Assume the terms of a sequence $\{a_n\}_{n=1}^{\infty}$ are given by the following formula:

$$a_n = \frac{1}{2n^3} + \frac{2^2}{2n^3} + \frac{3^2}{2n^3} + \cdots + \frac{n^2}{2n^3}.$$

Find the limit of the sequence or conclude that it diverges.

Hint: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

A) $\frac{2}{6}$.

B) $\frac{1}{6}$.

C) $\frac{2}{3}$.

D) 0.

E) Diverges.

○ A)

○ B)

○ C)

Sol^n : Given Sequce

$$a_n = \frac{1}{2n^3} + \frac{2^2}{2n^3} + \frac{3^2}{2n^3} + \cdots + \frac{n^2}{2n^3}$$

To compute $\lim_{n\to\infty} a_n = ?$

Now

$$a_n = \frac{1}{2n^3} + \frac{2^2}{2n^3} + \frac{3^2}{2n^3} + \cdots + \frac{n^2}{2n^3}$$

$$\Rightarrow \frac{1}{2n^3}\left(1 + 2^2 + 3^2 + \cdots + n^2\right)$$

$$\Rightarrow \frac{1}{2n^3} \sum_{i=1}^{n} i^2$$

$$\Rightarrow \frac{1}{2n^3} \frac{n(n+1)(2n+1)}{6}$$

Given Hint $= \left[\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}\right.$

$$= \frac{1}{12} \frac{(n+1)(2n+1)}{n^2}$$

$$\Rightarrow \frac{1}{12}\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right)$$

$$\therefore \lim_{n\to\infty} a_n = \frac{1}{12} \lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$$

$$\Rightarrow \frac{1}{12} \times 1 \times 2 = \boxed{\frac{1}{6}}$$

So Answer $= \boxed{\frac{1}{6}}$

Let R be the region in the plane enclosed by $y = x^5, y = 0$, and $x = 1$.
Find the volume of the solid formed by rotating R about the axis $x = 2$.

A) $V = \pi \int_0^1 \left(y^{\frac{2}{5}} - 1\right) dy.$

B) $V = \pi \int_0^1 \left(1 - \left(2 - y^{\frac{1}{5}}\right)^2\right) dy.$

C) $V = \pi \int_0^1 \left(\left(y^{\frac{1}{5}} + 2\right)^2 - 1\right) dy.$

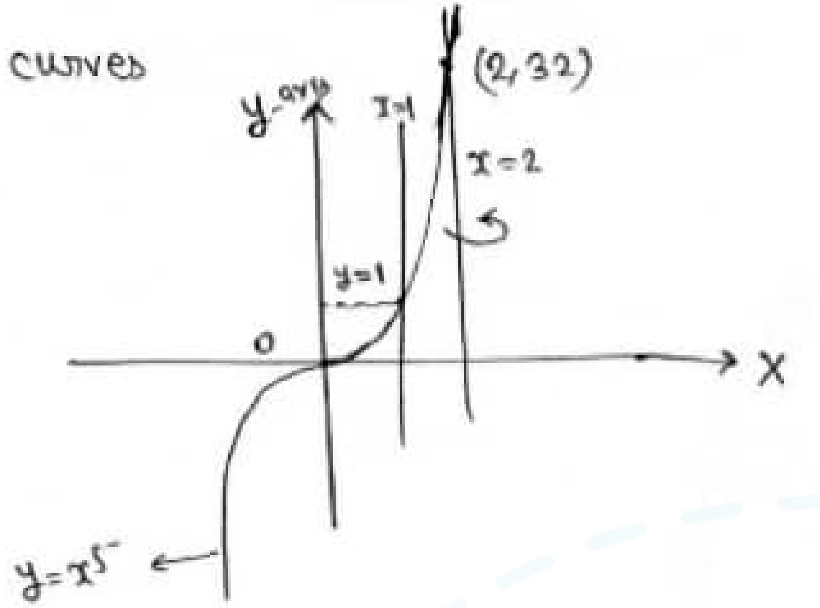
D) $V = \pi \int_0^1 \left(\left(2 - y^{\frac{1}{5}}\right)^2 - 1\right) dy.$

E) None of the above.

