If $f(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{5k}}{5^k k!}$, $g(x) = x^2 f'(5x)$, then the series representation of $g(x)$ is

A) $\sum_{k=1}^{\infty} \frac{(-1)^k k\, 5^{-k}\, x^{5k+1}}{k!}$

B) $\sum_{k=1}^{\infty} \frac{(-1)^k 5^{4k}\, x^{5k+1}}{k!}$

C) $\sum_{k=1}^{\infty} \frac{(-1)^k k\, 5^{4k}\, x^{5k}}{k!}$

D) $\sum_{k=1}^{\infty} \frac{(-1)^k k\, 5^{4k}\, x^{5k+1}}{k!}$

E) $\sum_{k=1}^{\infty} \frac{(-1)^k k\, 5^{k}\, x^{5k+1}}{k!}$

Select one:

- A
- B
- C
- D

$$f'(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (5k)(x)^{5k-1}}{5^k k!}$$

$$f'(5x) = \sum_{k=1}^{\infty} \frac{(-1)^k (5k)(5x)^{5k-1}}{5^k k!} = \sum_{k=1}^{\infty} \frac{(-1)^k k\, 5^{5k}\, x^{5k-1}}{5^k k!}$$

$$g(x) = x^2 f'(5x) = \sum_{k=1}^{\infty} \frac{(-1)^k k\, 5^{5k}\, x^{5k+1}}{5^k k!} \to g(x) = \sum_{k=1}^{\infty} \frac{(-1)^k k\, 5^{4k}\, x^{5k+1}}{k!}$$

$\boxed{\text{Answer is D}}$

The appropriate trigonometric substitution that solves the integral $\int \sqrt{x^2 - 4x + 13}dx$ is:

A) $x = 2 + 3\sec\theta$
B) $x = -2 + 3\tan\theta$
C) $x = 2 + 3\sin\theta$
D) $x = -2 + 3\sin\theta$
E) $x = 2 + 3\tan\theta$

Select one:

- ○ A
- ○ B
- ○ C
- ○ D
- ● E

$\sqrt{x^2 - 4x + 13} = \sqrt{(x-2)^2 + 9}$

الجواب يجب أن يتخلص من الجذر

$x = 2 + 3\tan\theta \rightarrow \sqrt{(2 + 3\tan\theta - 2)^2 + 9} = \sqrt{9\tan^2\theta + 9} = 3\sec\theta$

$\boxed{\text{Answer is E}}$

Consider the series $\sum_{n=0}^{\infty} a_n$ where $a_n = \frac{(-4)^n}{n\,(5)^{n+1}}$

Then $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

A) ∞

B) $\frac{5}{4}$

C) $\frac{4}{5}$

D) $\frac{-4}{5}$

E) 0

Select one:

- A
- B
- C
- D
- E

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{4^{n+1}}{(n+1)\,5^{n+2}} * \frac{(n)\,5^{n+1}}{4^n} = \lim_{n\to\infty}\frac{4}{5} * \frac{n}{n+1} = \frac{4}{5}$$

$\boxed{\text{Answer is C}}$

The interval of convergence the series $\sum_{n=0}^{\infty} \frac{7^{2n+5} x^{n+1}}{5^{3n+1}}$ is:

A) $\left[\frac{-125}{49}, \frac{125}{49}\right]$

B) $\left(\frac{-5}{7}, \frac{5}{7}\right)$

C) $\left[\frac{-5}{7}, \frac{5}{7}\right]$

D) $\left(\frac{-125}{49}, \frac{125}{49}\right)$

E) $(-\infty, \infty)$

Select one:

- A
- B
- C
- D
- E

Clear my choice

$$\sum_{n=0}^{\infty} \frac{7^{2n+5}x^{n+1}}{5^{3n+1}} = \sum_{n=0}^{\infty} \frac{7^{2n} * 7^5 * x^{n+1}}{5^{3n} * 5^1} = \frac{7^5}{5} \sum_{n=0}^{\infty} \frac{49^n x^{n+1}}{125^n}$$

