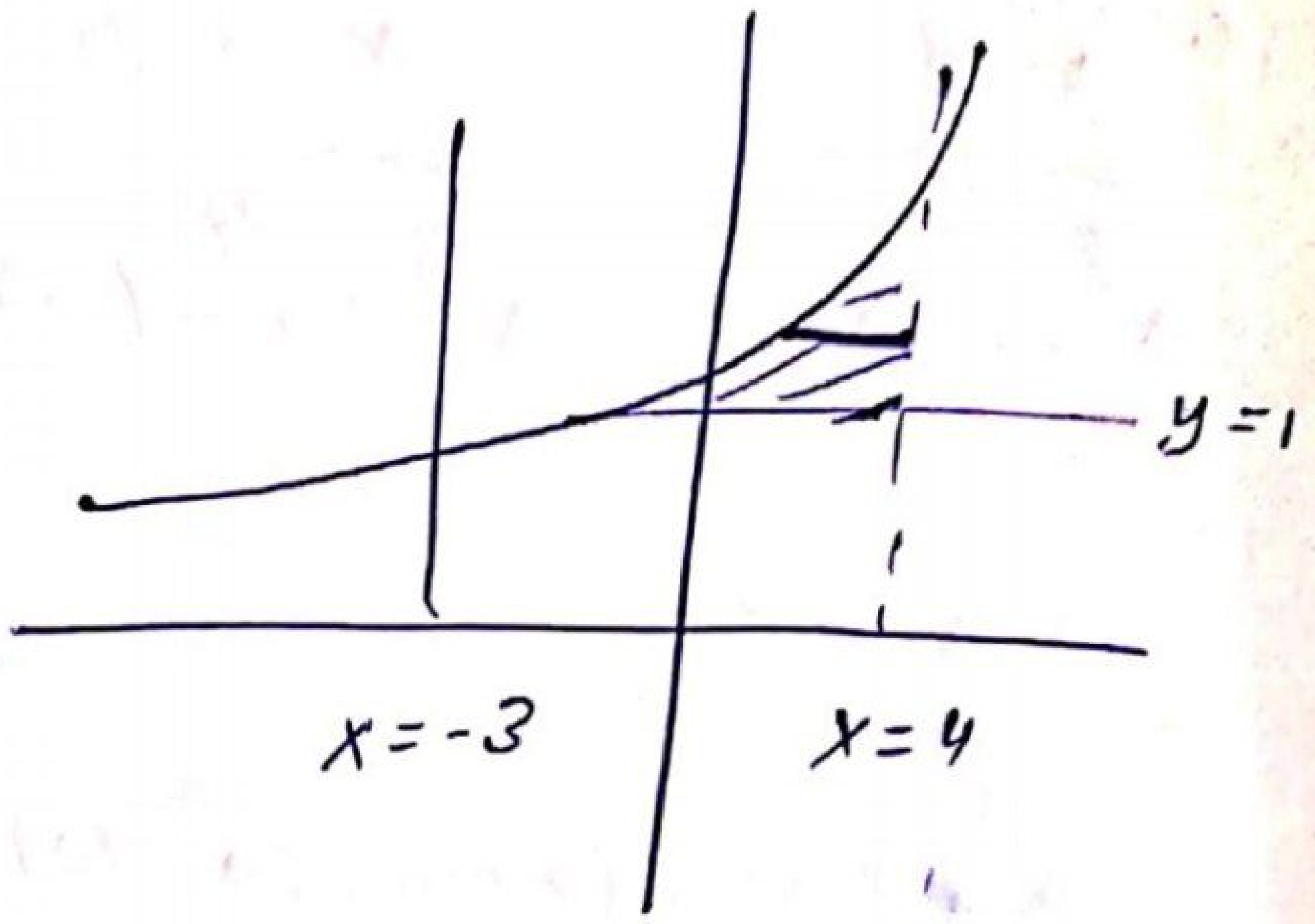
التكملة في الخلف ⟸

5) a) $r_1 = \ln y + 3$

$r_2 = 7$

$$v = \pi \int_{a_1}^{a_2} (r_2^2 - r_1^2)\, dy$$

$$= \pi \int_{1}^{e^4} [49 - (\ln + 3)^2]\, dy$$



b) $v = 2\pi \int r . h$

$$= 2\pi \int_{0}^{4} (x+3)(e^x - 1)\, dx$$

---

6) $\int \csc^n x dx = \int \csc x^{n-2} . \csc x^2 dx$

$u = \csc x^{n-2}$ $\quad$ $dv = \csc^2 x dx$

$du = -(n-2) \csc x^{n-3} . \csc x \cot x dx$ $\quad$ $v = -\cot x$

$\int \csc^n x dx = -\cot x \csc x^{n-2} - \int (n-2) \csc x^{n-2} \cot^2 x dx$

$\int \csc^n x dx = -\cot x \csc x^{n-2} - (n-2) \int \csc x^{n-2} (\csc^2 x - 1) dx$

$\int \csc^n x dx = -\cot x \csc x^{n-2} - (n-2) \int [\csc x^{n} - \csc x^{n-2}] dx$

$\int \csc^n x dx = -\cot x \csc x^{n-2} - \int (n-2) \csc^n x + \int (n-2) \csc x^{n-2} dx$

$\int \csc^n x dx [n-2+1] = -\cot x \csc x^{n-2} + (n-2) \int \csc x^{n-2} dx$

$(n-1) \int \csc^n x dx = -\cot x \csc x^{n-2} + (n-2) \int \csc x^{n-2} dx$

$$\int \csc^n x dx = \frac{-\cot x \csc x^{n-2}}{n-1} + \frac{n-2}{n-1} \int \csc x^{n-2} dx \quad \#$$

$$\int \csc^{n} x = -\csc x \cot x - (n-2)\int \csc^{n-2} \cdot \cot^{2} x \, dx$$

$$-(n-2)\int \csc^{(n-2)} x \left(\csc^{2} x - 1\right) dx$$

$$\int \csc^{n} x \, dx = -\csc x \cot x - (n-2)\left[\int \csc^{n} dx - \int \csc^{n-2} x \, dx\right]$$

$$(n-2+1)\int \csc^{n} x \, dx = -\csc x \cot x + (n-2)\int \csc^{n-2} x \, dx$$

$$\frac{(n-1)\int \csc^{n} x \, dx}{n-1} = \frac{-\csc x \cot x + (n-2)\int \csc^{n-2} x \, dx}{(n-1)}$$

$$\boxed{\int \csc^{n} x \, dx = \frac{-1}{(n-1)}\csc x \cot x + \frac{(n-2)}{(n-1)}\int \csc^{n-2} x \, dx}$$

#

43 seat number

$\frac{25}{25}$ Excellent

| | | |
|---|---|---|
| الجامعة الأردنية | 0301102 تفاضل وتكامل 2 | الاختبار الأول: 2017/3/16 |
| اسم الطالب: ..لما زكي درويش.................. | | مدرس المادة: د.بلال النابلسي |
| الرقم الجامعي:....................................... | | وقت المحاضرة: ١٢ الى ١ |

يتكون الامتحان من جزئين في ثلاث صفحات:

## Part I

14

**In questions 1 – 7, fill in the blanks to get correct statements (2 marks each)**

[1] The partial fractions decomposition for $\frac{x^2+5}{x^7-100x^3}$ is (Don't evaluate the constants)

$\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^3}+\frac{D}{x-\sqrt{10}}+\frac{E}{x+\sqrt{10}}+\frac{Fx+G}{x^2+10}$

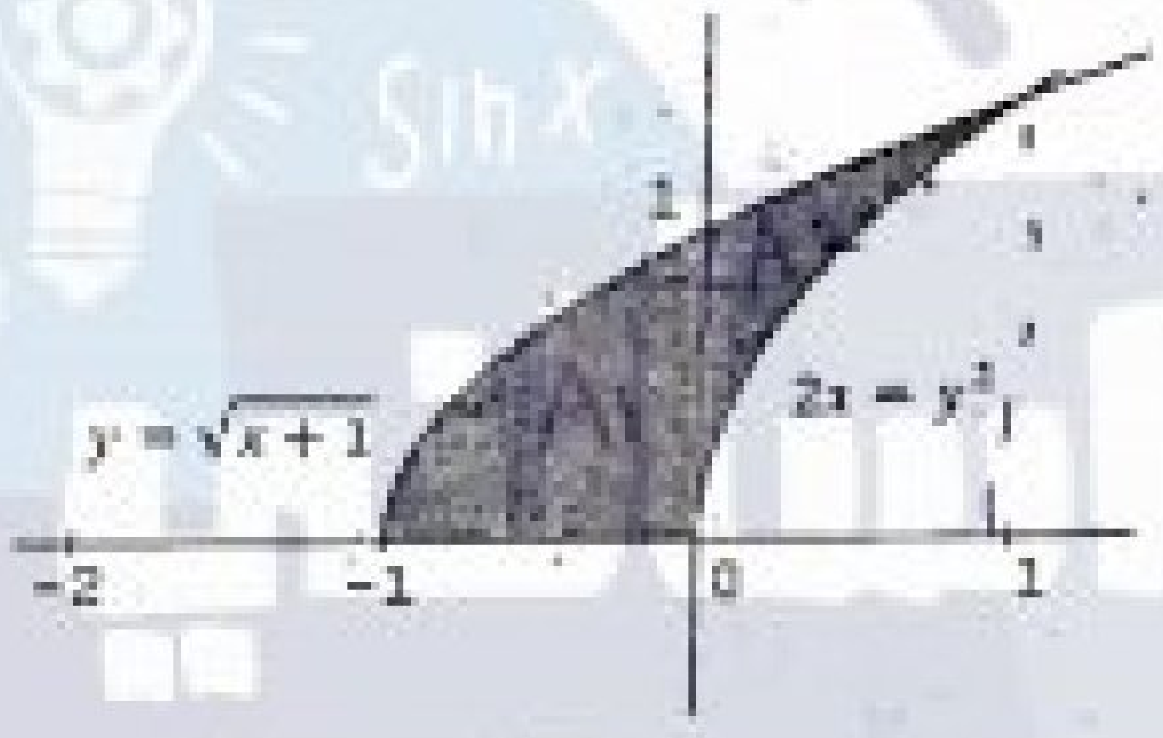
$\left(\frac{-6}{2}\right)^2$ $(x-3)^2-9$ $x-3=3\sec\theta$

[2] The trigonometric substitution that solves the integral $\int\sqrt{x^2-6x}\,dx$ is

$(x-3)=3\sec\theta$

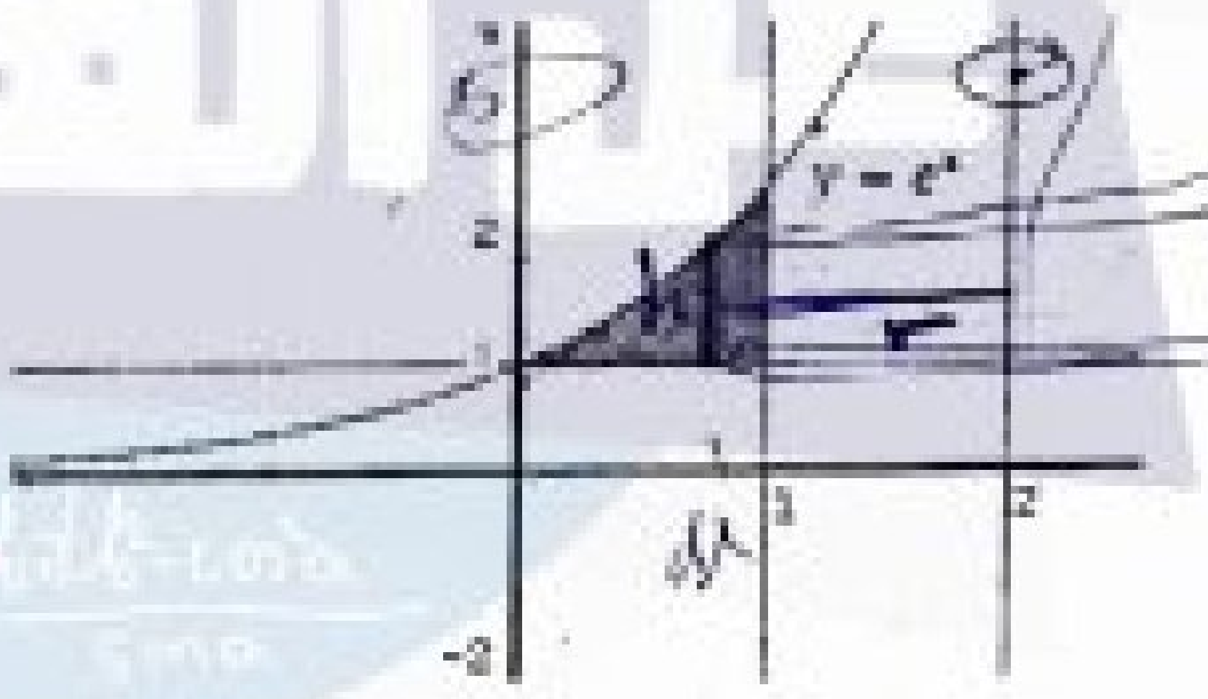
[3] The integral that evaluates the area of the shaded region in the companion figure is

$\int_{-1}^{0}\sqrt{x+1}\,dx+\int_{0}^{2}\left(\sqrt{x+1}-\sqrt{2x}\right)dx$



[4] The integral that represents the volume of the solid obtained by rotating the shaded region about $x = 2$ using the method of cylindrical shells is

$2\pi\int_{0}^{1}(2-x)\,(e^x-1)\,dx$



$2\pi\,(H)(R)$

$2\pi\,(2-x)(e^x-1)$

[5] $\int\frac{x^3}{\sqrt{1-x^8}}\,dx =$ $-\frac{1}{4}\sin^{-1}\sqrt{1-x^8}+C$

[6] Write down the improper integrals (but don't evaluate) using limits to solve

$\int_0^{\infty}\frac{e^x}{x-3}\,dx = \lim_{L\to3^-}\int_0^L\frac{e^x}{x-3}\,dx+\lim_{m\to3^+}\int_m^4\frac{e^x}{x-3}+\lim_{R\to\infty}\int_4^R\frac{e^x}{x-3}\,dx$

[7] $\int\sin(5x)\cos(2x)\,dx = \int\frac{1}{2}(\sin 7x+\sin 3x)\,dx = \boxed{\frac{1}{2}\left(\frac{-\cos 7x}{7}+\frac{-\cos 3x}{3}\right)+C}$

$\frac{1}{2}\left(\sin(5-2)x+\sin(7)x\right)$

$\sin(3)+\sin(7)$

1) $\dfrac{x^2+5}{x^7-100x^3} = \dfrac{x^2+5}{x^3(x^4-100)} = \dfrac{x^2+5}{x^3(x^2-10)(x^2+10)}$

$= \dfrac{x^2+5}{x^3(x-\sqrt{10})(x+\sqrt{10})(x^2+10)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x^3} + \dfrac{D}{x-\sqrt{10}} + \dfrac{E}{x+\sqrt{10}} + \dfrac{Fx+G}{x^2+10}$