- A)
- B)
- C)
- D)
- E)

Solution

Given curves



Washer method about the line $x=1$

Volume, $V = \pi \int_{y=a}^{b} (R^2 - r^2)\, dy$

Here $y = a = 0$, $y = b = 1$

$R = 2 - y^{1/5}$ and $r = 2 - 1 = 1$

$\left\{ \because y = x^5 \Rightarrow x = y^{1/5} \right\}$

$\therefore$ $$V = \pi \int_0^1 \left[(2 - y^{1/5})^2 - 1 \right] dy$$

$\therefore$ Hence option (D) is correct.

A solid is formed with a base that is a triangle with vertices at $(0,0)$, $(3,0)$ and $(0,1)$. Cross sections of this solid, perpendicular to the x −axis are squares. Find the volume of the solid.

A) $\frac{1}{2}$.

B) $\frac{1}{3}$.

C) 1.

D) $\frac{3}{2}$.

E) None of the above.

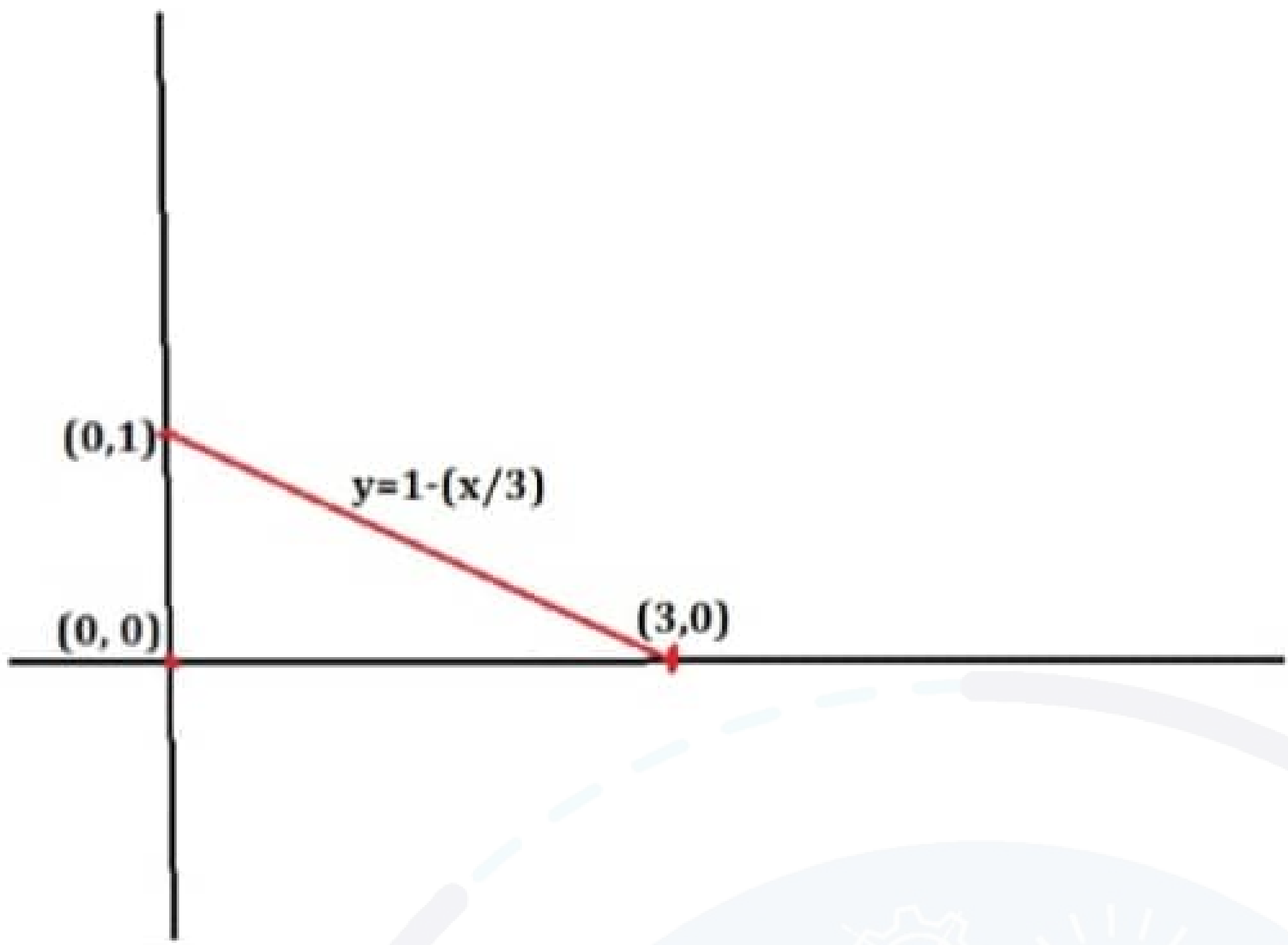
A)

