The integral that represents the volume of the solid obtained by rotating the region enclosed by $y = 3x$, $y = x + 2$, and $y = 0$ about x-axis is

A) $\int_{-2}^{0}(x+2)^2dx + \int_0^1 9x^2dx$

B) $\int_{-2}^{1}(x+2)^2dx + \int_0^1 4x^2dx$

C) $\int_{-2}^{0}(x+2)^2dx - \int_0^1(-8x^2+4x+4)dx$

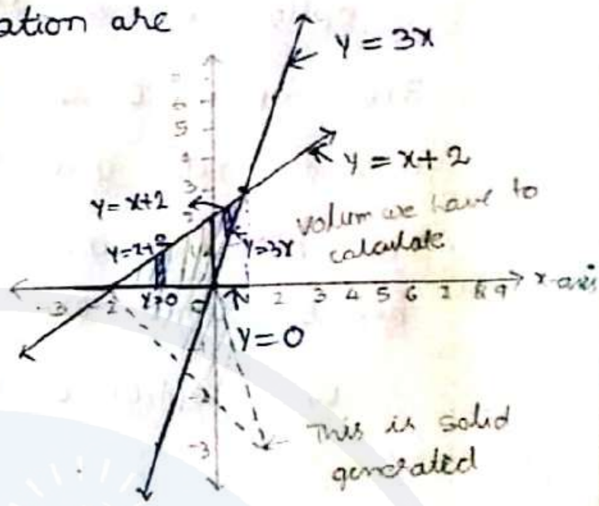
D) $\int_0^3(\frac{2y}{2}+2)dy$

E) $\int_{-2}^{1}(x+2)^2dx + \int_0^1(1-2x)^2dx$

Q. The integral that represents the volume of the solid obtained by rotating the region enclosed by $y = 3x$, $y = x+2$, and $y=0$ about x-axis is.

Sol:- **Step-1:-** Given line equation are

$y = 3x,$
$y = x+2$
$y = 0$



Step-2: solve for the intersection of points

$\Rightarrow 3x = x+2$

$2x = 2$

$x = 1 \Rightarrow y = 3$

Now the interval is from -2 to 1

Step-3:-

The volume of solid formed by revolving the region about the x-axis is [a,b] interval

$$V = \pi \int_a^b [f(x)^2 - g(x)^2] dx$$

$$V = \pi \left[\int_{-2}^{0} (x+2)^2 - (0)^2 dx - \int_0^1 [(x+2)^2 - (3x)^2] dx \right]$$

$$V = \pi \left[\int_{2}^{0} (x+2)^2 dx - \int_0^2 (x^2 + 4 + 4x - 9x^2) dx \right]$$

$$V = \pi \left[\int_{-2}^{0} (x+2)^2 dx - \int_0^2 (-8x^2 + 4x + 4) dx \right]$$

$\therefore$ option C is correct.

The option D is totally incorrect because it does not give volume.

The option B and E is the area as overlapped and the limits are not continious.

$\therefore$ only option C is correct.

The integral that represents the arc length of the curve $y = lnx$ from the point $(e, 1)$ to the point $(e^2, 2)$ is:

A) $\int_{e}^{e^2} \sqrt{x^2+1}\, dx$

B) $\int_{1}^{2} \frac{\sqrt{x^2+1}}{x}\, dx$

C) $\int_{e}^{e^2} \frac{lnx\sqrt{x^2+1}}{x}\, dx$

D) $\int_{e}^{e^2} \frac{\sqrt{x^2+1}}{x}\, dx$

E) $\int_{1}^{2} y\sqrt{e^{2y}+1}\, dy$

- A. -
- B. -
- C. -

1- $y = \ln x$, $(e,1)$ to $(e^2,2)$

$$\frac{dy}{dx} = \frac{1}{x}$$

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\frac{1}{x^2}} = \sqrt{\frac{x^2+1}{x^2}} = \frac{\sqrt{x^2+1}}{x}$$

Arc length $= \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$$= \int_e^{e^2} \sqrt{\frac{x^2+1}{x^2}}\, dx$$

$$= \int_e^{e^2} \frac{\sqrt{x^2+1}}{x}\, dx$$

Hence (D) $\int_e^{e^2} \frac{\sqrt{x^2+1}}{x}\, dx$

The integral that represents the volume of the solid obtain
region bounded by the curve $y = 5\cos^2(5x)$ and the lines

$y = 0, x = -\frac{\pi}{10}, x = \frac{\pi}{10}$ about the line $y = 6$ is:

A) $\pi \int_{-\frac{\pi}{10}}^{\frac{\pi}{10}} (120\cos^2(5x) - 50\cos^4(5x))dx$

B) $\pi \int_{-\frac{\pi}{10}}^{\frac{\pi}{10}} 10\cos^2(5x)dx$

C) $\pi \int_{-\frac{\pi}{10}}^{\frac{\pi}{10}} 5\cos^2(5x)dx$

D) $\pi \int_{-\frac{\pi}{10}}^{\frac{\pi}{10}} (60\cos^2(5x) - 25\cos^4(5x))dx$

E) $\pi \int_{-\frac{\pi}{10}}^{\frac{\pi}{10}} 25\cos^4(5x)dx$

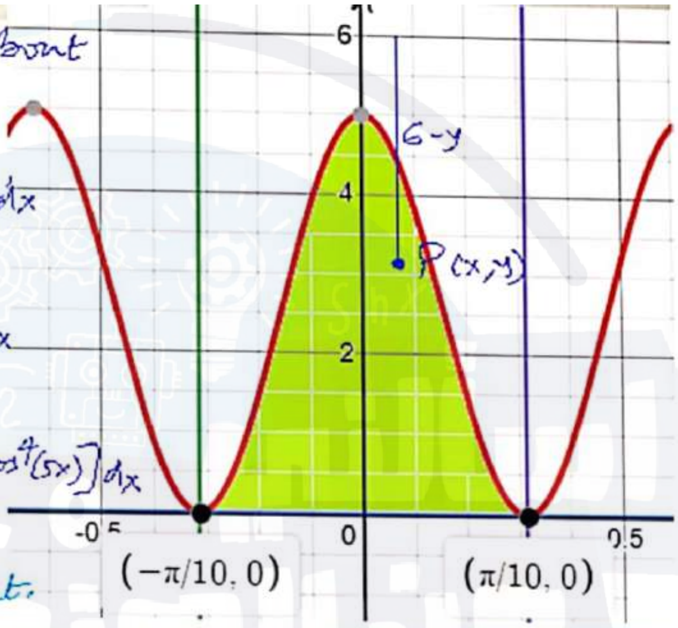
Volume of revolution about line $y = 6$ is:

$$V = \pi \int_{-\pi/10}^{\pi/10} \left[(6)^2 - (6-y)^2 \right] dx$$

$$= \pi \int_{-\pi/10}^{\pi/10} (12y - y^2)\, dx$$

$$= \pi \int_{-\pi/10}^{\pi/10} \left[60\cos^2(5x) - 25\cos^4(5x) \right] dx$$

$\Rightarrow$ Option (D) is correct.



A) $\pi \int_{-\frac{\pi}{10}}^{\frac{\pi}{10}} \left(120\cos^2(5x) - 50\cos^4(5x)\right) dx$

B) $\pi \int_{-\frac{\pi}{10}}^{\frac{\pi}{10}} 10\cos^2(5x)\, dx$

C) $\pi \int_{-\frac{\pi}{10}}^{\frac{\pi}{10}} 5\cos^2(5x)\, dx$

D) $\pi \int_{-\frac{\pi}{10}}^{\frac{\pi}{10}} \left(60\cos^2(5x) - 25\cos^4(5x)\right) dx$

E) $\pi \int_{-\frac{\pi}{10}}^{\frac{\pi}{10}} 25\cos^4(5x)\, dx$

(By cylindrical shells) The integral that repr
ume of the solid obtained by rotating the regic
$y = e^x$, $y = 2$ and $x = 0$ about $x = -1$ is:

