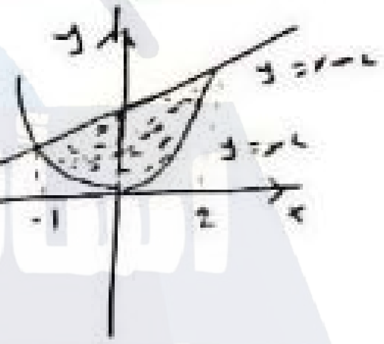
Q1:(5 pts) The base of a solid is the region bounded by $y = x^2$, $y = x + 2$. Find the volume of the solid if each cross section perpendicular to the y-axis is an equilateral triangle.

$x + 2 = x^2$
$x^2 - x - 2 = 0$
$(x-2)(x+1)$
$x = 2 \quad x = -1$



$$\cancel{\int_{-1}^{2} 2\pi\left(x^4 - (x+2)^2\right) dx}$$

$$\cancel{2\pi \int_{-1}^{2} (x^2 - x - 2)(x^2 + x + 2)\, dx}$$

Q2: (5 pts) Find the volume of the solid obtained by rotating the circle $(x-2)^2 + y^2 = 1$ about the line $x = -1$.

$x = -1$

The base is bounded by :

$y = x^2$, $y = x + 2$.

each cross section prependicular to the y-axis is an equilateral triangle :

$x^2 = x + 2$

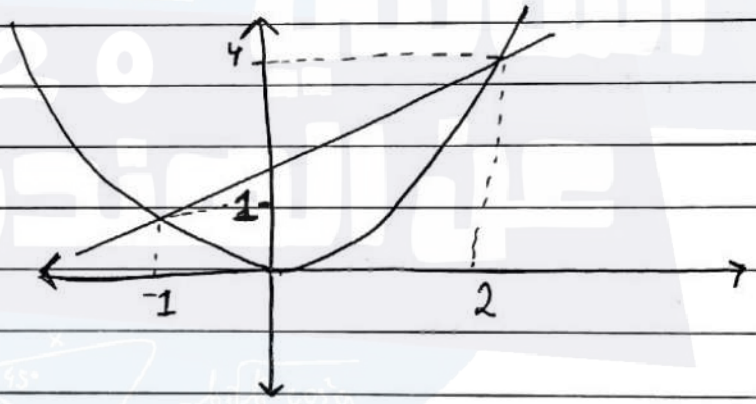
$x^2 - x - 2 = 0$

$(x + 1)(x - 2) = 0$

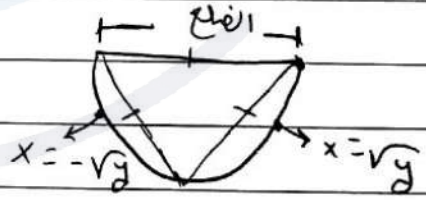
$x = -1$, 2 . نقاط التقاطع

when $x = -1$, $y = 1$

when $x = 2$, $y = 4$



From $y = 0$ to $y = 1$:



$A(y) = \frac{1}{2} \times (\text{الضلع})^2 \times \sin 60^\circ$

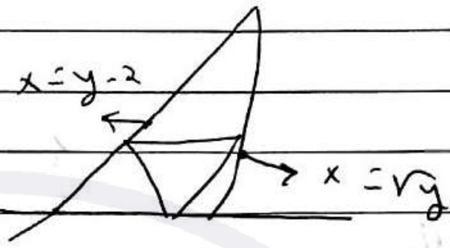
$= \frac{1}{2} \times (\sqrt{y} - -\sqrt{y})^2 \frac{\sqrt{3}}{2}$

$= \frac{1}{2} (2\sqrt{y})^2 \frac{\sqrt{3}}{2}$

$= \sqrt{3}\, y$

From $y=1$ to $y=4$:

$$A(y) = \frac{1}{2}\left(\sqrt{y} - y + 2\right)^2 \frac{\sqrt{3}}{2}$$



So:

$$V = \int_0^1 \sqrt{3}\, y\, dy + \int_1^4 \frac{\sqrt{3}}{4}\left(\sqrt{y} - y + 2\right)^2 dy$$

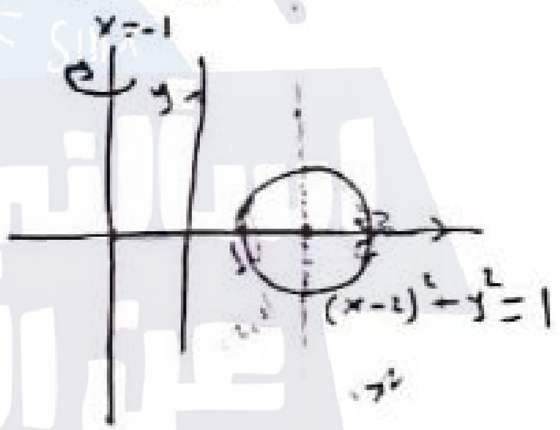
$$= \sqrt{3}\,\frac{y^2}{2}\Big|_0^1 + \frac{\sqrt{3}}{4}\int_1^4 -3y + y^2 + 4 - 2y\times\sqrt{y} + 4\sqrt{y}\; dy$$

$$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4}\left(\left(\frac{-3y^2}{2} + \frac{y^3}{3} + 4y - \frac{4y^2\sqrt{y}}{5} + \frac{8y\sqrt{y}}{3}\right)\Big|_1^4\right)$$

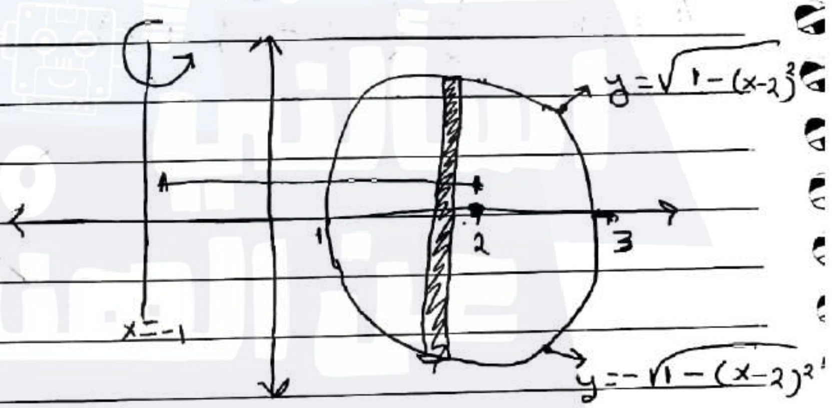
$$= \frac{\sqrt{3}}{2} + \frac{131\sqrt{3}}{120} = \frac{120\sqrt{3} + 262\sqrt{3}}{240}$$

$$= \frac{382\sqrt{3}}{240}.$$

Q2: (5 pts) Find the volume of the solid obtained by rotating the circle $(x-2)^2 + y^2 = 1$ about the line $x = -1$.



Q2) $(x-2)^2 + y^2 = 1$ about $x = -1$:



by shells :

$$V = 2\pi \int_a^b r\,h\,dx$$

$$= 2\pi \int_1^3 2\left(\sqrt{1-(x-2)^2}\right)(x+1)\,dx$$

Scanned with CamScanner

Q3: (5 pts) Find the area of the surface generated by rotating the curve $y = e^{2x}$, $0 \le x \le 1$ about the x-axis.

$$y = e^{2x}$$

$$\frac{dy}{dx} = 2e^{2x}$$

$$\left(\frac{dy}{dx}\right)^2 = 4e^{4x}$$

$$\int_0^1 2\pi e^{2x}\sqrt{1+4e^{4x}}$$

3

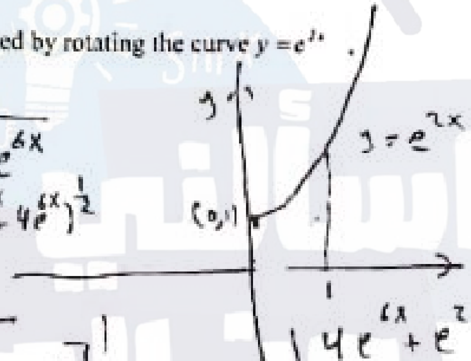
$$2\pi\int_0^1 \sqrt{e^{2x}+4e^{6x}}$$

$$2\pi\int_0^1 \cancel{\sqrt{4e^{6x}}} \quad \pi 2\int_0^1 (e^{2x}+4e^{6x})^{\frac{1}{2}}$$

$$\cancel{2\pi\int_0^1 \sqrt{4e^{6x}}}$$

$$2\pi \left.\frac{(e^{2x}+4e^{6x})^{\frac{3}{2}}}{(2e^{2x}+24e^{6x})\frac{3}{2}}\right]_0^1$$

$$= \frac{4\pi}{3}\frac{(e^2+4e^6)^{\frac{3}{2}}}{2e^2+24e^6} - \frac{4}{3}\pi\frac{(5)^{\frac{3}{2}}}{26}$$



$$4e^{6x}+e^{2x}$$

$$\left(\frac{b^{\frac{1}{4}}}{24}\right)^2 \quad \frac{1}{6} \quad \frac{1}{36}$$

Q4: Find the sum (if exists)

a) (3pts) $\sum \frac{1}{n^2+4n+3}$

$$\frac{1}{2}\sum^{\infty} \frac{1}{(n+1)} - \frac{1}{(n+3)} \quad \Big|\, n^2+4n+3$$

Q3] $y = e^{2x}$, $0 \leq x \leq 1$, about x-axis :

Surface area $= \int_0^1 2\pi (e^{2x}) \sqrt{1+(2e^{2x})^2}\, dx$

$= \int_0^1 2\pi e^{2x} \sqrt{1+4e^{4x}}\, dx$

$Z = e^{2x}$, $dx = \frac{dz}{2e^{2x}}$

$\int_1^{e^2} \frac{2\pi z \sqrt{1+\cancel{z}4z^2}\, dz}{2z} = \int_1^{e^2} \pi \sqrt{1+4z^2}\, dz.$

$Z = \frac{1}{2} \tan(t)$

$dz = \frac{1}{2} \sec^2(t)\, dt.$

$\pi \int \sqrt{1+\left(\frac{1}{4}\right)(4) \tan^2(t)} \left(\frac{1}{2}\right) \sec^2(t)\, dt.$

$= \frac{1}{2}\pi \int \sqrt{1+\tan^2 t} (\sec^2 t) = \frac{1}{2}\pi \int \sqrt{\sec^2 t}\ \sec^2 t\, dt$

$= \frac{1}{2}\pi \int \sec^3 t\, dt.$ { using formula.

$= \frac{1}{2}\pi \left(\frac{1}{2}\sec t \tan t + \frac{1}{2}\ln|\sec t + \tan t|\right)$

$= \frac{1}{2}\pi \left(z\sqrt{1+4z^2} + \frac{1}{2}\ln\left|\sqrt{1+4z^2} + 2z\right| \right) \Big|_1^{e^2}$

$= \frac{1}{2}\pi \left(e^2\sqrt{1+4e^4} + \frac{1}{2}\ln\left|\sqrt{1+4e^4} + 2e^2\right| - \sqrt{5} - \frac{1}{2}\ln\left|\sqrt{5}+2\right| \right)$

$$= \frac{4}{3}\pi \frac{(e^2+4e^6)^{\frac{3}{2}}}{2e^2+24e^6} - \frac{4}{3}\pi \frac{(\frac{5}{4})^{\frac{3}{2}}}{26}$$

Q4: Find the sum (if exists)

a) (3pts) $\sum_{n=1}^{\infty} \frac{1}{n^2+4n+3}$

$$\frac{1}{n^2+4n+3} = \frac{A}{n+3} + \frac{B}{n+1}$$

n^2+4n+3

$(n+3)(n+1)$

$1 = A(n+1) + B(n+3)$

$n=-1 \Rightarrow 1 = 2B \rightarrow B = \frac{1}{2}$

$n=-3 \Rightarrow 1 = -2A \rightarrow A = \frac{-1}{2}$

$$\frac{\frac{1}{2}}{n+1} - \frac{\frac{1}{2}}{n+3}$$

$$\frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{(n+1)} - \frac{1}{(n+3)}$$

by telscoping series

$$\frac{1}{2}\left(\frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \frac{1}{4} - \frac{1}{6} + \dots + \frac{1}{n+1} - \frac{1}{n+3}\right)$$

$$\frac{1}{2}\lim_{n\to\infty} \frac{1}{2} + \frac{1}{3} + \frac{1}{n+1} - \frac{1}{n+3}$$

$$= \frac{1}{2}\left(\frac{1}{2} + \frac{1}{3} + 0 + 0\right) = \frac{1}{2}\cdot\frac{5}{6} = \frac{5}{12}$$

3

b) (3pts) $\sum \frac{5(2^n)}{3^{n-1}} + \frac{1}{e^n}$

coseder:

so the series

Q4] a) $\sum_{n=1}^{\infty} \frac{1}{n^2+4n+3}$:

$$\frac{1}{n^2+4n+3} = \frac{A}{n+3} + \frac{B}{n+1}$$

$$1 = A(n+1) + B(n+3)$$

when $n=-1$: $1 = 2B \rightarrow B = \frac{1}{2}$

when $n=-3$: $1 = -2A \rightarrow A = \frac{-1}{2}$

so: $\boxed{\frac{\frac{-1}{2}}{n+3} + \frac{\frac{1}{2}}{n+1}}$

$\therefore$: $\frac{1}{2} \sum_{1}^{\infty} \frac{1}{n+1} - \frac{1}{n+3}$

$S_1 = \frac{1}{2} - \frac{1}{4}$

$S_2 = \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5}$

$S_3 = \frac{1}{2} - \cancel{\frac{1}{4}} + \frac{1}{3} - \frac{1}{5} + \cancel{\frac{1}{4}} - \frac{1}{6}$