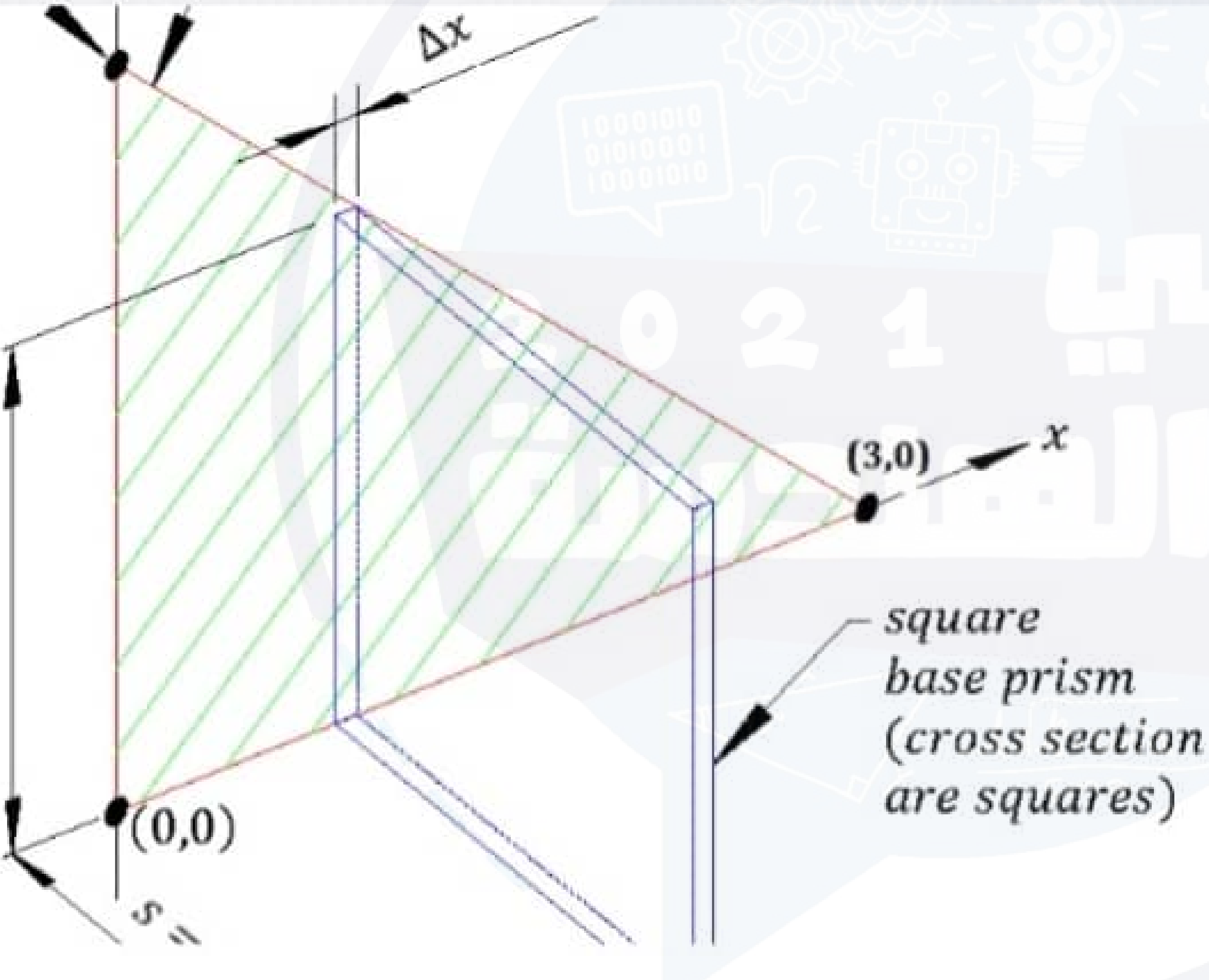
B)