A) $2\pi \int_0^{\ln 2} (x+1)(2-e^x)\,dx$

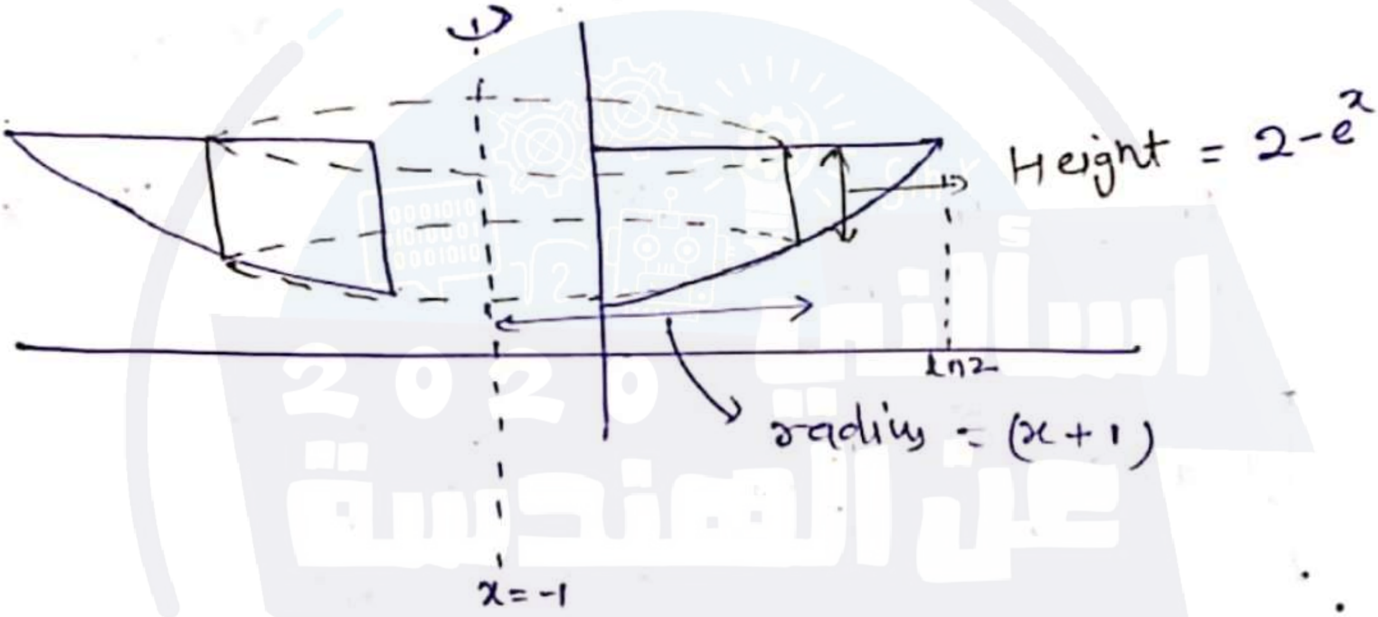
B) $2\pi \int_0^{\ln 2} (x-1)(2-e^x)\,dx$

C) $2\pi \int_0^{\ln 2} (1-x)(2-e^x)\,dx$

D) $2\pi \int_0^{\ln 2} x(2-e^x)\,dx$

E) $2\pi \int_0^{2} y \ln y \,dy$

$y = e^x, \quad y = 2, \quad x = 0 \quad$ about $x = -1$



the cross-sectional area of cylinder

$A(x) = 2\pi\,(\text{radius})\,(\text{height})$

$= 2\pi(x+1)(2-e^x)$

Volume $V = \int_{x=0}^{\ln 2} A(x)\,dx$

$= 2\pi \int_0^{\ln 2} (x+1)(2-e^x)\,dx \qquad (A)$

(By cylindrical shells) The integral that represents the volume of the solid obtained by rotating the region bounded by $y = e^x$, $y = 3$ and $x = 0$ about $x = -2$ is:

A) $2\pi \int_0^{ln3} x(3 - e^x)\, dx$

B) $2\pi \int_0^{ln3} (x - 2)(3 - e^x)\, dx$

C) $2\pi \int_0^{ln3} (2 - x)(3 - e^x)\, dx$

D) $2\pi \int_0^{3} y\, lny\, dy$

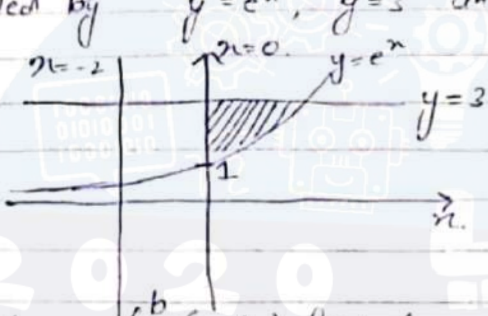
E) $2\pi \int_0^{ln3} (x + 2)(3 - e^x)\, dx$

○ A. -

○ B. -

○ C. -

Ans Volume of the solid obtained by rotating the region bounded by $y = e^x$, $y = 3$ and $x = 0$ about $x = -2$ is :-



$$V = 2\pi \int_a^b (x-h)\, f(x)\, dx$$

where, $h = -2$ (axis of rotation), $y = e^x$.