$S_4 = \frac{1}{2} + \frac{1}{3} - \cancel{\frac{1}{5}} - \frac{1}{6} + \cancel{\frac{1}{5}} - \frac{1}{7}$

$\vdots$

$S_n = \frac{1}{2} + \frac{1}{3} + \frac{1}{n+1} - \frac{1}{n+3}$

$$\lim_{n\to\infty} \frac{1}{2} + \frac{1}{3} + \frac{1}{n+1} - \frac{1}{n+3} = \frac{1}{2} + \frac{1}{3} + 0 + 0$$

$$= \frac{5}{6}$$

$$\frac{1}{2} \sum_{1}^{\infty} \frac{1}{n+1} - \frac{1}{n+3} = \left(\frac{1}{2}\right)\left(\frac{5}{6}\right) = \frac{5}{12}$$

$\dfrac{\frac{1}{2}}{n+1} - \dfrac{\frac{1}{2}}{n+3}$

$= \frac{1}{2}\left(\frac{1}{2} + \frac{1}{3} + 0 + 0\right) = \frac{1}{2} \cdot \frac{5}{6} = \frac{5}{12}$

b) (3pts) $\sum_{n=1}^{\infty} \frac{5(2^n)}{3^{n-1}} + \frac{1}{e^n}$

$= \sum_{n=1}^{\infty} \frac{5 \cdot 2^n}{3^{-1} \cdot 3^n} + \frac{1}{e^n}$

$= \sum_{n=1}^{\infty} 15\left(\frac{2}{3}\right)^n + \sum_{n=1}^{\infty}\left(\frac{1}{e}\right)^n$

by Geomatric Series

3

cosider: $\sum_{n=1}^{\infty}$

$\sum_{n=1}^{\infty} 15\left(\frac{2}{3}\right)^n$

$a = 15$

$r = \frac{2}{3} < 1$

$Sum = \frac{ar^n}{1-r}$

$= \frac{15 \cdot \frac{2}{3}}{1 - \frac{2}{3}}$

$= \frac{30}{1} = 30$

$\sum_{n=1}^{\infty}\left(\frac{1}{e}\right)^n$

$a = 1$

$r = \frac{1}{e} < 1$

$Sum = \frac{ar^n}{1-r}$

$= \frac{\frac{1}{e}}{1 - \frac{1}{e}}$

$= \frac{1}{e-1}$

So the Series $\sum_{n=1}^{\infty} \frac{5(2)^n}{3^{n-1}} + \frac{1}{e^n}$ is Convergent be Geomatric Series and $Sum = 30 + \frac{1}{e-1}$

$= \frac{30e - 29}{e-1}$

b) $\sum_{1}^{\infty} \frac{5(2^n)}{3^{n-1}} + \frac{1}{e^n}$:

$\equiv \sum_{1}^{\infty} \underbrace{\left(\frac{5}{3^{-1}}\right)\left(\frac{2}{3}\right)^n} + \frac{1}{e^n}$

$15 \left(\frac{2}{3}\right)^n$: $a = 15\left(\frac{2}{3}\right)$, $r = \frac{2}{3} < 1$

so: conv.

$\sum_{1}^{\infty} 15\left(\frac{2}{3}\right)^n = \frac{15\left(\frac{2}{3}\right)}{1-\frac{2}{3}}$

$= \frac{15 \cancel{x}\left(\frac{2}{3}\right)}{\frac{1}{3}} = 45\,(2/3) = 30$.

$\sum_{1}^{\infty} \left(\frac{1}{e}\right)^n$:

$a = \frac{1}{e}$, $r = \frac{1}{e} < 1$ conv.

$\sum_{1}^{\infty} \left(\frac{1}{e}\right)^n = \frac{\frac{1}{e}}{1-\frac{1}{e}} = \left(\frac{1}{e}\right)\frac{e}{e-1} = \frac{1}{e-1}$

so: $\sum_{1}^{\infty} 15\left(\frac{2}{3}\right)^n + \left(\frac{1}{e}\right)^n = 30 + \frac{1}{e-1}$

Q6: (9 pts) Test for convergence:

(a) $\sum_{n=1}^{\infty} \frac{1+\sin^2(n)}{5^n}$

$0 \leq \frac{1+\sin^2(n)}{5^n} \leq \frac{2}{5^n}$

$\sum b_n = \sum \frac{2}{5^n} = 2\left(\frac{1}{5}\right)^n$ is convergent by Geomatric Series

$\frac{1+\sin^2 n}{5^n} \leq 2\left(\frac{1}{5}\right)^n$

So. the Series $\sum_{n=1}^{\infty} \frac{1+\sin^2 n}{5^n}$ is convergent bey

Comparison

Comparation Test

3

Q6)

a) $\sum_{1}^{\infty} \frac{1+\sin^2(n)}{5^n}$:

$-1 \leq \sin(n) \leq 1 \longrightarrow 0 \leq \sin^2(n) \leq 1$

$\frac{1}{5^n} \leq \frac{1+\sin^2(n)}{5^n} \leq \frac{2}{5^n}$

$a = \frac{2}{5}$, $r = \frac{1}{5} < 1$

so conv.

while $\frac{2}{5^n}$ is conv, then $\frac{1+\sin^2(n)}{5^n}$

is conv. too by comparison test.

Section ______ Instructor: ______

Question 1 : Determine whether the sequence $\left\{\left(\frac{n+3}{n+1}\right)^n\right\}_{n=1}^{\infty}$ converges : *(4 marks)*

if so determine its limit .

$y = \left(\frac{n+3}{n+1}\right)^n$

$\ln y = n \ln \frac{n+3}{n+1}$

$= \lim_{n\to\infty} n \ln \frac{n+3}{n+1}$

$= \infty \cdot \ln\left(\lim_{n\to\infty} \frac{n+3}{n+1}\right)$

$= \infty \cdot \ln(1)$

$= \infty \cdot 0$!

لوبيتال

اشتقاق

$= \lim_{n\to\infty} \frac{\ln\left(\frac{n+3}{n+1}\right)}{n^{-1}} \quad n^{-1} \to \frac{1}{n} = \lim_{n\to\infty} \frac{\frac{(n+1)-(n+3)}{(n+1)^2} \Big/ \frac{n+3}{n+1}}{\frac{-1}{n^2}} = \lim_{n\to\infty} \frac{\frac{-2}{(n+3)(n+1)}}{\frac{-1}{n^2}}$

$= \lim_{n\to\infty} \frac{2 n^2}{(n+3)(n+1)} = \frac{n^2}{n^2} = \boxed{1}$

3

$\ln y = 1$

$\boxed{y = e^1}$ conv

Question 2 : Find the sum of the series $\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+2}\right) + \left(\frac{e}{\pi}\right)^n$ *(5 marks)*

$\boxed{1}\ \left(1 - \cancel{\frac{1}{3}}\right) + \left(\frac{e}{\pi}\right) + \boxed{2}\ \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\frac{e}{\pi}\right)^2 + \boxed{3}\ \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \left(\frac{e}{\pi}\right)^3 + \dots + \left(\cancel{\frac{1}{n}} - \frac{1}{n+2}\right) + \left(\frac{e}{\pi}\right)^n$

$\boxed{4}\ \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right) + \left(\frac{e}{\pi}\right)^4 + \boxed{5}\ \left(\cancel{\frac{1}{5}} - \frac{1}{7}\right) + \left(\frac{e}{\pi}\right)^5$

$S_n =$

3.5

$S_n = 1 + \frac{1}{2} - \frac{1}{n+2} + \left(\frac{e}{\pi}\right)^n$

$S_n = \frac{a}{1-(\ )}$

$\lim_{n\to\infty}\left(1 + \frac{1}{2} - \frac{1}{n+2} + \left(\frac{e}{\pi}\right)^n\right)$

$1 + \frac{1}{2} - 0 + \lim_{n\to\infty}\left(\frac{e}{\pi}\right)^n = \frac{2\times 1}{2\times 1} + \frac{1}{2} - 0 + 0 = \boxed{\frac{3}{2}}$

$\frac{2.7}{3.14} < 1$

الله ولي التوفيق

$Q_1)$ $\sum_{1}^{\infty} \left(\frac{n+3}{n+1}\right)^n$:

$$\lim_{n\to\infty} \left(\frac{\frac{n+3}{n}}{\frac{n+1}{n}}\right)^n = \lim_{n\to\infty} \frac{\left(1+\frac{3}{n}\right)^n}{\left(1+\frac{1}{n}\right)^n} = \frac{e^3}{e} = e^2$$

so it is conv.

$Q_2)$ $\sum_{1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right) + \left(\frac{e}{\pi}\right)^n$:

$$= \sum_{1}^{\infty} \frac{1}{n} - \frac{1}{n+2} + \sum_{1}^{\infty} \left(\frac{e}{\pi}\right)^n$$

$S_1 = 1 - \frac{1}{3}$

$S_2 = 1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4}$

$S_3 = 1 - \cancel{\frac{1}{3}} + \frac{1}{2} - \frac{1}{4} + \cancel{\frac{1}{3}} - \frac{1}{5}$

$S_4 = 1 + \frac{1}{2} - \quad - \frac{1}{5} + \frac{1}{4} - \frac{1}{6}$

$\vdots$

$S_n = 1 + \frac{1}{2} + \frac{1}{n} - \frac{1}{n+2}$

$\lim_{n\to\infty} S_n = 1 + \frac{1}{2} = 3/2$.

$\sum_{1}^{\infty} \left(\frac{e}{\pi}\right)^n$: $a = \frac{e}{\pi}$, r $\frac{e}{\pi} < 1$

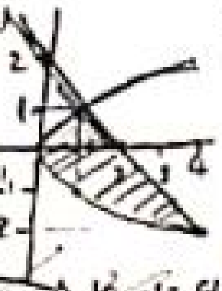
$$\sum_{1}^{\infty} \left(\frac{e}{\pi}\right)^n = \frac{\frac{e}{\pi}}{1-\frac{e}{\pi}} = \frac{e}{\pi} \times \frac{\pi}{\pi - e} = \frac{e}{\pi - e}.$$

so: sum $= \frac{3}{2} + \frac{e}{\pi - e}$

Question 3: (FOR THIS QUESTION SET UP THE INTEGRALS ONLY . DO NOT EVALUATE (لا تقوم بإيجاد التكاملات المناسبة مقابل كل فرع مع التبسيط اذا لزم ولا تجد تلك التكاملات اطلاقا

Given the curves : $C_1::x=y^2$ $C_2::x+y=2$ $C_3::y=x^2$ $C_4::y=3x$

1) Find the area between the two curve $C_1::x=y^2$ & $C_2::x+y=2$ (3 marks)



$\boxed{x=y^2}$ $\boxed{x+y=2}$

$y=\sqrt{x}$ $y=2-x$

$y^2=2-y$

$y^2+y-2=0$

$(y+2)(y-1)$ $\boxed{y=1}$ $\boxed{y=-2}$

$$\boxed{\int_{-2}^{1}(2-y)-y^2\,dy}$$

$$\int_{-2}^{1} 2-y-y^2\,dy$$

$$2y-\frac{y^2}{2}-\frac{y^3}{3}\Big|_{-2}^{1}$$

$\frac{16}{2}-\frac{1}{2}=\frac{61}{2}$

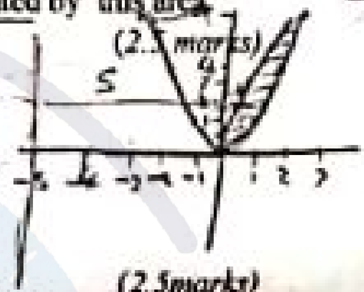
2) If the area between the two curves $C_3::y=x^2$ & $C_4::y=3x$ is rotated about the $\boxed{\text{line } x=-5}$. What is the resulting volume of the solid generated by this area.