Using ratio test: $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{49^{n+1}x^{n+2}}{125^{n+1}} * \frac{125^n}{49^n x^{n+1}}\right| = \left|\frac{49x}{125}\right|$

For the series to converge: $\left|\frac{49x}{125}\right| < 1 \longrightarrow \frac{-125}{49} < x < \frac{125}{49}$

Check endpoints: $x = \frac{-125}{49} \longrightarrow \frac{7^5}{5} \sum_{n=0}^{\infty} \frac{49^n (\frac{-125}{49})^{n+1}}{125^n}$, the series diverges

$x = \frac{125}{49} \longrightarrow \frac{7^5}{5} \sum_{n=0}^{\infty} \frac{49^n (\frac{125}{49})^{n+1}}{125^n}$, the series diverges

Answer is D

If $\sum_{n=0}^{\infty} a_n$, and $\sum_{n=0}^{\infty} b_n$ are two series with positive terms and $a_n \leq b_n$, then one of the following is true

A) If $\sum_{n=0}^{\infty} a_n$ converge, and $lim_{n\to\infty} \frac{a_n}{b_n} = 7$ then $\sum_{n=0}^{\infty} b_n$ converge

B) If $\sum_{n=0}^{\infty} a_n$ converge, and $lim_{n\to\infty} \frac{a_n}{b_n} = 7$ then $\sum_{n=0}^{\infty} b_n$ diverge

C) If $\sum_{n=0}^{\infty} a_n$ diverge, and $lim_{n\to\infty} \frac{a_n}{b_n} = \frac{1}{7}$ then $\sum_{n=0}^{\infty} b_n$ converge

D) If $\sum_{n=0}^{\infty} a_n$ converge, then $\sum_{n=0}^{\infty} b_n$ converge

E) If $\sum_{n=0}^{\infty} b_n$ diverge, then $\sum_{n=0}^{\infty} a_n$ diverge

Select one:

- A
- B
- C
- D
- E

Clear my choice

From LCT: Assume a_n is a convergent series and $a_n \leq b_n$

If $\lim_{n\to\infty} \frac{b_n}{a_n} = L \ (0 \leq L < \infty) \implies b_n$ is convergent

$\boxed{\text{Answer is A}}$

(By cylindrical shells) The integral that represents the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $y = 3$ and $x = 0$ about $y = 3$ is:

A) $2\pi \int_0^3 (3 + y)y^2 \, dy$

B) $2\pi \int_0^3 (3 - y)y \, dy$

C) $2\pi \int_0^3 (3 - y)y^2 \, dy$

D) $2\pi \int_0^3 (y - 3)y^2 \, dy$

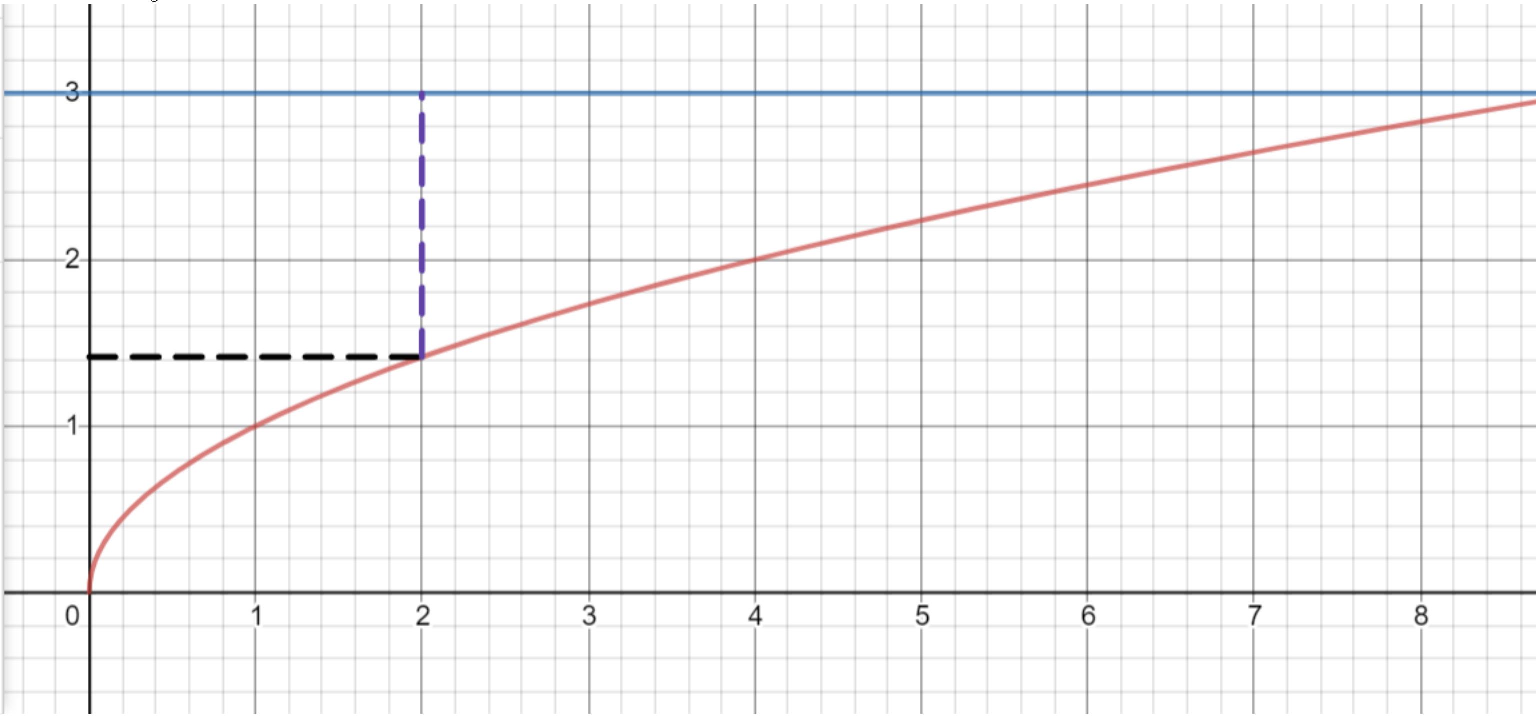
E) $2\pi \int_0^3 y^3 \, dy$

Select one:

- A
- B
- C
- D
- E

Cylindrical shells formula:

$$V = 2\pi \int_{y_1}^{y_2} rh\,dy$$



$r = 3 - y$ (The purple line, which is the distance from the axis of rotation)

$h = y^2$ (The black line, which is the distance from the y-axis)

$$V = 2\pi \int_{0}^{3} y^2(3 - y)dy$$

Answer is C

Which of the following series is converge:

A) $\sum_{n=1}^{\infty}\left(\frac{n+1}{n}\right)^{n}$

B) $\sum_{n=1}^{\infty}\left(\frac{3n^2+n+1}{2n^2+n+6}\right)^{n}$

C) $\sum_{n=1}^{\infty}\left(\frac{3n^3+n+1}{n^3+n+6}\right)^{n}$

D) $\sum_{n=1}^{\infty}\left(\frac{2n^2+3n+3}{5n^2+7n+3}\right)^{n}$

E) $\sum_{n=1}^{\infty}\left(\frac{n-3}{n}\right)^{n}$

Select one:

- ○ A
- ○ B
- ○ C
- ● D
- ○ E

Series B and C are divergent by root test

Series A: $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e \neq 0 \longrightarrow$ divergent by divergence test

Series E: $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(1 - \frac{3}{n}\right)^n = e^{-3} \neq 0 \longrightarrow$ divergent by divergence test

Series D: $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \frac{2n^2+3n+3}{5n^2+7n+3} = \frac{2}{5} < 1 \longrightarrow$convergent