---

2) $\int \sqrt{x^2-6x} = \int \sqrt{x^2-6x+9-9} = \int \sqrt{(x-3)^2-9}\, dx$

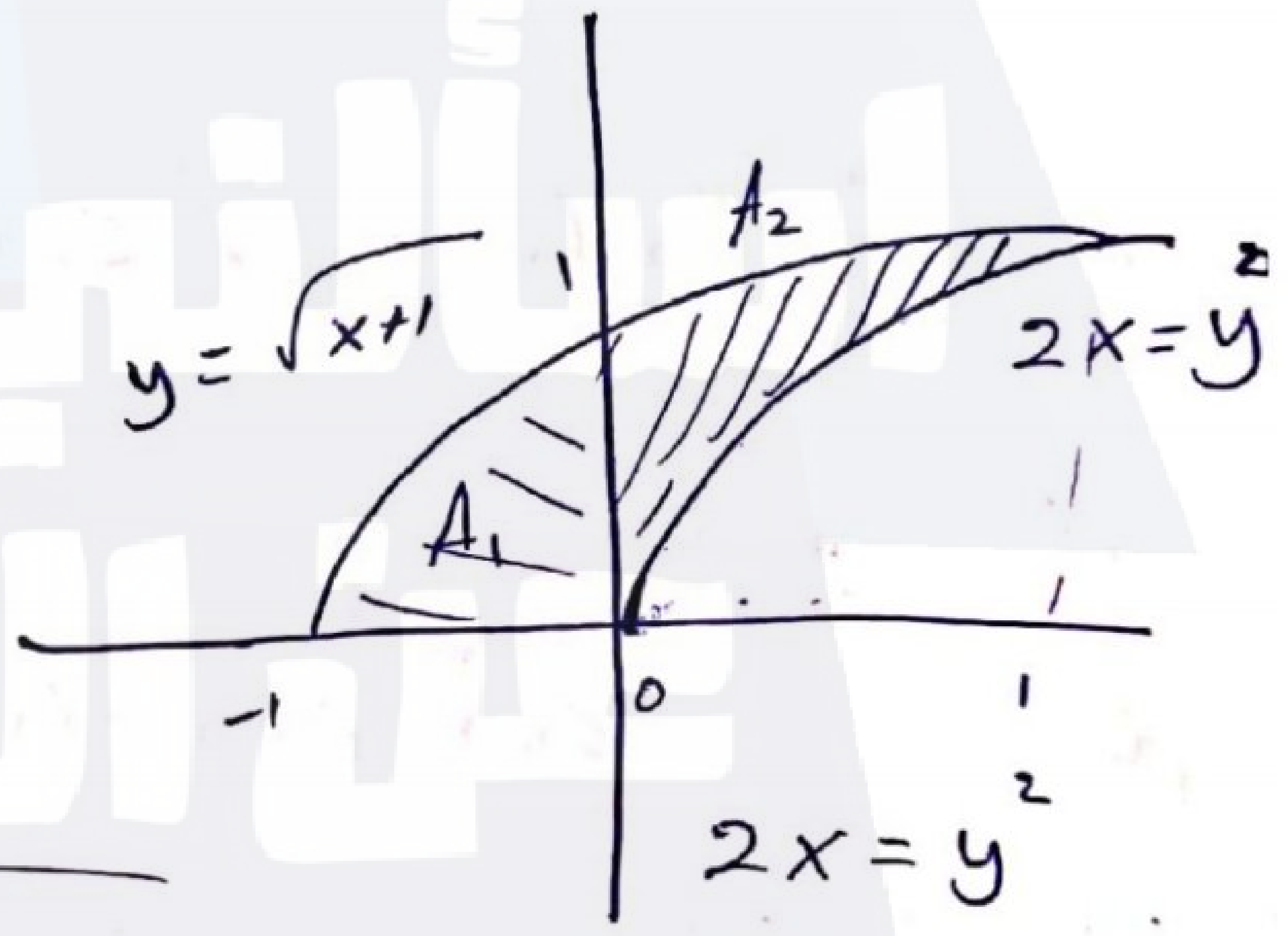
$x-3 = 3\sec\theta$

---

3) area $= A_1 + A_2$

$A_1 = \int_{-1}^{0} \sqrt{x+1}\, dx$

$A_2 = \int_{0}^{1} (\sqrt{x+1} - \sqrt{2x})\, dx$



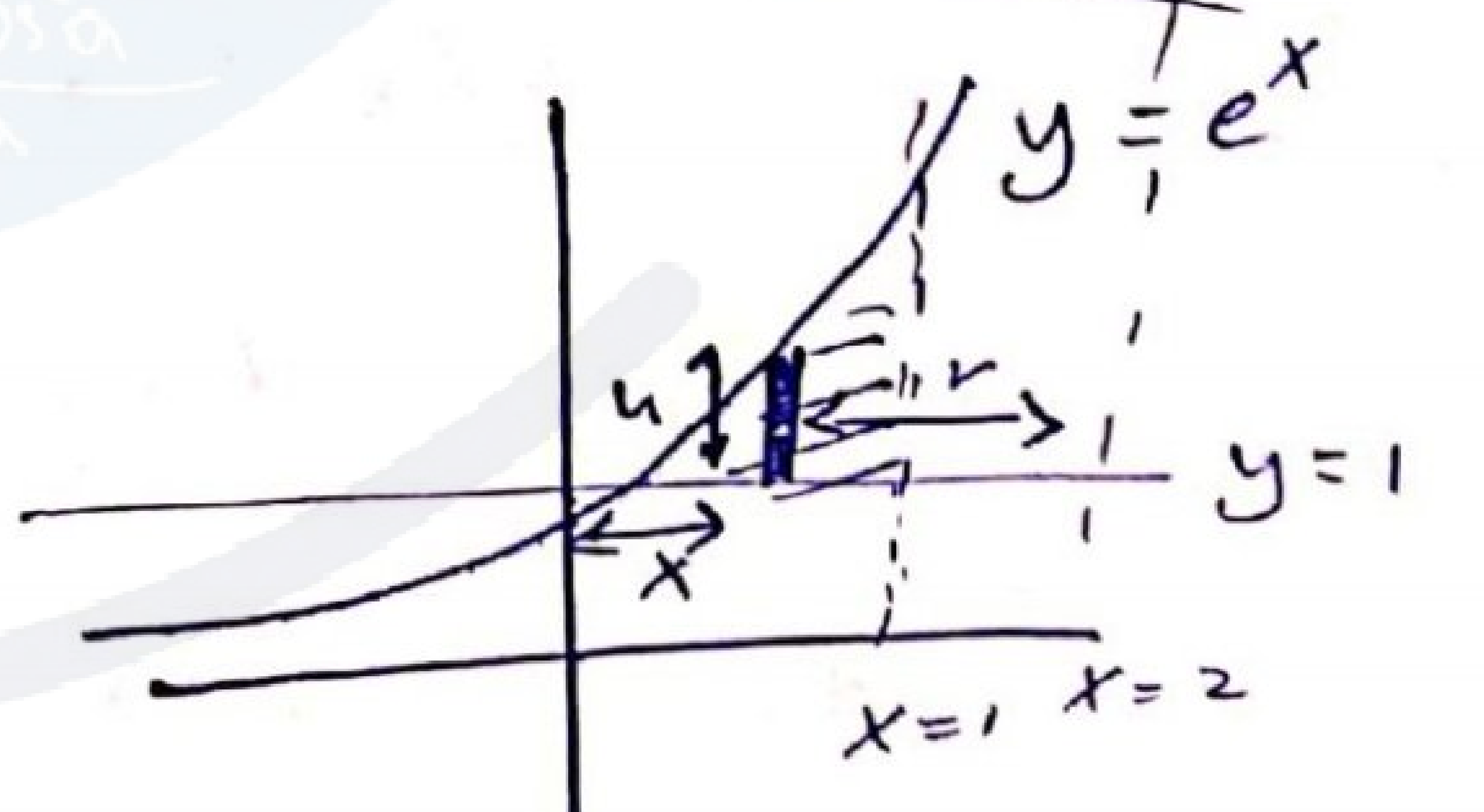
$\sqrt{2x} = y$

---

4) $v = 2\pi \int r . h$

$r = 2-x \quad , \quad h = e^x - 1$

$v = 2\pi \int_0^1 (2-x)(e^x-1)\, dx$



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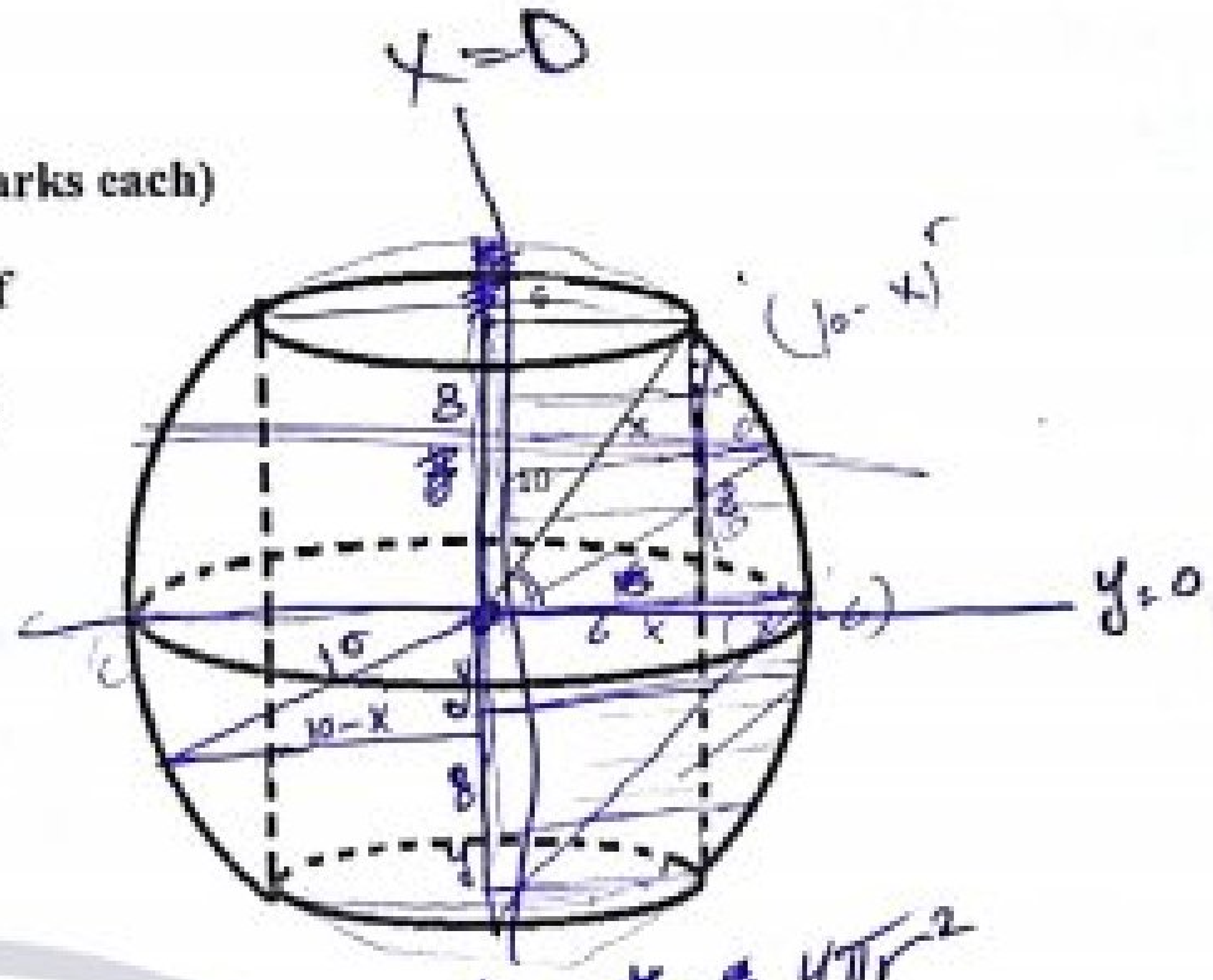
5) $\int \dfrac{x^3}{\sqrt{1-x^8}}\, dx \qquad u = x^4$
$\qquad du = 4x^3 dx$

$= \int \dfrac{\cancel{x^3} du}{4\cancel{x^3}\sqrt{1-u^2}} = \dfrac{1}{4} \int \dfrac{1}{\sqrt{1-u^2}} du = \dfrac{1}{4} \sin^{-1} u + C$

$= \dfrac{1}{4} \sin^{-1} x^4 + C$

## Part II

**In questions 8 – 10, give a detailed answer (4 marks each)**



Area $= 4\pi r^2$

[8] A sphere of radius 10 is drilled by a cylinder of radius 6. Find the volume of the resulting solid using the method of slicing by finding appropriate cross sections.

③

$\perp$ y axis

$$\int A_{sphere}\,dy - \int A_{cylinder}\,dy$$

$A_{sphere} = 4\pi r^2$

$= 4\pi(10-x)^2$

$A_{cylind} = 2\pi \cdot 6 \cdot 16$

$A_{cylind} = 2\pi\, 96$

$$\int_{-8}^{8} 4\pi(100-y^2)\,dy - \int_{-8}^{8} 192\pi\,dy$$

$$4\pi\left(100y - \frac{y^3}{3}\right)\Big]_{-8}^{8} - 192\pi y\Big]_{-8}^{8}$$

$100 = y^2 + (10-x)^2$

$-y^2 \quad -y^2$

$(100-y^2) = (10-x)^2$

[9] Evaluate $\int \frac{dx}{x\sqrt{16x+1}}$

④

$y = \sqrt{16x+1}$

$y^2 = 16x+1 \Rightarrow \frac{y^2-1}{16} = x$

$\frac{2y\,dy}{16} = dx$

$\frac{y}{8}\,dy = dx$

$$\int \frac{y\,dy}{8xy}$$

$$\int \frac{1}{8} \cdot \frac{\cancel{16}\ 2}{(y^2-1)}\,dy$$

$$\int \frac{2}{y^2-1}\,dy \Rightarrow \quad \frac{2}{y^2-1} = \frac{A}{y-1} + \frac{B}{y+1}$$

$2 = A(y+1) + B(y-1)$ $\quad$ $y=1$, $y=-1$

$1 = A$

$-1 = B$

$$\int \frac{2}{y^2-1}\,dy = \int \frac{1}{y-1}\,dy - \int \frac{1}{y+1}\,dy$$

$$= \ln|y-1| - \ln|y+1| + C$$

$$= \ln\left|\sqrt{16x+1} - 1\right| - \ln\left|\sqrt{16x+1} + 1\right| + C$$

6) $\int_{0}^{\infty} \frac{e^{x}dx}{x-3} = \int_{0}^{3} \frac{e^{x}dx}{x-3} + \int_{3}^{\infty} \frac{e^{x}dx}{x-3}$

$= \int_{0}^{3} \frac{e^{x}dx}{x-3} + \int_{3}^{5} \frac{e^{x}dx}{x-3} + \int_{5}^{\infty} \frac{e^{x}dx}{x-3}$ ⑳

$= \lim_{a\to 3^{-}} \int_{0}^{a} \frac{e^{x}dx}{x-3} + \lim_{b\to 3^{+}} \int_{b}^{5} \frac{e^{x}dx}{x-3} + \lim_{c\to \infty} \int_{5}^{c} \frac{e^{x}dx}{x-3}$

---

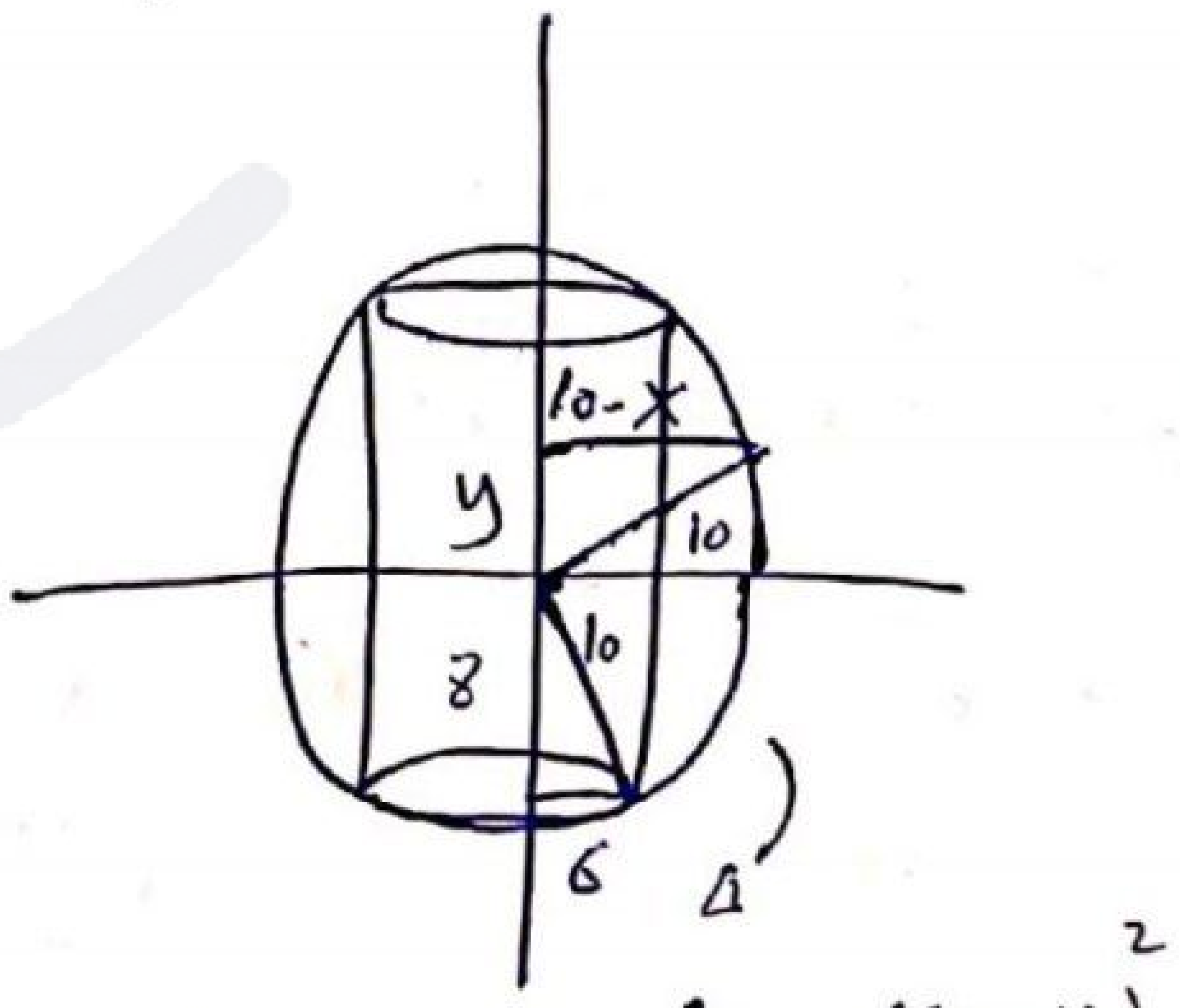
7) $\int \sin 5x \cos 2x \, dx = \int (\frac{1}{2} \sin 7x + \frac{1}{2} \sin 3x) dx$

$= -\frac{\cos 7x}{14} - \frac{\sin 3x}{6} + C$ ⑳

---

8) $v = v_{sphere} - v_{cylinder}$

$= \int area(sphere)\, dy - \int area(cylinder)\, dy$



$100 = y^{2} + (10-x)^{2}$

area (sphere) $= \pi (10-x^{2})^{2}$

$= \pi (10-x)^{2}$

$= \pi (100-y^{2})$

area (Cylinder) $= \pi r^{2} = \pi 36$

$v = \int_{-8}^{8} \pi (100-y^{2})\, dy - \int_{-8}^{8} 36\pi\, dy$

\#

㉑

4

~~(10)~~ (a) Use integration by parts to show that

$$\int_1^e x^n(\ln x)^m\,dx = \frac{e^{n+1}}{n+1} - \frac{m}{n+1}\int_1^e x^n(\ln x)^{m-1}\,dx$$