$\boxed{y=x^2}$ $\boxed{y=3x}$

$x^2=3x$

$x^2-3x=0$

$\boxed{x=0}$ $\boxed{x=3}$



i) Using Cylindrical Shell method (2.5 marks)

$$\int_0^3 2\pi h r\,dx \Rightarrow \boxed{\int_0^3 2\pi(3x-x^2)(x+5)\,dx}$$

$$2\pi\int_0^3(-2x^2-x^3+15x)\,dx$$

2.5

ii) Using Disks&Whasher (Slicing) Method (2.5marks)

$R=\sqrt{y}+5$ $r=\frac{y}{3}+5$ $x=\sqrt{y}$ $x=\frac{y}{3}$

$$\pi\int_0^9 R^2-r^2\,dy \Rightarrow \boxed{\pi\int_0^9(\sqrt{y}+5)^2-\left(\frac{y}{3}+5\right)^2dy}$$

$$\pi\int_0^9 \left(y+10\sqrt{y}+25\right)-\left(\frac{y^2}{9}+\frac{10y}{3}+25\right)dy \Rightarrow \boxed{\pi\int_0^9 -\frac{y^2}{9}+10\sqrt{y}-\frac{7}{3}y\,dy}$$

3) Find the perimeter (محيط) of the region bounded by the curves (2marks)

$C_3::y=x^2$ & $C_4::y=3x$

$x^2=3x$

$x^2-3x=0$

$\boxed{x=0}$ $\boxed{x=3}$

$y=(3)^2$ $\boxed{y=9}$ $\boxed{y=0}$

$y=x^2 \Rightarrow x=\sqrt{y}$

$x'=\frac{1}{2\sqrt{y}}$

$x'^2=\frac{1}{4y}$

$$\int_0^9\sqrt{1+y'^2}\,dx$$

$$\boxed{\int_0^9\sqrt{1+\frac{1}{4y}}\,dy} \Rightarrow \int_0^9\frac{\sqrt{4y+1}}{2\sqrt{y}}$$

4) If the curve in $C_3::y=x^2$ between the points (0,0) and (3,9) is rotated about $\boxed{x=-5}$ what is the resulting area of the surface of revolution . (2 marks)

y-axis $r=x=\sqrt{y}$

$y=x^2$ (0, 0) (3, 9)

$x=\sqrt{y}$

$x'=\frac{1}{2\sqrt{y}}$

$x'^2=\frac{1}{4y}$

$$\int_0^9 2\pi x\sqrt{1+x'^2}\,dy$$

$$2\pi\int_0^9\sqrt{y}\sqrt{1+\frac{1}{4\sqrt{y}}}\,dy$$

$$2\pi\int_0^9\sqrt{x}\,\frac{\sqrt{4y+1}}{2\sqrt{x}}\,dy$$

$$\boxed{\pi\int_0^9\sqrt{4y+1}\,dy}$$

1.5

Q3 $C_1: x = y^2$, $C_2: x + y = 2$, $C_3: y = x^2$

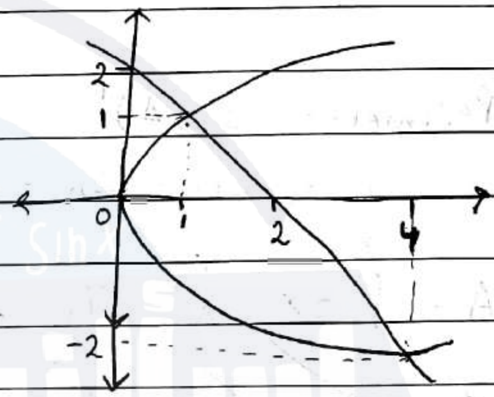
$y = 2 - x$

$x = \cancel{y}2 - y$

$C_4: y = 3x$

1 Area between C_1 , C_2 :

Intersection:



$2 - y = y^2 \rightarrow y^2 + y - 2 = 0$

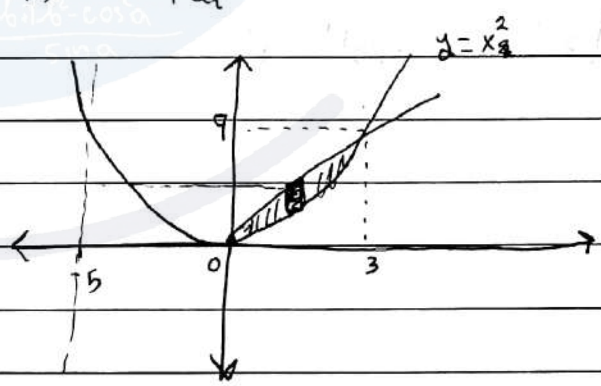
$(y+2)(y-1) = 0$

$y = -2$, $y = 1$

$x = 4$, $x = 1$

$$\int_{-2}^{1} 2 - y - y^2 \, dy$$

2 area between C_3 and C_4 is rotated about $x = -5$, Find volume by shells:



Intersection:

$3x = x^2 \rightarrow x(x-3) = 0$

$x = 0$, $x = 3$

$y = 0$, $y = 9$

$$V = 2\pi \int_0^3 (x+5)(3x - x^2) \, dx$$

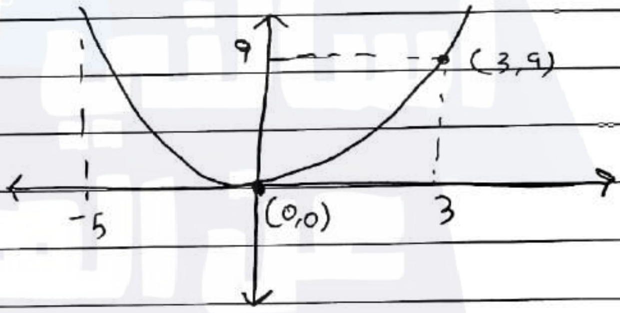
COLLEGE

ii) by disk and washer :

$$V = \pi \int_0^9 (\sqrt{y} + 5)^2 - \left(\frac{y}{3} + 5\right)^2 \, dy .$$

[3] غير مطلوب

$\boxed{4}$ area surface of rotating region between $y = x^2$ between $(0,0)$ $(3,9)$ about $x = -5$



$x = \sqrt{y}$

$\frac{dx}{dy} = \frac{1}{2\sqrt{y}}$

$\left(\frac{dx}{dy}\right)^2 = \frac{1}{4y}$

$S = 2\pi \int_0^9 (\sqrt{y} + 5) \sqrt{1 + \frac{1}{4y}} \, dy$.

Question 4: : Determine whether the series converges, converges conditionally or diverges and **state your test .**

1) $\sum_{k=1}^{\infty} \frac{(k+4)!}{4!\ k!\ 4^k}$

(3 marks)

Ratio test

$$\lim_{n\to\infty}\left|\frac{a_{(k+1)}}{a_n}\right| \Rightarrow \lim_{n\to\infty}\left|\frac{((k+1)+4)!}{4!(k+1)!4^{k+1}}\cdot\frac{4!\,k!\,4^k}{(k+4)!}\right|$$

$$\lim_{n\to\infty}\left|\frac{(k+5)\cancel{(k+4)!}}{\cancel{4^k}\cdot 4(k+1)\cancel{k!}}\cdot\frac{\cancel{4!}\cancel{4^k}}{\cancel{(k+4)!}}\right| \Rightarrow \lim_{n\to\infty}\left|\frac{(k+5)}{4(k+1)}\right| = \frac{\cancel{k}}{4\cancel{k}} = \boxed{\frac{1}{4}}$$

3.5

So ?

2) $\sum_{k=1}^{\infty} \frac{\tan^{-1}k}{1+k^2}$ (3 marks)

$$\frac{-\frac{\pi}{2}}{1+k^2} < \boxed{\frac{\tan^{-1}k}{1+k^2}} < \boxed{\frac{\frac{\pi}{2}}{1+k^2}} < \boxed{\frac{\frac{\pi}{2}}{k^2}}$$

conv by C.T — $p=2$ $p>1$ conv by P-series

$$\Rightarrow \sum_{k=1}^{\infty}\frac{\tan^{-1}k}{1+k^2} = \text{conv by C.T}$$

a_n اكبر من b_n علما

conv a_n اذا conv b_n

3

3) $\sum_{k=2}^{\infty} \frac{(-1)^{k+1}\ k^2}{k^3+1}$ (3 marks)

3+1

$a_k = \frac{k^2}{k^3+1}$ $b_n = \frac{k^2}{k^3} = \frac{1}{k}$ $p=1$ div by P-series

$\lim_{n\to\infty}\left(\frac{k^2}{k^3+1}\cdot k\right) = \frac{k^3}{k^3} = 1$ $0<1<\infty$ div by L.C.T

$\boxed{1}$ ✓ +ve موجبة $\therefore$ conv conditionally by Alternating test $\boxed{2}$ ✓

$\lim_{n\to\infty}\left|\frac{k^2}{k^3+1}\right| = 0$ by Alternating test $\boxed{3}$ ✓

$f(x) = \frac{x^2}{x^3+1} \Rightarrow f'(x) = \frac{(x^3+1)2x - x^2(3x^2)}{(x^3+1)^2} = \frac{2x^4+2x-3x^4}{(x^3+1)^2} = \frac{2x-x^4}{(x^3+1)^2}$ decresing

$\frac{2}{?}+\frac{4}{9}+\frac{9}{28}$ متناقص

Q4)

1) $\sum_{1}^{\infty} \frac{(k+4)!}{4!\, k!\, 4^k}$: by ratio test.

$$\lim_{k \to \infty} \left| \frac{a_{k+1}}{a_k} \right| = L.$$

$$\lim_{k \to \infty} \left| \frac{(k+5)!}{4!\,(k+1)!\,4^{k+1}} \times \frac{4!\,k!\,4^k}{(k+4)!} \right|$$

$$= \lim_{k \to \infty} \left| \frac{(k+5)}{(k+1)\,4} \right| = \lim_{k \to \infty} \left| \frac{k}{4k} \right| = \boxed{\frac{1}{4}} < 1$$

so : conv. abs.

2) $\sum_{1}^{\infty} \frac{\tan^{-1} k}{1+k^2}$:

$\frac{-\pi}{2} < \tan^{-1} k < \frac{\pi}{2} \Rightarrow \frac{\frac{-\pi}{2}}{1+k^2} < \frac{\tan^{-1} k}{1+k^2} < \frac{\frac{\pi}{2}}{1+k^2}$

$\frac{\frac{\pi}{2}}{1+k^2} < \frac{\frac{\pi}{2}}{k^2} \rightarrow \frac{\pi}{2}\left(\frac{1}{k}\right)^2$: $(p=2) > 1$

so it is conv by p-series

conv by comparison.

and while $\frac{\pi}{2}/1+k^2$ is conv, then $\frac{\tan^{-1} k}{1+k^2}$ is conv too by comparison test.

---

3) $\sum \frac{(-1)^{k+1} k^2}{k^3+1}$:

$\lim_{k\to\infty} \frac{k^2}{k^3+1} = \frac{1}{k} = \frac{1}{\infty} = 0$.

when $k=1$: $\frac{1}{2}$, when $k=2$: $\frac{4}{9}$

so it is decreasing

∴ conv by alternating .

$\sum \left|\frac{k^2}{k^3+1}\right|$: $a_k = \frac{k^2}{k^3+1}$, $b_k = \frac{1}{k}$, div by p-series

$\lim_{k\to\infty} \frac{k^2}{k^3+1} \times k = \lim_{k\to\infty} \frac{k^3}{k^3} = 1 \Rightarrow 0 < 1 < \infty$ so div.

so it is conv cond.

24

2019/4/17 الاختبار الثاني (تفاضل وتكامل 2) الجامعة الأردنية

مدرس المادة: د. منال غانم

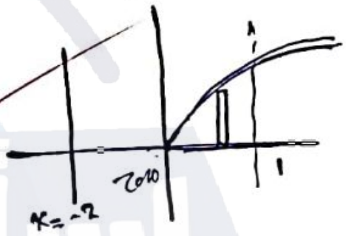
اسم الطالب: حمزة راضي سالم العرايرة

وقت المحاضرة: 8:00-9:00 ح ث ع

الرقم الجامعي: 0182885

Q2: (3 pts) (Set up the integral) that gives the volume of the solid generated by rotating the region bounded by $y=\sqrt{x}$, $x=1$, $y=0$ about $x=-2$ **Using Shell's method**.

$$V = \int_0^1 2\pi(x+2)(\sqrt{x}-\text{zero})\,dx$$



3

Q3: (5 pts) Find the area of the surface generated by rotating the curve $f(x)=\frac{x^3}{6}+\frac{1}{2x}$, $\frac{1}{2}\le x\le 1$ about the x-axis.

5

$$S = \int_{1/2}^{1} 2\pi\left(\frac{x^3}{6}+\frac{1}{2x}\right)\sqrt{1+(f'(x))^2}\,dx$$

$$1+(f'(x))^2 = 1+\left(\frac{x^2}{2}-\frac{1}{2x^2}\right)^2$$

$$= 1+\frac{x^4}{4}-\frac{1}{2}+\frac{1}{4x^4} = \frac{x^4}{4}+\frac{1}{2}+\frac{1}{4x^4} = \left(\frac{x^2}{2}+\frac{1}{2x^2}\right)^2$$

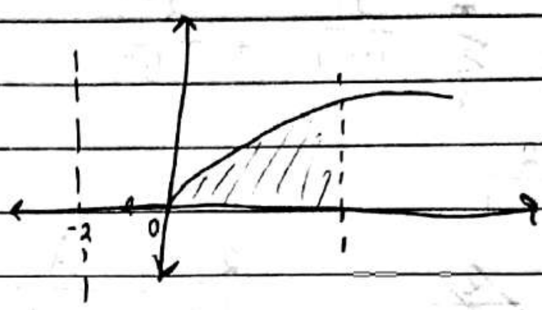
$$\rightarrow S = \int_{1/2}^{1} 2\pi\left(\frac{x^3}{6}+\frac{1}{2x}\right)\left(\frac{x^2}{2}+\frac{1}{2x^2}\right)dx = 2\pi\int_{1/2}^{1}\left(\frac{x^5}{12}+\frac{x}{12}+\frac{x}{4}+\frac{1}{4x^3}\right)dx$$

$$= 2\pi\left(\frac{x^6}{12*6}+\frac{x^2}{24}+\frac{x^2}{8}-\frac{1}{8x^2}\right)\Bigg]_{1/2}^{1}$$

$$= 2\pi\left(\frac{(1-(\frac{1}{2})^6)}{72}+\frac{(1-(\frac{1}{2})^2)}{24}+\frac{1-\frac{1}{4}}{8}-\frac{1}{8}(1-4)\right)$$

Q2)

$$V = \int_0^1 2\pi \, (x+2)\, (\sqrt{x}) \; dx$$



Q3) $f(x) = \frac{x^3}{6} + \frac{1}{2x}$, $\frac{1}{2} \leq x \leq 1$ about x-axis:

$$S = \int_{1/2}^{1} 2\pi \left(\frac{x^3}{6} + \frac{1}{2x}\right) \sqrt{1 + \left(\frac{x^2}{2} - \frac{2}{4x^2}\right)^2}$$

$$\Downarrow$$

$$1 + \frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4}$$

$$= \left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2$$

$$S = \int_{1/2}^{1} 2\pi \left(\frac{x^3}{6} + \frac{1}{2x}\right)\left(\frac{x^2}{2} + \frac{1}{2x^2}\right) dx$$

$$= \cancel{2}\pi \int_{1/2}^{1} \left(\frac{x^5}{\cancel{12}\,6} + \frac{x}{\cancel{12}\,6} + \frac{x}{\cancel{4}\,2} + \frac{1}{\cancel{4}x^3\,2}\right) dx$$

$$= \left(\frac{x^6}{36} + \frac{x^2}{12} + \frac{x^2}{4} - \frac{1}{4x^2}\right)\pi \Bigg|_{1/2}^{1}$$

$$= \left(\left(\frac{1}{36} + \frac{1}{12} + \frac{1}{4} - \frac{1}{4}\right) - \left(\frac{(1/2)^6}{36} + \frac{(1/2)^2}{12} + \frac{(1/2)^2}{4} - \frac{1}{4\left(\frac{1}{2}\right)^2}\right)\right)\pi$$

Scanned with CamScanner

Q4: (5pts) Find the sum (if exists)

$$\sum_{n=2}^{\infty} \frac{2}{n^2-1} + \frac{2^{n-1}}{5^n}$$

$$= \underbrace{\sum_{n=2}^{\infty} \frac{2}{(n-1)(n+1)}}_{\text{Telescoping}} \quad \boxed{+} \underbrace{\frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{2}{5}\right)^n}_{\text{geometric}}$$

1   2

#1 : $\sum_{n=2}^{\infty} \frac{2}{(n-1)(n+1)} \Rightarrow A(n+1) + B(n-1) = 2 \Rightarrow \sum_{n=2}^{\infty}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$

$\boxed{n=1 \rightarrow A=1 \quad n=-1 \rightarrow B=-1}$

$= \left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{3}} - \frac{1}{5}\right) + \dots + \left(\cancel{\frac{1}{n-1}} - \frac{1}{n+1}\right) = 1 - \frac{1}{n+1}$

$+ (\quad)$

$\rightarrow \lim_{n\to\infty}\left(1 - \frac{1}{n+1}\right) = \boxed{1}$

#2 : $\frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{2}{5}\right)^n = \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{2}{5}\right)^n - \frac{4}{9} = \frac{1}{2} * \frac{\frac{2}{5}}{1-\frac{2}{5}} - \frac{4}{9}$?