$3 = e^x$

$\therefore f(x) = 3 - e^x$. $\ln 3 = x = a$,

from limits $x = 0 \to \ln 3$, area. $b = 0$.

Under the graph will be under e^x but we want from $y = 3$ $\therefore f(x) = 3 - e^x$

$$V = 2\pi \int_0^{\ln 3} (x+2)(3-e^x)\, dx$$

The Integral that represents the area enclosed by y=x+1, y=2-x, y=0, x=0 is

A) $\int_0^1 (2-y)dy + \int_1^{3/2} (3-2y)dy$

B) $\int_0^1 (x+1)dx + \int_1^2 (2-x)dx$

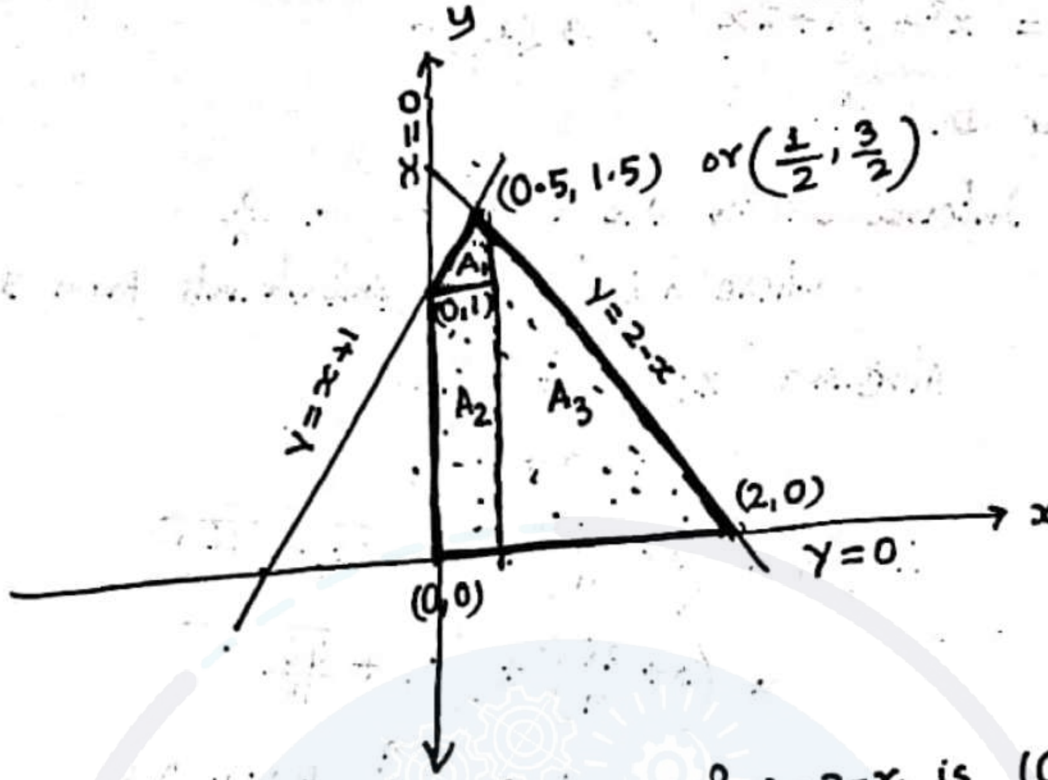
C) $\int_0^2 (2-x)dx$

D) $\int_0^{3/2} (3-2y)dy$

E) $\int_0^1 (2-y)dy + \int_1^{3/2} (y-1)dy$

○ A. -

○ B. -



The intersection point of $y=x+1$ & $y=2-x$ is $(0.5, 1.5)$

The Area enclosed by $y=x+1$, $y=2-x$, $y=0$, $x=0$ is shown above & is given by

$$A = A_1 + A_2 + A_3$$

$$= \int_1^{3/2} (1.5 - y)\,dy + \int_0^1 (0.5 - 0)\,dy + \int_0^{3/2} (1.5 - y)\,dy$$

$$= \int_1^{3/2} (1.5 - y)\,dy + \int_0^1 0.5\,dy + \int_0^1 (1.5 - y)\,dy + \int_1^{3/2} (1.5 - y)\,dy$$

$$= \int_0^1 (0.5 + 1.5 - y)\,dy + \int_1^{3/2} [(1.5 - y) + (1.5 - y)]\,dy$$

$$= \int_0^1 (2 - y)\,dy + \int_1^{3/2} (3 - 2y)\,dy$$

Answer 'A'

