$$\int_{3}^{\infty} \frac{2dx}{x^2 - 1} =$$

A. $\ln 3$

B. $\ln 2$

C. $\ln \frac{5}{3}$

D. $\ln \frac{3}{2}$

E. $\ln \frac{7}{5}$

- A. -
- B. -
- C. -
- D. -
- E. -

$$\frac{2}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$$
$$2 = A(x+1) + B(x-1)$$

$$x = 1 \longrightarrow \boxed{A = 1}$$

$$x = -1 \longrightarrow \boxed{B = -1}$$

$$\int_3^\infty \frac{2\,dx}{x^2-1} = \int_3^\infty \frac{dx}{x-1} - \int_3^\infty \frac{dx}{x+1}$$

$$\ln(x-1) - \ln(x+1)\Big|_3^\infty = \ln\left(\frac{x-1}{x+1}\right)\Big|_3^\infty = \ln(1) - \ln(\tfrac{2}{4}) = \ln 2$$

$\boxed{\text{Answer is B}}$

The Integral that represents the area enclosed by $y=x+2$, $y=4-x$, $y=0$, $x=0$ is

A) $\int_0^2 (x+2)dx + \int_2^4 (4-x)dx$

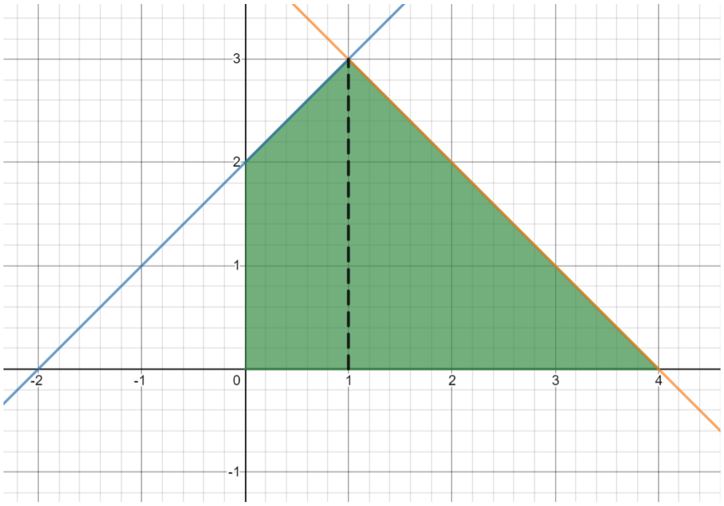
B) $\int_0^4 (4-x)dx$

C) $\int_0^4 (6-2y)dy$

D) $\int_0^2 (4-y)dy + \int_2^3 (6-2y)dy$

E) $\int_0^2 (4-y)dy + \int_2^4 (y-2)dy$

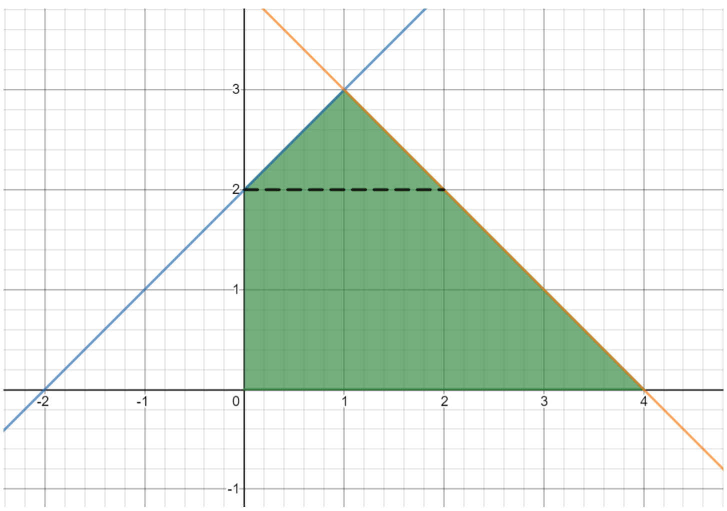
Area from x-axis



$x + 2 = 4 - x \rightarrow x = 1$

$$\text{Area} = \int_0^1 (x+2)\,dx + \int_1^4 (4-x)\,dx$$

Area from y-axis



Under the black line:

$$\int_0^2 (y-4) - 0\,dy = \int_0^2 (y-4)\,dy$$

Above the black line:

$y - 2 = 4 - y \rightarrow y = 3$

$$\int_2^3 ((4-y) - (y-2))\,dy = \int_2^3 (6-2y)\,dy$$

$$\text{Area} = \int_0^2 (y-4)\,dy + \int_2^3 (6-2y)\,dy$$

Answer is D

$$\int -12\ \sin 6x \sin 11x\, dx =$$

A) $-6 \sin 5x + 6 \sin 17x + C$

B) $6 \sin 5x - 6 \sin 17x + C$

C) $\frac{6}{5} \sin 5x - \frac{6}{17} \sin 17x + C$

D) $-30\sin 5x + 102 \sin 17x + C$

E) $\frac{-6}{5} \sin 5x + \frac{6}{17} \sin 17x + C$

Using: $\sin(x)\sin(y) = \dfrac{1}{2}(\cos(x-y) - \cos(x+y))$

$$\int -12\sin(6x)\sin(11x)\,dx = \int -6(\cos 5x - \cos 17x)\,dx = 6\left(\frac{\sin(17x)}{17} - \frac{\sin(5x)}{5}\right) + C$$

Answer is E

Which one of the following series is divergent?

(A) $\sum_{n=1}^{\infty} \frac{1}{5^n + n + 7}$

(B) $\sum_{n=2}^{\infty} \frac{1}{n^3 + \sqrt{n}}$

(C) $\sum_{n=1}^{\infty} \frac{5^n}{9^n + n^3}$

(D) $\sum_{n=1}^{\infty} \frac{5^n}{8^n + n}$

(E) $\sum_{n=6}^{\infty} \frac{1}{n - \sqrt[3]{n} - 2}$

○ A.-

For series (A), using LCT with $\boxed{b_n = \frac{1}{5^n}}$ it's convergent

For series (B), using LCT with $\boxed{b_n = \frac{1}{n^3}}$ it's convergent

For series (D), using LCT with $\boxed{b_n = \left(\frac{5}{9}\right)^n}$ it's convergent

For series (D), using LCT with $\boxed{b_n = \left(\frac{5}{8}\right)^n}$ it's convergent

For series (E), using LCT with $\boxed{b_n = \frac{1}{n}}$ it's divergent

$\boxed{\text{Answer is E}}$

If $f(x) = \Sigma_{k=0}^{\infty} \frac{(-1)^k\ x^{4k}}{4^k k!}$, $g(x) = x f'(4x)$, then the series representation of $g(x)$ is

A) $\Sigma_{k=1}^{\infty} \frac{(-1)^k\ k\ 4^k\ x^{4k}}{k!}$

B) $\Sigma_{k=1}^{\infty} \frac{(-1)^k\ 4^{3k}\ x^{4k+1}}{k!}$

C) $\Sigma_{k=1}^{\infty} \frac{(-1)^k\ k\ 4^{3k}\ x^{4k}}{k!}$

D) $\Sigma_{k=1}^{\infty} \frac{(-1)^k\ k\ 4^{-k}\ x^{4k}}{k!}$

E) $\Sigma_{k=1}^{\infty} \frac{(-1)^k\ 4^{3k}\ x^{4k}}{k!}$

$$f'(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (4k)(x)^{4k-1}}{4^k k!}$$

$$f'(4x) = \sum_{k=1}^{\infty} \frac{(-1)^k (4k)(4x)^{4k-1}}{4^k k!}$$

$$g(x) = xf'(4x) = \sum_{k=1}^{\infty} \frac{(-1)^k k\, 4^{4k}\, x^{4k}}{4^k k!} \rightarrow g(x) = \sum_{k=1}^{\infty} \frac{(-1)^k k\, 4^{3k}\, x^{4k}}{k!}$$

$\boxed{\text{Answer is C}}$

The power series representation of the function $x^6\left(\frac{1}{x+2}\right)$ is given by

(a) $\sum_{n=0}^{\infty} \frac{1}{2^{n+1}} x^{n+6}$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n-1}} x^{n+6}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} x^{n+6}$

(d) $\sum_{n=0}^{\infty} \frac{1}{2^{n+1}} x^{n+6}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} x^{n+6}$