$= \frac{1}{2} * \frac{\frac{2}{5}}{\frac{-3}{5}} - \frac{4}{9} = -\frac{1}{3} - \frac{4}{9} = \frac{-3}{9} - \frac{4}{9} = \boxed{\frac{-7}{9}}$

So $1 - \frac{7}{9} = \boxed{\frac{2}{9}}$

$\boxed{3}$

Q5: (3 pts) Find the limit of the sequence Find the limit of the sequence

$$\left\{\left(\frac{n}{n-3}\right)^n\right\}_{n=4}^{\infty}$$

Solution

$\boxed{2}$

$$\lim_{n\to\infty}\left(\frac{n}{n-3}\right)^n = e^{\lim_{n\to\infty} n \ln\frac{n}{n-3}} = e^{\lim_{n\to\infty}\frac{\ln\frac{n}{n-3}}{\frac{1}{n}}}$$

$$= e^{\lim_{n\to\infty}\frac{\ln n - \ln(n-3)}{\frac{1}{n}}} \xRightarrow{\text{lopital}} e^{\lim_{n\to\infty}\frac{\frac{1}{n} - \frac{1}{n-3}}{\frac{-1}{n^2}}} = e^{\lim \frac{\frac{n-3-n}{n(n-3)}}{\frac{-1}{n^2}}}$$

Finish

Q4) $\sum_{2}^{\infty} \frac{2}{n^2-1} + \frac{2^{n-1}}{5^n}$:

$\sum_{2}^{\infty} \frac{2}{n^2-1}$:

$$\frac{2}{n^2-1} = \frac{A}{n-1} + \frac{B}{n+1}$$

$2 = A(n+1) + B(n-1)$.

when $n=-1$: $2 = -2B \longrightarrow B = -1$.

when $n=1$: $2 = 2A \longrightarrow A = 1$ ..

$\frac{1}{n-1} - \frac{1}{n+1}$:

$S_2 = 1 - \frac{1}{3}$

$S_3 = 1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4}$

$S_4 = 1 - \cancel{\frac{1}{3}} + \frac{1}{2} - \frac{1}{4} + \cancel{\frac{1}{3}} - \frac{1}{5}$

$S_5 = 1 + \frac{1}{2} - \frac{1}{4} - \frac{1}{5} + \frac{1}{4} - \frac{1}{6}$

:

$S_n = 1 + \frac{1}{2} + \frac{1}{n-1} - \frac{1}{n+1}$

$\lim_{n\to\infty} S_n = 1 + \frac{1}{2} + 0 - 0 = \frac{3}{2}$.

$\sum_{2}^{\infty} \frac{1}{2}\left(\frac{2}{5}\right)^n$: $a = \frac{2}{25}$, $r = \frac{2}{5}$

$\sum_{2}^{\infty} \frac{1}{2}\left(\frac{2}{5}\right)^n = \frac{\frac{2}{25}}{1-\frac{2}{5}} = \frac{2}{25} \times \frac{5}{3} = \frac{2}{15}$.

So:

total sum

$\frac{3}{2} + \frac{2}{15}$

COLLEGE

Q5) $\lim_{n\to\infty} \left(\frac{n}{n-3}\right)^n = \lim_{n\to\infty} \left(\left(\frac{n-3}{n}\right)^n\right)^{-1}$

$= \lim_{n\to\infty} \left(\left(1-\frac{3}{n}\right)^n\right)^{-1} = \left(e^{-3}\right)^{-1} = e^{3}$.

Q6: (11 pts) Test for convergence:

(a) $\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n\sqrt{n}}$

$\sin^2(n) \leqslant 1$

$\to 0 < \frac{\sin^2(n)}{n^{\frac{3}{2}}} \leqslant \frac{1}{n^{\frac{3}{2}}}$

solution

let $b_n = \frac{1}{n^{\frac{3}{2}}}$

and we use comparison test

so $\sum b_n = \sum \frac{1}{n^{\frac{3}{2}}}$ and it's p-series since $p = \frac{3}{2} > 1$

so $\to \sum b_n$ is convergent $\to$

by using comparison test

$\sum a_n = \sum \frac{\sin^2(n)}{n\sqrt{n}}$ is convergent.

3.5

(b) $\sum_{n=1}^{\infty} \frac{\sqrt{n^3+n^2+1}}{n^2+4}$

solution

by using lim. comparison test $\Rightarrow$ let $b_n = \frac{1}{n^{\frac{1}{2}}}$ $\Rightarrow \lim_{n\to\infty} \frac{\sqrt{n^3+n^2+1}}{n^2+4} \times n^{\frac{1}{2}}$

$= \lim_{n\to\infty} \frac{n^{\frac{3}{2}}}{n^2} \times n^{\frac{1}{2}} = \boxed{1}$

4

so $\sum b_n$ is divergent, because it's p-series & $p = \frac{1}{2} < 1$

so $\sum a_n$ is also divergent.

(c) $\sum_{n=2}^{\infty} \left(1 - \frac{1}{n^2}\right)^n$

? ? ?

convergent

Q6 $\sum_{1}^{\infty} \frac{\sin^2(n)}{n\sqrt{n}}$.

$-1 \leq \sin(n) \leq 1 \implies 0 \leq \sin^2(n) \leq 1$

$0 \leq \frac{\sin^2(n)}{n\sqrt{n}} \leq \frac{1}{n\sqrt{n}}$

$\Rightarrow = \frac{1}{n^{\frac{3}{2}}} = \left(\frac{1}{n}\right)^{\frac{3}{2}}$

while $\frac{3}{2} > 1$

$\therefore$ conv by p-series.

$\sum_{1}^{\infty} \frac{1}{n\sqrt{n}}$ is conv, so $\sum_{1}^{\infty} \frac{\sin^2(n)}{n\sqrt{n}}$ is conv.

b) $\sum_{1}^{\infty} \frac{\sqrt{n^3+n^2+1}}{n^2+1}$:

$a_n = \frac{\sqrt{n^3+n^2+1}}{n^2+1}$, $b_n = \frac{\sqrt{n^3}}{n^2} = n^{\frac{-1}{2}} = \frac{1}{\sqrt{n}}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sqrt{n^3+n^2+1}}{n^2+1} \times \frac{\sqrt{n}}{1}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n^4}}{n^2} = \frac{n^2}{n^2} = 1 \Rightarrow 0 < 1 < \infty$$

now: $b_n = \left(\frac{1}{n}\right)^{\frac{1}{2}}$, $p = \frac{1}{2} < 1$

so b_n is div

b_n is div , so a_n is div too by lim comp.

c) $\cancel{\#1}$ $\sum_{2}^{\infty} \left(1 - \frac{1}{n^2}\right)^n$:

$\lim_{n\to\infty} e^{n \ln\left(1 - \frac{1}{n^2}\right)}$,

so: $\lim_{n\to\infty} \frac{\ln\left(1 - \frac{1}{n^2}\right)}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\frac{\frac{2}{n^3}}{\frac{n^2-1}{n^2}}}{\frac{-1}{n^2}}$

$= \lim_{n\to\infty} \frac{2}{n^3-n} \times -n^2$:

$= \lim_{n\to\infty} \frac{-2n^2}{n^3} = \lim_{n\to\infty} \frac{-2}{n} = 0$.

$e^0 = 1$

so: $\lim_{n\to\infty} a_n = 1 \neq 0$, so div by div test.

Q3: (5 pts) Find the area of the surface generated by rotating the curve $y = \frac{x^4}{2} + \frac{1}{16x^2}$, $1 \le x \le 2$ about the y-axis.

$f(x) = \frac{x^4}{2} + \frac{1}{16x^2}$

$f'(x) = \frac{4}{2}x^3 + \frac{-2}{16x^3}$

$f'(x) = \frac{4}{2}x^3 - \frac{1}{8x^3}$

$(f'(x))^2 = 4x^6 - \frac{1}{16x^6}$

$(f'(x))^2 + 1 = 4x^6 - \frac{1}{16x^6} + 1$

$(f'(x))^2 + 1 = 4(2x^3)^2 - \frac{1}{(4x^3)^2} + 1$

$(f'(x))^2 + 1 = \left(2x^3 - \frac{1}{4x^3}\right)^2$

area $= 2\pi \int_a^b x \cdot \sqrt{1 + (f'(x))^2}\, dx$

area $= 2\pi \int_1^2 x \cdot \sqrt{\left(2x^3 - \frac{1}{4x^3}\right)^2}\, dx$

area $= 2\pi \int_1^2 x \cdot \left(2x^3 - \frac{1}{4x^3}\right) dx$

Area $= 2\pi \int_1^2 2x^4 - \frac{1}{4x^2}\, dx$

$2\pi \cdot \frac{2x^5}{5} + \frac{2\pi}{4x}\left(\frac{4\pi(2)^5}{5} + \frac{2\pi}{8}\right) - \left(\frac{4\pi}{5} + \frac{2\pi}{4}\right)$

Q4: (6 pts) Find the sum (if exists)

(a) $\sum_{n=1}^{\infty} \frac{2^{n-1} + 5^n}{10^n} \rightarrow \sum_{n=1}^{\infty} \frac{2^{n-1}}{10^n} + \sum_{n=1}^{\infty} \frac{5^n}{10^n}$

Geometric Series

① $\sum_{n=1}^{\infty} \frac{2^n \cdot 2^{-1}}{10^n}$

$\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n \cdot \frac{1}{2}$ $\quad |r| = \left|\frac{1}{5}\right| = \frac{1}{5} < 1 \rightarrow$ Conv

Sum $= \frac{\frac{1}{2}}{1 - \frac{1}{5}} = \frac{\frac{1}{2}}{\frac{4}{5}} = \frac{5}{8}$

② $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ $\quad |r| = \left|\frac{1}{2}\right| = \frac{1}{2} < 1 \rightarrow$ Conv

Sum $= \frac{1}{1 - \frac{1}{2}} = 2$

$(1) + (2) = \frac{21}{8}$

$= \frac{5}{8} + \frac{16}{8} = \frac{21}{8}$

(b) $\sum_{n=1}^{\infty} \frac{1}{n^2 + 5n + 6}$ $\quad \sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)}$

$\sum_{n=1}^{\infty} \frac{A}{(n+2)} + \frac{B}{(n+3)}$ Partial Fraction

$A(n+3) + B(n+2) = 1$

$n = -3 \rightarrow \boxed{B = -1}$

$n = -2 \rightarrow \boxed{A = 1}$

$\sum_{n=1}^{\infty} \frac{1}{n+2} - \frac{1}{n+3}$

$\sum_{i=1}^{n} \frac{1}{i+2} - \frac{1}{i+3}$

$i = 1 \qquad i = 2$

$\left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \dots$

$i = n-1 \qquad i = n$

$\dots + \left(\frac{1}{n+1} - \frac{1}{n+2}\right) + \left(\frac{1}{n+2} - \frac{1}{n+3}\right)$

$S = \frac{1}{3} - \frac{1}{n+3}$

$\lim_{n \to \infty} \frac{1}{3} - \frac{1}{n+3} = \frac{1}{3}$ = Sum of Series

3

Q3] $y = \frac{x^4}{2} + \frac{1}{16x^2}$

Sol $y' = 2x^3 + \frac{-1(32x)}{(16)^2 x^4} = 2x^3 - \frac{1}{8x^3}$

$(y')^2 = 4x^6 - \frac{1}{2} + \frac{1}{64x^6}$

$(y')^2 + 1 = 4x^6 + \frac{1}{64x^6} + 1$

$= \left(2x^3 + \frac{1}{8x^3}\right)^2$

$S = 2\pi \int_1^2 x \sqrt{\left(2x^3 + \frac{1}{8x^3}\right)^2} \, dx$

$= 2\pi \int_1^2 2x^4 + \frac{1}{8x^2} \, dx$

$= 2\pi \left(\frac{2x^5}{5} - \frac{1}{8x} \right) \Big|_1^2$

$= 2\pi \left(\frac{64}{5} - \frac{1}{16} - \frac{2}{5} + \frac{1}{8} \right)$

Q4) a) $\sum_{1}^{\infty} \frac{2^{n-1} + 5^{n}}{10^{n}}$:

$= \sum_{1}^{\infty} \frac{1}{2}\left(\frac{1}{5}\right)^{n} + \sum_{1}^{\infty}\left(\frac{1}{2}\right)^{n}$

* $\sum_{1}^{\infty} \frac{1}{2}\left(\frac{1}{5}\right)^{n}$: $a = \frac{1}{10}$, $r = \frac{1}{5}$ $\left(-1 < \frac{1}{5} < 1\right)$

$\sum_{1}^{\infty} \frac{1}{2}\left(\frac{1}{5}\right)^{n} = \frac{1/10}{1 - 1/5} = \frac{1/10}{4/5} = \frac{1}{10} \times \frac{5}{4} = \frac{1}{8}$.