[redacted]

$dV = x^n$

$u = (\ln x)^m$

$V = \frac{x^{n+1}}{n+1}$

$du = m(\ln x)^{m-1}\cdot\frac{1}{x}$

$$(\ln x)^m\frac{x^{n+1}}{n+1} - \int\frac{m}{n+1}(\ln x)^{m-1}\frac{x^{n+1-1}}{x}$$

$$(\ln x)^m\frac{x^{n+1}}{n+1}\Big]_1^e - \frac{m}{n+1}\int x^n(\ln x)^{m-1}\,dx$$

$$= \boxed{\frac{e^{n+1}}{n+1} - \frac{m}{n+1}\int_1^e x^n(\ln x)^{m-1}\,dx}$$

(b) Use the formula in (a) to evaluate $\int_1^e x^6(\ln x)^2\,dx$

$$\int_1^e x^6(\ln x)^2 = \frac{e^7}{7} - \frac{2}{7}\int_1^e x^6(\ln x)\,dx$$

$$= \frac{e^7}{7} - \frac{2}{7}\left(\frac{e^7}{7} - \frac{1}{7}\int_1^e x^7\,dx\right)$$

$$= \frac{e^7}{7} - \frac{2}{7}\left(\frac{e^7}{7} - \frac{1}{7}\left(\frac{x^8}{8}\Big]_1^e\right)\right)$$

$$= \frac{e^7}{7} - \frac{2}{7}\left(\frac{e^7}{7} - \frac{1}{7}\left(\frac{e^8}{8} - \frac{1}{8}\right)\right)$$

$$= \frac{e^7}{7} - \frac{2}{7}\left(\frac{e^7}{7} - \frac{e^8}{54} + \frac{1}{54}\right).$$

9) $\int \frac{dx}{x\sqrt{16x+1}}$

$u = \sqrt{16x+1}$

$u^2 = 16x+1$

$2u\,du = 16\,dx$

$= \int \frac{2u\,du}{x \cdot u \cdot 16}$

$\frac{u^2-1}{16} = x$

$= \frac{1}{8}\int \frac{du}{x}$

$= \frac{1}{8}\int \frac{16\,du}{u^2-1} = \int \frac{2\,du}{(u-1)(u+1)} = 2\int \frac{A}{u-1} + 2\int \frac{B}{u+1}$

$* \; 16 = A(u+1) + B(u-1)$

$u = 1 \Rightarrow \boxed{A = 8}$

$u = -1 \Rightarrow \boxed{B = -8}$

$= 2\int \frac{8}{u-1}du + 2\int \frac{-8}{u+1}du$

$= 16\ln|u-1| - 16\ln|u+1| + c$

$= 16\ln|\sqrt{16x+1}-1|$ ~~[illegible]~~

$- 16\ln|\sqrt{16x+1}+1| + C$

---

10) a) $\int_1^e x^n(\ln x)^m\,dx$

$u = (\ln x)^m \qquad dv = x^n\,dx$

$du = \frac{m(\ln x)^{m-1}dx}{x} \qquad v = \frac{x^{n+1}}{n+1}$

$= \frac{(\ln x)^m \cdot x^{n+1}}{n+1} - \frac{m}{n+1}\int \frac{x^{n+1}\cdot(\ln x)^{m-1}}{x}dx$

$= \frac{(\ln x)^m x^{n+1}}{n+1} - \frac{m}{n+1}\int x^n(\ln x)^{m-1}dx$

#

b) $\int_1^e x^6(\ln x)^2\,dx$

$= \frac{e^7}{7} - \frac{2}{7}\int_1^e x^7 \ln x\,dx$

$= \frac{e^7}{7} - \frac{2}{7}\left[\frac{e^7}{7} - \frac{1}{7}\int_1^e x^7\,dx\right]$

$= \frac{e^7}{7} - \frac{2}{7}\left[\frac{e^7}{7} - \frac{1}{7}\frac{x^8}{8}\right]_1^e$

$= \frac{e^7}{7} - \frac{2}{7}\left[\frac{e^7}{7} - \frac{1}{7}\left(\frac{e^8}{8} - \frac{1}{8}\right)\right]$

**The University of Jordan**

**Student's Name:** ..................

**Student's Number:** ...............

**Department of Mathematics**

**Instructor's Name:** ..................

**Class Time:** ..........................

First Exam ◇ Calculus II (0301102) ◇ Summer 2016

Note: This exam is composed of 5 questions. You have 60 minutes to finish

**Question 1 [4 points]:** Evaluate $\int_0^1 \frac{dx}{x^2+3x+2}$.

**Question 2 [4 points]:** Determine whether the improper integral

$$\int_0^8 \frac{1}{\sqrt[3]{8-x}}dx$$

converges and, if so, determine its values.

**Question 3 [4 points]:** Find the area of the region(s) enclosed by

$$f(x) = (x-1)^3 \quad \text{and} \quad g(x) = x - 1.$$

**Question 4 [4 points]:** Suppose that $\int_0^{\pi} (ax+b)\cos x dx = 8$. What is the value of $a$?

**Question 5 [4 points]:** Evaluate $\int_0^1 \sqrt{36-x^2}\, dx$.

# # Calculus II ⇛ First Exam

كل الشكر للطالب
**معاذ امجد**
على حل الاسئله

① $\int_0^1 \frac{dx}{x^2+3x+2}$ . $\Rightarrow x^2+3x+2=(x+1)(x+2)$

$$\frac{1}{x^2+3x+2}=\frac{A}{x+1}+\frac{B}{x+2} \text{ then} \rightarrow A(x+2)+B(x+1)=1$$

$$\Rightarrow Ax+2A+Bx+B=1 \Rightarrow (A+B)x+2A+B=1$$

$$\left.\begin{matrix} A+B=0 \\ 2A+B=1 \end{matrix}\right\} \Rightarrow A=1 \text{ and } B=-1$$

$$\int_0^1 \frac{dx}{x^2+3x+2}=\int_0^1 \frac{dx}{x+1}-\int_0^1 \frac{dx}{x+2} \overset{!}{=} \left(\ln|x+1|-\ln|x+2|\right)\Big|_0^1$$

$$=\ln\left|\frac{x+1}{x+2}\right|\Big|_0^1=\ln\left(\frac{2}{3}\right)-\ln\left(\frac{1}{2}\right).$$

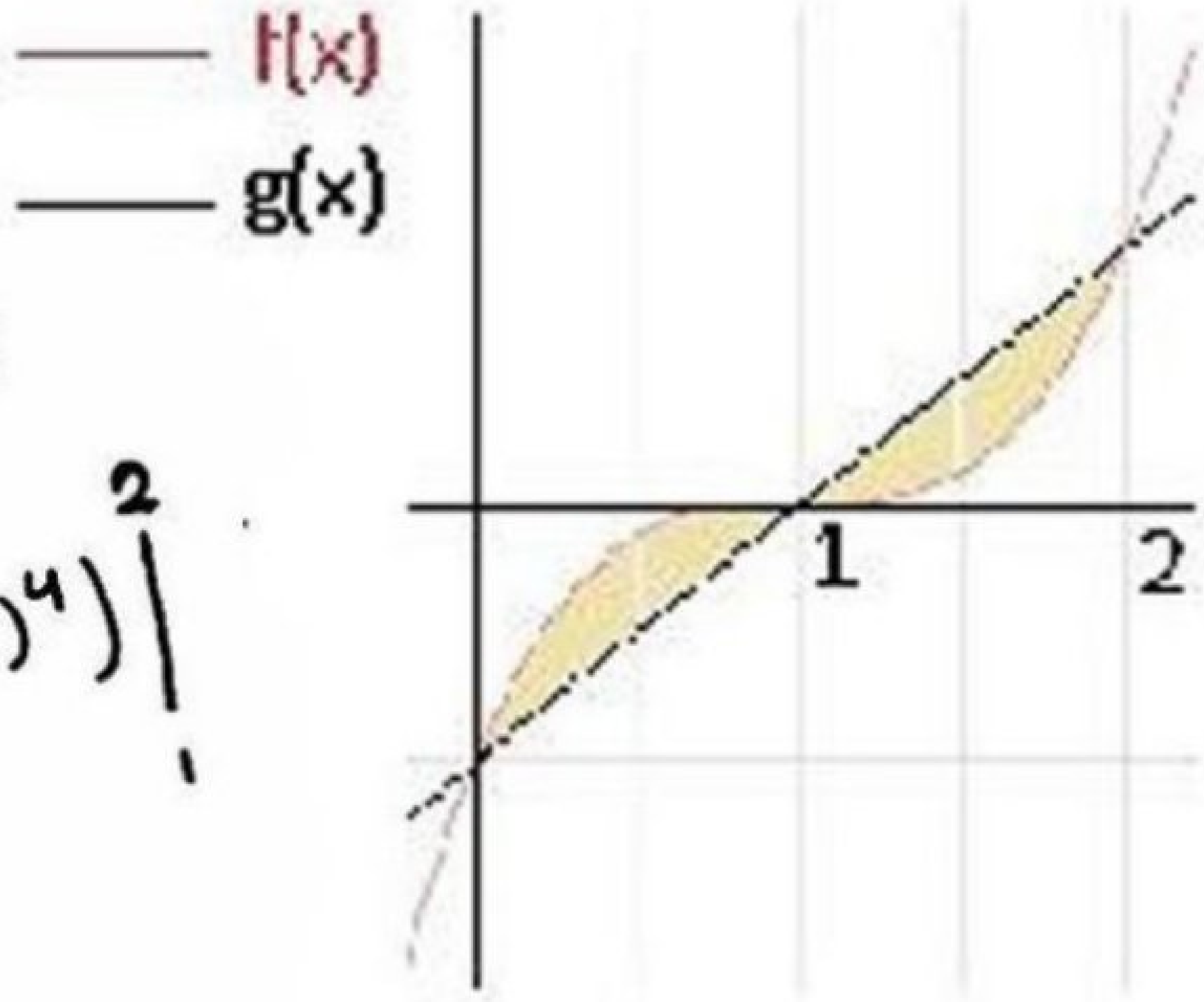
---

② $\int_0^8 \frac{1}{\sqrt[3]{8-x}} = \lim_{k\to\infty} \int_0^k (8-x)^{\frac{-1}{3}} . dx = \lim_{k\to\infty} \left[\frac{-3}{2}(8-x)^{\frac{2}{3}}\right]_0^k$

$$=\lim_{k\to\infty}\left[\frac{-3}{2}(8-k)^{\frac{2}{3}}+\left(\frac{3}{2}\right)(4)\right]=6$$

---

③ $f(x)=(x-1)^3$ . $g(x)=x-1$



$$\text{Area}=\int_0^1 [(x-1)^3-(x-1)].dx+\int_1^2 [(x-1)-(x-1)^3].dx$$

$$\text{Area}=\left(\frac{1}{4}(x-1)^4-\frac{1}{2}x^2+x\right)\Big|_0^1+\left(\frac{1}{2}x^2-x-\frac{1}{4}(x-1)^4\right)\Big|_1^2$$

$$\text{Area}=\frac{1}{2}$$

كل الشكر للطالب
**معاذ امجد**
على حل الاسئله

④ $\int_{0}^{\pi}(ax+b)\cos x.dx = 8$

$\Rightarrow f(x) = ax+b$ <u>and</u> $g'(x) = \cos(x)$

Integration by Parts gives →

$$= \int_{0}^{\pi}(ax+b)\cos x.\,dx = (ax+b)\sin x \Big|_{0}^{\pi} - \int_{0}^{\pi} a\sin x.\,dx.$$

$$= (ax+b)\sin x \Big|_{0}^{\pi} + a\cos x \Big|_{0}^{\pi} = -2a$$

$$\Rightarrow -2a = 8, \Rightarrow \boxed{a = -4}$$

---

⑤ $\int_{0}^{1}\sqrt{36-x^2}\,.dx \Rightarrow$

$x = 6\sin\theta.$

$dx = 6\cos\theta.\,d\theta$

$x = 0 \rightarrow \theta = 0$

$x = 1 \rightarrow \theta = \sin^{-1}(\frac{1}{6})$

$\Rightarrow 36 - x^2 = 36 - 36\sin^2\theta$

$= 36(1-\sin^2\theta)$

$= 36\cos^2\theta.$

<u>Then</u> $\int_{0}^{1}\sqrt{36-x^2}.\,dx = \int_{0}^{\sin^{-1}(\frac{1}{6})} 36\cos^2\theta.\,d\theta$

$$= 18\int_{0}^{\sin^{-1}(\frac{1}{6})}(1+\cos 2\theta).d\theta = 18\theta + 9\sin 2\theta \Big|_{0}^{\sin^{-1}(\frac{1}{6})}$$

$$= 18\theta + 9(2\sin\theta\cos\theta)\Big|_{0}^{\sin^{-1}(\frac{1}{6})}$$

$$= 18(\theta + \sin\theta\cos\theta)\Big|_{0}^{\sin^{-1}(\frac{1}{6})} = 18\left(\sin^{-1}(\tfrac{1}{6}) + (\tfrac{1}{6})(\tfrac{\sqrt{35}}{6})\right)$$

The university of Jordan Calculus II First exam 8 Mars 2016

اسم الطالب: الرقم الجامعي:

وقت المحاضرة: مدرس المادة:

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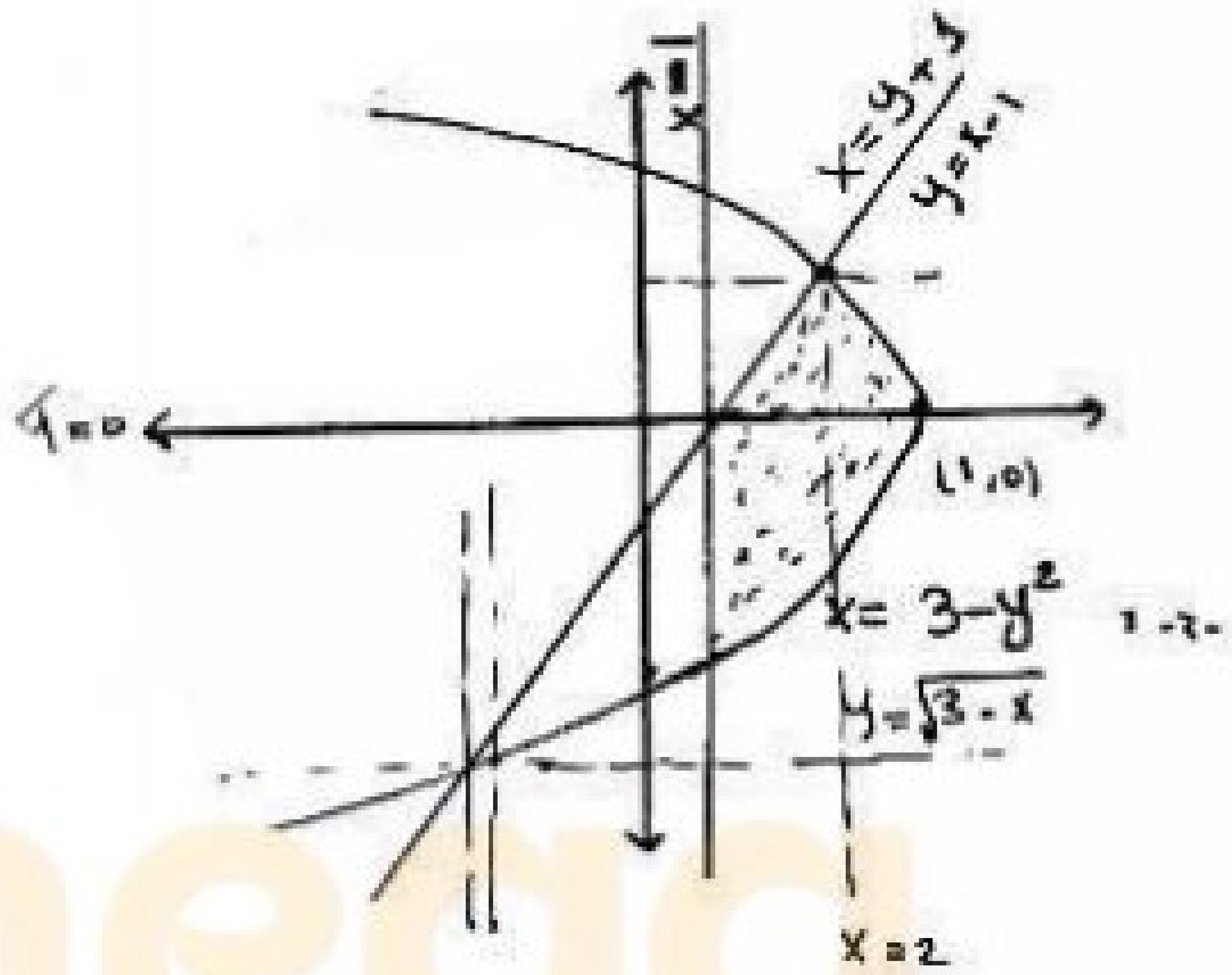
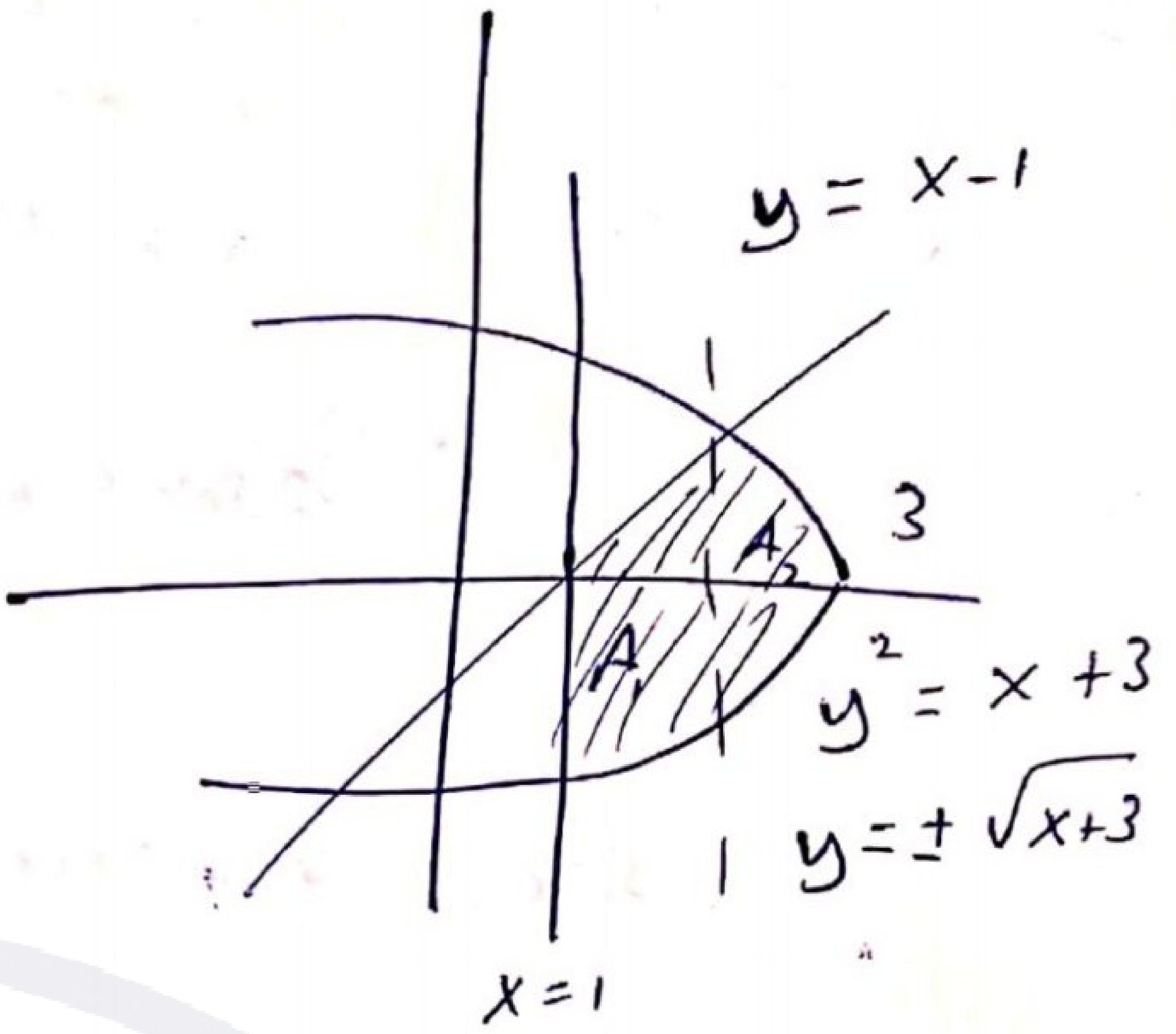
Q1: Find the area of the shaded region



