Q_{23} which of the following is div

a- $\sum_{n=1}^{\infty} \frac{1}{5^n n+7}$ conv by C.t or l.c.t $(b_n = \frac{1}{5^n})$

b- $\sum \frac{1}{n^3+\sqrt{n}}$ conv by l.c.t $(b_n = \frac{1}{n^3})$

c- $\sum \frac{5^n}{9^n+n^3}$ conv by l.c.t $(b_n = \frac{5^n}{9^n})$

d- $\sum \frac{5^n}{8^n+n}$ conv by l.c.t $(b_n = \frac{5^n}{8^n})$

e- $\sum \frac{1}{n-\sqrt[3]{n}-2}$ div by l.c.t $(b_n = \frac{1}{n})$

Q_{24} $f(x) = \frac{-3x^2+10}{x(x^4-9)}$, Partial fraction decomposition is

$(x^2-3)(x^2+3) = (x-\sqrt{3})(x+\sqrt{3})(x^2+3)$

$$\frac{a}{x} + \frac{b}{x-\sqrt{3}} + \frac{c}{x+\sqrt{3}} + \frac{dx+e}{x^2+3}$$

Q_{20} arc length for $y = x + \cos x$ from $(0,1)$ to $(\frac{\pi}{2}, \frac{\pi}{2})$

$$\frac{dy}{dx} = 1 - \sin x$$

$$\left(\frac{dy}{dx}\right)^2 = 1 - 2\sin x + \sin^2 x$$

$$\left(\frac{dy}{dx}\right)^2 + 1 = 2 - 2\sin x + \sin^2 x$$

$$S = \int_0^{\pi/2} \sqrt{\sin^2 x - 2\sin x + 2}\, dx$$

Q_{21} trig sub for $\int \sqrt{-5 + 6x - x^2}\, dx$

$$-(x^2 - 6x + 5)$$

$$-(x^2 - 6x + 9 - 9 + 5)$$

$$-((x-3)^2 - 4) = 4 - (x-3)^2$$

$$x - 3 = 2\sin\theta$$

$$\boxed{x = 2\sin\theta + 3}$$

Q_{22} $r = 2\sin\theta + 4\cos\theta$ represent

$$r^2 = 2r\sin\theta + 4r\cos\theta$$

$$x^2 + y^2 = 2y + 4x$$

$$\underbrace{x^2 - 4x + 4} - 4 + \underbrace{y^2 - 2y + 1} - 1 = 0$$

$$(x-2)^2 + (y-1)^2 = 5$$

Circle $c(2,1)$

Q_{18} $\int_1^\infty \frac{8}{x^2} = 8$, by Integral test

a- $\sum_{k=8}^{\infty} \frac{1}{k^2}$ div

b- $\sum_{k=1}^{\infty} \frac{8}{k^2}$ div

c- $\sum_{k=1}^{\infty} \frac{8}{k^2}$ converges and equals 8

d- $\sum_{k=8}^{\infty} \frac{1}{k^2}$ converges

e- $\sum_{k=1}^{\infty} \frac{8}{k^2}$ can't be tested by Integral test

← الجواب ليس c لان جواب التكامل لا يمثل sum للسيريز
والفايدة فقط اذا كانت conv او div

Q_{19} Find sum $\sum_{n=1}^{\infty} \frac{2^{n+1}}{7^n n!}$

$$2 \sum_{n=1}^{\infty} \frac{(2/7)^n}{n!} \qquad e^x = \sum_{n=1}^{\infty} \frac{x^n}{n!}$$

$$\text{Sum} = 2\left(e^{2/7} - \overset{a_0}{1}\right) = 2e^{2/7} - 2$$

Q_{14} the series $\sum \frac{(-1)^n}{n+1}$ is

→ $\sum \frac{(-1)^n}{n+1}$ $\lim \frac{1}{n+1} = 0$, a_n dec so conv by Alt

→ $\sum \frac{1}{n+1}$ $b_n = \frac{1}{n}$ div by P-S

$\lim \frac{a_n}{b_n} = 1$

a_n div by l.c.t

cond conv

Q_{15} $r = -3\sin\theta$, $r = 2+\sin\theta$, find intersection

$$2+\sin\theta = -3\sin\theta$$

$$\sin\theta = \frac{-1}{2}$$

$$\theta = \frac{7\pi}{6}, \frac{-\pi}{6}$$

$$r = 2 - \frac{1}{2} = \frac{3}{2} \qquad \left(\frac{3}{2}, \frac{7\pi}{6}\right), \left(\frac{3}{2}, \frac{-\pi}{6}\right)$$