* $\sum_{1}^{\infty} \left(\frac{1}{2}\right)^{n}$: $a = \frac{1}{2}$, $r = \frac{1}{2}$.

$\sum_{1}^{\infty} \left(\frac{1}{2}\right)^{n} = \frac{1/2}{1 - 1/2} = \frac{1/2}{1/2} = 1$

so: Total sum $= 1 + \frac{1}{8} = 9/8$.

b) $\sum_{1}^{\infty} \frac{1}{n^2+5n+6}$:

$$\frac{1}{n^2+5n+6} = \frac{A}{n+2} + \frac{B}{n+3}$$

$1 = A(n+3) + B(n+2)$

when $n=-3$: $1 = -B \Rightarrow B = -1$

when $n=-2$: $1 = A$

so: $\frac{1}{n+2} - \frac{1}{n+3}$

$S_1 = \frac{1}{3} - \frac{1}{4}$

$S_2 = \frac{1}{3} - \cancel{1/4} + \cancel{1/4} - 1/5$

$S_3 = \frac{1}{3} - \cancel{1/5} + \cancel{1/5} - 1/6$

$\vdots$

$S_n = \frac{1}{3} + \frac{1}{n+2} - \frac{1}{n+3}$

$$\lim_{n\to\infty} S_n = \frac{1}{3} + 0 + 0 = \frac{1}{3}$$

Scanned with CamScanner

(b) $\sum_{n=1}^{\infty} \frac{n+5}{\sqrt{n^3 - n + 1}}$ Using limit of comparison test $b_n = \frac{n}{n^{3/2}} = \frac{1}{n^{1/2}}$, $\sum b_n = \sum \frac{1}{n^{1/2}}$

P series

$\frac{1}{2} < 1$ div

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n+5}{(n^3 - n^3 + 1)} \cdot n^{1/2}$

$\lim_{n\to\infty} \frac{n+5}{n^{3/2} - n - n^{1/2}} = -1$ div

1

3

(c) $\sum_{n=1}^{\infty} n^2 \sin^2(\frac{2}{n})$ $b_n = \frac{2}{n}$ $\sum \frac{2}{n}$ n=1 div

$2n \sin^2(\frac{2}{n}) = \infty \cdot \infty$!!

$\lim_{n\to\infty} \frac{n^2 n \sin^2(\frac{2}{n})}{n} = \infty$ $\lim_{n\to\infty}$

(b) $\sum_{1}^{\infty} \frac{n+5}{\sqrt{n^3-n+1}}$:

$a_n = \frac{n+5}{\sqrt{n^3-n+1}}$, $b_n = \frac{n}{\sqrt{n^3}} = \frac{1}{\sqrt{n}}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{n+5}{\sqrt{n^3-n+1}} \times \frac{\sqrt{n}}{1} = \lim_{n\to\infty} \frac{n\sqrt{n}}{\sqrt{n^3}}$

$= \lim_{n\to\infty} \frac{n\sqrt{n}}{n\sqrt{n}} = 1$, so both conv or div.

$b_n = \left(\frac{1}{n}\right)^{\frac{1}{2}}$, $P = \frac{1}{2} < 1$, so b_n ~~conv~~ div by P-series.

$\therefore$ a_n is ~~conv~~ div by limit comp test.

(c) $\sum_{1}^{\infty} n^2 \sin^2\left(\frac{2}{n}\right)$:

$$\lim_{n\to\infty} n^2 \sin^2\left(\frac{2}{n}\right) = \left(\lim_{n\to\infty} n \sin\frac{2}{n}\right)^2$$

$$= \left(\lim_{n\to\infty} \frac{\sin\left(2\times\frac{1}{n}\right)}{\frac{1}{n}}\right)^2 = 2^2 = 4.$$

so the limit $\neq 0$, then it is div.

$\frac{17.5}{20}$

The University of Jordan	34	Second Exam Calculus II	Wednesday 5\7\2017
مدرس المادة: د. ... حليمة	اسم الطالب: عمر ... الشوا		
وقت المحاضرة: ١٢:٣٠	الرقم الجامعي:		

Q1: (4 marks) Find the arc length of the curve $x = \frac{1}{6}y^3 + \frac{1}{2}y^{-1}$, $1 \le y \le \sqrt{2}$.

$\left(\frac{dx}{dy}\right) = \frac{y^2}{2} + \left(-\frac{y^{-2}}{2}\right)$

$\left(\frac{dx}{dy}\right)^2 = \left(\frac{y^4}{2} - \frac{y^{-4}}{2} \cdot \frac{y^2}{2} + \frac{y^4}{4}\right)$

$= \frac{y^4}{2} - \frac{1}{2} + \frac{y^4}{4}$

$1 + \left(\frac{dx}{dy}\right)^2 = \frac{y^4}{2} + \frac{1}{2} + \frac{y^4}{4}$

$\int_1^{\sqrt{2}} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}$

$= \int_1^{\sqrt{2}} \sqrt{\frac{y^4}{2} + \frac{1}{2} + \frac{y^4}{4}} = \int_{\sqrt{2}} \left(\frac{y^4}{2} + \frac{1}{2} + \frac{y^4}{4}\right)^{\frac{1}{2}}$

3

$= \frac{y^5}{10} + \frac{y}{2} + \frac{y^5}{10} \Big]_1^{\sqrt{2}} = \left(\frac{(\sqrt{2})^5}{10} + \frac{\sqrt{2}}{2} + \frac{(\sqrt{2})^5}{2}\right) - \frac{14}{20}$

$\cancel{= \left(\frac{1}{10} + \frac{1}{2} + \frac{1}{10}\right) - \left(\frac{(\sqrt{2})^5}{10} + \frac{\sqrt{2}}{2} + \frac{(\sqrt{2})^5}{10}\right)}$

Q1) $x = \frac{1}{6} y^3 + \frac{1}{2} y^{-1}$:

$$\frac{dx}{dy} = \frac{1}{2} y^2 - \frac{1}{2y^2}$$

$$(dx/dy)^2 = \frac{1}{4} y^4 - \frac{1}{2} + \frac{1}{4y^4}$$

$$(dx/dy)^2 + 1 = \frac{1}{4} y^4 + \frac{1}{4y^4} + \frac{1}{2} = \left(\frac{1}{2} y^2 + \frac{1}{2y^2}\right)^2$$

$$L = \int_1^{\sqrt{2}} \sqrt{\left(\frac{1}{2} y^2 + \frac{1}{2y^2}\right)^2} \, dy = \int_1^{\sqrt{2}} \frac{1}{2} y^2 + \frac{1}{2} y^{-2} \, dy$$

$$= \left(\frac{1}{6} y^3 - \frac{1}{2} y^{-1}\right) \Big|_1^{\sqrt{2}}$$

$$= \frac{1}{6} (\sqrt{2})^3 - \frac{1}{2\sqrt{2}} - \frac{1}{6} + \frac{1}{2}$$

$\frac{1}{5}$ $\frac{25}{25} - \frac{3}{25}$ $\frac{22}{25}$

Q3: (6 marks) Find the sum of the following series :

(a) $\sum_{n=0}^{\infty} \frac{3^n}{5^{2n+1}}$ $\lim_{n\to\infty} \frac{3^n}{5^{2n} \cdot 5^1} = \frac{1}{5} \cdot \frac{3^n}{25^n} = \frac{1}{5} \left(\frac{3}{25}\right)^n$

$|r| < 1$

$\left|\frac{3}{25}\right| < 1$

conv. by geometric series.

3

$\text{sum} = \frac{\frac{1}{5}}{1 - \frac{3}{25}} = \frac{\frac{1}{5}}{\frac{22}{25}} \quad \frac{1}{\frac{22}{5}} = \boxed{\frac{5}{22}}$

(b) $\sum_{n=1}^{\infty} \left(\sin\left(\frac{2}{n+1}\right) - \sin\left(\frac{2}{n}\right)\right)$

$\lim_{n\to\infty} \left(\cancel{\sin 1} - \sin 2\right) + \left(\cancel{\sin\frac{2}{3}} - \cancel{\sin 1}\right) + \left(\cancel{\sin\frac{1}{2}} - \cancel{\sin\frac{2}{3}}\right)$

$+ \left(\sin\frac{2}{5} - \cancel{\sin\frac{1}{2}}\right) \cdots \left(\sin\frac{2}{n+1} - \sin\frac{2}{n}\right)$

3

$S_n = \lim_{n\to\infty} -\sin 2 + \sin\frac{2}{n+1}$ by telescopic series.

$= -\sin 2 + 0 = -\sin 2$

Q4: (2 marks) Determine whether the sequence $\lim_{n\to\infty} \left\{\ln\left(\frac{8n+1}{2n+7}\right)\right\}_{n=1}^{\infty}$ is convergent or divergent.

~~$b_n = \frac{8}{2} = \frac{8}{2} > 1$ div by geometric series.~~

$\sum_{n=1}^{\infty} \ln \frac{8n+1}{2n+7}$

2.5

$\boxed{\ln 4}$

conv

$\sum_{n=0}^{\infty} \ln \frac{8n+2}{2n+8}$

$= \frac{8\cancel{n}}{2\cancel{n}} = \frac{\cancel{8}^4}{\cancel{2}_1} = 4 > 1$ div by div test.

$\pm\infty$
D.N.E

Q3) a) $\sum_{0}^{\infty} \frac{3^n}{5^{2n+1}}$:

$= \sum_{0}^{\infty} \left(\frac{3}{25}\right)^n \left(\frac{1}{5}\right)$: $a = \frac{1}{5}$, $r = \frac{3}{25}$

$\sum_{0}^{\infty} \left(\frac{3}{25}\right)^n \left(\frac{1}{5}\right) = \frac{1}{5} \times \frac{25}{22}$

$= \frac{5}{22}$

b) $\sum_{1}^{\infty} \sin\left(\frac{2}{n+1}\right) - \sin\left(\frac{2}{n}\right)$:

$S_1 = \sin 1 - \sin 2$

$S_2 = \cancel{\sin 1} - \sin 2 + \sin\frac{2}{3} - \cancel{\sin 1}$

$S_3 = -\sin 2 + \cancel{\sin 2/3} + \sin 1/2 - \cancel{\sin 2/3}$

$S_4 = -\sin 2 + \cancel{\sin 1/2} + \sin 2/5 - \cancel{\sin 1/2}$

$\vdots$

$S_n = -\sin 2 + \sin\frac{2}{n+1} - \sin\left(\frac{2}{n}\right)$

$\lim_{n\to\infty} S_n = -\sin 2 + 0 + 0 = \underline{-\sin 2}$

Q4)

$$\lim_{n\to\infty} \ln \frac{8n+1}{2n+7} = \lim_{n\to\infty} \ln \frac{8n}{2n} = \ln 4$$

$\ln 4 \neq 0$

so: div by div test.

Q5: (6 marks) Test for convergence for the following

(a) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

① an continuous
② an decrease
③ +ve terms.

$\lim_{a\to\infty} \int_2^a \frac{2}{n(\ln n)^2}\, dn$

$u = \ln n$

$du = \frac{1}{n}\, dn$

$dn = n \cdot du$

$\lim_{a\to\infty} \int_2^a \frac{2}{\cancel{n}(u)^2} \cdot \cancel{n}\, du$

$\lim_{a\to\infty} \int_2^a \frac{2\, du}{(u)^2} =$

2

$\lim_{a\to\infty} \int_2^a 2 \cdot u^{-2} = 2u \cdot -u^{-1} \Big]_2^a$ $[(200)(-\infty)] -$ $[4 \cdot 4] =$ ~~Converge by~~ $\infty - 16 = \infty > 1$

check
$P > 2$ conv. by series
Converge by alternive serie.

(b) $\sum_{n=1}^{\infty} \frac{n!(3)^n}{(2n+1)!}$

$a_n = \left| \frac{a_{n+1}}{a_n} \right|$

$\left| \frac{(n+1)\, n! \cdot 3^{(n+1)}}{2(n+1) + n! + 1} \cdot \frac{(2n+1)!}{n!\, 3^n} \right|$

$\left| \frac{(n+1)\cancel{n!} + \cancel{3^n} \cdot 3}{(2n+2)(\cancel{2n+1})\cancel{n!} + 1} \cdot \frac{\cancel{(2n+1)n!}}{\cancel{n!\, 3^n}} \right|$

2

$= \left| \frac{(n+1)}{(2n+2)+1} \right| = \left| \frac{n+1}{2n+3} \right|$

$2n+3 > 2n \rightarrow \frac{1}{2n+3} < \frac{1}{2n}$

$(\frac{1}{2})^n$

$\left|\frac{1}{2}\right| < 1$

conv. by geo series

So if b_n is conv. by geo series
so $\sum_{n=1}^{\infty} \frac{n!(3)^n}{(2n+1)!}$ is conv. by limit comparison test.