Q2: Find (if exists) $\int_0^1 x(\ln x)dx$

1) $\text{area} = A_1 + A_2$



$$A_1 = \int_1^2 ((x-1) + \sqrt{3-x}\,)\,dx$$

$$A_1 = \frac{1}{2}x^2 - x - \frac{2}{3}(3-x)^{3/2}\Big|_1^2$$

$$A_1 = \frac{7}{6}$$

$$A_2 = \int_2^3 (\sqrt{3-x} + \sqrt{3-x}\,)\,dx$$

$$= -\frac{4}{3}(3-x)^{3/2}\Big|_2^3$$

$$A_2 = \frac{2\sqrt{2}}{3} \qquad \text{area} = \frac{7}{6} + \frac{2\sqrt{2}}{3}$$

$$x - 1 = \sqrt{x+3}$$

$$(x-1)^2 = x+3$$

$$x^2 - 2x + 1 = x + 3$$

$$x^2 - x + 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2, -1$$

---

2) $\int_0^1 x \ln x = \lim_{a \to 0} \int_a^1 x \ln x$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{dx}{x} \qquad v = \frac{x^2}{2}$

$$\frac{x^2}{2}\ln x\Big|_a^1 - \int_a^1 \frac{1}{2}x\,dx = -\frac{a^2}{2}\ln a - \frac{1}{4}x^2\Big|_a^1$$

$$= -\frac{a^2}{2}\ln a - \frac{1}{4} + \frac{a^2}{4} = 0 - \frac{1}{4} + 0 = \boxed{\frac{-1}{4}}$$

$$\lim_{a \to 0} -\frac{a^2}{2}\ln a = \lim_{a \to 0} \frac{-\ln a}{\frac{2}{a^2}} = \frac{\frac{-1}{a}}{\frac{-4}{a^3}} = 0$$

Q3: Find $\int \frac{x\,dx}{\sqrt{x^2-4x}}$

Q4: $\int \frac{1}{x}\left(\frac{x+2}{x-1}\right)^2 dx$

Q5: Find $\int \frac{(x-1)e^x}{x^2}dx$

3) $\int \frac{x\,dx}{\sqrt{x^2-4x}} = \int \frac{x\,dx}{\sqrt{x^2-4x+4-4}} = \int \frac{x\,dx}{\sqrt{(x-2)^2-4}}$

$= \int \frac{(2\sec\theta+2)}{\cancel{2}\tan\theta}\cdot\cancel{2}\sec\theta\tan\theta\,d\theta$

let $x-2 = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

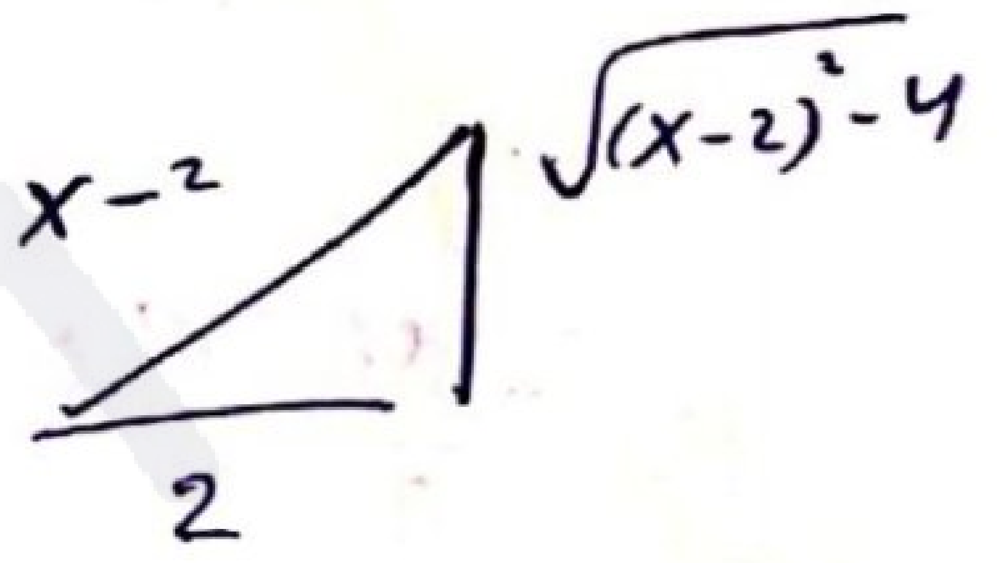
$x = 2\sec\theta + 2$

$= \int (2\sec^2\theta + 2\sec\theta)\,d\theta$

$= 2\tan\theta + 2\ln|\sec\theta+\tan\theta| + c$

$\sec\theta = \frac{x-2}{2}$

$= 2\frac{\sqrt{(x-2)^2-4}}{2} + 2\ln\left|\frac{x-2}{2} + \frac{\sqrt{(x-2)^2-4}}{2}\right| + c$



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4) $\int \frac{1}{x}\left(\frac{x+2}{x-1}\right)^2 dx = \int \frac{(x+2)^2}{x(x-1)^2}\,dx$ = [illegible]

$= \int \frac{A}{x}\,dx + \int \frac{B}{x-1}\,dx + \int \frac{C}{(x-1)^2}\,dx$

[illegible]

$(x+2)^2 = A(x-1)^2 + Bx(x-1) + Cx$

$x=-1 \Rightarrow \boxed{B=-3}$

$x=0 \Rightarrow \boxed{A=4}$

$x=1 \Rightarrow \boxed{C=9}$

$\Rightarrow = \int \frac{4}{x}\,dx + \int \frac{-3}{x-1}\,dx + \int \frac{9}{(x-1)^2}\,dx$

$= 4\ln x - 3\ln x-1 - \frac{9}{x-1} + c$

$$5)\ \int \frac{(x-1)e^x}{x^2}\,dx = \int \frac{xe^x - e^x}{x^2}\,dx = \int \frac{e^x}{x}\,dx - \int \frac{e^x}{x^2}\,dx$$

$$*\ \int \frac{e^x}{x^2}\,dx \Rightarrow \quad u = e^x \qquad dv = x^{-2}\,dx$$

$$du = e^x\,dx \qquad v = \frac{-1}{x}$$

$$\Rightarrow \int \cancel{\frac{e^x}{x}}\,dx + \frac{e^x}{x} - \int \cancel{\frac{e^x}{x}}\,dx = \frac{e^x}{x} + c$$

# # Calculus II

أفنان ابراهيم عبدالله مرعي

First exam «فصل ثاني 2015 / 2016»

**Q1 :** $y = x-1$

$x = 3 - y^2$

$x = 3 - (x-1)^2$

$x = 3 - (x^2 - 2x + 1)$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \ , \ x = -1$

لا نحتاجها

$\rightarrow$ area = A + B

$$\text{area} = \int_1^2 (x-1) - (-\sqrt{3-x})\,dx + \int_2^3 \sqrt{3-x} - (\sqrt{3-x})\,dx$$

$$= \frac{x^2}{2} - x + \frac{-2}{3}(3-x)^{\frac{3}{2}}\Big]_1^2 + \frac{-4}{3}(3-x)^{\frac{3}{2}}\Big]_2^3$$

$$= \frac{7}{6} + \frac{2\sqrt{2}}{3} \quad \#$$

**Q2:** $\int_0^1 x \ln(x)\,dx$ $\quad u = \ln(x) \quad dv = x$ $\quad$ « Improper Integral »

$du = \frac{1}{x}dx \quad v = \frac{x^2}{2}$

$$\int_0^1 x\ln(x) = \frac{x^2}{2}\ln(x)\Big]_0^1 - \int_0^1 \frac{x^2}{2}\cdot\frac{1}{x}dx$$

$$= \frac{x^2}{2}\ln(x)\Big]_0^1 - \frac{x^2}{4}\Big]_0^1 = \lim_{a\to 0}\frac{x^2}{2}\ln(x)\Big]_a^1 - \frac{1}{4}$$

$$= \lim_{a\to 0}\frac{a^2}{2}\ln(a) \qquad \text{use } \underline{\underline{L.H}}$$

$$= \lim_{a\to 0}\frac{\ln(a)}{\frac{2}{a^2}} = \lim_{a\to 0}\frac{\frac{1}{a}}{\frac{-4}{a^3}} = \text{Zero}$$

$$\Rightarrow \int_0^1 x\ln(x) = -\frac{1}{4} \quad \text{convergent.} \ \#$$

كل الشكر للطالبه
**افنان ابراهيم مرعي**
على حل الاسئله

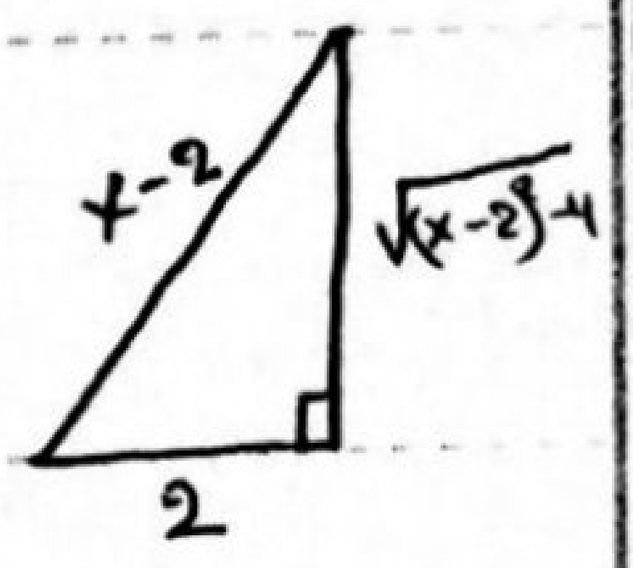
**Q3:** $\int \frac{x\,dx}{\sqrt{x^2-4x}}$

sol: ( إكمال مربع التمام ) $\Rightarrow \int \frac{x\,dx}{\sqrt{(x-2)^2-4}}$

let $x-2 = 2\sec\theta \Rightarrow x = 2\sec\theta + 2$ $\qquad dx = 2\sec\theta\tan\theta\,d\theta$

$(x-2)^2 = 4\sec^2\theta \Rightarrow \sqrt{(x-2)^2-4} = 2\tan\theta$.

$\sec\theta = \frac{x-2}{2}$



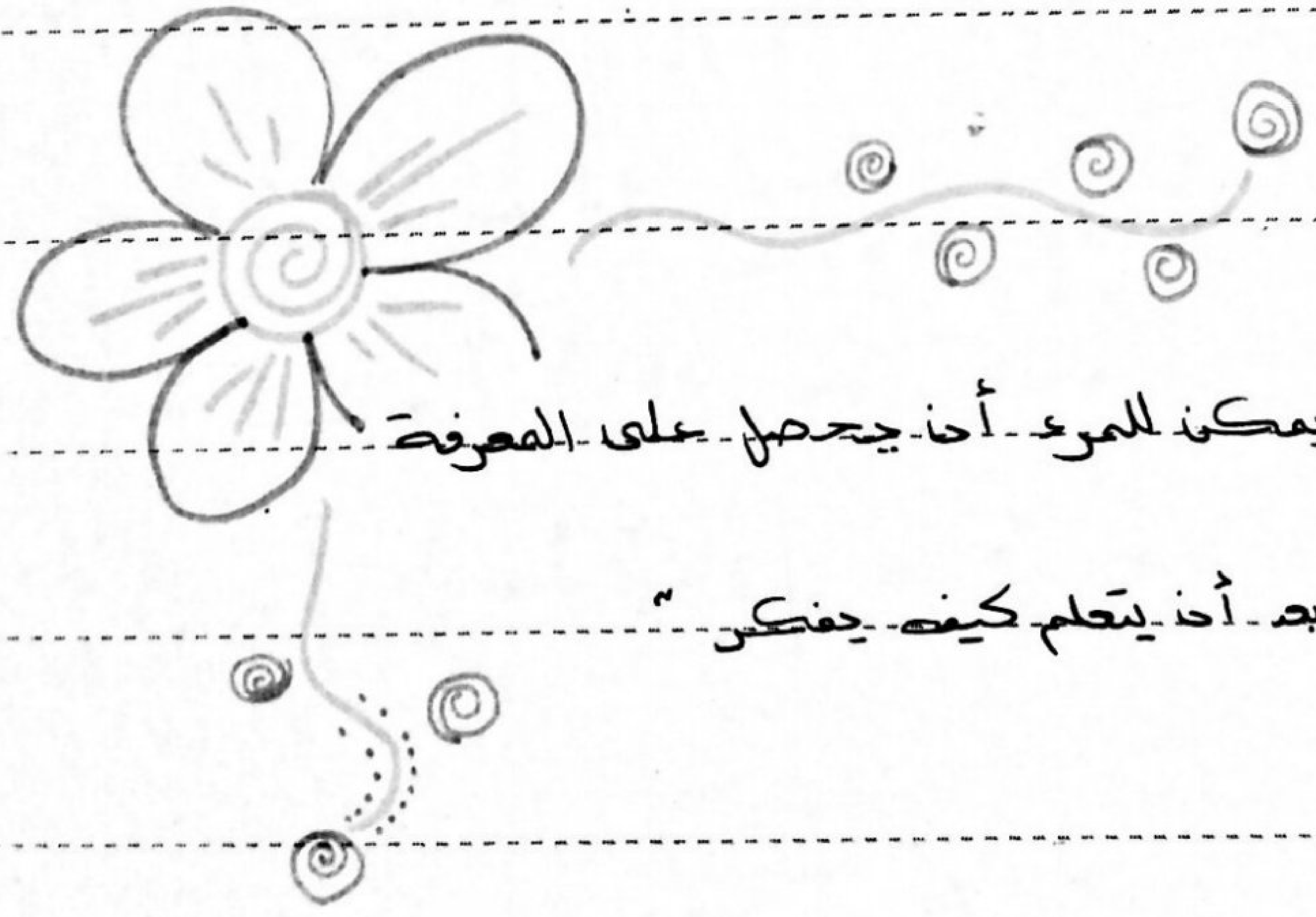
so, $\Rightarrow \int \frac{(2\sec\theta+2)}{\cancel{2}\,\cancel{\tan\theta}} \cdot \cancel{2}\sec\theta\,\cancel{\tan\theta}\,d\theta$ $\qquad \sin\theta = \frac{\sqrt{(x-2)^2-4}}{x-2}$