C)

D)

E)


(0,1)
y=1-(x/3)
(0, 0)
(3,0)


Δx
(3,0)
x
square
base prism
(cross section
are squares)
(0,0)

since, the cross – sections, cut

perpendicular to x – axis are s

Volume of the solid can be d

square base slides s and small

$$V = A\,\Delta x$$

$$V = s^2\,\Delta x \Rightarrow \mathbf{dv = s^2\ dx}$$

$$V = \int_0^3 s^2\ dx$$

$$V = \int_0^3 \left(1 - \frac{x}{3}\right)^2 dx$$

$$V = \int_0^3 \left(1 - \frac{2x}{3} + \frac{x^2}{9}\right) dx$$

$$V = \left[x - \frac{x^2}{3} + \frac{x^3}{27}\right]_0^3$$

$$V = 3 - 3 + 1$$

$$\mathbf{V = 1\ \ cubic\ \ units}$$

Option – C is the correc

Assume $\sum_{n=1}^{\infty} a_n$ is an infinite series with partial sums given

$$S_N = 4 + \frac{2}{N}.$$

What is a_5?

A) $-\frac{1}{10}$.

B) $\frac{1}{10}$.

C) $-\frac{2}{5}$.

D) $\frac{2}{5}$.

Given that $\sum_{n=1}^{\infty} a_n$ is an infinite series

with partial sums given $S_N = 1 + \frac{2}{N}$

To find a_5

Solution :- Since $S_n = 4 + \frac{2}{n}$

$\therefore \quad S_{n-1} = 4 + \frac{2}{n-1}$

Now we know that $a_n = S_n - S_{n-1}$

$\Rightarrow \quad a_n = 4 + \frac{2}{n} - 4 - \frac{2}{n-1}$

$\Rightarrow \quad a_n = \frac{2}{n} - \frac{2}{n-1}$

Putting $n = 5$ we get

$a_5 = \frac{2}{5} - \frac{2}{5-1}$

$\Rightarrow \quad a_5 = \frac{2}{5} - \frac{2}{4}$

$\Rightarrow \quad a_5 = \frac{8-10}{20}$

$\Rightarrow \quad a_5 = -\frac{1}{10}$

Hence $a_5 = \boxed{-\frac{1}{10}}$

option (A) is correct answer

Find the sum of the series:

$$\sum_{n=5}^{\infty} \frac{6}{n(n-3)}$$

or conclude that it diverges.

A) 0.

B) $\frac{11}{3}$.

C) $\frac{13}{3}$.

D) $\frac{13}{6}$.

E) Diverges.

A)

B)

C)

D)

E)

Step1

a)

$$\sum_{n=5}^{\infty} \frac{6}{n(n-3)}$$

We can write this as

$$\frac{6}{n(n-3)} = \frac{-2}{n} + \frac{2}{n-3} \Rightarrow \frac{2}{n-3} - \frac{2}{n}$$

$$\sum_{n=5}^{\infty} \left(\frac{2}{n-3} - \frac{2}{n} \right)$$

Step2

b)

$$= \left(1 - \frac{2}{5}\right) + \left(\frac{2}{3} - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{2}{7}\right) + \left(\frac{2}{5} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{2}{9}\right) + \left(\frac{2}{7} - \frac{1}{5}\right) + \left(\frac{1}{4} - \frac{2}{11}\right) + \left(\frac{2}{9} - \frac{1}{6}\right) + \left(\frac{1}{5} - \frac{2}{13}\right)$$

----

means

$$1 + \frac{2}{3} + \frac{1}{2}$$

$$\frac{6+4+3}{6} = \frac{13}{6}$$

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Flag question

Let $a > 1$. Find the area of the region bounded by $y = \sqrt{2(x-a)}$ and $y = x - a$.