Q5

a) $\sum_{2}^{\infty} \frac{1}{n(\ln n)^2}$: $a_n > 0$, cont , decreasing

$\int_{2}^{\infty} \frac{1}{x(\ln x)^2} dx = \lim_{L\to\infty} \int_{2}^{L} \frac{1}{x(\ln x)^2} dx$.

let : $u = \ln x \longrightarrow dx = x\, du$

$\int \frac{x}{x(u)^2} du = \int u^{-2} du = -u^{-1}$

$= \frac{-1}{\ln x} \Big|_{2}^{L} = \lim_{L\to\infty} \frac{-1}{\ln L} + \frac{1}{\ln 2} = \frac{1}{\ln 2}$.

so it is conv by Integral test .

b) $\sum_{1}^{\infty} \frac{n!(3)^n}{(2n+1)!}$: by Ratio test :

$$a_n = \frac{n!\,3^n}{(2n+1)!} \quad , \quad a_{n+1} = \frac{(n+1)!\,3^{n+1}}{(2n+3)!}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)(n)!\,3^n\,(3)}{(2n+3)(2n+2)(2n+1)!} \times \frac{(2n+1)!}{n!\,3^n}$$

$$= \lim_{n\to\infty} \frac{(n+1)\,(3)}{(2n+3)(2n+2)} = \lim_{n\to\infty} \frac{3n}{4n^2} = 0$$

so it is conv abs. by ratio test.

21/25

الجامعة الأردنية	الامتحان الثاني: تفاضل وتكامل 2	الخميس 2017/4/13
اسم الطالب: راما زكي محمد درويش		مدرس المادة: د. سلام النابلسي
الرقم الجامعي:		وقت المحاضرة: ١٢ ← ١

يتكون الامتحان من 6 أسئلة في 3 ورقات:

[1] (4 marks) Find the arc length of the curve $24xy = y^4 + 48$ from $y = 2$ to $y = 4$.

(4) $L = \int_2^4 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$

$\frac{24xy}{24y} = \frac{y^4+48}{24y}$

$x = \frac{y^3}{24} + \frac{2}{y}$

$\frac{dx}{dy} = \frac{3y^2}{24} - \frac{2}{y^2}$

$\left(\frac{dx}{dy}\right)^2 = \left(\frac{y^2}{8} - \frac{2}{y^2}\right)^2$

$= \frac{y^4}{64} - 2\cdot\frac{2}{y^2}\cdot\frac{y^2}{8} + \frac{4}{y^4}$

$\left(\frac{dx}{dy}\right)^2 + 1 = \frac{y^4}{64} - \frac{1}{2} + \frac{4}{y^4} + 1$

$L = \frac{y^4}{64} + \frac{1}{2} + \frac{4}{y^4} = \left(\frac{y^2}{8} + \frac{2}{y^2}\right)^2$

$\int_2^4 \sqrt{\left(\frac{y^2}{8} + \frac{2}{y^2}\right)^2}$

$\int_2^4 \frac{y^2}{8} + \frac{2}{y^2}\; y^{-2}\, dy$

$\left[\frac{y^3}{24} + \frac{2y^{-1}}{-1}\right]_2^4$

$\left[\frac{y^3}{24} - \frac{2}{y}\right]_2^4$

$= \frac{64}{24} - \frac{1}{2} - \left(\frac{8}{24} - 1\right)$

(1) [2] (2 marks) Set up the integral (**Do not Evaluate**) that gives the surface area of the solid that results when the region bounded by $x = \cosh y$, $1 \le y \le 2$ is revolved about $x - axis$.

$\boxed{\cosh^{-1} x = y}$

$\cosh x = \frac{e^x + e^{-x}}{2}$ } y

$\frac{e^x - e^{-x}}{2} = \frac{dy}{dx}$

$= 2\pi r \int \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$= 2\pi \cosh^{-1} x \int_{\cosh^{-1}(1) = y}^{\cosh^{-1}(2)} \sqrt{1+\left(\frac{e^x - e^{-x}}{2}\right)^2}$

$= 2\pi \int_0^{\cosh^{-1}(2)} \cosh^{-1} x \sqrt{1+\left(\frac{e^x - e^{-x}}{2}\right)^2}$

$\cosh x = \frac{e^x + e^{-x}}{2}$

$2 = \frac{e^x + e^{-x}}{2}$

$2e^x = e^{2x} + 1$

$e^{2x} - 2e^x + 1 = 0$

$(e^x - 1)(e^x - 1)$

$\ln e^x = \ln 1 = 0$

[3](a)(2 marks) Test for convergence $\sum_{=1}^{\infty} \frac{5}{(3-\sin n)^n}$

$\lim_{n\to\infty} \frac{5}{(3-\sin n.)^n} =$

0

By Root test

$$\lim_{n\to\infty} \left(\frac{5}{(3-\sin n)^n}\right)^{\frac{1}{n}}$$

$$\lim_{n\to\infty} \frac{5^{\frac{1}{n}}}{3-\sin n} = \frac{\lim_{n\to\infty} 5^{\frac{1}{n}}}{\lim_{n\to\infty} 3-\sin n} \cancel{= \frac{1}{\infty}} = 0 < 1$$

Conv. By root test

---

(4)

[4] (4 marks) Determine if the series $\sum_{n=1}^{\infty}(-1)^n \frac{(n!)^2}{(2n)!}$ converges absolutely, conditionally or diverges.

first $\boxed{\text{Absolute test}}$

$$\sum_{n=1}^{\infty} \left|(-1)^n \frac{(n!)^2}{(2n)!}\right| = \sum_{n=1}^{\infty} \left|\frac{(n!)^2}{(2n)!}\right|$$

By Ratio test

$$\lim_{n\to\infty} \frac{((n+1)!)^2}{(2(n+1))!} \cdot \frac{2n!}{(n!)^2}$$

$$\lim_{n\to\infty} \frac{(n+1)!\,(n+1)!}{(2n+2)!} \cdot \frac{2n!}{n!\;n!}$$

$$\lim_{n\to\infty} \frac{(n+1)\,\cancel{n!}\,(n+1)\cancel{n!}}{(2n+2)\,(2n+1)\,\cancel{(2n)!}} \cdot \frac{\cancel{2n!}}{\cancel{n!\,n!}}$$

$$\lim_{n\to\infty} \frac{(n+1)^2}{2(n+1)(2n+1)} = \lim_{n\to\infty} \frac{n+1}{2(2n+1)} = \frac{1}{4} < 1$$

Covn. by ratio test

by Absolute test $\sum_{n=1}^{\infty} \left|\frac{(n!)^2}{(2n)!}\right|$ conv. So $\sum_{n=1}^{\infty} (-1)^n \frac{(n!)^2}{(2n)!}$ is conv.

estimating the sum

$$\frac{1}{2(\ln 2)^2} \leq \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3} \leq \frac{1}{2(\ln 2)^2} + \frac{1}{2(\ln 2)^3}$$

$$\frac{1}{2(\ln 2)^2} \leq \sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^3} \leq \frac{1}{2(\ln 2)^2} + \frac{1}{2(\ln(2))^3}$$

Q3 (b)

* both are conv.

so $\boxed{\text{conv. absolutly}}$

3(b)(c) 3!

Q1) $24xy = y^4 + 48$

$$x = \frac{1}{24} y^3 + \frac{2}{y}$$

$$\frac{dx}{dy} = \frac{1}{8} y^2 - \frac{2}{y^2}$$

$$(dx/dy)^2 = \frac{1}{64} y^4 - \frac{1}{2} + \frac{4}{y^4}$$

$$(dx/dy)^2 + 1 = \left(\frac{1}{8} y^2 + \frac{2}{y^2}\right)^2$$

$$L = \int_2^4 \sqrt{\left(\frac{1}{8} y^2 + \frac{2}{y^2}\right)^2}\, dy = \int_2^4 \frac{1}{8} y^2 + \frac{2}{y^2}\, dy$$

$$= \left(\frac{y^3}{24} - \frac{2}{y}\right)\Bigg|_2^4 = \frac{64}{24} - \frac{1}{2} - \frac{8}{24} + 1$$

$$= \frac{56}{24} + \frac{12}{24} = \frac{68}{24}$$

Q2) $x = \cosh y \;,\; 1 \le y \le 2 \;,$ about x-axis:

$$S = \int_1^2 2\pi \cosh y \sqrt{1 + (\sinh y)^2} \; dy .$$

Q3) $\sum_{1}^{\infty} \frac{5}{(3-\sin n)^n}$:

$$\lim_{n \to \infty} |a_n|^{\frac{1}{n}} = 5 \lim_{n \to \infty} \frac{1}{3-\sin(n)} = 5 \times 0 = 0 < 1$$

so conv abs by Root test.

Q_4 $\sum_{1}^{\infty} (-1)^n \frac{(n!)^2}{(2n!)}$:

$$a_n = \frac{(n!)^2}{(2n)!} , \quad a_{n+1} = \frac{((n+1)!)^2}{(2n+2)!}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^2 (n!)^2}{(2n+2)(2n+1)(2n)!} \times \frac{(2n)!}{(n!)^2}$$

$$= \lim_{n\to\infty} \frac{(n+1)^2}{(2n+1)(2n+2)} = \lim_{n\to\infty} \frac{n^2}{4n^2} = \frac{1}{4} < 1$$

conv abs. by ratio test.

[5] (6 marks) Find the sum:

(a) $\sum_{k=1}^{\infty} \frac{4^{k-1}}{3^{2k+1}}$

2

$$\sum_{k=0}^{\infty} \frac{4^{k-1+1}}{3^{2k+2}} = \frac{4^k}{3^{2k} \cdot 3^2} = \frac{4^k}{9^k \cdot 3^2} = \sum_{k=0}^{\infty} \left(\frac{4}{9}\right)^k \cdot \frac{1}{9}$$

$\boxed{\text{geometric series}}$ $r = \frac{4}{9} < 1$

$$\text{Sum} = \frac{\frac{1}{9}}{1 - \frac{4}{9}} = \frac{1}{5}$$

(b) $\sum_{n=1}^{\infty} \left(3^{\frac{1}{n}} - 3^{\frac{1}{n+1}}\right)$

3 n^{th} partial

$$= \lim_{n\to\infty}\left(\sum_{k=1}^{n} \left(3^{\frac{1}{k}} - 3^{\frac{1}{k+1}}\right)\right)$$

↳ By telescoping method

$$\left(3^{1} - 3^{\frac{1}{2}}\right) + \left(3^{\frac{1}{2}} - 3^{\frac{1}{3}}\right) + \left(3^{\frac{1}{3}} - 3^{\frac{1}{4}}\right) + \cdots + \left(3^{\frac{1}{n-1}} - 3^{\frac{1}{n}}\right) + \left(3^{\frac{1}{n}} - 3^{\frac{1}{n+1}}\right)$$

$$\lim_{n\to\infty}\left(3 - 3^{\frac{1}{n+1}}\right) = 2$$

P-series, $p > 1$ conv

3

[6] (3 marks) Leonhard Euler proved that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. Use this to find the sums:

(a) $\sum_{n=2}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} - 1$

(b) $\sum_{n=2}^{\infty} \frac{1}{(n+1)^2}$

$$\frac{1}{9} + \frac{1}{16} + \frac{1}{25} - - - + \frac{1}{(n+1)^2} =$$

$$= \frac{\pi^2}{6} - 1 - \frac{1}{4}$$

a

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

$$\underset{n=1}{1} + \underset{n=2}{\frac{1}{4}} + \underset{n=3}{\frac{1}{9}} - - - + \frac{1}{n^2} = \frac{\pi^2}{6} - 1$$

b

$$\boxed{\underset{n=1}{1} + \underset{n=2}{\frac{1}{4}}} + \underset{n=3}{\frac{1}{9}} + \frac{1}{16} + \frac{1}{25} - - - + \frac{1}{n^2} = \frac{\pi^2}{6}$$

$$- - - = \frac{\pi^2}{6} - 1 - \frac{1}{4}$$

Q5]

a) $\sum_{1}^{\infty} \frac{4^{k-1}}{3^{2k+1}}$:

$= \sum_{1}^{\infty} \left(\frac{4}{9}\right)^{k} \left(\frac{1}{12}\right)$: $a = \frac{1}{27}$, $r = \frac{4}{9}$

$\frac{\frac{1}{27}}{1-\frac{4}{9}} = \frac{1}{27} \times \frac{9}{5} = \frac{1}{15}$

b) $\sum_{1}^{\infty}\left(3^{\frac{1}{n}} - 3^{\frac{1}{n+1}}\right)$:

$S_1 = 3 - 3^{\frac{1}{2}}$

$S_2 = 3 - 3^{\frac{1}{2}} + 3^{\frac{1}{2}} - 3^{\frac{1}{3}}$

$S_3 = 3 - 3^{\frac{1}{3}} + 3^{\frac{1}{3}} - 3^{\frac{1}{4}}$

$\vdots$

$S_n = 3 + \cancel{3^{1}} - 3^{\frac{1}{n+1}}$

$\lim_{n \to \infty} S_n = 3 + \cancel{3^{0}} - 3^{0} = \cancel{3}\, 2$

Q6)

a) $\sum_{1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \cdots = \frac{\pi^2}{6}$

so: $\sum_{2}^{\infty} \frac{1}{n^2} = \frac{1}{4} + \frac{1}{9} + \cdots = \frac{\pi^2}{6} - 1$

b) $\sum_{2}^{\infty} \frac{1}{(n+1)^2} = \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots = \frac{\pi^2}{6} - 1 - \frac{1}{4}$

$n = 1$

$n = 2$

Excellent

Jordan University
Mathematics Department
Calculus II, Second Exam, 22/8/2017

+1 $\frac{20.5}{20}$

$\frac{21-4}{4}$ $\frac{21-8}{4}$

Student's Name:: سجود عمر ظاهر اليوسف Student Number: [redacted]

Instructor's Name: د. عبد الرحمن يوسف . Lecture Time: [redacted]

1) (4 points) Find the area of the surface obtained by rotating the curve $y = \sqrt{5-x}$, $1 \le x \le 2$ about the x-axis.