(By cylindrical shells) The integral that represents the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = 6x$ about $x-axis$ is:

A) $2\pi \int_0^6 y(\sqrt{y} - \frac{y}{6})\, dy$

B) $2\pi \int_0^{36} (\sqrt{y} - \frac{y}{6})\, dy$

C) $2\pi \int_0^6 (36x^2 - x^4)\, dx$

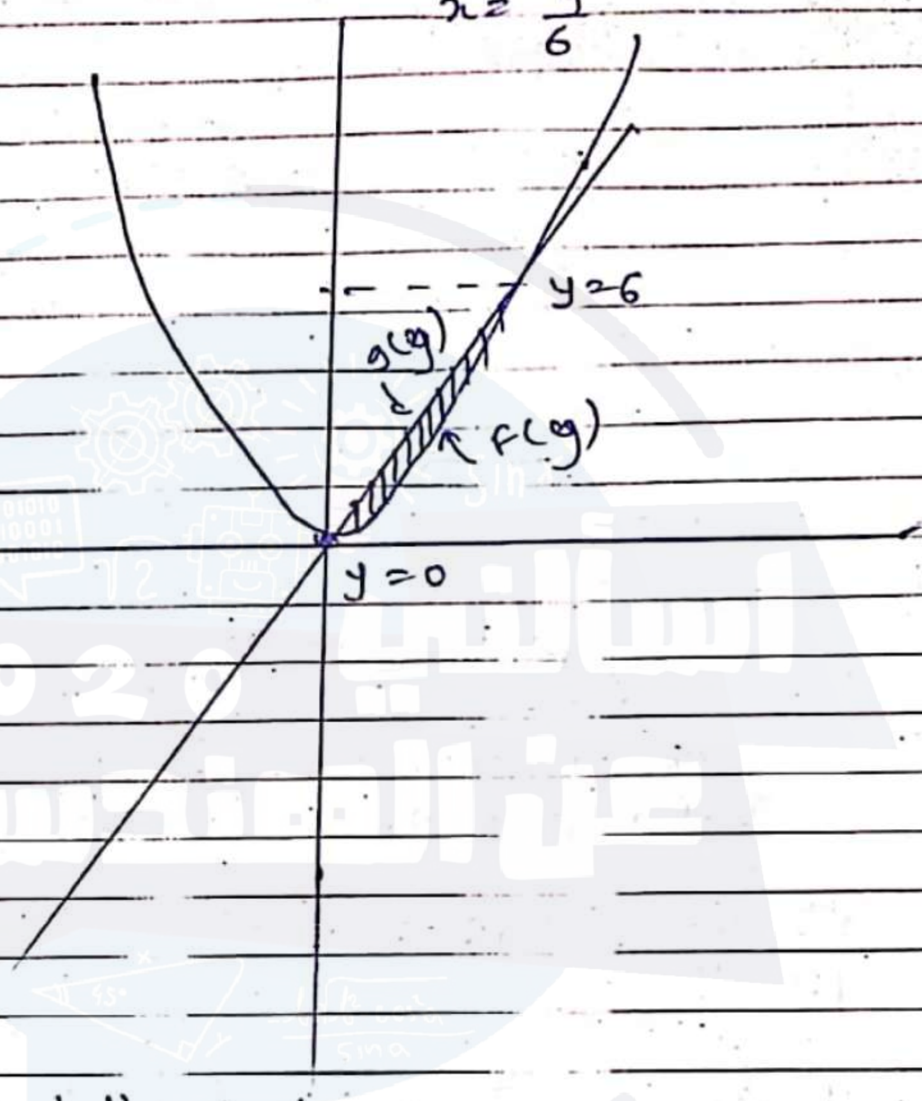
D) $2\pi \int_0^{36} y\sqrt{y}\, dy$

E) $2\pi \int_0^{36} y(\sqrt{y} - \frac{y}{6})\, dy$

Q.1)

$y = x^2$ and $y = 6x$

$x = \sqrt{y}$ $x = \frac{y}{6}$



∴ By ~~sheet~~ shell method

$$I = 2\pi \int_a^b y\,[f(y) - g(y)]\,dy$$

$$\therefore I = 2\pi \int_0^6 y\left[\sqrt{y} - \frac{y}{6}\right] dy$$

$$\therefore I = 2\pi \int_0^6 y\left(\sqrt{y} - \frac{y}{6}\right) dy$$ option A//

The area of the surface obtained by rotatin
$y = \sqrt{4 - x^2}$, $-1 \leq x \leq 1$, about the x -