$\cos\theta = \frac{2}{x-2}$

$= \int 2\sec^2\theta + 2\sec\theta\,d\theta$ $\qquad \tan\theta = \frac{\sqrt{(x-2)^2-4}}{2}$

$= 2\tan\theta + 2\ln|\sec\theta + \tan\theta| + C$

$= \frac{2\sqrt{(x-2)^2-4}}{2} + 2\ln\left|\frac{x-2}{2} + \frac{\sqrt{(x-2)^2-4}}{2}\right| + C$ #



"لا يمكن للمرء أن يحصل على المعرفة

إلا بعد أن يتعلم كيف يفكر"

**Q4:** $\int \frac{1}{x}\cdot\left(\frac{x+2}{x-1}\right)^2 dx$

sol: $\int \frac{1}{x}\cdot\left(\frac{x+2}{x-1}\right)^2 dx = \int \frac{(x+2)^2}{x(x-1)^2} dx = \int \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$

$C = \left.\frac{(x+2)^2}{x}\right|_{x=1} \Rightarrow C = 9$ & $A = \left.\frac{(x+2)^2}{(x-1)^2}\right|_{x=0} \Rightarrow A = 4$

$(x+2)^2 = A(x-1)^2 + Bx(x-1) + Cx$

$(x+2)^2 = 4(x-1)^2 + B(x)(x-1) + 9x$

$\boxed{\text{at } x=-1}$ $1 = 4(4) - B(-2) - 9 \Rightarrow B = -3$

$\Rightarrow \int\left(\frac{4}{x} - \frac{3}{(x-1)} + \frac{9}{(x-1)^2}\right) dx = 4\ln|x| + -3\ln|x-1| \frac{-9}{(x-1)} + C$ ⋇

**Q5:** $\int \frac{(x-1)e^x}{x^2} \cdot dx$

sol: $\int \frac{(x-1)e^x}{x^2} dx = \int\left(\frac{xe^x}{x^2} - \frac{e^x}{x^2}\right) dx = \int \frac{e^x}{x} dx - \int \frac{e^x}{x^2} dx$

$\Rightarrow \int \frac{e^x}{x^2} dx \Rightarrow$ let $u = e^x$ , $dv = x^{-2}$

$du = e^x dx \xleftarrow{\int} v = \frac{-1}{x}$

$= \frac{-e^x}{x} + \int \frac{e^x}{x} dx \Rightarrow$ نعوض هنا

$\int \frac{e^x}{x} - \left[-\frac{e^x}{x} + \int \frac{e^x}{x} dx\right] = \frac{e^x}{x} + C$ ⋇

الممسوحة ضوئيا بـ CamScanner

# Calculus 2

## First Exam

أسئلة اختبارات سابقة جديدة محلولة

إعداد

الاستاذ نادر ابومُغلي

**مركز نادر ابومُغلي الثقافي**

079/5545798 – 077/5695615 – 079/5000973

2012

University of Jordan
Department of Mathematics

**Calculus II** First Exam **Date 14/7/2011**

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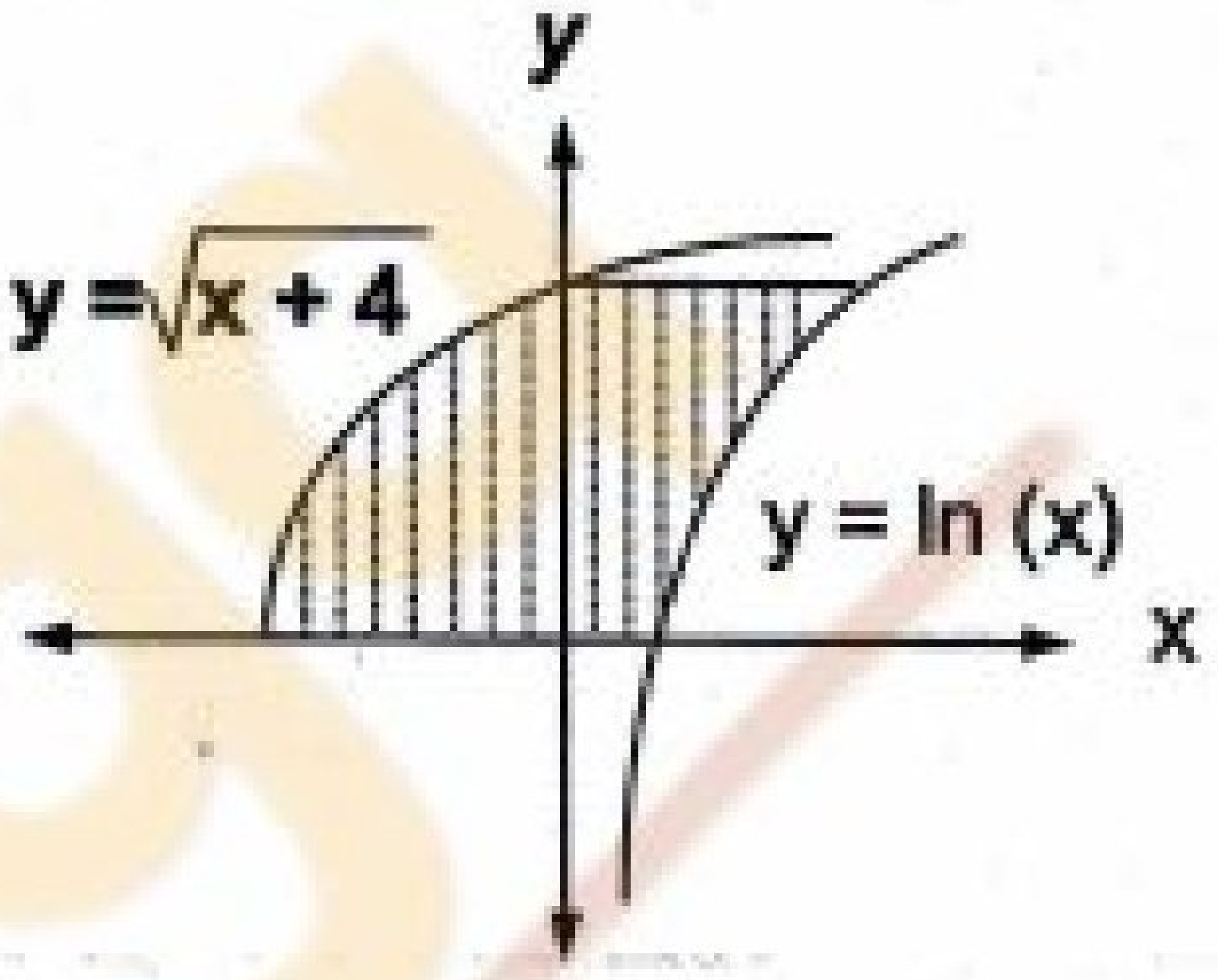
**Q1)** *Find* $\int \frac{1}{x^2} \tan^{-1} x \, dx$

**Q2)** *Find* $\int \frac{\sqrt{4-x}}{x} \, dx$

**Q3)** *Find the area of the shaded region*

**Q4)** *Find*



**Q4)** *Find the value of* $\int_1^3 \frac{dx}{x^2 - 2x}$ (if exist)

**Q5)** *Find* $\int \frac{dx}{\sqrt{e^{2x} + 2e^x}}$

1) $\int \frac{\tan^{-1} x \, dx}{x^2}$

$u = \tan^{-1} x \qquad dv = x^{-2} dx$

$du = \frac{dx}{1+x^2} \qquad v = \frac{-1}{x}$

$\frac{-\tan^{-1} x}{x} + \int \frac{dx}{x(x^2+1)}$

$\int \frac{dx}{x(x^2+1)} = \int \frac{A}{x} dx + \int \frac{Bx+C}{x^2+1} dx$

$1 = A(x^2+1) + (Bx+c)x$

at $x=0 \Rightarrow \boxed{A=1}$

at $x=1 \Rightarrow \quad 1 = 2 + B + c$

$B + c = -1 \quad —(1)$

at $x=-1 \Rightarrow \quad 1 = 2 + B - c \quad —(2)$

$B - c = -1$

$(1) + (2) \Rightarrow$

$2B = -2$

$\boxed{B=-1} \Rightarrow -1 - c = -1$

$\boxed{C=0}$

$\Rightarrow \frac{-\tan^{-1} x}{x} + \int \frac{dx}{x} + \int \frac{-x}{x^2+1} dx$

$= \frac{-\tan^{-1} x}{x} + \ln x - \frac{1}{2} \ln |x^2+1| + c$

2) $\int \frac{\sqrt{4-x}}{x} dx$ $\quad$ let $u = \sqrt{4-x}$

$u^2 = 4-x$

$2u\,du = -dx$

$= \int \frac{-u \cdot 2u\,du}{4-u^2}$

$= 2\int \frac{u^2}{u^2-4} du$ $\quad$ *

$$\begin{array}{r|l} & 1 \\ u^2-4 & u^2 \\ & -u^2 \mp 4 \\ \hline & 4 \end{array}$$

$\Rightarrow 2 \left[ \int du + \int \frac{4}{u^2-4} du \right]$

$= 2 \left( u + \int \frac{4}{(u-2)(u+2)} du \right)$

* $\frac{4}{(u-2)(u+2)} = \frac{A}{u-2} + \frac{B}{u+2}$

$4 = A(u+2) + B(u-2)$

$\boxed{u=2} \Rightarrow A=1$

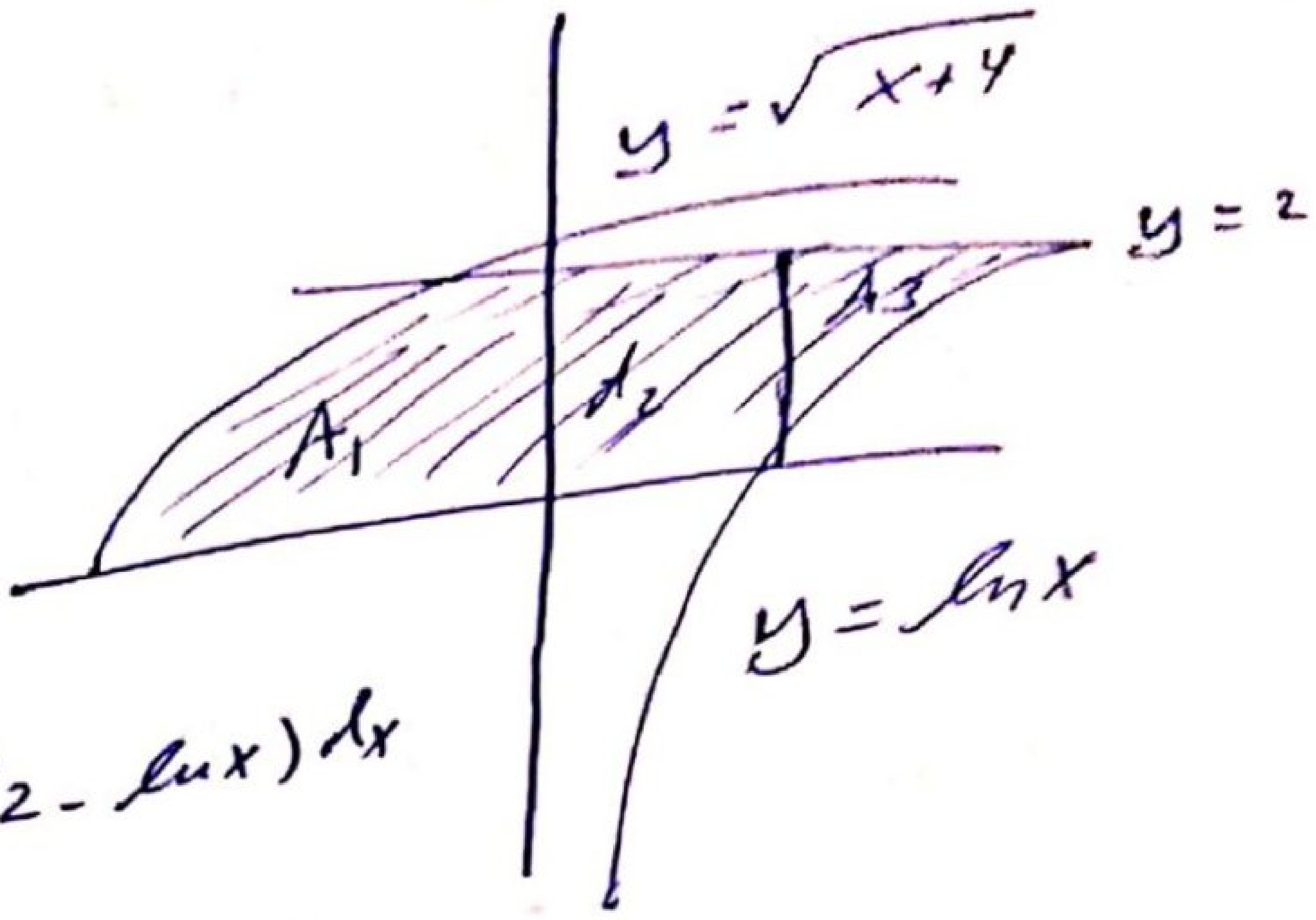
$\boxed{u=-2} \Rightarrow B=-1$

$= 2 \left( u + \int \frac{1}{u-2} du + \int \frac{-1}{u+2} du \right)$

$= 2 \left( u + \ln|u-2| - \ln|u+2| \right) + c$

$= 2 ( \sqrt{4-x} + \ln|\sqrt{4-x}-2| - \ln|\sqrt{4-x}+2| + c$

3)



$$A_1 = \int_{-4}^{0} \sqrt{4+x}\,dx$$

$$= \frac{2}{3}(x+4)^{3/2}\Big|_{-4}^{0}$$

$$A_1 = \frac{16}{3}$$

$$A_2 = \int_{0}^{1} 2dx = 2$$

$$A_3 = \int_{1}^{e^2} (2-\ln x)\,dx$$

$$= \int_{1}^{e^2} 2dx - \int_{1}^{e^2} \ln x\,dx$$

$$= 2(e^2-1) - \underbrace{\int_{1}^{e^2} \ln x\,dx}$$

$$= 2e^2 - 2 - 2e^2 + e^2 - 1$$

$$= \underline{\underline{e^2 - 3}}$$

$$* \int_{1}^{e^2} \ln x\,dx$$

$u = \ln x \qquad dv = dx$

$du = \frac{dx}{x} \qquad v = x$

$$x\ln x\Big|_{1}^{e^2} - \int_{1}^{e^2} dx = 2e^2 - e^2 + 1$$

---

$$4) \int_{1}^{3} \frac{dx}{x^2-2x} = \int_{1}^{2} \frac{dx}{x^2-2x} + \int_{2}^{3} \frac{dx}{x^2-2x}$$

$$* \lim_{a\to 2^-} \int_{1}^{a} \frac{dx}{x^2-2x} + \lim_{b\to 2^+} \int_{b}^{3} \frac{dx}{x^2-2x}$$

$$\int \frac{dx}{x^2-2x} = \int \frac{dx}{x(x-2)} = \int \frac{A}{x}dx + \int \frac{B}{x-2}$$

$1 = A(x-2) + Bx$

$x = 0 \Rightarrow A = -1/2$

$x = 2 \Rightarrow B = 1/2$

(يتبع)

$$\Rightarrow \lim_{a\to 2^-}\left(\int_1^a \frac{-\frac{1}{2}}{x}\,dx + \int_1^a \frac{\frac{1}{2}}{x-2}\,dx\right)$$

$$= \lim_{a\to 2^-}\left(-\frac{1}{2}\ln x\Big|_1^a + \frac{1}{2}\ln|x-2|\Big|_1^a\right)$$

$$= \lim_{a\to 2^-}\left(-\frac{1}{2}\ln a + \frac{1}{2}\ln(a-2)\right)$$

$$= -\frac{1}{2}\ln 2 + \frac{1}{2}\ln(2^- - 2)$$

$$= -\frac{1}{2}\ln 2 + \underline{\underline{\ln(0^-)}} \rightarrow \text{D.N.E}$$

$$= \text{div}$$

$$\text{div} \pm \text{div/con} = \underline{\underline{\text{div}}}$$

---

$$5)\ \int \frac{dx}{\sqrt{e^{2x}+2e^x}} \qquad u = e^x \qquad du = e^x dx \qquad = \int \frac{du}{u(\sqrt{u^2+2u+1-1}}$$