A) $\frac{2}{3}$.

B) $\frac{3}{2}$.

C) $\frac{8}{3}$.

D) $\frac{25}{6}$.

E) 6.

- A)
- B)
- C)
- D)
- E)

Clear my choice

Question 8

Not yet answered

Marked out of 2

Flag question

$$\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^{2n} =$$

A) e

a)

Soln

$y = \sqrt{2(x-a)}$ & $y = x - a$

$y^2 = 2(x-a)$ & $y = (x-a)$

find intersection points

$(x-a)^2 = 2(x-a)$

$(x-a) = 0 \qquad x = a$

$x - a = 2 \qquad x = a+2$

Step2

b)

find intersection points

$(x-a)^2 = 2(x-a)$

$(x-a) = 0 \qquad x = a$

$x - a = 2 \qquad x = a+2$

$\rightarrow$

$$A = \int_{x=a}^{x=a+2} \int_{y=x-a}^{y=\sqrt{2(x-a)}} dx\, dy$$

$$A = \int_a^{a+2} \left(\sqrt{2(x-a)} - (x-a)\right) dx$$

$$A = \frac{\sqrt{2}\,(x-a)^{3/2}}{3/2}\Big|_a^{a+2} - \frac{(x-a)^2}{2}\Big|_a^{a+2}$$

$$A = \frac{2\sqrt{2}}{3}(2)^{3/2} - \frac{(2)^2}{2}$$

$$A = \frac{2\sqrt{2}}{3} \times 2\sqrt{2} - 2 = \frac{4\times 2}{3} - 2$$

$$A = \frac{8}{3} - 2 = \frac{8-6}{3} = \frac{2}{3}$$

Question 1

Not yet answered

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Flag question

The value(s) of p for which the series $\sum_{n=1}^{\infty} \frac{n^3}{(1+n^4)^{-p}}$ is convergent using the integral test:

A) $p \geq 1.$

B) $p > 1.$

C) $p < 1.$

D) $p < -1.$

E) $p = -1.$

- A)
- B)
- C)
- D)
- E)

Clear my choice

Question 2

Not yet answered

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Flag question

Solution

Limit Comparison test:- If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are two positive term series such that $\lim_{n\to\infty} \frac{a_n}{b_n} = l$ (finite and non zero) then both series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converges or diverges together.

Given series is $\sum_{n=1}^{\infty} \frac{n^3}{(1+n^4)^{-p}} = \sum_{n=1}^{\infty} \frac{n^3}{n^{-4p}\left(1+\frac{1}{n^4}\right)}$

$$= \sum_{n=1}^{\infty} \frac{1}{n^{-4p-3}\left(1+\frac{1}{n^4}\right)}$$

Let $a_n = \frac{1}{n^{-4p-3}}$ and $b_n = \frac{1}{\left(1+\frac{1}{n^4}\right) n^{-4p-3}}$

Now $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{n^{-4p-3}}}{\frac{1}{\left(1+\frac{1}{n^4}\right) n^{-4p-3}}} = 1 \neq 0, \infty$