$y' = \frac{-1}{2\sqrt{5-x}}$

$(y')^2 = \frac{1}{4(5-x)}$

$A = 2\pi \int_1^2 f(x) \sqrt{1 + (f'(x))^2}\, dx$

$= 2\pi \int_1^2 \sqrt{5-x} \cdot \sqrt{1 + \frac{1}{4(5-x)}}\, dx$

$= 2\pi \int_1^2 \sqrt{(5-x) + \frac{\cancel{(5-x)}}{4\cancel{(5-x)}}}\, dx$

$= 2\pi \int_1^2 \sqrt{5 - x + \frac{1}{4}}\, dx$

$= 2\pi \int_1^2 \sqrt{\frac{21}{4} - x}\, dx$

$= 2\pi \cdot \left. \frac{-2(\frac{21}{4} - x)^{\frac{3}{2}}}{3} \right|_1^2$

$2\pi \left[-2\left[\left(\frac{21}{4} - \frac{24}{4} \right) - \left(\frac{21}{4} - \frac{4}{4} \right) \right] \right]$

$= 2\pi \left[-2 \left[\frac{13}{4} - \frac{15}{4} \right] \right]$

$= 2\pi \left[-2 \cdot \frac{-2}{4} \right]$

$= 2\pi$

3.5

3) (2 points) Find the limit of the following sequence

$\left\{ \left(\frac{2n+6}{2n} \right)^n \right\}_{n=1}^{\infty}$

$\lim_{n \to \infty} \left(\frac{2n+6}{2n} \right)^n$

$= \lim_{n \to \infty} \left(\frac{2n}{2n} + \frac{6}{2n} \right)^n$

$= \lim_{n \to \infty} \left(1 + \frac{3}{n} \right)^n = e^3$

2

Q1) $y = \sqrt{5-x}$, $1 \leq x \leq 2$:

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{5-x}} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{4(5-x)}$$

$$S = 2\pi \int_1^2 \sqrt{5-x} \sqrt{1 + \frac{1}{4(5-x)}} \, dx$$

$$= 2\pi \int_1^2 \sqrt{5-x + 1/4} \, dx$$

$$= 2\pi \int_1^2 \sqrt{21/4 - x} \, dx$$

$$= 2\pi \left(\frac{-2}{3} \sqrt{(21/4 - x)^3} \right) \Big|_1^2$$

$$= (2\pi)\left(\frac{-2}{3}\right)\left((13/4)^{3/2} - (17/4)^{3/2} \right)$$

Q3] $\lim_{n \to \infty} \left(\frac{2n+6}{2n}\right)^n$:

$= \lim_{n \to \infty} \left(1 + \frac{3}{n}\right)^n = e^3$

$GS \to a_n > 0$

2) Find the value of the sums

a) (3 points) $\sum_{n=1}^{\infty} \frac{(-1)^n\, 2^{2n}}{5^{n-1}}$

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^n 2^{2n}}{5^{n-1}} \right|$$

$$= \sum_{n=1}^{\infty} \left| \frac{4^n}{5^n} \cdot 5 \right|$$

$$= 5 \sum_{n=1}^{\infty} \left| \left(\frac{4}{5}\right)^n \right| :$$

conv by GS

$$5 \sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^n = \frac{5 \cdot \left(\frac{4}{5}\right)^1}{1 - \frac{4}{5}} = \frac{5 \cdot \frac{4}{5}}{\frac{1}{5}} = 4 \times 5 = 20$$

2

b) (3 points) $\sum_{n=2}^{\infty} \left(\frac{2}{n} - \frac{2}{n+2} \right)$

$$S_K = \left(1 - \cancel{\frac{2}{2}}\right) + \left(\frac{2}{3} - \cancel{\frac{2}{5}}\right) + \left(\cancel{\frac{1}{2}} - \frac{1}{3}\right) + \left(\cancel{\frac{2}{5}} - \frac{2}{7}\right) + \cdots +$$

$$+ \left(\frac{2}{K-3} - \cancel{\frac{2}{K-1}}\right) + \left(\frac{2}{K-2} - \cancel{\frac{2}{K}}\right) + \left(\cancel{\frac{2}{K-1}} - \frac{2}{K+1}\right) + \left(\cancel{\frac{2}{K}} - \frac{2}{K+2}\right)$$

$$S_K = \lim_{K \to \infty} 1 + \frac{2}{3} + \frac{2}{K+1} - \frac{2}{K+2} = \frac{5}{3}$$

3

4) Test the following series for convergence:

a) (3 points) $\sum_{n=1}^{\infty} \frac{(-1)^n\, 7^n}{(2n)!}$

by Ratio test

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{7^{n+1}}{(2n+2)!} \cdot \frac{(2n)!}{7^n} \right|$$

$$= \lim_{n \to \infty} \left| \frac{\cancel{7^n} \cdot 7}{(2n+2)(2n+1)\cancel{(2n)!}} \cdot \frac{\cancel{(2n)!}}{\cancel{7^n}} \right|$$

$$= \lim_{n \to \infty} \left| \frac{7}{(2n+2)(2n+1)} \right| = 0$$

3

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^n\, 7^n}{(2n)!}$ is convergent (abs conv) by Ratio Test

2

Scanned by CamScanner

Q2) a) $\sum_{1}^{\infty} \frac{(-1)^n 2^{2n}}{5^{n-1}} = \sum_{1}^{\infty} 5\left(\frac{-4}{5}\right)^n$ geo. series.

$a = -4 \;,\; r = \frac{-4}{5}$

$\sum_{1}^{\infty} \frac{(-1)^n 2^{2n}}{5^{n-1}} = \frac{-4}{1 + 4/5} = -4 \times \frac{5}{9} = \frac{-20}{9}$

COLLEGE

b) $\sum^{\infty}\left(\frac{2}{n}-\frac{2}{n+2}\right)$:

$S_2 = 1 - 1/2$

$S_3 = 1 - \frac{1}{2} + 2/3 - 2/5$

$S_4 = 1 - 1/2 + 2/3 - 2/5 + \cancel{1/2} - 1/3$

$S_5 = 1 + 2/3 - \cancel{2/5} - \frac{1}{3} + \cancel{2/5} - 2/7$

$\vdots$

$S_n = 1 + 2/3 - \frac{2}{n+2}$

$\lim_{n\to\infty} S_n = 1 + 2/3 - 0 = \frac{3}{3} + \frac{2}{3} = 5/3$.

Q4

a) $\sum^{\infty} \frac{(-1)^n \, 7^n}{(2n)!}$: by Ratio test.

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(7^n)\,7}{(2n+2)!} \times \frac{(2n)!}{7^n}$

$= \lim_{n\to\infty} \frac{7}{(2n+2)(2n+1)} = \frac{7}{4n^2} = 0$

so it is conv abs.

COLLEGE

b) (3 points). $\sum_{n=1}^{\infty} \frac{2^n+7}{5^n-4}$

let $b_n = \frac{2^n}{5^n} = \left(\frac{2}{5}\right)^n \leftarrow$ conv. by G.S

$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{(2^n+7)}{(5^n-4)} \cdot \frac{5^n}{2^n}$

$L = \lim_{n\to\infty} \frac{(5^n \cdot 2^n + 7 \cdot 5^n)}{(5^n \cdot 2^n + 4 \cdot 2^n)}$

$L = \lim_{n\to\infty} \frac{10^n + 7 \cdot 5^n}{10^n + 4 \cdot 2^n} = 1$ $\quad 0 < 1 < \infty$

2

$\because \sum_{n\geq 1}^{\infty} b_n$ is conv by G.S, & $0 < L < \infty$

$\therefore \sum_{n=1}^{\infty} \frac{2^n+7}{5^n-4}$ is also conv. by L.C.T

c) (2 points) $\sum_{k=2}^{\infty} \frac{2k+1}{5k+3}$

2

$\lim_{n\to\infty} \frac{2K+1}{5K+3} = \frac{2}{5} \neq 0$

$\therefore \sum_{n=2}^{\infty} \frac{2k+1}{5k+3}$ is div by limit of divergence test

d) (3 points) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt[4]{\ln(n)}}$

1) $\frac{1}{n\sqrt[4]{\ln(n)}}$ is +ve

2) $\frac{1}{n\sqrt[4]{\ln(n)}}$ is dec

$\therefore$ by Integral test

let $f(x) = \frac{1}{x\sqrt[4]{\ln(x)}}$

$\int_2^{\infty} \frac{1}{x(\ln(x))^{\frac{1}{4}}}dx = \lim_{L\to\infty} \int_2^{L} \frac{1}{x(\ln x)^{\frac{1}{4}}} dx$

$= \lim_{L\to\infty} \frac{4(\ln x)^{\frac{3}{4}}}{3} \Big|_2^L$

$= \lim_{L\to\infty} \frac{4}{3}(\ln L)^{\frac{3}{4}} - \lim_{L\to\infty} \frac{4}{3}(\ln(2))^{\frac{3}{4}}$

$= \infty - \frac{4}{3}\sqrt[4]{(\ln(2))^3} = \infty$ div

$\therefore \int_2^{\infty} \frac{1}{x\sqrt[4]{\ln(x)}}dx$ is div

$\therefore \sum_{n=2}^{\infty} \frac{1}{n\sqrt[4]{\ln(n)}}$ is div by Integral Test

3

3

b) $\sum_{1}^{\infty} \frac{2^n+7}{5^n-4}$:

$a_n = \frac{2^n+7}{5^n-4}$, $b_n = \left(\frac{2}{5}\right)^n \Rightarrow -1 < 2/5 < 1$

conv geo. series

$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{2^n+7}{5^n-4} \times \frac{5^n}{2^n}$

$= \lim_{n\to\infty} \frac{2^n \times 5^n}{5^n \times 2^n} = 1$

So a_n is conv by limit comp. test.

c) $\sum_{2}^{\infty} \frac{2k+1}{5k+3}$:

$\lim_{k\to\infty} \frac{2k+1}{5k+3} = \frac{2k}{5k} = \frac{2}{5} \neq 0$

So: div by div. test.

d) $\sum_{2}^{\infty} \frac{1}{n\sqrt[4]{\ln(n)}}$:

Positive, decreasing and cont.
∴ by Integral test:

$$\int_{2}^{\infty} \frac{1}{x\sqrt[4]{\ln x}} = \lim_{L\to\infty} \int_{2}^{L} \frac{1}{x\sqrt[4]{\ln x}}\, dx$$

let $u = \ln x \longrightarrow dx = x\,du$.

$$\int \frac{x}{x\sqrt[4]{u}}\, du = \int u^{-\frac{1}{4}}\, du = \frac{4}{3} u^{\frac{3}{4}}$$

$$= \frac{4}{3}\sqrt[4]{(\ln x)^3}\Big|_{2}^{L} = \frac{4}{3}\left(\left(\sqrt[4]{(\ln L)^3}\right) - \sqrt[4]{(\ln 2)^3}\right.$$

$$\lim_{L\to\infty} \frac{4}{3}\left(\left(\sqrt[4]{(\ln L)^3}\right) - \sqrt[4]{(\ln 2)^3}\right. = \infty$$

so it is div by integral test.

Scanned with CamScanner

The university of Jordan Calculus II Second exam 10 April 2016

الرقم الجامعي: اسم الطالب:

مدرس المادة: وقت المحاضرة:

Q1: Find the area of the surface obtained by rotating the curve

$y = \frac{1}{6}x^3 + \frac{1}{2}x^{-1}, \quad 1 \leq x \leq \sqrt{2}$ about x-axis.

Q2: The base of the solid S is the region bounded by $y = x^2 - 1$ and $y = 1 - x^2$. Find the volume of S if every cross section perpendicular to the x-axis is a right triangle(مثلث قائم الزاوية) with its hypotenuse(الوتر) on the base and one angle equal to 30°.