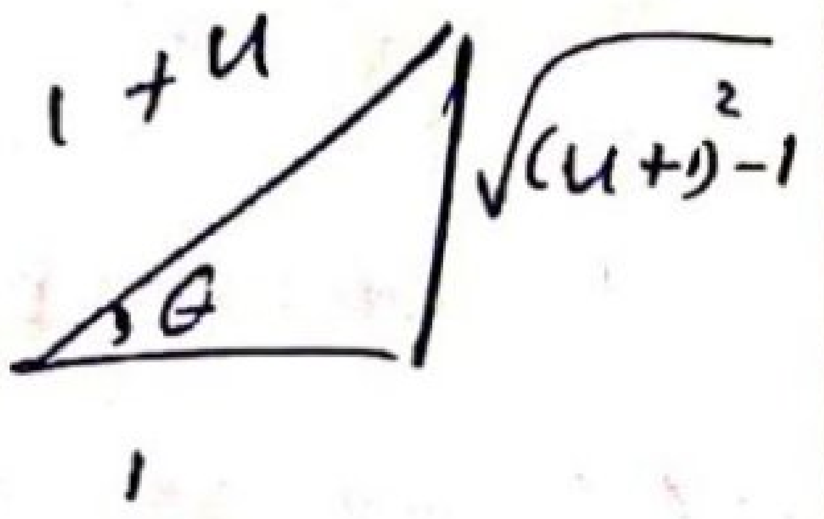
$$= \int \frac{du}{\sqrt{u^2+2u}}\cdot\frac{1}{u} = \int \frac{du}{u\sqrt{u^2+2u}}$$

$$= \int \frac{du}{u\sqrt{(u+1)^2-1}} \qquad u+1 = \sec\theta \qquad du = \sec\theta\tan\theta\, d\theta$$

$$= \int \frac{\sec\theta\tan\theta\, d\theta}{(\sec\theta-1)\cdot\tan\theta} = \int \frac{\sec\theta\, d\theta}{\sec\theta - 1} = \int \frac{\frac{1}{\cos\theta}\,d\theta}{\frac{1}{\cos\theta}-1}$$

$$= \int \frac{1}{1-\cos\theta}\,d\theta = \int \frac{(1+\cos\theta)\,d\theta}{\sin^2\theta} = \int \frac{1}{\sin^2\theta}\,d\theta + \int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

ضرب مرافق $\Delta$



$$= -\cot\theta - \csc\theta + C$$

put the answer in term of $\underline{u}$ ㉜

مركز نادر ابو مغلي الثقافي
عمان - الاردن -

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امتحان

14 - 7 - 2011

0795000973

مركز نادر ابو مغلي الثقافي
عمان - الاردن -

نادر ابو مغلي

1 $\int \frac{1}{x^2}\cdot\tan^{-1}x\ dx$

$u=\tan^{-1}x \Rightarrow du=\frac{1}{1+x^2}\,dx$

$dv=x^{-2}\cdot dx \rightarrow v=-x^{-1}=-\frac{1}{x}$

$\int u\cdot dv = u\cdot v - \int v\cdot du$

$=\left(\tan^{-1}x\right)\left(-\frac{1}{x}\right) - \int\left(\frac{-1}{x}\right)\left(\frac{1}{1+x^2}\,dx\right)$

$=\frac{-\tan^{-1}x}{x}+\int\frac{1}{x(1+x^2)}\,dx$

$\frac{1}{x(1+x^2)}=\frac{a}{x}+\frac{bx+c}{1+x^2}$

نوحد مقامات

$1=a(1+x^2)+(bx+c)(x)$

when $x=0$ $\Rightarrow 1=a(1)+0 \Rightarrow \boxed{a=1}$

لايجاد باقي المجاهيل

$1=a+ax^2+bx^2+cx$

$x^2$ معاملات : $0=a+b \Rightarrow 0=1+b \Rightarrow \boxed{b=-1}$

$x$ معاملات : $\boxed{0=c}$

$\int \frac{a}{x}+\frac{bx+c}{1+x^2}\ dx=$

$=\int\frac{1}{x}\,dx+\int\frac{-x}{1+x^2}\,dx$

$=\ln|x|-\frac{1}{2}\ln|1+x^2|$

$\frac{-\tan^{-1}x}{x}+\ln|x|-\frac{1}{2}\ln|1+x^2|+C$

2 $\int \sqrt{\frac{4-x}{x}}\,.dx$

Sol: $\int \frac{\sqrt{4-x}}{\sqrt{x}}\,dx$

Let $x=u^2$
$dx=2u\,du$

$\int \frac{\sqrt{4-u^2}}{u}\,2u\cdot du \quad = 2\int\sqrt{4-u^2}\,du$

Let $u=2\sin\theta$
$du=2\cos\theta\cdot d\theta$

$\sqrt{4-u^2}=\sqrt{4-4\sin^2\theta}$
$=\sqrt{4(1-\sin^2\theta)}$
$=\sqrt{4\cos^2\theta}=2\cos\theta$

$2\int\sqrt{4-u^2}\,.du$

$=2\int(2\cos\theta)(2\cos\theta)d\theta$

$=8\int\cos^2\theta\,.d\theta$

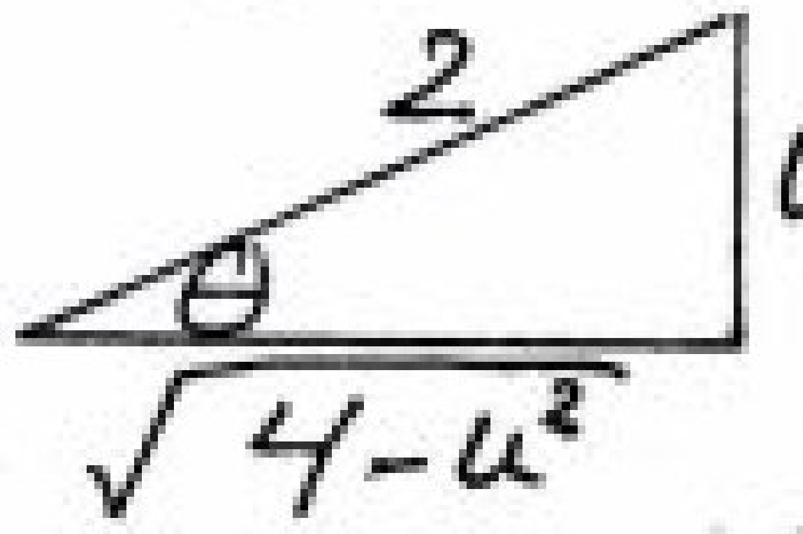
$=8\int\frac{1}{2}(1+\cos2\theta).d\theta$

$=4\left(\theta+\frac{\sin2\theta}{2}\right)+C \quad =4\left(\theta+\frac{2\sin\theta.\cos\theta}{2}\right)+C$

$=4\left(\theta+\sin\theta.\cos\theta\right)+C$

نحول بدلالة $u$

$u=2\sin\theta$
$\frac{u}{2}=\sin\theta$



$=4\left(\sin^{-1}\left(\frac{u}{2}\right)+\frac{u}{2}\cdot\frac{\sqrt{4-u^2}}{2}\right)+C$

$=4\sin^{-1}\left(\frac{u}{2}\right)+u\sqrt{4-u^2}+C$

نحول بدلالة $x$

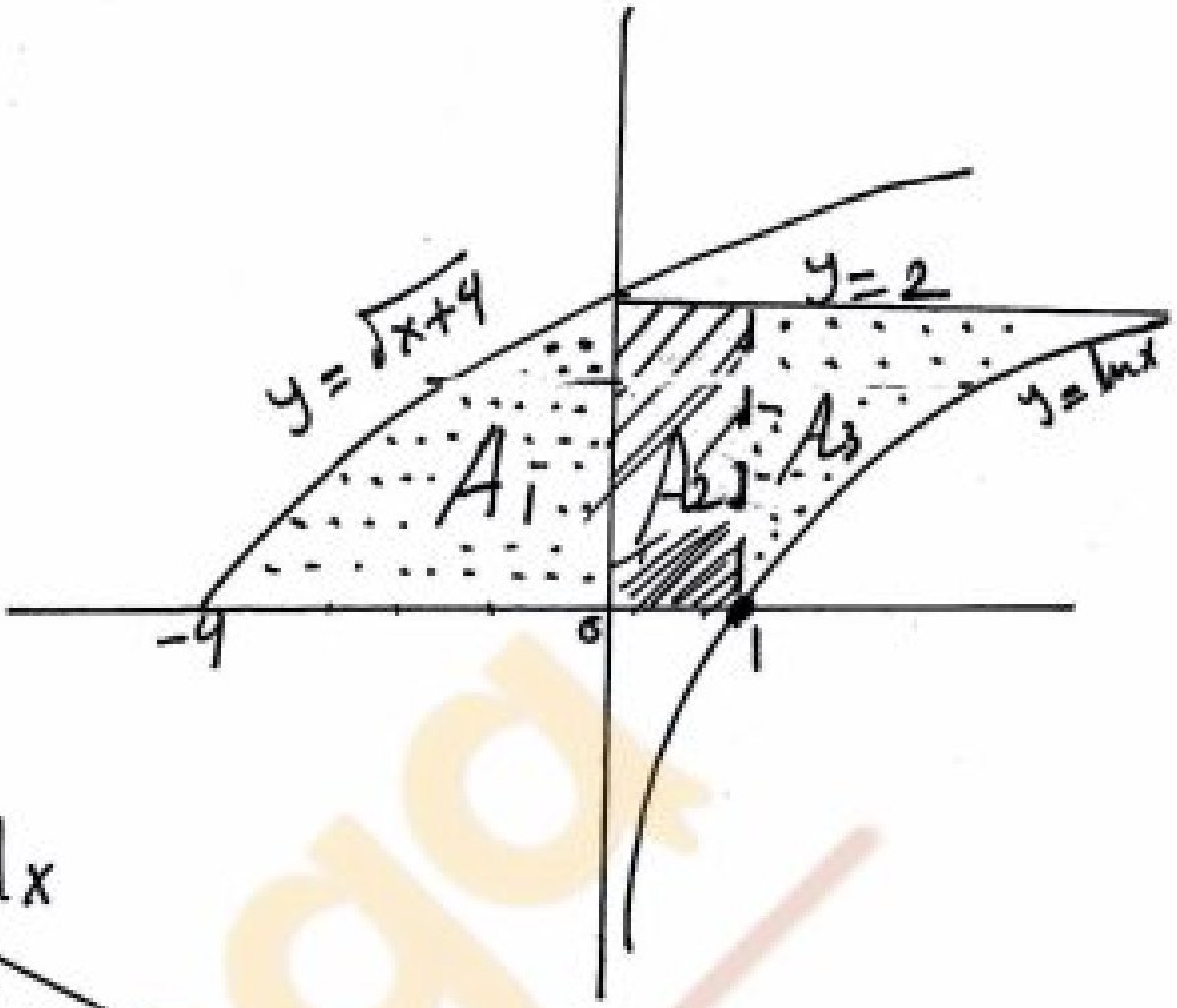
$4\sin^{-1}\left(\frac{\sqrt{x}}{2}\right)+\sqrt{x}\,\sqrt{4-x}+C$

3 $A_1 = \int_{-4}^{0} \sqrt{x+4} \cdot dx$

$= \frac{2}{3}(x+4)^{\frac{3}{2}} \Big|_{-4}^{0}$

$= \frac{2}{3}\left((4)^{\frac{3}{2}} - 0\right)$

$= \frac{16}{3}$



$A_2 = \int_{0}^{1} 2 \cdot dx = 2$

$A_3 = \int_{1}^{e^2} 2 - \ln x \cdot dx$

$= 2x - (x\ln x - x)$

$= 3x - x\ln x \Big|_{1}^{e^2}$

$= \left(3e^2 - (e^2 \cdot \ln e^2)\right) - (3 - 0)$

$= 3e^2 - 2e^2 \quad - 3$

$= e^2 - 3$

$\int \ln x \, dx$

by parts

$u = \ln x \rightarrow du = \frac{1}{x}dx$

$dv = dx \rightarrow v = x$

$u \cdot v - \int v \cdot du$

$\ln x \cdot x - \int x \cdot \frac{1}{x} dx$

$x\ln x - x$

$A = A_1 + A_2 + A_3$

$= \frac{16}{3} + 2 + e^2 - 3$

$= \frac{13}{3} + e^2$

تقاطع

$y = 2$ with

$y = \ln x$

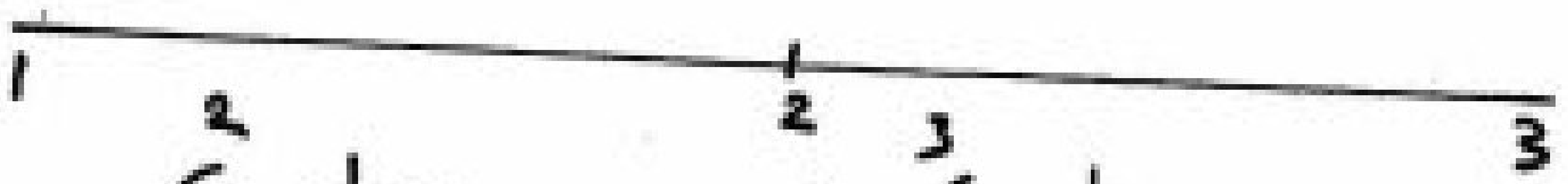
$2 = \ln x$

$e^2 = x$

نادر ابو مغلي

4 $\int_1^3 \frac{dx}{x^2-2x}$

$x=0$ , $x=2$ اصفار المقام



$\int_1^2 \frac{dx}{x^2-2x} + \int_2^3 \frac{dx}{x^2-2x}$

$\lim_{b\to 2^-}\int_1^b \frac{dx}{x^2-2x} + \lim_{c\to 2^+}\int_c^3 \frac{dx}{x^2-2x}$

$\int_1^b \frac{dx}{x^2-2x}$

$\frac{1}{x(x-2)} = \frac{a}{x} + \frac{b}{x-2}$

$1 = a(x-2) + b(x)$

when $x=0 \Rightarrow 1=-2a \Rightarrow a=-\frac{1}{2}$

$x=2 \Rightarrow 1 = 0+2b \Rightarrow b=\frac{1}{2}$

$\int \frac{-\frac{1}{2}}{x} + \frac{\frac{1}{2}}{x-2}\, dx = -\frac{1}{2}\ln|x| + \frac{1}{2}\ln|x-2|\Big]_1^b$

$= \left(-\frac{1}{2}\ln|b| + \frac{1}{2}\ln|b-2|\right) - (0)$

$\lim_{b\to 2^-} = -\frac{1}{2}\ln 2 + \frac{1}{2}(-\infty) = -\infty$ diverge

$\int_c^3 \frac{dx}{x^2-2x} = -\frac{1}{2}\ln|x| + \frac{1}{2}\ln|x-2|\Big]_c^3$

$= \left(-\frac{1}{2}\ln 3 + 0\right) - \left(-\frac{1}{2}\ln|c| + \frac{1}{2}\ln|c-2|\right)$

$\lim_{c\to 2^+} = -\frac{1}{2}\ln 3 + \frac{1}{2}\ln 2 - \frac{1}{2}\ln 0^+$

$= -\frac{1}{2}\ln 3 + \frac{1}{2}\ln 2 + \infty$

نادر ابومغلي

نادر ابو مغلي

5 $\int \frac{dx}{\sqrt{e^{2x}+2e^{x}}}$

Let $u=e^{x}$
$du=e^{x}.dx$
$du=u.dx$

$$\int \frac{\frac{1}{u}\,du}{\sqrt{u^2+2u}} = \int \frac{du}{u\sqrt{u^2+2u}}$$

$\sqrt{u^2+2u}$ by completing the square

$\sqrt{u^2+2u+1-1} = \sqrt{(u+1)^2-1}$

$$\int \frac{du}{u\sqrt{(u+1)^2-1}}$$

Let $u+1=\sec\theta$
$du=\sec\theta.\tan\theta.d\theta$
$\sqrt{(u+1)^2+1} = \sqrt{\sec^2\theta-1} = \sqrt{\tan^2\theta}$
$=\tan\theta$

$$\int \frac{\sec\theta.\tan\theta.d\theta}{(\sec\theta-1).\tan\theta}$$

$$\int \frac{\sec\theta}{\sec\theta-1}\,d\theta$$

$$\int \frac{\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}-1}\,d\theta$$

$$\int \frac{1}{1-\cos\theta}\,d\theta$$

$$\int \frac{1}{1-\cos\theta}\;\frac{1+\cos\theta}{1+\cos\theta}\,d\theta$$

$$\int \frac{1+\cos\theta}{1-\cos^2\theta} = \int \frac{1+\cos\theta}{\sin^2\theta}\,d\theta$$

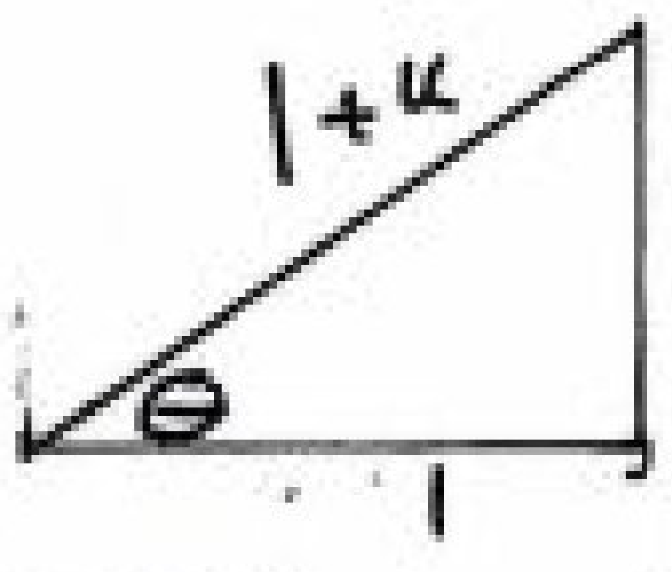
$$\int \frac{1}{\sin^2\theta}+\frac{\cos\theta}{\sin^2\theta}\;d\theta$$

$$\int \csc^2\theta+\cot\theta.\csc\theta\;d\theta$$

$$= -\cot\theta-\csc\theta\;+C$$

بدلالة $u$

$u+1=\sec\theta$
$\cos\theta=\frac{1}{u+1}$



$\cot\theta=\frac{1}{\sqrt{(1+u)^2-1}}$

$\csc\theta=\frac{1+u}{\sqrt{(1+u)^2-1}}$

$$\frac{-1}{\sqrt{(1+e^{x})^2-1}}-\frac{1+e^{x}}{\sqrt{(1+e^{x})^2-1}}+C$$

University of Jordan
Department of Mathematics

**Calculus II** **First Exam** **Date 3/11/2010**

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**Q1)** *Find the area between the two curves* $y^2 = -x + 4$ *and* $y = x + 2$