$\therefore \sum_{n=1}^{\infty} a_n$ and $\sum b_n$ both have same behaviour

Here $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n^{-4p-3}}$ converges by P-test

when $-4p-3 > 1 \Rightarrow -4p > 4$

$\Rightarrow \boxed{p < -1}$

$\therefore \sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^{-4p-3}\left(1+\frac{1}{n^4}\right)} = \sum \frac{n^3}{(1+n^4)^{-p}}$ converges

when $\boxed{p < -1}$

$\therefore$ (D) ✓

Question **2**

Not yet answered

Marked out of 3

Flag question

Given the following series

I. $\sum_{n=1}^{\infty}\left(\sqrt[n]{2}-1\right)^n$ II. $\sum_{n=1}^{\infty}\frac{(-9)^n}{n10^{n+1}}$

III. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n\ln n}$ IV. $\sum_{n=1}^{\infty}\left(\frac{-2n}{n+1}\right)^{5n}$

A) I and II diverge.
B) III converges conditionally.
C) III and IV converge absolutely.
D) IV converges conditionally.
E) III diverges.

- A)
- B)
- C)
- D)
- E)

Clear my choice

Next page

Previous activity

← Announcements

(I) $\sum_{n=1}^{\infty} (\sqrt[n]{2} - 1)^n$

Let $\sum_{n=1}^{\infty} u_n$ be the given series

Then $u_n = (\sqrt[n]{2} - 1)^n \quad \forall n \in \mathbb{N}$

$\Rightarrow u_n^{1/n} = \sqrt[n]{2} - 1$

$\Rightarrow \lim_{n\to\infty} u_n^{1/n} = \lim_{n\to\infty} (2^{1/n}) - 1$

$\Rightarrow \lim_{n\to\infty} u_n^{1/n} = 1 - 1 = 0 \quad [\because \lim_{n\to\infty} a^{1/n} = 1 \text{ if } a > 0]$

$\Rightarrow \lim_{n\to\infty} u_n^{1/n} = 0 < 1$

So, By Cauchy's Root test the given series is **convergent.**

(II) $\sum_{n=1}^{\infty} \frac{(-9)^n}{n\, 10^{n+1}} = \sum_{n=1}^{\infty} (-1)^n \left(\frac{9}{10}\right)^n \cdot \frac{1}{10n}$

Let $\sum_{n=1}^{\infty} u_n$ be the given series

Then $u_n = (-1)^n \left(\frac{9}{10}\right)^n \cdot \frac{1}{10n} \quad \forall n \in \mathbb{N}$

Now, $|u_n|^{1/n} = \frac{9}{10} \cdot \frac{1}{(10n)^{1/n}}$

$\Rightarrow \lim_{n\to\infty} |u_n|^{1/n} = \frac{9}{10} \cdot \frac{1}{\lim_{n\to\infty} (10)^{1/n} \cdot \lim_{n\to\infty} n^{1/n}}$

$\Rightarrow \lim_{n\to\infty} |u_n|^{1/n} = \frac{9}{10} < 1 \quad [\because \lim_{n\to\infty} n^{1/n} = 1,\ \lim_{n\to\infty} a^{1/n} = 1 \text{ if } a > 0]$

So, by Cauchy root test the given series **converges absolutely.**

(III) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n \ln n}$

Let $\sum_{n=1}^{\infty} u_n$ be the given series

Then $u_n = \frac{(-1)^n}{n \ln n}$ $n = 1, 2, \ldots$

Now, $|u_n| = \frac{1}{n \ln n}$ $n = 1, 2, 3, \ldots$

Now, $n \ln n < n$ for $n \geq 2$

$\Rightarrow \frac{1}{n} < \frac{1}{n \ln n}$ for $n \geq 2$

Since $\sum_{n=2}^{\infty} \left(\frac{1}{n}\right)$ is a divergent series, so by comparison test $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$ is divergent series.

Now, $\{u_n\} = \left\{\frac{1}{n \ln(n)}\right\}$ is a monotone decreasing sequence of positive real numbers

and $\lim_{n \to \infty} u_n = 0$

So, by Leibnitz's test $\sum_{n=1}^{\infty} u_n$ is convergent series.