- a. -
- b. -
- c. -
- d. -
- e. -

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

$$\frac{1}{1-(\frac{-x}{2})} = \sum_{k=0}^{\infty} \left(\frac{-x}{2}\right)^k = \frac{2}{x+2}$$

$$f(x) = \frac{x^6}{x+2} = \sum_{k=0}^{\infty} \frac{(-1)^n}{2^{n+1}} x^{n+6}$$

$$\boxed{\text{Answer is E}}$$

The sum of the series $\Sigma_{n=1}^{\infty}(7^{\frac{1}{n}} - 7^{\frac{1}{n+2}})$

A) $6 + \sqrt{7}$

B) $\sqrt{7}$

C) $7 + \sqrt{7}$

D) $5 + \sqrt{7}$

E) 7

Using partial sum:

$S_1 = 7^1 - 7^{1/3}$

$S_2 = (7^1 - 7^{1/3}) + (7^{1/2} - 7^{1/4})$

$S_3 = (7^1 - 7^{1/3}) + (7^{1/2} - 7^{1/4}) + (7^{1/3} - 7^{1/5}) = 7^1 + 7^{1/2} - 7^{1/4} - 7^{1/5}$

$S_n = 7 + 7^{1/2} - 7^{1/(n+1)} - 7^{1/(n+2)}$

$$\sum_{n=1}^{\infty}(7^{1/n} - 7^{1/(n+2)}) = \lim_{n\to\infty} S_n = 7 + \sqrt{7}$$

$\boxed{\text{Answer is C}}$

The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{2n+1}$ is

A. $(1,3]$

B. $(1,3)$

C. $[1,3)$

D. $[1,3]$

E. $(-\infty, \infty)$

Using ratio test:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}(x-2)^{n+1}}{2n+3} * \frac{2n+1}{(-1)^n(x-2)^n}\right| = |x-2|$$

For the series to converge: $|x-2| < 1 \longrightarrow 1 < x < 3$

Check endpoints: $x = -1 \longrightarrow \sum_{n=1}^{\infty}\frac{1}{2n+1} \Longrightarrow$ the series diverges by LCT with $b_n = \frac{1}{n}$

$x = 3 \longrightarrow \sum_{n=1}^{\infty}\frac{(-1)^n}{2n+1} \Longrightarrow$ the series converges by alternating test

$\boxed{\text{Answer is A}}$

Using the root test, the series $\sum_{n=1}^{\infty}(-1)^n\left(1-\frac{2}{n}\right)^{n^2}$

(A) The root test fails.

(B) Converges conditionally

(C) Diverges

(D) Converges absolutely

(E) None of the above.

- A. -
- B. -
- C. -
- D. -
- E. -

$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} (1 - \frac{2}{n})^n = e^{-2} < 1$, the series converges absolutely

$\boxed{\text{Answer is D}}$

$$\int (3x+5)e^{3x+5}dx =$$

A) $\frac{1}{3}e^{3x+5}(3x+4)+C$

B) $\frac{1}{3}e^{3x+5}(3x+6)+C$

C) $\frac{1}{3}e^{3x-5}(3x+6)+C$

D) $\frac{1}{3}e^{3x+5}(3x-4)+C$

E) $\frac{1}{3}e^{3x-5}(3x+4)+C$

- A. -
- B. -
- C. -
- D. -
- E. -

Using integration by parts:

$u = 3x + 5 \quad dv = e^{3x+5}$

$du = 3dx \qquad v = \frac{1}{3}e^{3x+5}$

$\int udv = uv - \int duv$

$$\int (3x+5)e^{3x+5}\,dx = \frac{1}{3}(3x+5)e^{3x+5} - \int e^{3x+5}\,dx = \frac{1}{3}(3x+4)e^{3x+5}$$

Answer is A

The sum of the series $\sum_{n=1}^{\infty}\left(\frac{5^{n+1}}{3^{n}\,(n!)}\right)$ is