**Q2)** *Find* $\displaystyle\int \frac{x \sin^{-1}(x)}{\sqrt{1-x^2}}\, dx$

**Q3 – a)** *Find* $\int \sin(3x)\cos(5x)\, dx$

**b)** *Find* $\displaystyle\int \frac{\sqrt{x^2-4}}{x}\, dx$

**Q4)** *Find the value of the following improper integrals* (if exist)

$$\int_e^{\infty} \frac{dx}{x(\ln x)^2}$$

**Q4)** *Find* $\displaystyle\int \frac{-2x^2+x+2}{x^2+x^4}\, dx$

1)

*point of intersection :

$4-y^2 = y-2$

$y^2+y-6=0$

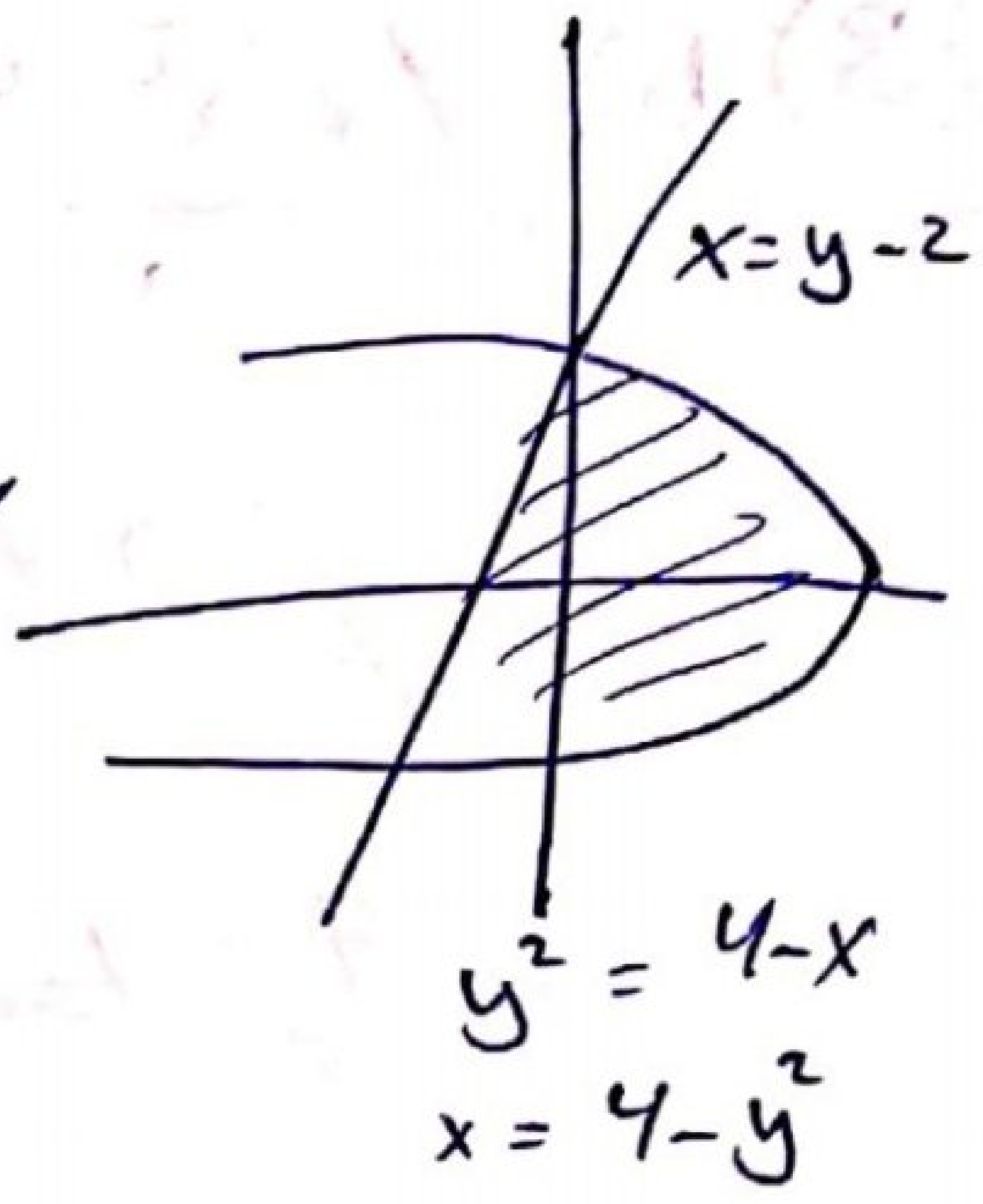
$(y+3)(y-2)=0$

$y=2,\ y=-3$

$A = \int_{-3}^{2} ((4-y^2)-(y-2))\,dy$

$= \int_{-3}^{2} (-y^2-y+6)\,dy$

$= \cancel{2} -y^3/_3 - y^2/_2 + 6y \Big|_{-3}^{2}$



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~~2) $\int \frac{\sqrt{x^2-4}}{x}\,dx$~~

2) $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx$ $\quad u = \sin^{-1}(x)$ $\quad dv = \frac{x}{\sqrt{1-x^2}}\,dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$ $\quad v = -\sqrt{1-x^2}$

$\Rightarrow -\sin^{-1}(x)\sqrt{1-x^2} + \int dx$

$= -\sin^{-1}(x)\sqrt{1-x^2} + x + c$

---

3) a) $\int \sin 3x \cos 5x\,dx = \int \frac{1}{2}\sin(8x) \mp \frac{1}{2}\sin(2x)\,dx$

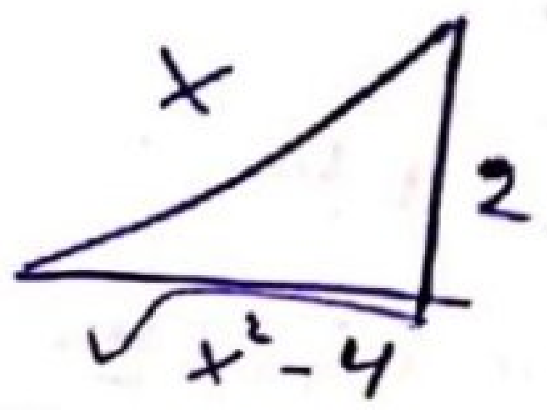
$= -\frac{1}{2}\cdot\frac{\cos 8x}{8} + \frac{1}{2}\cdot\frac{\cos 2x}{2} + c$

$= \frac{\cos 2x}{4} - \frac{\cos 8x}{16} + c$

3) b) $\int \frac{\sqrt{x^2-4}}{x}\,dx$ $\qquad x = 2\sec\theta$

$\qquad dx = 2\sec\theta\tan\theta\,d\theta$

$= \int \frac{2\tan\theta \cdot \cancel{2\sec\theta}\tan\theta\,d\theta}{\cancel{2\sec\theta}}$



$= 4\int \tan^2\theta\,d\theta = 4\int (\sec^2\theta - 1)\,d\theta$

$= 4(\tan\theta - \theta) + c$ $\qquad \frac{x}{2} = \sec\theta$

$= 4\left(\frac{2}{\sqrt{x^2-4}} - \sec^{-1}\left(\frac{x}{2}\right)\right) + c$ $\qquad$ ~~$\theta = \sec^{-1}(\frac{x}{2})$~~ $\qquad \theta = \sec^{-1}\left(\frac{x}{2}\right)$

4) $\int_e^\infty \frac{dx}{x(\ln x)^2} = \lim_{a\to\infty} \int_e^a \frac{dx}{x(\ln x)^2}$ $\qquad$ let $u = \ln x$

$\qquad du = \frac{dx}{x}$

$= \lim_{a\to\infty} \int_1^{\ln a} \frac{\cancel{x}\,du}{\cancel{x}\cdot u^2} = \int_1^{\ln a} u^{-2}\,du$

$= -\frac{1}{u}\Big|_1^{\ln a}$

$= \lim_{a\to\infty} \frac{-1}{\ln a} + 1 = 0 + 1 = 1$

5) $\int \frac{-2x^2+x+2}{x^4+x^2}\,dx = \int \frac{-2x^2+x+2}{x^2(x^2+1)} = \int \frac{A}{x} + \int \frac{b}{x^2} + \int \frac{Cx+D}{x^2+1}$

$-2x^2+x+2 = Ax(x^2+1) + b(x^2+1) + (Cx+D)x^2$

at $x=0 \Rightarrow$ $\boxed{b=2}$

$-2x^2+x+2 = Ax^3 + Ax + bx^2 + b + Cx^3 + Dx^2$

$x^3:\ 0 = A + C$

$x^2:\ -2 = b + D$

$x:\ \boxed{1 = A}$

$\boxed{C=-1}$ $\quad D = -4$

$= \int \frac{dx}{x} + \int 2x^{-2}\,dx + \int \frac{-x-4}{x^2+1}\,dx$

$= \ln x - \frac{2}{x} + \int \frac{-x}{x^2+1} - \int \frac{4}{x^2+1}\,dx$

$= \ln x - \frac{2}{x} - \frac{1}{2}\ln|x^2+1| - 4\tan^{-1}(x)$

$+ C$

3 - 11 - 2010

1

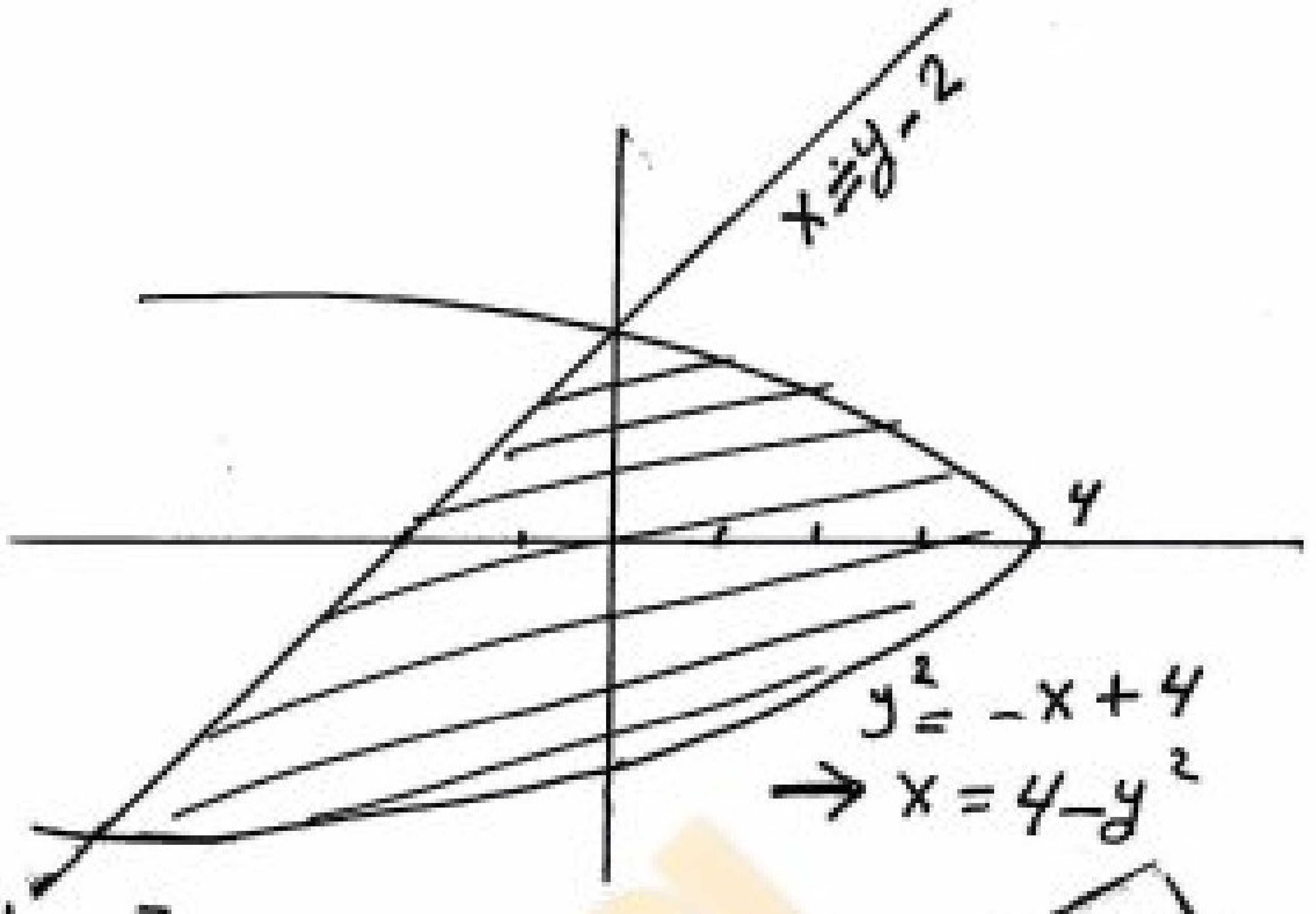
intersection

$4-y^2=y-2$

$0=y^2+y-6$

$0=(y+3)(y-2)$

$y=-3$ $y=2$



$\int_{-3}^{2}(4-y^2)-(y-2).dy =$

$\int_{-3}^{2} 6-y^2-y.dy = 6y-\frac{y^3}{3}-\frac{y^2}{2}\Big]_{-3}^{2}$

$=(12-\frac{8}{3}-2)-(-18+9-\frac{9}{2})$

---

3) ⓑ

$\int \frac{\sqrt{x^2-4}}{x}\,dx$

sol: $x=2\sec\theta$

$dx=2\sec\theta.\tan\theta\,d\theta$

$\sqrt{x^2-4}=\sqrt{4\sec^2\theta-4}$

$=\sqrt{4(\sec^2\theta-1)}$

$=\sqrt{4\tan^2\theta}$

$=2\tan\theta$

$\int\frac{2\tan\theta}{2\sec\theta}.2\sec\theta.\tan\theta\,d\theta$

$=2\int\tan^2\theta.\,d\theta$

$=2\int\sec^2\theta-1.\,d\theta$

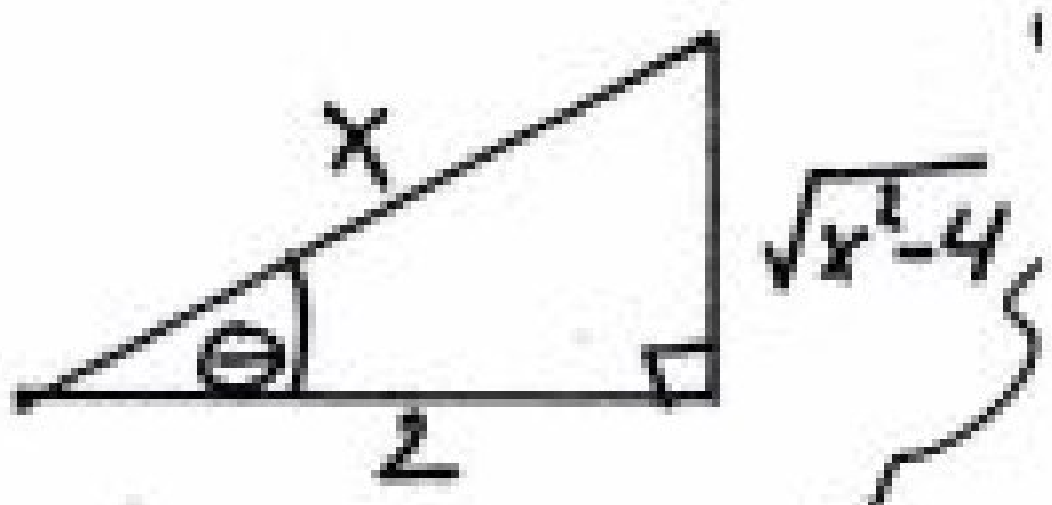
$=2(\tan\theta-\theta)+C$

نحول بدلالة x

$x=2\sec\theta$

$\sec\theta=\frac{x}{2}$

$\theta=\sec^{-1}(\frac{x}{2})$



$2(\tan\theta-\theta)+C$

$2(\frac{\sqrt{x^2-4}}{2}-\sec^{-1}(\frac{x}{2}))+C$

4 $\int_e^{\infty} \frac{dx}{x(\ln x)^2}$

sol: $\lim_{b\to\infty} \int_e^{b} \frac{dx}{x(\ln x)^2}$

Let $y = \ln x$
$du = \frac{1}{x}\,dx$
$x\,dy = dx$

$$\int_e^{b} \frac{dx}{x(\ln x)^2} = \int \frac{x\,dy}{x\,(y)^2} = \int \frac{1}{y^2}\,dy$$

$$= \int y^{-2}\cdot dy = -y^{-1} = -\frac{1}{y} = \frac{-1}{\ln x}$$

$$-\frac{1}{\ln x}\Big|_e^{b} = -\frac{1}{\ln b} - -\frac{1}{\ln e} = \frac{-1}{\ln b} + 1$$

$$\lim_{b\to\infty}\left(\frac{-1}{\ln b} + 1\right) = 0 + 1 = 1$$

---

5 $\int \frac{-2x^2+x+2}{x^4+x^2}\,dx$

$$\frac{-2x^2+x+2}{x^2(1+x^2)} = \frac{a}{x} + \frac{b}{x^2} + \frac{cx+d}{1+x^2}$$