A) 4π
B) 2π
C) 8π
D) 8
E) 4

Sol:-

Given $y = \sqrt{4-x^2}$; $-1 \le x < 1$

$\Rightarrow$ Surface Area about x-axis is given by

$$SA = 2\pi \int_a^b y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{4-x^2}\right) = \frac{1}{2\sqrt{4-x^2}}(-2x) = \frac{-x}{\sqrt{4-x^2}}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{x^2}{4-x^2} \quad \& \quad 1+\left(\frac{dy}{dx}\right)^2 = 1+\frac{x^2}{4-x^2} = \frac{4}{4-x^2}$$

$$SA = 2\pi \int_{-1}^{1} \sqrt{4-x^2}\sqrt{\frac{4}{4-x^2}}\, dx$$

$$= 2\pi \int_{-1}^{1} 2\sqrt{4-x^2}\,\frac{1}{\sqrt{4-x^2}}\, dx$$

$$= 4\pi \int_{-1}^{1} dx = 4\pi\,(x)_{-1}^{1} = 4\pi\,[1-(-1)]$$

$$= \underline{8\pi}.$$

$\therefore$ Surface Area = $\underline{8\pi}$

The integral that represents the arc length of the curve $y = lnx$ from the point $(e^{-1}, -1)$ to the point $(e, 1)$ is:

A) $\int_{-1}^{1} y\sqrt{e^{2y}+1}\, dy$

B) $\int_{-1}^{1} \frac{\sqrt{x^2+1}}{x}\, dx$

C) $\int_{e^{-1}}^{e} \frac{lnx\sqrt{x^2+1}}{x}\, dx$

D) $\int_{e^{-1}}^{e} \sqrt{x^2+1}\, dx$

E) $\int_{e^{-1}}^{e} \frac{\sqrt{x^2+1}}{x}\, dx$

Select one:

- A
- B
- C
- D
- E

we are given equation as

$$y = ln(x)$$

we can find derivative

$$\frac{dy}{dx} = \frac{1}{x}$$

now, we can use arc length formula

$$L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2} dx$$

now, we can set up integral

$$L = \int_{e^{-1}}^{e} \sqrt{1 + (\frac{1}{x})^2} dx$$

Hence,

option-E........Answer

Set up the Integral that represents the volume of the solid obtained by rotating the region bounded by the given curves about the specific line.

$x = 2\sqrt{y}$, x = 0 , y = 9 ; about the y axis

This is a problem where you will need to integrate with respect to y, rather than x.
Once the function is revolved around the y axis, imagine slicing it horizontally into many thin discs. the area of one disk would be πr^2, so the area at any point on the y axis would be $\pi(2\sqrt{y})^2 = 4\pi y$.
To find the volume of these disks, integrate with the boundaries y=0 (the x axis) and y=9

$$\int_0^9 4\pi y \, dy = 2\pi y^2 |_0^9 = 2\pi\left(81 - 0\right) = 162\pi$$

(By cylindrical shells) The integral that represents the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $y = 3$ and $x = 0$ about $y = 3$ is:

A) $2\pi \int_0^3 (3 + y)y^2 \, dy$

B) $2\pi \int_0^3 (3 - y)y \, dy$

C) $2\pi \int_0^3 (3 - y)y^2 \, dy$

D) $2\pi \int_0^3 (y - 3)y^2 \, dy$

E) $2\pi \int_0^3 y^3 \, dy$

Let thickness of the slice and shell is dy

Here the radius $r = 3 - y$

$h = 0 - y^2 = -y^2 \quad [\because y = \sqrt{x}$
$\therefore x = y^2]$

[Since the slice is taken at a value of y. So, as $y = \sqrt{x}$, $\therefore x = y^2$]

$\therefore$ volume of shell $= 2\pi r h \times dy = 2\pi (3-y)(-y^2)\, dy$

The resulting solid has a volume,

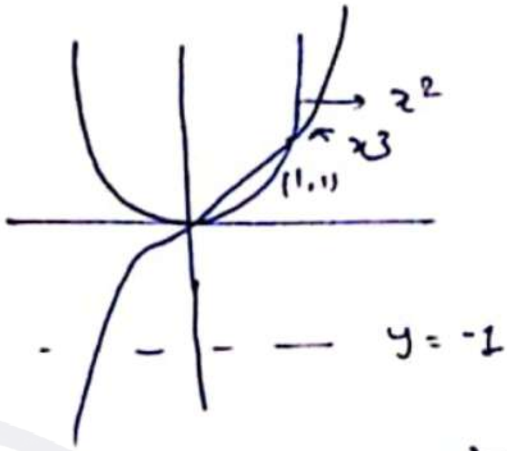
$$V = 2\pi \int_0^3 (3-y)(-y^2)\, dy$$

$$= 2\pi \int_0^3 (y-3) y^2\, dy \quad \text{(option D)}$$

Question 3. Find the volume of the solid that results when the region enclosed by $y = x^2$ and $y = x^3$ is revolved around the line $y = -1$.