(a) $e^{5/3} - 1$

(b) $\frac{1}{5}e^{5/3} - \frac{1}{5}$

(c) $5\,e^{5/3} - 5$

(d) $e^{5/3} - \frac{1}{3}$

(e) $e^{5/3} - \frac{1}{5}$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$5 \sum_{n=1}^{\infty} \frac{(5/3)^n}{n!} = 5 \left(\sum_{n=0}^{\infty} \frac{(5/3)^n}{n!} - 1 \right) = 5e^{5/3} - 5$$

$$\boxed{\text{Answer is C}}$$

The integral that represents the arc length of the curve $y = \ln x$ from the point $(e^4, 4)$ to the point $(e^5, 5)$ is:

A) $\int_{e^4}^{e^5} \frac{\ln x\sqrt{x^2+1}}{x}\, dx$

B) $\int_{4}^{5} \frac{\sqrt{x^2+1}}{x}\, dx$

C) $\int_{e^4}^{e^5} \frac{\sqrt{x^2+1}}{x}\, dx$

D) $\int_{e^4}^{e^5} \sqrt{x^2+1}\, dx$

E) $\int_{4}^{5} y\sqrt{e^{2y}+1}\, dy$

- A. -
- B. -
- C. -
- D. -
- E. -

Using arc length formula: $\int_a^b \sqrt{\left(\frac{dy}{dx}\right)^2 + 1}\, dx$

$$\frac{dy}{dx} = \frac{1}{x} \longrightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{x^2} = \longrightarrow \left(\frac{dy}{dx}\right)^2 + 1 = \frac{1}{x^2} + 1 = \frac{x^2+1}{x^2}$$

$$\int_{e^4}^{e^5} \frac{\sqrt{x^2+1}}{x}\, dx$$

Answer is C

The points of intersection between the Curves $r = 3\sin\theta$ and $r = 2 - \sin\theta$ are :

A) $(\frac{3}{2}, \frac{\pi}{6})$ and $(\frac{3}{2}, \frac{5\pi}{6})$

B) $(\frac{3}{2}, \frac{\pi}{6})$ and $(\frac{-3}{2}, \frac{\pi}{6})$

C) $(\frac{3}{2}, \frac{\pi}{3})$ and $(\frac{3}{2}, \frac{-\pi}{3})$

D) $(\frac{5}{2}, \frac{\pi}{6})$ and $(\frac{5}{2}, \frac{-\pi}{6})$

E) $(\frac{3}{2}, \frac{\pi}{6})$ and $(\frac{3}{2}, \frac{-\pi}{6})$

- ○ A. -
- ○ B. -
- ○ C. -
- ○ D. -
- ○ E. -

$$3\sin\theta = 2 - \sin\theta$$

$$\theta = \arcsin\left(\frac{1}{2}\right) \longrightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$\boxed{\text{Answer is A}}$

Suppose $a_n > 0$ and $\lim\limits_{n\to\infty} \dfrac{a_n}{(1/n^3)} = 3$. Which of the following is (always) correct?

(A) $\sum\limits_{n=1}^{\infty} a_n = \sum\limits_{n=1}^{\infty} \dfrac{3}{n^3}$

(B) $\sum\limits_{n=1}^{\infty} a_n = \sum\limits_{n=1}^{\infty} \dfrac{1}{3n^3}$

(C) $\sum\limits_{n=1}^{\infty} a_n$ is divergent

(D) $\sum\limits_{n=1}^{\infty} a_n$ is convergent

(E) $\sum\limits_{n=1}^{\infty} a_n = \sum\limits_{n=1}^{\infty} \dfrac{n^3}{3}$

From LCT:

If $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ $(0 \leq L < \infty)$ and b_n is convergent $\Longrightarrow a_n$ is convergent

And we have $b_n = \frac{1}{n^3}$, it's p-series with $p = 3$ so it's convergent

$\boxed{\text{Answer is D}}$

To determine whether the series $\sum_{n=1}^{\infty} \frac{\sqrt{n^4+1}}{n^5}$ is convergent or divergent using the limit Comparison test. What series should we use?

A. $\sum_{n=1}^{\infty} n^5$

B. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

C. $\sum_{n=1}^{\infty} \frac{1}{n}$

D. $\sum_{n=1}^{\infty} n$

E. $\sum_{n=1}^{\infty} n^3$

خذ اكبر قوة من البسط على اكبر قوة من المقام

$$b_n = \frac{\sqrt{n^4}}{n^5} = \frac{1}{n^3}$$

$\boxed{\text{Answer is B}}$