Q1) $y = \frac{1}{3}x^3 + \frac{1}{4}x^{-1}$

$y' = x^2 - \frac{1}{4}x^{-2}$

$(y')^2 = x^4 - \frac{1}{2} + \frac{1}{16}x^{-4}$

$(y')^2 + 1 = \left(x^2 + \frac{1}{4x^2}\right)^2$

$$S = \int_1^2 2\pi \left(\frac{x^3}{3} + \frac{1}{4x}\right)\sqrt{\left(x^2 + \frac{1}{4x^2}\right)^2}\, dx$$

$$= \int_1^2 2\pi \left(\frac{x^3}{3} + \frac{1}{4x}\right)\left(x^2 + \frac{1}{4x^2}\right) dx$$

$$= \int_1^2 2\pi \left(\frac{x^5}{3} + \frac{x}{3} + \frac{1}{16x^3}\right) dx$$

$$= 2\pi \left(\frac{x^6}{18} + \frac{x^2}{6} - \frac{1}{32x^2}\right)\Bigg|_1^2$$

$$= (2\pi)\left(\frac{16}{18} + \frac{4}{6} - \frac{1}{128} - \frac{1}{18} - \frac{1}{6} + \frac{1}{32}\right)$$

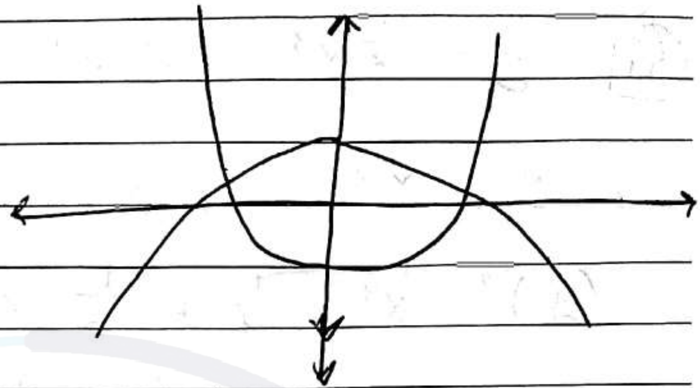
$$= 2\pi\left(\frac{15}{18} + \frac{3}{6} - \frac{1}{128} + \frac{1}{32}\right)$$

COLLEGE

Q2)

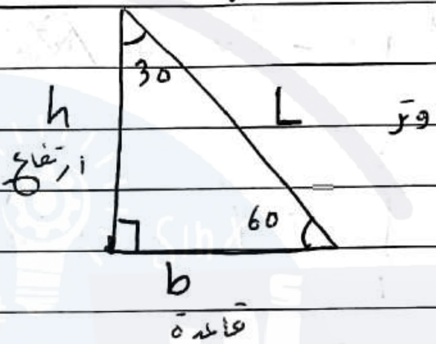
$$\sin 60^\circ = \frac{h}{L} = \frac{\sqrt{3}}{2}$$

$$h = \frac{\sqrt{3}}{2} L$$



$$\cos 60^\circ = \frac{b}{L} = \frac{1}{2}$$

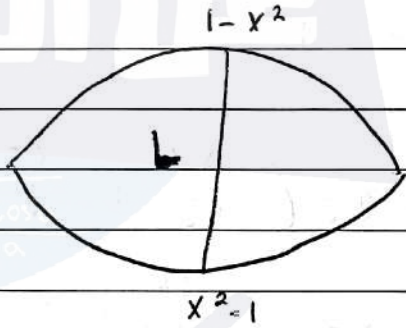
$$b = \frac{1}{2} L$$



كتبنا الأضلاع بدلالة الوتر.

$$A = \frac{1}{2}\left(\frac{1}{2}L\right)\left(\frac{\sqrt{3}}{2}L\right) = \frac{\sqrt{3}}{8} L^2$$

$$V = \int A(x)\, dx$$



$$V = \int_{-1}^{1} \frac{\sqrt{3}}{8}\left(2-2x^2\right)^2 dx$$

$$= \frac{\sqrt{3}}{8}\left(4x - \frac{8x^3}{3} + \frac{4x^5}{5}\right)\Big|_{-1}^{1}$$

$$= \frac{8\sqrt{3}}{15}$$

so $L = 1-x^2 - (x^2-1)$

$= 2 - 2x^2$

Intersection:

$1 - x^2 = x^2 - 1$

$x^2 = 1$

$x = \pm 1$

Q3: Find the sum

$$\sum_{n=1}^{\infty}\left(\frac{1}{(n+1)(n+3)}+\frac{5}{7^{n+2}}\right)$$

Q4: Set up the integral that gives the volume of the solid generated by revolving the region bounded by $y = x^3$, $y = 27, x = 0$ about the line $x = -3$.

(a) Using washer method, (Do not evaluate the integral).

(b) Using cylindrical shell method, (Do not evaluate the integral).

Q_3 $\sum_{1}^{\infty}\left(\frac{1}{(n+1)(n+3)} + \frac{5}{7^{n+2}}\right)$:

$1 = A(n+3) + B(n+1)$

$n=-3$: $1 = -2B \rightarrow B = -1/2$

$n=-1$: $1 = 2A \rightarrow A = 1/2$

$\sum_{1}^{\infty} \frac{1/2}{n+1} + \frac{-1/2}{n+3}$:

$S_1 = 1/4 - 1/8$

$S_2 = 1/4 - 1/8 + 1/6 - 1/10$

$S_3 = 1/4 - \cancel{1/8} + 1/6 - 1/10 + \cancel{1/8} - 1/12$

$S_4 = 1/4 + 1/6 - \cancel{1/10} - 1/12 + \cancel{1/10} - 1/14$.

$\vdots$

$S_n = 1/4 + 1/6 - \frac{1/2}{n+3}$

$\lim_{n\to\infty} S_n = 1/4 + 1/6 - 0 = \frac{6}{24} + \frac{4}{24} = \frac{10}{24}$

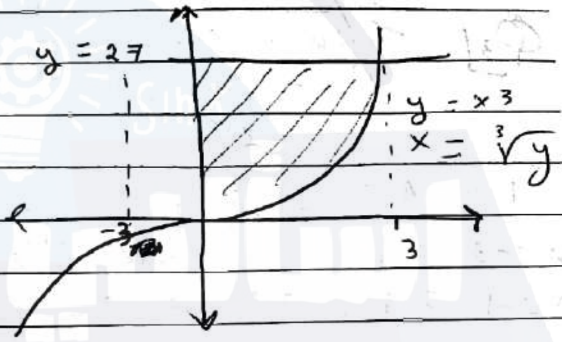
$* \sum_{1}^{\infty} 5/7^{n+2} = \left(\frac{5}{49}\right)\left(\frac{1}{7}\right)^n$

$a = \frac{5}{49\times 7}$, $r = 1/7$

$\sum_{1}^{\infty} \frac{5}{7^{n+2}} = \frac{5}{49\times 7} \times \frac{7}{6} = \frac{5}{49\times 6}$.

total : $\frac{10}{24} + \frac{5}{49\times 6}$

Q4)



a) $\int_0^{27} \pi \left(\left(\sqrt[3]{y} + 4\right)^2 - (-4)^2 \right) dy$.

b) $\int_0^{3} 2\pi \left(27 - x^3\right)(x+4)\, dx$.

Scanned with CamScanner

Q5 ⇒ Test for convergent ⇒

(a) $\sum_{n=1}^{\infty} ne^{-n^2}$

(b) $\sum_{n=1}^{\infty} \frac{(n+1)(5n+2)}{(3n+2)(n+3)(2n+7)(n+10)}$

Q6: A sequence $\{a_n\}_{n=1}^{\infty}$ whose sequence of partial sums is $\{S_n\}_{n=1}^{\infty}$, where $S_n = \frac{3n}{n+1}$, find a_n

Q5) a) $\sum_{1}^{\infty} n e^{-n^2}$:

$$\lim_{L\to\infty} \int_1^L n e^{-n^2} \, dn$$

$$u = n^2 \longrightarrow dn = du/2n$$

$$\int \frac{1}{2} e^{-u} \, du = \frac{-1}{2} e^{-u} = -1/2 \, e^{-n^2} \Big|_1^L$$

$$= \lim_{L\to\infty} \frac{-1}{2} \left(\frac{1}{L^{n^2}} - \frac{1}{e} \right) = \frac{1}{2e}$$

So conv by integral

COLLE

b) $a_n = \dfrac{(n+1)(5n+2)}{(3n+2)(n+3)(2n+7)(n+10)}$

$b_n = \dfrac{5n^2}{6n^4} = \dfrac{5}{6n^2} \rightarrow$ conv p-series

$\lim_{n\to\infty} \dfrac{a_n}{b_n} = \dfrac{5n^2 \times 6n^2}{5 \times 6n^4} = 1$.

so both conv by limit comp. test.

Q6]

$a_n = S_n - S_{n-1}$

$= \dfrac{3n}{n+1} - \dfrac{3n-3}{n} = \dfrac{3n^2 - 3n^2 + 3n - 3n + 3}{(n+1)(n)}$

$= \dfrac{3}{n(n+1)}$

The University of Jordan		Department of Mathematics
Student's Name:..................		Instructor's Name:..................
Student's Number:...............		Class Time: ....

Second Exam ◇ Calculus II (0301102) ◇ Summer 2016

Note: This exam is composed of 8 questions. You have 60 minutes to finish

Question 1 [5 points]: Use the **Washer method** to find the volume of the solid generated when the region bounded by $y = 2\sqrt{x-1}$ and $y = x - 1$ is revolved about the line $x = -1$.

Question 2 [3 points]: Test the following sequence for convergence.

$$\{a_n\} = \left\{\left(1 - \frac{9}{n^2}\right)^{\frac{n}{2}}\right\}_{n=1}^{\infty}$$

Question 4 [4 points]: Determine if the series is absolutely convergent, conditionally convergent or divergent.

$$\sum_{n=0}^{\infty} \frac{\sin n}{n^3}.$$

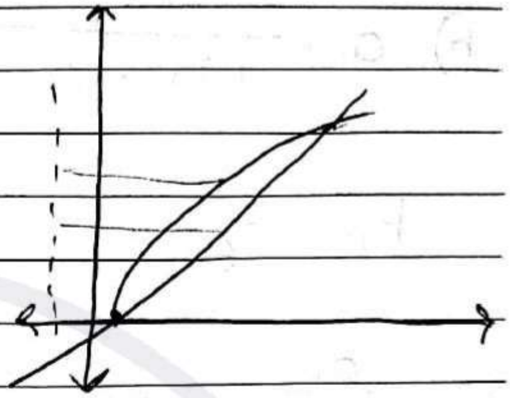
Question 5 [3 points]: Determine if the series converges or diverges. If the series converges find its value.

$$\sum_{n=0}^{\infty} 3^{2+n} 2^{1-3n}.$$

Q1

$y = x - 1 \rightarrow x = y + 1$

$y = 2\sqrt{x-1} \rightarrow x = y^2/4 + 1$



$y + 1 = y^2/4 + 1$

$4y = y^2 \rightarrow y^2 - 4y = 0$

$y(y-4) = 0$

$y = 0 \ , \ y = 4$

$$V = \pi \int_0^4 (y+2)^2 - (y^2/4 + 2)^2 \, dy$$

$$= \pi \int_0^4 4y - y^4/16 \, dy$$

$$= \pi \left(2y^2 - y^5/80\right) \Big|_0^4 = \frac{96\pi}{5}$$

Q2]

$$\lim_{n\to\infty}\left(1-\frac{9}{n^2}\right)^{\frac{n}{2}} = \left(1-\frac{3}{n}\right)^{\frac{n}{2}}\left(1+\frac{3}{n}\right)^{\frac{n}{2}}$$

$$= \sqrt{e^{-3} \times e^{3}}$$

$$= \sqrt{1} = 1$$

so conv.

Q4] $\sum_{0}^{\infty} \frac{\sin n}{n^3}$:

$\frac{\sin(n)}{n^3} \leq \frac{1}{n^3} \rightarrow$ conv p-series

$\frac{|\sin(n)|}{n^3} \leq \frac{1}{n^3} \rightarrow$ conv p-series

for $n>0$

so both conv, then it is conv abs.

Question 6 [4 points]: Determine if the series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{\sqrt{2n^2+4n+1}}{n^3+9}.$$

Question 7 [4 points]: Determine the radius of convergence and interval of convergence for the following power series.

$$\sum_{n=0}^{\infty} n!\,(2x+1)^n.$$

Question 8 [4 points]: Give a power series representation for **the integral of** the following function.

$$f(x) = \frac{x^4}{9+x^2}.$$

Q5) $\sum_{0}^{\infty} 3^{2+n} \, 2^{1-3n}$:

$$= \sum_{0}^{\infty} 9(3^n)\left(\frac{2}{8^n}\right) = \sum_{0}^{\infty} 18\left(\frac{3}{8}\right)^n .$$

$a = 18 \rightarrow r = \frac{3/8}{5}$

so conv.

$$\sum_{0}^{\infty} 3^{2+n} \, 2^{1-3n} = 18 \times \frac{8}{5}$$

$$= \frac{144}{5}$$

Q6) $a_n = \frac{\sqrt{2n^2+4n+1}}{n^3+9}$.

$b_n = \frac{\sqrt{2}}{n^2}$ } conv p-series

$$\lim_{n \to \infty} \frac{a_n}{b_n} = \frac{\sqrt{2}\, n}{n^3} \times \frac{n^2}{\sqrt{2}} = 1$$

so both conv by limit comp. test.

Scanned with CamScanner

Q7]
$$L = \lim_{n\to\infty} \left| \frac{(n+1)!\,(2x+1)^{n+1}}{n!\,(2x+1)^n} \right|$$

$$= \lim_{n\to\infty} \left| (2x+1)(n+1) \right|$$

$$2x+1 \neq 0$$
$$x \neq -1/2$$

interval of conv is $\left(\frac{-1}{2}\right)$
radius of conv is (0).

Q8]

$$f(x) = \frac{x^4}{9+x^2} = \frac{x^4}{9} \frac{1}{\left(1+\frac{1}{9}x^2\right)}$$

$$= \frac{x^4}{9} \cdot \frac{1}{1-\left(-\frac{1}{9}x^2\right)} = \frac{x^4}{9} \sum_0^{\infty} \left(\frac{-1}{9}x^2\right)^n$$

$$\int f(x)\,dx = \int \sum_0^{\infty} (-1)^n \left(1/9\right)^{n+1} (x)^{2n+1}\,dx$$

$$= \sum_0^{\infty} \frac{(-1)^n}{2n+5} \left(\frac{1}{9}\right)^{n+1} x^{2n+5} + C$$

Jordan University
Mathematics Department
Calculus II, Second Exam, 18/4/2015 II

Student's Name:

Student's Number:

Lecture Time:

3) (3 points) Use series to write the repeated decimal $0.4\overline{5}$ as a quotient of integers (fraction).

$0.4\bar{5} = 0.45555 \ldots$

$= 0.4 + 0.05 + 0.005 + \ldots$

$= 0.4 + \sum_{2}^{\infty} \frac{5}{(10)^n}$

$= 0.4 + \sum_{2}^{\infty} 5 \left(\frac{1}{10}\right)^n$

$a = 0.05$

$r = 0.1$

$0.4 + \sum_{2}^{\infty} 5 \left(\frac{1}{10}\right)^n = 0.4 + \frac{5}{100} \times \frac{10}{9}$

$= 0.4 + \frac{5}{90}$

$= 41/90$