نوحد مقامات

$$-2x^2+x+2 = a(x)(1+x^2) + b(1+x^2) + (cx+d)(x^2)$$

when $\boxed{x=0} \Rightarrow 2 = a(0) + b(1) + (d)(0) \Rightarrow \boxed{b=2}$

نعتمد على تساوي المعاملات

$$-2x^2+x+2 = ax + ax^3 + b + bx^2 + cx^3 + dx^2$$

$x^3$ معاملات $\Rightarrow \boxed{0 = a+c}$

$x^2$ معاملات $\Rightarrow -2 = b+d \Rightarrow -2 = 2+d \Rightarrow \boxed{-4=d}$

$x$ معاملات $\Rightarrow \boxed{1 = a}$

$a+c=0 \Rightarrow \boxed{c=-1}$

$$\int \frac{a}{x} + \frac{b}{x^2} + \frac{cx+d}{1+x^2} \quad dx$$

$$\int \frac{1}{x}\,dx + \int \frac{2}{x^2}\,dx + \int \frac{-x+-4}{1+x^2}\,dx$$

$$\int \frac{1}{x}\,dx + 2\int x^{-2}\cdot dx - \int \frac{x}{1+x^2}\,dx - 4\int \frac{1}{1+x^2}\,dx$$

$$\ln|x| \quad - 2x^{-1} \quad - \frac{1}{2}\ln|1+x^2| \quad -4\tan^{-1}x + C$$

The University of Jordan | First Exam | Second Semester
Department of Mathematics | Math 102 | 16.03.2011

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Name 0795000973

Section

Student Number 0795545798

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Evaluate the following integrals

1) $\int \mathrm{Ln}\,(x^2 + 4)\,dx$

2) $\int_1^\infty \frac{1}{x^{10} + x}\,dx$

3) $\int \frac{\sin x}{\sin x + \tan x}\,dx$

1) $\int \ln(x^2+4)\,dx$

$u = \ln(x^2+4) \qquad dv = dx$

$du = \frac{2x}{x^2+4} \qquad v = x$

$$\begin{array}{r|l} & 2 \\ \hline x^2+4 & 2x^2 \\ & -\cancel{2x^2} \mp 8 \\ \hline & 8 \end{array}$$

$= x\ln(x^2+4) - \int \frac{2x^2}{x^2+4}\,dx$

$= x\ln(x^2+4) - \int 2dx - \int \frac{8}{x^2+4}\,dx$

$= x\ln(x^2+4) - 2x - 4\tan^{-1}(\frac{x}{2}) + c$

---

2) $\int_1^\infty \frac{1}{x^{10}+x}\,dx = \lim_{a\to\infty} \int_1^a \frac{1}{x^{10}+x}\,dx$

$= \lim_{a\to\infty} \int_1^a \frac{1}{x^{10}(1+x^{-9})}\,dx = \lim_{a\to\infty} \int_1^a \frac{x^{-10}}{1+x^{-9}}\,dx = \lim_{a\to\infty} -\frac{1}{9}\ln|1+x^{-9}|\Big|_1^a$

$= \lim_{a\to\infty} -\frac{1}{9}\ln|\cancel{1+\underbrace{a^{-9}}_{0}}| + \frac{1}{9}\ln 2 = \frac{1}{9}\ln 2$

conv

---

3) $\int \frac{\sin x}{\sin x + \tan x}\,dx$ (Half angle - sup)

$u = \tan\frac{x}{2}$

$\sin x = \frac{2u}{1+u^2}$

$\tan x = \frac{2u}{1-u^2}$

$dx = \frac{2du}{1+u^2}$

$= \int \frac{\frac{2u}{1+u^2}}{\frac{2u}{1+u^2} + \frac{2u}{1-u^2}} \cdot \frac{2du}{1+u^2}$

$= \int \frac{4u\,du}{\frac{2u - \cancel{2u^3} + 2u + \cancel{2u^3}}{1-u^2}}$

$= \int \frac{\cancel{4u}\,du}{\cancel{4u}}(1-u^2)$

$= u - u^3/3 = \tan\frac{x}{2} - \tan^3\frac{x}{2} + c$

4) $\int \frac{x^2+1}{(x^2-2x+2)^2}\,dx$

5) $\int \frac{e^x}{(e^x-2)(e^{2x}+1)}\,dx$

$$4)\ \int \frac{x^2+1}{(x^2-2x+2)^2}\,dx = \int \frac{Ax+B}{x^2-2x+2}\,dx + \int \frac{Cx+D}{(x^2-2x+2)^2}\,dx$$

$$x^2+1 = (x^2-2x+2)(Ax+B) + Cx+D$$

$$x^2+1 = Ax^3 - 2Ax^2 + 2Ax + Bx^2 - 2Bx + 2B + Cx + D$$

$x^3:\ \boxed{0 = A}$ $\qquad x:\ 0 = 2\overset{0}{\cancel{A}} - 2B + C$

$x^2:\ 1 = -2\overset{0}{A} + B$ $\qquad \boxed{C=2}$

$\boxed{B=1}$

$C:\ 1 = -2B + D$

$\boxed{D=3}$

$$5)\ \int \frac{e^x}{(e^x-2)(e^{2x}+1)}\,dx \qquad \text{let } u = e^x \qquad du = e^x\,dx$$

$$= \int \frac{u\,du}{u(u-2)(u^2+1)} = \int \frac{du}{u^3-2u^2+u-2}$$

$$= \int \frac{du}{(u-2)(u^2+1)} = \int \frac{A}{u-2}\,du + \int \frac{Bx+C}{u^2+1}\,du$$

$$\begin{array}{r|l} & u^2+1 \\ \hline u-2 & u^3-2u^2+u-2 \\ & -u^3 \pm 2u^2 \\ \hline & u-2 \\ & -u \pm 2 \\ \hline & 0 \end{array}$$

$$1 = A(u^2+1) + (Bx+c)(u-2)$$

$u=2 \Rightarrow\ 1 = 5A$ $\qquad u=1 \Rightarrow 1 = \frac{2}{5} + (B \mp \frac{2}{5})(-2)$

$A = 1/5$

$\frac{3}{5} = -2B + \frac{4}{5}$

$B = \frac{1}{10}$

$u=0 \Rightarrow 1 = 1/5 - 2C$

$C = \frac{-2}{5}$

$$\Rightarrow \int \frac{\frac{1}{5}}{u-2}\,du + \int \frac{\frac{1}{10}u - \frac{2}{5}}{u^2+1}\,du$$

↳ $(\ln|u-2|)$

↳ $\int \frac{\frac{u}{10}}{u^2+1} - \int \frac{\frac{2}{5}}{u^2+1}$ ㊸

(u-sup) $\qquad$ ↳ $(\tan^{-1})$

**الجامعة الاردنية** **تفاضل وتكامل ٢** **الاختبلر للاول ٢٠١١/١١/١**

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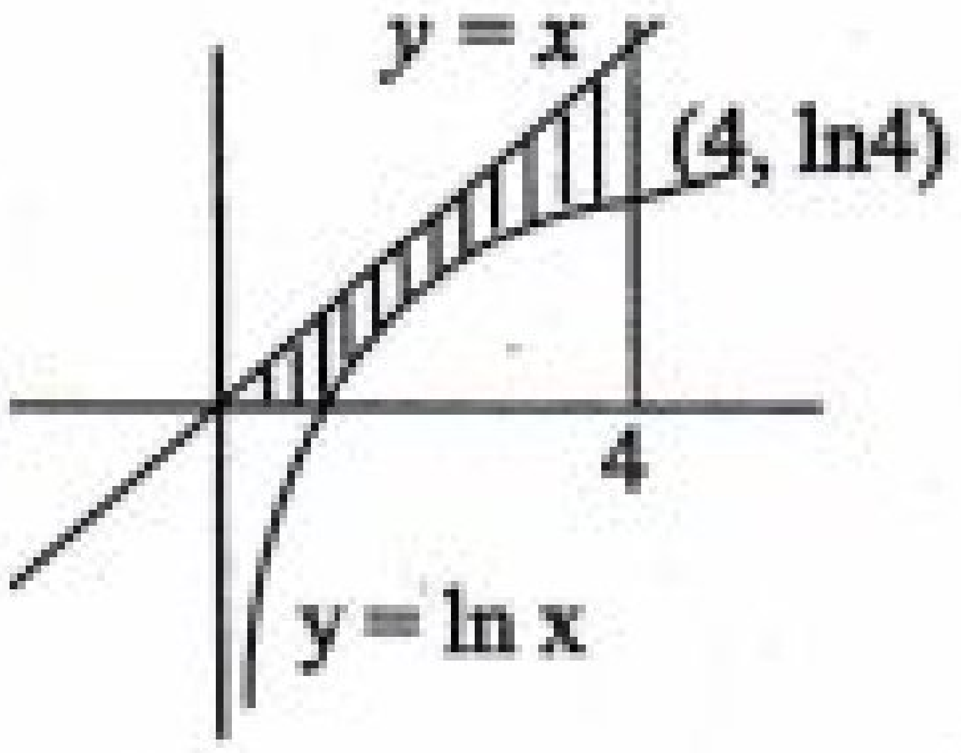
**[1]** ***Find the area of he shaded region***



**[2]** $$\int \frac{x^3\,dx}{\sqrt{x^2+4}}$$

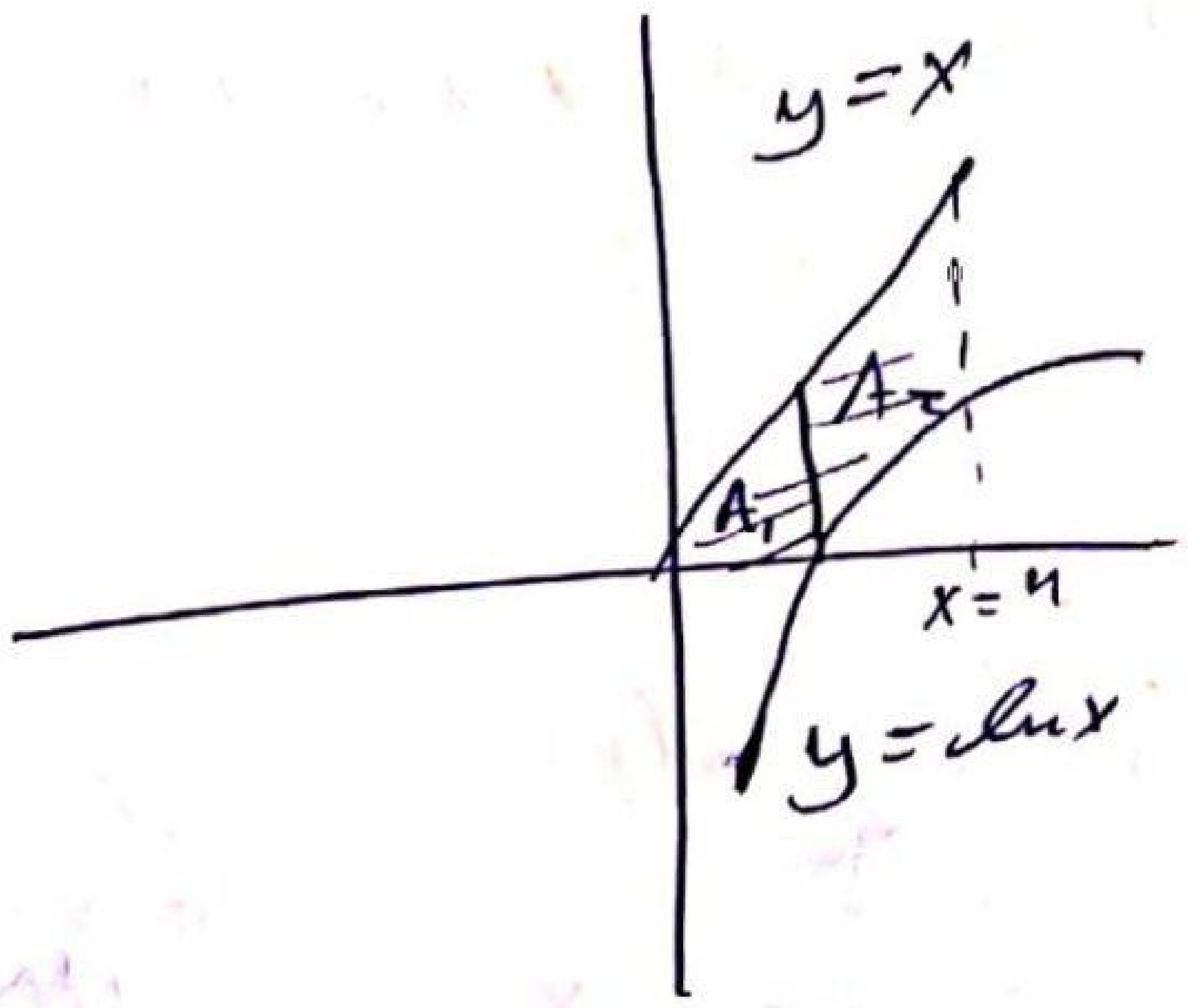
**[3]** $$\int \frac{x\tan^{-1}x\,dx}{\sqrt{1+4}}$$

**[4]** $$\int \frac{dx}{1-\cos x+\sin x}$$

**[5] (a)** ***Give the appropriate from the partial fraction decomposition of*** **(Don't find the constants)**

$$\frac{x^2-2x+5}{(x+1)^2(x^2-x+1)}$$

**(b)** ***Find*** **(if exist)** $$\int_0^{\frac{1}{8}} \frac{dx}{\sqrt[3]{x}-x}$$



1) $A = A_1 + A_2$

$A_1 = \int_0^1 x\,dx$

$= \frac{x^2}{2}\Big]_0^1 = \frac{1}{2}$

$A_2 = \int_1^4 (x - \ln x)\,dx$

$= \int_1^4 x\,dx - \int_1^4 \ln x\,dx \Rightarrow$ By parts $(u = \ln x \quad dv = \ln x)$

---

2) $\int \frac{x^3\,dx}{\sqrt{x^2+4}}$

$u = \sqrt{x^2+4}$

$u^2 = x^2+4$

$2u\,du = 2x\,dx$

$u\,du = x\,dx$

$\int \frac{x^3 \cdot u\,du}{\cancel{x} \cdot u}$

$\int (x^2)\,du = \int (u^2-4)\,du = \frac{u^3}{3} - 4u$

$= \frac{(x^2+4)^{3/2}}{3} - 4(x^2+4)^{1/2} + C$

---

4) $\int \frac{dx}{1-\cos x+\sin x}$

$u = \tan\frac{x}{2}$

$\sin x = \frac{2u}{1+u^2}$

$\cos x = \frac{1-u^2}{1+u^2}$

$\int \frac{\frac{2du}{u^2+1}}{1 - \frac{1-u^2}{u^2+1} + \frac{2u}{1+u^2}} = \int \frac{\frac{2du}{\cancel{1+u^2}}}{\frac{u^2+1-1+u^2+2u}{\cancel{1+u^2}}}$

$= \int \frac{\cancel{2}du}{\cancel{2}u^2+\cancel{2}u} = \int \frac{du}{u(u+2)} = \int \frac{A}{u}du + \int \frac{B}{u+2}$

$1 = A(u+2) + Bu$

$u=0 \Rightarrow A = 1/2$

$u=-2 \Rightarrow B = -1/2$

$= \int \frac{1}{2}u\,du + \int \frac{-\frac{1}{2}}{u+2}du$

$= \ln|u| - \frac{1}{2}\ln|u+2| + C$

$$5)\ a)\ \frac{x^2 \mp 2x + 5}{(x+1)^2\,(x^2-x+1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2-x+1}$$

---

$$b)\ \int_0^{1/8} \frac{dx}{\sqrt[3]{x}-x} = \lim_{a\to 0^+} \int_a^{1/8} \frac{dx}{\sqrt[3]{x}-x}$$

$$\int_a^{1/8} \frac{dx}{\sqrt[3]{x}-x} \qquad u=\sqrt[3]{x}$$

$$u^3 = x$$

$$3u^2\,du = dx$$

$$\int_{\sqrt[3]{a}}^{\frac{1}{2}} \frac{3u^2\,du}{u-u^3} = \int_{\sqrt[3]{a}}^{\frac{1}{2}} \frac{3u^2\,du}{u(1-u)(1+u)}$$

$$= \overset{0}{\cancel{\int_{\sqrt[3]{a}}^{\frac{1}{2}} \frac{A}{u}}} + \int_{\sqrt[3]{a}}^{\frac{1}{2}} \frac{B}{1-u} + \int_{\sqrt[3]{a}}^{\frac{1}{2}} \frac{C}{1+u}\,du$$

$$= \quad A=0,\ B=3/2,\ C=-3/2$$

$$= \lim_{a\to 0^+} \ -\frac{3}{2}\ln|1-u|\Big]_{\sqrt[3]{a}}^{\frac{1}{2}} + -\frac{3}{2}\ln|1+u|\Big]_{\sqrt[3]{a}}^{1/2}$$

$$=$$