Q_{16} $\sum \frac{5^n}{6^n - 1}$ using l.c.t find b_n

$$b_n = \frac{5^n}{6^n} = \left(\frac{5}{6}\right)^n$$

Q_{17} $y = \ln x$, $y = 1$, $x = e^2$ about x-axis, find volume

$r_1 = y = \ln x$

$r_2 = 1$

$$V = \pi \int_e^{e^2} \left((\ln x)^2 - 1\right) dx$$

Q_{11} $\int (4x+6)e^{4x+6}\,dx$

$$\begin{array}{cc} 4x+6 & e^{4x+6} \\ & \searrow^{+} \\ 4 & \frac{1}{4}e^{4x+6} \\ & \searrow^{-} \\ 0 & \frac{1}{16}e^{4x+6} \end{array}$$

$$= \frac{1}{4}(4x+6)\,e^{4x+6} - \frac{1}{4}e^{4x+6}$$

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$$= \frac{1}{4}e^{4x+6}\left(4x+6-1\right) = \frac{1}{4}e^{4x+6}(4x+5)$$

Q_{12} power series for $x^4\left(\frac{2}{x+2}\right)$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{2+x} = \frac{1}{2}\,\frac{1}{1-\left(\frac{-x}{2}\right)} = \frac{1}{2}\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{2^n}$$

$$\frac{2x^4}{x+2} = \cancel{2}\cdot\frac{1}{\cancel{2}}\sum_{n=0}^{\infty}\frac{(-1)^n x^{n+4}}{2^n}$$

Q_{13} $\sum_{k=1}^{\infty}\frac{1}{(\sin^4 k+4)k}$

by C.T $\sin^4 k \le 1$
$\quad_{+4}\qquad_{+4}$

$$(\sin^4 k+4).k \le 5k$$

$$\frac{1}{(\sin^4 k+4).k} \ge \frac{1}{5k} \to \text{div by P-S}$$

so a_k div by C.T

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Q_7 the appropriate trig sub for $\int \frac{dx}{(x^2-8x+20)^{3/2}}$ is

$x^2 - 8x + 16 - 16 + 20$

$(x-4)^2 + 4$

$x - 4 = 2\tan\theta$

$\boxed{x = 2\tan\theta + 4}$

Q_9 $\lim\limits_{n\to\infty} \frac{a_n}{2^n} = 6$ which of the following alway correct

a- $\sum a_n = \sum\limits_{n=1}^{\infty} \frac{6}{2^n}$ b- $\sum\limits_{n=1}^{\infty} a_n = \sum\limits_{n=1} 6\, 2^n$

c- $\sum a_n$ div D- $\sum a_n$ conv e. $\sum a_n = \sum \frac{2^n}{6}$

$\lim \frac{a_n}{2^n} = \lim \frac{a_n}{b_n} \rightarrow$ so $b_n = 2^n$

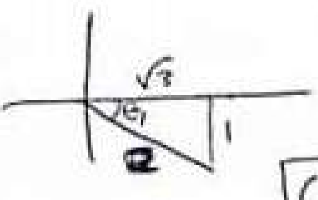
$\lim\limits_{n\to\infty} 2^n = \infty$ (div)

by l.c.t since b_n div so a_n is $\boxed{\text{div}}$

Q_{10} convert $(\sqrt{3}, -1)$ to polar

$r^2 = (\sqrt{3})^2 + (-1)^2 = 4$

$r = 2$



$\theta_1 = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$

so $\theta = \frac{-\pi}{6}$ or $\frac{7\pi}{6}$

$\boxed{\left(2, \frac{-\pi}{6}\right)}$

Q_5 $\sum_{n=1}^{\infty} \frac{(-1)^n (x-8)^n}{8n+1}$ find Interval of convergence

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim \left| \frac{\cancel{(x-8)^{n+1}}^{\,x-8}}{8(n+1)+1} \cdot \frac{8n+1}{\cancel{(x-8)^n}} \right| = |x-8|$$

for conv $|x-8| < 1$

$$\underset{+8}{-1} < \underset{+8}{x-8} < \underset{+8}{1}$$

$$7 < x < 9$$

for $x = 7$

$$\sum_{n=1}^{\infty} \frac{(-1)^n (-1)^n}{8n+1} = \sum \frac{1}{8n+1} \text{ div by l.c.t}$$

for $x = 9$

$$\sum \frac{(-1)^n}{8n+1} \text{ conv by alternating}$$

so $\boxed{(7,9]}$

Q_6 using root test the series $\sum_{n=1}^{\infty} (-1)^n \left(1+\frac{2}{n}\right)^{n^2}$ is

$$\lim_{n\to\infty} \left(\left(1+\frac{2}{n}\right)^{n^2}\right)^{1/n} = e^2 > 1 \text{ so } \boxed{\text{div}}$$

Q_8 Express $\int e^{-x^2}$ as power series

$\rightarrow e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$\rightarrow e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$

$$\rightarrow \int e^{-x^2} dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)\, n!} + c = c + \underset{n=0}{x} - \underset{n=1}{\frac{x^3}{3}} + \underset{n=2}{\frac{x^5}{5\,2!}} - \underset{n=3}{\frac{x^7}{7\,3!}}$$