$\boxed{\text{Answer is D}}$

The polar coordinate of the point $(1, -\sqrt{3})$ is

A) $(2, \frac{-\pi}{6})$

B) $(2, \frac{5\pi}{6})$

C) $(2, \frac{-\pi}{3})$

D) $(2, \frac{2\pi}{3})$

E) $(-2, \frac{\pi}{6})$

Select one:

- A
- B
- C
- D
- E

$$r = \sqrt{1^2 + (-\sqrt{3})^2} = 2$$

$$\theta = \arctan\left(\frac{-\sqrt{3}}{1}\right) = \frac{2\pi}{3} \text{ or } \frac{-\pi}{3}$$

الجواب حسب الربع، وبما أنه ال x موجب و y سالب إذا الزاوية في الربع الرابع

Answer is C

If $cos x = \Sigma_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$, then $\Sigma_{k=1}^{\infty} \frac{(-1)^k \pi^{2k+1}}{3^{2k+1}(2k)!} =$

A) $\frac{-3\pi}{2}$

B) $\frac{3\pi}{2}$

C) $\frac{\pi}{6}$

D) $\frac{-\pi}{2}$

E) $\frac{-\pi}{6}$

Select one:

- A
- B
- C
- D
- E

$$\sum_{k=1}^{\infty} \frac{(-1)^k \pi^{2k+1}}{3^{2k+1}(2k)!} = \frac{\pi}{3} \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{\pi}{3}\right)^{2k}}{(2k)!} = \frac{\pi}{3} \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\pi}{3}\right)^{2k}}{(2k)!} - 1 \right)$$

$$= \frac{\pi}{3} \left(\cos \frac{\pi}{3} - 1 \right) = \frac{-\pi}{6}$$

$\boxed{\text{Answer is E}}$

Which of the following series is conditionally converge:

A) $\sum_{n=1}^{\infty}(-1)^n \frac{2n+1}{3n^2+3n+1}$

B) $\sum_{n=1}^{\infty}(-1)^n \frac{2}{3n^2+3n+1}$

C) $\sum_{n=1}^{\infty}(-1)^n \frac{2n+1}{n^3+3n+1}$

D) $\sum_{n=1}^{\infty}(-1)^n \frac{3}{2^n+1}$

E) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{2^n-1}$

Select one:

- A
- B
- C
- D

For series (A),us alternating test:

1) Prove that a_n is decreasing, you can use the first derivative

2) $\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{2n+1}{3n^2+3n+1} = 0$

Since both conditions are true, the series is conditionally convergent

For series (B), using LCT with $\boxed{b_n = \frac{1}{n^2}}$ it's absolutely convergent

For series (C), using LCT with $\boxed{b_n = \frac{1}{n^2}}$ it's absolutely convergent

For series (D), using LCT with $\boxed{b_n = \frac{1}{2^n}}$ it's absolutely convergent

For series (E), using LCT with $\boxed{b_n = \frac{1}{2^n}}$ it's absolutely convergent

$\boxed{\text{Answer is A}}$

The sum of the series $\sum_{k=2}^{\infty} \frac{1-5^{k-1}}{6^{k-1}}$ is:

A) $\frac{24}{5}$

B) $\frac{26}{5}$

C) $\frac{20}{5}$

D) $\frac{-24}{5}$

E) $\frac{-8}{5}$

Select one:

- A
- B
- C
- D
- E

$$\sum_{n=2}^{\infty} \frac{1-5^{k-1}}{6^{k-1}} = \sum_{n=2}^{\infty} \frac{1}{6^{k-1}} - \sum_{n=2}^{\infty} \frac{5^{k-1}}{6^{k-1}} = 6 * \frac{1/36}{1-1/6} - \frac{6}{5} * \frac{25/36}{1-5/6} = \frac{-24}{5}$$

$$\boxed{\text{Answer is D}}$$

The integral that represents the volume of the solid obtained by rotating the region enclosed by $y = 4x$, $y = 2x + 2$, and $y = 0$ about x-axis is