$y = x^2$ $\quad y = x^3$ $\quad y = -1$

disk/washer method



inner radius

$= (1 + x^3)$

outer radius $= (1 + x^2)$

$$V = \int_0^1 \pi \left((\text{outer radius})^2 - (\text{inner rad})^2 \right) dx$$

$$V = \int_0^1 \pi \left[(1+x^2)^2 - (1+x^3)^2 \right] dx$$

$$V = \pi \int_0^1 (1 + x^4 + 2x^2 - 1 - x^6 - 2x^3)\, dx$$

$$V = \pi \int_0^1 (x^4 + 2x^2 - x^6 - 2x^3)\, dx$$

$$= \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} - \frac{x^7}{7} - \frac{2x^4}{4} \right]_0^1$$

$$= \pi \left[\frac{1}{5} + \frac{2}{3} - \frac{1}{7} - \frac{1}{2} \right]$$

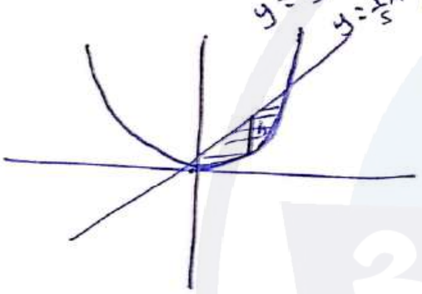
$$= \frac{47\pi}{210}$$

Ans $= \frac{47\pi}{210}$

By using cylindrical shell method, the integral that represents the volume of the solid generated by revolving the region enclosed by $x^2 = 5y$ and $y = \frac{x}{5}$ about y-axis is

Select one:

- A. $2\pi \int_0^{\frac{2}{5}} x\left(\frac{x^2}{5} - \frac{x}{5}\right)$
- B. $2\pi \int_0^1 x\left(\frac{x^2}{5} - \frac{x}{5}\right)$
- C. $2\pi \int_0^1 x\left(\frac{x}{5} - \frac{x^2}{5}\right)$



$h = \left(\frac{1}{5}x - \frac{1}{5}x^2\right)$

$r = x$

$$V = 2\pi \int_0^1 x\left(\frac{x}{5} - \frac{x^2}{5}\right) dx$$

C

The integral that represents the volume of the solid obtained by rotating the region bounded by the curve $y = 3e^{-2x^2}$ and the lines $y = 0, x = 0, x = 8$ about the x −axis is:

A) $18\pi \int_0^8 e^{-2x^4} dx$

B) $9\pi \int_0^8 e^{-4x^2} dx$

C) $9\pi \int_0^8 e^{-2x^4} dx$

D) $18\pi \int_0^8 e^{-4x^2} dx$

E) $6\pi \int_0^8 e^{-2x^2} dx$

Formula :

volume of revolution about x-axis

$$V = \int_a^b A(x)\,dx$$

Where $A(x) = \pi(\text{radius})^2$

Here, $y = 3e^{-2x^2}$ and $y = 0$

$$\therefore A(x) = \pi\left(3e^{-2x^2}\right)^2$$

$$= 9\pi e^{-4x^2}$$

Here, $x = 0$ to $x = 8$

$$\therefore V = \int_0^8 A(x)\,dx$$

$$= 9\pi \int_0^8 e^{-4x^2}\,dx$$

$\longrightarrow$ option (C)

$$\sum_{n=1}^{\infty}\left(e^{\frac{4}{n}} - e^{\frac{4}{n+1}}\right) =$$

(A) $e^1 - 1$

(B) $e^2 - 1$

(C) $e^3 - 1$

(D) $e^4 - 1$

(E) $e^5 - 1$

- A
- B
- C
- D ✓

$$\sum_{n=1}^{\infty} \frac{1+5^n}{7^n} =$$

(A) $\frac{17}{30}$

(B) $\frac{22}{24}$

(C) $\frac{27}{18}$

(D) $\frac{32}{12}$

(E) $\frac{37}{6}$

- ○ A
- ○ B
- ○ C
- ◉ D ✓

The integral for the area of the surface obtained by rotating the curve $y^2 = x+1,\ 0 \le x \le 3,\ y \ge 0$ about the x-axis is:

(a) $\int_0^3 \pi\sqrt{4x+1}dx$

(b) $\int_0^3 \pi\sqrt{4x+5}dx$

(c) $\int_0^3 \pi\sqrt{4x+9}dx$

(d) $\int_0^3 \pi\sqrt{4x+13}dx$

(e) $\int_0^3 \pi\sqrt{4x+17}dx$

○ A

◉ B ✓

Use the slicing method to find the volume of the solid whose base is the region inside the circle $x^2 + y^2 = 1$ if the cross sections taken perpendicular to the $y-$axis are squares

(A) $\frac{16}{3}$ (B) $\frac{16}{3}\pi$

(C) $\frac{8}{3}\pi$ (D) $\frac{8}{3}$

(E) $\frac{4}{3}$

A ✓

The integral that gives the volume when the region enclosed by $y = x^3$, $y = \sqrt{x}$ is revolved about the line $y = -2$. (Use cylindrical shell method).

(A) $\pi \int_0^1 (\sqrt{x} - x^3)^2 \, dx$

(B) $2\pi \int_0^1 (2 + x)(\sqrt{x} - x^3) \, dx$

(C) $2\pi \int_0^1 (2 + y)(y^2 - \sqrt[3]{y}) \, dy$

(D) $2\pi \int_0^1 (2 - y)(\sqrt[3]{y} - y^2) \, dy$

(E) $2\pi \int_0^1 (2 + y)(\sqrt[3]{y} - y^2) \, dy$

- A
- B
- C
- D
- E ✓

The limit of the sequence $a_n = \sqrt{\frac{2n^2}{8n^2+1}}$ is

A) $\frac{2}{3}$

B) $\frac{1}{4}$

C) $\frac{1}{2}$

D) $\frac{1}{8}$

E) $e^{\frac{1}{2}}$

- ○ A
- ○ B
- ◉ C ✓

The formula for the general term a_n of the sequence

$$\left\{0, \frac{3}{5}, \frac{8}{10}, \frac{15}{17}, \dots\right\}$$

A) $a_n = \frac{n^2+1}{n^2-1}$

B) $a_n = \frac{n^2-1}{n^2+1}$

C) $a_n = \frac{n^2+1}{n^2}$

D) $a_n = \frac{n^2+1}{n-1}$

E) $a_n = \frac{n^2}{n^2-1}$

A

B ✓

The arc length of $x = \frac{1}{2}(y^3 + 1)^2$ from $(0,-1)$ to $(2,1)$ is

(A) $\int_{-1}^{1} \sqrt{y^6 + 2y^3 + 2}\, dy$

(B) $\int_{0}^{2} \sqrt{1 + 9x^{10} + 18\, x^7 + 9x^4}\;\, dx$

(C) $\int_{-1}^{1} \sqrt{1 - 4y^{19} - 16y^7 - 16y^3}\;\, dy$

(D) $\int_{-1}^{1} \sqrt{1 + 9y^{10} + 18\, y^7 + 9y^4}\;\, dy$

(E) $\int_{0}^{2} \sqrt{x^6 + 2x^3 + 2}\, dx$

○ A

○ B

○ C

◉ D ✓

If the region enclosed by the $y-$axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the $y-$axis, the volume of the solid generated is

(A) π (B) $\frac{16\pi}{3}$

(C) $\frac{16\pi}{5}$ (D) $\frac{8\pi}{3}$

(E) $\frac{32\pi}{5}$

- A
- B
- C
- D ✗
- E

Correct answer

- E

The integral that finds the volume obtained by rotating the region enclosed by $y = x^2$ and $y = 8 - x^2$ about the line $x = 5$

(A) $2\pi \int_{-2}^{2} (5 - x)(8 - 2x^2)\, dx$

(B) $2\pi \int_{-2}^{2} (5 + x)(8 - 2x^2)\, dx$

(C) $2\pi \int_{-\sqrt{8}}^{\sqrt{8}} (5 - x)(8 - x^2)\, dx$

(D) $2\pi \int_{-2}^{2} (8 - 2x^2)\, dx$

(E) $2\pi \int_{-2}^{2} (5 - x)(8 + 2x^2)\, dx$

◉ A ✓