So, the given series is **conditionally convergent.**

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(iv) $\sum_{n=1}^{\infty} \left(\frac{-2n}{n+1}\right)^{5n}$

Let $\sum_{n=1}^{\infty} u_n$ be the given series

Then $u_n = \left(\frac{-2n}{n+1}\right)^{5n}$ $n \in \mathbb{N}$

$\Rightarrow u_n = \left\{\frac{-32n^5}{(n+1)^5}\right\}^n$, $n \in \mathbb{N}$

$\Rightarrow u_n = (-1)^n \cdot \left\{\frac{(32)\ n^5}{(n+1)^5}\right\}^n$, $n \in \mathbb{N}$

Now, $|u_n|^{1/n} = \frac{32 \cdot n^5}{(n+1)^5}$

$\Rightarrow \lim_{n\to\infty} |u_n|^{1/n} = 32 \lim_{n\to\infty} \frac{n^5}{(n+1)^5}$

$\Rightarrow \lim_{n\to\infty} |u_n|^{1/n} = 32 \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^5}$

$\Rightarrow \lim_{n\to\infty} |u_n|^{1/n} = 32 > 1$

So, $\sum_{n=1}^{\infty} u_n$ is divergent series, according to Cauchy's roots for series of arbitrary terms

(B) III converges conditionally

Not yet answered

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Flag question

Time left 0:43:38

Using the limit comparison test the appropriate b_n that is used to make the series $\sum_{n=1}^{\infty} \frac{ln(n)}{n^6}$ converges is:

A) $b_n = \frac{1}{n}$.

B) $b_n = \frac{1}{n^6}$.

C) $b_n = \frac{1}{n^7}$.

D) $b_n = n^{\frac{-1}{2}}$.

E) $b_n = n^{\frac{-11}{2}}$.

- A)
- B)
- C)
- D)
- E) (selected)

Clear my choice

Question 6

Not yet answered

Marked out of 3

Flag question

Limit comparison Test:-

Consider the two series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$.

where each series contains only positive terms

Suppose that $\rho = \lim_{n\to\infty} \frac{a_n}{b_n}$

i) If ρ is finite and positive, then either both series converge or both series diverge.

ii) If $\rho = 0$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges as well.

iii) If $\rho = \infty$ and $\sum_{n=1}^{\infty} b_n$ diverges then $\sum_{n=1}^{\infty} a_n$ diverges as well.

Step2

b)

Given that, $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^6}$. Let $a_n = \frac{\ln(n)}{n^6}$

Let $b_n = n^{-\frac{11}{2}} = \frac{1}{n^{\frac{11}{2}}}$

$\therefore \sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{11}{2}}}$ converges (since $\sum_{n=1}^{\infty} \frac{1}{n^p}$ convergent if $p>1$)

Now $\frac{a_n}{b_n} = \frac{\ln(n)}{n^6} \times n^{\frac{11}{2}}$

$= \frac{\ln(n)}{n^{\frac{1}{2}}}$

$\therefore \lim_{n\to\infty} \frac{\ln(n)}{n^{\frac{1}{2}}}$ ($\frac{\infty}{\infty}$ form)

$= \lim_{n\to\infty} \frac{1}{2n^{\frac{1}{2}}}$ (by L' Hospital rule)

$= 0$

$\therefore$ By limit comparison test the series $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^6}$ converges

$\therefore$ Thus (E) is correct option.

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C)

D)

E)

Clear my choice

Time left 0:19:58

Question 14

Not yet answered

Marked out of 2

Flag question

The sum of the series $\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{1-2n}$ is:

A) **1.**

B) **2.**

C) $\frac{1}{6}$.

D) $\frac{-2}{3}$.

E) **Diverges.**

A)

B)

C)

D)

E)

Clear my choice

Step1

a)

The given series is:

which can be rewritten as:

$$\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{-2n}$$
$$=\frac{1}{2}\sum_{n=1}^{\infty}\left(2^{-1}\right)^{-2n}$$
$$=\frac{1}{2}\sum_{n=1}^{\infty}2^{2n}$$
$$=\frac{1}{2}\sum_{n=1}^{\infty}4^{n}$$

Step2

b)

Now,

$\frac{1}{2}\sum_{n=1}^{\infty}4^{n}$ is a geometric series with a common ratio $|r|=|4|=4>1$.

Hence, the given series diverges.

So, option (E) is the correct option.