A) $\int_0^4 (\frac{y}{4} + 1)dy$

B) $\int_{-1}^0 (x + 2)^2 dx + \int_0^1 16x^2 dx$

C) $\int_{-1}^1 (2x + 2)^2 dx - \int_{-1}^1 16x^2 dx$

D) $\int_{-1}^1 (2x + 2)^2 dx + \int_0^1 (2 - 2x)^2 dx$

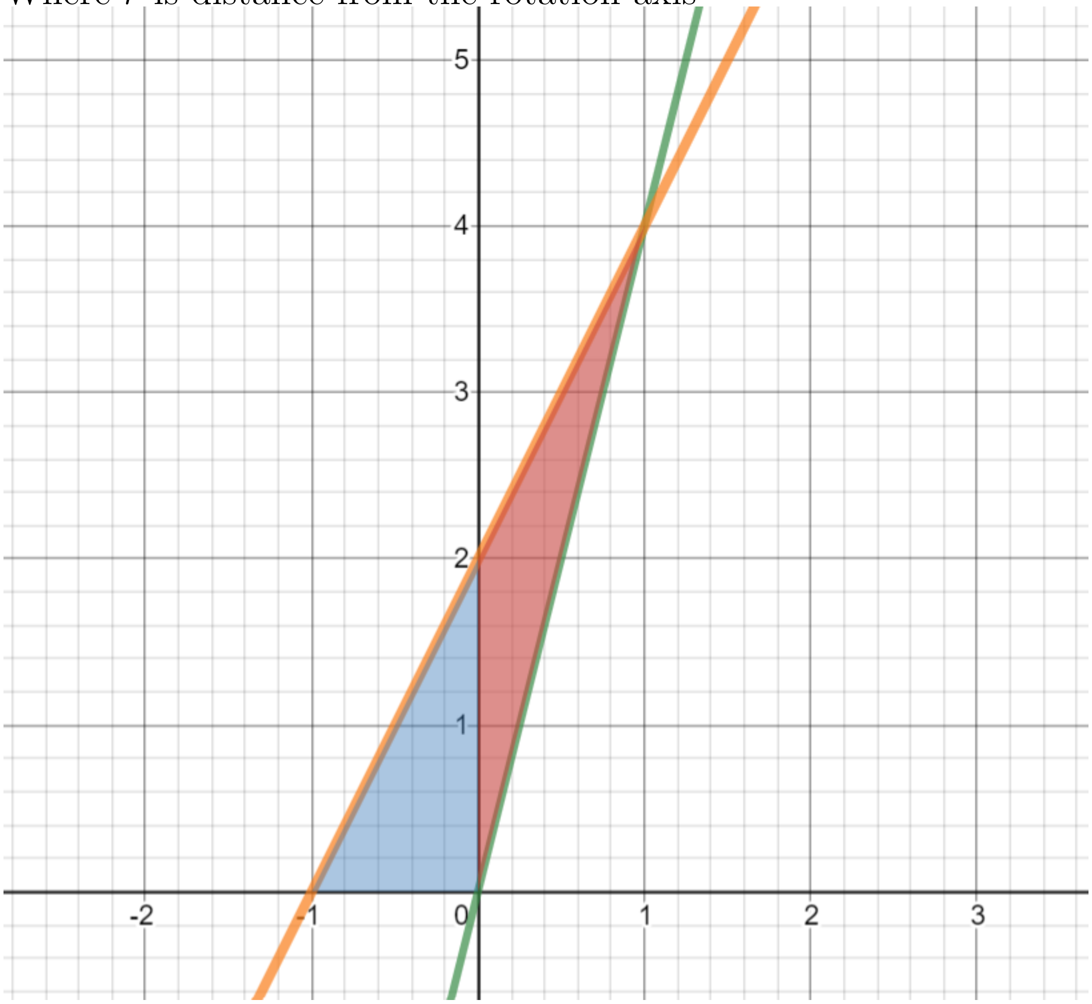
E) $\int_{-1}^0 (2x + 2)^2 dx + \int_0^1 (-12x^2 + 8x + 4)dx$

Select one

- A
- B
- C
- D
- E

Volume using disk method: $\pi \int_{x_1}^{x_2} [(r_2)^2 - (r_1)^2] dx$

Where r is distance from the rotation axis



Blue part:

$$V_1 = \pi \int_{-1}^{0} (2x+2)^2 dx$$

Red part:

$$V_2 = \pi \int_{0}^{1} [(2x+2)^2 - (4x)^2] dx = \pi \int_{0}^{1} (-12x^2 + 8x + 4) dx$$

$$V = V_1 + V_2 = \pi \int_{-1}^{0} (2x+2)^2 dx + \pi \int_{0}^{1} (-12x^2 + 8x + 4) dx$$

المطلوب في السؤال التكامل فقط، لهذا السبب π غير موجود في الاجابات

Answer is E

$$\int (3x+5)e^{3x+5}dx =$$

A) $\frac{1}{3}e^{3x+5}(3x+4)+C$

B) $\frac{1}{3}e^{3x+5}(3x+6)+C$

C) $\frac{1}{3}e^{3x-5}(3x+6)+C$

D) $\frac{1}{3}e^{3x+5}(3x-4)+C$

E) $\frac{1}{3}e^{3x-5}(3x+4)+C$

Select one:

- A
- B
- C
- D
- E

Using integration by parts:

$u = 3x + 5 \quad dv = e^{3x+5}$

$du = 3dx \qquad v = \frac{1}{3}e^{3x+5}$

$\int udv = uv - \int duv$

$$\int (3x+5)e^{3x+5}\,dx = \frac{1}{3}(3x+5)e^{3x+5} - \int e^{3x+5}\,dx = \frac{1}{3}(3x+4)e^{3x+5}$$

Answer is A

The power series representation of $f(x) = \frac{x}{x-5}$ is

A) $-\Sigma_{k=0}^{\infty} \frac{x^k}{5^{k+1}}, \quad |x| < 5$

B) $\Sigma_{k=0}^{\infty} \frac{x^{k+1}}{5^{k+1}}, \quad |x| < 5$

C) $-\Sigma_{k=0}^{\infty} \frac{x^{k+3}}{5^k}, \quad |x| < 5$

D) $-\Sigma_{k=0}^{\infty} \frac{x^{k+1}}{5^{k+1}}, \quad |x| < 5$

E) $\Sigma_{k=0}^{\infty} (-1)^k \frac{x^{k+1}}{5^{k+1}}, \quad |x| < 5$

Select one:

- A
- B
- C
- D
- E

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

$$\frac{1}{\frac{x}{5}-1} = -\sum_{k=0}^{\infty} \left(\frac{x}{5}\right)^k = \frac{5}{x-5}$$

$$f(x) = \frac{x}{x-5} = -\sum_{k=0}^{\infty} \frac{x^{k+1}}{5^{k+1}}$$

$$\boxed{\text{Answer is D}}$$

The curve $r = 2sin\theta - 6cos\theta$ represents

A) Circle with center $(-3, -1)$ and radius $\sqrt{10}$

B) Circle with center $(3, -1)$ and radius $\sqrt{10}$

C) Circle with center $(1, -3)$ and radius $\sqrt{10}$

D) Circle with center $(-3, 1)$ and radius $\sqrt{10}$

E) Circle with center $(-3, 1)$ and radius 2

Select one:

- A
- B
- C
- D
- E

Clear my choice

$r^2 = 2r\sin\theta - 6r\cos\theta$

$x^2 + y^2 = 2y - 6x$

$x^2 + 6x + y^2 - 2y = 0$

$x^2 + 6x + 9 + y^2 - 2y + 1 = 10$

$(x+3)^2 + (y-1)^2 = 10$

center is $(-3, 1)$ and radius is $\sqrt{10}$

OR: $r = 2a\cos\theta + 2b\sin\theta \rightarrow$ center is (a, b) and radius is $\sqrt{a^2 + b^2}$

$\boxed{\text{Answer is D}}$